



Antrouction pheticions on cryptotions

from substitutions to public keys

An introduction to Cryptography

Daniel Hutmacher

A large iceberg is shown floating in the ocean. The visible part above the water's surface is small compared to the massive portion submerged below. A white speech bubble with a black outline points from the left towards the top of the iceberg. Inside the bubble, the text "This presentation" is written in a black, sans-serif font.

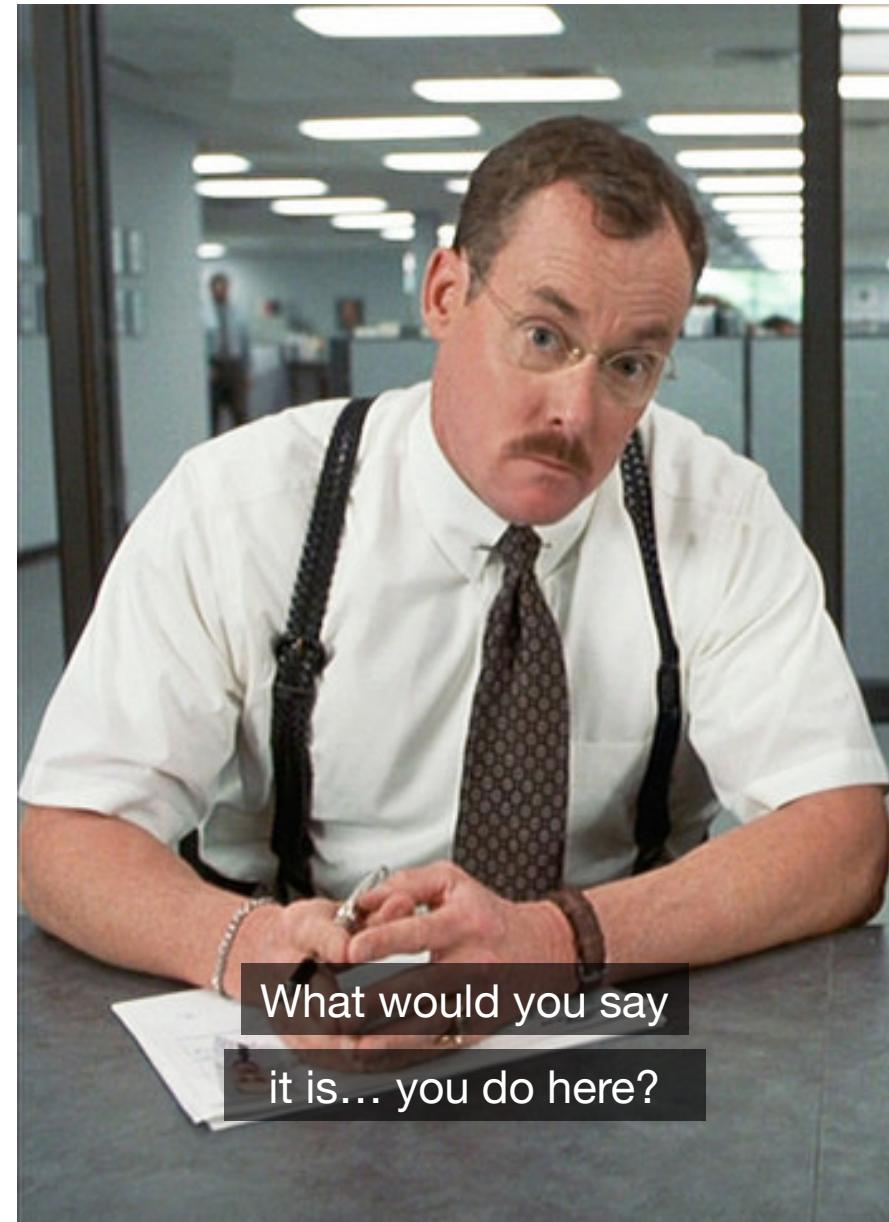
This presentation

Cryptography is pretty complex

Daniel Hutmacher

- SQL Server developer since 1999
- Actually a Data Platform MVP
- Consultant in my own business
- Organizer of Data Saturday Stockholm
- I run dataplatform.social and callfordataspeakers.com

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Twitter: @dhmacher



Caesar shift

Plaintext:

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a

Key:

Plaintext:

w	e		a	t	t	a	c	k		a	t		d	a	w	n								
x	f		b	u	u	b	d	l		b	u		e	b	x	o								

Ciphertext:

Substitution

Plaintext:

Key:

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
y	r	q	p	o	t	i	h	x	e	z	j	d	c	b	w	n	m	v	l	k	u	s	g	f	a

Plaintext:

Ciphertext:

w	e		a	t	t	a	c	k		a	t		d	a	w	n								
s	o		y	1	1	y	q	z		y	1		p	y	s	c								

Substitution ► Frequency analysis

Data source: https://en.wikipedia.org/wiki/Letter_frequency

	English	French	German	Spanish	Portuguese	Italian	Turkish	Swedish	Polish	Dutch	Danish	Icelandic	Finnish	Czech	Hungarian
e	12,7%	14,7%	16,4%	12,2%	12,6%	11,8%	8,9%	10,1%	7,9%	18,9%	15,5%	6,4%	8,0%	7,6%	11,6%
a	8,2%	7,6%	6,5%	11,5%	14,6%	11,7%	11,9%	9,4%	9,0%	7,5%	6,0%	10,1%	12,2%	8,4%	8,9%
n	6,7%	7,1%	9,8%	6,7%	4,4%	6,9%	7,5%	8,5%	5,6%	10,0%	7,2%	7,7%	8,8%	6,5%	6,8%
i	7,0%	7,5%	6,6%	6,2%	6,2%	10,1%	8,6%	5,8%	8,3%	6,5%	6,0%	7,6%	10,8%	6,1%	4,3%
r	6,0%	6,7%	7,0%	6,9%	6,5%	6,4%	6,7%	8,4%	4,6%	6,4%	9,0%	8,6%	2,9%	4,8%	2,7%
t	9,1%	7,2%	6,2%	4,6%	4,3%	5,6%	3,3%	7,7%	4,0%	6,8%	6,9%	5,0%	8,8%	5,7%	7,0%
s	6,3%	7,9%	7,3%	8,0%	6,8%	5,0%	3,0%	6,6%	4,3%	3,7%	5,8%	5,6%	7,9%	5,2%	7,0%
o	7,5%	5,8%	2,6%	8,7%	9,7%	9,8%	2,5%	4,5%	7,6%	6,1%	4,6%	2,2%	5,6%	6,7%	3,7%
l	4,0%	5,5%	3,4%	5,0%	2,8%	6,5%	5,9%	5,3%	2,1%	3,6%	5,2%	4,5%	5,8%	3,8%	6,7%
d	4,3%	3,7%	5,1%	5,0%	5,0%	3,7%	4,7%	4,7%	3,3%	5,9%	5,9%	1,6%	1,0%	3,5%	1,9%
m	2,4%	3,0%	2,5%	3,2%	4,7%	2,5%	3,8%	3,5%	2,9%	2,2%	3,2%	4,0%	3,2%	2,4%	3,8%
u	2,8%	6,3%	4,2%	3,9%	3,6%	2,8%	3,2%	1,9%	2,3%	2,0%	2,0%	4,6%	5,0%	2,2%	0,4%
k	1,8%	0,1%	1,4%	0,0%	0,0%	0,0%	4,7%	3,1%	3,4%	2,3%	3,4%	3,3%	5,0%	2,9%	4,9%
g	2,0%	0,9%	3,0%	1,8%	1,3%	1,6%	1,3%	2,9%	1,4%	3,4%	4,1%	4,2%	0,4%	0,1%	3,8%
c	2,8%	3,3%	2,7%	4,0%	3,9%	4,5%	1,0%	1,5%	4,0%	1,2%	0,6%	0,3%	0,7%	0,6%	
h	6,1%	0,9%	4,6%	2,0%	1,3%	0,1%	1,2%	2,1%	1,1%	2,4%	1,6%	1,9%	1,9%	1,4%	1,3%
v	1,0%	1,8%	0,8%	1,1%	1,6%	2,1%	1,0%	2,4%	0,0%	2,9%	2,3%	2,4%	2,3%	5,3%	2,3%
p	1,9%	2,5%	0,7%	2,5%	2,5%	3,1%	0,9%	1,8%	3,1%	1,6%	1,8%	0,8%	1,8%	1,9%	0,5%
b	1,5%	0,9%	1,9%	2,2%	1,0%	0,9%	2,8%	1,5%	1,5%	1,6%	2,0%	1,0%	0,3%	0,8%	1,9%
y	2,0%	0,7%	0,0%	1,4%	0,0%	0,0%	3,3%	0,7%	3,9%	0,0%	0,7%	0,9%	1,7%	1,0%	2,6%
z	0,1%	0,3%	1,1%	0,5%	0,5%	1,2%	1,5%	0,1%	5,6%	1,4%	0,0%	0,1%	1,6%	4,3%	
f	2,2%	1,1%	1,7%	0,7%	1,0%	1,2%	0,5%	2,0%	0,3%	0,8%	2,4%	3,0%	0,2%	0,1%	0,5%
j	0,3%	0,8%	0,3%	2,5%	0,9%	0,0%	0,0%	0,6%	2,3%	1,5%	0,7%	1,1%	2,0%	1,4%	1,5%
w	2,4%	0,0%	1,9%	0,0%	0,0%	0,0%		0,1%	4,5%	1,5%	0,1%	0,1%	0,1%	0,0%	
é		1,5%		0,4%	0,3%						0,6%		0,6%	4,3%	
á				0,5%	0,1%						1,8%		1,8%	0,9%	3,4%
ä				0,6%				1,8%					3,6%		
í				0,7%	0,1%			5,1%					1,6%		0,5%
ü				0,1%				0,0%					0,0%		

Frequency analysis

Ciphertext		English
a	0,5%	a 8,2%
b	1,3%	b 1,5%
c		c 2,8%
d	2,0%	d 4,3%
e	0,9%	e 12,7%
f	2,4%	f 2,2%
g	9,0%	g 2,0%
h	6,2%	h 6,1%
i	4,2%	i 7,0%
j		j 0,3%
k	1,4%	k 1,8%
l	8,0%	l 4,0%
m	6,5%	m 2,4%
n	2,0%	n 6,7%
o	3,1%	o 7,5%
p	1,1%	p 1,9%
q	0,8%	q
r	7,1%	r 6,0%
s	5,9%	s 6,3%
t	1,8%	t 9,1%
u	1,9%	u 2,8%
v	12,2%	v 1,0%
w	4,0%	w 2,4%
x	2,1%	x

dv'iv ml hgizmtvih gl olev
blf pmld gsv ifovh zmw hl wl r
z ufoo xlnnrgnvmg'h dszg r'n gsrmprmt lu
blf dlflowm'g tvg gsrh uiln zmb lgsvi tfb
r qfhg dzmmz gvoo blf sld r'n uvvormt
tlggz nzpv blf fmwvihgzmw
mvevi tlmmz trev blf fk
mvevi tlmmz ovg blf wldm
mvevi tlmmz ifm zilfmw zmw wvhvig blf
mvevi tlmmz nzpv blf xib
mvevi tlmmz hzb tllybyv
mvevi tlmmz gvoo z orv zmw sfig blf
dv'ev pmldm vzxs lgsvi uli hl olmt
blfi svzig'h yvvm zxsrmt, yfg blf'iv gll hsb gl hzb rg
rmhrwv, dv ylgs pmld dszg'h yvvm tlrmt lm
dv pmld gsv tznv zmw dv'iv tlmmz kozb rg
z mw ru blf zhp nv sld r'n uvvormt
wlm'g gvoo nv blf'iv gll yormw gl hvv
mvevi tlmmz trev blf fk
mvevi tlmmz ovg blf wldm
mvevi tlmmz ifm zilfmw zmw wvhvig blf
mvevi tlmmz nzpv blf xib
mvevi tlmmz hzb tllybyv
mvevi tlmmz gvoo z orv zmw sfig blf
mvevi tlmmz trev blf fk
mvevi tlmmz ovg blf wldm
mvevi tlmmz ifm zilfmw zmw wvhvig blf
mvevi tlmmz nzpv blf xib
mvevi tlmmz hzb tllybyv
mvevi tlmmz gvoo z orv zmw sfig blf
dv'ev pmldm vzxs lgsvi uli hl olmt
blfi svzig'h yvvm zxsrmt, yfg blf'iv gll hsb gl hzb rg
rmhrwv, dv ylgs pmld dszg'h yvvm tlrmt lm
dv pmld gsv tznv zmw dv'iv tlmmz kozb rg
r qfhg dzmmz gvoo blf sld r'n uvvormt
tlggz nzpv blf fmwvihgzmw

Frequency analysis

	Ciphertext	English
v	12,2%	e 12,7%
g	9,0%	t 9,1%
i	8,0%	a 8,2%
z	7,9%	o 7,5%
r	7,1%	i 7,0%
m	6,5%	n 6,7%
h	6,2%	s 6,3%
s	5,9%	h 6,1%
i	4,2%	r 6,0%
w	4,0%	d 4,3%
o	3,1%	l 4,0%
f	2,4%	c 2,8%
x	2,1%	u 2,8%
n	2,0%	m 2,4%
d	2,0%	w 2,4%
u	1,9%	f 2,2%
t	1,8%	g 2,0%
k	1,4%	y 2,0%
b	1,3%	p 1,9%
p	1,1%	k 1,8%
y	1,0%	b 1,5%
e	0,9%	v 1,0%
q	0,8%	j 0,3%
a	0,5%	z 0,1%

dv'iv ml hgizmtvih gl olev
blf pmld gsv ifovh zmw hl wl r
z ufoo xlnnrgnvmg'h dszg r'n gsrmprmt lu
blf dlflowm'g tvg gsrh uiln zmb lgsvi tfb
r qfhg dzmmz gvoo blf sld r'n uvvormt
tlggz nzpv blf fmwvihgzmw
mvevi tlmmz trev blf fk
mvevi tlmmz ovg blf wldm
mvevi tlmmz ifm zilfmw zmw wvhvig blf
mvevi tlmmz nzpv blf xib
mvevi tlmmz hzb tllybyv
mvevi tlmmz gvoo z orv zmw sfig blf
dv'ev pmldm vzxs lgsvi uli hl olmt
blfi svzig'h yvvm zxsrmt, yfg blf'iv gll hsb gl hzb rg
rmhrwv, dv ylgs pmld dszg'h yvvm tlrmt lm
dv pmld gsv tznv zmw dv'iv tlmmz kozb rg
z mw ru blf zhp nv sld r'n uvvormt
wlm'g gvoo nv blf'iv gll yormw gl hvv
mvevi tlmmz trev blf fk
mvevi tlmmz ovg blf wldm
mvevi tlmmz ifm zilfmw zmw wvhvig blf
mvevi tlmmz nzpv blf xib
mvevi tlmmz hzb tllybyv
mvevi tlmmz gvoo z orv zmw sfig blf
mvevi tlmmz trev blf fk
mvevi tlmmz ovg blf wldm
mvevi tlmmz ifm zilfmw zmw wvhvig blf
mvevi tlmmz nzpv blf xib
mvevi tlmmz hzb tllybyv
mvevi tlmmz gvoo z orv zmw sfig blf
dv'ev pmldm vzxs lgsvi uli hl olmt
blfi svzig'h yvvm zxsrmt, yfg blf'iv gll hsb gl hzb rg
rmhrwv, dv ylgs pmld dszg'h yvvm tlrmt lm
dv pmld gsv tznv zmw dv'iv tlmmz kozb rg
r qfhg dzmmz gvoo blf sld r'n uvvormt
tlggz nzpv blf fmwvihgzmw

Frequency analysis

	Ciphertext	English
v	12,2%	e 12,7%
g	9,0%	t 9,1%
i	8,0%	a 8,2%
z	7,9%	o 7,5%
r	7,1%	i 7,0%
m	6,5%	n 6,7%
h	6,2%	s 6,3%
s	5,9%	h 6,1%
i	4,2%	r 6,0%
w	4,0%	d 4,3%
o	3,1%	l 4,0%
f	2,4%	c 2,8%
x	2,1%	u 2,8%
n	2,0%	m 2,4%
d	2,0%	w 2,4%
u	1,9%	f 2,2%
t	1,8%	g 2,0%
k	1,4%	y 2,0%
b	1,3%	p 1,9%
p	1,1%	k 1,8%
y	1,0%	b 1,5%
e	0,9%	v 1,0%
q	0,8%	j 0,3%
a	0,5%	z 0,1%

we're na strongers ta lave
 pac knaw the rcles ond sa da i
 o fc11 uammitment's whot i'm thinking af
 pac wacldn't get this fram onp ather gcp
 i jcst wonno tell pac haw i'm feeling
 gatto moke pac cnderstond
 never ganno give pac cy
 never ganno let pac dawn
 never ganno rcn oracnd ond desert pac
 never ganno moke pac urp
 never ganno sop gaadbpe
 never ganno tell o lie ond hcrt pac
 we've knawn eouh ather far sa lang
 pacr heort's been ouhing, bct pac're taa shp ta sop it
 inside, we bath knaw whot's been gaing an
 we knew the gome ond we're ganno ylop it
 ond if pac osk me haw i'm feeling
 dan't tell me pac're taa blind ta see
 never ganno give pac cy
 never ganno let pac dawn
 never ganno rcn oracnd ond desert pac
 never ganno moke pac urp
 never ganno sop gaadbpe
 never ganno tell o lie ond hcrt pac
 never ganno give pac cy
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i	8,0%	a	8,2%
z	7,9%	o	7,5%
r	7,1%	i	7,0%
m	6,5%	n	6,7%
h	6,2%	s	6,3%
s	5,9%	h	6,1%
i	4,2%	r	6,0%
w	4,0%	d	4,3%
o	3,1%	l	4,0%
f	2,4%	c	2,8%
x	2,1%	u	2,8%
n	2,0%	m	2,4%
d	2,0%	w	2,4%
u	1,9%	f	2,2%
t	1,8%	g	2,0%
k	1,4%	y	2,0%
b	1,3%	p	1,9%
p	1,1%	k	1,8%
y	1,0%	b	1,5%
e	0,9%	v	1,0%
q	0,8%	j	0,3%
a	0,5%	z	0,1%

we're no strangers to love
 poc know the rcles and so do i
 a fc11 uommitment's what i'm thinking of
 poc wo1dn't get this from anp other gcp
 i jcst wanna tell poc how i'm feeling
 gotta make poc cnderstand
 never gonna give poc cy
 never gonna let poc down
 never gonna rcn arocnd and desert poc
 never gonna make poc ury
 never gonna sap goodbpe
 never gonna tell a lie and hcrt poc
 we've known eauh other for so long
 pocr heart's been auhing, bct poc're too shp to sap it
 inside, we both know what's been going on
 we know the game and we're gonna ylap it
 and if poc ask me how i'm feeling
 don't tell me poc're too blind to see
 never gonna give poc cy
 never gonna let poc down
 never gonna rcn arocnd and desert poc
 never gonna make poc ury
 never gonna sap goodbpe
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k	1,4%	y 2,0%
b	1,3%	p 1,9%
p	1,1%	k 1,8%
y	1,0%	b 1,5%
e	0,9%	v 1,0%
q	0,8%	j 0,3%
a	0,5%	z 0,1%

we're no strangers to love
 you know the rules and so do i
 a full commitment's what i'm thinking of
 you wouldn't get this from any other guy
 i just wanna tell you how i'm feeling
 gotta make you understand
 never gonna give you up
 never gonna let you down
 never gonna run around and desert you
 never gonna make you cry
 never gonna say goodbye
 never gonna tell a lie and hurt you
 we've known each other for so long
 our heart's been aching, but you're too shy to say it
 inside, we both know what's been going on
 we know the game and we're gonna ylap it
 and if you ask me how i'm feeling
 don't tell me you're too blind to see
 never gonna give you up
 never gonna let you down
 never gonna run around and desert you
 never gonna make you cry
 never gonna say goodbye
 never gonna tell a lie and hurt you
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r	7,1%
m	6,5%
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i	4,2%
w	4,0%
o	3,1%
f	2,4%
x	2,1%
n	2,0%
d	2,0%
u	1,9%
t	1,8%
k	1,4%
b	1,3%
p	1,1%
y	1,0%
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a	8,2%
o	7,5%
i	7,0%
n	6,7%
s	6,3%
h	6,1%
r	6,0%
d	4,3%
l	4,0%
c	2,8%
u	2,8%
m	2,4%
w	2,4%
f	2,2%
g	2,0%
y	2,0%
p	1,9%
k	1,8%
b	1,5%
v	1,0%
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z	0,1%

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we know the game and we're gonna play it
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gotta make you understand

Frequency analysis

Takeaway:

- The more predictable the cleartext is, the easier it is to crack.
- Frequency analysis requires a lot of text to make sense.
- This stuff was cracked 1200 years ago. Don't use it today.





Zimmermann's Gamble

Zimmermann's gamble



Arthur Zimmermann, German foreign secretary in 1916.

- For most of the first world war, the US tried to stay out of the conflict
- Following the sinking of the Lusitania in 1915, Germany pledged to only use their submarines on the surface.

