香港培正中學第三屆數學邀請賽
Pui Ching Middle School 3rd Invitational Mathematics Competition

個人賽（中二組）
Individual Event (Secondary 2)

時限：1 小時 30 分
Time allowed: 1 hour 30 minutes

參賽者須知：
Instructions to Contestants:

1. **本卷共設甲、乙兩部分，總分為** 100 **分**。
   This paper is divided into Section A and Section B. The total score is 100.

2. **除特別指明外，本卷內的所有數均為十進制**。
   Unless otherwise stated, all numbers in this paper are in decimal system.

3. **除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值**。
   Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted.

4. **把所有答案填在答題紙指定的空位上。毋須呈交計算步驟**。
   Put your answers on the spaces provided on the answer sheet. You are not required to hand in your steps of working.

5. **不得使用計算機**。
   The use of calculators is not allowed.

6. **本卷的附圖不一定依比例繪成**。
   The diagrams in this paper are not necessarily drawn to scale.

甲部（60 分）

Section A (60 marks)

第 1 至第 4 題，每題 3 分。	Questions 1 to 4 each carries 3 marks.
第 5 至第 8 題，每題 5 分。	Questions 5 to 8 each carries 5 marks.
第 9 至第 12 題，每題 7 分。	Questions 9 to 12 each carries 7 marks.

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1. **若 p 和 q 除以 2004 時的餘數分別為 1111 和 1234，則 $p+q$ 除以 2004 時的餘數是多少？**

   If p and q leave remainders of 1111 and 1234 respectively when divided by 2004, what is the remainder when $p+q$ is divided by 2004?

2. **小明在十月時儲蓄了一些金錢。在十一月的每一天的早上，他的媽媽都給他 15 元零用錢。每天他亦會花他所有金錢的總數的 10% 吃午餐。已知他在十一月結束時有 135 元，他在十一月共花了多少元吃午餐？**

   Peter saved some money in October. In each morning of November, his mother gave him $15 as pocket money. He then spent 10% of the total amount he had for lunch every day. Given that the amount he had was $135 at the end of November, how many dollars did he spend on lunch in November?

3. **設 A (1, 1)、B (9, 7) 和 C (7, 1) 為平面上的三點。若 D 是 AB 上的一點，使得 $AB \perp CD$，求 CD 的長度。**

   Let A (1, 1), B (9, 7) and C (7, 1) be three points on the plane. If D is a point on AB such that $AB \perp CD$, find the length of CD.

4. **乘積 1111111111×1111111111 中的數字之和是多少？**

   What is the sum of digits in the product 1111111111×1111111111?

5. **一個魔術師把一些球放進一個箱裏，其中每個球都寫上了一個小於 100 的正整數。每兩個球上的數字都不相同。然後他讓一位觀眾從箱中隨機抽出三個球。他保證無論觀眾抽了哪三個球，三個球上的數之和必可被 6 整除。箱中最多有多少個球？**

A magician put some balls in a box. On each ball a positive integer less than 100 was written. No two balls had the same number written. He then let a spectator draw three balls from the box randomly. He guaranteed that no matter which three balls were drawn, the sum of the numbers on the three balls is divisible by 6. At most how many balls were there in the box?

6. **設 $[x]$ 為不超過 x 的最大整數，例如 $[1.1] = 1$、$[6.9] = 6$ 和 $[5] = 5$。求下式的值：**

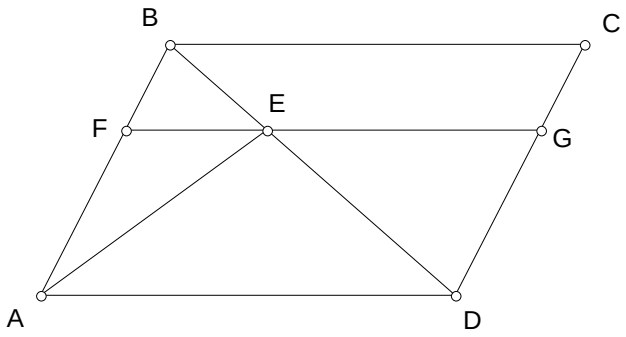
$$\left[\sqrt{1}\right]-\left[\sqrt{2}\right]+\left[\sqrt{3}\right]-\left[\sqrt{4}\right]+\left[\sqrt{5}\right]-\left[\sqrt{6}\right]+\cdots+\left[\sqrt{2003}\right]-\left[\sqrt{2004}\right]$$

