Collision Probability Assessment for Spherically Symmetric Debris Cloud Models

Kamyar Modjtahedzadeh*

www.spyderkam.com

December 10, 2025

Abstract

Interceptor missions create immediate debris hazards where follow-on missiles may transit expanding fragment clouds, raising the critical question of collision probability during the initial spherical phase lasting roughly 120 seconds after breakup. Current computational frameworks exhibit methodological gaps for this period, as traditional Poisson flux models assume fragment independence that fails under correlated gravitational evolution, while pure Monte Carlo propagation becomes prohibitively expensive for populations exceeding 10^8 fragments. This work presents a Monte Carlo-based Poisson process model that resolves this tension by treating collision events as independent in time rather than space, enabling accurate density sampling without individual orbit propagation. The methodology implements the NASA EVOLVE 4.0 breakup model's area-to-mass ratio distributions through piecewise statistical functions. Fragment positions are sampled from spherically symmetric density distributions using techniques that preserve the symmetric radial profile while ensuring computational tractability. The collision rate along arbitrary interceptor trajectories is computed through numerical integration of spatially varying number density weighted by effective cross-sectional area, with statistical confidence established through adaptive Monte Carlo sampling and Wilson score confidence intervals. The resulting algorithmic and statistical formulation enables real-time operational assessment for interceptor mission planning during the critical blackout period before debris cataloging becomes available.

Keywords: Orbital debris, Astrodynamics, Fragmentation simulation, NASA EVOLVE 4.0, Statistical sampling

*kam@spyderkam.com

1 Introduction

Interceptor missions create worst case debris scenarios where high-speed collisions fragment targets into numerous pieces, establishing an immediate danger zone around the intercept point. In multitarget engagement scenarios, follow-on interceptors may need to pass through this expanding debris field to reach additional targets, raising a critical operational question: What is the probability that a subsequent missile transiting the debris cloud will impact a fragment? Mission success depends on knowing whether follow-on interceptors can survive this transit. For the $\lesssim 120$ seconds of the debris cloud's initial configuration, which is spherical in geometry according to Vallado & Oltrogge (2017), there are significant methodological gaps in current collision probability frameworks [1]. While comprehensive techniques exist for long-term temporal evolution, the immediate post-fragmentation period remains underserved by operational computational methods. The most promising solutions to this problem have been recent boundary value problem (BVP) approaches from 2022 onward; having said that, many researchers explicitly call for improved approaches. The Monte Carlo-based Poisson process model proposed here directly addresses the gap by providing a novel approach for the computation of impact probability given debris cloud empirical parameters and interceptor trajectory.

1.1 Heavenly Debris Evolution

Understanding debris cloud evolution is fundamental to selecting appropriate collision probability methods. Jehn (1991) is foundational when it comes to identifying four phases with distinct computational requirements; its classifications are widely adopted in subsequent literature. As previously mentioned, the initial phase is spherical, and then after one quarter of the orbital period, ellipsoidal [1,2]. Immediately after fragmentation, fragments move together in a correlated pattern controlled by the Earth's gravity, making standard collision probability methods invalid. This initial spherical phase lasts minutes to hours depending on the orbit.

The second classified phase is toroidal, which is reached at approximately one orbital period. Within hours, differential orbital velocities spread the cloud into a torus shape centered on the original orbit. The third phase is the band formation: Over days to weeks, J_2 perturbation causes different orbital node regression rates, opening the torus into a latitude band around the Earth. The fourth and long-term phase has no geometric shape as the debris has dispersed into a random distribution around Earth by then. Fragments are removed individually by atmospheric drag based on their area-to-mass ratios, becoming part of the general orbital debris environment rather than maintaining any coherent cloud structure. Most operational tools target the second and/or third phase. The continuum method described by Letizia et al. (2016) note that it begins after "mean anomaly randomization," mean anomaly randomizes during the toroidal phase [3]. Only recent BVP-based methods specifically target the first phase.

1.1.1 Challenges of the Spherical Phase

There is a critical gap in current computational frameworks when it comes to the immediate post-fragmentation period. Jenkin (1996) provides the clearest statement of the fundamental problem: "Newly formed debris clouds resulting from orbital fragmentations are characterized by fragments with relative motion that is highly correlated by the central gravitational field, thereby eliminating any resemblance to a gas. While the use of the Poisson model in this context has been criticized, it generally has been used anyway due to the lack of a well-known and accepted alternative model" [4]. Vallado & Oltrogge (2017) state explicitly in their DREAD tool development, insufficient research and modeling have been done in this area to provide such actionable SSA, but the NASA Standard Breakup Model fails to conserve any of mass, momentum, or kinetic energy [1]. Furthermore, Liu et al. (2022) emphasize that the Poisson distribution is not applicable in the early phases, requiring further insight into analyzing the collision probability before the band formation [5].

Comprehensively, the core computational challenges for initial phase assessment include correlated fragment motion, non-uniform spatial distribution, computational burden, uncertainty propagation, and tracking limitations. Correlated fragment motion refers to how the independence assumptions underlying Poisson-based methods fail when fragments share a common gravitational evolution from a single source. Non-uniform spatial distribution refers to the fact that the continuum methods requiring randomized distributions are invalid during ellipsoidal and toroidal phases. Moreover, Monte Carlo methods are accurate but prohibitively expensive for real-time operational decisions, non-uniform breakup assumptions introduce significant errors in highly nonlinear early-phase dynamics, and small fragments remain untraceable, requiring statistical treatment without observational validation.

1.2 Existing Methods

1.2.1 Continuum and Density-Based Methods

Continuum approaches model debris clouds as fluids with continuously varying spatial density, enabling analytical solutions that reduce computational burden by orders of magnitude compared to discrete fragment propagation. Letizia et al. developed the primary continuum framework in a series of papers. Their 2015 paper derives cloud evolution analytically via the continuity equation under atmospheric drag, reducing computation time to under 10% of numerical propagation for equivalent fragment populations [6]. The 2016 extension models collision probability through spatial density integration [3]. The 2018 paper generalizes to full orbital perturbations; limitations include exponential atmospheric model, neglected eccentricity effects on drag, and required randomization of mean anomaly distribution [7]. Giudici et al. (2023) explicitly state: "Accurate and efficient continuum formulations have been developed to propagate clouds of fragments under atmospheric drag and J_2 perturbations, but a general model able to work under any dynamical regime has still to be found." [8].

It is noteworthy that Frey & Colombo (2021) presented transformation methods for probabilistic cloud evolution directly from NASA Standard Breakup Model distributions, enabling fully probabilistic fragmentation analysis without deterministic sampling [9]. Liu et al. (2022) extended analytical propagation for collisions near sun-synchronous orbits, introducing entropy metrics to model randomness during band formation, achieving high accuracy over 400-day propagation periods [5]. Continuum methods are valid for days to years after spatial distribution randomization, but they are not suitable for initial spherical phase analysis.

