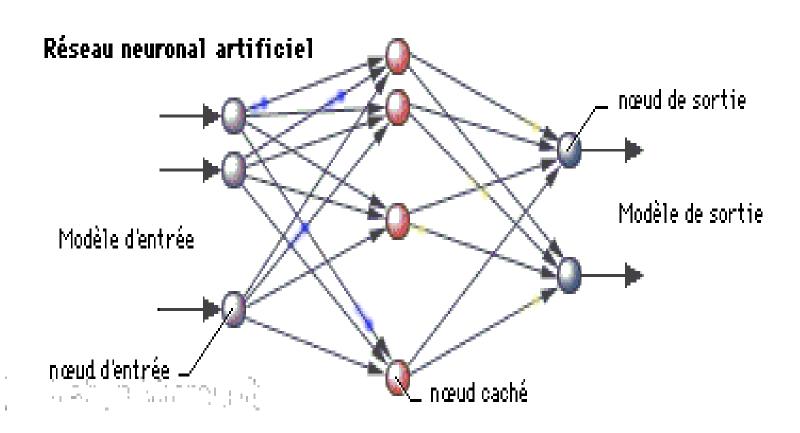
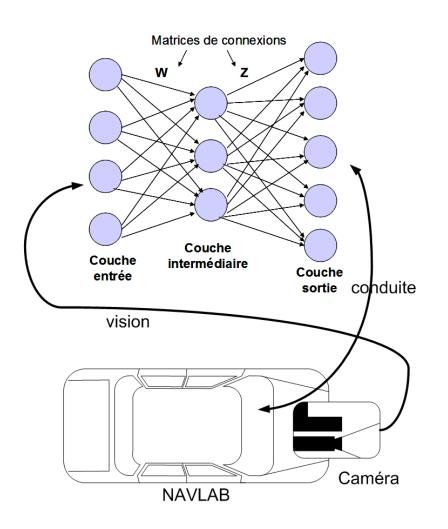
Neural Networks

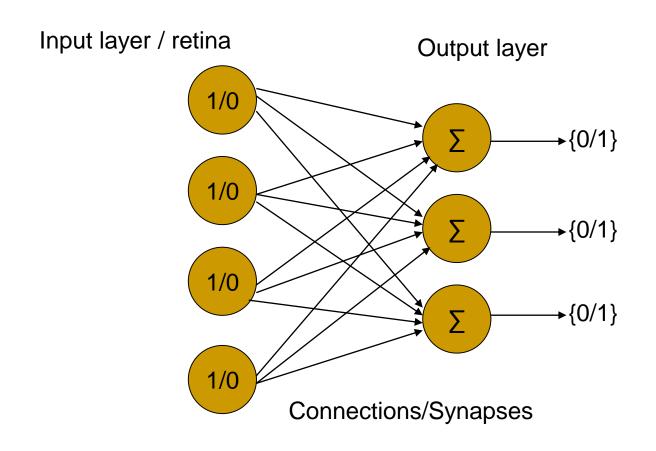


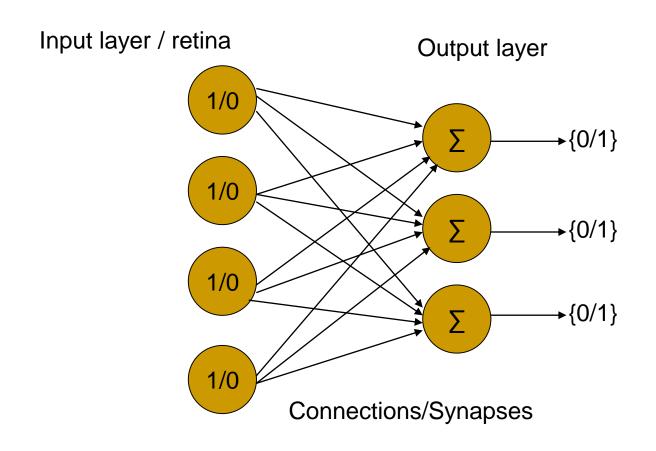


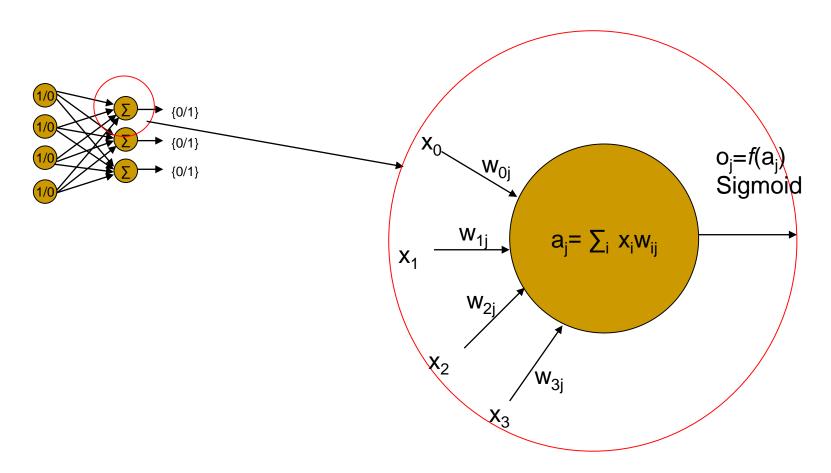
Plan

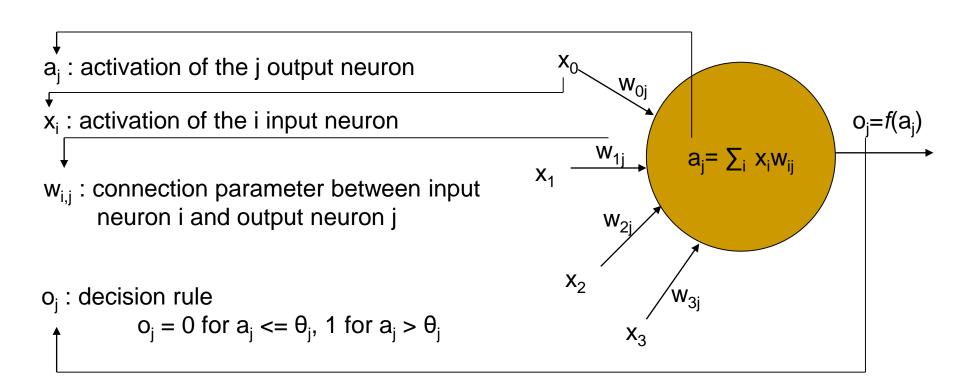
- Perceptron
 - Linear discriminant
- Associative memories
 - Hopfield networks
 - Chaotic networks
- Multilayer perceptron
 - Backpropagation

- Historically, the first neural net
- Inspired by human brain
- Proposed
 - By Rosenblatt
 - Between 1957 et 1961
- The brain was appearing as the best computer
- Goal: associated input patterns to recognition outputs
- Akin to a linear discriminant



