*MAT_LOW_DENSITY_SYNTHETIC_FOAM_{OPTION}

This is Material Type 179 (and 180 if the ORTHO option below is active) for modeling rate independent low density foams, which have the property that the hysteresis in the loading-unloading curve is considerably reduced after the first loading cycle. For this material we assume that the loading-unloading curve is identical after the first cycle of loading is completed and that the damage is isotropic, that is, the behavior after the first cycle of loading in the orthogonal directions also follows the second curve. The main application at this time is to model the observed behavior in the compressible synthetic foams that are used in some bumper designs. Tables may be used in place of load curves to account for strain rate effects.

Available options include:

<BLANK>

ORTHO

WITH_FAILURE

ORTHO_WITH_FAILURE

If the foam develops orthotropic behavior, that is, after the first loading and unloading cycle the material in the orthogonal directions are unaffected, then the ORTHO option should be used. If the ORTHO option is active the directionality of the loading is stored. This option requires additional storage for history variables related to the orthogonality and is slightly more expensive.

An optional failure criterion is included. A description of the failure model is provided below for material type 181, *MAT_SIMPLIFIED_RUBBER/FOAM.

Card Summary:

Card 1. This card is required.

MID	RO	E	LCID1	LCID2	HU	BETA	DAMP

Card 2. This card is required.

SHAPE	FAIL	BVFLAG	ED	BETA1	KCON	REF	TC

Card 3. This card is included if LCID < 0.

RFLAG	DTRT						

Card 4. This card is included if and only if the IF_FAILURE keyword option is used.

K	GAMA1	GAMA2	EH				

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	LCID1	LCID2	HU	BETA	DAMP
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	1.	none	0.05

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus. This modulus is used if the elongations are tensile as described for the *MAT_LOW_DENSITY_FOAM.
LCID1	Load curve or table ID describing nominal stress as a function of strain for the undamaged material (see Remark 2): GT.0: Load curve ID (see *DEFINE_CURVE) for nominal stress as a function of strain for the undamaged material. LT.0: -LCID1 is a table ID (see *DEFINE_TABLE) for nominal stress as a function of strain for the undamaged material as a function of strain rate
LCID2	Load curve or table ID. The load curve ID (see *DEFINE_CURVE) defines the nominal stress as a function of strain for the damaged material. The table ID (see *DEFINE_TABLE) defines the nominal stress as a function of strain for the damaged material as a function of strain rate. See Remark 2.
HU	Hysteretic unloading factor between 0.0 and 1.0 (default = 1.0, that is, no energy dissipation); see Figure M179-1 and Remarks 1 and 2.
BETA	Decay constant to model creep in unloading, β
DAMP	Viscous coefficient (.05 < recommended value < .50) to model damping effects. LT.0.0: |DAMP| is the load curve ID that defines the damping constant as a function of the maximum strain in

VARIABLE	DESCRIPTION
	compression defined as: $$\varepsilon_{max} = \max(1-\lambda_1, 1-\lambda_2, 1.-\lambda_3)\ .$$ In tension, the damping constant is set to the value corresponding to the strain at 0.0. The abscissa should be defined from 0.0 to 1.0.

Card 2	1	2	3	4	5	6	7	8
Variable	SHAPE	FAIL	BVFLAG	ED	BETA1	KCON	REF	TC
Type	F	F	F	F	F	F	F	F
Default	1.0	0.0	0.0	0.0	0.0	0.0	0.0	10^{20}

VARIABLE	DESCRIPTION
SHAPE	Shape factor for unloading. Active for nonzero values of the hysteretic unloading factor. Values less than one reduce the energy dissipation and greater than one increase dissipation; see also Figure M179-1 and Remarks 1 and 2.
FAIL	Failure option after cutoff stress is reached: EQ.0.0: Tensile stress remains at cut-off value, EQ.1.0: Tensile stress is reset to zero.
BVFLAG	Bulk viscosity activation flag: EQ.0.0: No bulk viscosity (recommended), EQ.1.0: Bulk viscosity active.
ED	Optional Young's relaxation modulus, E_d, for rate effects.
BETA1	Optional decay constant, β_1.
KCON	Stiffness coefficient for contact interface stiffness. If undefined, the maximum slope in the stress as a function of strain curve is used. When the maximum slope is used for the contact, the time step size for this material is reduced for stability. In some cases, Δt may be significantly smaller, so defining a reasonable stiffness is

VARIABLE	DESCRIPTION
	recommended.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword *INITIAL_FOAM_REFERENCE_GEOMETRY. EQ.0.0: Off EQ.1.0: On
TC	Tension cut-off stress

Additional card for LCID1 < 0.

Card 3	1	2	3	4	5	6	7	8
Variable	RFLAG	DTRT						
Type	F	F						
Default	0.0	0.0						

VARIABLE	DESCRIPTION
RFLAG	Rate type for input: EQ.0.0: LCID1 and LCID2 should be input as functions of true strain rate. EQ.1.0: LCID1 and LCID2 should be input as functions of engineering strain rate.
DTRT	Strain rate averaging flag: EQ.0.0: Use weighted running average. LT.0.0: Average the last 11 values. GT.0.0: Average over the last DTRT time units.

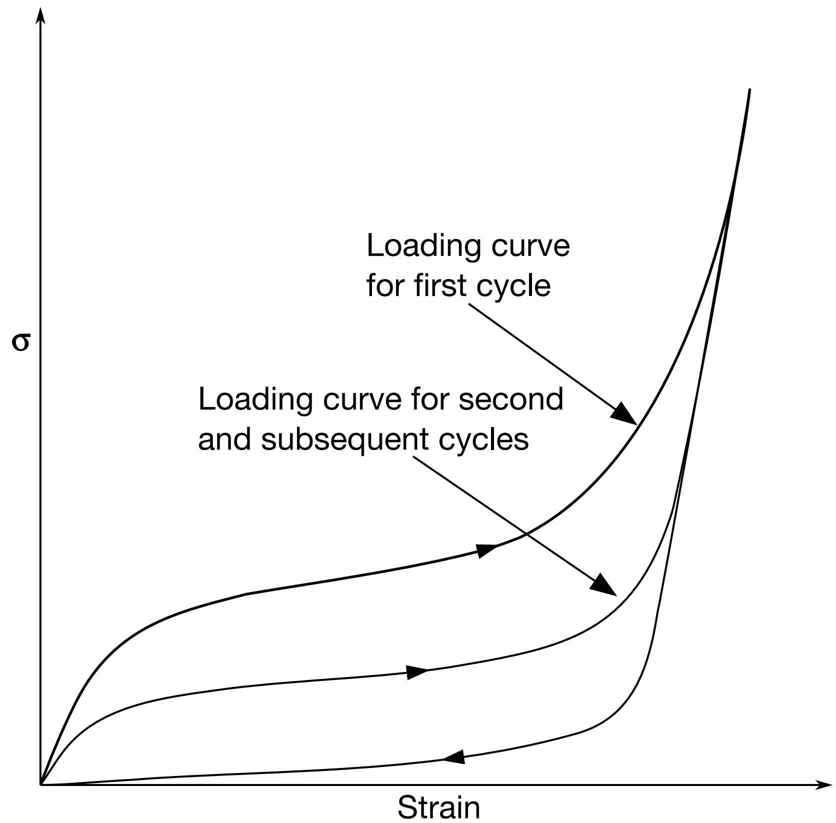


Figure M179-1. Loading and reloading curves.

Additional card for WITH_FAILURE keyword option.

Card 4	1	2	3	4	5	6	7	8
Variable	K	GAMA1	GAMA2	EH				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
K	Material failure parameter that controls the volume enclosed by the failure surface. LE.0.0: Ignore failure criterion; GT.0.0: Use actual K value for failure criterions.
GAMA1	Material failure parameter; see Figure M181-1.
GAMA2	Material failure parameter
EH	Damage parameter

Remarks:

1. **Uniaxial response.** This model is based on *MAT_LOW_DENSITY_FOAM. The uniaxial response is shown in Figure M179-1 with a large shape factor and small hysteretic factor. If the shape factor is not used, the unloading will occur on the loading curve for the second and subsequent cycles.

2. **Damage and hysteresis.** The damage is defined as the ratio of the current volume strain to the maximum volume strain, and it is used to interpolate between the responses defined by LCID1 and LCID2.

   HU defines a hysteretic scale factor that is applied to the stress interpolated from LCID1 and LCID2,

$$\sigma = \left[\mathrm{HU} + (1 - \mathrm{HU}) \times \min\left(1, \frac{e_{\mathrm{int}}}{e_{\mathrm{int}}^{\mathrm{max}}}\right)^{s}\right] \sigma(\mathrm{LCID1,LCID2})$$

   where e_{int} is the internal energy and S is the shape factor. Setting HU to 1 results in a scale factor of 1. Setting HU close to zero scales the stress by the ratio of the internal energy to the maximum internal energy raised to the power S, resulting in the stress being reduced when the strain is low.

MAT_SIMPLIFIED_RUBBER/FOAM_*{OPTION}

This is Material Type 181. This material model provides a rubber and foam model specified with a single uniaxial load curve or a family of uniaxial curves at discrete strain rates. Hysteretic unloading may optionally be modeled through a single uniaxial unloading curve or a two-parameter formulation. Specifying a Poisson's ratio greater than 0.0 and less than 0.49 activates the foam formulation. This material may be used with both shell and solid elements.

Available options include:

<BLANK>

WITH_FAILURE

LOG_LOG_INTERPOLATION

When the WITH_FAILURE keyword option is active, a strain-based failure surface is defined that is suitable for incompressible polymers. It models failure in both tension and compression. With LOG_LOG_INTERPOLATION, LS-DYNA interpolates the strain rate effect in the table TBID using log-log interpolation.

This material law has been developed at DaimlerChrysler, Sindelfingen, in collaboration with Paul Du Bois, LSTC, and Prof. Dave J. Benson, UCSD.

Card Summary:

Card 1. This card is required.

MID	RO	KM	MU	G	SIGF	REF	PRTEN

Card 2. This card is required.

SGL	SW	ST	LC/TBID	TENSION	RTYPE	AVGOPT	PR

Card 3. This card is included if the WITH_FAILURE keyword option is used.

K	GAMA1	GAMA2	EH				

Card 4. This card is optional. It must be included if Card 5 is included.

LCUNLD	HU	SHAPE	STOL	VISCO	HISOUT		

Card 5. This card is optional. Up to 12 cards in this format may be input. If fewer than 12 cards are input, the next keyword ("*") card terminates this input.

G*i*	BETA*i*	VFLAG					

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	KM	MU	G	SIGF	REF	PRTEN
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
KM	Linear bulk modulus (see PR on Card 2)
MU	Damping coefficient (0.05 < recommended value < 0.50; default is 0.10).
G	Shear modulus for frequency independent damping. Frequency independent damping is based on a spring and slider in series. The critical stress for the slider mechanism is SIGF defined below. For the best results, the value of G should be 250-1000 times greater than SIGF. See Remark 1.
SIGF	Limit stress for frequency independent, frictional damping. See Remark 1.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: Off EQ.1.0: On
PRTEN	The tensile Poisson's ratio for shells (optional). If PRTEN is zero, PR will serve as the Poisson's ratio for both tension and

VARIABLE	DESCRIPTION
	compression in shells. If PRTEN is nonzero, PR will serve only as the compressive Poisson's ratio for shells.

Card 2	1	2	3	4	5	6	7	8
Variable	SGL	SW	ST	LC/TBID	TENSION	RTYPE	AVGOPT	PR
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
SGL	Specimen gauge length
SW	Specimen width
ST	Specimen thickness
LC/TBID	Load curve ID or table ID (see *DEFINE_TABLE) giving the force as a function of the actual change in the gauge length. If SGL, SW, and ST are set to unity (1.0), then curve LC is also engineering stress versus engineering strain. If the table definition is used, a family of curves is defined for discrete strain rates. The curves should cover the complete range of expected response, including both compressive (negative values) and tensile (positive values) regimes.
TENSION	Parameter that controls how the rate effects are treated. Applicable to the table definition. EQ.-1.0: Rate effects are considered during tension and compression loading, but not during unloading. EQ.0.0: Rate effects are considered for compressive loading only. EQ.1.0: Rate effects are treated identically in tension and compression.
RTYPE	Strain rate type if a table is defined: EQ.0.0: True strain rate EQ.1.0: Engineering strain rate
AVGOPT	Averaging option for strain rates to reduce numerical noise:

VARIABLE	DESCRIPTION
	LT.0.0: $\lvert$AVGOPT$\rvert$ is a time window/interval over which the strain rates are averaged. This option is recommended because it is time step size independent and generally more stable.
	EQ.0.0: Simple average of 12 time steps
	EQ.1.0: Running average of last 12 averages
PR	Poisson ratio or viscosity coefficient:
	LE.0.0: An incompressible rubber material is assumed, using the Ogden strain-energy functional. PR is set to 0.495 internally for computing the time-step only and is not used otherwise. Compressibility is defined using KM. For PR < 0 in solid elements, an incrementally updated mean viscous stress develops according to the following equation with $\beta = \lvert \mathrm{PR} \rvert$ and K_m = KM (see Card 1): $$p^{n+1} = p^n e^{-\beta\Delta t} + K_m \dot{\varepsilon}_{kk} \left(\frac{1 - e^{-\beta\Delta t}}{\beta} \right) .$$
	GT.0.0.AND.LT.0.49: A foam material is assumed, using the Hill strain-energy function. PR gives Poisson's ratio. KM on Card 1 is only used for critical time step computation and contact penalty stiffness. Selective-reduced integration is *not* used for fully-integrated elements.
	GE.0.49.AND.LT.0.5: An incompressible rubber material is assumed, using the Ogden strain-energy functional. PR is used for computing the time-step only. Compressibility is defined using KM on Card 1. Selective-reduced integration is not used for fully-integrated elements.

Additional card required for WITH_FAILURE option. Otherwise skip this card.

Card 3	1	2	3	4	5	6	7	8
Variable	K	GAMA1	GAMA2	EH				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
K	Material failure parameter that controls the volume enclosed by the failure surface. LE.0.0: Ignore failure criterion. GT.0.0: Use actual K value for failure criterion (see Remark 2).
GAMA1	Material failure parameter, Γ_1; see Remark 2 and Figure M181-1.
GAMA2	Material failure parameter, Γ_2; see Remark 2.
EH	Damage parameter, h. See Remark 2.

Optional Parameter Card.

Card 4	1	2	3	4	5	6	7	8
Variable	LCUNLD	HU	SHAPE	STOL	VISCO	HISOUT		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
LCUNLD	Optional load curve (see *DEFINE_CURVE) giving the force as a function of actual length during unloading. The unload curve should cover exactly the same range as LC or the load curves of TBID and its end points should have identical values. In other words, the combination of LC and LCUNLD or the first curve of TBID and LCUNLD describes a complete cycle of loading and unloading. See also material *MAT_083.
HU	Hysteretic unloading factor between 0 and 1 (default = 1.0, meaning no energy dissipation). See also material *MAT_083 and Figure M57-1. This option is ignored if LCUNLD is used.

VARIABLE	DESCRIPTION
SHAPE	Shape factor for unloading. Active for nonzero values of the hysteretic unloading factor HU. Values less than one reduces the energy dissipation and greater than one increases dissipation. See also material *MAT_083 and Figure M57-1.
STOL	Tolerance in stability check. See Remark 3.
VISCO	Flag to invoke viscoelastic formulation. The viscoelastic formulation does not apply to shell elements and will be ignored for shells. See Remark 4. EQ.0.0: Purely elastic EQ.1.0: Viscoelastic formulation (solids only)
HISOUT	History output flag. EQ.0.0: Default EQ.1.0: Principal strains are written to history variables 25, 26, and 27.

Optional Viscoelastic Constants Cards. Up to 12 cards in format 5 may be input. A keyword card (with a "*" in column 1) terminates this input if fewer than 12 cards are used.

Card 5	1	2	3	4	5	6	7	8
Variable	G*i*	BETA*i*	VFLAG					
Type	F	F	I					
Default	none	none	0					

VARIABLE	DESCRIPTION
G*i*	Optional shear relaxation modulus for the i^{th} term. The G*i* and BETA*i* terms are used only for solid elements when VISCO = 1. See Remark 4.
BETA*i*	Optional decay constant if i^{th} term. See Remark 4.
VFLAG	Type of viscoelasticity formulation. This appears only on the first line defining G*i*, BETA*i*, and VFLAG. See Remark 4.

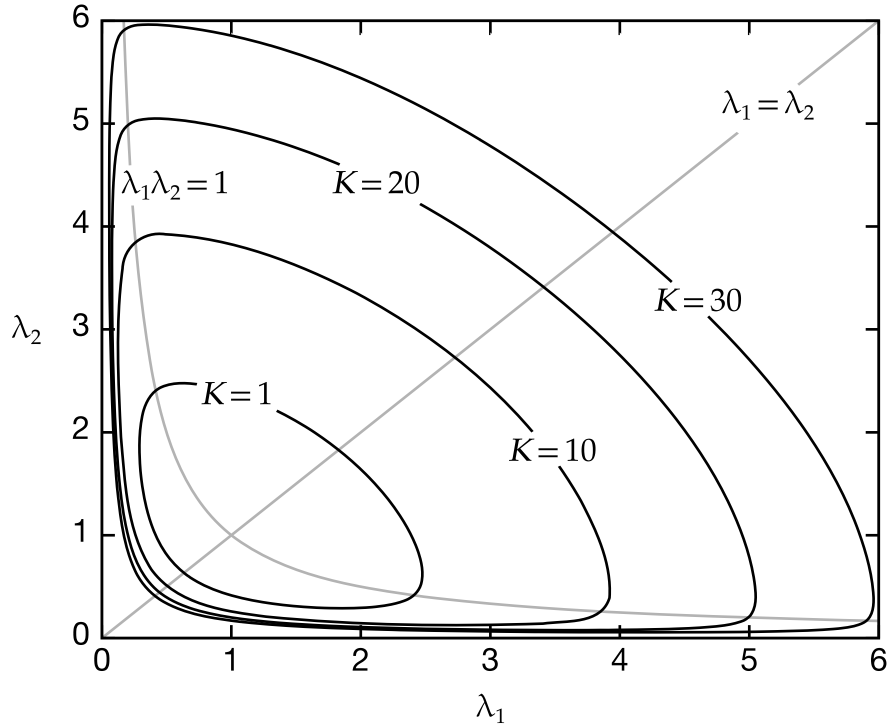
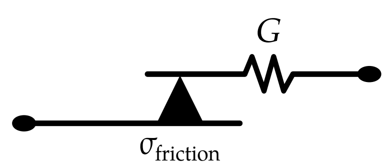


Figure M181-1. Failure surface for polymer for $\Gamma_1 = 0$ and $\Gamma_2 = 0.02$.

VARIABLE	DESCRIPTION
	EQ.0: Standard viscoelasticity formulation (default) EQ.1: Viscoelasticity formulation using instantaneous elastic stress

Remarks:

1. **Frequency-independent damping.** Frequency-independent damping is obtained by the having a spring and slider in series as shown in the following sketch:



2. **Failure criterion for polymers.** The general failure criterion for polymers is proposed by Feng and Hallquist as

$$f(I_1, I_2, I_3) = (I_1 - 3) + \Gamma_1(I_1 - 3)^2 + \Gamma_2(I_2 - 3) = K$$

where K is a material parameter which controls the size enclosed by the failure surface. I_1, I_2 and I_3 are the three invariants of right Cauchy-Green deformation tensor ($\mathbf{C}$):

$$
\begin{aligned}
I_1 &= C_{ii} = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \\
I_2 &= \frac{1}{2}(C_{ii}C_{jj} - C_{ij}C_{ij}) = \lambda_1^2\lambda_2^2 + \lambda_1^2\lambda_3^2 + \lambda_2^2\lambda_3^2 \\
I_3 &= \det(\mathbf{C}) = \lambda_1^2\lambda_2^2\lambda_3^2
\end{aligned}
$$

with λ_i as the stretch ratios in the three principal directions.

To avoid sudden failure and numerical difficulty, material failure, which is usually a time point, is modeled as a process of damage growth. In this case, the two threshold values are chosen as $(1 - h)K$ and K, where h (also called EH) is a small number chosen based on experimental results reflecting the range between damage initiation and material failure.

The damage is defined as function of f:

$$
D = \begin{cases} 0 & \text{if} \quad f \le (1-h)K \\ \frac{1}{2}\left[1 + \cos\frac{\pi(f-K)}{hK}\right] & \text{if} \quad (1-h)K < f < K \\ 1 & \text{if} \quad f \ge K \end{cases}
$$

With this definition, damage is first-order continuous, and the tangent stiffness matrix will be continuous. The reduced stress considering damage effect is

$$
\sigma_{ij} = (1 - D)\sigma_{ij}^o
$$

where σ_{ij}^o is the undamaged stress. Prior to final failure, material damage is assumed to be recoverable. Once material failure occurs, damage will become permanent.

3. **Stability of the stress-strain response.** Bad choices of curves for the stress-strain response may lead to an unstable model. LS-DYNA can check for stability given a certain tolerance with the field STOL. The check is done by examining the eigenvalues of the tangent modulus at selected stretch points. A warning message is issued if an eigenvalue is less than –STOL × BULK, where BULK indicates the bulk modulus of the material. For STOL < 0, the check is disabled. Otherwise, it should be chosen with care. A too small value may detect instabilities that are insignificant in practice. To avoid significant instabilities, we recommend using smooth curves. At best the curves should be continuously differentiable. For the incompressible case, a sufficient condition for stability is that the stress-stretch curve $S(\lambda)$ can be written as

$$
S(\lambda) = H(\lambda) - \frac{H\left(\frac{1}{\sqrt{\lambda}}\right)}{\lambda\sqrt{\lambda}}
$$

where $H(\lambda)$ is a function with $H(1) = 0$ and $H'(\lambda) > 0$.

4. **Viscoelasticity.** For solid elements, rate effects may also be taken into account through linear viscoelasticity by setting VISCO = 1.0. For VFLAG = 0, viscoelasticity is modeled through a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau \ ,$$

or in terms of the second Piola-Kirchhoff stress, $\mathbf{S}_0$, and Green's strain tensor, $\mathbf{E}_{\mathrm{RT}}$

$$S_{ij} = \int_0^t G_{ijkl}(t-\tau)\frac{\partial E_{kl}}{\partial \tau} d\tau \ .$$

Here $g_{ijkl}(t-\tau)$ and $G_{ijkl}(t-\tau)$ are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

The relaxation function is represented by six terms from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^{N} \alpha_m e^{-\beta t}$$

given by

$$g(t) = \sum_{i=1}^{n} G_i e^{-\beta_i t} \ .$$

This model is effectively a Maxwell fluid which consists of a dampers and springs in series. We characterize this in the input by shear moduli, G_i, and decay constants, β_i. An arbitrary number of terms may be used.

For VFLAG = 1, the viscoelastic term is

$$\sigma_{ij} = \int_0^t g_{ijkl}(t-\tau)\frac{\partial \sigma_{kl}^E}{\partial \tau} d\tau$$

where σ_{kl}^E is the instantaneous stress evaluated from the internal energy functional. The coefficients in the Prony series, therefore, correspond to normalized relaxation moduli instead of elastic moduli.

MAT_SIMPLIFIED_RUBBER_WITH_DAMAGE_*{OPTION}

Available options include (see Remark 1):

<BLANK>

LOG_LOG_INTERPOLATION

This is Material Type 183. This material model provides an incompressible rubber model defined by a single uniaxial load curve for loading (or a table if rate effects are considered) and a single uniaxial load curve for unloading. This model is similar to *MAT_181/*MAT_SIMPLIFIED_RUBBER/FOAM. This material may be used with both shell and solid elements.

This material law has been developed at DaimlerChrysler, Sindelfingen, in collaboration with Paul Du Bois, LSTC, and Prof. Dave J. Benson, UCSD.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	MU	G	SIGF		
Type	A	F	F	F	F	F		

Card 2	1	2	3	4	5	6	7	8
Variable	SGL	SW	ST	LC / TBID	TENSION	RTYPE	AVGOPT	
Type	F	F	F	F	F	F	F	

Card 3	1	2	3	4	5	6	7	8
Variable	LCUNLD	REF	STOL					
Type	F	F	F					

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).

VARIABLE	DESCRIPTION
RO	Mass density
K	Linear bulk modulus
MU	Damping coefficient
G	Shear modulus for frequency-independent damping. Frequency-independent damping is based on a spring and slider in series. The critical stress for the slider mechanism is SIGF defined below. For the best results, the value of G should be 250-1000 times greater than SIGF.
SIGF	Limit stress for frequency-independent, frictional damping.
SGL	Specimen gauge length
SW	Specimen width
ST	Specimen thickness
LC/TBID	Load curve ID or table ID (see *DEFINE_TABLE) defining the force as a function of actual change in the gauge length. If SGL, SW, and ST are set to unity (1.0), curve LC is also engineering stress as a function of engineering strain. If the table definition is used, a family of curves is defined for discrete strain rates. The curves should cover the complete range of expected responses, including both compressive (negative values) and tensile (positive values) regimes. See Remark 1.
TENSION	Parameter that controls how the rate effects are treated. It is applicable to the table definition. EQ.-1.0: Rate effects are considered during tension and compression loading, but not during unloading. EQ.0.0: Rate effects are considered for compressive loading only. EQ.1.0: Rate effects are treated identically in tension and compression.
RTYPE	Strain rate type if a table is defined: EQ.0.0: True strain rate EQ.1.0: Engineering strain rate

VARIABLE	DESCRIPTION
AVGOPT	Averaging option for determining strain rate to reduce numerical noise. EQ.0.0: Simple average of twelve time steps EQ.1.0: Running 12 point average
LCUNLD	Load curve (see *DEFINE_CURVE) defining the force as a function of actual change in the gauge length during unloading. The unloading curve should cover exactly the same range as LC (or as the first curve of table TBID) and its endpoints should have identical values, meaning the combination of LC (or the first curve of table TBID) and LCUNLD describes a complete cycle of loading and unloading.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY. EQ.0.0: Off EQ.1.0: On
STOL	Tolerance in stability check. See Remark 2.

Remarks:

1. **LOG_LOG_INTERPOLATION.** The LOG_LOG_INTERPOLATION option interpolates the strain rate effect in the table TBID using log-log interpolation.

2. **Stability.** A bad choice of curves for the stress-strain response may lead to an unstable model. STOL enables this check with its value setting the tolerance level. The check is done by examining the eigenvalues of the tangent modulus at selected stretch points and a warning message is issued if an eigenvalue is less than –STOL × BULK, where BULK indicates the bulk modulus of the material. STOL < 0 disables the check. When enabled, the value of STOL should be chosen with care because a too small value may detect instabilities that are insignificant in practice. To avoid significant instabilities it is recommended to use smooth curves. At best the curves should be continuously differentiable. In fact, for the incompressible case, a sufficient condition for stability is that the stress-stretch curve $S(\lambda)$ can be written as

$$S(\lambda) = H(\lambda) - \frac{H(\frac{1}{\sqrt{\lambda}})}{\lambda\sqrt{\lambda}}$$

where $H(\lambda)$ is a function with $H(1) = 0$ and $H'(\lambda) > 0$.

*MAT_COHESIVE_ELASTIC

This is Material Type 184. It is a simple cohesive elastic model for use with cohesive element formulations; see the field ELFORM in *SECTION_SOLID and *SECTION_-SHELL.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ROFLG	INTFAIL	ET	EN	FN_FAIL	FT_FAIL
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
ROFLG	Flag for whether density is specified per unit area or volume: EQ.0: Specifies the density is per unit volume (default) EQ.1: Specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero
INTFAIL	The number of integration points required for the cohesive element to be deleted. The value of INTFAIL may range from 1 to 4 with 1 the recommended value. LT.0.0: Employs a Newton-Cotes integration scheme, and the element will be deleted when |INTFAIL| integration points have failed. EQ.0.0: Employs a Newton-Cotes integration scheme, and the element will *not* be deleted even if it satisfies the failure criterion. GT.0.0: Employs a Gauss integration scheme, and the element will be deleted when INTFAIL integration points have failed.
ET	Stiffness in the plane of the cohesive element
EN	Stiffness normal to the plane of the cohesive element
FN_FAIL	Traction in the normal direction for tensile failure

VARIABLE	DESCRIPTION
FT_FAIL	Traction in the tangential direction for shear failure

Remarks:

This material cohesive model outputs three tractions having units of force per unit area into the d3plot database rather than the usual six stress components. The in-plane shear traction along the 1-2 edge replaces the x-stress, the orthogonal in plane shear traction replaces the y-stress, and the traction in the normal direction replaces the z-stress.

*MAT_COHESIVE_TH

This is Material Type 185. It is a cohesive model by Tvergaard and Hutchinson [1992] for use with cohesive element formulations; see the variable ELFORM in *SECTION_SOLID and *SECTION_SHELL. The implementation is based on the description of the implementation in the Sandia National Laboratory code, Tahoe [2003].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ROFLG	INTFAIL	SIGMAX	NLS	TLS	TLS2
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	LAMDA1	LAMDA2	LAMDAF	STFSF	ISW	ALPHA1	ALPHA2	
Type	F	F	F	F	I	F	F	

Additional card that may be used for XFEM shells; see *SECTION_SHELL_XFEM.

Card 3	1	2	3	4	5	6	7	8
Variable	DR	ALPHA3						
Type	F	F						

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
ROFLG	Flag for whether density is specified per unit area or volume. EQ.0: Specifies density per unit volume (default) EQ.1: Specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero

VARIABLE	DESCRIPTION
INTFAIL	The number of integration points required for the cohesive element to be deleted. The value of INTFAIL may range from 1 to 4 with 1 as the recommended value. LT.0.0: Employs a Newton-Cotes integration scheme. The element will be deleted when |INTFAIL| integration points have failed. EQ.0.0: Employs a Newton-Cotes integration scheme. The element will *not* be deleted even if it satisfies the failure criterion. GT.0.0: Employs a Gauss integration scheme. The element will be deleted when INTFAIL integration points have failed.
SIGMAX	Peak traction
NLS	Length scale (maximum separation) in the normal direction
TLS	Length scale (maximum separation) in the tangential direction
LAMDA1	Scaled distance to peak traction (Λ_1)
LAMDA2	Scaled distance to beginning of softening (Λ_2).
LAMDAF	Scaled distance for failure (Λ_{fail})
STFSF	Penetration stiffness multiplier. The penetration stiffness, *PS*, in terms of input parameters becomes: $$\mathrm{PS} = \frac{\mathrm{STFSF} \times \mathrm{SIGMAX}}{\mathrm{NLS} \times \left(\frac{\mathrm{LAMDA1}}{\mathrm{LAMDAF}}\right)}$$
TLS2	Length scale (maximum separation) in the tear direction (for XFEM shells only). See Remark 2.
ISW	Cohesive law for XFEM shells only (see Remark 2): EQ.-1: Initially rigid cohesive law (type I) EQ.-2: Initially rigid cohesive law (type II)
ALPHA1	Ratio of maximum mode II shear traction to normal traction (for XFEM shells only)

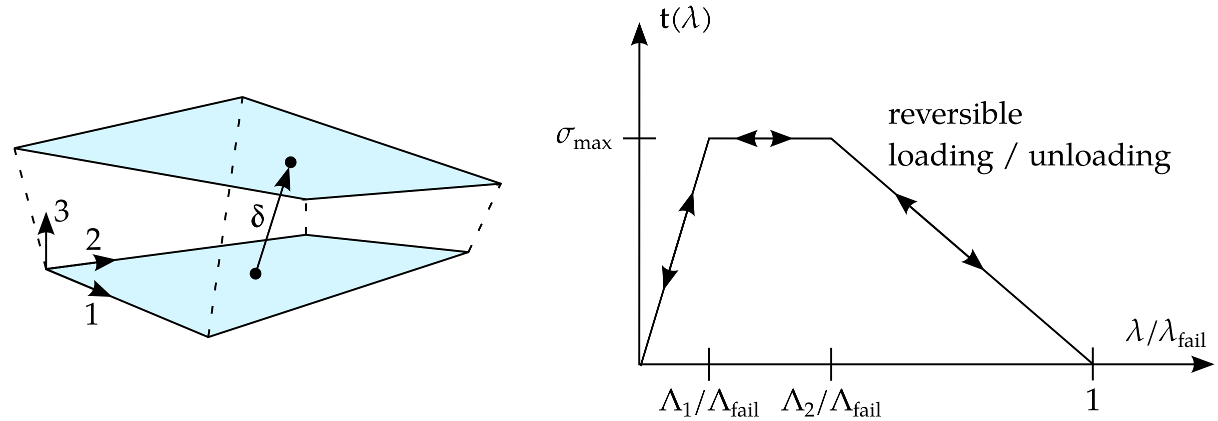


Figure M185-1. Relative displacement and trilinear traction-separation law

VARIABLE	DESCRIPTION
ALPHA2	Ratio of maximum mode III shear traction to normal traction (for XFEM shells only)
DR	Critical rotation scale (for XFEM shells only)
ALPHA3	Ratio of maximum bending moment to normal traction (for XFEM shells only)

Material Model:

In this cohesive material model, we use a dimensionless separation measure, λ, for the interaction between relative displacements in the normal (δ_3 - mode I) and tangential (δ_1, δ_2 - mode II) directions (see Figure M185-1 left):

$$\lambda = \sqrt{\left(\frac{\delta_1}{\mathrm{TLS}}\right)^2 + \left(\frac{\delta_2}{\mathrm{TLS}}\right)^2 + \left(\frac{\langle\delta_3\rangle}{\mathrm{NLS}}\right)^2}$$

The Macaulay brackets distinguish between tension ($\delta_3 \geq 0$) and compression ($\delta_3 < 0$). NLS and TLS are critical values, representing the maximum separations in the interface in the normal and tangential directions. For the stress calculation, we use a trilinear traction-separation law, given by (see Figure M185-1 right):

$$t(\lambda) = \begin{cases} \sigma_{\max}\dfrac{\lambda}{\Lambda_1/\Lambda_{\mathrm{fail}}} & \lambda < \Lambda_1/\Lambda_{\mathrm{fail}} \\ \sigma_{\max} & \Lambda_1/\Lambda_{\mathrm{fail}} < \lambda < \Lambda_2/\Lambda_{\mathrm{fail}} \\ \sigma_{\max}\dfrac{1-\lambda}{1-\Lambda_2/\Lambda_{\mathrm{fail}}} & \Lambda_2/\Lambda_{\mathrm{fail}} < \lambda < 1 \end{cases}$$

With this law, the traction drops to zero when $\lambda = 1$. A potential, ϕ, is defined as:

$$\phi(\delta_1, \delta_2, \delta_3) = \text{NLS} \times \int_0^\lambda t(\bar{\lambda})\ d\bar{\lambda}$$

Finally, the tangential components (t_1, t_2) and normal component (t_3) of the traction acting on the interface in the fracture process zone are given by:

$$t_{1,2} = \frac{\partial \phi}{\partial \delta_{1,2}} = \frac{t(\lambda)}{\lambda} \frac{\delta_{1,2}}{\text{TLS}} \frac{\text{NLS}}{\text{TLS}}, \qquad t_3 = \frac{\partial \phi}{\partial \delta_3} = \frac{t(\lambda)}{\lambda} \frac{\delta_3}{\text{NLS}}$$

which in matrix notation is

$$\begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \frac{t(\lambda)}{\lambda} \begin{bmatrix} \frac{\text{NLS}}{\text{TLS}^2} & 0 & 0 \\ 0 & \frac{\text{NLS}}{\text{TLS}^2} & 0 \\ 0 & 0 & \frac{1}{\text{NLS}} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}$$

In the case of compression ($\delta_3 < 0$), penetration is avoided by:

$$t_3 = \frac{\text{STFSF} \times \sigma_{\text{max}}}{\text{NLS} \times \Lambda_1 / \Lambda_{\text{fail}}} \delta_3$$

Loading and unloading follows the same path, that is, this model is completely reversible.

Remarks:

1. **Traction output to d3plot.** This cohesive material model outputs three tractions having units of force per unit area to the d3plot database rather than the usual six stress components. The in-plane shear traction,t_1, along the 1-2 edge replaces the x-stress, the orthogonal in-plane shear traction, t_2, replaces the y-stress, and the traction in the normal direction, t_3, replaces the z-stress.

2. **XFEM Shells.** For XFEM shells, TLS for δ_2 in the above equation is replaced by TLS2. Since the initially rigid cohesive law is used, an element fails only when the stress level reaches SIGMAX. Λ_1 is only used to define the penetration stiffness in case of a crack closing (compression).

*MAT_COHESIVE_GENERAL

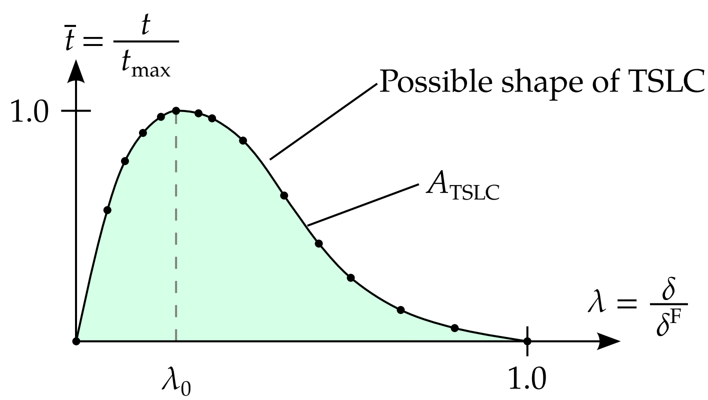
This is Material Type 186. It can be used only with cohesive element formulations; see the variable ELFORM in *SECTION_SOLID and *SECTION_SHELL. The material model allows you to choose from three general irreversible mixed-mode interaction cohesive formulations. It also includes an arbitrary normalized traction-separation law given by a load curve (TSLC). These three formulations are differentiated through the type of the effective separation parameter (TES). The interaction between fracture modes I and II is considered. Irreversible conditions are enforced with a damage formulation (unloading/reloading path pointing to/from the origin). See remarks for details.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ROFLG	INTFAIL	TES	TSLC	GIC	GIIC
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	XMU	T	S	STFSF	TSLC2			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
ROFLG	Flag for whether density is specified per unit area or volume: EQ.0: Specifies density per unit volume (default) EQ.1: Specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero
INTFAIL	Number of integration points required for a cohesive element to be deleted. The value of INTFAIL may range from 1 to 4 with 1 the recommended value. LT.0.0: Employs a Newton-Cotes integration scheme. The element will be deleted when |INTFAIL| integration

VARIABLE	DESCRIPTION
	points have failed. EQ.0.0: Employs a Newton-Cotes integration scheme. The element will *not* be deleted even if it satisfies the failure criterion. GT.0.0: Employs a Gauss integration scheme. The element will be deleted when INTFAIL integration points have failed.
TES	Type of effective separation parameter (ESP). EQ.0.0: A dimensional separation measure is used. For the interaction between modes I and II, a mixed-mode propagation criterion is given by a power law. See Remarks 1 and 2. EQ.1.0: A dimensional separation measure is used. For the interaction between modes I and II, a mixed-mode propagation criterion is given by the Benzeggagh-Kenane law [1996]. See Remarks 1 and 2. EQ.2.0: A dimensionless separation measure is used for the interaction between mode I displacements and mode II displacements (similar to *MAT_185, but with damage and general traction-separation law). See Remarks 1 and 3.
TSLC	Normalized traction-separation load curve ID. The curve must be normalized in both coordinates and must contain at least three points: $(0.0, 0.0)$, $(\lambda_0, 1.0)$, and $(1.0, 0.0)$. These points represent the origin, the peak, and the complete failure, respectively (see Figure M186-1). A platform can exist in the curve like the trilinear TSLC (see *MAT_185). See Remark 1.
GIC	Fracture toughness / energy release rate G_I^c for mode I
GIIC	Fracture toughness / energy release rate G_{II}^c for mode II
XMU	Exponent that appears in the power failure criterion (TES = 0.0) or the Benzeggagh-Kenane failure criterion (TES = 1.0). Recommended values for XMU are between 1.0 and 2.0. See Remark 2.
T	Peak traction in normal direction (mode I). See Remark 1.
S	Peak traction in tangential direction (mode II). See Remark 1.



	Mode I	Mode II
t_{max}	T	S
δ^F	$\frac{G_I^C}{A_{TSLC}T}$	$\frac{G_{II}^C}{A_{TSLC}S}$
G^C	G_I^C	G_{II}^C

Figure M186-1. Normalized traction-separation law

VARIABLE	DESCRIPTION
STFSF	Penetration stiffness multiplier for compression. Factor = (1.0 + STFSF) is used to scale the compressive stiffness, that is, no scaling is done with STFSF = 0.0 (recommended).
TSLC2	Normalized traction-separation load curve ID for Mode II. The curve must be normalized in both coordinates and must contain at least three points: $(0.0, 0.0)$, $(\lambda_0, 1.0)$, and $(1.0, 0.0)$, which represents the origin, the peak and the complete failure, respectively (see Figure M186-1). If not specified, TSLC is used for Mode II behavior as well. See Remark 1.

Remarks:

1. **Traction-separation behavior.** For all three formulations, the traction-separation behavior of this model is mainly given by G_I^c and T for normal mode I, G_{II}^c and S for tangential mode II, and an arbitrary normalized traction-separation load curve for both modes (see Figure M186-1). The maximum (or failure) separations are then given by:

$$\delta_I^F = \frac{G_I^c}{A_{TSLC} \times \mathrm{T}} \quad , \quad \delta_{II}^F = \frac{G_{II}^c}{A_{TSLC} \times \mathrm{S}}$$

Here A_{TSLC} is the area under the normalized traction-separation curve given with TSLC.

If TSLC2 is defined,

$$\delta_I^F = \frac{G_I^c}{A_{TSLC} \times \mathrm{T}} \quad , \quad \delta_{II}^F = \frac{G_{II}^c}{A_{TSLC2} \times \mathrm{S}}$$

Here A_{TSLC2} is the area under the normalized traction-separation curve given with TSLC2.

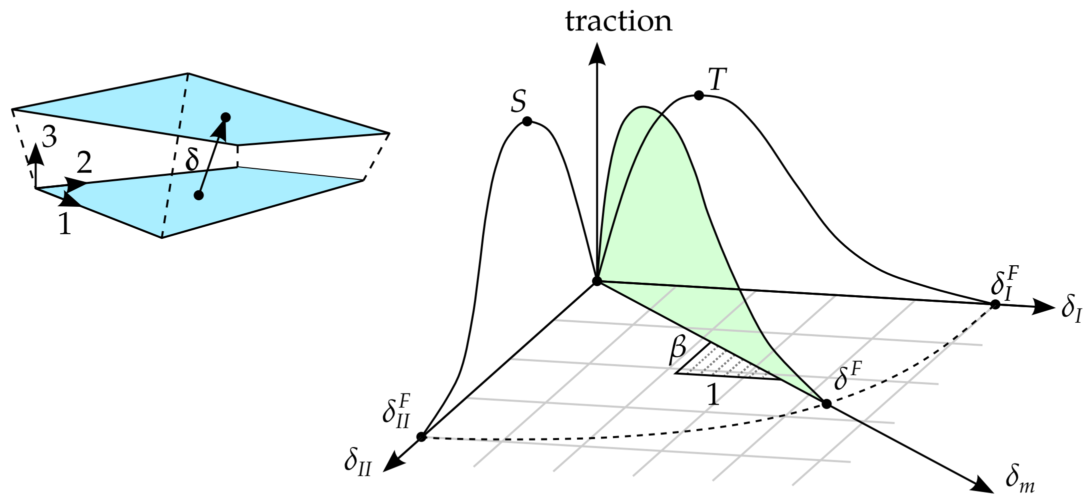


Figure M186-2. Mixed mode traction-separation law

2. **First and second mixed-mode interaction cohesive formulations (TES = 0.0 and 1.0).** For mixed-mode behavior, three different formulations are possible. We recommend TES = 0.0 with XMU = 1.0 as a first try. In this remark we will discuss the two formulations with dimensional separation measures.

   The total mixed-mode relative displacement δ_m is defined as $\delta_m = \sqrt{\delta_I^2 + \delta_{II}^2}$, where $\delta_I = \delta_3$ is the separation in normal direction (mode I) and $\delta_{II} = \sqrt{\delta_1^2 + \delta_2^2}$ is the separation in tangential direction (mode II). See Figure M186-2.

   The ultimate mixed-mode displacement δ^F (total failure) for the power law (TES = 0.0) is

$$\delta^F = \frac{1+\beta^2}{A_{\mathrm{TSLC}}}\left[\left(\frac{\mathrm{T}}{G_I^c}\right)^{\mathrm{XMU}} + \left(\frac{\mathrm{S}\times\beta^2}{G_{II}^c}\right)^{\mathrm{XMU}}\right]^{-\frac{1}{\mathrm{XMU}}}$$

   If TSLC2 is defined, this changes to:

$$\delta^F = (1+\beta^2)\left[\left(\frac{A_{\mathrm{TSLC}}\times\mathrm{T}}{G_I^c}\right)^{\mathrm{XMU}} + \left(\frac{A_{\mathrm{TSLC2}}\times\mathrm{S}\times\beta^2}{G_{II}^c}\right)^{\mathrm{XMU}}\right]^{-\frac{1}{\mathrm{XMU}}}$$

   Alternatively, for the Benzeggagh-Kenane law [1996] (TES = 1.0) δ^F is given by:

$$\delta^F = \frac{1+\beta^2}{A_{\mathrm{TSLC}}(\mathrm{T}+\mathrm{S}\times\beta^2)}\left[G_I^c + (G_{II}^c - G_I^c)\left(\frac{\mathrm{S}\times\beta^2}{T+\mathrm{S}\times\beta^2}\right)^{\mathrm{XMU}}\right]$$

If TSLC2 is defined, this changes to:

$$\delta^F = \frac{1+\beta^2}{A_{\text{TSLC}} \times \text{T} + A_{\text{TSLC2}} \times \text{S} \times \beta^2}\left[G_I^c + (G_{II}^c - G_I^c)\left(\frac{A_{\text{TSLC2}} \times \text{S} \times \beta^2}{A_{\text{TSLC}} \times T + A_{\text{TSLC2}} \times \text{S} \times \beta^2}\right)^{\text{XMU}}\right]$$

where $\beta = \delta_{II}/\delta_I$ is the "mode mixity". The larger the chosen exponent, XMU, is, the larger the fracture toughness will be in mixed-mode situations.

In this model, damage of the interface is considered, that is, irreversible conditions are enforced with loading/unloading paths coming from/pointing to the origin. This formulation is similar to *MAT_COHESIVE_MIXED_MODE (*MAT_138), but with the arbitrary traction-separation law TSLC.

3. **Third mixed-mode interaction cohesive formulations (TES = 2.0).** For TES = 2.0, we use a dimensionless effective separation parameter λ to model the interaction between relative displacements in normal (δ_3 - mode I) and tangential (δ_1,δ_2 - mode II) directions:

$$\lambda = \sqrt{\left(\frac{\delta_1}{\delta_{II}^F}\right)^2 + \left(\frac{\delta_2}{\delta_{II}^F}\right)^2 + \left\langle\frac{\delta_3}{\delta_I^F}\right\rangle^2}$$

Macaulay brackets distinguish between tension ($\delta_3 \geq 0$) and compression ($\delta_3 < 0$). δ_I^F and δ_{II}^F are critical values, representing the maximum separations in the interface in normal and tangential direction. For the stress calculation, the normalized traction-separation load curve TSLC is used:

$$t = t_{\text{max}} \times \bar{t}(\lambda)$$

This formulation is similar to *MAT_COHESIVE_TH (*MAT_185) but with an arbitrary traction-separation law and a damage formulation (that is, irreversible conditions are enforced with loading/unloading paths coming from/pointing to the origin).

*MAT_SAMP-1

Purpose: This is Material Type 187 (Semi-Analytical Model for Polymers). This material model uses an isotropic C-1 smooth yield surface to describe non-reinforced plastics. [Kolling, Haufe, Feucht, and Du Bois 2005] details the implementation.

This material law has been developed at DaimlerChrysler, Sindelfingen, in collaboration with Paul Du Bois and Dynamore, Stuttgart.

Card Summary:

Card 1. This card is required.

MID	RO	BULK	GMOD	EMOD	NUE	RBCFAC	NUMINT

Card 2. This card is required.

LCID-T	LCID-C	LCID-S	LCID-B	NUEP	LCID-P		INCDAM

Card 3. This card is required.

LCID-D	EPFAIL	DEPRPT	LCID_TRI	LCID_LC			

Card 4. This card is required.

MITER	MIPS		INCFAIL	ICONV	ASAF		NHSV

Card 5. This card is optional.

LCEMOD	BETA	FILT					

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	BULK	GMOD	EMOD	NUE	RBCFAC	NUMINT
Type	A	F	F	F	F	F	F	I/F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).

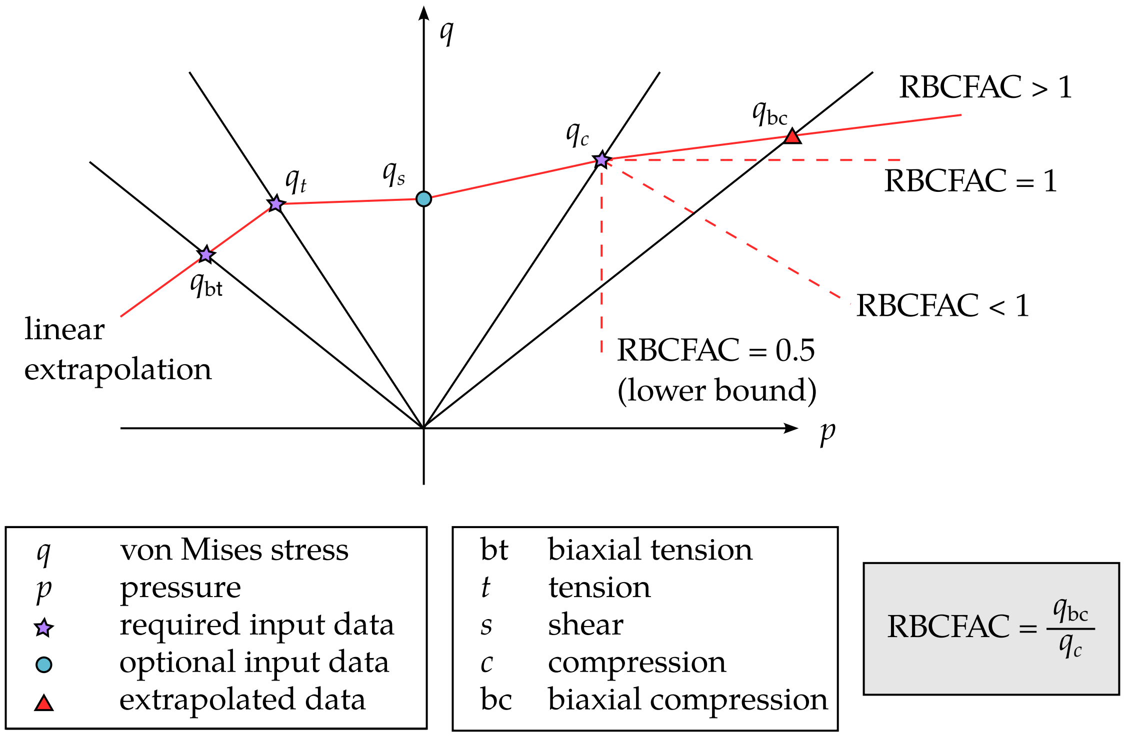


Figure M187-1. von Mises stress as a function of pressure

VARIABLE	DESCRIPTION
RO	Mass density
BULK	Optional bulk modulus used in the time step calculation for solids only
GMOD	Optional shear modulus used in the time step calculation for solids only
EMOD	Young's modulus
NUE	Poisson ratio
RBCFAC	Ratio of yield in biaxial compression as a function of yield in uniaxial compression. A nonzero RBCFAC with all four curves LCID-T, LCID-C, LCID-S, and LCID-B defined activates a piecewise-linear yield surface as shown in Figure M187-1. See Remark 3. The default is 0.
NUMINT	Number of integration points which must fail before the element is deleted. This option is available for shells and solids. LT.0.0: |NUMINT| is the percentage of integration points/layers which must fail before the shell element fails. For fully

VARIABLE	DESCRIPTION
	integrated shells, a layer fails if one integration point in the layer fails, and then the element fails if the given percentage of layers fails. Only available for shells.

Card 2	1	2	3	4	5	6	7	8
Variable	LCID-T	LCID-C	LCID-S	LCID-B	NUEP	LCID-P		INCDAM
Type	I	I	I	I	F	I		I

VARIABLE	DESCRIPTION
LCID-T	Load curve or table ID giving the yield stress as a function of plastic strain. These curves should be obtained from quasi-static and (optionally) dynamic uniaxial tensile tests. This input is mandatory, and the material model will not work unless at least one tensile stress-strain curve is given. If LCID-T is a table ID, the table values are plastic strain rates, and a curve of yield stress versus plastic strain must be given for each of those strain rates. If the first value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of plastic strain rate. When the highest plastic strain rate is several orders of magnitude greater than the lowest strain rate, it is recommended that the natural log of plastic strain rate be input in the table. See Remark 4.
LCID-C	Optional load curve ID giving the yield stress as a function of plastic strain. This curve should be obtained from a quasi-static uniaxial compression test.
LCID-S	Optional load curve ID giving the yield stress as a function of plastic strain. This curve should be obtained from a quasi-static shear test.
LCID-B	Optional load curve ID giving the yield stress as a function of plastic strain, this curve should be obtained from a quasi-static biaxial tensile test.
NUEP	Plastic Poisson’s ratio: an estimated ratio of transversal to longitudinal plastic rate of deformation under uniaxial loading should be given.
LCID-P	Load curve ID giving the plastic Poisson's ratio as a function of

VARIABLE	DESCRIPTION
	plastic strain during uniaxial tensile and uniaxial compressive testing. The plastic strain on the abscissa is negative for compression and positive for tension. It is important to cover both tension and compression. If LCID-P is given, NUEP is ignored.
INCDAM	Flag to control the damage evolution as a function of triaxiality: EQ.0: Damage evolution is independent of the triaxiality. EQ.1: An incremental formulation is used to compute the damage.

