## *MAT_IFPD

This is Material Type 319. It is for modeling fluid particles for incompressible free surface flow with incompressible SPG. It was developed to predict the shape evolution of solder joints during the electronic reflow process. See Pan et al 2020 for details.

**WARNING:** The *MAT_319 keyword name cannot be used in the input deck in R13. For R13, you must use *MAT_IFPD as the keyword name. For releases after September 2021, *MAT_319 can be used in the input deck.

**NOTE:** This material only works for ISPG element formulations set on *SECTION_FPD.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | DYNVIS | SFTEN | | | | |
| Type | A | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Fluid density |
| DYNVIS | Dynamic viscosity of the fluid |
| SFTEN | Surface tension coefficient of the fluid |

**References:**

Pan, X., Wu, C.T., and Hu, W. “Incompressible Smoothed Particle Galerkin (ISPG) Method for an Efficient Simulation of Surface Tension and Wall Adhesion Effects in the 3D Reflow Soldering Process,” *16th International LS-DYNA Users Conference* (2020).

## *MAT_COHESIVE_GASKET

This is Material Type 326 developed for analysis of gaskets. This material model can only be used with cohesive elements. Also, a gasket thickness must be set; see the variable ELFORM and GASKETT in *SECTION_SOLID.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | ROFLG | INTFAIL | | | | |
| Type | A | F | F | I | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LC | UC | ETEN | | | | | |
| Type | F | F | F | | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ETSR | | | | | | | |
| Type | F | | | | | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EMEM | PR | PS | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label be specified (see *PART). |
| RO | Mass density |

| VARIABLE | DESCRIPTION |
|---|---|
| ROFLG | Flag for whether density is specified per unit area or volume:<br>EQ.0: Density is per unit volume (default).<br>EQ.1: Density is per unit area for controlling the mass of cohesive elements with an initial volume of zero |
| INTFAIL | Quadrature rule. Note that this material has no failure<br>LE.0: 2 × 2 Newton-Cotes quadrature<br>GT.0: 2 × 2 Gaussian quadrature |
| LC | Main load curve ID defining the pressure as function of closure, $p = p(c)$ |
| UC | Table ID defining the unloading curves |
| ETEN | Tensile stiffness |
| ETSR | Transverse shear stiffness |
| EMEM | Membrane stiffness |
| PR | Membrane Poisson ratio |
| PS | Membrane plane stress or plain strain assumption:<br>EQ.0: Plane stress (default)<br>EQ.1: Plane strain |

**Remarks:**

1. **Cohesive Elements for Modeling Gaskets.** A gasket is a mechanical seal placed between two mating surfaces to prevent leakage. A gasket is typically thin in comparison to the length and width of its surface. This makes it cumbersome to model with solid elements, since these require good aspect ratios. Cohesive elements, however, are less sensitive to this kind of geometric quality and are, therefore, better suited for modeling gaskets. To use cohesive elements for modeling gaskets, the normal (the local 3-direction) of the cohesive element must be aligned with the gasket thickness direction, and the mid-surface of the cohesive element (the local 1-, and 2-direction) must coincide with the gasket surface.

2. **Material Model.** The strains pertaining to the normal are defined by: $\varepsilon_{13} = \delta_1$, $\varepsilon_{23} = \delta_2$, and $\varepsilon_{33} = \delta_3$. $\delta_i$ is the separation in local direction $i = 1,2,3$, meaning

the relative displacement between the top and bottom face of the cohesive element measured along local direction $i$. In particular, $-\varepsilon_{33}$ is the so-called gasket closure, $c$. The so-called membrane strains, $\varepsilon_{11}$, $\varepsilon_{12}$, and $\varepsilon_{22}$, in the plane orthogonal to the normal follow the usual definition $\varepsilon_{ij} = (\partial_j u_i + \partial_i u_j)/2$, where $u_i$, $i = 1,2$, is the local displacement of the mid-surface.

The cohesive gasket material model is comprised of the following three uncoupled material models:

a) Isotropic linear elastic membrane stress:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = D \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix}$$

with $D$ as the constitutive matrix for either plane strain or stress.

b) Isotropic linear elastic transverse stress:

$$\sigma_{i3} = E_{TSR}\delta_i, \qquad i = 1,2$$

c) A user defined pressure-closure relation of the form (see Remark 3):

$$\begin{aligned} -\sigma_{33} \equiv p = f(c), &\qquad c > 0 \\ \sigma_{33} = 0, &\qquad \text{otherwise} \end{aligned}$$

3. **Pressure-Closure Relation.** The pressure-closure relation, $f$, is an important feature of the material model. It can be used to include mechanical effects that are typical for gaskets, such as hysteresis. It consists of a main loading curve and one or more unloading curves.

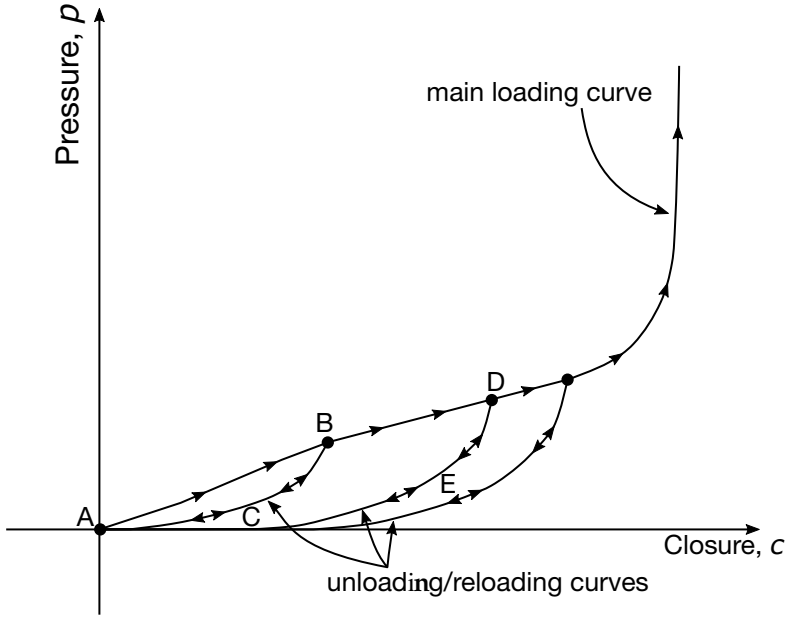


**Figure M326-1.** Schematic pressure-closure response

Figure M326-1 gives a schematic of a pressure-closure response. As the gasket is compressed the closure, $c$, increases and the pressure, $p$, follows the main loading curve from A to, say, B. Now, if the gasket for some reason is unloaded at B, $c$ will then decrease and $p$ follows the unloading curve BCA back to the initial configuration A. If the gasket is then reloaded it will follow the path ACB back to B. Then, when $c$ exceeds the closure value at point B, it will continue on the main loading curve until some new point D, where unloading takes place. The new unloading then follows the path DEA back to A. If unloading occurs between two unloading curves, interpolation is used to determine the pressure.

Which unloading path to follow and where to switch to the main loading curve is determined by keeping track of the maximum value of $c$, called $c_{\max}$, at the last unloading point. If the closure becomes negative, that is, the gasket is subject to tension rather than compression, its stiffness is given by $E_{TEN}$. This is mostly for numerical stability. The unloading curves must be input using *DEFINE_TABLE_2D, with the first dependency being maximum closure and the second closure, meaning $p = p(c_{max}, c)$. Also, the unloading curves should be in a normalized form giving zero pressure for zero closure and unit pressure for unit closure.

4. **History Variables.** This material model outputs the maximum closure as history variable #1 to the post-processing database. Therefore, NEIPH and NEIPS must be set in *DATABASE_EXTENT_BINARY.

## *MAT_ALE_VACUUM

See *MAT_VACUUM or *MAT_140.

***MAT_ALE_GAS_MIXTURE**_{OPTION}

Available settings of *OPTION* are:

<BLANK>

ADV

With *OPTION1* set to <BLANK>, this keyword is exactly the same as *MAT_GAS_MIXTURE or *MAT_148. See that manual page for the input format,

The keyword format described below is only for *OPTION1* set to ADV. It is exclusively used as a component of *AIRBAG_SALE. It was created to support more than 8 gas species.

**Data Cards for the ADV Keyword Option Only:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | IADIAB | RUNIV | NSPS | | | | |
| Type | A | I | F | I | | | | |
| Default | none | 0 | 0.0 | none | | | | |

Include NSPS instantiations of this card, one for each gas species.

| Cards 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MOLWT | CPMOL | B | C | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified. |
| IADIAB | Flag to turn on/off adiabatic compression logic for an ideal gas. See Remark 5 of *MAT_148.<br>EQ.0: Off (default) |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1: On |
| RUNIV | Universal gas constant in per-mole unit (8.31447 J/(mole × K)). |
| NSPS | Number of gas species |
| MOLWT | Molecular weight of each ideal gas in the mixture (mass-unit/mole). |
| CPMOL | Heat capacity at constant pressure in per-mole unit. These are nominal heat capacity values typically at STP. These are denoted by the variable *A* in the equation in Remark 2 of *MAT_148. |
| B | First-order coefficient for a temperature-dependent heat capacity at constant pressure. These are denoted by the variable *B* in the equation in Remark 2 of *MAT_148. |
| C | Second-order coefficient for a temperature-dependent heat capacity at constant pressure. These are denoted by the variable *C* in the equation in Remark 2 of *MAT_148. |

**Remarks:**

1. **Element energy update.** For the ADV keyword option, pressure work always gives the element energy update. Without the keyword option, the ideal gas gamma law, by default, gives the element energy update. However, this method can be changed to pressure work by setting PDV to 1. Please refer to Remark 6 of *MAT_GAS_MIXTURE for explanations.

