

香港培正中學第一屆數學邀請賽

Pui Ching Middle School 1st Invitational Mathematics Competition

個人賽 (中二組)

Individual Event (Secondary 2)

時限：1 小時

Time allowed: 1 hour

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

Unless otherwise stated, all answers should be given in exact numerals in their simplest form.
No approximation is accepted.

4. 所有答案填在答題紙指定的空位上。毋須呈交計算步驟。

Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.

5. 不得使用計算機。

The use of calculators is not allowed.

6. 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 5 題，每題 2 分。

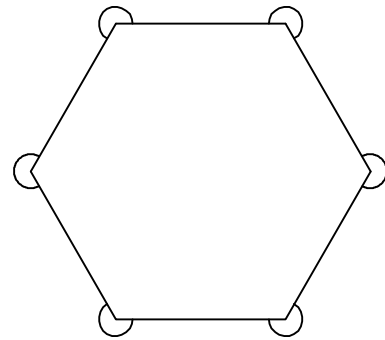
Questions 1 to 5 each carries 2 marks.

1. 100000000001 除以 6 時，餘數是多少？

What is the remainder when 100000000001 is divided by 6?

2. 求圖中畫有標記的六隻角之和。

Find the sum of the six marked angles in the figure.



3. 定義 $a * b = 3a + 4b + 5$ 。若 $x * 2 = 2002$ ，求 x 。

Define $a * b = 3a + 4b + 5$. If $x * 2 = 2002$, find x .

4. 小明和小芬各自想了一個數，已知小明想到的數比小芬的大。若兩數之和為 2002，兩數之差為 1001，則小芬想到的數是多少？

Michael and Fanny each thought of a number. It is known that Michael's number is larger than Fanny's. If the sum of the two numbers is 2002 and their difference is 1001, what is the number Fanny thought of?

5. 求 $7^1 + 7^2 + \cdots + 7^{2002}$ 的個位數字。

Find the unit digit of $7^1 + 7^2 + \cdots + 7^{2002}$.

第 6 至第 10 題，每題 4 分。

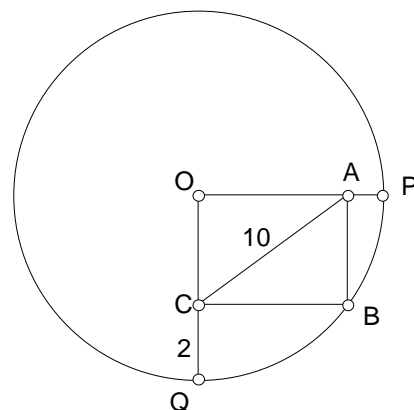
Questions 6 to 10 each carries 4 marks.

6. 有多少個五位正整數的所有數字均為奇數，且五位數字之積不是 5 的倍數？

How many 5-digit positive integers have the property that all their digits are odd and the product of the digits is not a multiple of 5?

7. 如圖所示， O 為圓形的圓心， B 為圓周上的一點，且 $OABC$ 為長方形。 OA 與 OC 延長後分別交圓於 P 及 Q 。若 $AC = 10$ ，且 $CQ = 2$ ，求 AP 。

In the figure, O is the centre of the circle, B is a point on the circumference and $OABC$ is a rectangle. OA and OC are produced to meet the circle at P and Q respectively. If $AC = 10$ and $CQ = 2$, find AP .



8. 一筆金錢，由甲、乙、丙、丁四人分享。甲、乙共分得總數的 40%，乙、丙共分得總數的 50%，甲、丙共分得總數的 60%，且丙分得 4200 元。問丁分得多少元？

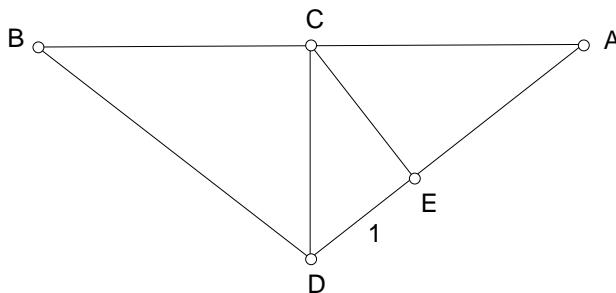
A sum of money was shared among A , B , C and D . A and B together got 40% of the total amount; B and C together got 50% of the total amount; A and C together got 60% of the total amount. It is known that C got \$4200. How many dollars did D get?

