AP Physics C Review

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May 3, 2017

Contents

Part I
Vectors

1 Basics

The Unit Vectors

$$\hat{i} + \hat{j} + \hat{k}$$

Alternate notation:

$$\vec{v} = \langle x, y, z \rangle$$

$\hat{i}$ = positive x direction
$\hat{j}$ = positive y direction
$\hat{k}$ = positive z direction

2 Vector Operations

2.1 Adding Vectors

To add 2 or more vectors together, simply add the separate components of each vector.
ex:

let $\vec{A} = 4.2\hat{i} - 1.5\hat{j}$
let $\vec{B} = -1.6\hat{i} + 2.9\hat{k}$
$\vec{A} + \vec{B} = 4.2\hat{i} - 1.6\hat{i} - 1.5\hat{j} + 2.9\hat{k} = 3.6\hat{i} - 1.5\hat{j} + 2.9\hat{k}$

2.2 Vector Multiplication

There are two ways to multiply vectors, one of which produces a scalar and the other another vector:

- Dot Product: $\vec{a} \cdot \vec{b} =$ Scalar
- Cross Product: $\vec{a} \times \vec{b} =$ Vector

2.2.1 Dot Product

The dot product takes two vectors as arguments and returns a scalar. It is defined in two waus:

- $\vec{a} \cdot \vec{b} = \|a\|\|b\| \cos(\theta)$
  Where $\|v\|$ is the length of a vector v $\left(\sqrt{v_x^2 + v_y^2 + v_z^2}\right)$, and θ is the angle between the two vectors.

- $\hat{a} \cdot \hat{b} = (a_x b_x + a_y b_y + a_z b_z)$

Properties Some basic properties of the dot product are as follows

- $\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1$

- $\hat{\imath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{\jmath} = \hat{k} \cdot \hat{\imath} = 0$

2.2.2 Cross Product

The cross product takes two vectors and produces a third that is perpindicular to both initial vectors, thus it must be performed in 3d space. This is the basis of the right hand rule, which will be expanded upon later:

- $\hat{\imath} \times \hat{\jmath} = \hat{k}$

- $\hat{\jmath} \times \hat{k} = \hat{\imath}$

- $\hat{k} \times \hat{\imath} = \hat{\jmath}$

In general, the cross product is defined as

$$\vec{a} \times \vec{b} = \|a\|\|b\| \sin\theta c$$

Where $\|v\|$ is the length of a vector $\vec{v}$ $\left(\sqrt{v_x^2 + v_y^2 + v_z^2}\right)$,$\theta$ is the angle between the two vectors, and c is a unit vector perpindicular to both a and b. This is not easily implemented though. Thus, the formula is also given by:

$$\vec{a} \times \vec{b} = \langle a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x \rangle$$

This isn't much better, but there is another way to determine the cross product:

$$\vec{a} \times \vec{b} = \begin{bmatrix} a_y & a_z \\ b_y & b_z \end{bmatrix} \hat{\imath} - \begin{bmatrix} a_x & a_z \\ b_x & b_z \end{bmatrix} \hat{\jmath} + \begin{bmatrix} a_x & a_y \\ b_x & b_y \end{bmatrix} \hat{k}$$

Where,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

Part II
Electrostatics and Magnetism

3 Field & Voltage Basics

3.1 Coulomb's Law

3.1.1 Force

The force that two charge particles exert on eacother is defined as:

$$\vec{F} = k\frac{q_1 q_2}{r^2}$$

where q_1 and q_2 are the charge values of each particle, r is the distance between them, and $k = 9e^9 \frac{Nm^2}{C^2} = \frac{1}{4\pi\epsilon_0}$ where ϵ_0 is the vaccuum permittivity$\left(\epsilon_0 = 8.854e^{-12} \frac{C^2}{Nm^2}\right)$

