# Gaussian Distribution of PID Cloud Fragments

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# Motivation

- Long term interest in the concept of *debris clouds*
- More stars in the observable universe than grains of sand on Earth
  - concept of *packing density*
- Goal: Find the probability of impact given an interceptor is flying through a cloud of debris.

# Presentation Outline

**Packing Density Model**

**Fragment Size and Spatial Distribution Considerations**

- Normal Distribution of Fragments ★
- Fragment Size Dependency

**Evolution of PID Cloud with Time**

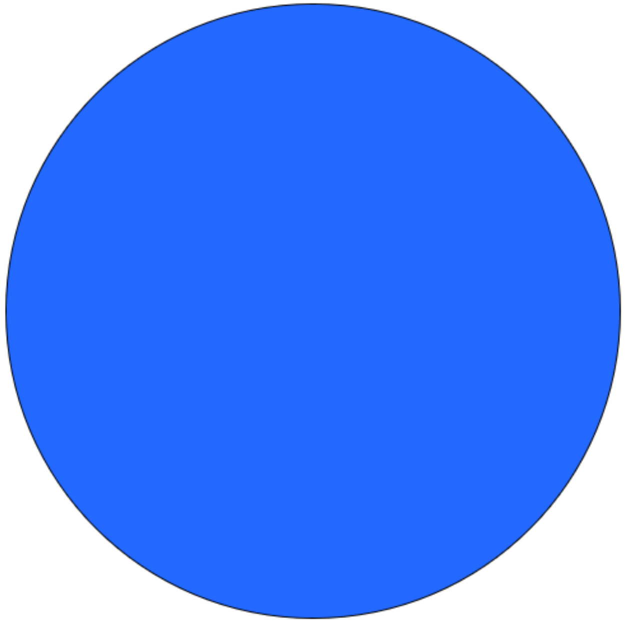
**Orbital Mechanics**

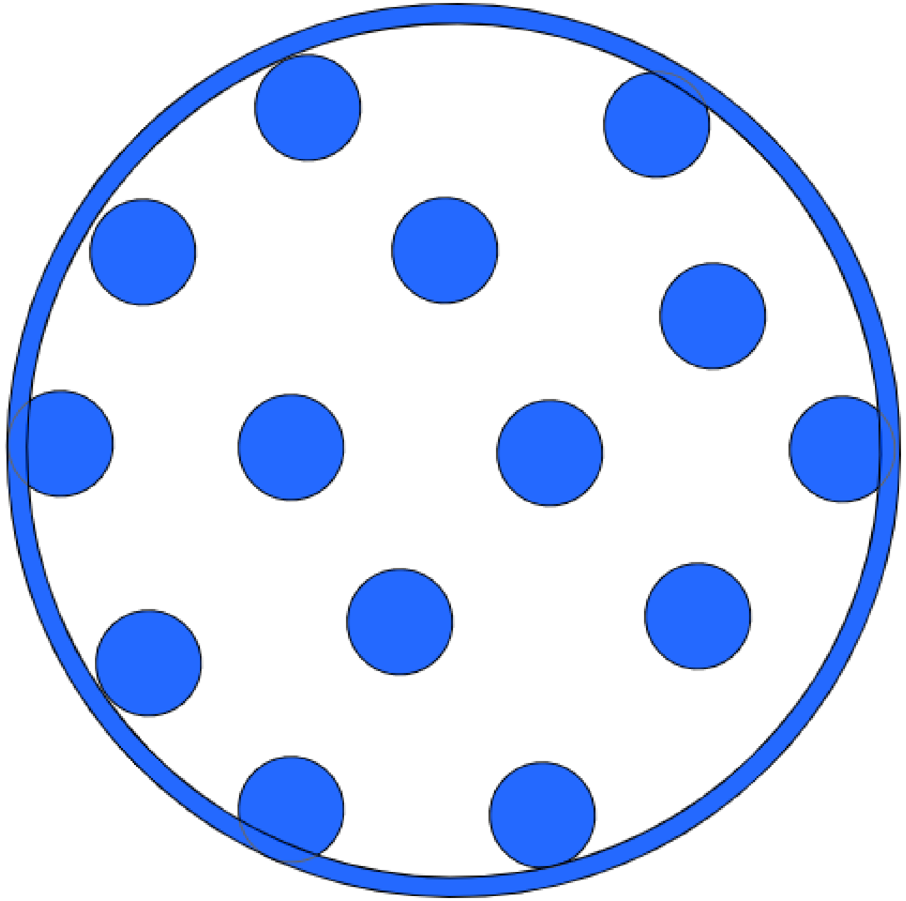
# I. Packing Density Model

# PID Cloud Formation

- A uniform solid sphere exists in space.
- It either explodes or collides with another body such as an interceptor and causes the sphere to fragment into equivalent pieces of post-impact debris (PID) with all the PID fragments being evenly spaced.
- The fragments forms a cloud resembling a sphere.

# Solid Sphere vs. PID Cloud





# Packing Density & Mass Conservation

- Packing density is a number that captures how much space is filled by stuff, like grains of sand in a spherical shell.

- The packing density of the cloud is always less than one, because the PID fragments don't fit together perfectly—there are always gaps or voids between them, so they can't fill the entire space like the original solid sphere did.

$$\begin{cases} \eta_c < 1 \\ \eta_s = 1 \end{cases}$$

- The mass of the solid sphere is the same as the mass of the debris cloud (mass conservation).