9. 求最小的質數 p ，使得 $2002 - p$ 和 $2002 + p$ 均為質數。

Find the smallest prime number p for which both $2002 - p$ and $2002 + p$ are prime.

10. 圖中， C 和 E 分別為 AB 和 AD 上的點，使得 $AD = BD$ ， $AC = BC$ ， $DE = 1$ ， $\angle DEC = 90^\circ$ ，且 $\angle ADB = 120^\circ$ 。求 AE 的長度。

In the figure, C and E are points on AB and AD respectively such that $AD = BD$, $AC = BC$, $DE = 1$, $\angle DEC = 90^\circ$ and $\angle ADB = 120^\circ$. Find the length of AE .



第 11 至第 15 題，每題 6 分。

Questions 11 to 15 each carries 6 marks.

11. 若把 $\left(1+\frac{1}{5}\right)\times\left(1+\frac{1}{5^2}\right)\times\left(1+\frac{1}{5^4}\right)\times\left(1+\frac{1}{5^8}\right)\times\left(1+\frac{1}{5^{16}}\right)\times\left(1+\frac{1}{5^{32}}\right)\times\left(1+\frac{1}{5^{64}}\right)$ 化成最簡分數，答案為 $\frac{p}{q}$ 。求 $4p-5q$ 的值。

When $\left(1+\frac{1}{5}\right)\times\left(1+\frac{1}{5^2}\right)\times\left(1+\frac{1}{5^4}\right)\times\left(1+\frac{1}{5^8}\right)\times\left(1+\frac{1}{5^{16}}\right)\times\left(1+\frac{1}{5^{32}}\right)\times\left(1+\frac{1}{5^{64}}\right)$ is expressed in the simplest fraction, the answer is $\frac{p}{q}$. Find the value of $4p-5q$.

12. 在 1 時 5 分 k 秒，時鐘上的時針和分針剛好重疊。求最接近 k 的整數。

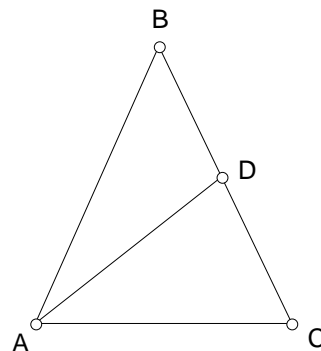
At 5 minutes and k seconds past one, the minute hand and hour hand of a clock overlap. Find the integer closest to k .

13. $ABCDEF$ 為六邊形，每隻內角均相等。若 $AB = 1$ ， $BC = CD = 3$ ，且 $DE = 2$ ，求 $ABCDEF$ 的周界。

$ABCDEF$ is a hexagon with each interior angle equal. If $AB = 1$, $BC = CD = 3$ and $DE = 2$, find the perimeter of $ABCDEF$.

14. 圖中， ABC 為三角形， $AB = BC$ ，且 D 為 BC 上的一點，使得 $BD = DA = AC = 1$ 。若 $CD = x$ ，且 $x^2 + x + k = 0$ ，求 k 。

In the figure, ABC is a triangle, $AB = BC$ and D is a point on BC such that $BD = DA = AC = 1$. If $CD = x$ and $x^2 + x + k = 0$, find k .



15. 甲、乙、丙、丁四個袋內共有 3456 顆糖果。現把甲袋內的糖果的二分之一放進乙袋，然後把乙袋內的糖果的三分之一放進丙袋，接著把丙袋內的糖果的四分之一放進丁袋，最後把丁袋內的糖果的五分之一放進甲袋，則四個袋內的糖果數目相等。問乙袋中原有多少顆糖果？

There are altogether 3456 candies in Bags A , B , C and D . Now one-half of the candies in Bag A are put into Bag B . Then one-third of the candies in Bag B are put into Bag C . After that one-fourth of the candies in Bag C are put into Bag D . Finally one-fifth of the candies in Bag D are put into Bag A . Then the four bags have the same number of candies. How many candies are there in Bag B originally?

第 16 至第 20 題，每題 8 分。

Questions 16 to 20 each carries 8 marks.

