

香港培正中學第二屆數學邀請賽

Pui Ching Middle School 2nd Invitational Mathematics Competition

個人賽（中二組）

Individual Event (Secondary 2)

時限：1 小時 30 分

Time allowed: 1 hour 30 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

Unless otherwise stated, all answers should be given in exact numerals in their simplest form.
No approximation is accepted.

4. 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。

Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.

5. 不得使用計算機。

The use of calculators is not allowed.

6. 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

1. 若 n 位數 $22\dots22$ 可被 9 整除，求 n 的最小可能值。

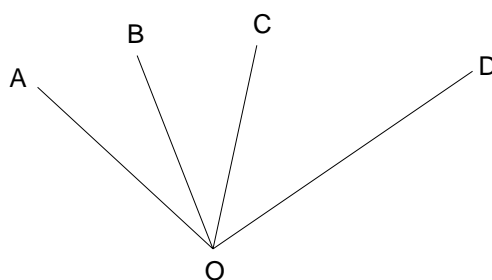
If the n -digit number $22\dots22$ is divisible by 9, find the smallest possible value of n .

2. 在一個圓形上畫 2003 條直徑，最多可把圓形分成幾份？

2003 diameters are drawn on a circle. What is the maximum number of regions formed?

3. 圖中， $\angle AOD = 120^\circ$ ， $3\angle AOB = \angle BOD$ ，且 $\angle AOC = 2\angle COD$ 。求 $\angle BOC$ 。

In the figure, $\angle AOD = 120^\circ$, $3\angle AOB = \angle BOD$ and $\angle AOC = 2\angle COD$. Find $\angle BOC$.



4. 數字之積為偶數的四位正整數共有多少個？

How many four-digit positive integers have their product of digits even?

第 5 至第 8 題，每題 4 分。

Questions 5 to 8 each carries 4 marks.

5. 求所有長和闊均為不大於 20 的整數的長方形（包括正方形）的面積的平均值。（註：把長方形的長闊對調，所得的長方形與原來的長方形視作不同。例如：長為 1、闊為 2 的長方形與長為 2、闊為 1 的長方形視作不同。）

Find the average of the areas of all rectangles (including squares) with integral lengths and breadths not more than 20. (Note: By switching the length and width the resulting rectangle is considered to be different from the original one. For instance, a rectangle of length 1 and width 2 is considered to be different from a rectangle of length 2 and width 1.)

6. 若把所有「3」字開首的正整數由小至大排列，即 3, 30, 31, 32, ...，第 2003 項是甚麼？

If all positive integers with leftmost digit 3 are listed in ascending order, i.e. 3, 30, 31, 32, ..., what will be the 2003rd term?

7. 小明從首 2003 個正整數中選取了所有是平方數或立方數的整數，然後把它們加起來。小明所得答案的個位數字是多少？

Peter selects those which are squares or cubes among the first 2003 positive integers and then adds them up. What is the unit digit of his result?

8. 直角坐標平面上畫有 2003 條直線，每條均平行於 x 軸或 y 軸。這些直線最多可把平面分成多少份？

2003 straight lines are drawn on the rectangular coordinate plane. Each of them is either parallel to the x -axis or the y -axis. What is the maximum number of regions formed by these lines?

第 9 至第 12 題，每題 5 分。

Questions 9 to 12 each carries 5 marks.

9. 將 $\frac{1}{998}$ 寫成循環小數時，小數點後第 25 位數字是甚麼？

When $\frac{1}{998}$ is written as a recurring decimal, what is the 25th digit after the decimal point?

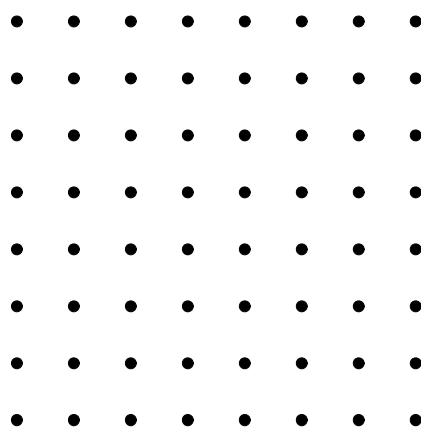
10. 設 x 為順序寫出 1 到 2003 所得的正整數， y 為把 x 的所有數字倒轉所得的正整數，即 $x=123456789101112\dots200120022003$ 及 $y=300220021002\dots211101987654321$ 。已知 s 為合成數，且為 $y-x$ 的因數。求 s 的最小可能值。

Let x be the positive integer formed by writing 1 to 2003 in order, and y be the positive integer formed by reversing the digits of x , i.e. $x=123456789101112\dots200120022003$ and $y=300220021002\dots211101987654321$. Given that s is a composite number and is a factor of $y-x$. Find the smallest possible value of s .

