

香港培正中學第一屆數學邀請賽

Pui Ching Middle School 1st Invitational Mathematics Competition

個人賽 (中四組)

Individual Event (Secondary 4)

時限：1 小時 30 分

Time allowed: 1 hour 30 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

Unless otherwise stated, all answers should be given in exact numerals in their simplest form.
No approximation is accepted.

4. 所有答案填在答題紙指定的空位上。毋須呈交計算步驟。

Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.

5. 不得使用計算機。

The use of calculators is not allowed.

6. 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 5 題，每題 2 分。

Questions 1 to 5 each carries 2 marks.

1. 若某球體的表面積及體積之數值均為 $k\pi$ ，求 k 的值。

If both the volume and the surface area of a sphere are numerically equal to $k\pi$, find the value of k .

2. 若 $\sin x^\circ = \cos x^\circ$ ，當中 $0 < x < 360$ ，求 x 所有可能值之和。

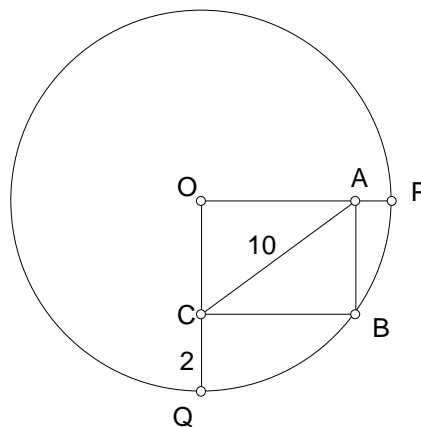
If $\sin x^\circ = \cos x^\circ$, where $0 < x < 360$, find the sum of all possible values of x .

3. 某年，X 國舉辦第二屆數學邀請賽。男參賽者的數目比第一屆增加了 20%，女參賽者的數目則比第一屆增加了 80%。若第二屆的參賽者總數比第一屆多 32%，則第二屆女參賽者的數目佔全部參賽者的幾分之幾？

In a certain year, Country X organised the second invitational mathematics competition. The number of male contestants was increased by 20% compared to the first competition while the number of female contestants was increased by 80% compared to the first competition. If total number of contestants in the second competition was 32% more than that in the first competition, what fraction of the total number of contestants was the number of female contestants in the second competition?

4. 如圖所示， O 為圓形的圓心， B 為圓周上的一點，且 $OABC$ 為長方形。 OA 與 OC 延長後分別交圓於 P 及 Q 。若 $AC = 10$ ，且 $CQ = 2$ ，求 AP 。

In the figure, O is the centre of the circle, B is a point on the circumference and $OABC$ is a rectangle. OA and OC are produced to meet the circle at P and Q respectively. If $AC = 10$ and $CQ = 2$, find AP .



5. 已知當 $\frac{1}{19}$ 寫成循環小數時是 $0.\dot{0}5263157894736842\dot{1}$ 。如果將 $\frac{200}{19}$ 寫成循環小數，小數點後第 2002 個位的數字是甚麼？

Given that when $\frac{1}{19}$ is written as a recurring decimal, it is $0.\dot{0}5263157894736842\dot{1}$. If $\frac{200}{19}$ is written as a recurring decimal, what is the 2002nd digit after the decimal point?

第 6 至第 10 題，每題 4 分。

Questions 6 to 10 each carries 4 marks.

6. 投擲三顆公平的骰子，問所得點數之積是 5 的倍數的概率是多少？

Three fair dice are thrown. What is the probability that the product of the numbers obtained is a multiple of 5?

7. 若 x 為實數，且 $\log_{10} x = \frac{1}{x}$ ，求 $(x^2)^{2x}$ 的值。

If x is a real number such that $\log_{10} x = \frac{1}{x}$, find the value of $(x^2)^{2x}$.

