*EOS

When defining an equation of state, the type of equation of state is specified by a corresponding 3-digit number in the command name, e.g., *EOS_004, or equivalently, by it's more descriptive designation, e.g., *EOS_GRUNEISEN. The equations of state can be used with a subset of the materials that are available for solid elements; see the MATERIAL MODEL REFERENCE TABLES in the beginning of the *MAT section of this Manual. *EOS_015 is linked to the type 2 thick shell element and can be used to model engine gaskets.

The meaning associated with particular extra history variables for a subset of material models and equations of state are tabulated at http://www.dynasupport.com/howtos-/material/history-variables. The first three extra history variables when using an equation of state are (1) internal energy, (2) pressure due to bulk viscosity, and (3) the element volume from the previous time step.

TYPE 1: *EOS_LINEAR_POLYNOMIAL

TYPE 2: *EOS_JWL

TYPE 3: *EOS_SACK_TUESDAY

TYPE 4: *EOS_GRUNEISEN

TYPE 5: *EOS_RATIO_OF_POLYNOMIALS

TYPE 6: *EOS_LINEAR_POLYNOMIAL_WITH_ENERGY_LEAK

TYPE 7: *EOS_IGNITION_AND_GROWTH_OF_REACTION_IN_HE

TYPE 8: *EOS_TABULATED_COMPACTION

TYPE 9: *EOS_TABULATED

TYPE 10: *EOS_PROPELLANT_DEFLAGRATION

TYPE 11: *EOS_TENSOR_PORE_COLLAPSE

TYPE 12: *EOS_IDEAL_GAS

TYPE 13: *EOS_PHASE_CHANGE

TYPE 14: *EOS_JWLB

*EOS

TYPE 15:   *EOS_GASKET

TYPE 16:   *EOS_MIE_GRUNEISEN

TYPE 19:   *EOS_MURNAGHAN

TYPE 21-30:   *EOS_USER_DEFINED

An additional option **TITLE** may be appended to all the ***EOS** keywords. If this option is used then an additional line is read for each section in 80a format which can be used to describe the equation of state. At present LS-DYNA does not make use of the title. Inclusion of title simply gives greater clarity to input decks.

Definitions and Conventions

In order to prescribe the boundary and/or initial thermodynamic condition, manual computations are often necessary. Conventions or definitions must be established to simplify this process. Some basic variables are defined in the following. Since many of these variables have already been denoted by different symbols, the notations used here are unique in this section only! They are presented to only clarify their usage. A corresponding SI unit set is also presented as an example.

First consider a few volumetric parameters since they are a measure of compression (or expansion).

Volume:

$$V \approx (\text{m}^3)$$

Mass:

$$M \approx (\text{Kg})$$

Current specific volume (per mass):

$$v = \frac{V}{M} = \frac{1}{\rho} \approx \left(\frac{m^3}{\text{Kg}} \right)$$

Reference specific volume:

$$v_0 = \frac{V_0}{M} = \frac{1}{\rho_0} \approx \left(\frac{m^3}{\text{Kg}} \right)$$

Relative volume:

$$v_r = \frac{V}{V_0} = \frac{(V/M)}{(V_0/M)} = \frac{v}{v_0} = \frac{\rho_0}{\rho}$$

Current normalized volume increment:

$$\frac{dv}{v} = \frac{v - v_0}{v} = 1 - \frac{1}{v_r} = 1 - \frac{\rho}{\rho_0}$$

A frequently used volumetric parameter is:

$$\mu = \frac{1}{v_r} - 1 = \frac{v_0 - v}{v} = -\frac{dv}{v} = \frac{\rho}{\rho_0} - 1$$

Sometimes another volumetric parameter is used:

$$\eta = \frac{v_0}{v} = \frac{\rho}{\rho_0}$$

Thus, the relation between μ and η is,

$$\mu = \frac{v_0 - v}{v} = \eta - 1$$

The following table summarizes these volumetric parameters.

VARIABLES	COMPRESSION	NO LOAD	EXPANSION
$v_r = \dfrac{v}{v_0} = \dfrac{\rho_0}{\rho}$	< 1	1	> 1
$\eta = \dfrac{1}{v_r} = \dfrac{v_0}{v} = \dfrac{\rho}{\rho_0}$	> 1	1	< 1
$\mu = \dfrac{1}{v_r} - 1 = \eta - 1$	> 0	0	< 0

V0 – Initial Relative Volume

There are 3 definitions of density that must be distinguished from each other:

$\rho_0 = \rho_{\text{ref}}$
   $= $ Density at nominal/reference state, usually non-stress or non-deformed state.
$\rho|_{t=0} = $ Density at time 0
   $\rho \ = $ Current density

Recalling the current relative volume

$$v_r = \frac{\rho_0}{\rho} = \frac{v}{v_0},$$

at time = 0 the relative volume is

$$v_{r0} = v_r|_{t=0} = \frac{\rho_0}{\rho|_{t=0}} = \frac{v|_{t=0}}{v_0}.$$

*EOS

Generally, the V0 input parameter in an *EOS card refers to this v_{r0}. ρ_0 is generally the density defined in the *MAT card. Hence, if a material is mechanically compressed at $t = 0$, V0, or v_{r_0}, the initial relative volume, may be computed and input accordingly ($v_0 \neq$ V0).

The "reference" state is a unique state with respect to which the material stress tensor is computed. Therefore v_0 is very critical in computing the pressure level in a material. Incorrect choice of v_0 would lead to incorrect pressure computed. In general, v_0 is chosen such that at zero compression or expansion, the material should be in equilibrium with its ambient surrounding. In many of the equations shown in the EOS section, μ is frequently used as a measure of compression (or expansion). However, the users must clearly distinguish between μ and v_{r0}.

E0 – Internal Energy

Internal energy represents the thermal energy state (temperature dependent component) of a system. One definition for internal energy is

$$E = MC_v T \approx \text{(Joule)}$$

Note that the capital "E" here is the absolute internal energy. It is not the same as that used in the subsequent *EOS keyword input, or some equations shown for each *EOS card. This internal energy is often defined with respect to a mass or volume unit.

Internal energy per unit mass (also called specific internal energy):

$$e = \frac{E}{M} = C_V T \approx \left(\frac{\text{Joule}}{\text{Kg}}\right)$$

Internal energy per unit <u>current</u> volume:

$$e_V = \frac{M}{V} C_V T = \rho C_V T = \frac{C_V T}{v} \approx \left(\frac{\text{Joule}}{\text{m}^3} = \frac{\text{N}}{\text{m}^2}\right)$$

Internal energy per unit <u>reference</u> volume:

$$e_{V_0} = \frac{M}{V_0} C_v T = \rho_0 C_v T = \frac{C_v T}{v_0} \approx \left(\frac{\text{Joule}}{\text{m}^3} = \frac{\text{N}}{\text{m}^2}\right)$$

e_{V_0} typically refers to the capital "E" shown in some equations under this "EOS" section. Hence the initial "<u>*internal energy per unit reference volume*</u>", E0, a keyword input parameter in the *EOS section can be computed from

$$e_{V_0}\big|_{t=0} = \rho_0 C_V T\big|_{t=0}$$

To convert from e_{V0} to e_V, simply divide e_{V0} by v_r

$$e_V = \rho C_V T = [\rho_0 C_V T]\frac{\rho}{\rho_0} = \frac{e_{V_0}}{v_r}$$

Equations of States (EOS)

A thermodynamic state of a homogeneous material, not undergoing any chemical reactions or phase changes, may be defined by two state variables. This relation is generally called an equation of state. For example, a few possible forms relating pressure to two other state variables are

$$P = P(\rho, T) = P(v, e) = P(v_r, e_V) = P(\mu, e_{V_0})$$

The last equation form is frequently used to compute pressure. The EOS for solid phase materials is sometimes partitioned into 2 terms, a cold pressure and a thermal pressure

$$P = P_c(\mu) + P_T(\mu, e_{V_0})$$

$P_c(\mu)$ is the cold pressure hypothetically evaluated along a 0-degree-Kelvin isotherm. This is sometimes called a 0-K pressure-volume relation or cold compression curve. $P_T(\mu, e_{V_0})$ is the thermal pressure component that depends on both volumetric compression and thermal state of the material.

Different forms of the EOS describe different types of materials and how their volumetric compression (or expansion) behaviors. The coefficients for each EOS model come from data-fitting, phenomenological descriptions, or derivations based on classical thermodynamics, etc.

Linear Compression

In low pressure processes, pressure is not significantly affected by temperature. When volumetric compression is within an elastic linear deformation range, a linear bulk modulus may be used to relate volume changes to pressure changes. Recalling the definition of an isotropic bulk modulus is [Fung 1965],

$$\frac{\Delta v}{v} = -\frac{P}{K}.$$

This may be rewritten as

$$P = K\left[-\frac{\Delta v}{v}\right] = K\mu.$$

The bulk modulus, K, thus is equivalent to C_1 in *EOS_LINEAR_POLYNOMIAL when all other coefficients are zero. This is a simplest form of an EOS. To initialize a pressure for such a material, only v_{r0} must be defined.

Initial Conditions

In general, a thermodynamic state must be defined by two state variables. The need to specify v_{r0} and/or $e_{V_0}|_{t=0}$ depends on the form of the EOS chosen. The user should review the equation term-by-term to establish what parameters to be initialized.

