TP GAA (Genetic Algorithms and Ant System)
Techniques of AI [INFO-H-410]
Correction

v1.0.1

Source files, code templates and corrections related to practical sessions can be found on the UV or on github (`https://github.com/iridia-ulb/INFOH410`).

Genetic Algorithms

Question 1. a) Based on the two candidate solutions 11101001000 and 00001010101, apply the following crossover procedures:

- Single-point crossover: 11111000000
- Two-point crossover: 00111110000
- Uniform crossover: 10011010011

b) Implement a genetic algorithm that starts with an initial population of randomly generated strings, and that returns a string as close as possible to the target string "Hello, World". You can use the provided template in which the `fitness`, `mutate`, and `crossover` functions are missing. Your goal is to implement these functions, together with the core logic of the algorithm.

Answer: (a) 11101001000 and 00001010101, give, spc: 00001001000, tpc: 11001011000, and uc: 01101001001

(b) see github

Ant system

Question 2. a) Is it possible to use AS with $\{\alpha = 0, \beta = 1\}$ or $\{\alpha = 1, \beta = 0\}$? What is the effect in both cases?

b) How does ρ relates with the exploration and exploitation performed by AS? What about α and β?

Answer: (a) Both options are possible, but maybe not a great idea. If AS is set with parameters $\alpha = 0$ and $\beta = 1$, pheromones will be neglected from the transition rule that ants employ to construct solutions, meaning that they will not be learning from high quality solutions found in previous iterations. On the other hand, if AS is set with $\alpha = 1$ and $\beta = 0$, ants solutions construction will be biased only by pheromone, which may end up in low quality solutions, since the heuristic information also allows to select high quality solution components, but from a greedy perspective. In a nutshell, AS performance is generally better when the algorithm can profit both from the numerical information learned in past iterations (pheromones) and from problem-dependent information (heuristic information).

(b) ρ controls the speed at which pheromone decreases. For large ρ values, the value of pheromones will decrease faster resulting in a more exploratory behavior, and for small ρ values, the value of pheromones will decrease slower leading to a more exploitative behavior. In the case of α and β, these two parameters control the relative influence of the pheromones and the heuristic information. When both values are large, the algorithm will behave more exploitative because ants will be biased towards solutions that have either high pheromone values or better heuristic information. Conversely, when both α and β are small, the probability associated to all feasible solution components will be more even, resulting in a more exploratory behavior.

Question 3. Assume the following symmetric Traveling Salesman Problem (TSP) instance:

	A	B	C	D	E
A	–	1	2	2	6
B	1	–	6	8	10
C	2	6	–	12	4
D	2	8	12	–	1
E	6	10	4	1	–

(a) TSP instance (tsp)

	A	B	C	D	E
A	–	0.56	0.66	0.60	0.50
B	0.56	–	0.60	0.56	0.60
C	0.66	0.60	–	0.50	0.56
D	0.60	0.56	0.50	–	0.66
E	0.50	0.60	0.56	0.66	–

(b) Pheromone matrix (τ) (iteration 1)

An Ant System algorithm is applied to this instance using $\alpha = 2$, $\beta = 1$, $\rho = 0.5$, $\#ants = 3$, $\eta_{ij} = 1/tsp_{ij}$ and $\tau_0 = 1$, after the first iteration the pheromone matrix (τ) is the one given in the figure above.

a) What is the meaning of the values in τ? Why $\tau_{C,D} = 0.5$?

b) Use this information to generate the first solution of iteration 2 starting from city D. Use random numbers: $\{0.80, 0.27, 0.88, 0.47, 0.05, 0.98, 0.23, 0.06\}$

c) Actually, the following solutions generated by the algorithm are $AEDCBA$ with $cost = 26$ and $DECBAD$ with $cost = 14$. Update the pheromone using this information (disregard the solution of the previous question).

d) After 12 iterations the pheromone matrix is the following:

	A	B	C	D	E
A	–	0.4285	0.0004	0.4285	0.0003
B	0.4285	–	0.4286	0.0003	0.0003
C	0.0004	0.4286	–	0.0003	0.4285
D	0.4285	0.0003	0.0003	–	0.4286
E	0.0003	0.0003	0.4285	0.4286	–

Figure 2: Pheromone matrix (τ) (iteration 12)

Would you advice to continue executing more iterations? Why?

Remember: the tour length is computed starting and ending in the same city.

e) Using the provided template, implement the `compute_probability_matrix`, `evaporate_pheromone`, `deposit_pheromone` and `get_next_city` functions of ACO for solving the TSP.

Answer: (a) $\tau_{i,j}$ indicates the desirability of adding edge i to the tour being constructed by an ant when in edge j. Since $\rho = 0.5$ and $\tau_0 = 1$, $\tau_{C,D} = 0.5$ indicates that the edge (C, D) was not part of the tour constructed by any ant.

(b) Using the random proportional rule of AS,

$$p^k_{i,j}(t) = \frac{\tau_{i,j}(t)^\alpha \cdot \eta^\beta_{i,j}}{\sum_{l \in N^k_i} \tau_{i,l}(t)^\alpha \cdot \eta^\beta_{i,l}},$$

we compute the probability of adding each feasible city, and using the roulette wheel mechanism and the set of random numbers previously generated, we select one of the city to be added to the tour.