16. 有一個牧羊人到草地上放羊。如果他放 40 隻羊，草會在 30 星期後被吃完；如果他放 100 隻羊，草則會在 5 星期後被吃完。已知每隻羊每天所吃的草的量也一樣，並且草地上的草會以均速生長。如果他希望草永遠不會被吃完，那麼他最多可放多少隻羊？

A shepherd is herding his sheep on a piece of grassland. If he herds 40 sheep, all grass will be eaten after 30 weeks; if he herds 100 sheep, all grass will be eaten after 5 weeks. Given that the amount of grass eaten by every sheep is the same every day and that the grass grows with a constant rate. If he hopes that the grass will not be eaten up forever, what is the maximum number of sheep he can herd?

17. 某個會有 2002 名會員。除該會的創會會員外，每人均需得到剛好一名已入會的會員推薦方能入會。每名會員獲發一張會員證。基於行政上的理由，每名非創會會員的會員證均需與推薦人的會員證同色，而不同創會會員之會員證的顏色不同。若一名會員曾經推薦新會員入會，則被推薦的人數必定為 12、30 或 42。此外，沒有會員於入會後退會。問該會的會員證至少有多少種不同的顏色？

A club has 2002 members. Except for its founders, each person must be nominated by exactly one existing member before he can join the club. Every member is issued a membership card. For administrative purpose, the membership card of each non-founding member must be of the same colour as that of one who nominated him, and the membership cards of different founders are of different colours. If a member of the club has ever nominated new members, the number of nominees must be 12, 30 or 42. Also, no member ever drops out after joining the club. What is the minimum number of different colours of the membership cards?

18. 一所飲品公司正推行一項以印花換領飲品的計劃。此飲品乃非賣品，但每 11 個印花可換領 1 瓶飲品。而且，每瓶飲品上皆有 1 個印花以作日後換領之用。如果某人希望換領 20020302 瓶飲品，那麼他最少需要多少個印花？

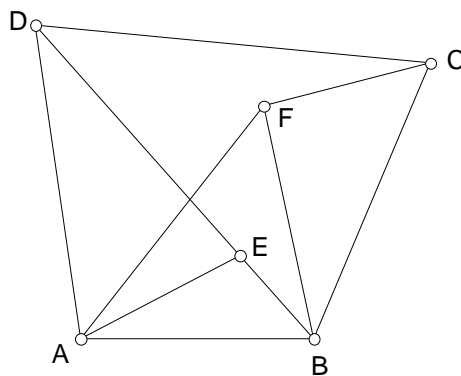
A beverage company is promoting a scheme of drink redemption by stamps. The drinks are not for sale, but every 11 stamps can redeem a bottle of drink. Moreover, there is a stamp on each bottle of drink for further redemption. If someone wants to redeem 20020302 bottles of drink, how many stamps does he need?

19. 現有 10 塊等邊三角形卡片和 n 塊正五邊形卡片 ($n > 0$)。每塊三角形和五邊形的卡片的邊長皆相等。已知全部卡片可以構成一個多面體 (這些卡片全都用作這個多面體的面)。求 n 的最小可能值。

There are 10 pieces of cardboard in the shape of an equilateral triangle and n pieces of cardboard in the shape of a regular pentagon ($n > 0$). The lengths of the sides of all the triangles and pentagons are the same. It is known that using all these pieces, a polyhedron can be constructed (all pieces of cardboard are used as faces of the polyhedron). Find the smallest possible value of n .

20. 如圖所示, E 為四邊形 $ABCD$ 對角線 BD 上的一點。若 $AB = 4$, $BC = 8$, $CD = 12$, $DB = 10$, $BF = 5$, $FA = 6$, $AE = 3$, 且 $EB = 2.5$, 求 CF 的長度。

In the figure, E is a point on diagonal BD of quadrilateral $ABCD$. If $AB = 4$, $BC = 8$, $CD = 12$, $DB = 10$, $BF = 5$, $FA = 6$, $AE = 3$ and $EB = 2.5$, find the length of CF .



全卷完

END OF PAPER

個人賽 (中二組) 答案

Individual Event (Secondary 2) Answers

1. 5

11. -1

2. 1440°

12. 27

3. 663

13. 15

4. 500.5

14. -1

5. 6

15. 648

6. 1024

16. 28

7. 4

17. 4

8. 3000

18. 200203021

9. 1999

19. 2

10. 3

20. 6