Card 3	1	2	3	4	5	6	7	8
Variable	LCID-D	EPFAIL	DEPRPT	LCID_TRI	LCID_LC			
Type	I	F	F	I	I			

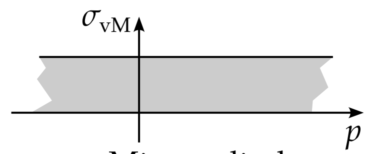
VARIABLE	DESCRIPTION
LCID-D	Load curve ID giving the damage parameter as a function of equivalent plastic strain during uniaxial tensile testing (history variable #2). By default, this option assumes that effective (i.e. undamaged) yield values are used in the load curves LCID-T, LCID-C, LCID-S and LCID-B. If LCID-D is given a negative value, true (meaning damaged) yield stress values can be used. In this case an automatic stress-strain recalibration (ASSR) algorithm is activated. The damage value must be defined in the range $0 \leq d < 1$. If EPFAIL and DEPRPT are given, the curve is used only until the effective plastic strain reaches EPFAIL.
EPFAIL	This parameter is the equivalent plastic strain at failure under uniaxial tensile loading (history variable #2). If EPFAIL is given as a negative integer, a load curve is expected that defines EPFAIL as a function of the plastic strain rate. The default value is 10^5.
DEPRPT	Increment of equivalent plastic strain under uniaxial tensile loading (history variable #2) between the failure and rupture points. Stresses will fade out to zero between EPFAIL and EPFAIL + DEPRPT. If DEPRPT is given a negative value, a curve definition is expected where DEPRPT is defined as a function of the triaxiality.
LCID_TRI	Load curve that specifies a factor that works multiplicatively on the

VARIABLE	DESCRIPTION
	value of EPFAIL depending on the triaxiality (that is, p/σ_{vM}). For a triaxiality of -1/3 a value of 1.0 should be specified.
LCID_LC	Load curve that specifies a factor that works multiplicatively on the value of EPFAIL depending on a characteristic element length, defined as the average length of spatial diagonals

Card 4	1	2	3	4	5	6	7	8
Variable	MITER	MIPS		INCFAIL	ICONV	ASAF		NHSV
Type	I	I		I	I	F		I

VARIABLE	DESCRIPTION
MITER	Maximum number of iterations in the cutting plane algorithm. The default is 400.
MIPS	Maximum number of iterations in the secant iteration performed to enforce plane stress (shell elements only). This variable is obsolete. A fixed three-step approach is used by default.
INCFAIL	Flag to control the failure evolution as a function of triaxiality: EQ.0: Failure evolution is independent of the triaxiality. EQ.1: Incremental formulation is used to compute the failure value. EQ.-1: The failure model is deactivated.
ICONV	Formulation flag: EQ.0: Default EQ.1: Yield surface is internally modified by increasing the shear yield until a convex yield surface is achieved.
ASAF	Safety factor used only if ICONV = 1. Values between 1 and 2 can improve convergence, however the shear yield will be artificially increased if this option is used. The default is 1.
NHSV	Number of history variables. Default is 22. Set to 28 if the "instability criterion" should be included in the output (see Remark 5). Note that NEIPS or NEIPH must also be set on *DATABASE_EX-

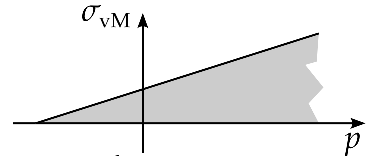
$$\left.\begin{array}{l}\text{LCID-C} = 0\\ \text{LCID-S} = 0\\ \text{LCID-B} = 0\end{array}\right\} \Rightarrow \begin{cases}\sigma_c = \sigma_t\\ \sigma_s = \dfrac{\sigma_t}{\sqrt{3}}\end{cases}$$



von Mises cylinder

$$\left.\begin{array}{l}\text{LCID-C} = 0\\ \text{LCID-S} \neq 0\\ \text{LCID-B} = 0\end{array}\right\} \Rightarrow \sigma_c = \frac{\sqrt{3}\sigma_t\sigma_s}{2\sigma_t - \sqrt{3}\sigma_s}$$

$$\left.\begin{array}{l}\text{LCID-C} \neq 0\\ \text{LCID-S} = 0\\ \text{LCID-B} = 0\end{array}\right\} \Rightarrow \sigma_s = \frac{2\sigma_c\sigma_t}{\sqrt{3}(\sigma_t + \sigma_c)}$$



Drucker-Prager cone

$$\left.\begin{array}{l}\text{LCID-C} = 0\\ \text{LCID-S} = 0\\ \text{LCID-B} \neq 0\end{array}\right\} \Rightarrow \begin{cases}\sigma_c = \dfrac{\sigma_t\sigma_b}{3\sigma_b - 2\sigma_t}\\ \sigma_s = \dfrac{\sigma_t\sigma_b}{\sqrt{3}(2\sigma_b - \sigma_t)}\end{cases}$$

Figure M187-2. Fewer than 3 load curves

VARIABLE	DESCRIPTION
	TENT_BINARY for the history variable data to be output.

Optional Card.

Card 5	1	2	3	4	5	6	7	8
Variable	LCEMOD	BETA	FILT					
Type	F	F	F					

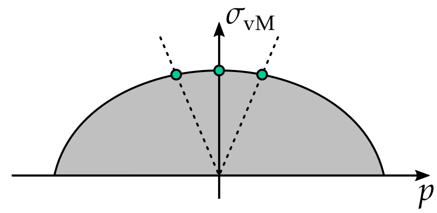
VARIABLE	DESCRIPTION
LCEMOD	Load curve ID defining Young's modulus as function of effective strain rate
BETA	Decay constant in viscoelastic law: $\dot{\sigma}(t) = -\beta \times \sigma(t) + E(\dot{\varepsilon}(t)) \times \dot{\varepsilon}(t)$ If LCEMOD > 0 is used, a non-zero value for BETA is mandatory.
FILT	Factor for strain rate filtering: $\dot{\varepsilon}_{n+1}^{avg} = (1 - \text{FILT}) \times \dot{\varepsilon}_{n+1}^{\text{cur}} + \text{FILT} \times \dot{\varepsilon}_n^{avg}$

$$\left.\begin{array}{l}\text{LCID-C} \neq 0 \\ \text{LCID-S} \neq 0 \\ \text{LCID-B} = 0\end{array}\right\} \Rightarrow \text{normal SAMP-1 behavior}$$

$$\left.\begin{array}{l}\text{LCID-C} \neq 0 \\ \text{LCID-S} = 0 \\ \text{LCID-B} \neq 0\end{array}\right\} \Rightarrow \sigma_s = \frac{1}{\sqrt{3}}\sqrt{\frac{3\sigma_b^2\sigma_c\sigma_t}{(2\sigma_b+\sigma_c)(2\sigma_b-\sigma_t)}}$$

$$\left.\begin{array}{l}\text{LCID-C} = 0 \\ \text{LCID-S} \neq 0 \\ \text{LCID-B} \neq 0\end{array}\right\} \Rightarrow \sigma_c = \frac{6(162\sigma_b^2\sigma_s^2+\sigma_b\sigma_s^2\sigma_t)}{6\sigma_b\sigma_s^2+323\sigma_b^2\sigma_t+3\sigma_s^2\sigma_t}$$

$$\left.\begin{array}{l}\text{LCID-C} \neq 0 \\ \text{LCID-S} \neq 0 \\ \text{LCID-B} \neq 0\end{array}\right\} \Rightarrow \text{overspecified, least square}$$



SAMP-1 yield surface defined through load curves

Figure M187-3. Three or more load curves

Load Curves:

Material SAMP-1 uses three yield curves internally to evaluate a quadratic yield surface. *MAT_SAMP-1 accepts four different kinds of yield curves, LCID-T, LCID-C, LCID-S, and LCID-B where data from tension tests (LCID-T) is always required, but the others are optional. If fewer than three curves are defined, as indicated by setting the missing load curve IDs to 0, the remaining curves are generated internally.

1. **Fewer than 3 load curves.** In the case of fewer than 3 load curves, a linear yield surface in the invariant space spanned by the pressure and the von Mises stress is generated using the available data. See Figure M187-2.

2. **Three or more load curves.** See Figure M187-3.

Remarks:

1. **Damage.** If the LCID-D is given, then a damage curve as a function of equivalent plastic strains acting on the stresses is defined as shown in Figure M187-4.

   Since the damaging curve acts on the yield values, the inelastic results are affected by the damage curve. As a means to circumvent this, the load curve LCID-D may be given a negative ID. This will lead to an internal conversion of from nominal to effective stresses (ASSR). While this conversion is possible for

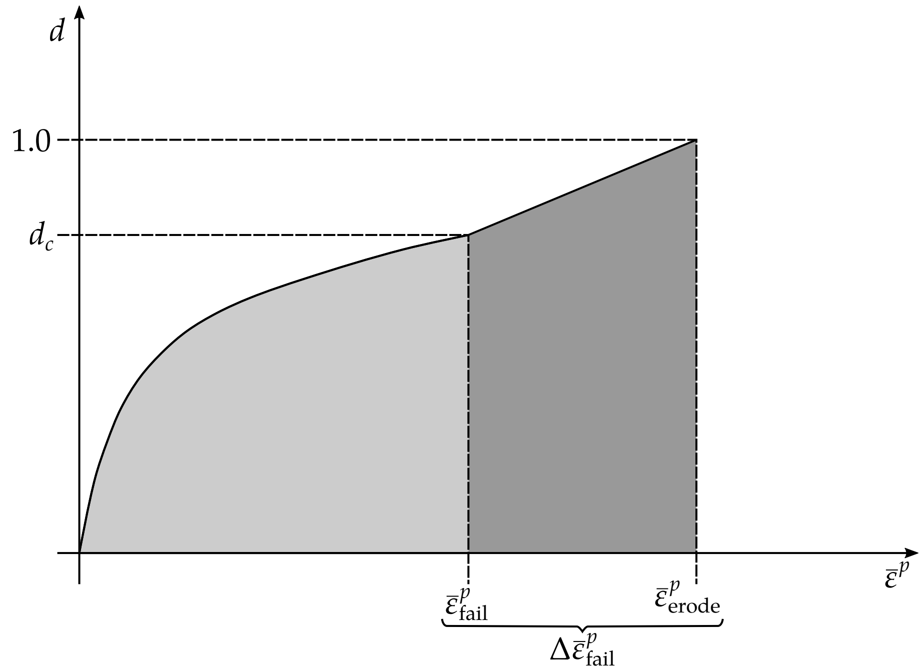


Figure M187-4. EPFAIL and DEPRPT defined the failure and fading behavior of a single element.

some combinations of yield curve definitions, plastic Poisson's ratio and damage curves, the corresponding inverse problem cannot be solved for all combinations. An error message is provided in this case.

2. **Unsolvable yield surface case.** Since the generality of arbitrary curve inputs allows unsolvable yield surfaces, SAMP may modify curves internally. This will always lead to warning messages at the beginning of the simulation run. One modification that is not allowed is negative tangents of the last two data points of any of the yield curves.

3. **RBCFAC.** If RBCFAC is nonzero and curves LCID-T, LCID-C, LCID-S, and LCID-B are specified, the yield surface in I_1-σ_{vm} -stress space is constructed such that a piecewise-linear yield surface is activated. This option can help promote the convergence of the plasticity algorithm. Figure M187-1 illustrates the effect of RBCFAC on behavior in biaxial compression.

4. **Dynamic amplification factor for yield stress.** If LCID-T is given as a table specifying strain-rate scaling of the yield stress, then the compressive, shear, and biaxial yield stresses are computed by multiplying their respective static values by the dynamic amplification factor (dynamic/static ratio) of the tensile yield stress.

5. **Instability criterion.** Instability at an integration point is a value between 0 and 1 indicating the integration point's proximity to damage start. If instability reaches 1, then damage starts and grows from 0 to 1. The element then

"ruptures". The choice of INCFAIL determines how instability is calculated. For INCFAIL = 0,

$$\text{instability} = (\text{equivalent plastic strain})/\text{EPFAIL}$$

6. **History variables.** This material has the following history variables. NEIPH or NEIPS on *DATABASE_EXTENT_BINARY must be set for the history data to be output.

History Variable #	Description
2	Plastic strain in tension/compression
3	Plastic strain in shear
4	Biaxial plastic strain
5	Damage
6	Volumetric plastic strain
16	Plastic strain rate in tension/compression
17	Plastic strain rate in shear
18	Biaxial plastic strain rate
28	Instability criterion (set NHSV = 28, see Remark 5)

*MAT_SAMP_LIGHT

Purpose: This is a slimmed-down form of Material Type 187. In contrast to the original SAMP-1 the options here are limited to rate-independent or rate-dependent flow in tension and compression as well as constant or variable plastic Poisson's ratio. Shear and biaxial test data are not incorporated. Damage and failure are not available here. *MAT_-ADD_EROSION or *MAT_ADD_DAMAGE can be included for damage and failure. But as in the original model, a viscoelastic extension can be activated.

This model is based on a complete re-coding of the plasticity algorithm. The efficiency was improved to the extent that the computing times should be shorter. As compared to *MAT_SAMP-1, results should differ only slightly. To achieve the most similar results when including rate effects, set RATEOP = 1 because it is equivalent to the viscoplastic formulation in *MAT_SAMP-1.

Card Summary:

Card 1. This card is required.

MID	RO			EMOD	NUE	LCEMOD	BETA

Card 2. This card is required.

LCID-T	LCID-C	CTFLG	RATEOP	NUEP	LCID-P	RFILTF	

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO			EMOD	NUE	LCEMOD	BETA
Type	A	F			F	F	I	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see*PART).
RO	Mass density
EMOD	Young's modulus
NUE	Poisson ratio

VARIABLE	DESCRIPTION
LCEMOD	Load curve ID defining Young's modulus as function of effective strain rate. LCEMOD ≠ 0 activates viscoelasticity (see Remark 3). The parameters BETA and RFILTF must be defined too.
BETA	Decay constant in viscoelastic law (see Remark 3). BETA has the unit [1/time]. If LCEMOD > 0 is used, a nonzero value for BETA is mandatory.

Card 2	1	2	3	4	5	6	7	8
Variable	LCID-T	LCID-C	CTFLG	RATEOP	NUEP	LCID-P	RFILTF	
Type	I	I	F	I	F	I	F	
Default	none	0	0	0	none	0	0.95	

VARIABLE	DESCRIPTION
LCID-T	Load curve or table ID giving the yield stress as a function of plastic strain. These curves should be obtained from quasi-static and (optionally) dynamic uniaxial tensile tests. This input is mandatory. If LCID-T is a table ID, the table values are effective strain rates, and a curve of yield stress as a function of plastic strain must be given for each of those strain rates. If the first value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of effective strain rate. When the highest effective strain rate is several orders of magnitude greater than the lowest strain rate, it is recommended that the natural log of strain rate be input in the table. See Remark 1.
LCID-C	Optional load curve (or table) ID giving the yield stress as a function of plastic strain (and strain rate). This curve (or table) should be obtained from uniaxial compression tests. If LCID-C is defined as a curve and LCID-T given as a table, then the rate dependence from the tension table is adopted in compression as well. See Remark 1.
CTFLG	Curve treatment flag (for LCID-T, LCID-C, and LCID-P) EQ.0: Rediscretized curves (default). We recommend this option with an appropriate value of LCINT for accurate

VARIABLE	DESCRIPTION
	resolution of the curves (see *DEFINE_CURVE and *CONTROL_SOLUTION). EQ.1: Original curve values from the input
RATEOP	Calculation of effective strain rate option: EQ.0: Original method for calculating the effective *total* strain rate. EQ.1: Viscoplastic formulation, meaning using effective plastic strain rate. Recommended option to achieve the best match with *MAT_SAMP-1. EQ.2: Improved method for calculating the effective total strain rate. This method gives a slightly closer match (compared to RATEOP = 0) to *MAT_SAMP-1.
NUEP	Plastic Poisson's ratio: an estimated ratio of transversal to longitudinal plastic rate of deformation under uniaxial loading should be given. Ignored if LCID-P is nonzero. See Remark 1.
LCID-P	Load curve ID giving the plastic Poisson's ratio as a function of an equivalent plastic strain measure during uniaxial tensile and uniaxial compressive testing. The plastic strain measure on the abscissa is negative for compression and positive for tension. It is important to cover both tension and compression. If LCID-P is given, NUEP is ignored. See Remark 1.
RFILTF	Smoothing factor on the effective strain rate (default is 0.95). The filtered strain rate is used for the rate-dependent plastic flow (LCID-T being a table) as well as for the viscoelasticity (LCEMOD > 0). See Remark 2. $$\dot{\varepsilon}_n^{\mathrm{avg}} = \mathrm{RFILTF} \times \dot{\varepsilon}_{n-1}^{\mathrm{avg}} + (1 - \mathrm{RFILTF}) \times \dot{\varepsilon}_n$$

Remarks:

1. **Yield surfaces.** In the case of one tensile load curve (or table) LCID-T, the yield surface is von Mises type with associated (NUEP = 0.5) or non-associated (else) plastic flow. In the case LCID-T and LCID-C are both defined, a Drucker-Prager type yield surface is used. The plastic flow direction again depends on the choice of NUEP/LCID-P.

   The yield condition is given by

$$F = \sigma^{\text{eff}} - \sigma^{Y,T} \leq 0$$

with the effective stress

$$\sigma^{\text{eff}} = (1 - \xi) \times \sigma^{\text{vM}} - 3\xi \times p$$

based on the von Mises stress σ^{vM}, the pressure p and the Drucker-Prager slope parameter

$$\xi = \frac{\sigma^{Y,C} - \sigma^{Y,T}}{2 \times \sigma^{Y,C}}$$

defined by the tensile and compressive yield stresses $\sigma^{Y,T}$ and $\sigma^{Y,C}$ (input with LCID-T and optionally LCID-C). The plastic potential

$$G = \sqrt{\left(\sigma^{\text{vM}^2} + \alpha \times p^2\right)}$$

with

$$\alpha = \frac{9}{2}\left(\frac{1 - 2\nu^p}{1 + \nu^p}\right)$$

and the plastic Poisson's ratio ν^p defines the direction of plastic flow. ν^pcan be input as a constant with NUEP or as a function of equivalent plastic strain with LCID-P. The elasto-plastic equations are solved with a classical predictor-corrector algorithm for the plastic strain increment $\Delta\varepsilon^p$ and stress $\boldsymbol{\sigma}^{n+1}$.

2. **Effective strain rate.** If tables are used for hardening, the rate dependence is defined by using an effective strain rate. To reduce the noise from the elastic portion of that strain rate, an averaged value is used. This is governed by the filtering parameter RFILTF as shown above.

3. **Nonlinear viscoelasticity**. For LCEMOD ≠ 0, viscoelasticity is activated. With this model, EMOD becomes the Young's modulus for equilibrium stress (E_{eq}). LCEMOD specifies the viscous Young's modulus which is a function of averaged strain rate ($E_v(\dot{\varepsilon}^{\text{avg}})$). BETA gives a constant decay parameter specifying the ratio $\frac{E_v(\dot{\varepsilon}^{\text{avg}})}{\eta(\dot{\varepsilon}^{\text{avg}})}$ where η is the viscosity. The viscoelastic stress-strain law is given by:

$$\dot{\sigma}_v = -\text{BETA} \times \sigma(t) + E_v(\dot{\varepsilon}^{\text{avg}}) \times \dot{\varepsilon}^{\text{avg}}$$

The one-dimensional viscoelastic behavior can be visualized by a generalized Maxwell cell as rheological model as shown in Figure M187-1.

We will be using the notation of the viscoelastic-plastic stress update. For simplicity we will discuss the one-dimensional case. Let $\Delta\varepsilon^{\text{vp}}$ be the viscoplastic strain increment and let σ_{eq} and σ_v be the equilibrium and viscous stress portions. The stress update is then:

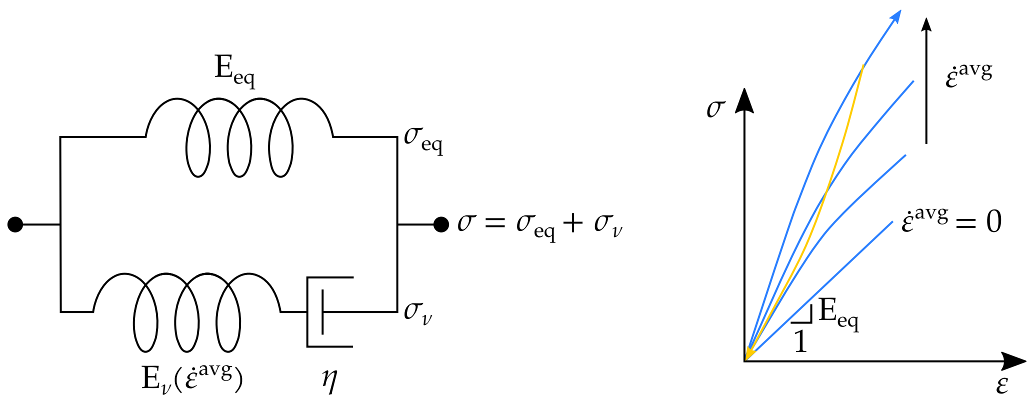


Figure M187-1. Viscoelastic model and strain rate dependent stress-strain curves.

$$\sigma^{n+1} = \sigma_{\mathrm{eq}}^{n+1} + \sigma_v^{n+1}$$

or with stress increments:

$$\sigma^{n+1} = \sigma^n + \Delta\sigma_{\mathrm{eq}} + \Delta\sigma_v.$$

In the elastic trial-step the equilibrium and viscous stress increments $\Delta\sigma_{\mathrm{eq}}^{\mathrm{tr}}$ and $\Delta\sigma_v^{\mathrm{tr}}$ are calculated as:

$$\Delta\sigma_{\mathrm{eq}}^{\mathrm{tr}} = E_{\mathrm{eq}} \times \Delta\varepsilon$$

and

$$\Delta\sigma_v^{\mathrm{tr}} = E_v(\dot{\varepsilon}^{\mathrm{avg}}) \frac{1 - e^{-\beta\Delta t}}{\beta\Delta t} (\dot{\varepsilon}^{\mathrm{avg}}\, \Delta t) - \left(1 - e^{-\beta\Delta t}\right)\sigma_v^n$$

And the yield condition (see Remark 1) is evaluated:

$$F\left(\sigma^{n+1,\mathrm{tr}}\right) = F(\sigma^n + \Delta\sigma_{\mathrm{eq}}^{\mathrm{tr}} + \Delta\sigma_v^{\mathrm{tr}}).$$

For $F \leq 0$ the trial stress state is the current stress, that is, $\sigma^{n+1} = \sigma^n + \Delta\sigma_{\mathrm{eq}}^{\mathrm{tr}} + \Delta\sigma_v^{\mathrm{tr}}$. Otherwise, the plastic strain increment $\Delta\varepsilon^p$ must be evaluated and the equilibrium and viscous stress increments are updated.

For $F\left(\sigma_{\mathrm{eq}}^{\mathrm{tr}}\right) > 0$:

$$\sigma_{\mathrm{eq}}^{n+1} = \sigma_{\mathrm{eq}}^{\mathrm{tr}} - E_{\mathrm{eq}}\Delta\varepsilon^p$$
$$\sigma_v^{n+1} = \sigma_v^{\mathrm{tr}} - E_v(\dot{\varepsilon}^{\mathrm{avg}}) \frac{1 - e^{-\beta\Delta t}}{\beta\Delta t}\Delta\varepsilon^p$$

For $F\left(\sigma_{\mathrm{eq}}^{\mathrm{tr}}\right) \leq 0$:

$$\sigma_{\mathrm{eq}}^{n+1} = \sigma_{\mathrm{eq}}^{\mathrm{tr}}$$

4. **History variables.** This material has the following extra history variables:

History Variable #	Description
1	Filtered deviatoric strain rate for viscoelasticity (LCEMOD > 0)
2	Volumetric plastic strain
3	Number of iterations
4	Current yield stress
5	Filtered total strain rate
7	Deviatoric plastic strain

*MAT_THERMO_ELASTO_VISCOPLASTIC_CREEP

This is Material Type 188. In this model, creep is described separately from plasticity using Garafalo's steady-state hyperbolic sine creep law or Norton's power law. Viscous effects of plastic strain rate are considered using the Cowper-Symonds model. Young's modulus, Poisson's ratio, thermal expansion coefficient, yield stress, material parameters of Cowper-Symonds model as well as the isotropic and kinematic hardening parameters are all assumed to be temperature dependent. Application scope includes: simulation of solder joints in electronic packaging, modeling of tube brazing process, creep age forming, etc.

Card Summary:

Card 1. This card is required.

MID	RO	E	PR	SIGY	ALPHA	LCSS	REFTEM

Card 2. This card is required.

QR1	CR1	QR2	CR2	QX1	CX1	QX2	CX2

Card 3. This card is required.

C	P	LCE	LCPR	LCSIGY	LCQR	LCQX	LCALPH

Card 4. This card is required.

LCC	LCP	LCCR	LCCX	CRPA	CRPB	CRPQ	CRPM

Card 5. This card is optional.

CRPLAW							

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	ALPHA	LCSS	REFTEM
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
SIGY	Initial yield stress
ALPHA	Thermal expansion coefficient
LCSS	Load curve ID or Table ID. The load curve defines effective stress as a function of effective plastic strain. The table defines for each temperature value a load curve ID referencing stress as a function of effective plastic strain for that temperature. The stress as a function of effective plastic strain curve for the lowest value of temperature is used if the temperature falls below the minimum value. Likewise, the stress as a function of effective plastic strain curve for the highest value of temperature is used if the temperature exceeds the maximum value. Card 2 is ignored with this option.
REFTEM	Reference temperature that defines thermal expansion coefficient

Card 2	1	2	3	4	5	6	7	8
Variable	QR1	CR1	QR2	CR2	QX1	CX1	QX2	CX2
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
QR1	Isotropic hardening parameter Q_{r1}
CR1	Isotropic hardening parameter C_{r1}
QR2	Isotropic hardening parameter Q_{r2}
CR2	Isotropic hardening parameter C_{r2}
QX1	Kinematic hardening parameter $Q_{\chi1}$

VARIABLE	DESCRIPTION
CX1	Kinematic hardening parameter $C_{\chi 1}$
QX2	Kinematic hardening parameter $Q_{\chi 2}$
CX2	Kinematic hardening parameter $C_{\chi 2}$

Card 3	1	2	3	4	5	6	7	8
Variable	C	P	LCE	LCPR	LCSIGY	LCQR	LCQX	LCALPH
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
C	Viscous material parameter C
P	Viscous material parameter P
LCE	Load curve for scaling Young's modulus as a function of temperature
LCPR	Load curve for scaling Poisson's ratio as a function of temperature
LCSIGY	Load curve for scaling initial yield stress as a function of temperature
LCQR	Load curve for scaling the isotropic hardening parameters QR1 and QR2 or the stress given by the load curve LCSS as a function of temperature
LCQX	Load curve for scaling the kinematic hardening parameters QX1 and QX2 as a function of temperature
LCALPH	Load curve for scaling the thermal expansion coefficient as a function of temperature

Card 4	1	2	3	4	5	6	7	8
Variable	LCC	LCP	LCCR	LCCX	CRPA	CRPB	CRPQ	CRPM
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
LCC	Load curve for scaling the viscous material parameter C as a function of temperature
LCP	Load curve for scaling the viscous material parameter P as a function of temperature
LCCR	Load curve for scaling the isotropic hardening parameters CR1 and CR2 as a function of temperature
LCCX	Load curve for scaling the kinematic hardening parameters CX1 and CX2 as a function of temperature
CRPA	Creep law parameter A GT.0.0: Constant value LT.0.0: Load curve ID = (-CRPA) which defines A as a function of temperature, $A(T)$
CRPB	Creep law parameter B GT.0.0: Constant value LT.0.0: Load curve ID = (-CRPB) which defines B as a function of temperature, $B(T)$
CRPQ	Creep law parameter $Q = E/R$ where E is the activation energy and R is the universal gas constant. GT.0.0: Constant value LT.0.0: Load curve ID = (-CRPQ) which defines Q as a function of temperature, $Q(T)$
CRPM	Creep law parameter m GT.0.0: Constant value LT.0.0: Load curve ID = (-CRPM) which defines m as a function of temperature, $m(T)$

Optional card 5

Card 5	1	2	3	4	5	6	7	8
Variable	CRPLAW							
Type	F							

VARIABLE	DESCRIPTION
CRPLAW	Creep law definition (see Remarks): EQ.0.0: Garofalo's hyperbolic sine law (default) EQ.1.0: Norton's power law

Remarks:

If LCSS is not defined, the uniaxial stress-strain curve has the form

$$\sigma(\varepsilon^p_{\mathrm{eff}}) = \sigma_0 + Q_{r1}[1 - \exp(-C_{r1}\varepsilon^p_{\mathrm{eff}})] + Q_{r2}[1 - \exp(-C_{r2}\varepsilon^p_{\mathrm{eff}})] + Q_{\chi 1}[1 - \exp(-C_{\chi 1}\varepsilon^p_{\mathrm{eff}})] + Q_{\chi 2}[1 - \exp(-C_{\chi 2}\varepsilon^p_{\mathrm{eff}})].$$

Viscous effects are accounted for using the Cowper-Symonds model, which scales the yield stress with the factor:

$$1 + \left(\frac{\dot{\varepsilon}^p_{\mathrm{eff}}}{C}\right)^{1/p} .$$

For CRPLAW = 0, the steady-state creep strain rate of Garafalo's hyperbolic sine equation is given by

$$\dot{\varepsilon}^c = A[\sinh(B\tau^e)]^m \exp\left(-\frac{Q}{T}\right) .$$

For CRPLAW = 1, the steady-state creep strain rate is given by Norton's power law equation:

$$\dot{\varepsilon}^c = A(\tau^e)^B t^m.$$

In the above, τ^e is the effective elastic stress in the von Mises sense, T is the temperature and t is the time. The following is a schematic overview of the resulting stress update. The multiaxial creep strain increment is given by

$$\Delta\varepsilon^c = \Delta\varepsilon^c \frac{3\boldsymbol{\tau}^e}{2\tau^e} ,$$

where $\boldsymbol{\tau}^e$ is the elastic deviatoric stress tensor. Similarly, the plastic and thermal strain increments are given by

$$\Delta\boldsymbol{\varepsilon}^p = \Delta\varepsilon^p \frac{3\boldsymbol{\tau}^e}{2\tau^e}$$
$$\Delta\boldsymbol{\varepsilon}^T = \alpha_{t+\Delta t}(T - T_{\text{ref}})\mathbf{I} - \boldsymbol{\varepsilon}_t^T$$

where α is the thermal expansion coefficient (note the definition compared to that of other materials). Adding it all together, the stress update is given by

$$\boldsymbol{\sigma}_{t+\Delta t} = \mathbf{C}_{t+\Delta t}\left(\boldsymbol{\varepsilon}_t^e + \Delta\boldsymbol{\varepsilon} - \Delta\boldsymbol{\varepsilon}^p - \Delta\boldsymbol{\varepsilon}^c - \Delta\boldsymbol{\varepsilon}^T\right) \ .$$

The plasticity is isotropic or kinematic but with a von Mises yield criterion, the subscript in the equation above indicates the simulation time of evaluation. Internally, this stress update requires the solution of a nonlinear equation in the effective stress, the viscoelastic strain increment and potentially the plastic strain increment.

*DEFINE_MATERIAL_HISTORIES can be used to output the viscoelastic (creep strain).

**DEFINE_MATERIAL_HISTORIES Properties*

Label	Attributes				Description
Effective Creep Strain	-	-	-	-	Viscoelastic strain ε^c, see above
Plastic Strain Rate	-	-	-	-	Effective plastic strain rate $\dot{\varepsilon}^p_{\text{eff}}$

*MAT_ANISOTROPIC_THERMOELASTIC

This is Material Type 189. This model characterizes elastic materials whose elastic properties are temperature-dependent.

It is available for solid elements, thick shell formulations 3, 5, and 7, and multi-material ALE solid elements. Note that it is not validated for multi-material ALE solid elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	TA1	TA2	TA3	TA4	TA5	TA6
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	C11	C12	C13	C14	C15	C16	C22	C23
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	C24	C25	C26	C33	C34	C35	C36	C44
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	C45	C46	C55	C56	C66	TGE	TREF	AOPT
Type	F	F	F	F	F	F	F	F

Card 5	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	MACF	
Type	F	F	F	F	F	F	F	

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	REF
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
TA*i*	Curve IDs defining the coefficients of thermal expansion for the six components of strain tensor as function of temperature.
C*ij*	Curve IDs defining the 6 × 6 symmetric constitutive matrix in material coordinate system as function of temperature. Note that 1 corresponds to the *a* material direction, 2 to the *b* material direction, and 3 to the *c* material direction.
TGE	Curve ID defining the structural damping coefficient as function of temperature.
TREF	Reference temperature for the calculation of thermal loads or the definition of thermal expansion coefficients.
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details): EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a**-direction. This option is for solid

VARIABLE	DESCRIPTION
	elements only.
	EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR
	EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.
	EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only.
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*).
XP, YP, ZP	Coordinates of point *p* for AOPT = 1 and 4
A1, A2, A3	Components of vector **a** for AOPT = 2
MACF	Material axes change flag for solid elements:
	EQ.-4: Switch material axes *b* and *c* before BETA rotation
	EQ.-3: Switch material axes *a* and *c* before BETA rotation
	EQ.-2: Switch material axes *a* and *b* before BETA rotation
	EQ.1: No change, default
	EQ.2: Switch material axes *a* and *b* after BETA rotation
	EQ.3: Switch material axes *a* and *c* after BETA rotation

VARIABLE	DESCRIPTION
	EQ.4: Switch material axes *b* and *c* after BETA rotation Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_-SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 6 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.
D1, D2, D3	Components of vector **d** for AOPT = 2
V1, V2, V3	Components of vector **v** for AOPT = 3 and 4.
BETA	Material angle in degrees for AOPT = 3. It may be overwritten on the element card; see *ELEMENT_SOLID_ORTHO.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: Off EQ.1.0: On

*MAT_FLD_3-PARAMETER_BARLAT

This is Material Type 190. This model was developed by Barlat and Lian [1989] for modeling sheets with anisotropic materials under plane stress conditions. This material allows the use of the Lankford parameters for the definition of the anisotropy. This particular development is due to Barlat and Lian [1989]. It has been modified to include a failure criterion based on the Forming Limit Diagram. The curve can be input as a load curve or calculated based on the n-value and sheet thickness.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	HR	P1	P2	ITER
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	M	R00	R45	R90	LCID	E0	SPI	P3
Type	F	F	F	F	I	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT	C	P	FLDCID	RN	RT	FLDSAFE	FLDNIPF
Type	F	F	F	I	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus, E
PR	Poisson's ratio, ν
HR	Hardening rule: EQ.1.0: Linear (default) EQ.2.0: Exponential (Swift) EQ.3.0: Load curve EQ.4.0: Exponential (Voce) EQ.5.0: Exponential (Gosh) EQ.6.0: Exponential (Hocket-Sherby)
P1	Material parameter: HR.EQ.1.0: Tangent modulus HR.EQ.2.0: k, strength coefficient for Swift exponential hardening HR.EQ.4.0: a, coefficient for Voce exponential hardening HR.EQ.5.0: k, strength coefficient for Gosh exponential hardening HR.EQ.6.0: a, coefficient for Hocket-Sherby exponential hardening
P2	Material parameter: HR.EQ.1.0: Yield stress HR.EQ.2.0: n, exponent for Swift exponential hardening

VARIABLE	DESCRIPTION
	HR.EQ.4.0: c, coefficient for Voce exponential hardening HR.EQ.5.0: n, exponent for Gosh exponential hardening HR.EQ.6.0: c, coefficient for Hocket-Sherby exponential hardening
ITER	Iteration flag for speed: EQ.0.0: Fully iterative EQ.1.0: Fixed at three iterations Generally, ITER = 0 is recommended. However, ITER = 1 is somewhat faster and may give acceptable results in most problems.
M	m, exponent in Barlat's yield surface
R00	R_{00}, Lankford parameter determined from experiments
R45	R_{45}, Lankford parameter determined from experiments
R90	R_{90}, Lankford parameter determined from experiments
LCID	Load curve ID for the load curve hardening rule (HR = 3.0)
E0	Material parameter HR.EQ.2.0: ε_0 for determining initial yield stress for Swift exponential hardening. The default value is 0.0. HR.EQ.4.0: b, coefficient for Voce exponential hardening HR.EQ.5.0: ε_0 for determining initial yield stress for Gosh exponential hardening. The default value is 0.0. HR.EQ.6.0: b, coefficient for Hocket-Sherby exponential hardening
SPI	If E0 is zero above and HR = 2.0: EQ.0.0: $\varepsilon_0 = \left(E/k\right)^{1/(n-1)}$ LE.0.2: $\varepsilon_0 = \mathrm{SPI}$ GT.0.2: $\varepsilon_0 = \left(\mathrm{SPI}/k\right)^{1/n}$
P3	Material parameter: HR.EQ.5.0: p, parameter for Gosh exponential hardening

VARIABLE	DESCRIPTION
	HR.EQ.6.0: n, exponent for Hocket-Sherby exponential hardening
AOPT	Material axes option (see *MAT_*OPTION*TROPIC_ELASTIC for a more complete description): EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA. EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR. EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal. LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
C	C in Cowper-Symonds strain rate model
P	p in Cowper-Symonds strain rate model. $p = 0.0$ for no strain rate effects.
FLDCID	Load curve ID defining the Forming Limit Diagram. Minor engineering strains in percent are defined as abscissa values and major engineering strains in percent are defined as ordinate values. The forming limit diagram is shown in Figure M39-1. In defining the curve, list pairs of minor and major strains starting with the leftmost point and ending with the rightmost point. See *DEFINE_CURVE. See Remark 2.
RN	Hardening exponent equivalent to the n-value in a power law hardening law. If the parameter FLDCID is not defined, this value in combination with the value RT can be used to calculate a forming limit curve to allow for failure. Otherwise it is ignored. See Remark 2.
RT	Sheet thickness used for calculating a forming limit curve. This

VARIABLE	DESCRIPTION
	value does not override the sheet thickness in any way. It is only used in conjunction with the parameter RN to calculate a forming limit curve if the parameter FLDCID is not defined. See Remark 2.
FLDSAFE	A safety offset of the forming limit curve. This value should be input as a percentage (such as 10, not 0.10). This safety margin will be applied to the forming limit curve defined by FLDCID or the curve calculated by RN and RT.
FLDNIPF	Numerical integration points failure treatment: GT.0.0: The number of element integration points that must fail before the element is deleted. By default, if one integration point has strains above the forming limit curve, the element is flagged for deletion. LT.0.0: The element is deleted when all integration points within a relative distance of –FLDNIPF from the mid-surface have failed (value between -1.0 and 0.0).
A1, A2, A3	Components of vector **a** for AOPT = 2.
V1, V2, V3	Components of vector **v** for AOPT = 3.
D1, D2, D3	Components of vector **d** for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card. See *ELEMENT_SHELL_BETA.

Remarks:

1. **Theoretical basis.** See *MAT_036 for the theoretical basis.

2. **Forming limit curve.** The forming limit curve can be input directly as a curve by specifying a load curve ID with the parameter FLDCID. When defining such a curve, the major and minor strains must be input as percentages. Alternatively, the parameters RN and RT can be used to calculate a forming limit curve. The use of RN and RT is not recommended for non-ferrous materials. RN and RT are ignored if a nonzero FLDCID is defined.

3. **History variable.** The first history variable is the maximum strain ratio defined by:

$$\frac{\varepsilon_{\mathrm{major}_{\mathrm{workpiece}}}}{\varepsilon_{\mathrm{major}_{\mathrm{fld}}}}$$

corresponding to $\varepsilon_{\mathrm{minor}_{\mathrm{workpiece}}}$. A value between 0 and 1 indicates that the strains lie below the forming limit curve. Values above 1 indicate that the strains are above the forming limit curve.

*MAT_SEISMIC_BEAM

This is Material Type 191. This material enables lumped plasticity to be developed at the "node 2" end of Belytschko-Schwer beams (resultant formulation). The plastic yield surface allows for interaction between the two moments and the axial force.

Card Summary:

Card 1. This card is required.

MID	RO	E	PR	ASFLAG	FTYPE	DEGRAD	IFEMA

Card 2. This card is required.

LCPMS	SFS	LCPMT	SFT	LCAT	SFAT	LCAC	SFAC

Card 3a. This card is included if and only if FTYPE = 1.

ALPHA	BETA	GAMMA	DELTA	A	B	FOFFS	

Card 3b. This card is included if and only if FTYPE = 2.

SIGY	D	W	TF	TW			

Card 3c. This card is included if and only if FTYPE = 4.

PHI_T	PHI_C	PHI_B					

Card 3d. This card is included if and only if FTYPE = 5.

ALPHA	BETA	GAMMA	DELTA	PHI_T	PHI_C	PHI_B	

Card 4. This card is included if and only if IFEM > 0.

PR1	PR2	PR3	PR4				

Card 5. This card is included if and only if IFEM > 0.

TS1	TS2	TS3	TS4	CS1	CS2	CS3	CS4

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	ASFLAG	FTYPE	DEGRAD	IFEMA
Type	A	F	F	F	F	I	I	I
Default	none	none	none	none	0.0	1	0	0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus.
PR	Poisson's ratio
ASFLAG	Axial strain definition for force-strain curves, degradation and FEMA output: EQ.0.0: True (log) total strain EQ.1.0: Change in length EQ.2.0: Nominal total strain EQ.3.0: FEMA plastic strain (= nominal total strain minus elastic strain)
FTYPE	Formulation type for interaction: EQ.1: Parabolic coefficients, axial load and biaxial bending (default) EQ.2: Japanese code, axial force and major axis bending. EQ.4: AISC utilization calculation but no yielding EQ.5: AS4100 utilization calculation but no yielding
DEGRAD	Flag for degrading moment behavior (see Remark 5): EQ.0: Behavior as in previous versions EQ.1: Fatigue-type degrading moment-rotation behavior

VARIABLE	DESCRIPTION
	EQ.2: FEMA-type degrading moment-rotation behavior
IFEMA	Flag for input of FEMA thresholds: EQ.0: No input EQ.1: Input of rotation thresholds only EQ.2: Input of rotation and axial strain thresholds

Card 2	1	2	3	4	5	6	7	8
Variable	LCPMS	SFS	LCPMT	SFT	LCAT	SFAT	LCAC	SFAC
Type	F	F	F	F	F	F	F	F
Default	none	1.0	LCMPS	1.0	none	1.0	LCAT	1.0

VARIABLE	DESCRIPTION
LCPMS	Load curve ID giving plastic moment as a function of plastic rotation at node 2 about the local *s*-axis. See *DEFINE_CURVE.
SFS	Scale factor on *s*-moment at node 2
LCPMT	Load curve ID giving plastic moment as a function of plastic rotation at node 2 about local the *t*-axis. See *DEFINE_CURVE.
SFT	Scale factor on *t*-moment at node 2
LCAT	Load curve ID giving axial tensile yield force as a function of total tensile (elastic + plastic) strain or of elongation. See ASFLAG above. All values are positive. See *DEFINE_CURVE.
SFAT	Scale factor on axial tensile force
LCAC	Load curve ID giving compressive yield force as a function of total compressive (elastic + plastic) strain or of elongation. See ASFLAG above. All values are positive. See *DEFINE_CURVE.
SFAC	Scale factor on axial tensile force

FTYPE 1 Card. This card 3 format is used when FTYPE = 1 (default).

Card 3a	1	2	3	4	5	6	7	8
Variable	ALPHA	BETA	GAMMA	DELTA	A	B	FOFFS	
Type	F	F	F	F	F	F	F	
Default	Rem 1	Rem 1	Rem 1	Rem 1	Rem 1	Rem 1	0.0	

VARIABLE	DESCRIPTION
ALPHA	Parameter to define yield surface
BETA	Parameter to define yield surface
GAMMA	Parameter to define yield surface
DELTA	Parameter to define yield surface
A	Parameter to define yield surface
B	Parameter to define yield surface
FOFFS	Force offset for yield surface (see Remark 2)

FTYPE 2 Card. This card 3 format is used when FTYPE = 2.

Card 3b	1	2	3	4	5	6	7	8
Variable	SIGY	D	W	TF	TW			
Type	F	F	F	F	F			
Default	none	none	none	none	none			

VARIABLE	DESCRIPTION
SIGY	Yield stress of material
D	Depth of section used to calculate interaction curve
W	Width of section used to calculate interaction curve

VARIABLE	DESCRIPTION
TF	Flange thickness of section used to calculate interaction curve
TW	Web thickness used to calculate interaction curve

FTYPE 4 Card. This card 3 format is used when FTYPE = 4.

Card 3c	1	2	3	4	5	6	7	8
Variable	PHI_T	PHI_C	PHI_B					
Type	F	F	F					
Default	0.8	0.85	0.9					

VARIABLE	DESCRIPTION
PHI_T	Factor on tensile capacity, ϕ_t
PHI_C	Factor on compression capacity, ϕ_c
PHI_B	Factor on bending capacity, ϕ_b

FTYPE 5 Card. This card 3 format is used when FTYPE = 5.

Card 3d	1	2	3	4	5	6	7	8
Variable	ALPHA	BETA	GAMMA	DELTA	PHI_T	PHI_C	PHI_B	
Type	F	F	F	F	F	F	F	
Default	none	none	1.4	none	1.0	1.0	1.0	

VARIABLE	DESCRIPTION
ALPHA	Parameter to define yield surface
BETA	Parameter to define yield surface
GAMMA	Parameter to define yield surface
DELTA	Parameter to define yield surface

VARIABLE	DESCRIPTION
PHI_T	Factor on tensile capacity, ϕ_t
PHI_C	Factor on compression capacity, ϕ_c
PHI_B	Factor on bending capacity, ϕ_b

FEMA Limits Card 1. Additional card for IFEMA > 0.

Card 4	1	2	3	4	5	6	7	8
Variable	PR1	PR2	PR3	PR4				
Type	F	F	F	F				
Default	0	0	0	0				

VARIABLE	DESCRIPTION
PR1 - PR4	Plastic rotation thresholds 1 to 4

FEMA Limits Card 2. Additional card for IFEMA = 2.

Card 5	1	2	3	4	5	6	7	8
Variable	TS1	TS2	TS3	TS4	CS1	CS2	CS3	CS4
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	TS1	TS2	TS3	TS4

VARIABLE	DESCRIPTION
TS1 - TS4	Tensile axial strain thresholds 1 to 4
CS1 - CS4	Compressive axial strain thresholds 1 to 4

Remarks:

1. **FTYPE 1.** Yield surface for formulation type 1 is of the form:

$$\psi = \left(\frac{M_s}{M_{ys}}\right)^{\alpha} + \left(\frac{M_t}{M_{yt}}\right)^{\beta} + A\left(\frac{F}{F_y}\right)^{\gamma} + B\left(\frac{F}{F_y}\right)^{\delta} - 1 \ ,$$

where

$$\begin{aligned} M_s, M_t &= \text{moments about the local } s \text{ and } t \text{ axes} \\ M_{ys}, M_{yt} &= \text{current yield moments} \\ F &= \text{axial force} \\ F_y &= \text{yield force; LCAC in compression or LCAT in tension} \\ \alpha, \beta, \gamma, \delta &= \text{input parameters; must be greater than or equal to 1.1} \\ A, B &= \text{input parameters} \end{aligned}$$

If α, β, γ, δ, A and B are *all* set to zero, then the following default values are used:

Field	Default Value
ALPHA	2.0
BETA	2.0
GAMMA	2.0
DELTA	4.0
A	2.0
B	-1.0

2. **FOFFS.** FOFFS offsets the yield surface parallel to the axial force axis. It is the compressive axial force at which the maximum bending moment capacity about the local *s*-axis (determined by LCPMS and SFS) and that about the local *t*-axis (determined by LCPMT and SFT) occur. For steel beams and columns, the value of FOFFS is usually zero. For reinforced concrete beams, columns and shear walls, the maximum bending moment capacity occurs corresponding to a certain compressive axial force, FOFFS. The value of FOFFS can be input as either positive or negative. Internally, LS-DYNA converts FOFFS to, and regards compressive axial force as, negative.

3. **FTYPE 4.** Interaction surface FTYPE 4 calculates a utilisation parameter using the yield force and moment data given on Card 2, but the elements remain elastic even when the forces or moments exceed yield values. This is done for consistency with the design code OBE AISC LRFD (2000). The utilization calculation is as follows:

$$\text{Utilization} = \frac{K_1 F}{\phi F_y} + \frac{K_2}{\phi_b}\left(\frac{M_s}{M_{ys}} + \frac{M_t}{M_{yt}}\right) \ ,$$

where M_s, M_t, M_{ys}, M_{yt}, and F_y are as defined in Remark 1. ϕ is PHI_T under and PHI_C under compression. K_1 and K_2 are as follow:

$$K_1 = \begin{cases} 0.5 & \frac{F}{\phi F_y} < 0.2 \\ 1.0 & \frac{F}{\phi F_y} \geq 0.2 \end{cases}$$

$$K_2 = \begin{cases} 1.0 & \frac{F}{\phi F_y} < 0.2 \\ 8/9 & \frac{F}{\phi F_y} \geq 0.2 \end{cases}$$

4. **FTYPE 5.** Interaction surface FTYPE 5 is similar to FTYPE 4 (calculates a utilization parameter using the yield data, but the elements do not yield). The equations are taken from Australian code AS4100. The user must select appropriate values of α, β, γ and δ using the various clauses of Section 8 of AS4100. It is assumed that the local *s*-axis is the major axis for bending.

$$\text{Utilization} = \max(U_1, U_2, U_3, U_4, U_5)$$

where

$$U_1 = \frac{F}{\beta \phi_c F_{yc}} \qquad \text{used for members in compression}$$

$$U_2 = \frac{F}{\phi_t F_{yt}} \qquad \text{used for members in tension}$$

$$U_3 = \left[\frac{M_s}{K_2 \phi_b M_{ys}}\right]^\gamma + \left[\frac{M_t}{K_1 \phi_b M_{yt}}\right]^\gamma \qquad \text{used for members in compression}$$

$$U_4 = \left[\frac{M_s}{K_4 \phi_b M_{ys}}\right]^\gamma + \left[\frac{M_t}{K_3 \phi_b M_{yt}}\right]^\gamma \qquad \text{used for members in tension}$$

$$U_5 = \frac{F}{\phi_c F_{yc}} + \frac{M_s}{\phi_b M_{ys}} + \frac{M_t}{\phi_b M_{yt}} \qquad \text{used for all members}$$

In the above, M_s, M_t, F, M_{ys}, M_{yt}, F_{yt} and F_{yc} are as defined in Remark 1. K_1, K_2, K_3, and K_4 are subject to a minimum value of 10^{-6} and defined as

$$K_1 = 1.0 - \frac{F}{\beta \phi_c F_{yc}}$$

$$K_2 = \min\left[K_1, \alpha\left(1.0 - \frac{F}{\delta \phi_c F_{yc}}\right)\right]$$

$$K_3 = 1.0 - \frac{F}{\phi_t F_{yt}}$$

$$K_4 = \min\left[\mathrm{K}_3, \alpha\left(1.0 + \frac{F}{\phi_t F_{yt}}\right)\right]$$

α, β, γ, δ, ϕ_t, ϕ_c, and ϕ_b are input parameters.

5. **DEGRAD.** The option for degrading moment behavior changes the meaning of the plastic moment-rotation curve as follows:

   a) If DEGRAD = 0 (not recommended), the x-axis points on the curve represent current plastic rotation (meaning total rotation minus the elastic component of rotation). This quantity can be positive or negative depending on the direction of rotation; during hysteresis the behavior will repeatedly follow backwards and forwards along the same curve. The curve should include negative and positive rotation and moment values. This option is retained so that results from existing models will be unchanged.

   b) If DEGRAD = 1, the x-axis points represent cumulative absolute plastic rotation. This quantity is always positive and increases whenever there is plastic rotation in either direction. Thus, during hysteresis, the yield moments are taken from points in the input curve with increasingly positive rotation. If the curve shows a degrading behavior (reducing moment with rotation), then, once degraded by plastic rotation, the yield moment can never recover to its initial value. This option can be thought of as having "fatigue-type" hysteretic damage behavior, where all plastic cycles contribute to the total damage.

   c) If DEGRAD = 2, the x-axis points represent the high-tide value (always positive) of the plastic rotation. This quantity increases only when the absolute value of plastic rotation exceeds the previously recorded maximum. If smaller cycles follow a larger cycle, the plastic moment during the small cycles will be constant, since the high-tide plastic rotation is not altered by the small cycles. Degrading moment-rotation behavior is possible. This option can be thought of as showing rotation-controlled damage and follows the FEMA approach for treating fracturing joints.

   DEGRAD applies also to the axial behavior. The same options are available as for rotation: DEGRAD = 0 gives unchanged behavior from previous versions; DEGRAD = 1 gives a fatigue-type behavior using cumulative plastic strain; and DEGRAD = 2 gives FEMA-type behavior, where the axial load capacity depends on the high-tide tensile and compressive strains. The definition of strain for this purpose is according to ASFLAG on Card 1 – it is expected that ASFLAG = 2 will be used with DEGRAD = 2. The "axial strain" variable plotted by post-processors is the variable defined by ASFLAG.

   The output variables plotted as "plastic rotation" have special meanings for this material model– note that hinges form only at Node 2. "Plastic rotation at End 1" is really a high-tide mark of absolute plastic rotation <u>at Node 2</u>, defined as follows:

   d) Current plastic rotation is the total rotation minus the elastic component of rotation.

e) Take the absolute value of the current plastic rotation, and record the maximum achieved up to the current time. This is the high-tide mark of plastic rotation.

If DEGRAD = 0, "Plastic rotation at End 2" is the current plastic rotation at Node 2. If DEGRAD = 1 or 2, "Plastic rotation at End 2" is the current total rotation at Node 2. The total rotation is a more intuitively understood parameter, such as for plotting hysteresis loops. However, with DEGRAD = 0, the previous meaning of that output variable has been retained such that results from existing models are unchanged.

FEMA thresholds are the plastic rotations at which the element is deemed to have passed from one category to the next, e.g. "Elastic", "Immediate Occupancy", "Life Safe", etc. The high-tide plastic rotation (maximum of Y and Z) is checked against the user-defined limits FEMA1, FEMA2, etc. The output flag is then set to 0, 1, 2, 3, or 4: 0 means that the rotation is less than FEMA1; 1 means that the rotation is between FEMA1 and FEMA2, and so on. By contouring this flag, it is possible to see quickly which joints have passed critical thresholds.

6. **Output.** For this material model, special output parameters are written to the d3plot and d3thdt files. The number of output parameters for beam elements is automatically increased to 20 (in addition to the six standard resultants) when parts of this material type are present. Some post-processors may interpret this data as if the elements were integrated beams with 4 integration points. Depending on the post-processor used, the data may be accessed as follows:

Extra Variable # (Integration Point 4 Description)	Data Description
16 (or Axial Stress)	FEMA rotation flag
17 (or XY Shear Stress)	Current utilization
18 (or ZX Shear Stress)	Maximum utilization to date
20 (or Axial Strain)	FEMA axial flag

"Utilization" is the yield parameter, where 1.0 is on the yield surface.

*MAT_SOIL_BRICK

Purpose: This is Material Type 192. It is intended for modeling over-consolidated clay.

Card Summary:

Card 1. This card is required.

MID	RO	RLAMBDA	RKAPPA	RIOTA	RBETA1	RBETA2	RMU

Card 2. This card is required.

RNU	RLCID	TOL	PGCL	SUB-INC	BLK	GRAV	THEORY

Card 3a. This card is included only if THEORY = 4 or 104.