11. 在一個 3×3 的方格表中，每格均填進 1 至 9 中的其中一個整數，而且填進每格的數字皆不同。我們然後把每行中數字最大的一格填上紅色，每行中數字最小的一格填上綠色。設 M 為三個紅色方格數字中最小的一個， m 為三個綠色方格數字中最大的一個。 $M - m$ 有多少個不同的可能值？

In a 3×3 table, each cell is filled with an integer between 1 and 9, while different cells are filled with different numbers. We then colour in red the cell in each row with the largest number, and colour in green the cell in each row with the smallest number. Let M be the smallest of the numbers in the three red-coloured cells, and m be the largest of the numbers in the three green-coloured cells. How many different possible values of $M - m$ are there?

12. 如圖所示，平面上有 64 點，它們以每行 8 點，每列 8 點的方式鋪成一個由 49 個全等的小正方形組成的大正方形。如果我們在這些點中任選其中 4 點成為一個正方形的頂點，正方形的面積有多少個不同的可能值？



As shown in the figure, 64 points are drawn on a plane, arranged in a way such that there are 8 points in each row and 8 points in each column, forming 49 congruent squares which comprise a large square. If we choose 4 points among them to be the vertices of a square, how many different possible areas of the squares are there?

第 13 至第 16 題，每題 6 分。

Questions 13 to 16 each carries 6 marks.

13. 對於正整數 n ，設 $A(n)$ 為 n 在二進制表示中「1」的數目。例如，23 以二進制表示為 10111，故 $A(23) = 4$ 。求 $A(1) + A(2) + \cdots + A(1024)$ 。

For positive integer n , let $A(n)$ denote the number of 1's in the base 2 representation of n . For example, 23 in base 2 representation is 10111, so $A(23) = 4$. Find the value of $A(1) + A(2) + \cdots + A(1024)$.

14. 若九位數 $\overline{123xyz789}$ 可被 999 整除，求 $x + y + z$ 的值。

If the nine-digit number $\overline{123xyz789}$ is divisible by 999, find the value of $x + y + z$.

15. 潘先生對小敏和小賢說：「我想了兩個正整數 a 和 b ，其中 $a > b$ 。」然後他秘密地將兩數之差（ $a - b$ ）告訴小敏，及將兩數之積（ ab ）告訴小賢。以下是他們之後的對話。

潘先生問小敏：「你知道 a 和 b 是甚麼嗎？」

小敏說：「不知道。」

然後潘先生問小賢：「你知道 a 和 b 是甚麼嗎？」

小賢說：「我知道啊。它們是 _____」

潘先生立即打斷了小賢的說話，並再問小敏：「現在你知道 a 和 b 是甚麼嗎？」

這時小敏回答：「聽過你和小賢剛才的對話後，我知道它們是甚麼了。它們的和 $a + b$ 大於 90，小於 100。潘先生，對嗎？」

潘先生說：「是啊，你們真聰明呢。」

假設小賢和小敏都是誠實和聰明的（即是說只要當答案可以確定時，他們一定知道答案），求 ab 。

Mr Poon told Dora and Ken: 'I thought of two positive integers a and b , where $a > b$.' He then secretly told Dora the difference of the two numbers (i.e. $a - b$) and Ken the product of the two numbers (i.e. ab). Their subsequent conversation is recorded below.

Mr Poon asked Dora, 'Do you know what a and b are?'

'No.' Dora answered.

Then Mr Poon asked Ken, 'Do you know what a and b are?'

'Yes, they are ...' Ken said.

Mr Poon interrupted Ken immediately and asked Dora again, 'Do you know what a and b are now?'

At that time, Dora said, 'Now I know what they are after listening to the conversation between Ken and you. Their sum (i.e. $a + b$) is greater than 90 but less than 100. Am I right, Mr Poon?'

'Yes. You are so clever!' Mr Poon said.

Assuming that Ken and Dora are honest and intelligent (it means that whenever the answer can be confirmed, they must know the answer), find ab .

16. 2003 位小朋友被編成 1 至 2003 號。他們每人最初有 20 顆糖果，然後按以下方法派給或拿走糖果：先派 3 顆糖果給每位編號是 1 的倍數的小朋友，再從每位編號是 2 的倍數的小朋友手上取走 1 顆糖果，然後派 3 顆糖果給每位編號是 3 的倍數的小朋友，再從每位編號是 4 的倍數的小朋友手上取走 1 顆糖果，如此類推，直至最後派 3 顆糖果給每位編號是 2003 的倍數的小朋友。最後有多少個小朋友比最初多了糖果（即有 21 顆糖果或以上）？

