Genetic Algorithms and Ant Colony Optimisation

Introduction: Optimisation

- Optimisation : find an extremum
- Extrema can be local / global
- In R^n (real numbers): methods with and without gradients
- Local :
  - With derivative (ok : space = R^n) ➔ gradient (possibly: first degree or even more)
  - Without derivative : select a point, explore the neighborood, take the best, do it again. (type hill climber, local search)
- Global :
  - = local with different initial conditions.
  - Method without derivatives → GA

- Combinatorial optimisation problems.
- Deterministic algorithms : Explore too much and take too much time ➔ meta-heuristiques : find rapidly a satisfactory solution
- Example : Scheduling problem, packing or ordering problems
- The classics of the classics : TSP
  - The travelling salesman problem
  - N cities
  - Find the shortest path going through each city only once
  - Benchmarking problems
  - Problems NP-complete (the time to find grows exponentially with the size of the problem (N! ~ N^(N+1/2)))

Genetic Algorithms: Introduction

- Evolutionary computing
- 1975 : John Holland → Genetic algorithms
- 1992 : John Koza → Genetic programming

Genetic algorithms

- Darwinian inspiration
- Evolution = optimisation:

Reproduction

- 2 genetic operators:
  - Cross-over (recombination)
  - Mutation

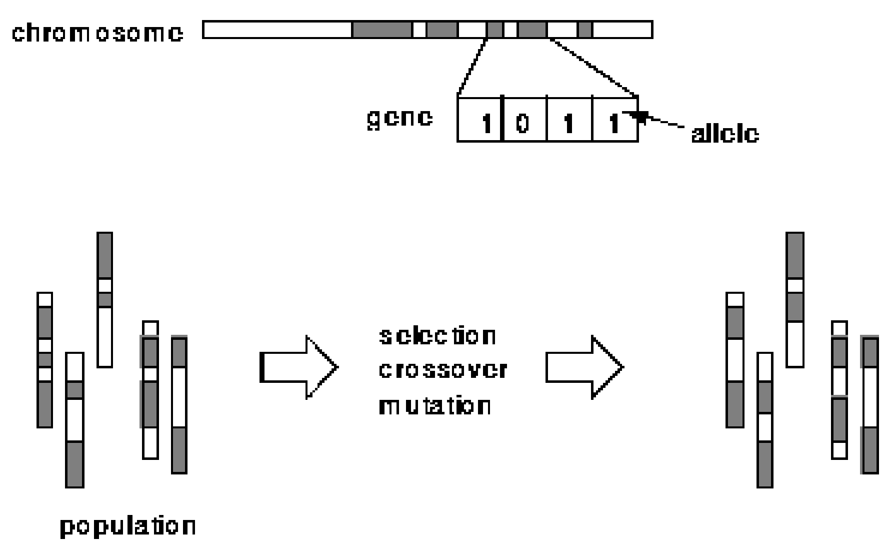
- Fitness

The standard algorithm

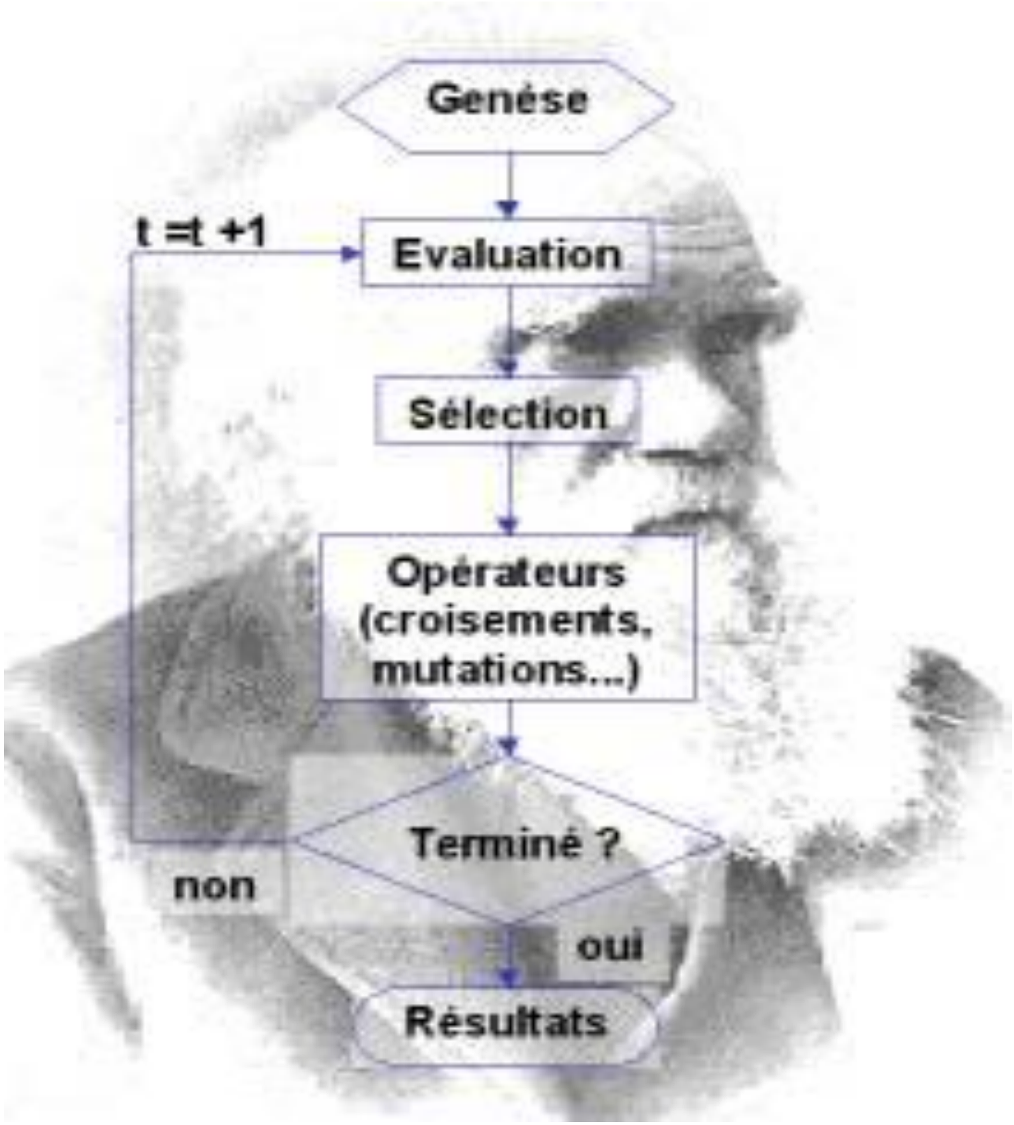
- Generate random population
- Repeat
  - Evaluate fitness *f(x)* for each individual of the population
  - Create a new population (to repeat until a stopping critetion)
    - Selection (according to fitness)
    - Crossover (according to probability of crossover)
    - Mutation (according to probability of mutation)
    - evaluate the new individuals in the population (replacement)
  - Replace the old population by the new (better) ones

- Until stop condition; return the best solution of the current population

