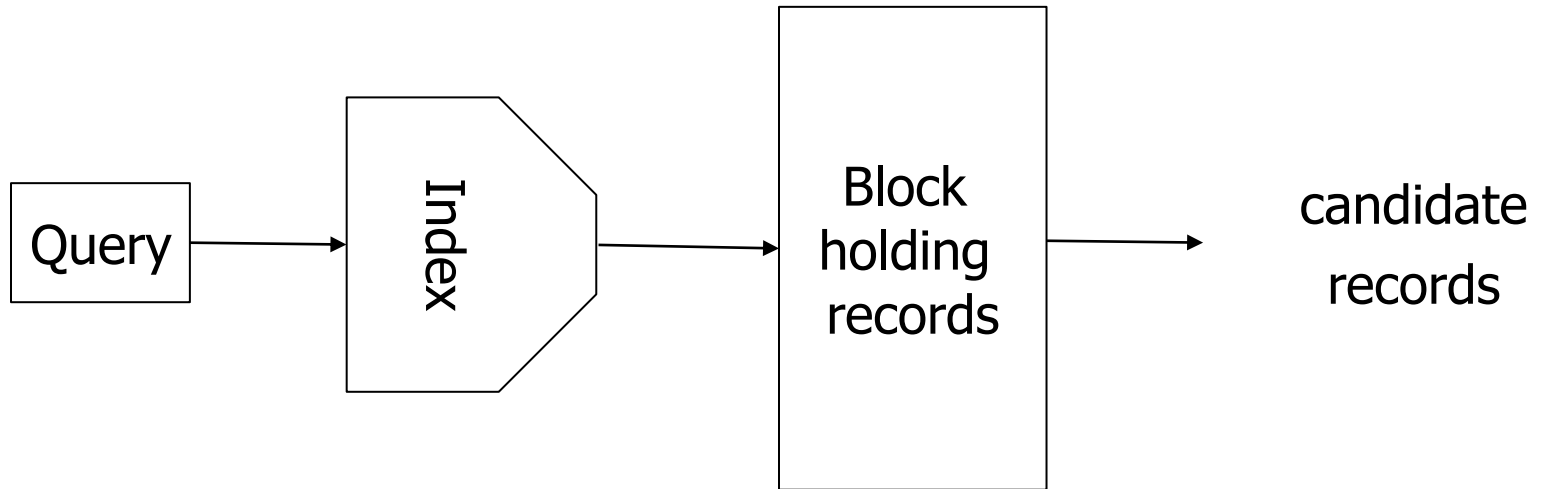


# **Indexing**

Hector Garcia-Molina  
Mahmoud Sakr

# Indexing



# Topics

- Conventional indexes
- B-trees
- Hashing schemes

## Sequential File

10	
20	

30	
40	

50	
60	

70	
80	

90	
100	

## Dense Index

## Sequential File

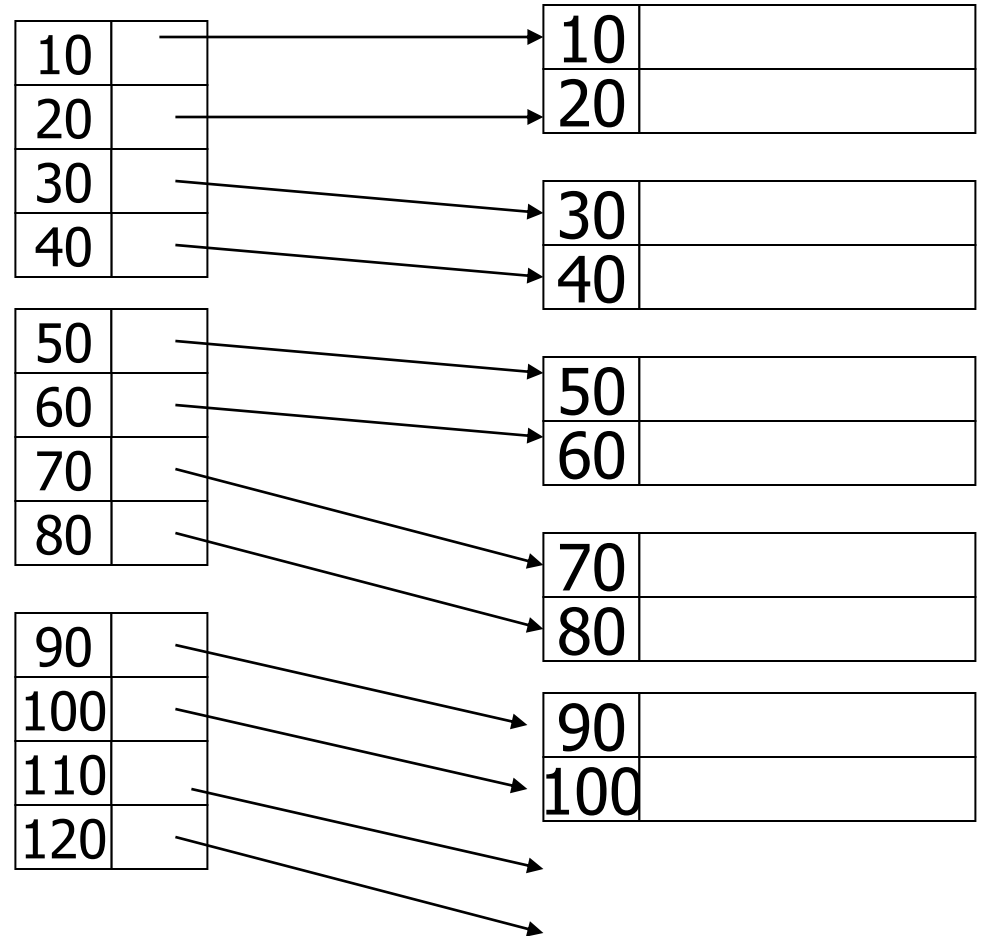
Dense Index = a pointer per key

How to search for a key= 30 ?

How to search for a key= 25 ?

Can we use a dense index on a non-sequential file ?

Why querying a dense index is more efficient than querying the sequential file ?



## Sparse Index

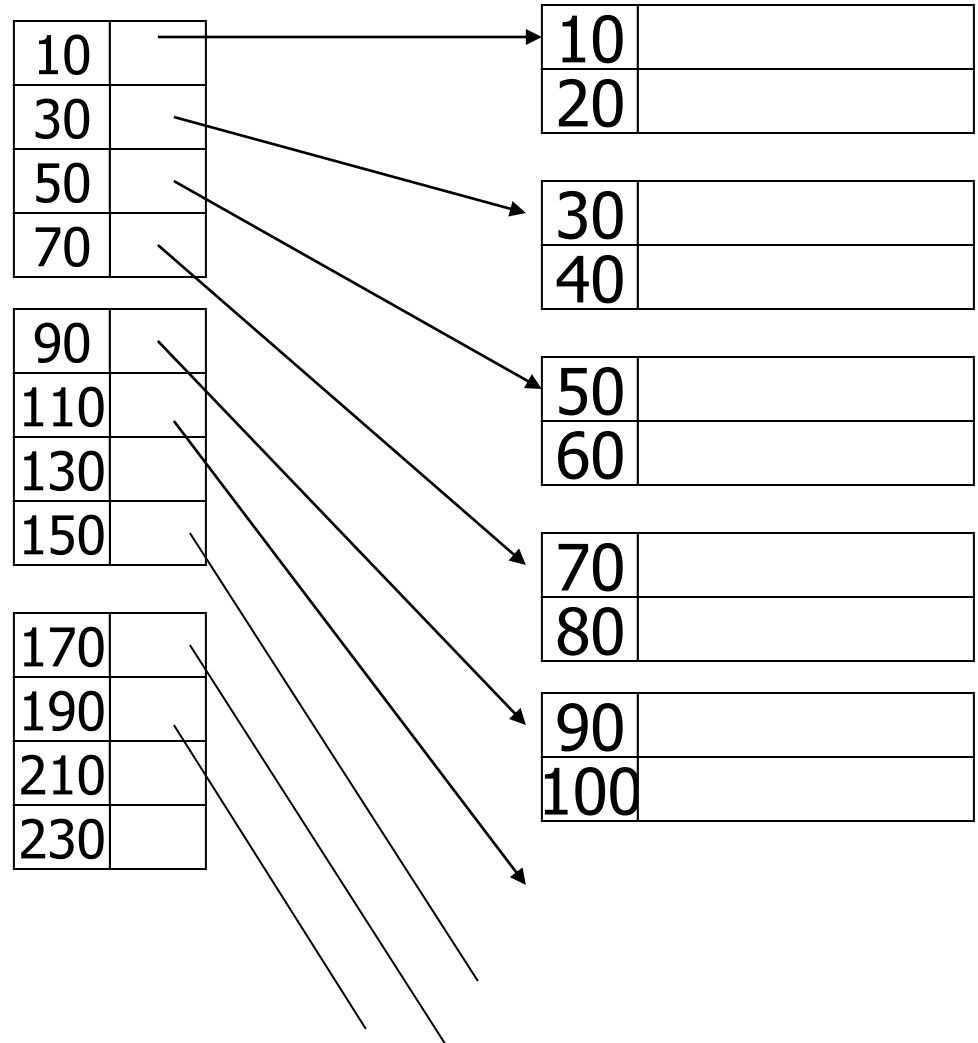
## Sequential File

Sparse index = a pointer per block

How to search for a key= 30 ?

How to search for a key= 25 ?

Can we use a sparse index on a non-sequential file ?



## Sparse 2nd level

## Sequential File

10	
90	
170	
250	

330	
410	
490	
570	

10	
30	
50	
70	

90	
110	
130	
150	

170	
190	
210	
230	

10	
20	

30	
40	

50	
60	

70	
80	

90	
100	

If the first level index has so many pages, adding a second level sparse index can speed up the search.

Do we benefit from a second level  
**dense** index?

# Sparse vs. Dense Tradeoff

- Sparse: Less index space per record  
can keep more of index in memory
- Dense: Can tell if any record exists  
without accessing file

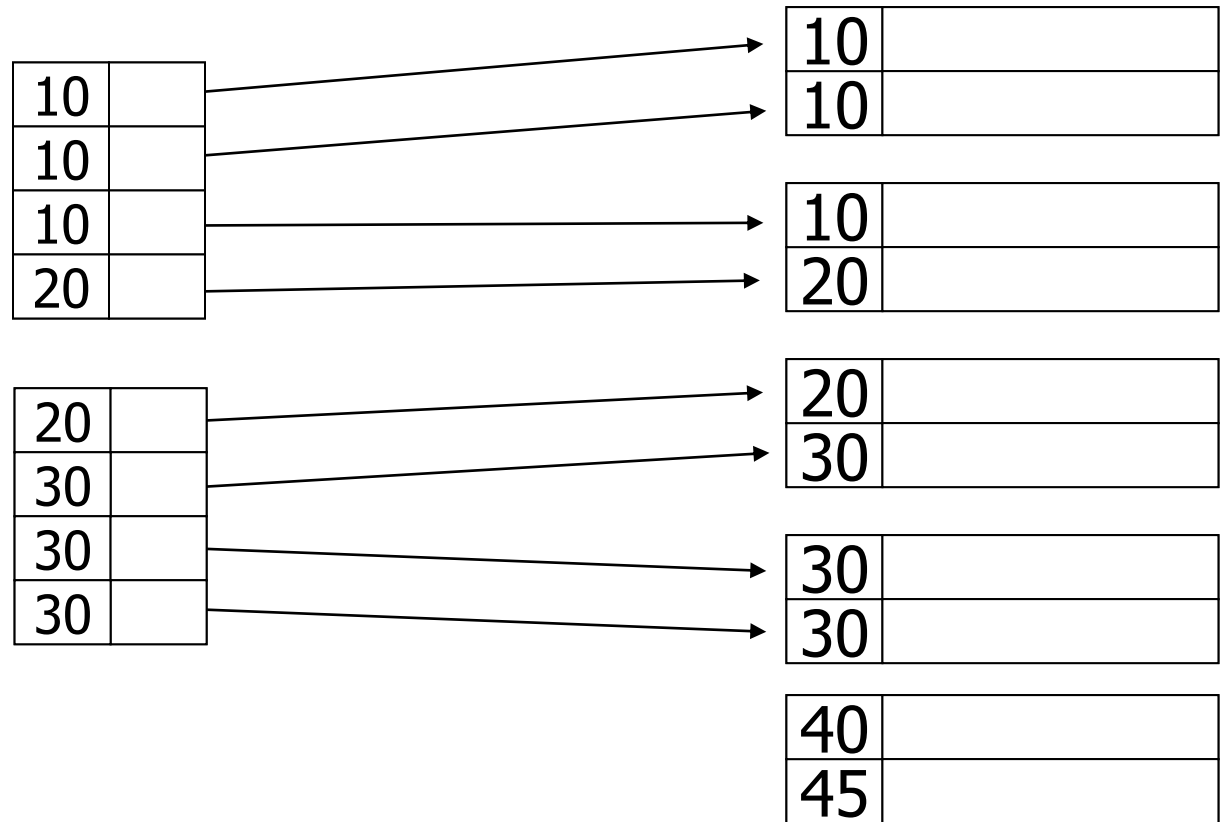


# Duplicate keys

Dense index, one way to implement?

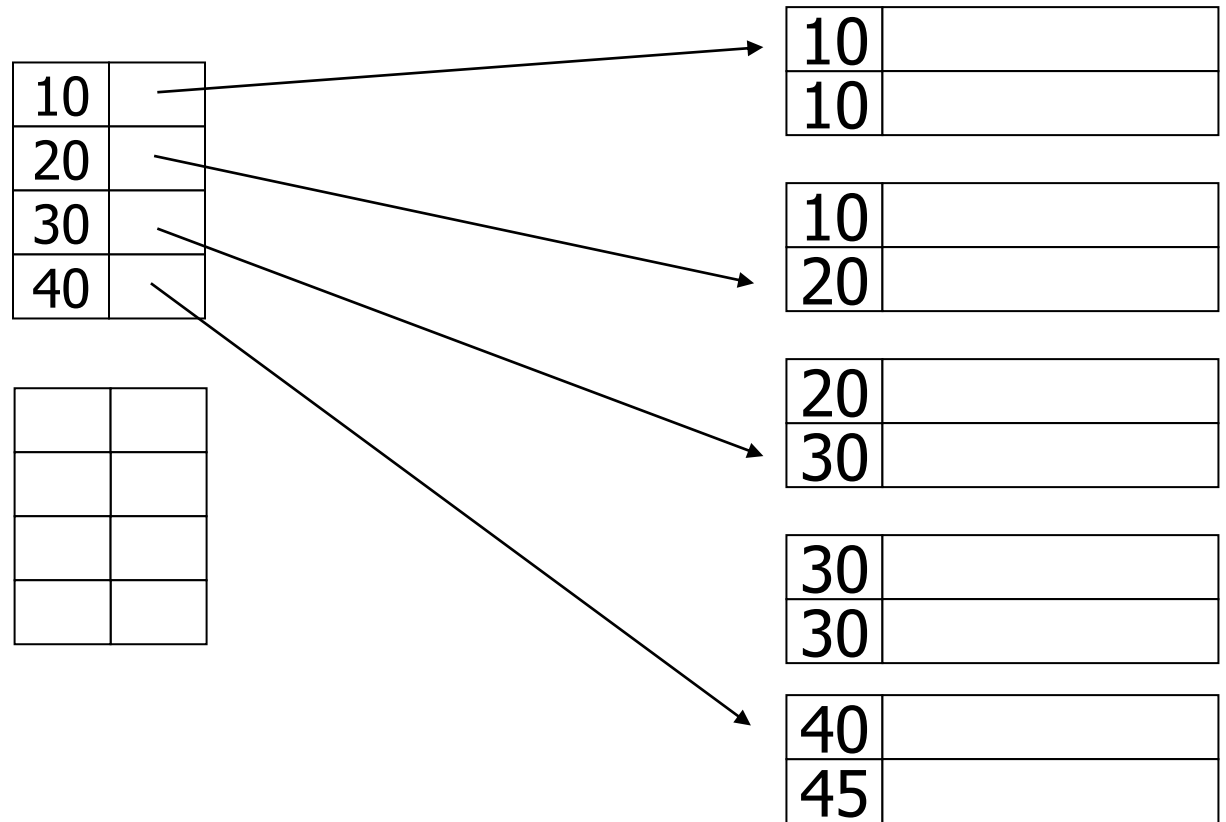
Dense index.

Can We do better ?



# Duplicate keys

Dense index, better way?

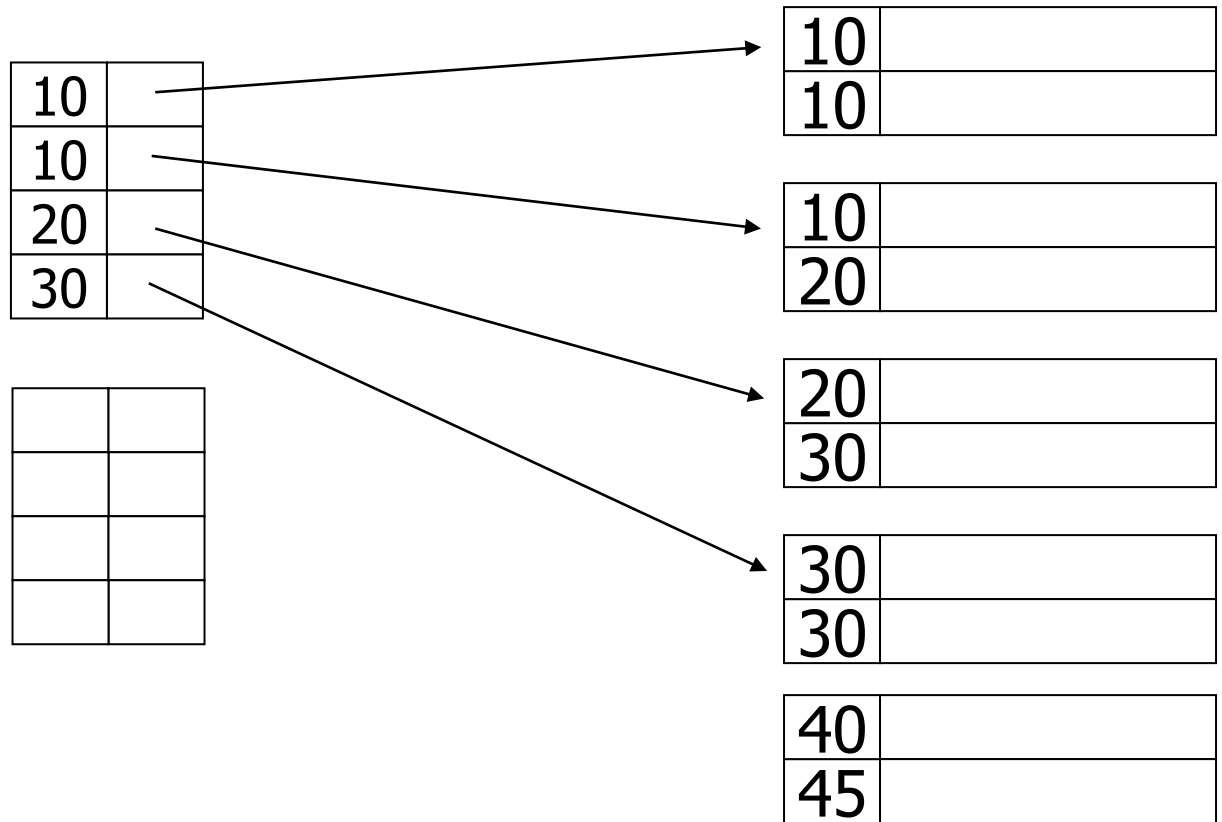


# Duplicate keys

## Sparse index, one way?

Sparse index - place  
key from block

How to search for 30 ?



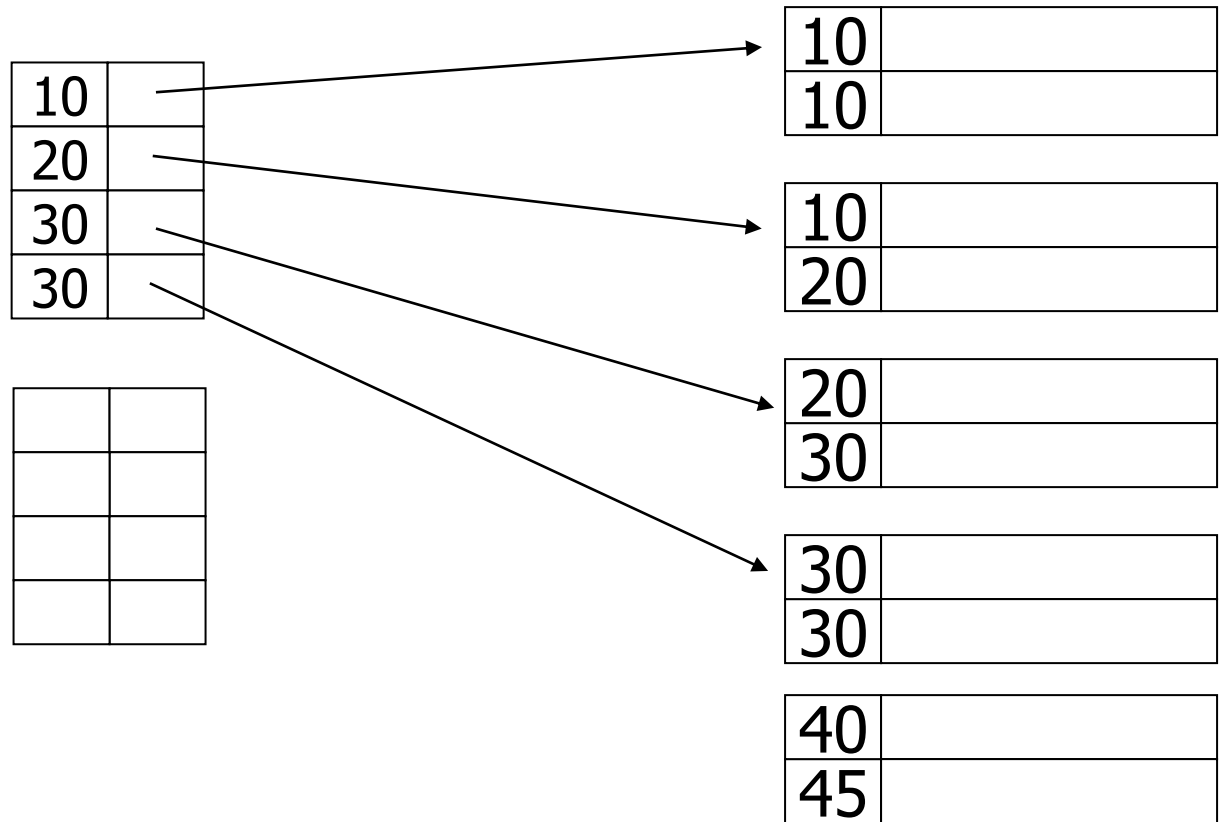
# Duplicate keys

## Sparse index, another way?

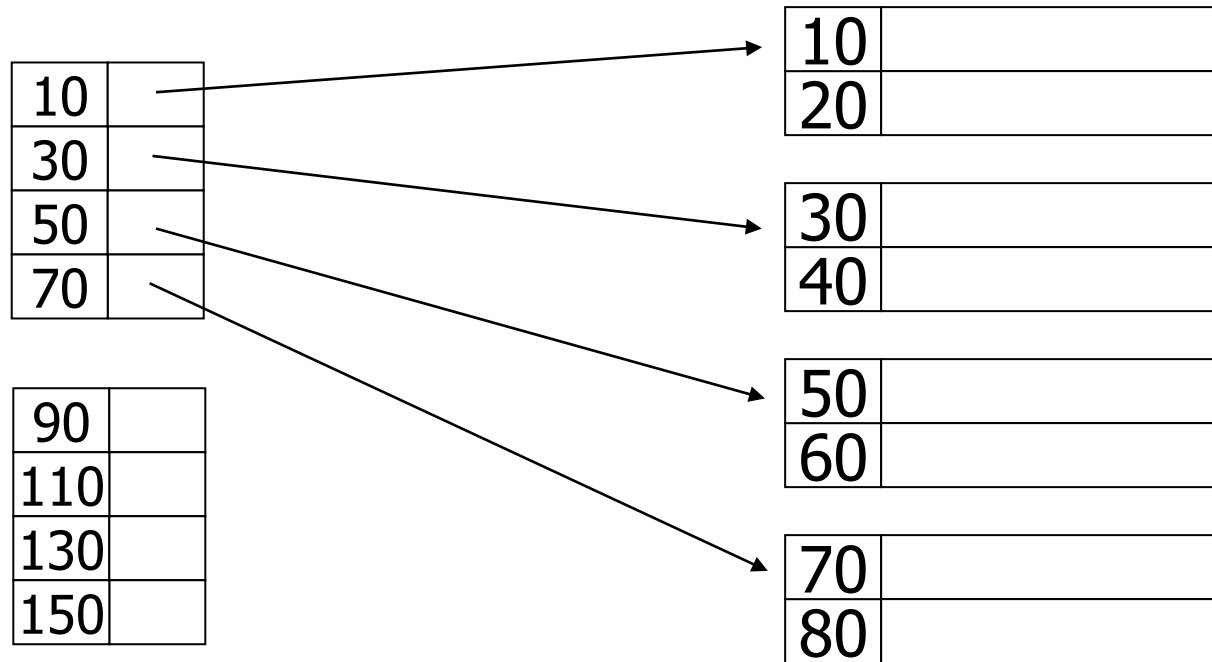
Sparse index - place  
first **new** key from block

How to search for 30 ?