3.1.2 Field

The electric field generated by a point charge of Q is given by

$$\vec{E} = \frac{\vec{F}}{q} = \frac{kQ}{r^2} = \frac{Q}{4\pi\epsilon_0 r^2}$$

3.1.3 Properties of Field

- Electric field may be defined as change in electric potential

3.2 Electric Potential

Electric Potential is defined as the work required to bring a positive unit charge (q) from a reference point to a specific point within an electric field. The reference point is often infinity, where there is effectively no influence from the field on a charge. The electric potential created by a point charge Q at distance r is given by:

$$V = \frac{kQ}{r} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

For constant fields(e.g. parallel plates), the electric potential at distance d from the postive place in and electric field E is given by

$$V = Ed$$

3.2.1 Electric Potenial and Work

The change in electric potential is simply the work W_F done by an external force divided by the charge q of the particle that has moved. This can also be seen as a simple change in electrostatic potential energy U_e. Thus, it is also the opposite of the work W_E done by Coulomb force.

$$\Delta V = \frac{W_F}{q} = \frac{-W_E}{q} = \frac{\Delta U_e}{q}$$

3.2.2 Equipotential Surfaces

The collection of all point within an electric field that have equal potential is called an equipotential surface. The surfaces always form on a plane perpindicular to electric field lines, and no work is done in moving along an equipotential surface, only between them.

4 Field and Potential Using Calculus

4.1 Ring Charge

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4.1.1 Setting Up The Integral

Since

$$E = \frac{kQ}{r^2}$$

thus,

$$\mathrm{d}E = \frac{k\mathrm{d}q}{r^2}$$

Since the horizontal components of the field cancel out, the only remaining parts are the vertical components. Thus, the effective field integral is:

$$\mathrm{d}E = \frac{k\cos\theta\mathrm{d}q}{r^2}$$

Since $\cos\theta = \frac{Z}{r}$,

$$\mathrm{d}E = \frac{kZ\mathrm{d}q}{r^3}$$

We want E, not dE, so we apply the integral:

$$\int \mathrm{d}E = \int \frac{kZ\mathrm{d}q}{r^3}$$

4.1.2 Integrating

$$E = \int \frac{kZ\mathrm{d}q}{r^3} = \frac{kZQ}{r^3}$$

Since $r = \sqrt{Z^2 + R^2}$,

$$E = \frac{kZQ}{(Z^2 + R^2)^{\frac{3}{2}}}$$

4.2 Part of a Ring

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4.2.1 Setting Up the Integral

$$\int \mathrm{d}E = \int \mathrm{kdq}r^2$$

Assuming the linear charge density λ is uniform, $\lambda = \frac{Q}{s} = \frac{\mathrm{d}q}{\mathrm{d}s}$ where s is the arc lenghth of the segment of the ring. Thus, $\mathrm{d}q = \lambda \mathrm{d}s$ Therefore:

$$E = \int \frac{k\lambda \mathrm{d}s}{r^2}$$

Since the components not parallel with the line drawn from the center of the arc cancel eachother out, the actual field that is given by

$$E = \int \frac{k\lambda \cos\theta \mathrm{d}s}{r^2}$$

We then need to put the equation in terms of θ so that it can be integrated. Since $s = r\theta$, $\mathrm{d}s = r\mathrm{d}\theta$ Thus:

$$E = \int_a^b \frac{k\lambda \cos\theta \mathrm{d}\theta}{r}$$

Where a and b are the angle bounds of the arc relative to the normal

4.2.2 Integration

$$E = \frac{k\lambda}{r}\int_a^b \cos\theta \mathrm{d}\theta$$

$$E = \frac{k\lambda}{r}(\sin b - \sin a)$$

5 Gauss' Law

Gauss' law states that the total electrical flux ϕ through the a Gaussian surface A is equal to the enclosed charge q_{enc} over the vacuum permitivity ϵ_0. Since $\phi = E_\perp A$, This is given by:

$$\oint E_\perp \mathrm{d}A = \frac{q_{enc}}{\epsilon_0}$$

Here is a quick reference for symbols of charge density:

- λ: Linear Charge Density
- σ: Area Charge Density
- ρ: Volumetric Charge Density

5.1 Field within Insulating Spheres

5.1.1 Uniform Charge Density

If the insulating sphere with radius R and a uniform volume charge density ρ, then calculus is not needed to determine the charge enclosed by a Gaussian sphere of radius r

$$\rho = \frac{Q}{V} = \frac{\mathrm{d}q}{\mathrm{d}V}$$

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{q_{enc}}{\frac{4}{3}\pi r^3}$$

$$q_{enc} = \frac{Qr^3}{R^3}$$

5.1.2 Non-Uniform Charge Density

If the charge density ρ varies as a function of r, then to find the charge enclosed you must use calculus.

$$\rho(r) = \frac{\mathrm{d}q}{\mathrm{d}V}$$

$$\rho(r)\mathrm{d}V = \mathrm{d}q$$

Since $V_r = \frac{4}{3}\pi r^3$, $\mathrm{d}V = 4\pi r^2 \mathrm{d}r$,

$$\rho(r) 4\pi r^2 \mathrm{d}r = \mathrm{d}q$$

$$Q_R = \int_0^R \rho(r) 4\pi r^2 \mathrm{d}r$$

Where R is the radius inside the nonuniform charge density that you are evaluating.

5.1.3 Field Inside of the Sphere

Using Gauss' Law, and the equations for the charge enclosed at a certain radius, we can find the field at a given point. Recall that the charge enclosed q_{enc} in a gaussian surface is:

- $\frac{Qr^3}{R^3}$ for a uniform charge density
- $\int_0^r \rho(x) 4\pi r^2 \mathrm{d}x$ for a nonuniform charge density $\rho(x)$
- Q whenever $r_{Gauss} > R_{Sphere}$

Applying this to Gauss' Law, we see that:

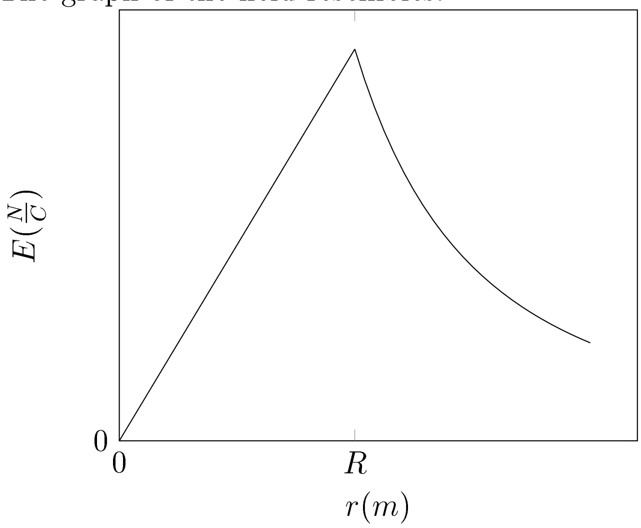
$$\oint E \mathrm{d}A = \frac{q_{enc}}{\epsilon_0}$$

Applying the dimensions of a sphere to this gives us:

$$E = \frac{q_{enc}}{4\pi r^2 \epsilon_0}$$

5.1.4 Field Graph

The graph of the field resembles:



5.2 Insulating Sphere with a Hole

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There are three distinct areas of this sphere:

- $r < a$
- $a < r < b$
- $b < r$

Where r is the radius of the Gaussian Sphere you are evaluating field at. The field must be evaluated separately at each area.