Zimmermann's gamble

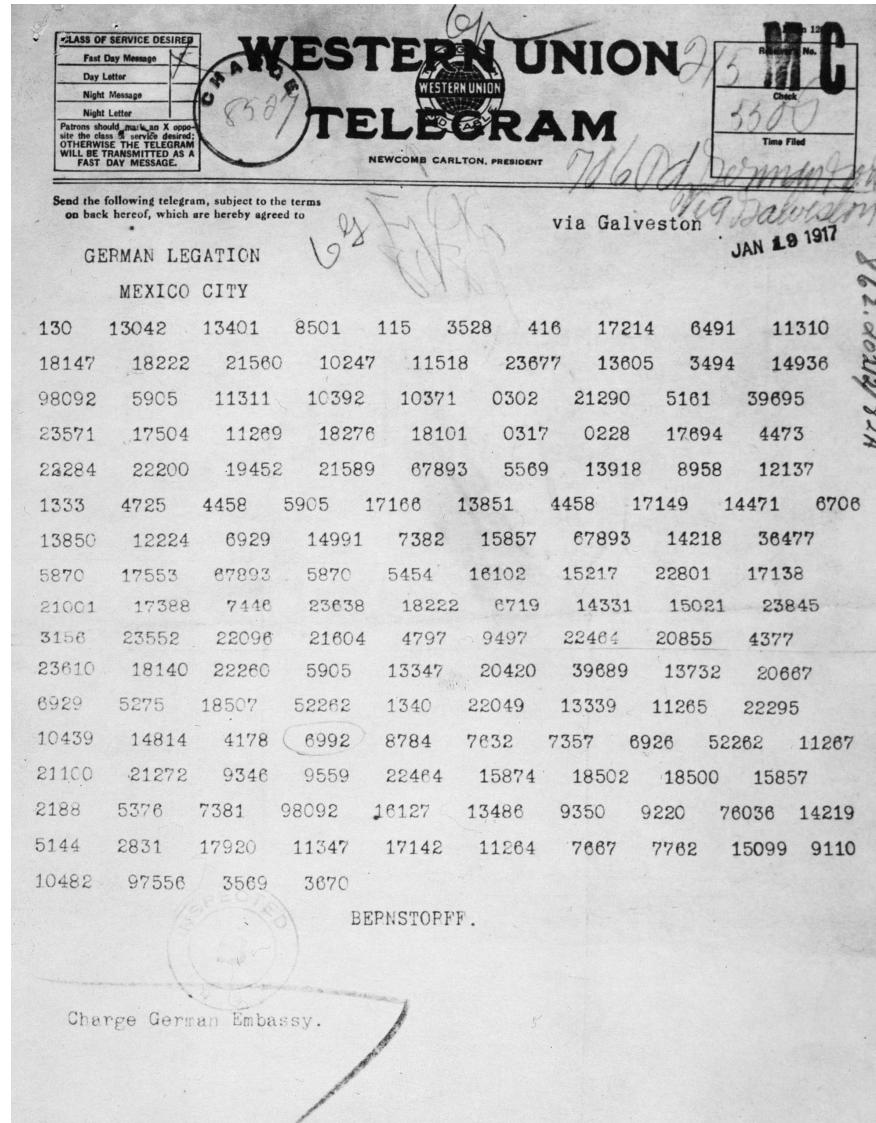


- In 1917, though, Germany thought an unrestricted submarine warfare could be decisive in winning the war.

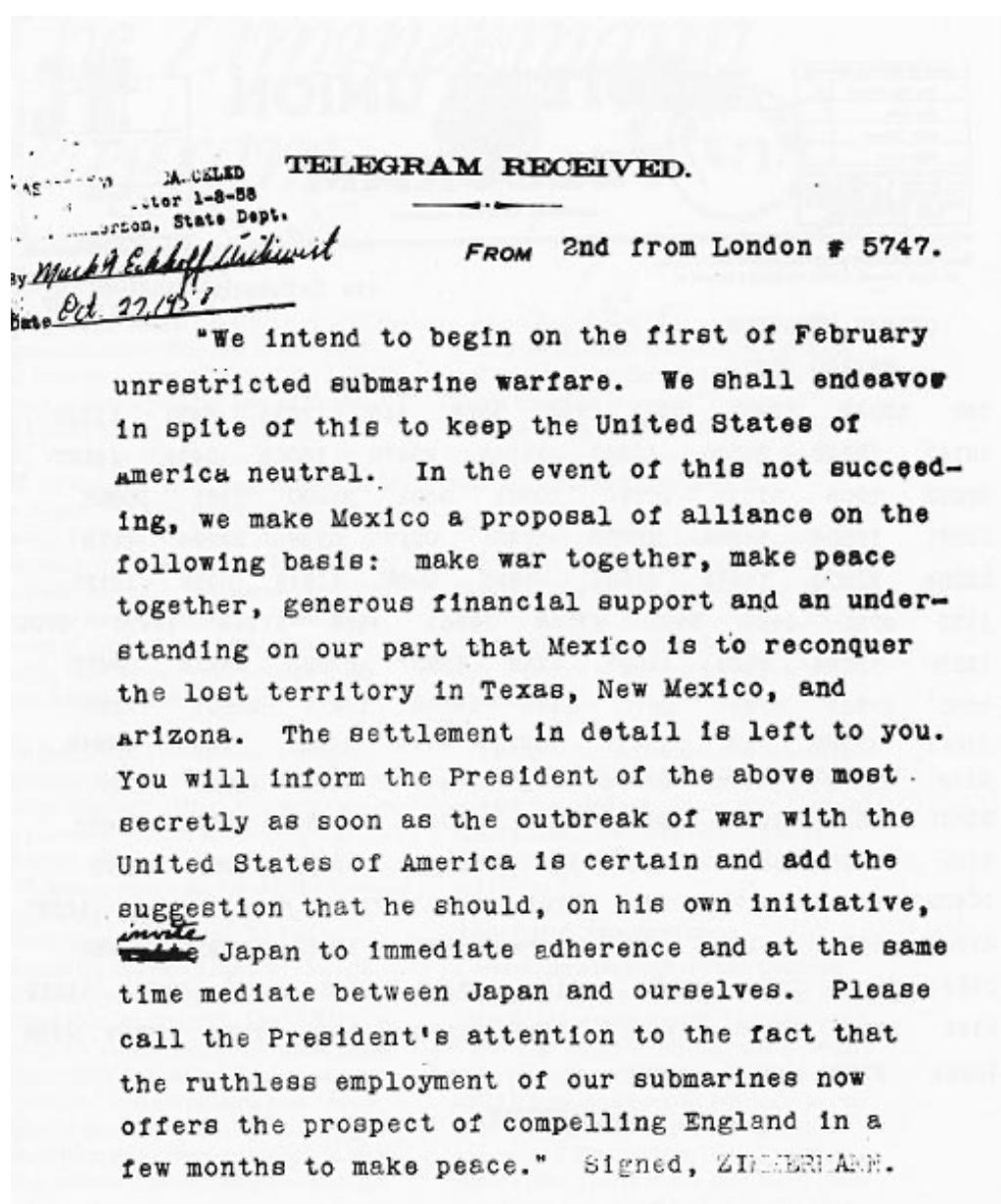
But they really did not want to pick a fight with the US.
So secretary Zimmermann hatched a plan.

Arthur Zimmermann, German foreign secretary in 1916.

Zimmermann's gamble



Zimmermann's gamble



Zimmermann's gamble



Arthur Zimmermann, German foreign secretary in 1916.

- Zimmermann publicly admitted in March 1917 that it was genuine.
- The publishing swayed US opinion to declare war only weeks later.

The Enigma

- A polyalphabetic substitution cipher
 - Three scramblers in series.
 - An additional reflector.
 - A plugboard that allows the user to additionally swap up to 6 characters.
- Frequency analysis is effectively useless.
- Reversible!
- The breaking of the Enigma was not made public until 1974 – more than 30 years after the fact.



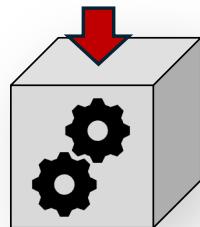


The advent of computers

Modern block ciphers

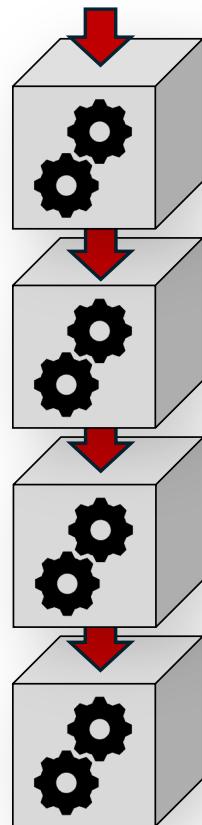
- With the adoption of computer communication, more complex block ciphers started emerging.
- In 1973 the DES algorithm, developed by IBM, was adopted as the national standard in the US.
- DES was, however, made deliberately weak with its 56-bit key.
- During a transitional period, triple-DES was considered adequate, although not great.
- AES has since become a globally accepted standard.

Anatomy of a modern block cipher



A substitution box (an “S-box”) processes the incoming data frame in a specific way.

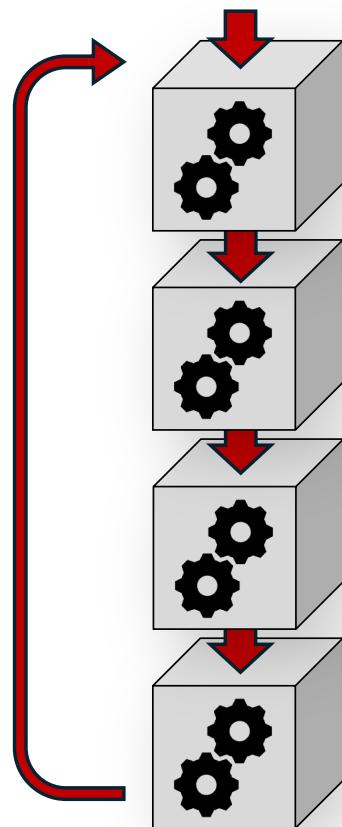
Anatomy of a modern block cipher



A substitution box (an “S-box”) processes the incoming data frame in a specific way.

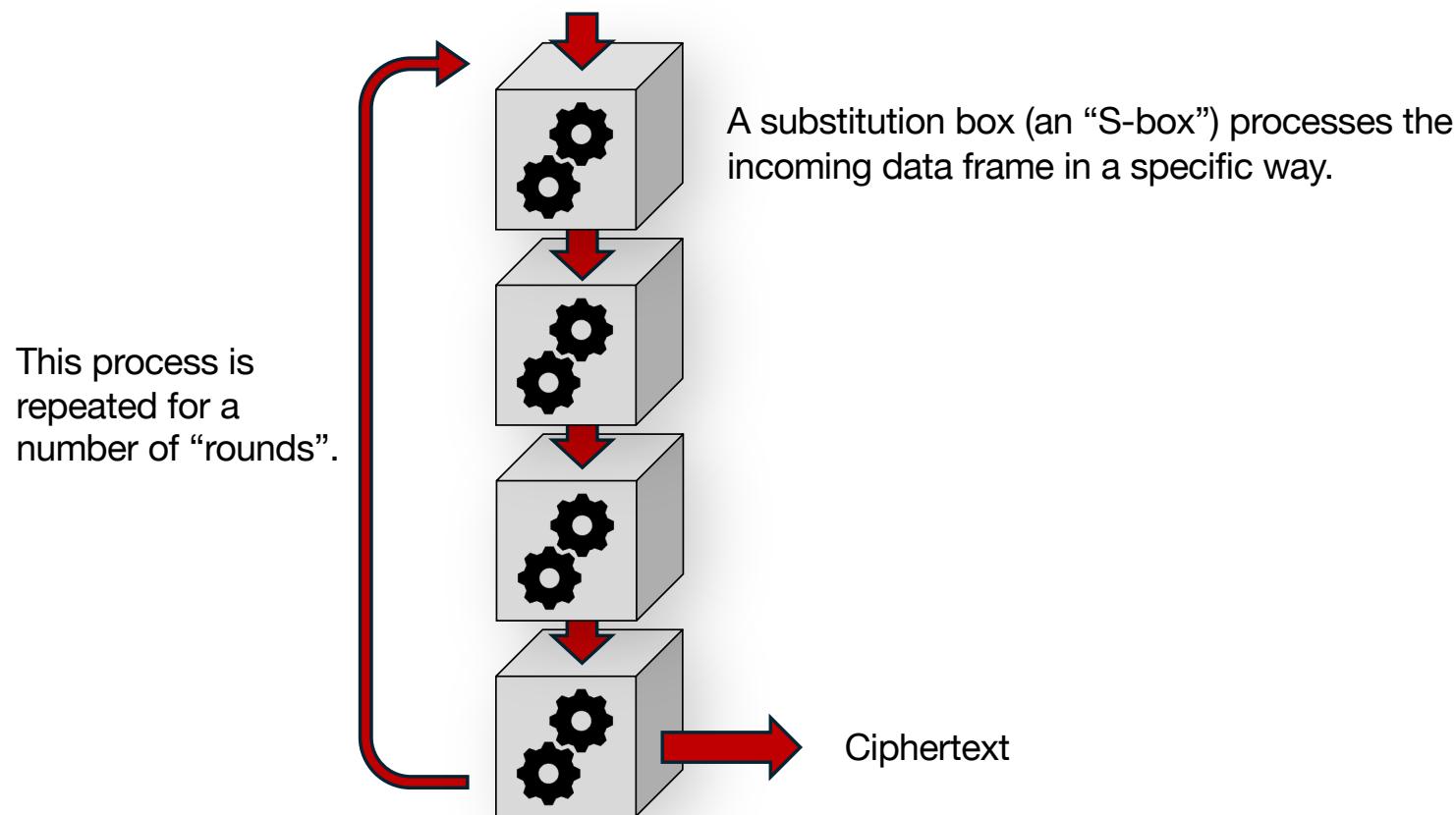
Anatomy of a modern block cipher

This process is repeated for a number of “rounds”.



A substitution box (an “S-box”) processes the incoming data frame in a specific way.

Anatomy of a modern block cipher



AES

Plaintext:

A	t	t	a	c	k		a	t		d	a	w	n	!	
41	74	74	61	63	6B	20	61	74	20	64	61	77	6E	21	00

Hex:

Password:

i	k	n	o	w	w	h	a	t	y	o	u	d	i	d	!
69	6B	6E	6F	77	77	68	61	74	79	6F	75	64	69	64	21

Hex:

AES

Data frame:

41	74	74	61
63	6b	20	61
74	20	64	61
77	6e	21	00



Plaintext:

A	t	t	a	c	k		a	t	d	a	w	n	!		
41	74	74	61	63	6B	20	61	74	20	64	61	77	6E	21	00

Hex:

Password:

i	k	n	o	w	w	h	a	t	y	o	u	d	i	d	!
69	6B	6E	6F	77	77	68	61	74	79	6F	75	64	69	64	21

Hex:

AES

Data frame:

41	74	74	61
63	6b	20	61
74	20	64	61
77	6e	21	00

Key 0:

69	77	74	64
6b	77	79	69
6e	68	6f	64
6f	61	75	21

Key 1:

91	E6	92	F6
28	5f	26	4f
93	Fb	94	F0
2c	4d	38	19

Key 2:

17	f1	63	95
a4	fb	dd	92
47	bc	28	d8
6e	23	1b	02

Key 3

5c	ac
c5	3e
30	8c
44	67

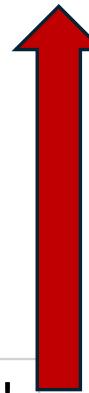
Key expansion

Turns a key into 10, 12 or 14 “round keys”, 16 bytes each.

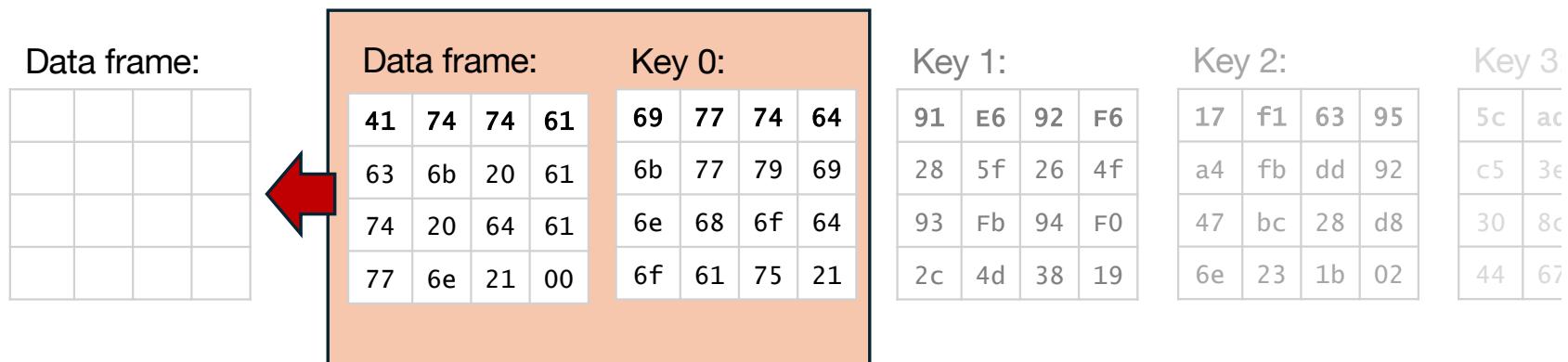
Password:

Hex:

i	k	n	o	w	w	h	a	t	y	o	u	d	i	d	!
69	6B	6E	6F	77	77	68	61	74	79	6F	75	64	69	64	21



AES



Key expansion

Turns a key into 10, 12 or 14 “round keys”, 16 bytes each.

AddRoundKey

XORs the data frame with the round key

SubBytes

ShiftRows

MixColumns

AES

Data frame:

Data frame:

41	74	74	61
63	6b	20	61
74	20	64	61

Key 0:

69	77	74	64
6b	77	79	69
6e	68	6f	64
6f	61	75	21

Key 1:

91	E6	92	F6
28	5f	26	4f
93	Fb	94	F0
2c	4d	38	19

Key 2:

17	f1	63	95
a4	fb	dd	92
c5	3e	30	8c
47	bc	28	d8
6e	23	1b	02

Key 3

5c	ad
c5	3e
30	8c
44	67

XOR: Either one or the other, but not both.

0x41 [] [] [] [] [] []

0x69 [] [] [] [] [] []

= 0x28 [] [] [] [] [] []

Key expansion turns a 16 byte key into 10, 12 or 14 “round keys”, 16 bytes each.

AddRoundKey

XORs the data frame with the round key

SubBytes

ShiftRows

MixColumns

AES

Data frame:

Data frame:

41	74	74	61
63	6b	20	61
74	20	64	61

Key 0:

69	77	74	64
6b	77	79	69
6e	68	6f	64
6f	61	75	21

Key 1:

91	E6	92	F6
28	5f	26	4f
93	Fb	94	F0
2c	4d	38	19

Key 2:

17	f1	63	95
a4	fb	dd	92
c5	3e	30	8c
47	bc	28	d8
6e	23	1b	02

Key 3

5c	ad
c5	3e
30	8c
44	67

XOR: Either one or the other, but not both.

0x41 0 1 0 0 0 0 1

0x69 0 1 1 0 1 0 0 1

= **0x28** [redacted]

Key expansion

turns a

, 12 or 14 “round keys”, 16 bytes each.

AddRoundKey

XORs the data frame with the round key

SubBytes

ShiftRows

MixColumns

AES

Data frame:

Data frame:

41	74	74	61
63	6b	20	61
74	20	64	61

Key 0:

69	77	74	64
6b	77	79	69
6e	68	6f	64
6f	61	75	21

Key 1:

91	E6	92	F6
28	5f	26	4f
93	Fb	94	F0
2c	4d	38	19

Key 2:

17	f1	63	95
a4	fb	dd	92
c5	3e	28	d8
30	8c	6e	02

5c	ad
c5	3e
30	8c
44	67

XOR: Either one or the other, but not both.

0x41 0 1 0 0 0 0 0 1

0x69 0 1 1 0 1 0 0 1

= 0x28 0 0 1 0 1 0 0 0

Key expansion

turns a

, 12 or 14 “round keys”, 16 bytes each.

AddRoundKey

XORs the data frame with the round key

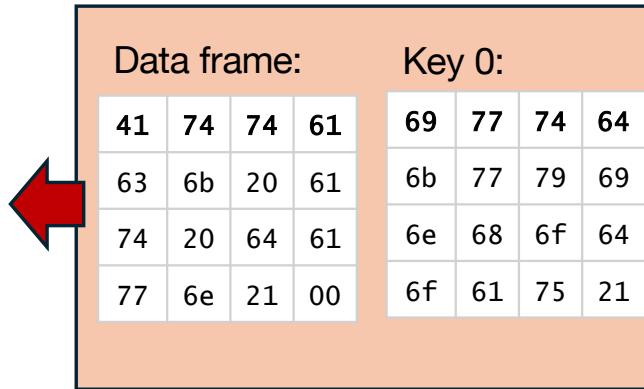
SubBytes

ShiftRows

MixColumns

AES

Data frame:			
28	03	00	05
08	1c	59	08
1a	48	0b	05
18	0f	54	21



Key 1:

91	E6	92	F6
28	5f	26	4f
93	Fb	94	F0
2c	4d	38	19

Key 2:

17	f1	63	95
a4	fb	dd	92
47	bc	28	d8
6e	23	1b	02

Key 3

5c	ac
c5	3e
30	8c
44	67

Key expansion

Turns a key into 10, 12 or 14 “round keys”, 16 bytes each.

