Swarm Intelligence

Ant System and Traveling Salesman Problem

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Outline

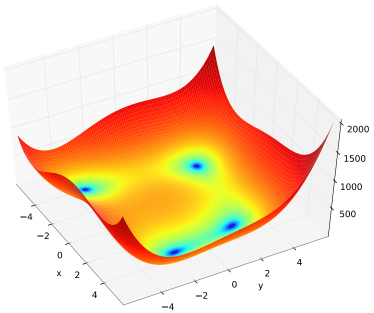
1. Concepts review
2. Traveling salesman problem
   - Problem definition
   - Examples
3. Ant System Algorithm
   - Description
   - Application to TSP
4. Class exercise
5. Practical exercise

Concept review

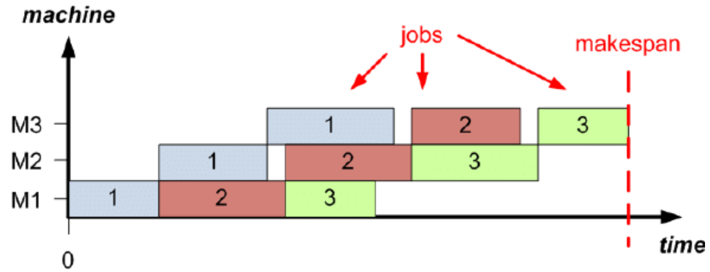
- **Optimization problems**

Problems in which we need to find the minimum or maximum value of an objective over a feasible region

- continuous



- discreet



- multi-objective, dynamic, etc.

Concept review

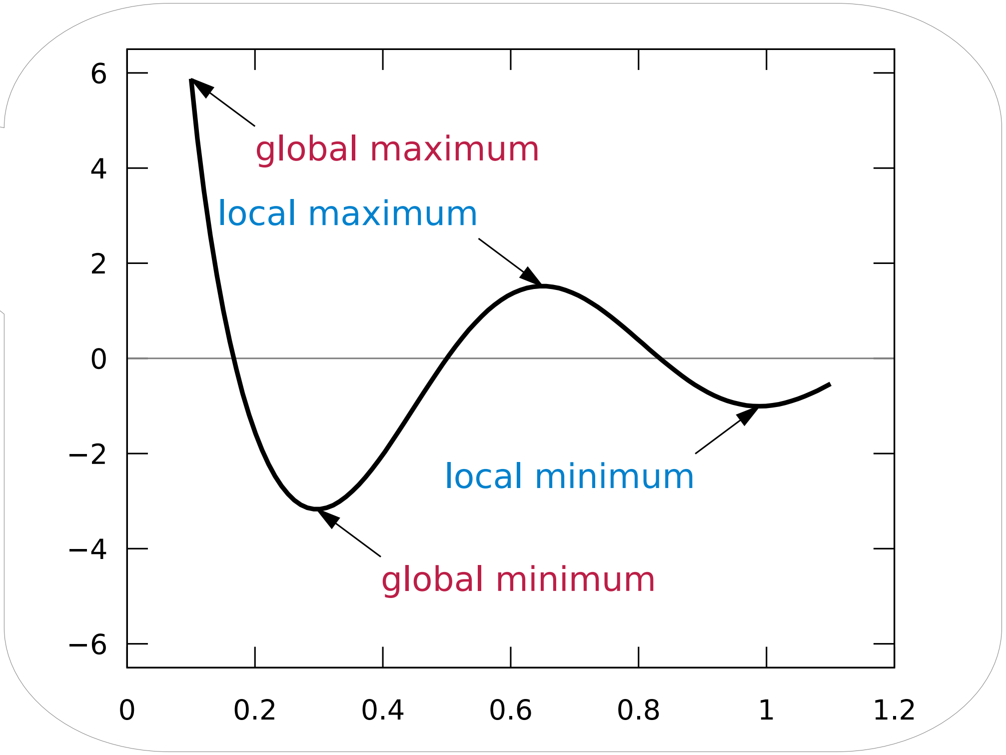
- Optimization problems
- **Objective function**

The mathematical expression that is to be optimized (maximized or minimized) in an optimization problem

- gives a measure of the quality of a solution

Concept review

- Optimization problems
- Objective function
- **Search space**
  - Local / global optima



Concept review

- Optimization problems
- Objective function
- Search space
  - Local / global optima
- **Searching**
  - Exact vs. approximation methods
  - Constructive vs. perturbative

Exact methods:
- guaranteed to eventually find the optimal solution, or to determine that no solution exists
- Impractical in most cases: exponential computational time

Approximation methods:
- no guarantee of finding the optimal solution
- provide good quality solutions in polynomial time