*EOS

For many of the EOS available, pressure is specified (given), and the user must make an assumption on either $e_{V_0}\big|_{t=0}$ or v_{r0}. Consider two possibilities (1) $T|_{t=0}$ is defined or assumed from which $e_{V_0}\big|_{t=0}$ may be computed, or (2) $\rho|_{t=0}$ is defined or assumed from which v_{r0} may be obtained.

When to Use EOS

For small strains considerations, a total stress tensor may be partitioned into a deviatoric stress component and a mechanical pressure.

$$\sigma_{ij} = \sigma'_{ij} + \frac{\sigma_{kk}}{3}\delta_{ij} = \sigma'_{ij} - P\delta_{ij}$$

$$P = -\frac{\sigma_{kk}}{3}$$

The pressure component may be written from the diagonal stress components.

Note that $\frac{\sigma_{kk}}{3} = \frac{[\sigma_{11}+\sigma_{22}+\sigma_{33}]}{3}$ is positive in tension while P is positive in compression.

Similarly, the total strain tensor may be partitioned into a deviatoric strain component (volume-preserving deformation) and a volumetric deformation.

$$\varepsilon_{ij} = \varepsilon'_{ij} + \frac{\varepsilon_{kk}}{3}\delta_{ij}$$

where $\frac{\varepsilon_{kk}}{3}$ is called the mean normal strain, and ε_{kk} is called the dilatation or volume strain (change in volume per unit initial volume)

$$\varepsilon_{kk} = \frac{V - V_0}{V_0}$$

Roughly speaking, a typical convention may refer to the relation $\sigma'_{ij} = f(\varepsilon'_{ij})$ as a "constitutive equation", and $P = f(\mu, e_{V_0})$ as an EOS. The use of an EOS may be omitted only when volumetric deformation is very small, and $|P| \ll |\sigma'_{ij}|$.

A Note About Contact When Using an Equation of State

When a part includes an equation of state, it is important that the initial geometry of that part not be perturbed by the contact algorithm. Such perturbation can arise due to initial penetrations in the contact surfaces but can usually be avoided by setting the variable IGNORE to 1 or 2 in the *CONTACT input or by using a segment based contact (SOFT = 2).

*EOS_LINEAR_POLYNOMIAL

This is Equation of State Form 1.

Purpose: Define coefficients for a linear polynomial EOS, and initialize the thermodynamic state of the material by defining E0 and V0 below.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	C0	C1	C2	C3	C4	C5	C6
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	E0	V0						
Type	F	F						

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).
C0	The 0th polynomial equation coefficient
C1	The 1st polynomial equation coefficient (when used by itself, this is the *elastic* bulk modulus, meaning it cannot be used for deformation that is beyond the elastic regime).
⋮	⋮
C6	The 6th polynomial equation coefficient
E0	Initial internal energy per unit reference volume (see the beginning of the *EOS section)
V0	Initial relative volume (see the beginning of the *EOS section)

Remarks:

1. **Pressure.** The linear polynomial equation of state is linear in internal energy. The pressure is given by:

    $$P = C_0 + C_1\mu + C_2\mu^2 + C_3\mu^3 + (C_4 + C_5\mu + C_6\mu^2)E.$$

    where terms $C_2\mu^2$ and $C_6\mu^2$ are set to zero if $\mu < 0$, $\mu = \rho/\rho_0 - 1$, and ρ/ρ_0 is the ratio of current density to reference density. ρ is a nominal or reference density defined in the *MAT_NULL card.

    The linear polynomial equation of state may be used to model gas with the gamma law equation of state. This may be achieved by setting:

    $$C_0 = C_1 = C_2 = C_3 = C_6 = 0$$

    and

    $$C_4 = C_5 = \gamma - 1$$

    where

    $$\gamma = \frac{C_p}{C_v}$$

    is the ratio of specific heats. Pressure for a perfect gas is then given by:

    $$P = (\gamma - 1)\frac{\rho}{\rho_0}E = (\gamma - 1)\frac{e_{V_0}}{v_r}$$

    E has the units of pressure (where $E = e_{V_0}$ and $v_r = \rho_0/\rho$).

2. **Initial Pressure.** When $C_0 = C_1 = C_2 = C_3 = C_6 = 0$, it does not necessarily mean that the initial pressure is zero ($P_0 \neq C_0$!). The initial pressure depends on the values of all the coefficients and on $\mu|_{t=0}$ and $E|_{t=0}$. The pressure in a material is computed from the whole equation above, $P = P(\mu, E)$. It is always preferable to initialize the initial condition based on $\mu|_{t=0}$ and $E|_{t=0}$. The use of $C_0 = C_1 = C_2 = C_3 = C_6 = 0$ must be done with caution as it may change the form and behavior of the material. The safest way is to use the whole EOS equation to manually check for the pressure value. For example, for an ideal gas, only C_4 and C_5 are nonzero; C_4 and C_5 are equal to $\gamma - 1$ while all other coefficients (C_0, C_1, C_2, C_3, and C_6) are zero to satisfy the perfect gas equation form.

3. **V0 and E0.** V0 and E0 defined in this card must be the same as the time-zero ordinates for the 2 load curves defined in the *BOUNDARY_AMBIENT_EOS card, if it is used. This is so that they would both consistently define the same initial state for a material.

*EOS_JWL_{OPTION}

This is Equation of State Form 2.

Available options are:

       \<BLANK\>

       AFTERBURN

Card Summary:

Card 1. This card is required.

EOSID	A	B	R1	R2	OMEG	E0	V0

Card 2a. This card is included if and only if the AFTERBURN keyword option is used and OPT = 1 or 2.

OPT	QT	T1	T2				

Card 2b. This card is included if and only if the AFTERBURN keyword option is used and OPT = 3.

OPT	Q0	QA	QM	QN	CONM	CONL	CONT

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	A	B	R1	R2	OMEG	E0	V0
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).
A	A, see Remarks.
B	B, see Remarks.
R1	R_1, see Remarks.

VARIABLE	DESCRIPTION
R2	R_2, see Remarks.
OMEG	ω, see Remarks.
E0	Detonation energy per unit initial volume and initial value for e_{V_0}. See Remarks.
V0	Initial relative volume, which gives the initial value for v_r. See Remarks.

Afterburn Card. Additional card for AFTERBURN option with OPT = 1 or 2.

Card 2a	1	2	3	4	5	6	7	8
Variable	OPT	QT	T1	T2				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
OPT	Afterburn option:
	EQ.0.0: No afterburn energy (Standard *EOS_JWL)
	EQ.1.0: Constant rate of afterburn energy added between times T1 and T2
	EQ.2.0: Linearly increasing rate of afterburn energy added between times T1 and T2
	EQ.3.0: Miller's extension for afterburn energy
QT	Afterburn energy per unit volume for simple afterburn
T1	Start time of energy addition for simple afterburn
T2	End time of energy addition for simple afterburn

Afterburn Card. Additional card for AFTERBURN option with OPT = 3.

Card 2b	1	2	3	4	5	6	7	8
Variable	OPT	Q0	QA	QM	QN	CONM	CONL	CONT
Type	F	F	F	F	F	F	F	F
Default	none	none	none	0.5	1/6	1.	1.	1.

VARIABLE	DESCRIPTION

OPT
Afterburn option

    EQ.0.0: No afterburn energy (Standard *EOS_JWL)

    EQ.1.0: Constant rate of afterburn energy added between times T1 and T2

    EQ.2.0: Linearly increasing rate of afterburn energy added between times T1 and T2

    EQ.3.0: Miller's extension for afterburn energy

Q0
Afterburn energy per unit volume for Miller's extension

QA
Energy release constant a for Miller's extension

QM
Energy release exponent m for Miller's extension

QN
Pressure exponent n for Miller's extension

CONM
Mass Conversion factors:

    GT.0.0: Mass conversion factor from model units to calibration units for Miller's extension

    LT.0.0: Use predefined factors to convert model units to published calibration units of g, cm, µs. Choices for model units are:

        EQ.-1.0: g, mm, ms

        EQ.-2.0: g, cm, ms

        EQ.-3.0: kg, m, s

        EQ.-4.0: kg, mm, ms

        EQ.-5.0: metric ton, mm, s

VARIABLE	DESCRIPTION
	EQ.-6.0: lbf-s^2/in, in, s
	EQ.-7.0: slug, ft, s
CONL	CONM.GT.0.0: Length conversion factor from model units to calibration units for Miller's extension
	CONM.LT.0.0: Ignored
CONT	CONM.GT.0.0: Time conversion factor from model units to calibration units for Miller's extension
	CONM.LT.0.0: Ignored

Remarks:

1. **Equation of State.** The JWL equation of state defines the pressure as

$$p = A\left(1 - \frac{\omega}{R_1 v_r}\right)e^{-R_1 v_r} + B\left(1 - \frac{\omega}{R_2}\right)e^{-R_2 v_r} + \frac{\omega e_{V_0}}{v_r},$$

and is usually used for detonation products of high explosives.

A, B, and E0 have units of pressure. R1, R2, OMEG, and V0 are dimensionless. We recommend using a unit system of grams, centimeters, and microseconds when a model includes high explosive(s). In this consistent unit system, pressure is in Mbar.

2. **Afterburn.** The AFTERBURN option allows the addition of afterburn energy Q to the calculation of pressure by replacing e_{V_0} in the above equation with $(e_{V_0} + Q)$, that is, the last term on the right-hand side becomes

$$\frac{\omega(e_{V_0} + Q)}{v_r}$$

The simple afterburn option adds the energy at a constant rate (OPT = 1) or a linearly-increasing rate (OPT = 2) between times T1 and T2 such that the total energy added per unit volume at time T2 is the specified energy QT.