How to search for 35 ?

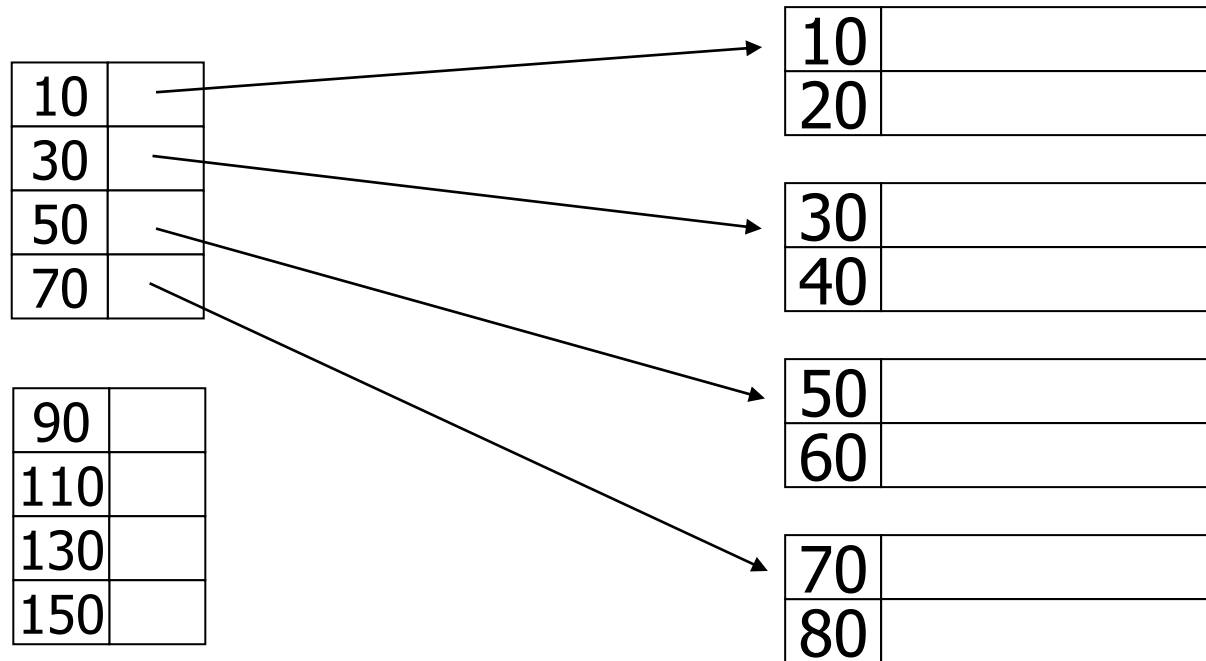


# Deletion from sparse index



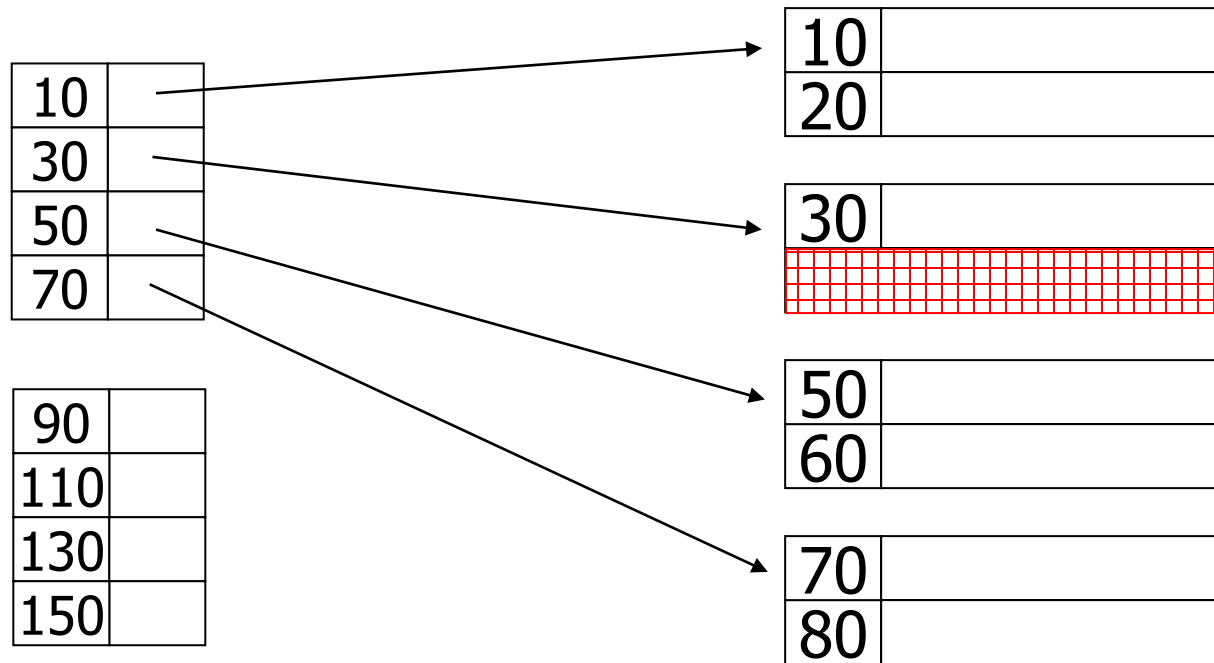
# Deletion from sparse index

– delete record 40



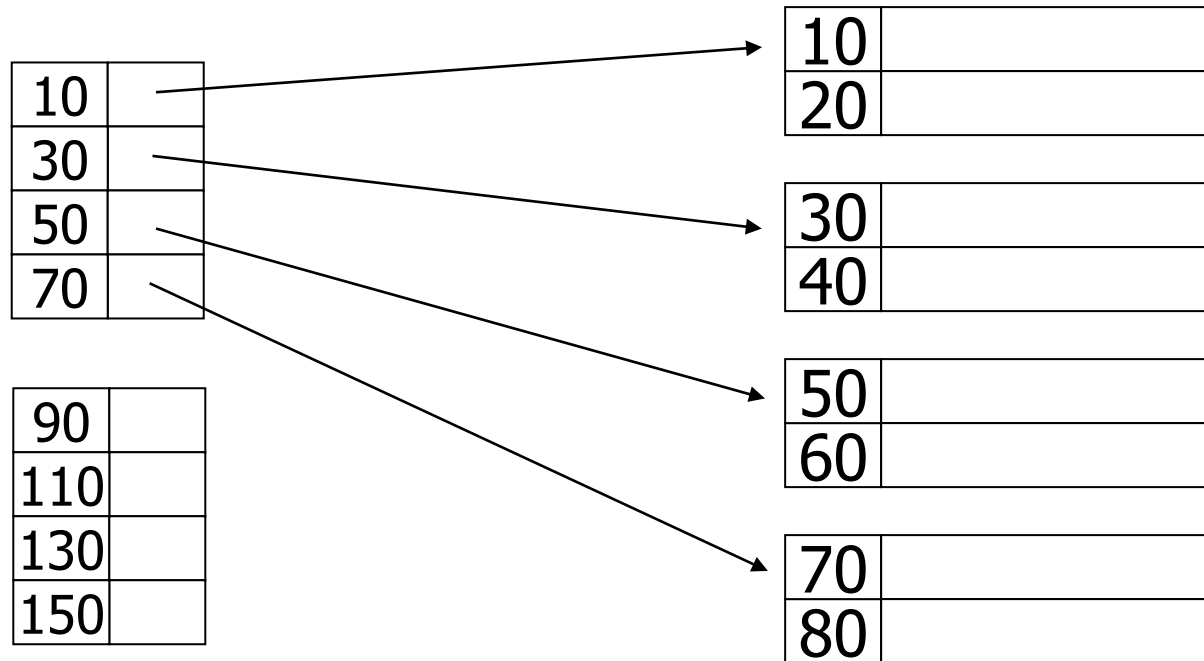
# Deletion from sparse index

– delete record 40



# Deletion from sparse index

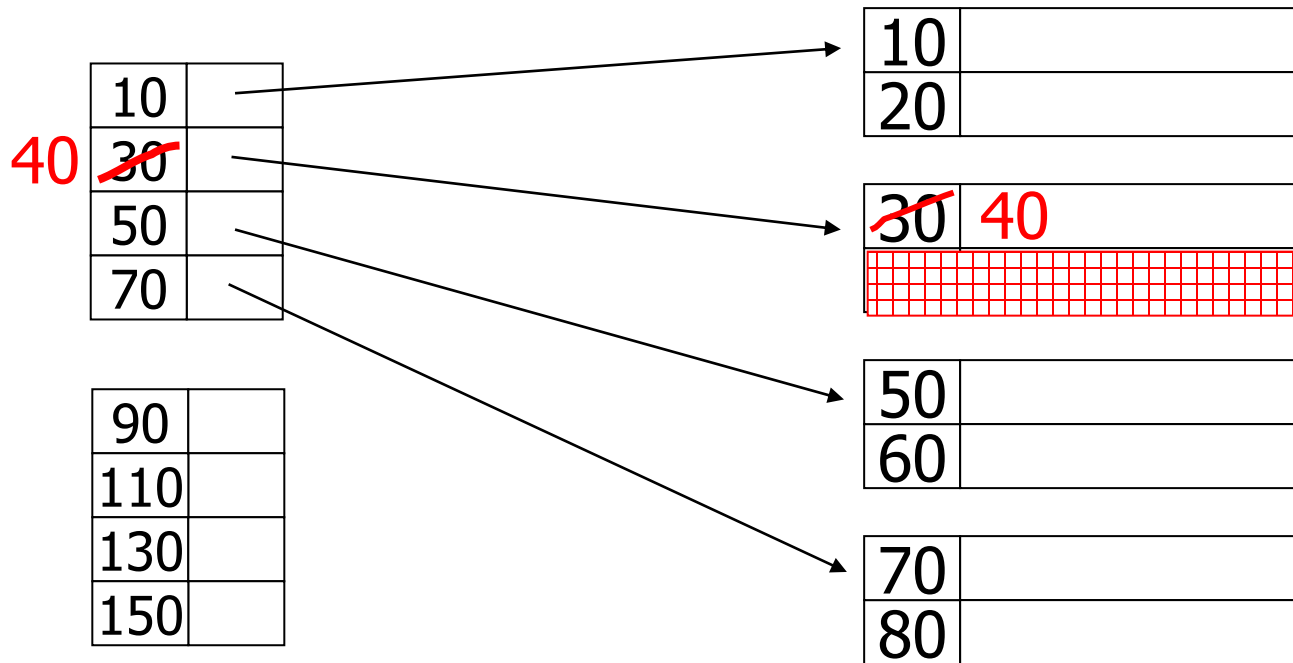
– delete record 30





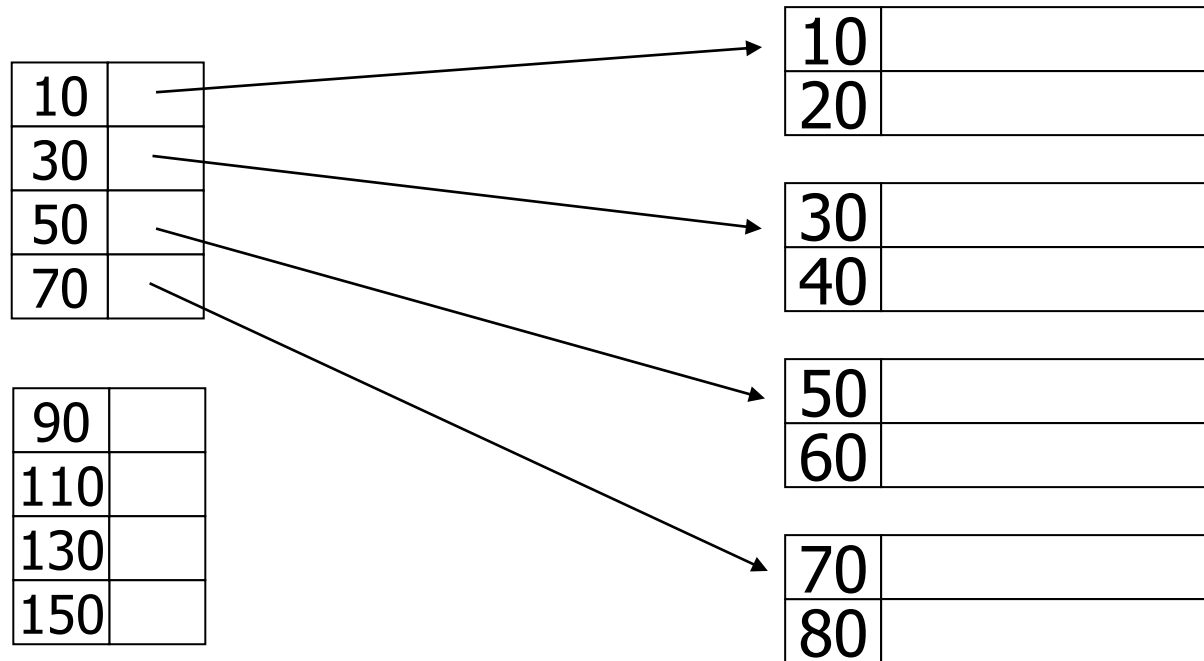
# Deletion from sparse index

– delete record 30



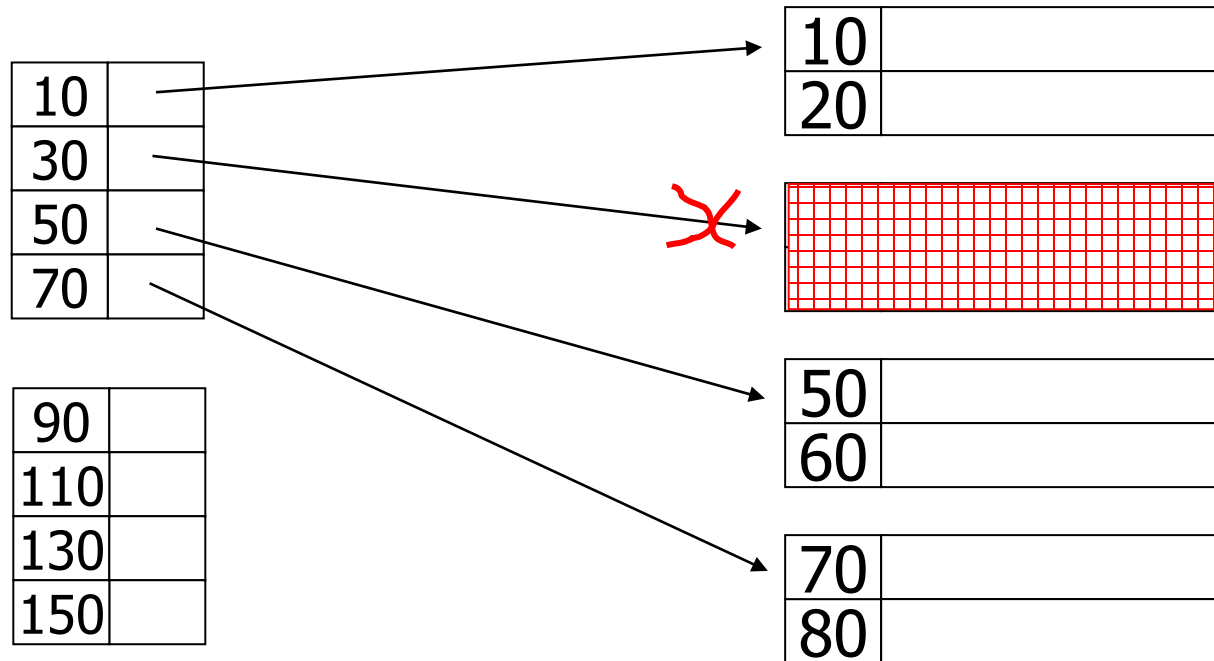
# Deletion from sparse index

– delete records 30 & 40



# Deletion from sparse index

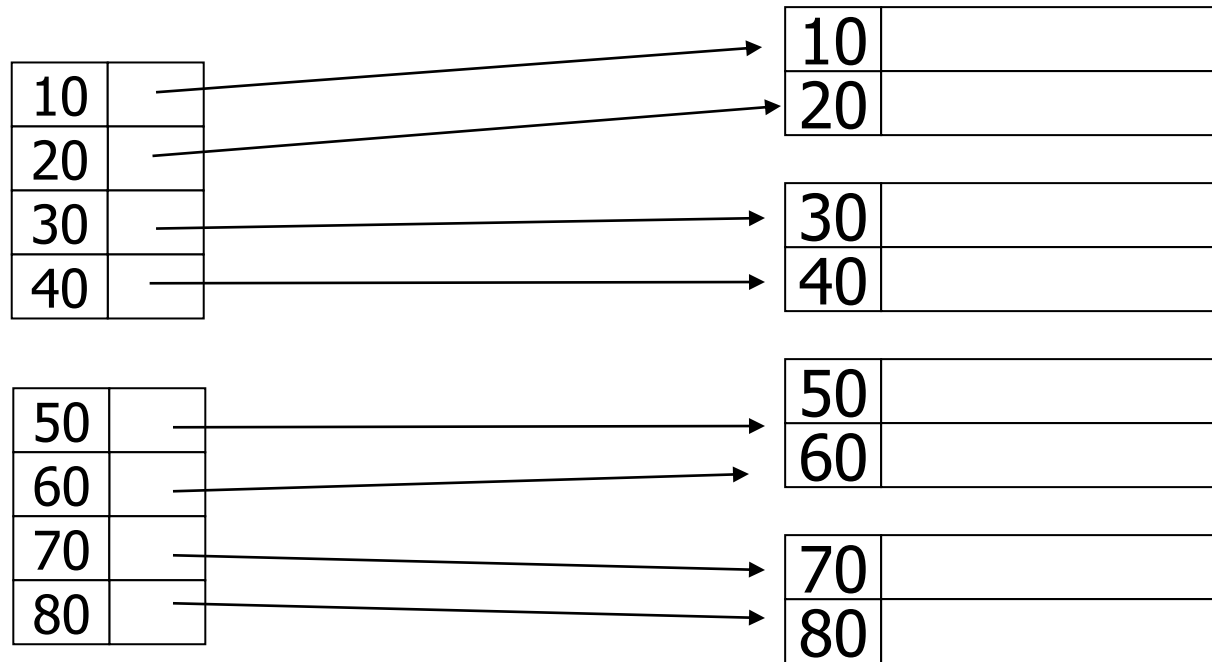
– delete records 30 & 40



- delete records 30 & 40

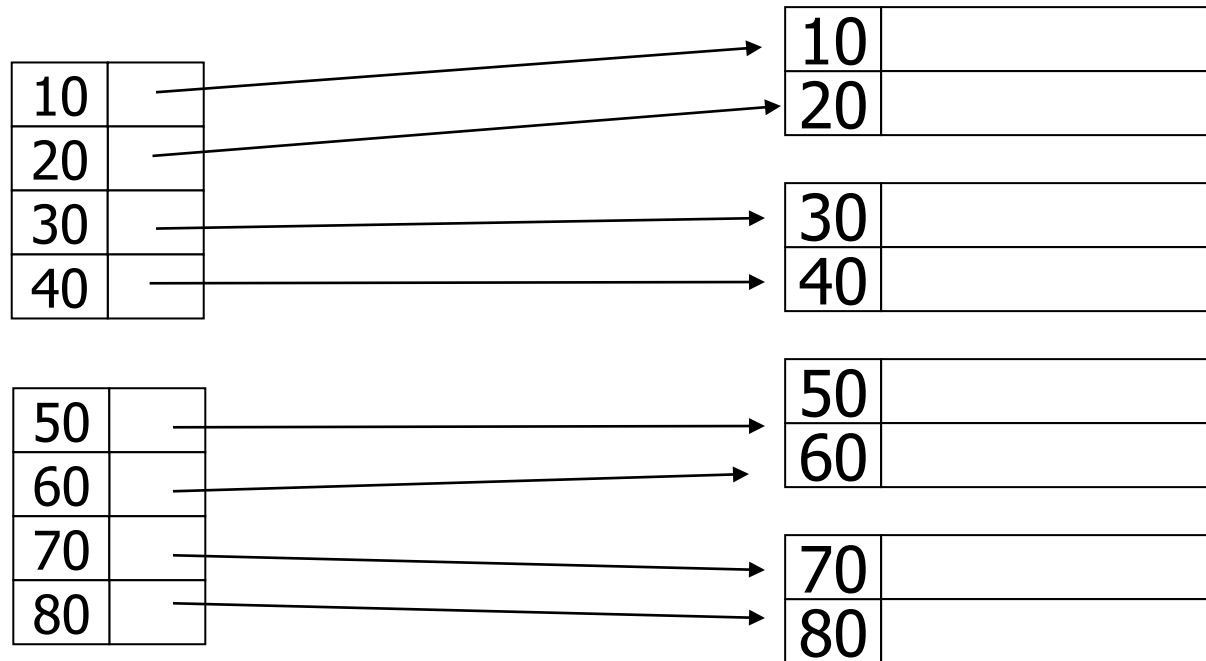


# Deletion from dense index



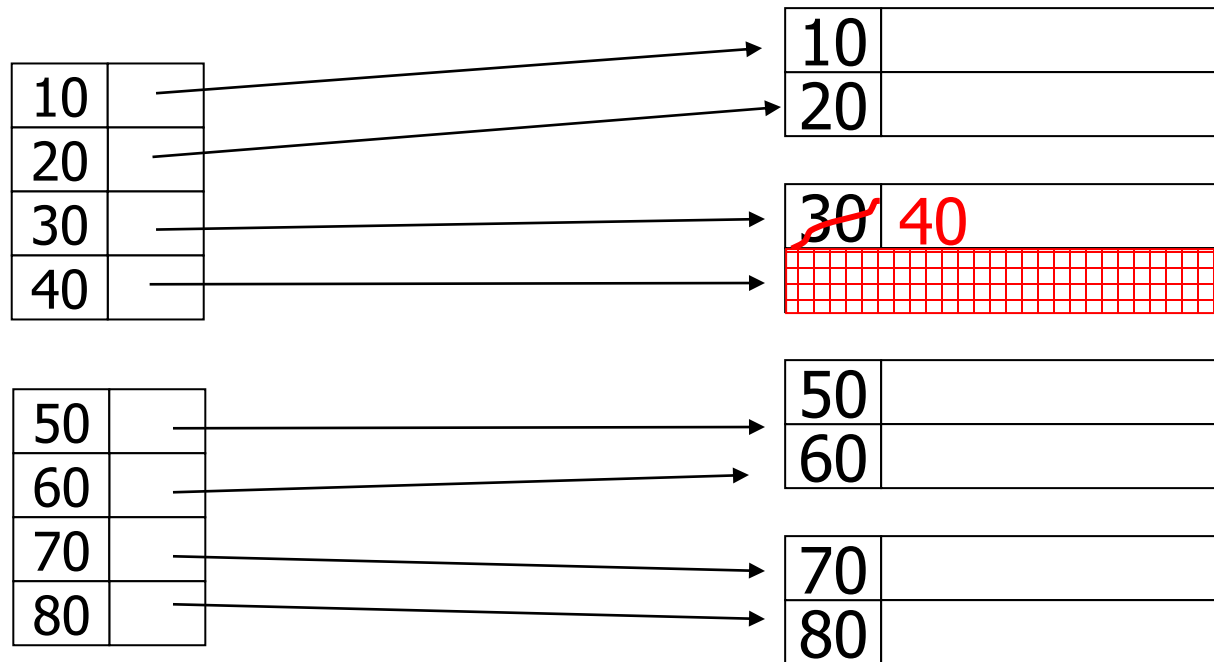
# Deletion from dense index

– delete record 30



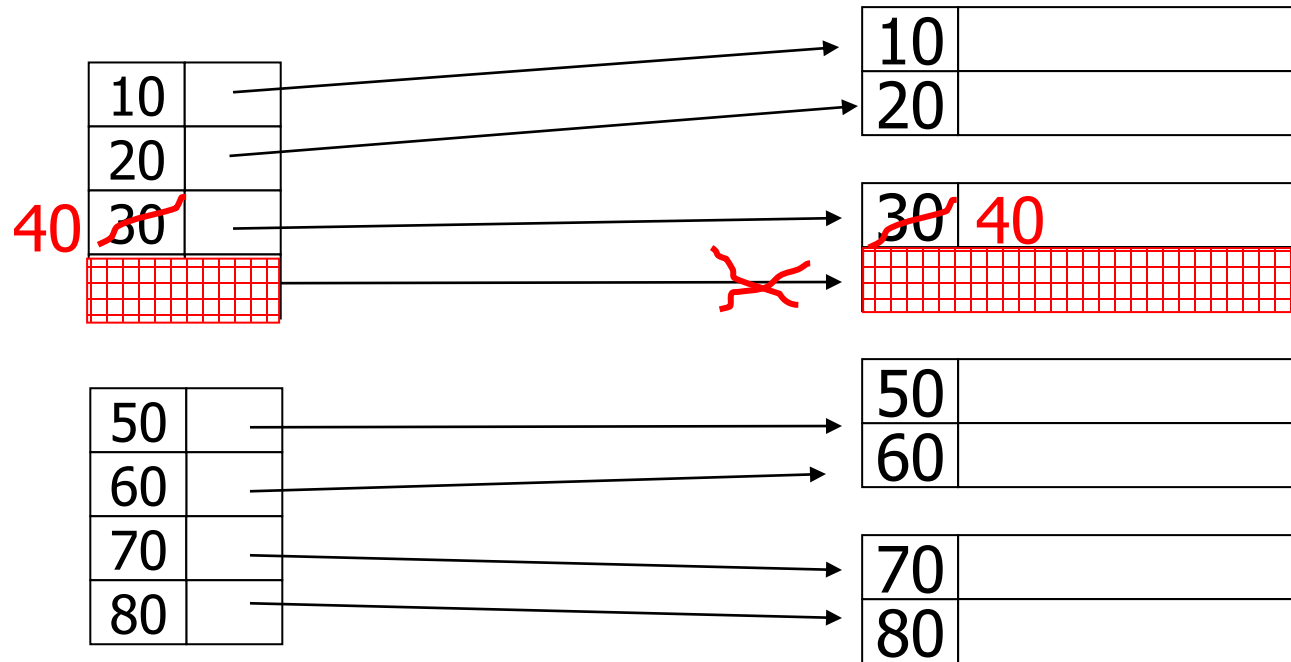