5.2.1 Within the Hole($r < a$)

Since there is no charge enclosed within the hole, there is no field withing the hole. Proof:

$$\oint E_{\perp} \mathrm{d}A = \frac{q_{enc}}{\epsilon_0} = 0$$

5.2.2 Between the Hole and the Edge of Sphere($a < r < b$)

Find the Charge Enclosed:

$$\rho = \frac{Q}{V} = \frac{q_{enc}}{V_{enc}}$$

$$V_{enc} = \frac{4}{3}\pi(r^3 - a^3)$$

$$\frac{Q}{\frac{4}{3}\pi(b^3 - r^3)} = \frac{q_{enc}}{\frac{4}{3}\pi(r^3 - a^3)}$$

$$\frac{Q(r^3 - a^3)}{b^3 - a^3} = q_{enc}$$

Apply Gauss' Law:

$$\epsilon_0 \oint E_{\perp} \mathrm{d}A = \frac{Q(r^3 - a^3)}{b^3 - a^3}$$

$$E 4\pi r^2 = \frac{Q(r^3 - a^3)}{\epsilon_0(b^3 - a^3)}$$

$$E = \frac{Q(r^3 - a^3)}{\epsilon_0 4\pi r^2(b^3 - a^3)}$$

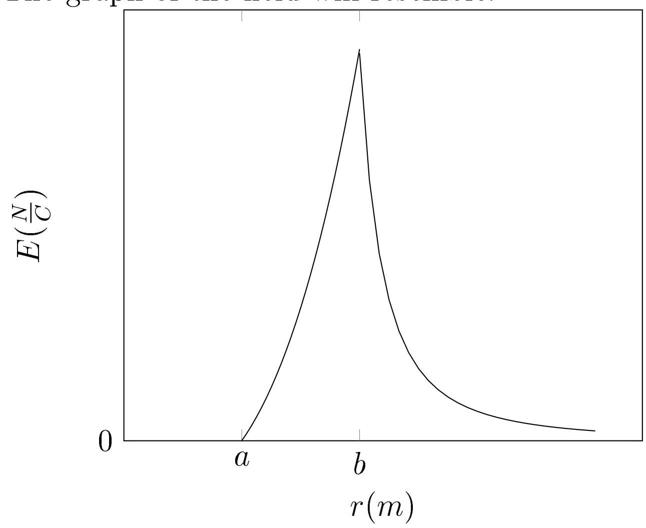
5.2.3 Outside of the Sphere($b < a$)

Since the only thing that matters in Gauss' Law is the enclosed charge, even if there is a hole in the center the sphere behaves like a solid sphere outside of the sphere.

$$E = \frac{Q}{\epsilon_0 4\pi r^2}$$

5.2.4 Field Graph

The graph of the field will resemble:



Where a is the radius of the hole, and b is the radius of the sphere

5.3 Field Within a Conducting Sphere

Since electrons repel eachother, if they are allowed to move, they will move to the furthest distance that they can get from eachother. For a conduction sphere, this is the outermost level of the sphere. Thus, a charged conducting sphere is for all intents and purposes, a shell of charge. Applying Gauss' law in this situation tells us that there is 0 field everywhere within the sphere, as there is no charge enclosed in any Gaussian surface made in the sphere. The field generated from the sphere beyond the sphere itself however, behaves the same way as that of an insulating sphere, as the charge enclosed is the same. There are a few things to note about conducting spheres:

- The charge will always stay totally on the outside shell of the sphere
- Even if there is a hole within the sphere, field will still be 0
- Field will compensate if there is charge inside of the conductor, say on the inner shell, so that there is no internal field no matter what.

5.4 Field at the Surface of a Conducting Surface

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Apply Gauss' law to situation:

$$\epsilon_0 \oint E_{\perp} \mathrm{d}A = q_{enc}$$

Since $\sigma = \frac{Q}{A} = \frac{\mathrm{d}q}{\mathrm{d}A}$,

$$\epsilon_0 E A_{Gauss} = \sigma A_{Gauss}$$

Thus:

$$E = \frac{\sigma}{\epsilon_0}$$

5.5 Field of a thin, Nonconducting sheet

INSERT PICTURE HERE

Apply Gauss' law to situation:

$$\epsilon_0 \oint E_{\perp} \mathrm{d}A = q_{enc}$$

Since $\sigma = \frac{Q}{A} = \frac{\mathrm{d}q}{\mathrm{d}A}$,

$$\epsilon_0 2 E A_{Gauss} = \sigma A_{Gauss}$$

Thus:

$$E = \frac{\sigma}{2\epsilon_0}$$

5.6 Field Between Insulating Plates

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5.7 Field of a Long Conducting Rod