- The GA lingo



chromosome
gene
1
0
1
1
allele
selection
crossover
mutation
population


Genèse
t =t +1
Evaluation
Sélection
Opérateurs
(croisements,
mutations...)
Terminé ?
non
oui
Résultats

Chromosones encoding

- Can be influenced by the problem to solve
- Examples:
  - Binary encoding
  - Permutation encoding (ordening problems) e.g. TSP problem)
  - Real value encoding (evolutionary strategies)
  - Tree encoding (genetic programming)

Binary Encoding

Chromosome A	101100101100101011100101
Chromosome B	111111100000110000011111

- Binary encoding is the most common, mainly because first works about GA used this type of encoding.In **binary encoding** every chromosome is a string of **bits**, **0** or **1**.

- **Example of Problem:** Knapsack problem

The problem: There are things with given value and size. The knapsack has given capacity. Select things to maximize the value of things in knapsack, but do not extend knapsack capacity.
Encoding: Each bit says, if the corresponding thing is in knapsack.

Permutation Encoding

Chromosome A	1 5 3 2 6 4 7 9 8
Chromosome B	8 5 6 7 2 3 1 4 9

- In **permutation encoding**, every chromosome is a string of numbers, which represents number in a **sequence**.

- **Example of Problem:** Traveling salesman problem (TSP)

The problem: There are cities and given distances between them.Travelling salesman has to visit all of them, but he does not to travel very much. Find a sequence of cities to minimize travelled distance. **Encoding:** Chromosome says order of cities, in which salesman will visit them.

Value Encoding

Chromosome A	1.2324 5.3243 0.4556 2.3293 2.4545
Chromosome B	ABDJEIFJDHDIERJFDLDFLFEGT
Chromosome C	(back), (back), (right), (forward), (left)

- In **value encoding**, every chromosome is a string of some values. Values can be anything connected to problem, form numbers, real numbers or chars to some complicated objects.

