香港培正中學第二屆數學邀請賽

Pui Ching Middle School 2nd Invitational Mathematics Competition

個人賽（中一組）

Individual Event (Secondary 1)

時限：1 小時 30 分

Time allowed: 1 hour 30 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分為 100 分。

    There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

    Unless otherwise stated, all numbers in this paper are in decimal system.

3. 除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。

    Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

4. 把所有答案填在答題紙指定的空位上。毋須呈交計算步驟。

    Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.

5. 不得使用計算機。

    The use of calculators is not allowed.

6. 本卷的附圖不一定依比例繪成。

    The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

1. 若 n 位數 11…11 可被 9 整除，求 n 的最小可能值。

   If the n-digit number 11…11 is divisible by 9, find the smallest possible value of n.

2. 陳先生參加了一個有獎遊戲。這個有獎遊戲的玩法如下：每位參加者需要回答獎券上的 10 道是非題。如果他能夠答對 5 題或以上則中獎，否則便算落空。陳先生最少需要參加遊戲多少次才可以確保最少有一次中獎？（註：每次獎券上的問題皆相同。）

   Mr Chan participates in a game. The rule of the game is as follows: Each player needs to answer 10 'True or False' questions on a ticket. If he can answer at least 5 of them correctly, he wins a prize. Otherwise, he loses. At least how many times must Mr Chan play this game so as to guarantee at least 1 prize? (Note: The same questions are set on every ticket.)

3. 某國家只有一元、三元、五元及七元四種硬幣。若不設找贖，共有多少種方法付款恰好十元？

   In a country, there are only four types of coins, $1, $3, $5 and $7. In how many ways can exactly $10 be paid if no change is allowed?

4. 若把所有小於 100 的奇質數乘起來，所得結果的個位數字是甚麼？

   If we multiply together all odd primes less than 100, what will be the unit digit of the result?

第 5 至第 8 題，每題 4 分。

Questions 5 to 8 each carries 4 marks.

5. 若把所有「3」字開首的正整數由小至大排列，即 3, 30, 31, 32, …，第 2003 項是甚麼？

   If all positive integers with leftmost digit 3 are listed in ascending order, i.e. 3, 30, 31, 32, …, what will be the 2003rd term?

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6. 設 $n \geq 3$ 爲正整數。若 $\left(1+\frac{1}{3}\right) \times \left(1+\frac{1}{4}\right) \times \left(1+\frac{1}{5}\right) \times \cdots \times \left(1+\frac{1}{n}\right) > 2003$，求 n 的最小可能值。

Let $n \geq 3$ be a positive integer. If $\left(1+\frac{1}{3}\right) \times \left(1+\frac{1}{4}\right) \times \left(1+\frac{1}{5}\right) \times \cdots \times \left(1+\frac{1}{n}\right) > 2003$, find the smallest possible value of n.

7. 小明、小強和小霖在猜一輛玩具車的售價。小明猜 \$123，小強猜 \$129，小霖猜 \$141。結果他們的估計中，誤差最少爲 \$4，最多爲 \$10。求玩具車的實際售價。

Alan, Peter and Tom each made a guess of the price of a toy car. Alan guessed \$123, Peter guessed \$129 and Tom guessed \$141. Among their guesses, the smallest error was \$4 and the largest error was \$10. Find the actual price of the toy car.

8. 一位老師把 2003 名學生分成若干組，並記下了每組的人數。他最多會記下多少個不同的正整數？

A teacher divides 2003 students into a certain number of groups, and the number of students in each group is recorded. At most how many different positive integers can be recorded?

第 9 至第 12 題，每題 5 分。

Questions 9 to 12 each carries 5 marks.

9. 求所有長和闊均爲不大於 10 的整數的長方形（包括正方形）的面積的平均值。（註：把長方形的長闊對調，所得的長方形與原來的長方形視作不同。例如：長爲 1、闊爲 2 的長方形與長爲 2、闊爲 1 的長方形視作不同。）

Find the average of the areas of all rectangles (including squares) with integral lengths and breadths not more than 10. (Note: By switching the length and width the resulting rectangle is considered to be different from the original one. For instance, a rectangle of length 1 and width 2 is considered to be different from a rectangle of length 2 and width 1.)

10. 平面上畫了 1000 對平行線。這些線最多可把平面分成多少份？

1000 pairs of parallel lines are drawn on a plane. What is the maximum number of regions formed by these lines?