1.2.2 Boundary Value Problem-Based Methods

The most significant methodological advances for short-term debris cloud assessment come from BVP-based approaches developed since 2022, which map collision geometry to initial velocity space. Shu et al. (2022) introduced higher-order BVPs for debris cloud hazard analysis using differential algebra techniques. Their method obtains transfer maps from initial velocity space to final position space, calculating debris density within spacecraft encounter volumes through Taylor expansions [10]. This work specifically targets hours-scale post-fragmentation scenarios. Parigini et al. (2024) applied automatic domain splitting with high-order Taylor expansions to short-term collision probability; their Taylor Monte Carlo method reduces computational cost while maintaining statistical rigor for first hours after fragmentation [11]. Not to mention, Morselli et al. (2015) developed foundational work combining differential algebra with Monte Carlo sampling for collision probability computation, using high-order series expansions to approximate trajectory propagation while maintaining sampling-based statistical confidence [12].

The BVP-based methods were further extended to GEO collision scenarios by Shu et al. (2025), modeling two debris clouds from collision as a single probability density function. They find that collision probability with millimeter-sized fragments may reach 1% within 36 hours for GEO fragmentation events; probability with fragments of characteristic length $\gtrsim 5\,\text{cm}$ reaches $\approx 10^{-5}$ [13]. Following the theoretical basis set forth by Jenkin (1996), Trombetta et al. (2025) proposed an operational integration with the GSOC Collision Avoidance System [4, 14]; their method maps primary position evolution to Initial Spread Velocity Space (ISVS) by solving multi-revolution Lambert problems and integrating NASA Breakup Model probability density functions over swept volumes [14]. With a final error of 0.7%, their approach specifically addresses the blackout period before debris cataloging in the first 6 hours after fragmentation.

1.2.3 Pure Monte Carlo-Based Methods

Despite BVP methods producing the most promising results lately, perhaps the most accurate approaches to finding the collision probability with fragments in a debris cloud come from Monte Carlo simulation methods. Validated against the Inter-Agency Space Debris Coordination Committee (IADC) studies, Jang et al. (2024) achieved computational efficiency in orbit propagation for handling large populations over centuries. They presented their novel Monte Carlo methods enhancing the MIT Orbital Capacity Analysis Tool (MOCAT) for long-term environment evolution [15]. Over and above that, Valencia et al. (2025) integrated NASA Standard Breakup Model with a stochastic Monte Carlo simulation for hypervelocity impact fragmentation, capturing inherent randomness, including ejecta effects. Their model predicts approximately 10^9 distinct debris fragments per hypervelocity impact event [16]. Please note that in a preceding work by Li et al. (2022), it is explicitly stated that: "Although such simulations are accurate when a large number of samples are used, these methods are perceived as computationally intensive, which limits their application in practice" [17]. For debris populations exceeding 100 million objects (millimeter-range fragments), deterministic approaches become prohibitively expensive.

1.3 Proposed Method: Poisson Process Model with Monte Carlo Density Sampling

The computational implementation presented in this letter addresses the methodological gap for spherical phase debris cloud assessment through a Monte Carlo-based Poisson process model that combines the statistical rigor of sampling methods with the physical accuracy of Poisson collision theory. While the introduction has highlighted limitations in both Poisson and Monte Carlo approaches, the present framework utilizes their respective strengths while avoiding their individual weaknesses. Traditional Poisson methods fail during the spherical phase because they assume fragment independence and uniform spatial distribution, neither of which holds in the estimations to be made. Pure Monte Carlo propagation of individual fragment trajectories achieves high accuracy but becomes computationally prohibitive for populations exceeding 10^8 fragments. The present approach resolves this tension by treating collision events as a Poisson process in time while using Monte Carlo sampling to accurately capture the non-uniform spatial density distribution given from spherically symmetric cloud models. Rather than propagating millions of individual fragment orbits, the method samples fragment positions from the spatial density function at each step in time along the interceptor trajectory. This sampling-based approach preserves the non-uniform spatial structure that invalidates classical Poisson methods while avoiding the computational expense of deterministic propagation. The Poisson process framework remains valid because collision events along the trajectory occur independently in infinitesimal time intervals, even though the spatial distribution at each instant is non-uniform. This distinction is critical—the method does not assume fragment independence in space—only statistical independence of collision events in time. Treating the spherical phase as maintaining quasi-static geometry throughout its duration, the computational efficiency achieved through density-based sampling enables real-time operational assessment for the first 120 seconds post-fragmentation, directly addressing the blackout period identified by Trombetta et al. (2025) while maintaining the statistical confidence of Monte Carlo methods [14]. The framework leverages empirical parameters from the NASA Standard Breakup Model and symmetric fragment dispersion, requiring only the interceptor trajectory and debris cloud characteristics as inputs. By focusing computational resources on sampling the density distribution rather than propagating individual orbits, the method achieves the accuracy necessary for operational decision-making within the time constraints of multitarget engagement scenarios.

This synthesis of Poisson process theory with Monte Carlo sampling represents a computationally tractable approach specifically designed for the operational constraints of interceptor mission planning. The following sections develop the mathematical foundation for this approach, beginning with an implementation of NASA's EVOLVE 4.0 Breakup Model and progressing through Poisson process theory to the complete collision probability calculation. The methodology yields a computational algorithm rather than a closed-form analytical expression for collision probability. This algorithmic approach arises from the combination of spherically symmetric density distributions with arbitrary interceptor trajectories, where analytical integration becomes intractable for general trajectory geometries. The algorithm's structure enables direct implementation in operational software systems while maintaining mathematical rigor through Monte Carlo convergence properties.

2 Distribution of Area-to-Mass Ratios

Based on the NASA breakup model's area-to-mass ratio distribution [18], a piecewise statistical function has been structured for the fragment's logarithm characteristic length (L_c) input and its cross-sectional area-to-mass ratio (A/M) output. The function partitions the domain into three regions based on fragment size:

$$\xi(x) = \begin{cases} \xi_{\text{small}}(\phi) & \text{if } \phi < \log(0.08) \\ \xi_{\text{inter}}(\phi) & \text{if } \log(0.08) \le \phi \le \log(0.11) \\ \xi_{\text{large}}(\phi) & \text{if } \phi > \log(0.11)\,, \end{cases} \tag{2.1}$$

where $\phi \equiv \log L_c$, ξ_{small} is the small fragment distribution and ξ_{large} is the large fragment distribution.

2.1 Interpolation of Middle Region

For the middle region ($8\,\text{cm} \le L_c \le 11\,\text{cm}$), the ξ function uses linear interpolation;

$$\xi_{\text{inter}}(\phi) = \xi_{\text{small}}(\phi) \cdot (1 - w(\phi)) + \xi_{\text{large}}(\phi) \cdot w(\phi)\,, \tag{2.2}$$

where the weight function is given by,

$$w(\phi) = \frac{\phi - \log(0.08)}{\log(0.11) - \log(0.08)}\,. \tag{2.3}$$

This creates a smooth transition between the statistical distributions for small and large fragments [18].[1]

2.2 Statistical Sampling Functions

Each of the base functions ξ_{small} and ξ_{large} perform statistical sampling from normal distributions in logarithmic space. The output value is ultimately transformed from logarithmic space back to linear space. This mathematical structure creates a continuous statistical model for A/M values that accommodates different physical regimes across the size spectrum of orbital debris fragments [18].