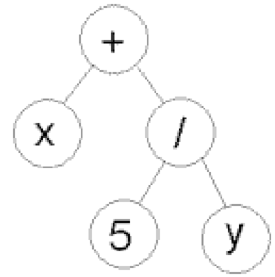
- **Example of Problem:** Finding weights for neural network

The problem: There is some neural network with given architecture. Find weights for inputs of neurons to train the network for wanted output.

Encoding: Real values in chromosomes represent corresponding weights for inputs.

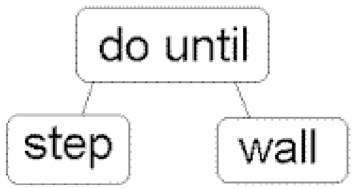
Tree Encoding

Chromosome A



(+ x (/ 5 y))

Chromosome B



(do_until step wall)

- In **tree encoding** every chromosome is a tree of some objects, such as functions or commands in programming language.Used in genetic programming
- **Example of Problem:** Finding a function from given values

The problem: Some input and output values are given. Task is to find a function, which will give the best (closest to wanted) output to all inputs.

Encoding: Chromosome are functions represented in a tree.

Crossover - Recombination

- C1: 1011|10001
- C2: 0110|11100
- → D1: 1011|11100
- → D2: 0110|10001

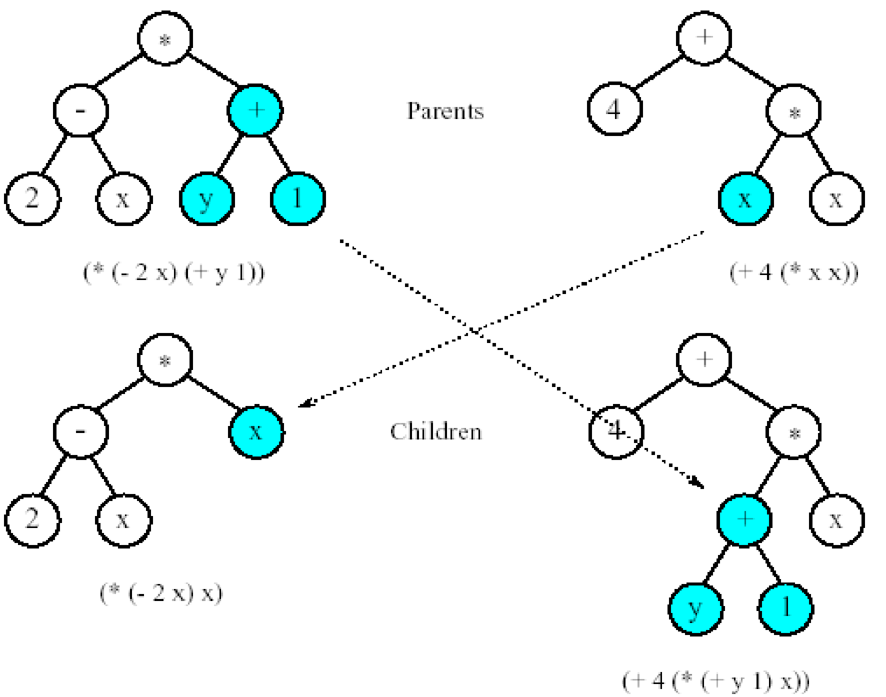
- Variants, many points of crossover

Crossover – Binary Encoding

- Single Point Crossover
  - 11001011 et 10011111→11001111
- Two Point Crossover
  - 11001011 et 10011111 → 11011111
- Uniform Crossover
  - 11001011 et 10011111 → 11011111
- Difference operators:
  - 11001011 AND 10011111 → 10001011

Crossover - variants

- Permutation encoding
  - Single Point Crossover
    - (**123456**789) et (**453689**721) → (123459768)
- Tree encoding