For the Miller's extension model (OPT = 3), the afterburn energy is added using a time-dependent growth term

$$\frac{d\lambda}{dt} = a(1 - \lambda)^m p^n, \qquad Q = \lambda Q_0 \ .$$

Here, m, n, and λ are dimensionless, with λ a positive fraction less than 1.0. The parameter a has units consistent with this growth equation, and Q_0 has units of pressure.

The values for Q_0, a, m, n published by Miller and Guirguis (1993) are calibrated in the units of g, cm, μs, with the consistent pressure unit of Mbar, though in principle any consistent set of units may be used for calibration. The factors CONM, CONL, and CONT convert the unit system of the model being analyzed to the calibration unit system in which the Miller's extension parameters are specified, such that a mass value in model units may be multiplied by CONM to obtain the corresponding value in calibration units. These conversion factors allow consistent evaluation of the growth equation in the calibrated units. For user convenience, predefined conversion factors are provided for converting various choices for the model units system to the calibration unit system used by Miller and Guirguis.

3. **History Variables.** When this equation of state is used with *MAT_HIGH_EX-PLOSIVE_BURN in which the variable BETA is set to 0 or 2, the absolute value of the history variable labeled as "effective plastic strain" is the explosive light-ing time. This lighting time takes into account shadowing if invoked (see *CON-TROL_EXPLOSIVE_SHADOW).

    There are four additional history variables for the JWL equation of state. Those history variables are internal energy, bulk viscosity in units of pressure, volume, and burn fraction, respectively. To output the history variables, set the variable NEIPH in *DATABASE_EXTENT_BINARY.

    The AFTERBURN option introduces an additional 5th history variable that records the added afterburn energy Q for simple afterburn (OPT = 1 or 2) but contains the growth term λ when using the Miller's extension model (OPT = 3).

*EOS_SACK_TUESDAY

This is Equation of State Form 3.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	A1	A2	A3	B1	B2	E0	V0
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).
Ai, Bi	Constants in the equation of state
E0	Initial internal energy
V0	Initial relative volume

Remarks:

The Sack equation of state defines pressure as

$$p = \frac{A_3}{V^{A_1}} e^{-A_2 V} \left(1 - \frac{B_1}{V}\right) + \frac{B_2}{V} E$$

and is used for detonation products of high explosives.

*EOS_GRUNEISEN

This is Equation of State Form 4.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	C	S1	S2	S3	GAMMA0	A	E0
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	V0	(not used)	LCID					
Type	F		I					

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).
C, Si, GAMMA0	Constants in the equation of state
A	First order volume correction coefficient
E0	Initial internal energy
V0	Initial relative volume
LCID	Load curve ID, which can be the ID of a *DEFINE_CURVE, *DEFINE_CURVE_FUNCTION, or *DEFINE_FUNCTION, defining the energy deposition rate as a function of time. If an energy leak rate is intended, do not specify a negative ordinate in LCID, rather, use the constant(s) in the equation of state, that is, set GAMMA0 and/or A to a negative value. If *DEFINE_FUNCTION is used, the input of the defined function is time.

Remarks:

The Gruneisen equation of state with cubic shock-velocity as a function of particle-velocity $v_s(v_p)$ defines pressure for compressed materials as

$$p = \frac{\rho_0 C^2 \mu \left[1 + \left(1 - \frac{\gamma_0}{2}\right)\mu - \frac{a}{2}\mu^2\right]}{\left[1 - (S_1 - 1)\mu - S_2 \frac{\mu^2}{\mu + 1} - S_3 \frac{\mu^3}{(\mu + 1)^2}\right]^2} + (\gamma_0 + a\mu)E$$

and for expanded materials as

$$p = \rho_0 C^2 \mu + (\gamma_0 + a\mu)E \ .$$

Here C is the intercept of the $v_s(v_p)$ curve (in velocity units); S_1, S_2, and S_3 are the unitless coefficients of the slope of the $v_s(v_p)$ curve; γ_0 is the unitless Gruneisen gamma; a is the unitless, first order volume correction to γ_0; and

$$\mu = \frac{\rho}{\rho_0} - 1 \ .$$

E denotes the internal energy, which is increased according to an energy deposition rate as a function of time curve (LCID).

*EOS_RATIO_OF_POLYNOMIALS

This is Equation of State Form 5.

Card Summary:

Card 1. This card is required.

EOSID							

Card 2. This card is required.

A10	A11	A12	A13

Card 3. This card is required.

A20	A21	A22	A23

Card 4. This card is required.

A30	A31	A32	A33

Card 5. This card is required.

A40	A41	A42	A43

Card 6. This card is required.

A50	A51	A52	A53

Card 7. This card is required.

A60	A61	A62	A63

Card 8. This card is required.

A70	A71	A72	A73

Card 9. This card is required.

A14	A24		

Card 10. This card is required.

ALPHA	BETA	E0	V0

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID							
Type	A							

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).

Card 2	1	2	3	4	5	6	7	8
Variable	A10		A11		A12		A13	
Type	F		F		F		F	

Card 3	1	2	3	4	5	6	7	8
Variable	A20		A21		A22		A23	
Type	F		F		F		F	

Card 4	1	2	3	4	5	6	7	8
Variable	A30		A31		A32		A33	
Type	F		F		F		F	

Card 5	1	2	3	4	5	6	7	8
Variable	A40		A41		A42		A43	
Type	F		F		F		F	

Card 6	1	2	3	4	5	6	7	8
Variable	A50		A51		A52		A53	
Type	F		F		F		F	

Card 7	1	2	3	4	5	6	7	8
Variable	A60		A61		A62		A63	
Type	F		F		F		F	

Card 8	1	2	3	4	5	6	7	8
Variable	A70		A71		A72		A73	
Type	F		F		F		F	

Card 9	1	2	3	4	5	6	7	8
Variable	A14		A24					
Type	F		F					

VARIABLE	DESCRIPTION
Aij	Polynomial coefficients

Card 10	1	2	3	4	5	6	7	8
Variable	ALPHA		BETA		E0		V0	
Type	F		F		F		F	

VARIABLE	DESCRIPTION
ALPHA	α
BETA	β
E0	Initial internal energy
V0	Initial relative volume

Remarks:

The ratio of polynomials equation of state defines the pressure as

$$p = \frac{F_1 + F_2 E + F_3 E^2 + F_4 E^3}{F_5 + F_6 E + F_7 E^2} (1 + \alpha\mu) \ ,$$

where

$$F_i = \sum_{j=0}^{n} A_{ij}\mu^j \ , \qquad n = \begin{cases} 4 & i < 3 \\ 3 & i \geq 3 \end{cases}$$

$$\mu = \frac{\rho}{\rho_0} - 1$$

In expanded elements F_1 is replaced by $F_1' = F_1 + \beta\mu^2$. By setting coefficient $A_{10} = 1.0$, the delta-phase pressure modeling for this material will be initiated. The code will reset it to 0.0 after setting flags.

*EOS_LINEAR_POLYNOMIAL_WITH_ENERGY_LEAK

This is Equation of State Form 6.

Purpose: Define coefficients for a linear polynomial EOS and initialize the thermodynamic state of the material by defining E0 and V0 below. Energy deposition is prescribed using a curve.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	C0	C1	C2	C3	C4	C5	C6
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	E0	V0	LCID					
Type	F	F	I					

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see*PART).
Ci	Constants in the equation of state
E0	Initial internal energy
V0	Initial relative volume
LCID	Load curve ID, which can be the ID of *DEFINE_CURVE, *DEFINE_CURVE_FUNCTION or *DEFINE_FUNCTION, defining the specific energy deposition rate as a function of time. If a specific energy leak rate is intended, do not specify a negative ordinate in LCID, rather, use the constant(s) in the equation of state, such as setting C4 to a negative value. If *DEFINE_FUNCTION is used, the input of the defined function is time. Note that the specific energy rate has units of energy per time and volume.

Remarks:

This polynomial equation of state, linear in the internal energy per initial volume, E, is given by

$$p = C_0 + C_1\mu + C_2\mu^2 + C_3\mu^3 + (C_4 + C_5\mu + C_6\mu^2)E$$

in which C_1, C_2, C_3, C_4, C_5, and C_6 are user defined constants and

$$\mu = \frac{1}{V} - 1 \ .$$

where V is the relative volume. In expanded elements, we set the coefficients of μ^2 to zero, that is,

$$C_2 = C_6 = 0$$

Internal energy, E, is increased according to an energy deposition rate as a function of time curve (LCID).