# Deletion from dense index

– delete record 30



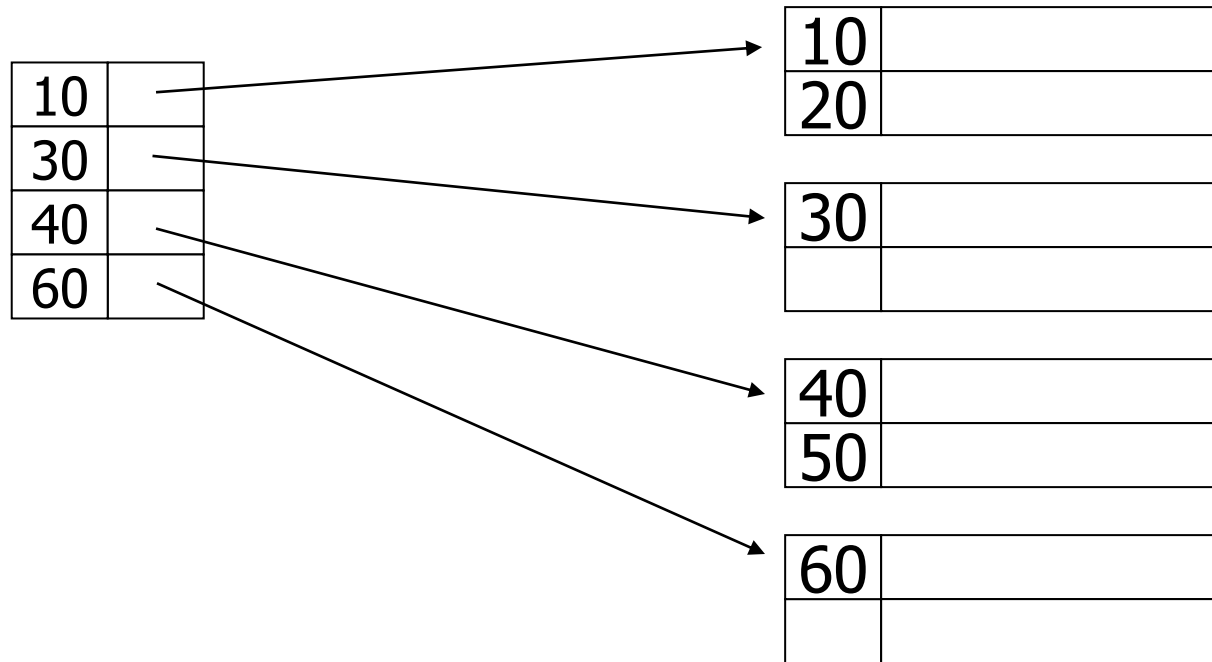
# Deletion from dense index

– delete record 30



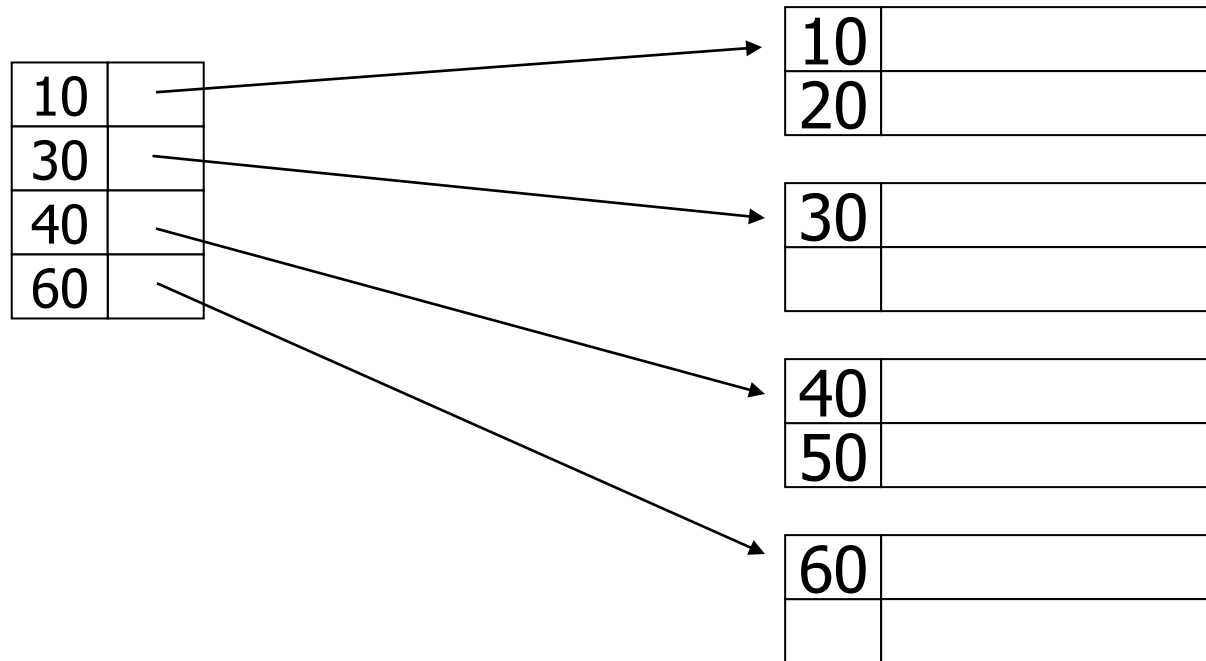


# Insertion, sparse index case



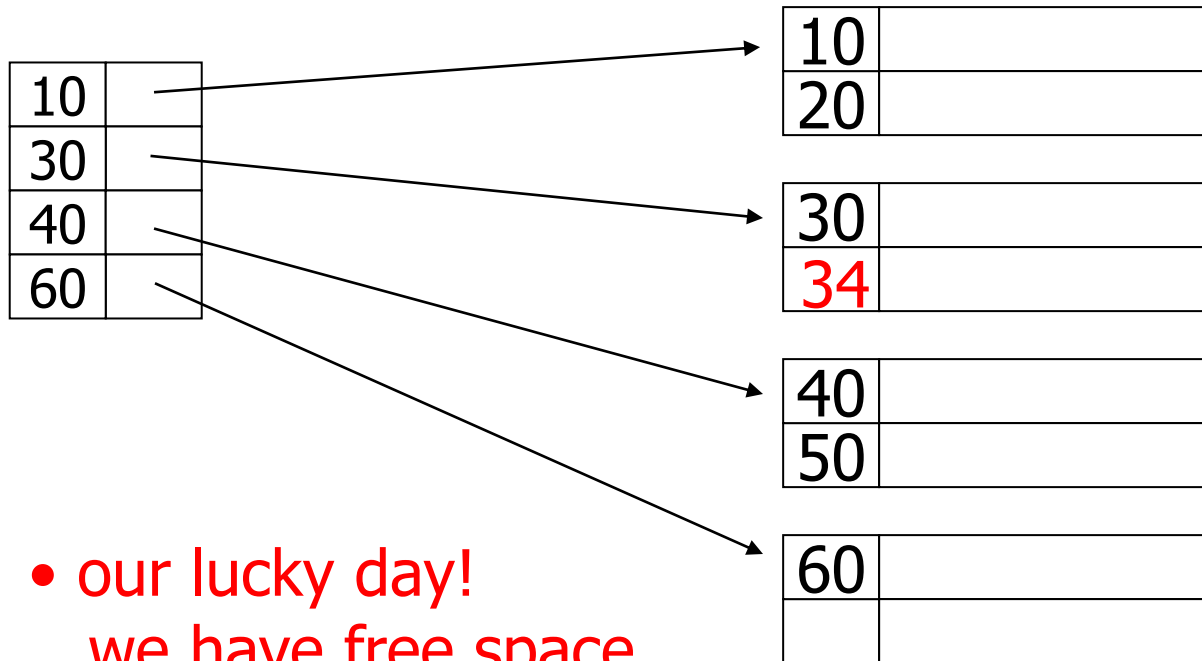
# Insertion, sparse index case

– insert record 34



# Insertion, sparse index case

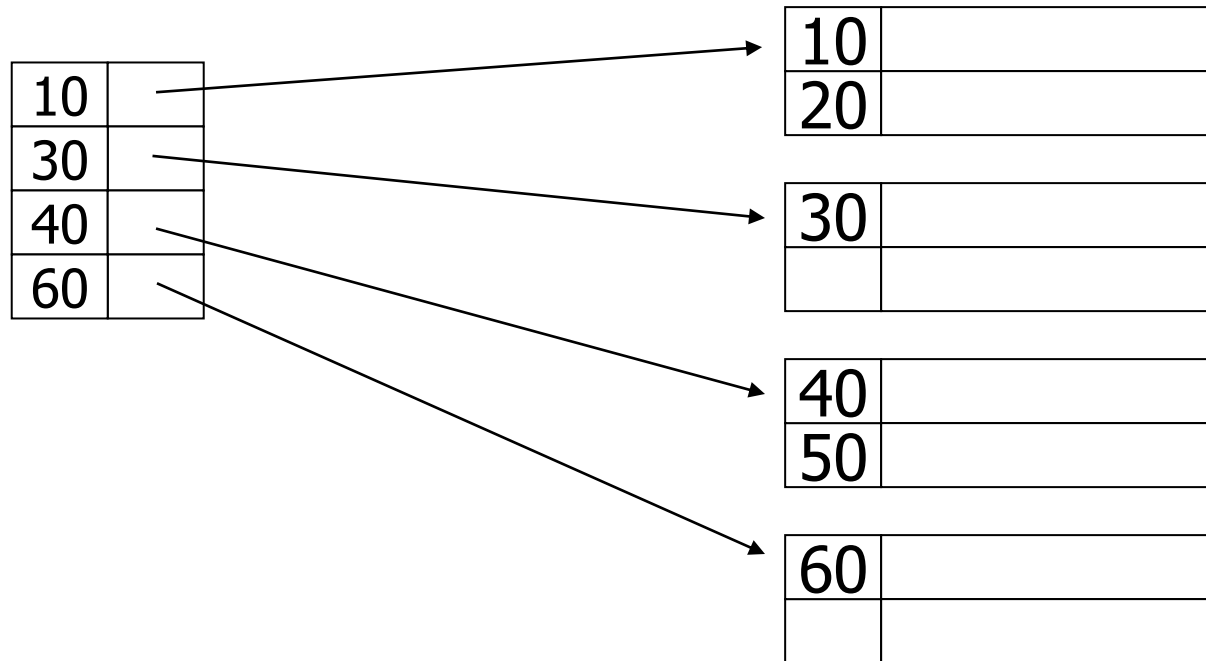
– insert record 34



- our lucky day!  
we have free space  
where we need it!

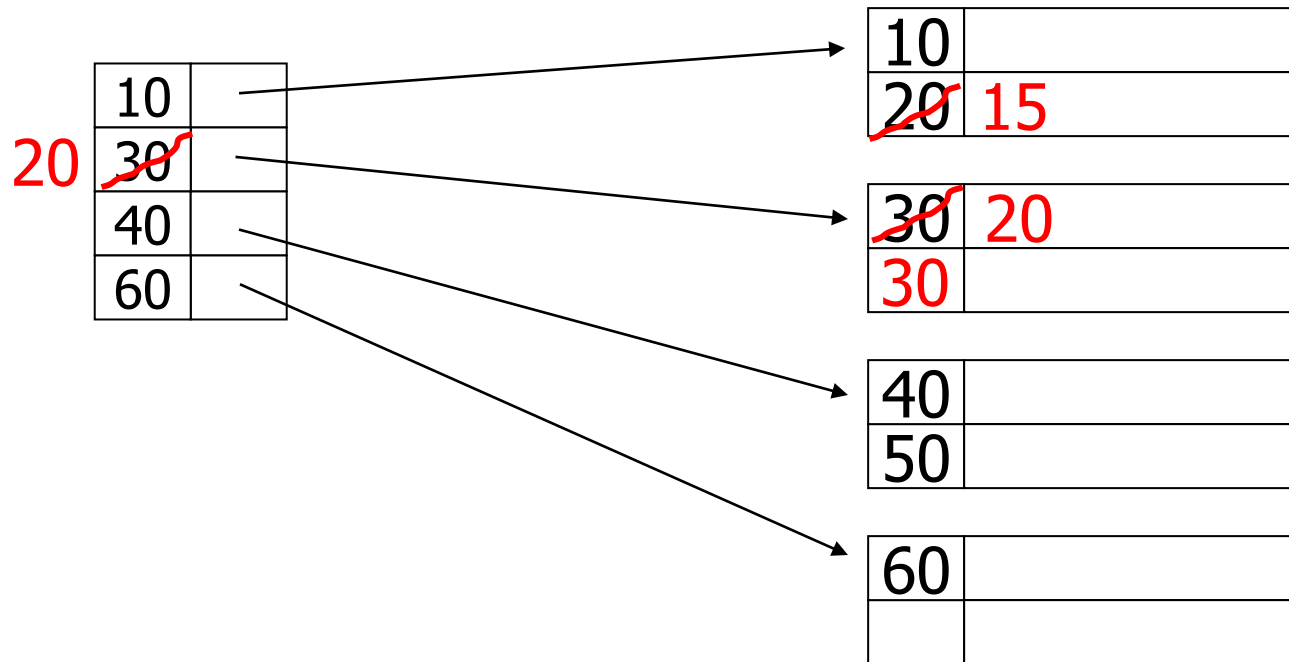
# Insertion, sparse index case

– insert record 15



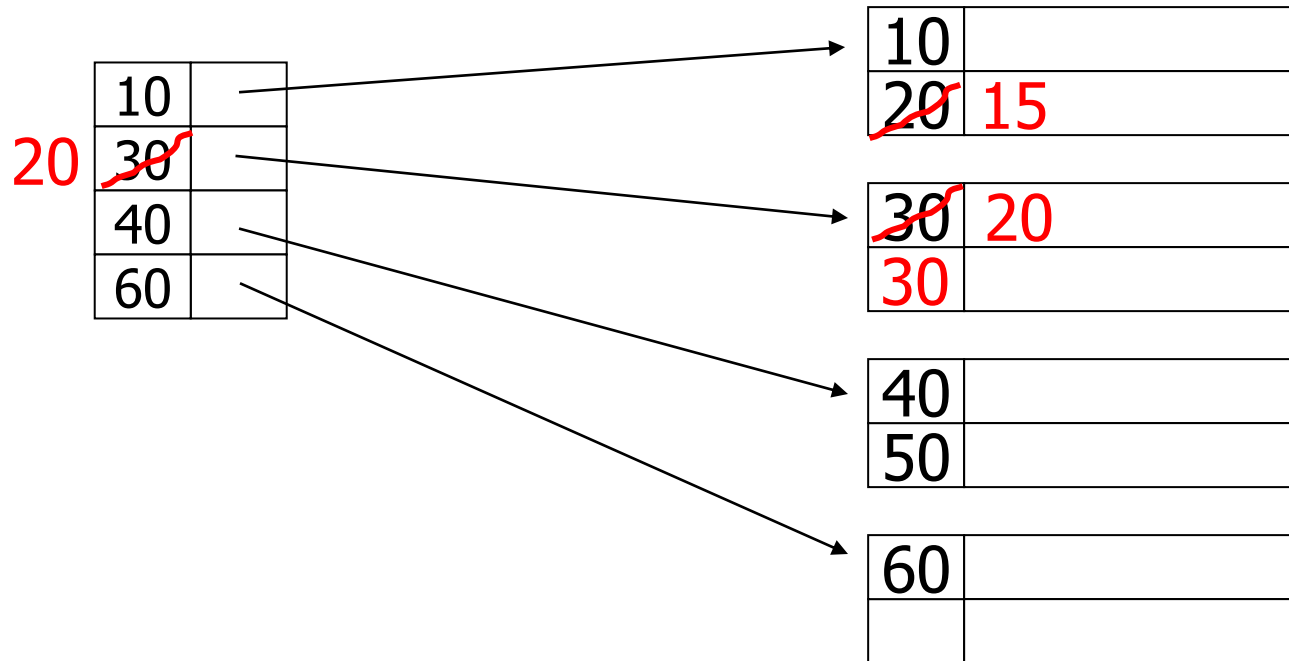
# Insertion, sparse index case

– insert record 15



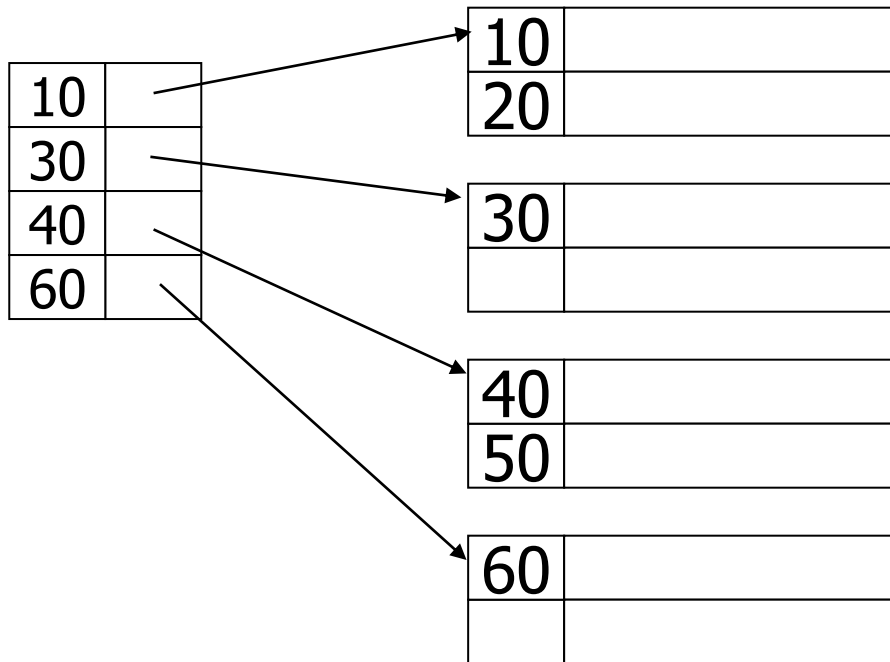
# Insertion, sparse index case

– insert record 15



# Insertion, sparse index case

– insert record 25




## Insertion, dense index case

- Similar
- Often more expensive . . .



