Swarm Intelligence — Class Exercises 1 (SOLVED)

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1. What kind of algorithm is Ant system (AS)? Does AS guarantees to eventually find the optimal solution or to determine that no solution exists?

   Answer: AS is an stochastic algorithm for the approximate solution of discrete optimization problems. This kind of algorithms do not guarantee to find the optimal solution, or solutions, to a problem or to eventually determine that no solution exists. However, for AS, as well as many other ACO algorithms, there are convergence proofs showing that for any small constant $\epsilon \geq 0$ and for a sufficiently large number of algorithm iterations t, the probability of finding an optimal solution at least once is $1 - \epsilon$ and that this probability tends to 1 for $t \rightarrow \infty$.

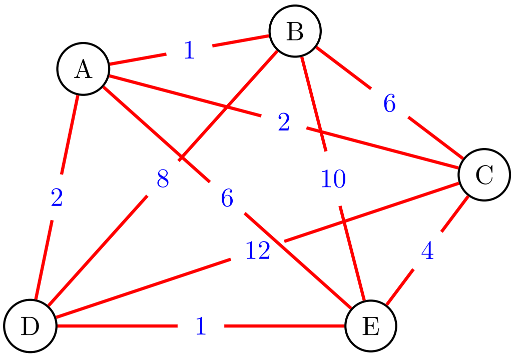
2. Is it possible to use AS with $\{\alpha = 0, \beta = 1\}$ or $\{\alpha = 1, \beta = 0\}$? What is the effect in each case?
   Answer: Although both options are possible, using these parameter settings may affect the optimization capabilities of AS. If AS is set with parameters $\alpha = 0$ and $\beta = 1$, pheromones will be neglected from the transition rule that ants employ to construct solutions, meaning that they will not be learning from the high-quality solutions found in previous iterations. On the other hand, if AS is set with $\alpha = 1$ and $\beta = 0$, ants solutions construction will be biased only by pheromones, which may result in low quality solutions as the heuristic information allows to include problem-dependent information that can be used greedily by the algorithm. In a nutshell, AS performance is generally better when the algorithm can profit both from the numerical information learned in past iterations (pheromones) and from problem-dependent information (heuristic information).

3. How ρ relates with the exploration and exploitation performed by AS? What about α and β?
   Answer: ρ controls the speed at which pheromone decreases. For large ρ values, the value of pheromones will decrease faster resulting in a more exploratory behavior, and for small ρ values, the value of pheromones will decrease slower leading to a more exploitative behavior. In the case of α and β, these two parameters control the relative influence of the pheromones and the heuristic information. When both values are large, the algorithm will behave more exploitative because ants will be biased towards solutions that have either high pheromone values or better heuristic information. Conversely, when both α and β are small, the probability

associated to all feasible solution components will be more even, resulting in a more exploratory behavior.

4. In class we used a minimization problem (TSP) as example. Consider now the problem of finding the maximum length tour: what modifications are needed to solve this problem using AS?
Answer: Two modifications are necessary. The first one concerns the heuristic information: instead of using $\eta_{ij} = 1/d_{ij}$ (as in the TSP), where d_{ij} is the distance or cost associated to edge (i, j), we use $\eta_{ij} = d_{ij}$. The second concerns the pheromone update: instead of using $\Delta\tau^k_{i,j} = 1/L_k$, we could use $\Delta\tau^k_{i,j} = L_k/|\pi|$, where $|\pi|$ is the number of solution components in the tour.

5. Assume the following symmetric TSP instance:



(a) Distance between each city.

	A	B	C	D	E
A	–	0.56	0.66	0.60	0.50
B	0.56	–	0.60	0.56	0.60
C	0.66	0.60	–	0.50	0.56
D	0.60	0.56	0.50	–	0.66
E	0.50	0.60	0.56	0.66	–

(b) Pheromone matrix (τ) at the end of iteration 1

An Ant System algorithm is applied to the TSP instance shown in Figure (a) using $\alpha = 2$, $\beta = 1$, $\rho = 0.5$, $\#ants = 3$, $\eta_{ij} = 1/d_{ij}$ and $\tau_0 = 1$. After the first iteration, the pheromone matrix (τ) is the one given above in Figure (b).

(i) What is the meaning of the values in τ? Why $\tau_{C,D} = 0.5$?
Answer: $\tau_{i,j}$ indicates the desirability of adding edge i to the tour being constructed by an ant when in edge j. Since $\rho = 0.5$ and $\tau_0 = 1$, $\tau_{C,D} = 0.5$ indicates that the edge (C, D) was not part of the tour constructed by any ant.