AddRoundKey

XORs the data frame with the round key

SubBytes

ShiftRows

MixColumns

AES

Data frame:

34	7b	63	6b
30	9c	cb	30
a2	52	2b	6b
ad	76	20	fd



63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
e0	32	3a	0a	49	6	24	5c	c2	d3	ac	62	91	95	e4	79
e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

Key 1:

91	E6	92	F6
28	5f	26	4f
93	Fb	94	F0
2c	4d	38	19

Key 2:

17	f1	63	95
a4	fb	dd	92
47	bc	28	d8
6e	23	1b	02

Key 3

5c	ac
c5	3e
30	8c
44	67

Key expansion

Turns a key into 10, 12 or 14 “round keys”, 16 bytes each.

AddRoundKey

XORs the data frame with the round key

SubBytes

Applies a substitution (S-box) using a static table

ShiftRows

MixColumns

AES

Data frame:

34	7b	63	6b
9c	cb	30	30
2b	6b	a2	52
fd	ad	76	20

Data frame:

	34	7b	63	6b
	30	9c	cb	30
a2	52	2b	6b	
ad	76	20	fd	



Key 1:

91	E6	92	F6
28	5f	26	4f
93	Fb	94	F0
2c	4d	38	19

Key 2:

17	f1	63	95
a4	fb	dd	92
47	bc	28	d8
6e	23	1b	02

Key 3:

5c	ac
c5	3e
30	8c
44	67

Key expansion

Turns a key into 10, 12 or 14 “round keys”, 16 bytes each.

AddRoundKey

XORs the data frame with the round key

SubBytes

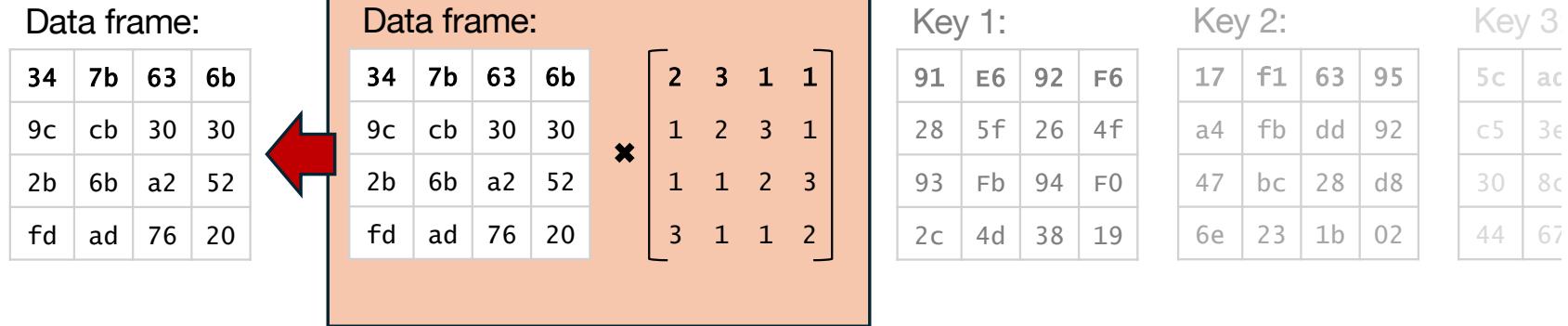
Applies a substitution (S-box) using a static table

ShiftRows

Shifts the rows in the data frame

MixColumns

AES



Key expansion

Turns a key into 10, 12 or 14 “round keys”, 16 bytes each.

AddRoundKey

XORs the data frame with the round key

SubBytes

Applies a substitution (S-box) using a static table

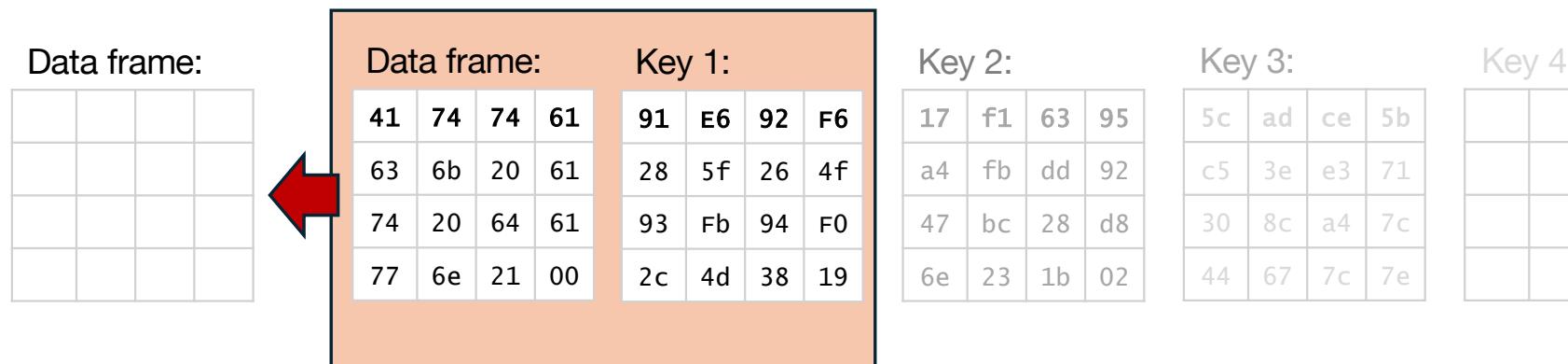
ShiftRows

Shifts the rows in the data frame

MixColumns

Uses matrix multiplication to create a substitution

AES



Repeat 9, 11, or 13 times

Key expansion

Turns a key into 10, 12 or 14 “round keys”, 16 bytes each.

AddRoundKey

XORs the data frame with the round key

SubBytes

Applies a substitution (S-box) using a static table

ShiftRows

Shifts the rows in the data frame

MixColumns

Uses matrix multiplication to create a substitution

AES

Used for:

- Data encryption for TLS (formerly SSL)
- Full disk encryption
- AES is ubiquitous - found in
 - software,
 - operating systems,
 - CPUs (including with native instructions),
 - PLCs, storage controllers, etc
 - USB flash drives
- The design allows for impressively high throughput

Even this thing has AES-256 encryption built in.



Hashing

Encryption

- Encryption generates “ciphertext”.

Hashing

- Hashing creates a “digest”.

Please pay the recipient \$100.



09urxmo298r2eofwehfi2ehfoi2efh

Please pay the recipient \$100.



0x4273a48237e9280c

Hashing

Encryption

- Encryption generates “ciphertext”.
- Encryption is *reversible*. One ciphertext generates exactly one plaintext.

Hashing

- Hashing creates a “digest”.
- Hashing is *not reversible*. Sometimes, the two plaintexts can generate the same digest

Please pay the recipient \$100.



09urxmo298r2eofwehfi2ehfoi2efh

Please pay the recipient \$100.



0x4273a48237e9280c

Hashing

Encryption

- Encryption generates “ciphertext”.
- Encryption is *reversible*. One ciphertext generates exactly one plaintext.
- Encryption uses a key or password

Hashing

- Hashing creates a “digest”.
- Hashing is *not reversible*. Sometimes, the two plaintexts can generate the same digest
- Hashing can use a “salt”

Please pay the recipient \$100.



09urxmo298r2eofwehfi2ehfoi2efh

Please pay the recipient \$100.



0x4273a48237e9280c

Hashing

Encryption

- Encryption generates “ciphertext”.
- Encryption is *reversible*. One ciphertext generates exactly one plaintext.
- Encryption uses a key or password
- Length of ciphertext is similar in length to plaintext

Please pay the recipient \$100.



09urxmo298r2eofwehfi2ehfoi2efh

Hashing

- Hashing creates a “digest”.
- Hashing is *not reversible*. Sometimes, the two plaintexts can generate the same digest
- Hashing can use a “salt”
- Digests are typically fixed-width

Please pay the recipient \$100.



0x4273a48237e9280c

Hashing

Hashing

passw0rd#1!



0x138d20ec87452a0f

Hashing

Hashing

passw0rd#1!



0x138d20ec87452a0f



0x138d20ec87452a0f



passw0rd#1!

There are “rainbow tables”
for common passwords,
allowing an attacker to
reverse-engineer hashes.

Hashing

Hashing

passw0rd#1!
↓
0x138d20ec87452a0f

0x138d20ec87452a0f
↓
passw0rd#1!

Hashing with salt

Salt
xyz
↓
0x39ac862c958d3460

Hashing

Hashing

passw0rd#1!
↓
0x138d20ec87452a0f



0x138d20ec87452a0f
↓
passw0rd#1!

Hashing with salt

Salt
xyz passw0rd#1!
↓
0x39ac862c958d3460

Different salt
abc passw0rd#1!
↓
0xaa1209752f83942b

Hashing

How are hashes used?

- git commits
- Storing passwords
- Detecting changes in code/text, etc
- Digital signatures

It gets more mathematical

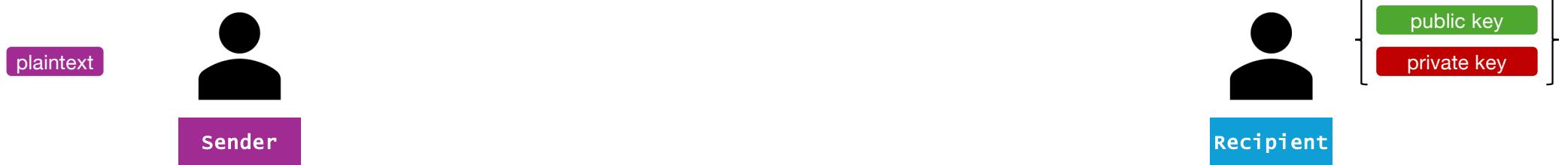
Asymmetric encryption

Also called “public key encryption”.

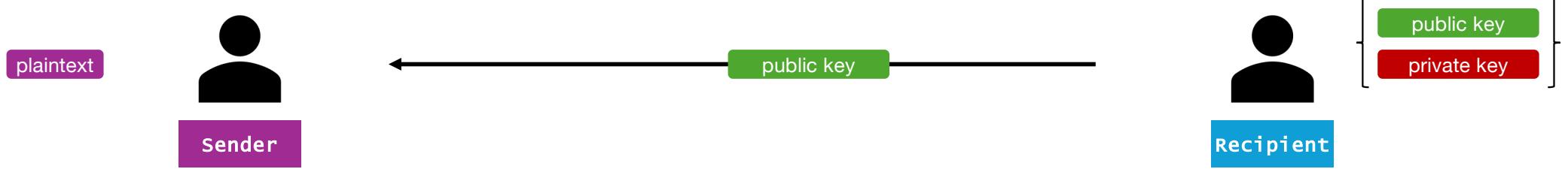
The two most common algorithms are

- RSA (Rivest, Shamir, Adleman)
- Elliptic Curve Encryption (ECC)

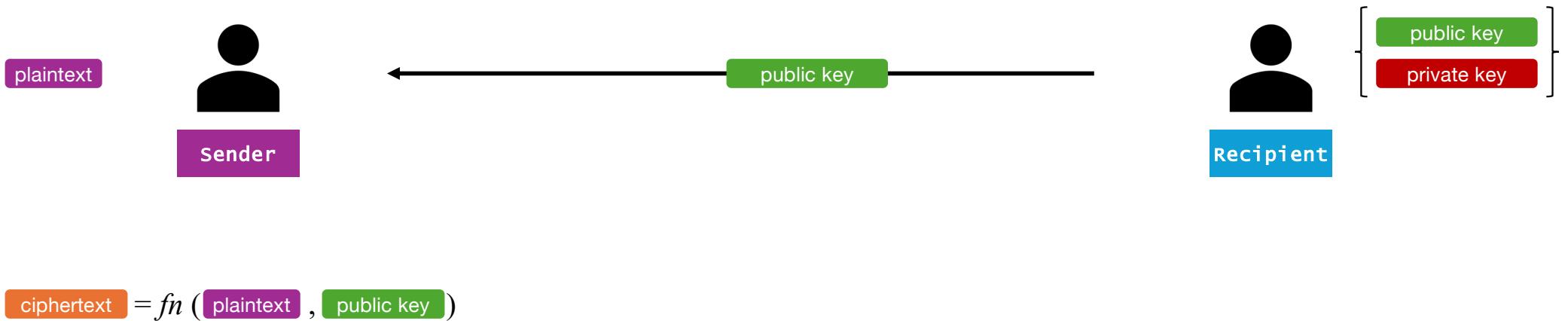
Asymmetric encryption



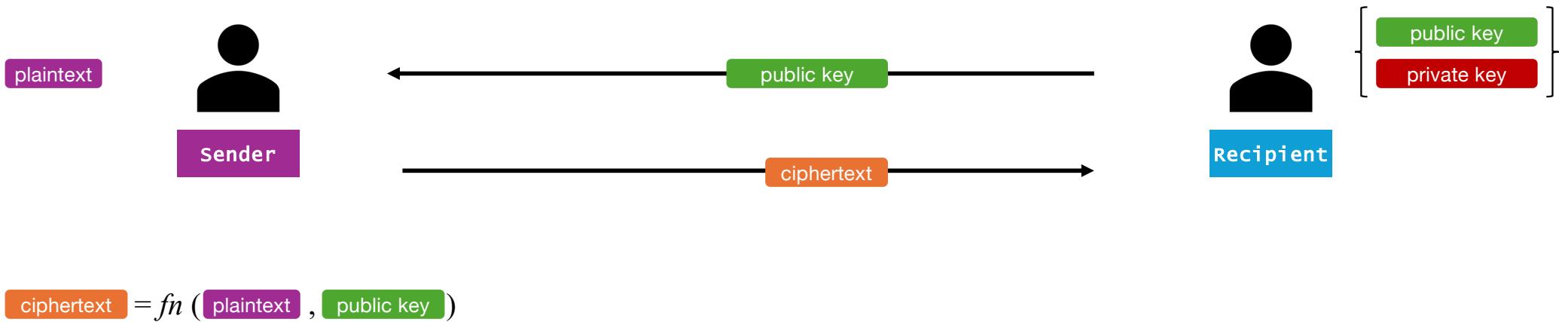
Asymmetric encryption



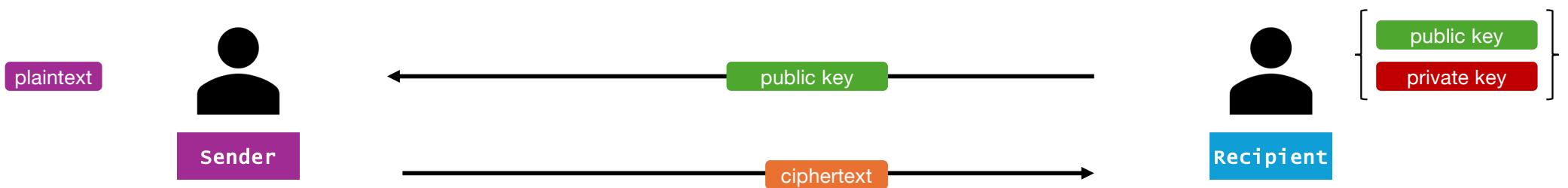
Asymmetric encryption



Asymmetric encryption



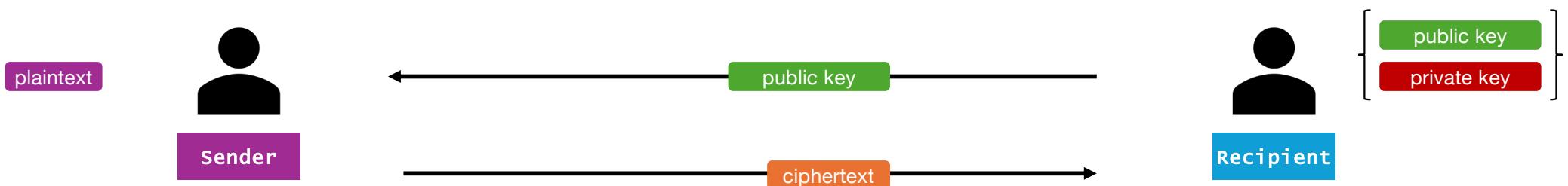
Asymmetric encryption



$$\text{ciphertext} = fn(\text{plaintext}, \text{public key})$$

$$\text{plaintext} = fn(\text{ciphertext}, \text{private key})$$

Asymmetric encryption



$$\text{ciphertext} = fn(\text{plaintext}, \text{public key})$$

$$\text{plaintext} = fn(\text{ciphertext}, \text{private key})$$

This "one-way" function is fairly simple math. But the reverse is very difficult.

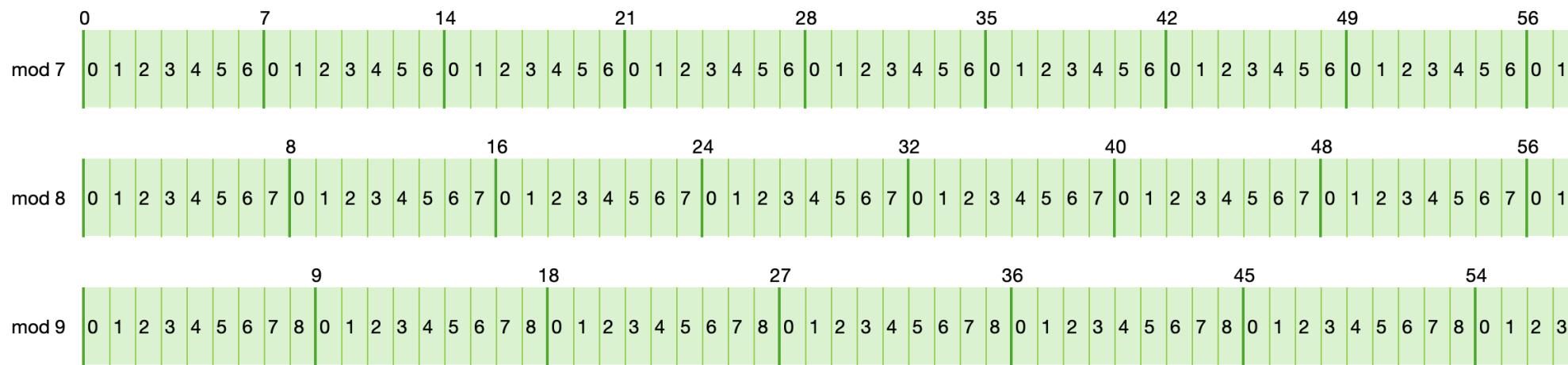
Jevons' number

"Can the reader say what two numbers multiplied together will produce the number 8,616,460,799? I think it unlikely that anyone but myself will ever know."

William Stanley Jevons
Principles of Science, 1874

Some required math basics

The **modulo** of an integer is the "remainder" when you divide the integer by the modulo.



Think of the modulo as an incrementing number that resets at intervals.

Some required math basics

A **prime** number is an integer that is evenly divisible by only itself and 1.