# Secondary indexes

Sequence  
field



30	
50	

20	
70	

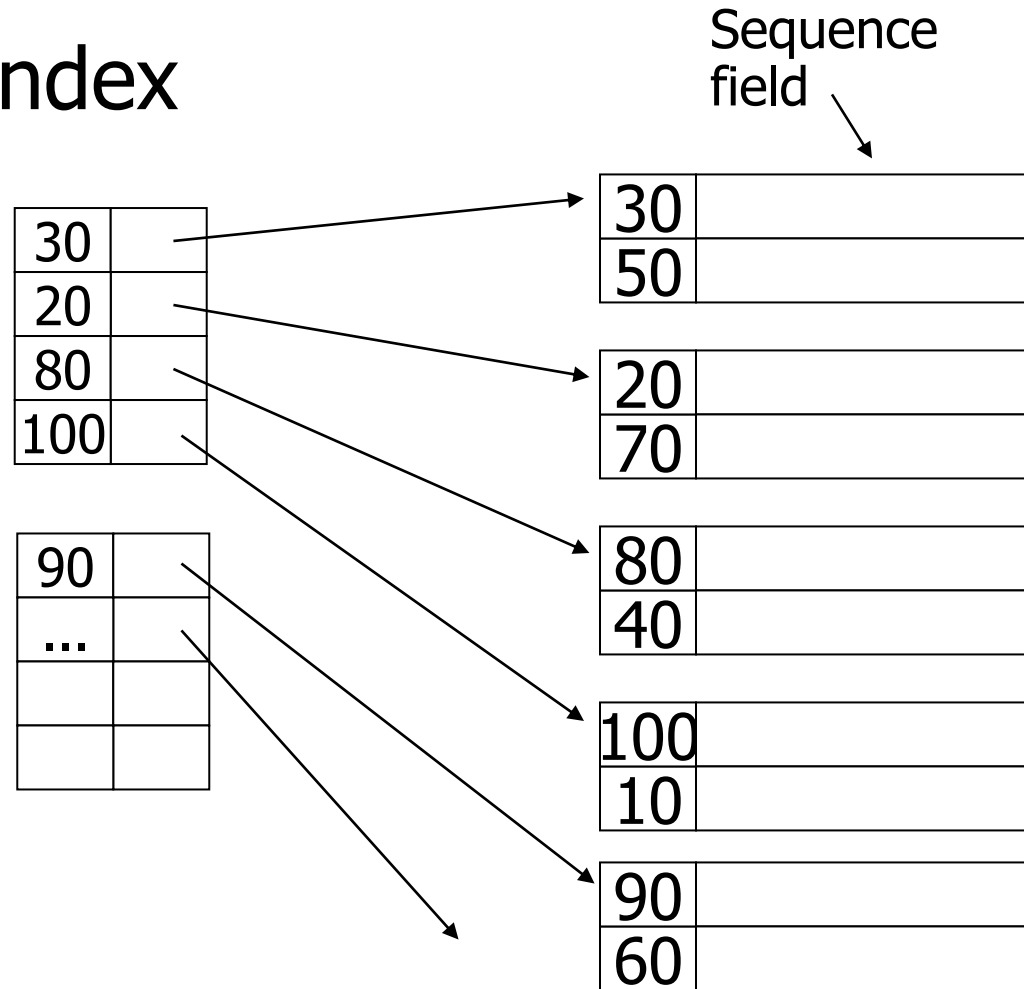
80	
40	

100	
10	

90	
60	

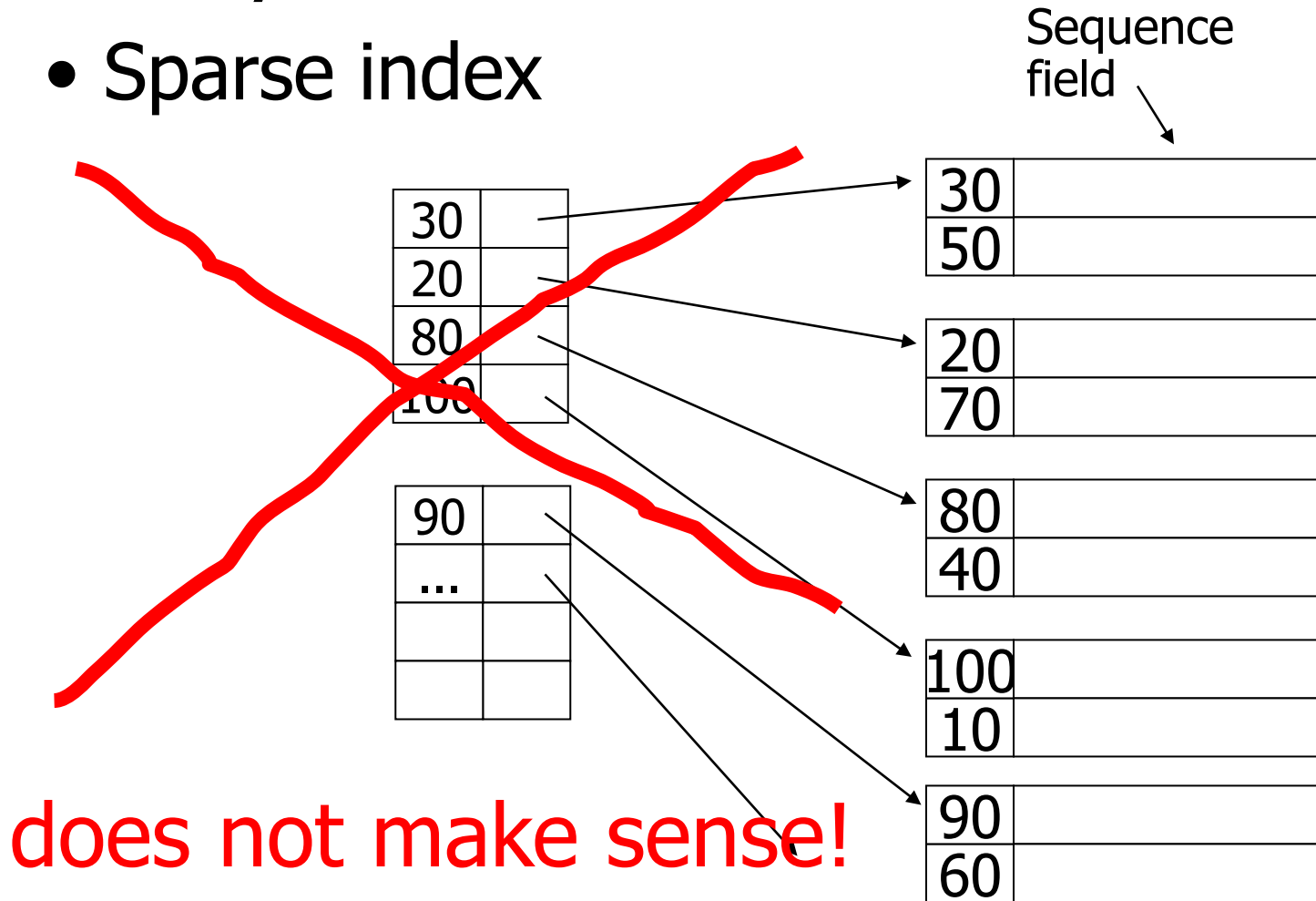
# Secondary indexes

- Sparse index



# Secondary indexes


- Sparse index



# Secondary indexes

- Dense index

Sequence  
field



30	
50	

20	
70	

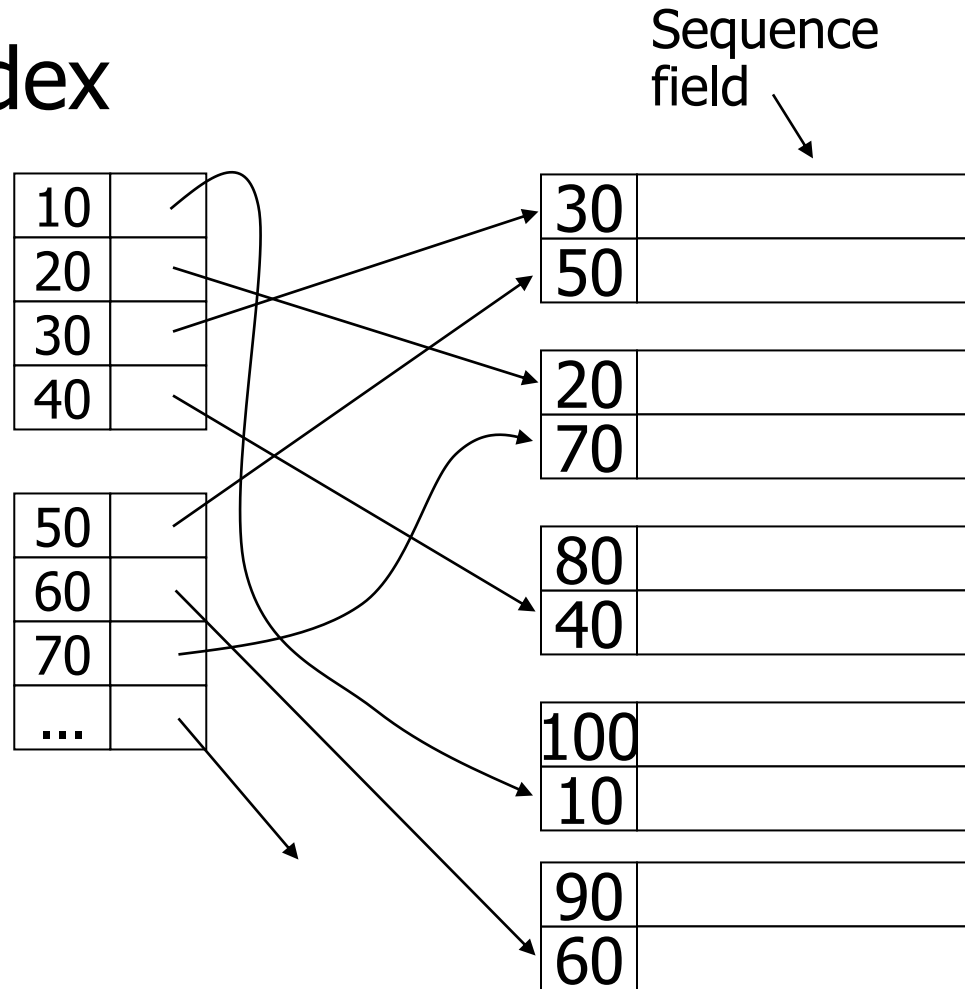
80	
40	

100	
10	

90	
60	

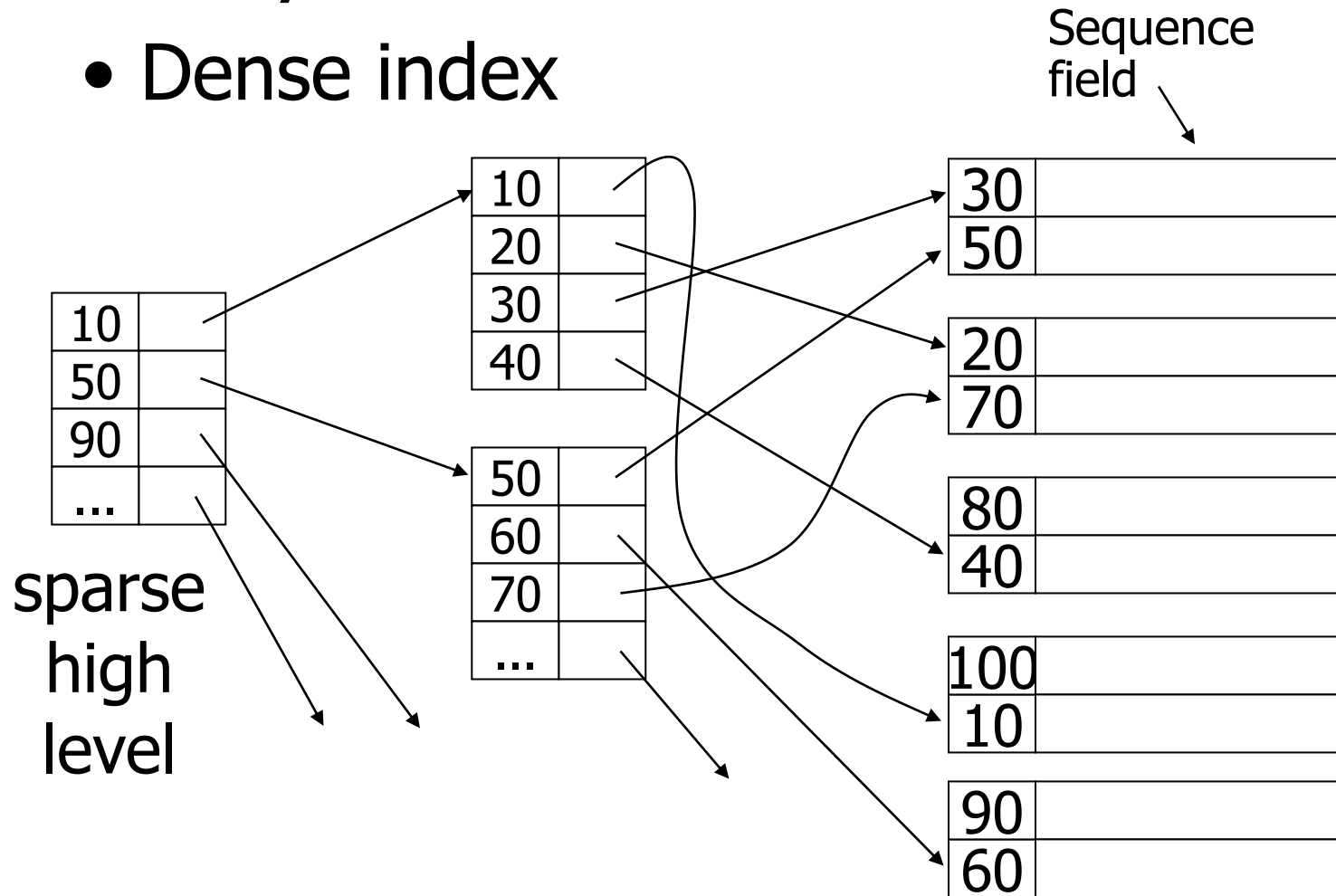
# Secondary indexes

- Dense index



# Secondary indexes

- Dense index



## With secondary indexes:

- Lowest level is dense
- Other levels are sparse

# Duplicate values & secondary indexes

20	
10	

20	
40	

10	
40	

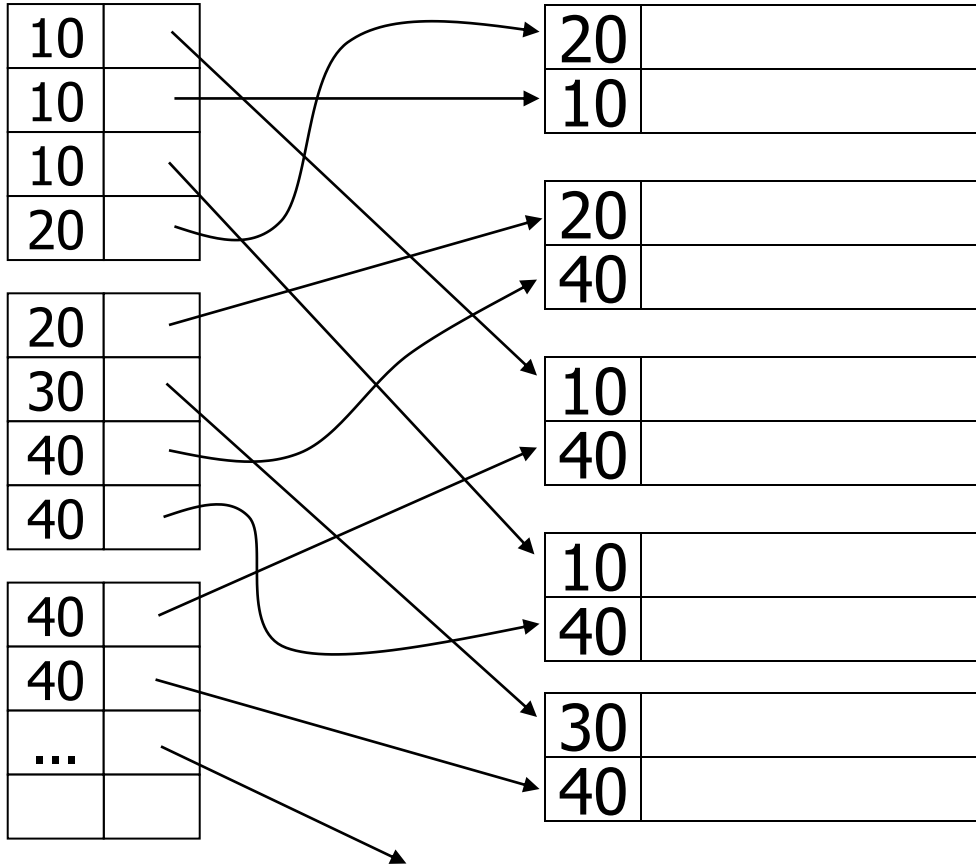
10	
40	

30	
40	



# Duplicate values & secondary indexes

one option...



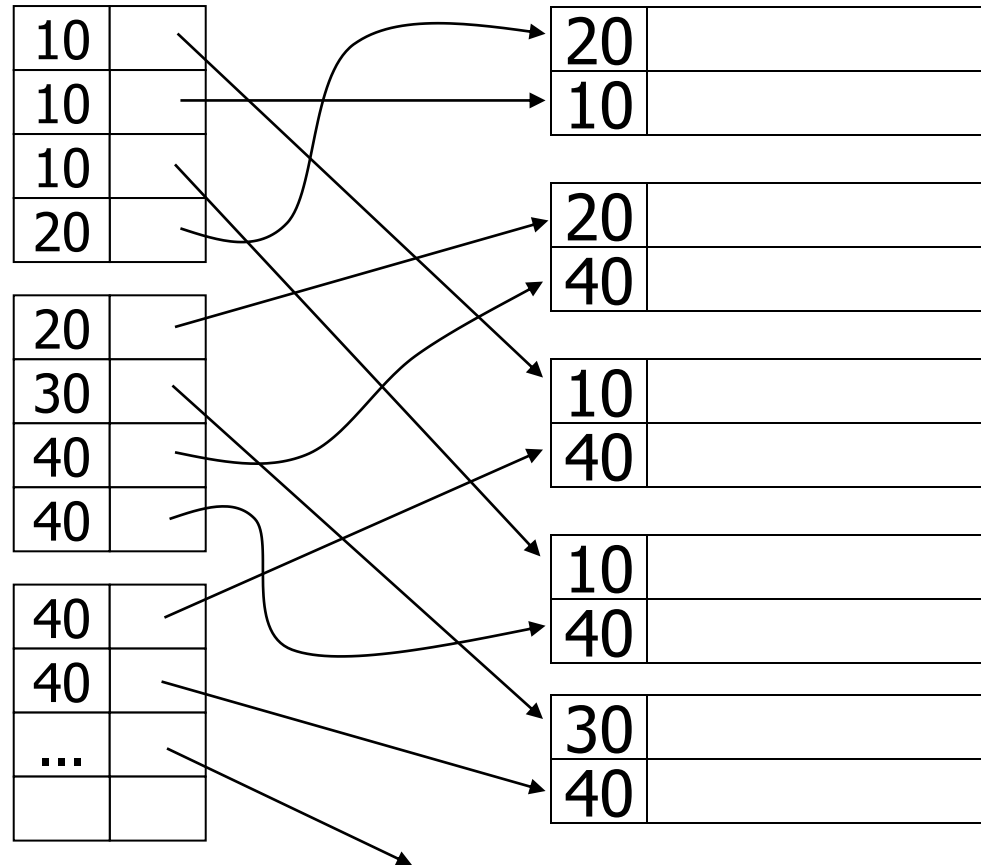
# Duplicate values & secondary indexes

one option...

Problem:

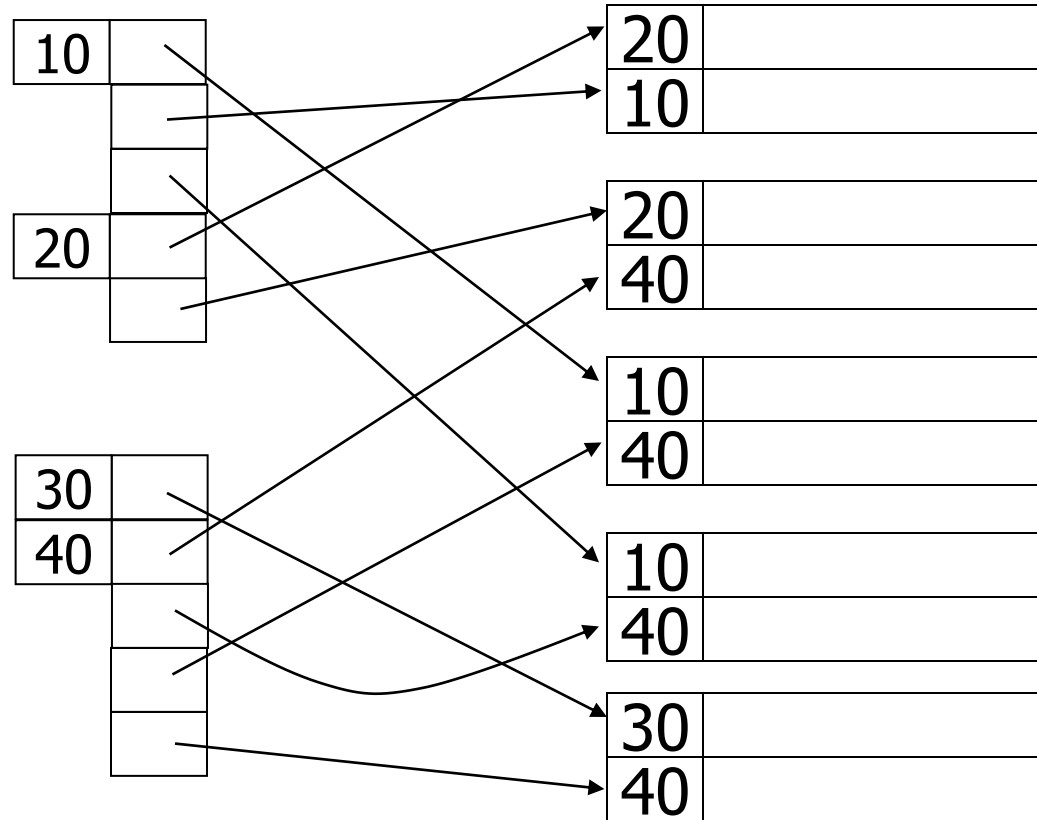
excess overhead!

