MATH 155 HWK2

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January 23, 2021

Question 1a

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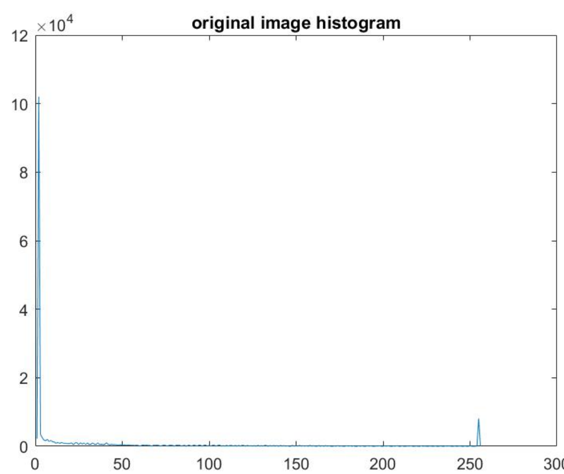
%% Question 1a: draw histogram

image=imread('Fig3.08(a).jpg');
pixelnum=size(image,1)*size(image,2);

HIm=uint8(zeros(size(image,1),size(image,2)));
freq=zeros(256,1);

for i=1:size(image,1)
    for j = 1:size(image,2)
        value=image(i,j);
        freq(value+1)=freq(value+1)+1;
    end
end

figure,plot(freq),title('original image histogram')
```

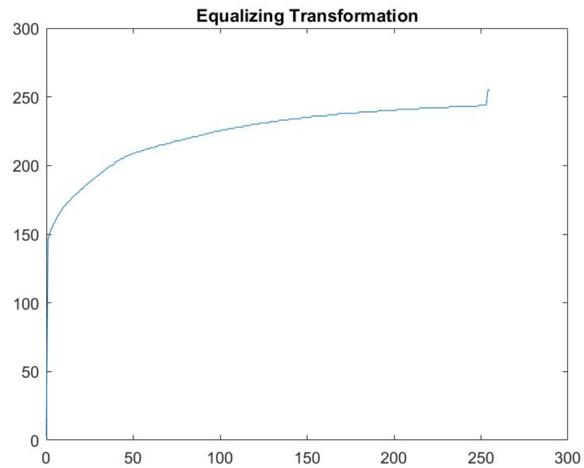


Question 1b

```
%% Question 1b: histogram equalization

[M,N]=size(image);
p = freq/(M*N);
T=@(r) round(255*sum(p(1:r+1)));
x=0:255; %
Tx=zeros(256,1); %to store the transformation on x

for j=1:256
    Tx(j) = T(j-1);
end
figure()
plot(x,Tx);
title('Equalizing Transformation')
```



Question 1c

```
%% Question 1a: draw histogram

HIm=uint8(HIm*255);

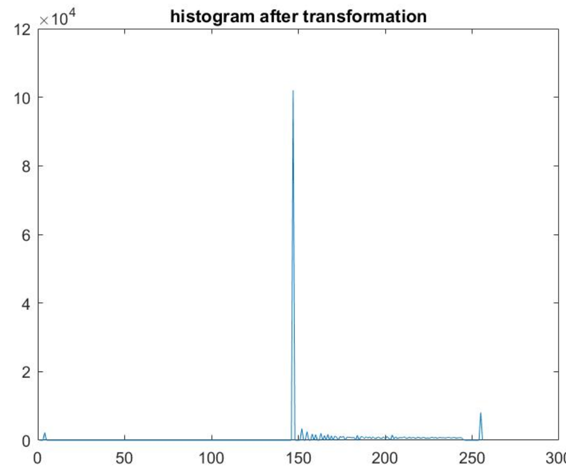
figure,imshow(HIm),title('transformed image');

freqn=zeros(256,1);

for i=1:size(HIm,1)
    for j = 1:size(HIm,2)
        value=HIm(i,j);
        freqn(value+1)=freqn(value+1)+1;
```

```
    end
end

figure,plot(freqn),title('histogram after transformation')
```



Question 2

Since $s_k = T(r_k) = \frac{1}{mn}\sum_{j=0}^{k} n_{rj}$ and since every value rk gets mapped to sk, so $n_{rk} = n_{sk}$. Thus, we have that $v_k = T(s_k) = \frac{1}{mn}\sum_{j=0}^{k} n_{sj} = \frac{1}{mn}\sum_{j=0}^{k} n_{rj}$ which shows that results are the same.

Question 3a

We first prove that for all $r \in (-\infty, \infty)$, $P_r(r) \geq 0$. Since for $r \in [0, L-1]$, $P_r(r) = \frac{6r+2}{3(L-1)^2+2(L-1)}$ and we know that $(L-1) \geq 0$,$r \geq 0$ and subsequently $(L-1)^2 \geq 0$, $6r+2 \geq 0$ and $2(L-1) \geq 0$. This shows that the fraction between these quantities is also ≥ 0. $P_r(r)$ for all other $r \geq 0$ is trivial as stated in the conditions.

Next, we will show that $\int_{-\infty}^{\infty} P_r(r)\,dr$.

$$\int_{-\infty}^{\infty} P_r(r)\,dr = \int_0^{L-1} P_r(r)\,dr = \int_0^{L-1} \frac{6r+2}{3(L-1)^2+2(L-1)}\,dr$$

$$= \frac{1}{3(L-1)^2+2(L-1)} \int_0^{3(L-1)} 6r + 2\,dr$$

$$= \frac{1}{3(L-1)^2+2(L-1)} \left[3^2 + 2r\right]_0^{L-1}$$

$$= \frac{1}{3(L-1)^2 + 2(L-1)} 3(L-1)^2 + 2(L-1) = 1$$

Question 3b

$$s = T(r) = (L-1) \int_0^r \frac{6r+2}{3(L-1)^2 + 2(L-1)} dr$$

$$\Rightarrow \frac{2}{3(L-1)+2} \int_0^r (3r+1) dr$$

$$\Rightarrow \frac{2}{3(L-1)+2} (\frac{3r^2}{2} + r)$$

$$\Rightarrow s = T(r) = \frac{3r^2 + 2r}{3(L-1)+2}$$

Question 3c

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right| = \frac{6r+2}{3(L-1)^2 + 2(L-1)} \left| \left[\frac{d}{dr} \frac{3r^2 + 2r}{3(L-1)+2} \right]^{-1} \right|$$

$$= \frac{6r+2}{3(L-1)^2 + 2(L-1)} \left| \left[\frac{6r+2}{3(L-1)+2} \right]^{-1} \right|$$

$$= \frac{1}{L-1}$$

Which signals a uniform PDF

Question 4

First, by the histogram equalization transformation:

$$s = T(r) = (L-1) \int_0^r (-2r+2) dr = (L-1)(-r^2 + 2r).$$

$$G(z) = (L-1) \int_0^z 2z dz = (L-1) z^2.$$

since $G(z) = s$, and

$$G(z) = (L-1)z^2,$$

$$s = (L-1)z^2.$$

$$z = \sqrt{\frac{s}{(L-1)}}$$

so z in terms of r is

$$z = \sqrt{(-r^2 + 2r)}$$