# Packing Density & Mass Conservation

- Mass conservation $\Longrightarrow \eta_s V_s = \eta_c V_c$:

$$\frac{R_c}{R_s} = \sqrt[3]{\frac{1}{\eta_c}}$$

- Implying that $R_s < R_c$, as expected.

# II. Spatial Distribution Considerations and Fragment Size

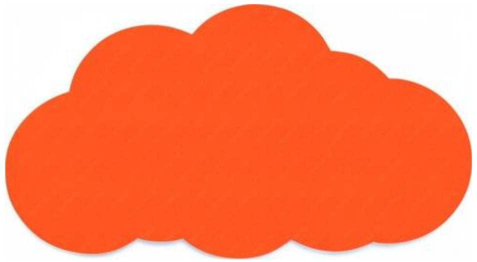
# Caveats of Packing Density Model

Assumes all debris fragments are identical.

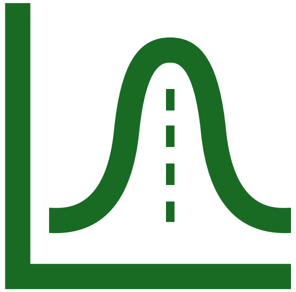
Assumes all fragments are evenly spaced.

# II.1 Gaussian Distribution

# Postulates



The fragments will be distributed at a peak relative to the cloud’s radius.



The fragment dispersion post-impact is symmetric about the peak of the distribution

# Normal Distribution Function

$$\rho = \rho_0 \exp\left[-\frac{(r - \mu R_\mathrm{c})^2}{2\sigma^2 R_\mathrm{c}^2}\right]$$

- $\rho_0$ is a normalization constant
- $r$ is the radial distance from the center
- $\mu R_\mathrm{c}$ is the peak of the distribution
- $\sigma$ is the standard deviation

# II.2 Size Dependency

# Characteristic Length

- Fragments have different sizes!
- The size is defined by each fragment's respective characteristic length ($L_c$)
- The parameters in the formula for $\rho$ are now size dependent;
  - $\rho_0 \rightarrow \rho_0(L_\text{c})$
  - $\mu \rightarrow \mu(L_\text{c})$
  - $\sigma \rightarrow \sigma(L_\text{c})$

# Size Dependency of Parameters

- $\rho_0(L_\mathrm{c})$ can be found by integrating the mass integral itself over the range of characteristic lengths.

- $\mu(L_\mathrm{c})$ is now defined for each specific $L_\mathrm{c}$.

- $\sigma(L_\mathrm{c})$:
  - $v \propto A/m$; smaller $L_\mathrm{c}$ implies higher $A/m$ and hence higher $v$.
  - $\sigma \propto v$ and velocity is inversely related to $L_\mathrm{c}$:

    $$\sigma(L_\mathrm{c}) \simeq \sigma_0 \cdot L_\mathrm{c}^{-\alpha}$$

    where $\sigma_0$ and $\alpha$ are observable constants.

# III. Time Evolution of Debris Cloud

# Transition from Sphere

$$\text{Spherical} \longrightarrow \text{Ellipsoidal} \longrightarrow \text{Toroidal} \longrightarrow \text{Shell}$$

- The cloud radius evolves with time.
- For short time periods ($\mathrm{t} \leq 120\ s$), this expansion is linear:

$$R_{\mathrm{c}}(t) = R_{\mathrm{c}}(0) + v_{\mathrm{expansion}} \cdot t$$

# Temporal Evolution of $\rho_0$

- The mass density’s normalization constant must evolve to preserve the total mass of the cloud as it expands.
- The increase in volume is proportional to the cube of the increase of $R_c$:

$$\rho_0(L_c, t) = \rho_0(L_c, 0) \cdot \left[\frac{R_c(0)}{R_c(t)}\right]^3$$

# IV. Orbital Mechanics

# Gravitational Acceleration

- The acceleration of a fragment in Earth's Newtonian gravitational field is given by :

$$\frac{d^2\mathbf{r}_{\text{grav}}}{dt^2} = \frac{-GM_{\oplus}}{r^2}\hat{r}$$

- The total acceleration would then be:

$$\mathbf{a}_{\text{total}} = \frac{d^2\mathbf{r}_{\text{grav}}}{dt^2} + \mathbf{a}_{\text{pert}}$$

where is the acceleration due to perturbation effects.

# Perturbation Forces

- The expression for the non-gravitational perturbation acceleration is:

$$\mathbf{a}_{\mathrm{pert}} = \mathbf{a}_{\mathrm{SRP}} + \mathbf{a}_{\mathrm{d}} + \mathbf{a}_{\mathrm{EM}}$$

  - SRP: solar radiation pressure
  - d: drag
  - EM: electromagnetic (neglected)

# Perturbation Impact on Dispersion

- The spatial distribution is affected by SRP forces such that:

$$\sigma(L_c, t) = \sigma(L_c, 0) + \gamma t L_c^{-\alpha}$$

  where $\gamma$ is an empirical parameter.

- The peak radial position, $\mu$, is unaffected by perturbations for short timescales.

# Final Mass Density

- The Gaussian distribution function with all its mapping is:

$$\rho(r, L_c, t) = \rho_0(L_c, t) \cdot \exp\left[-\frac{1}{2}\left(\frac{r - \mu(L_c)R_c(t)}{\sigma(L_c, t)R_c(t)}\right)^2\right]$$

# The End