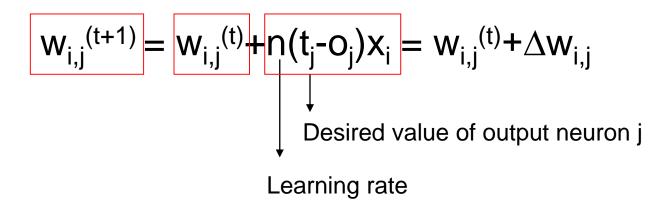






- Need an associated learning
 - Learning is supervised
 - Based on a couple input pattern and desired output
 - If the activation of output neuron is OK => nothing happens
 - Otherwise inspired by neurophysiological data
 - If it is activated : decrease the value of the connection
 - If it is unactivated: increase the value of the connection
 - Iterated until the output neurons reach the desired value

- Supervised learning
 - How to decrease or increase the connections?
 - Learning rule of Widrow-Hoff
 - Closed to Hebbian learning



Theory of linear discriminant

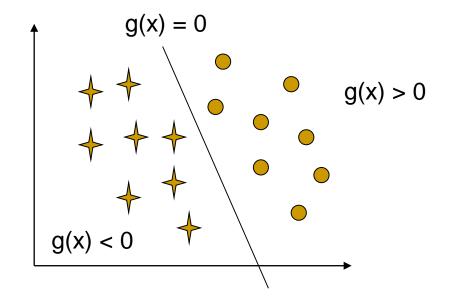
Compute:

$$g(x) = W^T x + W_o$$

And:

Choose:

class 1 if g(x) > 0 class 2 otherwise



But how to find W on the basis of the data?

Gradient descent:

$$\Delta W_i = -\eta \, \frac{\partial E}{\partial W_i}, \forall i$$

In general a sigmoid is used for the statistical interpretation: (0,1)

$$Y = 1/1 + \exp[-g(x)]$$

Easy to derive = Y(1-Y)

Class 1 if Y > 0.5 and 2 otherwise

The error could be least square: $(Y - Y_d)^2$

Or maximum likelihood: $-\sum Y_d \log Y + (1 - Y_d) \log(1 - Y)$

But at the end, you got the learning rule: $\Delta W = \eta \sum (Yd - Y)X_j$

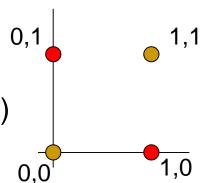
Perceptron limitations

Limitations

- Not always easy to learn
- But above all, cannot separate not linearly separable data

Why so ?

 The XOR kills NN researches for 20 years (Minsky and Papert were responsable)

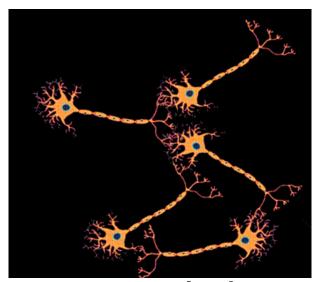


Consequence

- We had to wait for the magical hidden layer
- And for backpropagation

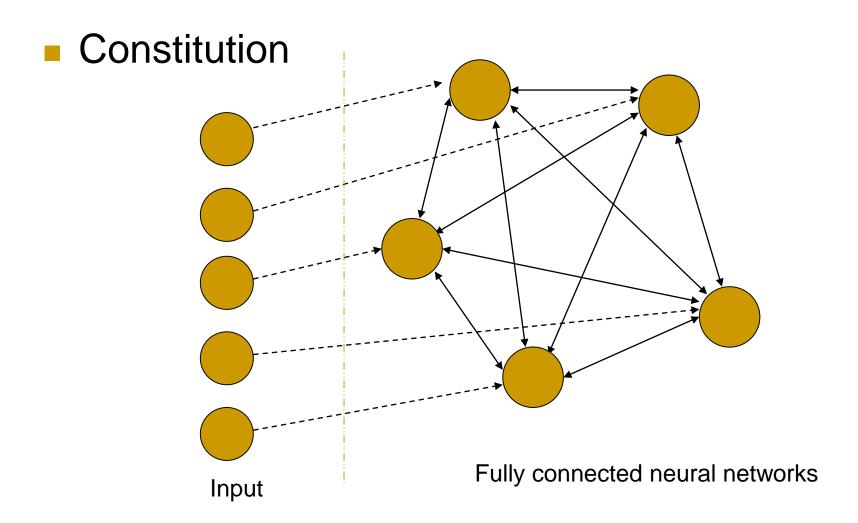
Associative memories

- Around 1970
- Two types
 - Hetero-associative
 - And auto-associative



- We will treat here only auto-associative
- Make an interesting connections between neurosciences and physics of complex systems
- John Hopfield

Auto-associative memories



Hopfield networks recurrent networks and memories

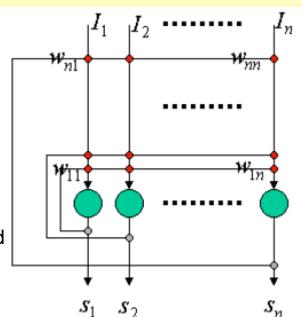
$$h_i(t) = \sum_{i=1}^{N} w_{ij} s_j(t) + \theta_i$$
$$s_i(t+1) = \operatorname{sgn}[h_i(t)]$$

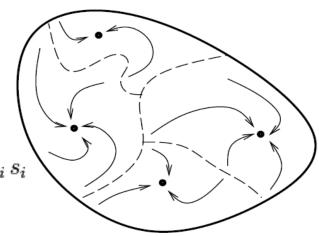
- 1. Statistical analysis using spin models
 - According to weight constrains (weights have to be symmetric), the dynamics iterates to stable patterns: fixed point attractors, i.e. 'memories'
- 2. Hebbian learning of the desired attractors

$$\{\xi^{\mu} = (\xi_1^{\mu}, \xi_2^{\mu} \dots \xi_N^{\mu}); 1 \le \mu \le M\}$$

$$w_{ij} = \frac{1}{N} \sum_{p=1}^{M} \xi_i^p \xi_j^p$$

3. Minimize the Energy Function
$$E = -\sum_{i < j} w_{ij} \, s_i \, s_j + \sum_i \theta_i \, s_i$$



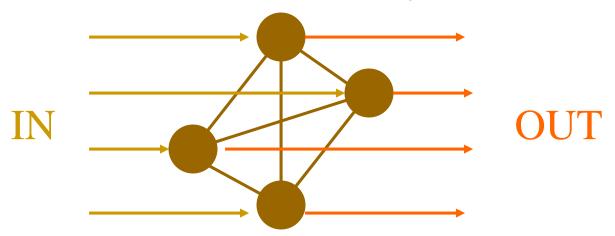


Associative memories



Hopfield -> DEMO

- Fully connected graphs
- Input layer = Output layer = Networks
- The connexions have to be symmetric