## *MAT_ALE_VISCOUS

This may also be referred to as MAT_ALE_03. This "fluid-like" material model is very similar to Material Type 9 (*MAT_NULL). It allows the modeling of non-Newtonian fluids with constant or variable viscosity. The variable viscosity is a function of an equivalent deviatoric strain rate. If inviscid material is modeled, the deviatoric or viscous stresses are zero, and the equation of state supplies the pressures (or diagonal components of the stress tensor). All *MAT_ALE_ cards apply only to ALE elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PC | MULO | MUHI | RK | | RN |
| Type | A | F | F | F | F | F | | F |
| Defaults | none | none | 0.0 | 0.0 | 0.0 | 0.0 | | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| PC | Pressure cutoff (≤ 0.0). See Remark 4. |
| MULO | Dynamic viscosity (see Remark 1):<br>EQ.0.0: Inviscid fluid is assumed.<br>GT.0.0: If MUHI = 0.0 or is not defined, then this is the traditional constant dynamic viscosity coefficient, $\mu$. Otherwise if MUHI > 0.0, then MULO and MUHI are the lower and upper dynamic viscosity limit values for a power-law-like variable viscosity model.<br>LT.0.0: -MULO is a load curve ID defining dynamic viscosity as a function of equivalent strain rate. |
| MUHI | Dynamic viscosity:<br>EQ.0.0: Only MULO is used to define the dynamic viscosity, default<br>LT.0.0: The viscosity can be defined by the user in the file dyn21.F with a routine called f3dm9ale_userdef1. The |

| VARIABLE | DESCRIPTION |
|---|---|
| | file is part of the general usermat package. Note that in this case MULO is a parameter for the subroutine. |
| | GT.0.0: This is the upper dynamic viscosity limit. This is defined only if RK and RN are defined for the variable viscosity case. |
| RK | Variable dynamic viscosity multiplier. See Remark 6. |
| RN | Variable dynamic viscosity exponent. See Remark 6. |

**Remarks:**

1. **Deviatoric viscous stress.** The null material must be used with an equation-of-state. Pressure cutoff is negative in tension. A (deviatoric) viscous stress of the form

$$\sigma'_{ij} = 2\mu\dot{\varepsilon}'_{ij}$$

$$\left[\frac{N}{m^2}\right] \sim \left[\frac{N}{m^2}s\right]\left[\frac{1}{s}\right]$$

is computed for nonzero $\mu$ where $\dot{\varepsilon}'_{ij}$ is the deviatoric strain rate. $\mu$ is the dynamic viscosity. For example, in SI unit system, $\mu$ has a unit of [Pa × s].

2. **Hourglass control issues.** The null material has no shear stiffness and hourglass control must be used with care. In some applications, the default hourglass coefficient might lead to significant energy losses. In general for fluid(s), the hourglass coefficient QM should be small (in the range $10^{-4}$ to $10^{-6}$ for the standard default IHQ choice).

3. **Null material properties.** Null material has no yield strength and behaves in a fluid-like manner.

4. **Numerical cavitation.** The pressure cut-off, PC, must be defined to allow for a material to "numerically" cavitate. In other words, when a material undergoes dilatation above certain magnitude, it should no longer be able to resist this dilatation. Since dilatation stress or pressure is negative, setting PC limit to a very small negative number would allow for the material to cavitate once the pressure in the material goes below this negative value.

5. **Issues with small values of viscosity exponent.** If the viscosity exponent is less than 1.0 (RN $< 1.0$), then RN $- 1.0 < 0.0$. In this case, at very low equivalent strain rate, the viscosity can be artificially very high. MULO is then used as the viscosity value.

6. **Empirical dynamic viscosity.** The empirical variable dynamic viscosity is typically modeled as a function of *equivalent shear rate* based on experimental data.

$$\mu(\dot{\bar{\gamma}}') = \mathrm{RK} \times \dot{\bar{\gamma}}'^{(\mathrm{RN}-1)}$$

For an incompressible fluid, this may be written equivalently as

$$\mu(\dot{\bar{\varepsilon}}') = \mathrm{RK} \times \dot{\bar{\varepsilon}}'^{(\mathrm{RN}-1)}$$

The "overbar" denotes a scalar equivalence. The "dot" denotes a time derivative or rate effect. And the "prime" symbol denotes deviatoric or volume preserving components. The *equivalent shear rate* components may be related to the basic definition of (small-strain) strain rate components as follows:

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \Rightarrow \dot{\varepsilon}'_{ij} = \dot{\varepsilon}_{ij} - \delta_{ij}\left(\frac{\dot{\varepsilon}_{kk}}{3}\right)$$
$$\dot{\gamma}_{ij} = 2\dot{\varepsilon}_{ij}$$

Typically, the 2nd invariant of the deviatoric strain rate tensor is defined as:

$$I_{2\dot{\varepsilon}'} = \frac{1}{2}\left[\dot{\varepsilon}'_{ij}\dot{\varepsilon}'_{ij}\right]$$

The equivalent (small-strain) deviatoric strain rate is defined as:

$$\dot{\bar{\varepsilon}}' \equiv 2\sqrt{I_{2\dot{\varepsilon}'}} = \sqrt{2\left[\dot{\varepsilon}'_{ij}\dot{\varepsilon}'_{ij}\right]} = \sqrt{4\left[\dot{\varepsilon}'_{12}{}^2 + \dot{\varepsilon}'_{23}{}^2 + \dot{\varepsilon}'_{31}{}^2\right] + 2\left[\dot{\varepsilon}'_{11}{}^2 + \dot{\varepsilon}'_{22}{}^2 + \dot{\varepsilon}'_{33}{}^2\right]}$$

In non-Newtonian literatures, the *equivalent shear rate* is sometimes defined as

$$\dot{\bar{\gamma}} \equiv \sqrt{\frac{\dot{\gamma}_{ij}\dot{\gamma}_{ij}}{2}} = \sqrt{2\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}} = \sqrt{4[\dot{\varepsilon}_{12}^2 + \dot{\varepsilon}_{23}^2 + \dot{\varepsilon}_{31}^2] + 2[\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{22}^2 + \dot{\varepsilon}_{33}^2]}$$

It turns out that, (a) for incompressible materials ($\dot{\varepsilon}_{kk} = 0$), and (b) the shear terms are equivalent when $i \neq j \rightarrow \dot{\varepsilon}_{ij} = \dot{\varepsilon}'_{ij}$, the *equivalent shear rate* is algebraically equivalent to the *equivalent (small-strain) deviatoric strain rate*.

$$\dot{\bar{\varepsilon}}' = \dot{\bar{\gamma}}'$$

## *MAT_ALE_MIXING_LENGTH

This may also be referred to as *MAT_ALE_04. This viscous "fluid-like" material model is an advanced form of *MAT_ALE_VISCOUS. It allows the modeling of fluid with constant or variable viscosity and a *one-parameter mixing-length turbulence model*. The variable viscosity is a function of an equivalent deviatoric strain rate. The equation of state supplies the pressures for the stress tensor. All *MAT_ALE_cards apply only to ALE elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PC | MULO | MUHI | RK | | RN |
| Type | A8 | F | F | F | F | F | | F |
| Defaults | none | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | 0.0 |

**Internal Flow Card.**

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCI | C0 | C1 | C2 | C3 | C4 | C5 | C6 |
| Type | F | F | F | F | F | F | F | F |
| Defaults | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

**External Flow Card.**

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCX | D0 | D1 | D2 | E0 | E1 | E2 | |
| Type | F | F | F | F | F | F | F | |
| Defaults | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| PC | Pressure cutoff (≤ 0.0) |
| MULO | Dynamic viscosity:<br>GE.0.0: if MUHI = 0.0 or is not defined, then this is the traditional constant dynamic viscosity coefficient, $\mu$. Otherwise if MUHI > 0.0, then MULO and MUHI are the lower and upper dynamic viscosity limit values.<br>LT.0.0: -MULO is a load curve ID defining dynamic viscosity as a function of equivalent strain rate. |
| MUHI | Upper dynamic viscosity limit (default = 0.0) if MULO > 0. This is defined only if RK and RN are defined for the variable viscosity case. |
| RK | Variable dynamic viscosity multiplier (see Remark 6 of MAT_ALE_VISCOUS). The viscosity is computed as $\mu(\overline{\dot{\varepsilon}'}) = \text{RK} \times \overline{\dot{\varepsilon}'}^{(\text{RN}-1)}$, where the equivalent deviatoric strain rate is $\overline{\dot{\varepsilon}'} = \sqrt{\frac{2}{3}\left[\dot{\varepsilon}'_{11}{}^2 + \dot{\varepsilon}'_{22}{}^2 + \dot{\varepsilon}'_{33}{}^2 + 2\left(\dot{\varepsilon}'_{12}{}^2 + \dot{\varepsilon}'_{23}{}^2 + \dot{\varepsilon}'_{31}{}^2\right)\right]}$ . |
| RN | Variable dynamic viscosity exponent (see RK) |
| LCI | Characteristic length, $l_{ci}$, of the internal turbulent domain |
| C0 - C6 | Internal flow mixing length polynomial coefficients. The one-parameter turbulent mixing length is computed as $l_m = l_{ci}\left[C_0 + C_1\left(1 - \frac{y}{l_{ci}}\right) + \cdots + C_6\left(1 - \frac{y}{l_{ci}}\right)^6\right]$ |
| LCX | Characteristic length, $l_{cx}$, of the external turbulent domain |
| D0 - D2 | External flow mixing length polynomial coefficients. If $y \le l_{cx}$, then the mixing length is computed as $l_m = \left[D_0 + D_1 y + D_2 y^2\right]$. |
| E0 - E2 | External flow mixing length polynomial coefficients. If $y > l_{cx}$, then the mixing length is computed as $l_m = \left[E_0 + E_1 y + E_2 y^2\right]$. |

**Remarks:**

1. **Deviatoric Viscous Stress.** The null material must be used with an equation of-state. Pressure cutoff is negative in tension. A (deviatoric) viscous stress of the form

$$\sigma'_{ij} = \mu\dot{\varepsilon}'_{ij}$$
$$\left[\frac{N}{m^2}\right] \approx \left[\frac{N}{m^2}s\right]\left[\frac{1}{s}\right]$$

is computed for nonzero $\mu$ where $\dot{\varepsilon}'_{ij}$ is the deviatoric strain rate. $\mu$ is the dynamic viscosity with unit of [Pa × s].

2. **Hourglass Control Issues.** The null material has no shear stiffness and hourglass control must be used with care. In some applications, the default hourglass coefficient might lead to significant energy losses. In general for fluid(s), the hourglass coefficient QM should be small (in the range $10^{-4}$ to $10^{-6}$ for the standard default IHQ choice).

3. **Null Material Properties.** The null material has no yield strength and behaves in a fluid-like manner.

4. **Numerical Cavitation.** The pressure cut-off, PC, must be defined to allow for a material to "numerically" cavitate. In other words, when a material undergoes dilatation above certain magnitude, it should no longer be able to resist this dilatation. Since dilatation stress or pressure is negative, setting PC limit to a very small negative number would allow for the material to cavitate once the pressure in the material goes below this negative value.