RVHHH						STRSUB	

Card 3b. This card is included only if THEORY = 204 or 304.

RVHHH						STRSUB	CRFLG

Card 3c. This card is included only if THEORY = 7 or 107.

EHEV	GHHGVH	PRHH	CAP				

Card 4. This card is included only if THEORY = 204 or 304 and CRFLG > 0.

SLRATIO	BETAC	EPSDOT1	EPSDOT2				

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	RLAMBDA	RKAPPA	RIOTA	RBETA1	RBETA2	RMU
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	1.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
RLAMBDA	Material coefficient, see Remark 1.
RKAPPA	Material coefficient, see Remark 1.
RIOTA	Material coefficient, see Remark 1.
RBETA1	Material coefficient, see Remark 1.
RBETA2	Material coefficient, see Remark 1.
RMU	Shape factor coefficient. This parameter will modify the shape of the yield surface used. A value of 1.0 implies a von Mises type surface, while 1.1 to 1.25 is more indicative of soils. The default value is 1.0. See Remarks 1 and 9.

Card 2	1	2	3	4	5	6	7	8
Variable	RNU	RLCID	TOL	PGCL	SUB-INC	BLK	GRAV	THEORY
Type	F	F	F	F	F	F	F	I
Default	none	none	0.0005	none	none	none	9.807	0

VARIABLE	DESCRIPTION
RNU	Poisson's ratio. See Remarks 1 and 2.
RLCID	Load curve ID (see *DEFINE_CURVE) consisting of up to 10 points defining nonlinear response in terms of stiffness degradation. The x-axis is strain ("string length"), and the y-axis is the ratio of secant stiffness to small-strain stiffness. See Remarks 3 and 12.
TOL	User defined tolerance for convergence checking. Default value is set to 0.0005 (recommended). See Remark 11.
PGCL	Pre-consolidation ground level. This parameter defines the maximum surface level (relative to $z = 0.0$ in the model) of the soil throughout geological history which is used calculate the

VARIABLE	DESCRIPTION
	maximum overburden pressure on the soil elements. See Remark 6.
SUB-INC	User defined strain increment size. A typical value is 0.005. This is the maximum strain increment permitted in the iteration scheme within the material model. If the value is exceeded, a warning is echoed to the d3hsp file. See Remark 11.
BLK	The elastic bulk stiffness of the soil which is used for contact stiffness only.
GRAV	The gravitational acceleration which is used to calculate the element stresses due to the overlying soil. Default is set to 9.807 m/s^2.
THEORY	Version of material subroutines used (see Remarks 7 and 8): EQ.0: 1995 version (default) EQ.4: 2003 version, load/unload initialization EQ.7: 2003 version, load/unload initialization, anisotropy from Ellison et al (2012) EQ.104: 2003 version, load/unload/reload initialization EQ.107: 2003 version, load/unload/reload initialization, anisotropy from Ellison et al (2012) EQ.204: 2015 version, load/unload initialization EQ.304: 2015 version, load/unload/reload initialization

Define Card 3a only if THEORY = 4 or 104. Omit otherwise.

Card 3a	1	2	3	4	5	6	7	8
Variable	RVHHH						STRSUB	
Type	F						F	
Default	0.0						0.001	

VARIABLE	DESCRIPTION
RVHHH	Anisotropy parameter: shear modulus in vertical planes divided by shear modulus in horizontal plane. If this field is blank or zero,

VARIABLE	DESCRIPTION
	isotropic behavior is assumed. See Remark 10.
STRSUB	Strain limit, used to determine whether subcycling within the material model is required (recommended value: 0.001)

Define Card 3b only if THEORY = 204 or 304. Omit otherwise.

Card 3b	1	2	3	4	5	6	7	8
Variable	RVHHH						STRSUB	CRFLG
Type	F						F	F
Default	0.0						0.001	0.0

VARIABLE	DESCRIPTION
RVHHH	Anisotropy parameter: shear modulus in vertical planes divided by shear modulus in horizontal plane. If this field is blank or zero, isotropic behavior is assumed. See Remark 10.
STRSUB	Strain limit, used to determine whether subcycling within the material model is required (recommended value: 0.001)
CRFLG	Creep flag: EQ.0: No creep EQ.24: Creep activated; see Card 4.

Define Card 3c only if THEORY = 7 or 107. Omit otherwise.

Card 3c	1	2	3	4	5	6	7	8
Variable	EHEV	GHHGVH	PRHH	CAP				
Type	F	F	F	F				
Default	0.0	0.0	0.0	0.0				

VARIABLE	DESCRIPTION
EHEV	Anisotropy parameter: Young's modulus in horizontal directions divided by Young's modulus in vertical direction. See Remarks 1 and 10.
GHHGVH	Anisotropy parameter: shear modulus in horizontal plane divided by shear modulus in vertical planes. See Remarks 1 and 10.
PRHH	Anisotropy parameter: Poisson's ratio in horizontal plane. See Remarks 1 and 10.
CAP	Anisotropy parameter. See Remarks 1 and 10.

Define Card 4 only if THEORY = 204 or 304 and CRFLG > 0. Omit otherwise.

Card 4	1	2	3	4	5	6	7	8
Variable	SLRATIO	BETAC	EPSDOT1	EPSDOT2				
Type	F	F	F	F				
Default	0.0	0.0	0.0	0.0				

VARIABLE	DESCRIPTION
SLRATIO	Creep parameter, see Remark 12
BETAC	Creep parameter, see Remark 12
EPSDOT1	Creep parameter: reference strain rate for volumetric strains. See Remark 12.
EPSDOT2	Creep parameter: reference strain rate for shear strains. See Remark 12.

Remarks:

1. **Material behavior and references.** The material model consists of up to 10 nested elasto-plastic yield surfaces defined in a transformed stress-strain space, termed "BRICK" space. The sizes of the yield surfaces are given in terms of BRICK strains and are called "string-lengths". Explanation of the input parameters and underlying concepts may be found in Simpson (1992), Lehane &

Simpson (2000), and Ellison et al (2012). The input parameters correspond to those described in Ellison et al as follows:

LS-DYNA	Ellison et al
RLAMBDA	λ
RKAPPA	κ
RIOTA	ι
RBETA1	β^G
RBETA2	β^Φ
RMU	μ
RNU	ν
EHEV	E_h/E_v
GHHGVH	G_{hh}/G_{vh}
PRHH	ν_{hh}
CAP	ζ

See Table 3 in Ellison et al for example input parameter values.

2. **Elastic stiffness.** The elastic bulk modulus is given by p'/ι, where p' is the current mean effective stress (compression positive), and the small-strain elastic shear modulus is calculated from the bulk modulus and Poisson's ratio RNU.

3. **Nonlinear stress-strain response.** The curve RLCID defines the nonlinear behavior in terms of secant stiffness degradation. The same curve is assumed to apply to all six BRICK stress-strain components. For example, shear in the xz-plane will follow the curve, such that the x-axis points correspond to shear angle γ_{xz}, while the y-axis contains G/G_{max}, where G is the secant stiffness (equal to τ_{xz}/γ_{xz}) and G_{max} is the small-strain elastic shear modulus.

4. **Model requirements.** This material type requires that the model be oriented such that the z-axis is defined in the upward direction. Compressive initial stress must be defined, using, for example, *INITIAL_STRESS_SOLID or *INITIAL_-STRESS_DEPTH. Stresses must remain compressive throughout the analysis.

5. **Units.** The recommended unit system is kiloNewtons, meters, seconds, tonnes. There are some built-in defaults that assume stress units of kN/m^2.

6. **Over-consolidated clays.** Over-consolidated clays have suffered previous loading to higher stress levels than are present at the start of the analysis due to phenomena such as ice sheets during previous ice ages, or the presence of soil

or rock that has subsequently been eroded. The maximum vertical stress during that time is assumed to be:

$$\sigma_{V,MAX} = RO \times GRAV \times (PGCL - Z_{el}) \ ,$$

where

RO, GRAV, and PGCL = input parameters
Z_{el} = z coordinate of center of element

Since that time, the material has been unloaded until the vertical stress equals the user-defined initial vertical stress. The previous load/unload history has a significant effect on the subsequent behavior. For example, the horizontal stress in an over-consolidated clay may be greater than the vertical stress.

7. **Initialization.** This material model initializes each element with a load/unload cycle under uniaxial vertical strain conditions. The element is loaded up to a vertical stress of $\sigma_{V,MAX}$ (defined in Remark 6 above) and then unloaded to the user-defined initial vertical stress $\sigma_{V,USER}$ (see *INITIAL_STRESS_SOLID or *INITIAL_STRESS_DEPTH). During this initialization cycle, the stresses and history variables are updated using the same constitutive behavior as during the main analysis. Therefore, the horizontal stress at the start of the analysis $\sigma_{H,ACTUAL}$ (as seen in the results files at time zero) is an output of the initialization process and will be different from the user-defined initial horizontal stress $\sigma_{H,USER}$; the latter is ignored. Optionally, initialization may be switched to a load/unload/reload cycle (see input settings of THEORY). In this case, the element is loaded up to a vertical stress of $\sigma_{V,MAX}$, unloaded to a stress $\sigma_{V,MIN}$ which is less than $\sigma_{V,USER}$, and then reloaded to $\sigma_{V,USER}$. The value of $\sigma_{V,MIN}$ is calculated automatically to try to minimize the difference between $\sigma_{H,ACTUAL}$ and $\sigma_{H,USER}$.

8. **Material subroutine version.** This material model is developed for a Geotechnical FE program (Oasys Ltd.'s SAFE) written by Arup. The default THEORY = 0 gives a vectorized version ported from SAFE in the 1990's. Since then the material model has been developed further in SAFE, with versions ported to LS-DYNA in 2003 (THEORY = 4 and 104) and 2015 (THEORY = 204 and 304); these are not vectorized and will run more slowly on most computer platforms. Nevertheless, the 2015 version is recommended. THEORY = 0, 4, and 104 are retained only for backward compatibility.

9. **Shape factor.** The shape factor for a typical soil would be 1.25. Do not use values higher than 1.35.

10. **Anisotropy.** Anisotropy is treated by applying stretch factors to the strain axis of the stress-strain curves for certain BRICK shear components. It may be defined by *either* using THEORY = 204 or 304 together with RVHHH on Card 3b *or* using THEORY = 7 or 107 with the parameters on Card 3c. Using THEORY = 4

or 104 with non-zero anisotropy parameters on Card 3a is permitted but not recommended. See Ellison et al (2012) for description of the anisotropy effects modelled by THEORY = 7/107 and the meaning of the parameters on Card 3c. If anisotropy is not required, use THEORY = 204 or 304 and leave Card 3b blank. “Vertical” and “horizontal” are defined in the global coordinate system with “vertical” being the global z-axis.

11. **TOL and SUBINC.** These parameters usually have little influence on the result. Smaller values may sometimes improve accuracy, at cost of greater run times.

12. **Creep.** Creep is implemented by scaling the strain “string lengths” (see RLCID) as a function of strain rate:

$$S = \mathrm{SLRATIO} \times S_{\mathrm{RLCID}} \left[1 + \mathrm{BETAC} \times ln \left(\frac{|\dot{\varepsilon}|}{\dot{\varepsilon}_{\mathrm{ref}}} + 1 \right) \right]$$

Where S is string length; S_{RLCID} are the string lengths in the curve RLCID; SLRATIO and BETAC are input parameters on Card 4; $|\dot{\varepsilon}|$ is strain rate; and $\dot{\varepsilon}_{\mathrm{ref}}$ is input parameter EPSDOT1 or EPSDOT2 for volumetric and shear strains, respectively. Note that SLRATIO × S_{RLCID} gives the string lengths for zero strain rate.

References:

[1] Simpson, B., “Retaining structures: displacement and design”, *Géotechnique*, Vol. 42, No. 4, 539-576, (1992).

[2] Lehane, B. & Simpson, B., “Modelling glacial till conditions using a Brick soil model”, *Can. Geotech. J*. Vol. 37, No. 5, 1078–1088 (2000).

[3] Ellison, K., Soga, K., & Simpson, B., “A strain space soil with evolving stiffness memory”, *Géotechnique,* Vol. 62, No. 7, 627-641 (2012).

*MAT_DRUCKER_PRAGER

This is Material Type 193. This material enables modeling soil effectively. The parameters used to define the yield surface are familiar geotechnical parameters (such as the angle of friction). The modified Drucker-Prager yield surface is used in this material model, enabling the shape of the surface to be distorted into a more realistic definition for soils.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	GMOD	RNU	RKF	PHI	CVAL	PSI
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	1.0	none	none	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	STR_LIM							
Type	F							
Default	0.005							

Card 3	1	2	3	4	5	6	7	8
Variable	GMODDP	PHIDP	CVALDP	PSIDP	GMODGR	PHIGR	CVALGR	PSIGR
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density

VARIABLE	DESCRIPTION
GMOD	Elastic shear modulus
RNU	Poisson's ratio
RKF	Failure surface shape parameter
PHI	Angle of friction (radians)
CVAL	Cohesion value
PSI	Dilation angle (radians)
STR_LIM	Minimum shear strength of material is given by STR_LIM × CVAL
GMODDP	Depth at which shear modulus (GMOD) is correct
PHIDP	Depth at which angle of friction (PHI) is correct
CVALDP	Depth at which cohesion value (CVAL) is correct
PSIDP	Depth at which dilation angle (PSI) is correct
GMODGR	Gradient at which shear modulus (GMOD) increases with depth
PHIGR	Gradient at which friction angle (PHI) increases with depth
CVALGR	Gradient at which cohesion value (CVAL) increases with depth
PSIGR	Gradient at which dilation angle (PSI) increases with depth

Remarks:

1. **Orientation.** This material type requires the model to be oriented such that the Z-axis is defined in the upward direction. The key parameters are defined such that they may vary with depth (i.e. the Z-axis).

2. **Shape factor.** The shape factor for a typical soil would be 0.8 but should not be pushed further than 0.75.

3. **STR_LIM.** If STR_LIM is set to less than 0.005, the value is reset to 0.005.

4. **Yield function.** The yield function is defined as:

$$t - p \times \tan\beta - d = 0$$

where:

Variable	Description
p	Hydrostatic pressure, $p = J_1/3$
t	$t = q/2\left(a - b(r/q)^3\right)$
q	von Mises stress, $q = \sqrt{3J_2}$
a	$a = 1 + 1/K$
b	$b = 1 - 1/K$
K	Input field RKF
r	$r = (27J_3/2)^{1/3}$
J_2	Second deviatoric stress invariant
J_3	Third deviatoric stress invariant
$\tan\beta$	$\tan\beta = 6\sin\varphi/(3 - \sin\varphi)$
d	$d = 6C\cos\varphi/(3 - \sin\varphi)$
φ	Input field PHI
C	Input field CVAL

5. **Executable precision.** We recommend using this material with a double precision executable.

6. **Output.** This remark applies to versions R14 and onwards. "Plastic Strain" is the deviatoric plastic strain, defined in the same way as for material types 3, 24, etc. Extra history variables may be requested for solid elements (NEIPH on *DATABASE_EXTENT_BINARY). They are described in the following table.

History Variable #	Description
1	Volumetric strain
2	Mobilized fraction (= 1 when on yield surface)
3	At-rest coefficient (ratio of horizontal stress to vertical stress)
4	Friction angle in radians (differs from input parameter PHI only if PHIDP and PHIGR are used)
5	Cohesion (differs from input parameter CVAL only if CVALDP and CVALGR are used)
6	Dilation angle in radians (differs from input parameter PSI only if PSIDP and PSIGR are used)
7	Shear modulus (differs from input parameter GMOD only if GMODDP and GMODGR are used)

*MAT_RC_SHEAR_WALL

Purpose: This is Material Type 194. It is for shell elements only. It uses empirically-derived algorithms to model the effect of cyclic shear loading on reinforced concrete walls. It is primarily intended for modeling squat shear walls but can also be used for slabs. Because the combined effect of concrete and reinforcement is included in the empirical data, crude meshes can be used. The model has been designed such that the minimum amount of input is needed: generally, only the variables on the first card need to be defined.

NOTE: This material does not support the specification of a material angle, β_i, for each through-thickness integration point of a shell.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	TMAX			
Type	A	F	F	F	F			
Default	none	none	none	0.0	0.0			

Include the following data if "Uniform Building Code" formula for maximum shear strength or tensile cracking are required – otherwise leave blank.

Card 2	1	2	3	4	5	6	7	8
Variable	FC	PREF	FYIELD	SIG0	UNCONV	ALPHA	FT	ERIENF
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Card 3	1	2	3	4	5	6	7	8
Variable	A	B	C	D	E	F		
Type	F	F	F	F	F	F		
Default	0.05	0.55	0.125	0.66	0.25	1.0		

Card 4	1	2	3	4	5	6	7	8
Variable	Y1	Y2	Y3	Y4	Y5			
Type	F	F	F	F	F			
Default	0.0	0.0	0.0	0.0	0.0			

Card 5	1	2	3	4	5	6	7	8
Variable	T1	T2	T3	T4	T5			
Type	F	F	F	F	F			
Default	0.0	0.0	0.0	0.0	0.0			

Card 6	1	2	3	4	5	6	7	8
Variable	AOPT							
Type	F							
Default	0.0							

Card 7	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		
Default	0.0	0.0	0.0	0.0	0.0	0.0		

Card 8	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label not must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
TMAX	Ultimate in-plane shear stress. If set to zero, LS-DYNA calculates TMAX based on the formulae in the Uniform Building Code, using the data on Card 2. See Remark 3.
FC	Unconfined compressive strength of concrete. It is used in the calculation of ultimate shear stress. Crushing behavior is not modeled.
PREF	Percent reinforcement. For example, if 1.2% of the material is reinforcement, enter 1.2.
FYIELD	Yield stress of reinforcement

VARIABLE	DESCRIPTION
SIG0	Overburden stress (in-plane compressive stress). It is used in the calculation of ultimate shear stress. Usually, SIG0 is left as zero.
UCONV	Unit conversion factor. UCONV is expected to be set such that, $$\text{UCONV} = \sqrt{1.0\ \text{PSI}\ \text{ in the model's stress units.}}$$ This factor is used to convert the ultimate tensile stress of concrete which is expressed as $\sqrt{\text{FC}}$ where FC is given in PSI. Therefore a unit conversion factor of $\sqrt{\text{PSI/Stress Unit}}$ is required. Examples: UCONV = 83.3 = $\sqrt{6894}$ if the stress unit is N/m^2; UCONV = 0.083 if the stress unit is MN/m^2 or N/mm^2
ALPHA	Shear span factor. See Remark 3.
FT	Cracking stress in direct tension. See Remark 5. The default is 8% of the cylinder strength.
ERIENF	Young's modulus of the reinforcement. It is used to calculate the post-cracked stiffness. See Remark 5.
A	Hysteresis constants determining the shape of the hysteresis loops
B	Hysteresis constants determining the shape of the hysteresis loops
C	Hysteresis constants determining the shape of the hysteresis loops
D	Hysteresis constants determining the shape of the hysteresis loops
E	Hysteresis constants determining the shape of the hysteresis loops
F	Strength degradation factor. After the ultimate shear stress has been achieved, F multiplies the maximum shear stress from the curve for subsequent reloading. F = 1.0 implies no strength degradation (default). F = 0.5 implies that the strength is halved for subsequent reloading.
Y1, Y2, …, Y5	Engineering shear strain points on stress-strain curve. By default, these are calculated from the values on Card 1. See Remark 3.
T1, T2, …, T5	Shear stress points on stress-strain curve. By default, these are calculated from the values on Card 1. See Remark 3.
AOPT	Material axes option (see *MAT_*OPTION*TROPIC_ELASTIC for more details):

VARIABLE	DESCRIPTION
	EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in Figure M2-1, and then rotated about the shell element normal by the angle BETA.
	EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.
	EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element (see Figure M2-1). **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword.
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
XP, YP, ZP	Coordinates of point P for AOPT = 1
A1, A2, A3	Components of vector **a** for AOPT = 2
V1, V2, V3	Components of vector **v** for AOPT = 3
D1, D2, D3	Components of vector **d** for AOPT = 2
BETA	Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA.

Remarks:

1. **Model limitations.** The element is linear elastic except for in-plane shear and tensile cracking effects. Crushing due to direct compressive stresses is modeled only insofar as there is an in-plane shear stress component. Using this model is not recommended when the nonlinear response to direct compressive or loads is important.

2. **In-plane shear stress.** Note that the in-plane shear stress t_{xy} is defined as the shear stress in the element's local xy-plane. This shear stress is not necessarily equal to the maximum shear stress in the plane. For example, if the principal

stresses are at 45 degrees to the local axes, t_{xy} is zero. Therefore, it is important to ensure that the local axes are appropriate. For a shear wall the local axes should be vertical or horizontal. By default, the local x-axis points from node 1 to node 2 of the element. It is possible to change the local axes by using AOPT > 0.

3. **TMAX and shear stress as a function of shear strain.** If TMAX is set to zero, the ultimate shear stress is calculated using a formula in the Uniform Building Code 1997, section 1921.6.5:

$$\mathrm{TMAX_{UBC}} = \mathrm{UCONV} \times \mathrm{ALPHA} \times \sqrt{\mathrm{FC}} + \mathrm{RO} \times \mathrm{FY}$$

where,

$$\begin{aligned}
\mathrm{UCONV} &= \text{unit conversion factor, see varriable list} \\
\mathrm{ALPHA} &= \text{aspect ratio} \\
&= 2.0 \text{ for } h/l \in (2.0, \infty) \text{ increases linearly to 3.0 for } h/l \in (2.0, 1.5) \\
\mathrm{FC} &= \text{unconfined compressive strength of concrete} \\
\mathrm{RO} &= \text{fraction of reinforcement} \\
&= (\text{percent reinforcement})/100 \\
\mathrm{FY} &= \text{yield stress of reinforcement}
\end{aligned}$$

To this we add shear stress due to the overburden to obtain the ultimate shear stress:

$$\mathrm{TMAX_{UBC}} = \mathrm{TMAX_{UBC}} + \mathrm{SIG0}$$

where

$$\mathrm{SIG0} = \text{in-plane compressive stress under static equilibrium conditions}$$

The UBC formula for ultimate shear stress is generally conservative (predicts that the wall is weaker than shown in test), sometimes by 50% or more. A less conservative formula is that of Fukuzawa:

$$\begin{aligned}
\mathrm{TMAX} = \max\left[\left(0.4 + \frac{A_c}{A_w}\right), 1\right] \times 2.7 \times \left(1.9 + \frac{M}{L_v}\right) \times \mathrm{UCONV} + \sqrt{\mathrm{FC}} + 0.5 \\
\times \mathrm{RO} \times \mathrm{FY} + \mathrm{SIG0}
\end{aligned}$$

where

$$\begin{aligned}
A_C &= \text{Cross-sectional area of stiffening features such as columns or flanges} \\
A_w &= \text{Cross-sectional area of wall} \\
M/L_v &= \text{Aspect ratio of wall height/length}
\end{aligned}$$

Other terms are as above. This formula is not included in the material model. TMAX should be calculated by hand and entered on Card 1 if the Fukuzawa formula is required.

Note that none of the available formulae, including Fukuzawa, predict the ultimate shear stress accurately for all situations. Variance from the experimental results can be as great as 50%.

The shear stress as a function of shear strain curve is then constructed automatically as follows, using the algorithm of Fukuzawa extended by Arup:

a) Assume ultimate engineering shear strain, $\gamma_u = 0.0048$

b) First point on curve, corresponding to concrete cracking, is at

$$\left(0.3 \times \frac{\text{TMAX}}{G}, 0.3 \times \text{TMAX}\right),$$

where G is the elastic shear modulus given by

$$G = \frac{E}{2(1+\nu)}.$$

c) Second point, corresponding to the reinforcement yield, is at

$$(0.5 \times \gamma_u, 0.8 \times \text{TMAX}).$$

d) Third point, corresponding to the ultimate strength, is at

$$(\gamma_u, \text{TMAX}).$$

e) Fourth point, corresponding to the onset of strength reduction, is at

$$(2\gamma_u, \text{TMAX}).$$

f) Fifth point, corresponding to failure is at

$$(3\gamma_u, 0.6 \times \text{TMAX}).$$

After failure, the shear stress drops to zero. The curve points can be entered by the user if desired, in which case they override the automatically calculated curve. However, it is anticipated that in most cases the default curve will be preferred due to ease of input.

4. **Hysteresis.** Hysteresis follows the algorithm of Shiga as for the squat shear wall spring (see *MAT_SPRING_SQUAT_SHEARWALL). The hysteresis constants which are defined in fields A, B, C, D, and E can be entered if desired, but it is generally recommended that the default values be used.

5. **Cracking.** Cracking in tension is checked for the local x and y directions only. Cracking is calculated separately from the in-plane shear. A trilinear response is assumed, with turning points at concrete cracking and reinforcement yielding. The three regimes are:

a) *Pre-cracking.* A linear elastic response is assumed using the overall Young's Modulus on Card 1.

b) *Cracking.* Cracking occurs in the local x or y directions when the tensile stress in that direction exceeds the concrete tensile strength FT (if not input on Card 2, this defaults to 8% of the compressive strength FC). Post-cracking, a linear stress-strain response is assumed up to reinforcement yield at a strain defined by reinforcement yield stress divided by reinforcement Young's Modulus.

c) *Post-yield.* A constant stress is assumed (no work hardening).

Unloading returns to the origin of the stress-strain curve. For compressive strains the response is always linear elastic using the overall Young's modulus on Card 1. If insufficient data is entered, no cracking occurs in the model. As a minimum, FC and FY are needed.

6. **History variables.** Extra variables are available for post-processing as follows:

History Variable #	Description
1	Current engineering shear strain
2	Shear status: 0, 1, 2, 3, 4, or 5. The shear status shows how far along the shear stress-strain curve each element has progressed. For instance, status 2 means that the element has passed the second point on the curve. These status levels correspond to performance criteria in building design codes such as FEMA.
3	Maximum direct strain so far in the local x-direction (for tensile cracking)
4	Maximum direct strain so far in the local y-direction (for tensile cracking)
5	Tensile status: EQ.0: Elastic EQ.1: Cracked EQ.2: Yielded

*MAT_CONCRETE_BEAM

This is Material Type 195 for beam elements. This model can define an elasto-plastic material with an arbitrary stress as a function of strain curve and arbitrary strain rate dependency. It supports failure based on a plastic strain or a minimum time step size. See Remarks below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	ETAN	FAIL	TDEL
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	10^{20}	10^{20}

Card 2	1	2	3	4	5	6	7	8
Variable	C	P	LCSS	LCSR				
Type	F	F	F	F				
Default	0	0	0	0				

Card 3	1	2	3	4	5	6	7	8
Variable	NOTEN	TENCUT	SDR					
Type	I	F	F					
Default	0	10^{15}	0.0					

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density

VARIABLE	DESCRIPTION
E	Young's modulus
PR	Poisson's ratio
SIGY	Yield stress
ETAN	Tangent modulus; ignored if LCSS > 0
FAIL	Failure flag: LT.0.0: User-defined failure subroutine is called to determine failure. EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved. GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.
TDEL	Minimum time step size for automatic element deletion.
C	Strain rate parameter, C; see Remarks below.
P	Strain rate parameter, p; see Remarks below.
LCSS	Load curve ID or Table ID. Load curve ID defining effective stress as a function of effective plastic strain. The table ID defines for each strain rate value a load curve ID giving the stress as a function of effective plastic strain for that rate; see Figure M16-1 Stress as a function of effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress as a function of effective plastic strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value. The strain rate parameters (C and p) and the curve ID, LCSR, are ignored if a Table ID is defined.
LCSR	Load curve ID defining strain rate scaling effect on yield stress
NOTEN	No-tension flag: EQ.0: Beam takes tension. EQ.1: Beam takes no tension. EQ.2: Beam takes tension up to value given by TENCUT.
TENCUT	Tension cutoff value

VARIABLE	DESCRIPTION
SDR	Stiffness degradation factor

Remarks:

The stress strain behavior may be treated using a bilinear stress strain curve through defining the tangent modulus, ETAN. An effective stress as a function of effective plastic strain curve (LCSS) may be input instead of defining ETAN. The cost is roughly the same for either approach. The most general approach is to use the table definition (LCSS) discussed below.

Three options to account for strain rate effects are possible.

1. Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1+\left(\frac{\dot{\varepsilon}}{C}\right)^{1/p}$$

   where $\dot{\varepsilon}$ is the strain rate. $\dot{\varepsilon}=\sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}$.

2. For complete generality a load curve (LCSR) to scale the yield stress may be input instead. In this curve the scale factor as a function of strain rate is defined.

3. If different stress as a function of strain curves can be provided for various strain rates, the option using the reference to a table (LCSS) can be used.

*MAT_GENERAL_SPRING_DISCRETE_BEAM

This is Material Type 196. This model permits elastic and elastoplastic springs with damping to be represented with a discrete beam element of type 6 by using six springs each acting about one of the six local degrees-of-freedom. For elastic behavior, a load curve defines force or moment as a function of displacement or rotation. For inelastic behavior, a load curve defines yield force or moment as a function of plastic deflection or rotation, which can vary in tension and compression.

The two nodes defining a beam may be coincident to give a zero length beam or offset to give a finite length beam. For finite length discrete beams, the absolute value of the field SCOOR in the *SECTION_BEAM input should be set to a value of 2.0, which causes the local *r*-axis to be aligned along the two nodes of the beam to give physically correct behavior. The distance between the nodes of a beam should not affect the behavior of this material model. A triad is used to orient the beam for the directional springs.

Card Summary:

Card 1. This card is required.

MID	RO						DOSPOT

Card 2. For each active degree of freedom include a pair of Cards 2 and 3. This data is terminated by the next keyword ("*") card or when all six degrees of freedom have been specified.

DOF	TYPE	K	D	CDF	TDF		

Card 3. For each active degree of freedom include a pair of Cards 2 and 3. This data is terminated by the next keyword ("*") card or when all six degrees of freedom have been specified.

FLCID	HLCID	C1	C2	DLE	GLCID		

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO						DOSPOT
Type	A	F						I

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density; see also volume in *SECTION_BEAM definition.
DOSPOT	Activate thinning of tied shell elements when SPOTHIN > 0 on *CONTROL_CONTACT. EQ.0: Spot weld thinning is inactive for shells tied to discrete beams that use this material (default). EQ.1: Spot weld thinning is active for shells tied to discrete beams that use this material.

Card 2	1	2	3	4	5	6	7	8
Variable	DOF	TYPE	K	D	CDF	TDF		
Type	I	I	F	F	F	F		

VARIABLE	DESCRIPTION
DOF	Active degree-of-freedom, a number between 1 and 6 inclusive. Each value of DOF can only be used once. The active degree-of-freedom is measured in the local coordinate system for the discrete beam element.
TYPE	Material behavior: EQ.0: elastic (default) EQ.1: inelastic
K	Elastic loading/unloading stiffness. This is required input for inelastic behavior.
D	Optional viscous damping coefficient
CDF	Compressive displacement at failure. Input as a positive number. After failure, no forces are carried. This option does not apply to zero length springs. EQ.0.0: inactive

VARIABLE	DESCRIPTION
TDF	Tensile displacement at failure. After failure, no forces are carried. EQ.0.0: inactive

Card 3	1	2	3	4	5	6	7	8
Variable	FLCID	HLCID	C1	C2	DLE	GLCID		
Type	F	F	F	F	F	I		

VARIABLE	DESCRIPTION
FLCID	Load curve ID (see *DEFINE_CURVE). TYPE.EQ.0: this curve defines force or moment as a function of deflection for nonlinear elastic behavior. TYPE.EQ.1: this curve defines the yield force as a function of plastic deflection. If the abscissa of the first point of the curve is 0. the force magnitude is identical in tension and compression, that is, only the sign changes. If not, the yield stress in the compression is used when the spring force is negative. The plastic displacement increases monotonically in this implementation. The load curve is required input.
HLCID	Optional load curve ID (see *DEFINE_CURVE) defining force as a function of relative velocity. If the origin of the curve is at (0,0), the force magnitude is identical for a given magnitude of the relative velocity, meaning only the sign changes.
C1	Damping coefficient
C2	Damping coefficient
DLE	Factor to scale time units
GLCID	Optional load curve ID (see *DEFINE_CURVE) defining a scale factor as a function of deflection for load curve ID, HLCID. If zero, a scale factor of unity is assumed.

Remarks:

1. **Elastic Behavior.** If TYPE = 0, elastic behavior is obtained. In this case, if the linear spring stiffness is used, the force, F, is given by:

$$F = \mathrm{K} \times \Delta L + \mathrm{D} \times \Delta\dot{L}\ \ .$$

But if the load curve ID is specified, the force is then given by:

$$F = K f(\Delta L)\left[1 + \mathrm{C1} \times \Delta\dot{L} + \mathrm{C2} \times \mathrm{sgn}(\Delta\dot{L})\ln\left(\max\left\{1.,\frac{|\Delta\dot{L}|}{\mathrm{DLE}}\right\}\right)\right] + \mathrm{D}\times\Delta\dot{L} \\ + g(\Delta L)h(\Delta\dot{L})\ \ .$$

In these equations, ΔL is the change in length

$$\Delta L = \text{current length} - \text{initial length}$$

For the first three degrees of freedom the fields on Cards 2 and 3 have dimensions as shown below. Being angular in nature, the next three degrees of freedom involve moment instead of force and angle instead of length but are otherwise identical.

$$[\mathrm{K}] = \begin{cases} \dfrac{[\text{force}]}{[\text{length}]} & \mathrm{FLCID} = 0 \\ \text{unitless} & \mathrm{FLCID} > 0 \end{cases}$$

$$[\mathrm{D}] = \frac{[\text{force}]}{[\text{velocity}]} = \frac{[\text{force}][\text{time}]}{[\text{length}]}$$

$$[\mathrm{FLCID}] = [\mathrm{GLCID}] = ([\text{length}],[\text{force}])$$

$$[\mathrm{HLCID}] = ([\text{velocity}],[\text{force}])$$

$$[\mathrm{C1}] = \frac{[\text{time}]}{[\text{length}]}$$

$$[\mathrm{C2}] = \text{unitless}$$

$$[\mathrm{DLE}] = \frac{[\text{length}]}{[\text{time}]}$$

2. **Inelastic Behavior.** If TYPE = 1, inelastic behavior is obtained. A trial force is computed as:

$$F^T = F^n + K \times \Delta\dot{L}(\Delta t)$$

and the yield force is taken from the load curve:

$$F^Y = F_y(\Delta L^{\text{plastic}})\ \ ,$$

where L^{plastic} is the plastic deflection, given by

$$\Delta L^{\text{plastic}} = \frac{F^T - F^Y}{S + K^{\max}}\ \ .$$

The maximum elastic stiffness is $K^{\max} = \max(K, 2 \times S^{\max})$, where S is the slope of FLCID. The trial force is checked against the yield force to determine F:

$$F = \begin{cases} F^Y & \text{if } F^T > F^Y \\ F^T & \text{if } F^T \le F^Y \end{cases}$$

The final force, which includes rate effects and damping, is given by:

$$F^{n+1} = F \times \left[1 + \mathrm{C1} \times \Delta\dot{L} + \mathrm{C2} \times \mathrm{sgn}(\Delta\dot{L}) \ln\left(\max\left\{1., \frac{|\Delta\dot{L}|}{\mathrm{DLE}}\right\}\right)\right] + \mathrm{D} \times \Delta\dot{L} \\ + g(\Delta L) h(\Delta\dot{L}) \ .$$

Unless the origin of the curve starts at $(0{,}0)$, the negative part of the curve is used when the spring force is negative where the negative of the plastic displacement is used to interpolate, F_y. The positive part of the curve is used whenever the force is positive.

3. **Cross-Sectional Area.** The cross-sectional area is defined on the section card for the discrete beam elements, See *SECTION_BEAM. The square root of this area is used as the contact thickness offset if these elements are included in the contact treatment.

4. **Rotational Displacement.** Rotational displacement is measured in radians.

*MAT_SEISMIC_ISOLATOR

This is Material Type 197 for discrete beam elements. Sliding (pendulum) and elastomeric seismic isolation bearings can be modeled, applying bi-directional coupled plasticity theory. The hysteretic behavior was proposed by Wen [1976] and Park, Wen, and Ang [1986]. The sliding bearing behavior is recommended by Zayas, Low and Mahin [1990]. The algorithm used for implementation was presented by Nagarajaiah, Reinhorn, and Constantinou [1991]. Further options for tension-carrying friction bearings are as recommended by Roussis and Constantinou [2006]. Element formulation type 6 *must* be used. Local axes are defined on *SECTION_BEAM; the default is the global axis system. The local z-axis is expected to be vertical. On *SECTION_BEAM SCOOR must be set to zero when using this material model, even if the element has non-zero initial length.

Card Summary:

Card 1. This card is required.

MID	RO	A	BETA	GAMMA	DISPY	STIFFV	ITYPE

Card 2. This card is required.

PRELOAD	DAMP	MX1	MX2	MY1	MY2	CDE	IEXTRA

Card 3. This card is used for ITYPE = 0, 2, or 5. Leave this card blank for all other settings of ITYPE.

FMAX	DELF	AFRIC	RADX	RADY	RADB	STIFFL	STIFFTS

Card 4a. This card is included for ITYPE = 1 or 4.

FORCEY	ALPHA	STIFFT	DFAIL				

Card 4b. This card is included for ITYPE = 2.

				FMAXYC	FMAXXT	FMAXYT	YLOCK

Card 4c. This card is included for ITYPE = 3.

FORCEY	ALPHA						

Card 4d. This blank card is included for all other settings of ITYPE (0 or5).

Card 5. This card is included for ITYPE = 3 only. Omit for other settings of ITYPE.

HTCORE	RCORE	TSHIM	ROLCL	ROSCS	THCST	YLE2	

Card 6. This card is included for ITYPE = 3 only. Omit for other settings of ITYPE.

PCRINI	DIAMB	FCAV0	CAVK	CAVTR	CAVA	PHIM	

Card 7. This card is included for ITYPE = 4 only. Omit for other settings of ITYPE.

BETA							

Card 8. This card is included for ITYPE = 5 only. Omit for other settings of ITYPE.

FYRIM	DFRIM						

Card 9. This card is included if and only if IEXTRA = 1.

KTHX	KTHY	KTHZ					

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	A	BETA	GAMMA	DISPY	STIFFV	ITYPE
Type	A	F	F	F	F	F	F	I
Default	none	none	1.0	0.5	0.5	none	none	0.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
A	Nondimensional variable - see below
GAMMA	Nondimensional variable - see below
BETA	Nondimensional variable - see below
DISPY	Yield displacement (length units - must be > 0.0)

VARIABLE	DESCRIPTION
STIFFV	Vertical stiffness (force/length units)
ITYPE	Type: EQ.0: sliding (spherical or cylindrical) EQ.1: elastomeric EQ.2: sliding (two perpendicular curved beams) EQ.3: lead rubber bearing EQ.4: high damping rubber bearing EQ.5: sliding with rim failure

Card 2	1	2	3	4	5	6	7	8
Variable	PRELOAD	DAMP	MX1	MX2	MY1	MY2	CDE	IEXTRA
Type	F	F	F	F	F	F	F	I
Default	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0

VARIABLE	DESCRIPTION
PRELOAD	Vertical preload not explicitly modeled (force units)
DAMP	Damping ratio (nondimensional)
MX1, MX2	Moment factor at ends 1 and 2 in local x-direction
MY1, MY2	Moment factor at ends 1 and 2 in local y-direction
CDE	Viscous damping coefficient (ITYPE = 1, 3 or 4)
IEXTRA	If IEXTRA = 1, optional Card 9 will be read

Sliding Isolator Card. This card is used for ITYPE = 0, 2, or 5. Leave this card *blank* for all other settings of ITYPE.

Card 3	1	2	3	4	5	6	7	8
Variable	FMAX	DELF	AFRIC	RADX	RADY	RADB	STIFFL	STIFFTS
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	10^{20}	10^{20}	10^{20}	STIFFV	0.0

VARIABLE	DESCRIPTION
FMAX	Maximum friction coefficient (dynamic)
DELF	Difference between maximum friction and static friction coefficient
AFRIC	Velocity multiplier in sliding friction equation (time/length units)
RADX	Radius for sliding in local x direction
RADY	Radius for sliding in local y direction
RADB	Radius of retaining ring
STIFFL	Stiffness for lateral contact against the retaining ring
STIFFTS	Stiffness for tensile vertical response (default = 0)

This card is included only for ITYPE = 1 or 4.

Card 4a	1	2	3	4	5	6	7	8
Variable	FORCEY	ALPHA	STIFFT	DFAIL	FMAXYC	FMAXXT	FMAXYT	YLOCK
Type	F	F	F	F	F	F	F	F
Default	none	0.0	0.5 × STIFFV	10^{20}	FMAX	FMAX	FMAX	0.0

VARIABLE	DESCRIPTION
FORCEY	Yield force

VARIABLE	DESCRIPTION
ALPHA	Ratio of post-yielding stiffness to pre-yielding stiffness
STIFFT	Stiffness for tensile vertical response (elastomeric isolator)
DFAIL	Lateral displacement at which the isolator fails

This card is included only for ITYPE = 2.

Card 4b	1	2	3	4	5	6	7	8
Variable					FMAXYC	FMAXXT	FMAXYT	YLOCK
Type					F	F	F	F
Default					FMAX	FMAX	FMAX	0.0

VARIABLE	DESCRIPTION
FMAXYC	Max friction coefficient (dynamic) for local y-axis (compression)
FMAXXT	Max friction coefficient (dynamic) for local x-axis (tension)
FMAXYT	Max friction coefficient (dynamic) for local y-axis (tension)
YLOCK	Stiffness locking the local y-displacement (optional -single-axis sliding)

This card is included only for ITYPE = 3.

Card 4c	1	2	3	4	5	6	7	8
Variable	FORCEY	ALPHA						
Type	F	F						
Default	none	0.0						

VARIABLE	DESCRIPTION
FORCEY	Yield force

VARIABLE	DESCRIPTION
ALPHA	Ratio of post-yielding stiffness to pre-yielding stiffness

Include this *blank* card for ITYPE = 0 or 5.

Card 4d	1	2	3	4	5	6	7	8
Variable								
Type								
Default								

Lead Rubber Bearing Card. This card is included for ITYPE = 3 only. Omit for other settings of ITYPE.

Card 5	1	2	3	4	5	6	7	8
Variable	HTCORE	RCORE	TSHIM	ROLCL	ROSCS	THCST	YLE2	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	none	none	

VARIABLE	DESCRIPTION
HTCORE	Height of lead core (length units)
RCORE	Radius of lead core (length units)
TSHIM	Total thickness of shim plates (length units)
ROLCL	Mass density times specific heat capacity of lead (units: $F.L^{-2}T^{-1}$)
ROLCS	Mass density times specific heat capacity of steel (units: $F.L^{-2}T^{-1}$)
THCST	Thermal conductivity of steel (units: $F.t^{-1}T^{-1}$)
YLE2	E_2 in temperature-dependent yield stress of lead (units: 1/Temperature)

Lead Rubber Bearing Card. This card is included for ITYPE = 3 only. Omit for other settings of ITYPE.

Card 6	1	2	3	4	5	6	7	8
Variable	PCRINI	DIAMB	FCAV0	CAVK	CAVTR	CAVA	PHIM	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	none	none	

VARIABLE	DESCRIPTION
PCRINI	Buckling capacity (force units)
DIAMB	External diameter of bearing (length units)
FCAV0	Tensile capacity limited by cavitation (force units)
CAVK	Cavitation parameter (units 1/length)
TR	Total thickness of rubber (length units)
CAVA	Strength degradation parameter (dimensionless)
PHIM	Maximum cavitation damage index (dimensionless)

High Damping Rubber Bearing Yield Card. This card is included for ITYPE = 4 only. Omit for other settings of ITYPE.

Card 7	1	2	3	4	5	6	7	8
Variable	BETA							
Type	F							
Default	0.0							

VARIABLE	DESCRIPTION
BETA	Quadratic factor for yield force

Rim Failure Card. This card is included for ITYPE = 5 only. Omit for other settings of ITYPE.

Card 8	1	2	3	4	5	6	7	8
Variable	FYRIM	DFRIM						
Type	F	F						
Default	10^{20}	10^{20}						

VARIABLE	DESCRIPTION
FYRIM	Radial force at failure of rim
DFRIM	Radial displacement of rim to failure after FYRIM is reached

Rotational Stiffness Card. Card 9 for IEXTRA = 1 only. Omit if IEXTRA=0.

Card 9	1	2	3	4	5	6	7	8
Variable	KTHX	KTHY	KTHZ					
Type	F	F	F					
Default	0	0	0					

VARIABLE	DESCRIPTION
KTHX	Rotational stiffness in local x direction (moment per radian)
KTHY	Rotational stiffness in local y direction (moment per radian)
KTHZ	Rotational stiffness in local z direction (moment per radian)

Remarks:

1. **Horizontal behavior of the isolator.** The horizontal behavior for all isolator types is governed by plastic history variables Zx and Zy that evolve according to equations given in the reference; A, GAMMA, BETA and DISPYt are the input parameters for this. The intention is to provide smooth build-up, rotation and reversal of forces in response to bidirectional displacement histories in the

horizontal plane. The theoretical model has been correlated to experiments on seismic isolators.

2. **Sliding surface for sliding isolator.** The RADX and RADY inputs for the sliding isolator are optional. If left blank, the sliding surface is assumed to be flat. A cylindrical surface is obtained by defining either RADX or RADY; a spherical surface can be defined by setting RADX = RADY. The effect of the curved surface is to add a restoring force proportional to the horizontal displacement from the center. As seen in elevation, the top of the isolator will follow a curved trajectory, lifting as it displaces away from the center.

3. **Vertical behavior of the isolator.** The vertical behavior for all types is linear elastic, but with different stiffnesses for tension and compression. By default, the tensile stiffness is zero for the sliding types. For the elastomeric type in the case of uplift, the tensile stiffness will be different from the compressive stiffness. For the sliding type, compression is treated as linear elastic, but no tension can be carried.

4. **Vertical preload.** Vertical preload can be modeled either explicitly (for example, by defining gravity), or by using the PRELOAD input. PRELOAD does not lead to any application of vertical force to the model. It is added to the compression in the element before calculating the friction force and tensile/compressive vertical behavior.

5. **Overview of ITYPE.** Various settings of ITYPE are described as follows.

   a) ITYPE = 0 is used to model a single (spherical) pendulum bearing. Triple pendulum bearings can be modelled using three of these elements in series, following the method described by Fenz and Constantinou 2008.

   b) ITYPE = 2 is intended to model uplift-prevention sliding isolators that consist of two perpendicular curved beams joined by a connector that can slide in slots on both beams. The beams are aligned in the local x and y axes, respectively. The vertical displacement is the sum of the displacements induced by the respective curvatures and slider displacements along the two beams. Single-axis sliding is obtained by using YLOCK to lock the local y displacement. To resist uplift, STIFFTS must be defined (recommended value: same as STIFFV). This isolator type allows for different friction coefficients on each beam as well as different values in tension and compression. The total friction, taking into account sliding velocity and the friction history functions, is first calculated using FMAX which applies to the local x-axis when in compression, and then scaled as necessary, such as by FMAXXT/FMAX (for the local x-axis when in tension) and by FMAXYC/FMAX or FMAXYT/FMAX for the y-axis as appropriate. For this reason, FMAX should not be zero.

c) ITYPE = 3 is used to model Lead Rubber Bearings (LRB), made of rubber with a lead core. Phenomenological models following Kumar et al. (2014) are incorporated to simulate the following salient behavior:

   i) The properties of the lead core may degrade in the short-term because of substantial internal heat generation from cyclic deformation.

   ii) Under larger lateral deformation, the rubber may experience net tension which will affect the compression and tension stiffness, and lead to potential vertical instability.

   iii) Cavitation may happen when the bearing is under excessive tension, resulting in permanent damage in the tensile capacity.

d) ITYPE = 4 is used to model higher damping rubber bearings. It differs from elastomeric bearing (ITYPE = 1) in that the time-varying yield force is a function of resultant horizontal displacement, governed by:

$$\text{Yield force} = \text{FORCEY}\left(1 + \text{BETA}\left(\frac{dx^2 + dy^2}{\text{DISPY}^2}\right)\right).$$

Here dx and dy are the displacements in the local x and y directions.

e) ITYPE = 5 is the same as ITYPE = 0 (spherical sliding bearing), except for the additional capability of yielding and failure of the rim, also called the retaining ring. The rim is intended to prevent the radial displacement of the slider exceeding RADB, but if sufficient radial force is applied, the rim can yield and then fail, leading to the slider falling off the supporting surface. When the rim fails, the isolator element is deleted.

6. **Damping.** DAMP is the fraction of critical damping for free vertical vibration of the isolator, based on the mass of the isolator (including any attached lumped masses) and its vertical stiffness. The viscosity is reduced automatically if it would otherwise infringe numerical stability. Damping is generally recommended:

   a) Oscillations in the vertical force have a direct effect on friction forces in sliding isolators.

   b) For isolators with curved surfaces, vertical oscillations can be excited as the isolator slides up and down the curved surface.

   It may occasionally be necessary to increase DAMP if these oscillations become significant.

7. **Rotational stiffness.** By default, this element has no rotational stiffness - a pin joint is assumed. However, if required, "offset moments" can be generated according to the vertical load multiplied by the lateral displacement of the isolator. This is invoked using MX1, MX2, MY1, MY2. The moment *about* the local x-axis (meaning the moment that is dependent on lateral displacement in the local y-direction) is reacted on nodes 1 and 2 of the element in the proportions MX1 and MX2, respectively. Similarly, moments about the local y-axis are reacted in the proportions MY1 and MY2. These inputs effectively determine the location of the pin joint.

   For example, consider an isolator installed between the top of the foundation of a building (Node 1 of the isolator element) and the base of a column of the superstructure (Node 2 of the isolator element). To model a pin at the base of the column and react the offset moment on the foundation, set MX1 = MY1 = 1.0 and MX2 = MY2 = 0.0. For the same model, MX1 = MY1 = 0.0 and MX2 = MY2 = 1.0 would imply a pin at the top of the foundation - all the moment is transferred to the column. Some isolator designs have the pin at the bottom for moments about one horizontal axis, and at the top for the other axis - these can be modeled by setting MX1 = MY2 = 1.0 and MX2 = MY1 = 0.0. MX1, MX2, MY1 and MY2 are all expected to be greater than or equal to 0 and less than or equal to 1. Also, if MX1 and MX2 are not both zero, then MX1 + MX2 is expected to equal 1.0, and similarly for MY1 and MY2. However, no error checks are performed to ensure this.

   Optionally, rotational stiffnesses that resist rotation of Node 2 relative to Node 1 may be defined on Card 9. These moments are applied equal and opposite on Nodes 1 and 2, irrespective of the settings of MX1, MX2, MY1 and MY2.

8. **Density.** Density should be set to a reasonable value, say 2000 to 8000 kg/m^3. The element mass will be calculated as density × volume (volume is entered on *SECTION_BEAM).

9. ***SECTION_BEAM input.** Note on values for *SECTION_BEAM:

   a) Set ELFORM to 6 (discrete beam).

   b) VOL (the element volume) might typically be set to 0.1 m^3.

   c) INER always needs to be non-zero. It will influence the solution only when the element has rotational stiffness, that is, when any of MX1, MX2, MY1, MY2, KTHX, KTHY or KTHZ are non-zero. A reasonable value might be 10-20 kg-m^2.

   d) CID can be left blank if the isolator is aligned in the global coordinate system, otherwise a coordinate system should be referenced.

e) By default, the isolator will be assumed to rotate with the average rotation of its two nodes. If the base of the column rotates slightly the isolator will no longer be perfectly horizontal: this can cause unexpected vertical displacements coupled with the horizontal motion. To avoid this, rotation of the local axes of the isolator can be eliminated by setting RRCON, SRCON, and TRCON to 1.0. This does not introduce any rotational restraint to the model, it only prevents the orientation of the isolator from changing as the model deforms.

f) SCOOR must be set to zero.

g) All other parameters on *SECTION_BEAM can be left blank.

10. **Post-processing note.** As with other discrete beam material models, the force described in some post-processors as "Axial" is really the force in the local x-direction; "Y-Shear" is really the force in the local y-direction; and "Z-Shear" is really the force in the local z-direction.

*MAT_JOINTED_ROCK

This is Material Type 198. Joints (planes of weakness) are assumed to exist throughout the material at a spacing small enough to be considered ubiquitous. The planes are assumed to lie at constant orientations defined on this material card. Up to three planes can be defined for each material. The base material is like *MAT_DRUCKER_PRAGER (*MAT_193). Input parameters for the base material are defined on Cards 1 through 3, while the joint planes are defined using Card 4. See *MAT_MOHR_COULOMB (*MAT_173) for a preferred alternative to this material model.

Card Summary:

Card 1. This card is required.

MID	RO	GMOD	RNU	RKF	PHI	CVAL	PSI

Card 2. This card is required.

STR_LIM	NPLANES	ELASTIC	LCCPDR	LCCPT	LCCJDR	LCCJT	LCSFAC

Card 3. This card is required.

GMODDP	PHIDP	CVALDP	PSIDP	GMODGR	PHIGR	CVALGR	PSIGR

Card 4. Include an instance of this card for each plane. Up to three planes may be defined.

DIP	STRIKE	CPLANE	FRPLANE	TPLANE	SHRMAX	LOCAL	

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	GMOD	RNU	RKF	PHI	CVAL	PSI
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	1.0	none	none	0.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
GMOD	Elastic shear modulus
RNU	Poisson's ratio
RKF	Failure surface shape parameter, see Remark 5.
PHI	Angle of friction (radians)
CVAL	Cohesion value (shear strength at zero normal stress)
PSI	Dilation angle (radians)

Card 2	1	2	3	4	5	6	7	8
Variable	STR_LIM	NPLANES	ELASTIC	LCCPDR	LCCPT	LCCJDR	LCCJT	LCSFAC
Type	F	I	I	I	I	I	I	I
Default	0.005	0	0	0	0	0	0	0

VARIABLE	DESCRIPTION
STR_LIM	Minimum shear strength of material is given by STR_LIM × CVAL (see Remark 6)
NPLANES	Number of joint planes (maximum of 3)
ELASTIC	Behavior of base material (see Remark 3): EQ.0: Nonlinear using all parameters on Cards 1 through 3 EQ.1: Linear elastic; only the joint planes are nonlinear
LCCPDR	Load curve for extra cohesion for base material (dynamic relaxation) as a function of time. See Remark 8.
LCCPT	Load curve for extra cohesion for base material (transient) as a function of time. See Remark 8.

VARIABLE	DESCRIPTION
LCCJDR	Load curve for extra cohesion for joints (dynamic relaxation) as a function of time. See Remark 8.
LCCJT	Load curve for extra cohesion for joints (transient) as a function of time. See Remark 8.
LCSFAC	Load curve giving a factor on strength as a function of time (see Remark 9).