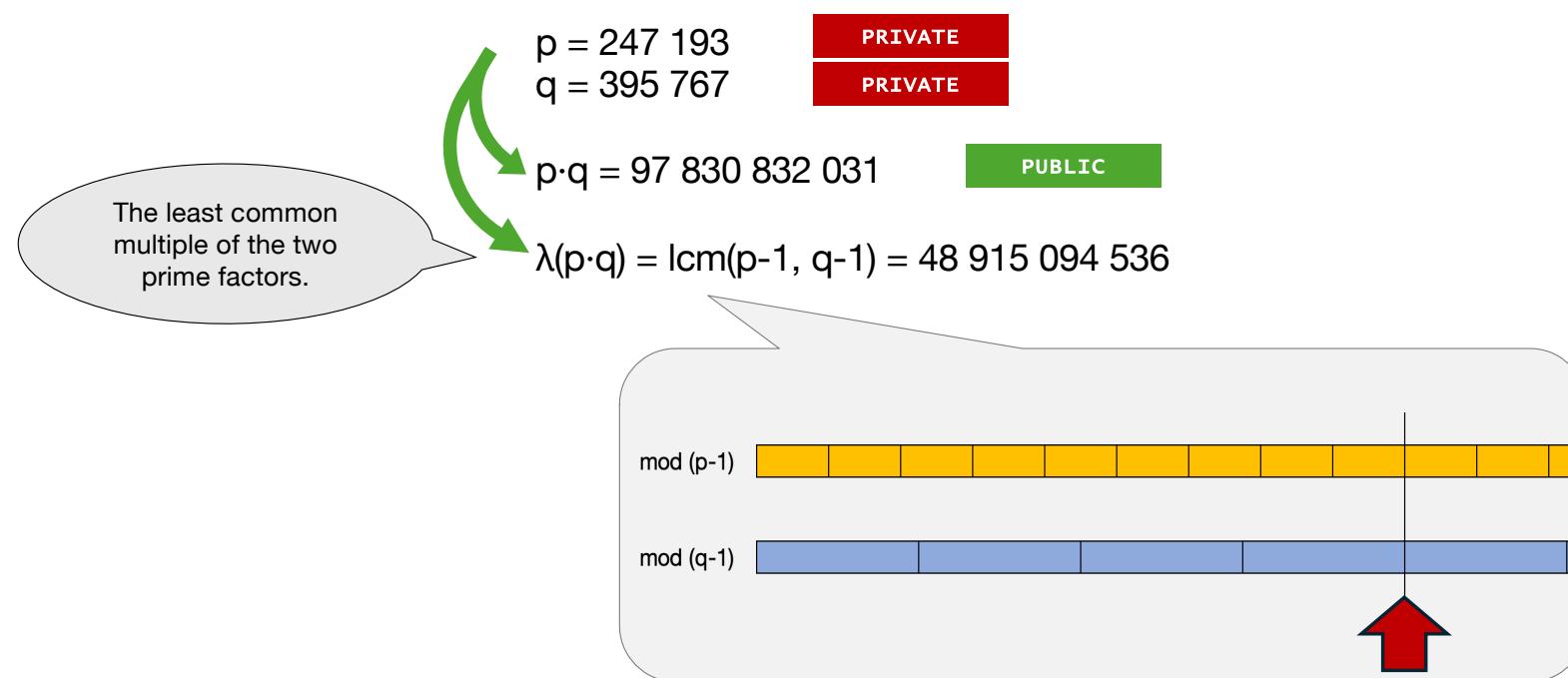
1	1 is prime	9	$9 = 3 \cdot 3$	17	17 is prime
2	2 is prime	10	$10 = 2 \cdot 5$	18	$18 = 2 \cdot 3 \cdot 3$
3	3 is prime	11	11 is prime	19	19 is prime
4	$4 = 2 \cdot 2$	12	$12 = 2 \cdot 2 \cdot 3$	20	$20 = 2 \cdot 2 \cdot 5$
5	5 is prime	13	13 is prime	21	$21 = 3 \cdot 7$
6	$6 = 2 \cdot 3$	14	$14 = 7 \cdot 2$	22	$22 = 11 \cdot 2$
7	7 is prime	15	$15 = 5 \cdot 3$	23	23 is prime
8	$8 = 2 \cdot 2 \cdot 2$	16	$16 = 2 \cdot 2 \cdot 2 \cdot 2$	24	$24 = 2 \cdot 2 \cdot 2 \cdot 3$

Every non-prime integer is a multiple of two or more prime numbers.

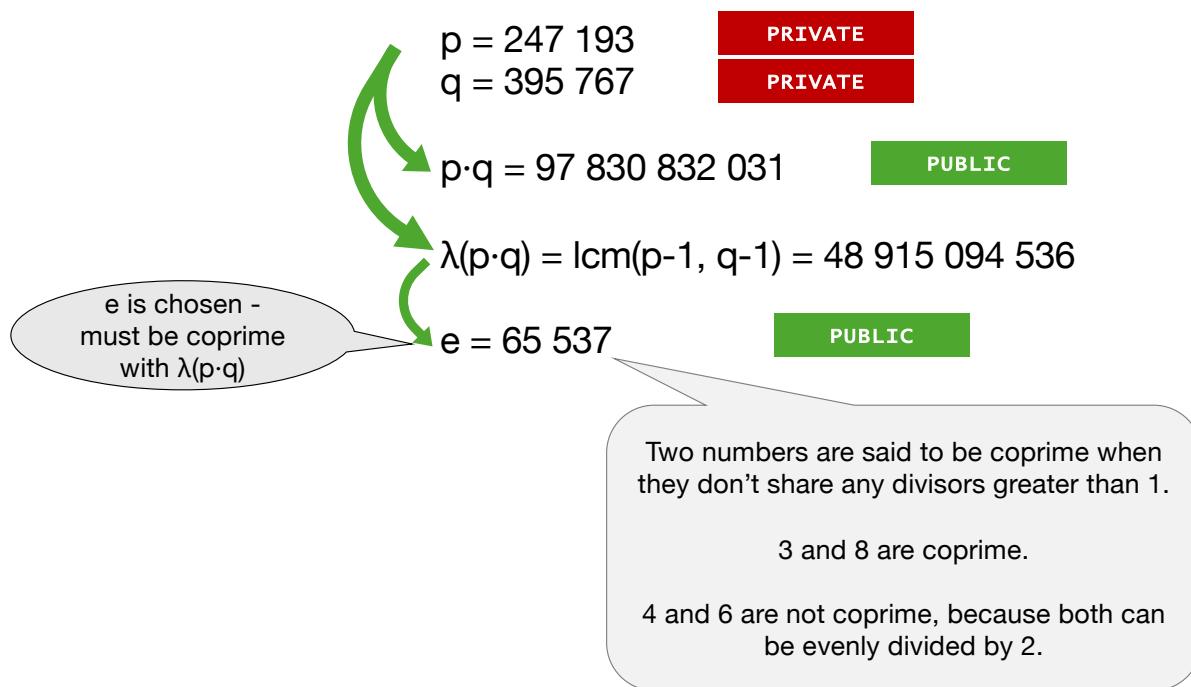
RSA



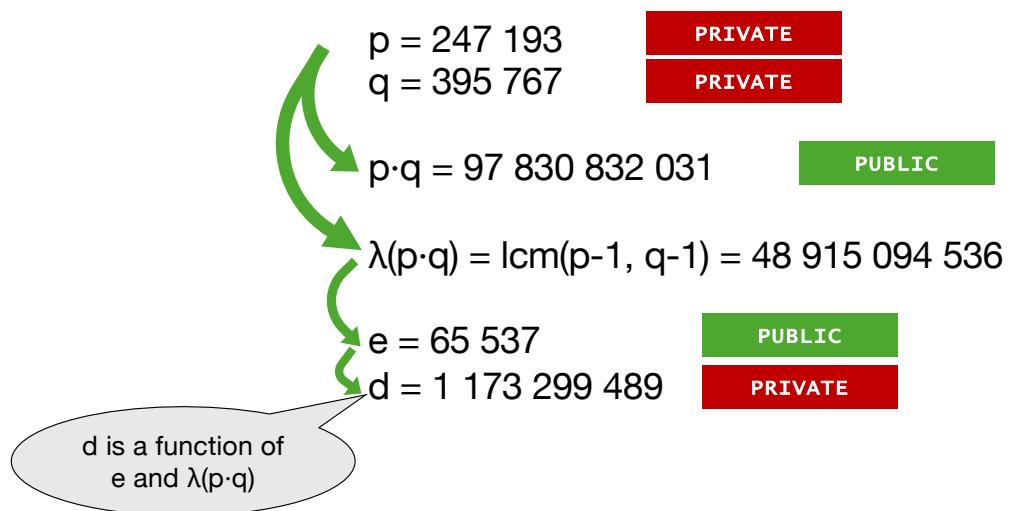
RSA



RSA



RSA



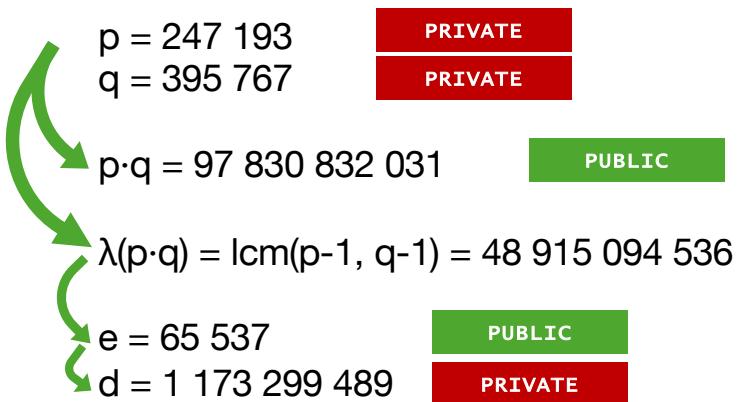
RSA

Public key

e p·q

Private key

d p·q



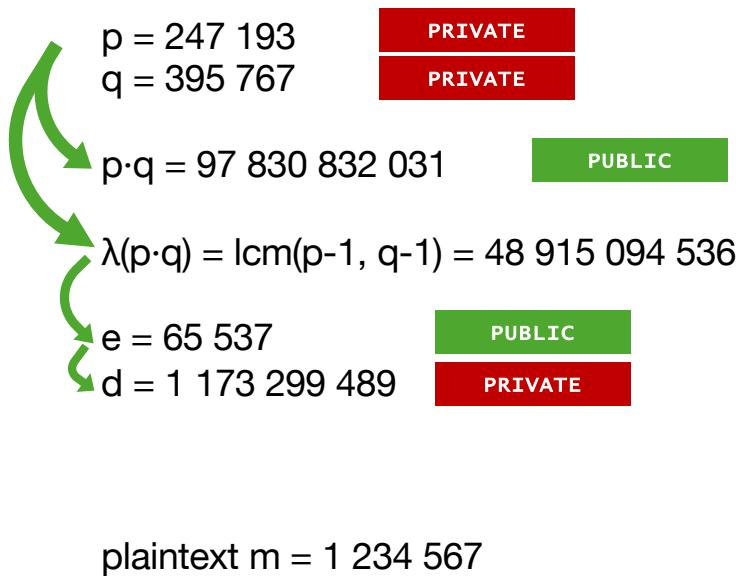
RSA

Public key

e p·q

Private key

d p·q



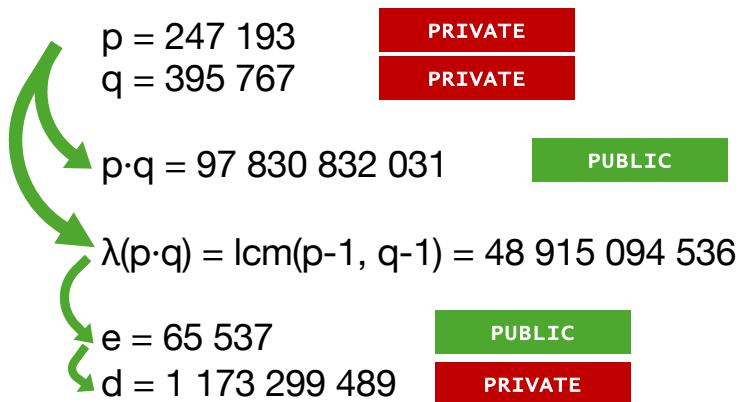
RSA

Public key

e p·q

Private key

d p·q



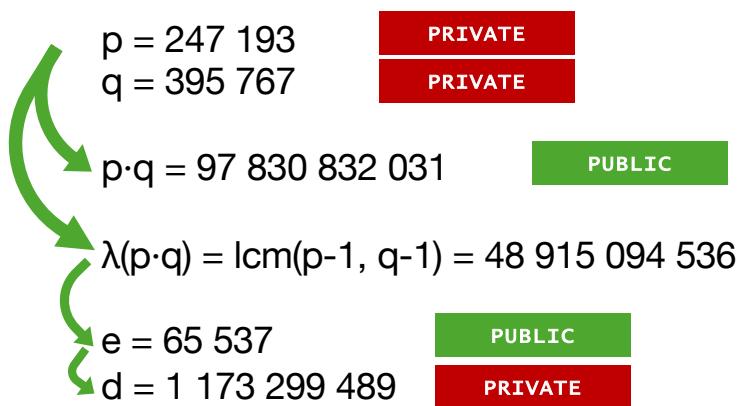
plaintext $m = 1\ 234\ 567$

Encryption: ciphertext $c = m^e \bmod p \cdot q = 65\ 699\ 382\ 295$

RSA

Public key
e p·q

Private key
d p·q



plaintext $m = 1\ 234\ 567$

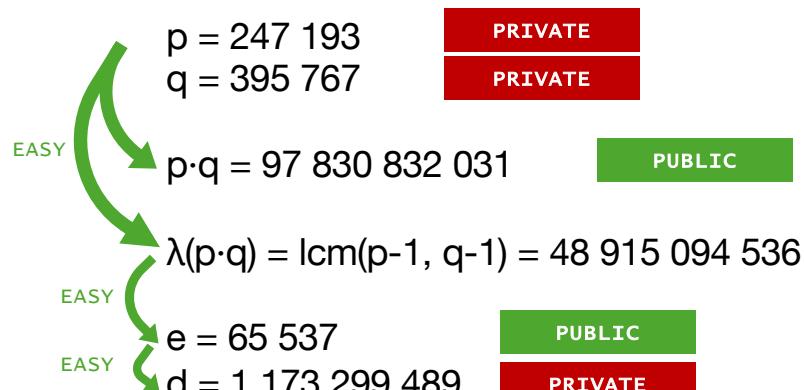
Encryption: ciphertext $c = m^e \bmod p \cdot q = 65\ 699\ 382\ 295$

Decryption: plaintext $m = c^d \bmod p \cdot q = 1\ 234\ 567$

RSA

Public key
e p·q

Private key
d p·q



plaintext m = 1 234 567

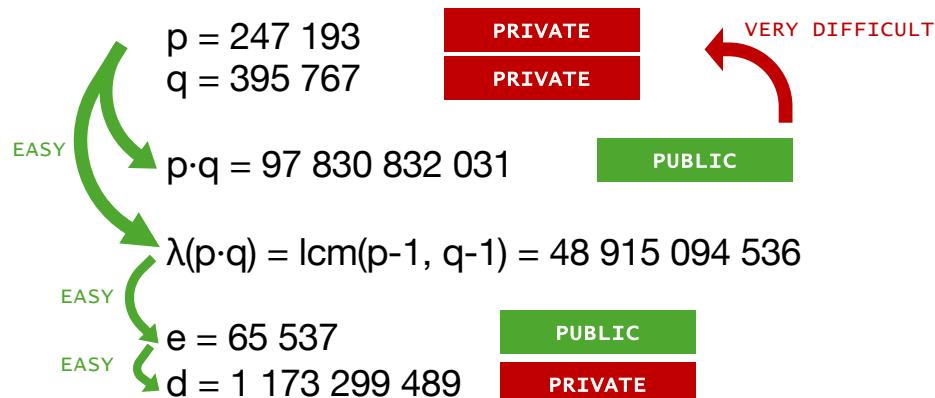
Encryption: ciphertext $c = m^e \pmod{p\cdot q} = 65 699 382 295$

Decryption: plaintext $m = c^d \pmod{p\cdot q} = 1 234 567$

RSA

Public key
e p·q

Private key
d p·q



plaintext m = 1 234 567

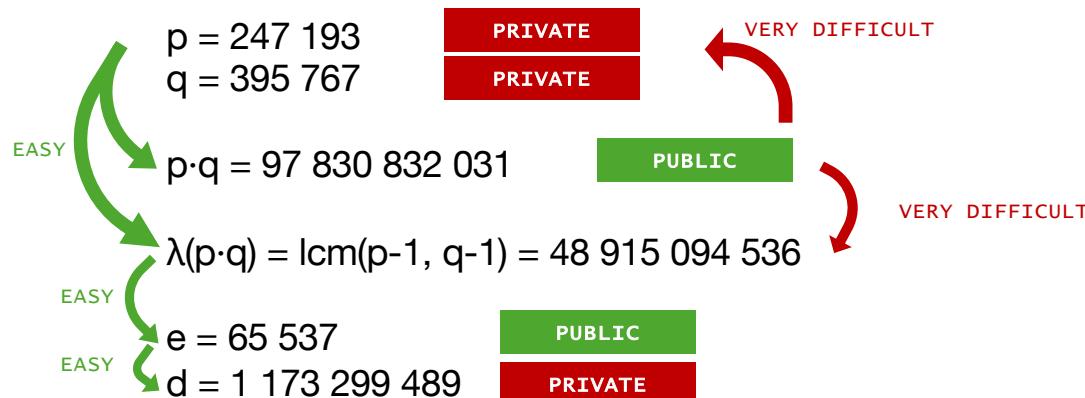
Encryption: ciphertext c = m^e mod p·q = 65 699 382 295

Decryption: plaintext m = c^d mod p·q = 1 234 567

RSA

Public key
e p·q

Private key
d p·q



plaintext $m = 1\ 234\ 567$

Encryption: ciphertext $c = m^e \bmod p \cdot q = 65\ 699\ 382\ 295$

Decryption: plaintext $m = c^d \bmod p \cdot q = 1\ 234\ 567$

RSA

Which two prime numbers multiply to form this result?

$p \cdot q = 307\ 557\ 980\ 540\ 279\ 294\ 232\ 889\ 703\ 319\ 188\ 128\ 660\ 178\ 473\ 934\ 208\ 428\ 941\ 529\ 507\ 290\ 014\ 700\ 153\ 668\ 237\ 193\ 840\ 016\ 959\ 648\ 364\ 895\ 607\ 412\ 313\ 512\ 951\ 227\ 128\ 221\ 045\ 224\ 559\ 205\ 896\ 270\ 620\ 516\ 337\ 082\ 808\ 739\ 837\ 588\ 961\ 254\ 353\ 255\ 830\ 027\ 840\ 166\ 730\ 656\ 182\ 059\ 939\ 220\ 104\ 108\ 619\ 578\ 923\ 418\ 753\ 353\ 272\ 689\ 302\ 172\ 594\ 198\ 324\ 164\ 702\ 665\ 735\ 456\ 123\ 372\ 112\ 931\ 626\ 325\ 928\ 229\ 001\ 407\ 813\ 673\ 728\ 165\ 736\ 146\ 453\ 855\ 297\ 160\ 675\ 344\ 337\ 682\ 232\ 580\ 861\ 895\ 550\ 087\ 017\ 268\ 994\ 973\ 859\ 666\ 439\ 292\ 146\ 128\ 040\ 198\ 542\ 069\ 704\ 950\ 256\ 297\ 882\ 300\ 231\ 190\ 908\ 477\ 379\ 997\ 770\ 136\ 082\ 327\ 312\ 952\ 935\ 689\ 262\ 892\ 629\ 588\ 356\ 961\ 572\ 827\ 315\ 993\ 037\ 083\ 976\ 288\ 531\ 844\ 687\ 385\ 055\ 499\ 903\ 044\ 511\ 450\ 039\ 012\ 753\ 955\ 234\ 146\ 300\ 919\ 159\ 567\ 440\ 161\ 336\ 043\ 261\ 154\ 130\ 838\ 051\ 072\ 922\ 104\ 944\ 728\ 496\ 403\ 982\ 179\ 433\ 674\ 737\ 196\ 227\ 320\ 743\ 852\ 387\ 755\ 112\ 827\ 166\ 448\ 480\ 967\ 725\ 082\ 114\ 732\ 889\ 029\ 922\ 825\ 155\ 341\ 072\ 387\ 200\ 684\ 641\ 384\ 860\ 543\ 585\ 552\ 397\ 175\ 025\ 316\ 086\ 544\ 491\ 738\ 636\ 866\ 945\ 634\ 642\ 543\ 579\ 069\ 450\ 381\ 289\ 326\ 765\ 766\ 927\ 135\ 738\ 548\ 449\ 509\ 733\ 934\ 196\ 182\ 156\ 369\ 067\ 294\ 637\ 972\ 434\ 294\ 530\ 583\ 620\ 547\ 004\ 745\ 403\ 650\ 343\ 545\ 789\ 619\ 780\ 447\ 563\ 019\ 123\ 590\ 591$

(roughly a 4096 bit key size)

RSA

Which two prime numbers

$p \cdot q = 307\ 557\ 980\ 540\ 279\ 294\ 895\ 607\ 412\ 313\ 512\ 951\ 227\ 120\ 059\ 939\ 220\ 104\ 108\ 619\ 578\ 673\ 728\ 165\ 736\ 146\ 453\ 855\ 950\ 256\ 297\ 882\ 300\ 231\ 190\ 844\ 687\ 385\ 055\ 499\ 903\ 044\ 51\ 403\ 982\ 179\ 433\ 674\ 737\ 196\ 2384\ 860\ 543\ 585\ 552\ 397\ 175\ 02\ 934\ 196\ 182\ 156\ 369\ 067\ 294\ 63$

(roughly a 4096 bit key size)



You

Which two prime numbers can be multiplied to form 307 557 980 540 279 294 232 889 703

319 188 128 660 178 473 934 208 428 941 529
507 290 014 700 153 668 237 193 840 016 959
648 364 895 607 412 313 512 951 227 128 221
045 224 559 205 896 270 620 516 337 082 808
739 837 588 961 254 353 255 830 027 840 166
730 656 182 059 939 220 104 108 619 578 923
418 753 353 272 689 302 172 594 198 324 164
702 665 735 456 123 372 112 931 626 325 928
229 001 407 813 673 728 165 736 146 453 855
297 160 675 344 337 682 232 580 861 895 550
087 017 268 994 973 859 666 439 292 146 128
040 198 542 069 704 950 256 297 882 300 231
190 908 477 379 997 770 136 082 327 312 952
935 689 262 892 629 588 356 961 572 827 315
993 037 083 976 288 531 844 687 385 055 499
903 044 511 450 039 012 753 955 234 146 300
919 159 567 440 161 336 043 261 154 130 838
051 072 922 104 944 728 496 403 982 179 433
674 737 196 227 320 743 852 387 755 112 827
166 448 480 967 725 082 114 732 889 029 922
825 155 341 072 387 200 684 641 384 860 543
585 552 397 175 025 316 086 544 491 738 636

0 016 959 648 364
166 730 656 182
229 001 407 813
198 542 069 704
083 976 288 531
104 944 728 496
387 200 684 641
548 449 509 733

RSA

Which two prime numbers

$p \cdot q = 307\ 557\ 980\ 540\ 279\ 294\ 895\ 607\ 412\ 313\ 512\ 951\ 227\ 120\ 059\ 939\ 220\ 104\ 108\ 619\ 578\ 926\ 673\ 728\ 165\ 736\ 146\ 453\ 855\ 295\ 950\ 256\ 297\ 882\ 300\ 231\ 190\ 908\ 844\ 687\ 385\ 055\ 499\ 903\ 044\ 514\ 403\ 982\ 179\ 433\ 674\ 737\ 196\ 238\ 384\ 860\ 543\ 585\ 552\ 397\ 175\ 029\ 934\ 196\ 182\ 156\ 369\ 067\ 294\ 63$

(roughly a 4096 bit key size)



Copilot

The two prime numbers that can be multiplied to form the given number are:

5915587277

X

519700308044732377919392591523925225

Please note that these are very large prime numbers and the multiplication of these two primes results in the given number. Prime numbers are numbers that have only two distinct positive divisors: 1 and itself. The concept of prime numbers is fundamental in number theory. They have several applications in computer science, cryptography, and other fields.