5. **Issues with Small Value of Viscosity Exponent.** If the viscosity exponent is less than 1.0 (RN $< 1.0$) then RN $- 1.0 < 0.0$. In this case, at very low equivalent strain rate, the viscosity can be artificially very high. MULO is then used as the viscosity value.

6. **Turbulent Viscosity.** Turbulence is treated simply by considering its effects on viscosity. Total effective viscosity is the sum of the laminar and turbulent viscosities, $\mu_{\text{eff}} = \mu_l + \mu_t$ where $\mu_{\text{eff}}$ is the effective viscosity, and $\mu_t$ is the turbulent viscosity. The turbulent viscosity is computed based on the Prandtl's Mixing Length Model,

$$\mu_t = \rho l_m^2 |\nabla \mathbf{v}| \ .$$

## *MAT_ALE_INCOMPRESSIBLE

See *MAT_160.

## *MAT_ALE_HERSCHEL

This may also be referred to as MAT_ALE_06. This is the Herschel-Buckley model. It is an enhancement to the power law viscosity model in *MAT_ALE_VISCOUS (*MAT_-ALE_03). Two additional input parameters, the yield stress threshold and critical shear strain rate, can be specified to model "rigid-like" material for low strain rates.

It allows the modeling of non-viscous fluids with constant or variable viscosity. The variable viscosity is a function of an equivalent deviatoric strain rate. All *MAT_ALE_ cards apply only to ALE element formulation.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PC | MULO | MUHI | RK | | RN |
| Type | A | F | F | F | F | F | | F |
| Defaults | none | none | 0.0 | 0.0 | 0.0 | 0.0 | | 0.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GDOTC | TAO0 | | | | | | |
| Type | F | F | | | | | | |
| Default | none | none | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| PC | Pressure cutoff (≤ 0.0); see Remark 4. |
| MULO | There are 4 possible cases (see Remark 1):<br>1. If MULO = 0.0, then an inviscid fluid is assumed.<br>2. If MULO > 0.0, and MUHI = 0.0 or is not defined, then this is the traditional constant dynamic viscosity coefficient $\mu$. |

| VARIABLE | DESCRIPTION |
|---|---|
| | 3. If MULO > 0.0, and MUHI > 0.0, then MULO and MUHI are lower and upper viscosity limit values for a power-law-like variable viscosity model.<br>4. If MULO is negative (for example, MULO = -1), then a user-input data load curve (with LCID = 1) defining dynamic viscosity as a function of equivalent strain rate is used. |
| MUHI | Upper dynamic viscosity limit (default = 0.0). This is defined only if RK and RN are defined for the variable viscosity case. |
| RK | $k$, consistency factor (see Remark 6) |
| RN | $n$, power law index (see Remark 6) |
| GDOTC | $\dot{\gamma}_c$, critical shear strain rate (see Remark 6) |
| TAO0 | $\tau_0$, yield stress (see Remark 6) |

**Remarks:**

1. **Viscous stress.** The null material must be used with an equation-of-state. Pressure cutoff is negative in tension. A (deviatoric) viscous stress of the form

$$\sigma'_{ij} = 2\mu\dot{\varepsilon}'_{ij}$$

$$\left[\frac{N}{m^2}\right] \sim \left[\frac{N}{m^2}s\right]\left[\frac{1}{s}\right]$$

is computed for nonzero $\mu$ where $\dot{\varepsilon}'_{ij}$ is the deviatoric strain rate. $\mu$ is the dynamic viscosity. For example, in SI unit system, $\mu$ has a unit of [Pa-s].

2. **Hourglass control.** The null material has no shear stiffness and hourglass control must be used with care. In some applications, the default hourglass coefficient might lead to significant energy losses. In general, for fluid(s), the hourglass coefficient QM should be small (in the range of $10^{-4}$ to $10^{-6}$ for the standard default IHQ choice).

3. **Yield strength.** Null material has no yield strength and behaves in a fluid-like manner.

4. **Pressure cut-off.** The pressure cut-off, PC, must be defined to allow for a material to "numerically" cavitate. In other words, when a material undergoes dilatation above a certain magnitude, it should no longer be able to resist this

dilatation. Since dilatation stress or pressure is negative, setting PC limit to a very small negative number would allow for the material to cavitate once the pressure in the material goes below this negative value.

5. **Viscosity.** If the viscosity exponent is less than 1.0, $RN < 1.0$, then $RN - 1.0 < 0.0$. In this case, at very low equivalent strain rate, the viscosity can be artificially very high. MULO is then used as the viscosity value.

6. **Herschel-buckley model.** The Herschel-Buckley model employs a large viscosity to model the "rigid-like" behavior for low shear strain rates ($\dot{\gamma} < \dot{\gamma}_c$).

$$\mu(\dot{\gamma}) = \tau_0 \frac{(2-\dot{\gamma}/\dot{\gamma}\_c)}{\dot{\gamma}_c} + k[(2-n) + (n-1)\frac{\dot{\gamma}}{\dot{\gamma}_c}]$$

A power law is used once the yield stress is passed.

$$\mu(\dot{\gamma}) = \frac{\tau_0}{\dot{\gamma}} + k(\frac{\dot{\gamma}}{\dot{\gamma}_c})^{n-1}$$

The shear strain rate is:

$$\dot{\gamma} \equiv \sqrt{\frac{\dot{\gamma}_{ij}\dot{\gamma}_{ij}}{2}} = \sqrt{2\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}} = \sqrt{4[\dot{\varepsilon}_{12}^2 + \dot{\varepsilon}_{23}^2 + \dot{\varepsilon}_{31}^2] + 2[\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{22}^2 + \dot{\varepsilon}_{33}^2]}$$

### *MAT_ISPG_CARREAU

This is Material Type 1 for ISPG. The Carreau model attempts to describe a wide range of fluids by establishing a curve-fit to piece together functions for both Newtonian and shear-thinning ($n < 1$) non-Newtonian laws.

**NOTE:** This material only works for ISPG element formulations set on *SECTION_ISPG. It may only be in the ISPG input deck included in the LS-DYNA input deck with *INCLUDE_ISPG.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | VISC0 | SFTEN | VISC_LIM | LAMBDA | N | |
| Type | A | F | F | F | F | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | TREF | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Fluid density |
| VISC0 | Zero-shear viscosity of the fluid, $\eta_0$ |
| SFTEN | Surface tension coefficient of the fluid |
| VISC_LIM | Infinite-shear viscosity, $\eta_\infty$ |
| LAMBDA | Time constant, $\lambda$ |
| N | Power-law index, $n$ |
| ALPHA | Ratio of the activation energy to thermodynamic constant |

| VARIABLE | DESCRIPTION |
|---|---|
| TREF | Reference temperature in Kelvin, $T_\alpha$. The defaults is 273.15 K. |

**Remarks:**

1. **Viscosity.** In the Carreau model, the viscosity is described as:

$$\eta = \eta_\infty + (\eta_0 - \eta_\infty)(1 + \dot{\gamma}^2\lambda^2)^{(n-1)/2}$$

where $\dot{\gamma} = \sqrt{\frac{1}{2}\mathbf{D}:\mathbf{D}}$ is the second invariant of the rate-of-deformation tensor $\mathbf{D} = \frac{1}{2}\left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial j}\right)\mathbf{e}_i \otimes \mathbf{e}_j$, $\lambda$ is a time constant, $n$ is the power-law index, and $\eta_0$ and $\eta_\infty$ are the zero- and infinite-shear viscosities, respectively. The total viscosity is calculated as

$$\mu = H(T)\eta(\dot{\gamma}) \ ,$$

where $H(T)$ is the temperature dependence. It is described by an Arrhenius law as:

$$H(T) = \exp\left[\alpha\left(\frac{1}{T - T_0} - \frac{1}{T_\alpha - T_0}\right)\right] \ .$$

where $\alpha$ is the ratio of the activation energy to the thermodynamic constant. $T_\alpha$ is a reference temperature in Kelvin with a default value of 273.15 K. $T_0$ is the temperature shift in Kelvin. It is hard-coded as 0 K. If the parameter $\alpha$ is set to 0, the temperature dependence will be ignored because $H(T) = 1$.

2. **Enabling temperature dependence.** To consider temperature dependence, use either *ISPG_CONTROL_SOLUTION or *LOAD_THERMAL_LOAD_CURVE. To obtain the temperature by solving the energy governing equations, enable a combined flow and thermal analysis with *ISPG_CONTROL_SOLUTION. Temperatures defined with *LOAD_THERMAL_LOAD_CURVE are applied only during a flow-only analysis *LOAD_THERMAL_LOAD_CURVE is ignored during a combined flow and thermal analysis.

### *MAT_ISPG_CROSSMODEL

This is Material Type 2 for ISPG. The Cross model attempts to describe the shear-rate dependence across the Newtonian region and the shear-thinning region.

> **NOTE:** This material only works for ISPG element formulations set on *SECTION_ISPG. It may only be in the ISPG input deck included in the LS-DYNA input deck with *INCLUDE_ISPG.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | VISC0 | SFTEN | | LAMBDA | n | |
| Type | A | F | F | F | | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | TREF | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Fluid density |
| VISC0 | Zero-shear viscosity of the fluid, $\eta_0$ |
| SFTEN | Surface tension coefficient of the fluid |
| LAMBDA | Natural time. $\lambda$ |
| N | Power-law index, $n$ |
| ALPHA | Ratio of the activation energy to thermodynamic constant, $\alpha$ |
| TREF | Reference temperature in Kelvin, $T_\alpha$. The default value is 273.15 K. |

**Remarks:**

1. **Viscosity.** The Cross model describes the viscosity as:

$$\eta = \frac{\eta_0}{1 + (\lambda\dot{\gamma})^{1-n}}$$

where $\dot{\gamma} = \sqrt{\frac{1}{2}\mathbf{D}:\mathbf{D}}$ is the second invariant of the rate-of-deformation tensor $\mathbf{D} = \frac{1}{2}\left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial j}\right)\mathbf{e}_i \otimes \mathbf{e}_j$ , $\lambda$ is a time constant, $n$ is the power-law index, and $\eta_0$ is the zero-shear-rate viscosity. The total viscosity is calculated as

$$\mu = H(T)\eta(\dot{\gamma})$$

where $H(T)$ is the temperature dependence. It is described by an Arrhenius law as:

$$H(T) = \exp\left[\alpha\left(\frac{1}{T - T_0} - \frac{1}{T_\alpha - T_0}\right)\right]$$

where $\alpha$ is the ratio of the activation energy to the thermodynamic constant. $T_\alpha$ is a reference temperature in Kelvin with a default value of 273.15 K. $T_0$ is the temperature shift in Kelvin. It is hardcoded as 0 K. If the parameter $\alpha$ is set to 0, the temperature dependence will be ignored because $H(T) = 1$.

