Weekly Summary

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Overview

PID regulator Review

Function of the parameters

PID gain	**P.O.**	**Settling time**	**Steady-state error**
Increasing K_p	Increases	-	Decreases
Increasing K_i	Increases	Increases	Zero steady-state error
Increasing K_d	Decreases	Decreases	-

Table: Table caption

PID regulator Review

Turning strategy

- Set K_I and K_D to 0
- Increasing K_P slowly until the output of the closed-loop system oscillates just on the edge of instability.
- Reduce K_p to achieve **quarter amplitude decay**.
- Increasing K_D to decrease setting time and overshoot.
- Increasing K_I to eliminate steady-state error.

SS and TF: Conversion

SS to TF

$$\begin{aligned} sX(s) &= AX(s) + BU(s) \\ Y(s) &= CX(s) + DU(s) \end{aligned} \tag{1}$$

$$H(s) = \frac{Y(s)}{U(s)} = C\Phi(s)B + D = C(sI - A)^{-1}B + D \tag{2}$$

SS and TF: Conversion

TF to SS

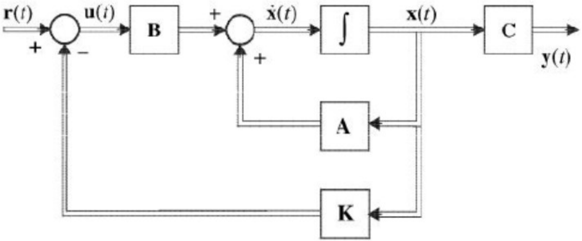
$$H(s) = \frac{Y(s)}{U(s)} = \frac{b_0}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n} \tag{3}$$

$$y^{(n)} + a_1 y^{(n-1)} + \cdots + a_{n-1}\dot{y} + a_n y = u$$

$$\begin{bmatrix} x_1 = y \\ x_2 = \dot{y} \\ \vdots \\ x_n = y^{(n-1)} \end{bmatrix} \rightarrow \begin{bmatrix} \dot{x}_1 = \dot{y} \\ \dot{x}_2 = \ddot{y} \\ \vdots \\ \dot{x}_n = y^{(n)} \end{bmatrix} \tag{4}$$

State Feedback

Goal: By design a proper feedback controller with gain K, we can place the eigenvalues in any positions we what.



$$\begin{aligned} \dot{x}(t) &= (\mathbf{A} - \mathbf{bk})x(t) + \mathbf{b}u(t) \\ y(t) &= \mathbf{C}x(t) \end{aligned} \tag{5}$$

State Feedback Controller Design

Method 1:Pole placement

Compute K according to the desired poles

Method 2:Solving Lyapunov equation

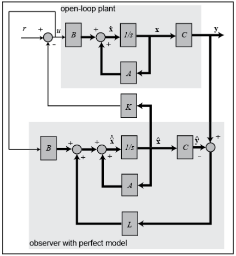
Compute K by solving a Lyapunov equation

Method 3:Linear quadratic method

Please refer to section 4

State Estimator

A state estimator or observer is **a device generates an estimate of the state**.



$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{b}u(t) + \mathbf{l}(y(t) - \mathbf{c}\hat{\mathbf{x}}(t)) \tag{6}$$

State Estimator Design

Method 1:Pole placement

Compute L according to the desired poles, normally we need to place the poles at least five times farther to the left than the dominant poles of the system to get a good observer dynamics.

Method 2:Solving Lyapunov equation

Compute L by solving a Lyapunov equation

Method 3:Linear quadratic estimator

Please refer to section 4

Riccati equation

Problem

Get the solution of

$$\frac{dy}{dx} = A(x)y^2 + B(x)y + C(x) \tag{7}$$

with a form of

$$y = y_1 + \frac{1}{v(x)} \tag{8}$$

Solution

- rewrite the quadratic first order problem to a linear first order problem
- Solve the rewritten problem

Linear Quadratic Optimization

In a control system, we want to balance the performance and actuator effort (energy) by setting up a cost function of the performance (x) and the effort (u):

$$J = \int_0^\infty \left(x^\top Q x + u^\top R u \right) dt \tag{9}$$

Solving the equation above, we can get a gain matrix K that produce the lowest cost given the dynamic system.

Linear Quadratic Regulator

Consider a continuous-time linear system, defined on $t \in [t_0, t_1]$, described by:

$$\dot{x} = Ax + Bu \tag{10}$$

with a quadratic cost function defined as:

$$J = x^T(t_1)F(t_1)x(t_1) + \int_{t_0}^{t_1} \left(x^T Q x + u^T R u + 2x^T N u \right) dt \tag{11}$$

the feedback control law that minimizes the value of the cost is:

$$u = -Kx \tag{12}$$

Linear Quadratic Regulator

where K is given by:

$$K = R^{-1}\left(B^T P(t) + N^T\right) \tag{13}$$

and P is found by solving the continuous time Riccati differential equation:

$$A^T P(t) + P(t)A - (P(t)B + N)R^{-1}\left(B^T P(t) + N^T\right) + Q = -\dot{P}(t) \tag{14}$$

Unfinished Work

- Specific procedures of solving the continuous Riccati differential equation in LQR
- How to design a linear quadratic estimator
- How to pick up proper noise covariance matrix in LQG

References

Chi-tsong Chen (2012)
Linear system theory and design
Oxford University Press

Dorf R C (2011)
Modern control systems
Pearson

The End