MATH 155 HWK 7

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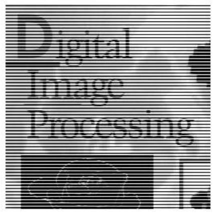
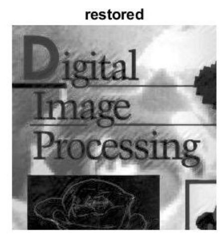
Question 1

I adopted my notch reject filter from a post on Stack Overflow: https://stackoverflow.com/questions/29235421/f proper-notch-filter-to-remove-pattern-from-image/29246091

```
fourier_trans2=fftshift(fft2(input));
normalize = @(img) (img - min(img(:))) / (max(img(:)) - ...
    min(img(:)));
gau_notch = @(v,mu,cov) ...
    1-exp(-0.5*sum((bsxfun(@minus,v,mu).*(cov\bsxfun(@minus,v,mu)))));

x_center=129;
y_center=129;

nx1=149.5-129;
nx2=165.5-129;
ny=157.5-129;

notch_filter=ones(M,N);

[y,x] = meshgrid(1:N, 1:M);
X = [y(:) x(:)].';
notch_filter = notch_filter .* ...
    reshape(gau_notch(X,[x_center+nx1;y_center+ny],eye(2)*25),[M,N]);
notch_filter = notch_filter .* ...
    reshape(gau_notch(X,[x_center+nx2;y_center+ny],eye(2)*25),[M,N]);
notch_filter = notch_filter .* ...
    reshape(gau_notch(X,[x_center-nx1;y_center-ny],eye(2)*25),[M,N]);
notch_filter = notch_filter .* ...
    reshape(gau_notch(X,[x_center-nx2;y_center-ny],eye(2)*25),[M,N]);

fourier_trans2=fourier_trans2.*notch_filter;
ifft_=ifft2(ifftshift(fourier_trans2));
restored=histeq(normalize(ifft_));

figure();
imshow(restored);title('restored')
```



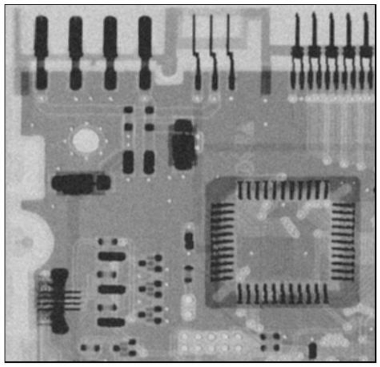


left: degraded; right: restored

Question 2

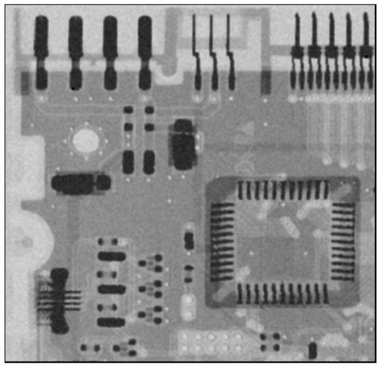
2a arithmetic mean filter

```
A=imread('Fig0507(b).tif');
A=im2double(A);
[M,N]=size(A);
B=zeros(M,N);

for i=2:M-2
    for j=2:N-2
        value=0;
        %loop through filter
            for x=1:3
                for y=1:3
                   value=A(i+x-1,j+y-1)+value;
                end
            end
        B(i,j)=value/9;
    end
end
figure();
imshow(B)
```



2b geometric mean

```
A=imread('Fig0507(b).tif');
A=im2double(A);
[M,N]=size(A);
B=zeros(M,N);

for i=2:M-2
    for j=2:N-2
        value=1;
        %loop through filter
            for x=1:3
                for y=1:3
                   value=A(i+x-1,j+y-1).*value;
                end
            end
        B(i,j)=value^(1/9);
    end
end
figure();
imshow(B)
```



2c

We can see that the arithmetic mean filter and the geometric mean filter aren't a lot different from each other, except for the darker areas where geometric mean filter performs a bit better. I used matlab to calculate SNR with the code below:

```
top=0;
bottom=0;
for i=1:M
    for j=1:N
        top=top+(B(i,j)^2);
        bottom=bottom+(A(i,j)-B(i,j))^2;
    end
end

SNR=10*log10(top/bottom);
SNR
```

The SNR for arithmetic mean filter is 1.1195, while the SNR for the geometric mean filter is 11.0637, further showing that there isn't a lot of difference between the two filters.