• 1 of 30

0 016 959 648 364
166 730 656 182
229 001 407 813
198 542 069 704
083 976 288 531
104 944 728 496
387 200 684 641
548 449 509 733

RSA

Which two prime numbers

$p \cdot q = 307\ 557\ 980\ 540\ 279\ 294\ 895\ 607\ 412\ 313\ 512\ 951\ 227\ 12059\ 939\ 220\ 104\ 108\ 619\ 578\ 92673\ 728\ 165\ 736\ 146\ 453\ 855\ 2950\ 256\ 297\ 882\ 300\ 231\ 190\ 90844\ 687\ 385\ 055\ 499\ 903\ 044\ 51403\ 982\ 179\ 433\ 674\ 737\ 196\ 2384\ 860\ 543\ 585\ 552\ 397\ 175\ 02934\ 196\ 182\ 156\ 369\ 067\ 294\ 63$

0 016 959 648 364
166 730 656 182
229 001 407 813
198 542 069 704
083 976 288 531
104 944 728 496
387 200 684 641
548 449 509 733

(roughly a 4096 bit key size)



RSA

Which two prime numbers multiply to form this result?

$p \cdot q = 307\ 557\ 980\ 540\ 279\ 294\ 232\ 889\ 703\ 319\ 188\ 128\ 660\ 178\ 473\ 934\ 208\ 428\ 941\ 529\ 507\ 290\ 014\ 700\ 153\ 668\ 237\ 193\ 840\ 016\ 959\ 648\ 364\ 895\ 607\ 412\ 313\ 512\ 951\ 227\ 128\ 221\ 045\ 224\ 559\ 205\ 896\ 270\ 620\ 516\ 337\ 082\ 808\ 739\ 837\ 588\ 961\ 254\ 353\ 255\ 830\ 027\ 840\ 166\ 730\ 656\ 182\ 059\ 939\ 220\ 104\ 108\ 619\ 578\ 923\ 418\ 753\ 353\ 272\ 689\ 302\ 172\ 594\ 198\ 324\ 164\ 702\ 665\ 735\ 456\ 123\ 372\ 112\ 931\ 626\ 325\ 928\ 229\ 001\ 407\ 813\ 673\ 728\ 165\ 736\ 146\ 453\ 855\ 297\ 160\ 675\ 344\ 337\ 682\ 232\ 580\ 861\ 895\ 550\ 087\ 017\ 268\ 994\ 973\ 859\ 666\ 439\ 292\ 146\ 128\ 040\ 198\ 542\ 069\ 704\ 950\ 256\ 297\ 882\ 300\ 231\ 190\ 908\ 477\ 379\ 997\ 770\ 136\ 082\ 327\ 312\ 952\ 935\ 689\ 262\ 892\ 629\ 588\ 356\ 961\ 572\ 827\ 315\ 993\ 037\ 083\ 976\ 288\ 531\ 844\ 687\ 385\ 055\ 499\ 903\ 044\ 511\ 450\ 039\ 012\ 753\ 955\ 234\ 146\ 300\ 919\ 159\ 567\ 440\ 161\ 336\ 043\ 261\ 154\ 130\ 838\ 051\ 072\ 922\ 104\ 944\ 728\ 496\ 403\ 982\ 179\ 433\ 674\ 737\ 196\ 227\ 320\ 743\ 852\ 387\ 755\ 112\ 827\ 166\ 448\ 480\ 967\ 725\ 082\ 114\ 732\ 889\ 029\ 922\ 825\ 155\ 341\ 072\ 387\ 200\ 684\ 641\ 384\ 860\ 543\ 585\ 552\ 397\ 175\ 025\ 316\ 086\ 544\ 491\ 738\ 636\ 866\ 945\ 634\ 642\ 543\ 579\ 069\ 450\ 381\ 289\ 326\ 765\ 766\ 927\ 135\ 738\ 548\ 449\ 509\ 733\ 934\ 196\ 182\ 156\ 369\ 067\ 294\ 637\ 972\ 434\ 294\ 530\ 583\ 620\ 547\ 004\ 745\ 403\ 650\ 343\ 545\ 789\ 619\ 780\ 447\ 563\ 019\ 123\ 590\ 591$

The correct answer is

$p = 5\ 357\ 543\ 035\ 931\ 336\ 604\ 742\ 125\ 245\ 300\ 009\ 052\ 807\ 024\ 058\ 527\ 668\ 037\ 218\ 751\ 941\ 851\ 755\ 255\ 624\ 680\ 612\ 465\ 991\ 894\ 078\ 479\ 290\ 637\ 973\ 364\ 587\ 765\ 734\ 125\ 935\ 726\ 428\ 461\ 570\ 217\ 992\ 288\ 787\ 349\ 287\ 401\ 967\ 283\ 887\ 412\ 115\ 492\ 710\ 537\ 302\ 531\ 185\ 570\ 938\ 977\ 091\ 076\ 523\ 237\ 491\ 790\ 970\ 633\ 699\ 383\ 779\ 582\ 771\ 973\ 038\ 531\ 457\ 285\ 598\ 238\ 843\ 271\ 083\ 830\ 214\ 915\ 826\ 312\ 193\ 418\ 602\ 834\ 035\ 927$

and

$q = 57\ 406\ 534\ 763\ 712\ 726\ 211\ 641\ 660\ 058\ 884\ 099\ 201\ 115\ 885\ 104\ 434\ 760\ 023\ 882\ 136\ 841\ 288\ 313\ 069\ 618\ 515\ 692\ 832\ 974\ 315\ 825\ 313\ 495\ 922\ 298\ 231\ 949\ 373\ 138\ 672\ 355\ 948\ 043\ 152\ 766\ 571\ 296\ 567\ 808\ 332\ 659\ 269\ 564\ 994\ 572\ 656\ 140\ 000\ 344\ 389\ 574\ 120\ 022\ 435\ 714\ 463\ 495\ 031\ 743\ 122\ 390\ 807\ 731\ 823\ 194\ 181\ 973\ 658\ 513\ 020\ 233\ 176\ 985\ 452\ 498\ 279\ 081\ 199\ 404\ 472\ 314\ 802\ 811\ 655\ 824\ 768\ 082\ 110\ 985\ 166\ 340\ 672\ 084\ 454\ 492\ 229\ 252\ 801\ 189\ 742\ 403\ 957\ 029\ 450\ 467\ 388\ 250\ 214\ 501\ 358\ 353\ 312\ 915\ 261\ 004\ 066\ 118\ 140\ 645\ 880\ 633\ 941\ 658\ 603\ 299\ 497\ 698\ 209\ 063\ 510\ 889\ 929\ 202\ 021\ 079\ 926\ 591\ 625\ 770\ 444\ 716\ 951\ 045\ 960\ 277\ 478\ 891\ 794\ 836\ 019\ 580\ 040\ 978\ 608\ 315\ 291\ 377\ 690\ 212\ 791\ 863\ 007\ 764\ 174\ 393\ 209\ 716\ 027\ 254\ 457\ 637\ 891\ 941\ 312\ 587\ 717\ 764\ 400\ 411\ 421\ 385\ 408\ 982\ 726\ 881\ 092\ 425\ 574\ 515\ 033$

RSA

Which two prime numbers multiply to form this result?

$p \cdot q = 307\ 557\ 980\ 540\ 279\ 29$
895 607 412 313 512 951 227
059 939 220 104 108 619 578
673 728 165 736 146 453 855
950 256 297 882 300 231 190
844 687 385 055 499 903 044
403 982 179 433 674 737 196
384 860 543 585 552 397 175
934 196 182 156 369 067 294

The correct answer is

$p = 5\ 357\ 543\ 035\ 931\ 336\ 60$
637 973 364 587 765 734 125
076 523 237 491 790 970 633

and

$q = 57\ 406\ 534\ 763\ 712\ 726\ 2$
922 298 231 949 373 138 672 555 948 043 152 766 571 290 567 808 552 059 209 564 994 572 656 140 000 544 589 574 120 022 435 714 463 495
031 743 122 390 807 731 823 194 181 973 658 513 020 233 176 985 452 498 279 081 199 404 472 314 802 811 655 824 768 082 110 985 166 340
672 084 454 492 229 252 801 189 742 403 957 029 450 467 388 250 214 501 358 353 312 915 261 004 066 118 140 645 880 633 941 658 603 299
497 698 209 063 510 889 929 202 021 079 926 591 625 770 444 716 951 045 960 277 478 891 794 836 019 580 040 978 608 315 291 377 690 212
791 863 007 764 174 393 209 716 027 254 457 637 891 941 312 587 717 764 400 411 421 385 408 982 726 881 092 425 574 515 033

If you guessed correctly,

just shake your head and smile.

And never, ever tell a soul.



7 193 840 016 959 648 364
027 840 166 730 656 182
325 928 229 001 407 813
128 040 198 542 069 704
993 037 083 976 288 531
072 922 104 944 728 496
341 072 387 200 684 641
135 738 548 449 509 733
590 591

2 465 991 894 078 479 290
531 185 570 938 977 091
418 602 834 035 927

92 832 974 315 825 313 495
120 022 435 714 463 495
031 743 122 390 807 731 823 194 181 973 658 513 020 233 176 985 452 498 279 081 199 404 472 314 802 811 655 824 768 082 110 985 166 340
672 084 454 492 229 252 801 189 742 403 957 029 450 467 388 250 214 501 358 353 312 915 261 004 066 118 140 645 880 633 941 658 603 299
497 698 209 063 510 889 929 202 021 079 926 591 625 770 444 716 951 045 960 277 478 891 794 836 019 580 040 978 608 315 291 377 690 212
791 863 007 764 174 393 209 716 027 254 457 637 891 941 312 587 717 764 400 411 421 385 408 982 726 881 092 425 574 515 033

RSA

RSA has a published list of prime factors, some of which have prize money attached.

The largest RSA number factored so far is "RSA-250", equivalent of a 829 bit key, which was factored in 2020.

The computation effort is roughly equivalent to 2700 CPU core years on a 2.1 GHz Xeon.

The patent number for RSA, 4 405 829, is actually prime.

Elliptic Curve (ECC)

Way, waaaaay too complicated for an introductory session.

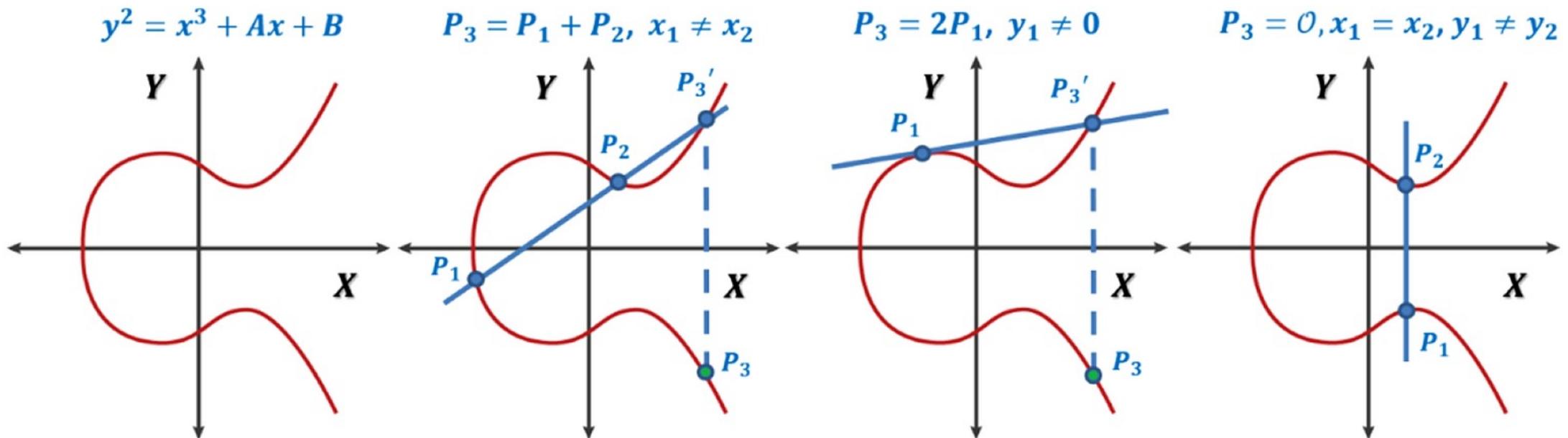


Image source: <https://www.nature.com/articles/s41598-022-17045-x>

RSA & ECC: Asymmetric encryption

How is asymmetric encryption used?

- Key exchange for TLS (formerly SSL), SSH
- Blockchains, crypto currencies
- Secure signatures

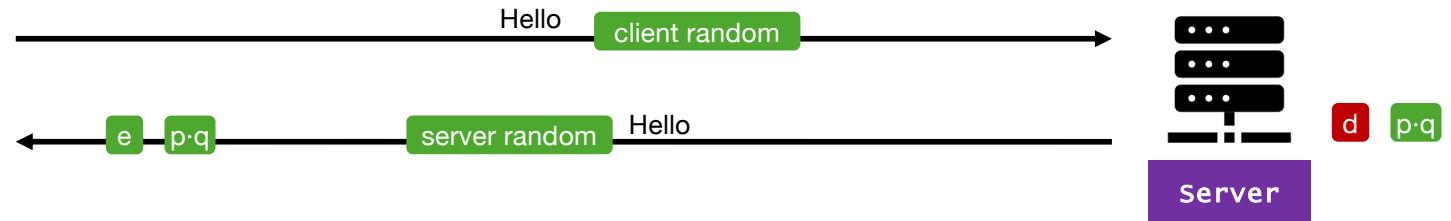
Asymmetric algorithms are typically only used to encrypt the key or hash digest. The data is encrypted using much faster symmetric encryption.

Big picture

Asymmetric



Client

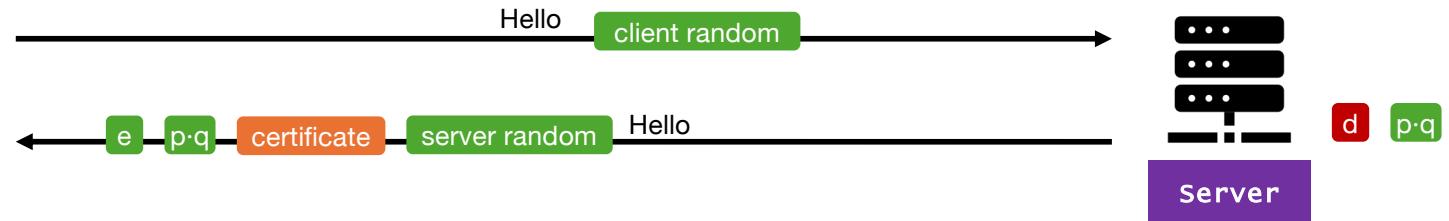


Big picture

Asymmetric



Client

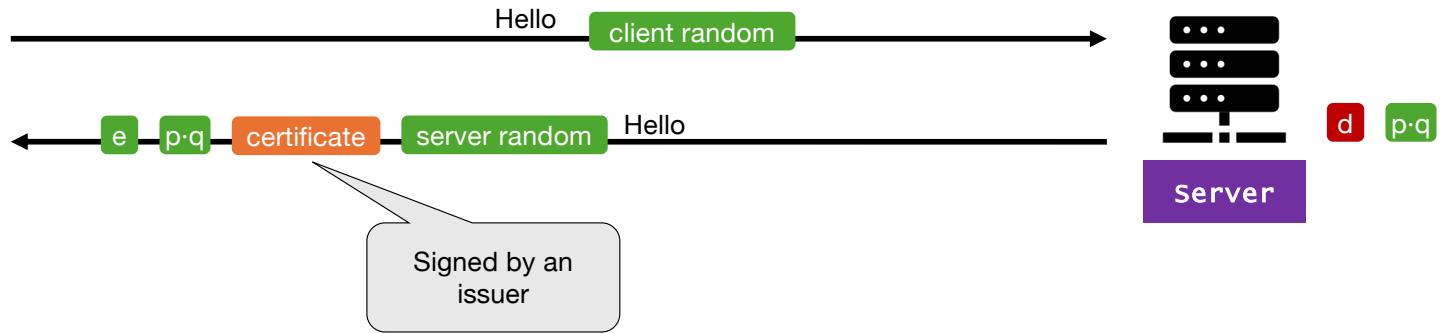


Asymmetric

Big picture

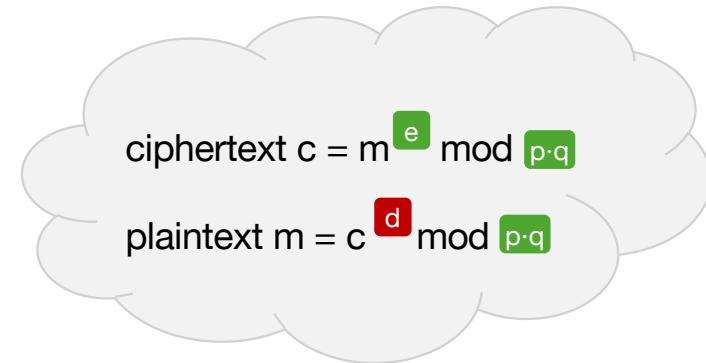


Client

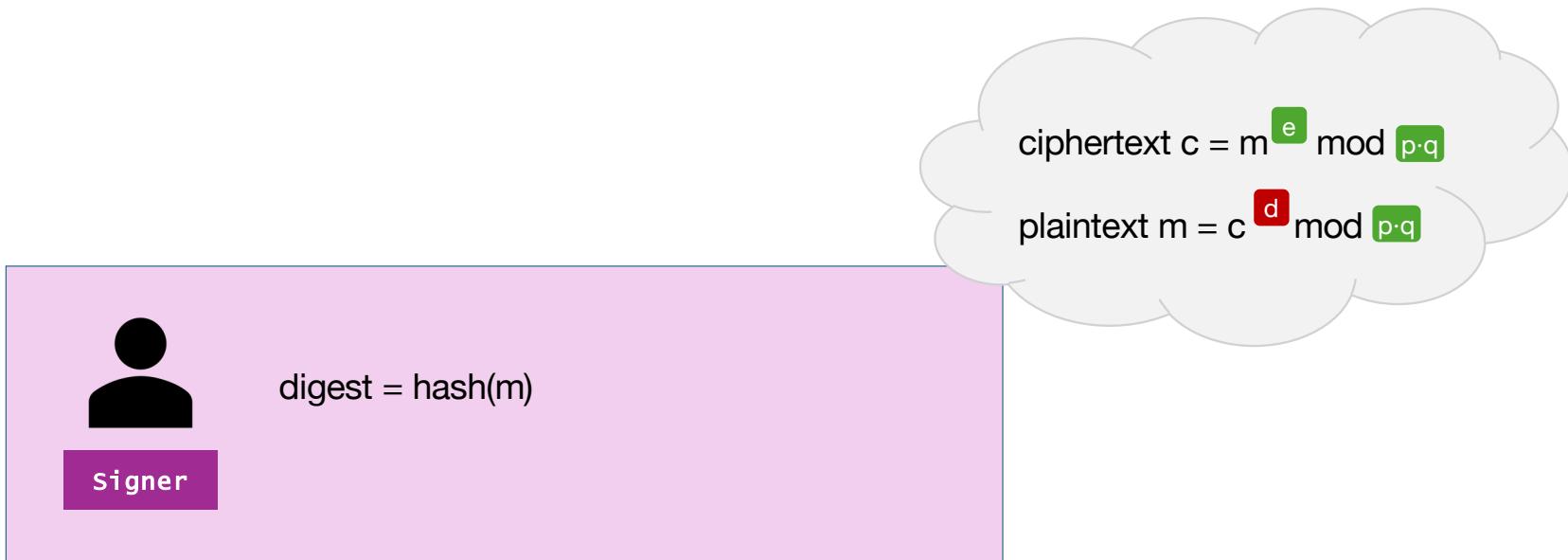


Signatures

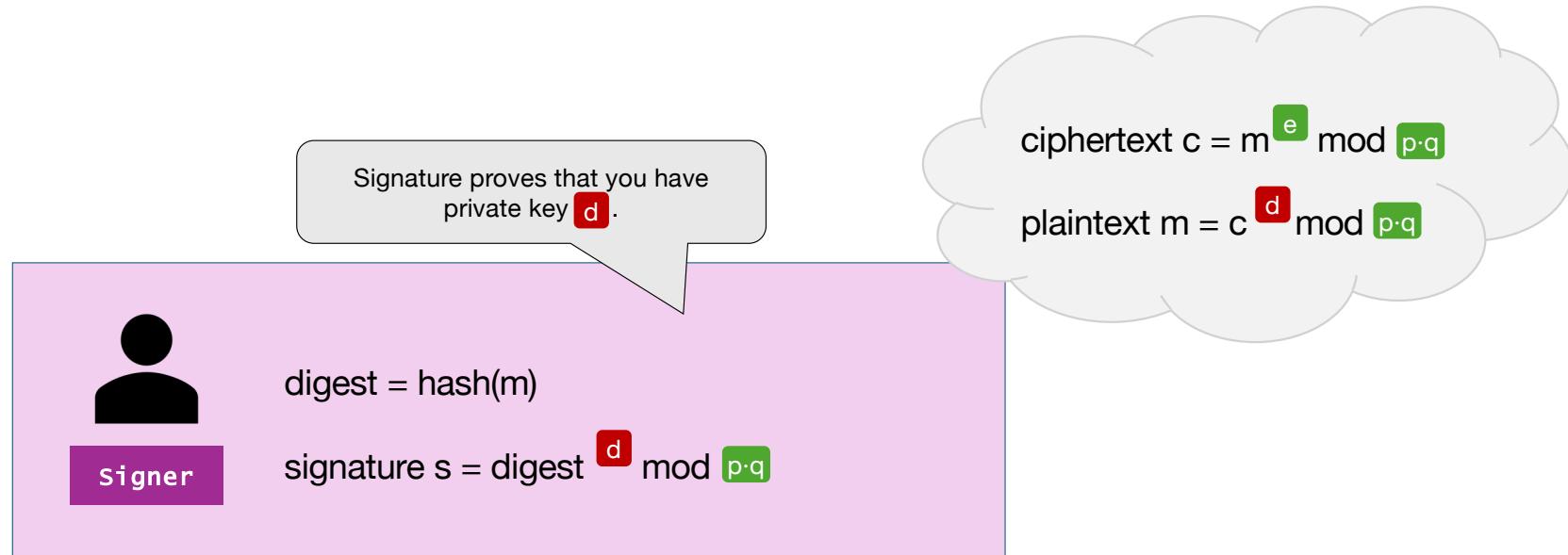
Signing uses the same function as encryption and decryption, but e and d are swapped.