Card 3	1	2	3	4	5	6	7	8
Variable	GMODDP	PHIDP	CVALDP	PSIDP	GMODGR	PHIGR	CVALGR	PSIGR
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Remark	4	4	4	4	4	4	4	4

VARIABLE	DESCRIPTION
GMODDP	Z-coordinate at which GMOD is correct
PHIDP	Z-coordinate at which PHI is correct
CVALDP	Z-coordinate at which CVAL is correct
PSIDP	Z-coordinate at which PSI is correct
GMODGR	Gradient of GMOD as a function of Z-coordinate (usually negative)
PHIGR	Gradient of PHI as a function of Z-coordinate
CVALGR	Gradient of CVAL as a function of Z-coordinate (usually negative)
PSIGR	Gradient of PSI as a function of Z-coordinate

Repeat Card 4 for each plane (maximum of 3 planes):

Card 4	1	2	3	4	5	6	7	8
Variable	DIP	STRIKE	CPLANE	FRPLANE	TPLANE	SHRMAX	LOCAL	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	1.e20	0.0	

VARIABLE	DESCRIPTION
DIP	Angle of the plane in degrees below the horizontal
STRIKE	Plan view angle (degrees) of downhill vector drawn on the plane
CPLANE	Cohesion for shear behavior on plane
FRPLANE	Friction angle for shear behavior on plane (degrees)
TPLANE	Tensile strength across plane (generally zero or very small)
SHRMAX	Max shear stress on plane (upper limit, independent of compression)
LOCAL	Axes (see Remark 10) EQ.0: DIP and STRIKE are with respect to the global axes. EQ.1: DIP and STRIKE are with respect to the local element axes.

Remarks:

1. **Joint plane orientations.** The joint plane orientations are defined by the angle of a “downhill vector” drawn on the plane, that is, the vector is oriented within the plane to obtain the maximum possible downhill angle. DIP is the angle of this line below the horizontal. STRIKE is the plan-view angle of the line (pointing downhill) measured clockwise from the global Y-axis about the global Z-axis. Note that DIP and STRIKE can also be with respect to the local element axes. See Remark 10 for details.

2. **Rigid body motion.** The joint planes rotate with the rigid body motion of the elements, irrespective of whether their initial definitions are in the global or local axis system.

3. **Elastic only behavior.** The full facilities of the modified Drucker Prager model for the base material can be used – see description of material type 193. Alternatively, to speed up the calculation, the ELASTIC flag can be set to 1, in which case the yield surface will not be considered, and only RO, GMOD, RNU, GMODDP, GMODGR, and the joint planes will be used.

4. **Model orientation.** This material type requires that the model is oriented such that the Z-axis is defined in the upward direction. The key parameters are defined such that they may vary with depth (i.e. the Z-axis), see Card 3. If Card 3 is left blank, the material properties do not vary with depth.

5. **Shape factor RKF.** The shape factor for a typical soil would be around 0.8. Values less than 0.75 should not be used.

6. **STR_LIM.** If STR_LIM is set to less than 0.005, the value is reset to 0.005.

7. **Correction to Drucker Prager model.** A correction has been introduced into the Drucker Prager model, such that the yield surface never infringes the Mohr-Coulomb criterion. Thus, the model does not give the same results as a "pure" Drucker Prager model.

8. **Load curves giving extra cohesion.** The load curves LCCPDR, LCCPT, LCCJDR, and LCCJT allow additional cohesion to be specified as a function of time. This cohesion is in addition to that specified in the material parameters. This feature is intended for use during the initial stages of an analysis to allow application of gravity or other loads without cracking or yielding, and for the cracking or yielding then to be introduced in a controlled manner. This is done by specifying extra cohesion that exceeds the expected stresses initially, then declining to zero. If no curves are specified, no extra cohesion is applied.

9. **LCSFAC.** The load curve giving a factor on strength applies simultaneously to the cohesion and tan PHI of the base material and all joints. This feature is intended for reducing the strength of the material gradually to explore factors of safety. If no curve is present, a constant factor of 1 is assumed. Values much greater than 1.0 may cause problems with stability.

10. **Masonry and joint planes.** Joint planes are generally defined in the global axis system if they are taken from survey data, and the material represents rock. For this case, set LOCAL = 0. In other cases, it may be more convenient to define the joint plane angles, DIP and STRIKE, relative to the element local axis system (to do this, set LOCAL = 1). For example, this material model can be used to represent masonry with the weak planes representing the mortar joint. In this situation, these joints may be parallel to the local element axes throughout the mesh.

    The choice of defining the joint angles relative to global versus local coordinates is available only for solid elements. For thick shell elements (*ELEMENT_-

TSHELL), DIP and STRIKE are always relative to the element's local axis, and the setting of LOCAL is ignored.

11. **Extra history variables.** Extra history variables may be plotted (see NEIPH on *DATABASE_EXTENT_BINARY). They are described in the following table:

History Variable #	Description
1	Mobilized strength fraction for base material
2	At-rest coefficient (defined as horizontal stress divided by vertical stress, where "horizontal stress" is the average of the stresses in the global X and Y directions, and "vertical stress" is in the global Z direction).
4 – 6	Crack opening strains for planes 1 through 3
7 – 9	Crack accumulated engineering shear strain for planes 1 through 3
10 – 12	Current shear utilization for planes 1 through 3
13 – 15	Maximum shear utilization to date for planes 1 through 3

*MAT_BARLAT_YLD2004

This is Material Type 199. This model was developed by Aretz and Barlat [2004] and Barlat et al. [2005]. It incorporates a yield criterion called Barlat 2004-18p, where up to 18 material parameters are used to define anisotropy for a full 3D stress state. This model is currently available for solid elements and thick shell formulations 3, 5 and 7.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR				
Type	A	F	F	F				

Card 2	1	2	3	4	5	6	7	8
Variable	CP12	CP13	CP21	CP23	CP31	CP32		
Type	F	F	F	F	F	F		

Card 3	1	2	3	4	5	6	7	8
Variable	CPP12	CPP13	CPP21	CPP23	CPP31	CPP32		
Type	F	F	F	F	F	F		

Card 4	1	2	3	4	5	6	7	8
Variable	CP44	CP55	CP66	CPP44	CPP55	CPP66		
Type	F	F	F	F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	AOPT	A	LCSS					
Type	F	F	I					

Card 6	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	MACF	
Type	F	F	F	F	F	F	I	

Card 7	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus LT.0.0: -E is either a load curve ID for Young's modulus as a function of plastic strain or a table ID for Young's modulus as a function of plastic strain and temperature.
PR	Poisson's ratio
CPij	9 coefficients c'_{ij} of the first linear transformation matrix $\mathbf{C}'$
CPPij	9 coefficients c''_{ij} of the second linear transformation matrix $\mathbf{C}''$
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details): EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR

VARIABLE	DESCRIPTION
	EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.
	EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only.
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*).
A	Flow potential exponent a
LCSS	Load curve ID or table ID for (isotropic) hardening:
	GT.0: If LCSS is a load curve, then yield stress $\overline{\sigma}$ is a function of plastic strain. If LCSS is a table, then yield stress $\overline{\sigma}$ is a function of plastic strain and plastic strain rate.
	LT.0: If -LCSS is a load curve, then yield stress $\overline{\sigma}$ is a function of plastic strain. If -LCSS is a table, then yield stress $\overline{\sigma}$ is a function of plastic strain and temperature.
XP YP ZP	Define coordinates of point *P* for AOPT = 1 and 4
A1, A2, A3	Components of vector **a** for AOPT = 2
MACF	Material axes change flag for solid elements:
	EQ.-4: Switch material axes b and c before BETA rotation
	EQ.-3: Switch material axes a and c before BETA rotation

VARIABLE	DESCRIPTION
	EQ.-2: Switch material axes a and b before BETA rotation EQ.1: No change, default EQ.2: Switch material axes a and b after BETA rotation EQ.3: Switch material axes a and c after BETA rotation EQ.4: Switch material axes b and c after BETA rotation
	Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_-SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 7 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.
V1, V2, V3	Components of vector **v** for AOPT = 3
D1, D2, D3	Components of vector **d** for AOPT = 2
BETA	Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SOLID_ORTHO and *ELEMENT_TSHELL_BETA.

Remarks:

The 3D yield condition for this material can be written as (see Barlat et al. [2005])

$$\begin{aligned}\phi &= \phi(\tilde{\mathbf{S}}', \tilde{\mathbf{S}}'') \\ &= |\tilde{S}'_1 - \tilde{S}''_1|^a + |\tilde{S}'_1 - \tilde{S}''_2|^a + |\tilde{S}'_1 - \tilde{S}''_3|^a + |\tilde{S}'_2 - \tilde{S}''_1|^a + |\tilde{S}'_2 - \tilde{S}''_2|^a \\ &\qquad + |\tilde{S}'_2 - \tilde{S}''_3|^a + |\tilde{S}'_3 - \tilde{S}''_1|^a + |\tilde{S}'_3 - \tilde{S}''_2|^a + |\tilde{S}'_3 - \tilde{S}''_3|^a \\ &= 4\bar{\sigma}^a\end{aligned}$$

Here $\tilde{S}'_i$ and $\tilde{S}''_i$ (i = 1,2,3) are the 6 principal values, a is the flow potential exponent, and $\bar{\sigma}$ is the effective uniaxial yield stress (defined with LCSS). The diagonal tensors $\tilde{\mathbf{S}}' = \mathrm{diag}(\tilde{S}'_1, \tilde{S}'_2, \tilde{S}'_3)$ and $\tilde{\mathbf{S}}'' = \mathrm{diag}(\tilde{S}''_1, \tilde{S}''_2, \tilde{S}''_3)$ contain the principal values of $\tilde{\mathbf{s}}'$ and $\tilde{\mathbf{s}}''$. $\tilde{\mathbf{s}}'$ and $\tilde{\mathbf{s}}''$ result from two linear transformations of the deviatoric portion of the Cauchy stress, **s**:

$$\begin{aligned}\tilde{\mathbf{s}}' &= \mathbf{C}'\mathbf{s} \\ \tilde{\mathbf{s}}'' &= \mathbf{C}''\mathbf{s}\end{aligned}$$

$\mathbf{C}'$ and $\mathbf{C}''$ have the following form:

$$\mathbf{C}' = \begin{bmatrix} 0 & -c'_{12} & -c'_{13} & 0 & 0 & 0 \\ -c'_{21} & 0 & -c'_{23} & 0 & 0 & 0 \\ -c'_{31} & -c'_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c'_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c'_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c'_{66} \end{bmatrix}$$

$$\mathbf{C}'' = \begin{bmatrix} 0 & -c''_{12} & -c''_{13} & 0 & 0 & 0 \\ -c''_{21} & 0 & -c''_{23} & 0 & 0 & 0 \\ -c''_{31} & -c''_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c''_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c''_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c''_{66} \end{bmatrix}$$

Each transformation matrix requires 9 coefficients that must be defined on Cards 2, 3, and 4 of this material model input. For identification of all 18 coefficients, uniaxial tests in several directions, biaxial tests, and crystal plasticity models (for out-of-plane properties) are needed. See Barlat et al. [2005] for more details and examples for parameters sets.

Note that the sequence of stress tensor components in LS-DYNA is as follows

$$\mathbf{s} = \begin{bmatrix} s_{xx} \\ s_{yy} \\ s_{zz} \\ s_{xy} \\ s_{yz} \\ s_{zx} \end{bmatrix}$$

meaning matrix entry "44" is linked to stress component "xy", "55" belongs to "yz", and "66" refers to "zx". If compared to the paper from Barlat et al. [2005] that means the following relations hold (each equation: LS-DYNA parameters on the left, Barlat coefficients on the right):

$$\mathrm{CP44} = c'_{66} \qquad \mathrm{CP55} = c'_{44} \qquad \mathrm{CP66} = c'_{55}$$
$$\mathrm{CPP44} = c''_{66} \qquad \mathrm{CPP55} = c''_{44} \qquad \mathrm{CPP66} = c''_{55}$$

For example, the following input would correspond to the parameters in Table 2 of that paper for 6111-T4 aluminum alloy:

```
*MAT_BARLAT_YLD2004
...
$     CP12      CP13      CP21      CP23      CP31      CP32
  1.241024  1.078271  1.216463  1.223867  1.093105  0.889161
$    CPP12     CPP13     CPP21     CPP23     CPP31     CPP32
  0.775366  0.922743  0.765487  0.793356  0.918689  1.027625
$     CP44      CP55      CP66     CPP44     CPP55     CPP66
  1.349094  0.501909  0.557173  0.589787  1.115833  1.112273
...
```

*MAT_BARLAT_YLD2004_27P

This is Material Type 199_27P. This model is a straightforward extension of material type 199. Aretz et al. [2010] developed the extension. It consists of a yield criterion called Barlat 2004-27p, where up to 27 material parameters define anisotropy for a 3D stress state. This model is currently available for solid elements and thick shell formulations 3, 5, and 7.

Card Summary:

Card 1. This card is required.

MID	RO	E	PR				

Card 2. This card is required.

CP12	CP13	CP21	CP23	CP31	CP32		

Card 3. This card is required.

CPP12	CPP13	CPP21	CPP23	CPP31	CPP32		

Card 4. This card is required.

CPPP12	CPPP13	CPPP21	CPPP23	CPPP31	CPPP32		

Card 5. This card is required.

CP44	CP55	CP66	CPP44	CPP55	CPP66		

Card 6. This card is required.

CPPP44	CPPP55	CPPP66					

Card 7. This card is required.

AOPT	A	LCSS					

Card 8. This card is required.

XP	YP	ZP	A1	A2	A3	MACF	

Card 9. This card is required.

V1	V2	V3	D1	D2	D3	BETA	

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR				
Type	A	F	F	F				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus LT.0.0: -E is either a load curve ID for Young's modulus as a function of plastic strain or a table ID for Young's modulus as a function of plastic strain and temperature.
PR	Poisson's ratio

Card 2	1	2	3	4	5	6	7	8
Variable	CP12	CP13	CP21	CP23	CP31	CP32		
Type	F	F	F	F	F	F		

Card 3	1	2	3	4	5	6	7	8
Variable	CPP12	CPP13	CPP21	CPP23	CPP31	CPP32		
Type	F	F	F	F	F	F		

Card 4	1	2	3	4	5	6	7	8
Variable	CPPP12	CPPP13	CPPP21	CPPP23	CPPP31	CPPP32		
Type	F	F	F	F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	CP44	CP55	CP66	CPP44	CPP55	CPP66		
Type	F	F	F	F	F	F		

Card 6	1	2	3	4	5	6	7	8
Variable	CPPP44	CPPP55	CPPP66					
Type	F	F	F					

VARIABLE	DESCRIPTION
CP*ij*	9 coefficients c'_{ij} of the first linear transformation matrix $\mathbf{C}'$
CPP*ij*	9 coefficients c''_{ij} of the second linear transformation matrix $\mathbf{C}''$
CPPP*ij*	9 coefficients c'''_{ij} of the second linear transformation matrix $\mathbf{C}'''$

Card 7	1	2	3	4	5	6	7	8
Variable	AOPT	A	LCSS					
Type	F	F	I					

VARIABLE	DESCRIPTION
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):

VARIABLE	DESCRIPTION
	EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.
	EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR
	EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the element's keyword input or input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA, depending on the value of MACF.
	EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector, **v**, and an originating point, P, which define the centerline axis. This option is for solid elements only.
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*).
A	Flow potential exponent a
LCSS	Load curve ID or table ID for (isotropic) hardening:
	GT.0: If LCSS is a load curve, yield stress, $\overline{\sigma}$, is a function of plastic strain. If LCSS is a table, $\overline{\sigma}$ is a function of plastic strain and plastic strain rate.
	LT.0: If -LCSS is a load curve, yield stress, $\overline{\sigma}$, is a function of plastic strain. If -LCSS is a table, $\overline{\sigma}$ is a function of plastic strain and temperature.

Card 8	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	MACF	
Type	F	F	F	F	F	F	I	

VARIABLE	DESCRIPTION
XP YP ZP	Define coordinates of point, *P*, for AOPT = 1 and 4
A1, A2, A3	Components of vector **a** for AOPT = 2
MACF	Material axes change flag for solid elements: EQ.-4: Switch material axes *b* and *c* before BETA rotation EQ.-3: Switch material axes *a* and *c* before BETA rotation EQ.-2: Switch material axes *a* and *b* before BETA rotation EQ.1: No change, default EQ.2: Switch material axes *a* and *b* after BETA rotation EQ.3: Switch material axes *a* and *c* after BETA rotation EQ.4: Switch material axes *b* and *c* after BETA rotation Figure M2-2 indicates when LS-DYNA applies MACF during the procedure to obtain the final material axes. If you define BETA on *ELEMENT_SOLID_{*OPTION*}, LS-DYNA uses that BETA for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 9 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.

Card 9	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3
D1, D2, D3	Components of vector **d** for AOPT = 2

VARIABLE	DESCRIPTION
BETA	Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SOLID_ORTHO and *ELEMENT_TSHELL_BETA.

Remarks:

We can write the 3D yield condition for this material as (see Aretz et al. [2010]):

$$
\begin{aligned}
\phi &= \phi(\tilde{\mathbf{S}}', \tilde{\mathbf{S}}'', \tilde{\mathbf{S}}''') \\
&= |\tilde{S}'_1 - \tilde{S}''_1|^a + |\tilde{S}'_1 - \tilde{S}''_2|^a + |\tilde{S}'_1 - \tilde{S}''_3|^a + |\tilde{S}'_2 - \tilde{S}''_1|^a + |\tilde{S}'_2 - \tilde{S}''_2|^a + |\tilde{S}'_2 - \tilde{S}''_3|^a \\
&\qquad + |\tilde{S}'_3 - \tilde{S}''_1|^a + |\tilde{S}'_3 - \tilde{S}''_2|^a + |\tilde{S}'_3 - \tilde{S}''_3|^a + |\tilde{S}'''_1 - \tilde{S}'''_2|^a + |\tilde{S}'''_2 - \tilde{S}'''_3|^a + |\tilde{S}'''_3 - \tilde{S}'''_1|^a \\
&= 6\bar{\sigma}^a
\end{aligned}
$$

Here $\tilde{S}'_i$, $\tilde{S}''_i$ and $\tilde{S}'''_i$ ($i = 1, 2, 3$) are the nine principal values, a is the flow potential exponent, and $\bar{\sigma}$ is the effective uniaxial yield stress (defined with LCSS). The diagonal tensors $\tilde{\mathbf{S}}' = \mathrm{diag}(\tilde{S}'_1, \tilde{S}'_2, \tilde{S}'_3)$, $\tilde{\mathbf{S}}'' = \mathrm{diag}(\tilde{S}''_1, \tilde{S}''_2, \tilde{S}''_3)$ and $\tilde{\mathbf{S}}''' = \mathrm{diag}(\tilde{S}'''_1, \tilde{S}'''_2, \tilde{S}'''_3)$ contain the principal values of $\tilde{\mathbf{s}}'$, $\tilde{\mathbf{s}}''$ and $\tilde{\mathbf{s}}'''$. $\tilde{\mathbf{s}}'$, $\tilde{\mathbf{s}}''$ and $\tilde{\mathbf{s}}'''$ result from three linear transformations of the deviatoric portion of the Cauchy stress, $\mathbf{s}$:

$$
\begin{aligned}
\tilde{\mathbf{s}}' &= \mathbf{C}'\mathbf{s} \\
\tilde{\mathbf{s}}'' &= \mathbf{C}''\mathbf{s} \\
\tilde{\mathbf{s}}''' &= \mathbf{C}'''\mathbf{s}
\end{aligned}
$$

$\mathbf{C}'$, $\mathbf{C}''$, and $\mathbf{C}'''$ have the following form:

$$
\mathbf{C}' = \begin{bmatrix}
0 & -c'_{12} & -c'_{13} & 0 & 0 & 0 \\
-c'_{21} & 0 & -c'_{23} & 0 & 0 & 0 \\
-c'_{31} & -c'_{32} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c'_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c'_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & c'_{66}
\end{bmatrix}
$$

$$
\mathbf{C}'' = \begin{bmatrix}
0 & -c''_{12} & -c''_{13} & 0 & 0 & 0 \\
-c''_{21} & 0 & -c''_{23} & 0 & 0 & 0 \\
-c''_{31} & -c''_{32} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c''_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c''_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & c''_{66}
\end{bmatrix}
$$

$$
\mathbf{C}''' = \begin{bmatrix}
0 & -c'''_{12} & -c'''_{13} & 0 & 0 & 0 \\
-c'''_{21} & 0 & -c'''_{23} & 0 & 0 & 0 \\
-c'''_{31} & -c'''_{32} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c'''_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c'''_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & c'''_{66}
\end{bmatrix}
$$

Each transformation matrix requires nine coefficients input on Cards 2, 3, 4, 5, and 6. You must identify the 27 coefficients from the results of uniaxial tests in several directions, biaxial tests, and crystal plasticity models (for out-of-plane properties). See Barlat et al. [2005] and Aretz et al. [2010] for more details and examples of parameter sets.

Note that the sequence of stress tensor components in LS-DYNA is as follows

$$\mathbf{s} = \begin{bmatrix} s_{xx} \\ s_{yy} \\ s_{zz} \\ s_{xy} \\ s_{yz} \\ s_{zx} \end{bmatrix}$$

meaning matrix entry “44” is linked to stress component “xy”, “55” belongs to “yz”, and “66” refers to “zx”. If compared to the paper from Aretz et al. [2010] that means the following relations hold (each equation: LS-DYNA parameters on the left, Barlat coefficients on the right):

$$\begin{aligned} \mathrm{CP44} &= c'_{66} & \mathrm{CP55} &= c'_{44} & \mathrm{CP66} &= c'_{55} \\ \mathrm{CPP44} &= c''_{66} & \mathrm{CPP55} &= c''_{44} & \mathrm{CPP66} &= c''_{55} \\ \mathrm{CPPP44} &= c'''_{66} & \mathrm{CPPP55} &= c'''_{44} & \mathrm{CPPP66} &= c'''_{55} \end{aligned}$$

For example, the following input corresponds to the anisotropy parameters of the paper from Aretz et al. [2010] for AA3104-H19 aluminum alloy:

```
*MAT_BARLAT_YLD2004_27P
...
$     CP12      CP13      CP21      CP23      CP31      CP32
  0.606220   1.40199  0.367381  0.382048-0.0338334  0.821313
$    CPP12     CPP13     CPP21     CPP23     CPP31     CPP32
   1.47683  0.607440   1.02276  0.529721  0.870353  0.487571
$   CPPP12    CPPP13    CPPP21    CPPP23    CPPP31    CPPP32
 -0.334771 -0.488257  0.559627   1.11301  0.436153  0.808785
$     CP44      CP55      CP66     CPP44     CPP55     CPP66
  0.882890   1.00000   1.00000  0.972130   1.00000   1.00000
$   CPPP44    CPPP55    CPPP66
  0.898776   1.00000   1.00000
...
```

*MAT_STEEL_EC3

This is Material Type 202. Tables and formulae from Eurocode 3 are used to derive the mechanical properties and their variation with temperature, although these can be overridden by user-defined curves. It is currently available only for Hughes-Liu beam elements. This material model is intended for modelling structural steel in fires.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY			
Type	A	F	F	F	F			
Default	none	none	none	none	none			

Card 2	1	2	3	4	5	6	7	8
Variable	LC_E	LC_PR	LC_AL	TBL_SS	LC_FS			
Type	F	F	F	F	F			
Default	0.0	0.0	0.0	0.0	0.0			

Card 3 *must* be included but left blank.

Card 3	1	2	3	4	5	6	7	8
Variable								
Type								
Default								

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density

VARIABLE	DESCRIPTION
E	Young's modulus – a reasonable value must be provided even if LC_E is also input. See Remark 2.
PR	Poisson's ratio
SIGY	Initial yield stress, σ_{y0}
LC_E	Optional load curve ID giving Young's modulus as a function of temperature (overrides E and factors from EC3)
LC_PR	Optional load curve ID giving Poisson's ratio as a function of temperature (overrides PR)
LC_AL	Optional load curve ID giving alpha as a function of temperature (over-rides thermal expansion data from EC3)
TBL_SS	Optional table ID containing stress-strain curves at different temperatures (overrides curves from EC3)
LC_FS	Optional load curve ID giving failure strain as a function of temperature

Remarks:

1. **Eurocode 3 and Required Input.** By default, only E, PR and SIGY must be defined. The Young's Modulus, E, will be scaled by a factor that is a function of temperature as specified in EC3. The factor is 1.0 at low temperatures. Eurocode 3 (EC3) Section 3.2 specifies the stress-strain behaviour of carbon steels at temperatures between 20°C and 1200°C. The stress-strain curves given in EC3 are scaled within the material model such that the maximum stress at low temperatures is SIGY (see Figure below). By default, the thermal expansion coefficient as a function of temperature will be as specified in EC3 Section 3.4.1.1.

2. **Optional input.** LC_E, LC_PR and LC_AL are optional; they should have temperature on the x-axis and the material property on the y-axis, with the points in order of increasing temperature. If defined (that is, nonzero), they override E, PR, and the relationships from EC3. However, a reasonable value for E should always be included, since these values will be used for purposes such as contact stiffness calculation.

   TBL_SS is also optional. It overrides SIGY and the stress-strain relationships from EC3. If present, TBL_SS must be the ID of a *DEFINE_TABLE. The field VALUE on the *DEFINE_TABLE should contain the temperature at which each stress-strain curve is applicable; the temperatures should be in ascending order.

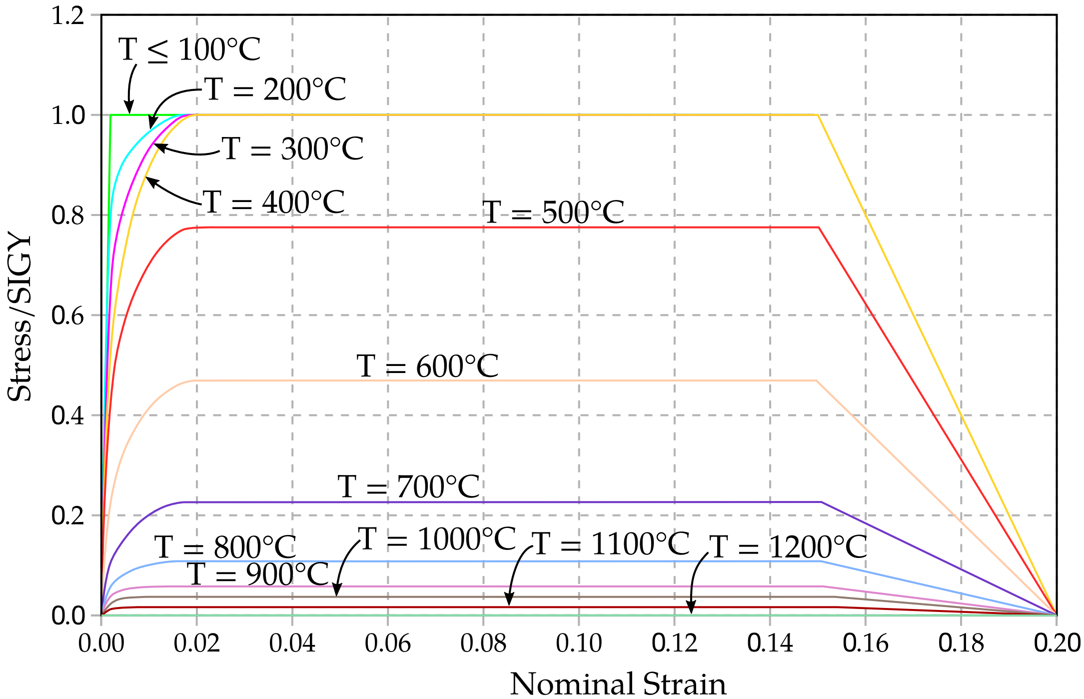


Figure M202-1. Stress-strain curves at various temperatures

The curves that follow the temperature values have plastic strain on the *x*-axis and yield stress on the *y*-axis as per other LS-DYNA elasto-plastic material models. As with all instances of *DEFINE TABLE, the curves containing the stress-strain data must immediately follow the *DEFINE_TABLE input data and must be in the correct order (that is, the same order as the temperatures).

3. **Temperature.** Temperature can be defined by any of the *LOAD_THERMAL methods. The temperature does not have to start at zero: the initial temperature will be taken as a reference temperature for each element, so non-zero initial temperatures will not cause thermal shock effects.

4. **Extra history variables.** Temperature is output by this material model as Extra History Variable 1. This can be useful for checking the input in cases where temperature varies across the different integration points, as is the case with *LOAD_THERMAL_VARIABLE_BEAM

*MAT_HYSTERETIC_REINFORCEMENT

This is Material Type 203. It is intended as an alternative reinforcement model for layered reinforced concrete shell or beam elements, for use in seismic analysis where the nonlinear hysteretic behavior of the reinforcement is important. *PART_COMPOSITE or *INTEGRATION_BEAM should be used to define some integration points as a part made of *MAT_HYSTERETIC_REINFORCEMENT, while other integration points have concrete properties using *MAT_CONCRETE_EC2. When using beam elements, ELFORM = 1 is required.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	YM	PR	SIGY	LAMDA	SBUCK	POWER
Type	A	F	F	F	F	F	F	F
Default	none	0.0	0.0	0.0	0.0	0.0	SIGY	0.5

Card 2	1	2	3	4	5	6	7	
Variable	FRACX	FRACY	LCTEN	LCCOMP	AOPT	EBU	DOWNSL	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.1	

Card 3	1	2	3	4	5	6	7	
Variable	DBAR	FCDOW	LCHARD	UNITC	UNITL			
Type	F	F	F	F	F			
Default	0.0	0.0	0.0	1.0	1.0			

Card 4	1	2	3	4	5	6	7	8
Variable	EPDAM1	EPDAM2	DRESID					
Type	F	F	F					
Default	0.0	0.0	0.0					

Additional Card for AOPT ≠ 0.

Card 5	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		
Default				0.0	0.0	0.0		

Additional Card for AOPT ≠ 0.

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
YM	Young's modulus
PR	Poisson'srRatio
SIGY	Yield stress

VARIABLE	DESCRIPTION
LAMDA	Slenderness ratio
SBUCK	Initial buckling stress (should be positive)
POWER	Power law for Bauschinger effect (non-dimensional)
FRACX	Fraction of reinforcement at this integration point in local *x*-direction
FRACY	Fraction of reinforcement at this integration point in local *y*-direction
LCTEN	Optional curve providing the factor on SIGY as a function of plastic strain (tension)
LCCOMP	Optional curve providing the factor on SBUCK as a function of plastic strain (compression)
AOPT	Material axes option (see *MAT_*OPTION*TROPIC_ELASTIC): EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in Figure M2-1. The **a**-direction is from node 1 to node 2 of the element. The **b**-direction is orthogonal to the **a**-direction and is in the plane formed by nodes 1, 2, and 4. The material axes are then rotated about the normal vector to the surface of the shell by the angle BETA. EQ.2.0: Globally orthotropic with material axes determined by vectors **a** and **d** input below, as with *DEFINE_COORDINATE_VECTOR. EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element (see Figure M2-1). **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. LT.0.0: |AOPT| is a coordinate system ID (see *DEFINE_COORDINATE_OPTION).
EBU	Optional buckling strain. If defined, it overrides LAMBDA.

VARIABLE	DESCRIPTION
DOWNSL	Initial downslope of the buckling curve as a fraction of the Young's modulus (dimensionless)
DBAR	Reinforcement bar diameter used for dowel action. See Remark 7.
FCDOW	Concrete compressive strength used for dowel action. See Remark 7. This field has units of stress.
LCHARD	Characteristic length for dowel action (length units)
UNITC	Factor to convert model stress units to MPa. For instance, if the model units are Newtons and meters, UNITC = 10^{-6}. [UNITC] = 1/[STRESS].
UNITL	Factor to convert model length units to millimeters. For example, if the model units are meters, UNITL = 1000. [UNITL] = 1/[LENGTH].
EPDAM1	Accumulated plastic strain at which hysteretic damage begins
EPDAM2	Accumulated plastic strain at which hysteretic damage is complete
DRESID	Residual factor remaining after hysteretic damage
A1, A2, A3	Components of vector **a** for AOPT = 2
V1, V2, V3	Components of vector **v** for AOPT = 3
D1, D2, D3	Components of vector **d** for AOPT = 2
BETA	Angle for AOPT = 0 and 3

Remarks:

1. **Material directions.** The reinforcement is treated as bars, acting independently in the local material x and y directions. By default, the local material x-axis is the element's x-axis (parallel to the line from Node 1 to Node 2), but this direction may be overridden using AOPT or the element's BETA angles.

2. **Using this material with shell and beam elements.** For shell elements, the reinforced concrete section should be defined using *PART_COMPOSITE with some integration points being reinforcement (referencing a material ID using this material model) and others being concrete (using, for example, *MAT_CONCRETE_EC2). By default, shear strains in the local xy, yz, and zx directions are unresisted by this material model, so it should not be used alone (without

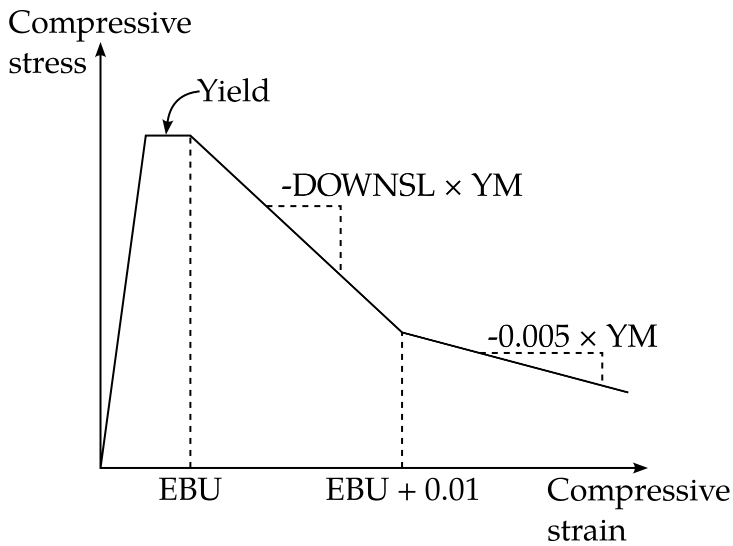


Figure M203-1. Buckling behavior using EBU and DOWNSL

concrete). The area fractions of reinforcement in the local x and y-directions at each integration point are given by the thickness fraction of the integration point in the *PART_COMPOSITE definition times the fractions FRACX and FRACY on Card 2 above.

For beam elements, *INTEGRATION_BEAM defines the material at the integration point (e.g. reinforcement or concrete). Due to a limitation of *INTEGRATION_BEAM, this material model can be paired only with *MAT_CONCRETE_EC2 within the same *INTEGRATION_BEAM.

3. **Stress-strain response.** The tensile response is elastic-perfectly plastic, using yield stress SIGY. Optionally, load curves may be used to describe the stress-strain response in tension (LCTEN) and compression (LCCOMP). Either, neither, or both curves may be defined. If present, LCTEN overrides the perfectly plastic tensile response, and LCCOMP overrides the buckling curve. The tensile and compressive plastic strains are considered independent of each other.

4. **Bar buckling.** To define bar buckling, set either the slenderness ratio LAMDA or the initial buckling strain EBU with downslope DOWNSL. If bar buckling is not defined, the bars simply yield in compression. If both ways are defined, the buckling behavior defined by EBU and DOWNSL overrides LAMDA.

   The slenderness ratio LAMDA determines buckling behavior and is defined as,

$$\frac{kL}{r},$$

   where k depends on end conditions, and

   L = unsupported length of reinforcement bars
   r = radius of gyration which for round bars is equal to (bar radius)/2.

Users are expected to determine LAMDA accounting for the expected crack spacing.

Figure M203-1 shows the alternative buckling behavior defined by EBU and DOWNSL.

5. **Hysteresis response.** Reloading after a change of load direction follows a Bauschinger-type curve, leading to the hysteresis response shown below:

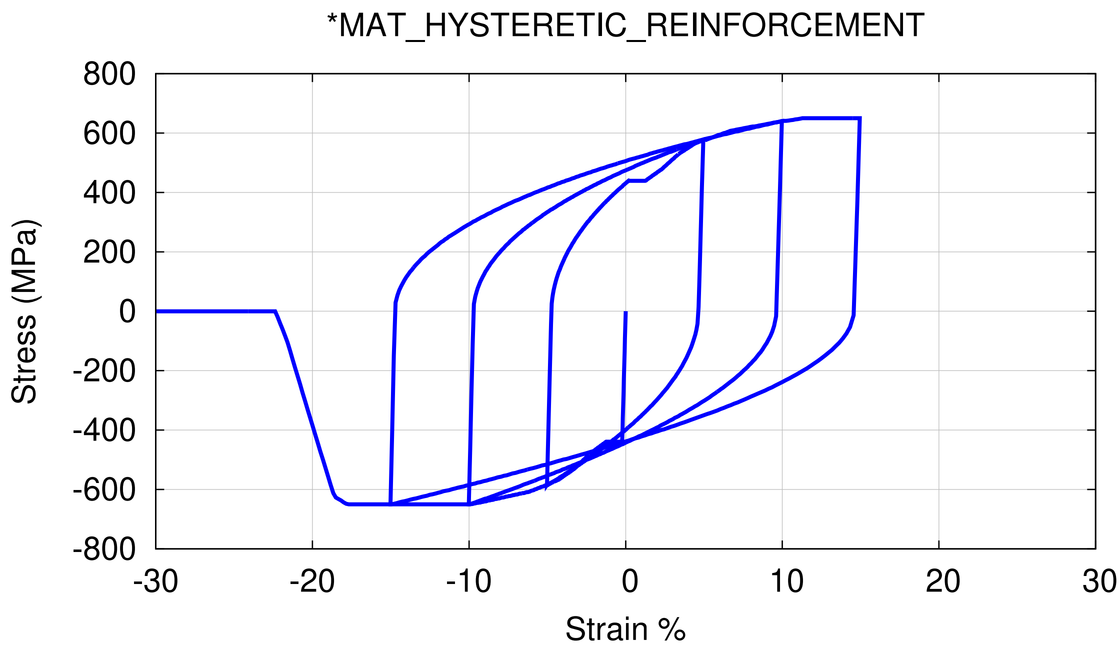


Figure M203-2. Example hysteretic response

6. **Damage modeling.** Two types of damage accumulation may be modeled. Damage based on ductility (strain) can be modeled using the curves LCTEN and LCCOMP. At high strain, these curves show reducing stress with increasing strain.

   To model damage based on hysteretic energy accumulation, use the parameters EPDAM1, EPDAM2, and DRESID. The damage is a function of accumulated plastic strain. For this purpose, plastic strain increments are always treated as positive in both tension and compression, and buckling strain also counts towards the accumulated plastic strain. The material has its full stiffness and strength until the accumulated plastic strain reaches EPDAM1. Between plastic strains EPDAM1 and EPDAM2, the stiffness and strength fall linearly with accumulated plastic strain, reaching a factor DRESID at plastic strain EPDAM2.

7. **Dowel action.** The data on Card 3 defines the shear stiffness and strength and is optional. Shear resistance is assumed to occur by dowel action. The bars bend

locally to the crack and crush the concrete. An elastic-perfectly-plastic relation is assumed for all shear components (in-plane and through-thickness). The assumed (smeared) shear modulus and yield stress applicable to the reinforcement bar cross-sectional area are as follows, based on formulae derived from experimental data by El-Ariss, Soroushian, and Dulacska:

$$G[\mathrm{MPa}] = 8.02E^{0.25}F_c^{0.375}L_{\mathrm{char}}D_b^{0.75}$$
$$\tau_y = 1.62\sqrt{F_c S_y}$$

where,

$$E = \text{steel Young's modulus in MPa}$$
$$F_c = \text{compressive strength of concrete in MPa}$$
$$L_{\mathrm{char}} = \text{characteristic length of shear deformation in mm}$$
$$D_b = \text{bar diameter in mm}$$
$$S_y = \text{steel yield stress in MPa.}$$

The input parameters should be given in model units. For instance, DBAR and LCHAR are in model length units, and FCDOW is in model stress units. These units are converted internally using UNITL and UNITC.

8. **Element erosion.** By default, no erosion occurs. Elements are deleted only if EPDAM1 and EPDAM2 are nonzero, DRESID is zero, and the accumulated plastic strain reaches EPDAM2. If FRACX and FRACY are both nonzero, i.e., if there is reinforcement in both local directions, elements only erode when the condition above has been reached in both local directions.

9. **Output.** The output stresses, as for all other LS-DYNA material models, are by default in the global coordinate system. They are scaled by the reinforcement fractions FRACX and FRACY. The plastic strain output is the accumulated plastic strain (increments always treated as positive) and is the greater such value of the two local directions. Extra history variables are available as follows:

History Variable #	Description
1	Reinforcement stress in the local x-direction (not scaled by FRACX)
2	Reinforcement stress in the local y-direction (not scaled by FRACY)
3	Total strain in the local x-direction
4	Total strain in the local y-direction
5	Accumulated plastic strain in the local x-direction
6	Accumulated plastic strain in the local y-direction
7	Shear stress (dowel action) in local xy

History Variable #	Description
8	Shear stress (dowel action) in local xz
9	Shear stress (dowel action) in local yz
10	Maximum (high-tide) total strain in the local x-direction
11	Maximum (high-tide) total strain in the local y-direction

*MAT_DISCRETE_BEAM_POINT_CONTACT

This is Material Type 205. It is used for discrete beam elements only (ELFORM = 6). It simulates contact forces between a point (Node 2) and an imaginary flat surface (fixed relative to Node 1). The beam elements may have nonzero initial length. On *SECTION_BEAM, SCOOR must be set to -13, the triad rotation option. The triad rotation option ensures that the axis system remains fixed in the imaginary surface.

Card Summary:

Card 1. This card is required.

MID	RO	STIFF	FRIC	DAMP	DMXPZ	LIMPZ	

Card 2. This card is required.

DMXPX	DMXNX	DMXPY	DMXNY	LIMPX	LIMNX	LIMPY	LIMNY

Card 3. This card is required.

KROTX	KROTY	KROTZ	TKROT	FBONDH	FBONDT	DBONDH	DBONDT

Card 4. This card is optional.

LCZ	DAMPZ	STIFFH	FRMAX	DAMPH	GAP0	AFAC	

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	STIFF	FRIC	DAMP	DMXPZ	LIMPZ	
Type	A	F	F	F	F	F	F	
Default	none	none	none	0.0	0.0	10^{20}	0.0	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
STIFF	Stiffness (Force/length units)
FRIC	Friction coefficient (dimensionless)

VARIABLE	DESCRIPTION
DAMP	Damping factor (dimensionless), in the range 0 to 1. We suggest a value of 0.5.
DMXPZ	Displacement limit in positive local z-direction (uplift)
LIMPZ	Action when Node 2 passes DMXPZ: EQ.0: Element is deleted. EQ.1: Further displacement is resisted by stiffness STIFF.

Card 2	1	2	3	4	5	6	7	8
Variable	DMXPX	DMXNX	DMXPY	DMXNY	LIMPX	LIMNX	LIMPY	LIMNY
Type	F	F	F	F	F	F	F	F
Default	10^{20}	DMXPX	DMXPX	DMXPY	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION
DMXPX	Displacement limit in positive local x-direction
DMXNX	Displacement limit in negative local x-direction
DMXPY	Displacement limit in positive local y-direction
DMXNY	Displacement limit in negative local y-direction
LIMPX	Action when Node 2 passes DMXPX: EQ.0: Element is deleted. EQ.1: Further displacement is resisted by stiffness STIFF.
LIMNX	Action when Node 2 passes DMXNX: EQ.0: Element is deleted. EQ.1: Further displacement is resisted by stiffness STIFF.
LIMPY	Action when Node 2 passes DMXPY: EQ.0: Element is deleted. EQ.1: Further displacement is resisted by stiffness STIFF.

VARIABLE	DESCRIPTION
LIMNY	Action when Node 2 passes DMXNY: EQ.0: Element is deleted. EQ.1: Further displacement is resisted by stiffness STIFF.

Card 3	1	2	3	4	5	6	7	8
Variable	KROTX	KROTY	KROTZ	TKROT	FBONDH	FBONDT	DBONDH	DBONDT
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	10^{20}	10^{20}

VARIABLE	DESCRIPTION
KROTX	Rotational stiffness about local x
KROTY	Rotational stiffness about local y
KROTZ	Rotational stiffness about local z
TKROT	Time at which rotational stiffness becomes active
FBONDH	Force to break initial bond in plane of contact surface
FBONDT	Force to break initial bond in tension, normal to contact surface
DBONDH	Displacement over which bond force in the plane of the contact surface reduces from FBONDH to zero
DBONDT	Displacement over which bond force normal to the contact surface reduces from FBONDT to zero

This card is optional

Card 4	1	2	3	4	5	6	7	8
Variable	LCZ	DAMPZ	STIFFH	FRMAX	DAMPH	GAP0	AFAC	
Type	I	F	F	F	F	F	F	
Default	0	0.0	STIFF	∞	0.0	0.0	1.0	

VARIABLE	DESCRIPTION
LCZ	Optional load curve ID giving force-displacement for compression in local z-direction (abscissa: displacement; ordinate: force). The load curve must be defined in the positive quadrant, meaning that the compressive force values should be defined as positive values.
DAMPZ	Viscous damping coefficient in local z-direction (applied in addition to DAMP) (force/velocity units)
STIFFH	Elastic stiffness in local x- and y-directions
FRMAX	Upper limit on friction force
DAMPH	Viscous damping coefficient in local x- and y-directions (applied in addition to DAMP) (force/velocity units).
GAP0	Initial gap in local z-direction (length units)
AFAC	Scale factor applied to all stiffnesses and forces

Remarks:

1. **Model Description.** This material model simulates contact between a point (Node 2 of the beam element) and an imaginary flat surface which is fixed relative to Node 1. In these remarks we call the imaginary surface the "contact surface" – this does *not* refer to *CONTACT. The local axes are determined by CID on *SECTION_BEAM. The imaginary surface is in the local xy-plane passing through the initial coordinates of Node 2. The local z-axis points outwards from the surface. The surface translates and rotates with Node 1. SCOOR must be set to -13. The elements may have nonzero length.

   When Node 2 moves in the negative local z-direction relative to Node 1 (penetration into the contact surface), the motion is resisted by stiffness STIFF and the force generated is described here as the contact force. By default, uplift (Node 2 moving in the positive local z-direction relative to Node 1) is not resisted. If uplift greater than DMXPZ occurs, either the element is deleted (if LIMPZ = 0) or further uplift is resisted by STIFF (if LIMPZ = 1).

   Sliding on the surface (motion of Node 2 in the local x- and y-directions) is resisted by friction. The maximum friction force is given by FRIC times the contact force, with an upper limit of FRMAX if that parameter is nonzero. When one of the displacement limits, DMXPX, DMXNX, DMXPY, or DMXNY, is reached, the default behavior is for Node 2 to fall off the edge of the contact surface, and the element is deleted (see Figure M205-1). Optionally, the input fields LIMPX, LIMNX, LIMPY, and LIMNY can be used to change the behavior to "hard limits" using stiffness STIFF – these represent contact with a hard surface perpendicular to the surface on which Node 2 slides. In that case, the limit distances, DMXPX, DMXNX, DMXPY, and

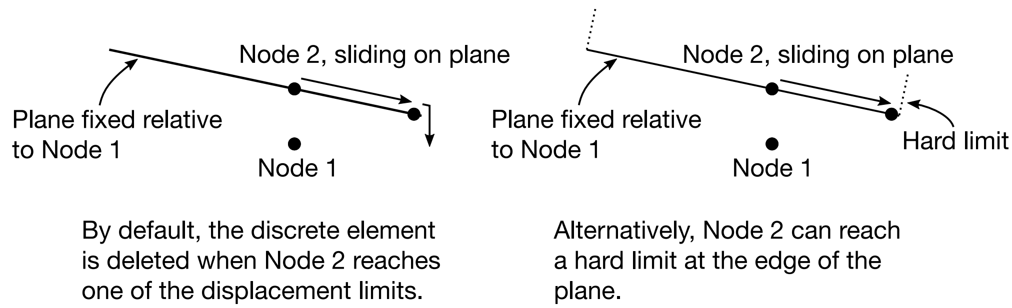


Figure M205-1. Illustration of the two different actions that can be applied to the element when Node 2 reaches the edge of the plane. The left schematic depicts the default behavior in which the element is deleted when Node 2 reaches the edge. The right image illustrates Node 2 reaching a hard limit. When it reaches this limit, a contact force is applied.

DMXNY, represent the initial gap between the point (Node 2) and the hard surface.

Optionally, an initial bond strength can be defined. The bond forces are in addition to any contact and friction forces. After breakage of the bond, contact and sliding can continue to occur.

If LCZ is defined, compressive loading in the contact direction follows LCZ while unload/reload is linear with stiffness STIFF. The value of STIFF must not be less than the maximum slope of any segment of the curve LCZ.

You can also define moment stiffnesses through KROTX, KROTY and KROTZ. Optionally, these stiffnesses can be set to become active at a certain time (TKROT). If TKROT is nonzero, the moment stiffness will be zero before that time and during any dynamic relaxation. If TKROT is left zero, the moment stiffness will be active from the start of the analysis including during dynamic relaxation.

Damping is applied to the force normal to the surface, to the bond forces, and to any forces generated by "hard limits" (LIMPX etc.), but not to sliding. Two damping methods are available: DAMP and DAMPZ/DAMPH. DAMP is recommended for general use where the exact amount of damping is unimportant, and the requirement is simply to remove unwanted oscillations. DAMPZ and DAMPH are available for cases where particular values of viscous damping coefficient are required. When LCZ is also defined and the response is following the load curve (meaning not during unloading/reloading), the damping coefficient DAMPZ is scaled to the ratio of the slope of LCZ to STIFF. The slope of LCZ is the gradient at the current point on the load curve.

2. ***SECTION_BEAM Input.** Note on values for *SECTION_BEAM:

   a) Set ELFORM to 6 (discrete beam)

b) CID can be left blank if the contact surface is aligned in the global XY-plane, otherwise a coordinate system should be referenced.

c) SCOOR must be set to -13.

3. **Output.** Beam "axial" or "X" force is the force in the local x-direction. "Shear-Y" or "Y" force is the force in the local y-direction. "Shear-Z" or "Z" force is the force in the local z-direction, normal to the contact surface.

   Other output is written to the d3plot and d3thdt files in the places where post-processors expect to find the stress and strain at the first integration point for integrated beams:

Integration Point	Post-Processing Data Component	Actual Meaning
1	Axial stress	Displacement in the local x-direction
1	XY shear stress	Displacement in the local y-direction
1	ZX shear stress	Displacement in the local z-direction
1	Plastic strain	Minimum overlap, meaning the minimum value so far during the analysis of the remaining displacement before Node 2 falls of the surface in the x- or y-directions
1	Axial strain	Bond damage

*MAT_SOIL_SANISAND

This is Material Type 207. It is supported for solid elements only. It is intended for modelling sands and sandy soils under monotonic and cyclic loading conditions. Cyclic shear loading leads to dilative and contractive volumetric behavior based on critical state soil mechanics. When used together with LS-DYNA's pore fluid analysis capabilities, liquefaction and related phenomena can be modelled. See Remarks below.

Card Summary:

Card 1. This card is required.

MID	RO	G0	K0NU	PREF	RHOC	THETA	X

Card 2. This card is required.

EIN	ALPHAC	E0	LAMBDA	XI	NB	H0	CH

Card 3. This card is required.

P0	CC	ND	A0				

Card 4. This card is required.

ANISO	KH	ZMAX	CZ				

Card 5. This card is required.

PATM	M	N	V				

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	G0	K0NU	PREF	RHOC	THETA	X
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.37	none	none

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
G0	Shear-modulus-related dimensionless term G_0. See Remark 6.
K0NU	Elastic constant (see Remark 6): GT.0.0: Bulk modulus term K_0 LT.0.0: Absolute value is Poisson's ratio, ν.
PREF	Reference pressure in Limiting Compression Curve, associated with unity void ratio. See Remark 10.
RHOC	Exponent in Limiting Compression Curve, ρ_c. See Remark 10.
THETA	Exponent in transitional compression behavior, θ. See Remark 10.
X	Material constant X. See Remark 10.

Card 2	1	2	3	4	5	6	7	8
Variable	EIN	ALPHAC	E0	LAMBDA	XI	NB	H0	CH
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	0.7	none	none	none

VARIABLE	DESCRIPTION
EIN	Initial void ratio
ALPHAC	Critical surface angle in q-p space, α_c^c. See Remark 8.
E0	Material constant in Critical State Line, e_0. See Remark 8.
LAMBDA	Material constant in Critical State Line, λ. See Remark 8.
XI	Material constant in Critical State Line, ξ. See Remark 8.
NB	Bounding surface parameter, n^b. See Remark 8.

VARIABLE	DESCRIPTION
H0	Kinematic hardening parameter, h_0. See Remark 7.
CH	Kinematic hardening parameter, c_h. See Remark 7.

Card 3	1	2	3	4	5	6	7	8
Variable	P0	CC	ND	A0				
Type	F	F	F	F				
Default	none	0.778	none	none				

VARIABLE	DESCRIPTION
P0	Initial value of yield surface parameter, p_0. See Remark 7.
CC	Ratio of critical surface angle in extension to critical surface angle in compression, c. See Remark 8.
ND	Dilatancy surface parameter, n_d. See Remark 8.
A0	Dilatancy parameter, A_0

Card 4	1	2	3	4	5	6	7	8
Variable	ANISO	KH	ZMAX	CZ				
Type	F	F	F	F				
Default	0.333	1.0	none	none				

VARIABLE	DESCRIPTION
ANISO	Inherent fabric anisotropy measure. See Remark 12.
KH	Material constant, k_h, in dependence of hardening parameters on inherent fabric anisotropy. See Remark 12.
ZMAX	Material constant for fabric change effect, $z_{\max}$. See Remark 11.

VARIABLE	DESCRIPTION
CZ	Material constant for fabric change effect, c_z. See Remark 11.

Card 5	1	2	3	4	5	6	7	8
Variable	PATM	M	N	V				
Type	F	F	F	F				
Default	none	0.05	20.0	1000.0				

VARIABLE	DESCRIPTION
PATM	Atmospheric pressure, p_{atm}
M	Yield surface constant, m. See Remark 7.
N	Yield surface constant, n. See Remark 7.
V	Flow rule constant, V

Remarks:

1. **References.** SANISAND (an acronym for Simple ANIsotropic SAND) is a family of constitutive models within the frameworks of critical state soil mechanics and bounding surface plasticity. Although the overall principles remain the same, different research teams have developed versions of SANISAND with different details. The material model implemented in LS-DYNA as *MAT_SANISAND is based on Dafalias and Manzari [2004] with high-pressure yield surface cap from Taiebat and Dafalias [2008] and options for inherent fabric anisotropy taken from Dafalias, Papadimitriou, and Li [2004]. The references give the constitutive model equations both in “triaxial formulation” (in which there are only two stress variables q and p), and in “multi-axial formulation” (in which the deviatoric stress-related terms are tensors). The LS-DYNA implementation follows the multi-axial formulation, but, in these Remarks, equations are given in triaxial formulation for ease of understanding the principles.