2. **Enabling temperature dependence.** To consider temperature dependence, use either *ISPG_CONTROL_SOLUTION or *LOAD_THERMAL_LOAD_CURVE. To obtain the temperature by solving the energy governing equations, enable a combined flow and thermal analysis with *ISPG_CONTROL_SOLUTION. Temperatures defined with *LOAD_THERMAL_LOAD_CURVE are applied only during a flow-only analysis *LOAD_THERMAL_LOAD_CURVE is ignored during a combined flow and thermal analysis.

### *MAT_ISPG_ISO_NEWTONIAN

This is Material Type 3 for ISPG. This material type models the Newtonian flow behavior of an incompressible free surface flow. We developed it to predict the shape evolution of solder joints during the electronic reflow process. See Pan et al. 2020 for details.

> **NOTE:** This material only works for ISPG element formulations set on *SECTION_ISPG. It may only be in the ISPG input deck included in the LS-DYNA input deck with *INCLUDE_ISPG.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | DYNVIS | SFTEN | | | | |
| Type | A | F | F | F | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | TREF | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Fluid density |
| DYNVIS | Dynamic viscosity of the fluid, $\eta$ |
| SFTEN | Surface tension coefficient of the fluid |
| ALPHA | Ratio of the activation energy to thermodynamic constant, $\alpha$ |
| TREF | Reference temperature in Kelvin, $T_\alpha$. The default value is 273.15 K. |

**Remarks:**

1. **Viscosity.** The total viscosity is calculated as

$$\mu = H(T)\eta$$

where $H(T)$ is the temperature dependence. It is described by an Arrhenius law as:

$$H(T) = \exp\left[\alpha\left(\frac{1}{T - T_0} - \frac{1}{T_\alpha - T_0}\right)\right]$$

where $\alpha$ is the ratio of the activation energy to the thermodynamic constant. $T_\alpha$ is a reference temperature in Kelvin with a default value of 273.15 K. $T_0$ is the temperature shift in Kelvin. It is hardcoded as 0 K. If the parameter $\alpha$ is set to 0, the temperature dependence will be ignored because $H(T) = 1$.

2. **Enabling temperature dependence.** To consider temperature dependence, use either *ISPG_CONTROL_SOLUTION or *LOAD_THERMAL_LOAD_CURVE. To obtain the temperature by solving the energy governing equations, enable a combined flow and thermal analysis with *ISPG_CONTROL_SOLUTION. Temperatures defined with *LOAD_THERMAL_LOAD_CURVE are applied only during a flow-only analysis *LOAD_THERMAL_LOAD_CURVE is ignored during a combined flow and thermal analysis.

**References:**

Pan, X., Wu, C.T., and Hu, W. "Incompressible Smoothed Particle Galerkin (ISPG) Method for an Efficient Simulation of Surface Tension and Wall Adhesion Effects in the 3D Reflow Soldering Process," 16th International LS-DYNA Users Conference (2020).

## *MAT_ISPG_CROSS_CASTRO_MACOSKO

This is Material Type 4 for ISPG. The Cross Castro Macosko model attempts to describe the effects of shear rate, temperature, and degree of cure on the viscosity of a reactive fluid. The degree of cure is described by the Kama-Sourour model, which is the most used model to describe the curing kinetics of thermoset compounds.

> **NOTE:** This material only works for ISPG element formulations set on *SECTION_ISPG. It may only be in the ISPG input deck included in the LS-DYNA input deck with *INCLUDE_ISPG.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | B | SFTEN | TAU | N | | |
| Type | A | F | F | F | F | F | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TREF | | | | | | | |
| Type | F | | | | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHAG | C1 | C2 | | | | | |
| Type | F | F | F | | | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | M_KS | N_KS | A1 | A2 | T1 | T2 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Fluid density |
| B | Coefficient for calculating the zero-shear viscosity of the fluid, $B$ |
| SFTEN | Surface tension coefficient of the fluid |
| TAU | Shear stress at the transition from Newtonian to non-Newtonian flow, $\tau^*$ |
| N | Power-law index, $n$ |
| TREF | Reference temperature for calculating the zero-shear viscosity, $T_{\text{ref}}$ |
| ALPHAG | Gel point at which flow is no longer possible, $\alpha_g$ |
| C1 | Experimental constant, $c_1$ |
| C2 | Experimental constant, $c_2$ |
| M_KS | Reaction order in the Kamal-Sourour model, $m_{\text{ks}}$ |
| N_KS | Reaction order in the Kamal-Sourour model, $n_{\text{ks}}$ |
| A1 | Fitted rate coefficient, $A_1$ |
| A2 | Fitted rate coefficient, $A_2$ |
| T1 | Activation temperature, $T_1$ |
| T2 | Activation temperature, $T_2$ |

**Remarks:**

1. **Viscosity.** In the Cross Castro Macosko model, the viscosity is described as:

$$\eta(T,\dot{\gamma},\alpha) = \frac{\eta_0(T)}{1+\left(\frac{\eta_0(T)\dot{\gamma}}{\tau^*}\right)^{1-n}}\left(\frac{\alpha_g}{\alpha_g-\alpha}\right)^{(c_1+c_2\alpha)}$$

Here, $\eta_0(T)$ is the temperature-dependent zero-shear viscosity of the fluid:

$$\eta_0(T) = B\exp\left(\frac{T_{\text{ref}}}{T}\right) ,$$

$\dot{\gamma}$ is the second invariant of the rate of deformation tensor, **D**:

$$\dot{\gamma} = \sqrt{\frac{1}{2}\mathbf{D}:\mathbf{D}} \ ,$$

and $\alpha$ is the degree of cure. $B$, $T_{\text{ref}}$, $\alpha_g$, $c_1$, and $c_2$ are input parameters.

2. **Degree of cure.** The Kamal-Sourour model describes the conversion rate of $\alpha$:

$$\frac{d\alpha}{dt} = (k_1 + k_2\alpha^{m_{\text{ks}}})(1 - \alpha^{n_{\text{ks}}})$$

with

$$k_1 = A_1 \exp\left(-\frac{E_1}{RT}\right)$$
$$k_2 = A_2 \exp\left(-\frac{E_2}{RT}\right)$$

Here, $m_{\text{ks}}$ and $n_{\text{ks}}$ are the reaction orders, $k_1$ and $k_2$ are the Arrhenius rate constants, $A_1$ and $A_2$ are fitted rate coefficients, $E_1$ and $E_2$ are the activation energies, $R$ is the universal gas law constant of 8.314 $\text{J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$, and $T$ is the cure temperature. The activation temperature is obtained by dividing the activation energy by the universal gas law constant, such as $T_1 = E_1/R$.

3. **Enabling temperature dependence.** To consider temperature dependence, use either *ISPG_CONTROL_SOLUTION or *LOAD_THERMAL_LOAD_CURVE. To obtain the temperature by solving the energy governing equations, enable a combined flow and thermal analysis with *ISPG_CONTROL_SOLUTION. Temperatures defined with *LOAD_THERMAL_LOAD_CURVE are applied only during a flow-only analysis *LOAD_THERMAL_LOAD_CURVE is ignored during a combined flow and thermal analysis.

## *MAT_SPH_VISCOUS

This may also be referred to as *MAT_SPH_01. This "fluid-like" material model is very similar to Material Type 9 (*MAT_NULL). It models viscous fluids with constant or variable viscosity. The variable viscosity is a function of an equivalent deviatoric strain rate. If an inviscid material is modeled, the deviatoric or viscous stresses are zero, and the equation of state supplies the pressures (or diagonal components of the stress tensor).

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PC | MULO | MUHI | RK | RC | RN |
| Type | A | F | F | F | F | F | F | F |
| Defaults | none | none | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| PC | Pressure cutoff (≤ 0.0). See Remark 4. |
| MULO | Dynamic viscosity (see Remark 1):<br>EQ.0.0: Inviscid fluid is assumed.<br>GT.0.0: If MUHI = 0.0 or is not defined, then this is the traditional constant dynamic viscosity coefficient, $\mu$. Otherwise, if MUHI > 0.0, then MULO and MUHI are the lower and upper dynamic viscosity limit values for a power-law-like variable viscosity model.<br>LT.0.0: -MULO is a load curve ID defining dynamic viscosity as a function of equivalent strain rate. |
| MUHI | Dynamic viscosity:<br>EQ.0.0: Only MULO is used to define the dynamic viscosity, default<br>LT.0.0: The viscosity can be defined by the user in the file dyn21.F with a routine called f3dm9ale_userdef1. The file is part of the general usermat package. Note that in |

| VARIABLE | DESCRIPTION |
|---|---|
| | this case MULO is a parameter for the subroutine.<br>GT.0.0: This is the upper dynamic viscosity limit. This is defined only if RK and RN are defined for the variable viscosity case. |
| RK | Variable dynamic viscosity multiplier. See Remark 6. |
| RC | Cross viscosity model:<br>RC.GT.0.0: Use the Cross viscosity model which overwrites all other options. The values of MULO, MUHI, RK, and RN are used in the Cross viscosity model. See Remark 7.<br>RC.LE.0.0: Use a viscosity model based on the above fields. See Remark 6. |
| RN | Variable dynamic viscosity exponent. See Remark 6. |

**Remarks:**

1. **Deviatoric viscous stress.** This material must be used with an equation of state. Pressure cutoff is negative in tension. A (deviatoric) viscous stress of the form

$$\sigma'_{ij} = 2\mu\dot{\varepsilon}'_{ij}$$

$$\left[\frac{N}{m^2}\right] \sim \left[\frac{N}{m^2}s\right]\left[\frac{1}{s}\right]$$

is computed for nonzero $\mu$ where $\dot{\varepsilon}'_{ij}$ is the deviatoric strain rate. $\mu$ is the dynamic viscosity. For example, in the SI unit system, $\mu$ has units of [Pa × s].

2. **Hourglass control issues.** This material has no shear stiffness and hourglass control must be used with care. In some applications, the default hourglass coefficient might lead to significant energy losses. In general, for fluid(s), the hourglass coefficient QM should be small (in the range of $10^{-4}$ to $10^{-6}$ for the standard default IHQ choice).