8. 若 n 為正整數，使得 $1 + 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{2n} - 225$ ，求 n 。

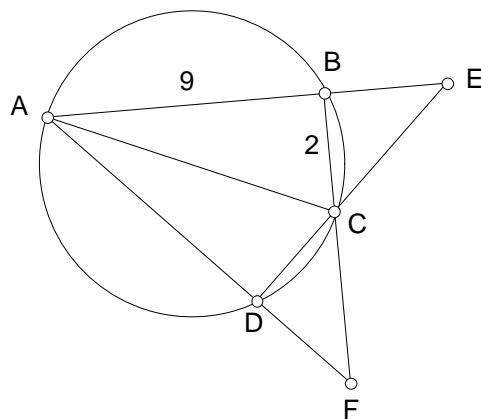
If n is a positive integer such that $1 + 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{2n} - 225$, find n .

9. 符合 $x - y = x^2 + xy + y^2$ 的非負整數 (x, y) 共有多少對？

How many pairs of non-negative integers (x, y) satisfy $x - y = x^2 + xy + y^2$?

10. 圖中， ABE 、 DCE 、 ADF 及 BCF 為直線， $\angle BEC = \angle CFD$ ， $AB = 9$ ，且 $BC = 2$ 。求 AC 的長度。

In the figure, ABE , DCE , ADF and BCF are straight lines, $\angle BEC = \angle CFD$, $AB = 9$ and $BC = 2$. Find the length of AC .



第 11 至第 15 題，每題 6 分。

Questions 11 to 15 each carries 6 marks.

11. 一所飲品公司正推行一項以印花換領飲品的計劃。此飲品乃非賣品，但每 11 個印花可換領 1 瓶飲品。而且，每瓶飲品上皆有 1 個印花以作日後換領之用。如果某人希望換領 20020302 瓶飲品，那麼他最少需要多少個印花？

A beverage company is promoting a scheme of drink redemption by stamps. The drinks are not for sale, but every 11 stamps can redeem a bottle of drink. Moreover, there is a stamp on each bottle of drink for further redemption. If someone wants to redeem 20020302 bottles of drink, how many stamps does he need?

12. 在一個正方形內畫 2002 點，並把其中一些點用線段連起，再沿這些線段把正方形切割成一些三角形，而這些三角形的頂點均為正方形的頂點或該 2002 點中的一些點。問最多可得到多少個三角形？

2002 points are drawn inside a square. Some of these points are connected by line segments and the square is cut along these line segments into a number of triangles, each of which has vertices among either the 2002 points or the vertices of the square. What is the largest number of triangles obtained?

13. 若 x 為實數，求 $|x-1|-|x-2|+|x-3|-|x-4|+\cdots+|x-2001|-|x-2002|$ 的最小可能值。

If x is a real number, find the minimum possible value of

$$|x-1|-|x-2|+|x-3|-|x-4|+\cdots+|x-2001|-|x-2002|.$$

14. 求 $3^{2002}+5^{2002}$ 除以 64 時的餘數。

Find the remainder when $3^{2002}+5^{2002}$ is divided by 64.

15. 求 $\frac{\tan 1^\circ}{1+\tan 1^\circ}+\frac{\tan 2^\circ}{1+\tan 2^\circ}+\frac{\tan 3^\circ}{1+\tan 3^\circ}+\cdots+\frac{\tan 89^\circ}{1+\tan 89^\circ}$ 的值。

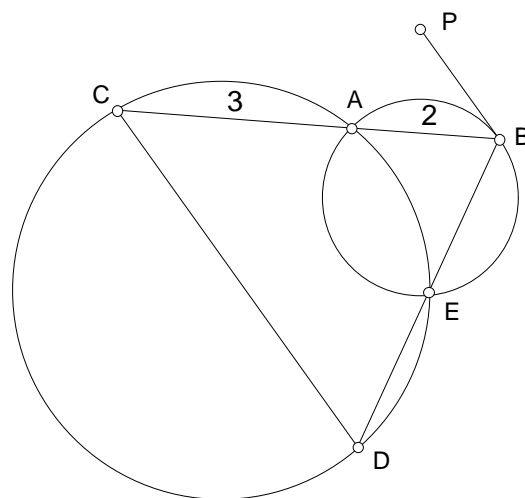
Find the value of $\frac{\tan 1^\circ}{1+\tan 1^\circ}+\frac{\tan 2^\circ}{1+\tan 2^\circ}+\frac{\tan 3^\circ}{1+\tan 3^\circ}+\cdots+\frac{\tan 89^\circ}{1+\tan 89^\circ}$.