Concept review

- Optimization problems
- Objective function
- Search space
  - Local / global optima
- **Searching**
  - Exact vs. approximation methods
  - Constructive vs. perturbative

Constructive heuristics:
- start from an empty solution and iteratively add new components until a solution is complete

Perturbative algorithms:
- require an initial solution
- apply a perturbation (i.e., a variation to one or more of the solution components) to create a new solution

Concept review

- Optimization problems
- Objective function
- Search space
  - Local / global optima
- Searching
  - Exact vs. approximation methods
  - Constructive vs. perturbative
- **Exploration and exploitation**

Traveling Salesman Problem

- Given a set of cities, a salesman needs to find the shortest possible tour that takes him through all cities just once and then back home

Traveling Salesman Problem

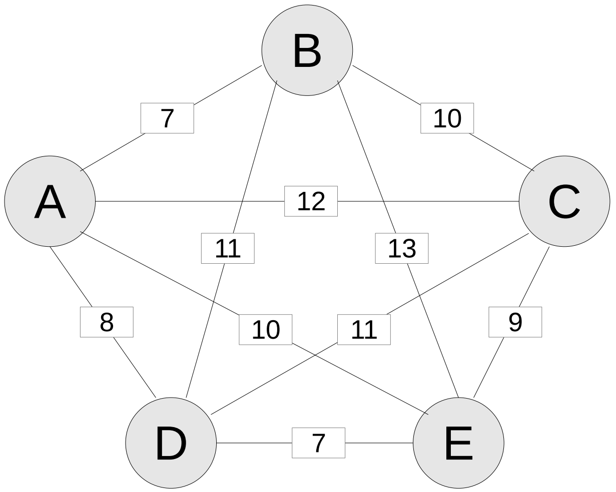
The traveling salesman problem (TSP) is:

- a classical **combinatorial optimization problem**
- **NP hard**
- the problem to which the Ant System algorithm was first applied
- popular test bed for new algorithms

Traveling Salesman Problem
Formal definition

The TSP can be modeled as a graph ***G(N,E)*** where:

- ***N*** is the set of nodes representing the cities
- ***E*** is the set of edges
- Each edge has a cost ***d*** associated to it
  - d_{ij}, in this case, is the distance from city ***i*** to city ***j***



Traveling Salesman Problem
Formal definition

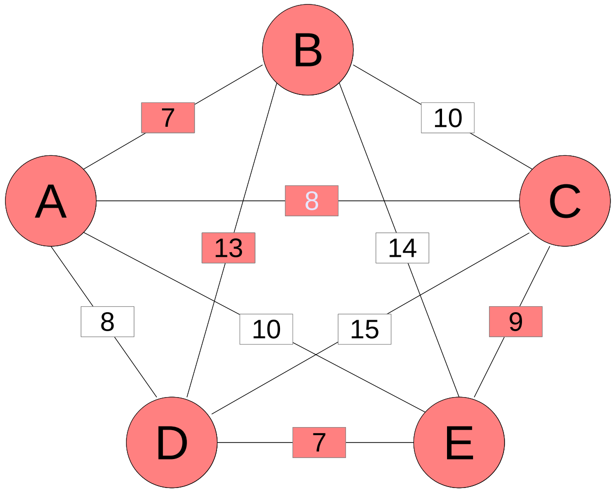
The goal is to find a Hamiltonian tour in a graph G(N,E) that minimizes the following function

$$f(\pi)=\sum_{i=1}^{n-1} d_{\pi(i)\pi(i+1)} + d_{\pi(n)\pi(1)} ,$$

where ***π*** *is a permutation of the nodes,* ***n*** *is the number of nodes in G, and* ***d*** *is the distance or cost measure associated to an edge*

Traveling Salesman Problem
First attempt to solve – constructive heuristic

- The **nearest neighborhood heuristic** is a simple greedy-type construction heuristic
  - It starts from a randomly chosen city
  - Greedy rule: select the closest city that is not yet visited



- Initial city: C
- Closest city: A cost: 8
- Closest city: B cost: 7
- Closest city: D cost: 13
- Closest city: E cost: 7
- Return city cost: 9

Total: 44

Traveling Salesman Problem

First attempt to solve – constructive heuristic

- The nearest neighbor algorithm is ***easy to implement*** and ***executes quickly***
- Usually the last a few edges added are extremely large, due to the “*greedy*" nature of the algorithm
- In some cases, it even constructs the unique worst possible tour
- How to generate a tour more intelligently?
  - **Learn from the previous constructions**!