3. **Null material properties.** This material has no yield strength and behaves in a fluid-like manner because it is based on the null material.

4. **Numerical cavitation.** The pressure cut-off, PC, must be defined to allow for a material to "numerically" cavitate. In other words, when a material undergoes dilatation above certain magnitude, it should no longer be able to resist this dilatation. Since dilatation stress or pressure is negative, setting PC limit to a very

small negative number would allow for the material to cavitate once the pressure in the material goes below this negative value.

5. **Issues with small values of viscosity exponent.** If the viscosity exponent is less than 1.0 (RN < 1.0), then RN − 1.0 < 0.0. In this case, at very low equivalent strain rate, the viscosity can be artificially very high. MULO is then used as the viscosity value.

6. **Empirical dynamic viscosity.** The empirical variable dynamic viscosity is typically modeled as a function of *equivalent shear rate* based on experimental data:

$$\mu(\dot{\bar{\gamma}}') = \mathrm{RK} \times \dot{\bar{\gamma}}'^{(\mathrm{RN}-1)} \ .$$

For an incompressible fluid, this may be written equivalently as

$$\mu(\dot{\bar{\varepsilon}}') = \mathrm{RK} \times \dot{\bar{\varepsilon}}'^{(\mathrm{RN}-1)} \ .$$

The "overbar" denotes a scalar equivalence, the "dot" denotes a time derivative or rate effect, and the "prime" symbol denotes deviatoric or volume preserving components. The *equivalent shear rate* components may be related to the basic definition of (small-strain) strain rate components as follows:

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \Rightarrow \dot{\varepsilon}'_{ij} = \dot{\varepsilon}_{ij} - \delta_{ij}\left(\frac{\dot{\varepsilon}_{kk}}{3}\right)$$
$$\dot{\gamma}_{ij} = 2\dot{\varepsilon}_{ij}$$

Typically, the 2nd invariant of the deviatoric strain rate tensor is defined as:

$$I_{2\dot{\varepsilon}'} = \frac{1}{2}\left[\dot{\varepsilon}'_{ij}\dot{\varepsilon}'_{ij}\right] \ .$$

The equivalent (small-strain) deviatoric strain rate is defined as:

$$\dot{\bar{\varepsilon}}' \equiv 2\sqrt{I_{2\dot{\varepsilon}'}} = \sqrt{2\left[\dot{\varepsilon}'_{ij}\dot{\varepsilon}'_{ij}\right]} = \sqrt{4\left[\dot{\varepsilon}'_{12}{}^2 + \dot{\varepsilon}'_{23}{}^2 + \dot{\varepsilon}'_{31}{}^2\right] + 2\left[\dot{\varepsilon}'_{11}{}^2 + \dot{\varepsilon}'_{22}{}^2 + \dot{\varepsilon}'_{33}{}^2\right]} \ .$$

In non-Newtonian literatures, the *equivalent shear rate* is sometimes defined as

$$\dot{\bar{\gamma}} \equiv \sqrt{\frac{\dot{\gamma}_{ij}\dot{\gamma}_{ij}}{2}} = \sqrt{2\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}} = \sqrt{4[\dot{\varepsilon}_{12}^2 + \dot{\varepsilon}_{23}^2 + \dot{\varepsilon}_{31}^2] + 2[\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{22}^2 + \dot{\varepsilon}_{33}^2]} \ .$$

It turns out that when (a) the material is incompressible material ($\dot{\varepsilon}_{kk} = 0$) and (b) the shear terms are equivalent (when $i \neq j \rightarrow \dot{\varepsilon}_{ij} = \dot{\varepsilon}'_{ij}$), the *equivalent shear rate* is algebraically equivalent to the *equivalent (small-strain) deviatoric strain rate*:

$$\dot{\bar{\varepsilon}}' = \dot{\bar{\gamma}}' \ .$$

7. **Cross viscous model.** The Cross viscous model is one of simplest and most used model for shear-thinning behavior. With shear-thinning behavior, the fluid's viscosity decreases with increasing local shear rate, $\dot{\bar{\gamma}}$. Thus, using the Cross viscous model, the dynamic viscosity $\mu$ is defined as a function of $\dot{\bar{\gamma}}$:

$$\mu(\dot{\bar{\gamma}}') = \mathrm{MUHI} + (\mathrm{MULO} - \mathrm{MUHI})/(1.0 + \mathrm{RK} \times \dot{\bar{\gamma}}')^{\mathrm{RN}-1} \ .$$

Here RK and RN are two positive fitting parameters, and MULO and MUHI are the limiting values of the viscosity at low and high shear rates, respectively. RK, RN, MULO and MUHI are fields from the keyword input.

## *MAT_SPH_INCOMPRESSIBLE_FLUID

This may also be referred to as *MAT_SPH_02. This material is only used for the implicit incompressible SPH formulation (FORM = 13 in *CONTROL_SPH).

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | MU | GAMMA1 | GAMMA2 | STENS | | |
| Type | A | F | F | F | F | F | | |
| Defaults | none | none | 0.0 | 0.0 | 0.0 | 0.0 | | |

This card is optional.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CP | LAMBDA | | | | | | |
| Type | F | F | | | | | | |
| Default | 0.0 | 0.0 | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| MU | Dynamic viscosity<br>LT.0.0: \|MU\| is a load curve of dynamic viscosity as a function of temperature. See *DEFINE_CURVE. |
| GAMMA1 | Numerical surface tension coefficient. For water, we recommend a coefficient of $\gamma_1 = 1000\ \mathrm{m/s^2}$. GAMMA1 is only used if IMAT = 0 in *CONTROL_SPH_INCOMPRESSIBLE. |
| GAMMA2 | Numerical surface tension coefficient. For water, we recommend a coefficient of $\gamma_2 = 1\ \mathrm{m/s^2}$. GAMMA2 is only used if IMAT = 0 in |

| VARIABLE | DESCRIPTION |
|---|---|
| | *CONTROL_SPH_INCOMPRESSIBLE. |
| STENS | Physical surface tension coefficient. It is only used if IMAT = 1 in *CONTROL_SPH_INCOMPRESSIBLE. |
| CP | Fluid specific heat. It is used to calculate heat transfer coefficients if IHTC = 1 in *CONTROL_SPH_INCOMPRESSIBLE. |
| LAMBDA | Fluid thermal conductivity. It is used to calculate heat transfer coefficients if IHTC = 1 in *CONTROL_SPH_INCOMPRESSIBLE. |

### Remarks:

The surface tension coefficients, GAMMA1 and GAMMA2, are purely numerical and are based on a normalized version of the algorithm presented in [1]. If IMAT = 1 in *CONTROL_SPH_INCOMPRESSIBLE, surface tension is calculated based on the physical surface tension properties of the fluid.

### References:

[1] Akinci, N., Akinci, G. & Teschner, M. (2013). Versatile surface tension and adhesion for SPH fluids. ACM Transactions on Graphics (TOG) 32.6 182.

## *MAT_SPH_INCOMPRESSIBLE_STRUCTURE

This may also be referred to as *MAT_SPH_03. This material is only used for the implicit incompressible SPH formulation (FORM = 13 in *CONTROL_SPH) and should be assigned to structures sampled with the *DEFINE_SPH_MESH_SURFACE keyword.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | BETA | ROUGH | ADH | | | |
| Type | A | F | F | F | F | | | |
| Defaults | none | none | 0.0 | 0.0 | 0.0 | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density. This should be set to the rest density of the fluid. The actual mass of the structure will be calculated from the parent surfaces sampled with the *DEFINE_SPH_MESH_SURFACE keyword. |
| BETA | Numerical surface adhesion coefficient. For water, a value of $\beta = 1000\ \mathrm{m/s^2}$ is recommended. Only used if IMAT = 0 in *CONTROL_SPH. |
| ROUGH | Surface roughness coefficient. A friction force between the structure and the fluid is generated based on the viscosity of the fluid scaled by this coefficient. A value between 0.0 and 10.0 is usually recommended. |
| ADH | Surface adhesion scaling coefficient. It is only used if IMAT = 1 in *CONTROL_SPH. An attractive force between fluid and structure is calculated based on surface tension forces in the fluid and then scaled by ADH. |

**Remarks:**

The surface adhesion coefficient is purely numerical and is based on a normalized version of the algorithm presented in [1].

## References:

[1] Akinci, N., Akinci, G. & Teschner, M. (2013). Versatile surface tension and adhesion for SPH fluids. ACM Transactions on Graphics (TOG) 32.6 182.

## *MAT_SPRING_ELASTIC

This is Material Type 1 for discrete elements (*ELEMENT_DISCRETE). This model provides a translational or rotational elastic spring located between two nodes. Only one degree of freedom is connected.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | K | | | | | | |
| Type | A | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| K | Elastic stiffness (force/displacement) or (moment/rotation) |

**Remarks:**

Rotational displacement is measured in radians.

### *MAT_DAMPER_VISCOUS

This is Material Type 2 for discrete elements (*ELEMENT_DISCRETE). This material provides a linear translational or rotational damper located between two nodes. Only one degree of freedom is then connected.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | DC | | | | | | |
| Type | A | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| DC | Damping constant (force/displacement rate) or (moment/rotation rate) |

**Remarks:**

Rotational displacement is measured in radians.

## *MAT_SPRING_ELASTOPLASTIC

This is Material Type 3 for discrete elements (*ELEMENT_DISCRETE). This material provides an elastoplastic translational or rotational spring with isotropic hardening located between two nodes. Only one degree of freedom is connected.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | K | KT | FY | | | | |
| Type | A | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| K | Elastic stiffness (force/displacement) or (moment/rotation) |
| KT | Tangent stiffness (force/displacement) or (moment/rotation) |
| FY | Yield (force) or (moment) |

**Remarks:**

Rotational displacement is measured in radians.

## *MAT_SPRING_NONLINEAR_ELASTIC

This is Material Type 4 for discrete elements (*ELEMENT_DISCRETE). This material provides a nonlinear elastic translational and rotational spring with arbitrary force as a function of displacement and moment as a function of rotation, respectively. Optionally, strain rate effects can be considered through a velocity dependent scale factor or defining a table of curves. With the spring located between two nodes, only one degree of freedom is connected.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | LCD | LCR | | | | | |
| Type | A | I | I | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| LCD | Load curve ID (see *DEFINE_CURVE) describing force as a function of displacement or moment as a function of rotation relationship. The load curve *must* define the response in the negative and positive quadrants and pass through point (0,0). Negative data point(s) must come first in the curve definition, where negative values represent compression in the case of a translational spring.<br>LCD may also be a table ID (see *DEFINE_TABLE). The table gives for each loading rate a load curve ID defining the force-displacement (or moment-rotation) curve. Values between the data points are computed by linear interpolation. If a table ID is specified, LCR will be ignored. |
| LCR | Optional load curve describing scale factor on force or moment as a function of relative velocity or rotational velocity, respectively. |

**Remarks:**

Rotational displacement is measured in radians.