2. **Pore pressure build-up and liquefaction.** Pore water pressure build-up occurs when the “soil skeleton” contracts (for example, under cyclic shearing), leaving a greater proportion of the external load to be supported by the pore water and a lesser proportion acting on the soil skeleton, that is, the effective stress is reduced. Since sandy soils are frictional materials, reduced effective stress leads

to reduced shear stiffness and reduced shear strength. It ultimately leads to liquefaction of the soil. Therefore, accurate modelling of pore pressure build-up and liquefaction effects depend critically on accurate dilation/contraction behavior of the material model, which is a key aspect of the SANISAND material model.

3. **Modeling pore pressure.** Pore pressure is not included in the material model itself. As with other LS-DYNA soil models, the material model represents the effective stress (soil skeleton) behavior. Pore pressure can be modelled with *CONTROL_PORE_FLUID and *BOUNDARY_PORE_FLUID. The analysis type should be set to Undrained or Time Dependent Consolidation (ATYPE = 1 or 3, respectively) in order for pore pressure changes to occur in response to the dilation or contraction of the soil.

4. **Void ratio.** Soils consist of solid particles with voids between them. Many of the equations governing the behavior of this material model are given in terms of the void ratio, e. The volume of voids divided by the volume of solids gives the void ratio. It is a measure of how loosely or tightly packed the particles are (the lower the void ratio, the more tightly packed). The void ratio at the start of the analysis is given by input parameter EIN. The void ratio varies during the analysis: volumetric strains are assumed to relate to changes of void volume, while the volume of solids remains unchanged.

5. **Stress ratio.** Most elasto-plastic constitutive models are described in terms of their stress-strain behavior. For frictional materials, such as sandy soils, shear strength is considered proportional to pressure. Thus, it is more appropriate to define the equations in terms of stress ratio rather than stress. Stress ratio in triaxial formulation is a scalar quantity η equal to q/p (where q is Von Mises stress and p is pressure). In the multi-axial formulation, the stress ratio is a tensor quantity equal to the deviatoric stress tensor divided by pressure. Stress ratios described in the references include the “dilatancy stress ratio” (also called the “phase transformation line”) which marks the boundary between contractive and dilative response to shear strains, the “critical state stress ratio” (see Remark 8), and the “bounding stress ratio” at which shear failure occurs.

6. **Elastic properties.** The elastic shear modulus, G, is given by:

$$G = G_0 p_{\mathrm{atm}} \frac{(2.97 - e)^2}{1 + e} \left(\frac{p}{p_{\mathrm{atm}}}\right)^{1/2}$$

where $G_0$and p_{atm} are the input parameters G0 and PATM, p is the current pressure, and e is the current void ratio. G0 is dimensionless; PATM has stress units.

If input parameter K0NU is positive, then the bulk modulus, K, is given by:

$$K = K_0 p_{\mathrm{a}} \frac{1 + e}{e} \left(\frac{p}{p_{\mathrm{atm}}}\right)^{2/3}$$

where K_0 is the input parameter K0NU. If input parameter K0NU is negative, then the bulk modulus, K, is given by:

$$K = \frac{2(1+\nu)}{3(1-2\nu)}G$$

where $\nu = -$K0NU.

7. **Yield surface and hardening.** The yield surface is defined as a narrow cone of semi-angle m centered on a back stress ratio α which, together with the hardening rule, enables realistic nonlinear unload/reload behavior under cyclic loading. The yield surface is closed at the high pressure end by a cap-like feature that models grain crushing using parameter p_0, as described in Taiebat and Dafalias [2008]:

$$f = (q - p\alpha)^2 - m^2 p^2 \left[1 - \left(\frac{p}{p_0}\right)^n\right] = 0$$

where m and n are input parameters M and N (see Card 5), and p_0 has an initial value given by input parameter P0. Description of the flow rule and evolution of α and p_0 are given in the references. The yield surface as described in Dafalias and Manzari [2004] is the same as above except that it lacks the term in square brackets, which corresponds to a yield surface cap at high pressure. When P0 is set to a high value compared to the expected pressure, the yield surface in this material model becomes the same as that of Dafalias and Manzari [2004].

Hardening is described by a "stress ratio hardening modulus", H, which is the rate of change of stress ratio η with deviatoric plastic strain:

$$H = \frac{d\eta}{d\varepsilon_p} = \frac{G_0|h_0|(1 - c_h e)(\alpha_b - \eta)}{R}\left(\frac{p}{p_{atm}}\right)^{-1/2}.$$

Here h_0 is the input parameter H0, c_h is the input parameter CH, $\alpha_b - \eta$ is the "distance" (in stress ratio terms) between the current stress ratio and its image on the bounding surface for loading in the current loading direction. R is defined as given in Equations 5, 6, and 24 of Dafalias and Manzari [2004] and can be considered as the "distance" (in stress ratio terms) between the current stress ratio and the stress ratio where the current plastic loading cycle began (such as at the most recent loading direction reversal). This term is zero on initial loading and immediately following a stress reversal, giving an instantaneously infinite hardening slope and a smooth transition between elastic and plastic behavior.

If the anisotropy parameter ANISO has a non-default value, anisotropy effects are included as per Equation 15 in Dafalias, Papadimitriou and Li [2004].

8. **Critical state.** The critical state is a combination of values of stress ratio, pressure, and void ratio at which the application of shear strain causes no change to any of those three parameters. It is assumed that soil starting from any initial state, if subjected to sufficient shear strain, will tend towards and eventually

reach a critical state. In MAT_SANISAND, the critical state void ratio, e_c, is defined as follows:

$$e_{\mathrm{c}} = e_0 - \lambda(p/p_{\mathrm{a}})^{\xi} \ .$$

In the above, e_0, λ, ξ, and p_{a} are input parameters E0, LAMBDA, XI and PATM, respectively. If the anisotropy parameter ANISO has a non-default value, e_0 in the above equation is replaced by $e_0 \exp(-A)$ where A is the lode angle-dependent state parameter described in Dafalias, Papadimitriou and Li [2004].

The state parameter ψ is given by $e - e_c$, which may be thought of as the distance from the critical state in terms of void ratio.

The critical state stress ratio, α_c^c, is a constant given by the input parameter ALPHAC. If the material is on the critical state line in e-p space according to the above equation, that is, when $\psi = 0$, then both the dilatancy stress ratio, α^d, and the bounding stress ratio, α^b, coincide with the critical state stress ratio. These stress ratios vary with ψ in the following manner:

$$\alpha^d = c\alpha_c^c \exp(n^d \psi)$$
$$\alpha^b = c\alpha_c^c \exp(-n^b \psi)$$

where n^d and n^b are the input parameters ND and NB, respectively, and $c = 1$ for compressive loading. For extension loading, c is equal to the input parameter CC.

9. **Dilation and contraction.** The volumetric plastic strain increment is linked to the deviatoric plastic strain increment through a dilation angle, D. D is defined such that the volumetric plastic strain is contractive at stress ratios below α^d and dilative at stress ratios above α^d:

$$D = \frac{\dot{\epsilon}_v^p}{\dot{\epsilon}_q^p} = sA_d(\alpha^d - \alpha) \ .$$

In the above equation, $\dot{\epsilon}_v^p$ and $\dot{\epsilon}_v^p$ are the volumetric and deviatoric plastic strain increments, s takes the value 1 or –1 according to loading direction, and A_d is the input parameter A0. If the fabric change parameters ZMAX and CZ are defined (see Remark 11), A_d is scaled according to Equation 12 of Dafalias and Manzari [2004].

10. **Limiting Compression Curve (LCC).** The maximum pressure that the material can sustain is governed by the LCC, which is analogous to a yield curve in pressure-void ratio space. The LCC consists of points $(p_{\mathrm{LCC}}, e_{\mathrm{LCC}})$ such that:

$$\log e_{\mathrm{LCC}} = -\rho_c \log\left(\frac{p_{\mathrm{LCC}}}{p_{\mathrm{REF}}}\right) \ .$$

Here ρ_c and p_{REF} are the input parameters RHOC and PREF. The transition from elastic volumetric response to plastic volumetric deformation along the LCC is a smooth function governed by input parameters THETA, and X. See Taiebat and Dafalias [2008] for further details.

11. **Fabric change.** Input parameters ZMAX and CZ control the fabric change effect described by Dafalias and Manzari [2004]. During cyclic shearing, in the dilatant phase the contact surfaces between the sand particles are re-oriented in a manner that greatly increases the contractive tendency upon subsequent reversal of the shearing direction. If this effect is not modelled, simulations of cyclic shearing may tend to stabilize at a nonzero effective pressure in cases where experiments would show the effective pressure diminishing to zero.

12. **Fabric anisotropy.** Input parameters ANISO and KH control the fabric anisotropy effect described by Dafalias, Papadimitriou and Li [2004]. The default, given by ANISO = 1/3 and KH=1, is isotropic behavior. Non-default values cause the plastic hardening behavior and the critical state line to depend on a lode-angle, leading to a better match of experimental results under a wide variety of stress states. ANISO = 0 corresponds to a fabric formation where particles lie entirely on the global XY-plane. ANISO = 1 implies a fabric formation where particles are oriented entirely parallel to the vertical global Z-direction. It is expected that the most common cases will be in the range of 0 < ANISO < 1/3, i.e., with a preference toward horizontal orientations.

13. **Output.** The current void ratio, e, is output in place of plastic strain.

References:

[1] Taiebat M. and Dafalias, Y. F. “SANISAND: simple anisotropic sand plasticity model.” *International Journal for Numerical and Analytical Methods in Geomechanics*, 32(8), 915–948 (2008).

[2] Dafalias,Y. F. and Manzari, M.T. “Simple plasticity sand model accounting for fabric change effects.” *Journal of Engineering Mechanics,* 130(6), 622–634 (2004).

[3] Dafalias, Y. F., Papadimitriou, A. G., and Li, X. S. “Sand plasticity model accounting for inherent fabric anisotropy.” *Journal of Engineering Mechanics,*130(11), 1319–1333 (2004).

*MAT_BOLT_BEAM

This is Material Type 208 for use with beam elements using ELFORM = 6 (discrete beam). The beam elements must have nonzero initial length so that the directions in which tension and compression act can be distinguished. See Remarks 1 and 2.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	KAX	KSHR	blank	blank	FPRE	TRAMP
Type	A	F	F	F			F	F
Default	none	none	0.0	0.0			0.0	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	LCAX	LCSHR	FRIC	CLEAR	DAFAIL	DRFAIL	DAMAG	T0PRE
Type	I	I	F	F	F	F	F	F
Default	0	0	0.0	0.0	10^{20}	10^{20}	0.1	0.0

Card 3	1	2	3	4	5	6	7	8
Variable	DACFAIL	AXSHFL	HOLSHR	IAXIS				
Type	F	I	I	I				
Default	10^{20}	0	0	1				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
KAX	Axial elastic stiffness (Force/Length units)

VARIABLE	DESCRIPTION
KSHR	Shear elastic stiffness (Force/Length units)
FPRE	Preload force
TRAMP	Time duration during which preload is ramped up
LCAX	Load curve giving axial load as a function of plastic displacement (x-axis = displacement (length units), y-axis = force). See Remark 4.
LCSHR	Load curve ID or table ID giving lateral load as a function of plastic displacement (x-axis - displacement (length units), y-axis - force). In the table case, each curve in the table represents lateral load as a function of displacement at a given (current) axial load, meaning the values in the table are axial forces. See Remark 4.
FRIC	Friction coefficient resisting sliding of bolt head/nut (non-dimensional)
CLEAR	Radial clearance (gap between bolt shank and the inner diameter of the hole) (length units). See Remark 5.
DAFAIL	Axial tensile displacement at which failure is initiated (length units)
DRFAIL	Radial displacement at which failure is initiated (excludes clearance)
DAMAG	Failure is completed at $$(\mathrm{DAFAIL\ or\ DRFAIL\ or\ DACFAIL}) \times (1 + \mathrm{DAMAG})$$
T0PRE	Time at which preload application begins
DACFAIL	Axial compressive displacement at which failure is initiated (positive value, length units)
AXSHFL	Flag to determine effect on axial response of increase of length of element due to shear displacement. In this context, shear displacement excludes sliding within the clearance gap. See Remark 6. EQ.0: Shear-induced length increase treated as axial load. EQ.1: Shear-induced length increase is ignored.

VARIABLE	DESCRIPTION
HOLSHR	Flag for hole enlargement due to shear (see Remark 7): EQ.0: Hole does not enlarge due to shear deformation. NE.0: Shear deformation after bolt contacts the inner diameter of the hole enlarges the hole.
IAXIS	Flag to determine how the bolt axis relates to the beam element local axes. See Remark 2. EQ.1: Each element creates own axes, bolt axis (N1-N2 direction) is local x (default) EQ.2: Each element creates own axes, bolt axis (N1-N2 direction) is local y EQ.3: Each element creates own axes, bolt axis (N1-N2 direction) is local z EQ.4: Axis system defined by CID on *SECTION_BEAM, bolt axis is local x EQ.5: Axis system defined by CID on Section card, bolt axis is local y EQ.6: Axis system defined by CID on Section card, bolt axis is local z

Remarks:

1. **Bolted Joint Geometry.** The element represents a bolted joint. The nodes of the beam should be thought of as representing the points at the centers of the holes in the plates that are joined by the bolt.

2. **Local Axes.** There are three options for defining the bolt axis direction and local axis system for elements of this material type. Here, "bolt axis" means the direction in which tension is applied, while shear forces are perpendicular to the bolt axis. "Local axes" means the axis system in which the element's deformations are calculated and its forces are output. By default, the bolt axis coincides with the local x-axis, although that can be changed using the input parameter IAXIS.

   a) **Local axes defined by a *DEFINE_COORDINATE_NODES *that has FLAG set to 1.*** The coordinate system is referenced either as CID on *SECTION_BEAM or through PARAM3 on *ELEMENT_BEAM_THICKNESS. The behavior is then as follows:

i) The local axis system is defined by the three nodes of the *DEFINE_COORDINATE_NODES throughout the analysis, as described in the Remarks under *DEFINE_COORDINATE_NODES. The initial direction given by Nodes 1 and 2 of the beam element and the rotation of the local system based on SCOOR from *SECTION_BEAM are both irrelevant in this case.

ii) By default, the axial direction of the bolt coincides with the local x-axis of the *DEFINE_COORDINATE_NODES. Tension (positive force) is generated when Node 2 displaces in the positive local x direction relative to Node 1. Care is needed to ensure that the beam element topology is defined with Node 1 and Node 2 the correct way around to generate tension in the expected direction – for example, having Node 2 initially offset in the positive local x direction from Node 1 will mean that tension is generated when the beam element elongates in the local x-direction.

iii) The axial direction of the bolt can be oriented along the local y- or z-axis instead of the local x-axis by setting IAXIS to 2 or 3, respectively.

iv) For bolts that are differently oriented, you will need to *either* separate them into different *PARTs (so that the different *DEFINE_COORDINATE_NODES can be referenced by different *SECTION_BEAM cards) *or* have PARAM3 on *ELEMENT_BEAM_THICKNESS reference the needed *DEFINE_COORDINATE_NODES coordinate system.

b) **Bolt axis defined by the initial orientation of the beam element.** To obtain this behavior, set CID = 0 on *SECTION_BEAM (and set PARAM3 = 0 if using *ELEMENT_BEAM_THICKNESS), and position Nodes 1 and 2 of each beam element with a nonzero distance between them, aligned with the axis of the bolt. Note that the behavior described below will also occur if IAXIS < 4 and CID is nonzero, but the referenced coordinate system is a *DEFINE_COORDINATE_SYSTEM or a *DEFINE_COORDINATE_NODES with FLAG = 0 (i.e. it does not fulfill all the conditions for a) above).

i) Each beam element automatically creates its own axis system where the bolt axis is initially in the direction defined by Node 1 to Node 2. Note that this behavior is different than that for other Discrete Beam material types.

ii) During the analysis, the local axis system rotates as defined by SCOOR on *SECTION_BEAM. For example, if SCOOR = -13, the axes rotate with Node 1. We recommend using SCOOR = -13 or +13.

iii) The bolt axis is always initially coincident with the Node 1 to Node 2 direction, but IAXIS = 1, 2 or 3 controls whether the bolt axis is labelled as the local *x*-, *y*- or *z*-axis. This setting has no effect on the analysis, only on the output of results for post-processing. This can be useful when post-processing discrete beams of different material types so that, for example, the beam *z*-force has a similar meaning across the different material types.

c) **Local axes initially defined by a *DEFINE_COORDINATE_*OPTION* with rotation of the axes controlled by SCOOR**. To obtain this behavior, the coordinate system should be specified by either *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_NODES with FLAG = 0. Reference the coordinate system as CID on *SECTION_BEAM (or as PARAM3 on *ELEMENT_BEAM_THICKNESS) and set IAXIS to 4, 5 or 6. The behavior is as follows:

i) The local axis system defined with CID on *SECTION_BEAM (or PARAM3 on *ELEMENT_BEAM_THICKNESS) is used. The initial direction Node 1 to Node 2 has no influence on the local axes or the bolt axis direction.

ii) During the analysis, the local axis system rotates as defined by SCOOR on *SECTION_BEAM. For example, if SCOOR = -13, the axes rotate with Node 1. We recommend using SCOOR = -13 or +13.

iii) The bolt axis is oriented along the local *x*-, *y*- or *z*-axis according to whether IAXIS = 4, 5 or 6, respectively.

iv) Care is needed to ensure that the beam element topology is defined with Node 1 and Node 2 the correct way around to generate tension in the expected direction – for example, if IAXIS = 4, having Node 2 initially offset in the positive local x direction from Node 1 will mean that tension is generated when the beam element elongates in the local *x* direction.

v) For bolts that are differently oriented, you will need to *either* separate them into different *PARTs (so that the different *DEFINE_COORDINATE_NODES can be referenced by different *SECTION_BEAM cards) *or* have PARAM3 on *ELEMENT_BEAM_THICKNESS reference the needed *DEFINE_COORDINATE_NODES coordinate system.

3. **Axial Response.** The axial response is tensile only. If the element shortens in the bolt axis direction, instead of generating a compressive axial load, a gap is assumed to develop between the bolt head (or nut) and the surface of the plate.

Contact between the bolted surfaces must be modelled separately, such as using *CONTACT or another discrete beam element.

4. **Yield Force Curves.** Curves LCAX and LCSHR give yield force as a function of plastic displacement for the axial and shear directions, respectively. The force increments are calculated from the elastic stiffnesses, subject to the yield force limits given by the curves.

5. **Sliding Shear Displacement.** CLEAR allows the bolt to slide in shear, resisted by friction between bolt head/nut and the surfaces of the plates, from the initial position at the center of the hole. CLEAR is the total sliding shear displacement before contact occurs between the bolt shank and the inside surface of the hole. Sliding shear displacement is not included in the displacement used for LCSHR. LCSHR is intended to represent the behavior after the bolt shank contacts the edge of the hole.

6. **Shear Deformation and Axial Tension.** If KSHR and LCSHR represent deformation and rotation of the bolt itself, AXSHFL = 0, the default setting, is recommended. On the other hand, if KSHR and LCSHR represent deformation of the bearing surfaces, AXSHFL = 1 is recommended, and HOLSHR = 1 is likely to be appropriate too (see Remark 7).

   The explanation is as follows. Consider the case where a shear displacement is applied to the bolted joint, while the plates are constrained to remain the same distance apart. Mechanisms by which a bolted joint can displace in the shear direction include:

   a) the bolt sliding within a clearance gap;

   b) rotation, bending and shearing of the bolt itself; and

   c) deformation of the bearing surfaces while the bolt itself remains almost rigid and perpendicular to the plates.

   The tension in the bolt might be expected to increase when mechanism (b) occurs because the length of the bolt itself must increase (in this example, the plates are held the same distance apart). Tension in the bolt would not increase with mechanisms (a) or (c). In the LS-DYNA model, applying shear while holding the plates the same distance apart will cause the element to lengthen. When calculating the axial force in the bolt, *MAT_BOLT_BEAM always ignores any lengthening due to mechanism (a) but is unable to distinguish between (b) and (c) – KSHR and LCSHR could represent either or both types of mechanism. AXSHFL tells *MAT_BOLT_BEAM whether to treat these shear deformations as contributing to changes of length to the bolt itself, and hence whether axial tension will be generated.

7. **Hole Enlargement.** If HOLSHR is nonzero, shear deformation beyond that necessary to close the clearance gap enlarges the hole in the plates and does not deform the bolt itself. The force-deformation relation of this mechanism is still governed by LCSHR, and the deformation (that is, enlargement of the hole) is tracked separately in each of the local $-y$, $+y$, $-z$, and $+z$ directions. Thus, for example, enlargement of the hole in the positive Y direction has no effect on the position of the edge of the hole in the negative Y direction. When HOLSHR is used, AXSHFL should be set to 1.

8. **Output.** Beam "axial" or "X" force is the axial force in the beam. "shear-Y" and "shear-Z" are the shear forces. Other output is written to the d3plot and d3thdt files in the places where post-processors expect to find the stress and strain at the first two integration points for integrated beams.

Integration Point	Post-Processing Data Component	Actual Meaning
1	Axial Stress	Change of length
1	*XY* Shear stress	Sliding shear displacement in local y
1	*ZX* Shear stress	Sliding shear displacement in local Z
1	Plastic strain	Resultant shear sliding displacement
1	Axial strain	Axial plastic displacement
2	Axial Stress	Shear plastic displacement excluding sliding
2	XY Shear stress	Not used
2	ZX Shear stress	Not used
2	Plastic strain	Not used
2	Axial strain	Not used

*MAT_HYSTERETIC_BEAM

This is Material Type 209. It can be used only with resultant beam elements (ELFORM = 2). It is intended for modelling buildings in seismic analysis and is similar to *MAT_191 but with increased capabilities. Plastic hinges can form at both ends of the element, and plasticity options are available for axial and shear behavior as well as bending. The yield surface incorporates moment-axial interaction. Advanced features implemented for this material include hinge locations and pinching effect (Card 3), asymmetry and shear failure (Card 4), Bauschinger effect (Card 5), stiffness degradation (Cards 6 and 7), and FEMA flags (Cards 8, 9, and 10).

Card Summary:

Card 1. This card is required.

MID	RO	E	PR	IAX	ISURF	IHARD	IFEMA

Card 2. This card is required.

LCPMS	SFS	LCPMT	SFT	LCAT	SFAT	LCAC	SFAC

Card 3. This card is required.

ALPHA	BETA	GAMMA	F0	PINM	PINS	HLOC1	HLOC2

Card 4. This card is required.

DELTAS	KAPPAS	DELTAT	KAPPAT	LCSHS	SFSHS	LCSHT	SFSHT

Card 5. This card is required.

HARDMS	GAMMS	HARDMT	GAMMT	HARDAT	GAMAT	HARDAC	GAMAC

Card 6. This card is required.

OMGMS1	OMGMS2	OMGMT1	OMGMT2	OMGAT1	OMGAT2	OMGAC1	OMGAC2

Card 7. This card is required.

RUMS	RUMT	DUAT	DUAC	LAM1	LAM2	SOFT1	SOFT2

Card 8. Include this card if IFEMA > 0.

PRS1	PRS2	PRS3	PRS4	PRT1	PRT2	PRT3	PRT4

Card 9. Include this card if IFEMA > 1.

TS1	TS2	TS3	TS4	CS1	CS2	CS3	CS4

Card 10. Include this card if IFEMA > 2.

SS1	SS2	SS3	SS4	ST1	ST2	ST3	ST4

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	IAX	ISURF	IHARD	IFEMA
Type	A	F	F	F	I	I	I	I
Default	none	none	none	none	1	1	2	0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young’s modulus
PR	Poisson’s ratio
IAX	Abscissa definition for axial yield force as a function of inelastic deformation/strain curves (LCAT and LCAC on Card 2): EQ.1: Plastic deformation (change in length) EQ.2: Nominal plastic strain, that is, $\frac{\text{plastic deformation}}{\text{initial length}}$
ISURF	Yield surface type for interaction (see Remark 2): EQ.1: Simple power law (default) EQ.2: Power law based on resultant moment EQ.3: Skewed yield surface version of ISURF = 2 (see Remark 5)

VARIABLE	DESCRIPTION
IHARD	Hardening type during cyclic response (see Remark 6): EQ.1: Cumulative absolute deformation EQ.2: Peak deformation EQ.3: Peak deformation, yield-oriented EQ.4: Peak deformation, peak-oriented
IFEMA	Flag for input of FEMA thresholds (Cards 8, 9 and 10; see Remarks 9 and 11): EQ.0: No input EQ.1: Input of rotation thresholds only EQ.2: Input of rotation and axial strain thresholds EQ.3: Input of rotation, axial strain and shear strain thresholds

Card 2	1	2	3	4	5	6	7	8
Variable	LCPMS	SFS	LCPMT	SFT	LCAT	SFAT	LCAC	SFAC
Type	I	F	I	F	I	F	I	F
Default	none	1.0	LCPMS	SFS	none	1.0	LCAT	SFAT

VARIABLE	DESCRIPTION
LCPMS	Load curve ID (See *DEFINE_CURVE) giving normalized yield moment as a function of plastic rotation at hinges about the local s-axis. All values are positive.
SFS	Representative yield moment for plastic hinges about local the s-axis (scales the normalized moment from LCPMS)
LCPMT	Load curve ID (See *DEFINE_CURVE) giving normalized yield moment as a function of plastic rotation at hinges about the local t-axis. All values are positive.
SFT	Representative yield moment for plastic hinges about local the t-axis (scales the normalized moment from LCPMT)
LCAT	Load curve ID (See *DEFINE_CURVE) giving normalized axial

VARIABLE	DESCRIPTION
	tensile yield force as a function of inelastic deformation/strain. See IAX above for definition of deformation/strain. All values are positive. See *DEFINE_CURVE.
SFAT	Representative tensile strength (scales the normalized force from LCAT)
LCAC	Load curve ID (See *DEFINE_CURVE) giving normalized axial compressive yield force as a function of inelastic deformation/strain. See IAX above for definition of deformation/strain. All values are positive. See *DEFINE_CURVE.
SFAC	Representative compressive strength (scales the normalized force from LCAC)

Card 3	1	2	3	4	5	6	7	8
Variable	ALPHA	BETA	GAMMA	F0	PINM	PINS	HLOC1	HLOC2
Type	F	F	F	F	F	F	F	F
Default	2.0	2.0	2.0	0.0	1.0	1.0	0.0	0.0

VARIABLE	DESCRIPTION
ALPHA	Parameter to define moment-axial yield surface: GT.0.0: Yield surface parameter ALPHA (must not be < 1.1); see Remark 2. LT.0.0: User-defined yield surface for the local *s*-axis. |ALPHA| is the load curve ID giving the yield locus. The abscissa is the moment about the local *s*-axis; the ordinate is the axial force (tensile positive). See Remark 4.
BETA	Parameter to define moment-axial yield surface: GT.0.0: Yield surface parameter BETA (must not be < 1.1); see Remark 2. LT.0.0: User-defined yield surface for the local *t*-axis. |BETA| is the load curve ID giving the yield locus. Abscissa is moment about the local *t*-axis; the ordinate is the axial force

VARIABLE	DESCRIPTION
	(tensile positive). See Remark 4.
GAMMA	Parameter to define yield surface which must not be < 1.1 (see Remark 2)
F0	Force at which maximum yield moment is achieved (tensile positive; for reinforced concrete, a negative (compressive) value would be entered).
PINM	Pinching factor for flexural hysteresis (for IHARD = 3 or 4 only). See Remark 7.
PINS	Pinching factor for shear hysteresis (for IHARD = 3 or 4 only). See Remark 7.
HLOC1	Location of plastic Hinge 1 from Node 1 (see Remark 1): GE.0.0: HLOC1 is the distance of Hinge 1 to Node 1 divided by element length. LT.0.0.AND.GT.-1.0: -HLOC1 is the distance of Hinge 1 to Node 1 divided by element length; deactivate shear yielding. EQ.-1.0: deactivate Hinge 1. EQ.-10.0: deactivate shear yielding; Hinge 1 is located at Node 1. EQ.-11.0: deactivate Hinge 1 and shear yielding.
HLOC2	Location of plastic Hinge 2 from Node 2 (see Remark 1): GE.0.0: HLOC2 is the distance of Hinge 2 to Node 2 divided by element length. LT.0.0.AND.GT.-1.0: HLOC2 is the distance of Hinge 2 to Node 2 divided by element length; deactivate shear yielding. EQ.-1.0: deactivate Hinge 2. EQ.-10.0: deactivate shear yielding; Hinge 2 is located at Node 2. EQ.-11.0: deactivate Hinge 2 and shear yielding.

Card 4	1	2	3	4	5	6	7	8
Variable	DELTAS	KAPPAS	DELTAT	KAPPAT	LCSHS	SFSHS	LCSHT	SFSHT
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	none	1.0	LCSHS	SFSHS

VARIABLE	DESCRIPTION
DELTAS	Parameter to define the skew for yield surface (ISURF = 3); see Remark 2.
KAPPAS	Parameter to define the skew for yield surface (ISURF = 3); see Remark 2.
DELTAT	Parameter to define the skew for yield surface (ISURF = 3); see Remark 2.
KAPPAT	Parameter to define the skew for yield surface (ISURF = 3); see Remark 2.
LCSHS	Load curve ID (see *DEFINE_CURVE) giving yield shear force as a function of inelastic shear strain (shear angle) in the local *s*-direction (see Remark 10).
SFSHS	Scale factor on yield shear force in the local *s*-direction (scales the force from LCSHS): GT.0.0: Constant scale factor LT.0.0: User-defined interaction with axial force. |SFSHS| is the load curve ID giving scale factor as a function of normalized axial force (tensile is positive). The normalization uses SFAT for tensile force and SFAC for compressive force. For example, point $(-1.0, 0.5)$ on the curve defines a scale factor of 0.5 for compressive force of -SFAC.
LCSHT	Load curve ID (see *DEFINE_CURVE) giving yield shear force as a function of inelastic shear strain (shear angle) in the local *t*-direction (see Remark 10).
SFSHT	Scale factor on yield shear force in the local *t*-direction (scales the force from LCSHS).

VARIABLE	DESCRIPTION
	GT.0.0: Constant scale factor
	LT.0.0: User-defined interaction with axial force. |SFSHT| is the load curve ID giving scale factor as a function of normalized axial force (tensile is positive). The normalization uses SFAT for tensile force and SFAC for compressive force. For example, point $(-1.0, 0.5)$ on the curve defines a scale factor of 0.5 for compressive force of -SFAC.

Card 5	1	2	3	4	5	6	7	8
Variable	HARDMS	GAMMS	HARDMT	GAMMT	HARDAT	GAMAT	HARDAC	GAMAC
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	HARDMS	GAMMS	0.0	0.0	HARDAT	GAMAT

VARIABLE	DESCRIPTION
HARDMS	Kinematic hardening modulus for moment about the local *s*-axis (see Remark 8)
GAMMS	Kinematic hardening limit for moment about the local *s*-axis (see Remark 8)
HARDMT	Kinematic hardening modulus for moment about the local *t*-axis
GAMMT	Kinematic hardening limit for moment about the local *t*-axis
HARDAT	Kinematic hardening modulus for tensile axial force
GAMAT	Kinematic hardening limit for tensile axial force
HARDAC	Kinematic hardening modulus for compressive axial force
GAMAC	Kinematic hardening limit for compressive axial force

Card 6	1	2	3	4	5	6	7	8
Variable	OMGMS1	OMGMS2	OMGMT1	OMGMT2	OMGAT1	OMGAT2	OMGAC1	OMGAC2
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	OMGMS1	OMGMS2	0.0	0.0	OMGAT1	OMGAT2

VARIABLE	DESCRIPTION
OMGMS1	Damage evolution parameter ω_{s1} for moment about the local s-axis (see Remark 9)
OMGMS2	Damage evolution parameter ω_{s2} for moment about the local s-axis
OMGMT1	Damage evolution parameter ω_{t1} for moment about the local t-axis
OMGMT2	Damage evolution parameter ω_{t2} for moment about the local t-axis
OMGAT1	Damage evolution parameter ω_{at1} for tensile force
OMGAT2	Damage evolution parameter ω_{at2} for tensile force
OMGAC1	Damage evolution parameter ω_{ac1} for compressive force
OMGAC2	Damage evolution parameter ω_{ac2} for compressive force

Card 7	1	2	3	4	5	6	7	8
Variable	RUMS	RUMT	DUAT	DUAC	LAM1	LAM2	SOFT1	SOFT2
Type	F	F	F	F	F	F	F	F
Default	10^{20}	RUMS	10^{20}	DUAT	0.0	LAM1	3.0	4.0

VARIABLE	DESCRIPTION
RUMS	Ultimate plastic rotation about s-axis for damage calculation (see Remark 9)
RUMT	Ultimate plastic rotation about t-axis for damage calculation

VARIABLE	DESCRIPTION
DUAT	Ultimate tensile plastic deformation/strain for damage calculation. See IAX above in Card 1 and Remark 9.
DUAC	Ultimate compressive plastic deformation/strain for damage calculation. See IAX above in Card 1.
LAM1	Damage evolution parameter
LAM2	Damage evolution parameter
SOFT1	Threshold index at which softening starts (see Remark 9) LE.4.0: Threshold index for start of softening, see Cards 8 thru 10 EQ.5.0: Softening and element deletion are disabled.
SOFT2	Threshold index at which the element is fully softened and to be removed (ignored if SOFT1 = 5)

Plastic Rotation Thresholds Card. Define Card 8 only if IFEMA > 0 (see Remarks 9 and 11).

Card 8	1	2	3	4	5	6	7	8
Variable	PRS1	PRS2	PRS3	PRS4	PRT1	PRT2	PRT3	PRT4
Type	F	F	F	F	F	F	F	F
Default	10^{20}	2×10^{20}	3×10^{20}	4×10^{20}	PRS1	PRS2	PRS3	PRS4

VARIABLE	DESCRIPTION
PRS1 – PRS4	Plastic rotation thresholds 1 to 4 about s-axis
PRT1 – PRT4	Plastic rotation thresholds 1 to 4 about t-axis

Plastic Axial Strains Threshold Card. Define Card 9 only if IFEMA > 1 (see Remarks 9 and 11).

Card 9	1	2	3	4	5	6	7	8
Variable	TS1	TS2	TS3	TS4	CS1	CS2	CS3	CS4
Type	F	F	F	F	F	F	F	F
Default	10^{20}	2×10^{20}	3×10^{20}	4×10^{20}	TS1	TS2	TS3	TS4

VARIABLE	DESCRIPTION
TS1 – TS4	Tensile plastic axial deformation/strain thresholds 1 to 4
CS1 – CS4	Compressive plastic axial deformation/strain thresholds 1 to 4

Plastic Shear Strains Threshold Card. Define Card 10 only if IFEMA > 2 (see Remarks 9 and 11).

Card 10	1	2	3	4	5	6	7	8
Variable	SS1	SS2	SS3	SS4	ST1	ST2	ST3	ST4
Type	F	F	F	F	F	F	F	F
Default	10^{20}	2×10^{20}	3×10^{20}	4×10^{20}	SS1	SS2	SS3	SS4

VARIABLE	DESCRIPTION
SS1 – SS4	Plastic shear strain thresholds 1 to 4 in the *s*-direction
ST1 – ST4	Plastic shear strain thresholds 1 to 4 in the *t*-direction

Remarks:

1. **Plastic hinge locations.** Two plastic hinges can be developed at user-specified locations. The default plastic hinge locations are at the ends of the beam element. See Figure M209-1.

2. **Yield surface**. Axial/moment interaction is defined according to the setting of ISURF on Card 1 (see also Remark 4).

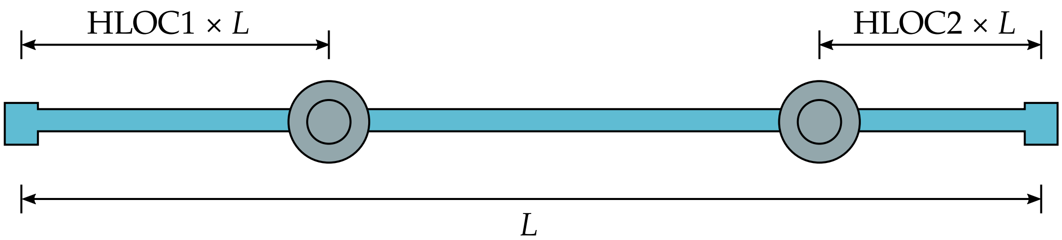


Figure M209-1. Plastic hinge locations for HLOC1 > 0.0 and HLOC2 > 0.0

a) ISURF = 1 (default, simple power law):

$$\psi = \left|\frac{M_s - m_s}{M_{ys}}\right|^{\alpha} + \left|\frac{M_t - m_t}{M_{yt}}\right|^{\beta} + \left|\frac{F-f - F_0}{F_y - F_0}\right|^{\gamma} - 1$$

b) ISURF = 2 (power law based on resultant moment):

$$\psi = \left[\left(\frac{M_s - m_s}{M_{ys}}\right)^2 + \left(\frac{M_t - m_t}{M_{yt}}\right)^2\right]^{\frac{\alpha}{2}} + \left|\frac{F-f - F_0}{F_y - F_0}\right|^{\gamma} - 1$$

c) ISURF = 3 (skew yield surface version of ISURF = 2; see Remark 5):

$$\psi = \left\{\left[\frac{(M_s - m_s) + \delta_s(F-f - F_0)}{(1-\delta_s\kappa_s)M_{ys}}\right]^2 + \left[\frac{(M_t - m_t) + \delta_t(F-f - F_0)}{(1-\delta_t\kappa_t)M_{yt}}\right]^2\right\}^{\frac{\alpha}{2}} + \left|\frac{(F-f - F_0) + \kappa_s(M_s - m_s) + \kappa_t(M_t - m_t)}{(1-\delta_s\kappa_s - \delta_t\kappa_t)(F_y - F_0)}\right|^{\gamma} - 1$$

In the above equations,

Variable	Definition
M_s and M_t	Moments about the s- and t-axes, respectively
F	Axial force
M_{ys} and M_{yt}	Current yield moments
F_y	Current axial yield force, where $$F_y = \begin{cases} F_{yt} & \text{for } (F-f) \ge F_0 \\ -F_{yc} & \text{for } (F-f) < F_0 \end{cases}$$
F_{yt} and F_{yc}	Current tensile and compressive strengths, respectively
m_s, m_t and f	Current moments and forces that determine the center of the yield surface. They are closely related to the Bauschinger effect or kinematic hardening discussed below.

Variable	Definition
α, β, and γ	Input parameters ALPHA, BETA, and GAMMA on Card 3 which are real numbers ≥ 1.1 unless ALPHA and BETA are < 0 (see Remark 4)
δ_s and δ_t	Input parameters DELTAS and DELTAT (length units) on Card 4 for skew of yield surface in the local *s*- and *t*-directions, respectively
κ_s and κ_t	Input parameters KAPPAS and KAPPAT (1/length units) on Card 4 for skew of yield surface in the local *s*- and *t*-directions, respectively
F_0	Input parameter F0 on Card 3 (see Remark 3)

3. **Force offset**. The input parameter F0 offsets the yield surface parallel to the axial force axis. It is the axial force at which the maximum bending moment capacity occurs and is treated as tensile if F0 is positive, or compressive if F0 is negative. The same axial force offset F0 is used for both the local axes (*s* and *t*). For steel components, the value of F0 is usually zero. For reinforced concrete components, F0 should be input as negative, corresponding to the compressive axial force at which the moment capacities are maximum.

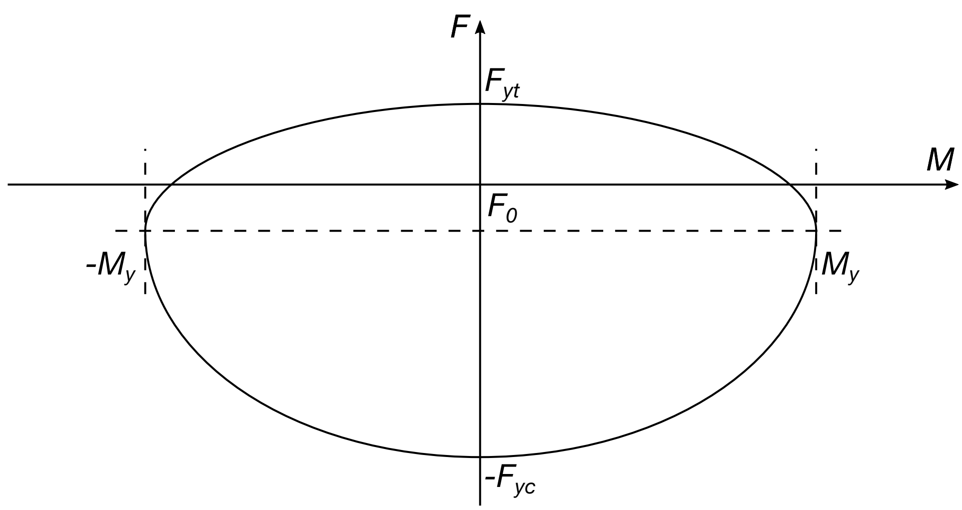


Figure M209-2. Effect of force offset on the yield surface

4. **User-defined yield surface shape.** Optionally, you may provide curves defining the shape of the axial-moment yield surface. When ALPHA and BETA are less than 0.0 and GAMMA is equal to 0.0, the absolute values of ALPHA and BETA are the IDs of the load curves (see *DEFINE_CURVE) that define the yield loci in M_s- F and M_t- F planes. The program will automatically find the set of parameters ALPHA, BETA, GAMMA, SFS, SFT, SFAT, SFAC, F0, DELTAS,

KAPPAS, DELTAT and KAPPAT that best fits the yield loci and the yield surface type ISURF.

5. **Skew yield surface.** Reinforced concrete sections with asymmetric reinforcement have a skew yield surface, meaning that the bending moment capacities at zero axial force are different in positive and negative bending, and the maximum axial capacity occurs at a nonzero bending moment. Furthermore, the axial load at which maximum biaxial bending moment occurs depends on the angle of the bending axis. This can be modelled with ISURF = 3, where DELTAS and DELTAT control the slope of the line connecting peak tensile and compressive strength vertices, and KAPPAS and KAPPAT control the slope of the line connecting peak moment vertices (which lies in the balance plane).

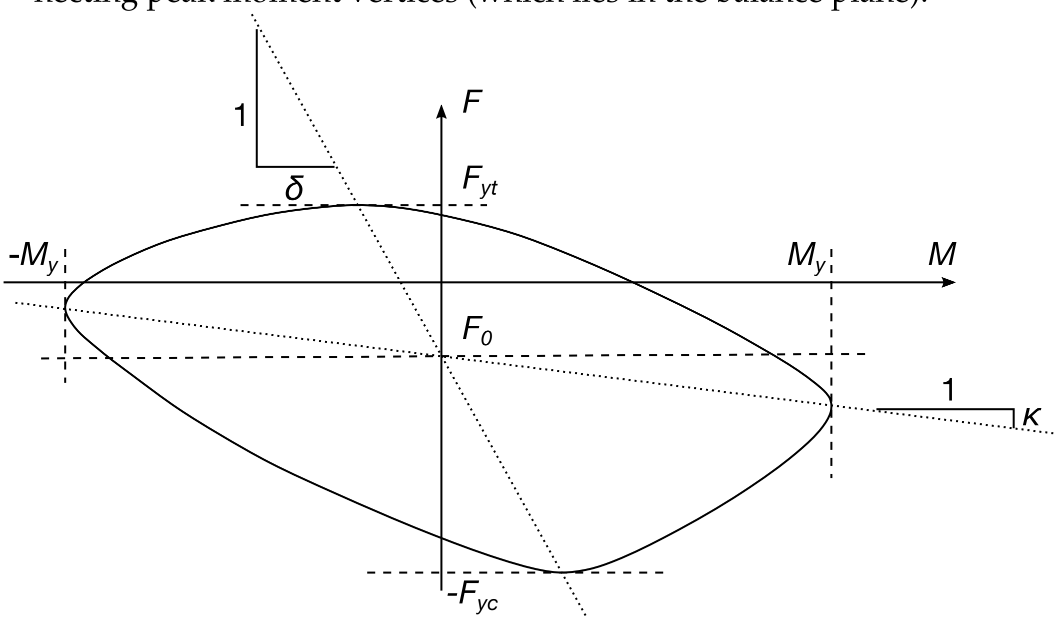


Figure M209-3. Example of a skew yield surface

6. **Hardening behavior during cyclic deformation (hysteresis).** The input parameter IHARD determines how the force as a function of deformation and moment as a function of rotation curves on Card 2 are applied during cyclic deformation. In this case, "deformation" includes both axial deformation and rotation at plastic hinges. If IHARD = 1, the abscissa represents cumulative absolute plastic deformation. This quantity is always positive. It increases whenever there is deformation in either direction. Thus, during hysteresis, the yield moments are taken from points in the input curve with increasingly positive deformation. If the curve shows a degrading behavior (reducing strength with deformation), then, once degraded by plastic deformation, the yield force or moment can never recover to its initial value. This option can be described as "fatigue-type" hysteretic behavior, where all plastic cycles contribute to the degradation. In the axial direction, plastic deformation is accumulated separately for tensile and compressive deformations.

If IHARD = 2, 3 or 4, the abscissa represents the peak absolute value of the plastic deformation. This quantity increases only when the absolute value of plastic deformation exceeds the previously recorded maximum. This option can be described as "high-tide" hardening behavior and follows the FEMA approach. In particular, IHARD of 3 and 4 reproduce the yield-oriented and peak-oriented hysteresis, respectively, as shown below.

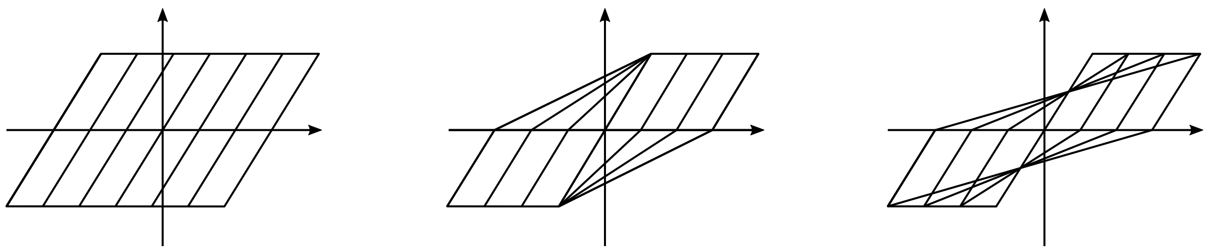


Figure M209-4. Example hardening curves

7. **Pinching.** Pinched-shape hysteresis loops are seen in experiments on reinforced concrete members. They are caused by stiffness changes due to cracks opening and closing. This effect on the flexure response may be simulated using input parameter PINM. The default, PINM = 1.0, gives no pinching. The pinch points are given by moments and rotations illustrated in the schematic below. Input parameter PINS has the same effect on shear hysteresis as PINM does on flexure hysteresis. See Figure M209-5.

8. **Kinematic hardening.** Kinematic hardening (Bauschinger effect, whereby an increase in tensile yield strength occurs at the expense of compressive yield

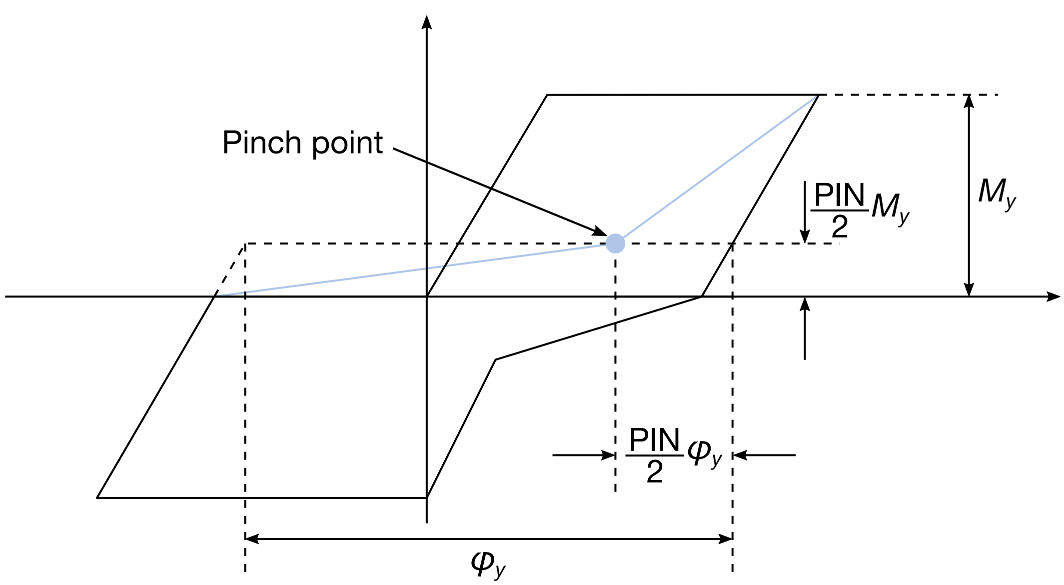


Figure M209-5. Example hysteresis curve with pinch point strength) is modelled by shift of the yield surface and is controlled by input parameters HARDxx and GAMxx, where xx is MS, MT, AT, and AC for moment about s-axis, moment about t-axis, axial tension and axial compression, respectively. HARDxx is the rate at which the yield surface shifts, in units of force/displacement or force/strain for axial response (depending on the setting of IAX) and in units of moment/rotation for flexure response. GAMxx is defined such that HARDxx/GAMxx is the maximum force or moment by which the yield surface can shift.

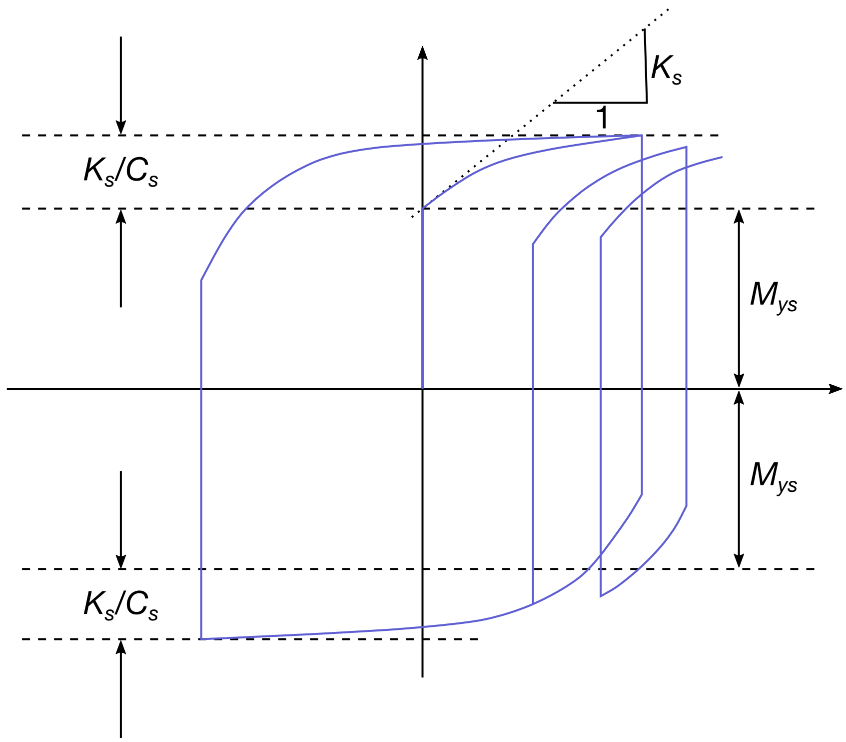


Figure M209-6. Example hysteresis curve with kinematic hardening for the moment about the s-axis. Here K_s is HARDMS and C_s is GAMMS.

9. **Degradation, damage, and element erosion.** Stiffness and strength degradation are modelled using a damage approach. The damaged fraction of the material does not contribute to the forces, the moments or the stiffness. Damage is calculated in two stages:

   a) a single damage parameter based on passing FEMA thresholds, and

   b) component-specific damage (where “component” means axial tension, axial compression, bending about s-axis and bending about t-axis).

   The force or moment for component xx, $F_{xx,\mathrm{actual}}$, is calculated as:

$$F_{xx,\mathrm{actual}} = (1 - D_{xx})(1 - D_{\mathrm{FEMA}})F_{xx,\ \mathrm{nominal}} \ .$$

xx can be AT for axial tension, AC for axial compression, MS for bending about the s-axis, MT for bending about the t-axis. Here, $F_{xx,\mathrm{nominal}}$ is the force or moment calculated in the absence of damage for component xx. D_{xx} is the component-specific damage fraction for component xx, and D_{FEMA} is the damage calculated from passing FEMA thresholds (see Remark 11).

The component-specific damage may be defined to be dependent on cumulative plastic deformation, on peak deformation, or a combination of both:

$$D_{xx}(t) = 1 - \left[1 - \omega_{1xx}\frac{\Delta_{xx,peak}}{\Delta_{xx,ult}}\right]^{\lambda_1}\left[1 - \omega_{2xx}\frac{\Delta_{xx,accum}}{\Delta_{xx,ult}}\right]^{\lambda_2} .$$

Here $\omega_{1xx}$are $\omega_{2xx}$are the input parameters OMGxx1 and OMGxx2. $\Delta_{xx,\mathrm{peak}}$ is the peak deformation (i.e., axial displacement, axial strain or rotation depending on xx and IAX) that has occurred to date; $\Delta_{xx,\mathrm{accum}}$ is the accumulated plastic deformation; and $\Delta_{xx,\mathrm{ult}}$ is the input parameter DUAT, DUAC, RUMS or RUMT. λ_1 and λ_2 are the input parameters LAM1 and LAM2. Setting these to zero disables dependence on peak deformation and cumulative plastic deformation, respectively.

The damage D_{FEMA} is calculated using input parameters SOFT1 and SOFT2, taking the most damaged component including shear as well as axial and moment components. For example, if SOFT1 = 3 and SOFT2 = 4, and the most damaged component has reached a FEMA index of 3.25 (meaning one quarter of the way from threshold 3 to threshold 4), then $D_{\mathrm{FEMA}} = 0.25$. When the most damaged component reaches a FEMA index of 4.0, D_{FEMA} reaches zero and the element will be deleted.

By default, SOFT1 = 3 and SOFT2 = 4. Thus, softening and element removal can occur even if the input parameters SOFT1 and SOFT2 have not been set by the user. This damage mechanism can be switched off by setting SOFT1 = 5. In that case, D_{FEMA} is always zero irrespective of which thresholds are passed, and elements will not be deleted.