11. 某間超級市場只有一台收銀機。現有十名客人在排隊，其中一人選購了 1 件貨品，一人選購了 2 件貨品，如此類推，購物最多的客人選購了 10 件貨品。已知每選購一件貨品付款需時一分鐘（例如：若選購了 5 件貨品，付款過程便需時 5 分鐘）。設 a_1、a_2、$\cdots$、a_{10} 分別代表十名客人的輪候時間（以分鐘為單位），輪候時間包括繳付自己所購貨品的時間。求 $a_1 + a_2 + \cdots + a_{10}$ 的最小可能值。

A supermarket has only one cashier counter. Now 10 customers queue up for paying. One of them purchases 1 piece of goods, one purchases 2 pieces of goods, and so on, and one purchases 10 pieces of goods. It is known that the paying process takes 1 minute for each piece of good bought (for instance, the paying process will take 5 minutes if one buys 5 pieces of goods). Let a_1, a_2, …, a_{10} denote the waiting time (in minutes) for the 10 customers. Waiting time includes the time paying for one's own goods. Find the minimum possible value of $a_1 + a_2 + \cdots + a_{10}$.

12. 在一個 3×3 的方格表中，每格均填進 1 至 9 中的其中一個整數，而且填進每格的數字皆不同。我們然後把每行中數字最大的一格填上紅色，每行中數字最小的一格填上綠色。設 M 為三個紅色方格數字中最小的一個，m 為三個綠色方格數字中最大的一個。$M - m$ 有多少個不同的可能值？

In a 3×3 table, each cell is filled with an integer between 1 and 9, while different cells are filled with different numbers. We then colour in red the cell in each row with the largest number, and colour in green the cell in each row with the smallest number. Let M be the smallest of the numbers in the three red-coloured cells, and m be the largest of the numbers in the three green-coloured cells. How many different possible values of $M - m$ are there?

第 13 至第 16 題，每題 6 分。

Questions 13 to 16 each carries 6 marks.

13. $ABCD$ 是一個平行四邊形，E 為平面上任意一點。若 $\triangle ABE$ 及 $\triangle DEC$ 的面積分別為 5 及 2，求 $ABCD$ 的最小可能面積。

$ABCD$ is a parallelogram and E is an arbitrary point on the plane. If the areas of $\triangle ABE$ and $\triangle DEC$ are 5 and 2 respectively, find the minimum possible area of $ABCD$.

14. 潘先生對小敏和小賢說：「我想了兩個正整數 a 和 b，其中 $a > b$。」然後他秘密地將兩數之差（$a - b$）告訴小敏，及將兩數之積（ab）告訴小賢。以下是他們之後的對話。

　　　　潘先生問小敏：「你知道 a 和 b 是甚麼嗎？」

　　　　小敏說：「不知道。」

　　　　然後潘先生問小賢：「你知道 a 和 b 是甚麼嗎？」

　　　　小賢說：「我知道啊。它們是……」

　　　　潘先生立即打斷了小賢的說話，並再問小敏：「現在你知道 a 和 b 是甚麼嗎？」

　　　　這時小敏回答：「聽過你和小賢剛才的對話後，我知道它們是甚麼了。它們的和 $a + b$ 大於 90，小於 100。潘先生，對嗎？」

　　　　潘先生說：「是啊，你們真聰明呢。」

假設小賢和小敏都是誠實和聰明的（即是說只要當答案可以確定時，他們一定知道答案），求 ab。

Mr Poon told Dora and Ken: 'I thought of two positive integers a and b, where $a > b$.' He then secretly told Dora the difference of the two numbers (i.e. $a - b$) and Ken the product of the two numbers (i.e. ab). Their subsequent conversation is recorded below.

　　Mr Poon asked Dora, 'Do you know what a and b are?'

　　'No.' Dora answered.

　　Then Mr Poon asked Ken, 'Do you know what a and b are?'

　　'Yes, they are …' Ken said.

　　Mr Poon interrupted Ken immediately and asked Dora again, 'Do you know what a and b are now?'

　　At that time, Dora said, 'Now I know what they are after listening to the conversation between Ken and you. Their sum (i.e. $a + b$) is greater than 90 but less than 100. Am I right, Mr Poon?'

　　'Yes. You are so clever!' Mr Poon said.

Assuming that Ken and Dora are honest and intelligent (it means that whenever the answer can be confirmed, they must know the answer), find ab.

15. 小美參選學生會主席，得票率（準確至小數點後一個位）為 99.3%。問最少有幾人投了票？

Amy joined the presidential election of the Student Union and obtained 99.3% of the votes, correct to 1 decimal place. What is the smallest possible number of voters?