## *MAT_DAMPER_NONLINEAR_VISCOUS

This is Material Type 5 for discrete elements (*ELEMENT_DISCRETE). This material provides a viscous translational damper with an arbitrary force as a function of velocity dependency or a rotational damper with an arbitrary moment as a function of rotational velocity dependency. With the damper located between two nodes, only one degree of freedom is connected.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | LCDR | | | | | | |
| Type | A | I | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| LCDR | Load curve ID defining force as a function of rate-of-displacement relationship or a moment as a function of rate-of-rotation relationship. The load curve *must* define the response in the negative and positive quadrants and pass through point (0,0). |

**Remarks:**

Rotational displacement is measured in radians.

## *MAT_SPRING_GENERAL_NONLINEAR

This is Material Type 6 for discrete elements (*ELEMENT_DISCRETE). This material provides a general nonlinear translational or rotational spring with arbitrary loading and unloading definitions. Optionally, hardening or softening can be defined. With the spring located between two nodes, only one degree of freedom is connected.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | LCDL | LCDU | BETA | TYI | CYI | | |
| Type | A | I | I | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| LCDL | Load curve or table ID giving force/torque as a function of displacement/rotation (curve) or as a function of velocity and displacement/rotation (table) for loading; see Figure MS6-1. |
| LCDU | Load curve or table ID giving force/torque as a function of displacement/rotation (curve) or as a function of velocity and displacement/rotation (table) for unloading; see Figure MS6-1. |
| BETA | Hardening parameter, $\beta$:<br>EQ.0.0: Tensile and compressive yield with strain softening (negative or zero slope allowed in the force as a function of displacement load curves). TYI and CYI are not implemented for this option.<br>NE.0.0: Kinematic hardening without strain softening<br>EQ.1.0: Isotropic hardening without strain softening |
| TYI | Initial yield force in tension ( > 0) |
| CYI | Initial yield force in compression ( < 0) |

**Remarks:**

1. **Load Curves.** Load curve points are in the format (displacement, force) or (rotation, moment). The points must be in order starting with the most negative

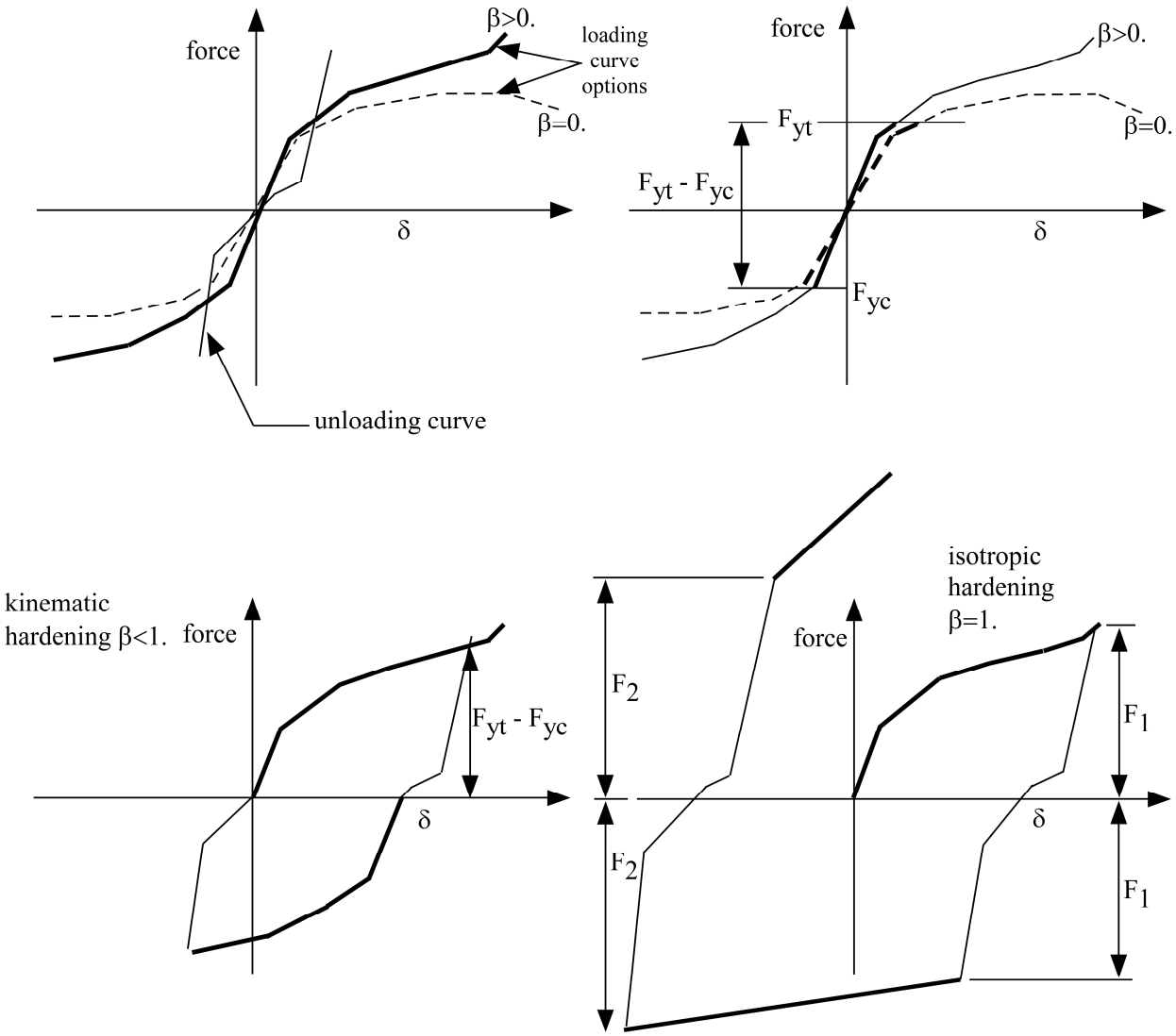


**Figure MS6-1.** General Nonlinear material for discrete elements

(compressive) displacement or rotation and ending with the most positive (tensile) value. The curves need not be symmetrical.

The displacement origin of the "unloading" curve is arbitrary since it will be shifted as necessary as the element extends and contracts. On reverse yielding the "loading" curve will also be shifted along the displacement re or. rotation axis.

2. **Initial Tensile and Compressive Yield Forces.** The initial tensile and compressive yield forces (TYI and CYI) define a range within which the element remains elastic (meaning the "loading" curve is used for both loading and unloading). If at any time the force in the element exceeds this range, the element is deemed to have yielded, and at all subsequent times the "unloading" curve is used for unloading.

3. **Rotational Displacement.** Rotational displacement is measured in radians.

## *MAT_SPRING_MAXWELL

This is Material Type 7 for discrete elements (*ELEMENT_DISCRETE). This material provides a three-parameter Maxwell viscoelastic translational or rotational spring. Optionally, a cutoff time with a remaining constant force/moment can be defined.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | K0 | KI | BETA | TC | FC | COPT | |
| Type | A | F | F | F | F | F | F | |
| Default | none | none | none | none | $10^{20}$ | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| K0 | $K_0$, short-time stiffness |
| KI | $K_\infty$, long-time stiffness |
| BETA | Decay parameter, $\beta$ |
| TC | Cut off time. After this time, a constant force/moment is transmitted. |
| FC | Force/moment after cutoff time |
| COPT | Time implementation option:<br>EQ.0.0: Incremental time change<br>NE.0.0: Continuous time change |

**Remarks:**

1. **Stiffness.** The time-varying stiffness, $K(t)$, may be described in terms of the input parameters as

$$K(T) = K_\infty + (K_0 - K_\infty)\exp(-\beta t) \ .$$

This equation was implemented by Schwer [1991] as either a continuous function of time or incrementally following the approach of Herrmann and Peterson

[1968]. The continuous function of time implementation has the disadvantage of the energy absorber's resistance decaying with increasing time even without deformation. The advantage of the incremental implementation is that an energy absorber must undergo some deformation before its resistance decays, meaning there is no decay until impact, even in delayed impacts. The disadvantage of the incremental implementation is that very rapid decreases in resistance cannot be easily matched.

2. **Rotational displacement.** Rotational displacement is measured in radians.

## *MAT_SPRING_INELASTIC

This is Material Type 8 for discrete elements (*ELEMENT_DISCRETE). This material provides an inelastic tension or compression only, translational or rotational spring. Optionally, a user-specified unloading stiffness can be taken instead of the maximum loading stiffness.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | LCFD | KU | CTF | | | | |
| Type | A | I | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| LCFD | Load curve ID describing arbitrary force/torque as a function of displacement/rotation relationship. This curve must be defined in the positive force-displacement quadrant regardless of whether the spring acts in tension or compression. |
| KU | Unloading stiffness (optional). The maximum of KU and the maximum loading stiffness in the force/displacement or the moment/rotation curve is used for unloading. |
| CTF | Flag for compression/tension:<br>EQ.-1.0: Tension only<br>EQ.1.0: Compression only (default) |

**Remarks:**

Rotational displacement is measured in radians.

