Question 1

$$\mathcal{F}\left[f(x+1,y)-f(x,y)\right]$$

$$\mathcal{F}\left[f(x+1,y)-f(x,y)\right] = \mathcal{F}\left[f(x-(-1),y-0)-f(x,y)\right] \tag{1}$$

$$= \mathcal{F}\left[f(x-(-1),y-0)\right] - \mathcal{F}\left[f(x,y)\right] \tag{2}$$

$$= F(u,v) \star e^{-j2\pi\left(\frac{-u}{M}\right)} - F(u,v) \tag{3}$$

$$= F(u,v)\left[e^{-j2\pi\left(\frac{-u}{M}\right)} - 1\right] \tag{4}$$

so

$$H(u,v) = e^{-j2\pi\left(\frac{-u}{M}\right)} - 1$$

Question 2

by definitions

$$f(x) \star h(x) = \sum_{m=0}^{M-1} f(m)h(x-m) \quad .$$

and from the question one we have that

$$\mathcal{F}\left[h(x-m)\right] = \sum_{x=0}^{M-1} h(x-m)e^{-j2\pi\left(\frac{ux}{M}\right)} = H(u)e^{-j2\pi\left(\frac{um}{M}\right)}$$

(5)

so

$$\mathcal{F}\left[f(x) \star h(x)\right] = \sum_{x=0}^{M-1}\left[\sum_{m=0}^{M-1} f(m)h(x-m)\right] e^{-j2\pi\left(\frac{ux}{M}\right)} \tag{6}$$

$$= \sum_{m=0}^{M-1} f(m)\left[\sum_{x=0}^{M-1} h(x-m)e^{-j2\pi\left(\frac{ux}{M}\right)}\right] \tag{7}$$

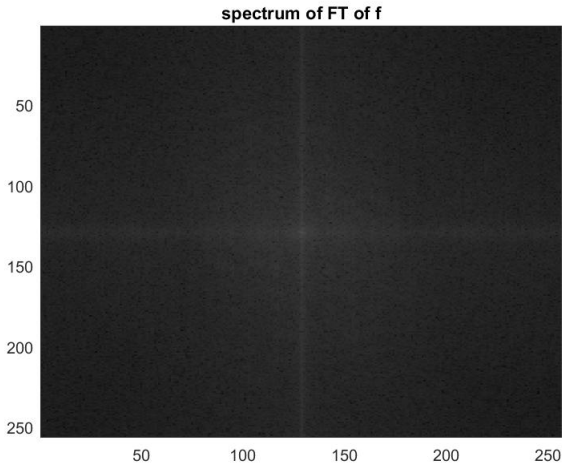
$$= \sum_{m=0}^{M-1} f(m)H(u)e^{-j2\pi\left(\frac{um}{M}\right)} \tag{8}$$

$$= H(u)\sum_{m=0}^{M-1} f(m)e^{-j2\pi\left(\frac{um}{M}\right)} \tag{9}$$

$$= F(u)H(u) \tag{10}$$

Question 3

```
f=imread('Fig5.26a.jpg');

[M N] = size(f);

B=double(f);

% multiply f by (-1)^(x+y) to shift the center
for i = 1:M,
   for j = 1:N,
         d=(i-1)+(j-1);
         C(i,j)=B(i,j)*(-1)^d;
   end
end

% compute the DFT of f*(-1)^{x+y}
fourier_trans=fft2(C);

% compute the spectrum
D=abs(fourier_trans);

c=5;
for i = 1:M,
             for j = 1:N,

                 E(i,j)=c*log(1+D(i,j));
             end
          end

% plot the spectrum (transformed by log for visualisation purposes)
figure
image(E); colormap(gray); title('spectrum of FT of f');
```



The spectrum is displayed here
To obtain the average intensity, we would just sum up all the intensity of the

input image and divide by the total number of pixels of the input image.

```
%% for average

scum=0;
for i = 1:M
   for j = 1:N
         scum=B(i,j)+scum;
   end
end

ave=scum/(M*N);
ave
```

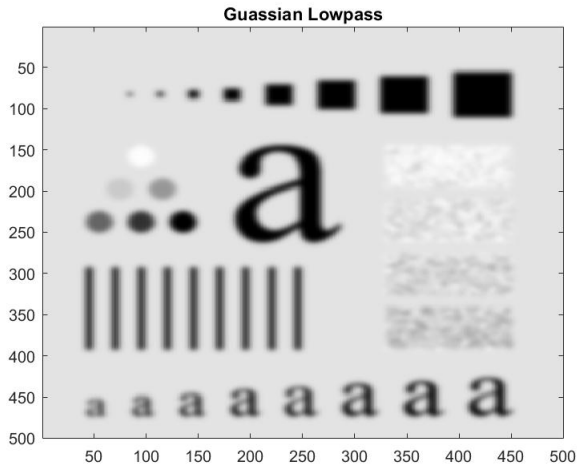
The average is 138.0044 as reported in Matlab

Question 4a

The code for guassian lowpass filter, according to the example given in class, is

```
A=imread('Fig4.45(a).jpg');
[M N] = size(A);
A=double(A);

after=shifting(A);

F=fft2(after);

FH=GLF(F,25);

g=real(ifft2(FH));
g=shifting(g);

figure()
image(g); colormap(gray(256));
title('Guassian Lowpass');

function FH=GLF(F,D0)
    [M,N]=size(F);
    % we need a distance function D(u,v)
    D=@(u,v,M,N) sqrt( (u-M/2)^2+ (v-N/2)^2 );

    FH=zeros(M,N);
    for u=1:M
        for v=1:N
            FH(u,v) = F(u,v)*exp(-(D(u,v,M,N))^2/(2*(D0^2)));
        end
    end
end

function g=shifting(f)
    g=zeros(size(f));
    for x=1:size(f,1)
```

```
34          for y=1:size(f,2)
35              g(x,y) = f(x,y)*(-1)^(x+y);
36          end
37      end
38  end
```



Question4b

```
1  A=imread('Fig4.45(a).jpg');
2  [M N] = size(A);
3  A=double(A);
4
5  after=shifting(A);
6
7  F=fft2(after);
8
9  FH=GHF(F,25);
10
11 g=real(ifft2(FH));
12 g=shifting(g);
13
14 figure()
15 image(g); colormap(gray(256));
16 title('Guassian Lowpass');
17
18 function FH=GHF(F,D0)
19     [M,N]=size(F);
20     % we need a distance function D(u,v)
21     D=@(u,v,M,N) sqrt( (u-M/2)^2+ (v-N/2)^2 );
22
23     FH=zeros(M,N);
24     for u=1:M
25         for v=1:N
```

```
26              FH(u,v) = F(u,v)*(1-exp(-(D(u,v,M,N))^2/(2*(D0^2))));
27          end
28      end
29  end
30
31  function g=shifting(f)
32      g=zeros(size(f));
33      for x=1:size(f,1)
34          for y=1:size(f,2)
35              g(x,y) = f(x,y)*(-1)^(x+y);
36          end
37      end
38  end
```