個人賽（中一組）

Individual Event (Secondary 1)

時限：**1** 小時 **30** 分

Time allowed: 1 hour 30 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設甲、乙兩部分，總分為 100 分。

    This paper is divided into Section A and Section B. The total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

    Unless otherwise stated, all numbers in this paper are in decimal system.

3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

    Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

4. 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。

    Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.

5. 不得使用計算機。

    The use of calculators is not allowed.

6. 本卷的附圖不一定依比例繪成。

    The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 4 題，每題 3 分。	Questions 1 to 4 each carries 3 marks.
第 5 至第 8 題，每題 5 分。	Questions 5 to 8 each carries 5 marks.
第 9 至第 12 題，每題 7 分。	Questions 9 to 12 each carries 7 marks.

1. 若 n 除以 2004 時的餘數為 1234，則 $2n$ 除以 2004 時的餘數是多少？

    If n leaves a remainder of 1234 when divided by 2004, what is the remainder when $2n$ is divided by 2004?

2. 3600 個正方形盒子排成了一個邊長為 60 個盒子的正方形。小明在其中一個盒子裏放了一個金幣。小芳想找出金幣。她每次可以選一個盒子，並打開它和（最多八個）圍繞著它的盒子。她最少要選多少個盒子才可保證找到金幣？

    3600 square boxes were aligned as a square with a length of 60 boxes. Alan put a coin in one of the boxes. Betty wanted to find the coin. Every time she could choose one of the boxes, open it and the (at most eight) boxes surrounding it. At least how many boxes must Betty choose to guarantee that she must find the coin?

3. 數字之和為偶數的三位正整數（例如：123、790）共有多少個？

    How many 3-digit positive integers have the sum of their digits even (e.g. 123, 790)?

4. 小明在十月時儲蓄了一些金錢。在十一月的每一天的早上，他的媽媽都給他 15 元零用錢。每天他亦會花他所有金錢的總數的 10% 吃午餐。已知他在十一月結束時有 135 元，他在十一月共花了多少元吃午餐？

    Peter saved some money in October. In each morning of November, his mother gave him $15 as pocket money. He then spent 10% of the total amount he had for lunch every day. Given that the amount he had was $135 at the end of November, how many dollars did he spend on lunch in November?

5. 在 24 至 2004 之間，正因數之積為奇數的正整數有多少個？

How many positive integers between 24 and 2004 are there such that the product of their positive factors is odd?

6. 在一個遊戲中，2004 名參加者排成一行，他們由左至右被編號為 1 至 2004。遊戲開始時，各參加者分別由左至右順序叫出「A、B、C、D、E、A、B、C、D、E、…」。叫出「E」的參加者為輸家，他們需要離開，不能再參加遊戲。餘下的各得 1 分，並重新再分別由左至右順序叫出「A、B、C、D、E、A、B、C、D、E、…」。跟之前一樣，叫出「E」的參加者需要離開，餘下的各再得 1 分，然後遊戲再重新開始。遊戲一直繼續，直至餘下的參加者少於 5 人。問編號 1002 的參加者得到多少分？

In a game 2004 players line up in a row. They are numbered from 1 to 2004 from left to right. To start the game, the players call out 'A, B, C, D, E, A, B, C, D, E, …' from left to right in order. Those who call out 'E' lose and they have to leave. The remaining players get 1 point each, and they start to call out 'A, B, C, D, E, A, B, C, D, E, …' from left to right in order again. As before, those who call out 'E' lose and have to leave; those remaining get 1 more point each and the game starts again. The game is repeated until fewer than 5 players remain. How many points does the player numbered 1002 get?

7. 小明用木棒砌三角錐體。他有兩種分別長 10 cm 和 21 cm 的木棒。他最多可以砌出多少個不同形狀的三角錐體？（若兩個三角錐體可旋轉至對應邊長度相同，則兩個三角錐體的形狀視為相同。）

Peter tries to build tetrahedra with wooden sticks. He only has wooden sticks of lengths 10 cm and 21 cm. What is the largest number of tetrahedra of different shapes he can build? (Two tetrahedra are regarded to have the same shape if we can suitably rotate them to make the lengths of the corresponding edges the same.)