- disk space
- search time



# Duplicate values & secondary indexes

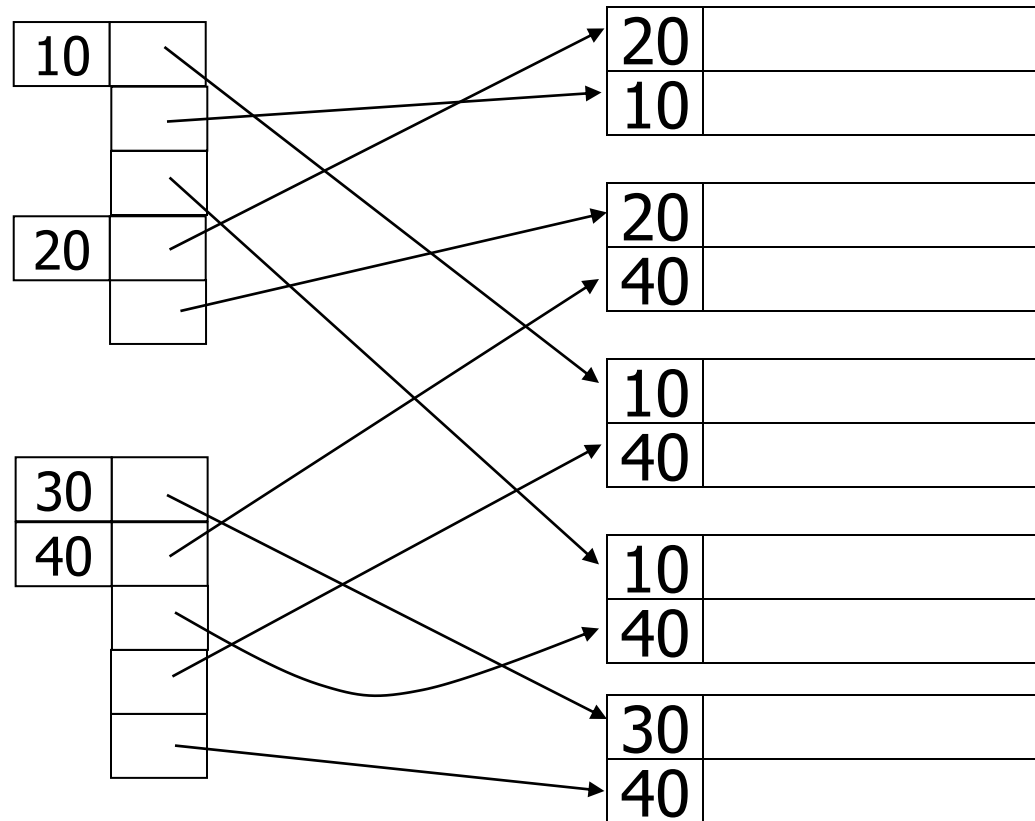
another option...



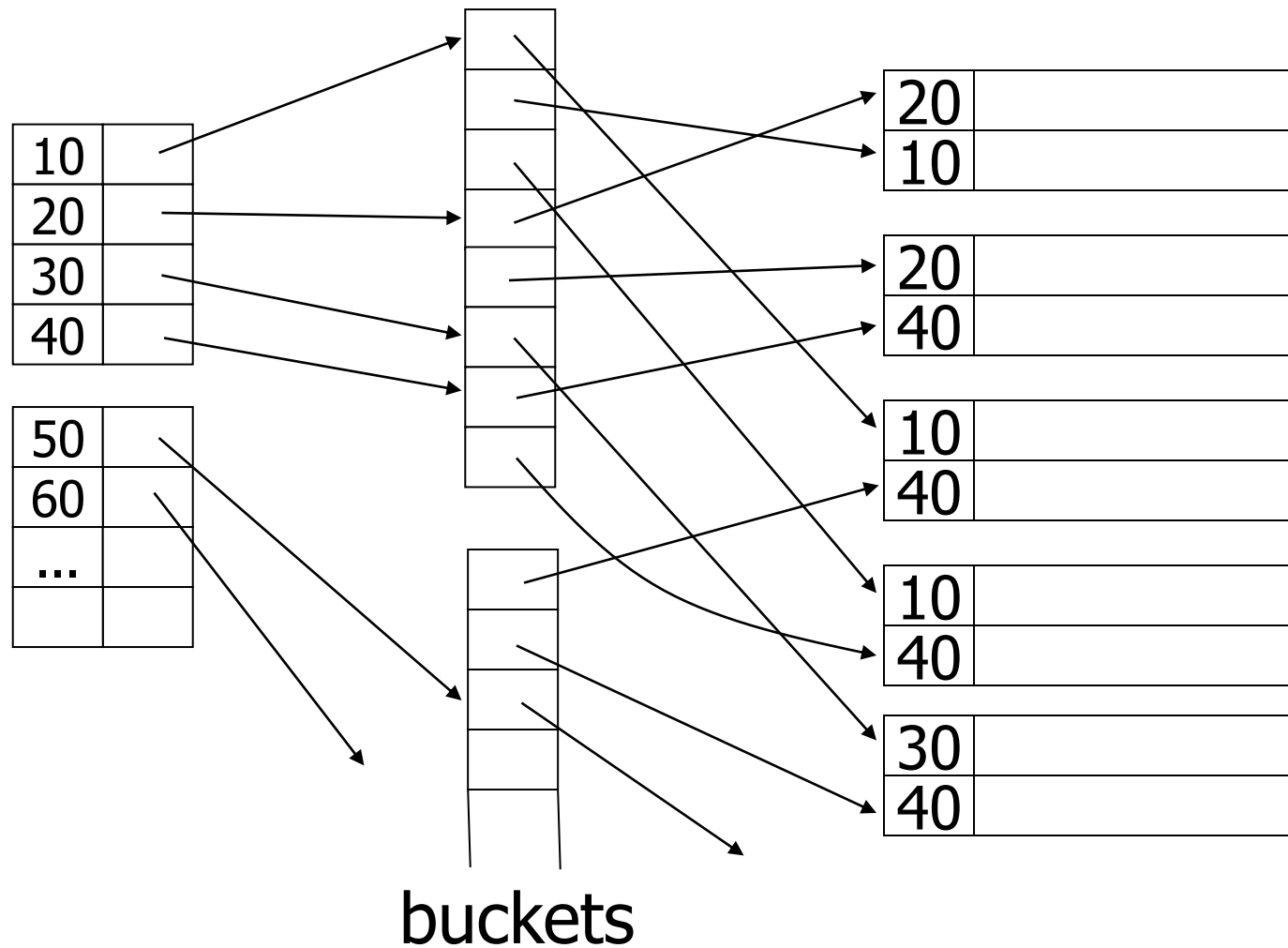
# Duplicate values & secondary indexes

another option...

Problem:  
variable size  
records in  
index!



# Duplicate values & secondary indexes



# Why “bucket” idea is useful

## Indexes

Name: primary

Dept: secondary

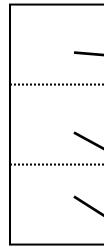
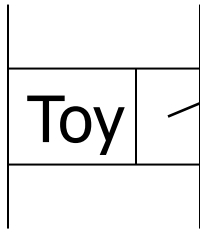
Floor: secondary

## Records

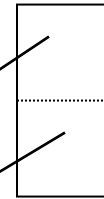
EMP (name,dept,floor,...)

Query: Get employees in  
(Toy Dept)  $\wedge$  (2nd floor)

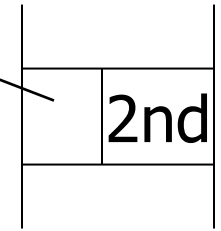
Dept. index



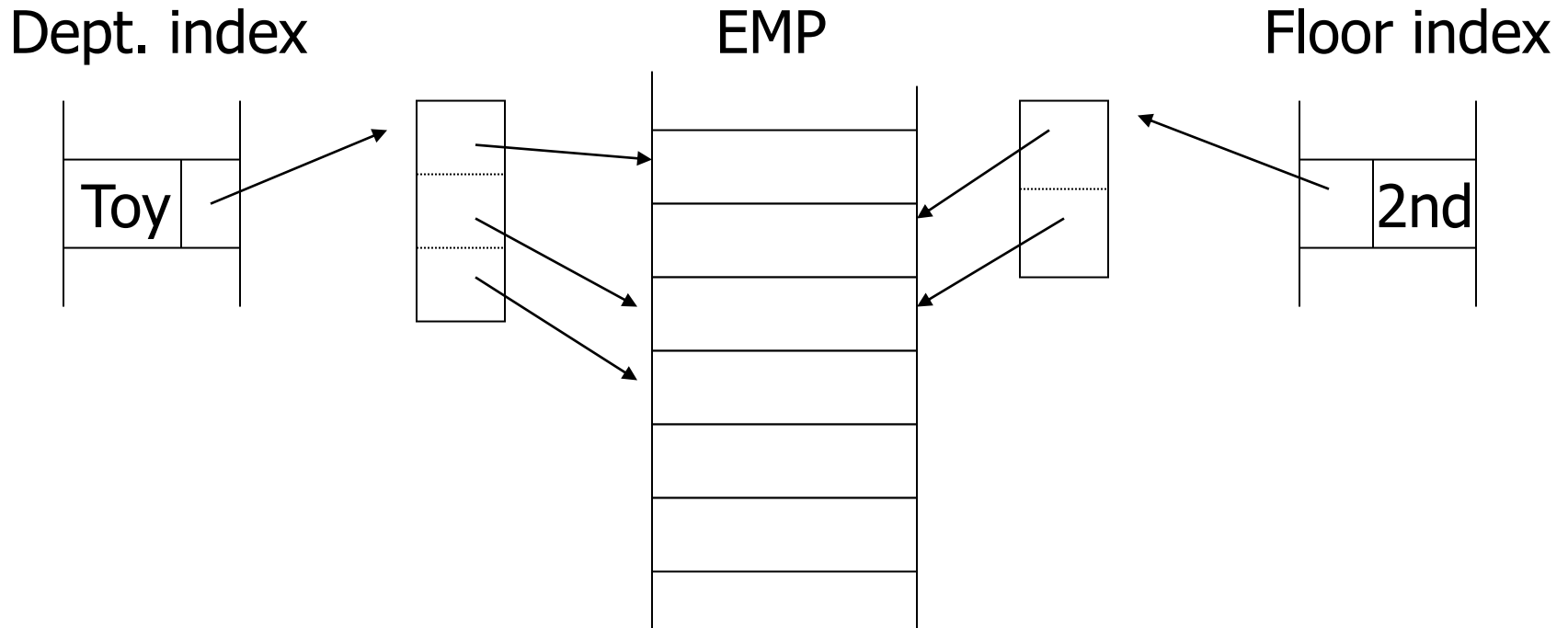
EMP

Floor index



Query: Get employees in  
(Toy Dept)  $\wedge$  (2nd floor)



→ Intersect toy bucket and 2nd Floor bucket to get set of matching EMP's



# Summary so far

- Conventional index
  - Basic Ideas: sparse, dense, multi-level...
  - Duplicate Keys
  - Deletion/Insertion
  - Secondary indexes

# Conventional indexes

## Advantage:

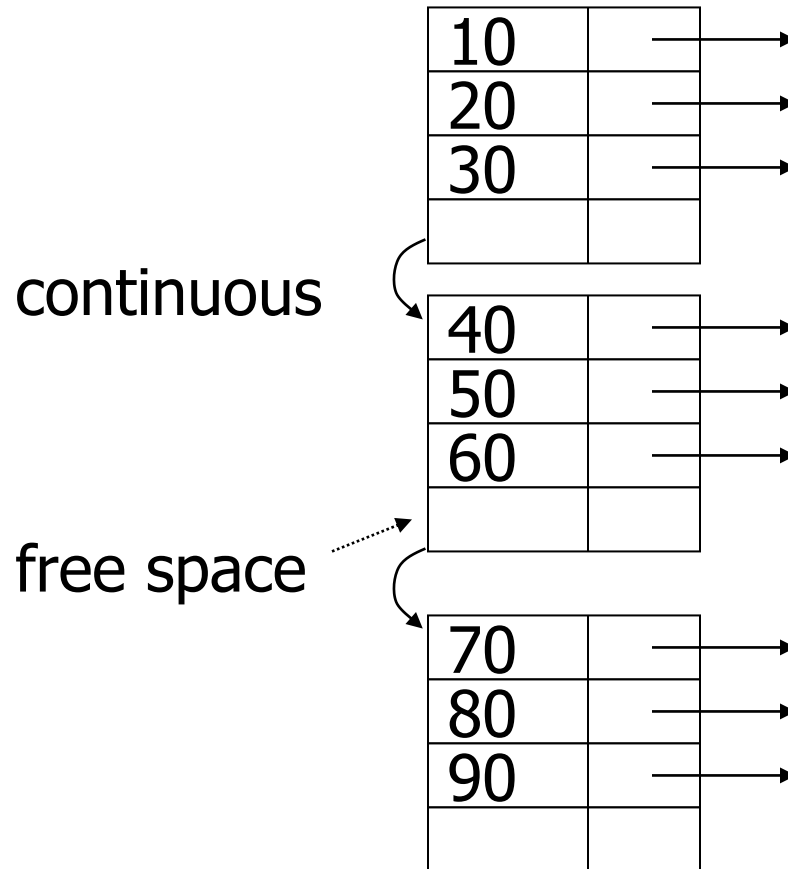
- Simple
- Index is sequential file  
good for scans

## Disadvantage:

- Inserts expensive, or
- Lose sequentiality & balance

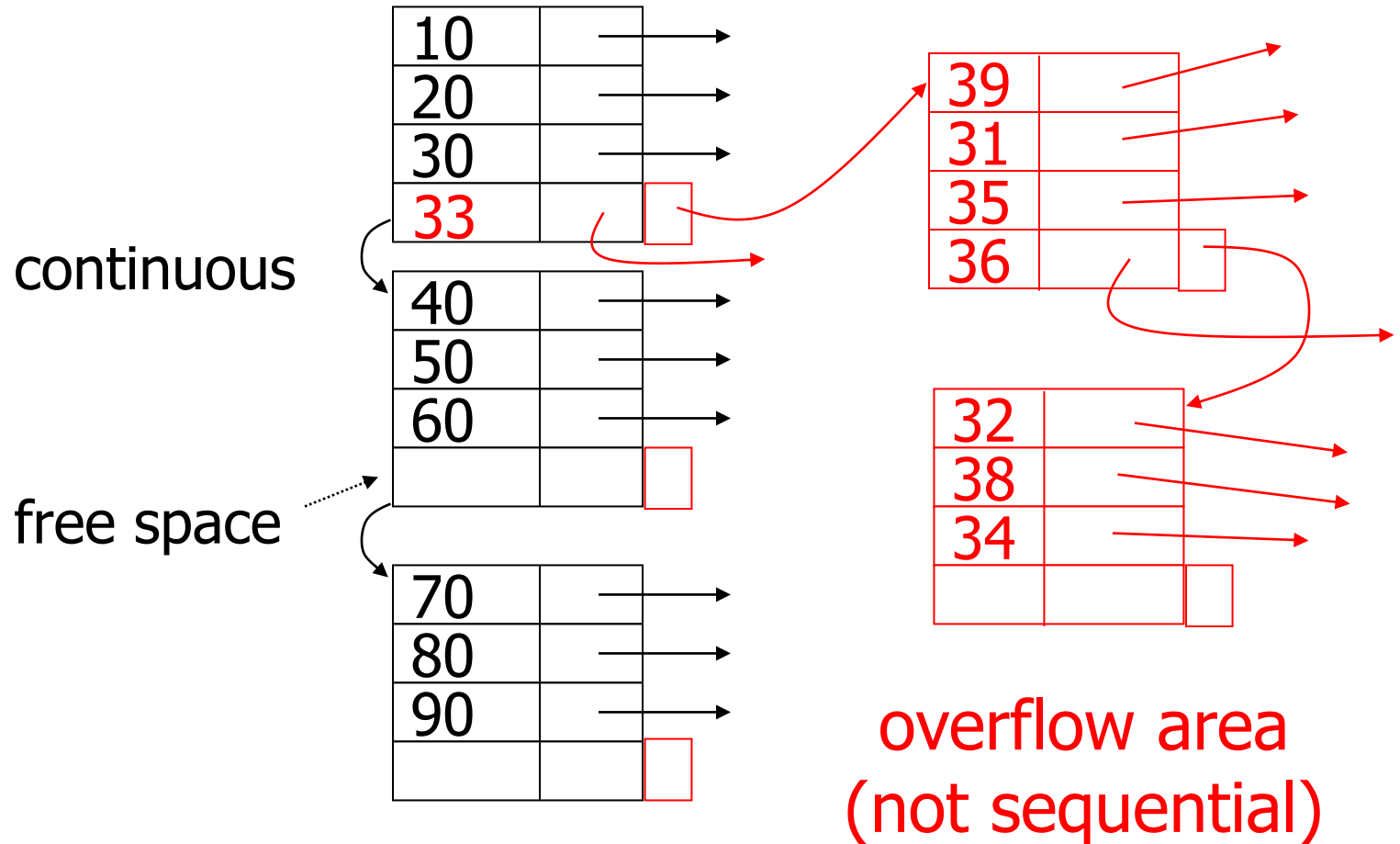
# Example

## Index (sequential)



# Example

## Index (sequential)



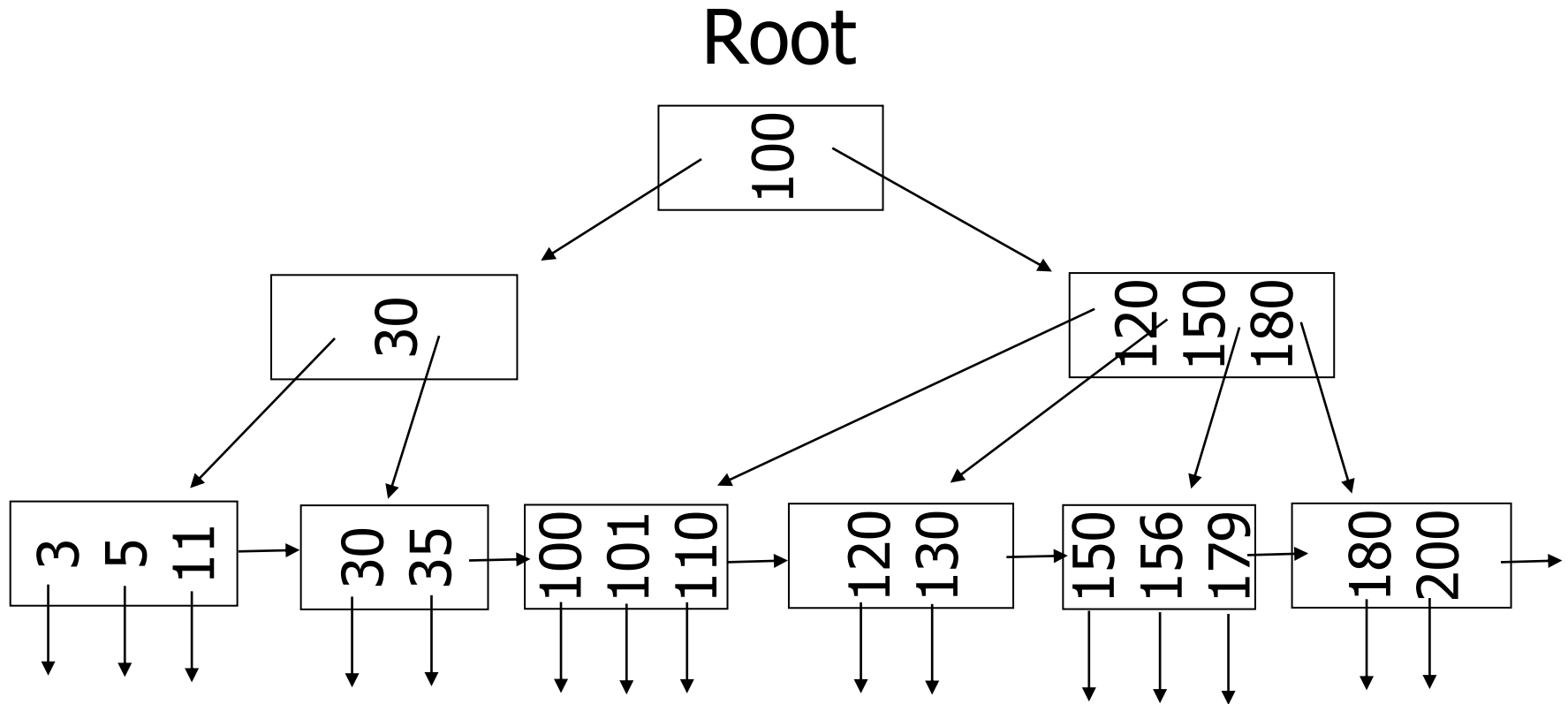
## Outline:

- Conventional indexes
- B-Trees  $\Rightarrow$  NEXT
- Hashing schemes

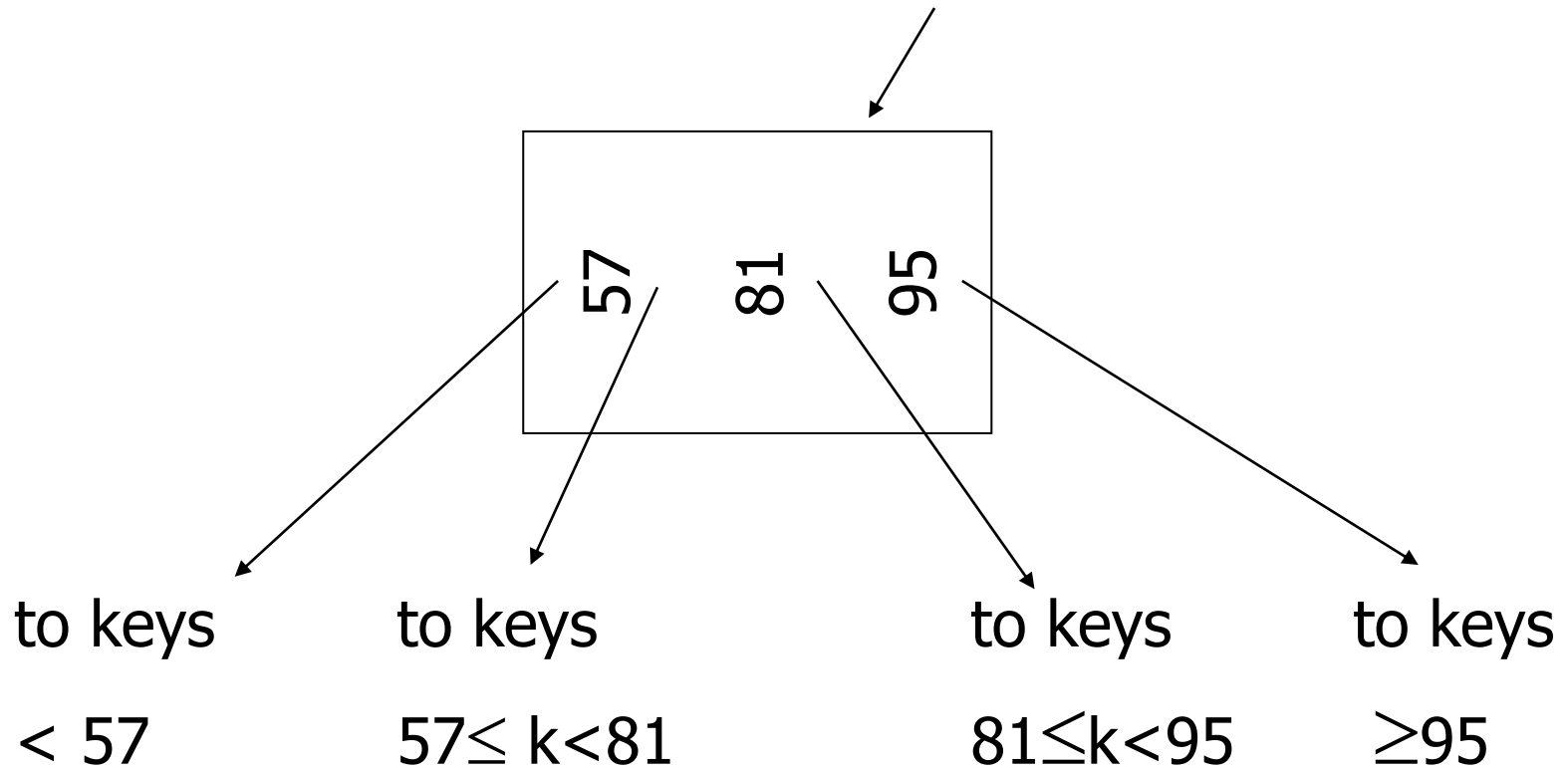
- NEXT: Another type of index
  - Give up on sequentiality of index
  - Try to get “balance”

# B+Tree Example

n=3

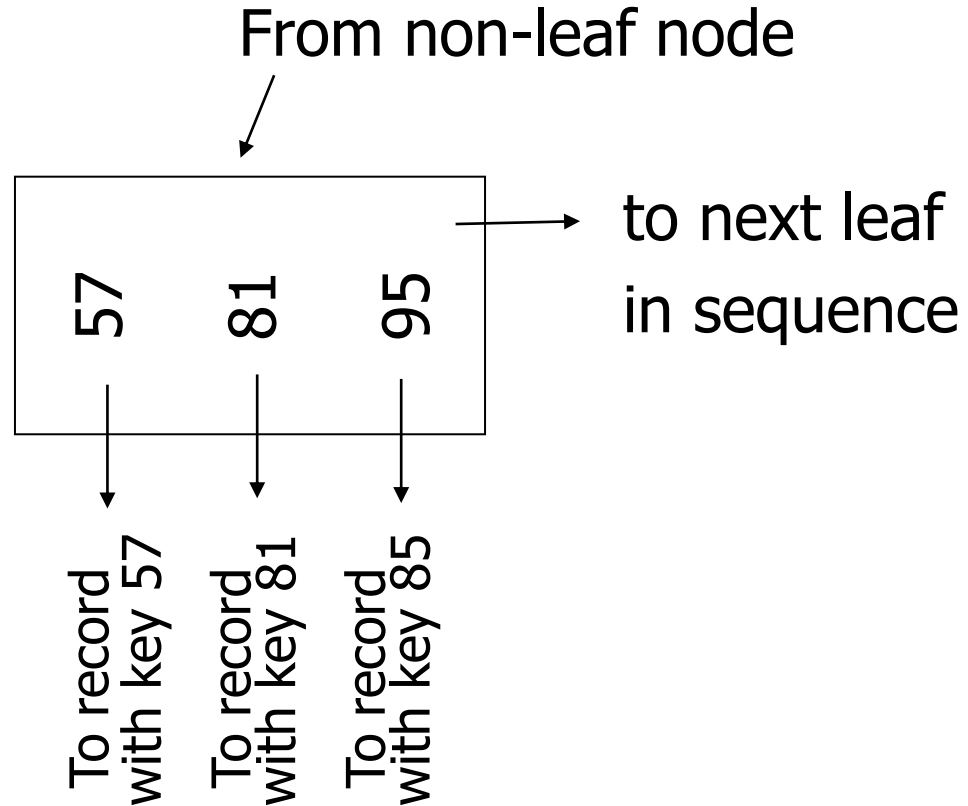


# Sample non-leaf





# Sample leaf node:



Size of nodes: { n+1 pointers  
n keys (fixed)

# Don't want nodes to be too empty

- Use at least

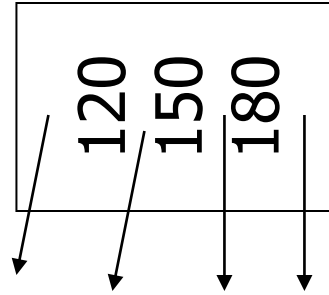
Non-leaf:  $\lceil (n+1)/2 \rceil$  pointers

Leaf:  $\lfloor (n+1)/2 \rfloor$  pointers to data

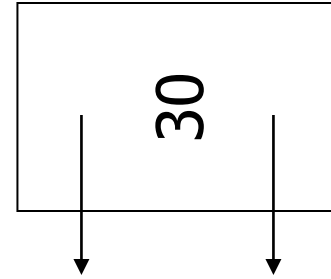
$n=3$

Non-leaf

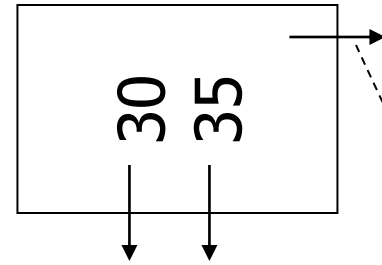
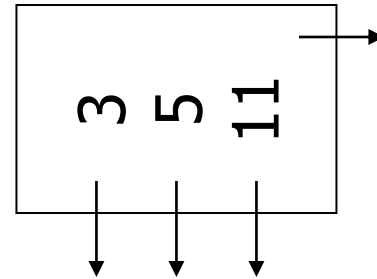
Full node



min. node



Leaf



counts even if null

## B+tree rules \_\_\_\_\_ tree of order $n$

- (1) All leaves at same lowest level  
(balanced tree)
- (2) Pointers in leaves point to records  
except for “sequence pointer”

### (3) Number of pointers/keys for B+tree

	Max ptrs	Max keys	Min ptrs→data	Min keys
Non-leaf (non-root)	$n+1$	$n$	$\lceil (n+1)/2 \rceil$	$\lceil (n+1)/2 \rceil - 1$
Leaf (non-root)	$n+1$	$n$	$\lfloor (n+1)/2 \rfloor$	$\lfloor (n+1)/2 \rfloor$
Root	$n+1$	$n$	1	1

