

**第五屆培正數學邀請賽**

**5th Pui Ching Invitational Mathematics Competition**

**初賽（中二組）**

**Heat Event (Secondary 2)**

**時限：1 小時 15 分**

**Time allowed: 1 hour 15 minutes**

**參賽者須知：**

**Instructions to Contestants:**

1. 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

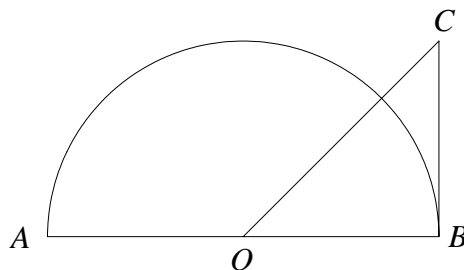
The diagrams in this paper are not necessarily drawn to scale.

1. 若把分數  $\frac{3}{8}$  的分子和分母都加上  $n$ ，則所得的分數會與  $\frac{1}{2}$  等值。求  $n$ 。 (3 分)

If  $n$  is added to both the numerator and the denominator of  $\frac{3}{8}$ , the result obtained will be equal to  $\frac{1}{2}$ . Find  $n$ . (3 marks)

2. 圖中， $\triangle OBC$  是等腰直角三角形，其中  $BO = BC$ 。 $AB$  是以  $O$  為圓心的半圓的直徑。若  $AB = 24$ ，而全圖覆蓋範圍的面積是  $a + b\pi$ ，其中  $a$  和  $b$  是整數，求  $a + b$  的值。

In the figure,  $\triangle OBC$  is a right-angled isosceles triangle with  $BO = BC$ .  $AB$  is a diameter of the semi-circle with centre  $O$ . If  $AB = 24$  and the figure covers an area of  $a + b\pi$  where  $a$  and  $b$  are integers, find the value of  $a + b$ .



(3 分)

(3 marks)

3. 設  $a = 6x - 48$ 、 $b = 7x - 112$ 。已知  $a$  和  $b$  當中較大的一個不是負數，求  $x$  的最小可能值。

(4 分)

Let  $a = 6x - 48$  and  $b = 7x - 112$ . Given that the larger number among  $a$  and  $b$  is not negative, find the smallest possible value of  $x$ . (4 marks)

4. 318 名小朋友排成一個圈玩遊戲。遊戲開始前，其中一位小朋友手持皮球。在遊戲中，當小朋友手上持有皮球時，他必須將球傳給其中一位與他相鄰的小朋友。在第 2006 次傳球後，肯定不會手持皮球的小朋友有多少名？

(4 分)

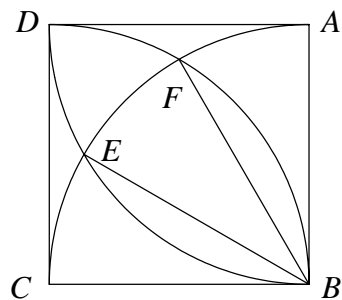
318 children were arranged in a circle to play a game. A ball was held by one of the children before the game. When a child held a ball during the game, he must pass the ball to one of his adjacent children. After the 2006th pass, how many children must not be holding the ball? (4 marks)

5. 小穎想了一個正整數  $n$ ，並寫下了它所有小於 100 的正倍數。她發現在這些數當中，「8」這個數字出現了剛好四次。求  $n$  的值。

(4 分)

Diana thought of a positive integer  $n$  and wrote down all its positive multiples less than 100. She found that the digit '8' occurs exactly four times. Find the value of  $n$ . (4 marks)

6. 如圖所示， $ABCD$  是正方形，而三段圓弧的圓心分別是  $A$ 、 $B$  和  $C$ ，半徑都與正方形的邊長相同。以  $B$  為圓心的圓弧分別交以  $A$ 、 $C$  為圓心的圓弧於  $E$  和  $F$ 。若  $\angle EBF = x^\circ$ ，求  $x$ 。



(4 分)

As shown in the figure,  $ABCD$  is a square. The centres of the three arcs are  $A$ ,  $B$  and  $C$  while their radii are the same as the side length of the square. The arc centred at  $B$  meets the arc centred at  $A$  and  $C$  at  $E$  and  $F$  respectively. If  $\angle EBF = x^\circ$ , find  $x$ .