*EOS_IGNITION_AND_GROWTH_OF_REACTION_IN_HE

This is Equation of State Form 7.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	A	B	XP1	XP2	FRER	G	R1
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	R2	R3	R5	R6	FMXIG	FREQ	GROW1	EM
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	AR1	ES1	CVP	CVR	EETAL	CCRIT	ENQ	TMP0
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	GROW2	AR2	ES2	EN	FMXGR	FMNGR		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).
A	Product JWL constant (see second equation in Remarks)
B	Product JWL constant (see second equation in Remarks)
XP1	Product JWL constant (see second equation in Remarks)

VARIABLE	DESCRIPTION
XP2	Product JWL constant (see second equation in Remarks)
FRER	Constant in ignition term of reaction equation
G	ωC_v of product
R1	Unreacted JWL constant (see first equation in Remarks)
R2	Unreacted JWL constant (see first equation in Remarks)
R3	ωC_v of unreacted explosive
R5	Unreacted JWL constant (see first equation in Remarks)
R6	Unreacted JWL constant (see first equation in Remarks)
FMXIG	Maximum F for ignition term
FREQ	Constant in ignition term of reaction equation
GROW1	Constant in growth term of reaction equation
EM	Constant in growth term of reaction equation
AR1	Constant in growth term of reaction equation
ES1	Constant in growth term of reaction equation
CVP	Heat capacity of reaction products
CVR	Heat capacity of unreacted HE
EETAL	Constant in ignition term of reaction equation
CCRIT	Constant in ignition term of reaction equation
ENQ	Heat of reaction
TMP0	Initial temperature (°K)
GROW2	Constant in completion term of reaction equation
AR2	Constant in completion term of reaction equation
ES2	Constant in completion term of reaction equation
EN	Constant in completion term of reaction equation

VARIABLE	DESCRIPTION
FMXGR	Maximum F for growth term
FMNGR	Minimum F for completion term

Remarks:

Equation of State Form 7 is used to calculate the shock initiation (or failure to initiate) and detonation wave propagation of solid high explosives. It should be used instead of the ideal HE burn options whenever there is a question whether the HE will react, there is a finite time required for a shock wave to build up to detonation, and/or there is a finite thickness of the chemical reaction zone in a detonation wave. At relatively low initial pressures (<2-3 GPa), this equation of state should be used with material type 10 for accurate calculations of the unreacted HE behavior. At higher initial pressures, material type 9 can be used. A JWL equation of state defines the pressure in the <u>unreacted</u> explosive as

$$P_e = r_1 e^{-r_5 V_e} + r_2 e^{-r_6 V_e} + r_3 \frac{T_e}{V_e}, \qquad (r_3 = \omega_e C_{v_r})$$

where V_e and T_e are the relative volume and temperature, respectively, of the unreacted explosive. Another JWL equation of state defines the pressure in the <u>reaction products</u> as

$$P_p = ae^{-xp_1 V_p} + be^{-xp_2 V_p} + \frac{gT_p}{V_p}, \qquad (g = \omega_p C_{v_p})$$

where V_p and T_p are the relative volume and temperature, respectively, of the reaction products. As the chemical reaction converts unreacted explosive to reaction products, these JWL equations of state are used to calculate the mixture of unreacted explosive and reaction products defined by the fraction reacted F ($F = 0$ implies no reaction, $F = 1$ implies complete reaction). The temperatures and pressures are assumed to be equal ($T_e = T_p, p_e = p_p$) and the relative volumes are additive, that is,

$$V = (1 - F)V_e + FV_p$$

The chemical reaction rate for conversion of unreacted explosive to reaction products consists of three physically realistic terms: an ignition term in which a small amount of explosive reacts soon after the shock wave compresses it; a slow growth of reaction as this initial reaction spreads; and a rapid completion of reaction at high pressure and temperature. The form of the reaction rate equation is

$$\frac{\partial F}{\partial t} = \overbrace{\text{FREQ} \times (1-F)^{\text{FRER}}(V_e^{-1} - 1 - \text{CCRIT})^{\text{EETAL}}}^{\text{Ignition}} + \overbrace{\text{GROW1} \times (1-F)^{\text{ES1}}F^{\text{AR1}}p^{\text{EM}}}^{\text{Growth}}$$
$$+ \underbrace{\text{GROW2} \times (1-F)^{\text{ES2}}F^{\text{AR2}}p^{\text{EN}}}_{\text{Completion}}$$

The ignition rate is set equal to zero when $F \geq$ FMXIG, the growth rate is set equal to zero when $F \geq$ FMXGR, and the completion rate is set equal to zero when $F \leq$ FMNGR.

Details of the computational methods and many examples of one and two dimensional shock initiation and detonation wave calculation can be found in the references (Cochran and Chan [1979], Lee and Tarver [1980]). Unfortunately, sufficient experimental data has been obtained for only two solid explosives to develop very reliable shock initiation models: PBX-9504 (and the related HMX-based explosives LX-14,LX-10,LX-04, etc.) and LX-17 (the insensitive TATB-based explosive). Reactive flow models have been developed for other explosives (TNT, PETN, Composition B, propellants, etc.) but are based on very limited experimental data.

When this EOS is used with *MAT_009, history variables 4, 7, 9, and 10 are temperature, burn fraction, $1/V_e$, and $1/V_p$, respectively. When used with *MAT_010, those histories variables are incremented by 1, that is, history variables 5, 8, 10, and 11 are temperature, burn fraction, $1/V_e$, and $1/V_p$, respectively. See NEIPH in *DATABASE_EXTENT_BINARY if these output variables are desired in the databases for post-processing.

*EOS_TABULATED_COMPACTION

This is Equation of State Form 8.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	GAMMA	E0	V0	LCC	LCT	LCK	LCID
Type	A	F	F	F	I	I	I	I

Parameter Card Pairs. Include one pair of the following two cards for each of VAR = ε_{v_i}, C_i, T_i, and K_i. These cards consist of four additional pairs for a total of 8 additional cards.

Card 3	1	2	3	4	5	6	7	8	9	10
Variable	[VAR]1		[VAR]2		[VAR]3		[VAR]4		[VAR]5	
Type	F		F		F		F		F	

Card 4	1	2	3	4	5	6	7	8	9	10
Variable	[VAR]6		[VAR]7		[VAR]8		[VAR]9		[VAR]10	
Type	F		F		F		F		F	

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).
GAMMA	γ (unitless); see equation in Remarks.
E0	Initial internal energy per unit reference volume (force per unit area)
V0	Initial relative volume (unitless)
LCC	Load curve defining tabulated function C. See equation in Remarks. The abscissa values of LCC, LCT and LCK must be negative of the volumetric strain in monotonically *increasing* order, in

VARIABLE	DESCRIPTION
	contrast to the convention in *EOS_009. The definition can extend into the tensile regime.
LCT	Load curve defining tabulated function T. See equation in Remarks.
LCK	Load curve defining tabulated bulk modulus
LCID	Load curve ID, which can be the ID of *DEFINE_CURVE, *DEFINE_CURVE_FUNCTION or *DEFINE_FUNCTION, defining the energy deposition rate as a function of time. If an energy leak rate is intended, do not specify a negative ordinate in LCID, rather, use the constant(s) in the equation of state, that is, set GAMMA to a negative value. If *DEFINE_FUNCTION is used, the input of the defined function is time.
$\varepsilon_{v1}, \varepsilon_{v2}, \ldots, \varepsilon_{v_N}$	Volumetric strain, $\ln(V)$. The first abscissa point, EV1, must be 0.0 or positive if the curve extends into the tensile regime with subsequent points *decreasing* monotonically.
$C_1, C_2, \ldots, C_N$	$C(\varepsilon_V)$ (units = force per unit area); see equation in Remarks.
$T_1, T_2, \ldots, T_N$	$T(\varepsilon_V)$ (unitless) ; see equation in Remarks.
$K_1, K_2, \ldots, K_N$	Bulk unloading modulus (units = force per unit area)

Remarks:

The tabulated compaction model is linear in the internal energy E, which is increased according to an energy deposition rate as a function of time curve (LCID). Pressure is defined by

$$p = C(\varepsilon_V) + \gamma \, T(\varepsilon_V)E$$

in the loading phase. The volumetric strain, ε_V, is given by the natural logarithm of the relative volume, V. Unloading occurs along the unloading bulk modulus to the pressure cutoff. The minimum bulk unloading modulus must be defined by K_1. The pressure cutoff, a tension limit, is defined in the material model definition. Reloading always follows the unloading path to the point where unloading began and continues on the loading path; see Figure EOS8-1. Up to 10 points and as few as 2 may be used when defining the tabulated functions. LS-DYNA will extrapolate to find the pressure if necessary.
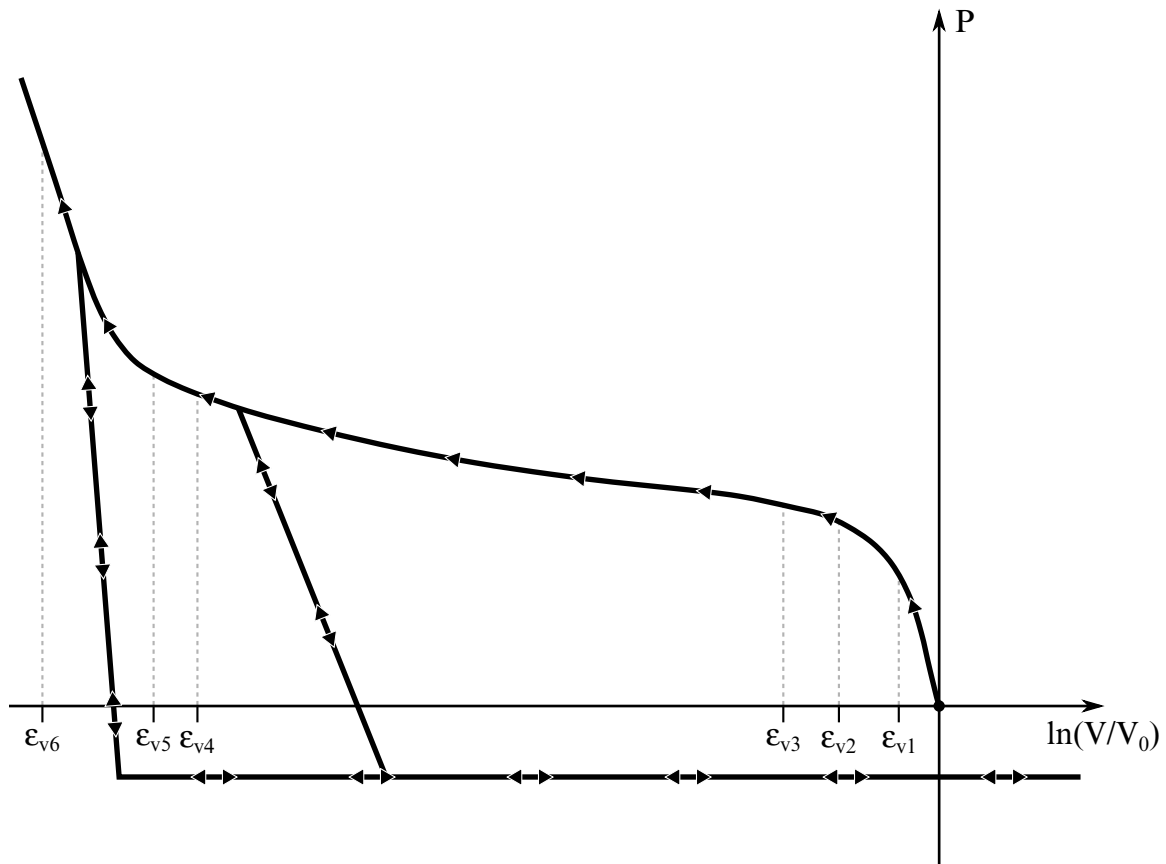
Figure EOS8-1. Pressure as a function of volumetric strain curve for Equation of State Form 8 with compaction. In the compacted states the bulk unloading modulus depends on the peak volumetric strain. Volumetric strain values should be input with correct sign (negative in compression) and in descending order. Pressure is positive in compression.