It is again an hebbian learning rule

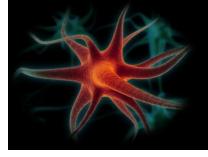
Associative memories



<u>Hopfield</u>

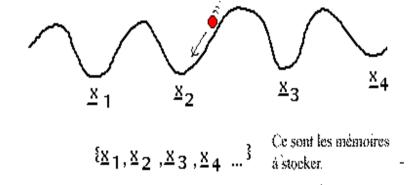
- The newtork becomes a dynamical machine
- It has been shown to converge into a fixed point
- This fixed point is a minimal of a Lyapunov energy
- These fixed point are used for storing «patterns »
- Discrete time and asynchronous updating
 - > input in {-1,1}
 - $> x_i \rightarrow sign(\Sigma_j w_{ij}x_j)$

Mémoires associatives



<u>Hopfield</u>

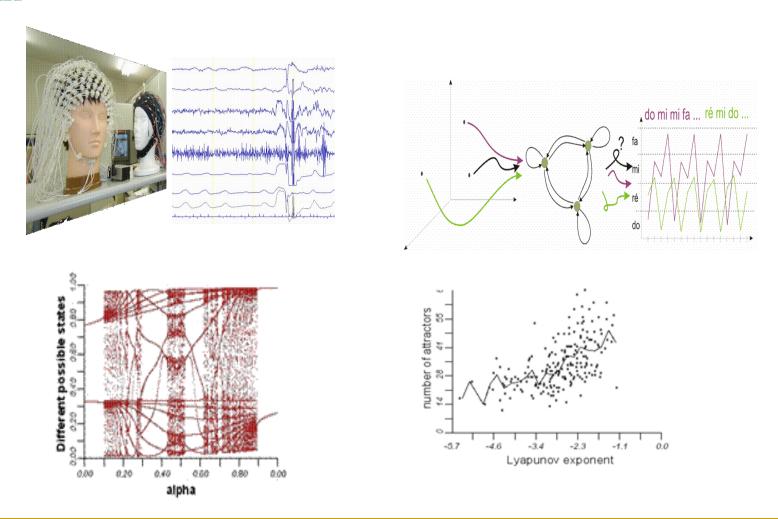
The learning is done by Hebbian learning



Over all patterns to learn:

$$\Delta W_{ij} = \sum_{patterns} X_i^p X_j^p$$

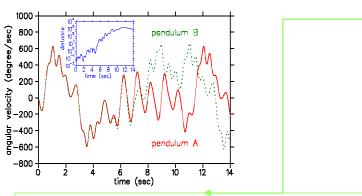
My researches: Chaotic encoding of memories in brain



What is "chaos"?

Chaos – is a *aperiodic* long-time behavior arising in a *deterministic* dynamical system that exhibits a *sensitive dependence on initial*

exhibits conditions.



The nearby trajectories separate exponentially fast

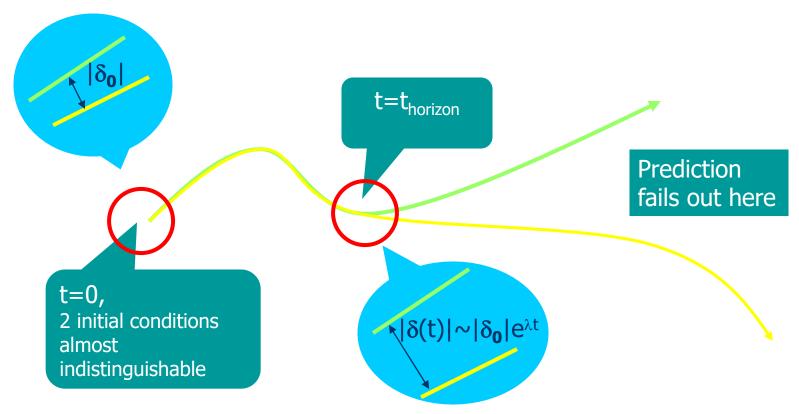


Lyapunov Exponent > 0

Trajectories which do not settle down to fixed points, periodic orbits or quasiperiodic orbits as $t\rightarrow\infty$

The system has no random or noisy inputs or parameters – the irregular behavior arises from system's nonliniarity

Demonstration

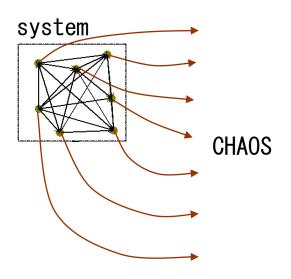


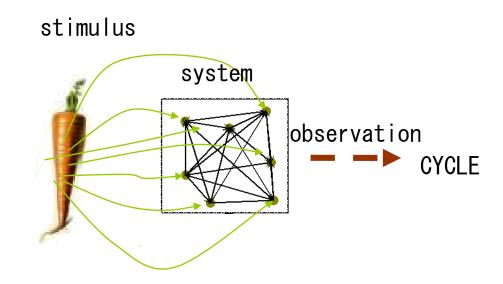
• No matter how hard we work to reduce measurement error, we cannot predict longer than a few multiples of $1/\lambda$.

