Exercise class: Probabilistic inference and Bayesian networks (solutions)

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Week 7

1. **Inference by enumeration**

   (a) We find:

$$P(a|b,\neg c) = \frac{P(a,b,\neg c)}{P(b,\neg c)} = \frac{27+9}{27+9+18+6} = \frac{36}{60} = \frac{3}{5}$$

   (b) We find:

	a		$\neg a$	
	b	$\neg b$	b	$\neg b$
c	$\frac{3}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{4}{32}$
$\neg c$	$\frac{9}{32}$	$\frac{2}{32}$	$\frac{6}{32}$	$\frac{4}{32}$

2. **Naive Bayes**
   Using the Naive Bayes assumption, we find:

$$\begin{aligned} P(\textit{iOS}|\textit{white},\textit{british}) &= \alpha \cdot P(\textit{white}|\textit{iOS}) \cdot P(\textit{british}|\textit{iOS}) \cdot P(\textit{iOS}) \\ &= \alpha \cdot 0.5 \cdot 0.1 \cdot 0.2 \\ &= 0.01 \cdot \alpha \end{aligned}$$

$$\begin{aligned} P(\textit{android}|\textit{white},\textit{british}) &= \alpha \cdot P(\textit{white}|\textit{android}) \cdot P(\textit{british}|\textit{android}) \cdot P(\textit{android}) \\ &= \alpha \cdot 0.15 \cdot 0.15 \cdot 0.75 \\ &= 0.016875 \cdot \alpha \end{aligned}$$

$$\begin{aligned} P(\textit{windows}|\textit{white},\textit{british}) &= \alpha \cdot P(\textit{white}|\textit{windows}) \cdot P(\textit{british}|\textit{windows}) \cdot P(\textit{windows}) \\ &= \alpha \cdot 0.1 \cdot 0.2 \cdot 0.05 \\ &= 0.001 \cdot \alpha \end{aligned}$$

   and thus

$$\alpha = \frac{1}{0.01 + 0.016875 + 0.001} = \frac{1}{0.027875}$$

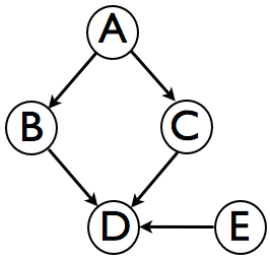
   which allows us to conclude

$$P(\textit{iOS}|\textit{white},\textit{british}) = \frac{0.01}{0.027875} = \frac{1}{2.7875} \approx 0.3587$$

3. **Markov blanket**

(a) The Markov blanket of F is given by $\{C, I, E, B, D, H\}$.

(b) A possible solution is given by:



(c) A possible solution is given by: