Hong Kong Mathematics Olympiad 2002-2003
Heat Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 f 是一函數，使對所有整數 m 及 n，$f(m)$ 是整數及 $f(mn)=f(m)f(n)$。已知當 $9>m>n$ 時，$f(m)>f(n)$，且 $f(2)=3$ 及 $f(6)>22$，求 $f(3)$ 的值。

   Let f be a function such that for all integers m and n, $f(m)$ is an integer and $f(mn)=f(m)f(n)$. It is given that $f(m)>f(n)$ when $9>m>n$, $f(2)=3$ and $f(6)>22$, find the value of $f(3)$.

2. 若 $P=\frac{1}{4}$，求 $P\log_2 P$ 的值。

   If $P=\frac{1}{4}$, find the value of $P\log_2 P$.

3. 若 $0\le x\le 1$，求 $\left[\log_{10}\left(\frac{99999x+1}{1000}\right)\right]^2$ 的最大值。

   If $0\le x\le 1$, find the maximum value of $\left[\log_{10}\left(\frac{99999x+1}{1000}\right)\right]^2$.

4. 已知 0 及 1 是二次方程 $a(x+1)(x+2)+b(x+2)(x+3)+c(x+3)(x+1)=0$ 的根。若 $k=\frac{a}{b}$，求 k 的值。

   Given that a quadratic equation $a(x+1)(x+2)+b(x+2)(x+3)+c(x+3)(x+1)=0$ has roots 0 and 1, and $k=\frac{a}{b}$, find the value of k.

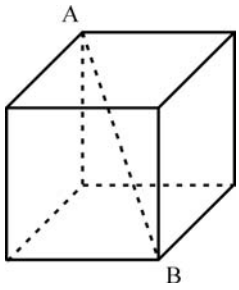
5. 課室內有 n 個人，若每個人恰好跟其他人各握手一次，則共有 28 次握手，求 n 的值。

There are n persons in the classroom. If each person in the classroom shakes hands exactly once with each other person in the classroom and there are altogether 28 handshakes. Find the value of n.

6. 若對任意 $0 < x < \frac{\pi}{2}$，$\cot\frac{1}{4}x - \cot x \equiv \frac{\sin kx}{\left(\sin\frac{1}{4}x\right)(\sin x)}$，其中 k 是一常數，求 k 的值。

If for any $0 < x < \frac{\pi}{2}$, $\cot\frac{1}{4}x - \cot x \equiv \frac{\sin kx}{\left(\sin\frac{1}{4}x\right)(\sin x)}$, where k is a constant, find the value of k.

7.

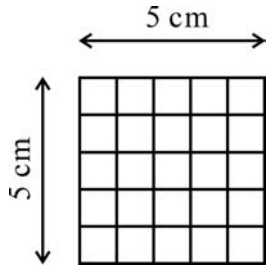


圖一 (Figure 1)

圖一中，正方體的對角線 AB 的長度是 $\sqrt{12}$ cm。若該正方體的體積是 M cm^3，求 M 的值。

In Figure 1, AB is a diagonal of the cube and $AB = \sqrt{12}$ cm. If the volume of the cube is M cm^3, find the value of M.

8.



圖二 (Figure 2)

圖二中，一個面積爲 25 cm^2 的正方形被分成 25 個邊長爲 1 cm 的小正方形。若圖中共有 K 個不同的正方形，求 K 的值。

In Figure 2, a square with area equal to 25 cm^2 is divided into 25 small squares with side length equal to 1 cm. If the total number of different squares in the figure is K, find the value of K?

9. 已知六位數 $N = x1527y$ 是 4 的倍數，且 N 被 11 除餘 5。求 $x + y$ 的值。

It is given that the 6-digits number $N = x1527y$ is a multiple of 4, and the remainder is 5 when N is divided by 11. Find the value of $x + y$.

10. 一個三角形的三邊長分別是 7.5 cm、 11 cm 和 x cm。若 x 爲整數， 求 x 的最小值。

The sides of a triangle have lengths 7.5 cm, 11 cm and x cm respectively. If x is an integer, find the minimum value of x.

*** *全卷完* ***

*** ***End of Paper*** ***