Let $[x]$ be the greatest integer not exceeding x. For example, $[1.1] = 1$, $[6.9] = 6$ and $[5] = 5$. Find the value of the following expression:

$$\left[\sqrt{1}\right]-\left[\sqrt{2}\right]+\left[\sqrt{3}\right]-\left[\sqrt{4}\right]+\left[\sqrt{5}\right]-\left[\sqrt{6}\right]+\cdots+\left[\sqrt{2003}\right]-\left[\sqrt{2004}\right]$$

7. **圖中，$ABCD$ 為平行四邊形，F 和 G 分別是 AB 和 CD 上的點，使 $FG \parallel AD$。FG 與 BD 交於 E。若 ΔAEF 和梯形 $BCGE$ 的面積分別是 1 和 5，求 $ABCD$ 的面積。**

In the figure, $ABCD$ is a parallelogram. F and G are points on AB and CD respectively such that $FG \parallel AD$. FG intersects BD at E. If the areas of ΔAEF and trapezium $BCGE$ are 1 and 5 respectively, find the area of $ABCD$.



8. **小雄有** 100 **個紅球、**200 **個藍球和** 250 **個黃球。他得到一部神奇的機器，可以讓他輸入一些球後輸出更多的球。它的使用方法如下：**

Sam has 100 red balls, 200 blue balls and 250 yellow balls. He has got a magic machine, in which some balls can be input to output more balls. It operates as follows:

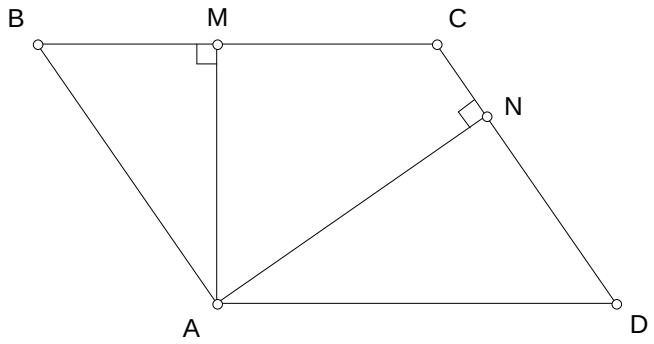
輸入 Input	**輸出** Output
2 **個紅球和** 2 **個藍球** 2 red balls and 2 blue balls	7 **個黃球和** 1 **個紅球** 7 yellow balls and 1 red ball
4 **個黃球和** 1 **個藍球** 4 yellow balls and 1 blue ball	5 **個紅球和** 2 **個黃球** 5 red balls and 2 yellow balls

小雄想產生盡量多的球，顏色不拘。他最多可擁有多少個球？

Sam wants to produce as many balls as he can, the colours of which do not matter. How many balls can he have at most?

9. *ABCD* **為平行四邊形，** *M* **和** *N* **分別是** *BC* **和** *CD* **上的點，使得** $AM \perp BC$ **和** $AN \perp CD$**。若** $AB = 13$**，**$BM = 5$**，**$MC = 9$**，求** *MN* **的長度。**

ABCD is a parallelogram. *M* and *N* are points on *BC* and *CD* respectively such that $AM \perp BC$ and $AN \perp CD$. If $AB = 13$, $BM = 5$ and $MC = 9$, find the length of *MN*.

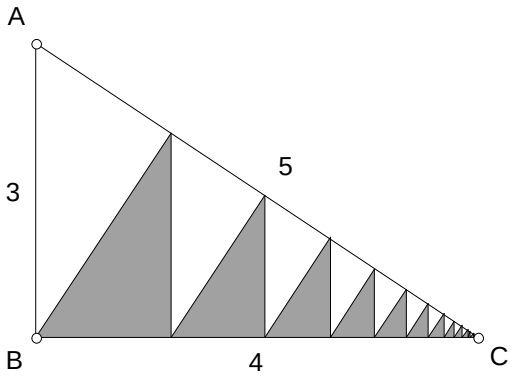


10. **一種常用的日期表示法是以「年 / 月 / 日」六位數字形式寫出日期，例如：2004 年 3 月 7 日寫成 04/03/07。由於 $04+03=07$，我們說這天是「好日子」。一般來說，若某天在以上的日期表示法中，代表「年」、「月」、「日」的三個兩位數中其中一個等於另外兩個之和，則那天稱為「好日子」。那麼，在二十一世紀中（2001 年 1 月 1 日至 2100 年 12 月 31 日），「好日子」共有多少天？**