(4 marks)

7. 123 名學生排成一列。已知任何連續三名學生的平均年齡都是 20 歲。求全部學生的年齡總和。

(4 分)

123 students were aligned in a row. It is known that the average age of any three consecutive students was 20. Find the sum of ages of all students.

(4 marks)

8. 在一個電視遊戲節目裏，100 名參賽者需於「紅」和「黃」之中二擇其一。他們作出選擇後仍有一次改變主意的機會。下表顯示了遊戲的統計數據，可是部分數字卻遺失了，只能以「？」顯示。根據表裏的資料，有多少名參賽者開始時選了「黃」，並且沒有改變主意？

(4 分)

In a TV game show, 100 contestants were asked to choose either 'red' or 'yellow'. After making the choice, they had a chance to change their options. The table below shows the statistics of the game. However, some data are missing, so they are only displayed with '?'. From the information in the table, how many contestants chose 'yellow' at first and did not change their options?

(4 marks)

第一次選擇 First option	紅 Red	?
	黃 Yellow	?
改變主意後 After the change of options	紅 Red	56
	黃 Yellow	?
改變主意的人數 Number of contestants changing their options		12
由紅轉黃的人數 Number of contestants changing from red to yellow		?
由黃轉紅的人數 Number of contestants changing from yellow to red		7

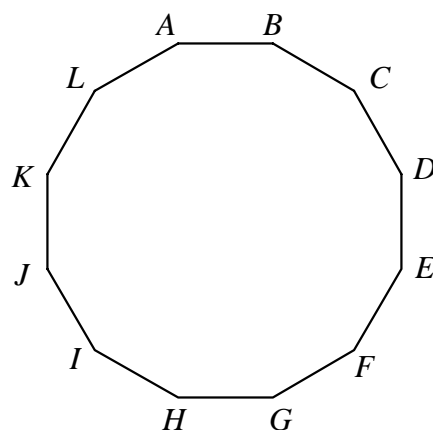
9. 小芳用了 7 塊長為 4、闊為 3 的長方形紙片拼成了一個大長方形。已知那些紙片沒有重疊，求大長方形的周界的可能值之和。 (5 分)

Amy used 7 pieces of rectangular cardboard with length 4 and width 3 to form a big rectangle. Given that there is no overlapping of cardboard, find the sum of all possible values of the perimeter of the big rectangle. (5 marks)

10. 數列 1、4、9、16、... 把所有正平方數由小至大排列。如果將數列內所有個位數字為 4 的數刪掉後，第 2006 項便是  $k^2$ ，其中  $k$  是正整數。求  $k$  的值。 (5 分)

The sequence 1, 4, 9, 16, ... lists all positive square numbers in ascending order. If all terms with unit digit 4 are removed, the 2006th term is  $k^2$  where  $k$  is a positive integer. Find the value of  $k$ . (5 marks)

11. 小德和小培在圖中的正十二邊形的跑道上跑步。他們同時在 A 點出發，並各自以均速沿順時針方向跑。他們在起跑後 1 小時於 I 點首次相遇。設於起跑後  $k$  小時他們首次在 A 點相遇，求  $k$  的值。



Sam and Tom ran on a track in the shape of a regular 12-sided polygon. They started running from A and each ran with constant speeds along the clockwise direction. They first met at I one hour after they started running. Suppose they met at A for the first time  $k$  hours after they started running, find the value of  $k$ .

(5 分)

(5 marks)

12. 某幢 2006 層的樓宇裏的電梯只有兩個按鈕「上」和「下」。當「上」被按一次時，電梯便會上升三層。（若當時電梯已在最頂的三層，那麼它便會停在頂層，即 2006 樓。）當「下」被按一次時，電梯便會下降四層。（若當時電梯已在最底的四層，那麼它便會停在底層，即 1 樓。）已知電梯的速度是每秒 1 層。假如小剛希望由 1000 樓前往 2000 樓，他最少需時多少秒？ (5 分)

An elevator in a 2006-storey building has only 2 buttons, 'Up' and 'Down'. When 'Up' is pressed once, the elevator goes up 3 storeys. (If the elevator is at the top 3 floors, then it stops at the top floor, i.e. the 2006th floor.) When 'Down' is pressed once, the elevator goes down 4 storeys. (If the elevator is at the bottom 4 floors, then it stops at the bottom floor, i.e. the 1st floor). It is known that the speed of the elevator is 1 storey per second. Suppose John wants to go to the 2000th floor from the 1000th floor by the elevator. At least how many seconds does he need? (5 marks)

13. 某數學比賽共設有 5 份試卷，每份 20 題。已知部分題目在多於一份試卷裏重覆使用。假設每題最多只在 3 份試卷出現，5 份試卷裏最少共有多少道不同的題目？

(5 分)

There are 5 papers in a mathematical competition and 20 questions are set in each paper. It is known that some questions are used repeatedly in more than one paper. Given that each question only appears in at most 3 papers, at least how many different questions are there in these 5 papers?