*EOS_TABULATED

This is Equation of State Form 9.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	GAMA	E0	V0	LCC	LCT		
Type	A	F	F	F	I	I		

Parameter Card Pairs. Include one pair of the following two cards for each of VAR = ε_{V_i}, C_i, T_i. These cards consist of three additional pairs for a total of 6 additional cards. These cards are not required if LCC and LCT are specified.

Card 2	1	2	3	4	5	6	7	8	9	10
Variable	[VAR]1		[VAR]2		[VAR]3		[VAR]4		[VAR]5	
Type	F		F		F		F		F	

Card 3	1	2	3	4	5	6	7	8	9	10
Variable	[VAR]6		[VAR]7		[VAR]8		[VAR]9		[VAR]10	
Type	F		F		F		F		F	

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).
GAMA	γ, (unitless) see equation in Remarks.
E0	Initial internal energy per unit reference volume (force per unit area).
V0	Initial relative volume (unitless).

VARIABLE	DESCRIPTION
LCC	Load curve defining tabulated function C. See equation in Remarks. The abscissa values of LCC and LCT must *increase* monotonically. The definition can extend into the tensile regime.
LCT	Load curve defining tabulated function T. See equation in Remarks.
$\varepsilon_{V_1}, \varepsilon_{V_2}, \ldots, \varepsilon_{V_N}$	Volumetric strain, $\ln(V)$, where V is the relative volume. The first abscissa point, EV1, must be 0.0 or positive if the curve extends into the tensile regime with subsequent points *decreasing* monotonically.
$C_1, C_2, \ldots, C_N$	Tabulated points for function C (force per unit area).
$T_1, T_2, \ldots, T_N$	Tabulated points for function T (unitless).

Remarks:

The tabulated equation of state model is linear in internal energy. Pressure is defined by

$$P = C(\varepsilon_V) + \gamma T(\varepsilon_V) E$$

The volumetric strain, ε_V is given by the natural logarithm of the relative volume V. Up to 10 points and as few as 2 may be used when defining the tabulated functions. LS-DYNA will extrapolate to find the pressure if necessary.

*EOS_PROPELLANT_DEFLAGRATION

This is Equation of State Form 10. It has been added to model airbag propellants.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	A	B	XP1	XP2	FRER		
Type	A	F	F	F	F	F		

Card 2	1	2	3	4	5	6	7	8
Variable	G	R1	R2	R3	R5			
Type	F	F	F	F	F			

Card 3	1	2	3	4	5	6	7	8
Variable	R6	FMXIG	FREQ	GROW1	EM			
Type	F	F	F	F	F			

Card 4	1	2	3	4	5	6	7	8
Variable	AR1	ES1	CVP	CVR	EETAL	CCRIT	ENQ	TMP0
Type	F	F	F	F	F			

Card 5	1	2	3	4	5	6	7	8
Variable	GROW2	AR2	ES2	EN	FMXGR	FMNGR		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).
A	Product JWL coefficient
B	Product JWL coefficient
XP1	Product JWL coefficient
XP2	Product JWL coefficient
FRER	Unreacted Co-volume
G	Product ωC_v
R1	Unreacted JWL coefficient
R2	Unreacted JWL coefficient
R3	Unreacted ωC_v
R5	Unreacted JWL coefficient
R6	Unreacted JWL coefficient
FMXIG	Initial Fraction Reacted F_0
FREQ	Initial Pressure P_0
GROW1	First burn rate coefficient
EM	Pressure Exponent (1st term)
AR1	Exponent on F (1st term)
ES1	Exponent on $(1 - F)$ (1st term)
CVP	Heat capacity C_v for products
CVR	Heat capacity C_v for unreacted material
EETAL	Extra, not presently used
CCRIT	Product co-volume
ENQ	Heat of Reaction

VARIABLE	DESCRIPTION
TMP0	Initial Temperature (298°K)
GROW2	Second burn rate coefficient
AR2	Exponent on F (2nd term)
ES2	Exponent on $(1 - F)$ (2nd term)
EN	Pressure Exponent (2nd term)
FMXGR	Maximum F for 1st term
FMNGR	Minimum F for 2nd term

Remarks:

A deflagration (burn rate) reactive flow model requires an unreacted solid equation of state, a reaction product equation of state, a reaction rate law and a mixture rule for the two (or more) species. The mixture rule for the standard ignition and growth model [Lee and Tarver 1980] assumes that both pressures and temperatures are completely equilibrated as the reaction proceeds. However, the mixture rule can be modified to allow no thermal conduction or partial heating of the solid by the reaction product gases. For this relatively slow process of airbag propellant burn, the thermal and pressure equilibrium assumptions are valid. The equations of state currently used in the burn model are the JWL, Gruneisen, the van der Waals co-volume, and the perfect gas law, but other equations of state can be easily implemented. In this propellant burn, the gaseous nitrogen produced by the burning sodium azide obeys the perfect gas law as it fills the airbag but may have to be modeled as a van der Waal's gas at the high pressures and temperatures produced in the propellant chamber. The chemical reaction rate law is pressure, particle geometry and surface area dependent, as are most high-pressure burn processes. When the temperature profile of the reacting system is well known, temperature dependent Arrhenius chemical kinetics can be used.

Since the airbag propellant composition and performance data are company private information, it is very difficult to obtain the required information for burn rate modeling. However, Imperial Chemical Industries (ICI) Corporation supplied pressure exponent, particle geometry, packing density, heat of reaction, and atmospheric pressure burn rate data which allowed us to develop the numerical model presented here for their NaN_3 + Fe_2O_3 driver airbag propellant. The deflagration model, its implementation, and the results for the ICI propellant are presented in [Hallquist, et.al., 1990].

The unreacted propellant and the reaction product equations of state are both of the form:

$$p = Ae^{-R_1V} + Be^{-R_2V} + \frac{\omega C_v T}{V - d} \ ,$$

where p is pressure (in Mbars), V is the relative specific volume (inverse of relative density), ω is the Gruneisen coefficient, C_v is heat capacity (in Mbars -cc/cc°K), T is the temperature in °K, d is the co-volume, and A, B, R_1 and R_2 are constants. Setting $A = B = 0$ yields the van der Waal's co-volume equation of state. The JWL equation of state is generally useful at pressures above several kilobars, while the van der Waal's is useful at pressures below that range and above the range for which the perfect gas law holds. Additionally, setting $A = B = d = 0$ yields the perfect gas law. If accurate values of ω and C_v plus the correct distribution between "cold" compression and internal energies are used, the calculated temperatures are very reasonable and thus can be used to check propellant performance.

The reaction rate used for the propellant deflagration process is of the form:

$$\frac{\partial F}{\partial t} = \underbrace{Z(1 - F)^y F^x p^w}_{0<F<F_{\text{limit1}}} + \underbrace{V(1 - F)^u F^r p^s}_{F_{\text{limit2}}<F<1}$$

where F is the fraction reacted ($F = 0$ implies no reaction, $F = 1$ is complete reaction), t is time, and p is pressure (in Mbars), r, s, u, w, x, y, F_{limit1} and F_{limit2} are constants used to describe the pressure dependence and surface area dependence of the reaction rates. Two (or more) pressure dependant reaction rates are included in case the propellant is a mixture or exhibited a sharp change in reaction rate at some pressure or temperature. Burning surface area dependencies can be approximated using the $(1 - F)^y F^x$ terms. Other forms of the reaction rate law, such as Arrhenius temperature dependent $e^{-E/RT}$ type rates, can be used, but these require very accurate temperatures calculations. Although the theoretical justification of pressure dependent burn rates at kilobar type pressures is not complete, a vast amount of experimental burn rate as a function of pressure data does demonstrate this effect and hydrodynamic calculations using pressure dependent burn accurately simulate such experiments.