[1] It is called ξ_{inter} instead of ξ_{middle} because this function does not represent a unique statistical distribution for medium-sized fragments. Instead, it performs an interpolation (weighted blending) between the small and large fragment distributions. The name describes its mathematical operation rather than just the size category.

Without interpolation, a fragment at 7.9 cm would use entirely different statistical parameters than one at 8.1 cm, creating an unrealistic discontinuity in physical properties. Linear interpolation creates a gradual transition: At exactly 8 cm almost entirely small fragment distribution is used, at 9.5 cm (midpoint) weighting of both distributions is used, and at exactly 11 cm almost the entirety of large fragment distribution is used. This represents the physical reality that fragmentation processes produce a continuous spectrum of fragment properties rather than discrete categories with sharp boundaries.

2.2.1 Small Fragment Distribution

The small fragment distribution function $\xi_{\text{small}}(\phi)$ implements a unimodal log-normal sampling model:

$$\xi_{\text{small}}(\phi) = 10^{\mathcal{N}_\chi} , \tag{2.4}$$

where $\mathcal{N}_\chi \sim \text{Normal}(\overline{\chi}, \sigma_\chi)$ represents a random sample from a normal distribution, $\overline{\chi}$ is the mean of $\chi \equiv \log \frac{A}{M}$, and σ_χ is the standard deviation of χ. The aforementioned normal distribution is:

$$\frac{1}{\sigma_\chi \sqrt{2\pi}} \exp\left[-\frac{(\chi - \overline{\chi})^2}{2\sigma_\chi^2}\right] , \tag{2.5}$$

in Johnson et al. (2001), this normal distribution models the logarithm of area-to-mass ratios [18]. In other words, $\log(A/M)$ peaks and is symmetric around that peak value of χ.

Moreover, $\overline{\chi}$ and σ_χ follow piecewise functions. The expression for $\overline{\chi}(\phi)$ is:

$$\overline{\chi}(\phi) = \begin{cases} -0.3 & \text{if } \phi \leq -1.75 \\ -0.3 - 1.4(\phi + 1.75) & \text{if } -1.75 < \phi < -1.25 \\ -1.0 & \text{if } \phi \geq -1.25 \ , \end{cases} \tag{2.6}$$

and for the standard deviation of χ:

$$\sigma_\chi(\phi) = \begin{cases} 0.2 & \text{if } \phi \leq -3.5 \\ 0.2 + 0.1333(\phi + 3.5) & \text{if } \phi > -3.5 \ . \end{cases} \tag{2.7}$$

2.2.2 Large Fragment Distribution

The large fragment distribution implements a bimodal log-normal sampling model [18];

$$\xi_{\text{large}}(\phi) = \begin{cases} 10^{\mathcal{N}_{\chi_1}} & \text{with probability } w_l(\phi) \\ 10^{\mathcal{N}_{\chi_2}} & \text{with probability } 1 - w_l(\phi) \,, \end{cases} \tag{2.8}$$

where the weighting parameter, w_l, determines how the bimodal distribution splits fragments between two different normal distributions. The term *bimodal* captures the fact that larger fragments tend to come from two different populations with distinct physical properties: Fragments with higher and lower A/M and values. Particles with higher area-to-mass ratios—represented by $10^{\mathcal{N}_{\chi_1}}$—are typically less dense and from planar material components while those with lower ratios—$10^{\mathcal{N}_{\chi_2}}$—usually more dense and from compact material. This approach is consistent with the physical reality that parent bodies are composed of materials with varying densities and structural properties.

All parameters are piecewise functions of ϕ. For upper stage fragments, the weight parameter:

$$w_l(\phi) = \begin{cases} 1.0 & \text{if } \phi \leq -1.4 \\ 1.0 - 0.3571(\phi + 1.4) & \text{if } -1.4 < \phi < 0 \\ 0.5 & \text{if } \phi \geq 0 \,, \end{cases} \tag{2.9}$$

the first distribution mean:

$$\overline{\chi}_1(\phi) = \begin{cases} -0.45 & \text{if } \phi \leq -0.5 \\ -0.45 - 0.9 \cdot (\phi + 0.5) & \text{if } -0.5 < \phi < 0 \\ -0.9 & \text{if } \phi \geq 0 \,, \end{cases} \tag{2.10}$$

the first distribution standard deviation:

$$\sigma_{\chi_1}(\phi) = 0.55 \,, \tag{2.11}$$

the second distribution mean:

$$\overline{\chi}_2(\phi) = -0.9 \,, \tag{2.12}$$

and the second distribution standard deviation:

$$\sigma_{\chi_2}(\phi) = \begin{cases} 0.28 & \text{if } \phi \leq -1.0 \\ 0.28 - 0.1636 \cdot (\phi + 1) & \text{if } -1.0 < \phi < 0.1 \\ 0.1 & \text{if } \phi \geq 0.1 \,. \end{cases} \tag{2.13}$$

3 Sampling in 3D Space with Spherically Symmetric Dispersion

Given the *spherically symmetric* volumetric mass density ($\tilde{\rho}$) of a debris cloud, fragment positions can be generated that must follow that distribution.[2] The statistical sampling here requires consideration of the spherical geometry to accurately represent the spatial distribution of fragments.

3.1 Spherical Volume Element in Probability Sampling

The spherical volume element is:

$$d\mathcal{V} = r^2 \sin\theta \, dr d\theta d\varphi \,. \tag{3.1}$$

The normalized probability density function (PDF)—i.e., probability per unit volume—is:

$$\mathcal{P}(r) = \frac{\tilde{\rho}(r)}{M_{\text{total}}} \,, \tag{3.2}$$

where M_{total} is the total mass of all the fragments. The probability of finding a fragment in a specific volume element is then proportional to $\tilde{\rho} d\mathcal{V}$;[3]

$$\mathbb{P}_{r,\theta,\varphi} \propto \tilde{\rho}(r) \cdot r^2 \sin\theta \,. \tag{3.3}$$

The statistical approach presented here reflects techniques developed in the NASA EVOLVE 4.0 framework for modeling fragmentation debris clouds in orbital environments [18].