(5 marks)

14. 如果某正整數可以寫成兩個不同的合成數之和，我們便稱它為「好數」。例如，因為  $140 = 63 + 77$ ，而 63 和 77 是不同的合成數，所以 140 是「好數」。有多少個正整數不是「好數」？

(6 分)

A positive integer is said to be 'good' if it can be written as the sum of two distinct composite numbers. For instance, 140 is 'good' since  $140 = 63 + 77$ , where 63 and 77 are distinct composite numbers. How many positive integers are not 'good'?

(6 marks)

15. 黑板上寫了 1、2、3、...、2006 這 2006 個不同的整數。後來小輝把黑板上的兩個數  $x$  和  $y$  (其中  $x \geq y$ ) 擦掉，再在黑板寫上它們的差  $x - y$ 。當他重覆上述的動作 2005 次後，黑板上便只剩下一個數。這個數的最大可能值是甚麼？

(6 分)

On a blackboard 2006 distinct integers 1, 2, 3, ..., 2006 were written. After that, Peter erased 2 numbers  $x$  and  $y$  on the blackboard (where  $x \geq y$ ) at a time and wrote their difference  $x - y$  on the blackboard. After repeating this process 2005 times, exactly one number remained on the blackboard. What is the largest possible value of this number?

(6 marks)

16. 在所示的算式中，每個字母代表一個 1 至 9 的不同數字。求  $A + B + C$  的值。

(6 分)

In the calculation shown, each letter represents a different digit from 1 to 9. Find the value of  $A + B + C$ .

$$\begin{array}{r} A \quad B \\ \times \quad A \\ \hline C \quad B \quad B \end{array}$$

(6 marks)

17. 設  $m$  和  $n$  是兩個五位正整數，其中  $m > n$ 。已知  $m$  和  $n$  的 10 個數字都不相同。求  $m - n$  的最小值。

(6 分)

Let  $m$  and  $n$  be two five-digit positive integers with  $m > n$ . It is known that the 10 digits of  $m$  and  $n$  are all different. Find the smallest possible value of  $m - n$ .

(6 marks)

18. 在圖中的  $4 \times 4$  方格表裏，每個方格都填上了一個整數。包含了剛好一個平方數的長方形（包括正方形）有多少個？

In the figure, each square in the  $4 \times 4$  square grid is labelled with an integer. How many rectangles (including squares) contain exactly one square number?

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

(7 分)

(7 marks)

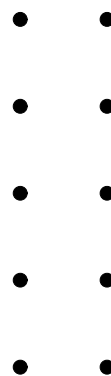
19. 某城市每 5 年的元旦便進行一次人口普查。王氏夫婦是該市的市民。他們二人在 1990 年結婚，婚後育有幾名孩子。已知在 1991 年、1996 年、2001 年和 2006 年的四次人口普查裏，王氏夫婦和他們的孩子的年齡總和皆是 8 的倍數。王氏夫婦二人最少有多少名孩子？（假設王家各人都活過 2006 年。）

(7 分)

In a certain city, a census is conducted every 5 years on the New Year day. Wong's couple are citizens of that city. They got married in 1990 and gave birth to several children after marriage. It is known that in the four censuses in 1991, 1996, 2001 and 2006, the sums of the ages of the Wong's couple and their children are all multiples of 8. At least how many children does Wong's couple have? (Assume all members in the Wong's family are alive in 2006.)

(7 marks)

20. 圖中有兩列點，每列五點。現在要用三條直線線段把左列的其中三點與右列的其中三點連起來（沒有兩條線段有相同的端點）。如果三條線段各不相交，有多少種連線方法？



(7 分)

There are two columns of five points in the figure. It is required to connect three points on the left with three points on the right with three segments of straight lines (no two of which have a common endpoint). If the three segments do not intersect, in how many ways can the points be connected?

(7 marks)

全卷完

END OF PAPER