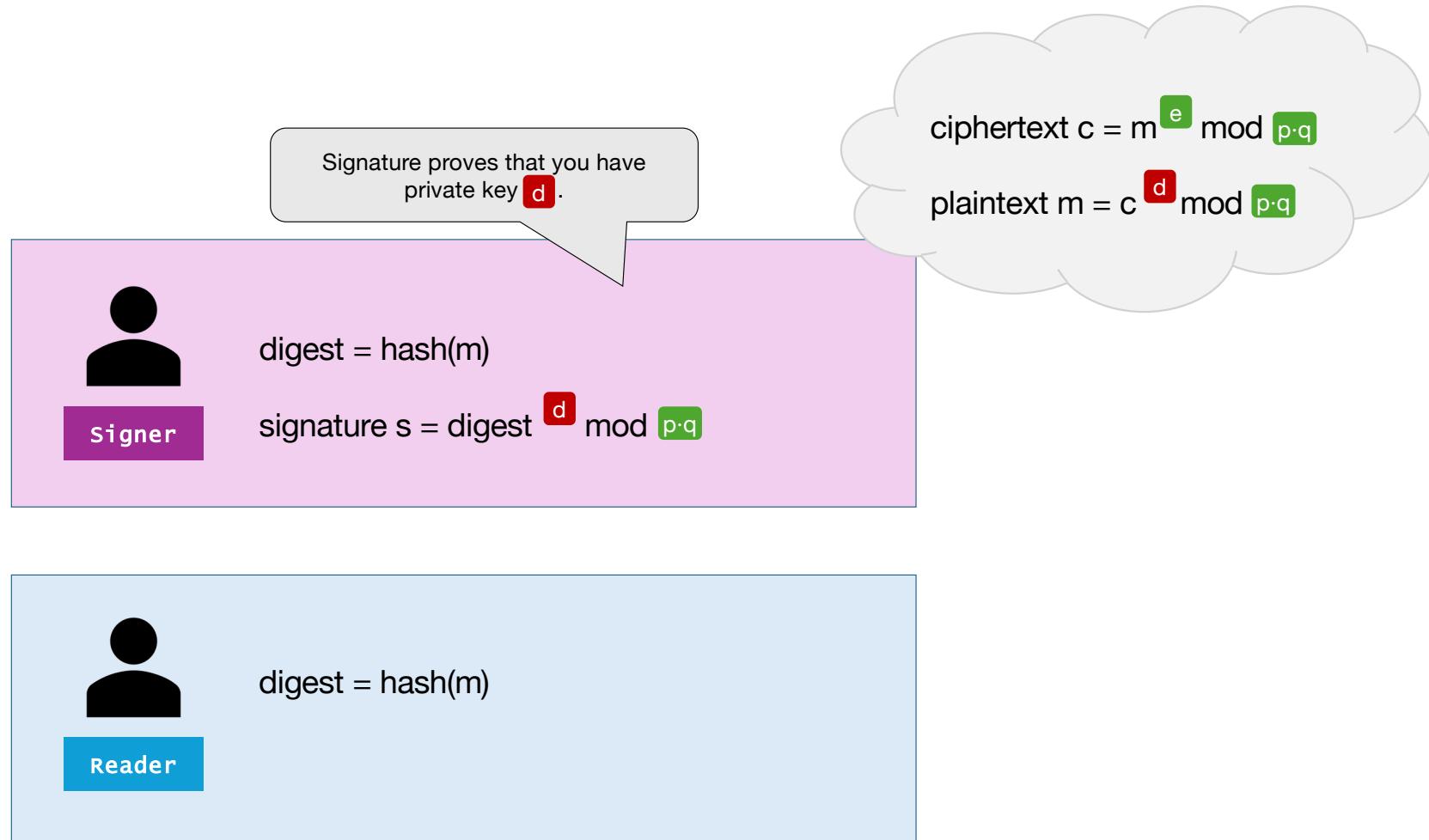
Signatures



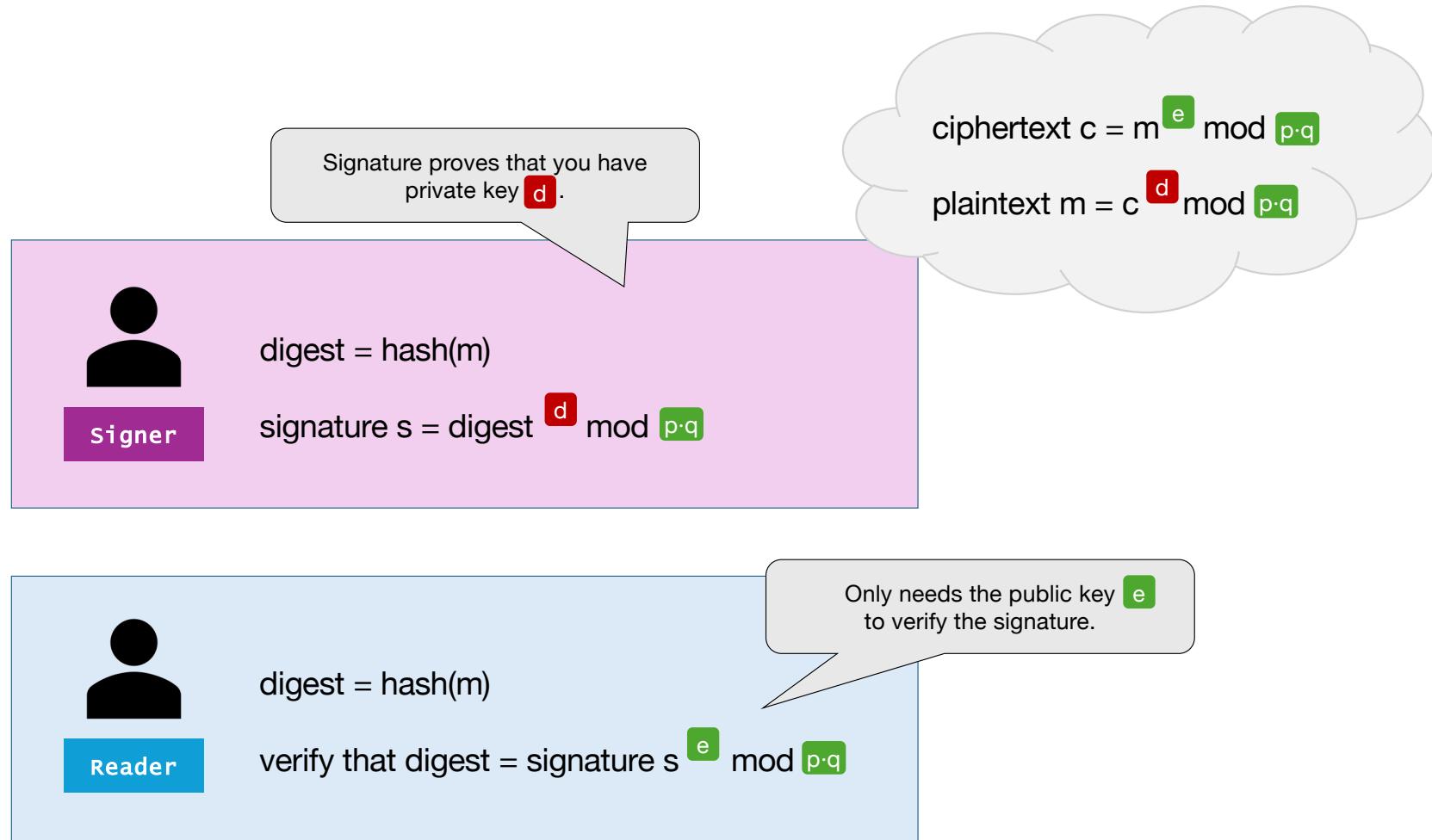
Signatures



Signatures



Signatures

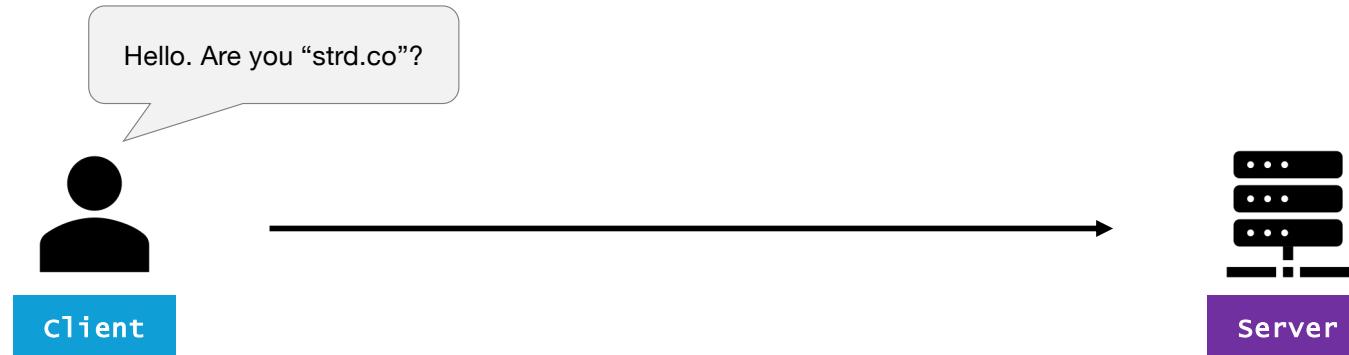


Signatures

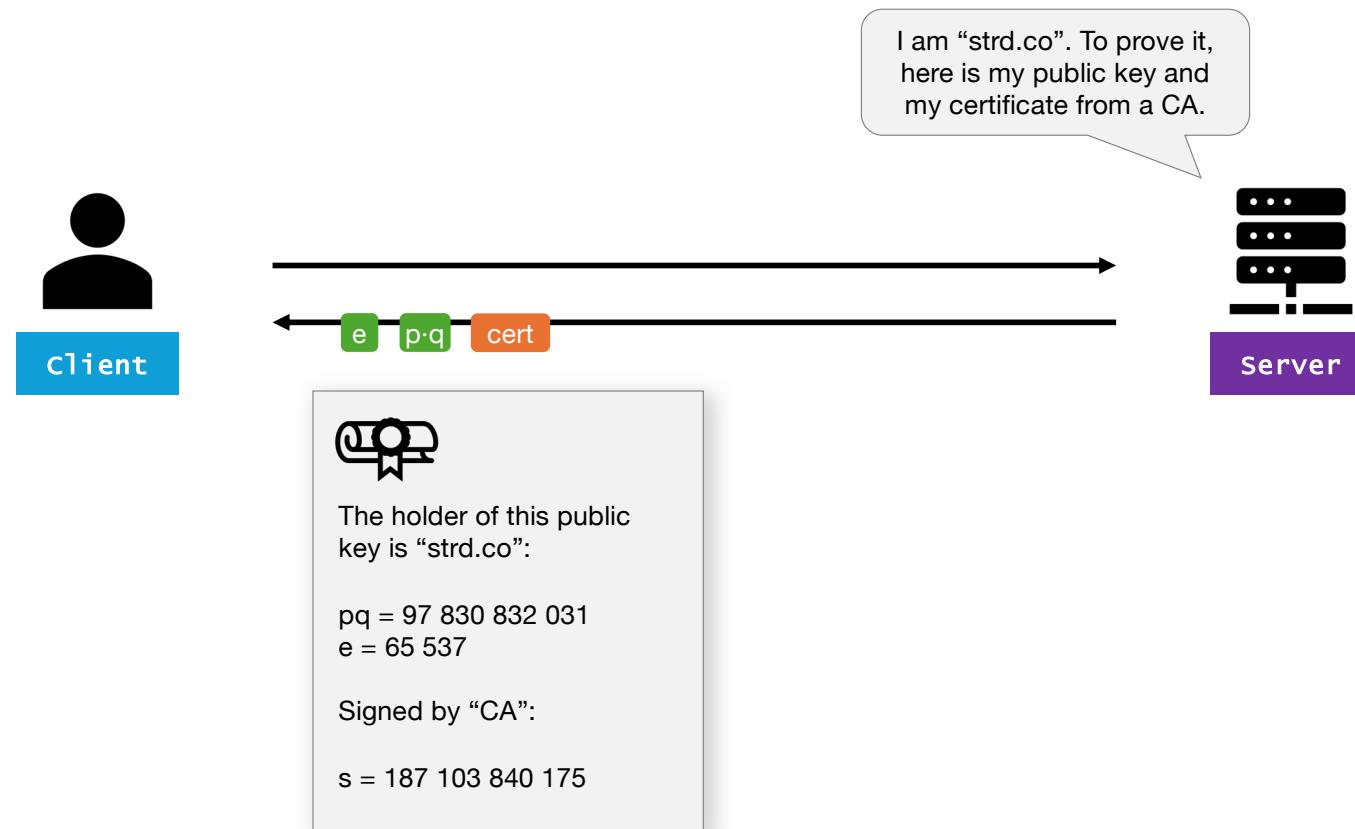
How are cryptographic signatures used?