3.2 Marginal and Conditional PDFs

To sample a three-dimensional point in spherical coordinates, the marginal PDFs for the radial and angular components.[4]

3.2.1 Radial PDF

Following (3.3), normalizing the radial probability—i.e., the radial component of $\mathbb{P}_{r,\theta,\varphi}$—requires dividing it by the radial segment of the total mass [19];

$$\int_0^\infty \mathbb{P}_r(r) \, dr = \int_0^\infty \frac{\tilde{\rho}(r) \cdot r^2}{\int_0^\infty \tilde{\rho}(r') \cdot r'^2 \, dr'} \, dr = 1 \,. \tag{3.4}$$

The marginal PDF for r is obtained by integrating $\mathbb{P}$ over θ and φ:

$$\begin{aligned}
\mathcal{P}_r(r) &= \int_0^{2\pi} \int_0^{\pi} \mathbb{P}_{r,\theta,\varphi} \, d\theta \, d\varphi & (3.5)\\
&= \int_0^{2\pi} \int_0^{\pi} \frac{\tilde{\rho}(r) \cdot r^2 \sin\theta}{4\pi \int_0^\infty \tilde{\rho}(r') \cdot r'^2 \, dr'} \, d\theta \, d\varphi & (3.6)\\
&= \frac{\tilde{\rho}(r) \cdot r^2}{\int_0^\infty \tilde{\rho}(r') \cdot r'^2 \, dr'} \,. & (3.7)
\end{aligned}$$

The *radial PDF*, which shows that the probability density for the radial distance is proportional to $\tilde{\rho} \times r^2$, reflecting the increasing volume of spherical shells at larger r.

[2] The tilde notation is used to explicitly specify spherical symmetry.
[3] See Appendix B for more details.
[4] A *marginal PDF* is the probability density function of one or more variables from a joint distribution, obtained by averaging out (integrating) the other variables. In the context of sampling fragment positions in a 3D debris cloud, where positions are defined by spherical coordinates (r, θ, φ), the marginal PDF for the radial distance r describes the likelihood of a fragment being at a certain distance from the origin, ignoring the angular coordinates θ and φ. Its derived by summing up the probabilities over all possible angles, focusing only on the radial part of the distribution.

3.2.2 Angular PDFs

The joint angular PDF, conditioned on r, is:

$$\mathcal{P}_{\theta,\varphi}(r) = \frac{\mathbb{P}_{r,\theta,\varphi}}{\mathbb{P}_r(r)} \tag{3.8}$$

$$= \left(\frac{\tilde{\rho}(r) \cdot r^2 \sin\theta}{4\pi \int_0^\infty \tilde{\rho}(r') \cdot r'^2 \, dr'} \right) \Big/ \left(\frac{\tilde{\rho}(r) \cdot r^2}{\int_0^\infty \tilde{\rho}(r') \cdot r'^2 \, dr'} \right) \tag{3.9}$$

$$= \frac{\sin\theta}{4\pi} \,. \tag{3.10}$$

Line (3.9) can be factored using separation of variables;

$$\mathcal{P}_{\theta,\varphi} = \mathcal{P}_\theta(\theta) \cdot \mathcal{P}_\varphi(\varphi) \,, \tag{3.11}$$

where $\mathcal{P}_\theta$ and $\mathcal{P}_\varphi$ are given by,

$$\mathcal{P}_\theta(\theta) = \frac{\sin\theta}{2} \qquad \text{for } \theta \in [0,\, \pi] \tag{3.12}$$

$$\mathcal{P}_\varphi(\varphi) = \frac{1}{2\pi} \qquad \text{for } \varphi \in [0,\, 2\pi) \,. \tag{3.13}$$

The sine term in (3.12) indicates that θ is not uniformly distributed; instead, the polar angle distribution is weighted by the solid angle element.

3.3 Sampling Procedure

To sample a position in spherical coordinates, each coordinate must be generated according to its respective PDF.

3.3.1 Azimuthal Angle

The azimuthal angle φ follows a uniform distribution from (3.13);

$$\varphi \sim \text{Uniform}[0,\, 2\pi) \tag{3.14}$$

$$\mathcal{U}_1 \sim \text{Uniform}[0,\, 1] \,, \tag{3.15}$$

where Uniform refers to the uniform probability distribution.[5] φ can then be written in terms of $\mathcal{U}_1$:

$$\varphi = 2\pi\mathcal{U}_1 \,. \tag{3.16}$$

3.3.2 Polar Angle

The polar angle's PDF is given by (3.12). To sample θ, use the cumulative distribution function (CDF):

$$F_\theta(\theta) = \int_0^\theta \mathcal{P}_\theta(\theta') \, d\theta' = \frac{1-\cos\theta}{2} \,, \tag{3.17}$$

then set $F_\theta(\theta) = \mathcal{U}_2$ where $\mathcal{U}_2 \sim \text{Uniform}[0,1]$,

$$\theta = \arccos(1 - 2\mathcal{U}_2) \,. \tag{3.18}$$

Alternatively, since $\cos\theta \in [-1,\, 1]$ and $\mathcal{U}_2$ follow a uniform distribution on $[0,\, 1]$, a standard linear transformation for converting a uniform random variable from one range to another can be used. The formula to be used is:

$$\mathcal{X} = a + (b-a)\mathcal{U} \,, \tag{3.19}$$

where $\mathcal{X}$ is uniformly distributed on $[a,\, b]$ [20,21].[6] Since a and b are respectively equal to -1 and $+1$, $\mathcal{X} \to \cos\theta = 2\mathcal{U}_2 - 1$; and so,

$$\theta = \arccos(2\mathcal{U}_2 - 1) \,. \tag{3.20}$$

This ensures that the angular distribution is isotropic over the sphere.

[5] $\mathcal{U} \sim \text{Uniform}[0,1]$ means that $\mathcal{U}$ has an equal probability of taking any value between 0 and 1.

[6] In general, a uniform random variable $\mathcal{U}$ that is distributed on any interval $[c,\, d]$ can be transformed into a uniform distribution on $[a,\, b]$ using a modified formula:

$$\mathcal{X} = a + (b-a)\frac{\mathcal{U} - c}{d - c} \,.$$

This formula first normalizes $\mathcal{U}$ to $[0,\, 1]$ and then scales it to $[a,\, b]$ [22,23].

3.3.3 Radial Distance

The PDF for r is:

$$\mathcal{P}_r(r) = \frac{\tilde{\rho}(r) \cdot r^2}{\int_0^\infty \tilde{\rho}(r') \cdot r'^2 \, dr'} \qquad \text{for } r \in [0, \infty), \tag{3.21}$$

sampling from this PDF depends on the form of $\tilde{\rho}(r)$. Sampling from this PDF depends on the form of $\tilde{\rho}(r)$. While several specialized methods exist, two fundamental approaches are particularly relevant: the inverse CDF method and rejection sampling.

Inverse CDF Method First, compute the CDF:

$$F_r(r) = \int_0^r \mathcal{P}_r(r') \, dr' = \frac{\int_0^r \tilde{\rho}(r') \cdot r'^2 \, dr'}{\int_0^\infty \tilde{\rho}(r'') \cdot r''^2 \, dr''}, \tag{3.22}$$

then, solve $F_r(r) = U_3$ for r. This requires an analytical or numerical inverse, which may be complex for arbitrary $\tilde{\rho}(r)$ [22, 23].

Rejection Sampling If the inverse CDF is intractable, use rejection sampling. First, choose a proposal distribution $q(r)$; e.g., a Gaussian or exponential, that is easy to sample from and bounds the target PDF. Next, sample $r \sim q(r)$ and $\mathcal{U} \sim \text{Uniform}[0, 1]$. Then, accept r if,

$$\mathcal{U} \leq \frac{\mathcal{P}_r(r)}{\beta \cdot q(r)}, \tag{3.23}$$

where β is a constant such that $\mathcal{P}_r(r) \leq \beta \cdot q(r)$ for all r [21, 23].