# Insert into B+tree

(a) simple case

- space available in leaf

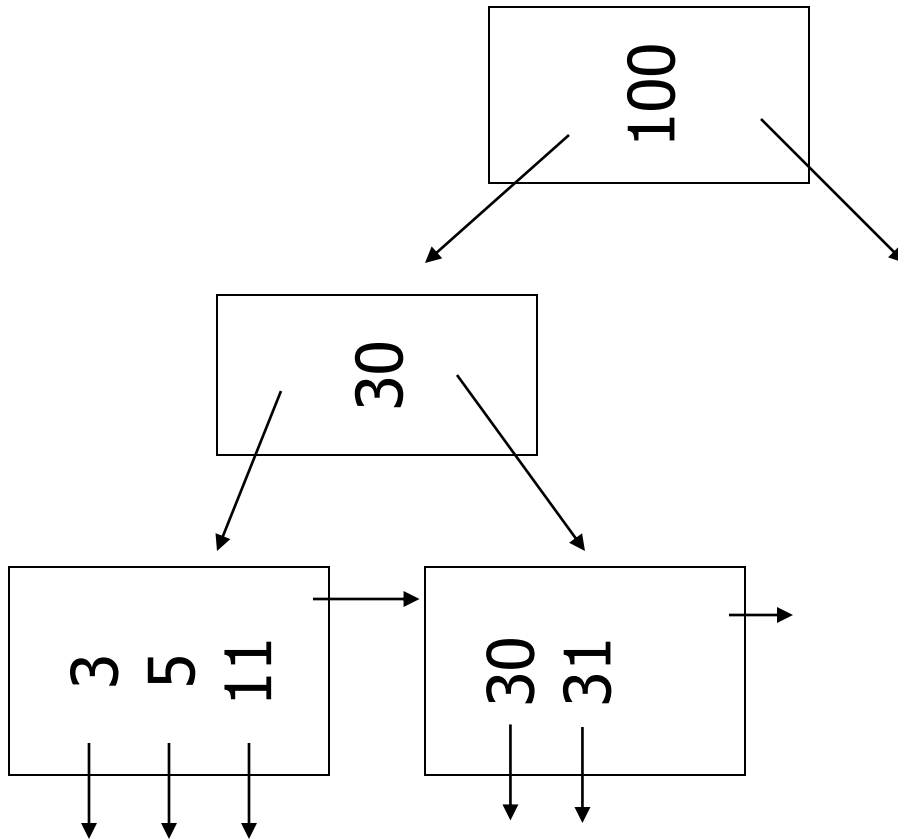
(b) leaf overflow

(c) non-leaf overflow

(d) new root

(a) Insert key = 32

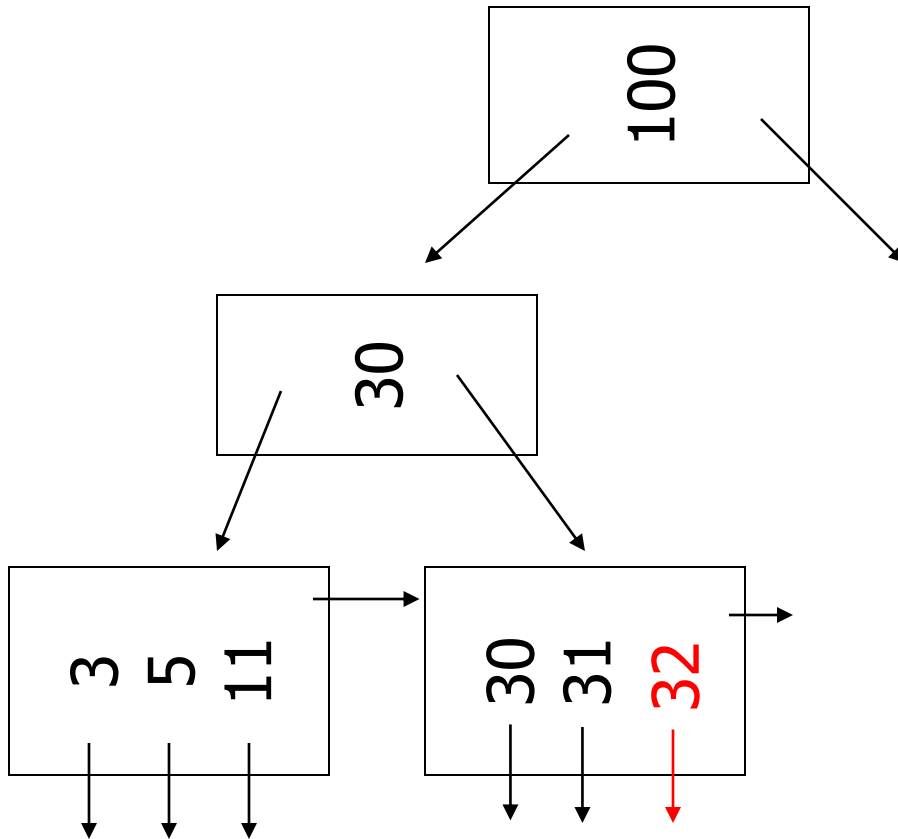
n=3





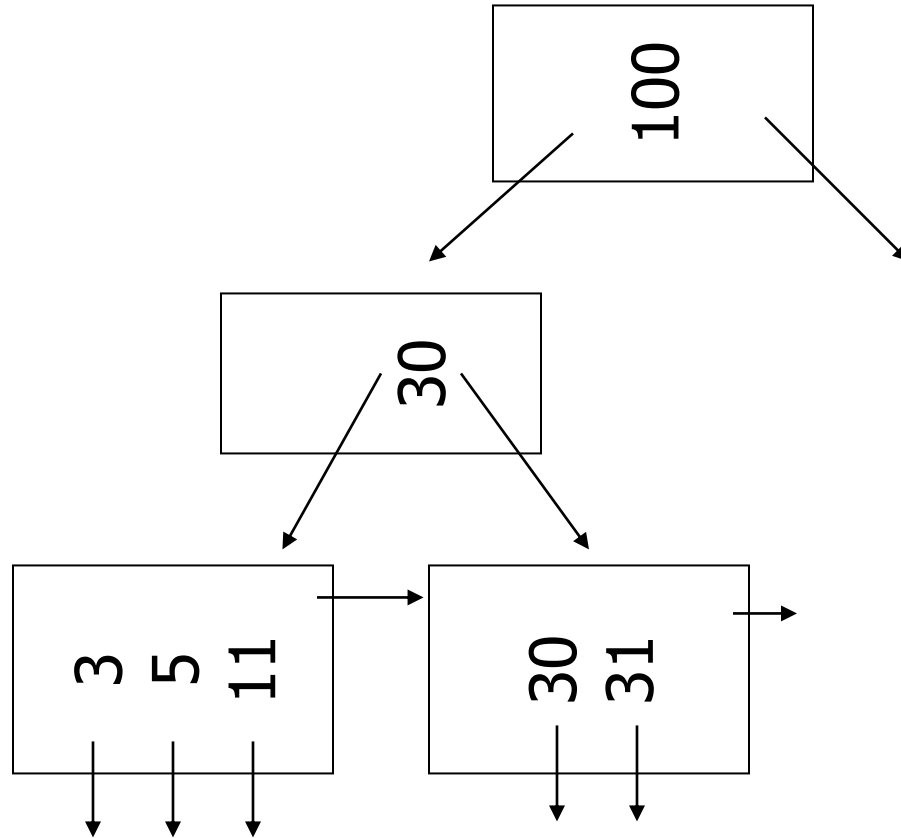
(a) Insert key = 32

n=3



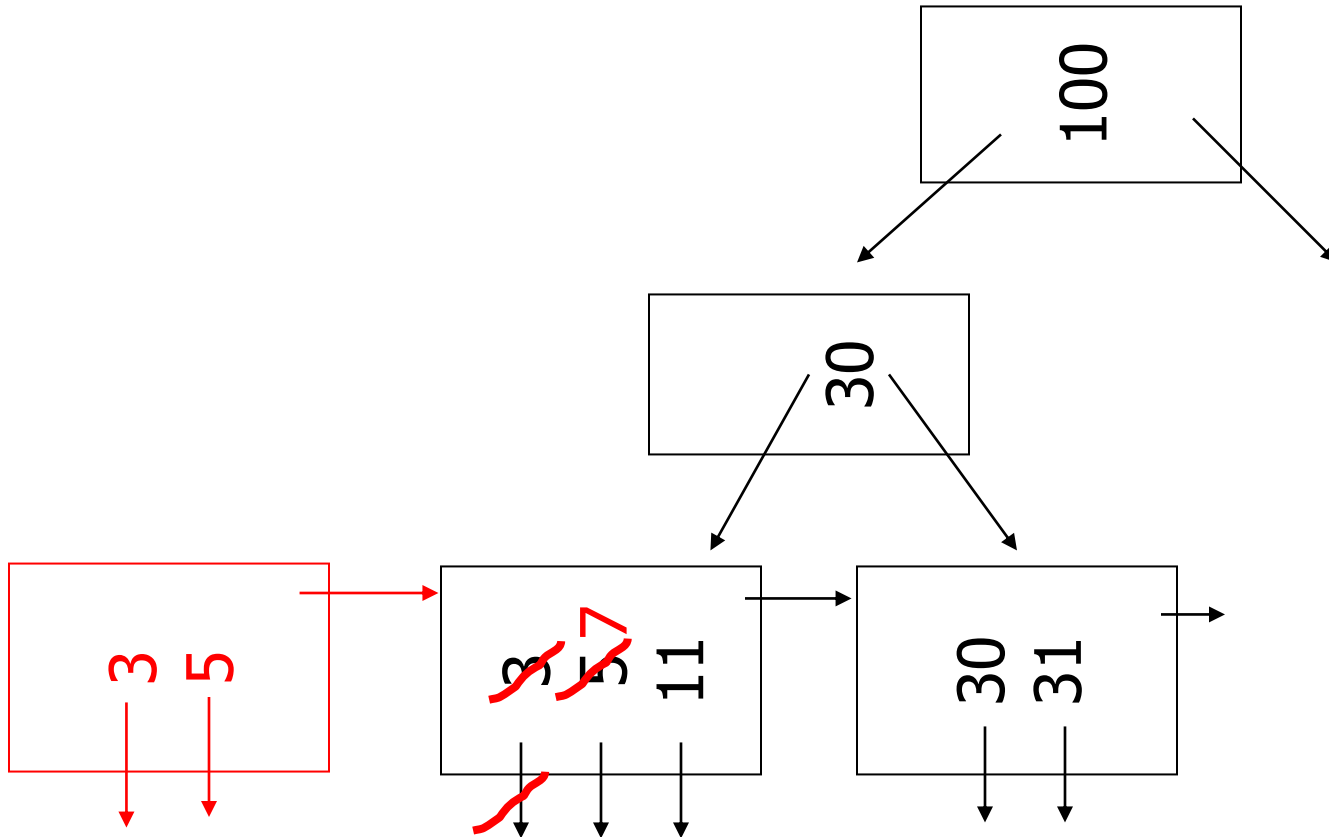
(a) Insert key = 7

n=3



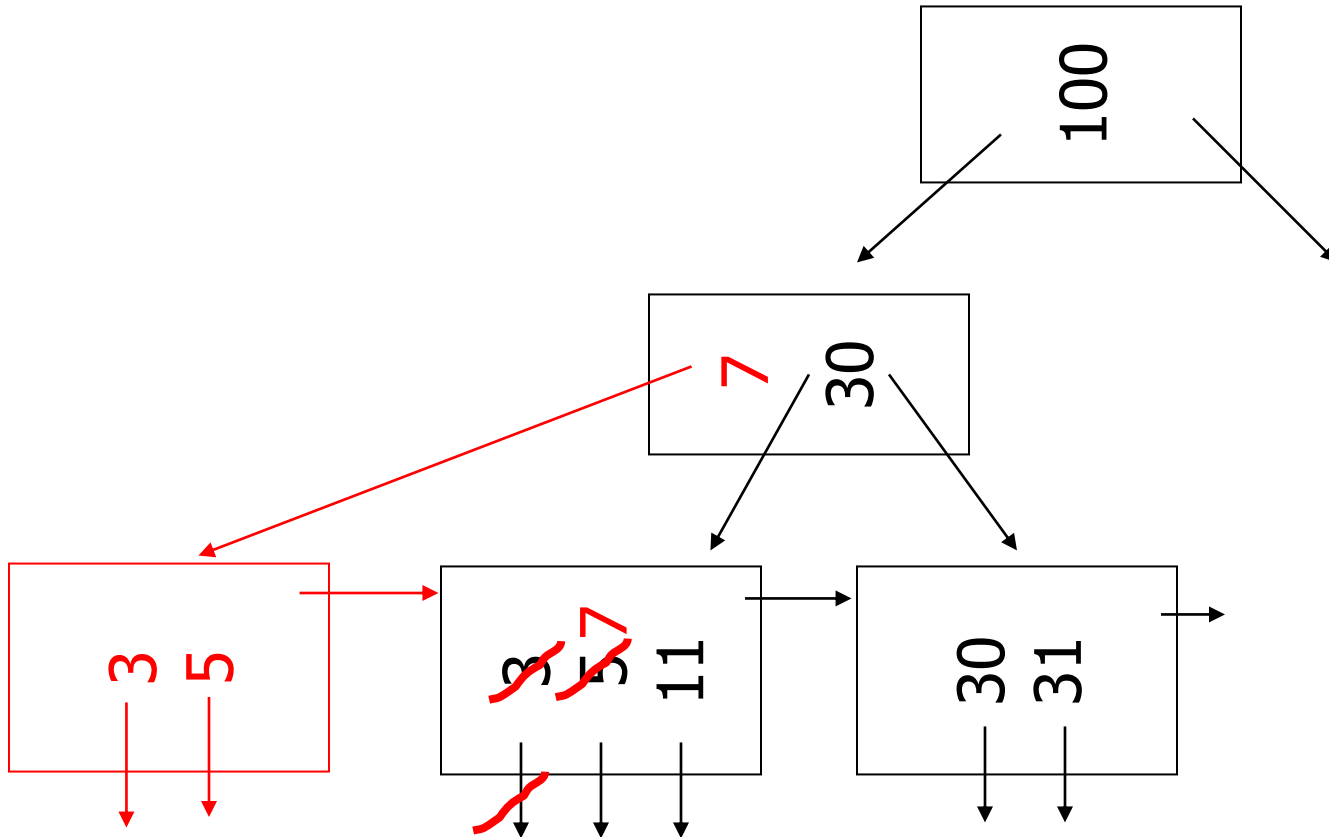
(a) Insert key = 7

n=3



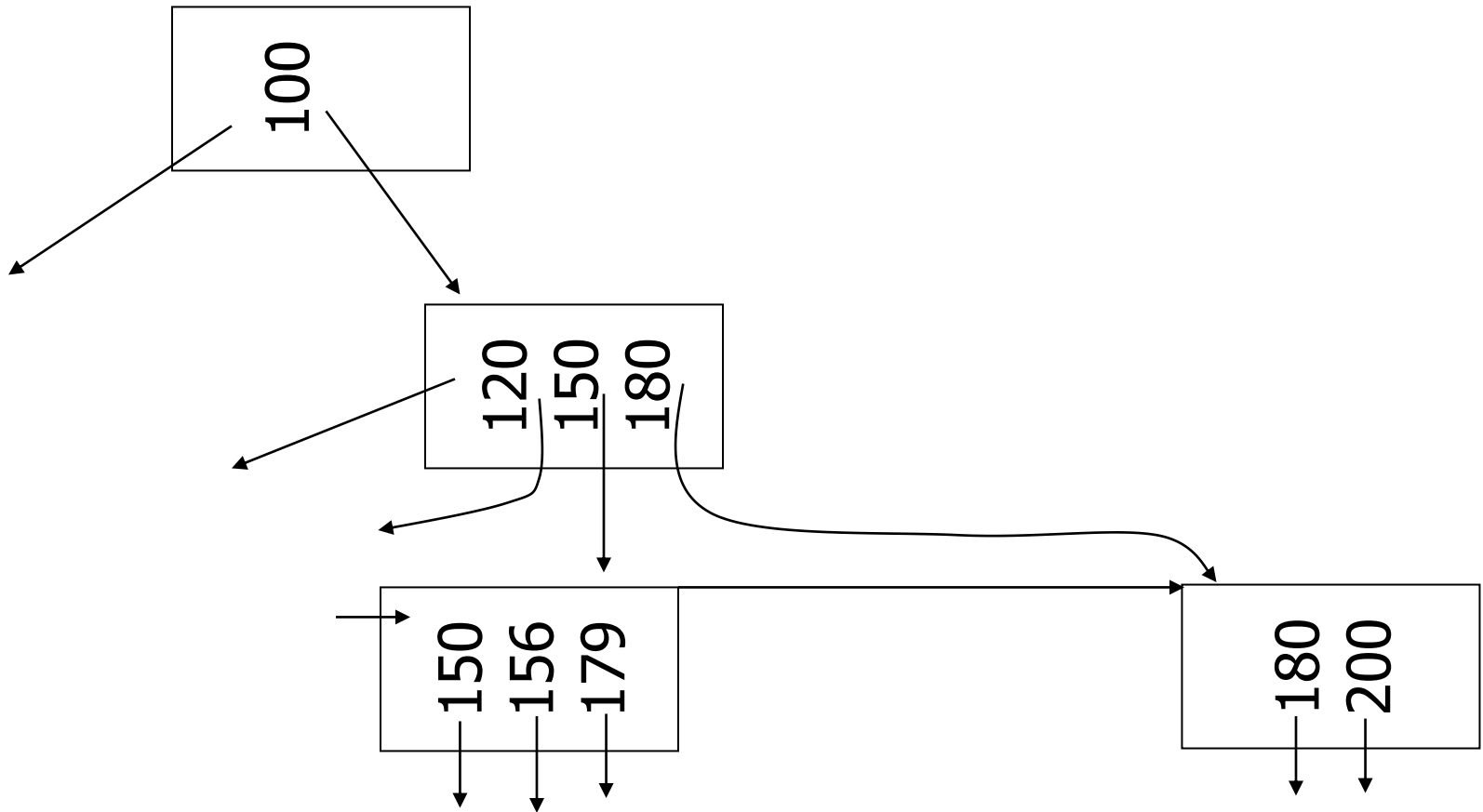
(a) Insert key = 7

n=3



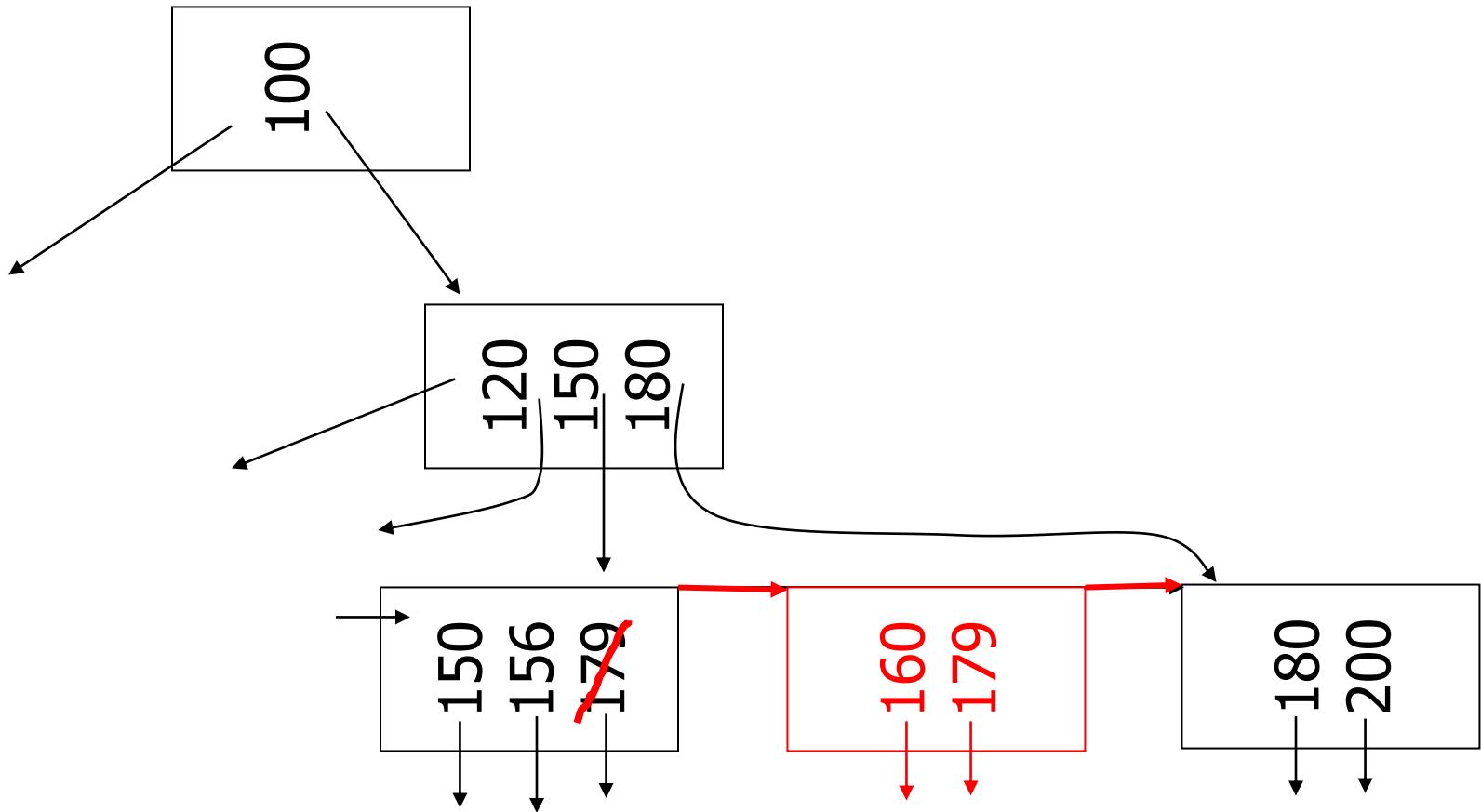
(c) Insert key = 160

n=3



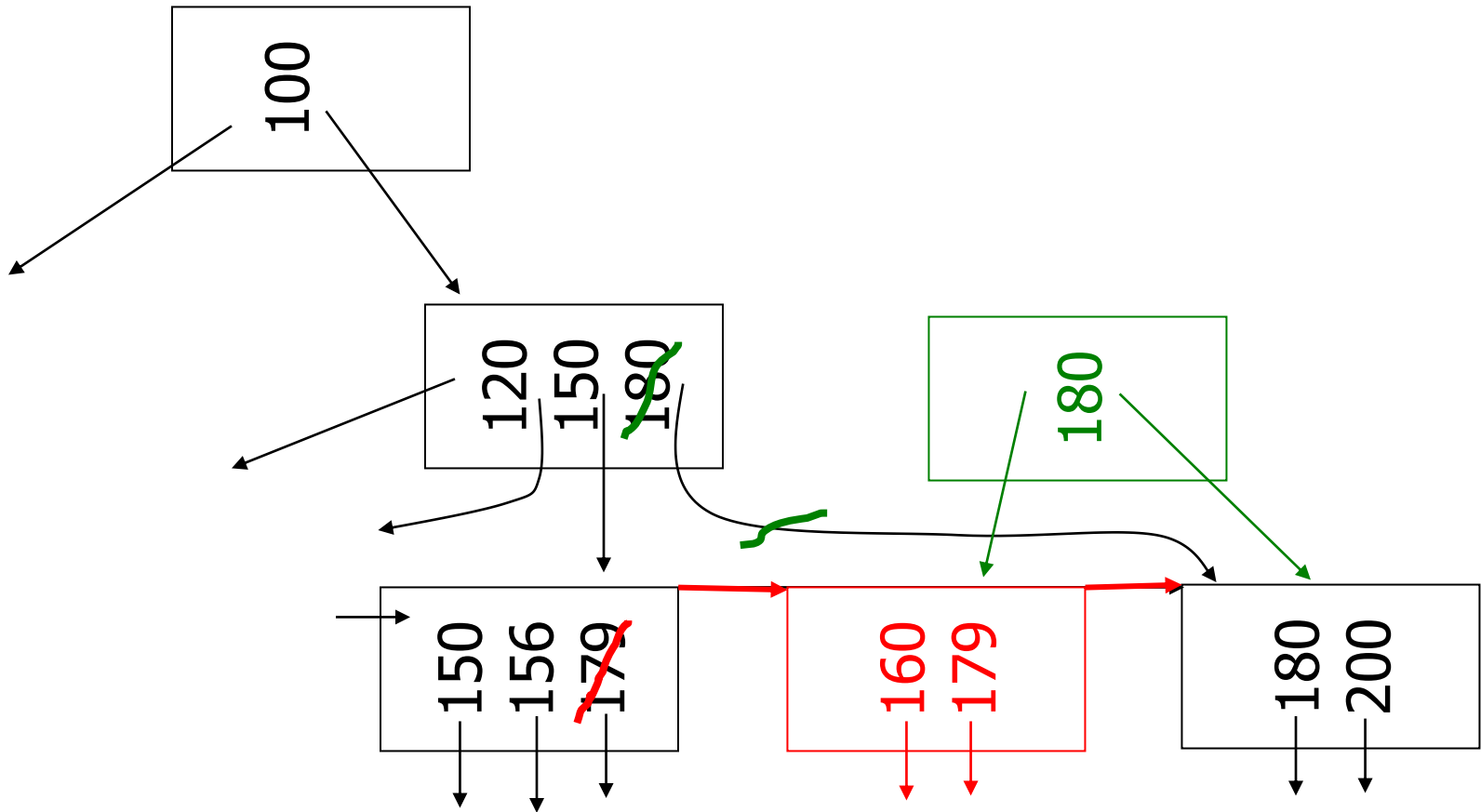
(c) Insert key = 160

n=3



(c) Insert key = 160

n=3



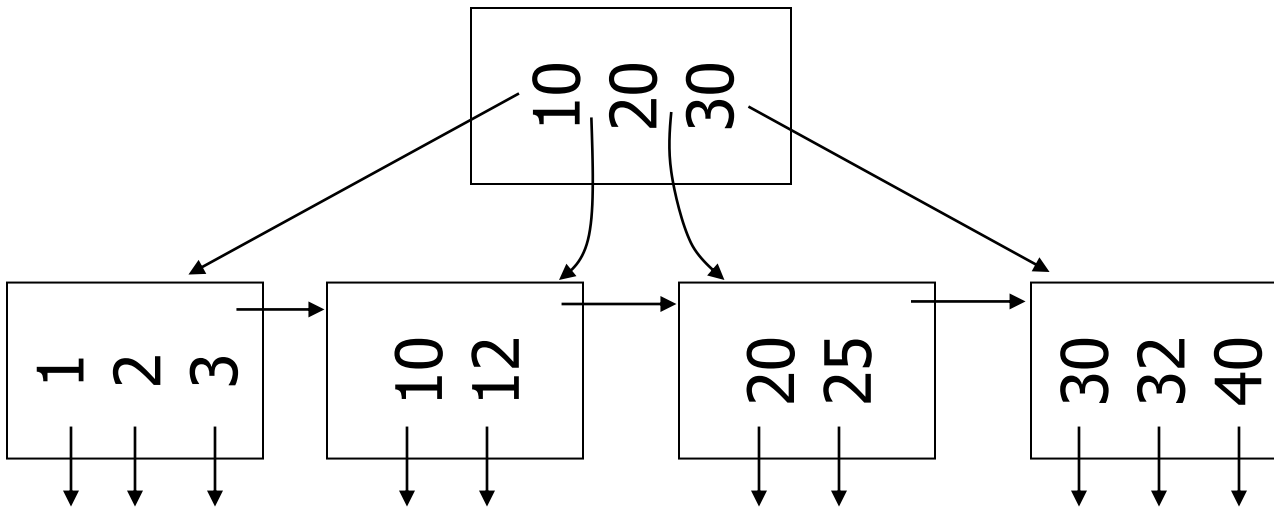
n=3





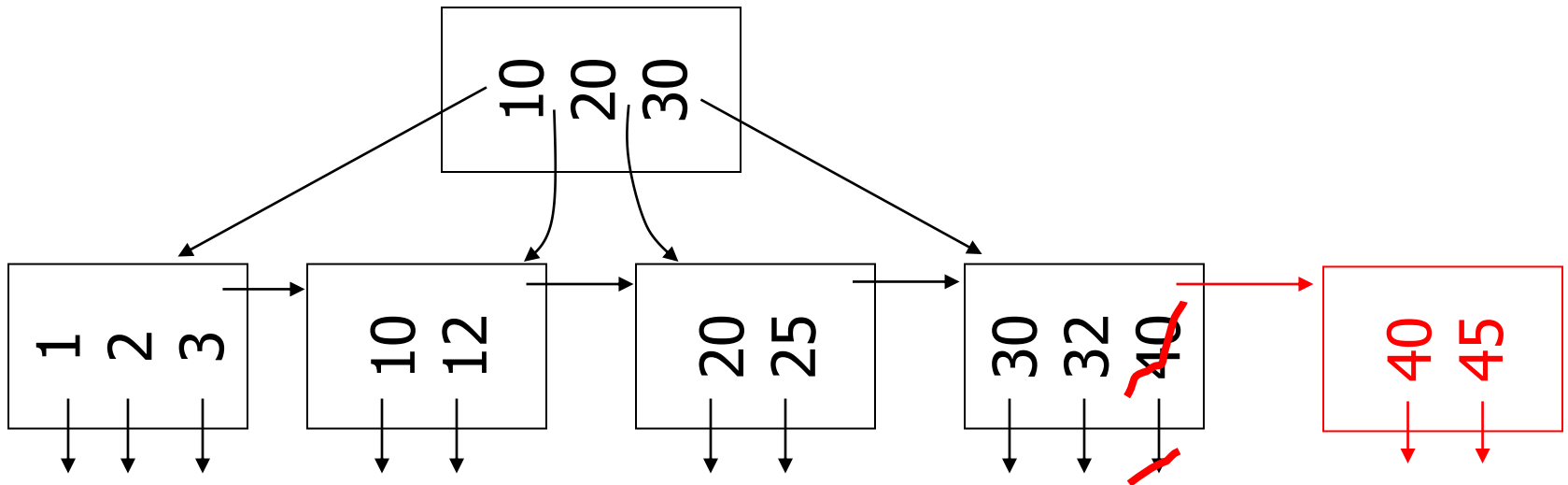
(d) New root, insert 45

n=3



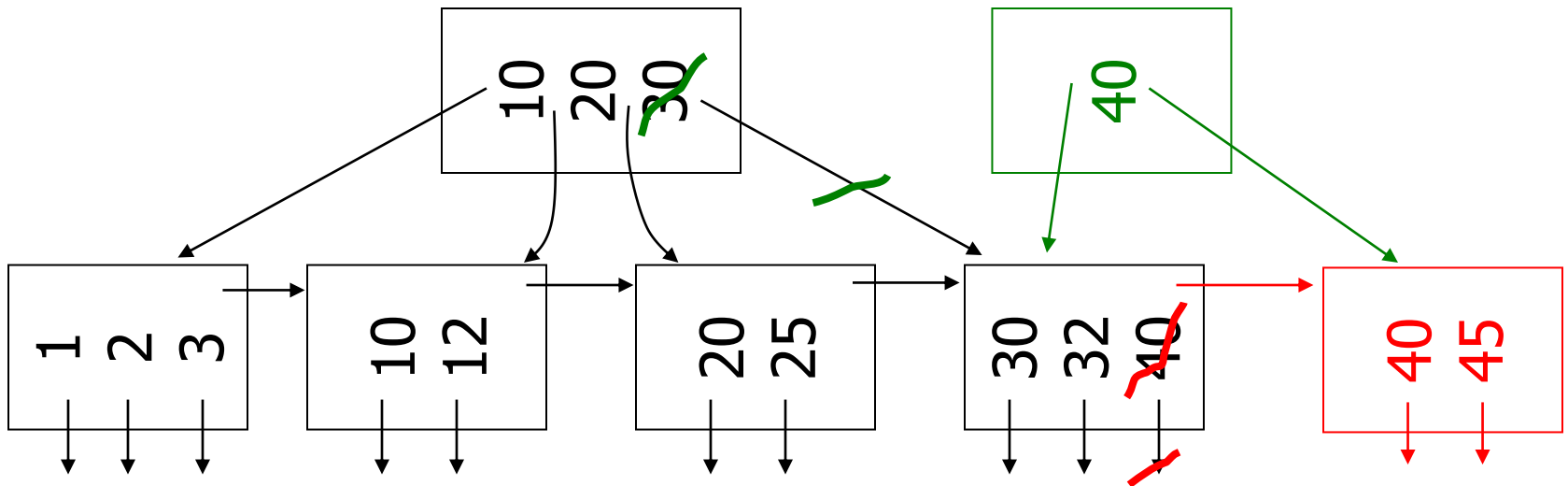
(d) New root, insert 45

n=3



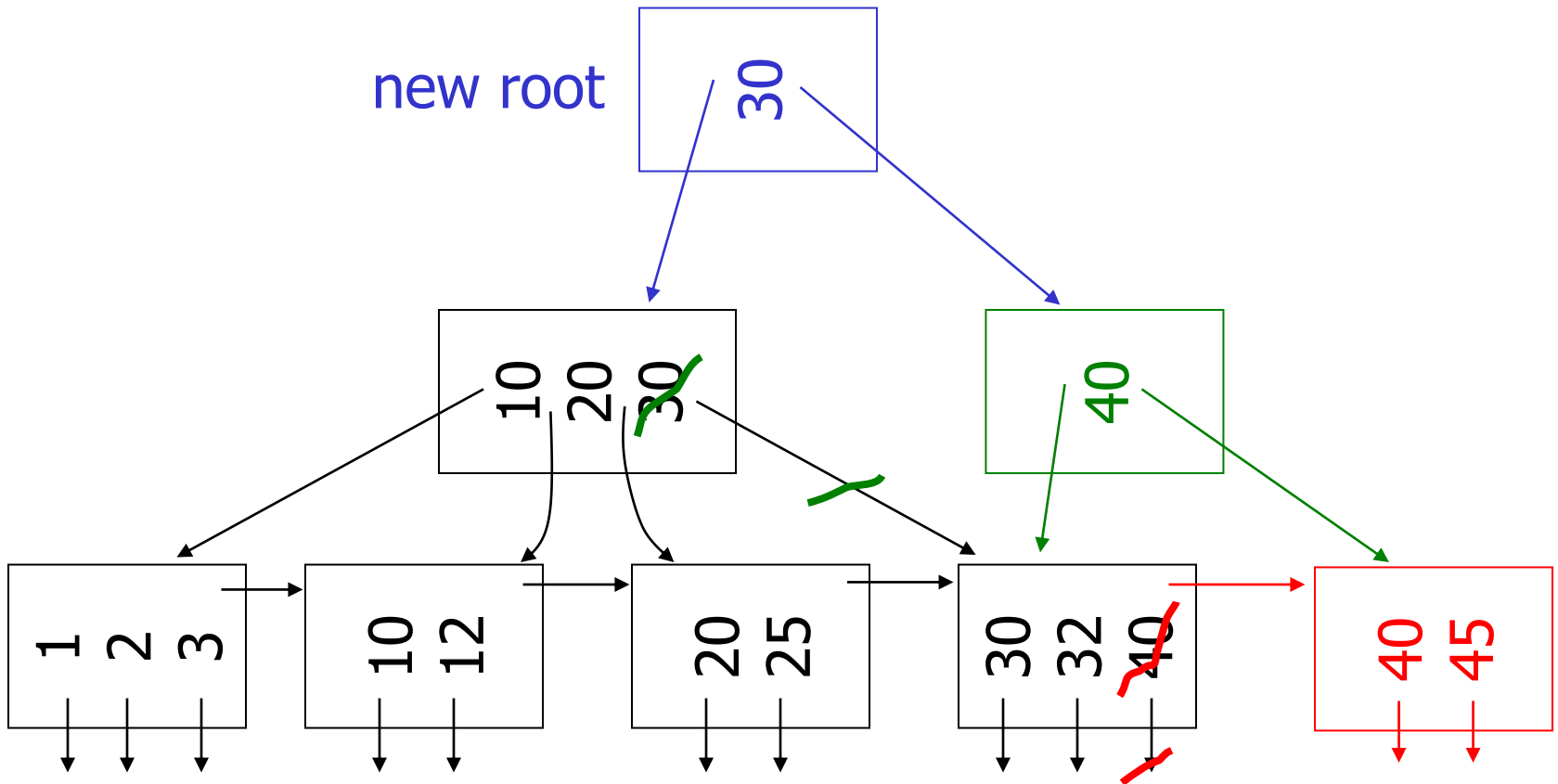
(d) New root, insert 45

n=3



(d) New root, insert 45

n=3



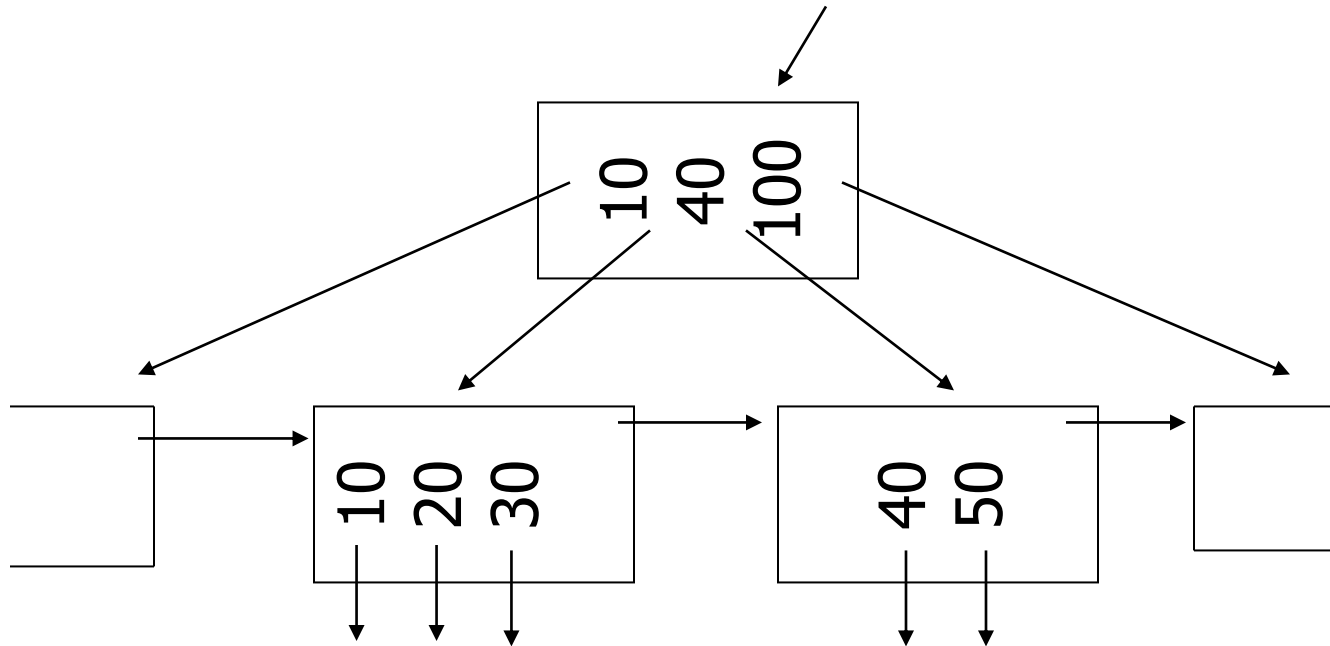
## Deletion from B+tree

- (a) Simple case - no example
- (b) Coalesce with neighbor (sibling)
- (c) Re-distribute keys
- (d) Cases (b) or (c) at non-leaf