- Code signing
- DNSSEC
- Signing certificates

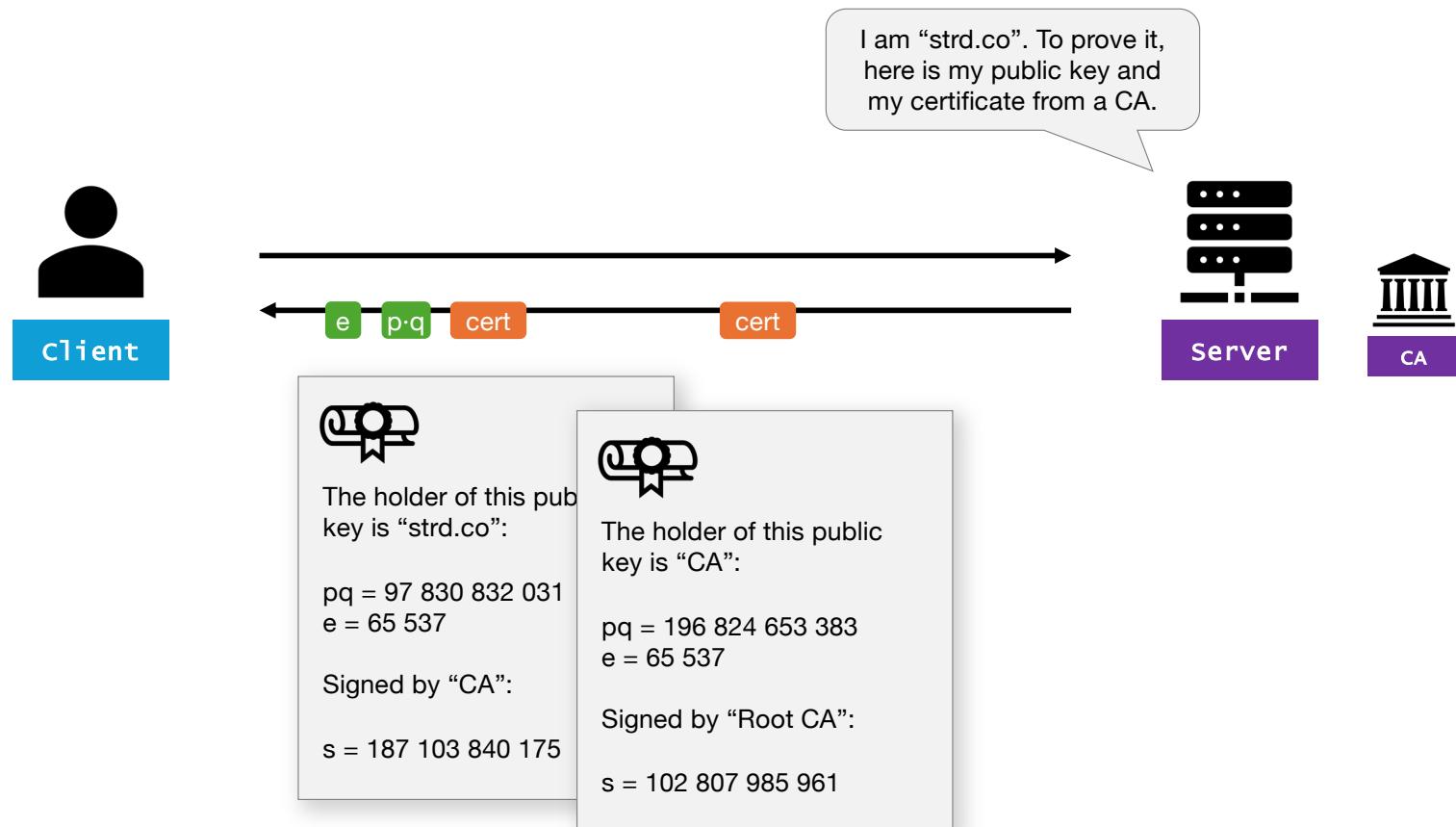
Certificate chains



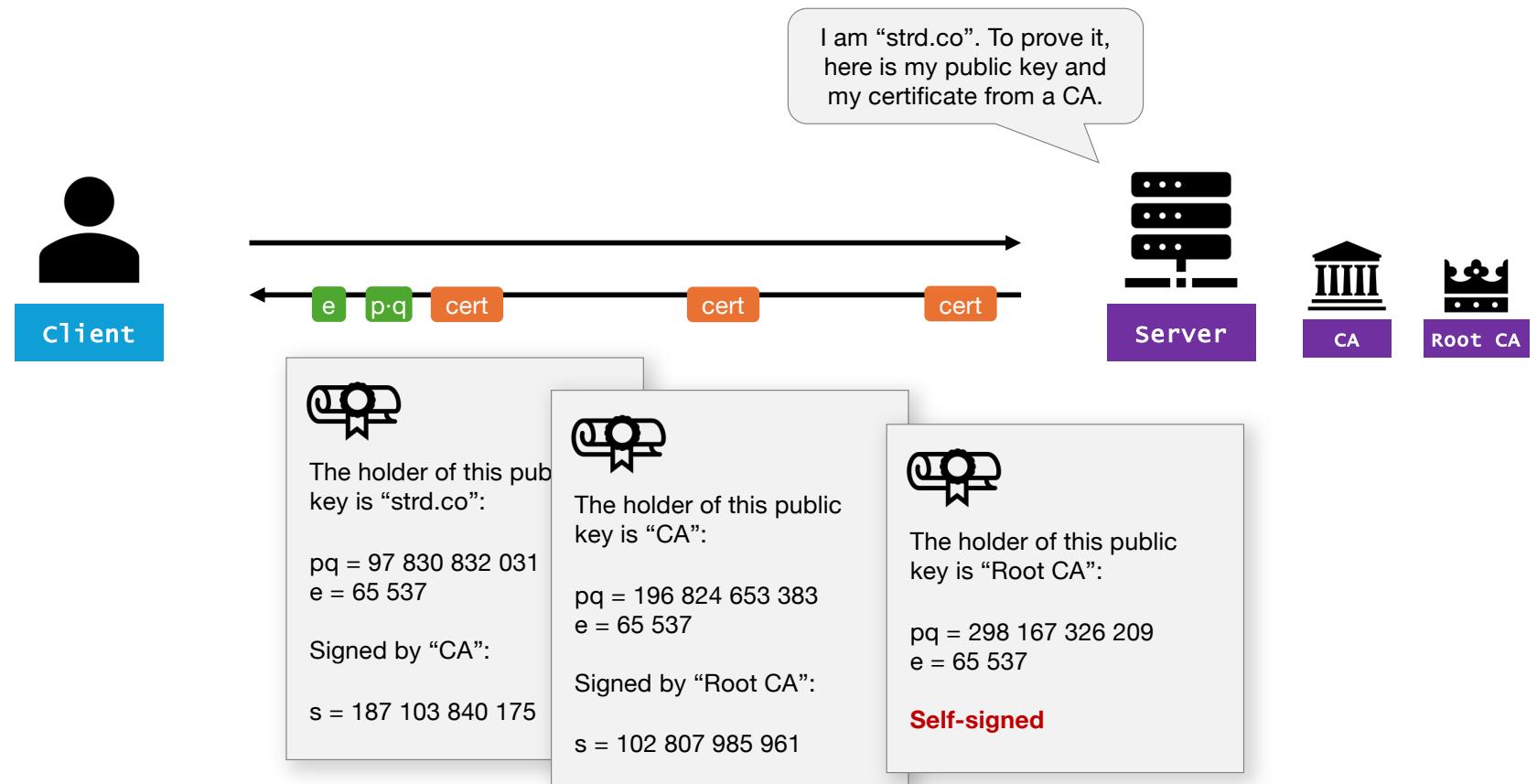
Certificate chains



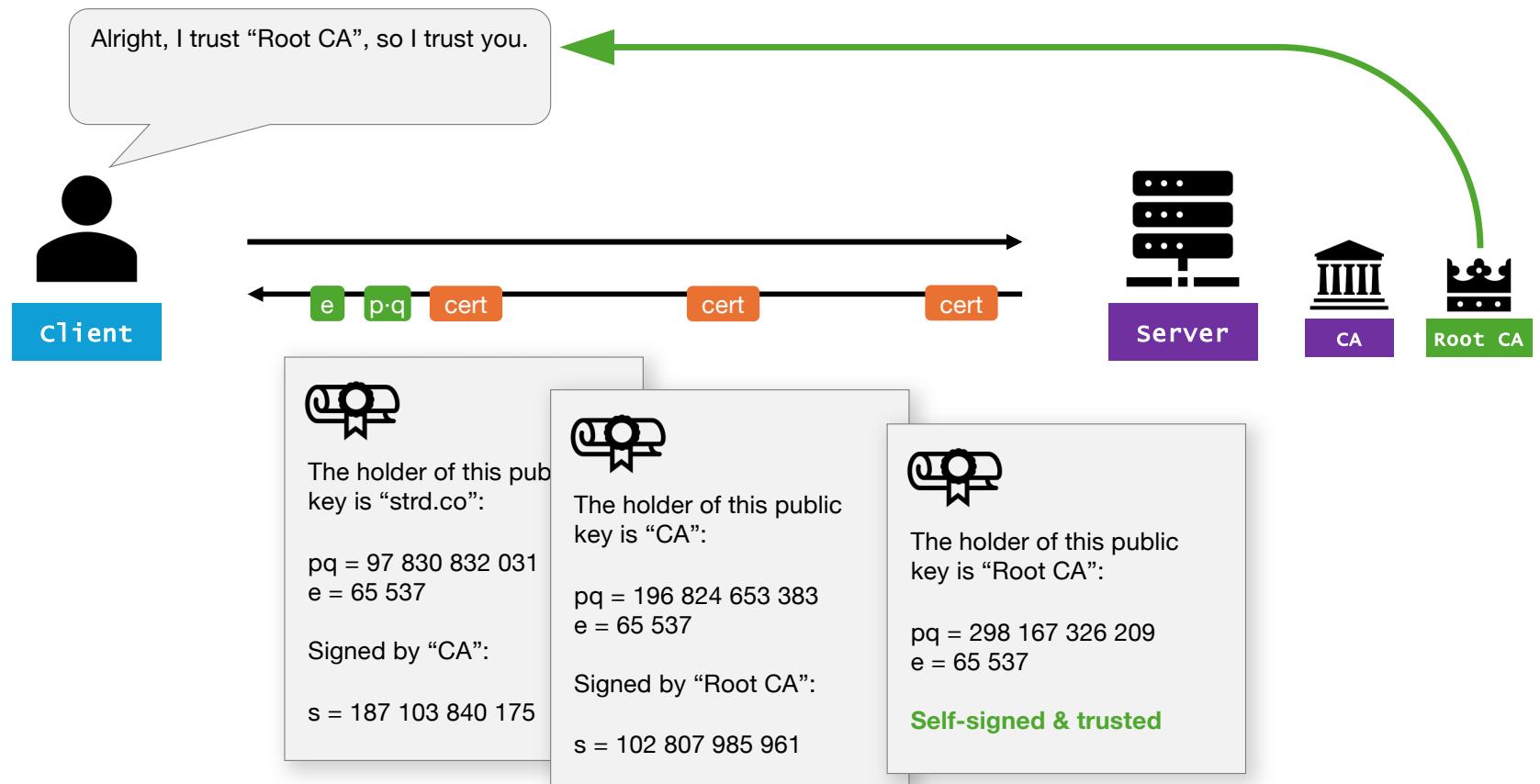
Certificate chains



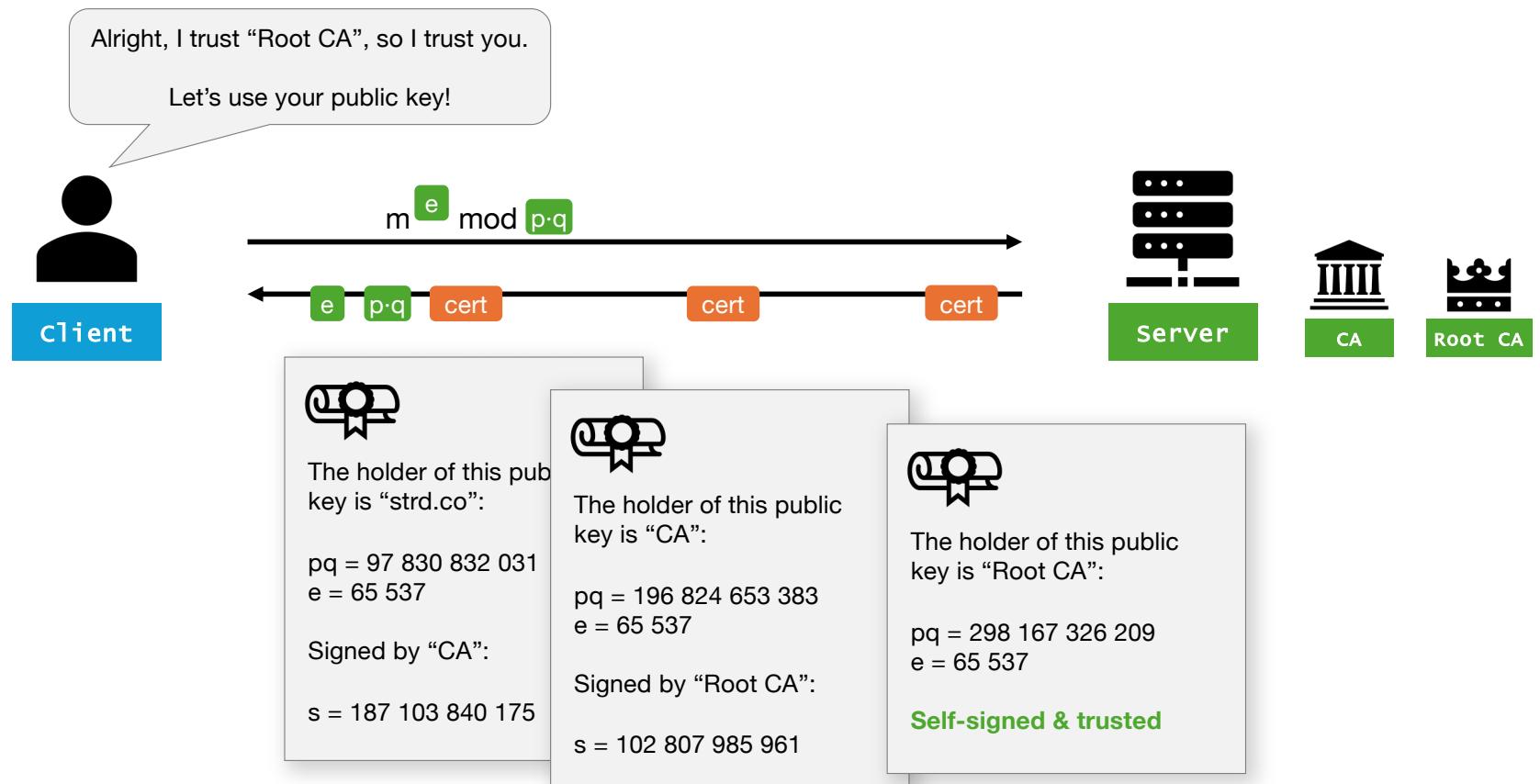
Certificate chains



Certificate chains



Certificate chains



Certificate chains

- A certificate is a “digital identity card”. The holder can prove that it has a matching private key without publishing it.
- Just like identity cards, certificates have authorized issuers.
- Certificates can be self-issued (“self-signed”).
- Certificates mitigate man-in-the-middle (MITM) attacks.

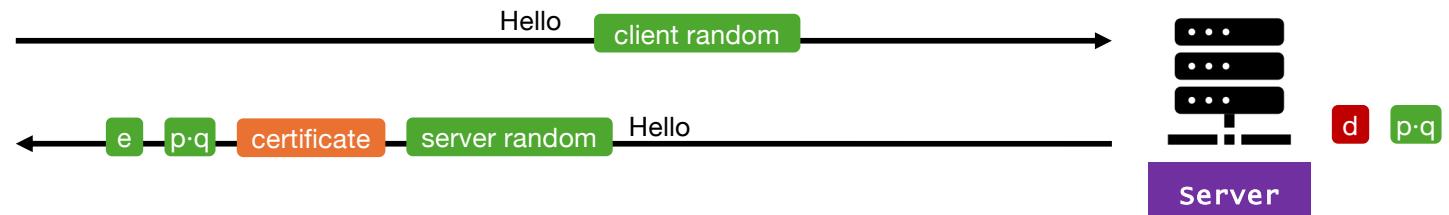
Big picture

Asymmetric



Client

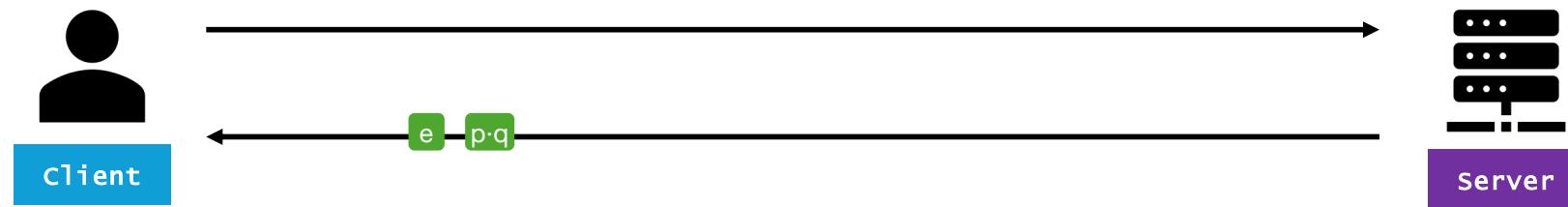
Validate certificate



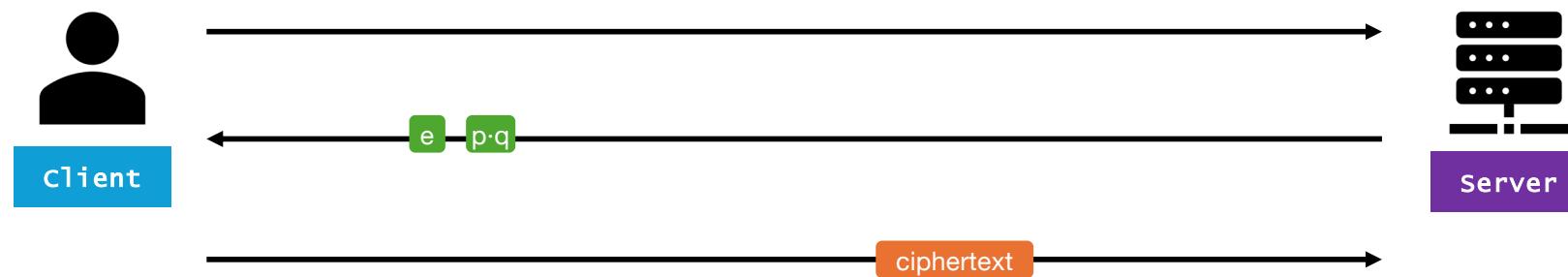
MITM



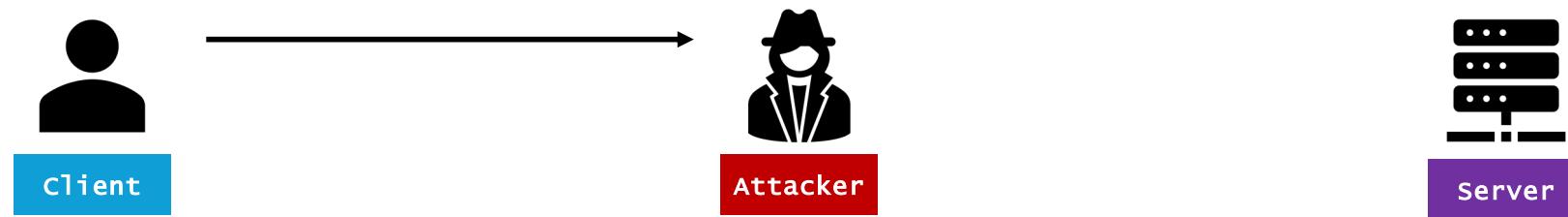
MITM



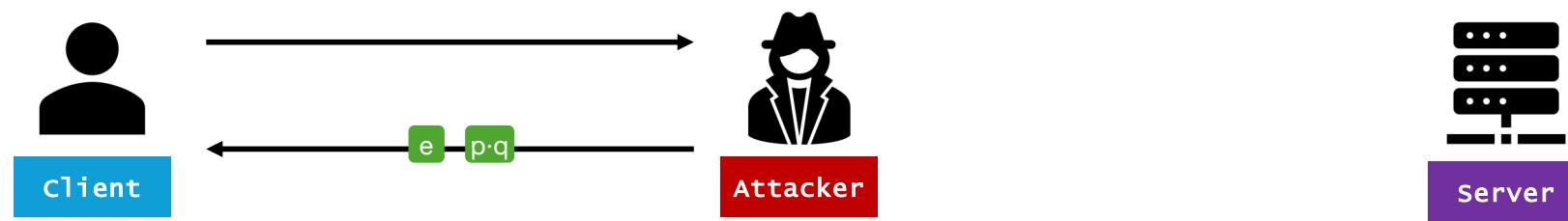
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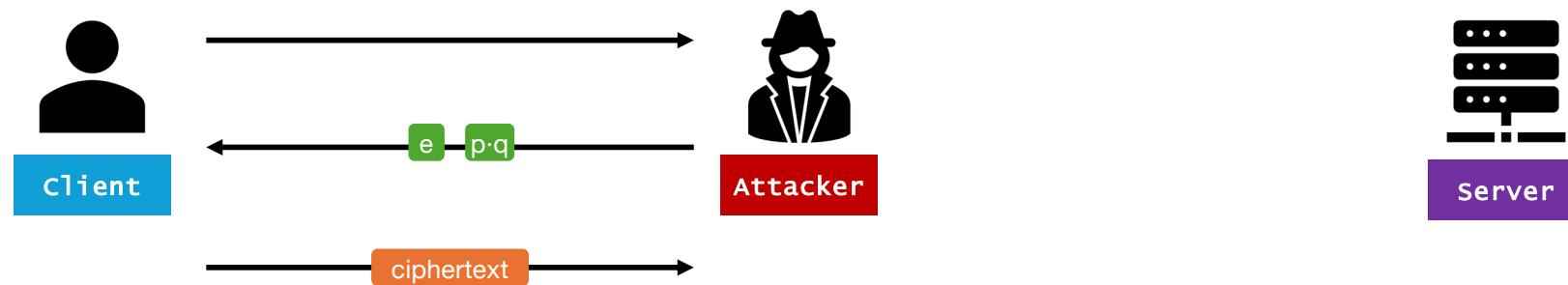
MITM



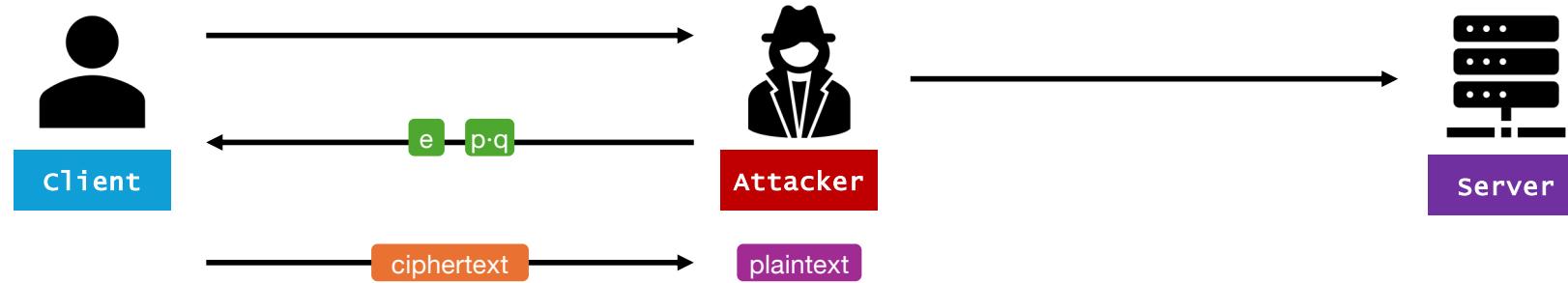
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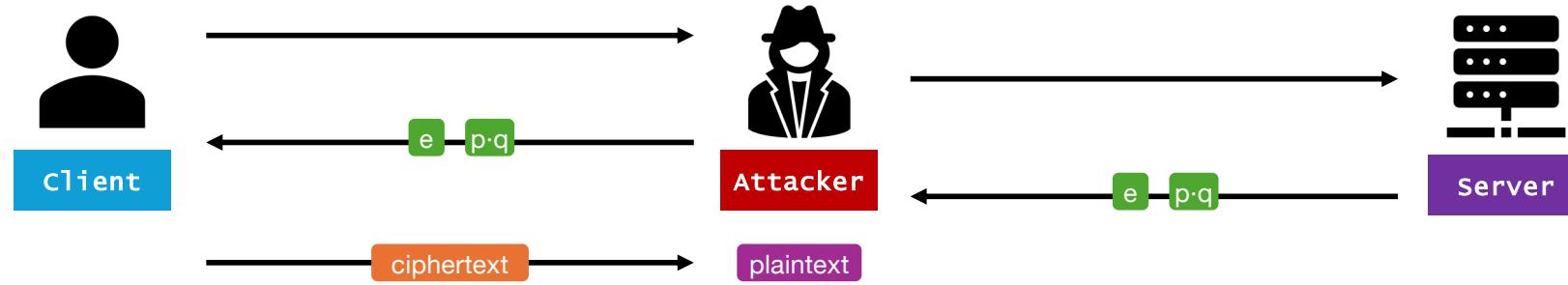
MITM



