

第五屆培正數學邀請賽
5th Pui Ching Invitational Mathematics Competition

初賽（中四組）
Heat Event (Secondary 4)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分為 100 分。

There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均為十進制。

Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數（包括 0 和 9999）。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

All answers are integers between 0 and 9999 (including 0 and 9999). Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

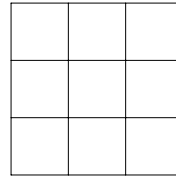
The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

The diagrams in this paper are not necessarily drawn to scale.

1. 求 7777777 的最小質因數。 (2 分)
Find the smallest prime factor of 7777777. (2 marks)
2. 一個長方形的面積是 24，周界是 20。求長方形最長的一條邊的長度。 (2 分)
A rectangle has area 24 and perimeter 20. Find the length of the longest side of the rectangle. (2 marks)
3. 小明把一些大於 2006 的質數乘起來，得到的答案為 S 。若 S 的個位數字是 T ，問 T 有多少個不同的可能值？ (3 分)
Mike multiplied together some prime numbers greater than 2006 and obtained the answer S . If the unit digit of S is T , how many different possible values of T are there? (3 marks)
4. A 和 B 是平面上給定的兩點。若 C 是平面上滿足條件 $AB = AC$ 和 $\angle BAC = 90^\circ$ 的一點，那麼 C 的位置有多少個不同的可能性？ (3 分)
 A and B are two given points on the plane. If C is a point on the plane such that $AB = AC$ and $\angle BAC = 90^\circ$, how many different possible positions for C are there? (3 marks)
5. 設 n 為大於 1 的整數。若 $n + \sqrt{n} + \sqrt[3]{n}$ 是整數，求 n 的最小可能值。 (3 分)
Let n be an integer greater than 1. If $n + \sqrt{n} + \sqrt[3]{n}$ is an integer, find the smallest possible value of n . (3 marks)
6. 若 x 是實數，且 $x^2 - 7x + 10 \leq 0$ ，求 $x^2 - 6x + 2006$ 的最大值。 (4 分)
If x is a real number such that $x^2 - 7x + 10 \leq 0$, find the maximum value of $x^2 - 6x + 2006$. (4 marks)
7. 現有一個八重旋轉對稱圖形。若每次繞著旋轉對稱中心把圖形順時針旋轉 12° ，則最少要轉多少次才可使影像與原圖形重合？ (4 分)
There is a figure possessing rotational symmetry of order 8. If we rotate the figure clockwise by 12° through the centre of rotational symmetry each time, what is the minimum number of rotations needed so that the image will coincide with the original figure? (4 marks)

8. 圖中是一個 3×3 的方格表。現在我們要把每個方格都填上紅色、綠色或藍色，令每直行和橫行的三個方格的顏色都不相同。我們共有多少種填色方法？



(5 分)

The figure shows a 3×3 grid. We want to colour each cell in red, green or blue in a way such that the three cells in each row and in each column receive different colours.

How many different ways of colouring are there?

(5 marks)

9. 設 k 為實數。若聯立方程 $\begin{cases} 4x - 3y = 3 \\ 6x + ky = 4 \end{cases}$ 無解，求最接近 k^2 的整數。 (5 分)

Let k be a real number. If the system of equations $\begin{cases} 4x - 3y = 3 \\ 6x + ky = 4 \end{cases}$ has no solution,

find the integer closest to k^2 .

(5 marks)

10. 一位魔術師聲稱他找到了一個「魔幻數」。這個「魔幻數」其實是一個整數 k ，只要觀眾任意選出一個整數 h ，方程 $x^2 + hx + k = 2006$ 總有兩個不同的實數解。求 k 的最大可能值。 (5 分)

A magician claims that he has found a 'magic number'. It is an integer k such that no matter which integer h the audience picks, the equation $x^2 + hx + k = 2006$ will always have two distinct real solutions. Find the largest possible value of k .

(5 marks)

11. 在一次數學測驗中，每位學生的得分都是 0 至 100 的整數（包括 0 和 100），合格分數則為 40。某班的平均得分是 53。後來，一名得到 97 分的學生退學，結果餘下學生的平均得分變成 51。那麼，班中最多有幾人不合格？ (6 分)

In a mathematics test, the score of each student is an integer between 0 and 100 (inclusive), and the passing score is 40. In a certain class, the average score of the students is 53. Later, a student who got 97 marks in the test withdrew from school and the average score of the remaining students of the class became 51. At most how many students in that class have failed the test?

(6 marks)

12. 某城市的電話號碼全為八位數字，而所有電話的鍵盤款式均如圖所示。該城市並規定所有電話號碼的相鄰數字，必須為電話鍵盤內的相鄰數字（例如：85256321）。若某電話號碼有四個不同的數字，求該電話號碼八個數字之和的最大可能值。

7	8	9
4	5	6
1	2	3

(6 分)

In a city, all telephone numbers have 8 digits and all telephones have a keyboard in the form as shown. Furthermore, consecutive digits in a telephone number must be adjacent digits on the keyboard (e.g. 85256321). If a telephone number consists of four different digits, find the greatest possible value of the sum of the 8 digits of the telephone number.

