Exercise class: Bayesian networks (solutions)

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Week 9

1. (a) The edge from A to B can be removed, because:

$$P(b|a) = \frac{P(a,b)}{P(a)} = \frac{45}{50} = 0.9$$
$$P(b|\neg a) = \frac{P(\neg a,b)}{P(\neg a)} = 0.9$$
$$P(b) = 0.9$$

Hence we have $P(b|a) = P(b|\neg a) = P(b)$ and thus also $P(\neg b|a) = P(\neg b|\neg a) = P(\neg b)$ which means that B is independent of A.

(b) The conditional probability table corresponding with A is defined by:

$$P(a) = 0.5$$

The conditional probability table corresponding with B is defined by (taking into account that we can remove the edge from A to B):

$$P(b) = 0.9$$

The conditional probability table corresponding with C is defined by:

$$P(c|a) = \frac{P(a,c)}{P(a)} = \frac{35}{50} = 0.7$$
$$P(c|\neg a) = \frac{P(\neg a,c)}{P(\neg a)} = \frac{15}{50} = 0.3$$

The conditional probability table corresponding with D is defined by:

$$P(d|b,c) = \frac{PL(b,c,d)}{P(b,c)} = \frac{27}{45} = \frac{3}{5} = 0.6$$
$$P(d|b,\neg c) = \frac{PL(b,\neg c,d)}{P(b,\neg c)} = \frac{9}{45} = \frac{1}{5} = 0.2$$
$$P(d|\neg b,c) = \frac{PL(\neg b,c,d)}{P(\neg b,c)} = \frac{15}{50} = \frac{3}{10} = 0.3$$
$$P(d|\neg b,\neg c) = \frac{PL(\neg b,\neg c,d)}{P(\neg b,\neg c)} = \frac{25}{50} = \frac{1}{2} = 0.5$$

2. (a) We find that:

$$P(a) = 0.012 + 0.028 + 0.288 + 0.072 = 0.4$$
$$P(a|b) = \frac{P(a,b)}{P(b)} = \frac{0.012 + 0.028}{0.012 + 0.028 + 0.108 + 0.252} = \frac{0.04}{0.4} = 0.1$$

and hence B is not independent of A, which means that network structure (i) cannot be used.

(b) Can be used as a complete network can model any probability distribution.

(c) We find that:

$$\begin{aligned}
P(c|b) &= \frac{P(b,c)}{P(b)} = \frac{0.012 + 0.108}{0.4} = 0.3 \\
P(c|a,b) &= \frac{P(a,b,c)}{P(a,b)} = \frac{0.012}{0.04} = 0.3 \\
P(c|\neg a,b) &= \frac{P(\neg a,b,c)}{P(\neg a,b)} = \frac{0.108}{0.108 + 0.252} = 0.3 \\
P(c|\neg b) &= \frac{P(\neg b,c)}{P(\neg b)} = \frac{0.288 + 0.192}{0.6} = 0.8 \\
P(c|a,\neg b) &= \frac{P(a,\neg b,c)}{P(a,\neg b)} = \frac{0.288}{0.288 + 0.072} = 0.8 \\
P(c|\neg a,\neg b) &= \frac{P(\neg a,\neg b,c)}{P(\neg a,\neg b)} = \frac{0.192}{0.192 + 0.048} = 0.8
\end{aligned}$$

In other words, C is conditionally independent of A given B, which means that the network structure (iii) can be used.

(d) We find:

$$\begin{aligned}
P(b|a) &= \frac{P(a,b)}{P(a)} = \frac{0.04}{0.4} = 0.1 \\
P(b|a,c) &= \frac{P(a,b,c)}{P(a,c)} = \frac{0.012}{0.012 + 0.288} = 0.04
\end{aligned}$$

In other words, B is not conditionally independent of C given A, which means that the network structure (iv) cannot be used.