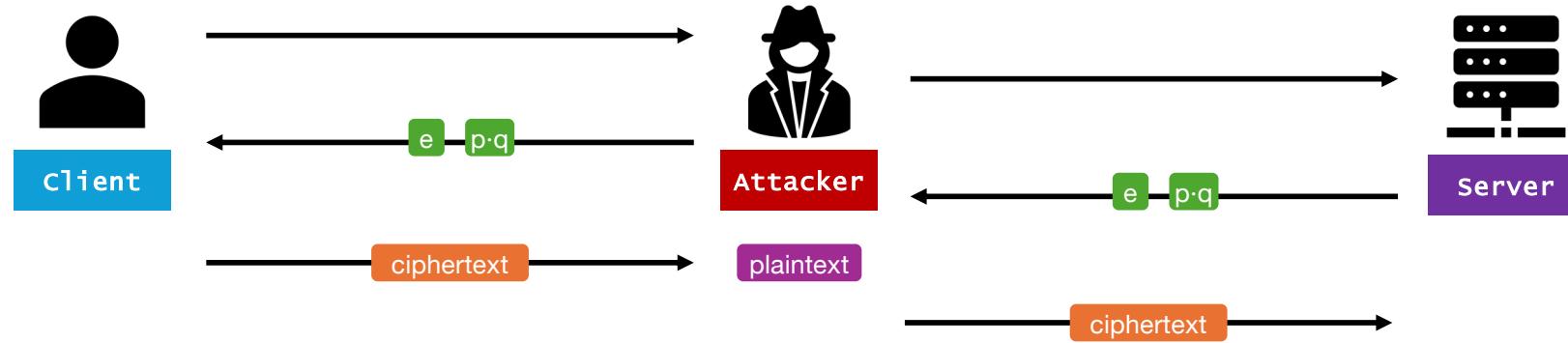
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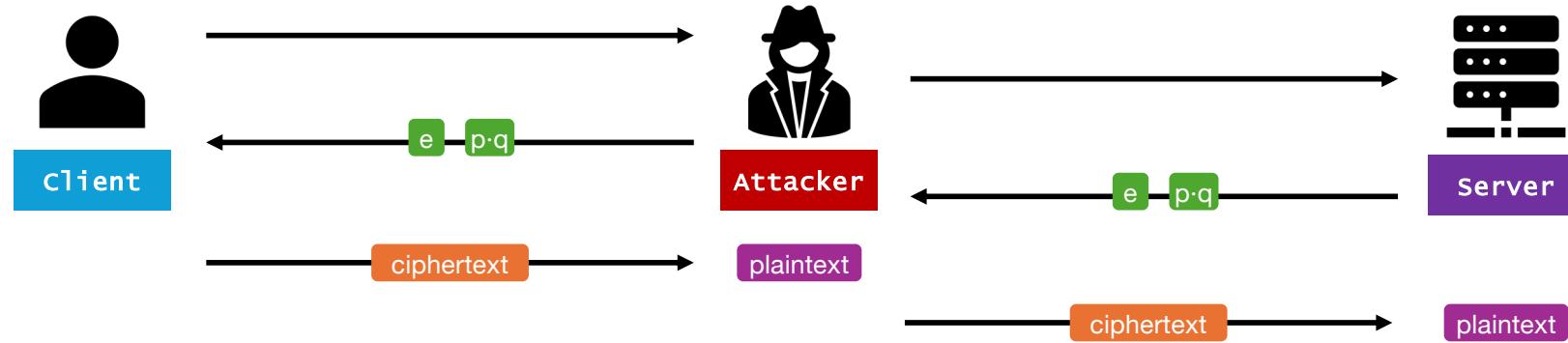
MITM



MITM

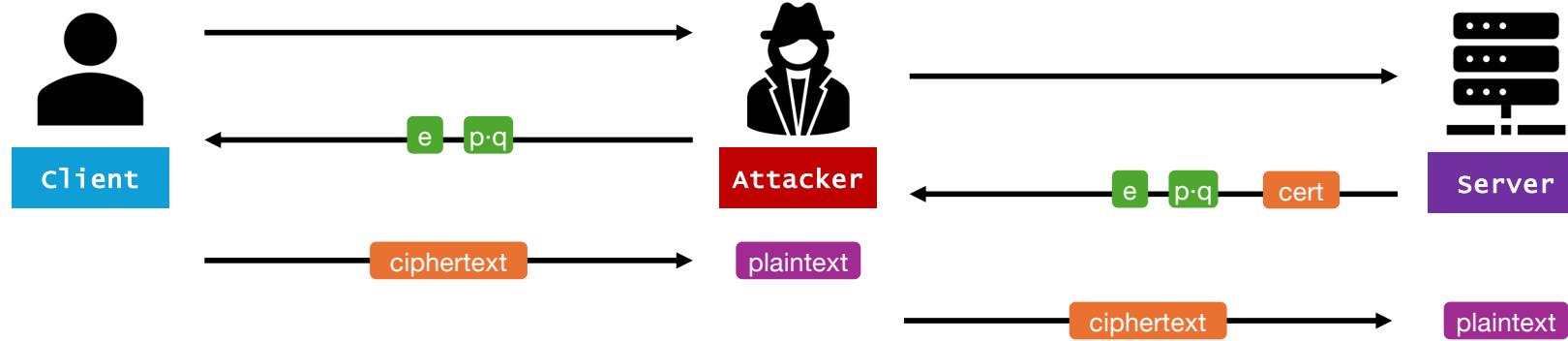


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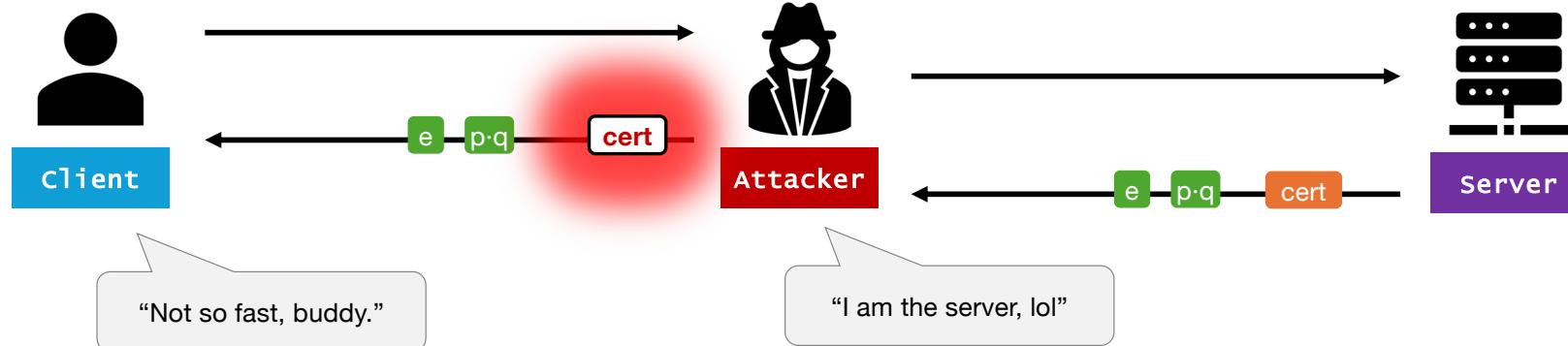
MITM

MITM attacks don't work when the client checks for a certificate:



MITM

MITM attacks don't work when the client checks for a certificate:



MITM

- Your certificate chain is only as strong as its weakest link.
- Be careful with the “trust certificate” checkbox.
- Modern firewalls inspect TLS traffic using a MITM pattern.

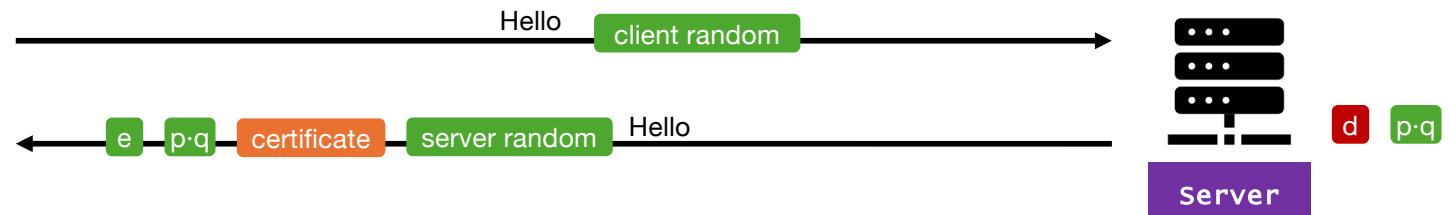
Big picture

Asymmetric



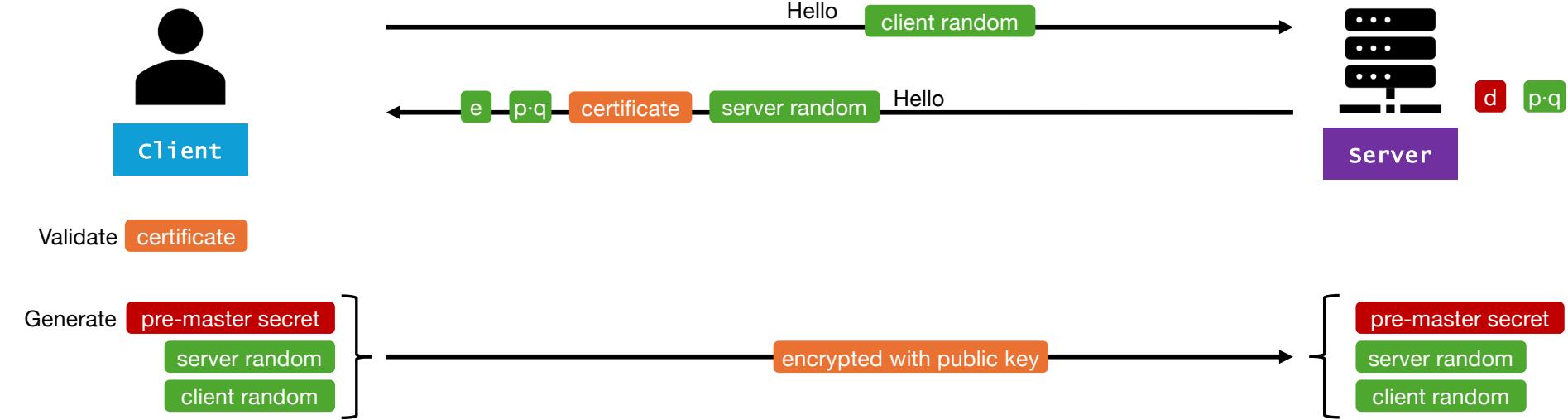
Client

Validate certificate



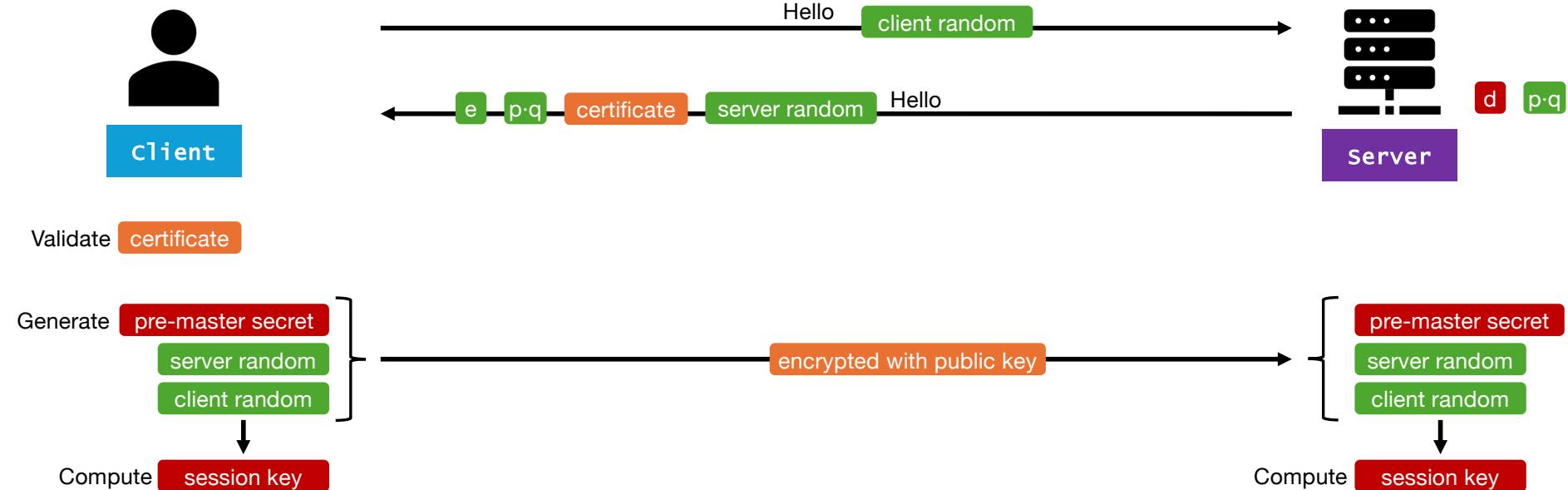
Big picture

Asymmetric

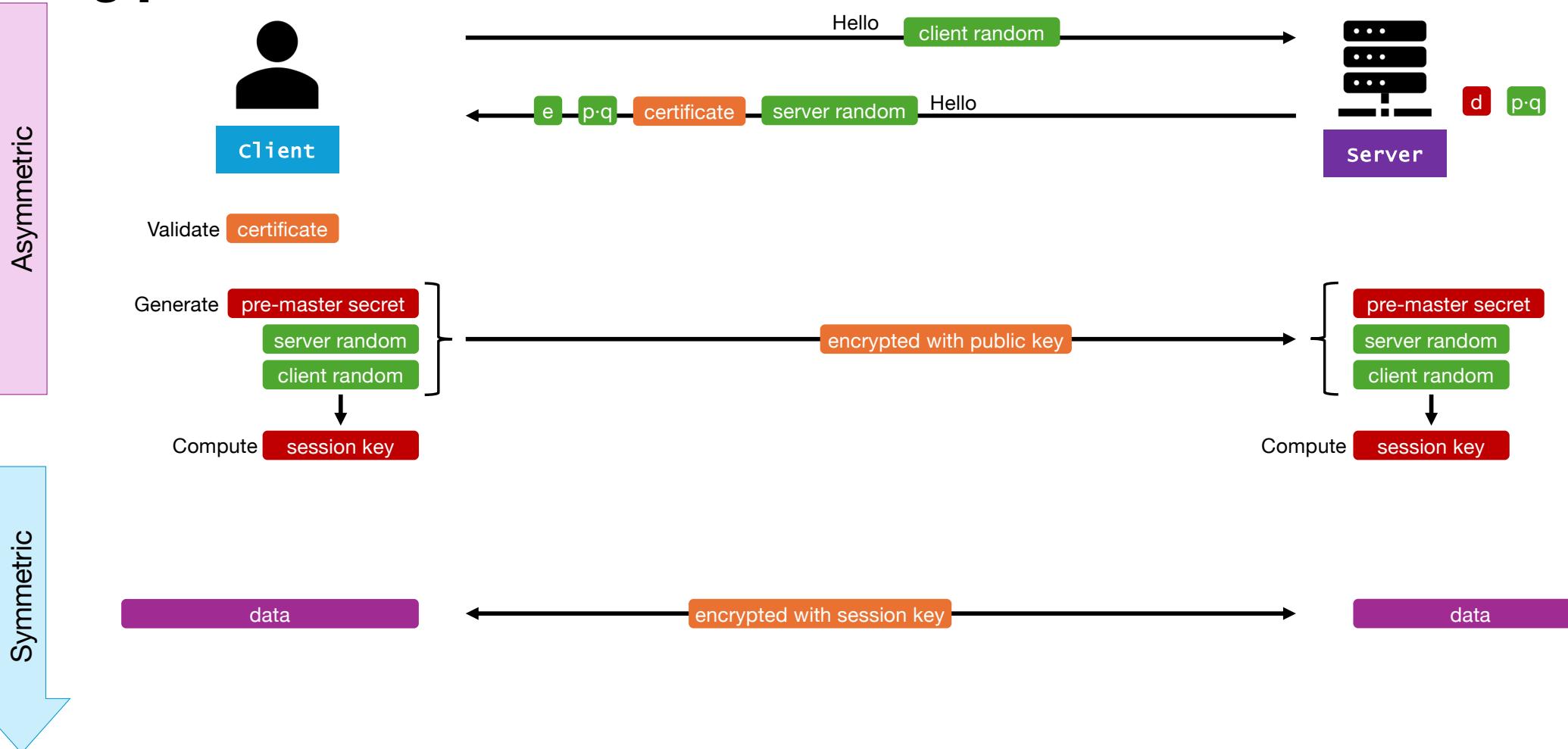


Big picture

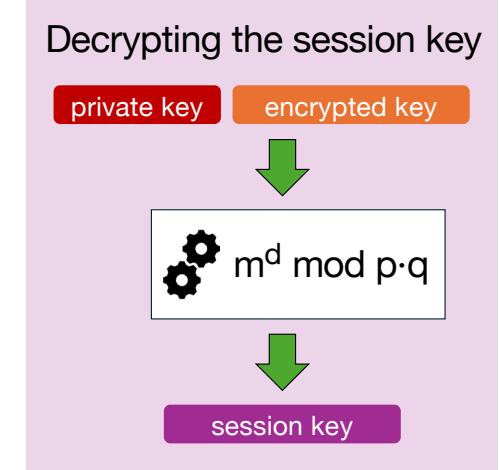
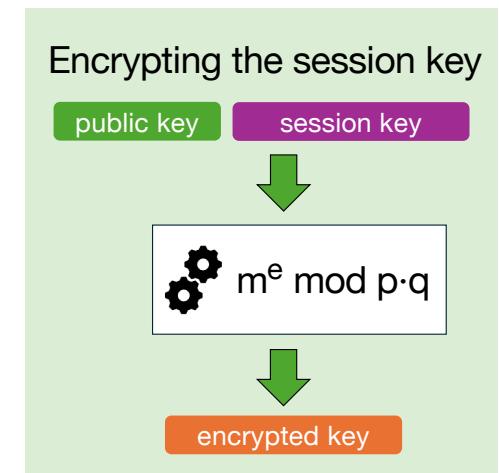
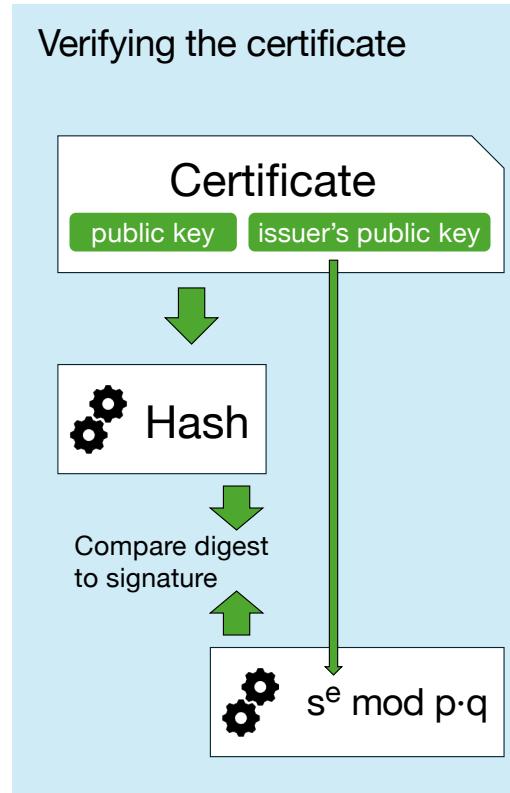
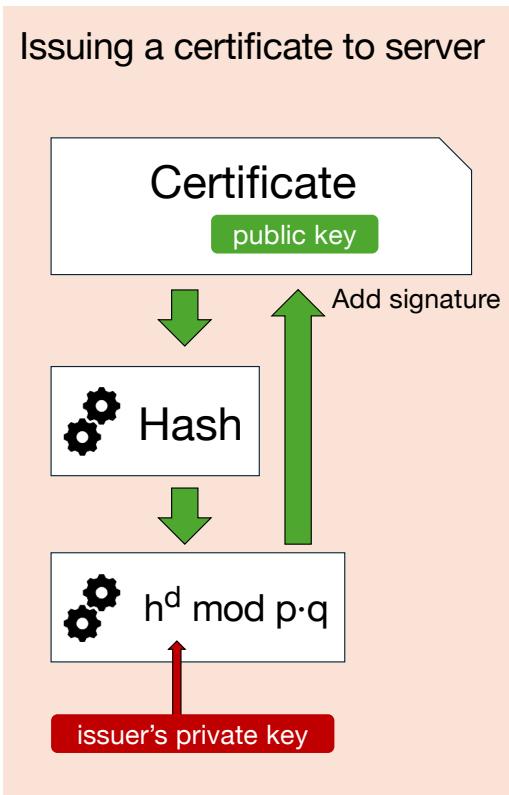
Asymmetric



Big picture



All the moving parts



Mmmkay

Thanks for staying awake!

Feel free to reach out with
questions or comments.

Email: daniel@strd.co
Blog: sqlsunday.com
Bluesky: [@dhma.ch](https://bluesky.social/@dhma.ch)
Twitter: [@dmacher](https://twitter.com/dhmacher)



Tools, links, docs

AES:

- AES visualizer: http://advancedcomputing.org/aes_visualizer.html

RSA:

- Interactive RSA calculation: <https://www.henryschnale.org/2022/03/14/rsa.html>
- RSA numbers: https://en.wikipedia.org/wiki/RSA_numbers
- Big number calculator: <https://defuse.ca/big-number-calculator.htm>
- Prime factorization tool:
<http://www.javascripter.net/math/calculators/primefactorscalculator.htm>
- Fermat attack on RSA (when p and q are close to the square root):
<https://fermatattack.secvuln.info/>