In absence of information

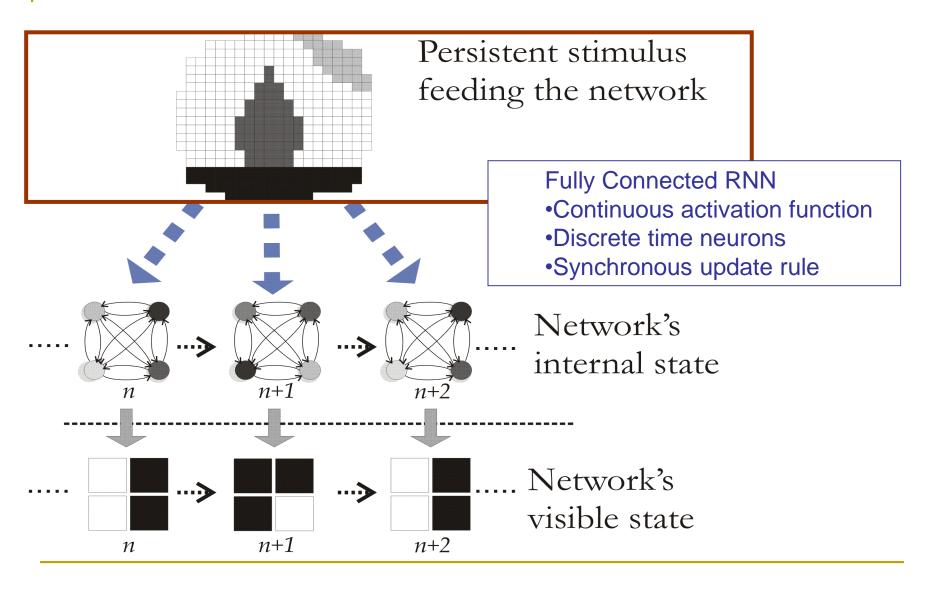
Storing information

observation

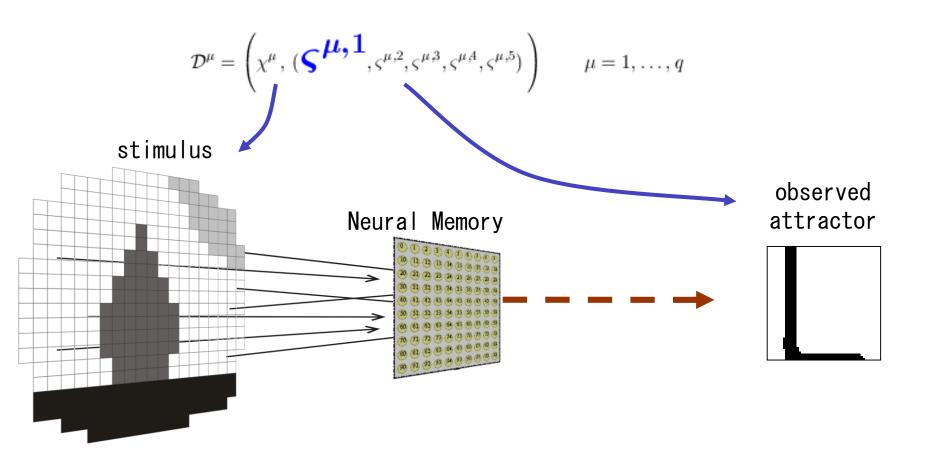




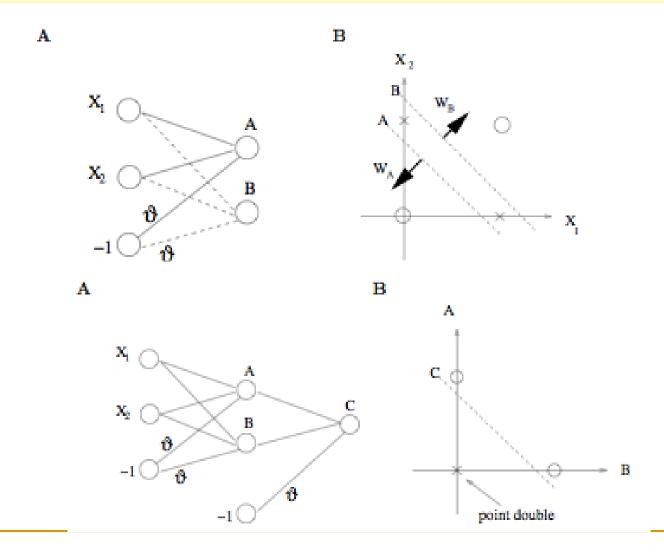
The model



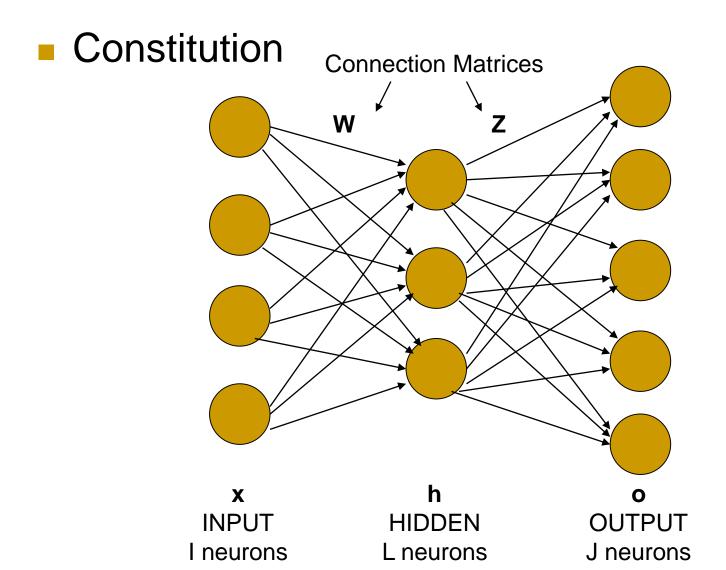
The "out-supervised" learning task



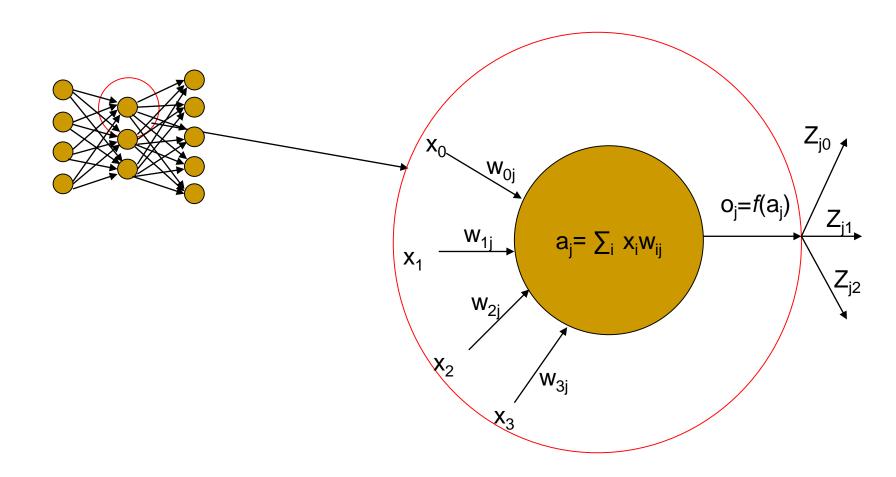
Multi-Layers Perceptron the XOR problem



Multilayer perceptron



Multilayer Perceptron



Error backpropagation

- Learning algorithm
- How it proceeds :
 - Inject an input
 - Get the output
 - Compute the error with respect to the desired output
 - Propagate this error back from the output layer to the input layer of the network
 - Just a consequence of the chaining derivative of the gradient descent