Example for a Gaussian Dispersion For a hypothetical Gaussian volumetric mass density,

$$\tilde{\rho}(r) = \rho_0 \exp\left[-0.5\left(\frac{r - \mu R}{\sigma R}\right)^2\right], \tag{3.24}$$

rejection sampling is often used. Rejection sampling is optimal for this radial PDF due to the mathematical intractability of its CDF inverse [22]. The product structure of the geometric factor (r^2) and Gaussian term creates a distribution ideally suited for rejection techniques [23], avoiding computationally expensive numerical integration while maintaining accuracy [21]. This approach is particularly efficient for three-dimensional physical simulations [20], and the bounded nature of the cloud's radius facilitates the construction of appropriate envelope functions [23], making rejection sampling both mathematically sound and computationally practical for debris cloud modeling. The radial PDF subsequently becomes:

$$\mathcal{P}_r(r) \propto r^2 \exp\left[-0.5\left(\frac{r - \mu R}{\sigma R}\right)^2\right]. \tag{3.25}$$

In computational implementation, two approaches can be used based on required accuracy and performance constraints. A pure rejection sampling implementation requires defining an envelope distribution $q(r)$ from which samples are easier to draw, typically a Gaussian centered at μR with a standard deviation σR. The algorithm then iteratively samples $r_{\text{proposed}} \sim q(r)$ and accepts this sample with *acceptance probability*:

$$\frac{r_{\text{proposed}}^2 \exp\left[-0.5\left(\frac{r_{\text{proposed}} - \mu R}{\sigma R}\right)^2\right]}{\beta \cdot q(r_{\text{proposed}})},$$

where β is chosen such that this ratio is always ≤ 1. When an independently generated quantity $\mathcal{U}$ is less than this ratio, accept the sample; otherwise, try again. This ensures that the accepted samples follow the desired target distribution.

For applications where computational efficiency is prioritized, *rectification* can be employed by directly sampling from a normal distribution with mean μR and standard deviation σR, then adjusting for non-negativity constraints by truncating negative values;

$$\mathcal{N}_r \sim \text{Normal}(\mu R, \sigma R) \tag{3.26}$$

$$r = \max(0, \mathcal{N}_r). \tag{3.27}$$

Rectification is a necessary physical correction as it will set $r \to 0$ if r is computed to be negative. From a mathematical perspective, this creates a modified distribution that has the same probability density as the normal distribution for all positive values and has a point mass (spike) at exactly $r = 0$ that accumulates all the probability that would have been spread across negative values. Since the normal distribution has support on $(-\infty, +\infty)$, directly sampling $\mathcal{N}_r$ produces a non-zero probability of generating negative values, which would be physically meaningless for radial distances. The likelihood of negative samples depends on the ratio $\mu R/\sigma R$. While a proper truncated normal distribution would be mathematically more precise [23], the rectified Gaussian approximation represents a computationally efficient approximation that preserves the essential statistical properties of the distribution while ensuring physical validity [24].

Anyhow, this approximation preserves the Gaussian profile while ensuring physically meaningful, non-negative radial distances. The azimuthal angle is sampled uniformly, just as in (3.14), and the polar angle is sampled with proper weighting by the solid angle element:

$$\cos\theta \sim \text{Uniform}[-1, 1] \tag{3.28}$$

$$\theta = \arccos(\cos\theta), \tag{3.29}$$

the complete position vector in Cartesian coordinates is then given by the standard coordinate transformation;

$$\vec{r} = (r\sin\theta\cos\varphi,\, r\sin\theta\sin\varphi,\, r\cos\theta). \tag{3.30}$$

This rectified Gaussian approximation method produces statistically accurate fragment distributions while maintaining reasonable simulation performance.

4 Numerical Size Distribution

4.1 Counts by Size Category

To determine the number of fragments within specific size ranges, use the cumulative distribution function (N). The number of fragments between characteristic lengths L_1 and L_2, where $L_1 < L_2$, is $N(L_1) - N(L_2)$.

The practical bounds for all size categories are set to $L_{\min} = 0.001\,\text{m}$ and $L_{\max} = 1.0\,\text{m}$. Using the cumulative distribution function for *collisions*,

$$\Upsilon_{\text{small}} = 13482.1 M_{\text{parent}}^{0.75} \tag{4.1}$$

$$\Upsilon_{\text{medium}} = 3.15 M_{\text{parent}}^{0.75} \tag{4.2}$$

$$\Upsilon_{\text{large}} = 4.26 M_{\text{parent}}^{0.75}, \tag{4.3}$$

where Υ is the symbol for *number of*.

4.1.1 Percentage Per Category (Collision)

The total number of fragments is:

$$\Upsilon_{\text{total}} = \Upsilon_{\text{small}} + \Upsilon_{\text{medium}} + \Upsilon_{\text{large}} = 13489.5 M_{\text{parent}}^{0.75}. \tag{4.4}$$

The percentage distributions are:

- Small fragments: 99.95%
- Medium fragments: 0.023%
- Large fragments: 0.032%

4.1.2 Intra-Category Distribution

It has been established how the fragments are distributed based on their classification; however, the three size categories also have internal distributions. The small, medium, and large fragments themselves follow the same respective power law distribution pattern. Within any subrange size category, the fragments are *not* uniformly distributed as smaller fragments within any classification are more numerous than larger ones. For any subrange within any category, from L_a to L_b, where $0.001\,\text{m} \le L_a < L_b \le 1.0\,\text{m}$, the number of fragments is:

$$\Upsilon_{\text{subrange}} = N(L_a) - N(L_b) = 0.1 M_{\text{parent}}^{0.75}\left(L_a^{-1.71} - L_b^{-1.71}\right). \tag{4.5}$$

4.2 Discrete Fragment Count

4.2.1 Size Step Configuration

The power law exponent (-1.71) means fragment counts change *much more rapidly* at smaller sizes. Small fragments need fine resolution to capture this steep variation, while longer fragments change more gradually. Allocate computational resources where matter most; high resolution in where counts change rapidly (small), moderate resolution in the transition zone (medium), and standard resolution where changes are gradual (large).

4.2.2 Quantized Piece Count Estimation

To estimate the number of fragments for any particular characteristic length, the following formula is used:

$$\Upsilon(L_c) = N(L_c) - N(L_c + \varepsilon), \tag{4.6}$$

where ε is an infinitesimal increment. With this formulation, $\Upsilon(L_c) \approx \varepsilon \cdot n(L_c)$, where $n = -dN/dL_c$ is the differential size distribution function. Loop through discrete L_c values with assigned ΔL_c size steps and calculate the number of pieces for each characteristic length with the above formula.

