Short title

Shiqi Duar

Review the PID regulato

Review the state-space, state-feedback and estimator SS and TF: conversion

Quickly review the Riccati equation used for design the

LQG regulato

Problem Description Linear Quadratic Regulator

Weekly Summary

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November 16, 2019

Overview

Short title

Shiqi Duai

Review the PID regulator

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Quickly review the Riccati equation used for design the LQG regulator

Problem
Description
Linear Quadrati

1 Review the PID regulator

- 2 Review the state-space, state-feedback and estimator
 - SS and TF: conversion
 - State feedback and estimation
- 3 Quickly review the Riccati equation used for design the LQG regulator
- 4 LQG regulator
 - Problem Description
 - Linear Quadratic Regulator

PID regulator Review

Short title

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LQG regulato

Problem
Description
Linear Quadrati

Function of the parameters

PID gain	P.O.	Settling time	Steady-state error
Increasing K_p Increasing K_i	Increases Increases	- Increases	Decreases Zero steady- state error
Increasing K_d	Decreases	Decreases	_

Table: Table caption

PID regulator Review

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LQG regulato

Turning strategy

- Set K_I and K_D to 0
- Increasing K_P slowly until the output of the closed-loop system oscillates just on the edge of instability.
- Reduce K_p to achieve quarter amplitude decay.
- Increasing K_D to decrease setting time and overshoot.
- Increasing K_I to eliminate steady-state error.

SS and TF: Conversion

Short title

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SS and TF: conversion State feedbag

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LQG regulato

Problem Description Linear Quadrat Regulator SS to TF

$$sX(s) = AX(s) + BU(s)$$

$$Y(s) = CX(s) + DU(s)$$
(1)

$$H(s) = \frac{Y(s)}{U(s)} = C\Phi(s)B + D = C(sI - A)^{-1}B + D$$
 (2)

SS and TF: Conversion

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LQG regulator

Problem Description Linear Quadrat TF to SS

$$H(s) = \frac{Y(s)}{U(s)} = \frac{b_0}{s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n}$$

$$y^{(n)} + a_1 y^{(n-1)} + \dots + a_{n-1} \dot{y} + a_n y = u$$
(3)

$$\begin{bmatrix} x_1 = y \\ x_2 = \dot{y} \\ \vdots \\ x_n = y^{(n-1)} \end{bmatrix} \rightarrow \begin{bmatrix} \dot{x}_1 = \dot{y} \\ \dot{x}_2 = \ddot{y} \\ \vdots \\ \dot{x}_n = y^{(n)} \end{bmatrix}$$

$$(4)$$

State Feedback

Short title

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Review the PID regulator

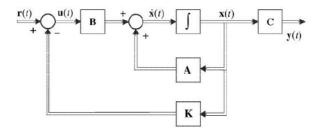
Review the state-space, state-feedback and estimator SS and TF:

State feedback

Quickly review the Riccati equation used for design the LQG regulato

LQG regulato

Problem Description Linear Quadrati Goal: By design a proper feedback controller with gain K, we can place the eigenvalues in any positions we what.



$$\dot{x}(t) = (\mathbf{A} - \mathbf{b}\mathbf{k})x(t) + \mathbf{b}u(t)$$

$$y(t) = \mathbf{C}x(t)$$
(5)

State Feedback Controller Design

Short title

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State feedback and estimation

Quickly review the Riccati equation used for design the LQG regulator

Problem
Description

Method 1:Pole placement

Compute K according to the desired poles

Method 2:Solving Lyapunov equation

Compute K by solving a Lyapunov equation

Method 3:Linear quadratic method

Please refer to section 4

State Estimator

Short title

Shiqi Duar

Review the PID regulato

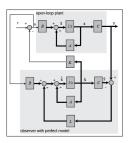
state-space, state-feedba and estimate SS and TF:

State feedback

Quickly review the Riccati equation used for design the LQG regulato

LQG regulato

Problem Description Linear Quadrati A state estimator or observer is a device generates an estimate of the state.



$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{b}u(t) + \mathbf{I}(y(t) - \mathbf{c}\hat{\mathbf{x}}(t))$$
 (6)

State Estimator Design

Short title

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Problem
Description
Linear Quadratic

Method 1:Pole placement

Compute L according to the desired poles, normally we need to place the poles at least five times farther to the left than the dominant poles of the system to get a good observer dynamics.

Method 2:Solving Lyapunov equation

Compute L by solving a Lyapunov equation

Method 3:Linear quadratic estimator

Please refer to section 4

Riccati equation

Short title

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Problem Description **Problem**

Get the solution of

$$\frac{dy}{dx} = A(x)y^2 + B(x)y + C(x) \tag{7}$$

with a form of

$$y = y_1 + \frac{1}{v(x)} \tag{8}$$

Solution

- rewrite the quadratic first order problem to a linear first order problem
- Solve the rewritten problem

Linear Quadratic Optimization

Short title

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LQG regulato

Problem Description Linear Quadra Regulator In a control system, we want to balance the performance and actuator effort (energy) by setting up a cost function of the performance (x) and the effort (u):

$$J = \int_0^\infty \left(x^\top Q x + u^\top R u \right) dt \tag{9}$$

Solving the equation above, we can get a gain matrix K that produce the lowest cost given the dynamic system.

Linear Quadratic Regulator

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LQG regulator
Problem
Description
Linear Quadratic
Regulator

Consider a continuous-time linear system, defined on $t \in [t_0, t_1]$, described by:

$$\dot{x} = Ax + Bu \tag{10}$$

with a quadratic cost function defined as:

$$J = x^{T}(t_{1})F(t_{1})x(t_{1}) + \int_{t_{0}}^{t_{1}} (x^{T}Qx + u^{T}Ru + 2x^{T}Nu) dt$$
(11)

the feedback control law that minimizes the value of the cost is:

$$u = -Kx \tag{12}$$

Linear Quadratic Regulator

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LQG regulator
Problem
Description
Linear Quadratic
Regulator

where K is given by:

$$K = R^{-1} \left(B^T P(t) + N^T \right) \tag{13}$$

and P is found by solving the continuous time Riccati differential equation:

$$A^{T}P(t)+P(t)A-(P(t)B+N)R^{-1}\left(B^{T}P(t)+N^{T}\right)+Q=-\dot{P}(t)$$
(14)

Unfinished Work

Short title

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Review the PID regulator

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Quickly review the Riccati equation used for design the LQG regulator

LQG regulator
Problem
Description
Linear Quadratic
Regulator

- Specific procedures of solving the continuous Riccati differential equation in LQR
- How to design a linear quadratic estimator
- How to pick up proper noise covariance matrix in LQG

References

Short title

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LQG regulato

Linear Quadratic Regulator



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Modern control systems

Pearson

Short title

Shiqi Duan

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_QG regulator

Problem
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Regulator

The End