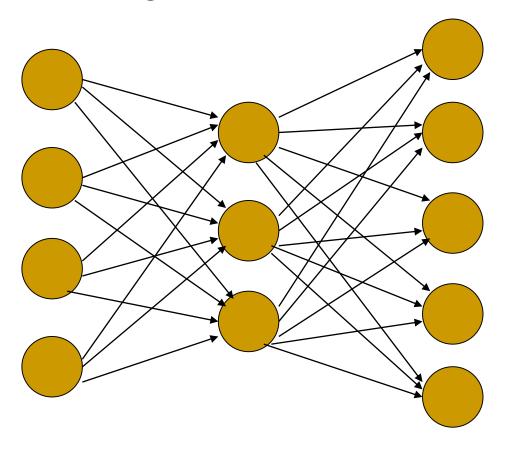
- Select a derivable transfert function
 - Classicaly used : The logistics

$$f(x) = \frac{1}{1 + e^{-x}}$$

And its derivative

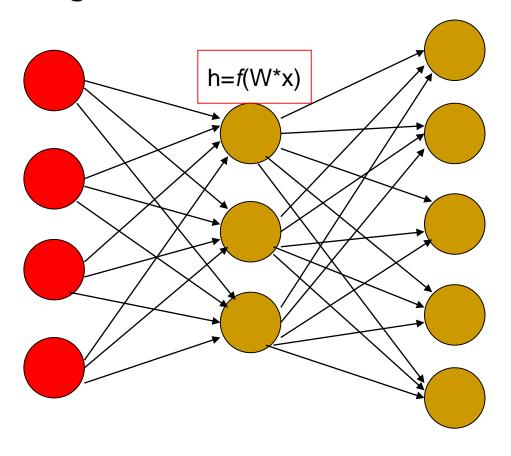
$$f'(x) = f(x)[1 - f(x)]$$

The algorithm

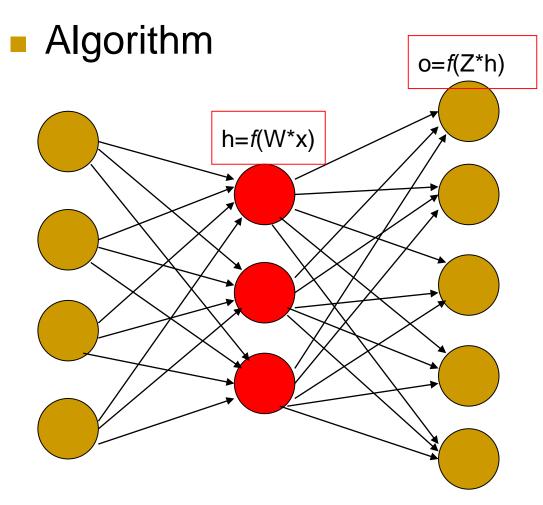


1. Inject an entry

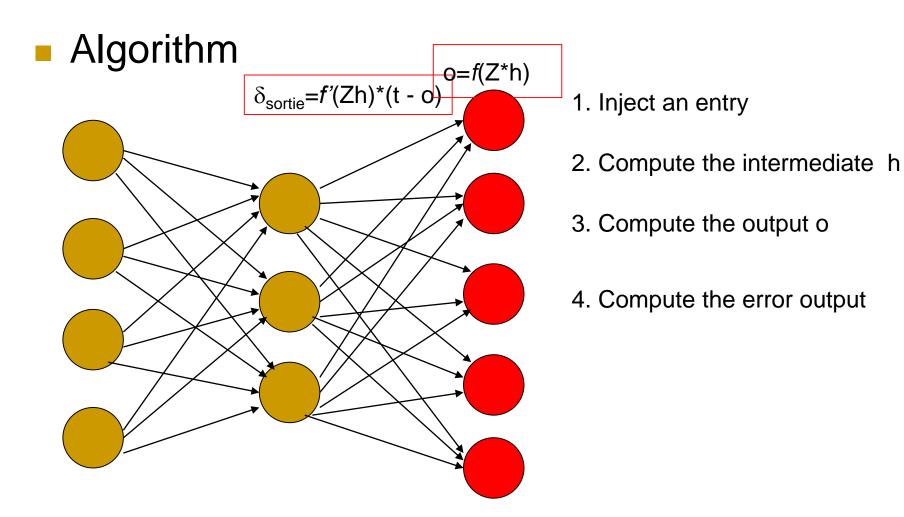
Algorithm



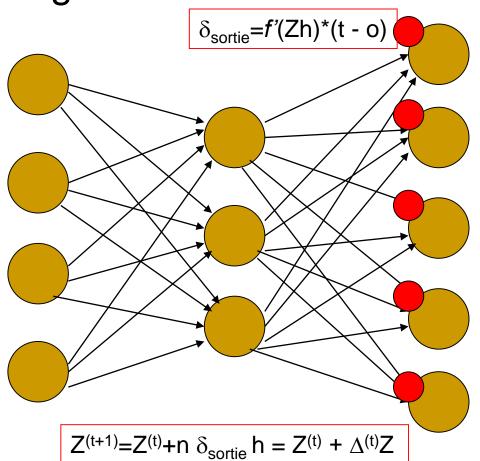
- 1. Inject an entry
- 2. Compute the intermediate h



- 1. Inject an entry
- 2. Compute the intermediate h
- 3. Compute the output o

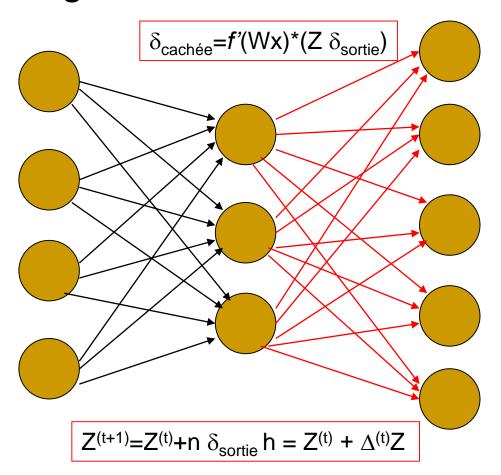


Algorithm



- 1. Inject an entry
- 2. Compute the intermediate h
- 3. Compute the output o
- 4. Compute the error output
- 5. Adjust Z on the basis of the error

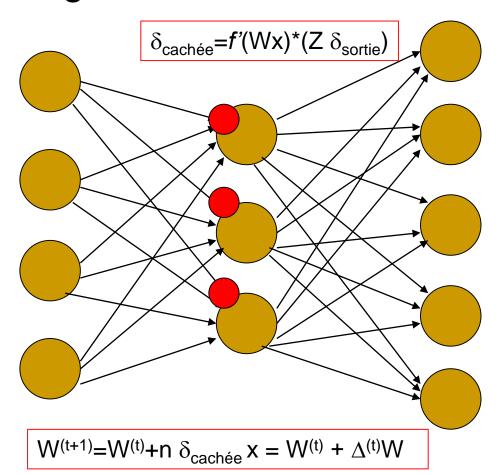
Algorithm



- 1. Inject an entry
- 2. Compute the intermediate h
- 3. Compute the output o
- 4. Compute the error output
- 5. Adjust Z on the basis of the error
- 6. Compute the error on the hidden layer