5 Impact Probability at Event Zero

Given the parent mass, initial cloud radius, cumulative distribution function, $\tilde{\rho}(r, L_c)$, and hit distance threshold, the impact probability will be calculated such that a trajectory passing through random entry and exit points on the cloud sphere at time $t = 0$ will encounter at least one fragment within distance ℓ. A fundamental result in stochastic processes from *Poisson process theory* is the application of a *non-homogeneous Poisson process* [25]; since encounters occur randomly at rate (per unit length) $\Lambda(\mathcal{L})$ along path $\mathcal{L}$, then the number of encounters follows a Poisson distribution with parameter $\int \Lambda \, d\mathcal{L}'$. The probability of at least one encounter is $1 - \mathbb{P}_{\text{avoid}} = 1 - \exp[-\int \Lambda \, d\mathcal{L}']$ [25, 26]. The total impact probability is computed as the weighted sum over all fragment size categories:

$$\mathbb{P}_{\text{impact}} = \sum_{L_c = L_{\min}}^{L_{\max}} f \cdot \mathbb{P}_{\text{impact}}(L_c), \tag{5.1}$$

where f represents the fraction of fragments given from percentage distributions per size category, as in §4.1.1. For a specific characteristic length, the probability of impact along a trajectory is [3]:

$$\mathbb{P}_{\text{impact}}(L_c, \mathcal{L}) = 1 - \exp\left[-\int_{\mathcal{L}} \Lambda(L_c, \mathcal{L}') \, d\mathcal{L}' \right]. \tag{5.2}$$

The collision rate at position r along trajectory $\mathcal{L}$ is [27–31]:

$$\Lambda(r, \mathcal{L}) = \rho_N(r, \mathcal{L}) \cdot \pi \ell^2, \tag{5.3}$$

$\rho_N(r)$ is the number density of fragments at radial distance r and $\pi \ell^2$ represents the effective impact cross-sectional area. The number density is related to the mass density through,

$$\rho_N(r, L_c) = \frac{\tilde{\rho}(r, L_c)}{\overline{M}(L_c)} \frac{dN}{dL_c}, \tag{5.4}$$

where $\overline{M}(L_c)$ is the mean mass of all fragments with a characteristic length L_c.

5.1 Monte Carlo Implementation

5.1.1 Estimator Design and Variance Analysis

The impact probability is estimated through repeated sampling [20–22]:

$$\hat{\mathbb{P}}_{\text{impact}} = \frac{1}{\Upsilon_{\text{trials}}} \sum_{j=1}^{\Upsilon_{\text{trials}}} I_j, \tag{5.5}$$

where I_j is an indicator function:

$$I_j = \begin{cases} 1 & \text{if trajectory } j \text{ encounters any fragment} \\ 0 & \text{otherwise.} \end{cases} \tag{5.6}$$

This indicator function captures whether a given trajectory j results in a collision, effectively turning each trial into a binary outcome. The reliability of this approximation depends on the number of trials and the inherent variability of the sampling process, which is quantified through the variance of the estimator.

The variance of the Monte Carlo estimator is [21, 22]:

$$\mathrm{Var}(\mathbb{P}) = \frac{\mathbb{P}(1-\mathbb{P})}{\Upsilon_{\text{trials}}} . \tag{5.7}$$

The confidence interval using the Wilson score method is:

$$\mathbb{P} \in \left[\frac{\hat{\mathbb{P}} + \frac{\zeta^2}{2\Upsilon} - \zeta\sqrt{\frac{\hat{\mathbb{P}}(1-\hat{\mathbb{P}})}{\Upsilon} + \frac{\zeta^2}{4\Upsilon^2}}}{1+\frac{\zeta^2}{\Upsilon}}, \frac{\hat{\mathbb{P}} + \frac{\zeta^2}{2\Upsilon} + \zeta\sqrt{\frac{\hat{\mathbb{P}}(1-\hat{\mathbb{P}})}{\Upsilon} + \frac{\zeta^2}{4\Upsilon^2}}}{1+\frac{\zeta^2}{\Upsilon}} \right] , \tag{5.8}$$

where ζ is the standard normal quantile (z-score) [20–22, 32].

5.1.2 Adaptive Sample Size with Sequential Refinement

Since $\mathbb{P}$ is unknown a priori, an adaptive sampling approach is employed [21, 22]. Starting with an initial batch of $\Upsilon_0 = 10^4$ trials, the required sample size is updated after each batch;

$$\Upsilon_{\text{next}} = \frac{\zeta^2\left(1-\hat{\mathbb{P}}_{\text{current}}\right)}{\hat{\mathbb{P}}_{\text{current}}\,\epsilon^2} . \tag{5.9}$$

where ϵ is the tolerance. The sampling continues until convergence is achieved :

$$\frac{\mathbb{P}_{\text{upper}} - \mathbb{P}_{\text{lower}}}{\hat{\mathbb{P}}_{\text{current}}} < \epsilon , \tag{5.10}$$

where $\mathbb{P}_{\text{lower}}$ and $\mathbb{P}_{\text{upper}}$ come from the confidence interval formula in (5.8).[7] As more trials are added, the $\sqrt{\Upsilon}$ in the Wilson formula makes the interval tighter, reducing the relative width until it is less than the tolerance [20–22].

5.2 Importance Sampling

Instead of uniform sampling on the sphere, use importance sampling where samples are drawn from a biased distribution (called the importance distribution) that prioritizes "important" regions of the sample spacethose more likely to contribute to the quantity being estimated, and then adjust for this bias by weighting the samples [21, 22, 31]. To sample entry and exit points this way, use the following non-normalized PDF:

$$\mathcal{P}(\mathbf{p}_{\text{entry}}, \mathbf{p}_{\text{exit}}) \propto \exp\left[-\frac{\ell_{\min}^2(\mathbf{p}_{\text{entry}}, \mathbf{p}_{\text{exit}})}{2\sigma_{\text{IS}}^2} \right] , \tag{5.11}$$

where $\ell_{\min}$ is a function that calculates the minimum distance between the trajectory defined by the entry and exit points and the peak density sphere at radius μR_{c} and σ_{IS} is the importance sampling standard deviation, a tuning parameter that controls how strongly the sampling is biased toward trajectories passing near the peak density radius [21, 22].

Furthermore, the importance-sampled estimator becomes,

$$\hat{\mathbb{P}}_{\text{IS}} = \frac{1}{\Upsilon_{\text{trials}}} \sum_{j=1}^{\Upsilon_{\text{trials}}} I_j w_j , \tag{5.12}$$

where the weight is [21, 22]:

$$w_j = \frac{h_j(\mathbf{p}_{\text{entry}}, \mathbf{p}_{\text{exit}})}{\mathcal{P}_j(\mathbf{p}_{\text{entry}}, \mathbf{p}_{\text{exit}})} , \tag{5.13}$$

with h representing the uniform distribution on the sphere. Without weights, estimates would be biased high because trajectories near the peak (where fragments are dense) are oversampled. The weights correct this by giving oversampled trajectories proportionally lower contributions [22, 33]. This approach works because it provides an unbiased estimate of the true probability, even though sampling is done non-uniformly [21, 22, 33].[8]

[7] The symbol for tolerance, ϵ, not to be confused with ε, which is used for an infinitesimal increment of characteristic length in Equation (4.6).

