# Initial report of 1976 U.S. Atmospheric Model methodology

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# 1 Background

"The USSA mathematical model divides the atmosphere into layers with an assumed linear distribution of absolute temperature $T$ against geopotential altitude $h$. The other two values (pressure $P$ and density $\rho$) are computed by simultaneously solving the equations resulting from:

$$\frac{dP}{dh} = -\rho g \tag{1}$$

$$P = \rho R_{\text{specific}} T \tag{2}$$

where $R_{\text{specific}}$ is the specific gas constant for dry air and $g$ is the standard gravitational acceleration." Equation (1) evaluates the vertical pressure variation with the mean sea level as a boundary condition ($P_0 = 101{,}325$ Pa) and Equation (2) is the ideal gas law in molar form [1].

Combining equations (1) and (2) to find an expression for the pressure:

$$\ln P \Big|_{P_0}^{P_1} = -\frac{g}{R_{\text{specific}}} \int_{h_0}^{h_1} \frac{dh}{T(h)} \tag{3}$$

where the minimum geopotential altitude above sea level is set to $h_0 = -1{,}524\,\text{m}$. From The Engineer ToolBox, various temperatures have been given for their corresponding geopotential altitudes in Table 1 [2].

Table 1: USSA1976 Temperature for Geopotential Altitude - Imperial (BG) Units

| $h$ [ft] | $T$ [°F] |
|---|---|
| $-5000$ | 76.84 |
| 0 | 59 |
| 5000 | 41.17 |
| 10000 | 23.36 |
| 15000 | 5.55 |
| 20000 | $-12.26$ |
| 25000 | $-30.05$ |
| 30000 | $-47.83$ |
| 35000 | $-65.61$ |
| 40000 | $-69.70$ |
| 45000 | $-69.70$ |
| 50000 | $-69.70$ |
| 60000 | $-69.70$ |
| 70000 | $-67.42$ |
| 80000 | $-61.98$ |
| 90000 | $-56.54$ |
| 100000 | $-51.10$ |
| 150000 | 19.40 |
| 200000 | $-19.78$ |
| 250000 | $-88.77$ |

# 2 The Implemented Model

## 2.1 Calculating Temperature, Pressure, and Density

To find an expression for temperature as a function of geopotential altitude, use the data from Table 1. Start by converting the data into SI units, plot $T$ against $h$, and then fit a curve to the data as in Figure 1. And so, $T(h) =$

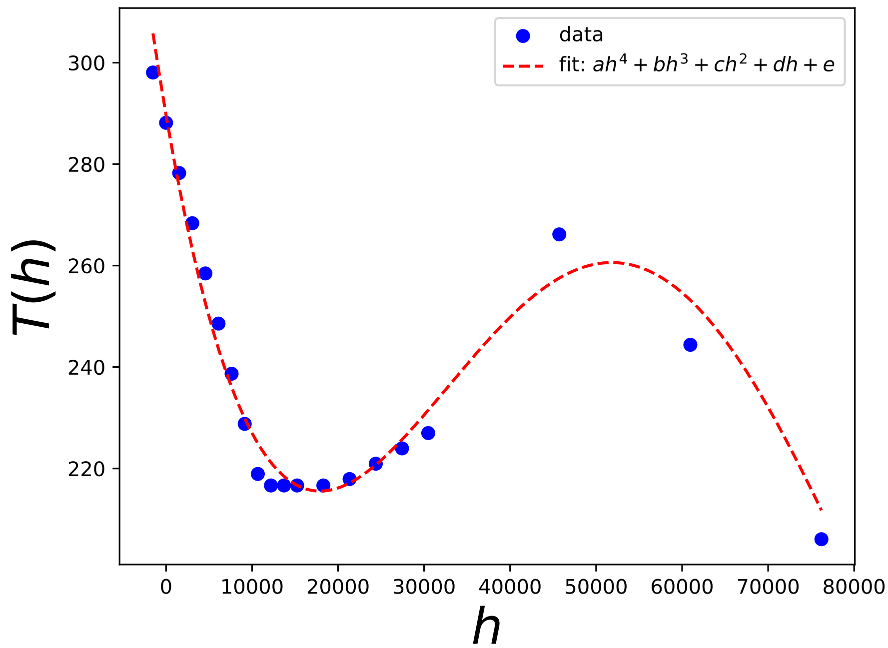


Figure 1: $T(h)$ vs. $h$; A scatter plot of temperature as a function of geopotential altitude given from the data in Table 1. A fourth degree polynomial has been fitted to the data in order to find a mathematical expression for $T(h)$. Please note that the fit would have more (and actual) physical meaning if it was not a generic polynomial since a polynomial of a large enough degree will virtually always fit to any function. The parameters which are found from the fit are: $a = 0.000000000000000027$, $b = -0.000000000006074933$, $c = 0.000000422889669793$, $d = -0.00991893662487261$, and $e = 289.661267963331567898$.[1]

