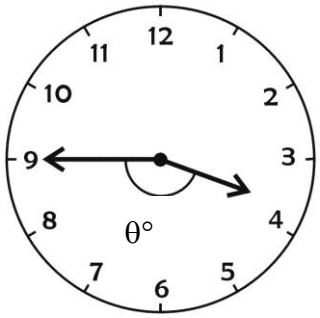
Hong Kong Mathematics Olympiad 2006-2007
Heat Event (Individual)

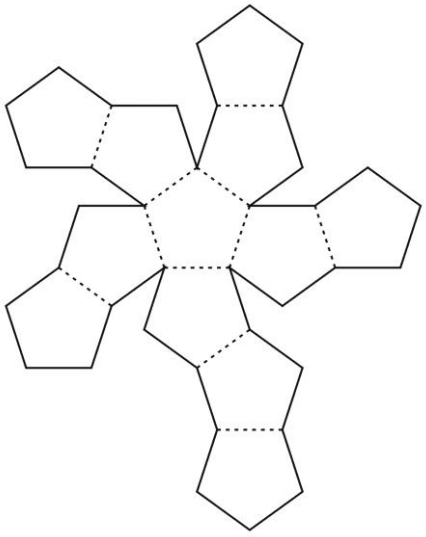
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 如圖一，時鐘顯示着三時四十五分。若時針與分針的交角是 $\theta°$，求 θ 的值。
In Figure 1, a clock indicates the time 3: 45. If the angle between the hour-hand and the minute-hand is $\theta°$, find the value of θ.



圖一
Figure 1

2. 如圖二的摺紙圖樣能摺出一個正多面體。若該正多面體有 y 條棱，求 y 的值。
In Figure 2, there is a paper net that can be wrapped into a regular polyhedron. If this polyhedron has y edges, find the value of y.



圖二
Figure 2

3. 在 4 本英文書、6 本中文書及 9 本日文書中任取兩本。已知這兩本書是相同語言的。若有 X 個不同的選擇，求 X 的值。

Among 4 English books, 6 Chinese books and 9 Japanese books, two books are selected. It is found that they are of the same language. If there are X such choices, find the value of X.

4. 設 r_1 和 r_2 是方程 $(x-2006)(x-2007)=2007$ 的兩個實根。若 r 是方程 $(x-r_1)(x-r_2)=-2007$ 較小的實根，求 r 的值。

Let r_1 and r_2 be the two real roots of the equation $(x-2006)(x-2007)=2007$. If r is the smaller real root of the equation $(x-r_1)(x-r_2)=-2007$, find the value of r.

5. 已知 α 及 β 是方程 $x^2-5^{2007}x+5^{1000}=0$ 的根。若 $s=\log_{25}\frac{\alpha^2}{\beta}+\log_{25}\frac{\beta^2}{\alpha}$，求 s 的值。

Given that α and β are the roots of the equation $x^2-5^{2007}x+5^{1000}=0$.

If $s=\log_{25}\frac{\alpha^2}{\beta}+\log_{25}\frac{\beta^2}{\alpha}$, find the value of s.

6. 對任意實數 a、b、c 及 d，定義運算 $*$：

$(a,\ b)*(c,\ d)=(ad+bc,\ bd)$。

若 $(x,\ y)=\left(1,\ \frac{3}{7-\sqrt{5}}\right)*\left(8+\sqrt{5},\ 3\right)$ 及 $a=\frac{x}{y}$，求 a 的值。

For any real number a, b, c and d, we define the operation $*$:

$(a,\ b)*(c,\ d)=(ad+bc,\ bd)$.

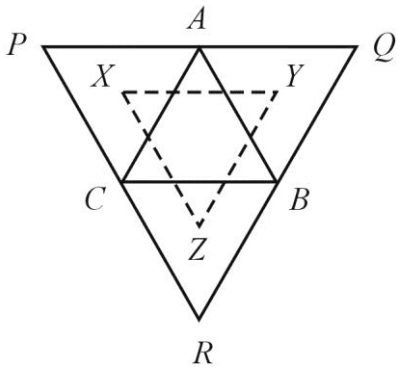
If $(x,\ y)=\left(1,\ \frac{3}{7-\sqrt{5}}\right)*\left(8+\sqrt{5},\ 3\right)$ and $a=\frac{x}{y}$, find the value of a.

7. 已知 $\sin\alpha-\cos\alpha=\frac{1}{5}$ 及 $0°<\alpha<180°$。若 $\tan\alpha=B$，求 B 的值。

Given that $\sin\alpha-\cos\alpha=\frac{1}{5}$ and $0°<\alpha<180°$. If $\tan\alpha=B$, find the value of B.

8. 如圖三，ΔPAC、ΔQBA、ΔRCB 及 ΔABC 皆是等邊三角形。點 X、Y 及 Z 分別為 ΔPAC、ΔQBA 及 ΔRCB 的內心。若 PA 的長度是10 cm 及 ΔXYZ 的周界是 w cm，求 w 的值。(註：三角形的內心為該三角形三條內角平分線的交點。)

In Figure 3, ΔPAC, ΔQBA, ΔRCB and ΔABC are equilateral triangles. The points X, Y and Z are the incentre of ΔPAC, ΔQBA and ΔRCB respectively. If the length of PA is 10 cm and the perimeter of ΔXYZ is w cm, find the value of w. (Remark: the incentre of a triangle is the point of intersection of the three interior angle bisectors of the triangle.)



圖三

Figure 3

9. 設 $f(x)=\frac{1}{2}\left(4x^2-60x+9+\left|4x^2-60x+9\right|\right)$，若 $k=f(1)+f(2)+f(3)+\cdots+f(15)+f(16)$，求 k 的值。

Let $f(x)=\frac{1}{2}\left(4x^2-60x+9+\left|4x^2-60x+9\right|\right)$. If $k=f(1)+f(2)+f(3)+\cdots+f(15)+f(16)$, find the value of k.

10. 在平面上點 P 的坐標是 $(-3, 4)$。以 $(0, 0)$ 為中心，點 P 順時針方向旋轉 $45°$ 後，再沿 y-軸反射到達點 $Q=(x, y)$。若 $z=x+y$，求 z 的值。

The coordinates of point P on the plane is $(-3, 4)$. After rotating $45°$ clockwise about the centre $(0, 0)$ and reflecting along the y-axis, the point P reaches the point $Q=(x, y)$. If $z=x+y$, find the value of z.

******* *全卷完* *******

******* ***End of Paper*** *******