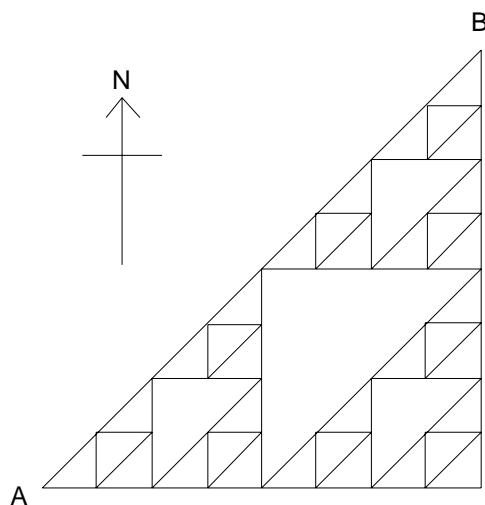
2003 children are labelled 1 to 2003. Each of them has 20 candies at the beginning. Candies are then given to or taken away from them as follows. 3 candies are given to each child with a label which is a multiple of 1, then 1 candy is taken away from each child with a label which is a multiple of 2, then 3 candies are given to each child with a label which is a multiple of 3, then 1 candy is taken away from each child with a label which is a multiple of 4, and so on, till finally 3 candies are given to each child with a label which is a multiple of 2003. How many children can get extra candies (i.e. have 21 candies or more) in the end?

第 17 至第 20 題，每題 7 分。

Questions 17 to 20 each carries 7 marks.

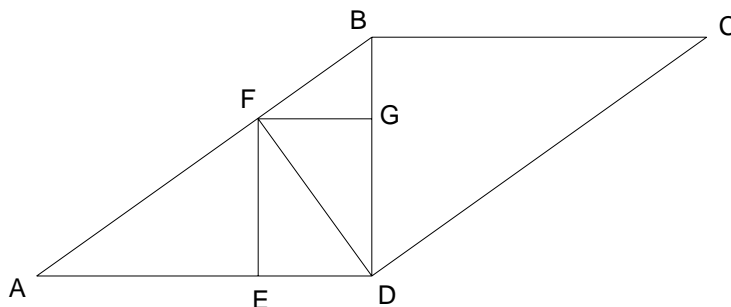
17. 圖中所示為「三角城」的街道圖。若某人欲於 A 點走到 B 點，但只可以沿著街道向東、北或東北走，他可以有多少種走法？

The figure below shows the street map of ‘Triangle City’. If a person at A wants to go to B but is only allowed to go along the streets towards north, east or northeast, in how many different possible ways can he travel?



18. 圖中， $ABCD$ 為平行四邊形。 E 是 AD 上的一點， F 是 AB 上的一點， G 是 BD 上的一點，使得 $DEFG$ 為長方形。若 $BG = 1$ ， $CD = 8$ ，且 $FD \perp CD$ ，求 AE 的長度。

In the figure, $ABCD$ is a parallelogram. E is a point on AD , F is a point on AB and G is a point on BD such that $DEFG$ is a rectangle. If $BG = 1$, $CD = 8$ and $FD \perp CD$, find the length of AE .

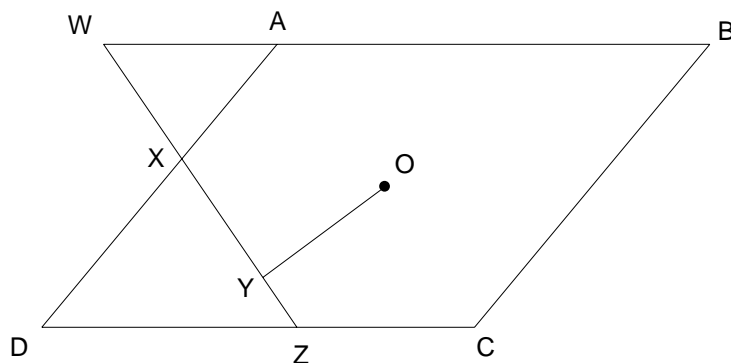


19. 15 名學生（中一級至中五各級三名）參加一個工作坊。他們分成三人一組，並規定每組中任何兩位組員最多只可以相差一級。共有多少種不同的方法把學生分組？

15 students, three from each of secondary 1 to secondary 5, take part in a workshop. They are to be divided into groups of three, and it is required that any two members in the same group differ by at most one form. In how many different ways can we divide the students into groups?

20. 如圖所示， $ABCD$ 為菱形， X 及 Z 分別是 AD 及 DC 上的點。 O 是菱形內的一點， Y 為 O 到 XZ 的垂足，且 O 與 AB 、 BC 、 CD 和 DA 的距離均等於 OY 。若 ZX 與 BA 延長後交於 W ，且 $WX = 5$ ， $XY = 3$ 及 $YZ = 2$ ，求 $ABCD$ 的面積。

In the figure, $ABCD$ is a rhombus. X and Z are points on AD and DC respectively. O is a point inside the rhombus, Y is the foot of the perpendicular from O to XZ and the distances from O to AB , BC , CD and DA are all equal to OY . If ZX and BA are produced to meet at W , $WX = 5$, $XY = 3$ and $YZ = 2$, find the area of $ABCD$.



全卷完

END OF PAPER

個人賽（中二組）答案

Individual Event (Secondary 2) Answers

1. 9

13. 5121

2. 4006

14. 15

3. 50°

15. 97

4. 8375

16. 1753

5. 110.25

17. 1806

6. 30891

18. $3\sqrt{3}$

7. 0

19. 280

8. 1005006

20. 80

9. 2

10. 6

11. 8

12. 18