8. 圖中，*ABCD* 為平行四邊形，*F* 和 *G* 分別是 *AB* 和 *CD* 上的點，使 *FG // AD*。*FG* 與 *BD* 交於 *E*。若 △*AEF* 和梯形 *BCGE* 的面積分別是 1 和 5，求 *ABCD* 的面積。

In the figure, *ABCD* is a parallelogram. *F* and *G* are points on *AB* and *CD* respectively such that *FG // AD*. *FG* intersects *BD* at *E*. If the areas of △*AEF* and trapezium *BCGE* are 1 and 5 respectively, find the area of *ABCD*.
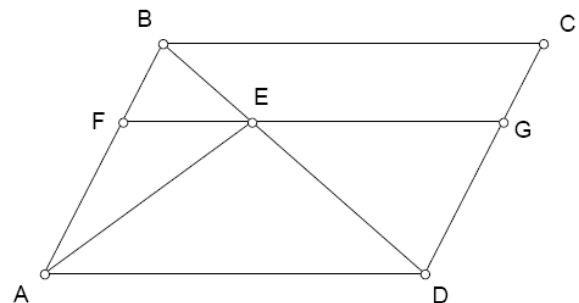
9. 圖中，ΔABC 的邊長為 3、4、5，其中 B 是直角。從 B 開始重覆畫垂線到 AC 和 BC。求陰影部分的面積。

   In the figure, ΔABC with side lengths 3, 4, 5 is right-angled at B. Starting from B, perpendiculars are drawn to sides AC and BC repeatedly. Find the area of the shaded region.
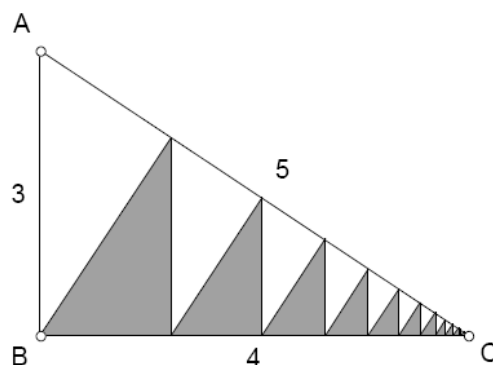
10. 如圖所示，384 個大小相同的正方形整齊地排成 64 列，每列 6 個，並用直線互相連起。每個正方形均被塗上紅色或綠色，使得沒有兩列的塗色完全相同。現在我們把紅色的正方形移走，那麼餘下的綠色正方形被分成若干個互不相連的部分。設最多正方形的一部分有 S 個正方形，求 S 的最小可能值。

   In the figure, 384 squares of the same size are arranged regularly in 64 columns with 6 squares in each column. The squares are connected together by straight lines. Each square is coloured red or green in a way such that no two columns have identical colouring. Now we remove the red squares, so that the remaining green squares are split into several mutually disconnected parts. Suppose that the part with the largest number of squares has S squares, find the smallest possible value of S.

   64 列
   64 columns
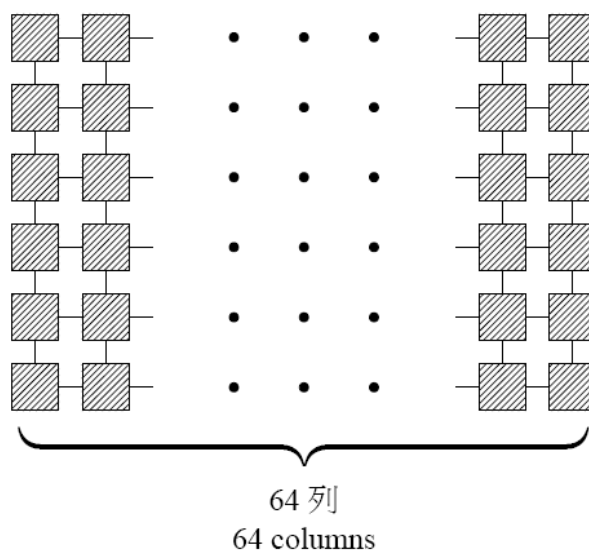
11. 在 ΔABC 中，D 是 AC 上的一點，E 是 AB 上的一點，使得 $BC^2 = AB \times BE = AC \times CD$。若 $BC = 12$，$BD = 16$，求 CE 的長度。