A usual way of writing dates is the 'YY/MM/DD' method of expressing a date as a six-digit number. For instance, 7th March 2004 is denoted as 04/03/07. Since $04+03=07$, we say that this is a 'good day'. In general, a day is said to be a 'good day' if, among the three two-digit numbers representing 'year', 'month' and 'day' in the above representation, one of them is equal to the sum of the other two. How many 'good days' are there in the 21st century (from 1st January 2001 to 31st December 2100)?

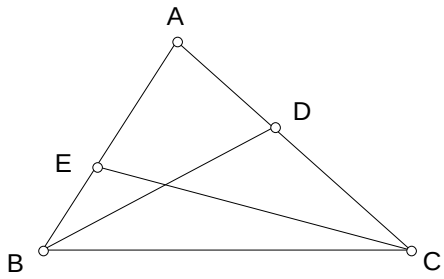
11. **圖中，ΔABC 的邊長為 3、4、5，其中 B 是直角。從 B 開始重覆畫垂線到 AC 和 BC。求陰影部分的面積。**

In the figure, ΔABC with side lengths 3, 4, 5 is right-angled at B. Starting from B, perpendiculars are drawn to sides AC and BC repeatedly. Find the area of the shaded region.



12. **在 ΔABC 中，D 是 AC 上的一點，E 是 AB 上的一點，使得 $BC^2 = AB\times BE = AC\times CD$。若 $BC = 12$，$BD = 16$，求 CE 的長度。**

In ABC, D is a point on AC and E is a point on AB such that $BC^2 = AB\times BE = AC\times CD$. If $BC = 12$ and $BD = 16$, find the length of CE.

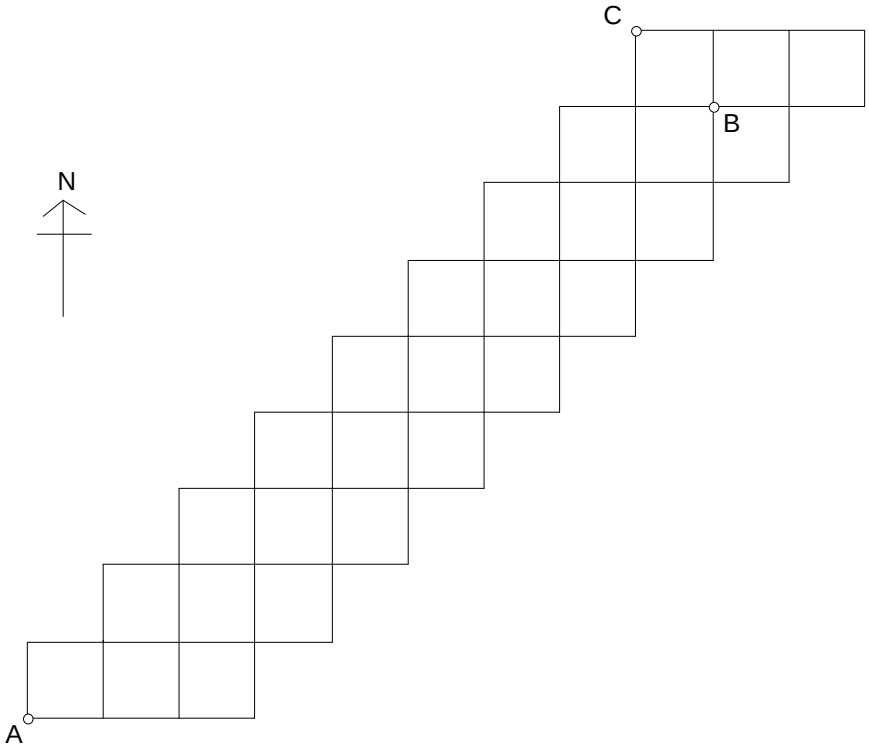


乙部（40 分）

Section B (40 marks)

13. **下圖所示為「正方形城」的街道圖。每個小正方形的邊長均代表實際距離 1 公里。陳先生在 *A* 點開設了一間薄餅店，提供薄餅速遞服務。**

The figure below is the street map of 'Square City'. The side length of each small square represents an actual distance of 1 km. Mr Chan operates a pizza restaurant at point *A* and provides pizza delivery service.



