Hong Kong Mathematics Olympiad 2001-2002 Heat Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms. 除非特別聲明,答案須用數字表達,並化至最簡。

1. 袋中有數字卡 9 張,其數字分別為 1 至 9。若隨機一次抽出 3 張,求被抽出的卡的數字全是奇數的概率。(答案以分數表達,並化至最簡。)

There are 9 cards, numbered from 1 to 9, in a bag. If 3 cards are drawn together at random, find the probability that all are odd. (Express your answer in the simplest fraction.)

2. 已知 $a^3=150b$,且 a 和 b 都是正整數。求 b 的最小值。

Given $a^3 = 150 b$, and a, b are positive integers, find the least value of b.

3. 已知 $\cos 15^\circ = \frac{\sqrt{a} + \sqrt{b}}{4}$,且 $a \cdot b$ 是自然數。若 a + b = y,求 y 的值。

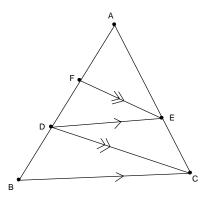
Suppose $\cos 15^\circ = \frac{\sqrt{a} + \sqrt{b}}{4}$, and a, b are natural numbers. If a + b = y, find the value of y.

4. 把數字 2, 3, 4, 5 組成沒有重複數字的四位數, 求這些四位數的和。

Each of the digits 2, 3, 4, 5 can be used once and once only in writing a four-digit number. Find the sum of all such numbers.

5. 在 $\triangle ABC$,DE//BC,FE//DC,AF = 2,FD = 3 和 DB = X。求X的值。

In $\triangle ABC$, DE//BC, FE//DC, AF = 2, FD = 3 and DB = X. Find the value of X.



6. 若一圓內接四邊形的四邊長度為9,10,10和21,求該圓內接四邊形的面積。

If the lengths of the sides of a cyclic quadrilateral are 9, 10, 10 and 21 respectively, find the area of the cyclic quadrilateral.

If
$$\frac{(a-b)(c-d)}{(b-c)(d-a)} = 3$$
, find the value of $\frac{(a-c)(b-d)}{(a-b)(c-d)}$.

When the expression $x^3 + kx^2 + 3$ is divided by x + 3, the remainder is 2 less than when divided by (x+1). Find the value of k.

9. 已知圓形上的某扇形的周界為 18。當圓的半徑為 r 時,該扇形的面積達至最大值,求 r 的值。

Given that the perimeter of a sector of a circle is 18. When the radius is r, the area of the sector attains the maximum value, find the value of r.

10. 已知
$$f\left(x+\frac{1}{x}\right) = x^2 + \frac{1}{x^2}$$
 , 求 $f(5)$ 的值。

Given
$$f\left(x+\frac{1}{x}\right) = x^2 + \frac{1}{x^2}$$
, find the value of $f(5)$.