## *MAT_SPRING_TRILINEAR_DEGRADING

This is Material Type 13 for discrete elements (*ELEMENT_DISCRETE). This material allows concrete shearwalls to be modeled as discrete elements under applied seismic loading. It represents cracking of the concrete, yield of the reinforcement, and overall failure. Under cyclic loading, the stiffness of the spring degrades, but the strength does not.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | DEFL1 | F1 | DEFL2 | F2 | DEFL3 | F3 | FFLAG |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| DEFL1 | Deflection at the point where concrete cracking occurs |
| F1 | Force corresponding to DEFL1 |
| DEFL2 | Deflection at the point where reinforcement yields |
| F2 | Force corresponding to DEFL2 |
| DEFL3 | Deflection at complete failure |
| F3 | Force corresponding to DEFL3 |
| FFLAG | Failure flag |

## *MAT_SPRING_SQUAT_SHEARWALL

This is Material Type 14 for discrete elements (*ELEMENT_DISCRETE). This material allows squat shear walls to be modeled using discrete elements. The behavior model captures concrete cracking, reinforcement yield, and ultimate strength followed by degradation of strength finally leading to collapse.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | A14 | B14 | C14 | D14 | E14 | LCID | FSD |
| Type | A | F | F | F | F | F | I | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| A14 | Material coefficient *A* |
| B14 | Material coefficient *B* |
| C14 | Material coefficient *C* |
| D14 | Material coefficient *D* |
| E14 | Material coefficient *E* |
| LCID | Load curve ID referencing the maximum strength envelope curve |
| FSD | Sustained strength reduction factor |

**Remarks:**

Material coefficients *A*, *B*, *C*, and *D* are empirically defined constants for setting the shape of the polynomial curves that govern the cyclic behavior of the discrete element. The loading and unloading paths use different polynomial relationships, allowing energy absorption through hysteresis. Coefficient *E* determines the "jump" from the loading path to the unloading path (or vice versa) when a full hysteresis loop is not completed. The load curve referenced is used to define the force-displacement characteristics of the shear wall under monotonic loading. The polynomials defining the cyclic behavior refer to this curve. On the second and subsequent loading/unloading cycles, the shear wall has reduced strength. The variable FSD is the sustained strength reduction factor.

## *MAT_SPRING_MUSCLE

This is Material Type S15 for discrete elements (*ELEMENT_DISCRETE). This material is a Hill-type muscle model with activation. The LS-DYNA implementation is due to Dr. J. A. Weiss.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | L0 | VMAX | SV | A | FMAX | TL | TV |
| Type | A | F | F | F | F | F | F | F |
| Default | none | 1.0 | none | 1.0 | none | none | 1.0 | 1.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FPE | LMAX | KSH | | | | | |
| Type | F | F | F | | | | | |
| Default | 0.0 | none | none | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| L0 | Initial muscle length, $L_0$ |
| VMAX | Maximum CE shortening velocity, $V_{\max}$ |
| SV | Scale factor, $S_v$, for $V_{\max}$ as a function of active state:<br>LT.0.0: Absolute value gives load curve ID.<br>GE.0.0: Constant value of 1.0 is used. |
| A | Activation level as a function of time function $a(t)$:<br>LT.0.0: Absolute value gives load curve ID.<br>GE.0.0: Constant value of A is used. |

| VARIABLE | DESCRIPTION |
|---|---|
| FMAX | Peak isometric force, $F_{max}$ |
| TL | Active tension as a function of length function, $f_{TL}(L)$:<br>LT.0.0: Absolute value gives load curve ID.<br>GE.0.0: Constant value of 1.0 is used. |
| TV | Active tension as a function of velocity function, $f_{TV}(V)$:<br>LT.0.0: Absolute value gives load curve ID.<br>GE.0.0: Constant value of 1.0 is used. |
| FPE | Normalized force, $f_{PE}$, as a function of length for parallel elastic element:<br>LT.0.0: Absolute value gives load curve ID.<br>EQ.0.0: Exponential function is used (see Remarks).<br>GT.0.0: Constant value of 0.0 is used. |
| LMAX | Relative length when $F^{PE}$ reaches $F_{max}$. Required if FPE = 0.0 above. See Remarks. |
| KSH | Constant, $K_{sh}$, governing the exponential rise of $F^{PE}$. Required if FPE = 0.0 above. See Remarks. |

**Remarks:**

The material behavior of the muscle model is adapted from the original model proposed by Hill [1938]. Reviews of this model and extensions can be found in Winters [1990] and Zajac [1989]. The most basic Hill-type muscle model consists of a contractile element (CE) and a parallel elastic element (PE) (Figure MS15-1). An additional series elastic element (SEE) can be added to represent tendon compliance.

The main assumptions of the Hill model are that the contractile element is entirely stress free and freely distensible in the resting state and is described exactly by Hill's equation (or some variation). When the muscle is activated, the series and parallel elements are elastic, and the whole muscle is a simple combination of identical sarcomeres in series and parallel. The main criticism of Hill's model is that the division of forces between the parallel elements and the division of extensions between the series elements is arbitrary and cannot be made without introducing auxiliary hypotheses. However, these criticisms apply to *any* discrete element model. Despite these limitations, the Hill model has

become extremely useful for modeling musculoskeletal dynamics, as illustrated by its widespread use today.

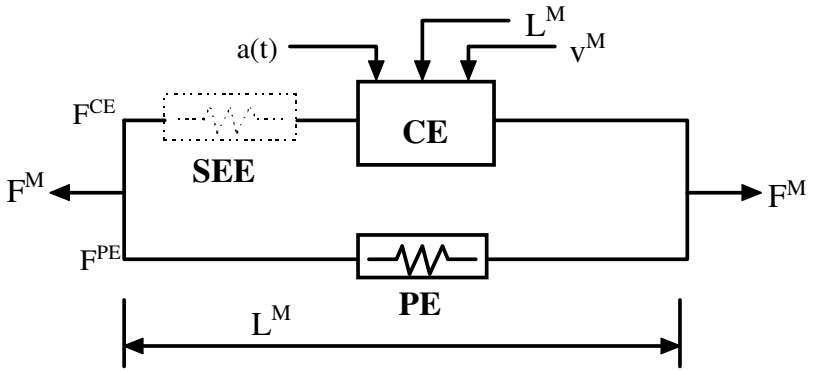


**Figure MS15-1.** Discrete model for muscle contraction dynamics, based on a Hill-type representation. The total force is the sum of passive force $F^{PE}$ and active force $F^{CE}$. The passive element (PE) represents energy storage from muscle elasticity, while the contractile element (CE) represents force generation by the muscle. The series elastic element (SEE), shown in dashed lines, is often neglected when a series tendon compliance is included. Here, $a(t)$ is the activation level, $L^M$ is the length of the muscle, and $V^M$ is the shortening velocity of the muscle.

When the contractile element (CE) of the Hill model is inactive, the entire resistance to elongation is provided by the PE element and the tendon load-elongation behavior. As activation is increased, force then passes through the CE side of the parallel Hill model, providing the contractile dynamics. The original Hill model accommodated only full activation - this limitation is circumvented in the present implementation by using the modification suggested by Winters (1990). The main features of his approach were to realize that the CE force-velocity input force equals the CE tension-length output force. This yields a three-dimensional curve to describe the force-velocity-length relationship of the CE. If the force-velocity $y$-intercept scales with activation, then given the activation, length and velocity, the CE force can be determined.

Without the SEE, the total force in the muscle FM is the sum of the force in the CE and the PE because they are in parallel:

$$F^M = F^{PE} + F^{CE} \quad .$$

The relationships defining the force generated by the CE and PE as a function of $L^M$ (length of the muscle), $V^M$ (shortening velocity of the muscle) and $a(t)$ are often scaled by $F_{max}$, the peak isometric force (p. 80, Winters 1990), $L_0$, the initial length of the muscle (p. 81, Winters 1990), and $V_{max}$, the maximum unloaded CE shortening velocity (p. 80, Winters 1990). From these, dimensionless length and velocity can be defined as:

$$L = L^M / L_0$$
$$V = \frac{V^M}{V_{\max} \times S_v[a(t)]}$$

Here, $S_v$ scales the maximum CE shortening velocity $V_{\max}$ and changes with activation level $a(t)$. This has been suggested by several researchers, that is, Winters and Stark [1985]. The activation level specifies the level of muscle stimulation as a function of time. Both have values between 0 and 1. The functions $S_v[a(t)]$ and $a(t)$ are specified using load curves in LS-DYNA, but the default values of $S_v = 1$ and $a(t) = 0$ can also be used. Note that $L$ is always positive and that $V$ is positive for lengthening and negative for shortening.

The relationship between $F^{CE}$, $V$ and $L$ was proposed by Bahler et al. [1967]. A three-dimensional relationship between these quantities is now considered standard for computer implementations of Hill-type muscle models [Winters 1990]. It can be written in dimensionless form as:

$$F^{CE} = a(t) \times F_{\max} \times f_{TL}(L) \times f_{TV}(V) \, .$$

Here, $f_{TL}(L)$ and $f_{TV}(V)$ are the tension-length and tension-velocity functions for active skeletal muscle. Thus, if current values of $L^M$, $V^M$, and $a(t)$ are known, then $F^{CE}$ can be determined (Figure MS15-1).

If FPE = 0.0, the force in the parallel elastic element, $F^{PE}$, is determined directly from the current length of the muscle using an exponential relationship [Winters 1990]:

$$f_{PE} = \frac{F^{PE}}{F_{MAX}} = \begin{cases} 0 & L \le 1 \\ \dfrac{1}{\exp(K_{sh}) - 1} \left\{ \exp\left[ \dfrac{K_{sh}}{L_{max}} (L - 1) \right] - 1 \right\} & L > 1 \end{cases}$$

Here, $L_{max}$ is the dimensionless length at which the force $F_{max}$ occurs, and $K_{sh}$ is a dimensionless shape parameter controlling the rate of rise of the exponential. Alternatively, the user can define a custom $f_{PE}$ curve giving tabular values of normalized force as a function of dimensionless length as a load curve.

For computation of the total force developed in the muscle $F^M$, the functions for the tension-length $f_{TL}(L)$ and force-velocity $f_{TV}$ relationships used in the Hill element must be defined. These relationships have been available for over 50 years but have been refined to allow for behavior such as active lengthening. The active tension-length curve $f_{TL}(L)$ describes the fact that isometric muscle force development is a function of length, with the maximum force occurring at an optimal length. According to Winters, this optimal length is typically around $L = 1.05$, and the force drops off for shorter or longer lengths, approaching zero force for $L = 0.4$ and $L = 1.5$. Thus the curve has a bell-shape. Because of the variability in this curve between muscles, the user must specify the function $f_{TL}(L)$ using a load curve, specifying pairs of points representing the normalized force (with values between 0 and 1) and normalized length $L$. See Figure MS15-2.

