第四屆培正數學邀請賽

4th Pui Ching Invitational Mathematics Competition

初賽（中一組）

Heat Event (Secondary 1)

時限：1 小時 15 分

Time allowed: 1 hour 15 minutes

參賽者須知：

Instructions to Contestants:

1. 本卷共設 20 題，總分爲 100 分。

   There are 20 questions in this paper and the total score is 100.

2. 除特別指明外，本卷內的所有數均爲十進制。

   Unless otherwise stated, all numbers in this paper are in decimal system.

3. 所有答案皆是 0 至 9999 之間的整數。依照答題紙上的指示填寫答案，毋須呈交計算步驟。

   All answers are integers between 0 and 9999. Follow the instructions on the answer sheet to enter the answers. You are not required to hand in your steps of working.

4. 不得使用計算機。

   The use of calculators is not allowed.

5. 本卷的附圖不一定依比例繪成。

   The diagrams in this paper are not necessarily drawn to scale.

第 1 至第 4 題，每題 3 分。

Questions 1 to 4 each carries 3 marks.

1. 一名懶惰的學生考試前沒有溫習，他以爲三角形的面積等於「底 × 高」。在考試中，其中一題要求學生找出某個三角形的面積 。這名學生得到的答案是 2468。那麼該題的正確答案是多少？

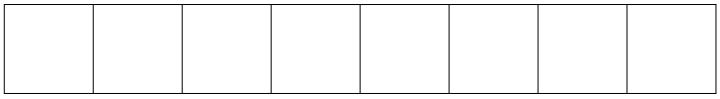
   A lazy student does not study for the examination and he thinks that the area of a triangle is equal to base × height. In the examination, a question asks for the area of a triangle. The student gets the answer 2468. What is the correct answer to the question?

2. 某次數學比賽中，其中一題要求參賽者把某個不等於零的數除以 6。小芬誤把「除」字看作「乘」字。那麼，小芬的答案是正確答案的多少倍？

   In a mathematical competition, a question asks contestants to divide a non-zero number by 6. Mary wrongly reads the word 'divide' as 'multiply'. How many times the correct answer is Mary's answer?

3. 現有八個方格，並要把每個塗上紅色或黃色。那麼至少有多少格的顏色相同？

   There are 8 cells and each one is to be coloured red or yellow. What is the least number of cells which are of the same colour?



4. 777777770000000044 有多少個小於 10 的正因數？

   How many positive factors of 777777770000000044 are smaller than 10?

第 5 至第 8 題，每題 4 分。

Questions 5 to 8 each carries 4 marks.

5. 在 2005 年中，有多少天的「月」和「日」均爲質數？

   In the year 2005, how many days are there such that the 'month' and the 'day' are both prime?

6. ΔABC 的面積是 2005。直線 L_1 穿過 A 且平行於 BC，直線 L_2 穿過 B 且平行於 AC，直線 L_3 穿過 C 且平行於 AB。由 L_1 、 L_2 和 L_3 三條直線圍出的三角形的面積是多少？

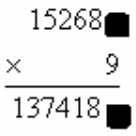
The area of ΔABC is 2005. The straight line L_1 passes through A and is parallel to BC; the straight line L_2 passes through B and is parallel to AC; the straight line L_3 passes through C and is parallel to AB. Find the area of the triangle bounded by the three straight lines L_1, L_2 and L_3.

7. 設 $S = 7 + 77 + 777 + 7777 + \cdots + 7777777777$ 。求 S 的最後四位數字。

Let $S = 7 + 77 + 777 + 7777 + \cdots + 7777777777$. Find the last four digits of S.

8. 小明在習作簿上寫上了一條正確的乘式 ，後來乘式中其中兩個個位數字卻被妹妹塗污了（見圖）。求兩個被塗污了的數字之積。

Alan put down a correct multiplication in his exercise book, and two unit digits in the multiplication were later crossed out by his sister, as in the figure. Find the product of the two digits which were crossed out.



第 9 至第 12 題，每題 5 分。

Questions 9 to 12 each carries 5 marks.

9. 某三角形三條邊的邊長分別是 2005、5002 和 n，其中 n 是正整數。問 n 有多少個不同的可能值？

A triangle has side lengths 2005, 5002 and n, where n is a positive integer. How many different possible values of n are there?

10. 某班有 n 名學生。已知任何三名學生中，均最少有兩人的出生月份不同。求 n 的最大可能值。

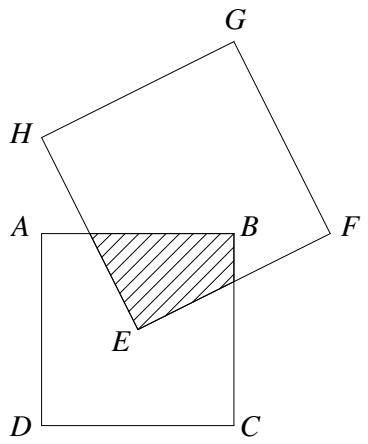
There are n students in a class. It is known that among any three students, we can always find at least two who were born in different months of the year. Find the largest possible value of n.