In ABC, D is a point on AC and E is a point on AB such that $BC^2 = AB \times BE = AC \times CD$. If $BC = 12$ and $BD = 16$, find the length of CE.
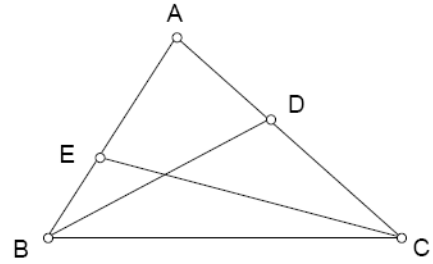
12. 一種常用的日期表示法是以「年／月／日」六位數字形式寫出日期，例如：2004 年 3 月 7 日寫成 04/03/07。由於 $04 + 03 = 07$，我們說這天是「好日子」。一般來說，若某天在以上的日期表示法中，代表「年」、「月」、「日」的三個兩位數中其中一個等於另外兩個之和，則那天稱為「好日子」。那麼，在二十一世紀中（2001 年 1 月 1 日至 2100 年 12 月 31 日），「好日子」共有多少天？

A usual way of writing dates is the 'YY/MM/DD' method of expressing a date as a six-digit number. For instance, 7th March 2004 is denoted as 04/03/07. Since $04 + 03 = 07$, we say that this is a 'good day'. In general, a day is said to be a 'good day' if, among the three two-digit numbers representing 'year', 'month' and 'day' in the above representation, one of them is equal to the sum of the other two. How many 'good days' are there in the 21st century (from 1st January 2001 to 31st December 2100)?

13. 下圖所示為「正方形城」的街道圖。每個小正方形的邊長均代表實際距離 1 公里。陳先生在 *A* 點開設了一間薄餅店，提供薄餅速遞服務。

    The figure below is the street map of 'Square City'. The side length of each small square represents an actual distance of 1 km. Mr Chan operates a pizza restaurant at point *A* and provides pizza delivery service.
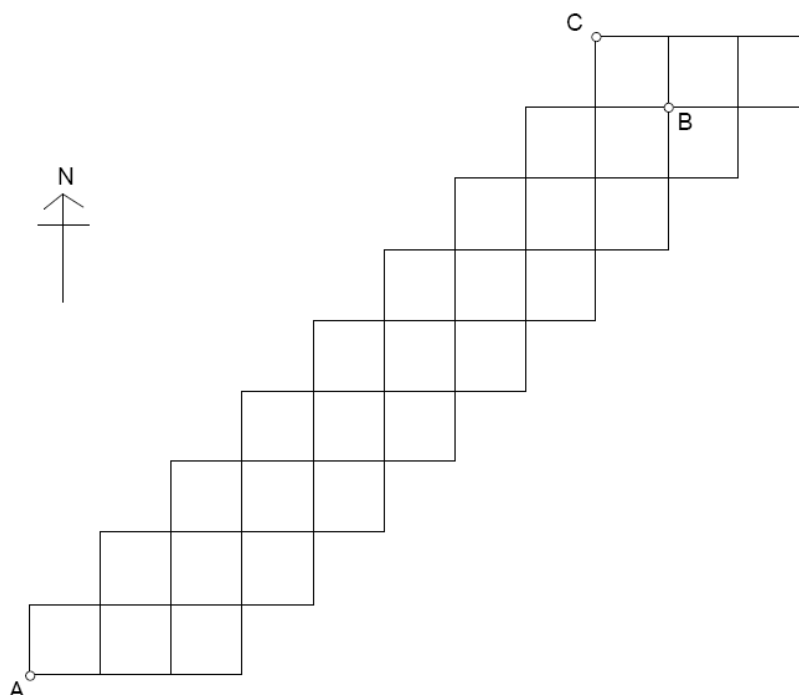
    (a) 某天，陳先生分別接到在 *B* 點和 *C* 點的兩個訂單。他從 *A* 點出發，需要沿街道把薄餅送到 *B* 點和 *C* 點（次序不拘），並返回 *A* 點。那麼，他所走的距離最短是多少公里？ （2分）

    One day Mr Chan received two orders at points *B* and *C* respectively. Starting from point *A*, he had to deliver the pizzas to points *B* and *C* (the order of which does not matter) via the streets and then return to point *A*. What is the minimum distance (in km) that he must travel? (2 marks)

