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1) $f(t) = \begin{cases} A & \text{if } 0 \le t \le k \\ 0 & \text{otherwise} \end{cases}$

$F(u) = \int_0^k Ae^{-j2\pi ut}dt = \frac{-A}{j2\pi u}[e^{-j2\pi uk} - e^0]$

$= \frac{-A}{j2\pi u}[e^{-j2\pi uk} - 1]$

since $e^{-j2\pi uk} = \cos(-2\pi uk) + j\sin(2\pi uk)$

at $F(0) = \frac{-A}{j2\pi u}[\underbrace{\cos(-2\pi uk)}_{1} + \underbrace{j\sin(-2\pi uk)}_{0} \underbrace{-1}_{-1}]$

$= \frac{-A}{j2\pi u}[0] = 0.$

2) a)

$F(u) = \sum_{x=0}^{M-1} f(x)e^{-i2\pi ux/M}$

$f(x) = \frac{1}{M}\sum_{x=0}^{M-1} F(u)e^{i2\pi ux/M}$

$= \frac{1}{M}\sum_{u=0}^{M-1}\underbrace{\sum_{y=0}^{M-1} f(y)e^{-i2\pi uy/M}e^{i2\pi ux/M}}_{\begin{cases} M & \text{if } r=x \\ 0 & \text{otherwise} \end{cases}}$ so

$= \frac{1}{M}\cdot M\sum_{y=0}^{M-1} f(y)$

$\Rightarrow f(x) = f(y)$

b) $F(u) = \sum_{y=0}^{M-1} f(y)e^{-i2\pi iuy/M}$

$= \sum_{y=0}^{M-1}\frac{1}{M}\sum_{u=0}^{M-1} F(u)e^{2\pi iux/M}e^{-i2\pi iuy/M}$

$= \frac{1}{M}\sum_{y=0}^{M-1}\sum_{u=0}^{M-1} F(u)e^{2\pi iux/M}e^{-i2\pi iuy/M}.$

$= \frac{1}{M}M\sum_{y=0}^{M-1} F(u) \Rightarrow F(u) = F(u)$

(3) $f(x) = f(x+kM)$

$\mathcal{F}[f(x+kM)] = \frac{1}{M}\sum_{u=0}^{M-1} F(u)e^{j2\pi\left[\frac{(u+kM)x}{M}\right]}$

$= \frac{1}{M}\sum_{u=0}^{M-1} F(u)e^{j2\pi\left(\frac{ux}{M}\right)}\underbrace{e^{j2\pi uk}}_{1}$ by euler's identity

since $e^{j2\pi uk} = 1$

$= \frac{1}{M}\sum_{u=0}^{M-1} F(u)e^{j2\pi\left(\frac{ux}{M}\right)} = f(x)$

(4) show that 2D Fourier transform is linear

$F(\mu,v) = \mathcal{F}[f(t,z)]$

$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,z)e^{-j2\pi(\mu t+vz)}dt\,dz$

$\mathcal{F}[af(t,z) + g(t,z)]$

$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[af(t,z) + g(t,z)]e^{-j2\pi(ut+vz)}dt\,dz$

$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} af(t,z)e^{-j2\pi(ut+vz)} + g(t,z)e^{-j2\pi(ut+vz)}dt\,dz$

$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} af(t,z)e^{-j2\pi(ut+vz)}dt\,dz$

$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(t,z)e^{-j2\pi(ut+vz)}dt\,dz$

$= a\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,z)e^{-j2\pi(ut+zv)}dt\,dz$

$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(t,z)e^{-j2\pi(ut+vz)}dt\,dz$

$= a\mathcal{F}[f(t,z)] + \mathcal{F}[g(t,z)]$

thus it is linear.

5)

$$h(x,y) = \iint H(\mu,v)e^{j2\pi(\mu x+vy)}$$

$$h(-x,-y) = \iint H(\mu,v)e^{-j2\pi(\mu x+vy)}$$

$$h(x,y) = \iint \overline{H(\mu,v)e^{-j2\pi(\mu x+vy)}}$$

$$\Rightarrow \overline{h(-x,-y)}$$

$$h(x,y) = \iint \overline{H(\mu,v)}e^{j2\pi(\mu x+vy)}$$

$$\overline{h(x,y)} = \iint H(\mu,v)e^{-j2\pi(\mu x+vy)} = h(-x,-y)$$

since $h(x,y) = \overline{h(x,y)}$ let $s=-\mu$ and $t=-v$

$$\overline{h(x,y)} = \iint H(s,t)e^{j2\pi(sx+ty)}\,ds\,dt$$

$$= h(x,y)$$

since $\overline{h(x,y)} = \overline{h(x,y)}$ and
$h(x,y) = h(-x,-y)$ and
$h(-x,-y) = \overline{h(x,y)}$

so $h(x,y) = \overline{h(x,y)} = \overline{h(-x,-y)} = h(-x,-y)$