Backpropagation

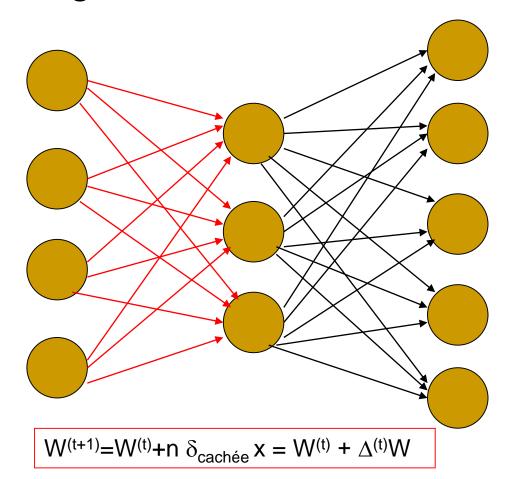
Algorithm



- 1. Inject an entry
- 2. Compute the intermediate h
- 3. Compute the output o
- 4. Compute the error output
- 5. Adjust Z on the basis of the error
- 6. Compute the error on the hidden layer
- 7. Adjust W on the basis of this error

Backpropagation

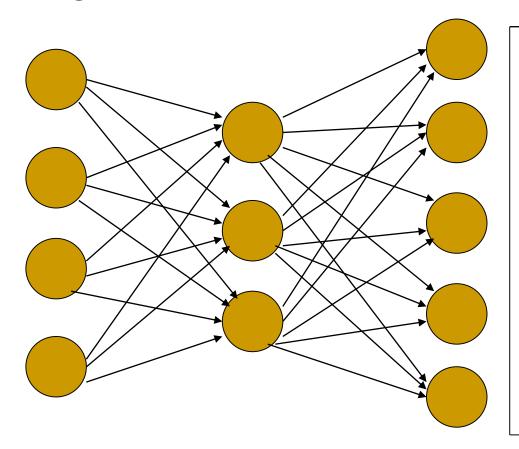
Algorithm



- 1. Inject an entry
- 2. Compute the intermediate h
- 3. Compute the output o
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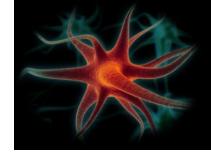
Backpropagation

Algorithm

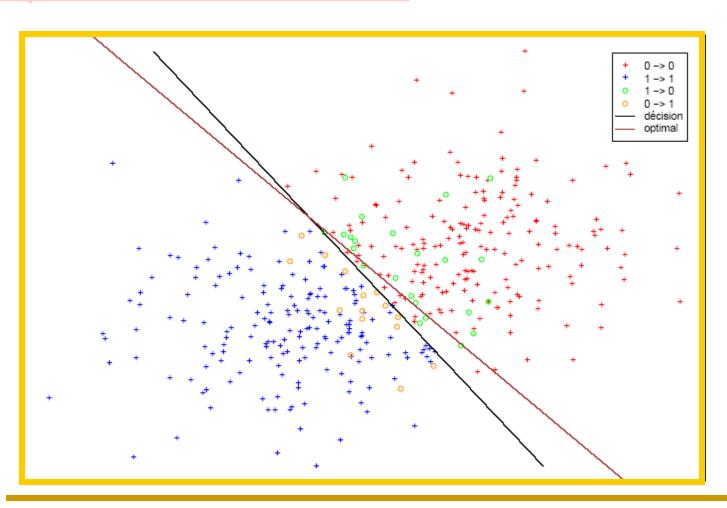


- ↑1. Inject an entry
 - 2. Compute the intermediate h
 - 3. Compute the output o
- 4. Compute the error output
- 5. Adjust Z on the basis of the error
- 6. Compute the error on the hidden layer
- 7. Adjust W on the basis of this error