(b) 若陳先生只可沿街道向北方或東方走，則由 A 點走到 B 點有多少種不同的走法？　（5分）

If Mr Chan can only go northward or eastward via the streets, in how many different ways can he travel from point A to point B?　（5 marks）

(c) 陳先生發現，薄餅店位於城市的西南端，地點並不理想。一旦接到偏遠的東北部的訂單，送貨的成本高昂，並不划算。因此，他決定把現有的薄餅店關閉，另覓兩個地點開設兩間新店。每間新店必須位於兩條街道的交匯處（即街道圖中小正方形的頂點），並且要求從城市內街道上的任何一點接到訂單時，均可從其中一間新店沿街道送貨，其距離不超過 6 公里。那麼，兩間新店的選址有多少個不同的組合？　（6分）

Mr Chan finds that the pizza restaurant, situated in the southwestern tip of the city, is not at an ideal location. Whenever orders from the remote northeastern area are received, high costs of transportation will be involved and thereby greatly reducing cost-effectiveness. As a result, he decides to close down the current restaurant and chooses two other locations for two new restaurants. Each new restaurant must be at the intersection of two streets (i.e. the vertices of the small squares in the street map), and it is required that whenever orders at any point on the streets of the city are received, delivery service can be provided from one of the new restaurants, and the distance of delivery is at most 6 km along the streets. How many different combinations are there for the locations of the two new restaurants?　（6 marks）

(d) 為了方便起見，陳先生把街道圖上的每個小正方形塗上紅色、黃色或綠色，使得同一橫行或直行中，沒有兩個小正方形的顏色相同。那麼，街道圖共有多少種不同的填色方法？　（7分）

For convenience, Mr Chan colours each small square on the street map in red, yellow or green, in a way such that no two small squares in the same row or column are assigned the same colour. How many different colouring schemes are there?　（7 marks）

14. *ABCD* 是一個長 p、闊 q 的長方形，並被分成 pq 個邊長為 1 的小正方形。這些小正方形的頂點稱為「格點」。圖中顯示了 $p = 5$、$q = 6$ 時的情況。當 p 和 q 的值有所不同時，*ABCD* 的形狀和大小亦會隨之而改變。
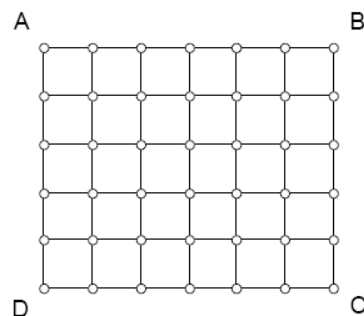
ABCD is a rectangle of length p and width q. It is divided into pq small squares of side length 1. Vertices of these small squares are said to be 'lattice points'. The figure shows the case with $p = 5$ and $q = 6$. The shape and size of *ABCD* vary as the values of p and q change.

(a) 當 $p = 99$，$q = 2004$ 時，圖中會有多少個格點？ （3 分）

   If $p = 99$ and $q = 2004$, how many lattice points will there be in the figure? (3 marks)

(b) 設 $p = 48$，$q = 72$。直線 *AC* 上有多少個格點（包括 *A* 和 *C*）？ （4 分）

   Suppose $p = 48$ and $q = 72$. How many lattices points are there on the straight line *AC* (including *A* and *C*)? (4 marks)

(c) 設 $p = 3$，$q = 4$。現要選取四個格點，以組成長方形（包括正方形）。那麼，共有多少種不同的選法？ （6 分）

   Suppose $p = 3$ and $q = 4$. Now we want to form rectangles (including squares) by picking four lattice points. How many different choices are there? (6 marks)

(d) 小明沿直線 *AC* 把長方形剪開。他發現剛好有 2004 個小正方形被分割成兩部分。把 *ABCD* 的周界記為 S。已知 S 有多個不同的可能值。在這些可能值中，求最接近 5000 的一個。 （7 分）

   Peter cuts the rectangle along the straight line *AC*. He finds that exactly 2004 of the small squares are split into two parts. Denote the perimeter of *ABCD* by S. It is known that S has many different possible values. Among these possible values, find the one that is closest to 5000. (7 marks)

全卷完

END OF PAPER