The deflagration reactive flow model is activated by any pressure or particle velocity increase on one or more zone boundaries in the reactive material. Such an increase creates pressure in those zones and the decomposition begins. If the pressure is relieved, the reaction rate decreases and can go to zero. This feature is important for short duration, partial decomposition reactions. If the pressure is maintained, the fraction reacted eventually reaches one and the material is completely converted to product molecules. The deflagration front rates of advance through the propellant calculated by this model for several propellants are quite close to the experimentally observed burn rate versus pressure curves.

To obtain good agreement with experimental deflagration data, the model requires an accurate description of the unreacted propellant equation of state, either an analytical fit to experimental compression data or an estimated fit based on previous experience with

similar materials. This is also true for the reaction products equation of state. The more experimental burn rate, pressure production and energy delivery data available, the better the form and constants in the reaction rate equation can be determined.

Therefore, the equation used in the burn subroutine for the pressure in the unreacted propellant is

$$P_u = \text{R1} \times e^{-\text{R5} \times V_u} + \text{R2} \times e^{-\text{R6} \backslash times\, V_u} + \frac{\text{R3} \times T_u}{V_u - \text{FRER}} \, ,$$

where V_u and T_u are the relative volume and temperature respectively of the unreacted propellant. The relative density is obviously the inverse of the relative volume. The pressure P_p in the reaction products is given by:

$$P_p = \text{A} \times e^{-\text{XP1} \times V_p} + \text{B} \times e^{-\text{XP2} \times V_p} + \frac{G \times T_p}{V_P - \text{CCRIT}} \, .$$

As the reaction proceeds, the unreacted and product pressures and temperatures are assumed to be equilibrated ($T_u = T_p = T$, $P = P_u = P_p$) while the relative volumes are additive:

$$V = (1 - F)V_u + FV_p$$

where V is the total relative volume. Other mixture assumptions can and have been used in different versions of DYNA2D/3D. The reaction rate law has the form:

$$\frac{\partial F}{\partial t} = \text{GROW1} \times (P + \text{FREQ})^{\text{EM}} (F + \text{FMXIG})^{\text{AR1}} (1 - F + \text{FMIXG})^{\text{ES1}}$$
$$+ \text{GROW2} \times (P + \text{FREQ})^{\text{EN}} (F + \text{FMIXG})^{\text{AR2}} (1 - F + \text{FMIXG})^{\text{ES2}} \, .$$

If F exceeds FMXGR, the GROW1 term is set equal to zero, and, if F is less than FMNGR, the GROW2 term is zero. Thus, two separate (or overlapping) burn rates can be used to describe the rate at which the propellant decomposes.

This equation of state subroutine is used together with a material model to describe the propellant. In the airbag propellant case, a null material model (type #10) can be used. Material type 10 is usually used for a solid propellant or explosive when the shear modulus and yield strength are defined. The propellant material is defined by the material model and the unreacted equation of state until the reaction begins. The calculated mixture states are used until the reaction is complete and then the reaction product equation of state is used. The heat of reaction, ENQ, is assumed to be a constant and the same at all values of F but more complex energy release laws could be implemented.

History variables 4 and 7 are temperature and burn fraction, respectively. See NEIPH in *DATABASE_EXTENT_BINARY if these output variables are desired in the databases for post-processing.

*EOS_TENSOR_PORE_COLLAPSE

This is Equation of State Form 11.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	NLD	NCR	MU1	MU2	IE0	EC0	
Type	A	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).
NLD	Virgin loading load curve ID
NCR	Completely crushed load curve ID
MU1	Excess Compression required before any pores can collapse
MU2	Excess Compression point where the Virgin Loading Curve and the Completely Crushed Curve intersect
IE0	Initial Internal Energy
EC0	Initial Excess Compression

Remarks:

The pore collapse model described in the TENSOR manual [23] is no longer valid and has been replaced by a much simpler method. This is due in part to the lack of experimental data required for the more complex model. It is desired to have a close approximation of the TENSOR model in the LS-DYNA code to enable a quality link between them. The TENSOR model defines two curves, the virgin loading curve and the completely crushed curve, as shown in Figure EOS11-1, as well as the excess compression point required for pore collapse to begin, μ_1, and the excess compression point required to completely crush the material, μ_2. From this data and the maximum excess compression the material has attained, $u_{\max}$, the pressure for any excess compression, μ, can be determined.

Unloading occurs along the virgin loading curve until the excess compression surpasses μ_1. After that, the unloading follows a path between the completely crushed curve and the virgin loading curve. Reloading will follow this curve back up to the virgin loading
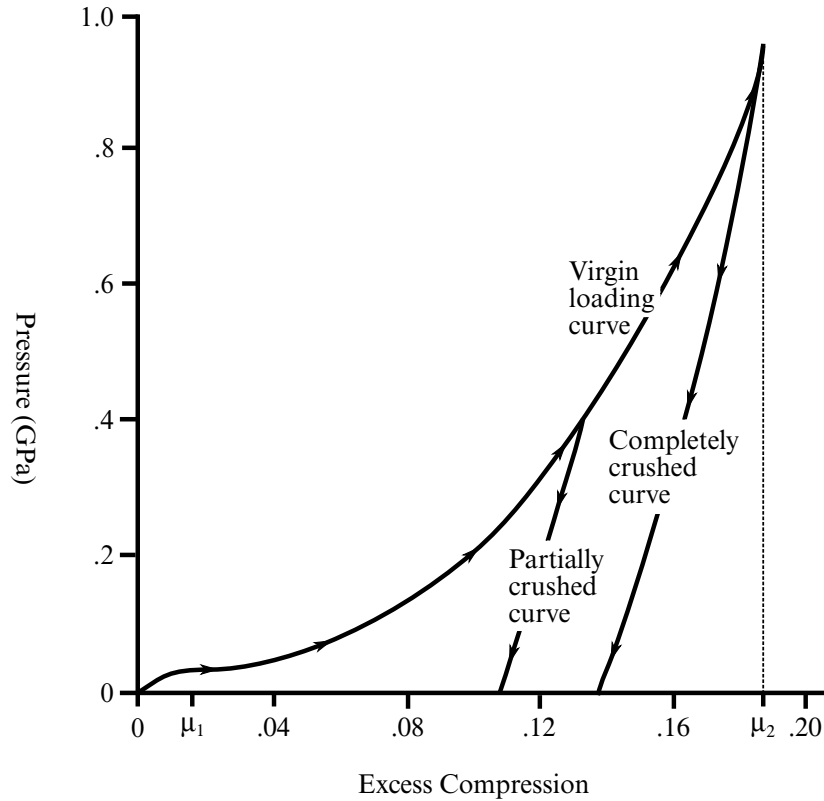
Figure EOS11-1. Pressure versus compaction curve

curve. Once the excess compression exceeds μ_2, then all unloading will follow the completely crushed curve.

For unloading between μ_1 and μ_2 a partially crushed curve is determined by:

$$p_{\mathrm{pc}}(\mu) = p_{\mathrm{cc}} \overbrace{\left[\frac{(1 + \mu_B)(1 + \mu)}{1 + \mu_{\max}} - 1 \right]}^{\mu_a} ,$$

where

$$\mu_B = P_{\mathrm{cc}}^{-1}(P_{\max})$$

and the subscripts "pc" and "cc" refer to the partially crushed and completely crushed states, respectively. This is more readily understood in terms of the relative volume, V.

$$V = \frac{1}{1 + \mu}$$

$$P_{\mathrm{pc}}(V) = P_{\mathrm{cc}}\left(\frac{V_B}{V_{\min}} V \right)$$

This representation suggests that for a fixed

$$V_{\min} = \frac{1}{\mu_{\max} + 1}$$

the partially crushed curve will separate linearly from the completely crushed curve as *V* increases to account for pore recovery in the material.

The bulk modulus *K* is determined to be the slope of the current curve times one plus the excess compression

$$K = \frac{\partial P}{\partial \mu}(1 + \mu) \ .$$

The slope $\frac{\partial P}{\partial \mu}$ for the partially crushed curve is obtained by differentiation as:

$$\frac{\partial p_{\text{pc}}}{\partial \mu} = \frac{\partial p_{\text{cc}}}{\partial x}\bigg|_{x = \frac{(1+\mu_b)(1+\mu)}{1+\mu_{\text{max}}} - 1} \left(\frac{1 + \mu_b}{1 + \mu_{\text{max}}}\right) \ .$$

Simplifying,

$$K = \frac{\partial P_{\text{cc}}}{\partial \mu_a}\bigg|_{\mu_{\text{a}}} (1 + \mu_a) \ ,$$

where

$$\mu_a = \frac{(1 + \mu_B)(1 + \mu)}{(1 + \mu_{\text{max}})} - 1 \ .$$

The bulk sound speed is determined from the slope of the completely crushed curve at the current pressure to avoid instabilities in the time step.

The virgin loading and completely crushed curves are modeled with monotonic cubic-splines. An optimized vector interpolation scheme is then used to evaluate the cubic-splines. The bulk modulus and sound speed are derived from a linear interpolation on the derivatives of the cubic-splines.