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Applying Gauss' Law:

$$\epsilon_0 \oint E_{\perp} \mathrm{d}A = q_{enc}$$

Since the surface area of a cylinder excluding the two faces is given by $2\pi rh$, where r is the radius of the cylinder and h is its length:

$$\epsilon_0 E 2\pi rh = q_{enc}$$

Since $\lambda = \frac{Q}{h} = \frac{\mathrm{d}q}{h}$,

$$\epsilon_0 E 2\pi rh = \lambda h$$

$$E = \frac{\lambda}{\epsilon_0 2\pi r}$$

5.8 Electric Potential Using Gauss

$$\Delta V = -\int E\mathrm{d}x$$

Thus, the electric potential at a point around a charged sphere can be found using Gauss' law. For example, for a conducting sphere of charge Q, the electric potential can be found at a point r fron its center outside of the shelll like so:

$$\Delta V = -\int_{\infty}^{r} \frac{Q}{4\pi\epsilon_0 r^2}\mathrm{d}x$$

$$V_r - V_\infty = -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{r}$$

$$V_r = -\frac{Q}{4\pi\epsilon_0}\left(-\frac{1}{r} + \frac{1}{\infty}\right)$$

$$V_r = \frac{Q}{4\pi\epsilon 0 r}$$

Which is the same expression that was gained from Coulomb's law.

6 Circuits

6.1 Current

$$I = \frac{Q}{t} = \frac{\mathrm{d}q}{\mathrm{d}t}$$

Thus,

$$\int I\mathrm{d}t = Q$$

6.1.1 Current Density

Current density J is given by:

$$J = (ne)\,V_d$$

Where n is a material specific value to the material of the wire(e.g. the n of copper is $8.5 * 10^{28}\frac{electrons}{m^3}$), e is the charge of an electron, and V_d is the drift velocity of the particles. The drift direction relative to current density can by found by:

- Postitive charge: J and V_d in same direction
- Negative charge: J and V_d in opposite directions

6.1.2 Resistivity, Current Density, and Electric Field

Resistivity ρ is given by:

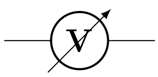
$$\rho = \frac{E}{J}$$

Where E is the electric field within the wire that is generated by the voltage across the wire

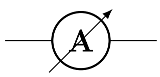
6.2 Meters

In order to measure the voltage and current in a circuit, voltmeters and ammeters respectively are used.

A voltmeter:

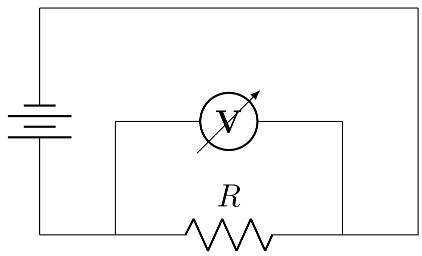


An ammeter:



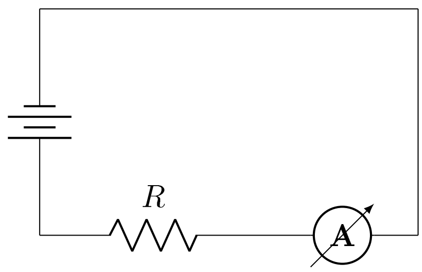
6.2.1 Voltmeter

To effectively measure the voltage across a circuit element, a voltmeter must have the most resistance possible(in an ideal world infinite resistance), and be placed in parallel with the element that is being measured, like so:



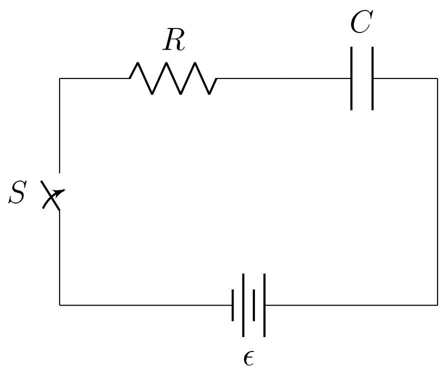
6.2.2 Ammeter

To effectively measure the current through a circuit element, an ammeter must be placed in serieswith the element, like so:



6.3 RC Circuits

RC Circuits are simply circuits that include both one or more resistors and one or more capacitors. The resistance can be generated by any power using element of the circuit(e.g a resistor, lightbulb, etc). They are for the most part examined over time and as the capacitor builds up charge. For example, here is a basic RC circuit:



There are a few important things to remember about RC circuits. In the following, let t be the amount of time that has passed since switch S has been closed.

- When $t = 0$(When the charge on the capacitor is 0) the capacitor acts as a wire
- When $t = \infty$ (When the charge is at it's max) the capacitor acts like a break in the circuit

Some of the effects of this are as follows:

- When $t = 0$, $q_{\text{Cap}} = 0$, $I = \max$, $V_{\text{res}} = \max$, and $V_{\text{cap}} = 0$
- When $t = \infty$, $q_{\text{Cap}} = \max$, $I = 0$, $V_{\text{res}} = 0$, and $V_{\text{cap}} = \max$
- As $t \to \infty$, $I \to 0$

6.3.1 Charge as a Function of Time

To find the amount of charge on a capacitor and the voltage across a capacitor at a time t after a circuit is closed, Kirchoff's law must be used:

$$V_0 = V_{elements} = \frac{Q}{C} + IR$$

Where V_0 is the voltage of the battery, C is the capacitance of the capacitor, Q is the current charge on the capacitor, and R is the resistance of the circuit. Since current is really just change in charge over time($\frac{\mathrm{d}Q}{\mathrm{d}t}$), the above can be rewritten as:

$$V_0 - \frac{Q}{C} - \frac{\mathrm{d}Q}{\mathrm{d}t}R$$

$$0 = \frac{\mathrm{d}Q}{\mathrm{d}t}R = V_0 - \frac{Q}{C}$$

$$\frac{\mathrm{d}Q}{\mathrm{d}t}R = -\frac{1}{RC}\left(Q - V_0C\right)$$

$$\frac{\mathrm{d}Q}{Q - V_0C} = -\frac{\mathrm{d}t}{RC}$$

Then integrate to get Q and t on their respective sides

$$\int_0^{Q_t} \frac{\mathrm{d}Q}{Q - V_0C} = -\frac{1}{RC}\int_0^t \mathrm{d}t$$

Perform the actual integration:

$$\ln\left(Q - V_0C\right)\Big]_0^{Q_t} = -\frac{t}{RC}$$

$$\ln\left(Q_t - V_0C\right) - \ln\left(-V_0\right) = -\frac{t}{RC}$$

$$\ln\left(\frac{Q_t - V_0C}{-V_0C}\right) = -\frac{t}{RC}$$

$$\frac{Q_t - V_0C}{-V_0C} = \mathrm{e}^{-\frac{t}{RC}}$$

$$Q_t - V_0C = -V_0C\mathrm{e}^{-\frac{t}{RC}}$$

Thus, the charge Q at time t aftwer current begins to flow to the capacitor of capacitance C through resistance R is given by:

$$Q_t = V_0C\left(1 - \mathrm{e}^{-\frac{t}{RC}}\right)$$

Both sides can be divided by C to give us the equation for voltage in respect for time

$$V = V_0\left(1 - \mathrm{e}^{-\frac{t}{RC}}\right)$$

6.3.2 Discharging Capacitor Time Function

When a capacitor becomes the source of emf, it discharges over time, losing its charge. To derive this, we use Kirchoff's again by have the V_0 be 0.