Neural network

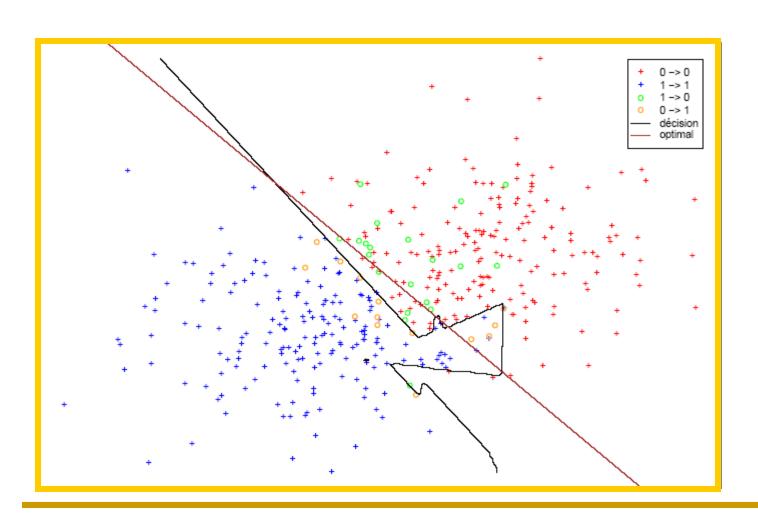


Simple linear discriminant



Neural networks

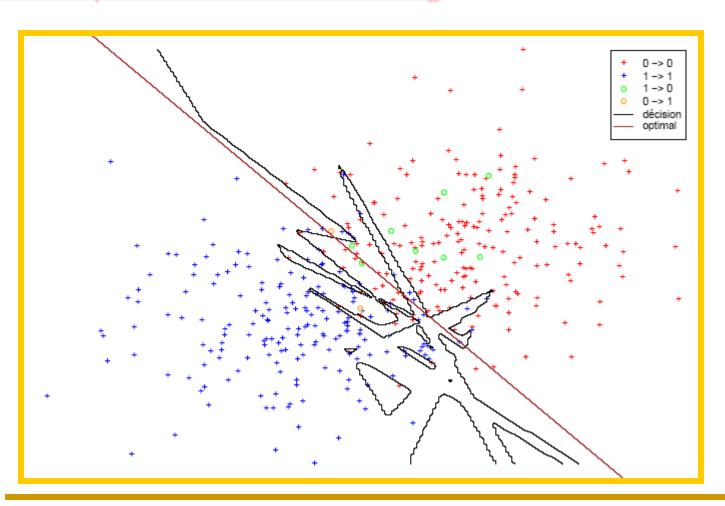
Few layers - Little learning

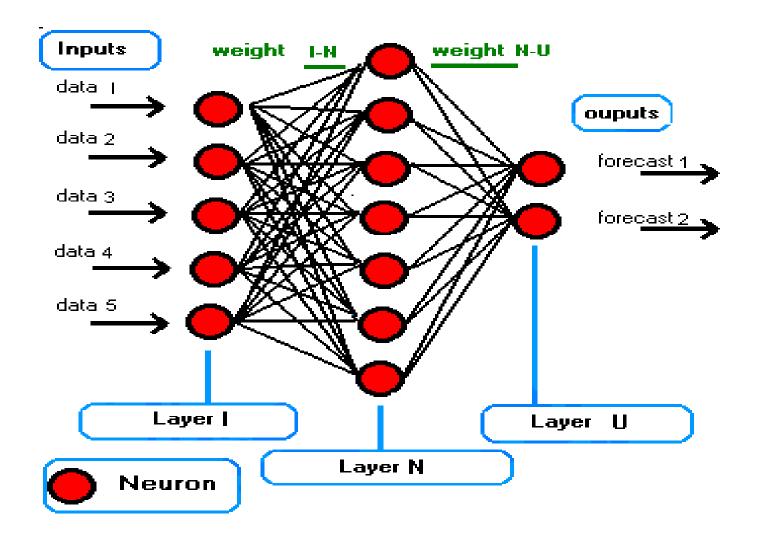


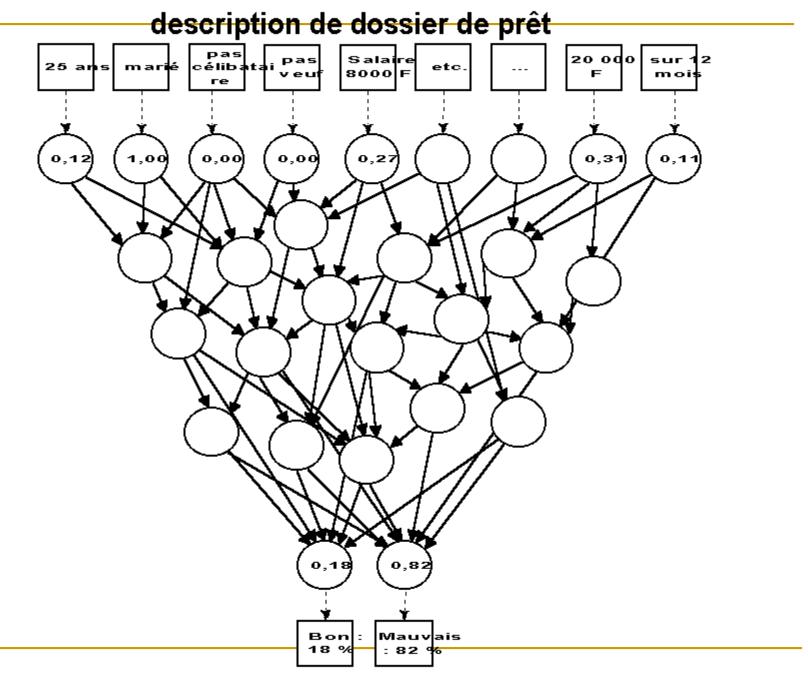
Neural networks



More layers - More learning

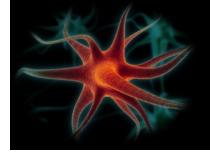






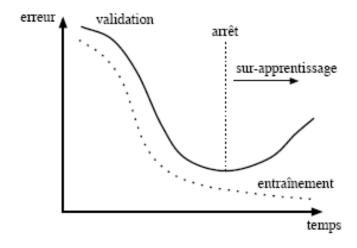
suggestion de décision

Neural networks

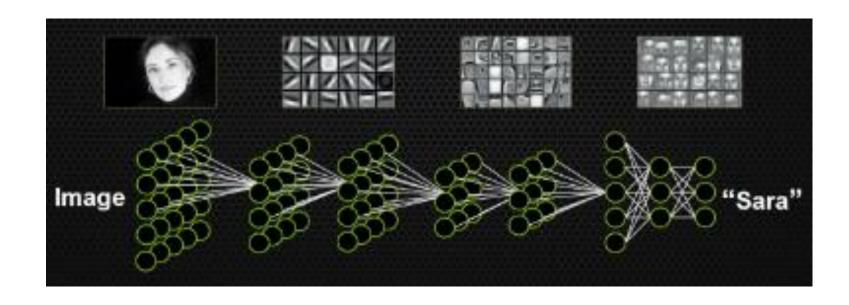


Tricks

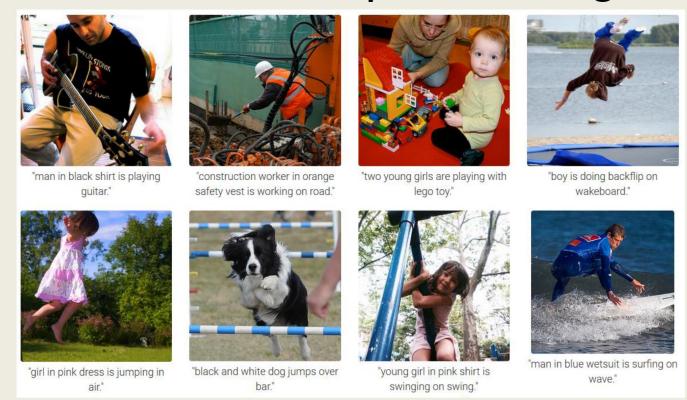
- Favour simple NN (you can add the structure in the error)
- Few layers are enough (theoretically only one)
- Exploit cross validation...



The revenge of Neural Networks: Deep Learning



Automatic description of images



Andrej Karpathy and Li Fei-Fei, http://cs.stanford.edu/people/karpathy/deepimagesent/

Beyond the Multi-Layer Perceptron

You can learn any function with one hidden layer, but it's not the best way to do it

Convolution layer for images:

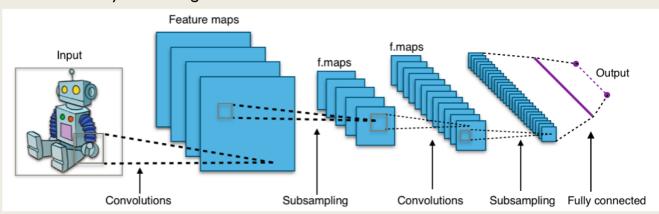


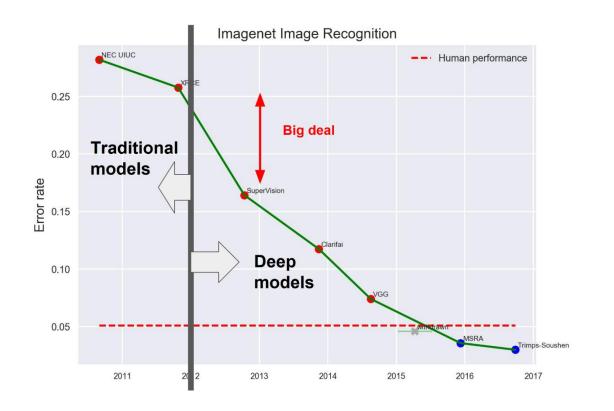
Image by Aphex34 - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=45679374

5 weird tricks to improve training

- How to initialize the model
- How to choose a nonlinearity
- How to avoid over-fitting
- How to pre-process the data