$ah^4 + bh^3 + ch^2 + dh + e$ with the coefficients given in the caption of Figure 1. With the data given in Table 1, this expression is assumed to only work if $-1{,}524\ \text{m} \leq h \leq 76{,}200\ \text{m}$. Equation (3) can now be written as:

$$P(h) = P_0 \exp\left[-\frac{g}{R_{\text{specific}}} \int_{h_0}^{h \leq 76200} \frac{dh}{T(h)}\right] \tag{4}$$

where $R_{\text{specific}} = 287.052874\ \text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$ and $g$ is negative [3,4]; with the currently given expression for the integrand this equation must be solved numerically.[2] Once the pressure is found then the density can be obtained with Equation (2) for each geopotential altitude given into the fourth degree polynomial for $T(h)$.

## 2.2 The Instantaneous Drag Force

The drag force formula is given by,

$$\vec{F}_{\text{d}} = \frac{1}{2}\rho\,\vec{v}^{\,2} C_{\text{d}} A \tag{5}$$

[1]Here, please do not mix up $e$ with Euler's constant.

[2]It is interesting to note that if the maximum value of $h$ is fed into (4) then the numerical value for the integral in the RHS becomes $323.7972921077598 \pm 1.173024627261422 \times 10^{-10}$.

where $C_\mathrm{d}$ is the drag coefficient and $A$ is the cross-sectional area. The method which computes drag takes in ECEF position,[3] velocity, and ballistic parameters ($C_\mathrm{d},\ A,\ m$) to output an acceleration due to force vector. The acceleration due to drag can be found using Newton's second law; $\vec{a}_\mathrm{d} = \vec{F}_\mathrm{d}/m$.

# 3 An Example

An object of mass $m = 100\,\mathrm{kg}$ is traveling with a velocity of $\vec{v} = (v_x, v_y, v_h) = (20, 100, 3000)\,\mathrm{m/s}$ at an ECEF position of $(0, 0, 10000)$ m where the latter component is the altitude. The drag coefficient is 1 and the cross-sectional area of the object is $100\,\mathrm{m}^2$. From the applied model for atmosphere here and the fourth degree polynomial for $T(h)$ its temperature is $226.956$ K and it follows that its pressure is $21846.097$ Pa and the density of air at that altitude is $0.335\,\mathrm{kg/m}^3$. The applied drag force at that altitude would be $\vec{F}_\mathrm{d} = (1.677 \times 10^5, 1.509 \times 10^8, 6.707 \times 10^3)$ N; ergo, the length of the acceleration due to drag would be $|\vec{a}_\mathrm{d}| = 1508978.973\,\mathrm{m/s}^2$.[4]

# Bibliography

[1] Wikipedia, *U.S. Standard Atmosphere*, retrieved on 01/22/2024 from `https://en.wikipedia.org/wiki/U.S._Standard_Atmosphere`.

[2] The Engineering ToolBox, *U.S. Standard Atmosphere vs. Altitude*, retrieved on 01/22/2024 from `https://www.engineeringtoolbox.com/standard-atmosphere-d_604.html`.

[3] Wikipedia, *Gas constant*, retrieved on 01/18/2024 from `https://en.wikipedia.org/wiki/Gas_constant`.

[4] Wikipedia, *Vertical pressure variation*, retrieved on 01/18/2024 from `https://en.wikipedia.org/wiki/Vertical_pressure_variation`.

[5] National Oceanic and Atmospheric Administration, National Aeronautics and Space Administration, United States Air Force, *U.S Standard Atmosphere, 1976*, US Department of Commerce/NOAA/NESDIS, October 1976.

[6] Ralph L. Carmichael, Public Domain Aeronautical Software, *Geopotential & Geometric Altitude*, retrieved on 01/22/2024 from `https://www.pdas.com/hydro.pdf`.

[7] Wikipedia, *Earth-centered, Earth-fixed coordinate system*, retrieved on 01/23/2024 from `https://en.wikipedia.org/wiki/Earth-centered,_Earth-fixed_coordinate_system`.

[3] The altitude is the only relevant coordinate when it comes to this specific calculation as Equation (5)'s only parameter dependent on position is $\rho$.
[4] Although it does change with altitude, $g = 9.81\,\mathrm{m/s}^2$ is used for all altitudes in this implementation.