[8] The weights could also be normalized first such that the normalized weight is equal to $w_j/\sum_{i=1}^{\Upsilon} w_i$; however, the non-normalized weight is numerically more stable [21, 22].

6 Outlook

The computational implementation presented in this work addresses a documented gap in debris cloud collision probability assessment during the initial spherical phase. While the literature demonstrates mature capabilities for long-term environment evolution and cataloged debris conjunction assessment, the immediate post-fragmentation period of roughly 120 seconds has remained methodologically underserved. The fundamental challenge identified by Jenkin (1996) persists—fragment motion during this phase exhibits strong gravitational correlation that violates the independence assumptions underlying traditional Poisson flux models—yet computational alternatives have been limited by the prohibitive expense of propagating populations exceeding one hundred million fragments [4]. The Monte Carlo-based Poisson process model developed here resolves this tension through a synthesis that preserves statistical rigor while achieving computational tractability. By treating collision events as a Poisson process in time rather than space, the method maintains validity even under highly correlated fragment distributions. The critical distinction lies in recognizing that collision events along an interceptor trajectory occur independently in infinitesimal time intervals, regardless of spatial correlation within the debris cloud at any given instant. This framework permits the use of Monte Carlo sampling to evaluate non-uniform (and uniform) spatial densities at each trajectory point without requiring propagation of individual fragment orbits. The approach thus captures the physical accuracy of density-based methods while avoiding the computational burden that Li et al. (2022) identify as the primary limitation of pure Monte Carlo trajectory propagation [17].

The algorithmic structure presented in §5 enables direct operational implementation for interceptor mission planning. The method requires only debris cloud characteristics from the NASA Standard Breakup Model, established parameters for a spherically symmetric distribution of fragments, and the interceptor trajectory as inputs. Computational efficiency derives from focusing resources on density sampling rather than orbit propagation, making real-time assessment feasible within the operational constraints of multitarget engagement scenarios. This addresses the blackout period identified by Trombetta et al. (2025) when debris remains uncataloged and traditional conjunction assessment proves ineffective [14]. The methodology represents a significant advance over current practice in several respects. First, it provides a computationally tractable alternative to pure Monte Carlo propagation for populations where deterministic approaches become prohibitive. Second, it offers a physically justified architecture that avoids the inappropriate spatial independence assumptions underlying traditional Poisson flux models that have been criticized since Jenkin (1996)'s assessment but continue to be applied due to a lack of alternatives [4]. Third, the method targets the specific operational question relevant to interceptor missions—transit survival probability through expanding debris fields during multitarget engagements. The convergence properties demonstrated through the Wilson score interval construction in §5 establish statistical confidence bounds appropriate for operational decision-making. The quasi-static geometric assumption for the spherical phase ensures that density-based sampling remains physically accurate throughout the roughly 120 second assessment window by estimating the real portion of the trajectory, $\mathcal{L}$, to be linear. On that note, due to the quasi-static nature of the model, the impact probability at event zero is estimated to be the same as the impact probability across the duration of the spherical phase of the debris cloud.

The computational efficiency of the sampling-based approach makes it a viable foundation for operational collision avoidance systems addressing the immediate post-fragmentation period, which remains challenging for existing methodologies. Future development of this work should include an incorporation of size-dependent density stratification from the complete NASA EVOLVE 4.0 model, which would enable specific risk assessment based on fragment sizes. It should also explore the numerical algorithm for the parameters of explosion rather than collision, as explosion may provide more realistic spherically symmetric models for the breakup of astronomical objects. Furthermore, validation against hypervelocity impact test data as it becomes available would refine confidence in the underlying spherically symmetric density dispersion assumptions. The mathematical framework underlying this implementation—Poisson process theory applied to inhomogeneous spatial densities—demonstrates that operational collision probability tools need not sacrifice physical accuracy for computational tractability. The binomial approximation for rare collision events, coupled with Monte Carlo convergence properties, provides both the statistical rigor necessary for mission-critical decisions and the computational performance required for real-time assessment. By recognizing that the fundamental independence assumption applies to collision events in time rather than fragment positions in space, the methodology resolves the conceptual tension between correlated fragment dynamics and statistical collision modeling that has persisted since early debris cloud probability assessments. This statistical framework establishes a computationally feasible path forward for operational tools addressing the immediate post-fragmentation period where traditional methods prove inadequate.

Acknowledgments

The author thanks Robert A. Amborski of Boeing Intelligence & Analytics for the initial conversation that motivated this work. Robert identified the operational gap regarding collision probability assessment for missile systems transiting post-impact debris clouds during the so-called blackout period. Robert observed that fragments spread into a Gaussian-like distribution during the initial phase. This research was conducted independently by the author across affiliations with Boeing Intelligence & Analytics, Lockheed Martin, and Northrop Grumman. No specific funding supported this work despite the reliance on these organization's non-proprietary resources.

References

[1] Vallado, D. A. & Oltrogge, D. L. (2017). Three-Dimensional Volumetric Risk Assessment for Fragmentation Event Debris Fields. *Journal of Geophysical Research: Space Physics*, **122**(11), 11213–11228.

[2] Jehn, R. (1991). Dispersion of Debris Cloud from In-Orbit Fragmentation Events. *ESA Journal*, **15**, 63–77.

[3] Letizia, F., Colombo, C., & Lewis, H. G. (2016). Collision Probability Due to Space Debris Clouds Through a Continuum Approach. *Journal of Guidance, Control, and Dynamics*, **39**(10), 2240–2249.

[4] Jenkin, A. B. (1996). Probability of Collision During the Early Evolution of Debris Clouds. *Acta Astronautica*, **38**(4), 525–538.

[5] Liu, Y., Jiang, Y., & Li, H. (2022). Analytical Propagation of Space Debris Density for Collisions near Sun-Synchronous Orbits. *Space: Science & Technology*, **2022**, 9825763.

[6] Letizia, F., Colombo, C., & Lewis, H. G. (2015). Analytical Model for the Propagation of Small-Debris-Object Clouds After Fragmentations. *Journal of Guidance, Control, and Dynamics*, **38**(8), 1478–1491.

[7] Letizia, F. (2018). Extension of the Density Approach for Debris Cloud Propagation. *Journal of Guidance, Control, and Dynamics*, **41**(12), 2651–2657.

[8] Giudici, L., Trisolini, M., & Colombo, C. (2023). Probabilistic multi-dimensional debris cloud propagation subject to non-linear dynamics. *Advances in Space Research*, **71**(8), 3589-3608.

[9] Frey, S. & Colombo, C. (2021). Transformation of satellite breakup distribution for probabilistic orbital collision hazard analysis. *Journal of Guidance, Control, and Dynamics*, **44**(1), 88–105.

[10] Shu, P., Yang, Z., Luo, Y.-Z., & Sun, Z.-J. (2022). Collision Probability of Debris Clouds Based on Higher-Order Boundary Value Problems. *Journal of Guidance, Control, and Dynamics*, **45**(8), 1512–1522.

