

# MATH 155 HWK 7

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## Question 1 an 2

Math 155

1a)  $f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux + vy)} du dv$

1b)  $\frac{df}{dx} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} j2\pi u F(u, v) e^{j2\pi(ux + vy)} du dv$   
 $\Rightarrow \frac{df}{dx dz} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (j2\pi u) (j2\pi v) F(u, v) e^{j2\pi(ux + vz)} du dz$   
 $\mathcal{F} \frac{df}{dx dz} = \mathcal{F}(\mathcal{F}^{-1}(F(u, v) (j2\pi u) (j2\pi v)))$   
 $= F(u, v) (2\pi j)^2 uv$

2

$$\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$F(u) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi ut} dt$$

$$= \int_{-\infty}^{\infty} \sin(2\pi \mu_0 t) e^{-j2\pi ut} dt$$

$$= \frac{j}{2} \int_{-\infty}^{\infty} [e^{j2\pi \mu_0 t} - e^{-j2\pi \mu_0 t}] e^{-j2\pi ut} dt$$

$$= \frac{j}{2} \int_{-\infty}^{\infty} e^{j2\pi \mu_0 t} e^{-j2\pi ut} dt - \frac{j}{2} \int_{-\infty}^{\infty} e^{-j2\pi \mu_0 t} e^{-j2\pi ut} dt$$

$$\mathcal{F}(f(t) e^{j2\pi \mu_0 t}) = F(u - \mu_0) \text{ so}$$

$$\mathcal{F}(1) e^{j2\pi \mu_0 t} = \delta(u - \mu_0)$$

$$\mathcal{F}(1) = 1$$

$$\checkmark F(u) = \frac{j}{2} [\delta(u + \mu_0) - \delta(u - \mu_0)]$$

### Question 3

$$\begin{aligned}
 3) \quad F(u,v) &= \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(\frac{ux}{M} + \frac{vy}{N})} \\
 \sin(2\pi(u_0x + v_0y)) &= \frac{e^{j2\pi(u_0x + v_0y)} - e^{-j2\pi(u_0x + v_0y)}}{2j} \\
 &= \frac{j}{2} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [e^{j2\pi(u_0x + v_0y)} - e^{-j2\pi(u_0x + v_0y)}] e^{-j2\pi(\frac{ux}{M} + \frac{vy}{N})} \\
 &= \frac{j}{2} \sum_x \sum_y e^{j2\pi(u_0x + v_0y)} e^{-j2\pi(\frac{ux}{M} + \frac{vy}{N})} - \left(\frac{j}{2}\right) \sum_x \sum_y e^{-j2\pi(u_0x + v_0y)} e^{-j2\pi(\frac{ux}{M} + \frac{vy}{N})} \\
 &= \frac{j}{2} \sum_x \sum_y e^{j2\pi(\frac{Mu_0 - u}{M}x + \frac{Nv_0 - v}{N}y)} - \left(\frac{j}{2}\right) \sum_x \sum_y e^{-j2\pi(\frac{Mu_0 + u}{M}x + \frac{Nv_0 + v}{N}y)} \\
 &= \frac{j}{2} F(u)(u - Mu_0, v - Nv_0) - \left(\frac{j}{2}\right) F(u)(u + Mu_0, v + Nv_0) \\
 &= \frac{j}{2} [F(u + Mu_0, v + Nv_0) - F(u - Mu_0, v - Nv_0)]
 \end{aligned}$$

### Question 4

#### 0.1 4a

The program is provided as

```

1 function [output] = sinnoise(input,A,u0,v0)
2     [M,N] = size(input);
3     noise=zeros(M,N);
4     for i= 1:M
5         for j= 1:N
6             noise(i,j)=A*sin((2*pi*u0*i)+(2*pi*v0*j));
7         end
8     end
9     output=zeros(M,N);
10
11     for j = 1:M
12         for l=1:N
13             output(j,l)=noise(j,l)+input(j,l);
14         end
15     end
16 end

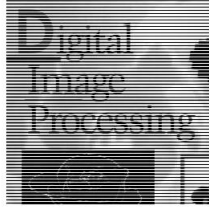
```

#### 4b

```

1 A=imread('Fig5.26a.jpg');
2 input=im2double(A);
3 [M,N]=size(A);
4 output=sinnoise(input,200,0.25,0);

```



4c

```
1  for i = 1:M
2      for j = 1:N
3          d=(i-1)+(j-1);
4          D(i,j)=output(i,j)*(-1)^d;
5      end
6  end
7
8  fourier_trans=fft2(D);
9
10 Q=abs(fourier_trans);
11
12 c=5;
13 for i = 1:M
14     for j = 1:N
15
16         E(i,j)=c*log(1+Q(i,j));
17     end
18 end
19
20 figure
21 image(E); colormap(gray); title('spectrum of degraded image');
```