16. 2003 位小朋友被編成 1 至 2003 號。他們每人最初有 20 顆糖果，然後按以下方法派給或拿走糖果：先派 3 顆糖果給每位編號是 1 的倍數的小朋友，再從每位編號是 2 的倍數的小朋友手上取走 1 顆糖果，然後派 3 顆糖果給每位編號是 3 的倍數的小朋友，再從每位編號是 4 的倍數的小朋友手上取走 1 顆糖果，如此類推，直至最後派 3 顆糖果給每位編號是 2003 的倍數的小朋友。最後有多少個小朋友比最初多了糖果（即有 21 顆糖果或以上）？

2003 children are labelled 1 to 2003. Each of them has 20 candies at the beginning. Candies are then given to or taken away from them as follows. 3 candies are given to each child with a label which is a multiple of 1, then 1 candy is taken away from each child with a label which is a multiple of 2, then 3 candies are given to each child with a label which is a multiple of 3, then 1 candy is taken away from each child with a label which is a multiple of 4, and so on, till finally 3 candies are given to each child with a label which is a multiple of 2003. How many children can get extra candies (i.e. have 21 candies or more) in the end?


第 17 至第 20 題，每題 7 分。

Questions 17 to 20 each carries 7 marks.

17. 小明選了十個連續正整數，然後分別把它們除以某正整數 n。他發現所得的十個餘數之和為 2003。求 n 的最小可能值。

Peter picked ten consecutive positive integers and divided each of them by a positive integer n. He found that the sum of the ten remainders obtained is equal to 2003. Find the smallest possible value of n.
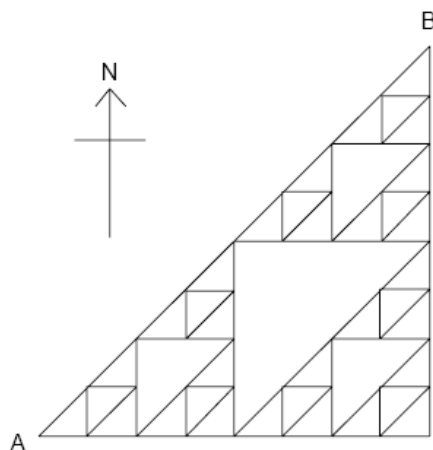
18. 在 ΔABC 中，$\angle A = 2\angle B$，$BC = 4$，且 $CA = 3$。求 AB 的長度。

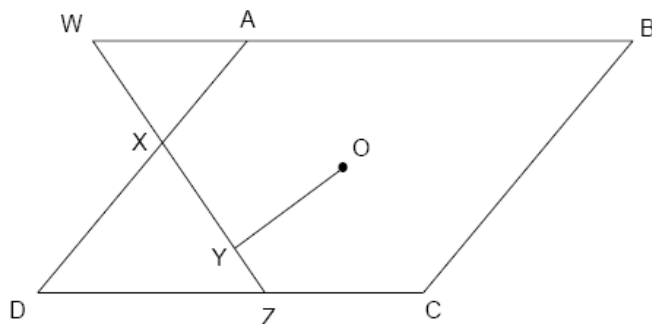In ΔABC, $\angle A = 2\angle B$, $BC = 4$ and $CA = 3$. Find the length of AB.
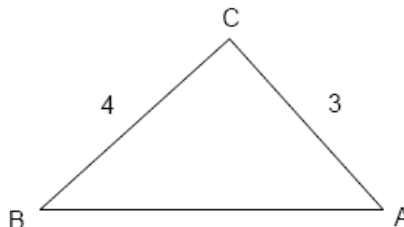
19. 圖中所示為「三角城」的街道圖。若某人欲於 A 點走到 B 點，但只可以沿著街道向東、北或東北走，他可以有多少種走法？

The figure below shows the street map of 'Triangle City'. If a person at A wants to go to B but is only allowed to go along the streets towards north, east or northeast, in how many different possible ways can he travel?

20. 如圖所示，ABCD 為菱形，X 及 Z 分別是 AD 及 DC 上的點。O 是菱形內的一點，Y 為 O 到 XZ 的垂足，且 O 與 AB、BC、CD 和 DA 的距離均等於 OY。若 ZX 與 BA 延長後交於 W，且 WX = 5，XY = 3 及 YZ = 2，求 ABCD 的面積。

In the figure, ABCD is a rhombus. X and Z are points on AD and DC respectively. O is a point inside the rhombus, Y is the foot of the perpendicular from O to XZ and the distances from O to AB, BC, CD and DA are all equal to OY. If ZX and BA are produced to meet at W, WX = 5, XY = 3 and YZ = 2, find the area of ABCD.

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END OF PAPER