10. **Shear behavior.** Nonlinear shear behavior is controlled using input parameters LCSHS, LCSHT, SFSHS and SFSHT. By default, the shear yield surface is independent of the yield surface for axial and flexure and takes the following form:

$$\psi_s = \left(\frac{V_s}{V_{ys}}\right)^2 + \left(\frac{V_t}{V_{yt}}\right)^2 - 1 .$$

Here, V_s and V_t are the current shear forces in the local s- and t-directions. V_{ys} and V_{yt} are the current yield shear forces in the local s- and t-directions

The shear yield forces are functions of plastic shear strain (that is, shear angle). The plastic shear strain can be either peak or cumulative, depending on IHARD.

Optionally, the shear strengths can be user-defined functions of the axial force; this is obtained by setting SFSHS and SFSHT to negative values.

11. **FEMA thresholds.** FEMA thresholds are used in performance-based earthquake engineering to classify the response according to the level of deformation. The thresholds are the divisions between regimes such as “Elastic”, “Immediate Occupancy”, “Life Safe”, etc. Output parameters indicate the status of each element with respect to these regimes. The thresholds are defined by input parameters PRS*n*, PRT*n*, TS*n*, CS*n*, SS*n*, ST*n* where $n = 1$, 2, 3, and 4 for the different regimes. PRS and PRT are plastic rotation about the *s* and *t* axes, TS and CS are tensile and compressive strain or deformation according to the setting of IAX, and SS and ST are shear strains in the *s* and *t* directions. The corresponding output parameters, described as “FEMA flags” (see Remark 12) are “high-tide” values indicating which thresholds have been passed during the analysis, for example 3.25 means that the maximum deformation that has occurred to date exceeds threshold 3 and is one quarter of the way from threshold 3 to threshold 4. See also the influence of SOFT1 and SOFT2 described in Remark 9.

12. **Output.** In addition to the six resultants written for all beam elements, this material model writes a further 50 extra history variables to the d3plot and d3thdt files, given in the table below. The data is written in the same position in these files as where integrated beams write the stresses and strains at integration points requested by BEAMIP on *DATABASE_EXTENT_BINARY. Therefore, some post-processors may interpret this data as if the elements were integrated beams with 10 integration points, and in that case, the data may be accessed by selecting the appropriate integration point and data component:

Integration point	1	2	3	4	5	6	7	8	9	10	
Extra (history) variable	1	6	11	16	21	26	31	36	41	46	XX(RR) axial stress
	2	7	12	17	22	27	32	37	42	47	XY(RS) shear stress
	3	8	13	18	23	28	33	38	43	48	ZX(TR) shear stress
	4	9	14	19	24	29	34	39	44	49	EPS
	5	10	15	20	25	30	35	40	45	50	XX(RR) axial strain

For example, extra history variable 16 is located at the position that would normally be XX(RR) axial stress for integration point 4.

Extra Variable	Description
1	Total axial deformation/strain
2	Hysteretic bending energy at plastic Hinge 1
3	Hysteretic bending energy at plastic Hinge 2
4	Plastic rotation about *s*-axis at Hinge 1

Extra Variable	Description
5	Plastic rotation about *s*-axis at Hinge 2
6	Plastic rotation about *t*-axis at Hinge 1
7	Plastic rotation about *t*-axis at Hinge 2
8	Bending moment about *s*-axis at node 1
9	Bending moment about *s*-axis at node 2
10	Bending moment about *t*-axis at node 1
11	Bending moment about *t*-axis at node 2
12	Hysteretic axial deformation energy
13	Internal energy
14	N/A
15	Axial plastic deformation
16	FEMA rotation flag
17	Current utilization
18	Peak utilization
19	FEMA shear flag
20	FEMA axial flag
21	Peak plastic tensile axial deformation/strain
22	Peak plastic compressive axial deformation/strain
23	Peak plastic rotation about *s*-axis at Hinge 1
24	Peak plastic rotation about *s*-axis at Hinge 2
25	Peak plastic rotation about *t*-axis at Hinge 1
26	Peak plastic rotation about *t*-axis at Hinge 2
27	Cumulative plastic tensile axial deformation/strain
28	Cumulative plastic compressive axial deformation/strain
29	Cumulative plastic rotation about *s*-axis at Hinge 1
30	Cumulative plastic rotation about *s*-axis at Hinge 2
31	Cumulative plastic rotation about *t*-axis at Hinge 1
32	Cumulative plastic rotation about *t*-axis at Hinge 2
33	Axial tensile damage
34	Axial compressive damage
35	Flexural damage about *s*-axis at Hinge 1

Extra Variable	Description
36	Flexural damage about *s*-axis at Hinge 2
37	Flexural damage about *t*-axis at Hinge 1
38	Flexural damage about *t*-axis at Hinge 2
39	Plastic shear strain in *s*-direction
40	Plastic shear strain in *t*-direction
41	Peak plastic shear strain in *s*-direction
42	Peak plastic shear strain in *t*-direction
43	Cumulative plastic shear strain in *s*-direction
44	Cumulative plastic shear strain in *t*-direction
45	Current axial-flexural utilization at Hinge 1
46	Peak axial-flexural utilization at Hinge 1
47	Current axial-flexural utilization at Hinge 2
48	Peak axial-flexural utilization at Hinge 2
49	Current shear utilization
50	Peak shear utilization

*MAT_SPR_JLR

This is Material Type 211. This material model was written for Self-Piercing Rivets (SPR) connecting aluminum sheets. Each SPR should be modeled by a single hexahedral (8-node solid) element, fixed to the sheet either by direct meshing or by tied contact. Pre- and post-processing methods are the same as for solid-element spot welds using *MAT_SPOTWELD. On *SECTION_SOLID, set ELFORM = 1.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	HELAS	TELAS		
Type	A	F	F	F	F	F		
Default	none	none	none	none	0.0	0.0		

Cards 2 and 3 define the input for the “Head” end of the SPR

Card 2	1	2	3	4	5	6	7	8
Variable	LCAXH	LCSHH	LCBMH	SFAXH	SFSHH	SFBMH		
Type	I	I	I	F	F	F		
Default	none	none	none	1.0	1.0	1.0		

Card 3	1	2	3	4	5	6	7	8
Variable	DFAKH	DFSHH	RFBMH	DMFAXH	DMFSHH	DMFBMH		
Type	F	F	F	F	F	F		
Default	Rem 13	Rem 13	Rem 13	0.1	0.1	0.1		

Cards 4 and 5 define the inputs for the "Tail" end of the SPR

Card 4	1	2	3	4	5	6	7	8
Variable	LCAXT	LCSHT	LCBMT	SFAXT	SFSHT	SBFMT		
Type	F	F	F	F	F	F		
Default	none	none	none	1	1	1		

Card 5	1	2	3	4	5	6	7	8
Variable	DFAXT	DFSHT	RFBMT	DFMAXT	DMFSHT	DMFBMT		
Type	F	F	F	F	F	F		
Default	Rem 13	Rem 13	Rem 13	0.1	0.1	0.1		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus, used only for contact stiffness calculation.
PR	Poisson's ratio, used only for contact stiffness calculation.
HELAS	SPR head end behavior flag: EQ.0.0: Nonlinear EQ.1.0: Elastic (use first two points of the load curves).
TELAS	SPR tail end behavior flag: EQ.0.0: Nonlinear EQ.1.0: Elastic (use the first two points of the load curves).
LCAXH	Load curve ID (see *DEFINE_CURVE) giving axial force as a function of deformation (head)

VARIABLE	DESCRIPTION
LCSHH	Load curve ID (see *DEFINE_CURVE) giving shear force as a function of deformation (head)
LCBMH	Load curve ID, see *DEFINE_CURVE, giving moment as a function of rotation (head)
SFAXH	Scale factor on axial force from curve LCAXH
SFSHH	Scale factor on shear force from curve LCSHH
SFBMH	Scale factor on bending moment from curve LCBMH
DFAXH	Optional displacement to start of softening in axial load (head)
DFSHH	Optional displacement to start of softening in shear load (head)
RFBMH	Optional rotation (radians) to start of bending moment softening (head)
DMFAXH	Scale factor on DFAXH
DMFSHH	Scale factor on FFSHH
DMFBMH	Scale factor on RFBMH
LCAXT	Load curve ID (see *DEFINE_CURVE) giving axial force as a function of deformation (tail)
LCSHT	Load curve ID (see *DEFINE_CURVE) giving shear force as a function of deformation (tail)
LCBMT	Load curve ID (see *DEFINE_CURVE) giving moment as a function of rotation (tail)
SFAXT	Scale factor on axial force from curve LCAXT
SFSHT	Scale factor on shear force from curve LCSHT
SFBMT	Scale factor on bending moment from curve LCBMT
DFAXT	Optional displacement to start of softening in axial load (tail)
DFSHT	Optional displacement to start of softening in shear load (tail)
RFBMT	Optional rotation (radians) to start of bending moment softening (tail)

VARIABLE	DESCRIPTION
DMFAXT	Scale factor on DFAXT
DMFSHT	Scale factor on FFSHT
DMFBMT	Scale factor on RFBMT

Remarks:

1. **SPR geometry.** "Head" is the end of the SPR that fully perforates a sheet. "Tail" is the end that is embedded within the thickness of a sheet.

   The sheet planes are defined at the head by the quadrilateral defined by nodes N1-N2-N3-N4 of the solid element; and at the tail by the quadrilateral defined by nodes N5-N6-N7-N8. It is essential that the nodes N1 to N4 are fixed to the head sheet (e.g. by direct meshing or tied contact): the element has no stiffness to resist relative motion of nodes N1 to N4 in the plane of the head sheet. Similarly, nodes N5 to N8 must be fixed to the tail sheet

   The tail of the SPR is defined as a point in the tail sheet plane, initially at the center of the element face. The head of the SPR is initially at the center of the head sheet plane. The SPR axis is defined as the line joining the tail to the head. Thus, the axis of the SPR would typically be coincident with the solid element local z-axis if the solid is a cuboid. It is the user's responsibility to ensure that each solid element is oriented correctly.

   During the analysis, the head and tail will always remain in the plane of the sheet but may move away from the centers of the sheet planes if the shear forces in these planes are sufficient.

2. **Young's modulus and Poisson's ratio.** E and PR are used only to calculate contact stiffness. They are not used by the material model.

3. **Axes.** Deformation is in length units and is on the x-axis. Force is on the y-axis. Rotation is in radians on the x-axis. Moment is on the y-axis.

4. **Load curve assumptions.** All the load curves are expected to start at (0,0). "Deformation" means the total deformation including both elastic and plastic components, similarly for rotation.

5. **"High tide" algorithm.** A "high tide" algorithm is used to determine the deformation or rotation to be used as the x-axis of the load curves when looking up the current yield force or moment. The "high tide" is the greatest displacement or rotation that has occurred so far during the analysis.

6. **Elastic stiffness.** The first two points of the load curve define the elastic stiffness, which is used for unloading.

7. **HELAS and TELAS.** If HELAS > 0, the remainder of the head load curves after the first two points is ignored and no softening or failure occurs. The same applies for TELAS and the tail load curves.

8. **Deformation and rotation of the SPR.** Axial deformation is defined as change of length of the line between the tail and head of the SPR. This line also defines the direction in which the axial force is applied.

   Shear deformation is defined as motion of the tail and head points, in the sheet planes. This deformation is not necessarily perpendicular to axial deformation. Shear forces in these planes are controlled by the load curves LCSHT and LCSHH.

   Rotation at the tail is defined as rotation of the tail-to-head line relative to the normal of the tail sheet plane; and for the head, relative to the normal of the head sheet plane.

9. **Element formulation.** Although ELFORM = 1 is used in the input data, *MAT_SPR_JLR is really a separate unique element formulation. The usual stress/force and hourglass calculations are bypassed, and deformations and nodal forces are calculated by a method unique to *MAT_SPR_JLR; for example, a single *MAT_SPR_JLR element can carry bending loads.

10. **Hourglass.** *HOURGLASS inputs are irrelevant to *MAT_SPR_JLR.

11. **SWFORC file.** Output to the swforc file works in the same way as for spotwelds. Although inside the material model the load curves LCSHT and LCSHH control "shear" forces in the sheet planes, in the swforc file the quoted shear force is the force normal to the axis of the SPR.

12. **Softening.** Before an element fails, it enters a "softening" regime in which the forces, moments and stiffnesses are ramped down as displacement increases (this avoids sudden shocks when the element is deleted). For example, for axial loading at the head, softening begins when the maximum axial displacement exceeds DFAXH. As the displacement increases beyond that point, the load curve will be ignored for that deformation component. The forces, moments and stiffnesses are ramped down linearly with increasing displacement and reach zero at displacement = DFAXH × (1 + DMFAXH) when the element is deleted. The softening factor scales all the force and moment components at both head and tail. Thus, all the force and moment components are reduced when any one displacement component enters the softening regime. For example, if DFAXT = 3.0mm, and DMFAXT = 0.1, then softening begins when axial displacement of the tail reaches 3.0 mm and final failure occurs at 3.3 mm.

13. **Initial softening displacements/rotations.** If the inputs DFAXH, DFSHH, RFBMH, DFAXT, DFSHT, and RFBMT are non-zero, these values must be within the abscissa values of the relevant curve, such that the curve force/moment value is greater than zero at the defined start of softening.

    If the inputs are left blank or zero, they will be calculated internally as follows:

    a) Final failure will occur at the displacement or rotation (DFAIL) at which the load curve reaches zero (determined, if necessary, by extrapolation from the last two points).

    b) Displacement or rotation at which softening begins is then back-calculated. For example, $\mathrm{DFAXT} = \mathrm{DFAIL}/(1 + \mathrm{DMGAXT})$.

    c) If DMGAXT is left blank or zero, it defaults to 0.1.

    d) If the load curve does not drop to zero, and the final two points have a zero or positive gradient, no failure or softening will be caused by that displacement component.

14. **Output stress.** Output stresses (in the d3plot and time-history output files) are set to zero.

15. **Displacement ratio.** The output variable "displacement ratio" (or rotation ratio for bending), *R*, is defined as follows. See also the Figure M211-1.

    a) *R = 0.0 to 1.0*. The maximum force or moment on the input curve has not yet been reached. *R* is proportional to the maximum force or moment reached so far, with 1.0 being the point of maximum force or moment on the input curve.

    b) *R = 1.0 to 2.0*. The element has passed the point of maximum force but has not yet entered the softening regime. *R* rises linearly with displacement (or rotation) from 1.0 when maximum force occurs to 2.0 when softening begins.

    c) *R = 2.0 to 3.0*. Softening is occurring. *R* rises linearly with displacement from 2.0 at the onset of softening to 3.0 when the element is deleted.

    The displacement (or rotation) ratio is calculated separately for axial, shear, and bending at the tail and head (see Remark 16 below). The output listed by post-processors as "plastic strain" is actually the maximum displacement or rotation ratio of any displacement or rotation component at head or tail. This same variable is also output as "Failure" in the spotweld data in the swforc file (or the swforc section of the binout file).

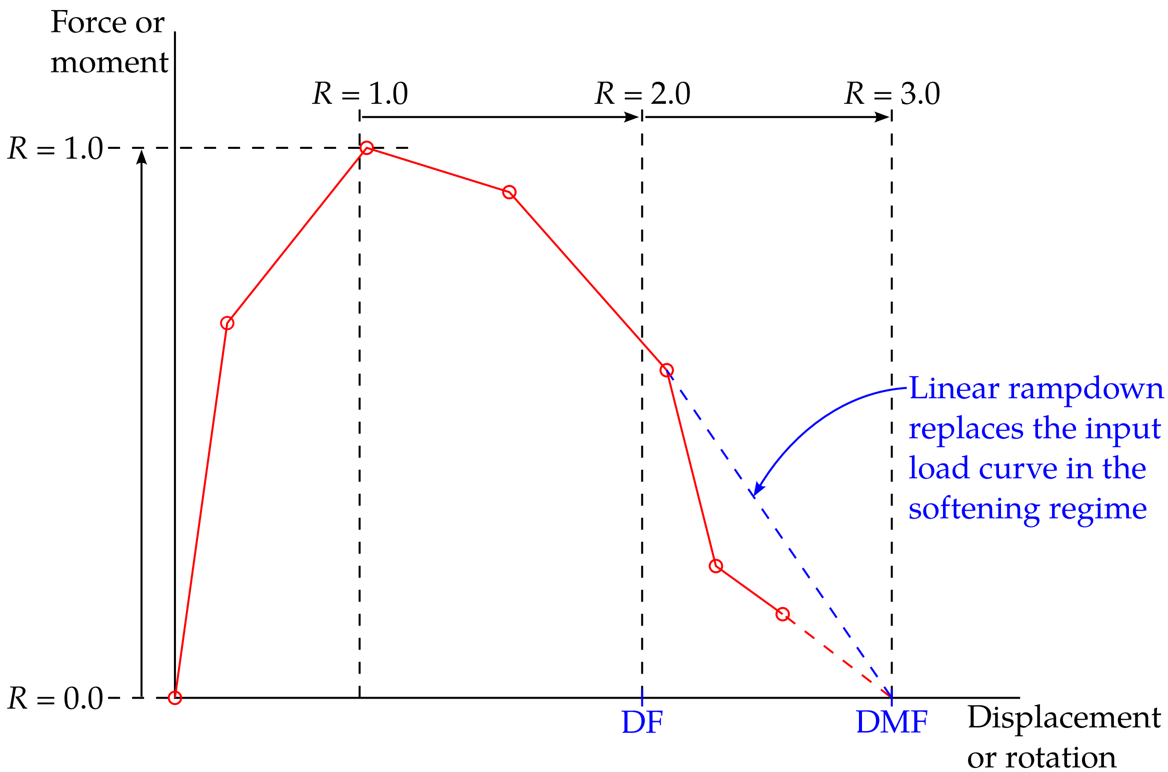


Figure M211-1. Output variable "displacement ratio" (or rotation ratio for bending)

16. **Additional history variables.** The additional history variables are listed in the table below.

History Variable #	Description
1	Failure time (used for swforc file)
2	Softening factor used internally to prevent abrupt failure.
3	Displacement ratio – axial, head
4	Displacement ratio – axial, tail
5	Displacement ratio – shear, head
6	Displacement ratio – shear, tail
7	Rotation ratio – bending, head
8	Rotation ratio – bending, tail
9	Used for swforc output
10	Shear force in "beam" x-axis
11	Shear force in "beam" y-axis
12	Axial force in "beam" z-axis (along "beam")

History Variable #	Description
13	Moment about "beam" x-axis at head
14	Moment about "beam" y-axis at head
15	Moment about "beam" z-axis at head (torsion – should be zero)
16	"Beam" length
17	Moment about "beam" x-axis at tail
18	Moment about "beam" y-axis at tail
19	Moment about "beam" z-axis at tail (torsion – should be zero)
20	Isoparametric coordinate of head of "beam" (s)
21	Isoparamteric coordinate of head of "beam" (t)
22	Isoparametric coordinate of tail of "beam" (s)
23	Isoparametric coordinate of tail of "beam" (t)
24	Timestep
25	Plastic displacement – axial, head
26	Plastic displacement – axial, tail
27	Plastic rotation – head
28	Plastic rotation – tail
29	Plastic displacement – shear in sheet axes, head
30	Plastic displacement – shear in sheet axes, tail
31	Global x-component of the "beam" x-axis
32	Global y-component of the "beam" x-axis
33	Global z-component of the "beam" x-axis
34	Shear displacement – local x-axis, head
35	Shear displacement – local y-axis, head
36	Shear displacement – local x-axis, tail
37	Shear displacement – local y-axis, tail
38	Total displacement – axial
39	Current rotation (radians) – head, local x-axis
40	Current rotation (radians) – head, local y-axis
41	Current rotation (radians) – tail, local x-axis

History Variable #	Description
42	Current rotation (radians) – tail, local y-axis

*MAT_COMPOSITE_TABULATED_PLASTICITY_DAMAGE

(V1.3.7) This is Material Type 213, an orthotropic, visco-elastic-plastic material with temperature and rate dependencies. It has a modular architecture supporting viscoelastic deformations, viscoplastic deformations [1-4, 9, 12], damage [6, 7], failure [8] and probabilistic analysis [5]. It is available for solid, thick, and thin shell elements. Thick shell elements with ELFORM = 1, 2, or 6 follow the thin shell input format, while those with ELFORM = 3, 5, or 7 follow the solid element input format. For thick shells, we recommend using ELFORM = 1 or 5.

Card Summary:

Card 1. This card is required.

MID	RO	EA	EB	EC	PRBA	PRCA	PRCB

Card 2. This card is required.

GAB	GBC	GCA	PTOL	AOPT	MACF	FILT	VEVP

Card 3. This card is required.

XP	YP	ZP	A1	A2	A3		

Card 4. This card is required.

V1	V2	V3	D1	D2	D3	BETA	TCSYM

Card 5. This card is required.

H11	H22	H33	H12	H23	H13	H44	H55

Card 6. This card is required.

H66	LT1	LT2	LT3	LT4	LT5	LT6	LT7

Card 7. This card is required.

LT8	LT9	LT10	LT11	LT12	YSC	DFLAG	DC

Card 8a.1. Include this card if FTYPE = 0.

FTYPE							

Card 8a.2. Include this card as a blank line if FTYPE = 0.

Card 8b.1. Include this card if FTYPE = 1 (Puck Failure Criterion).

FTYPE	FV0	FV1	FV2	FV3	FV4	FV5	FV6

Card 8b.2. Include this card if FTYPE = 1 (Puck Failure Criterion).

FV7	FV8	FV9	FV10	FV11	FV12	FV13	FV14

Card 8c.1. Include this card if FTYPE = 2 (Tsai-Wu Failure Criterion).

FTYPE		FV1	FV2	FV3	FV4	FV5	FV6

Card 8c.2. Include this card if FTYPE = 2. (Tsai-Wu Failure Criterion).

FV7	FV8	FV9	FV10	FV11	FV12	FV13	FV14

Card 8d.1. Include this card if FTYPE = 3 (Generalized Tabulated Failure Criterion).

FTYPE		FV1	FV2	FV3			

Card 8d.2. Include this card as a blank line if FTYPE = 3 (Generalized Tabulated Failure Criterion).

Card 9. BETA values only need to be specified when VEVP = 1 or 2.

BETA11	BETA22	BETA33	BETA44	BETA55	BETA66	BETA12	BETA23

Card 10. BETA values only need to be specified when VEVP = 1 or 2.

BETA13	CP	TQC	TEMP	PMACC			

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EA	EB	EC	PRBA	PRCA	PRCB
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
EA	E_a, Young's modulus in the a-direction
EB	E_b, Young's modulus in the b-direction
EC	E_c, Young's modulus in the c-direction
PRBA	ν_{ba}, (elastic) Poisson's ratio, *ba* (see Remark 9)
PRCA	ν_{ca}, (elastic) Poisson's ratio, *ca* (see Remark 9)
PRCB	ν_{cb}, (elastic) Poisson's ratio, *cb* (see Remark 9)

Card 2	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	PTOL	AOPT	MACF	FILT	VEVP
Type	F	F	F	F	F	I	F	I
Default	none	none	none	10^{-6}	0.0	0	0.0	0

VARIABLE	DESCRIPTION
GAB	G_{ab}, shear modulus *ab*-plane
GBC	G_{bc}, shear modulus *bc*-plane
GCA	G_{ca}, shear modulus *ca*-plane
PTOL	Yield function tolerance used during plastic multiplier calculations
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description): EQ.0.0: Locally orthotropic with material axes determined by element nodes. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA. EQ.1.0: Locally orthotropic with material axes determined by a

VARIABLE	DESCRIPTION
	point, P, in space and the global location of the element center. This option is for solid elements only.
	EQ.2.0: Globally orthotropic with material axes determined by vectors defined below
	EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.
	EQ.4.0: Locally orthotropic in cylindrical coordinate system with the material axes determined by a vector, **v**, and an originating point, P, defining the centerline axis. This option is for solid elements only.
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE-_COORDINATE_VECTOR).
MACF	Material axes change flag for solid elements:
	EQ.-4: Switch material axes b and c before BETA rotation
	EQ.-3: Switch material axes a and c before BETA rotation
	EQ.-2: Switch material axes a and b before BETA rotation
	EQ.1: No change, default
	EQ.2: Switch material axes a and b after BETA rotation
	EQ.3: Switch material axes a and c after BETA rotation
	EQ.4: Switch material axes b and c after BETA rotation
	Figure M2-2 indicates when LS-DYNA applies MACF during the

VARIABLE	DESCRIPTION
	process to obtain the final material axes. If BETA on *ELEMENT_-SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 4 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.
FILT	Factor for strain rate filtering (optional): $$\dot{\varepsilon}_{i+1}^{\mathrm{avg}} = (1 - \mathrm{FILT}) \times \dot{\varepsilon}_{i+1}^{\mathrm{cur}} + \mathrm{FILT} \times \dot{\varepsilon}_{i}^{\mathrm{avg}}$$ where i is the previous time step. The value of FILT is between 0 and 1
VEVP	Flag to control viscoelastic, viscoplastic behavior: EQ.0: Viscoplastic only with no rate effects in elastic region (default) EQ.1: Viscoelastic, viscoplastic (see Cards 9 and 10) EQ.2: Viscoelastic only (see Cards 9 and 10)

Card 3	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		
Default	0.0	0.0	0.0	0.0	0.0	0.0		

VARIABLE	DESCRIPTION
XP, YP, ZP	Coordinates of point P for AOPT = 1 and 4
A1, A2, A3	Components of vector **a** for AOPT = 2

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	TCSYM
Type	F	F	F	F	F	F	F	I
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3 and 4
D1, D2, D3	Components of vector **d** for AOPT = 2
BETA	Angle in degrees of a material rotation about the c-axis, available for AOPT = 0 (shells only) and AOPT = 3 (all element types). This angle may be overridden on the element card; see *ELEMENT_-SHELL_BETA and *ELEMENT_SOLID_ORTHO.
TCSYM	Flag for handling tension-compression asymmetry in all three material directions: EQ.0: Do not adjust user-defined data. EQ.1: Compute and use average of tension and compression elastic moduli in adjusting the stress-strain curve. See Remark 7. EQ.2: Use compression modulus as user-defined tension modulus in adjusting the stress-strain curve. See Remark 7. EQ.3: Use tension modulus as user-defined compression modulus in adjusting the stress-strain curve. See Remark 7. EQ.4: Use user-defined tensile curve as the compressive curve overriding the user-defined compressive curve. This implies that the normal stress-strain curves are symmetric including yield values. EQ.5: Use user-defined compressive curve as the tensile curve overriding the user-defined tensile curve. This implies that the normal stress-strain curves are symmetric including yield values.

Card 5	1	2	3	4	5	6	7	8
Variable	H11	H22	H33	H12	H23	H13	H44	H55
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	3.0	3.0

VARIABLE	DESCRIPTION
H*ij*	Plastic flow rule coefficients. See Remark 1.

Card 6	1	2	3	4	5	6	7	8
Variable	H66	LT1	LT2	LT3	LT4	LT5	LT6	LT7
Type	F	I	I	I	I	I	I	I
Default	3.0	none	none	none	none	none	none	none

VARIABLE	DESCRIPTION
H*ij*	Plastic flow rule coefficients. See Remark 1.
LT1	Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *a*-direction tension test. See Remarks 2, 8 and 9.
LT2	Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *b*-direction tension test. See Remarks 2, 8 and 9.
LT3	Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *c*-direction tension test. See Remarks 2, 8 and 9. Not required if used with thin shell elements.
LT4	Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *a*-direction compression test. See Remarks 2, 8 and 9.

VARIABLE	DESCRIPTION
LT5	Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *b*-direction compression test. See Remarks 2, 8 and 9.
LT6	Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *c*-direction compression test. See Remarks 2, 8 and 9. Not required if used with thin shell elements.
LT7	Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *ab*-plane shear test. See Remarks 2, 8 and 9.

Card 7	1	2	3	4	5	6	7	8
Variable	LT8	LT9	LT10	LT11	LT12	YSC	DFLAG	DC
Type	I	I	I	I	I	I	F	I
Default	none	none	none	none	none	none	0.0	none

VARIABLE	DESCRIPTION
LT8	Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *bc*-plane shear test. See Remarks 2, 8 and 9. Not required if used with thin shell elements.
LT9	Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *ac*-plane shear test. See Remarks 2, 8 and 9. Not required if used with thin shell elements.
LT10	Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the 45° off axis *ab*-plane tension or compression test. See Remarks 2, 8 and 9. Optional for both solid and shell elements.
LT11	Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the 45° off axis *bc*-plane tension or compression test. See Remarks 2, 8 and

VARIABLE	DESCRIPTION
	9. Not required if used with thin shell elements. Optional for solid elements.
LT12	Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the 45° off axis *ac*-plane tension or compression test. See Remarks 2, 8 and 9. Not required if used with thin shell elements. Optional for solid elements.
YSC	Load curve ID containing the stress-strain curve IDs and associated initial yield strain values. See Remark 3.
DFLAG	Damage formulation flag (see Remark 12): EQ.0: Based on effective stress (default) EQ.1: Based on corrected plastic strain
DC	Curve ID that specifies which components of the damage model are active. It contains the damage parameter ID and the corresponding damage as a function of total strain curve ID or Table3D ID. Set this value to zero if damage should not be included in the analysis. See Remark 4.

No Failure Card. The following two cards are included if FTYPE = 0. Card 8a.2 must be included as a blank line

Card 8a.1	1	2	3	4	5	6	7	8
Variable	FTYPE							
Type	I							

Card 8a.2	1	2	3	4	5	6	7	8
Variable								
Type								

VARIABLE	DESCRIPTION
FTYPE	Failure criterion type (see Remarks 5 and 6): EQ.0: No failure considered (default) EQ.1: Puck Failure Criterion (PFC) (solid elements only) EQ.2: Tsai-Wu Failure Criterion (TWFC) EQ.3: Generalized Tabulated Failure Criterion (GTFC)

PFC Card. The following two cards are included if FTYPE = 1.

Card 8b.1	1	2	3	4	5	6	7	8
Variable	FTYPE	FV0	FV1	FV2	FV3	FV4	FV5	FV6
Type	I	F	F	F	F	F	F	F

Card 8b.2	1	2	3	4	5	6	7	8
Variable	FV7	FV8	FV9	FV10	FV11	FV12	FV13	FV14
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
FTYPE	Failure criterion type (see Remarks 5 and 6): EQ.0: No failure considered (default) EQ.1: Puck Failure Criterion (PFC) (solid elements only) EQ.2: Tsai-Wu Failure Criterion (TWFC) EQ.3: General-ized Tabulated Failure Criterion (GTFC)
FV0	Γ_f, fiber mode fracture energy (a-direction)
FV1	Post-peak residual damage in the a-direction for tension. Value must be a real number between 0 and 1.
FV2	Post-peak residual damage in the a-direction for compression. Value must be a real number between 0 and 1.

VARIABLE	DESCRIPTION
FV3	Post-peak residual damage in the b and c-directions for tension. Value must be a real number between 0 and 1.
FV4	Post-peak residual damage in the b and c-directions for compression. Value must be a real number between 0 and 1.
FV5	Post-peak residual damage in shear. Value must be a real number between 0 and 1.
FV6	magnification factor, m_f
FV7	Slope parameter, p_{ba}^t
FV8	Slope parameter, p_{ba}^c
FV9	Slope parameter, p_{bb}^t
FV10	Slope parameter, p_{bb}^c
FV11	Fiber Poisson's ratio, ν_{ab}^f
FV12	Fiber Young's modulus, E_a^f
FV13	Inter-fiber mode I fracture energy, Γ_1
FV14	Inter-fiber mode II fracture energy, Γ_2

TWFC Card. The following two cards are included if FTYPE = 2.

Card 8c.1	1	2	3	4	5	6	7	8
Variable	FTYPE		FV1	FV2	FV3	FV4	FV5	FV6
Type	I		F	F	F	F	F	F

Card 8c.2	1	2	3	4	5	6	7	8
Variable	FV7	FV8	FV9	FV10	FV11	FV12	FV13	FV14
Type	F	F	F	F	F	F	I	I

VARIABLE	DESCRIPTION
FTYPE	Failure criterion type (see Remarks 5 and 6): EQ.0: No failure considered (default) EQ.1: Puck Failure Criterion (PFC) (solid elements only) EQ.2: Tsai-Wu Failure Criterion (TWFC) EQ.3: Generalized Tabulated Failure Criterion (GTFC)
FV1	$\hat{\sigma}_{aa}^{T}$, tensile failure stress in the a-direction
FV2	$\hat{\sigma}_{aa}^{C}$, compressive failure stress in the a-direction (input as a positive value)
FV3	$\hat{\sigma}_{bb}^{T}$, tensile failure stress in the b-direction
FV4	$\hat{\sigma}_{bb}^{C}$, compressive failure stress in the b-direction (input as a positive value)
FV5	$\hat{\sigma}_{cc}^{T}$, tensile failure stress in the c-direction
FV6	$\hat{\sigma}_{cc}^{C}$, compressive failure stress in the c-direction (input as a positive value)
FV7	$\hat{\sigma}_{ab}$, shear failure stress in the ab-plane
FV8	$\hat{\sigma}_{bc}$, shear failure stress in the bc-plane
FV9	$\hat{\sigma}_{ac}$, shear failure stress in the ac-plane
FV10	$\hat{\sigma}_{ab}^{45}$, failure stress in the 45° off axis ab-plane
FV11	$\hat{\sigma}_{bc}^{45}$, failure stress in the 45° off axis bc-plane
FV12	$\hat{\sigma}_{ac}^{45}$, failure stress in the 45° off axis ac-plane
FV13	Optional curve ID that defines orientation-dependent erosion strain for all nine stress strain curves (3 tension, 3 compression, and 3 shear). The ordinate values in the load curve define the various erosion strains in the following order:

Ordinate Value Descriptions
Tensile erosion strain in the a-direction, ε_{aaT}
Compressive erosion strain in the a-direction, ε_{aaC}

VARIABLE	DESCRIPTION

Tensile erosion strain in the *b*-direction, ε_{bbT}
Compressive erosion strain in the *b*-direction, ε_{bbC}
Tensile erosion strain in the *c*-direction, ε_{ccT}
Compressive erosion strain in the *c*-direction, ε_{ccC}
In-plane shear erosion strain in the *ab*-plane, ε_{ab}
Out-of-plane shear erosion strain in the *bc*-plane, ε_{bc}
Out-of-plane shear erosion strain in the *ac*-plane, ε_{bc}

FV14 — Optional curve ID that defines orientation-dependent post-peak residual strength (PPRD) for all nine stress strain curves (3 tension, 3 compression, and 3 shear). The ordinate values in the load curve define the various residual strength in the following order:

Ordinate Value Descriptions
Tensile post-peak residual strength in the *a*-direction, PPRD_{aaT}
Compressive post-peak residual strength in the *a*-direction, PPRD_{aaC}
Tensile post-peak residual strength in the *b*-direction, PPRD_{bbT}
Compressive post-peak residual strength in the *b*-direction, PPRD_{bbC}
Tensile post-peak residual strength in the *c*-direction, PPRD_{ccT}
Compressive post-peak residual strength in the *c*-direction, PPRD_{ddC}
In-plane shear post-peak residual strength in the *ab*-plane, PPRD_{ab}
Out-of-plane shear post-peak residual strength in the *bc*-plane, PPRD_{bc}
Out-of-plane shear post-peak residual strength in the *ac*-plane, PPRD_{ac}

GTFC Card. The following two cards are included if FTYPE = 3. Card 8d.2 must be included as a blank line.

Card 8d.1	1	2	3	4	5	6	7	8
Variable	FTYPE		FV1	FV2	FV3			
Type	I		F	F	F			

Card 8d.2	1	2	3	4	5	6	7	8
Variable								
Type								

VARIABLE	DESCRIPTION
FTYPE	Failure criterion type (see Remarks 5 and 6): EQ.0: No failure considered (default) EQ.1: Puck Failure Criterion (PFC) (solid elements only) EQ.2: Tsai-Wu Failure Criterion (TWFC) EQ.3: General-ized Tabulated Failure Criterion (GTFC)

FV1

In-plane and out-of-plane interaction term, n, used to compute d:

$$d = \begin{cases} \max(d_1, d_2) & \text{if } n = 0.0 \\ (d_1^n + d_2^n)^{1/n} & \text{otherwise} \end{cases}$$

where, $d_i = \varepsilon^{\mathrm{eq}} / \varepsilon_{\mathrm{fail}}$, $i = 1, 2$. $i = 1$ corresponds to the in-plane mode, while $i = 2$ corresponds to the out-of-plane mode. Here,

$$\varepsilon^{\mathrm{eq}} = \begin{cases} \sqrt{\varepsilon_{11}^2 + \varepsilon_{22}^2 + 2\varepsilon_{12}^2} & \text{for in-plane} \\ \sqrt{\varepsilon_{33}^2 + 2\varepsilon_{13}^2 + 2\varepsilon_{23}^2} & \text{for out-of-plane} \end{cases}$$

An element is eroded if d reaches a value of 1.0 for solid elements. For thin shell elements, an element is eroded if d_1 reaches a value of 1.0 since only the in-plane mode of failure is considered. n is not required for thin shell elements.

FV2

Table ID for the table containing in-plane $(\theta_{\mathrm{IP}}, \varepsilon_{\mathrm{fail}}^{\mathrm{eq}})$ values with respect to the specified a-direction stress. Here,

VARIABLE	DESCRIPTION
	$$\theta_{\mathrm{IP}} = \cos^{-1}\left(\frac{\sigma_{22}}{\sqrt{\sigma_{22}^2 + \sigma_{12}^2}}\right)$$
FV3	Table ID for the table containing out-of-plane $(\theta_{\mathrm{OOP}}, \varepsilon_{\mathrm{fail}}^{\mathrm{eq}})$ values with respect to the specified normal *c*-direction stress. Here, $$\theta_{OOP} = \cos^{-1}\left(\frac{\sigma_{13}}{\sqrt{\sigma_{13}^2 + \sigma_{23}^2}}\right)$$

Viscoelasticity Card. BETA values only need to be specified when VEVP = 1 or 2.

Card 9	1	2	3	4	5	6	7	8
Variable	BETA11	BETA22	BETA33	BETA44	BETA55	BETA66	BETA12	BETA23
Type	F	F	F	F	F	F	F	F
Default	0.001	0.001	0.001	0.001	0.001	0.001	↓	↓

VARIABLE	DESCRIPTION
BETA11	Decay constant for the relaxation matrix of the viscoelastic law in 1-direction (default = 0.001). It must be greater than or equal to zero.
BETA22	Decay constant for the relaxation matrix of the viscoelastic law in 2-direction (default = 0.001). It must be greater than or equal to zero.
BETA33	Decay constant for the relaxation matrix of the viscoelastic law in 3-direction (default = 0.001). This field is *not* required for thin shell elements. It must be greater than or equal to zero.
BETA44	Decay constant for the relaxation matrix of the viscoelastic law in 12-shear (default = 0.001). It must be greater than or equal to zero.
BETA55	Decay constant for the relaxation matrix of the viscoelastic law in 23-shear (default = 0.001). This field is *not* required for thin shell elements. It must be greater than or equal to zero.
BETA66	Decay constant for the relaxation matrix of the viscoelastic law in 13-shear (default = 0.001). This field is *not* required for thin shell elements. It must be greater than or equal to zero.

VARIABLE	DESCRIPTION
BETA12	Decay constant for the relaxation matrix of the viscoelastic law 12-coupling (default = (BETA11 + BETA22)/2). It must be greater than or equal to zero.
BETA23	Decay constant for the relaxation matrix of the viscoelastic law 23-coupling (default = (BETA22 + BETA33)/2). This field is *not* required for thin shell elements. It must be greater than or equal to zero.

Viscoelasticity Card. BETA values only need to be specified when VEVP = 1 or 2.

Card 10	1	2	3	4	5	6	7	8
Variable	BETA13	CP	TQC	TEMP	PMACC			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION
BETA13	Decay constant for the relaxation matrix of the viscoelastic law 13-coupling (default = (BETA11 + BETA33)/2). This field is *not* required for thin shell elements. It must be greater than or equal to zero.
CP	Specific heat capacity (per unit mass)
TQC	Taylor-Quinney Coefficient
TEMP	This is the reference (or initial) temperature used to obtain the corresponding stress-strain curves.
PMACC	Plastic multiplier computational accuracy (see Remark 10): EQ.0: Use up to a maximum of 1000 increments (default) EQ.N: Specify a positive value, N, greater than 1 as the maximum number of increments. An error message is issued if a converged solution cannot be found.

Remarks:

1. **Flow rule coefficients.** Flow rule coefficients (FRCs) are determined using the plastic Poisson's ratios. H33, H55 and H66 are not required for shell elements.

Details on how to compute FRCs can be found in [10]. Setting all the FRCs to zero invokes a simplified *MAT_213 material model. See Remark 13.

2. **Temperature-strain rate test result tables.** A minimum of two sets of (strain rate-temperature) curves are needed. If the material is not temperature and rate sensitive, make the two sets of table data identical (note that MAT_213 does not support DEFINE_TABLE_2D, meaning *DEFINE_TABLE must be used instead). If the material is rate and temperature sensitive, the curve corresponding to the smallest total strain rate for the given reference temperature (TEMP in Card 10) is assumed to be the quasi-static, room temperature (QS-RT) curve and influences the viscoelastic-plastic computations.

   An example TABLE_3D (LT*i*) structure for 3 total strain rates and 3 temperatures for tension in the *a*-direction test is shown below. The total strain rates are converted within LS-DYNA into effective plastic strain rate (EPSR) for each of the input stress-strain curves. The EPSR value assigned for each stress-strain curve is used for yield stress interpolation.

<table>
<tr><td rowspan="10">Tension
a-direction</td><td colspan="2">DEFINE_TABLE_3D
(Temperature)</td><td colspan="2">DEFINE_TABLE
(Total Strain Rate)</td></tr>
<tr><td rowspan="9">Table 1</td><td rowspan="3">Table 2: 10°C</td><td rowspan="3">Table 2</td><td>Curve 1 (10^{-3}/s)</td></tr>
<tr><td>Curve 2 (1/s)</td></tr>
<tr><td>Curve 3 (10/s)</td></tr>
<tr><td rowspan="3">Table 3: 20°C</td><td rowspan="3">Table 3</td><td>Curve 4 (10^{-3}/s)</td></tr>
<tr><td>Curve 5 (1/s)</td></tr>
<tr><td>Curve 6 (10/s)</td></tr>
<tr><td rowspan="3">Table 4: 50°C</td><td rowspan="3">Table 4</td><td>Curve 7 (10^{-3}/s)</td></tr>
<tr><td>Curve 8 (1/s)</td></tr>
<tr><td>Curve 9 (10/s)</td></tr>
</table>

   Restrictions/assumptions about the input data are as follows:

   a) For normal (tension and compression) and shear curve data: Use positive stress and positive strain values in the curve data.

b) For off-axis curve data: Use positive stress and positive strain values in the curve data if the off-axis test is a tension test. Use negative stress and positive strain values in the curve data if the off-axis test is a compressive test. The same combination of tension-compression tests is assumed for all *MAT_213 cards used in a specific model. For instance, if the LT10-LT11-LT12 combination is tension-compression-compression for one set *MAT_-213 data, then it is assumed that all other *MAT_213 data in the model use tension-compression-compression data.

c) All shear strain values are tensorial, not engineering (total strain rate input must be tensorial for shear component).

d) For an elastic component, meaning *a*-direction in a unidirectional composite, set the initial yield strain value (in YSC) greater than the failure strain (last strain value in the curve).

3. **YSC.** Curve of initial yield strain values (YSC) must list curves in ascending order as abscissa values with the corresponding yield strains given as the ordinate values. An example YSC data is shown below for a case with two sets of (temperature, strain rate) data.

Load Curve	Yield Strain	Curves
LC1	$\varepsilon_{y^0}^{LC1}$	Curve 1 (10°C, 10^{-3}/s)
LC2	$\varepsilon_{y^0}^{LC2}$	Curve 2 (10°C, 10^{-3}/s)
LC3	$\varepsilon_{y^0}^{LC3}$	Curve 3 (10°C, 10^{-3}/s)
LC4	$\varepsilon_{y^0}^{LC4}$	Curve 4 (10°C, 10^{-3}/s)
LC5	$\varepsilon_{y^0}^{LC5}$	Curve 5 (10°C, 10^{-3}/s)
LC6	$\varepsilon_{y^0}^{LC6}$	Curve 6 (10°C, 10^{-3}/s)
LC7	$\varepsilon_{y^0}^{LC7}$	Curve 7 (10°C, 10^{-3}/s)
LC8	$\varepsilon_{y^0}^{LC8}$	Curve 8 (10°C, 10^{-3}/s)
LC9	$\varepsilon_{y^0}^{LC9}$	Curve 9 (10°C, 10^{-3}/s)
LC10	$\varepsilon_{y^0}^{LC10}$	Curve 10 (10°C, 10^{-3}/s)
LC11	$\varepsilon_{y^0}^{LC11}$	Curve 11 (10°C, 10^{-3}/s)
LC12	$\varepsilon_{y^0}^{LC12}$	Curve 12 (10°C, 10^{-3}/s)

Load Curve	Yield Strain	Curves
LC13	$\varepsilon_{y^0}^{LC13}$	Curve 1 (20°C, 10^{-3}/s)
LC14	$\varepsilon_{y^0}^{LC14}$	Curve 2 (20°C, 10^{-3}/s)
LC15	$\varepsilon_{y^0}^{LC15}$	Curve 3 (20°C, 10^{-3}/s)
LC16	$\varepsilon_{y^0}^{LC16}$	Curve 4 (20°C, 10^{-3}/s)
LC17	$\varepsilon_{y^0}^{LC17}$	Curve 5 (20°C, 10^{-3}/s)
LC18	$\varepsilon_{y^0}^{LC18}$	Curve 6 (20°C, 10^{-3}/s)
LC19	$\varepsilon_{y^0}^{LC19}$	Curve 7 (20°C, 10^{-3}/s)
LC20	$\varepsilon_{y^0}^{LC20}$	Curve 8 (20°C, 10^{-3}/s)
LC21	$\varepsilon_{y^0}^{LC21}$	Curve 9 (20°C, 10^{-3}/s)
LC22	$\varepsilon_{y^0}^{LC22}$	Curve 10 (20°C, 10^{-3}/s)
LC23	$\varepsilon_{y^0}^{LC23}$	Curve 11 (20°C, 10^{-3}/s)
LC24	$\varepsilon_{y^0}^{LC24}$	Curve 12 (20°C, 10^{-3}/s)

4. **Damage curve.** Include in this curve only the active damage parameter ID and the corresponding curve or Table3D ID. Note that damage data can be rate and temperature dependent and are used with all relevant input stress-strain curves in MAT_213 V1.3.6 and later versions. The damage parameter ID definitions are shown in the following table. For thin shell elements, only in-plane damage is considered and only parameters 1, 2, 4, 5, 7, 13, 15, 16, 18, 21, 23, 24, 26, 37, 38, 40, 42, 45, 46, 48, 50, 61, 62, 64, 65 are active.

Damage Parameter ID	Damage Parameter	Damage Parameter ID	Damage Parameter	Damage Parameter ID	Damage Parameter
1	$d_{aa_T}^{aa_T}(\varepsilon_{aa_T})$	29	$d_{bb_T}^{aa_T}(\varepsilon_{cc_T})$	57	$d_{cc_C}^{bb_C}(\varepsilon_{cc_C})$
2	$d_{bb_T}^{bb_T}(\varepsilon_{bb_T})$	30	$d_{cc_T}^{bb_T}(\varepsilon_{cc_T})$	58	$d_{cc_C}^{ab}(\varepsilon_{cc_C})$
3	$d_{cc_T}^{cc_T}(\varepsilon_{cc_T})$	31	$d_{cc_T}^{aa_C}(\varepsilon_{cc_T})$	59	$d_{cc_C}^{bc}(\varepsilon_{cc_C})$
4	$d_{aa_C}^{aa_C}(\varepsilon_{aa_C})$	32	$d_{cc_T}^{bb_C}(\varepsilon_{cc_T})$	60	$d_{cc_C}^{ac}(\varepsilon_{cc_C})$
5	$d_{bb_C}^{bb_C}(\varepsilon_{bb_C})$	33	$d_{cc_T}^{cc_C}(\varepsilon_{cc_T})$	61	$d_{ab}^{aa_T}(\varepsilon_{ab})$

Damage Parameter ID	Damage Parameter	Damage Parameter ID	Damage Parameter	Damage Parameter ID	Damage Parameter
6	$d_{cc_C}^{cc_C}(\varepsilon_{cc_C})$	34	$d_{cc_T}^{ab}(\varepsilon_{cc_T})$	62	$d_{ab}^{bb_T}(\varepsilon_{ab})$
7	$d_{ab}^{ab}(\varepsilon_{ab})$	35	$d_{cc_T}^{bc}(\varepsilon_{cc_T})$	63	$d_{ab}^{cc_T}(\varepsilon_{ab})$
8	$d_{bc}^{bc}(\varepsilon_{bc})$	36	$d_{cc_T}^{ac}(\varepsilon_{cc_T})$	64	$d_{ab}^{aa_C}(\varepsilon_{ab})$
9	$d_{ac}^{ac}(\varepsilon_{ac})$	37	$d_{aa_C}^{aa_T}(\varepsilon_{aa_T})$	65	$d_{ab}^{bb_C}(\varepsilon_{ab})$
10	$d_{oab}^{oab}(\varepsilon_{oab})$	38	$d_{aa_C}^{bb_T}(\varepsilon_{aa_C})$	66	$d_{ab}^{cc_C}(\varepsilon_{ab})$
11	$d_{obc}^{obc}(\varepsilon_{obc})$	39	$d_{aa_C}^{cc_T}(\varepsilon_{aa_C})$	67	$d_{ab}^{bc}(\varepsilon_{ab})$
12	$d_{oac}^{oac}(\varepsilon_{oac})$	40	$d_{aa_C}^{bb_C}(\varepsilon_{aa_C})$	68	$d_{ab}^{ac}(\varepsilon_{ab})$
13	$d_{aa_T}^{bb_T}(\varepsilon_{aa_T})$	41	$d_{aa_C}^{cc_C}(\varepsilon_{aa_C})$	69	$d_{bc}^{aa_T}(\varepsilon_{bc})$
14	$d_{aa_T}^{cc_T}(\varepsilon_{aa_T})$	42	$d_{aa_C}^{ab}(\varepsilon_{aa_C})$	70	$d_{bc}^{bb_T}(\varepsilon_{bc})$
15	$d_{aa_T}^{aa_C}(\varepsilon_{aa_T})$	43	$d_{aa_C}^{bc}(\varepsilon_{aa_C})$	71	$d_{bc}^{cc_T}(\varepsilon_{bc})$
16	$d_{aa_T}^{bb_C}(\varepsilon_{aa_T})$	44	$d_{aa_C}^{ac}(\varepsilon_{aa_C})$	72	$d_{bc}^{aa_c}(\varepsilon_{bc})$
17	$d_{aa_T}^{cc_C}(\varepsilon_{aa_T})$	45	$d_{bb_C}^{aa_T}(\varepsilon_{bb_C})$	73	$d_{bc}^{bb_c}(\varepsilon_{bc})$
18	$d_{aa_T}^{ab}(\varepsilon_{aa_T})$	46	$d_{bb_C}^{bb_T}(\varepsilon_{bb_C})$	74	$d_{bc}^{cc_c}(\varepsilon_{bc})$
19	$d_{aa_T}^{bc}(\varepsilon_{aa_T})$	47	$d_{bb_C}^{cc_T}(\varepsilon_{bb_C})$	75	$d_{bc}^{ab}(\varepsilon_{bc})$
20	$d_{aa_T}^{ac}(\varepsilon_{aa_T})$	48	$d_{bb_C}^{aa_C}(\varepsilon_{bb_C})$	76	$d_{bc}^{ac}(\varepsilon_{bc})$
21	$d_{bb_T}^{aa_T}(\varepsilon_{bb_T})$	49	$d_{bb_C}^{cc_C}(\varepsilon_{bb_C})$	77	$d_{ac}^{aa_T}(\varepsilon_{ac})$
22	$d_{bb_T}^{cc_T}(\varepsilon_{bb_T})$	50	$d_{bb_C}^{ab}(\varepsilon_{bb_C})$	78	$d_{ac}^{bb_T}(\varepsilon_{ac})$
23	$d_{bb_T}^{aa_C}(\varepsilon_{bb_T})$	51	$d_{bb_C}^{bc}(\varepsilon_{bb_C})$	79	$d_{ac}^{cc_T}(\varepsilon_{ac})$
24	$d_{bb_T}^{bb_C}(\varepsilon_{bb_T})$	52	$d_{bb_C}^{ac}(\varepsilon_{bb_C})$	80	$d_{ac}^{aa_C}(\varepsilon_{ac})$
25	$d_{bb_T}^{cc_C}(\varepsilon_{bb_T})$	53	$d_{cc_C}^{aa_T}(\varepsilon_{cc_C})$	81	$d_{ac}^{bb_C}(\varepsilon_{ac})$
26	$d_{bb_T}^{ab}(\varepsilon_{bb_T})$	54	$d_{cc_C}^{bb_T}(\varepsilon_{cc_C})$	82	$d_{ac}^{cc_C}(\varepsilon_{ac})$
27	$d_{bb_T}^{bc}(\varepsilon_{bb_T})$	55	$d_{cc_C}^{cc_T}(\varepsilon_{cc_C})$	83	$d_{ac}^{ab}(\varepsilon_{ac})$
28	$d_{bb_T}^{ac_C}(\varepsilon_{bb_T})$	56	$d_{cc_C}^{aa_C}(\varepsilon_{cc_C})$	84	$d_{ac}^{bc}(\varepsilon_{ac})$

a) *Example for rate and temperature independent damage data*. To include damage information only for $d_{bb_C}^{bb_C}(\varepsilon_{bb_C})$ (uncoupled b-direction compression) and $d_{ab}^{ab}(\varepsilon_{ab})$ (uncoupled shear a-b), the following input can be used:

```
*DEFINE_CURVE
$$ Curve of Damage Index and Corresponding Damage Curves
$$ a-Damage Index    o-Damage Curve (damage vs.  total strain)
$#    lcid      sidr       sfa       sfo      offa      offo    dattyp
       101         0     0.000     0.000     0.000     0.000         0
$#                 a                   o
                   5                  25
                   7                  26
```

A typical damage curve has the total strain in the loading direction as abscissa values with the corresponding damage value given as the ordinate values as shown below. The final strain value in the curve must correspond to the final strain in the corresponding QS-RT input stress-strain curve.