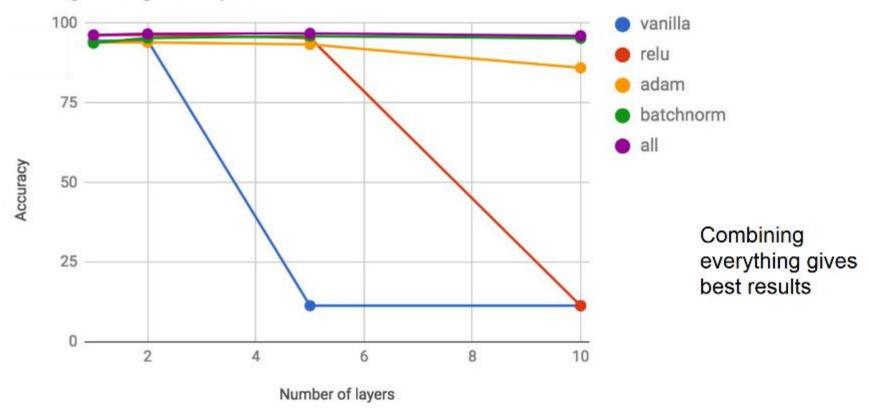


Theano is a good alternative to TensorFlow



Breakthrough

Testing training techniques



Deep in time

52

Beyond the Multi-Layer Perceptron

Gated memory for sequences:

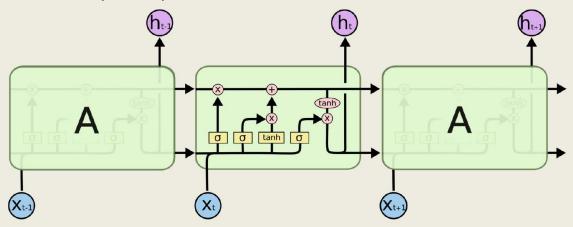
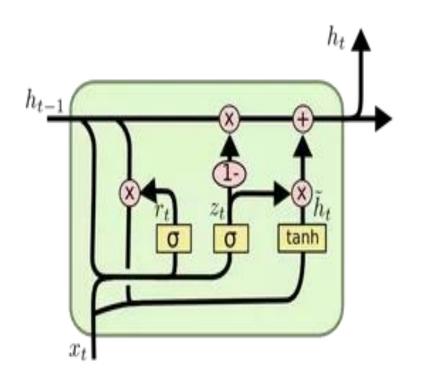


Image by Chris Olah, http://colah.github.io/posts/2015-08-Understanding-LSTMs/ (great explanation of modern recurrent neural nets)

The Neural Network Zoo: http://www.asimovinstitute.org/neural-network-zoo/

GRU RNN



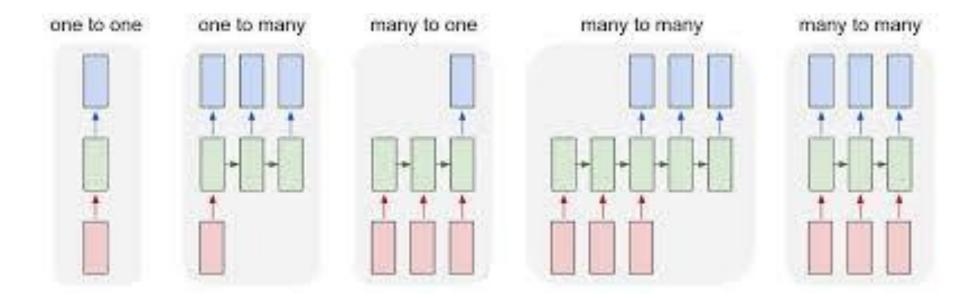
$$z_t = \sigma (W_z \cdot [h_{t-1}, x_t])$$

$$r_t = \sigma (W_r \cdot [h_{t-1}, x_t])$$

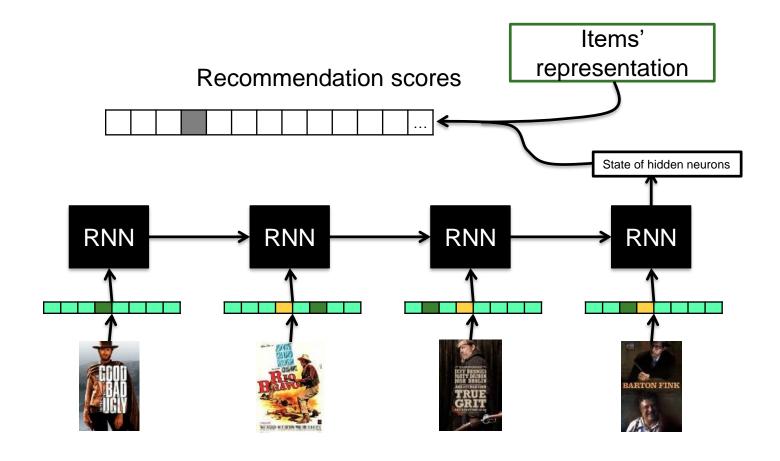
$$\tilde{h}_t = \tanh (W \cdot [r_t * h_{t-1}, x_t])$$

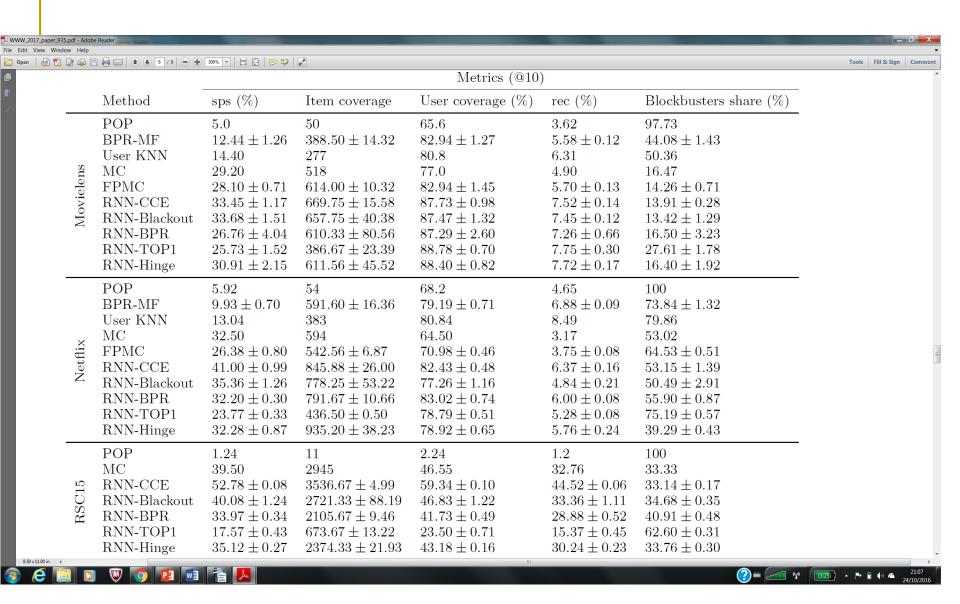
$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

Different usages



RNN: Algorithmes prédictifs





In Proceedings of NIPS 2016 and UMAP 2017