This is done as follows:
Initial city: D

Next city:

	j =A	j =B	j =C	j =D	j =E
probabilities ($p^{k=1}_{D,j}(t = 2)$)	0.27	0.06	0.031	0	0.64
roulette wheel ($rnd = 0.80$)	0.27	0.32	0.36	0	**1**

Next city:

	j =A	j =B	j =C	j =D	j =E
probabilities ($p^{k=1}_{E,j}(t = 2)$)	0.266	0.23	0.50	0	0
roulette wheel ($rnd = 0.27$)	0.266	**0.49**	1	0	0

Next city:

	j =A	j =B	j =C	j =D	j =E
probabilities ($p^{k=1}_{A,j}(t = 2)$)	0.84	0	0.16	0	0
roulette wheel ($rnd = 0.88$)	0.84	0	**1**	0	0

The first solution constructed at iteration $t = 2$ is: $DEBCAD$

(c) Using the pheromone update rule of AS,

$$\tau_{i,j}(t) = (1 - \rho) \cdot \tau_{i,j}(t - 1) + \sum_{k=1}^{m} \Delta\tau^k_{i,j},$$

we get:

$(AB) = (0.5) \cdot 0.56 + (1/26) + (1/14) = 0.38989011$
$(AC) = (0.5) \cdot 0.66 = 0.33$
$(AD) = (0.5) \cdot 0.60 + (1/14) = 0.371428571$
$(AE) = (0.5) \cdot 0.50 + (1/26) = 0.288461538$
$(BC) = (0.5) \cdot 0.60 + (1/26) + (1/14) = 0.4098901$
$(BD) = (0.5) \cdot 0.56 = 0.28$
$(BE) = (0.5) \cdot 0.60 = 0.3$
$(CD) = (0.5) \cdot 0.50 + (1/26) = 0.288461538$
$(CE) = (0.5) \cdot 0.56 + (1/14) = 0.351428571$
$(DE) = (0.5) \cdot 0.66 + (1/26) + (1/14) = 0.43989011$

(d) At this point the algorithm is stagnated, which means that the probability of constructing a set of solutions different from the one constructed in the last iteration is quite low. In this example, the algorithm will be constructing solutions

$$
\begin{aligned}
&ABCEDA,\\
&ADECBA,\\
&BADECB,\\
&BDECAB,\\
&CBADEC,\\
&CEDABC,\\
&DABCED,\\
&DECBAD,\\
&ECBADE, \text{ and}\\
&EDABCE
\end{aligned}
$$

over and over again.

(e) see github for implementation

Data: Pop_{size}, Num_{gen}, $Crossover_{proc}$, $Mutation_{proc}$

for $i \in 1, .., Pop_{size}$ **do**
  Population[i] $\leftarrow$ RandomSolution()
end
Evaluate(Population)
$S_{best} \leftarrow$ GetBestSolution(Population)
for $gen \in 1, .., Num_{gen}$ **do**
  Parents $\leftarrow$ Selection(Population)
  Children $\leftarrow \phi$
  for $Parent_1,\ Parent_2 \in Parents$ **do**
    $Children_1$, $Children_2$ $\leftarrow$ Crossover($Parent_1$, $Parent_2$, $Crossover_{proc}$)
    $Children_1$ $\leftarrow$ Mutate($Children_1$, $Mutation_{proc}$)
    $Children_2$ $\leftarrow$ Mutate($Children_2$, $Mutation_{proc}$)
  end
  Evaluate(Children)
  $P_{best} \leftarrow$ GetBestSolution(Children)
  if $P_{best} > S_{best}$ **then**
    $S_{best} \leftarrow P_{best}$
  end
  Population $\leftarrow$ Children
end

Algorithm 1: Pseudocode for a simple **genetic algorithm**. Pop_{size} is the number of individuals in the population, Num_{gen} is the number of generations, $Crossover_{proc}$ is a crossover procedure, and $Mutation_{proc}$ is a mutation procedure.

Data: n, m, α, β, ρ, τ_0

while *!termination()* **do**
  for $k \in 1, .., m$ **do**
    ants[k] [1] $\leftarrow$ SelectRandomCity()
    for $i \in 2, n$ **do**
      ants[k] [i] $\leftarrow$ ASDecisionRule(ants, i)
    end
    ants[k] [$n + 1$] $\leftarrow$ ants[k] [1]
  end
  UpdatePheromones(ants)
end

Algorithm 2: Pseudocode for the **Ant System algorithm**. n is the size of the problem, m the number of ants, α and β are parameters of the solution construction procedure ASDecisionRule (see Eq. 1), ρ is the parameter of the pheromone update procedure UpdatePheromones (see Eq. 2), and τ_0 is the initial value of the pheromones

Reminder

$$ASDecisionRule \rightarrow p_{ij}^k(t) = \frac{[\tau_{ij}]^\alpha \cdot [\eta_{ij}]^\beta}{\sum_{l \in N_i^k} [\tau_{il}]^\alpha \cdot [\eta_{il}]^\beta}, \qquad \text{if} \quad j \in N_i^k \tag{1}$$

$$
\begin{aligned}
UpdatePheromones \rightarrow \tau_{ij}(t) &= (1-\rho)\cdot\tau(t-1) + \sum_{k=1}^{m}\Delta\tau_{ij}^{k} \\
\Delta\tau_{ij}^{k} &= \frac{1}{L_k}, \qquad \text{if } arc(i,j) \text{ is used by ant k on its tour}
\end{aligned}
\tag{2}
$$

Found an error? Let us know: `https://github.com/iridia-ulb/INFOH410/issues`