Total Strain	Damage
0.0	0.0
0.01	0.0
0.02	0.0
0.03	0.05
0.04	0.08
0.05	0.12
0.06	0.17
0.07	0.23
0.08	0.3

b) *Example for rate and temperature dependent damage data.* To include damage information for three different strain rates (0.0001/s, 0.001/s and 325/s) at temperature 36℃ for $d_{bb_T}^{bb_T}(\varepsilon_{bb_T})$ (uncoupled b-direction tension) only, the following input can be used.

```
*DEFINE_CURVE
$$ a-damage parameter "ID"    o-temperature dependent damage - TABLE 3D ID
$#    lcid      sidr       sfa       sfo      offa      offo    dattyp
       101         0     0.000     0.000     0.000     0.000         0
$#                a1                  o1
                   2                1001
*DEFINE_TABLE_3D
$$ a-temperature   o- strain rate dependent damage - TABLE ID
$#    tbid       sfa      offa
      1001         0     0.000
$#             value             tableid
                36.0               10001
*DEFINE_TABLE
$$ Damage table for temperature 36
$$ a-strain rate   o- Damage curve ID
```

```
$#    tbid       sfa      offa
     10001         0     0.000
$#             value            curveid
              0.0001             100001
              0.0010             100002
             325.000             100003
*DEFINE_CURVE
$$ Damage curve for strain rate 0.0001
$$ a-total strain    o-damage parameter
$#    lcid      sidr       sfa       sfo      offa      offo    dattyp
    100001         0     0.000     0.000     0.000     0.000         0
$#                a1                  o1
             0.00000             0.00000
             0.00631             0.00000
             0.00631             0.00170
             0.00640             0.00680
               ...
             0.00800             0.68000
             0.01000             0.68000
*DEFINE_CURVE
$$ Damage curve for strain rate 0.0010
$$ a-total strain    o-damage parameter
$#    lcid      sidr       sfa       sfo      offa      offo    dattyp
    100002         0     0.000     0.000     0.000     0.000         0
$#                a1                  o1
             0.00000             0.00000
             0.01000             0.00000
             0.01005             0.00170
             0.01010             0.00680
               ...
             0.01100             0.68000
             0.01200             0.68000
*DEFINE_CURVE
$$ Damage curve for strain rate 325.000
$$ a-total strain    o-damage parameter
$#    lcid      sidr       sfa       sfo      offa      offo    dattyp
    100003         0     0.000     0.000     0.000     0.000         0
$#                a1                  o1
             0.00000             0.00000
             0.01100             0.00000
             0.01105             0.00170
             0.01110             0.00680
               ...
             0.01200             0.68000
             0.01400             0.68000
```

5. **Failure criterion.** Use Cards 8*n*.1 and 8*n*.2 for the failure criterion to be included in the failure model. The failure criterion and associated values are given as (see [8] for details)

   a) *Puck Failure Criterion (PFC).* For this criterion, FTYPE = 1 and FV0, …, FV14 are the magnitudes of fracture energy, damage, magnification factor, slope parameters, and material parameters for the fiber.

   b) *Tsai-Wu Failure Criterion (TWFC).* For this criterion, FTYPE = 2 and FV1, ..., FV12 are the magnitudes of the failure stresses $\hat{\sigma}_{aa}^{T}$, $\hat{\sigma}_{aa}^{C}$, $\hat{\sigma}_{bb}^{T}$, $\hat{\sigma}_{bb}^{C}$, $\hat{\sigma}_{cc}^{T}$, $\hat{\sigma}_{cc}^{C}$, $\hat{\sigma}_{ab}$, $\hat{\sigma}_{bc}$, $\hat{\sigma}_{ac}$, $\hat{\sigma}_{ab}^{45}$, $\hat{\sigma}_{bc}^{45}$, and $\hat{\sigma}_{ac}^{45}$ and FV13 and FV14 are erosion-related values and residual strength values

c) *Generalized Tabulated Failure Criterion (GTFC)*. For solid elements with FTYPE = 3, FV1 is n, the in-plane and out-of-plane interaction term. FV2 and FV3 are the Table IDs of the two tables for the in-plane and the out-of-plane $(\theta, \varepsilon_{fail})$ values that define the in-plane and out-of-plane failure surfaces, respectively. For the in-plane failure surface, the table contains the a-direction stress (S11) value-curve ID pairs. For the out-of-plane failure surface, the table contains the normal c-direction stress (S33) value-curve ID pairs. For thin shell elements, only FV2 is used with the preceding definition and the element fails when the damage is 1. There is no data in card 8d.2.

A partial example is shown below for solid elements.

```
$# Card 8d.1
$#   FTYPE       FV0       FV1       FV2       FV3
         3                 2.0      9013      9014
$# Card 8d.2
$#

. . .

$$ theta - equivalent failure strain (efs)
*DEFINE_TABLE
$#    tbid       sfa      offa
      9013         0     0.000
$#             value             curveid
                 0.0               90131
            366000.0               90132
*DEFINE_CURVE
$$ theta - equivalent failure strain for S11 = 0.0
$#    lcid      sidr       sfa       sfo      offa
offo    dattyp
     90131         0     0.000     0.000     0.000
0.000         0
$#                a1                  o1
            -180.000                0.02
             180.000                0.02
*DEFINE_CURVE
$$ theta - equivalent failure strain for S11 = 366000.0
$#    lcid      sidr       sfa       sfo      offa
offo    dattyp
     90132         0     0.000     0.000     0.000
0.000         0
$#                a1                  o1
            -180.000                0.02
             180.000                0.02
```

Repeat the *DEFINE_TABLE for Table ID 9014 with a set of normal c-direction stress (S33) values-curve ID pairs, followed by *DEFINE_CURVE for all the theta-equivalent failure strain curves for different normal c-direction stress (S33) data..

6. **Element erosion.** Element is eroded if failure occurs at any one Gauss point. Note that *DEFINE_ELEMENT_EROSION_SHELL is required for thin shell element erosion; the number of integration points needed to fail to erode the element is defined there.

7. **Adjusting stress-strain curves.** The user-defined stress-strain curves are adjusted when using TCSYM = 1, 2, and 3 as follows. If $E_a^{T_0}$ and $E_a^{C_0}$ represent the original a-direction (1-direction) elastic tensile and compressive moduli respectively, then the modified elastic moduli are computed as

   a) $TCSYM = 1.$ $E_a^T = E_a^C = 0.5\left(E_a^{T_0} + E_a^{C_0}\right)$

   b) $TCSYM = 2.$ $E_a^T = E_a^{T_0}$ and $E_a^C = E_a^{T_0}$

   c) $TCSYM = 3.$ $E_a^T = E_a^{C_0}$ and $E_a^C = E_a^{C_0}$.

   Let $R_a^T = E_a^T/E_a^{T_0}$ and $R_a^C = E_a^C/E_a^{C_0}$. The adjusted tensile strain is then computed as the original tensile strain divided by R_a^T, the adjusted compressive strain is computed as the original compressive strain divided by R_a^C, the adjusted tensile yield strain is computed as the original tensile yield strain divided by R_a^T, and the adjusted compressive yield strain is computed as the original compressive yield strain divided by R_a^C. The same process is then applied to the other two normal directions.

8. **Curve discretization.** For this material, only LCINT on *CONTROL_SOLUTION (not with *DEFINE_CURVE) can be used to specify the number of discretized points for the input curves. The default value is 100.

9. **Poisson's ratios.** If necessary, the input Poisson's ratios are adjusted internally in LS-DYNA to satisfy the criteria described in [11]. However, the user may enter all the Poisson's Ratios to be zero in which case Poisson's Ratio checks are not carried out.

10. **Plastic multiplier.** The plastic multiplier computations involve finding the root of the yield function. The root is computed numerically, not analytically. The first step is to find the interval bounding the root. The value of N set in field PMACC on Card 10 controls the discretization of the interval to find the bound. The larger the value of N, the more accurate the bound. However, the computational time is likely to increase with larger values of N.

11. **Stochastic variation.** A stochastic variation can be added using keyword *DEFINE_STOCHASTIC_VARIATION_PROPERTIES. There are six quantities (see table below) which can be varied, so Card 2 in *DEFINE_STOCHASTIC_VARIATION_PROPERTIES must be defined six times and assumes that the quantities being varied are in the order specified.

Material Properties to be Varied in Predefined Order
E_a
G_{ab}
G_{bc}
G_{ca}
In-plane failure radius
Out-of-plane failure radius

12. **Damage formulation.** By default, the damage calculations are carried out using effective stress as the internal state variable for tracking growth of damage parameters d_{cd}^{ab}. An alternate formulation is available where the damage parameters are taken as functions of directional plastic strains to track damage growth. Details of both the formulations are available in [7].

13. **Simplified material model**. Inputting all the FRCs (see Remark 1) as zero invokes a simplified material model, resulting essentially in a linear analysis. Rate and temperature dependencies are not supported. If needed, damage and failure models can be turned on with additional input. This simplified material model is available for all solid and shell elements and for thick shell formulation 5.

External Files Generated by MAT_213:

Two sets of external files are generated which contain information connected with the input stress-strain curves. The first set of files have a naming convention of "MAT_213__input_curve__stress-strain__curve_id_*i*.plt" Here, i is the i^{th} load curve input into *MAT_213. Each of these files contain LCINT (see Remark 8) stress-strain curve data points. An example of the set of files generated is shown below.

File Name	Load Curve
MAT_213__input_curve__stress-strain__curve_id_1.plt	LC1
MAT_213__input_curve__stress-strain__curve_id_2.plt	LC2
MAT_213__input_curve__stress-strain__curve_id_3.plt	LC3

File Name	Load Curve
MAT_213__input_curve__stress-strain__curve_id_4.plt	LC4
MAT_213__input_curve__stress-strain__curve_id_5.plt	LC5
MAT_213__input_curve__stress-strain__curve_id_6.plt	LC6
MAT_213__input_curve__stress-strain__curve_id_7.plt	LC7
MAT_213__input_curve__stress-strain__curve_id_8.plt	LC8
MAT_213__input_curve__stress-strain__curve_id_9.plt	LC9
MAT_213__input_curve__stress-strain__curve_id_10.plt	LC10
MAT_213__input_curve__stress-strain__curve_id_11.plt	LC11
MAT_213__input_curve__stress-strain__curve_id_12.plt	LC12
MAT_213__input_curve__stress-strain__curve_id_13.plt	LC13
MAT_213__input_curve__stress-strain__curve_id_14.plt	LC14
MAT_213__input_curve__stress-strain__curve_id_15.plt	LC15
MAT_213__input_curve__stress-strain__curve_id_16.plt	LC16
MAT_213__input_curve__stress-strain__curve_id_17.plt	LC17
MAT_213__input_curve__stress-strain__curve_id_18.plt	LC18
MAT_213__input_curve__stress-strain__curve_id_19.plt	LC19
MAT_213__input_curve__stress-strain__curve_id_20.plt	LC20
MAT_213__input_curve__stress-strain__curve_id_21.plt	LC21
MAT_213__input_curve__stress-strain__curve_id_22.plt	LC22
MAT_213__input_curve__stress-strain__curve_id_23.plt	LC23
MAT_213__input_curve__stress-strain__curve_id_24.plt	LC24

The second set of files have a naming convention of "MAT_213__modified_curve__-stress-pl_strain__curve_id_*i*.plt". As above, i is the i^{th} load curve input into *MAT_213. Each of these files contains LCINT stress-effective plastic strain curve data points. We urge you to use these plot files to check if the stress-effective plastic strain curves for each of the 12 components intersect or not when rate and temperature sensitive data are input. Intersecting curves or intersecting extrapolated curves for a component are likely to lead to inconsistent results. An example of the set of files generated is shown below.

File Name	Load Curve
MAT_213__modified_curve__stress-pl_strain__curve_id_1.plt	LC1
MAT_213__modified_curve__stress-pl_strain__curve_id_2.plt	LC2

File Name	Load Curve
MAT_213__modified_curve__stress-pl_strain__curve_id_3.plt	LC3
MAT_213__modified_curve__stress-pl_strain__curve_id_4.plt	LC4
MAT_213__modified_curve__stress-pl_strain__curve_id_5.plt	LC5
MAT_213__modified_curve__stress-pl_strain__curve_id_6.plt	LC6
MAT_213__modified_curve__stress-pl_strain__curve_id_7.plt	LC7
MAT_213__modified_curve__stress-pl_strain__curve_id_8.plt	LC8
MAT_213__modified_curve__stress-pl_strain__curve_id_9.plt	LC9
MAT_213__modified_curve__stress-pl_strain__curve_id_10.plt	LC10
MAT_213__modified_curve__stress-pl_strain__curve_id_11.plt	LC11
MAT_213__modified_curve__stress-pl_strain__curve_id_12.plt	LC12
MAT_213__modified_curve__stress-pl_strain__curve_id_13.plt	LC13
MAT_213__modified_curve__stress-pl_strain__curve_id_14.plt	LC14
MAT_213__modified_curve__stress-pl_strain__curve_id_15.plt	LC15
MAT_213__modified_curve__stress-pl_strain__curve_id_16.plt	LC16
MAT_213__modified_curve__stress-pl_strain__curve_id_17.plt	LC17
MAT_213__modified_curve__stress-pl_strain__curve_id_18.plt	LC18
MAT_213__modified_curve__stress-pl_strain__curve_id_19.plt	LC19
MAT_213__modified_curve__stress-pl_strain__curve_id_20.plt	LC20
MAT_213__modified_curve__stress-pl_strain__curve_id_21.plt	LC21
MAT_213__modified_curve__stress-pl_strain__curve_id_22.plt	LC22
MAT_213__modified_curve__stress-pl_strain__curve_id_23.plt	LC23
MAT_213__modified_curve__stress-pl_strain__curve_id_24.plt	LC24

Each of the aforementioned "MAT_213__modified_curve__stress-pl_strain__curve_-id_*i*.plt" files have similar headings

```
Curveplot
MAT213 mod crv i (EPSR =        xxxx.xxx)
effective plastic strain
stress
stress curve
stress #pts=     "LCINT"
```

List of History Variables for Solid Elements (LS-PrePost):

History Variable #	Symbols	Description
15	c_1^d	Damage in *a*-direction, tension
16	c_2^d	Damage in *b*-direction, tension
17	c_3^d	Damage in *c*-direction, tension
18	c_4^d	Damage in *a*-direction, compression
19	c_5^d	Damage in *b*-direction, compression
20	c_6^d	Damage in *c*-direction, compression
21	c_7^d	Damage in *ab*-plane, shear
22	c_8^d	Damage in *bc*-plane, shear
23	c_9^d	Damage in *a*c-plane, shear
24	d	Failure term (FTYPE = 3)
25	ε_{aaT}^p	Tensile plastic strain in *a*-direction (FTYPE ≠ 1)
26	ε_{bbT}^p	Tensile plastic strain in *b*-direction (FTYPE ≠ 1)
27	r_{IP}^f	Equivalent failure strain for in-plane mode (FTYPE = 3)
28	r_{IP}	Equivalent strain for in-plane mode (FTYPE = 3)
29	θ_{IP}	Failure angle for in-plane mode (FTYPE = 3)
30	r_{OOP}^f	Equivalent failure strain for out-of-plane mode (FTYPE = 3)
31	r_{OOP}	Equivalent strain for out-of-plane mode (FTYPE = 3)
32	θ_{OOP}	Failure angle for out-of-plane mode (FTYPE = 3)
33	ε_{ccT}^p	Tensile plastic strain in *c*-direction (FTYPE ≠ 1)
34	ε_{aaC}^p	Compressive plastic strain in *a*-direction (FTYPE ≠ 1)
35	ε_{bbC}^p	Compressive plastic strain in *b*-direction (FTYPE ≠ 1)
36	ε_{ccC}^p	Compressive plastic strain in *c*-direction (FTYPE ≠ 1)
37	ε_{ab}^p	Plastic tensorial strain in *ab*-plane (FTYPE ≠ 1)
24	ε_{aa}^0	Strain at failure onset in *a*-direction (FTYPE = 1)
25	ε_{aa}^f	Strain for erosion in *a*-direction (FTYPE = 1)

History Variable #	Symbols	Description
26	ε_{bb}^{0}	Strain at failure onset in *b*-direction (FTYPE = 1)
27	ε_{bb}^{f}	Strain for erosion in *b*-direction (FTYPE = 1)
28	ε_{cc}^{0}	Strain at failure onset in *c*-direction (FTYPE = 1)
29	ε_{cc}^{f}	Strain for erosion in *c*-direction (FTYPE = 1)
30	ε_{ab}^{0}	Tensorial shear strain at failure onset in *ab*-plane (FTYPE = 1)
31	ε_{ab}^{f}	Tensorial shear strain for erosion in *ab*-plane (FTYPE = 1)
32	ε_{bc}^{0}	Tensorial shear strain at failure onset in *bc*-plane (FTYPE = 1)
33	ε_{bc}^{f}	Tensorial shear strain for erosion in *bc*-plane (FTYPE = 1)
34	ε_{ac}^{0}	Tensorial shear strain at failure onset in *ac*-plane (FTYPE = 1)
35	ε_{ac}^{f}	Tensorial shear strain for erosion in *ac*-plane (FTYPE = 1)
36	FF	Flag for fiber-fracture
37	IFF	Flag for inter-fiber-fracture
38	T	Temperature
39	$\dot{\varepsilon}_{aa_T}$	Tensile strain rate in *a*-direction
40	$\dot{\varepsilon}_{bb_T}$	Tensile strain rate in *b*-direction
41	$\dot{\varepsilon}_{cc_T}$	Tensile strain rate in *c*-direction
42	$\dot{\varepsilon}_{aa_C}$	Compressive strain rate in *a*-direction
43	$\dot{\varepsilon}_{bb_C}$	Compressive strain rate in *b*-direction
44	$\dot{\varepsilon}_{cc_C}$	Compressive strain rate in *c*-direction
45	$\dot{\varepsilon}_{ab}$	Tensorial shear strain rate in *ab*-plane
46	$\dot{\varepsilon}_{bc}$	Tensorial shear strain rate in *bc*-plane
47	$\dot{\varepsilon}_{ac}$	Tensorial shear strain rate in *ac*-plane
48	$\sigma_{aa_T}^{\mathrm{eff}}$ or $\varepsilon_{aa_T}^{\mathrm{cp}}$	DFLAG.EQ.0: Effective tensile stress in the *a*-direction DFLAG.EQ.1: Corrected tensile plastic strain the *a*-direction

History Variable #	Symbols	Description
49	$\sigma_{bb_T}^{\mathrm{eff}}$ or $\varepsilon_{bb_T}^{\mathrm{cp}}$	DFLAG.EQ.0: Effective tensile stress in the *b*-direction DFLAG.EQ.1: Corrected tensile plastic strain the *b*-direction
50	$\sigma_{cc_T}^{\mathrm{eff}}$ or $\varepsilon_{cc_T}^{\mathrm{cp}}$	DFLAG.EQ.0: Effective tensile stress in the *c*-direction DFLAG.EQ.1: Corrected tensile plastic strain the *c*-direction
51	$\sigma_{aa_C}^{\mathrm{eff}}$ or $\varepsilon_{aa_C}^{\mathrm{cp}}$	DFLAG.EQ.0: Effective compressive stress in the *a*-direction DFLAG.EQ.1: Corrected compressive plastic strain the *a*-direction
52	$\sigma_{bb_C}^{\mathrm{eff}}$ or $\varepsilon_{bb_C}^{\mathrm{cp}}$	DFLAG.EQ.0: Effective compressive stress in the *b*-direction DFLAG.EQ.1: Corrected compressive plastic strain the *b*-direction
53	$\sigma_{cc_C}^{\mathrm{eff}}$ or $\varepsilon_{cc_C}^{\mathrm{cp}}$	DFLAG.EQ.0: Effective compressive stress in the *c*-direction DFLAG.EQ.1: Corrected compressive plastic strain the *c*-direction
54	$\sigma_{ab}^{\mathrm{eff}}$ or $\varepsilon_{ab}^{\mathrm{cp}}$	DFLAG.EQ.0: Effective shear stress in the *ab*-plane DFLAG.EQ.1: Corrected plastic tensorial strain the *ab*-plane
55	$\sigma_{bc}^{\mathrm{eff}}$ or $\varepsilon_{bc}^{\mathrm{cp}}$	DFLAG.EQ.0: Effective shear stress in the *bc*-plane DFLAG.EQ.1: Corrected plastic tensorial strain the *bc*-plane
56	$\sigma_{ac}^{\mathrm{eff}}$ or $\varepsilon_{ac}^{\mathrm{cp}}$	DFLAG.EQ.0: Effective shear stress in the *ac*-plane DFLAG.EQ.1: Corrected plastic tensorial strain the *ac*-plane
57	λ	Effective plastic strain
58	ε_{aa}	Strain in *a*-direction
59	ε_{bb}	Strain in *b*-direction
60	ε_{cc}	Strain in *c*-direction
61	ε_{ab}	Tensorial shear strain in *ab*-plane
62	ε_{bc}	Tensorial shear strain in *bc*-plane

History Variable #	Symbols	Description
63	ε_{ac}	Tensorial shear strain in *ac*-plane
64	$\sigma^{y}_{aa_T}$	Yield stress in tension *a*-direction
65	$\sigma^{y}_{bb_T}$	Yield stress in tension *b*-direction
66	$\sigma^{y}_{cc_T}$	Yield stress in tension *c*-direction
67	$\sigma^{y}_{aa_C}$	Yield stress in compression *a*-direction
68	$\sigma^{y}_{bb_C}$	Yield stress in compression *b*-direction
69	$\sigma^{y}_{cc_C}$	Yield stress in compression *c*-direction
70	σ^{y}_{ab}	Yield stress in shear *ab*-plane
71	σ^{y}_{bc}	Yield stress in shear *bc*-plane
72	σ^{y}_{ac}	Yield stress in shear *ac*-plane
73	T	Current cycle
74	σ^{e}_{aa}	Equilibrium stress in *a*-direction
75	σ^{e}_{bb}	Equilibrium stress in *b*-direction
76	σ^{e}_{cc}	Equilibrium stress in *c*-direction
77	σ^{e}_{ab}	Equilibrium shear stress in *ab*-plane
78	σ^{e}_{bc}	Equilibrium shear stress in *bc*-plane
79	σ^{e}_{ac}	Equilibrium shear stress in *ac*-plane
80	σ^{v}_{aa}	Viscous stress in *a*-direction
81	σ^{v}_{bb}	Viscous stress in *b*-direction
82	σ^{v}_{cc}	Viscous stress in *c*-direction
83	σ^{v}_{ab}	Viscous shear stress in *ab*-plane
90	σ^{v}_{bc}	Viscous shear stress in *bc*-plane
91	σ^{v}_{ac}	Viscous shear stress in *ac*-plane
92	$\dot{\lambda}$	Effective plastic strain rate
93	ε^{p}_{bc}	Plastic tensorial strain in *bc*-plane (FTYPE $\neq$ 1)
94	ε^{p}_{ac}	Plastic tensorial strain in *ac*-plane (FTYPE $\neq$ 1)

List of History Variables for Thin Shell Elements (LS-PrePost):

History Variable #	Symbols	Description
13	d_{max}	Maximum damage parameter
14	c_1^d	Damage in *a*-direction, tension
15	c_2^d	Damage in *b*-direction, tension
16	c_4^d	Damage in *a*-direction, compression
17	c_5^d	Damage in *b*-direction, compression
18	c_7^d	Damage in *a*-*b* plane, shear
19	d	Failure term (FTYPE = 3)
20	r_{IP}	Equivalent strain for in-plane mode
21	θ_{IP}	Failure angle for in-plane mode
22	F	Flag for failure of integration point. "1" if $d \geq 1$
23	T	Temperature
24	$\dot{\lambda}$	Effective plastic strain rate
25	$\dot{\varepsilon}_{aa_T}$	Tensile strain rate in the *a*-direction
26	$\dot{\varepsilon}_{bb_T}$	Tensile strain rate in the *b*-direction
27	$\dot{\varepsilon}_{aa_C}$	Compressive strain rate in the *a*-direction
28	$\dot{\varepsilon}_{bb_C}$	Compressive strain rate in the *b*-direction
29	$\dot{\varepsilon}_{ab}$	Tensorial shear strain rate in the *ab*-plane
30	$\varepsilon_{aa_T}^p$	Tensile plastic strain in the *a*-direction
31	$\varepsilon_{bb_T}^p$	Tensile plastic strain in the *b*-direction
32	$\varepsilon_{cc_T}^p$	Tensile plastic strain in the *c*-direction
33	$\varepsilon_{aa_C}^p$	Compressive plastic strain in the *a*-direction
34	$\varepsilon_{bb_C}^p$	Compressive plastic strain in the *b*-direction
35	$\varepsilon_{cc_C}^p$	Compressive plastic strain in the *c*-direction
36	ε_{ab}^p	Plastic tensorial shear strain in the *ab*-plane
37	λ	Effective plastic strain
38	ε_{aa}	Strain in *a*-direction
39	ε_{bb}	Strain in *b*-direction
40	ε_{cc}	Strain in *c*-direction

History Variable #	Symbols	Description
41	ε_{ab}	Tensorial shear strain in *ab*-plane
42	ε_{bc}	Tensorial shear strain in *bc*-plane
43	ε_{ac}	Tensorial shear strain in *ac*-plane
44	σ_{aa}	Stress in *a*-direction
45	σ_{bb}	Stress in *b*-direction
46	σ_{ab}	Shear stress in *ab*-plane
47	σ_{bc}	Shear stress in *bc*-plane
48	σ_{ac}	Shear stress in *ac*-plane
49	σ_{aa}^{e}	Equilibrium stress in *a*-direction
50	σ_{bb}^{e}	Equilibrium stress in *b*-direction
51	σ_{ab}^{e}	Equilibrium shear stress in *ab*-plane
52	σ_{aa}^{v}	Viscous stress in *a*-direction
53	σ_{bb}^{v}	Viscous stress in *b*-direction
54	σ_{ab}^{v}	Viscous shear stress in *ab*-plane
55	$\varepsilon_{aa_T}^{\mathrm{cp}}$	Corrected plastic strain in *a*-direction, tension
56	$\varepsilon_{bb_T}^{\mathrm{cp}}$	Corrected plastic strain in *b*-direction, tension
57	$\varepsilon_{aa_C}^{\mathrm{cp}}$	Corrected plastic strain in *a*-direction, compression
58	$\varepsilon_{bb_C}^{\mathrm{cp}}$	Corrected plastic strain in *b*-direction, compression
59	$\sigma_{aa_T}^{y}$	Yield stress in tension *a*-direction
60	$\sigma_{bb_T}^{y}$	Yield stress in tension *b*-direction
61	$\sigma_{aa_C}^{y}$	Yield stress in compression *a*-direction
62	$\sigma_{bb_C}^{y}$	Yield stress in compression *b*-direction
63	σ_{ab}^{y}	Yield stress in shear *ab*-plane
64	$\sigma_{ab}^{y,45}$	Yield stress in 45° off-axis *ab*-plane
65	E_p	Dissipated plastic energy

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[9] T. Achstetter (2019). Development of a composite material shell-element model for impact applications", PhD Dissertation, George Mason University.

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[12] N. Holt, B. Khaled, L. Shyamsunder and S. Rajan (2021). T800-F3900 Composite Stacked Ply Laminate Testing and Modeling Using MAT_213, DOT/FAA/TC-21/56, https://www.tc.faa.gov/its/worldpac/techrpt/tc21-56.pdf

*MAT_DRY_FABRIC

This is Material Type 214. This material model can be used to model high strength woven fabrics, such as Kevlar® 49, with transverse orthotropic behavior for use in structural systems where high energy absorption is required (Bansal et al., Naik et al., Stahlecker et al.). The major applications of the model are for the materials used in propulsion engine containment system, body armor and personal protections.

Woven dry fabrics are described in terms of two principal material directions, longitudinal warp and transverse fill yarns. The primary failure mode in these materials is the breaking of either transverse or longitudinal yarn. An equivalent continuum formulation is used and an element is designated as having failed when it reaches some critical value for strain in either directions. A linearized approximation to a typical stress-strain curve is shown in Figure M214-1 and to a typical engineering shear stress-strain curve is shown in the figure corresponding to the GAB*i* field in the variable list. Note that the principal directions are labeled *a* for the warp and *b* for the fill, and the direction *c* is orthogonal to *a* and *b*.

The material model is available for membrane elements and it is recommended to use a double precision version of LS-DYNA.

Card Summary:

Card 1. This card is required.

MID	RO	EA	EB	GAB1	GAB2	GAB3	

Card 2. This card is required.

GBC	GCA	GAMAB1	GAMAB2				

Card 3. This card is required.

AOPT					A1	A2	A3

Card 4. This card is required.

V1	V2	V3	D1	D2	D3	BETA	

Card 5. This card is required.

EACRF	EBCRF	EACRP	EBCRP				

Card 6. This card is required.

EASF	EBSF	EUNLF	ECOMF	EAMAX	EBMAX	SIGPOST	

Card 7. This card is required.

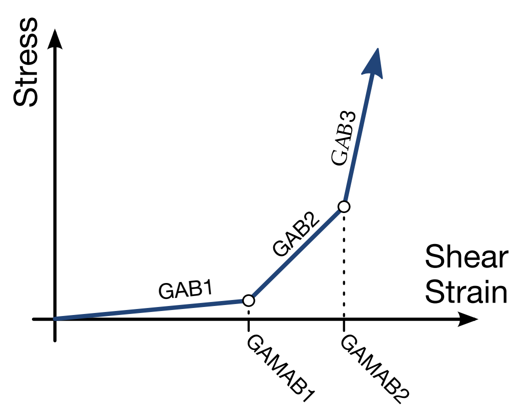
CCE	PCE	CSE	PSE	DFAC	EMAX	EAFAIL	EBFAIL

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EA	EB	GAB1	GAB2	GAB3	
Type	A	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	GBC	GCA	GAMAB1	GAMAB2				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Continuum equivalent mass density
EA	Modulus of elasticity in the longitudinal (warp) direction, which corresponds to the slope of segment AB in Figure M214-1
EB	Modulus of elasticity in the transverse (fill) direction, which corresponds to the slope of segment of AB in Figure M214-1
GAB*i* / GAMAB*i*	Shear stress-strain behavior is modeled as piecewise linear in three segments. *See the figure to the right*. The shear moduli GAB*i* correspond to the slope of the i^{th} segment. The start and end points for the segments are specified in the GAMAB[1-2] fields.



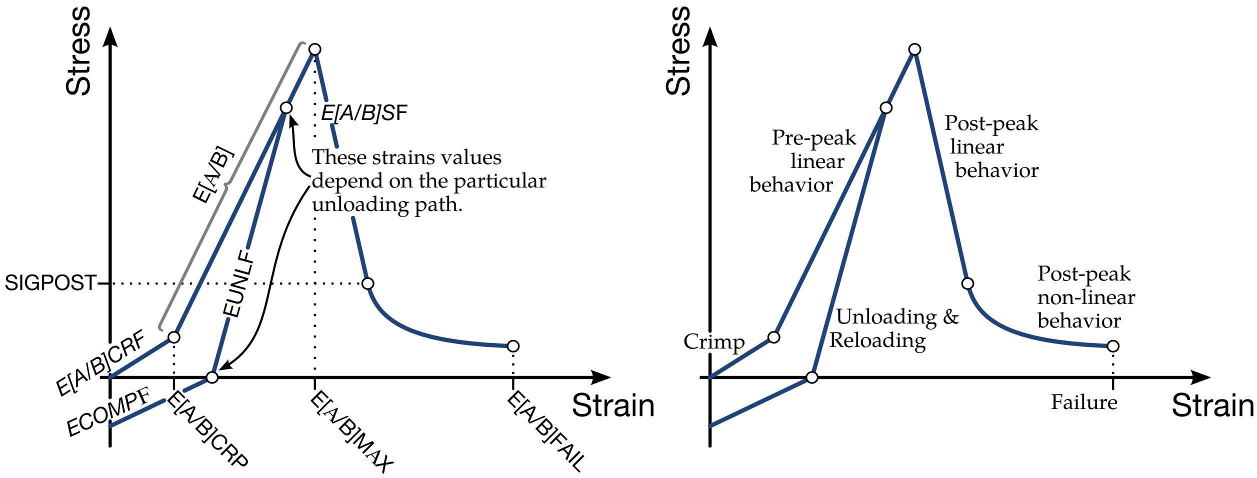


Figure M214-1. Stress – Strain curve for *MAT_DRY_FABRIC. This curve models the force-response in the longitudinal and transverse directions.

VARIABLE	DESCRIPTION
GBC	G_{bc}, shear modulus in *bc* direction
GCA	G_{ca}, shear modulus in *ca* direction

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT		XP	YP	ZP	A1	A2	A3
Type	F		F	F	F	F	F	F

VARIABLE	DESCRIPTION
AOPT	Material axes option. See *MAT_*OPTION*TROPIC_ELASTIC for a more complete description: EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the element normal by an angle BETA EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by

VARIABLE	DESCRIPTION
	the cross product of the vector **v** with the element normal
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
A1, A2, A3	Components of vector **a** for AOPT = 2

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3
D1, D2, D3	Components of vector **d** for AOPT = 2
BETA	Material angle in degrees for AOPT = 0 and 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA.

Card 5	1	2	3	4	5	6	7	8
Variable	EACRF	EBCRF	EACRP	EBCRP				
Type	F	F	F	F				
Remarks	2	2						

VARIABLE	DESCRIPTION
EACRF	Factor for crimp region modulus of elasticity in longitudinal direction (see Figure M214-1): $E_{a,\mathrm{crimp}} = E_{a,\mathrm{crimpfac}} E, \qquad E_{a,\mathrm{crimpfac}} = \mathrm{EACRF}$

VARIABLE	DESCRIPTION
EBCRF	Factor for crimp region modulus of elasticity in transverse direction (see Figure M214-1): $$E_{b,\mathrm{crimp}} = E_{b,\mathrm{crimpfac}}E, \qquad E_{b,\mathrm{crimpfac}} = \mathrm{EBCRF}$$
EACRP	Crimp strain in longitudinal direction (see Figure M214-1), $\varepsilon_{a,\mathrm{crimp}}$
EBCRP	Crimp strain in transverse direction (see Figure M214-1), $\varepsilon_{b,\mathrm{crimp}}$

Card 6	1	2	3	4	5	6	7	8
Variable	EASF	EBSF	EUNLF	ECOMF	EAMAX	EBMAX	SIGPOST	
Type	F	F	F	F	F	F	F	
Remarks	2	2	2	2				

VARIABLE	DESCRIPTION
EASF	Factor for post-peak region modulus of elasticity in longitudinal direction (see Figure M214-1): $$E_{a,\mathrm{soft}} = E_{a,\mathrm{softfac}}E, \qquad E_{a,\mathrm{softfac}} = \mathrm{EASF}$$
EBSF	Factor for post-peak region modulus of elasticity in transverse direction (see Figure M214-1): $$E_{b,\mathrm{soft}} = E_{b,\mathrm{softfac}}E, \qquad E_{b,\mathrm{softfac}} = \mathrm{EBSF}$$
EUNLF	Factor for unloading modulus of elasticity (see Figure M214-1): $$E_{\mathrm{unload}} = E_{\mathrm{unloadfac}}E, \qquad E_{\mathrm{unloadfac}} = \mathrm{EUNLF}$$
ECOMPF	Factor for compression zone modulus of elasticity (see Figure M214-1): $$E_{\mathrm{comp}} = E_{\mathrm{compfac}}E, \qquad E_{\mathrm{compfac}} = \mathrm{ECOMPF}$$
EAMAX	Strain at peak stress in longitudinal direction (see Figure M214-1), $\varepsilon_{a,\max}$
EBMAX	Strain at peak stress in transverse direction (see Figure M214-1), $\varepsilon_{b,\max}$

VARIABLE	DESCRIPTION
SIGPOST	Stress value in post-peak region at which nonlinear behavior begins (see Figure M214-1), σ_{post}

Card 7	1	2	3	4	5	6	7	8
Variable	CCE	PCE	CSE	PSE	DFAC	EMAX	EAFAIL	EBFAIL
Type	F	F	F	F	F	F	F	F
Remarks	1	1	1	1	2	3	2, 3	2, 3

VARIABLE	DESCRIPTION
CCE	Strain rate parameter C, Cowper-Symonds factor for modulus. If zero, rate effects are not considered.
PCE	Strain rate parameter P, Cowper-Symonds factor for modulus. If zero, rate effects are not considered.
CSE	Strain rate parameter C, Cowper-Symonds factor for stress to peak / failure. If zero, rate effects are not considered.
PSE	Strain rate parameter P, Cowper-Symonds factor for stress to peak / failure. If zero, rate effects are not considered.
DFAC	Damage factor, d_{fac}
EMAX	Erosion strain of element, ε_{max}
EAFAIL	Erosion strain in longitudinal direction (see Figure M214-1), $\varepsilon_{a,\text{fail}}$
EBFAIL	Erosion strain in transverse direction (see Figure M214-1), $\varepsilon_{b,\text{fail}}$

Remarks:

1. **Strain rate effects.** Strain rate effects are accounted for using a Cowper-Symonds model which scales the stress according to the strain rate:

$$\sigma^{\text{adj}} = \sigma\left(1 + \frac{\dot{\varepsilon}}{C}\right)^{\frac{1}{P}} .$$

In the above equation $\boldsymbol{\sigma}$ is the quasi-static stress, $\boldsymbol{\sigma}^{\text{adj}}$ is the adjusted stress accounting for strain rate $\dot{\varepsilon}$, and C (CCE) and P (PCE) are the Cowper-Symonds factors which must be determined experimentally for each material.

The model captures the non-linear strain rate effects in many materials. With its less than unity exponent, $1/p$, this model captures the rapid increase in material properties at low strain rate, while increasing less rapidly at very high strain rates. Because stress is a function of strain rate the elastic stiffness also is:

$$\mathbf{E}^{\text{adj}} = \mathbf{E}\left(1 + \frac{\dot{\varepsilon}}{C}\right)^{\frac{1}{P}}$$

where $\mathbf{E}^{\text{adj}}$ is the adjusted elastic stiffness. Additionally, the strains to peak and strains to failure are assumed to follow a Cowper-Symonds model with, *possibly different*, constants

$$\varepsilon^{\text{adj}} = \varepsilon\left(1 + \frac{\dot{\varepsilon}}{C_s}\right)^{\frac{1}{P_s}} \ ,$$

where, ε^{adj} is the adjusted effective strain to peak stress or strain to failure, and C_s and P_s are CSE and PSE respectively.

2. **Stress-strain beyond peak stress.** When strained beyond the peak stress, the stress decreases linearly until it attains a value equal to SIGPOST, at which point the stress-strain relation becomes nonlinear. In the non-linear region the stress is given by

$$\sigma = \sigma_{\text{post}}\left[1 - \left(\frac{\varepsilon - \varepsilon_{[a/b],\text{post}}}{\varepsilon_{[a/b],\text{fail}} - \varepsilon_{[a/b],\text{post}}}\right)^{d_{\text{fac}}}\right] \ ,$$

where σ_{post} and $\varepsilon_{\text{post}}$ are, respectively, the stress and strain demarcating the onset of nonlinear behavior. The value of SIGPOST is the same in both the transverse and longitudinal directions, whereas $\varepsilon_{\text{a,post}}$ and $\varepsilon_{\text{b,post}}$ depend on direction and are derived internally from EASF, EBSF, and SIGPOST. The failure strain, $\varepsilon_{[a/b],\text{fail}}$, specifies the onset of failure and differs in the longitudinal and transverse directions. Lastly the exponent, d_{fac}, determines the shape of nonlinear stress-strain curve between $\varepsilon_{\text{post}}$ and $\varepsilon_{[a/b],\text{fail}}$.

3. **Element erosion.** The element is eroded if either (a) or (b) is satisfied:

   a) $\varepsilon_a > \varepsilon_{a,\text{fail}}$ and $\varepsilon_b > \varepsilon_{b,\text{fail}}$

   b) $\varepsilon_a > \varepsilon_{\max}$ and $\varepsilon_b > \varepsilon_{\max}$.

*MAT_4A_MICROMEC

This is Material Type 215. A micromechanical material that distinguishes between a fiber/inclusion and a matrix material, developed by 4a engineering GmbH. It is available for shell, thick shell, and solid elements. Useful hints and an input example can be found in [1]. More theory and application notes are provided in [2].

This material is intended for anisotropic composite materials, especially for short (SFRT) and long fiber thermoplastics (LFRT). The matrix behavior is modeled with an isotropic elasto-viscoplastic von Mises model. The fiber/inclusion behavior is transverse isotropic elastic. This material model can be used for classical endless fiber composites.

The inelastic homogenization for describing the composite deformation behavior is based on:

- Mori Tanaka Meanfield Theory [3,4]
- ellipsoidal inclusions using Eshelby's solution [5,6]
- orientation averaging [7]
- a linear fitted closure approximation to determine the 4th order fiber orientation tensor out of the user provided 2nd order fiber orientation tensor.

The software product 4a micromec can calculate and export the thermo-elastic composite properties [8].

Failure/damage of the composite can be considered with:

- a ductile damage initiation and evolution model for the matrix (DIEM)
- fiber failure with a maximum stress criterion

References [9] and [10] provide more details on the material characterization.

The (fiber) orientation can be defined either for the whole material using Cards 2 and 3 or elementwise using *ELEMENT_(T)SHELL_BETA or *ELEMENT_SOLID_ORTHO. The manufacturing process highly influences the mechanical properties of SFRT and LFRT in injection molded parts. By mapping the fiber orientation from the process simulation to the structural analysis the local anisotropy can be considered [11,12]. The fiber orientation, length and volume fraction can therefore as well be defined for each integration point by using *INITIAL_STRESS_(T)SHELL(SOLID) [2]. Details on the history variables that can be initialized (extra history variables 9-18) can be found in the output section.

Card Summary:

Cards 2 through 4 specify fiber orientation. They may be overwritten with may be overwritten by *INITIAL_STRESS_(T)SHELL/SOLID. Cards 5 and 6 are for specifying parameters for the fiber/inclusion material. Cards 7 through 9 give properties associated with the matrix material.

Card 1. This card is required.

MID	MMOPT	BUPD			FAILM	FAILF	NUMINT

Card 2. This card is required.

AOPT	MACF	XP	YP	ZP	A1	A2	A3

Card 3. This card is required.

V1	V2	V3	D1	D2	D3	BETA	

Card 4. This card is required.

FVF		FL	FD		A11	A22	

Card 5. This card is required.

ROF	EL	ET	GLT	PRTL	PRTT		

Card 6. This card is required.

XT						SLIMXT	NCYRED

Card 7. This card is required.

ROM	E	PR					

Card 8. This card is required.

SIGYT	ETANT			EPS0	C		

Card 9. This card is required.

LCIDT				LCDI	UPF		NCYRED2

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	MMOPT	BUPD			FAILM	FAILF	NUMINT
Type	A	F	F			F	F	F
Default	none	0.0	0.01			0.0	0.0	1.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
MMOPT	Option to define micromechanical material behavior: EQ.0.0: Elastic EQ.1.0: Elastic-plastic
BUPD	Tolerance for update of Strain-Concentration Tensor
FAILM	Option for matrix failure using a ductile DIEM model. See sections Damage Initiation and Damage Evolution in the manual page for *MAT_ADD_DAMAGE_DIEM for a description of ductile damage initialization (DITYP = 0) based on stress triaxiality and a linear damage evolution (DETYP = 0) type. Also see fields LCDI and UPF on Card 9. LT.0.0: |FAILM| is effective plastic matrix strain at failure. When the matrix plastic strain reaches this value, the element is deleted from the calculation. EQ.0.0: Only visualization (triaxiality of matrix stresses) EQ.1.0: Active DIEM (triaxiality of matrix stresses) EQ.10.0: Only visualization (triaxiality of composite stresses) EQ.11.0: Active DIEM (triaxiality of composite stresses)
FAILF	Option for fiber failure: EQ.0.0: Only visualization (equivalent fiber stresses) EQ.1.0: Active (equivalent fiber stresses)

VARIABLE	DESCRIPTION
NUMINT	Number or percentage of failed integration points prior to element deletion (default value is 1): GT.0.0: Number of integration points which must fail before element is deleted. LT.0.0: Applies only to shells. |NUMINT| is the percentage of layers which must fail before an element fails. For shell formulations with 4 integration points per layer, the layer is considered failed if any of the integration points in the layer fails.

Card 2	1	2	3	4	5	6	7	8
Variable	AOPT	MACF	XP	YP	ZP	A1	A2	A3
Type	F	F	F	F	F	F	F	F
Default	0.0	0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description): EQ.0.0: Locally orthotropic with material axes determined by element nodes. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA. EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center. This option is for solid elements only. EQ.2.0: Globally orthotropic with material axes determined by vectors defined below EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is

VARIABLE	DESCRIPTION
	determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.
	EQ.4.0: Locally orthotropic in cylindrical coordinate system with the material axes determined by a vector, **v**, and an originating point, *P*, defining the centerline axis. This option is for solid elements only.
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE-_COORDINATE_VECTOR).
	The fiber orientation information may be overwritten using *INITIAL_STRESS_(T)SHELL/SOLID
MACF	Material axes change flag for solid elements:
	EQ.-4: Switch material axes *b* and *c* before BETA rotation
	EQ.-3: Switch material axes *a* and *c* before BETA rotation
	EQ.-2: Switch material axes *a* and *b* before BETA rotation
	EQ.1: No change, default
	EQ.2: Switch material axes *a* and *b* after BETA rotation
	EQ.3: Switch material axes *a* and *c* after BETA rotation
	EQ.4: Switch material axes *b* and *c* after BETA rotation
	Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_-SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 3 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.
XP, YP, ZP	Coordinates of point *p* for AOPT = 1 and 4
A1, A2, A3	Components of vector **a** for AOPT = 2

Card 3	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

VARIABLE	DESCRIPTION
V1, V2, V3	Define components of vector **v** for AOPT = 3 and 4.
D1, D2, D3	Define components of vector **d** for AOPT = 2.
BETA	Angle in degrees of a material rotation about the *c*-axis, available for AOPT = 0 (shells and tshells only) and AOPT = 3 (all element types). This angle may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, and *ELEMENT_SOLID_ORTHO.

Card 4	1	2	3	4	5	6	7	8
Variable	FVF		FL	FD		A11	A22	
Type	F		F	F		F	F	
Default	0.0		0.0	1.0		1.0	0.0	

VARIABLE	DESCRIPTION
FVF	Fiber volume or mass fraction: GT.0.0: Fiber volume fraction LT.0.0: |FVF| is the fiber mass fraction.
FL	Fiber length unless FD = 1. If FD = 1, then it is the aspect ratio (may be overwritten by *INITIAL_STRESS_(T)SHELL/SOLID)
FD	Fiber diameter

VARIABLE	DESCRIPTION
A11	Value of first principal fiber orientation (may be overwritten by *INITIAL_STRESS_(T)SHELL/SOLID)
A22	Value of second principal fiber orientation (may be overwritten by *INITIAL_STRESS_(T)SHELL/SOLID)

Card 5	1	2	3	4	5	6	7	8
Variable	ROF	EL	ET	GLT	PRTL	PRTT		
Type	F	F	F	F	F	F		
Default	0.0	0.0	0.0	0.0	0.0	0.0		

VARIABLE	DESCRIPTION
ROF	Mass density of fiber
EL	E_L, Young's modulus of fiber in the longitudinal direction
ET	E_T, Young's modulus of fiber in the transverse direction
GLT	G_{LT}, shear modulus LT
PRTL	ν_{TL}, Poisson's ratio TL
PRTT	ν_{TT}, Poisson's ratio TT

Card 6	1	2	3	4	5	6	7	8
Variable	XT						SLIMXT	NCYRED
Type	F						F	F
Default	0.0						0.0	10

VARIABLE	DESCRIPTION
XT	Fiber tensile strength in the longitudinal direction

VARIABLE	DESCRIPTION
SLIMXT	Factor to determine the minimum stress limit in the fiber after stress maximum (fiber tension)
NCYRED	Number of cycles for stress reduction from maximum to minimum (fiber tension)

Card 7	1	2	3	4	5	6	7	8
Variable	ROM	E	PR					
Type	F	F	F					
Default	0.0	0.0	0.0					

VARIABLE	DESCRIPTION
ROM	Mass density of matrix
E	Young's modulus of matrix
PR	Poisson's ratio of matrix

Card 8	1	2	3	4	5	6	7	8
Variable	SIGYT	ETANT			EPS0	C		
Type	F	F			F	F		
Default	0.0	0.0			0.0	0.0		

VARIABLE	DESCRIPTION
SIGYT	Yield stress of matrix in tension
ETANT	Tangent modulus of matrix in tension, ignore if LCIDT > 0 is defined.
EPS0	Quasi-static threshold strain rate (Johnson-Cook model) for bilinear hardening

VARIABLE	DESCRIPTION
C	Johnson-Cook constant for bilinear hardening

Card 9	1	2	3	4	5	6	7	8
Variable	LCIDT				LCDI	UPF		NCYRED2
Type	F				F	F		F
Default	0				0	0.0		1

VARIABLE	DESCRIPTION
LCIDT	Load curve ID or table ID for defining effective stress as a function of effective plastic strain in tension of matrix material (Table to include strain-rate effects, viscoplastic formulation.)
LCDI	Curve/table for ductile damage initiation parameter. The definitions depend on if the element is a shell or solid. **Shell elements.** LCDI can be a load curve or table ID. A load curve represents plastic strain at onset of damage as function of stress triaxiality. A table represents plastic strain at onset of damage as function of stress triaxiality and plastic strain rate. **Solid elements.** LCDI can be a load curve, table, or 3D table ID. A load curve represents plastic strain at onset of damage as function of stress triaxiality. A table represents plastic strain at onset of damage as function of stress triaxiality and lode angle. A 3D table represents plastic strain at onset of damage as function of stress triaxiality, lode angle and plastic strain rate.
UPF	Damage evolution parameter GT.0.0: Plastic displacement at failure, u_f^p LT.0.0: |UPF| is a table ID for u_f^p as a function of triaxiality and damage
NCYRED2	In case of matrix failure (IFAILM = 1 or 11), number of cycles for stress reduction of fiber stresses until the integration point will be marked as failed.

Output:

For this material, "Plastic Strain" is the equivalent plastic strain in the matrix. Extra history variables may be requested for (t)shell (NEIPS) and solid (NEIPH) elements with *DATABASE_EXTENT_BINARY. Extra history variables 1 through 8 are intended for post-processing while 9 through18 are intended for initialization with *INITIAL_-STRESS_(T)SHELL/SOLID. They have the following meaning:

History Variable #	Description
1	Equivalent plastic strain rate of matrix
2	Triaxiality of matrix, $\eta = -p/q$
3	Lode parameter of matrix, $\xi = -\frac{27J_3}{2q}$
4	Damage initiation, d, of matrix (Ductile Criteria)
5	Damage evolution, D, of matrix
6	Fiber reserve factor
7	Fiber damage variable
8	Fiber stress reduction variable (NCYRED2)
9	Value of first principal fiber orientation, A11
10	Value of second principal fiber orientation, A22
11	For shells, $\cos\alpha$ where α is the in-plane angle between the material coordinate system and the element coordinate system. For solids, q_{11} where q_{11} is the x-direction component of the first orientation direction in the element coordinate system.
12	For shells, -sin α where α is the in-plane angle between the material coordinate system and the element coordinate system. For solids, q_{12} where q_{12} is the y-direction component of the first orientation direction in the element coordinate system.
13	For shells, unused. For solids, q_{13} where q_{13} is the z-direction component of the first orientation direction in the element coordinate system.
14	For shells, unused. For solids, q_{31} where q_{31} is the x-direction component of the third orientation direction in the element coordinate system.
15	For shells, unused. For solids, q_{32} where q_{32} is the y-direction component of the third orientation direction in the element coordinate system.

History Variable #	Description
16	For shells, unused. For solids, q_{33} where q_{33} is the z-direction component of the third orientation direction in the element coordinate system.
17	Fiber volume fraction, FVF
18	Fiber length, FL

Material Orientation:

Figure 4 of Reference 13 shows the 2nd order orientation tensor for which there are eigenvectors and corresponding eigenvalues. The coordinate system based on the eigenvectors is the material coordinate system. The values $q_{11},\ldots, q_{33}$ for solids (history variables 11 through 16) and the values $\cos(\alpha)$ and -sin α for shells (history variables 11 and 12) specify this material coordinate system with respect to the element coordinate system. The values a_1 and a_2 (A11 and A22 of history variables 9 and 10) shown in the figure represent the eigenvalues, or in other words, the lengths of the ellipsoid. Thus. history variables 9 and 10 give the shape of the ellipsoid while history variables 11 through 16 give the orientation.

References:

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[8] http://micromec.4a.co.at

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[11] Reithofer et al., *Short and long fiber reinforced thermoplastics material models in LS-DYNA*, 10th European LS-DYNA Conference, Würzburg 2015.

[12] http://fibermap.4a.co.at

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*MAT_ELASTIC_PHASE_CHANGE

This is Material Type 216, a generalization of Material Type 1, for which material properties change on an element-by-element basis upon crossing a plane in space. This is an isotropic hypoelastic material and is available *only* for shell element types.

Phase 1 Properties.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R01	E1	PR1				
Type	A	F	F	F				
Default	none	none	none	0.0				

Phase 2 Properties.

Card 2	1	2	3	4	5	6	7	8
Variable		R02	E2	PR2				
Type		F	F	F				
Default		none	none	0.0				

Transformation Plane Card.

Card 3	1	2	3	4	5	6	7	8
Variable	X1	Y1	Z1	X2	Y2	Z2	THKFAC	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	none	1.0	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO*i*	Mass density for phase *i*
E*i*	Young's modulus for phase *i*
PR*i*	Poisson's ratio for phase *i*
X1, Y1, Z1	Coordinates of a point on the phase transition plane
X2, Y2, Z2	Coordinates of a point that defines the exterior normal with the first point
THKFAC	Scale factor applied to the shell thickness after the phase transformation

Phases:

The material properties for each element are initialized using the data for the first phase. After the center of the element passes through the transition plane defined by the two points, the material properties are irreversibly changed to the second phase.

The plane is defined by two points. The first point, coordinates X1, Y1, and Z1, lies on the plane. The second point, coordinates X2, Y2, and Z2, defines the exterior normal as a unit vector in the direction from the first point to the second point.

Remarks:

This hypoelastic material model may not be stable for finite (large) strains. If large strains are expected, a hyperelastic material model, such as *MAT_002 or *MAT_217, would be more appropriate.

*MAT_ORTHOTROPIC_ELASTIC_PHASE_CHANGE

This is Material Type 217. It is a generalization of the orthotropic version of Material Type 2 for which material properties change on an element-by-element basis upon crossing a plane in space.

This material is valid *only* for shells. The stress update is incremental. The elastic constants are formulated in terms of Cauchy stress and true strain.

Phase 1 Material Parameters Card 1.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R01	EA1	EB1	EC1	PRBA1	PRCA1	PRCB1
Type	A	F	F	F	F	F	F	F

Phase 1 Material Parameters Card 2.

Card 2	1	2	3	4	5	6	7	8
Variable	GAB1	GBC1	GCA1	AOPT1				
Type	F	F	F	F				

Local Coordinate System Card 1 (phase 1).

Card 3	1	2	3	4	5	6	7	8
Variable				A11	A21	A31		
Type				F	F	F		

Local Coordinate System Card 2 (phase 1).

Card 4	1	2	3	4	5	6	7	8
Variable	V11	V21	V31	D11	D21	D31	BETA1	
Type	F	F	F	F	F	F	F	

Phase 2 Material Parameters Card 1.

