

AP Physics C Review

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Part I

Vectors

1 Basics

The Unit Vectors

$$\hat{i} + \hat{j} + \hat{k}$$

Alternate notation:

$$\vec{v} = \langle x, y, z \rangle$$

\hat{i} = positive x direction

\hat{j} = positive y direction

\hat{k} = positive z direction

2 Vector Operations

2.1 Adding Vectors

To add 2 or more vectors together, simply add the separate components of each vector.

ex:

$$\text{let } \vec{A} = 4.2\hat{i} - 1.5\hat{j}$$

$$\text{let } \vec{B} = -1.6\hat{i} + 2.9\hat{k}$$

$$\vec{A} + \vec{B} = 4.2\hat{i} - 1.6\hat{i} - 1.5\hat{j} + 2.9\hat{k} = 3.6\hat{i} - 1.5\hat{j} + 2.9\hat{k}$$

2.2 Vector Multiplication

There are two ways to multiply vectors, one of which produces a scalar and the other another vector:

- Dot Product: $\vec{a} \cdot \vec{b} = \text{Scalar}$
- Cross Product: $\vec{a} \times \vec{b} = \text{Vector}$

2.2.1 Dot Product

The dot product takes two vectors as arguments and returns a scalar. It is defined in two ways:

- $\vec{a} \cdot \vec{b} = \|a\| \|b\| \cos(\theta)$
Where $\|v\|$ is the length of a vector v ($\sqrt{v_x^2 + v_y^2 + v_z^2}$), and θ is the angle between the two vectors.
- $\hat{a} \cdot \hat{b} = (a_x b_x + a_y b_y + a_z b_z)$

Properties Some basic properties of the dot product are as follows

- $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$
- $\hat{i} \cdot \hat{j} = \hat{k} \cdot \hat{j} = \hat{k} \cdot \hat{i} = 0$

2.2.2 Cross Product

The cross product takes two vectors and produces a third that is perpendicular to both initial vectors, thus it must be performed in 3d space. This is the basis of the right hand rule, which will be expanded upon later:

- $\hat{i} \times \hat{j} = \hat{k}$
- $\hat{j} \times \hat{k} = \hat{i}$
- $\hat{k} \times \hat{i} = \hat{j}$

In general, the cross product is defined as

$$\vec{a} \times \vec{b} = \|a\| \|b\| \sin \theta \vec{c}$$

Where $\|v\|$ is the length of a vector \vec{v} ($\sqrt{v_x^2 + v_y^2 + v_z^2}$), θ is the angle between the two vectors, and \vec{c} is a unit vector perpendicular to both a and b . This is not easily implemented though. Thus, the formula is also given by:

$$\vec{a} \times \vec{b} = \langle a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x \rangle$$

This isn't much better, but there is another way to determine the cross product:

$$\vec{a} \times \vec{b} = \begin{bmatrix} a_y & a_z \\ b_y & b_z \end{bmatrix} \hat{i} - \begin{bmatrix} a_x & a_z \\ b_x & b_z \end{bmatrix} \hat{j} + \begin{bmatrix} a_x & a_y \\ b_x & b_y \end{bmatrix} \hat{k}$$

Where,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

Part II

Electrostatics and Magnetism