# 第四屆培正數學邀請賽

# 4th Pui Ching Invitational Mathematics Competition

## 初賽（中四組）

## Heat Event (Secondary 4)

**時限：1 小時 15 分**

**Time allowed: 1 hour 15 minutes**

**參賽者須知：**

**Instructions to Contestants:**

1. 本卷共設 20 題，總分爲 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

   All answers are integers between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

   The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

   The diagrams in this paper are not necessarily drawn to scale.

**第 1 至第 4 題，每題 3 分。**

**Questions 1 to 4 each carries 3 marks.**

1. 現有兩個平行四邊形，第一個的底是第二個的 48 倍，第二個的高是第一個的 16 倍。若第一個平行四邊形的面積是 768，第二個平行四邊形的面積是多少？

   There are two parallelograms. The base of the first parallelogram is 48 times the second's while the height of the second parallelogram is 16 times the first's. If the first parallelogram has area 768, what is the area of the second parallelogram?

2. 某班有 48 位同學，英文得 80 分以上的有 36 人，數學得 80 分以上的有 25 人，兩科都得 80 分以上的有 20 人。問兩科都不超過 80 分的有多少人？

   There are 48 students in a class. Among them, 36 got more than 80 marks in English, 25 got more than 80 marks in Mathematics, and 20 got more than 80 marks in both subjects. How many students got 80 marks or less in both subjects?

3. 求最小的四位數 $n$，使 $n$、$n+2$、$n+4$ 三個數的平均值是 11 的倍數。

   Find the smallest four-digit integer $n$ for which the average of the three numbers $n$, $n+2$ and $n+4$ is a multiple of 11.

4. 某直角三角形的三條邊分別長 2005、5002 和 $x$。那麼 $x$ 有多少個不同的可能值？

   A right-angled triangle has side lengths 2005, 5002 and $x$. How many different possible values does $x$ have?

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**第 5 至第 8 題，每題 4 分。**

**Questions 5 to 8 each carries 4 marks.**

5. 約翰正在做功課，功課的題目是要找 $a - b$ 的值，其中 $a$ 及 $b$ 爲整數。但約翰實在太累了，他把題目中的「–」號誤看成「÷」號。幸運地，他得到了同樣的答案。求 $ab$。

   John was doing his homework. The question asks for the value of $a - b$ where $a$ and $b$ are integers. But John was very tired; he misread the '–' sign in the question as the '÷' sign. Luckily, he got the same answer. Find $ab$.

6. 設 [$x$] 代表不超過 $x$ 的最大整數，例如 [1.1] = 1、[6.9] = 6 和 [5] = 5。若 [$y$] = 3，求 [2005$y$] 的最大可能值。

Let [$x$] denote the greatest integer not exceeding $x$. For example, [1.1] = 1, [6.9] = 6 and [5] = 5. If [$y$] = 3, find the greatest possible value of [2005$y$].

7. 若把 $0.2\dot{3}$（即 0.2333…）化成最簡分數，分子是多少？

If $0.2\dot{3}$ (i.e. 0.2333…) is converted into a fraction in lowest terms, what is the numerator?

8. 已知 $p$ 是一個小於 200 的質數，而且 $p$ 的其中一個數字是 5。求 $p$ 的所有可能值之和。

Given that $p$ is a prime less than 200 and one of the digit(s) of $p$ is 5. Find the sum of all possible values of $p$.

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**第 9 至第 12 題，每題 5 分。**

**Questions 9 to 12 each carries 5 marks.**

9. 某張圓桌的桌面面積爲 $72\pi$。桌上放了一本菱形的書 ，它的四隻角均落在圓桌的周界上。求該書封面的面積。

The surface of a circular table has area $72\pi$. A book in the shape of a rhombus is placed on the table so that all four of its corners lie on the circumference of the circular table. Find the area of the cover of the book.

10. 有三種電線，每條分別長 3 米、4 米和 5 米。現要用這三種電線連接兩個相隔 48 米的地方，並規定每種電線最少要用一條，而且所用電線的總長度必須剛好爲 48 米。問最少要用最少條電線？

There are three types of wires, of lengths 3 m, 4 m and 5 m respectively. We need to connect two places 48 m apart with these wires. It is required that each type of wires must be used at least once and the total length of the wires used must be exactly 48 m. Find the minimum number of wires that must be used.

11. 設 $a_1 = 1$，且當 $n \geq 2$ 時，有 $a_n = a_{n-1}^{\ 2} + 1$。求 $a_{1000}$ 除以 10 時的餘數。

Let $a_1 = 1$ and $a_n = a_{n-1}^{\ 2} + 1$ for $n \geq 2$. Find the remainder when $a_{1000}$ is divided by 10.