Mutation

- D1: 101111100
- D2: 011010001
- →M1: 100111100
- →M2: 001010101

- variants

Mutation - Variants

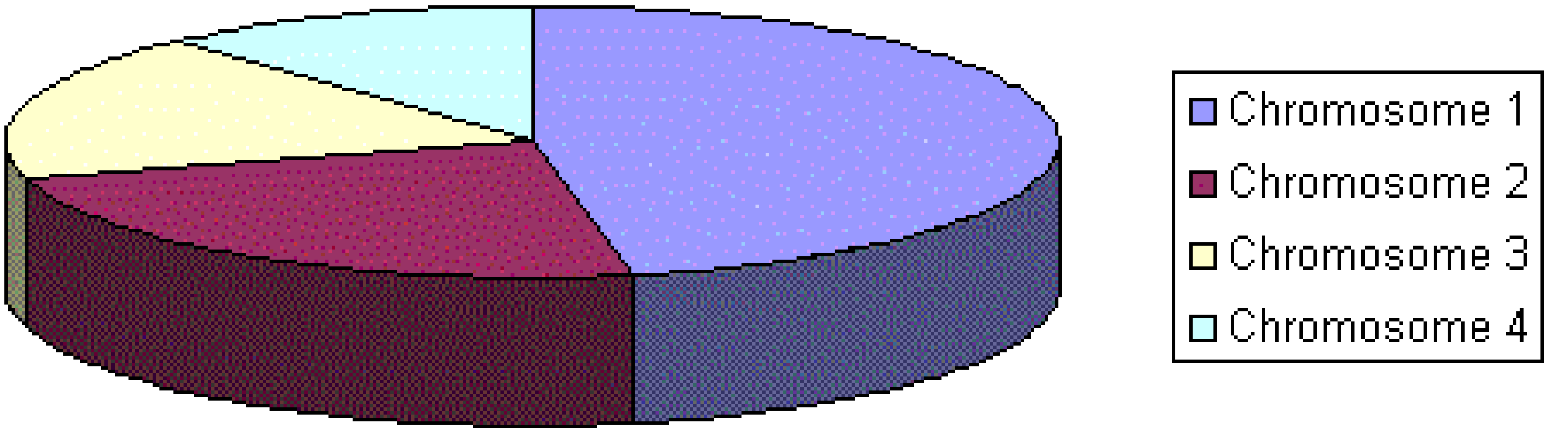
- Binary Encoding
  - Bit inversion 101111100→111111100
- Permutation Encoding
  - Order changing (123456897)→(183456297)
- Value Encoding
  - +/- one number (1.29 5.68 2.86 4.11 5.55) → (1.29 5.68 2.73 4.22 5.55)
- Tree Encoding: (ex)-change nodes