第 16 至第 20 題，每題 8 分。

Questions 16 to 20 each carries 8 marks.

16. 圖中， BAC 及 BED 為直線， PB 切圓 BAE 於 B ， CD 為圓 $ACDE$ 的直徑，且 $\angle PBA = 60^\circ$ 。若 $AB = 2$ ， $AC = 3$ ，求 BD 的長度。

In the figure, BAC and BED are straight lines, PB is tangent to circle BAE at B , CD is a diameter of circle $ACDE$, and $\angle PBA = 60^\circ$. If $AB = 2$, $AC = 3$, find the length of BD .



17. 設 x 和 y 為正整數。當 x 除以 y 時，商為 4321，餘數為 1234。當 x 除以 $2y$ 時，餘數的最小可能值是多少？

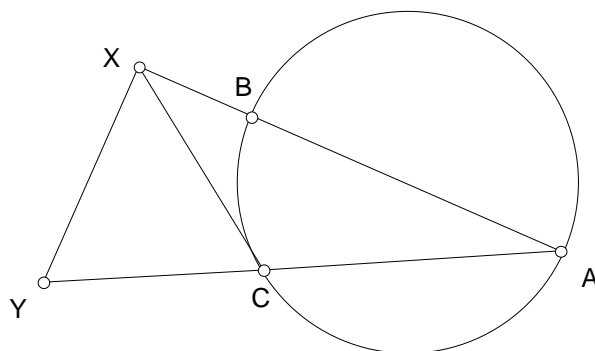
Let x and y be positive integers. When x is divided by y , the quotient is 4321 and the remainder is 1234. What is the smallest possible remainder when x is divided by $2y$?

18. 有一個牧羊人到草地上放羊。如果他放 40 隻羊，草會在 30 星期後被吃完；如果他放 100 隻羊，草則會在 5 星期後被吃完。已知每隻羊每天所吃的草的量也一樣，並且草地上的草會以均速生長。如果他希望草永遠不會被吃完，那麼他最多可放多少隻羊？

A shepherd is herding his sheep on a piece of grassland. If he herds 40 sheep, all grass will be eaten after 30 weeks; if he herds 100 sheep, all grass will be eaten after 5 weeks. Given that the amount of grass eaten by every sheep is the same every day and that the grass grows with a constant rate. If he hopes that the grass will not be eaten up forever, what is the maximum number of sheep he can herd?

19. 圖中, ABX 及 ACY 為直線, XC 切圓於 C , $XC = XY$, 且 $AX \perp XY$. 若 $AB = 2$, $BX = 1$, 求 AY 的長度。

In the figure, ABX and ACY are straight lines, XC touches the circle at C , $XC = XY$ and $AX \perp XY$. If $AB = 2$, $BX = 1$, find the length of AY .



20. 董先生參加某國家的總統選舉, 得票率 (準確至小數點後一個位) 為 66.6%。問董先生最少得到多少票?

Mr Tung joined the presidential election of a country and obtained 66.6% of the total votes, correct to 1 decimal place. What is the minimum number of votes Mr Tung obtained?

全卷完

END OF PAPER

個人賽（中四組）答案

Individual Event (Secondary 4) Answers

1. 36

11. 200203021

2. 270

12. 4006

3. $\frac{3}{11}$

13. -1001

4. 4

14. 34

5. 3

15. 44.5

6. $\frac{91}{216}$

16. $\sqrt{31}$

7. 10000

17. 2469

8. 4

18. 28

9. 2

19. $2\sqrt{3}$

10. $\sqrt{85}$

20. 191