Hong Kong Mathematics Olympiad 2004-2005
Heat Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 p 和 q 是正整數且 $\frac{96}{35} > \frac{p}{q} > \frac{97}{36}$，求 q 最小可能的值。

   Suppose p, q are positive integers and $\frac{96}{35} > \frac{p}{q} > \frac{97}{36}$, find the smallest possible value of q.

2. 已知 $x = 2005$ 及 $y = |4x^2 - 5x + 9| - 4|x^2 + 2x + 2| + 3x + 7$，求 y 的值。

   Given that $x = 2005$ and $y = |4x^2 - 5x + 9| - 4|x^2 + 2x + 2| + 3x + 7$, find the value of y.

3. 若 x 是實數且滿足 $\left(\sqrt{5+2\sqrt{6}}\right)^x + \left(\sqrt{5-2\sqrt{6}}\right)^x = 10$，求 x 的最小可能的值。

   If x is a real number satisfying the equation $\left(\sqrt{5+2\sqrt{6}}\right)^x + \left(\sqrt{5-2\sqrt{6}}\right)^x = 10$, find the smallest possible value of x.

4. 設 t 為實數且滿足 $(1+\sin t)(1+\cos t) = \frac{5}{4}$。若 $N = \sin t + \cos t$，求 N 的值。

   Let t be a real number satisfying $(1+\sin t)(1+\cos t) = \frac{5}{4}$. If $N = \sin t + \cos t$, find the value of N.

5. 如圖一，*ABCDEF* 是由六個正方形所組成的 “L 形” 圖案。*HAK* 是一直綫，陰影部分的面積是 *ABCDEF* 的面積的 $\frac{1}{2}$。若各小正方形的邊長是 1 cm，*HK* 的長度是 m cm，求 m 的值。

In Figure 1, *ABCDEF* is a “L shape” figure formed by six squares. *HAK* is a straight line and the area of the shaded region is equal to $\frac{1}{2}$ of the area of *ABCDEF*. If the length of each small square is 1 cm and the length of *HK* is *m* cm, find the value of *m*.

圖一

Figure 1

6. 設 n 是自然數，直綫 $nx+(n+1)y=\sqrt{2}$ 與兩坐標軸所圍成的三角形的面積是 S_n。若 $K=S_1+S_2+\cdots+S_{2005}$，求 K 的值。

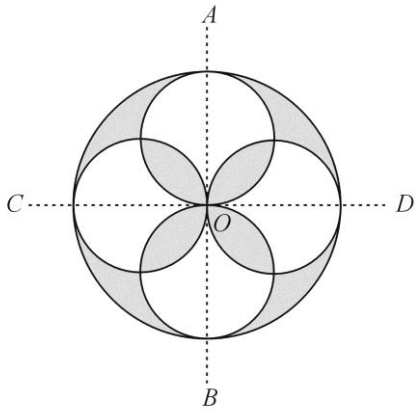
Let n be a natural number, the area of the triangle bounded by the line $nx+(n+1)y=\sqrt{2}$ and the two coordinate axes is S_n. If $K=S_1+S_2+\cdots+S_{2005}$, find the value of *K*.

7. 設 $[x]$ 表示不大於 x 的最大整數，例如 $[2.5]=2$。若 $M=\sum_{n=1}^{1024}\left[\log_2 n\right]$，求 M 的值。

Let $[x]$ be the largest integer not greater than *x*, for example, $[2.5]=2$. If $M=\sum_{n=1}^{1024}\left[\log_2 n\right]$, find the value of *M*.

8. 如圖二，*AB* 垂直於 *CD*，其交點 *O* 是大圓的圓心，而四個小圓的圓心分別在 *AB* 和 *CD* 上。已知大圓的半徑是 1 cm，四個小圓的半徑是 $\frac{1}{2}$ cm。若陰影部分的面積是 $R\text{ cm}^2$，求 R 的值。(取 $\pi = 3$)

In Figure 2, *AB* is perpendicular to *CD*, their intersection point *O* is the center of the large circle and the centers of the four circles lies on either *AB* or *CD*. Given also that the radius of the large circle is 1 cm and the radius of each of the four small circles is $\frac{1}{2}$ cm. If the area of the shaded region is $R\text{ cm}^2$, find the value of *R*. (take $\pi = 3$)



圖二

Figure 2

9. 已知 $60^a = 3$ 及 $60^b = 5$。若 $R = 12^{\frac{1-a-b}{2(1-b)}}$，求 R 的值。

Given that $60^a = 3$ and $60^b = 5$. If $R = 12^{\frac{1-a-b}{2(1-b)}}$, find the value of *R*.

10. 已知 2005 年 1 月 29 日是星期六，那麼 2008 年 1 月 29 日是星期幾？

Given that 29th January 2005 is Saturday, on what day is 29th January 2008?

******* *全卷完* *******

******* ***End of Paper*** *******