Ant System

- **Ant System** is a basic ant-based algorithm
- Ants visit the cities sequentially until they build a complete tour
- Transition from city $\boldsymbol{i}$ to $\boldsymbol{j}$ depends on:
  - Heuristic information ($\boldsymbol{\eta}$), that is numerical information associated to the edge cost (distance)
  - Pheromones ($\boldsymbol{\tau}$), that is the information acquired by the ants throughout the algorithm execution and that represents the learned desirability to visit city $\boldsymbol{i}$ when in city $\boldsymbol{j}$

Ant System
Stochastic solution construction

- Use memory to remember partial tours
- Being at a city ***i*** choose next city ***j*** probabilistically among feasible neighboring cities
- Probabilistic choice depends on:
  - pheromone trails $\boldsymbol{\tau_{ij}}$
  - heuristic information $\boldsymbol{\eta_{ij} = 1/d_{ij}}$
- Random proportional rule to compute the probability of moving from city ***j*** to city ***i***

$$p_{ij}^{k}(t)=\frac{[\tau_{ij}(t)]^{\alpha}\cdot[\eta_{ij}]^{\beta}}{\sum_{l\in N_i^k}[\tau_{il}(t)]^{\alpha}\cdot[\eta_{il}]^{\beta}}, \quad if \quad j\in N_i^k$$

Ant System
Pheromone update rule

- Use pheromone evaporation to avoid unlimited increase of pheromone trails and allow **forgetting** of earlier choices
  - Pheromone evaporation rate $0 < \rho \leq 1$

- Use pheromone deposit to positive feedback, reinforcing components of good solutions
  - Better solutions give more feedback

Ant System
Pheromone update rule

- Example of pheromone update

$$\tau_{ij}(t) = (1-\rho)\cdot\tau_{ij}(t-1) + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$$

$$\Delta\tau_{ij}^{k} = \frac{1}{L_k}, \quad if \quad edge(i,j) \quad is \quad used \quad by \quad ant \quad k \quad on \quad its \quad tour$$

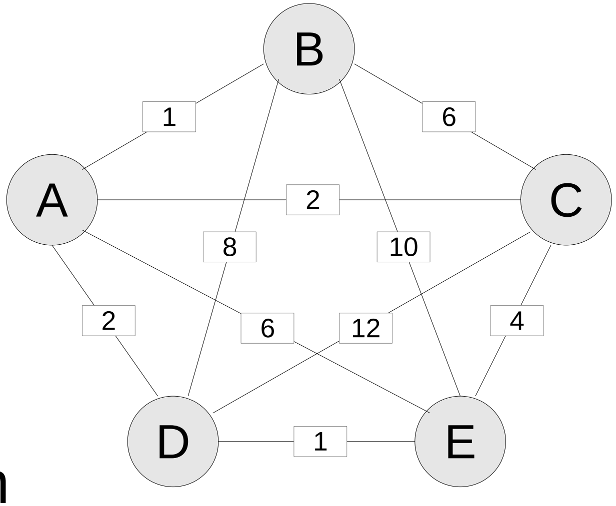
  - ***L_k*:** Tour length of ant *k*
  - ***m***: *number of ants*

Ant System
Simple pseudo code

```
While !termination()
  For k = 1 To m Do #m number of ants
     ants[k][1] ← SelectRandomCity()
    For i = 2 To n Do #n number of cities
      ants[k][i] ← ASDecisionRule(ants, i)
    EndFor
    ants[k][n+1] ← ants[k][1] #to complete the tour
  EndFor
  UpdatePheromone(ants)
EndWhile
```

Ant System
Simple example

- For our example with #ants=3, $\alpha=2$, $\beta=1$, $\rho=0.5$ and $\tau_0=1$



  - Heuristic Information

ηij	A	B	C	D	E
A	-	1/1	½	½	1/6
B	1/1	-	1/6	1/8	1/10
C	½	1/6	-	1/12	¼
D	½	1/8	1/12	-	1/1
E	1/6	1/10	¼	1/1	-

  - Pheromone trails

τij	A	B	C	D	E
A	-	0.56	0.66	0.60	0.50
B	0.56	-	0.60	0.56	0.60
C	0.66	0.60	-	0.50	0.56
D	0.60	0.56	0.50	-	0.66
E	0.50	0.60	0.56	0.66	-

Ant System
Simple example

- Ant #1 starts from city D (random), selection probabilities

$$p_{ij}^{k}(t)=\frac{[\tau_{ij}(t)]^{\alpha}\cdot[\eta_{ij}]^{\beta}}{\sum_{l\in N_i^k}[\tau_{il}(t)]^{\alpha}\cdot[\eta_{il}]^{\beta}}$$

pij	A	B	C	D	E
D	0.264	0.059	0.031	0.000	0.646

[0, 0.264, 0.323, 0.354, 1]