(a) **某天，陳先生分別接到在 *B* 點和 *C* 點的兩個訂單。他從 *A* 點出發，需要沿街道把薄餅送到 *B* 點和 *C* 點（次序不拘），並返回 *A* 點。那麼，他所走的距離最短是多少公里？** （2 分）

One day Mr Chan received two orders at points *B* and *C* respectively. Starting from point *A*, he had to deliver the pizzas to points *B* and *C* (the order of which does not matter) via the streets and then return to point *A*. What is the minimum distance (in km) that he must travel? (2 marks)

(b) **若陳先生只可沿街道向北方或東方走，則由 *A* 點走到 *B* 點有多少種不同的走法？** **（5 分）**

If Mr Chan can only go northward or eastward via the streets, in how many different ways can he travel from point *A* to point *B*? (5 marks)

(c) **陳先生發現，薄餅店位於城市的西南端，地點並不理想。一旦接到偏遠的東北部的訂單，送貨的成本高昂，並不划算。因此，他決定把現有的薄餅店關閉，另覓兩個地點開設兩間新店。每間新店必須位於兩條街道的交匯處（即街道圖中小正方形的頂點），並且要求從城市內街道上的任何一點接到訂單時，均可從其中一間新店沿街道送貨，其距離不超過 6 公里。那麼，兩間新店的選址有多少個不同的組合？** **（6 分）**

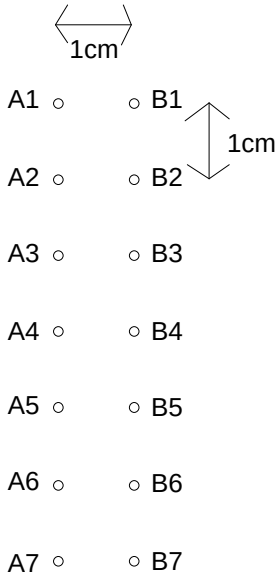
Mr Chan finds that the pizza restaurant, situated in the southwestern tip of the city, is not at an ideal location. Whenever orders from the remote northeastern area are received, high costs of transportation will be involved and thereby greatly reducing cost-effectiveness. As a result, he decides to close down the current restaurant and chooses two other locations for two new restaurants. Each new restaurant must be at the intersection of two streets (i.e. the vertices of the small squares in the street map), and it is required that whenever orders at any point on the streets of the city are received, delivery service can be provided from one of the new restaurants, and the distance of delivery is at most 6 km along the streets. How many different combinations are there for the locations of the two new restaurants? (6 marks)

(d) **為了方便起見，陳先生把街道圖上的每個小正方形塗上紅色、黃色或綠色，使得同一橫行或直行中，沒有兩個小正方形的顏色相同。那麼，街道圖共有多少種不同的填色方法？** **（7 分）**

For convenience, Mr Chan colours each small square on the street map in red, yellow or green, in a way such that no two small squares in the same row or column are assigned the same colour. How many different colouring schemes are there? (7 marks)