11. 已知 x、y 和 z 是互不相同的正整數，而且當中任何兩個數之積均可被餘下的一個數整除。求 $x+y+z$ 的最小值。

Given x, y and z are three pairwise different positive integers such that the product of any two of them is divisible by the remaining number. Find the minimum value of $x+y+z$.

12. 圖中，*ABCD* 和 *EFGH* 是正方形，邊長分別爲 24 和 32。*E* 是 *ABCD* 的中心。求陰影部分的面積。

In the figure, *ABCD* and *EFGH* are squares with side lengths 24 and 32 respectively. *E* is the centre of *ABCD*. Find the area of the shaded region.



第 13 至第 16 題，每題 6 分。

Questions 13 to 16 each carries 6 marks.

13. 我們有時會以「年 / 月 / 日」的形式表示日期。如果某天的「月」比「日」大，我們稱它爲「美好的日子」。例如 2005/03/01 和 2005/04/03 都是「美好的日子」，2005/01/01 和 2005/03/12 則不是「美好的日子」。小芬只會在每個「美好的日子」看電視一次，其他日子都不會看。在 2005/02/01，她首次看電視。在 2025/11/09，她第 n 次看電視。求 n。

We sometimes use the format 'Year/Month/Day' to represent dates. If the 'month' is greater than the 'day' in a day, we call it a 'nice day'. For example, 2005/03/01 and 2005/04/03 are 'nice days' while 2005/01/01 and 2005/03/12 are not. Sammy only watches television once during 'nice days' but not on other days. The first time she watched television was on 2005/02/01. On 2025/11/09, she will be watching television for the n-th time. Find n.

14. 設 $[x]$ 爲不超過 x 的最大整數，例如 $[1.1] = 1$、$[6.9] = 6$ 和 $[5] = 5$。若 n 是正整數，$\left[\frac{n}{2005}\right]\times\left[\frac{5002}{n}\right]$ 有多少個不同的可能值？

Let $[x]$ be the greatest integer not exceeding x. For example, $[1.1] = 1$, $[6.9] = 6$ and $[5] = 5$. If n is a positive integer, how many different possible values does $\left[\frac{n}{2005}\right]\times\left[\frac{5002}{n}\right]$ have?

15. 對於正整數 n，若 n 與 15 的最大公因數**不是** 1，那麼 n 便稱爲「好數」。若把所有「好數」從小至大排列，第 2005 個是甚麼？

A positive integer n is said to be 'good' if the H.C.F. of n and 15 is NOT equal to 1. If all the 'good' numbers are arranged in ascending order, what is the 2005th term?

16. 某個以十進制寫成的兩位數有一種特別的性質：如果我們把它的兩位數字倒轉，所得的數剛好是原數寫成十六進制時的表達式。求這個兩位數（答案以十進制表示）。

A two-digit denary number has a special property: when the two digits are reversed, the number obtained is precisely the hexadecimal representation of the original number. What is the two-digit number (in decimal notation)?

第 17 至第 20 題，每題 7 分。

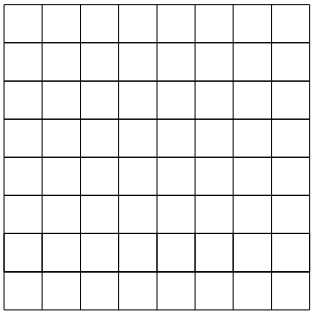
Questions 17 to 20 each carries 7 marks.

17. 一個正方體的每個面上均寫有一個不同的正整數。若任何兩個相鄰的面上的兩數的最大公因數均爲 1，求六個面上的六個數之和的最小可能值。

A different positive integer is written on each face of a cube. Whenever two faces are adjacent, the H.C.F. of the two numbers on the faces is equal to 1. What is the minimum sum of the six numbers on the six faces?

18. 在圖中的 8×8 方格表中，可以找到多少個由三個小方格組成的  、  、  或  形狀？

In the 8×8 table shown, how many  ,  ,  or  shapes made up of three small squares can be found?



19. 某多邊形的周界是 12，面積是 A。該多邊形每邊的長度都是整數，而且每隻內角都是 90° 或 270°。求 A 的所有可能值之和。

A polygon has perimeter 12 and area A. Each side of the polygon is an integer, and each of its interior angles is either 90° or 270°. Find the sum of all possible values of A.

20. 若干個互不相同的正整數之和爲 25，而它們之積是一個立方數。求該立方數。

Several pairwise distinct positive integers have sum 25, and their product is a cubic number. Find the cubic number.

全卷完

END OF PAPER