(ii) Using the information shown in Figure (a) and (b), and random numbers (rnd) $\{0.76, 0.80, 0.27, 0.88, 0.47, 0.05, 0.98, 0.23, 0.06\}$, generate the first solution of iteration 2.
Note: use the same roulette wheel mechanism of the random proportional rule to select the initial city assuming uniform probabilities for all cities.
Answer: Using the random proportional rule of AS,

$$p^k_{i,j}(t) = \frac{\tau_{i,j}(t)^\alpha \cdot \eta^\beta_{i,j}}{\sum_{l \in N^k_i} \tau_{i,l}(t)^\alpha \cdot \eta^\beta_{i,l}},$$

we compute the probability of adding each feasible city, and using the roulette wheel mechanism and the set of random numbers previously generated, we select one of the city to be added to the tour. This is done as follows:

Initial city:

	i =A	i =B	i =C	i =D	i =E
probabilities ($p_i^{k=1}(t=2)$)	0.2	0.2	0.2	0.2	0.2
roulette wheel ($rnd = 0.76$)	0.2	0.4	0.6	0.8	1

Next city:

	j =A	j =B	j =C	j =D	j =E
probabilities ($p_{D,j}^{k=1}(t=2)$)	0.27	0.06	0.031	0	0.64
roulette wheel ($rnd = 0.80$)	0.27	0.32	0.36	0	1

Next city:

	j =A	j =B	j =C	j =D	j =E
probabilities ($p_{E,j}^{k=1}(t=2)$)	0.266	0.23	0.50	0	0
roulette wheel ($rnd = 0.27$)	0.266	0.49	1	0	0

Next city:

	j =A	j =B	j =C	j =D	j =E
probabilities ($p_{A,j}^{k=1}(t=2)$)	0.84	0	0.16	0	0
roulette wheel ($rnd = 0.88$)	0.84	0	1	0	0

The first solution constructed at iteration $t = 2$ is: $DEBCAD$

(iii) The following solutions generated by the algoritm are $AEDCBA$ with $cost = 26$ and $DECBAD$ with $cost = 14$. Update the pheromone using this information.

Answer: Using the pheromone update rule of AS,

$$\tau_{i,j}(t) = (1-\rho)\cdot\tau_{i,j}(t-1) + \sum_{k=1}^{m}\Delta\tau_{i,j}^{k},$$

we get:

$(AB) = (0.5)\cdot 0.56 + (1/26) + (1/14) = 0.38989011$
$(AC) = (0.5)\cdot 0.66 = 0.33$
$(AD) = (0.5)\cdot 0.60 + (1/14) = 0.371428571$
$(AE) = (0.5)\cdot 0.50 + (1/26) = 0.288461538$
$(BC) = (0.5)\cdot 0.60 + (1/26) + (1/14) = 0.4098901$
$(BD) = (0.5)\cdot 0.56 = 0.28$
$(BE) = (0.5)\cdot 0.60 = 0.3$
$(CD) = (0.5)\cdot 0.50 + (1/26) = 0.288461538$
$(CE) = (0.5)\cdot 0.56 + (1/14) = 0.351428571$
$(DE) = (0.5)\cdot 0.66 + (1/26) + (1/14) = 0.43989011$

(iv) Figure 2 shows the pheromone matrix after 12 iterations. Would you advice to continue executing more iterations? Why?

Answer: At this point the algorithm is stagnated, which means that the probability of constructing a set of solutions different from

	A	B	C	D	E
A	–	0.4285	0.0004	0.4285	0.0003
B	0.4285	–	0.4286	0.0003	0.0003
C	0.0004	0.4286	–	0.0003	0.4285
D	0.4285	0.0003	0.0003	–	0.4286
E	0.0003	0.0003	0.4285	0.4286	–

Figure 2: Pheromone matrix (τ) (iteration 12)

the one constructed in the last iteration is quite low. In this example, the algorithm will be constructing solutions

$$ABCEDA,$$
$$ADECBA,$$
$$BADECB,$$
$$BDECAB,$$
$$CBADEC,$$
$$CEDABC,$$
$$DABCED,$$
$$DECBAD,$$
$$ECBADE, \text{ and}$$
$$EDABCE$$

over and over again.