Exercise class: Bayesian networks (solutions)

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Week 8

1. **Candy flavours**

   (a) The networks (ii) and (iii) can both correctly model the problem.

   (b) The network (iii) is the best representation, as it is minimal (fewer edges means that more independence assumptions have been made explicit).

   (c) We find

$$\begin{aligned}
&P(\textit{Wrapper} = \textit{red}) \\
&\quad = P(\textit{Wrapper} = \textit{red} | \textit{Flavour} = \textit{strawberry}) \cdot P(\textit{Flavour} = \textit{strawberry}) \\
&\qquad + P(\textit{Wrapper} = \textit{red} | \textit{Flavour} = \textit{lime}) \cdot P(\textit{Flavour} = \textit{lime}) \\
&\quad = 0.8 \cdot 0.7 + 0.1 \cdot 0.3 = 0.59
\end{aligned}$$

   (d) We find

$$\begin{aligned}
&P(\textit{Flavour} = \textit{strawberry} | \textit{Shape} = \textit{round}, \textit{Wrapper} = \textit{red}) \\
&\quad = \alpha \cdot P(\textit{Flavour} = \textit{strawberry}, \textit{Shape} = \textit{round}, \textit{Wrapper} = \textit{red}) \\
&\quad = \alpha \cdot P(F = s) \cdot P(S = r | F = s) \cdot P(W = r | F = s) \\
&\quad = \alpha \cdot 0.7 \cdot 0.8 \cdot 0.8 = \alpha \cdot 0.448 \\
&P(\textit{Flavour} = \textit{lime} | \textit{Shape} = \textit{round}, \textit{Wrapper} = \textit{red}) \\
&\quad = \alpha \cdot P(F = l) \cdot P(S = r | F = l) \cdot P(W = r | F = l) \\
&\quad = \alpha \cdot 0.3 \cdot 0.1 \cdot 0.1 = \alpha \cdot 0.003
\end{aligned}$$

   So $\alpha = \frac{1}{0.448+0.003} \approx 2.2172$ and

$$P(\textit{Flavour} = \textit{strawberry} | \textit{Shape} = \textit{round}, \textit{Wrapper} = \textit{red}) = \frac{0.448}{0.448 + 0.003} \approx 0.9933$$

2. **Politics**

   (a) (i) is not asserted because there is an edge from $\mathbf{H}$ to $\mathbf{S}$.
   (ii) is asserted since by definition of the Bayesian network, E is conditionally independent from H (which is a non-descendant) given the value of its only parent P.
   (iii) is not asserted: the network structure can only be used to derive information about conditional independence, not about the lack of conditional independence. To verify whether $P(E) \neq P(E|H)$, the corresponding probabilities would have to be evaluated using the conditional probability tables.

   (b) $P(h, s, \neg p, \neg e) = P(h)P(s|h)P(\neg p|h, s)P(\neg e|\neg p) = 0.1 \times 0.3 \times 0.1 \times 0.9 = 0.00027$

(c) We find

$$\begin{aligned}
P(e|h) = \alpha \cdot P(e,h) &= \alpha \cdot \sum_{s^* \in \{T,F\}} \sum_{p^* \in \{T,F\}} P(e,h,S=s^*,P=p^*) \\
&= \alpha \cdot P(h) \cdot \sum_{s^* \in \{T,F\}} \sum_{p^* \in \{T,F\}} P(S=s^*|h) \cdot P(P=p^*|h,S=s^*) \cdot P(e|P=p^*) \\
&= \alpha \cdot 0.1(0.3 \cdot (0.9 \cdot 0.6 + 0.1 \cdot 0.1) + 0.7 \cdot (0.5 \cdot 0.6 + 0.5 \cdot 0.1)) = \alpha \cdot 0.041
\end{aligned}$$

and

$$\begin{aligned}
P(\neg e|h) = \alpha \cdot P(\neg e,h) &= \alpha \cdot \sum_{s^* \in \{T,F\}} \sum_{p^* \in \{T,F\}} P(\neg e,h,S=s^*,P=p^*) \\
&= \alpha \cdot P(h) \cdot \sum_{s^* \in \{T,F\}} \sum_{p^* \in \{T,F\}} P(S=s^*|h) \cdot P(P=p^*|h,S=s^*) \cdot P(\neg e|P=p^*) \\
&= \alpha \cdot 0.1(0.3 \cdot (0.9 \cdot 0.4 + 0.1 \cdot 0.9) + 0.7 \cdot (0.5 \cdot 0.4 + 0.5 \cdot 0.9)) = \alpha \cdot 0.059
\end{aligned}$$

It follows that $\alpha = 10$ and $P(e|h) = 0.41$.

3. Only $P(b|a,c) = P(b|a)$ can be derived from the network.