- Select a city → rand 0.80
  - City **E** selected

pij	A	B	C	D	E
E	0.267	0.227	0.506	0.000	0.000

[0, 0.267, 0.494, 1]

- Select a city → rand 0.27
  - City **B** selected

pij	A	B	C	D	E
B	0.843	0.000	0.157	0.000	0.000

[0, 0.843, 1]

- Select a city → rand 0.88
  - City **C** selected

Ant System
Simple example

- First iteration we can have:
  - Ant #1: D-E-B-C-A-D
  - Ant #2: A-E-D-C-B-A
  - Ant #3: D-E-C-B-A-D
- Update the pheromone using this tours

$$\tau_{ij}(t)=[1-\rho]\cdot\tau(t-1)+\sum_{k=1}^{m}\Delta\tau_{ij}^{k}$$

- And then iterate

tij	A	B	C	D	E
A	-	0.39	0.38	0.42	0.29
B	0.39	-	0.46	0.28	0.35
C	0.38	0.46	-	0.29	0.35
D	0.42	0.28	0.29	-	0.49
E	0.29	0.35	0.35	0.49	-

Ant System

Class exercise #1 – test your knowledge of AS

- Open the file TSP_ANT_SYSTEM-class_exercise.pdf and answer the five points of the exercise
  - The goal of this exercise is for you to determine how well you understood the main concepts of Ant System and the way the algorithm works
  - Once you finished the exercise, compare and discuss your answers with one of your classmates

Ant System

Implementation exercise #1 – implementation of AS

- Open the file Implementation_Exercise1.pdf and solve point 1, which consists in implementing Ant System according to the provided template in C++
- The following slides give a practical view of the Ant System algorithm procedures.

Ant System Algorithm
Solution Construction

```
Procedure ConstructSolutions ()
   For k = 1 To m Do                     #m number of ants
      For i = 1 To n Do                  #n number of cities
         ant[k].visited[i] ← false
      EndFor
   EndFor
   step ← 1
   For k = 1 To m Do
      r ← random{1, . . . , n}
      ant[k].tour [step] ← r
      ant[k].visited [r] ← true
   EndFor
   While (step < n) Do
      step ← step + 1
      For k = 1 To m Do
         ASDecisionRule(k, step)
      EndFor
   EndWhile
   For k = 1 To m Do
      ant[k].tour [n+1] ← ant[k].tour[1]
      ant[k].tour length ← ComputeTourLength(k)
   EndFor
EndProcedure
```

Ant System Algorithm

Decision Rule

```
Procedure ASDecisionRule(k, i)
   #k ant identifier
   #i counter for construction step
   c ← ant[k].tour[i-1]
   sum_prob = 0.0
   For j = 1 To n Do
      If ant[k].visited[j] Then
         selection_prob[j] ← 0.0
      Else
         selection_prob[j] ← choice_info[c][j]
         sum_prob ← sum_prob + selection_prob[j]
      EndIf
   EndFor
   r ← random[0, sum_prob]
   j ← 1
   p ← selection_prob[j]
   While (p < r ) Do
      j ← j + 1
      p ← p + selection_prob[j]
   EndWhile
   ant[k].tour[i] ← j
   ant[k].visited[j] ← true
EndProcedure
```

Ant System Algorithm

Pheromone Update

```
Procedure ASPheromoneUpdate ()
    Evaporate()
    For k = 1 To m Do
        DepositPheromone(k)
    EndFor
    ComputeChoiceInformation()
EndProcedure
```

Ant System Algorithm

Pheromone Update – evaporation procedure

```
Procedure Evaporate
  For i = 1 To n Do
    For j = i To n Do
      pheromone[i][j] ← (1−ρ)·pheromone[i][j]
      pheromone[j][i] ← pheromone[i][j]
      #pheromones are symmetric
    EndFor
  EndFor
EndProcedure
```

Ant System Algorithm
Pheromone Update – deposit pheromone

```
Procedure DepositPheromone(k)
   #k ant identifier
   Δτ ← 1/ant[k].tour_length
   For i = 1 To n Do
      j ← ant[k].tour[i]
      l ← ant[k].tour[i+1]
      pheromone[j][l] ← pheromone[j][l] + Δτ
      pheromone[l][j] ← pheromone[j][l]
   EndFor
 EndProcedure
```

Ant System

Implementation exercise #1 – analysis of AS parameters

- Open the file Implementation_Exercise1.pdf and solve point 3 to 5, which consist in testing and analyzing the behavior of the algorithm.
  - Modify some parameters:
    - Number of ants
    - α, β, ρ
  - What effect can you appreciate?
  - What is the reason?