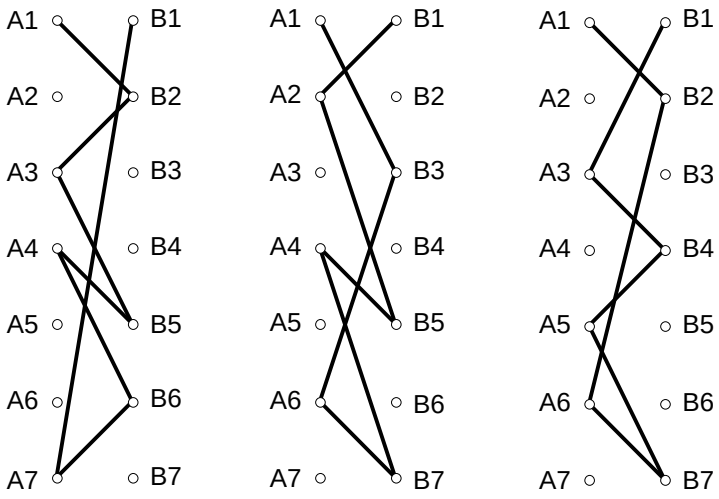
14. **小月要為一隻鞋穿鞋帶。鞋上有兩列、每列七個鞋帶孔，這些鞋帶孔整齊地在平面上排成一個長方形，相鄰的鞋帶孔相隔 1 厘米（見圖一）。小月穿鞋帶時，每次均會由 A1 孔開始穿，穿到 B 列的某一個孔，再穿回 A 列的另一個孔，如此類推，一直穿回 B1 孔為止。此外，為了節省時間起見，除第一行（A1 和 B1 孔）外，鞋帶只會同時穿過同一行的兩個孔中的剛好一個（即不會同時穿過 A2 和 B2，不會同時穿過 A3 和 B3，如此類推）。鞋帶只可以穿過每個孔最多一次。圖二所示的是一些符合上述規則的穿鞋帶方法。**

Mary is tying her shoelace. There are 2 columns of holes, each with 7 holes on it. The holes are evenly distributed on a plane in a rectangular shape, with each of the holes at a distance of 1 cm from its neighbours (see Figure 1). When Mary ties the shoelace, she always ties the shoelace through hole A1 first, then to a hole on column B, and then back to a hole on column A and so on, until she returns to hole B1. In order to save time, Mary will only tie the shoelace through exactly one of the two holes on the same row except for the first row (holes A1 and B1), i.e. the shoelace will not go through both A2 and B2, nor both A3 and B3, etc. The shoelace can pass through each hole at most once. Figure 2 shows some possible configurations of the shoelace under the above rules.



圖一

Figure 1



圖二

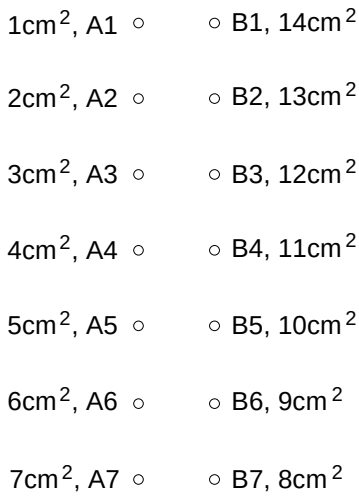
Figure 2

(a) **小月有多少種不同的方法穿鞋帶？（只要穿孔的次序相同，兩種穿鞋帶的方法便視為相同。）** **（4 分）**

In how many different ways can Mary tie the shoelace? (Two ways are regarded to be the same if the orders by which the shoelace passes through the holes are the same in both ways.) (4 marks)

(b) **圖中所示為各鞋帶孔的面積。若鞋帶所穿過的孔的面積之和是 S cm^2，那麼 S 除以 15 時的餘數有多少個不同的可能值？** **（6 分）**

The figure shows the areas of the holes. Suppose that the sum of the areas of the holes which the shoelace passes through is S cm^2. How many different remainders are possible when S is divided by 15? (6 marks)



(c) **圖中所示為穿過各鞋帶孔所需的時間。那麼，小月穿鞋帶最少需要多少秒？** **（4 分）**

The figure shows the time needed to tie the shoelace through each of the holes. What is the minimum amount of time (in seconds) that Mary needs to tie the shoelace? (4 marks)

(d) **求由 A1 孔到 B1 孔的鞋帶的長度的最小值。** **（6 分）**

Find the minimum length of the shoelace from hole A1 to hole B1. (6 marks)

全卷完

END OF PAPER

個人賽（中二組）答案

Individual Event (Secondary 2) Answers

1. 341

2. 450

3. $\frac{18}{5}$

4. 82

13 (a) 36

13 (b) 6561

13 (c) 87

13 (d) 1542

5. 17

6. –22

7. $\frac{32}{3}$

8. 950

14 (a) 720

14 (b) 10

14 (c) 51

14 (d) $5\sqrt{5}+2\sqrt{2}$

9. $\frac{180}{13}$

10. 730

11. $\frac{96}{41}$

12. 9