12. 每當小明的鬧鐘的時針和分針成 $20.05°$ 時，它便會響起來。每天小明的鬧鐘會響多少次？

Paul's clock rings whenever its hour hand and minute hand form an angle of $20.05°$. How many times does Paul's clock ring every day?

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**第 13 至第 16 題，每題 6 分。**

**Questions 13 to 16 each carries 6 marks.**

13. 對於一個四位數 $\overline{abcd}$ ，若 $\overline{ab}$ 、$\overline{ac}$ 、$\overline{ad}$ 、$\overline{bc}$ 、$\overline{bd}$ 和 $\overline{cd}$ 均爲質數，則稱這個四位數爲「怪數」。求最大的「怪數」。

A four-digit integer $\overline{abcd}$ is said to be a 'strange number' if $\overline{ab}$, $\overline{ac}$, $\overline{ad}$, $\overline{bc}$, $\overline{bd}$ and $\overline{cd}$ are all prime numbers. Find the largest 'strange number'.

14. 首 2005 個正整數中，有多少個可寫成四個連續正整數之和？

How many of the first 2005 positive integers can be written as the sum of four consecutive positive integers?

15. 一個凸 $n$ 邊形剛好有 $9n+36$ 條對角線。求 $n$。

A convex $n$-sided polygon has exactly $9n+36$ diagonals. Find $n$.

16. 某次測驗有兩部分，各設 20 題。在甲部中，同學只需填上答案，每題答對可得 3 分。在乙部中，每題答對可得 6 分，但同學需同時解釋其答案，從而每題答對的問題可另獲 0 分、1 分或 2 分的有效傳意分數。志強在測驗中取得 $n$ 分，其中 $n$ 不超過 100。那麼 $n$ 有多少個不同的可能值？

In a test there are two sections, each with 20 questions. In Section A, students are only required to give the answers, and 3 marks will be awarded for each correctly answered question. In Section B, 6 marks will be given for each correct answer, but students need also give explanations, for which they will be awarded an additional score of 0, 1 or 2 marks in each correctly answered question for effective communication. Jason takes the test and gets a total of $n$ marks. Given that $n$ does not exceed 100, how many possible values of $n$ are there?

**第 17 至第 20 題，每題 7 分。**

**Questions 17 to 20 each carries 7 marks.**

17. 若 $\alpha$ 和 $\beta$ 爲實數，且 $\frac{1}{\alpha+1}+\frac{1}{\beta+1}=3$ 及 $\frac{1}{\alpha+2}+\frac{1}{\beta+2}=1$，求 $\alpha^2+\beta^2$ 的值。

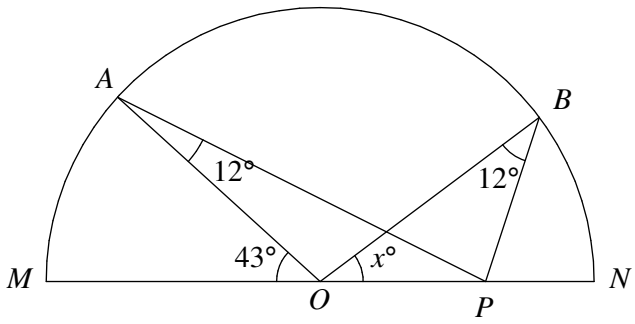
If $\alpha$ and $\beta$ are real numbers such that $\frac{1}{\alpha+1}+\frac{1}{\beta+1}=3$ and $\frac{1}{\alpha+2}+\frac{1}{\beta+2}=1$, find the value of $\alpha^2+\beta^2$.

18. 有些五位正整數具有一種特別的性質 ：任意從中選取三個數字 ，它們之和都是 3 的倍數。具這種特別性質的五位正整數共有多少個？

Some five-digit positive integers possess a special property: the sum of any three of its digits is divisible by 3. How many such five-digit positive integers are there?

19. 如圖所示 ，在圓心爲 $O$，直徑爲 $MN$ 的半圓上有不同的兩點 $A$ 和 $B$。$P$ 在 $ON$ 上，使 $\angle OAP = \angle OBP = 12°$。若 $\angle AOM = 43°$，$\angle BON = x°$，求 $x$。

In the figure, $A$ and $B$ are two distinct points on a semi-circle with centre $O$ and diameter $MN$. $P$ is a point on $ON$ such that $\angle OAP = \angle OBP = 12°$. If $\angle AOM = 43°$ and $\angle BON = x°$, find $x$.




20. 小美、小芬、小吉和小玲各自寫下了一個質數。已知這四個數之積是它們之和的十倍。求四數之積。

Amy, Betty, Cathy and Dorothy each writes down a prime number. The product of these numbers is 10 times their sum. Find the product of the four numbers.

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全卷完

**END OF PAPER**