Selection

- By roulette wheel
- By rank
- By tournement
- Steady-State

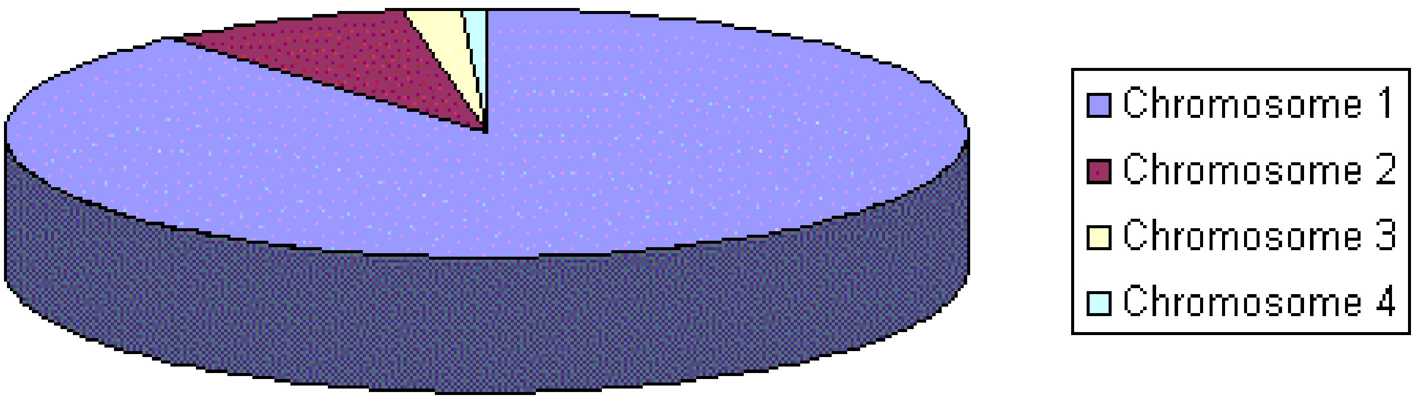
Roulette wheel

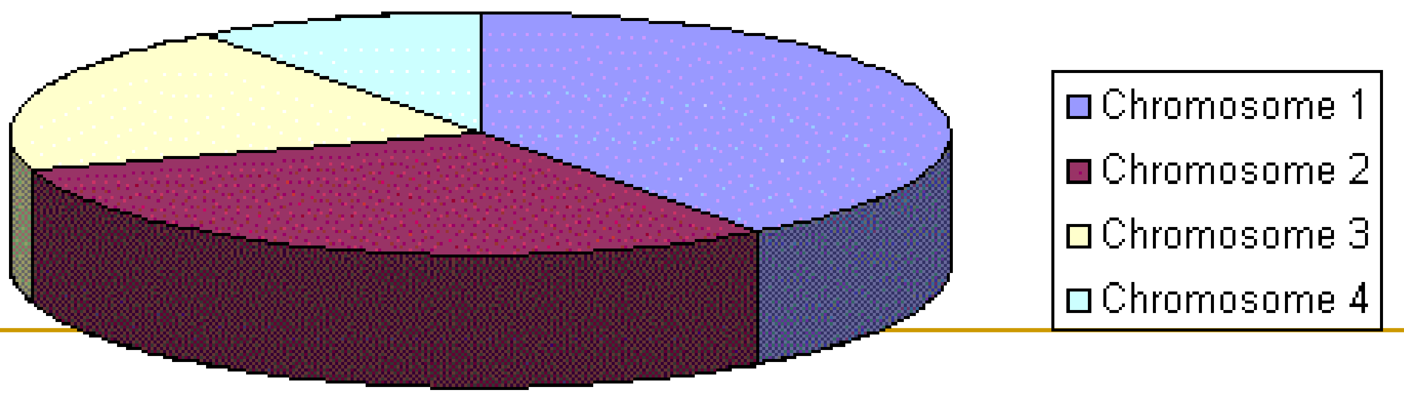
- Selection according to fitness



Selection by rank

- Sorting of the population ($n \rightarrow 1$)





Selection by tournament

- Size *k*
- Take randomly *k* individuals
- Make them compete and select the best

Elitism

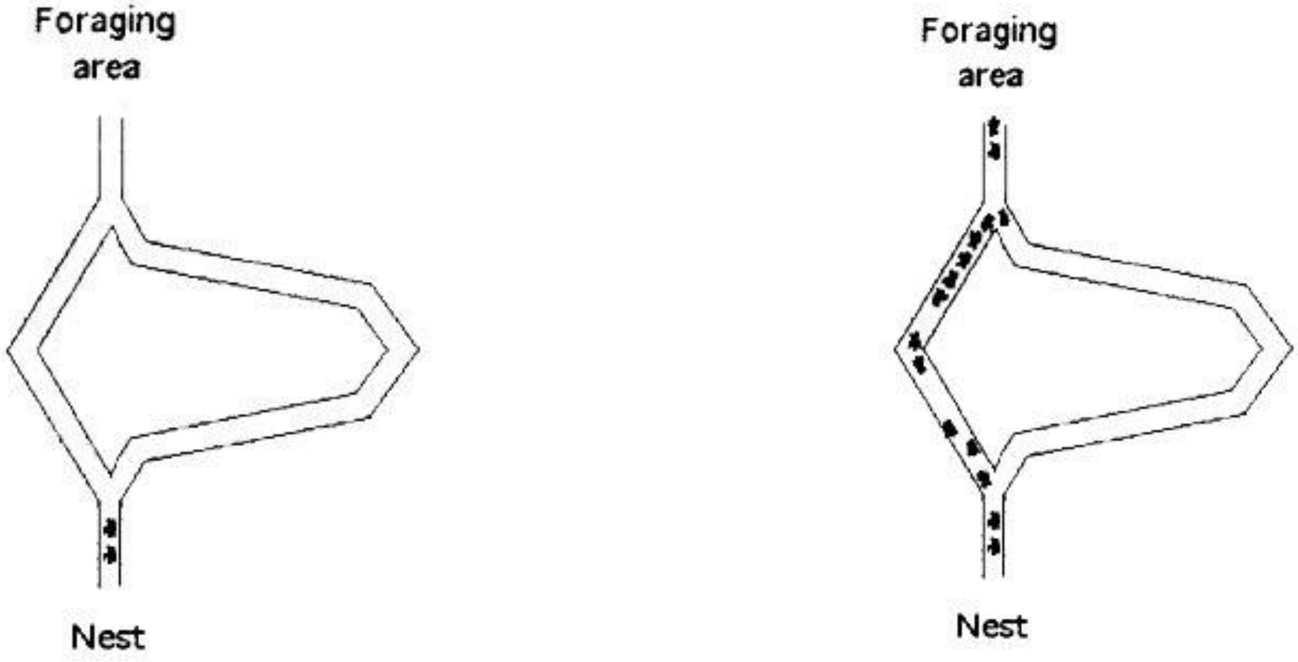
- → Elitism: copy the single or many bests in the population then construct the remaining ones by genetic operations