(6 marks)

13. 已知 B 是正整數。小華做二次方程的練習題時，發現 $x^2 + 2Bx = 0$ 、 $x^2 + 2Bx + 1 = 0$ 、 $x^2 + 2Bx + 2 = 0$ 、 $x^2 + 2Bx + 3 = 0$ 、 $x^2 + 2Bx + 12 = 0$ 這 13 條方程都有實數解，而且剛好有 3 條的兩個解都是整數。求 B 的值。

(6 分)

Given that B is a positive integer. When working on exercises in quadratic equations, Roy found that all 13 equations $x^2 + 2Bx = 0$, $x^2 + 2Bx + 1 = 0$, $x^2 + 2Bx + 2 = 0$, $x^2 + 2Bx + 3 = 0$, ..., $x^2 + 2Bx + 12 = 0$ have real roots and exactly 3 of them have both roots being integers. Find the value of B .

(6 marks)

14. 某個奇數 n 有 123 個正因數。問 $4n$ 有多少個正因數？

(6 分)

An odd number n has 123 positive factors. How many positive factors does $4n$ have?

(6 marks)

15. 圖中顯示一條乘式，但當中有些數字留空了。求乘積（即最底一行）的最後四位數字。

$$\begin{array}{r}
 9 \\
 \times \\
 \hline
 9 \\
 \hline
 9
 \end{array}$$

(6 分)

The figure shows a multiplication, but some digits are left out. Find the last four digits of the product (i.e. the last row).

(6 marks)

16. 圖中所示為一個 9×3 的方格陣，它被分成三個 3×3 的方格陣。現要在每格填上 1 至 9 的其中一個數字，使得每個數字在各直行皆出現一次，而且每個數字在三個 3×3 的方格陣中皆出現一次。那麼，有多少種方法填滿留空了的方格？

The figure shows a 9×3 grid, divided into three 3×3 grids. Now we want to fill each cell with one of the numbers from 1 to 9. Each number must appear once in each column and each number must appear once in each of the three 3×3 grids. How many different ways are there to fill the remaining squares?

1	4	7
2	5	8
3	6	9
4	7	1
5	8	2

(6 分)

(6 marks)

17. 有兩個正整數，它們之和為 145。若把它們的最小公倍數除以它們的最大公因數，商是 168。求兩數之差。

(7 分)

Two positive integers have sum 145. When their L.C.M. is divided by their H.C.F., the quotient is 168. Find the difference between the two numbers.

(7 marks)

18. 在一個神奇的罐子裏有 113 個紅球和 72 個綠球。罐子上有兩個按鈕甲和乙，只要按下它們便可以改變罐子裏球的數量。它們的作用如下：

按鈕甲：增加 4 個紅球和除去 5 個綠球；

按鈕乙：除去 6 個紅球和增加 5 個綠球。

如果按下某個按鈕時，某種顏色的球少於它可除去的球，則那種顏色的所有球會全被除去。假如我們希望罐子裏剛好有 60 個紅球和 60 個綠球，我們最少需要按下按鈕多少次？

(7 分)

In a magic can there are 113 red balls and 72 green balls. On the can there are two buttons, A and B. By pressing the buttons one can alter the number of balls in the can. The buttons work as follows:

Button A: Produce 4 red balls and remove 5 green balls

Button B: Remove 6 red balls and produce 5 green balls

If there are not enough balls of a certain colour to remove when a button is pressed, then the can will simply remove all balls of that colour. In order to get exactly 60 red balls and 60 green balls in the can, what is the minimum total number of times the two buttons should be pressed?

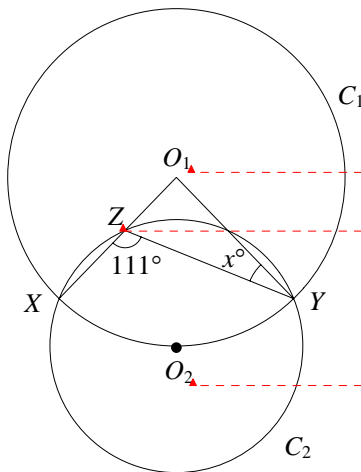
(7 marks)

19. 設 $[x]$ 代表不超過 x 的最大整數，例如 $[1.1] = 1$ 、 $[6.9] = 6$ 和 $[5] = 5$ 。若 $[y] = 3$ 、 $[y^2] = 11$ 、 $[y^3] = 41$ ，問 $[y^6]$ 有多少個不同的可能值？ (7 分)

Let $[x]$ denote the greatest integer not exceeding x . For example, $[1.1] = 1$, $[6.9] = 6$ and $[5] = 5$. If $[y] = 3$, $[y^2] = 11$ and $[y^3] = 41$, how many different possible values of $[y^6]$ are there? (7 marks)

20. C_1 和 C_2 是兩個圓，圓心分別是 O_1 和 O_2 ，其中 O_2 位於 C_1 的圓周上。這兩個圓相交於 X 點及 Y 點。若 C_1 的半徑 O_1X 與 C_2 再相交於 Z ，且 $\angle XZY = 111^\circ$ 、 $\angle O_1YZ = x^\circ$ ，求 x 。

C_1 and C_2 are two circles with centres O_1 and O_2 respectively, where O_2 lies on the circumference of C_1 . These two circles intersect at the points X and Y . If the radius O_1X of C_1 intersects C_2 again at Z and $\angle XZY = 111^\circ$, $\angle O_1YZ = x^\circ$, find x .



(7 分)

(7 marks)

全卷完

END OF PAPER