1 Question 3

3a

A pepper noise is a sudden dark pixel. Suppose we have a rather constant intensity A around the sudden pepper noise, so applying the contraharmonic formula would become approximately (assuming the dark pixel has an intensity of 0 or a small value negligible)

$$\hat{f}(x,y) = \frac{(mn-1)A^{Q+1}}{(mn-1)A^{Q}}$$

With a positive Q, when Q increase $\hat{f}(x,y)$ approaches A so we successfully get rid of the pepper noise.

3b

With a salt noise,we have a sudden bright spot. Suppose we have a bright spot with intensity (L-1), then the formula becomes

$$\hat{f}(x,y) = \frac{(mn-1)A^{Q+1} + (L-1)^{Q+1}}{(mn-1)A^{Q} + (L-1)^{Q}}$$

When Q is negative, since $(mn-1)A^Q$ is greater than $(L-1)^Q$, $(L-1)^Q$ will get reduced to almost 0 while $(mn-1)A^Q$ can remain. Thus, when Q is negative it is effective in reducing salt noises with high intensity value by reducing it to a very small value and preserving the intensity around it.

3c

Selecting the wrong polarity could either emphasize salt noises or pepper noises instead of reducing them. As we have seen before, positive Q reduces pepper noises and negative Q reduces to salt noises. But if we picked the wrong Q, we could apply a positive Q to a salt noise (which intensify it) or a negative Q to a pepper noise (which intensify it).

3d

When we have Q=-1, we have that

$$\hat{f}(x,y) = \frac{\sum_{s,t \in S_{x,y}} g(s,t)^0}{\sum_{s,t \in S_{x,y}} g(s,t)^{-1}} \tag{1}$$

$$= \frac{mn}{\sum_{s,t \in S_{x,y}} g(s,t)^{-1}}. \tag{2}$$

which turns the contraharmonic filter into a harmonic filter. Harmonic filter is good for reducing salt noise and Gaussian noise, but not good for pepper noise.

3e

If we have a constant area with intensity A we have that

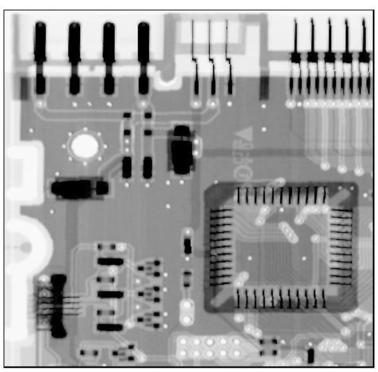
$$\hat{f}(x,y) = \frac{(mn)A^{Q+1}}{(mn)A^{Q}} = A$$

So regardless of the sign of Q, the result is A and the resulting value remains constant.

Question 4

4a

```
A=imread('Fig5.08(a).jpg');
A=im2double(A);
[M,N]=size(A);
B=zeros(M,N);

Q=1.5;
for i=2:M-2
    for j=2:N-2
        top=0;
        bottom=0;
        %loop through filter
            for x=1:3
                for y=1:3
                    top=(A(i+x-1,j+y-1)^(Q+1))+top;
                    bottom=(A(i+x-1,j+y-1)^(Q))+bottom;
                end
            end
        B(i,j)=top/bottom;
    end
end
figure();
imshow(B)
```



4b

```
A=imread('Fig5.08(b).jpg');
A=im2double(A);
[M,N]=size(A);
B=zeros(M,N);
```

```
Q=-1.5;
for i=2:M-2
    for j=2:N-2
        top=0;
        bottom=0;
        %loop through filter
            for x=1:3
                for y=1:3
                   top=(A(i+x-1,j+y-1)^(Q+1))+top;
                   bottom=(A(i+x-1,j+y-1)^(Q))+bottom;
                end
            end
        B(i,j)=top/bottom;
    end
end
figure();
imshow(B)
```