So many parameters

- Crossover probability
- Mutation probability
- Population size

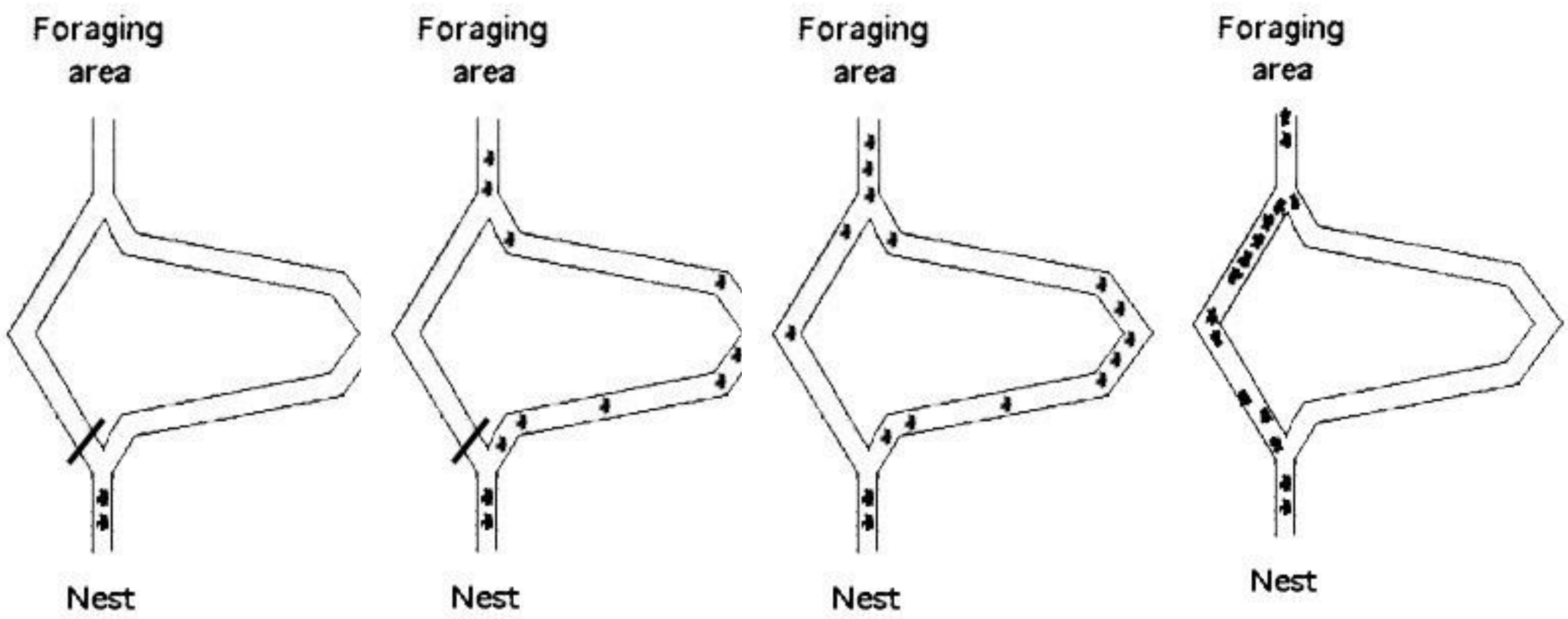
Ant Colony

In biology:



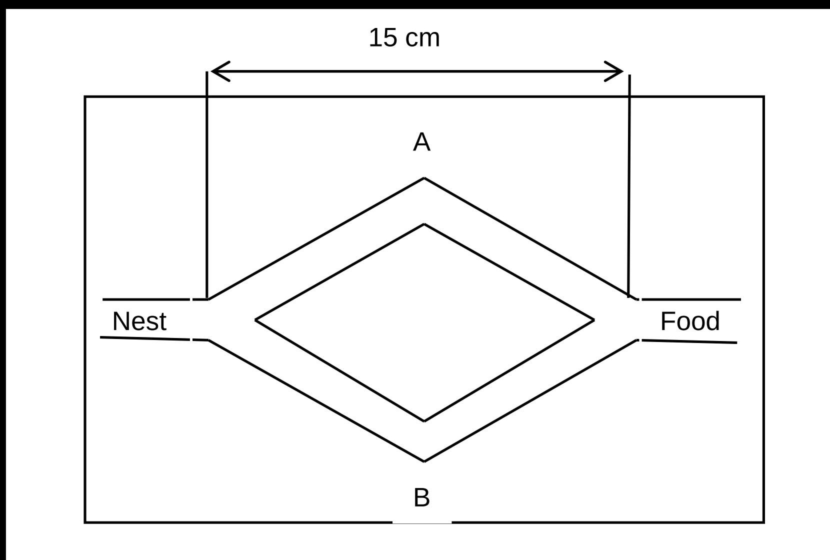
Ant Colony

Adaptivity

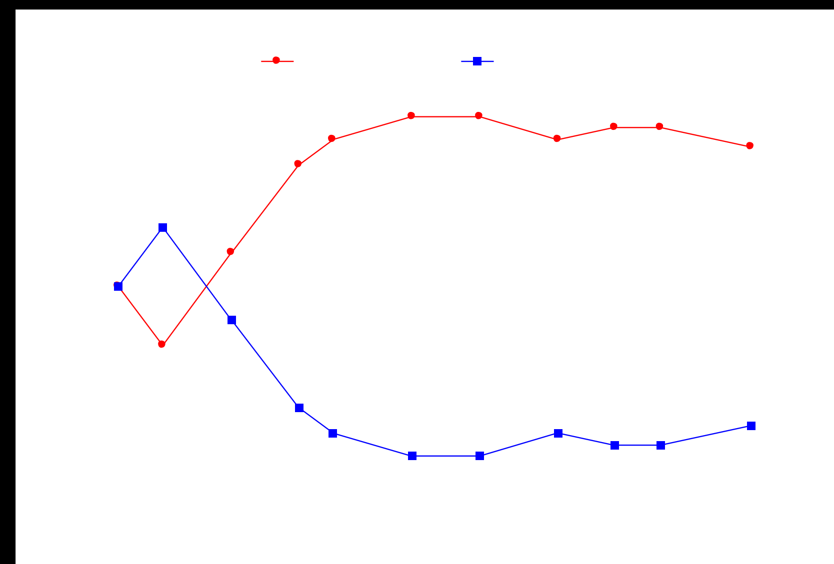


Ants Foraging Behavior
Example: The Double Bridge Experiment

Goss et al., 1989, Deneubourg et al., 1990



Simple bridge



% of ant passages on
the two branches

Ant Colony

Navigation

- At first: random
- Using pheromones as previous search experience

Recruitment (communication)

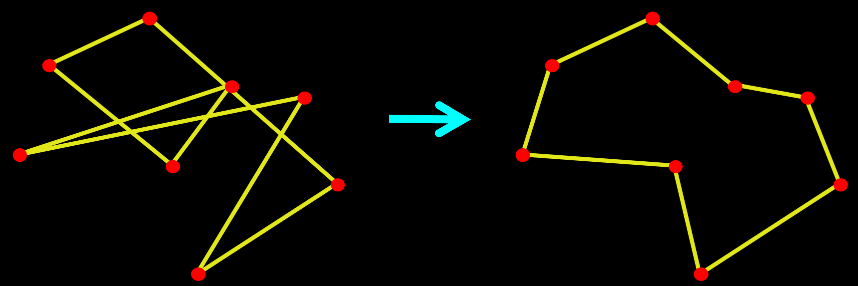
- Indirect via the environment

[Ants Trail]

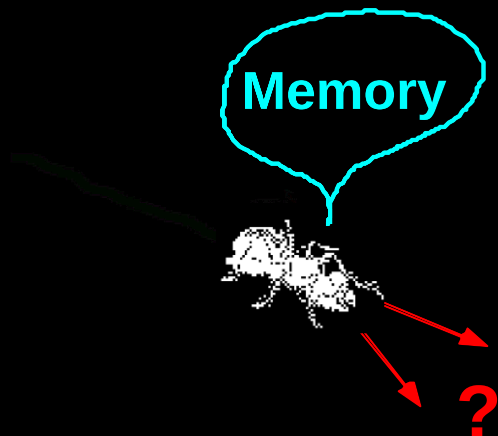
Ant System Applied to the TSP

Ant System is the ancestor of all Ant Colony Optimization algorithms

Dorigo, Maniezzo, Colorni, 1991
Dorigo & Gambardella, 1996



Pheromone trail depositing



Probabilistic rule to choose the path

Ant Algorithms

In computer science:

Decay over time

Pheromone update

$$\tau_{ij}(t+1) = (1-\rho)\cdot\tau_{ij}(t) + \sum_{k=1}^{m}\Delta\tau_{ij}^{k}(t) \qquad \forall(i,j)$$

The better the solution found by the ant, the more pheromone

$$\Delta\tau_{ij}^{k}(t) = \begin{cases} 1/L^{k}(t) & \text{if arc } (i,j) \text{ is used by ant } k \\ 0 & \text{otherwise} \end{cases}$$

Probability of selecting node j in i

$$p_{ij}^{k}(t) = \frac{[\tau_{ij}(t)]^{\alpha}\cdot[\eta_{ij}]^{\beta}}{\sum_{l\in\mathcal{N}_i^k}[\tau_{il}(t)]^{\alpha}\cdot[\eta_{il}]^{\beta}} \qquad \text{if } j\in\mathcal{N}_i^k$$