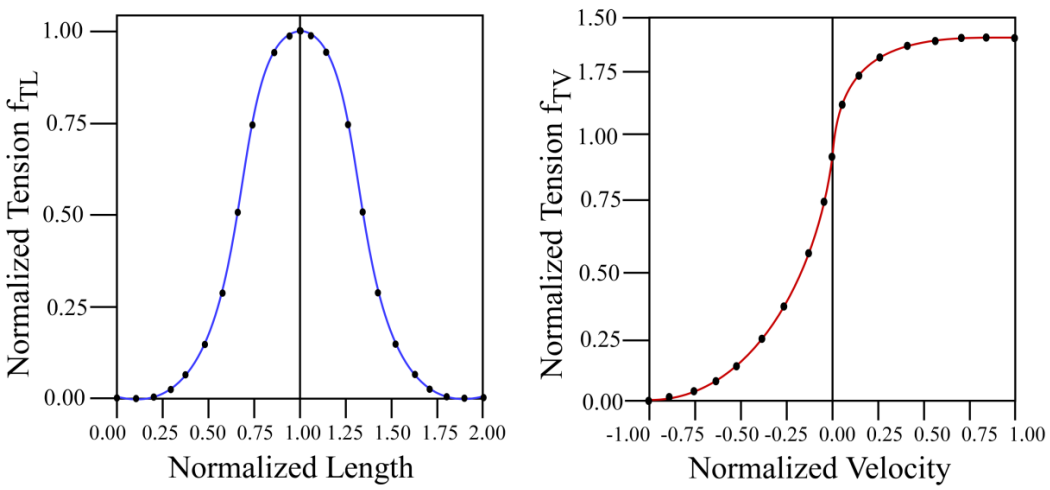


**Figure MS15-2.** Typical normalized tension-length (TL) and tension-velocity (TV) curves for skeletal muscle.

The active tension-velocity relationship $f_{TV}(V)$ used in the muscle model is mainly due to the original work of Hill. Note that the dimensionless velocity $V$ is used. When $V = 0.0$, the normalized tension is typically chosen to have a value of 1.0. When $V$ is greater than or equal to 0.0, muscle lengthening occurs. As $V$ increases, the function is typically designed so that the force increases from a value of 1.0 and asymptotes towards a value near 1.4 ass shown in Figure MS15-2. When $V$ is less than zero, muscle shortening occurs and the classic Hill equation hyperbola is used to drop the normalized tension to 0.0 as shown in Figure MS15-2. The user must specify the function $f_{TV}(V)$ using a load curve, specifying pairs of points representing the normalized tension (with values between 0.0 and 1.0) and normalized velocity $V$.

## *MAT_SEATBELT_{OPTION}

This is Material Type B01. It defines a seat belt material.

Available options include:

2D

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | MPUL | LLCID | ULCID | LMIN | CSE | DAMP | E |
| Type | A | F | I | I | F | F | F | F |
| Default | 0 | 0.0 | 0 | 0 | 0.0 | 0.0 | 0.1 | 0.0 |

**Bending/Compression Parameter Card.** Additional card for E > 0.0.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A | I | J | AS | F | M | R | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 2*I | A | $10^{20}$ | $10^{20}$ | 0.05 | |

**2D Card.** Additional 1st card for the 2D keyword option.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P1DOFF | FORM | ECOAT | TCOAT | SCOAT | EB | PRBA | PRAB |
| Type | I | I | F | F | F | F | F | F |
| Default | 0 | 0 | 0.0 | 0.0 | 0.0 | -0.1 | 0.3 | PRBA |

**2D Card.** Optional 2nd card for the 2D keyword option.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | | | | | | | |
| Type | F | | | | | | | |
| Default | ↓ | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Belt material number. A unique number or label must be specified (see *PART). |
| MPUL | Mass per unit length |
| LLCID | Curve or table ID for loading. LLCID can be either a single curve (force as a function of engineering strain) or a table defining a set of strain-rate dependent load curves. |
| ULCID | Load curve identification for unloading (force as a function of engineering strain) |
| LMIN | Minimum length (for elements connected to slip rings and retractors); see Remark 4. |
| CSE | Compressive stress elimination option which applies to shell elements only, available since r137465/dev for non-zero FORM. The old recommended option of CSE = 2, available since R8, still works if and only if FORM = 0. For non-zero FORM:<br>EQ.0.0: don't eliminate compressive stresses in shell fabric.<br>EQ.1.0: eliminate compressive stresses in shell fabric. |
| DAMP | Optional Rayleigh damping coefficient, which applies to shell elements only. A coefficient value of 0.10 is the default corresponding to 10% of critical damping. Sometimes smaller or larger values work better. |
| E | Young's modulus for bending/compression stiffness, when positive, the optional card is invoked. See Remark 5. |
| A | Cross sectional area for bending/compression stiffness; see remarks. |

| VARIABLE | DESCRIPTION |
|---|---|
| I | Area moment of inertia for bending/compression stiffness; see Remark 5. |
| J | Torsional constant for bending/compression stiffness; see Remark 5. |
| AS | Shear area for bending/compression stiffness; see Remark 5. |
| F | Maximum force in compression/tension; see Remark 5. |
| M | Maximum torque; see Remark 5. |
| R | Rotational mass scaling factor; see Remark 5. |
| P1DOFF | Part ID offset for internally created 1D, bar-type, belt parts for 2D seatbelt of this material, that is, the IDs of newly created 1D belt parts will be P1DOFF + 1, P1DOFF + 2, … If zero, the maximum ID of user-defined parts is used as the part ID offset. |
| FORM | Formulation of the translated fabric material; see FORM of *MAT_FABRIC for details. FORM = 0 was used since R8 and non-zero FORM is available since r137465/dev. |
| ECOAT | Young's modulus of coat material for FORM = -14; see *MAT_FABRIC for details.<br>EQ.0.0: ECOAT is the Young's modulus determined by LS-DYNA.<br>GT.0.0: ECOAT is the Young's modulus to be used for coat material.<br>LT.0.0: \|ECOAT\| is the ratio of coat material's Young's modulus to that of the fabric shell which is determined by LS-DYNA. |
| TCOAT | Thickness of coat material for FORM = -14; see *MAT_FABRIC for details. |
| SCOAT | Yield stress of coat material for FORM = -14; see *MAT_FABRIC for details. If not defined, the coat material is assumed to be elastic. |
| EB | Young's modulus along transverse direction; see *MAT_FABRIC for details. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: The Young's modulus along transverse direction is 10% of the Young's determined by LS-DYNA based on the loading curve, LLCID.<br>LT.0.0: \|EB\| is the ratio of Young's modulus along the transverse direction to the Young's modulus determined by LS-DYNA based on the loading curve, LLCID.<br>GT.0.0: EB is the Young's modulus along the transverse direction. |
| PRBA (PRAB) | Minor (Major) Poisson's ratio *ba* (*ab*) direction |
| GAB | Shear modulus in the *ab* direction. Set to a very small value for an isotropic elastic material; see *MAT_FABRIC. If defined to be zero, a default value of $EA/(2 \times (1 + PRBA))$ will be used where EA is the Young's modulus along the longitudinal direction and is set to 1% of the Young's modulus determined by LS-DYNA according to the loading curve, LLCID. |

**Remarks:**

1. **Loading and Unloading.** Each belt material defines stretch characteristics and mass properties for a set of belt elements. The user enters a load curve for loading, the points of which are (Strain, Force). Strain is defined as engineering strain, that is,

$$\text{Strain} = \frac{\text{current length}}{\text{initial length}} - 1.0$$

Another similar curve is entered to describe the unloading behavior. Both load curves should start at the origin (0,0) and contain positive force and strain values only. The belt material is tension only with zero forces being generated whenever the strain becomes negative. The first non-zero point on the loading curve defines the initial yield point of the material. On unloading, the unloading curve is shifted along the strain axis until it crosses the loading curve at the "yield" point from which unloading commences. If the initial yield has not yet been exceeded or if the origin of the (shifted) unloading curve is at negative strain, the original loading curves will be used for both loading and unloading. If the strain is less than the strain at the origin of the unloading curve, the belt is slack and no force is generated. Otherwise, forces will then be determined by the unloading curve for unloading and reloading until the strain again exceeds yield after which the loading curves will again be used.

2. **Damping.** A small amount of damping is automatically included. This reduces high frequency oscillation, but, with realistic force-strain input characteristics and loading rates, does not significantly alter the overall forces-strain performance. The damping forced opposes the relative motion of the nodes and is limited by stability:

$$D = \frac{0.1 \times \text{ mass} \times \text{relative velocity}}{\text{time step size}}$$

In addition, the magnitude of the damping force is limited to one-tenth of the force calculated from the force-strain relationship and is zero when the belt is slack. Damping forces are not applied to elements attached to sliprings and retractors.

3. **Nodal Masses.** MPUL, the mass per unit length, is used to calculate the nodal masses during initialization.

4. **Minimum Length.** LMIN, the "minimum" length, controls the shortest length allowed in any element. It also determines when an element passes through sliprings or is absorbed into the retractors. A large LMIN causes elements to easily pass through the sliprings. A small LMIN leads to a smaller time step and possible instability for 2D belts. One tenth of a typical initial element length is a good choice for a 1D belt. For a 2D belt, a larger value of 0.3 can be used for better robustness and a larger time step.

5. **Bending and Compression Stiffness for 1D Elements.** Since one-dimensional elements do not possess any bending or compression stiffness, dynamic analysis is mandatory during an implicit analysis that includes belts. However, one dimensional belt elements *can* be used in implicit statics by associating them with bending/compression properties with the first optional card. Two-dimensional belt elements are not supported with this feature.

   To achieve bending and compression stiffness in one-dimensional belts, the belt element is overlayed with a Belytschko-Schwer beam element (see *SECTION_BEAM, ELFORM = 2, for a more comprehensive description of fields A, I, J and AS) with circular cross section. These elements have 6 degrees of freedom including rotational degrees of freedom. The material used in this context is an elastic-ideal-plastic material where the elastic part is governed by the Young's modulus, $E$. Two yield values, F (the maximum compression/tension force) and M (the maximum torque), are used as upper bounds for the resultants. The bending/compression forces and moments from this contribution are accumulated to the force from the seatbelt itself. Since the main purpose is to eliminate the singularities in bending and compression, it is recommended to choose the bending and compression properties in the optional card carefully so as to not significantly influence the overall response.

For the sake of completeness, this feature is also supported by the explicit integrator; therefore, a rotational nodal mass is needed. Each of the two nodes of an element gets a contribution from the belt that is calculated as RMASS = R × (MASS/2) × I/A, where MASS indicates the total translational mass of the belt element and R is a scaling factor input by the user. The translational mass is not modified. The bending and compression properties do not affect the stable time step. If the belts are used *without* sliprings, then incorporating this feature is virtually equivalent to adding Belytschko-Schwer beams on top of conventional belt elements as part of the modelling strategy. If sliprings *are* used, this feature is necessary to properly support the flow of material through the sliprings and swapping of belt elements across sliprings. Retractors cannot be used with this feature.