[11] Parigini, C., Algethamie, R., & Armellin, R. (2024). Short-Term Collision Probability Caused by Debris Cloud. *Journal of Guidance, Control, and Dynamics*, **47**(5), 874–886.

[12] Morselli, A., Armellin, R., Di Lizia, P., & Bernelli-Zazzera, F. (2015). A high order method for orbital conjunctions analysis: Monte Carlo collision probability computation. *Advances in Space Research*, **55**(1), 311–333.

[13] Shu, P., Yang, Z., Luo, Y.-Z., Zhang, J., & Li, S. (2025). Short-term evolution and risks of debris cloud stemming from collisions in geostationary orbit. *Acta Astronautica*, **228**, 486–493.

[14] Trombetta, A., Flohrer, T., Krag, H., & Schildknecht, T. (2025). Debris-Cloud Collision Risk Assessment with GSOC Collision Avoidance System. *9th European Conference on Space Debris*, ESA.

[15] Jang, D., Gusmini, D., Siew, P. M., D'Ambrosio, A., Servadio, S., Machuca, P., & Linares, R. (2024). New Monte Carlo Model for the Space Environment. *Journal of Spacecraft and Rockets*.

[16] Valencia, S., Sanchez-Perez, J. M., & Vasile, M. (2025). Monte Carlo Stochastic Simulation of Hypervelocity Impact-Induced Fragmentation in FACSAT-2 CubeSat Debris. *Journal of Spacecraft and Rockets*.

[17] Li, Z., Hu, R., Zhao, L., & Tang, G. (2022). A review of space-object collision probability computation methods. *Astrodynamics*, **6**, 95–120.

[18] Johnson, N. L., Krisko, P.H., Liou, J.-C., & Anz-Meador, P.D. (2001). NASA's New Breakup Model of EVOLVE 4.0. *Advances in Space Research*, **28**(9), 1377–1384.

[19] Ellgen, P. (2020). Probability Density Functions for Velocity Components in Spherical Coordinates. *Thermodynamics and Chemical Equilibrium*, LibreTexts Chemistry.

[20] Davis-Ross, K. & Chunn, J. (2021). An Introduction to Probability and Simulation. *Chapman and Hall/CRC*.

[21] Law, A. M. (2015). Simulation Modeling and Analysis (5th ed.). *McGraw-Hill*.

[22] Ross, S. M. (2019). Introduction to Probability Models (12th ed.). *Academic Press*.

[23] Devroye, L. (1986). Non-Uniform Random Variate Generation. *Springer-Verlag*.

[24] Gentle, J. E. (2003). Random Number Generation and Monte Carlo Methods (2nd ed.). *Springer*.

[25] Haight, F. A. (1967). Handbook of the Poisson Distribution. *John Wiley & Sons*.

[26] Duderstadt, J. J. & Hamilton, L. J. (1976). Nuclear Reactor Analysis. *John Wiley & Sons*.

[27] Chapman, S. & Cowling, T. G. (1970). The Mathematical Theory of Non-uniform Gases (3rd ed.). *Cambridge University Press*.

[28] McQuarrie, D. A. (2000). Statistical Mechanics. *University Science Books*.

[29] Perkins, D. H. (2000). Introduction to High Energy Physics (4th ed.). *Cambridge University Press*.

[30] Grün, E., Zook, H. A., Fechtig, H., & Giese, R. H. (1985). Collisional Balance of the Meteoritic Complex. *Icarus*, **62**(2), 244–272.

[31] Bird, G. A. (1994). Molecular Gas Dynamics and the Direct Simulation of Gas Flows. *Oxford University Press*.

[32] Wilson, E. B. (1927). Probable Inference, the Law of Succession, and Statistical Inference. *Journal of the American Statistical Association*, **22**(158), 209–212.

[33] Kloek, T. & van Dijk, H. K. (1978). Efficient Estimation of Income Distribution Parameters. *Journal of Econometrics*, **8**(1), 61–74.

Appendix A: Area-to-Mass Ratio from Characteristic Length

The relationship between L_c and A/M in the NASA Breakup Model is not a simple direct formula, but rather a complex set of statistical distributions. First, L_c is related to cross-sectional area by:

$$A = \begin{cases} 0.540424 L_c^2 & \text{if } L_c < 0.00167\,\text{m} \\ 0.556945 L_c^{2.0047077} & \text{otherwise.} \end{cases} \tag{A.1}$$

Then, A/M follows statistical distributions based on L_c [18].

Appendix B: PDF Proportionality to Spherically Symmetric Dispersion

Individual fragments follow deterministic orbital paths. The transition from the statistical cloud model to individual fragment trajectories requires mapping the density distribution to specific initial state vectors. The probability is directly related to the mass density, $\mathcal{P}(\vec{r}, L_c, t) \propto \tilde{\rho}(|\vec{r}|, L_c, t)$. For a fragment with characteristic length L_c at time t after impact, its position vector $\vec{r}$ corresponds to the probability density function:

$$\mathcal{P}(\vec{r}, L_c, t) = \frac{\tilde{\rho}(|\vec{r}|, L_c, t)}{4\pi \int_0^{R_c(t)} \tilde{\rho}(r, L_c, t) r^2\, dr} . \tag{B.1}$$

where $R_c(t)$ is the radius of the cloud at time t. This normalized probability density function relates the spherically symmetric mass density dispersion to the spatial distribution of fragments across three-dimensional space. In this formulation, the radial distribution follows the spherically symmetric model while the angular components of $\vec{r}$ may vary according to impact dynamics.

Equation (B.1) represents the relationship between $\tilde{\rho}(r, L_c, t)$ and a probability density function for fragment positions in three-dimensional space. The volumetric mass density function describes how fragment mass is distributed radially within the cloud; to create a probability density function in space, ensure it integrates to one over all space, $\iiint_{\mathcal{V}} \mathcal{P} d\mathcal{V} = 1$. In spherical coordinates this becomes:

$$4\pi \int_0^{R_c(t)} \mathcal{P}(r, L_c, t)\, r^2\, dr = 1 . \tag{B.2}$$

For the probability density function to be proportional to the mass density, dividing $\tilde{\rho}$ by the total mass normalizes this distribution:

$$\mathcal{P}(\vec{r}, L_c, t) = \frac{\tilde{\rho}(|\vec{r}|, L_c, t)}{M_{\text{total}}(t)} , \tag{B.3}$$

where $M_{\text{total}}(t)$ is a constant function;

$$M_{\text{total}}(t) = 4\pi \int_0^{R_c(t)} \tilde{\rho}(r, L_c, t) r^2\, dr . \tag{B.4}$$

This normalized probability distribution provides the mathematical foundation for accurately positioning fragments according to their physical distribution while maintaining statistical validity through proper conversion of mass density into sampling weights. A sampling weight is a numerical value assigned to each potential position in space that determines its probability of being selected during random sampling. In this context, positions with higher mass density receive higher weights, making them more likely to be chosen when generating fragment positions.