Heuristic information

Ants Movie

Assumes optimisation problem represented as a graph problem

Ant Algorithms

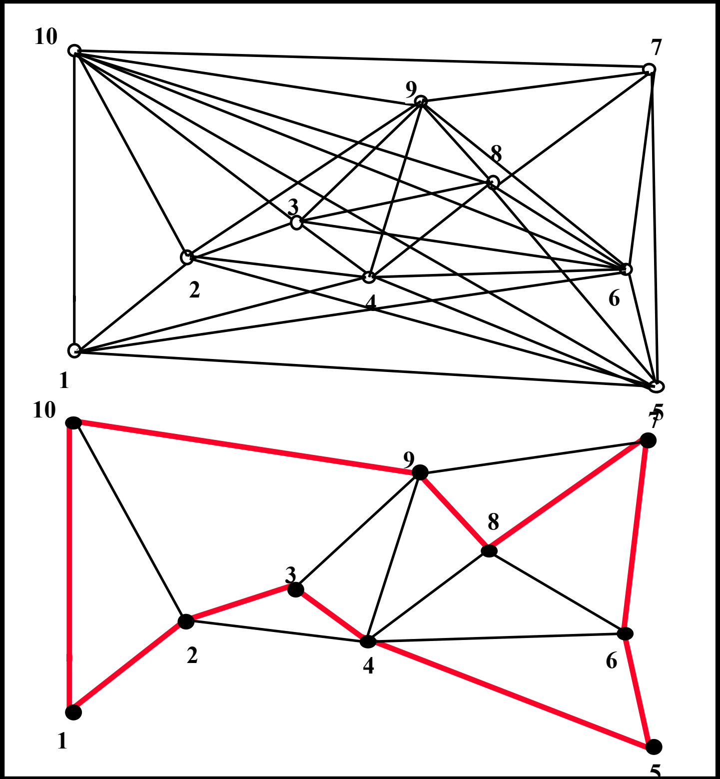
For all iterations

For all ants

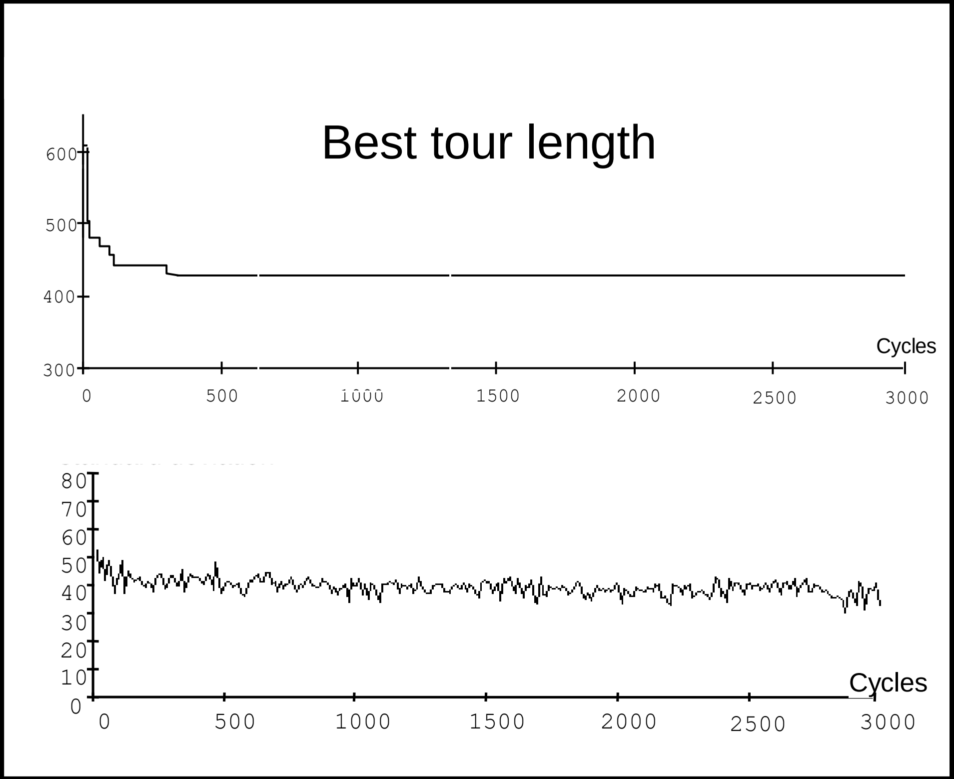
choose and perform action
(i.e. choose next node to visit)

Update pheromone

Ant System (AS): Some Results



Evolution of trail distribution



Tour length std deviation