*EOS_IDEAL_GAS

Purpose: This is Equation of State Form 12 for modeling ideal gas. It is an alternate approach to using *EOS_LINEAR_POLYNOMIAL with C4 = C5 = $(\gamma - 1)$ to model ideal gas. This has a slightly improved energy accounting algorithm.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	CVMASS	CPMASS	B	C	T0	V0	VCO
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	ADIAB							
Type	F							

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).
CVMASS	Nominal constant-volume specific heat coefficient, $C_{v,\mathrm{mass}}$ (see Remark 1)
CPMASS	Nominal constant-pressure specific heat coefficient, $C_{p,\mathrm{mass}}$ (see Remark 1)
B	Linear coefficient, B, for the variations of C_v and C_p as a function of T (see Remark 1)
C	Quadratic coefficient, C, for the variations of C_v and C_p as a function of T (see Remark 1)
T0	Initial temperature
V0	Initial relative volume (see the beginning of the *EOS section)
VCO	Van der Waals covolume

VARIABLE	DESCRIPTION
ADIAB	Adiabatic flag: EQ.0.0: Off EQ.1.0: On; ideal gas follows adiabatic law

Remarks:

1. **Pressure.** The pressure in the ideal gas law is defined as

    $$p = \rho(C_p - C_v)T$$
    $$C_p = C_{p,\text{mass}} + BT + CT^2$$
    $$C_v = C_{v,\text{mass}} + BT + CT^2$$

    where C_p and C_v are the specific heat capacities at constant pressure and at constant volume, respectively. ρ is the density. The relative volume is defined as

    $$v_r = \frac{V}{V_0} = \frac{(V/M)}{(V_0/M)} = \frac{v}{v_0} = \frac{\rho_0}{\rho} \,,$$

    where ρ_0 is a nominal or reference density defined in the *MAT_NULL card. The initial pressure can then be manually computed as

    $$P|_{t=0} = \rho|_{t=0}(C_P - C_V)T|_{t=0}$$
    $$\rho|_{t=0} = \left\{\frac{\rho_0}{v_r|_{t=0}}\right\}$$
    $$P|_{t=0} = \left\{\frac{\rho_0}{v_r|_{t=0}}\right\}(C_P - C_V)T|_{t=0}$$

    The initial relative volume, $v_r|_{t=0}$ (V0), initial temperature, $T|_{t=0}$(T0), and heat capacity information are defined in the *EOS_IDEAL_GAS input. Note that the "reference" density is typically a density at a non-stressed or nominal stress state. The initial pressure should always be checked manually against simulation result.

2. **Energy Conservation.** With adiabatic flag on, the adiabatic state is conserved, but *exact* internal energy conservation is sacrificed.

3. **Deviation from Ideal Gas Model.** The ideal gas model is good for low density gas only. Deviation from the ideal gas behavior may be indicated by the compressibility factor defined as

    $$Z = \frac{Pv}{RT}$$

    When Z deviates from 1, the gas behavior deviates from ideal.

4. **Initial Temperature and Initial Relative Volume.** V0 and T0 defined in this card must be the same as the time-zero ordinates for the 2 load curves defined in the *BOUNDARY_AMBIENT_EOS card, if it is used. This is so that they both would consistently define the same initial state for a material.

*EOS_PHASE_CHANGE

This is Equation of State Form 13. This EOS was designed for phase change from liquid to vapor, based on the homogeneous Schmidt model.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	RHOL	RHOV	CL	CV	GAMAL	PV	KL
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	E0	V0						
Type	F	F						

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).
RHOL	Density of liquid
RHOV	Density of saturated vapor
CL	Speed of sound of liquid
CV	Speed of sound of vapor
GAMAL	Gamma constant of liquid
PV	Pressure of vapor
KL	Bulk compressibility of liquid
E0	Initial internal energy
V0	Initial relative volume or initial pressure. GT.0.0: Initial relative volume. Default = 1 LT.0.0: Initial pressure.

Remarks:

This model is barotropic, so the pressure is only a function of density change. Details of the model can be found in Souli et al. [2014]. Examples of applications for this model are simulations involving water hammer or fuel injection. The ambient pressure should normally be set to atmospheric pressure.

Example input for water in the MKS system (m, kg, s):

```
*EOS_PHASE_CHANGE
$    EOSID      RHOL      RHOV        CL        CV        PV
         1 9.9742e+2  2.095e-2   1492.00    425.00     1.e+5
```

*EOS_JWLB

This is Equation of State Form 14. The JWLB (Jones-Wilkens-Lee-Baker) equation of state, developed by Baker [1991] and further described by Baker and Orosz [1991], describes the high pressure regime produced by overdriven detonations while retaining the low pressure expansion behavior required for standard acceleration modeling. The derived form of the equation of state is based on the JWL form due to its computational robustness and asymptotic approach to an ideal gas at high expansions. Additional exponential terms and a variable Gruneisen parameter have been added to adequately describe the high-pressure region above the Chapman-Jouguet state.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	A1	A2	A3	A4	A5		
Type	A	F	F	F	F	F		

Card 2	1	2	3	4	5	6	7	8
Variable	R1	R2	R3	R4	R5			
Type	F	F	F	F	F			

Card 3	1	2	3	4	5	6	7	8
Variable	AL1	AL2	AL3	AL4	AL5			
Type	F	F	F	F	F			

Card 4	1	2	3	4	5	6	7	8
Variable	BL1	BL2	BL3	BL4	BL5			
Type	F	F	F	F	F			

Card 5	1	2	3	4	5	6	7	8
Variable	RL1	RL2	RL3	RL4	RL5			
Type	F	F	F	F	F			

Card 6	1	2	3	4	5	6	7	8
Variable	C	OMEGA	E	V0				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
EOSID	Equation of state identification. A unique number or label must be specified (see *PART).
Ai	Equation of state coefficient A_i. See below.
Ri	Equation of state coefficient R_i. See below.
ALi	Equation of state coefficient $A_{\lambda i}$. See below.
BLi	Equation of state coefficient $B_{\lambda i}$. See below.
RLi	Equation of state coefficient $R_{\lambda i}$. See below.
C	Equation of state coefficient C. See below.
OMEGA	Equation of state coefficient ω. See below.
E	Energy density per unit volume
V0	Initial relative volume

Remarks:

The JWLB equation-of-state defines the pressure as

$$p = \sum_{i=1}^{5} A_i \left(1 - \frac{\lambda}{R_i V} \right) e^{-R_i V} + \frac{\lambda E}{V} + C \left(1 - \frac{\lambda}{\omega} \right) V^{-(\omega+1)}$$

$$\lambda = \sum_{i=1}^{5}(A_{\lambda i}V + B_{\lambda i})e^{-R_{\lambda i}V} + \omega$$

where V is the relative volume, E is the energy per unit initial volume, and A_i, R_i, $A_{\lambda i}$, $B_{\lambda i}$, $R_{\lambda i}$, C, and ω are input constants defined above.

JWLB input constants for some common explosives as found in Baker and Stiel [1997] are given in the following table.

	TATB	LX-14	PETN	TNT	Octol 70/30
ρ_0 (g/cc)	1.800	1.821	1.765	1.631	1.803
E_0 (Mbar)	.07040	.10205	.10910	.06656	.09590
D_{CJ} (cm/µs)	.76794	.86619	.83041	.67174	.82994
P_{CJ} (Mbar)	.23740	.31717	.29076	.18503	.29369
A_1 (Mbar)	550.06	549.60	521.96	490.07	526.83
A_2 (Mbar)	22.051	64.066	71.104	56.868	60.579
A_3 (Mbar)	.42788	2.0972	4.4774	.82426	.91248
A_4 (Mbar)	.28094	.88940	.97725	.00093	.00159
R_1	16.688	34.636	44.169	40.713	52.106
R_2	6.8050	8.2176	8.7877	9.6754	8.3998
R_3	2.0737	20.401	25.072	2.4350	2.1339
R_4	2.9754	2.0616	2.2251	.15564	.18592
C (Mbar)	.00776	.01251	.01570	.00710	.00968
ω	.27952	.38375	.32357	.30270	.39023
$A_{\lambda 1}$	1423.9	18307.	12.257	.00000	.011929
$B_{\lambda 1}$	14387.	1390.1	52.404	1098.0	18466.
$R_{\lambda 1}$	19.780	19.309	43.932	15.614	20.029
$A_{\lambda 2}$	5.0364	4.4882	8.6351	11.468	5.4192
$B_{\lambda 2}$	-2.6332	-2.6181	-4.9176	-6.5011	-3.2394
$R_{\lambda 2}$	1.7062	1.5076	2.1303	2.1593	1.5868

*EOS_GASKET

This is Equation of State Form 15. This EOS works with solid elements and thick shell formulations ELFORM = 2, 3, 5 and 7 to model the response of gaskets. For the thick shell formulation #2 only, it is completely decoupled from the shell material, meaning in the local coordinate system of the shell, this model defines the normal stress, σ_{zz}, and does not change any of the other stress components. The model is a reduction of the *MAT_-GENERAL_NONLINEAR_6DOF_DISCRETE_BEAM.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	LCID1	LCID2	LCID3	LCID4			
Type	A	I	I	I	I			

Card 2	1	2	3	4	5	6	7	8
Variable	UNLOAD	K	DMPF	TFS	CFS	LOFFSET	IVS	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).
LCID1	Load curve for loading
LCID2	Load curve for unloading
LCID3	Load curve for damping as a function of volumetric strain rate
LCID4	Load curve for scaling the damping as a function of the volumetric strain
UNLOAD	Unloading option (see Figure EOS15-1): EQ.0.0: Loading and unloading follow loading curve EQ.1.0: Loading follows loading curve, unloading follows unloading curve. The unloading curve ID if undefined is taken as the loading curve. EQ.2.0: Loading follows loading curve, unloading follows