## (b) Coalesce with sibling

– Delete 50

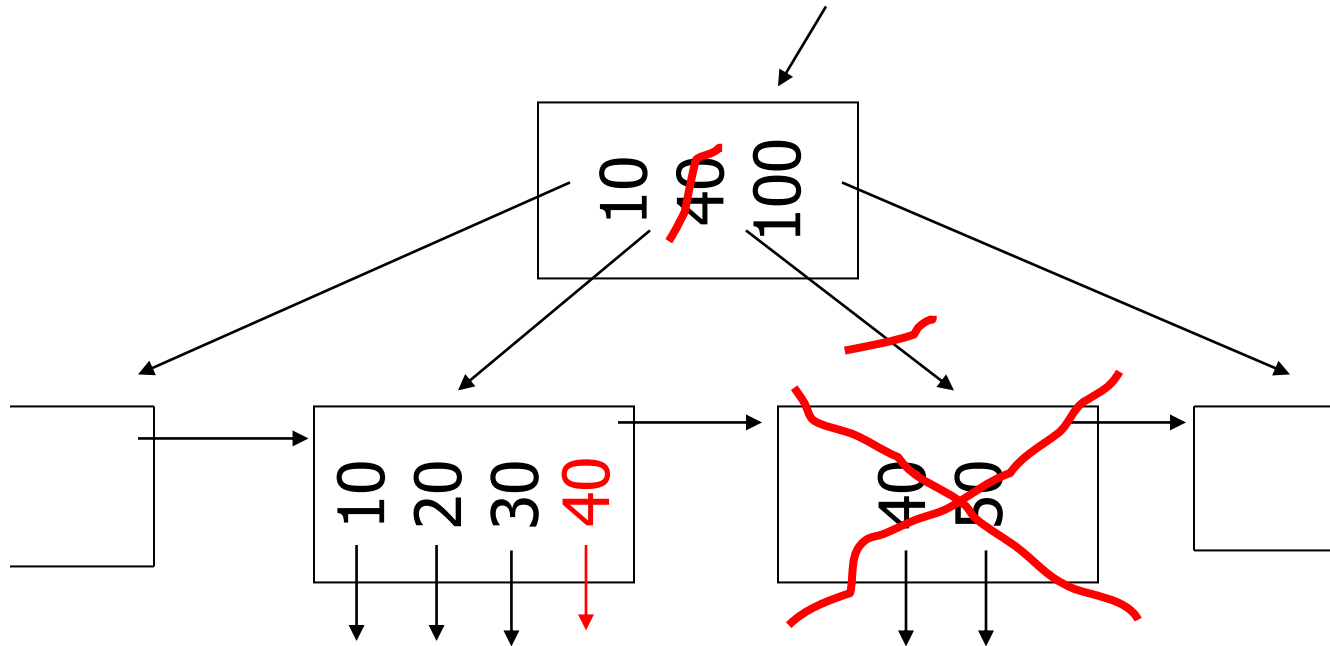
n=4



## (b) Coalesce with sibling

– Delete 50

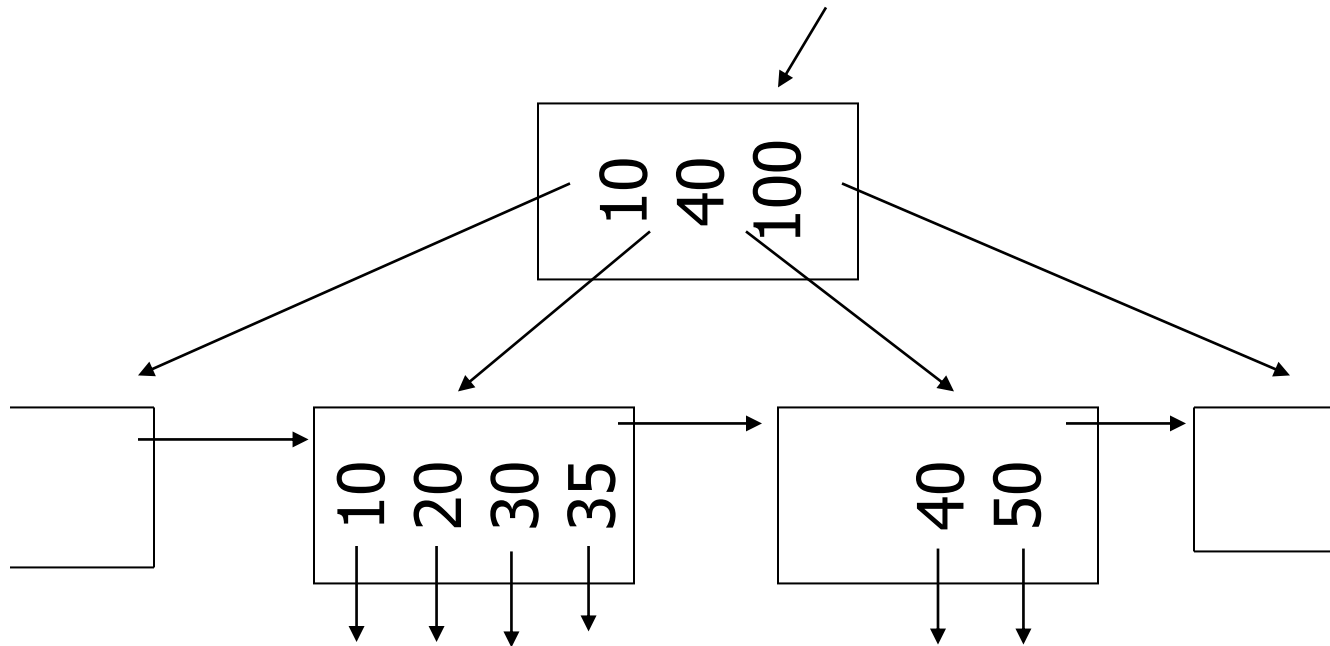
n=4



## (c) Redistribute keys

– Delete 50

n=4

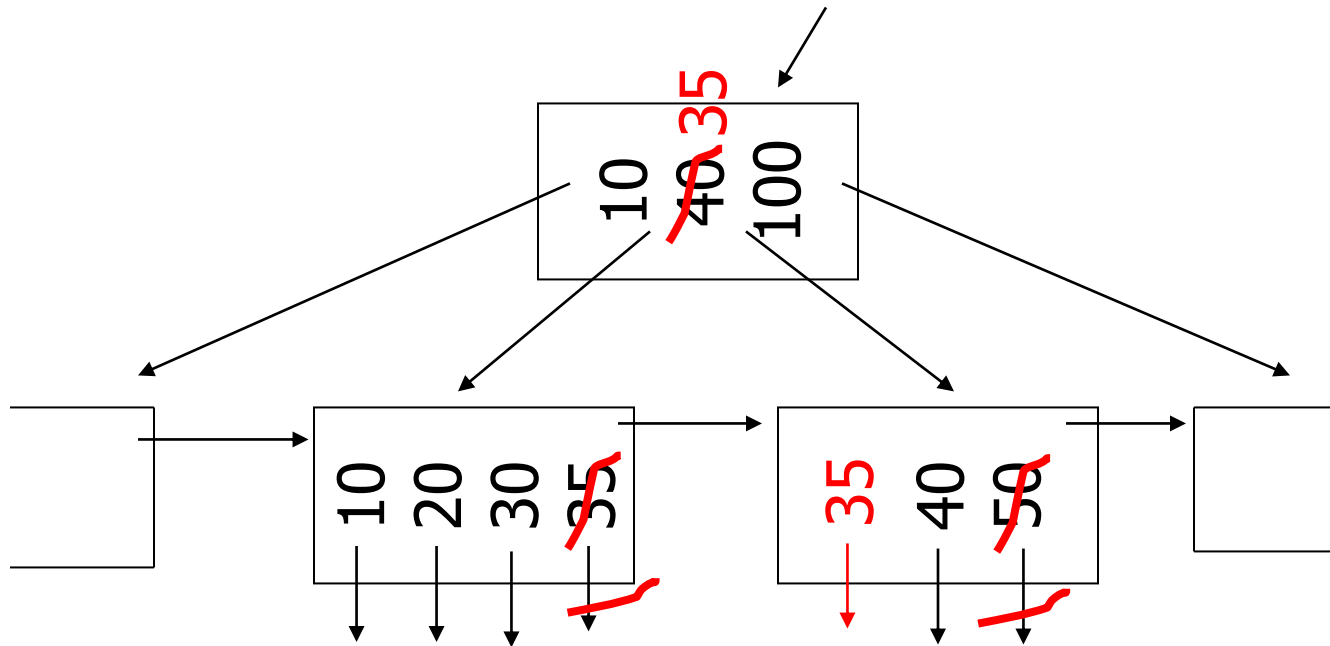




## (c) Redistribute keys

– Delete 50

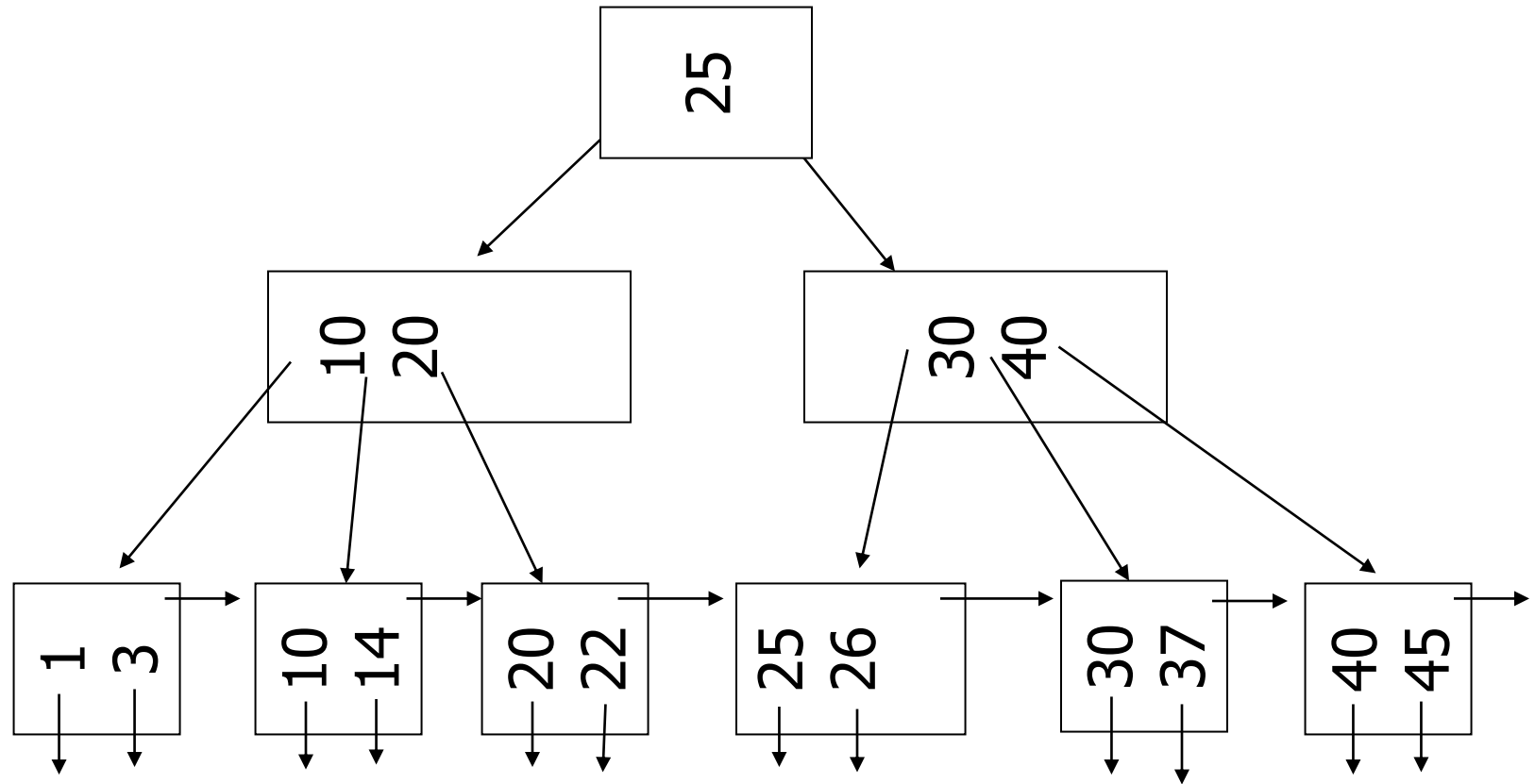
n=4



## (d) Non-leaf coalesce

– Delete 37

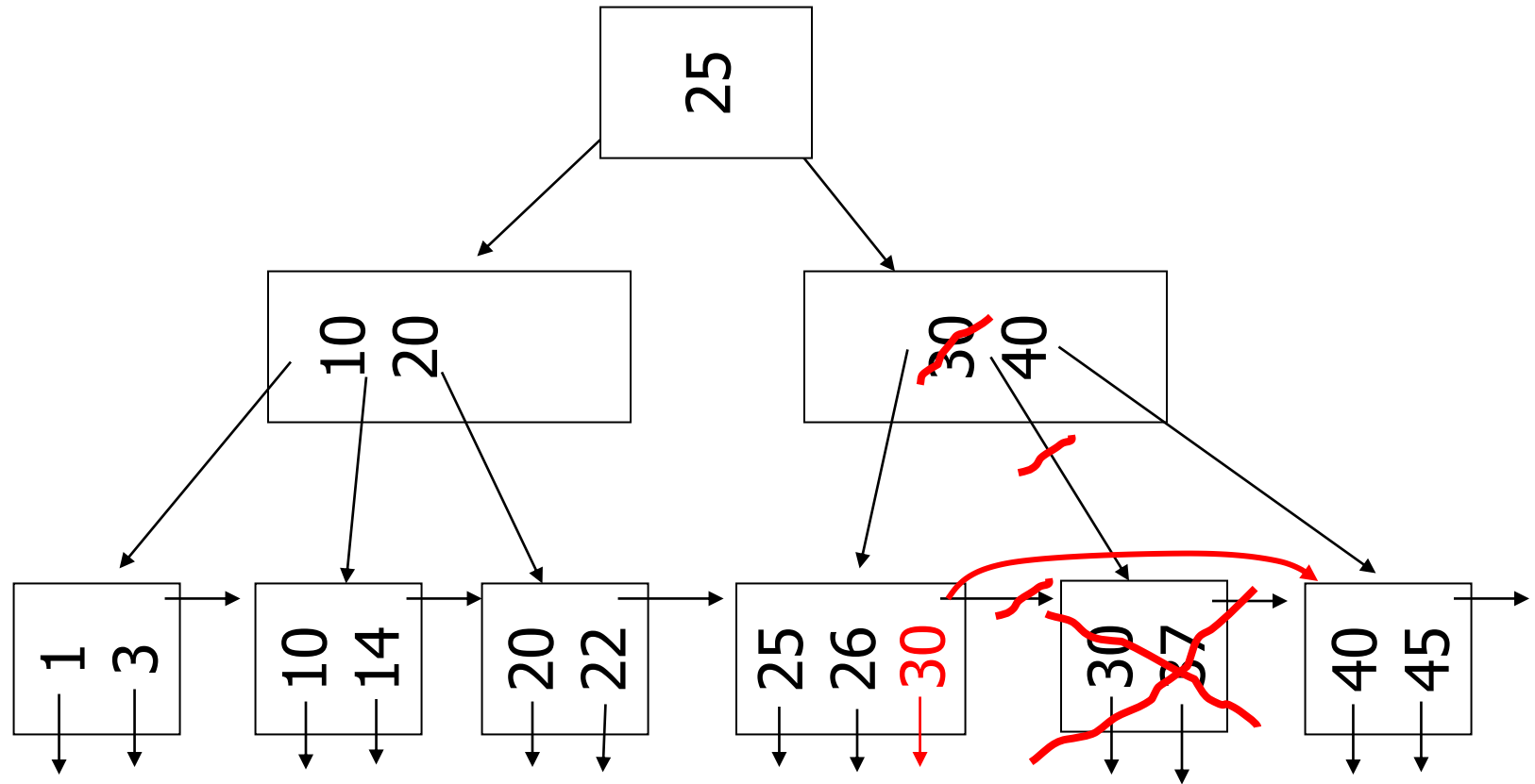
n=4



# (d) Non-leaf coalesce

– Delete 37

n=4



$n=4$

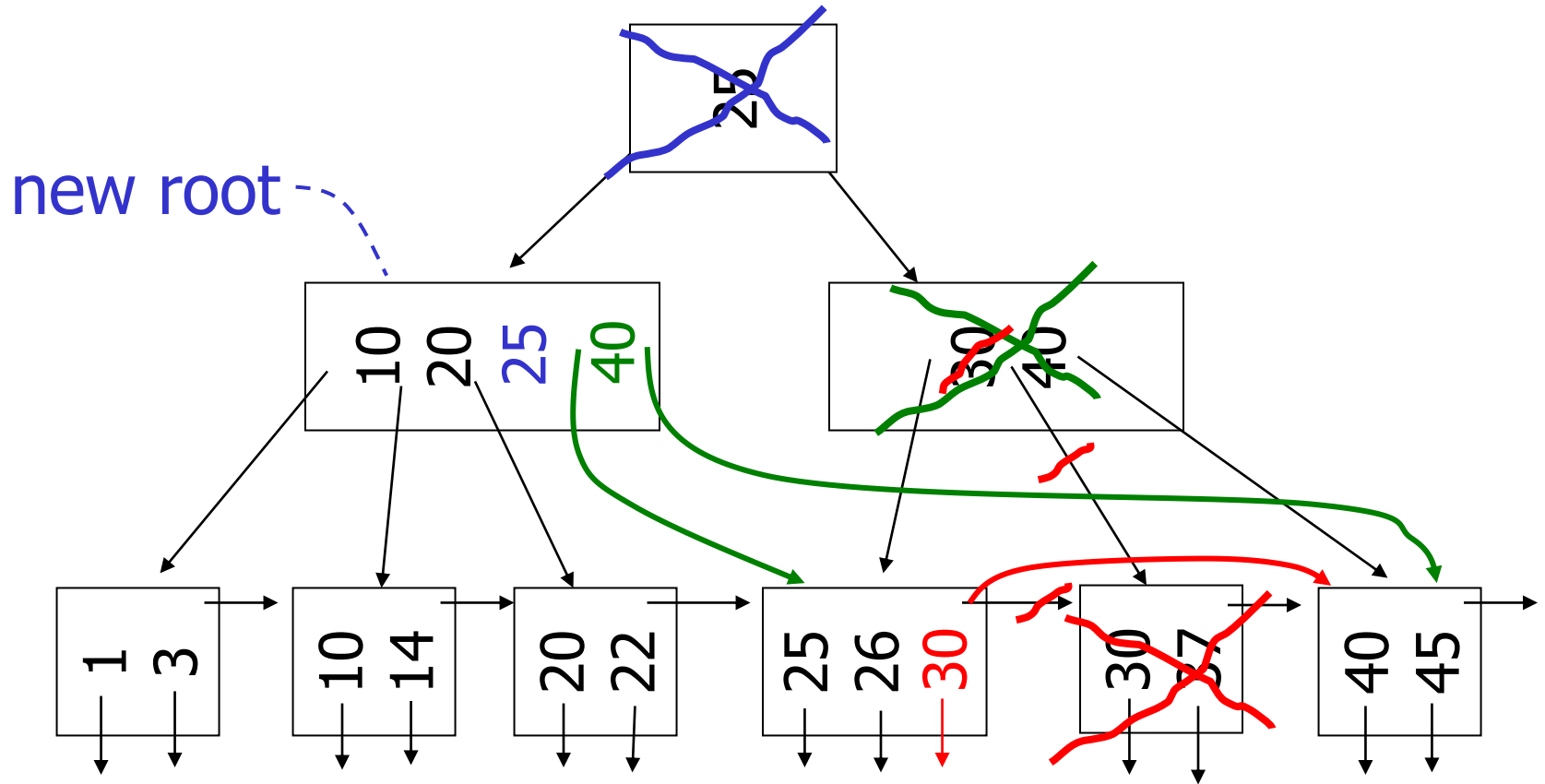
$n=4$



# (d) Non-leaf coalesce

– Delete 37

n=4



## B+tree deletions in practice

- Often, coalescing is not implemented
  - Too hard and not worth it!

# Outline/summary

- Conventional Indexes
  - Sparse vs. dense
  - Primary vs. secondary
- B trees
- Hashing schemes (recommended reading, not mandatory)

The slides in this lecture are taken from:

- Hector Garcia-Molina, CS 245: Database System Principles, Notes 4: Indexing.

## Reading

- Héctor García-Molina, Jeffrey Ullman, and Jennifer Widom. Database Systems: The Complete Book