Synthesis For Oracle Algorithms

Target DeutschJozsa Program

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Definition deutsch_jozsa' {n} (U : base_ucom n) : base_ucom n :=
  X 0 ; npar n U_H ; U; npar n U_H.
```

Given an oracle $f : \{0,1\}^n \to \{0,1\}$, we need to find a program P that the corresponding α_P satisfies

$$\sum_{i=0}^{2^n-1} f(i) = 2^{n-1} \to \alpha_P(n+1, |0\rangle^{n+1}, |0\rangle^{n+1}) = 0$$

and

$$\sum_{i=0}^{2^n-1} f(i) = 2^n \vee \sum_{i=0}^{2^n-1} f(i) = 0 \to \alpha_P(n+1, |0\rangle^{n+1}, |0\rangle^{n+1}) = \pm 1$$

When we meet the target program in the search, after SMT solver's simplication, its α is an expression

$$\frac{1}{2^n} \sum_{i=0}^{2^n-1} If(f(i) == 0, 1, -1) \qquad //(-1)^{f(i)}$$

To verify the two target propositions, we need a hint function from user or we encode it the synthesizer: $\lambda a.2 \cdot a - 1$. Then we use SMT solver to verify

$$\forall i \in \{0,1\}^n, 2 \cdot f(i) - 1 = If(f(i) == 0, 1, -1)$$

Then we can calculate the final expression to

$$\frac{1}{2^n} \sum_{i=0}^{2^n-1} If(f(i) == 0, 1, -1) = 2 \cdot \sum_{i=0}^{2^n-1} f(i) - 2^{n-1}$$

and verify the two target propositions easily.