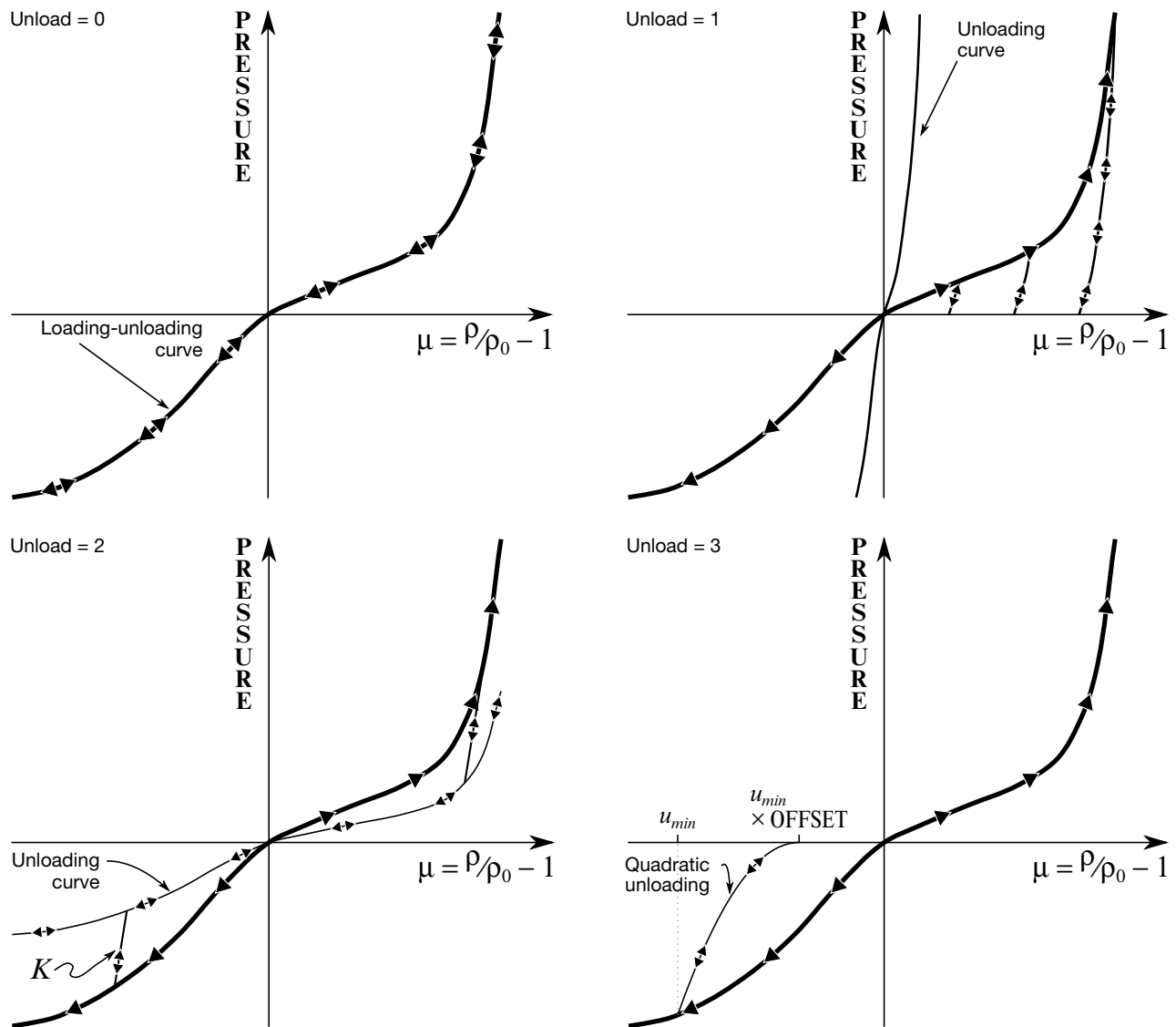| VARIABLE	DESCRIPTION
	unloading stiffness, K, to the unloading curve. The loading and unloading curves may only intersect at the origin of the axes.
	EQ.3.0: Quadratic unloading from peak displacement value to a permanent offset.
K	Unloading stiffness, for UNLOAD = 2 only
DMPF	Damping factor for stability. Values in the neighborhood of unity are recommended. The damping factor is properly scaled to eliminate time step size dependency.

Figure EOS15-1. Load and unloading behavior.

VARIABLE	DESCRIPTION
TFS	Tensile failure strain
CFS	Compressive failure strain
OFFSET	Offset factor between 0 and 1.0 to determine permanent set upon unloading if the UNLOAD = 3.0. The permanent sets in compression and tension are equal to the product of this offset value and the maximum compressive and tensile displacements, respectively.
IVS	Initial volume strain

*EOS_MIE_GRUNEISEN

This is Equation of State Form 16, a Mie-Gruneisen form with a $p - \alpha$ compaction model.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	GAMMA	A1	A2	A3	PEL	PCO	N
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

Card 2	1	2	3	4	5	6	7	8
Variable	ALPHA0	E0	V0					
Type	F	F	F					
Default	none	none	none					

VARIABLE	DESCRIPTION
EOSID	Equation of state identification. A unique number or label must be specified (see *PART).
GAMMA	Gruneisen gamma
A*i*	Hugoniot polynomial coefficient
PEL	Crush pressure
PCO	Compaction pressure
N	Porosity exponent
ALPHA0	Initial porosity
E0	Initial internal energy
V0	Initial relative volume

Remarks:

The equation of state is a Mie-Gruneisen form with a polynomial Hugoniot curve and a $p - \alpha$ compaction model. First, we define a history variable representing the porosity α that is initialised to $\alpha_0 > 1$. The evolution of this variable is given as

$$\alpha(t) = \max \left\{ 1, \min \left[\alpha_0, \min_{s \leq t} \left(1 + (\alpha_0 - 1) \left[\frac{p_{\text{comp}} - p(s)}{p_{\text{comp}} - p_{el}} \right]^N \right) \right] \right\} ,$$

where $p(t)$ indicates the pressure at time t. For later use, we define the cap pressure as

$$p_c = p_{\text{comp}} - (p_{\text{comp}} - p_{el}) \left[\frac{\alpha - 1}{\alpha_0 - 1} \right]^{\frac{1}{N}} .$$

The remainder of the EOS model is given by the equations

$$p(\rho, e) = \Gamma \alpha \rho e + p_H(\eta) \left[1 - \frac{1}{2} \Gamma \eta \right]$$

$$p_H(\eta) = A_1 \eta + A_2 \eta^2 + A_3 \eta^3$$

together with

$$\eta(\rho) = \frac{\alpha \rho}{\alpha_0 \rho_0} - 1 .$$

*EOS_MURNAGHAN

This is Equation of State Form 19. This EOS was designed to model incompressible fluid flow with SPH or ALE elements.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	GAMMA	K0	V0				
Type	A	F	F	F				

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).
GAMMA, K0	Constants in the equation of state
V0	Initial relative volume

Remarks:

The Murnaghan equation of state defines pressure as

$$p = k_0 \left[\left(\frac{\rho}{\rho_0} \right)^\gamma - 1 \right] \ .$$

To model fluid flows accurately, γ is often set to 7, and k_0 is chosen such that

$$c_0 = \sqrt{\frac{\gamma k_0}{\rho_0}} \geq 10 v_{\max} \quad ,$$

where $v_{\max}$ is the maximum expected fluid flow velocity. This will ensure low compressibility while allowing for a relatively large time step size.

*EOS_USER_DEFINED

These are Equations of State 21-30. The user can supply his own subroutines. See also Appendix B. The keyword input must be used for the user interface with data.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	EOST	LMC	NHV	IVECT	E0	V0	BULK
Type	A	I	I	I	I	F	F	F

Define LMC material parameters using 8 parameters per card.

Card 2	1	2	3	4	5	6	7	8
Variable	P1	P2	P3	P4	P5	P6	P7	P8
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).
EOST	User equation of state type (21-30 inclusive). A number between 21 and 30 has to be chosen.
LMC	Length of material constant array which is equal to the number of material constants to be input. (LMC ≤ 48)
NHV	Number of history variables to be stored, see Appendix B.
IVECT	Vectorization flag (on = 1). A vectorized user subroutine must be supplied.
E0	Initial internal energy
V0	Initial relative volume
BULK	Bulk modulus. This value is used in the calculation of the contact surface stiffness.
Pi	Material parameters $i = 1, \dots ,$LMC.

*EOS_USER_LIBRARY

This is Equation of State Form 42.

Purpose: Select a material ID defined in a library called seslib, and initialize the thermodynamic state of the material by defining E0 and V0 below. seslib must be in the working directory.

Card 1	1	2	3	4	5	6	7	8
Variable	EOSID	SESMID						
Type	A	I						

Card 2	1	2	3	4	5	6	7	8
Variable	E0	V0						
Type	F	F						

VARIABLE	DESCRIPTION
EOSID	Equation of state ID. A unique number or label must be specified (see *PART).
SESMID	Material ID
E0	Initial internal energy per unit reference volume (see the beginning of the *EOS section)
V0	Initial relative volume (see the beginning of the *EOS section)