$$0 = IR + \frac{Q}{C}$$

$$-\frac{Q}{RC} = \frac{\mathrm{d}Q}{\mathrm{d}t}$$

$$-\frac{\mathrm{d}t}{RC} = \frac{\mathrm{d}Q}{Q}$$

$$-\frac{1}{RC}\int_0^t \mathrm{d}t = \int_{Q_{max}}^{Q_t} \frac{\mathrm{d}Q}{Q}$$

$$-\frac{t}{RC} = \ln\left(Q_t\right) - \ln\left(Q_{max}\right)$$

$$-\frac{t}{RC} = \ln\left(\frac{Q_t}{Q_{max}}\right)$$

$$e^{-\frac{t}{RC}} = \frac{Q_t}{Q_{max}}$$

$$Q_t = Q_{max}e^{-\frac{t}{RC}}$$

By again dividing both sided by capacitance, it is possible to get the voltage over the capacitor as a function of time:

$$V_t = V_0 e^{-\frac{t}{RC}}$$

7 Capacitors

The capacitance of a capacitor is given by:

$$C = \frac{Q}{V}$$

The unit for capacitance is the Farad(F).
The energy that a capacitor is storing can be given by 3 equations:

- $U = \frac{q^2}{2C}$
- $U = \frac{1}{2}QV$
- $U = \frac{CV^2}{2}$

7.1 Finding the Capacitance of Different Capacitors

The capacitance of a capcitor relies only on the geometry of the capacitor. The first step to finding the capacitance of a capacitor is to find the electric potential difference across the plates using Gauss, and then simply divide the charge on one plate by that voltage difference, which cancels out all non-geometric variables.

7.1.1 Parallel Plate Capacitor

First find the charge on one plate

$$E = \frac{\sigma}{\epsilon_0}$$

$$E = \frac{Q}{\epsilon_0 A}$$

$$Q = E\epsilon_0 A$$

Then find the potential difference

$$V = Ed$$

Then apply those two to the capacitance equation

$$C = \frac{Q}{V} = \frac{E\epsilon_0 A}{Ed} = \frac{\epsilon_0 A}{d}$$

7.1.2 Cylindrical Capacitor

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Find the potential difference across the plates:

$$V = -\int_a^b E\mathrm{d}r$$

Since for a cylinder the field between shells $E = \frac{q_{enc}}{2\pi r h\epsilon_0}$,

$$V = \frac{q_{enc}}{2\pi h\epsilon_0}\int_a^b \frac{\mathrm{d}r}{r}$$

$$V = \frac{q_{enc}}{2\pi h\epsilon_0}\left[\ln(r)\right]_a^b$$

$$V = \frac{q_{enc}\ln(\frac{a}{b})}{2\pi h\epsilon_0}$$

Apply this to the capacitance equation:

$$C = \frac{q_{enc}}{\frac{q_{enc}\ln(\frac{a}{b})}{2\pi h\epsilon_0}}$$

Simplify:

$$C = k\frac{2\pi h\epsilon_0}{\ln(\frac{a}{b})}$$

Where k is the dielectric constant of the material between the plates

7.1.3 Spherical Capacitor

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Find the potential difference across the plates:

$$V = -\int_a^b E \mathrm{d}r$$

Since the field between two charge spherical shells if given by $E = \frac{q_{enc}}{4\pi r^2 \epsilon_0}$,

$$V = \frac{q_{enc}}{4\pi\epsilon_0}\int_a^b \frac{\mathrm{d}r}{r^2}$$

$$V = \frac{q_{enc}}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_a^b$$

$$V = \frac{q_{enc}}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

Apply the capacitance equation

$$C = \frac{q_{enc}}{\frac{q_{enc}}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)}$$

Simplify

$$C = \frac{4\pi\epsilon_0}{\frac{1}{a}-\frac{1}{b}}$$

Part III
Mechanics

8 Kinematics Review

Some quick identities

- $v = \frac{\mathrm{d}x}{\mathrm{d}t}$
- $a = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \frac{\mathrm{d}v}{\mathrm{d}t}$