Card 5	1	2	3	4	5	6	7	8
Variable		R02	EA2	EB2	EC2	PRBA2	PRCA2	PRCB2
Type		F	F	F	F	F	F	F

Phase 2 Material Parameters Card 2.

Card 6	1	2	3	4	5	6	7	8
Variable	GAB2	GBC2	GCA2					
Type	F	F	F					

Local Coordinate System Card 1 (phase 2).

Card 7	1	2	3	4	5	6	7	8
Variable				A12	A22	A32		
Type				F	F	F		

Local Coordinate System Card 2 (phase 2).

Card 8	1	2	3	4	5	6	7	8
Variable	V12	V22	V32	D12	D22	D32	BETA2	
Type	F	F	F	F	F	F	F	

Transformation Plane Card.

Card 9	1	2	3	4	5	6	7	8
Variable	X1	Y1	Z1	X2	Y2	Z2	THKFAC	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	none	1.0	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO*i*	Mass density for phase *i*
EA*i*	E_a, Young's modulus in *a*-direction for phase *i*
EB*i*	E_b, Young's modulus in *b*-direction for phase *i*
EC*i*	E_c, Young's modulus in *c*-direction phase *i* (nonzero value required but not used for shells)
PRBA*i*	ν_{ba}, Poisson's ratio in the *ba* direction for phase *i*
PRCA*i*	ν_{ca}, Poisson's ratio in the *ca* direction for phase *i*
PRCB*i*	ν_{cb}, Poisson's ratio in the *cb* direction for phase *i*
GAB*i*	G_{ab}, shear modulus in the *ab* direction for phase *i*
GBC*i*	G_{bc}, shear modulus in the *bc* direction for phase *i*
GCA*i*	G_{ca}, shear modulus in the *ca* direction for phase *i*
AOPT*i*	Material axes option for phase *i* (see *MAT_*OPTION*TROPIC for more details): EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in part (a) of Figure M2-1. The **a**-direction is from node 1 to node 2 of the element. The **b**-direction is orthogonal to the **a**-direction and is in the plane formed by nodes 1, 2, and 4. The material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.

VARIABLE	DESCRIPTION
	EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.
	EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then, **a** and **b** are rotated about **c** by an angle BETA.
	LT.0.0: |AOPT| is a coordinate system ID (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
A1*i*, A2*i*, A3*i*	Define components of the i^{th} phase's vector **a** for AOPT = 2
V1*i*, V2*i*, V3*i*	Define components of the i^{th} phase's vector **v** for AOPT = 3
D1*i*, D2*i*, D3*i*	Define components of the i^{th} phase's vector **d** for AOPT = 2
BETA*i*	Material angle of i^{th} phase in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA.
X1, Y1, Z1	Coordinates of a point on the phase transition page
X2, Y2, Z2	Coordinates of a point that defines the exterior normal with the first point
THKFAC	Scale factor applied to the shell thickness after the phase transformation.

Phases:

The material properties for each element are initialized using the data for the first phase. After the center of the element passes through the transition plane defined by the two points, the material properties are irreversibly changed to the second phase.

The plane is defined by two points. The first point, defined by the coordinates X1, Y1, and Z1, lies on the plane. The second point, defined by the coordinates X2, Y2, and Z2, define the exterior normal as a unit vector in the direction from the first point to the second point.

Material Formulation:

The material law that relates stresses to strains is defined as:

$$\mathbf{C} = \mathbf{T}^{\mathrm{T}} \mathbf{C}_L \mathbf{T}$$

where $\mathbf{T}$ is a transformation matrix, and $\mathbf{C}_L$ is the constitutive matrix defined in terms of the material constants of the orthogonal material axes, $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$. The inverse of $\mathbf{C}_L$ is defined as:

$$\mathbf{C}_L^{-1} = \begin{bmatrix} \frac{1}{E_a} & -\frac{v_{ba}}{E_b} & -\frac{v_{ca}}{E_c} & 0 & 0 & 0 \\ -\frac{v_{ab}}{E_a} & \frac{1}{E_b} & -\frac{v_{cb}}{E_c} & 0 & 0 & 0 \\ -\frac{v_{ac}}{E_a} & -\frac{v_{bc}}{E_b} & \frac{1}{E_c} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{ab}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{bc}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{ca}} \end{bmatrix}$$

where,

$$\frac{v_{ab}}{E_a} = \frac{v_{ba}}{E_b}, \frac{v_{ca}}{E_c} = \frac{v_{ac}}{E_a}, \frac{v_{cb}}{E_c} = \frac{v_{bc}}{E_b}.$$

*MAT_MOONEY-RIVLIN_PHASE_CHANGE

This is Material Type 218. It is a generalization of Material Type 27, for which material properties change on an element-by-element basis upon crossing a plane in space.

Phase 1 Card 1.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R01	PR1	A1	B1	REF		
Type	A	F	F	F	F	F		

Phase 1 Card 2.

Card 2	1	2	3	4	5	6	7	8
Variable	SGL1	SW1	ST1	LCID1				
Type	F	F	F	F				

Phase 2 Card 1.

Card 3	1	2	3	4	5	6	7	8
Variable		R02	PR2	A2	B2			
Type		F	F	F	F			

Phase 2 Card 2.

Card 4	1	2	3	4	5	6	7	8
Variable	SGL2	SW2	ST2	LCID2				
Type	F	F	F	F				

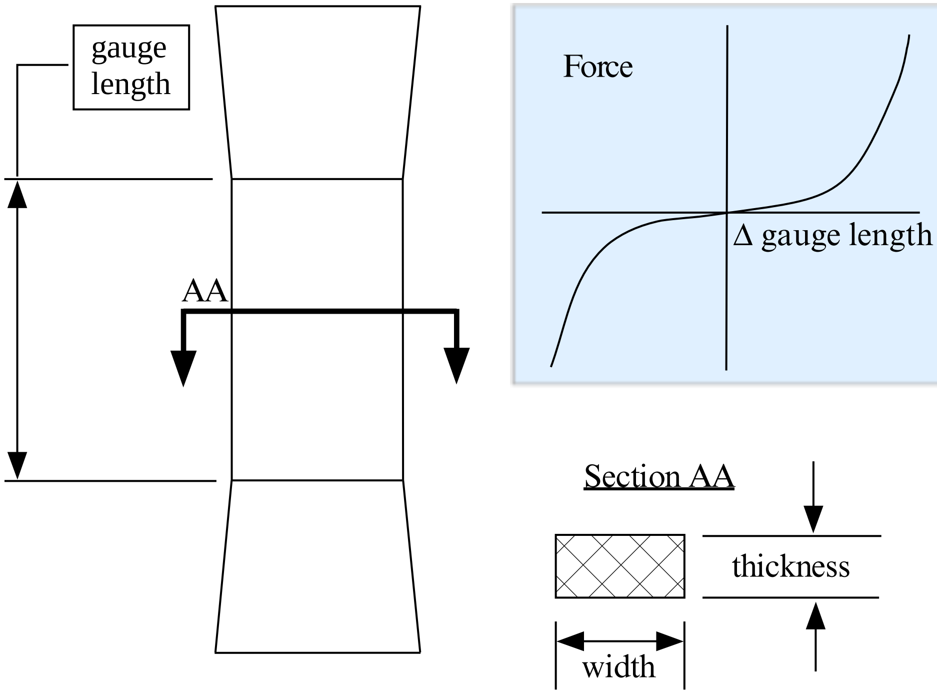

Figure M218-1. Uniaxial specimen for experimental data

Transformation Plane Card.

Card 5	1	2	3	4	5	6	7	8
Variable	X1	Y1	Z1	X2	Y2	Z2	THKFAC	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	none	1.0	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO*i*	Mass density for phase *i*.
PR*i*	Poisson's ratio where *i* indicates the phase. A value between 0.49 and 0.5 is recommended. Smaller values may not work.
A*i*	Constant for the i^{th} phase. See the literature and the equations defined in Material Formulation.

VARIABLE	DESCRIPTION
B*i*	Constant for the i^{th} phase. See the literature and the equations defined in Material Formulation.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details). EQ.0.0: Off, EQ.1.0: On.

If A = B = 0.0, then a least squares fit is computed from tabulated uniaxial data via a load curve. The following information should be defined:

VARIABLE	DESCRIPTION
SGL*i*	Specimen gauge length l_0 for the i^{th} phase, see Figure M218-1.
SW*i*	Specimen width for the i^{th} phase, see Figure M218-1.
ST*i*	Specimen thickness for the i^{th} phase, see Figure M218-1.
LCID*i*	Curve ID for the i^{th} phase (see *DEFINE_CURVE) giving the force as a function of actual change, ΔL, in the gauge length. See also Figure M218-2 for an alternative definition.
X1, Y1, Z1	Coordinates of a point on the phase transition plane.
X2, Y2, Z2	Coordinates of a point that defines the exterior normal with the first point.
THKFAC	Scale factor applied to the shell thickness after the phase transformation.

Phases:

The material properties for each element are initialized using the data for the first phase. After the center of the element passes through the transition plane defined by the two points, the material properties are irreversibly changed to the second phase.

The plane is defined by two points. The first point, defined by the coordinates X1, Y1, and Z1, lies on the plane. The second point, defined by the coordinates X2, Y2, and Z2, define the exterior normal as a unit vector in the direction from the first point to the second point.

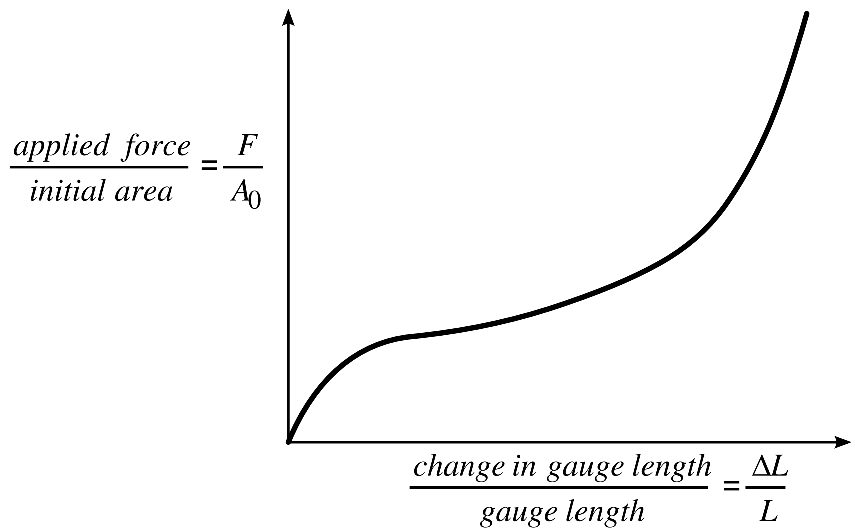

Figure M218-2 The stress as a function strain curve can be used instead of the force as a function of the change in the gauge length by setting the gauge length, thickness, and width to unity (1.0) and defining the engineering strain in place of the change in gauge length and the nominal (engineering) stress in place of the force.

Material Formulation:

The strain energy density function is defined as:

$$W = A(I-3) + B(II-3) + C\left(III^{-2}-1\right) + D(III-1)^2$$

where

$$\begin{aligned}
C &= 0.5\,A + B \\
D &= \frac{A(5v-2) + B(11v-5)}{2(1-2v)} \\
\nu &= \text{Poisson's ratio} \\
2(A+B) &= \text{shear modulus of linear elasticity} \\
I, II, III &= \text{invariants of right Cauchy-Green Tensor C.}
\end{aligned}$$

The load curve definition that provides the uniaxial data should give the change in gauge length, ΔL, as a function of the corresponding force. In compression, both the force and the change in gauge length must be specified as negative values. In tension the force and change in gauge length should be input as positive values. The principal stretch ratio in the uniaxial direction, λ_1, is then given by

$$\lambda_1 = \frac{L_0 + \Delta L}{L_0}$$

with L_0 being the initial length and L being the actual length.

Alternatively, the stress as a function strain curve can also be input by setting the gauge length, thickness, and width to unity (1.0) and defining the engineering strain in place of the change in gauge length and the nominal (engineering) stress in place of the force. See Figure M218-1.

The initialization phase performs a least squares fit to the experimental data. The d3hsp file provides a comparison between the fit and the actual input. It is a good idea to visually check to make sure it is acceptable. The coefficients A and B are also printed in the output file. It is also advised to use the material driver (see Appendix K) for checking out the material model.

*MAT_CODAM2

This is Material Type 219. This material model is the second generation of the UBC Composite Damage Model (CODAM2) for solid, shell, and thick shell elements developed at The University of British Columbia. The model is a sub-laminate-based continuum damage mechanics model for fiber reinforced composite laminates made up of transversely isotropic layers. The material model includes an optional non-local averaging and element erosion.

Card Summary:

Card 1. This card is required.

MID	RO	EA	EB		PRBA		PRCB

Card 2. This card is required.

GAB			NLAYER	R1	R2	NFREQ	

Card 3. This card is required.

XP	YP	ZP	A1	A2	A3	AOPT	

Card 4. This card is required.

V1	V2	V3	D1	D2	D3	BETA	MACF

Card 5. For each of the NLAYER layers specify on angle. Include as many cards as needed to set NLAYER values.

ANGLE1	ANGLE2	ANGLE3	ANGLE4	ANGLE5	ANGLE6	ANGLE7	ANGLE8

Card 6. This card is required.

IMATT	IFIBT	ILOCT	IDELT	SMATT	SFIBT	SLOCT	SDELT

Card 7. This card is required.

IMATC	IFIBC	ILOCC	IDELC	SMATC	SFIBC	SLOCC	SDELC

Card 8. This card is required.

ERODE	ERPAR1	ERPAR2	RESIDS				

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EA	EB		PRBA		PRCB
Type	A	F	F	F		F		F
Default	none	none	none	none		none		none

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
EA	E_a, Young's modulus in a-direction. This is the modulus along the direction of fibers.
EB	E_b, Young's modulus in b-direction. This is the modulus transverse to fibers.
PRBA	ν_{ba}, Poisson's ratio, *ba* (minor in-plane Poisson's ratio).
PRCB	ν_{cb}, Poisson's ratio, *cb* (Poisson's ratio in the plane of isotropy).

Card 2	1	2	3	4	5	6	7	8
Variable	GAB			NLAYER	R1	R2	NFREQ	
Type	F			I	F	F	I	
Default	none			0	0.0	0.0	0	

VARIABLE	DESCRIPTION
GAB	G_{ba}, Shear modulus, *ab* (in-plane shear modulus).
NLAYER	Number of layers in the sub-laminate excluding symmetry. As an example, in a $[0/45/-45/90]_{3s}$, NLAYER = 4.

VARIABLE	DESCRIPTION
R1	Non-local averaging radius
R2	Currently not used
NFREQ	Number of time steps between update of neighbor list for nonlocal smoothing. EQ.0: do only one search at the start of the calculation.

Card 3	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	AOPT	
Type	F	F	F	F	F	F	I	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0	

VARIABLE	DESCRIPTION
XP, YP, ZP	Coordinates of point p for AOPT = 1 and 4
A1, A2, A3	Components of vector **a** for AOPT = 2
AOPT	Material axes option (see *MAT_OPTIONTROPIC_ELASTIC, particularly the Material Directions section, for details): EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA. EQ.1.0: Locally orthotropic with material axes determined by a point, P, in space and the global location of the element center. This is the **a**-direction. This option is for solid elements only. EQ.2.0: Globally orthotropic with material axes determined by vectors **a** and **d**, as with *DEFINE_COORDINATE_VECTOR. EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between

VARIABLE	DESCRIPTION
	the inner surface and outher surface defined by the first four nodes and the last four nodes of the connectivity, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then, **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.
	EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, P, which define the centerline axis. This option is for solid elements only.
	LT.0.0: |AOPT| is a coordinate system ID (see *DEFINE_COORDINATE_OPTION).

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	MACF
Type	F	F	F	F	F	F	F	I
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3 and 4
D1, D2, D3	Components of vector **d** for AOPT = 2
BETA	Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. BETA may be overriden on the element card. See *ELEMENT_-SHELL_BETA or *ELEMENT_SOLID_ORTHO.
MACF	Material axes change flag for solid elements: EQ.-4: Switch material axes b and c before BETA rotation

VARIABLE	DESCRIPTION
	EQ.-3: Switch material axes *a* and *c* before BETA rotation EQ.-2: Switch material axes *a* and *b* before BETA rotation EQ.1: No change, default EQ.2: Switch material axes *a* and *b* after BETA rotation EQ.3: Switch material axes *a* and *c* after BETA rotation EQ.4: Switch material axes *b* and *c* after BETA rotation Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_-SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 4 rotates the axes. For all other values of AOPT, the material axes switch as specified by MACF, but no BETA rotation is performed.

Angle Cards. For each of the NLAYER layers specify on angle. Include as many cards as needed to set NLAYER values.

Card 5	1	2	3	4	5	6	7	8
Variable	ANGLE1	ANGLE2	ANGLE3	ANGLE4	ANGLE5	ANGLE6	ANGLE7	ANGLE8
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
ANGLE*i*	Rotation angle in degrees of the layers with respect to the material axes. Input one for each layer.

Card 6	1	2	3	4	5	6	7	8
Variable	IMATT	IFIBT	ILOCT	IDELT	SMATT	SFIBT	SLOCT	SDELT
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
IMATT	Initiation strain for damage in the matrix (transverse) under tensile conditions
IFIBT	Initiation strain for damage in the fiber (longitudinal) under tensile conditions
ILOCT	Initiation strain for the anti-locking mechanism. This parameter should be equal to the saturation strain for the fiber damage mechanism under tensile conditions.
IDELT	Not working in the current version. It can be used for visualization purposes only.
SMATT	Saturation strain for damage in the matrix (transverse) under tensile conditions
SFIBT	Saturation strain for damage in the fiber (longitudinal) under tensile conditions
SLOCT	Stuation strain for the anti-locking mechanism under tensile conditions. The recommended value for this parameter is ILOCT + 0.02.
SDELT	Not working for the current version. It can be used for visualization purposes only.

Card 7	1	2	3	4	5	6	7	8
Variable	IMATC	IFIBC	ILOCC	IDELC	SMATC	SFIBC	SLOCC	SDELC
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
IMATC	Initiation strain for damage in matrix (transverse) under compressive condition
IFIBC	Initiation strain for damage in the fiber (longitudinal) under compressive condition
ILOCC	Initiation strain for the anti-locking mechanism. This parameter should be equal to the saturation strain for the fiber damage mechanism under compressive condition.

VARIABLE	DESCRIPTION
IDELC	Initiation strain for delamination. Not working in the current version. Can be used for visualization purpose only.
SMATC	Saturation strain for damage in matrix (transverse) under compressive condition
SFIBC	Saturation strain for damage in the fiber (longitudinal) under compressive condition
SLOCC	Saturation strain for the anti-locking mechanism under compressive condition. The recommended value for this parameter is ILOCC + 0.02.
SDELC	Delamination strain. Not working in the current version. Can be used for visualization purpose only.

Card 8	1	2	3	4	5	6	7	8
Variable	ERODE	ERPAR1	ERPAR2	RESIDS				
Type	I	F	F	F				
Default	0	none	none	0.0				

VARIABLE	DESCRIPTION
ERODE	Erosion Flag (see Element Erosion in the remarks) EQ.0: erosion is turned off. EQ.1: non-local strain based erosion criterion EQ.2: local strain based erosion criterion EQ.3: use both ERODE = 1 and ERODE = 2 criteria.
ERPAR1	The erosion parameter #1 used in ERODE types 1 and 3. ERPAR1 ≥ 1.0. The recommended value of ERPAR1 is 1.2.
ERPAR2	The erosion parameter #2 used in ERODE types 2 and 3. The recommended value is five times SLOCC defined in Card 7.
RESIDS	Residual strength for layer damage

Model Description:

CODAM2 is developed for modeling the nonlinear, progressive damage behavior of laminated fiber-reinforced plastic materials. The model is based on the work by (Forghani, 2011; Forghani et al. 2011a; Forghani et al. 2011b) and is an extension of the original model, CODAM (Williams et al. 2003).

Briefly, the model uses a continuum damage mechanics approach and the following assumptions have been made in its development:

1. The material is an orthotropic medium consisting of a number of repeating units through the thickness of the laminate, called sub-laminates. For example, [0/±45/90] is in a $[0/\pm45/90]_{8S}$ laminate.

2. The nonlinear behavior of the composite sub-laminate is only caused by damage evolution. Nonlinear elastic or plastic deformations are not considered.

Formulation:

The in-plane secant stiffness of the damaged laminate is represented as the summation of the effective contributions of the layers in the laminate as shown.

$$\mathbf{A}^d = \sum t_k \mathbf{T}_k^{\mathrm{T}} \mathbf{Q}_k^d \mathbf{T}_k$$

where $\mathbf{T}_k$ is the transformation matrix for the strain vector, and $\mathbf{Q}_k^d$ is the in-plane secant stiffness of k^{th} layer in the principal orthotropic plane, and t_k is the thickness of the k^{th} layer of an n-layered laminate.

A physically-based and yet simple approach has been employed here to derive the damaged stiffness matrix. Two reduction coefficients, R_f and R_m, that represent the reduction of stiffness in the longitudinal (fiber) and transverse (matrix) directions have been employed. The shear modulus has also been reduced by the matrix reduction parameter. The major and minor Poisson's ratios have been reduced by R_f and R_m respectively. A sub-laminate-level reduction, R_L, is incorporated to avoid spurious stress locking in the damaged zone. This would lead to an effective reduced stiffness matrix $\mathbf{Q}_k^d$. The reduction coefficients are equal to 1 in the undamaged condition and gradually decrease to 0 for a saturated damage condition.

$$\mathbf{Q}_k^d = R_L \begin{bmatrix} \frac{(R_f)_k E_1}{1-(R_f)_k (R_m)_k \nu_{12}\nu_{21}} & \frac{(R_f)_k (R_m)_k \nu_{12} E_2}{1-(R_f)_k (R_m)_k \nu_{12}\nu_{21}} & 0 \\ \frac{(R_f)_k (R_m)_k \nu_{12} E_2}{1-(R_f)_k (R_m)_k \nu_{12}\nu_{21}} & \frac{(R_m)_k E_2}{1-(R_f)_k (R_m)_k \nu_{12}\nu_{21}} & 0 \\ 0 & 0 & (R_m)_k G_{12} \end{bmatrix} = \mathbf{Q}_k^{d\,\mathrm{T}} \ ,$$

where E_1, E_2, ν_{12}, ν_{21}, and G_{12} are the elastic constants of the lamina.

Damage Evolution:

In CODAM2, the evolution of damage mechanisms is expressed in terms of equivalent strain parameters. The equivalent strain function that governs the fiber stiffness reduction parameter is written in terms of the longitudinal normal strains by

$$\varepsilon_{f,k}^{\text{eq}} = \varepsilon_{11,k}, \qquad k = 1, \ldots, n$$

The equivalent strain function that governs the matrix stiffness reduction parameter is written in an interactive form in terms of the transverse and shear components of the local strain:

$$\varepsilon_{m,k}^{\text{eq}} = \text{sign}(\varepsilon_{22,k}) \sqrt{(\varepsilon_{22,k})^2 + \left(\frac{\gamma_{12,k}}{2}\right)^2}, \qquad k = 1, \ldots, n$$

The sign of the transverse normal strain plays a very important role in the initiation and growth of damage since it indicates the compressive or tensile nature of the transverse stress. Therefore, the equivalent strain for the matrix damage carries the sign of the transverse normal strain.

Evolution of the overall damage mechanism (anti-locking) is written in terms of the maximum principal strains:

$$\varepsilon_L^{\text{eq}} = \max[\text{prn}(\varepsilon)] \ .$$

Within the framework of non-local strain-softening formulations adopted here, all damage modes, be it intra-laminar (i.e. fiber and matrix damage) or overall sub-laminate modes are considered to be a function of the non-local (averaged) equivalent strain defined as:

$$\bar{\varepsilon}_\alpha^{\text{eq}} = \int_{\Omega_{\mathbf{X}}} \varepsilon_\alpha^{\text{eq}}(\mathbf{x}) w_\alpha(\mathbf{X} - \mathbf{x}) d\Omega \ ,$$

where the subscript α denotes the mode of damage: fiber ($\alpha = f$) and matrix ($\alpha = m$) damage in each layer, k, within the sub-laminate or associated with the overall sub-laminate, namely, locking ($\alpha = L$). Thus, for a given sub-laminate with n layers, $\varepsilon_\alpha^{\text{eq}}$ and $\bar{\varepsilon}_\alpha^{\text{eq}}$ are vectors of size $2n + 1$. $\mathbf{X}$ represents the position vector of the original point of interest and $\mathbf{x}$ denotes the position vector of all other points (Gauss points) in the averaging zone denoted by Ω. In classical isotropic non-local averaging approach, this zone is taken to be spherical (or circular in 2D) with a radius of r (named R1 in the material input card). The parameter, r, which affects the size of the averaging zone, introduces a length scale into the model that is linked directly to the predicted size of the damage zone. Averaging is done with a bell-shaped weight function, w_α, evaluated by

$$w_\alpha = \left[1 - \left(\frac{d}{r}\right)^2\right]^2 \ ,$$

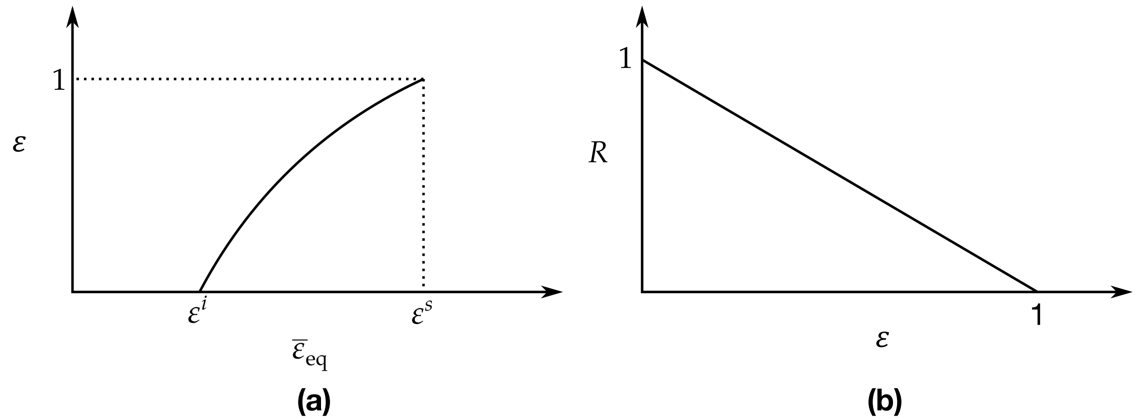

Figure M219-1. Illustrations of (a) damage parameter and (b) reduction parameter.

where d is the distance from the integration point of interest to another integration point within the averaging zone.

The damage parameters, ω_α, are calculated as a function of the corresponding averaged equivalent strains. In CODAM2 the damage parameters are assumed to grow as a hyperbolic function of the damage potential (non-local equivalent strains) such that when used in conjunction with stiffness reduction factors that vary linearly with the damage parameters they result in a linear strain-softening response (or a bilinear stress-strain curve) for each mode of damage

$$\omega_\alpha = \frac{\left(\left|\bar{\varepsilon}_\alpha^{\text{eq}}\right| - \varepsilon_\alpha^i\right)}{\left(\varepsilon_\alpha^s - \varepsilon_\alpha^i\right)} \frac{\varepsilon_\alpha^s}{\left|\bar{\varepsilon}_\alpha^{eq}\right|}, \qquad \left|\bar{\varepsilon}_\alpha^{\text{eq}}\right| - \varepsilon_\alpha^i > 0$$

where superscripts i and s denote, respectively, the damage initiation and saturation values of the strain quantities to which they are assigned. The initiation and saturation parameters are defined in Cards 6 and 7. Damage is considered to be a monotonically increasing function of time, t, such that

$$\omega_\alpha = \max_{\tau < t}(\omega_\alpha^\tau) \ ,$$

where ω_α^t is the value of ω_α for the current time (load state), and ω_α^τ represents the state of damage at previous times $\tau \le t$.

Damage is applied by scaling the layer stress by reduction parameters

$$R_\alpha = 1 - \omega_\alpha$$

where $\alpha = f$ and $\alpha = m$. The layer stresses are summed and then then scaled by reduction parameter

$$R_L = 1 - \omega_L \ .$$

Figures M219-1 (a) and (b) show the relationship between the damage and reduction parameters

If the parameter RESIDS > 0, damage in the layers is limited such that

$$R_f = \max(\mathrm{RESIDS}, 1 - \omega_f)$$
$$R_m = \max(\mathrm{RESIDS}, 1 - \omega_m)$$

Element Erosion:

When ERODE > 0, an erosion criterion is checked at each integration point. Shell elements and thick shell elements will be deleted when the erosion criterion has been met at all integration points. Brick elements will be deleted when the erosion criterion is met at any of the integration points. For ERODE = 1, the erosion criterion is met when maximum principal strain exceeds either SLOCT × ERPAR1 for elements in tension, or SLOCC × ERPAR1 for elements in compression. Elements are in tension when the magnitude of the first principal strain is greater than the magnitude of the third principal strain and in compression when the third principal strain is larger. When R1 > 0, the ERODE = 1 criterion is checked using the non-local (averaged) principal strain. For ERODE = 2, the erosion criterion is met when the local (non-averaged) maximum principal strain exceeds ERPAR2. For ERODE = 3, both of these erosion criteria are checked. For visualization purposes, the ratio of the maximum principal strain over the limit is stored in the location of plastic strain which is written by default to the elout and d3plot files.

History Variables:

History variables for CODAM2 are enumerated in the following tables. To include them in the d3plot database, use NEIPH (solids) or NEIPS (shells) on *DATABASE_EXTENT_-BINARY. For solid elements, add 4 to the variable numbers in the table because the first 6 history variables are reserved.

Damage parameters

History Variable #	Description
3	Overall (anti-locking) Damage.
4	Delamination Damage (for visualization only)
5	Fiber damage in the first layer
6	Matrix damage in the first layer
7	Fiber damage in the second layer
8	Matrix damage in the second layer
⋮	⋮
3 + 2 × NLAYER	Fiber damage in the last layer

History Variable #	Description
4 + 2 × NLAYER	Matrix damage in the last layer

Equivalent Strains used to evaluate damage (averaged if R1 > 0)

History Variable #	Description
5 + 2 × NLAYER	$\varepsilon_R^{\mathrm{eq}}$
6 + 2 × NLAYER	$\varepsilon_{f,1}^{\mathrm{eq}}$
7 + 2 × NLAYER	$\varepsilon_{m,1}^{\mathrm{eq}}$
8 + 2 × NLAYER	$\varepsilon_{f,2}^{\mathrm{eq}}$
9 + 2 × NLAYER	$\varepsilon_{m,2}^{\mathrm{eq}}$
$\vdots$	$\vdots$
4 + 4 × NLAYER	$\varepsilon_{f,n}^{\mathrm{eq}}$
5 + 4× NLAYER	$\varepsilon_{f,n}^{\mathrm{eq}}$

Total Strain

History Variable #	Description
6 + 4 × NLAYER	ε_x
7 + 4 × NLAYER	ε_y
8 + 4 × NLAYER	ε_z
9 + 4 × NLAYER	γ_{xy}
10 + 4 × NLAYER	γ_{yz}
11 + 4 × NLAYER	γ_{zx}

*MAT_RIGID_DISCRETE

This is Material Type 220. It is a rigid material for shells or solids. Unlike *MAT_020, a *MAT_220 part can be discretized into multiple disjoint pieces and have each piece behave as an independent rigid body. The inertia properties for the disjoint pieces are determined directly from the finite element discretization.

Nodes of a *MAT_220 part cannot be shared by any other rigid part. A *MAT_220 part may share nodes with deformable structural and solid elements.

This material option can be used to model granular material where the grains interact through an automatic single surface contact definition. Another possible use includes modeling bolts as rigid bodies where the bolts belong to the same part ID. This model eliminates the need to represent each rigid piece with a unique part ID.

Card	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR				
Type	A	F	F	F				
Default	none	none	none	none				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio

*MAT_ORTHOTROPIC_SIMPLIFIED_DAMAGE

This is Material Type 221. It is an orthotropic material with optional simplified damage and optional failure for composites. This model is valid for 3D solid elements, for thick shell formulations 3, 5, and 7, and for SPH elements. The elastic behavior is the same as *MAT_022. Nine damage variables are defined such that damage is different in tension and compression. These damage variables are applicable to E_a, E_b, E_c, G_{ab}, G_{bc} and G_{ca}. In addition, nine failure criteria on strains are available. When failure occurs, elements are deleted (erosion). Failure depends on the number of integration points failed through the element. See the material description below.

Card Summary:

Card 1. This card is required.

MID	RO	EA	EB	EC	PRBA	PRCA	PRCB

Card 2. This card is required.

GAB	GBC	GCA		AOPT	MACF		

Card 3. This card is required.

XP	YP	ZP	A1	A2	A3		

Card 4. This card is required.

V1	V2	V3	D1	D2	D3	BETA	

Card 5. This card is required.

NERODE	NDAM	EPS1TF	EPS2TF	EPS3TF	EPS1CF	EPS2CF	EPS3CF

Card 6. This card is required.

EPS12F	EPS23F	EPS13F	EPSD1T	EPSC1T	CDAM1T	EPSD2T	EPSC2T

Card 7. This card is required.

CDAM2T	EPSD3T	EPSC3T	CDAM3T	EPSD1C	EPSC1C	CDAM1C	EPSD2C

Card 8. This card is required.

EPSC2C	CDAM2C	EPSD3C	EPSC3C	CDAM3C	EPSD12	EPSC12	CDAM12

Card 9. This card is required.

EPSD23	EPSC23	CDAM23	EPSD31	EPSC31	CDAM31		

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EA	EB	EC	PRBA	PRCA	PRCB
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
EA	E_a, Young's modulus in a-direction
EB	E_b, Young's modulus in b-direction
EC	E_c, Young's modulus in c-direction
PRBA	ν_{ba}, Poisson ratio
PRCA	ν_{ca}, Poisson ratio
PRCB	ν_{cb}, Poisson ratio

Card 2	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA		AOPT	MACF		
Type	F	F	F		F	I		
Default	none	none	none		0.0	0		

VARIABLE	DESCRIPTION
GAB	G_{ab}, Shear modulus
GBC	G_{bc}, Shear modulu
GCA	G_{ca}, Shear modulus
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details): EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. EQ.1.0: Locally orthotropic with material axes determined by a point, P, in space and the global location of the element center; this is the **a**-direction. This option is for solid elements only. EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF. EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, P, which define the centerline axis. This option is for solid elements only. LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*).

VARIABLE	DESCRIPTION
MACF	Material axes change flag for solid elements: EQ.-4: Switch material axes b and c before BETA rotation EQ.-3: Switch material axes a and c before BETA rotation EQ.-2: Switch material axes a and b before BETA rotation EQ.1: No change, default EQ.2: Switch material axes a and b after BETA rotation EQ.3: Switch material axes a and c after BETA rotation EQ.4: Switch material axes b and c after BETA rotation Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_-SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 4 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.

Card 3	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		
Default	0.0	0.0	0.0	0.0	0.0	0.0		

VARIABLE	DESCRIPTION
XP, YP, ZP	Coordinates of point p for AOPT = 1 and 4
A1, A2, A3	Components of vector **a** for AOPT = 2

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3 and 4
D1, D2, D3	Components of vector **d** for AOPT = 2
BETA	Material angle in degrees for AOPT = 3. It may be overridden on the element card, see *ELEMENT_SOLID_ORTHO.

Card 5	1	2	3	4	5	6	7	8
Variable	NERODE	NDAM	EPS1TF	EPS2TF	EPS3TF	EPS1CF	EPS2CF	EPS3CF
Type	I	I	F	F	F	F	F	F
Default	0	0	10^{20}	10^{20}	10^{20}	-10^{20}	-10^{20}	-10^{20}

VARIABLE **DESCRIPTION**

NERODE

Element erosion flag. For multi-integration point elements, each of the failure strains mentioned below for NERODE ≥ 2 need only occur in one integration point to trigger element erosion. For NERODE values 6 to 11, which require more than one failure strain be reached, those failure strains need not occur in the same integration point.

EQ.0: No erosion (default)

EQ.1: Erosion occurs when one failure strain is reached in all integration points.

EQ.2: Erosion occurs when one failure strain is reached.

EQ.3: Erosion occurs when a tension or compression failure strain in the *a*-direction is reached.

VARIABLE	DESCRIPTION
	EQ.4: Erosion occurs when as a tension or compression failure strain in the *b*-direction is reached.
	EQ.5: Erosion occurs when a tension or compression failure strain in the *c*-direction is reached.
	EQ.6: Erosion occurs when tension or compression failure strain in both the *a*- and *b*-directions are reached.
	EQ.7: Erosion occurs when tension or compression failure strain in both the *b*- and *c*-directions are reached.
	EQ.8: Erosion occurs when tension or compression failure strain in both the *a*- and *c*-directions are reached.
	EQ.9: Erosion occurs when tension or compression failure strain in all 3 directions are reached.
	EQ.10: Erosion occurs when tension or compression failure strain in both the *a*- and *b*-directions is reached and either of the out-of-plane failure shear strains (*bc* or *ac*) is reached.
	EQ.11: Erosion occurs when tension failure strain in either the *a*- or *b*-directions is reached and either of the out-of-plane failure shear strains (*bc* or *ac*) is reached.
NDAM	Damage flag: EQ.0: No damage (default) EQ.1: Damage in tension only (null for compression) EQ.2: Damage in tension and compression
EPS1TF	Failure strain in tension along the *a*-direction
EPS2TF	Failure strain in tension along the *b*-direction
EPS3TF	Failure strain in tension along the *c*-direction
EPS1CF	Failure strain in compression along the *a*-direction
EPS2CF	Failure strain in compression along the *b*-direction
EPS3CF	Failure strain in compression along the *c*-direction

Card 6	1	2	3	4	5	6	7	8
Variable	EPS12F	EPS23F	EPS13F	EPSD1T	EPSC1T	CDAM1T	EPSD2T	EPSC2T
Type	F	F	F	F	F	F	F	F
Default	10^{20}	10^{20}	10^{20}	0.	0.	0.	0.	0.

VARIABLE	DESCRIPTION
EPS12F	Failure shear strain in the *ab*-plane
EPS23F	Failure shear strain in the *bc*-plane
EPS13F	Failure shear strain in the *ac*-plane
EPSD1T	Damage threshold in tension along the *a*-direction, ε_{1t}^s
EPSC1T	Critical damage threshold in tension along the *a*-direction, ε_{1t}^c
CDAM1T	Critical damage in tension along the *a*-direction, D_{1t}^c
EPSD2T	Damage threshold in tension along the *b*-direction, ε_{2t}^s
EPSC2T	Critical damage threshold in tension along the *b*-direction, ε_{2t}^c

Card 7	1	2	3	4	5	6	7	8
Variable	CDAM2T	EPSD3T	EPSC3T	CDAM3T	EPSD1C	EPSC1C	CDAM1C	EPSD2C
Type	I	I	F	F	F	F	F	F
Default	0.	0.	0.	0.	0.	0.	0.	0.

VARIABLE	DESCRIPTION
CDAM2T	Critical damage in tension along the *b*-direction, D_{2t}^c
EPSD3T	Damage threshold in tension along the *c*-direction, ε_{3t}^s
EPSC3T	Critical damage threshold in tension along the *c*-direction, ε_{3t}^c

VARIABLE	DESCRIPTION
CDAM3T	Critical damage in tension along the *c*-direction, D_{3t}^c
EPSD1C	Damage threshold in compression along the *a*-direction, ε_{1c}^s
EPSC1C	Critical damage threshold in compression along the *a*-direction, ε_{1c}^c
CDAM1C	Critical damage in compression along the *a*-direction, D_{1c}^c
EPSD2C	Damage threshold in compression along the *b*-direction, ε_{2c}^s

Card 8	1	2	3	4	5	6	7	8
Variable	EPSC2C	CDAM2C	EPSD3C	EPSC3C	CDAM3C	EPSD12	EPSC12	CDAM12
Type	F	F	F	F	F	F	F	F
Default	0.	0.	0.	0.	0.	0.	0.	0.

VARIABLE	DESCRIPTION
EPSC2C	Critical damage threshold in compression along the *b*-direction, ε_{2c}^c
CDAM2C	Critical damage in compression along the *b*-direction, D_{2c}^c
EPSD3C	Damage threshold in compression along the *c*-direction, ε_{3c}^s
EPSC3C	Critical damage threshold in compression along the *c*-direction, ε_{3c}^c
CDAM3C	Critical damage in compression along the *c*-direction, D_{3c}^c
EPSD12	Damage threshold for shear in the *ab*-plane, ε_{12}^s
EPSC12	Critical damage threshold for shear in the *ab*-plane, ε_{12}^c
CDAM12	Critical damage for shear in the *ab*-plane, D_{12}^c

Card 9	1	2	3	4	5	6	7	8
Variable	EPSD23	EPSC23	CDAM23	EPSD31	EPSC31	CDAM31		
Type	F	F	F	F	F	F		
Default	0.	0.	0.	0.	0.	0.		

VARIABLE	DESCRIPTION
EPSD23	Damage threshold for shear in the *bc*-plane, ε_{23}^{s}
EPSC23	Critical damage threshold for shear in the *bc*-plane, ε_{23}^{c}
CDAM23	Critical damage for shear in the *bc*-plane, D_{23}^{c}
EPSD31	Damage threshold for shear in the *ac*-plane, ε_{31}^{s}
EPSC31	Critical damage threshold for shear in the *ac*-plane, ε_{31}^{c}
CDAM31	Critical damage for shear in the *ac*-plane, D_{31}^{c}

Remarks:

If $\varepsilon_k^c < \varepsilon_k^s$, no damage is considered. Failure occurs only when failure strain is reached.

Failure can occur along the 3 orthotropic directions, in tension, in compression and for shear behavior. Nine failure strains drive the failure. When failure occurs, elements are deleted (erosion). Under the control of the NERODE flag, failure may occur either when only one integration point has failed, when several integration points have failed or when all integrations points have failed.

Damage applies to the 3 Young's moduli and the 3 shear moduli. Damage is different for tension and compression. Nine damage variables are used: d_{1t}, d_{2t}, d_{3t}, d_{1c}, d_{2c}, d_{3c}, d_{12}, d_{23}, d_{13}. The damaged flexibility matrix is:

$$
\text{-}S^{\mathrm{dam}} = \begin{pmatrix}
\frac{1}{E_a(1-d_{1[t,c]})} & \frac{-v_{ba}}{E_b} & \frac{-v_{ca}}{E_c} & 0 & 0 & 0 \\
\frac{-v_{ba}}{E_b} & \frac{1}{E_b(1-d_{2[t,c]})} & \frac{-v_{cb}}{E_c} & 0 & 0 & 0 \\
\frac{-v_{ca}}{E_c} & \frac{-v_{cb}}{E_c} & \frac{1}{E_c(1-d_{3[t,c]})} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{ab}(1-d_{12})} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{bc}(1-d_{23})} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{ca}(1-d_{31})}
\end{pmatrix}
$$

The nine damage variables are calculated as follows:

$$
d_k = \max\left(d_k, D_k^c \left\langle \frac{\varepsilon_k - \varepsilon_k^s}{\varepsilon_k^c - \varepsilon_k^s} \right\rangle_+ \right)
$$

with $k = 1t, 2t, 3t, 1c, 2c, 3c, 12, 23, 31.$

$$
\langle \quad \rangle_+ \text{ is the positive part: } \langle x \rangle_+ = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}.
$$

Damage in compression may be deactivated with the NDAM flag. In this case, damage in compression is null, and only damage in tension and for shear behavior are taken into account.

The nine damage variables may be post-processed through additional variables. The number of additional variables for solids written to the d3plot and d3thdt databases is input by the optional *DATABASE_EXTENT_BINARY card as variable NEIPH. These additional variables are tabulated below:

History Variable	Description	Value	LS-PrePost History Variable
d_{1t}	damage in traction along *a*	0 - no damage $0 < d_k < D_k^c$ - damage	plastic strain
d_{2t}	damage in traction along *b*		1
d_{3t}	damage in traction along *c*		2
d_{1c}	damage in compression along *a*		3
d_{2c}	damage in compression along *b*		4
d_{3c}	damage in compression along *c*		5
d_{12}	shear damage in *ab*-plane		6
d_{23}	shear damage in *bc*-plane		7
d_{13}	shear damage in *ac*-plane		8

The first damage variable is stored in the place of effective plastic strain. The eight other damage variables may be plotted in LS-PrePost as element history variables.

*MAT_TABULATED_JOHNSON_COOK_{OPTION}

This is Material Type 224. This type models an elasto-viscoplastic material with arbitrary stress versus strain curve(s) and arbitrary strain rate dependency. Plastic heating causes the temperature to increase adiabatically and material softening. Optional plastic failure strain can be a function of triaxiality, strain rate, temperature, and/or element size. Please take careful note of the sign convention for triaxiality, as illustrated in Figure M224-1. This material model resembles the original Johnson-Cook material (see *MAT_015) but with the possibility of general tabulated input parameters.

An equation of state (*EOS) is optional for solid elements, tshell formulations 3 and 5, and 2D continuum elements. It is invoked by setting EOSID to a nonzero value in *PART. If an equation of state is used, the material model gives only the deviatoric stresses, and the equation of state provides the pressure.

Available options include:

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LOG_INTERPOLATION

With LOG_INTERPOLATION, the strain rate effect in table LCK1 is interpolated with logarithmic interpolation.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	CP	TR	BETA	NUMINT
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	1.0	1.0

Card 2	1	2	3	4	5	6	7	8
Variable	LCK1	LCKT	LCF	LCG	LCH	LCI	BFLG	
Type	I	I	I	I	I	I	I	
Default	0	0	0	0	0	0	0	

This card is optional.

Card 3	1	2	3	4	5	6	7	8
Variable	FAILOPT	NUMAVG	NCYFAIL	ERODE	LCPS			
Type	I	I	I	I	I			
Default	0	1	1	0	0			

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus: GT.0.0: Constant value is used. LT.0.0: -E gives curve ID for temperature dependence.
PR	Poisson's ratio
CP	Specific heat (superseded by heat capacity in *MAT_THERMAL_*OPTION* if a coupled thermal/structural analysis)
TR	Room temperature
BETA	Fraction of plastic work converted into heat (supersedes FWORK in *CONTROL_THERMAL_SOLVER if a coupled thermal/structural analysis): EQ.0.0: Defaults to 1.0. GT.0.0: Constant value is used. LT.0.0: -BETA gives a curve ID for strain rate dependence, a table ID for strain rate and temperature dependence, a 3-dimensional table ID for temperature (TABLE_3D), strain rate (TABLE) and plastic strain (CURVE) dependence, or a 4-dimensional table ID for triaxiality (TABLE_4D), temperature (TABLE_3D), strain rate (TABLE) and plastic strain (CURVE) dependence. Please see the description of BFLG below for an alternative interpretation of TABLE_3D arguments.

VARIABLE	DESCRIPTION
NUMINT	GT.0.0: Number of integration points that must fail before the element is deleted. Available for shells and solids. LT.0.0: -NUMINT is the percentage of integration points/layers that must fail before the shell element fails. For fully integrated shells, a layer fails if one integration point fails. Then, the given percentage of layers must fail before the element fails. It is only available for shells. EQ.-200: Turns off erosion for shells and solids. Not recommended unless used in conjunction with *CONSTRAINED_TIED_NODES_FAILURE.
LCK1	Load curve ID, table ID, or 3D table ID. The load curve gives effective stress as a function of effective plastic strain. The table gives for each plastic strain rate value a load curve ID specifying the (isothermal) effective stress as a function of effective plastic strain for that rate. As in *MAT_024, natural logarithmic strain rates can be used by setting the *first* strain rate to a negative value. See Remark 1. If referring to a three-dimensional table ID, the yield stress can be a function of temperature (TABLE_3D), plastic strain rate (TABLE), and plastic strain (CURVE). LCKT is ignored in that case.
LCKT	Table ID defining for each temperature value a load curve ID giving the (quasi-static) effective stress versus effective plastic strain for that temperature. See Remark 1.
LCF	Load curve ID or table ID. The load curve ID defines plastic failure strain (or scale factor – see Remark 2) as a function of triaxiality. The table ID defines for each Lode parameter a load curve ID giving the plastic failure strain versus triaxiality for that Lode parameter. See Remark 2 for a description of the combination of LCF, LCG, LCH, and LCI.
LCG	Load curve ID defining plastic failure strain (or scale factor – see Remark 2) as a function of plastic strain rate (*The curve should not extrapolate to zero or failure may occur at low strain*). If the *first* abscissa value in the curve corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for all abscissa values. See Remark 2 for a description of the combination of LCF, LCG, LCH, and LCI.

VARIABLE	DESCRIPTION
LCH	Load curve ID defining plastic failure strain (or scale factor – see Remark 2) as a function of temperature. See Remark 2 for a description of the combination of LCF, LCG, LCH, and LCI.
LCI	Load curve ID, table ID, or table_3D ID. The load curve ID defines plastic failure strain (or scale factor – see Remark 2) as a function of element size. The table ID defines for each triaxiality a load curve ID giving the plastic failure strain versus element size for that triaxiality. If referring to a three-dimensional table ID, plastic failure strain can be a function of the Lode parameter (TABLE_3D), triaxiality (TABLE), and element size (CURVE). See Remark 2 for a description of the combination of LCF, LCG, LCH, and LCI.
BFLG	Flag for treatment of case BETA < 0 with TABLE_3D (available for solid elements only): EQ.0: Dissipation factor β is a function of temperature, strain rate, and plastic strain (as described above). EQ.1: Dissipation factor β is a function of maximum shear strain (TABLE_3D), strain rate (TABLE), and element size (CURVE).
FAILOPT	Flag for additional failure criterion F_2 (see Remark 3). EQ.0: Off (default) EQ.1: On
NUMAVG	Number of time steps for the running average of the plastic failure strain in the additional failure criterion. The default is 1 (no averaging). See Remark 3.
NCYFAIL	Number of time steps that the additional failure criterion must be met before element deletion. The default is 1. See Remark 3.
ERODE	Erosion flag (only for solid elements): EQ.0: Default, element erosion is allowed. EQ.1: Element does not erode; deviatoric stresses set to zero when element fails. EQ.2: Element does not erode. The stress response is uncoupled from material damage. We intend this option for forging simulations with 3D r-adaptivity.
LCPS	Table ID with first principal stress limit as a function of plastic

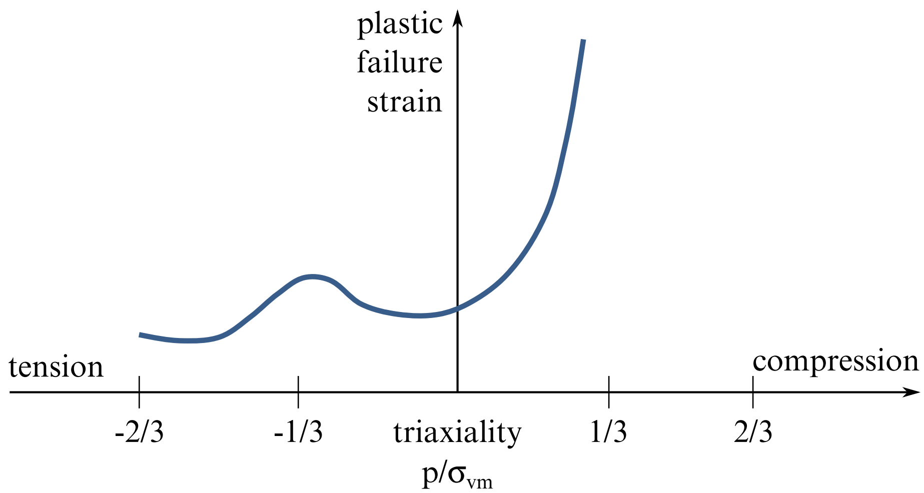

Figure M224-1. Typical failure curve for metal sheet, modeled with shell elements.

VARIABLE	DESCRIPTION
	strain (curves) and plastic strain rate (table). This option is for post-processing purposes only and gives an indication of areas in the structure where failure is likely to occur. History variable #17 is 1.0 for integration points that have exceeded the limit; otherwise, it has a value of 0.0.

Remarks:

1. **Flow stress.** The flow stress σ_y is expressed as a function of plastic strain ε_p, plastic strain rate $\dot{\varepsilon}_p$ and temperature T through the following formula (using load curves/tables LCK1 and LCKT):

$$s_y = k_1(\varepsilon_p, \dot{\varepsilon}_p)\frac{k_t(\varepsilon_p, T)}{k_t(\varepsilon_p, T_R)}$$

Note that T_R is a material parameter and should correspond to the temperature used when performing the room temperature tensile tests. If simulations are to be performed with an initial temperature T_I deviating from T_R, then this temperature should be set using *INITIAL_STRESS_SOLID/SHELL by setting history variable #14 for solid elements or history variable #10 for shell elements.

2. **Plastic failure strain.** Optional plastic failure strain is defined as a function of triaxiality p/σ_{vm}, Lode parameter, plastic strain rate $\dot{\varepsilon}_p$, temperature T and initial element size l_c (square root of element area for shells and volume over maximum area for solids) by

$$\varepsilon_{\mathrm{pf}} = f\left(\frac{p}{\sigma_{\mathrm{vm}}}, \frac{27J_3}{2\sigma_{\mathrm{vm}}^3}\right) g(\dot{\varepsilon}_p) h(T) i\left(l_c, \frac{p}{\sigma_{\mathrm{vm}}}\right)$$

using load curves/tables LCF, LCG, LCH, and LCI. If more than one of these four variables, LCF, LCG, LCH, and LCI, are defined, be aware that the net plastic failure strain is essentially the product of multiple functions, as shown in the above equation. This means that one and only one of the variables LCF, LCG, LCH, and LCI can point to a curve(s) that has plastic strain along the curve ordinate. The remaining nonzero variable(s) LCF, LCG, LCH, and LCI should point to a curve(s) that has a unitless scaling factor along the curve ordinate.

A typical failure curve LCF for a metal sheet, modeled with shell elements is shown in Figure M224-1. Triaxiality should be monotonically increasing in this curve. A reasonable range for triaxiality is -2/3 to 2/3 if shell elements are used (plane stress). For 3-dimensional stress states (solid elements), the possible range of triaxiality goes from -∞ to +∞, but to get a good resolution in the internal load curve discretization (depending on parameter LCINT of *CONTROL_SOLUTION) you should define lower limits, e.g. -1 to 1 if LCINT = 100 (default).

3. **Failure criterion.** The default failure criterion of this material model depends on plastic strain evolution $\dot{\varepsilon}_p$ and on plastic failure strain $\varepsilon_{\mathrm{pf}}$ and is obtained by accumulation over time:

$$F = \int \frac{\dot{\varepsilon}_p}{\varepsilon_{pf}} dt$$

where element erosion takes place when $F \geq 1$. This accumulation provides load-path-dependent treatment of failure. The value of F is stored as history variable #8 for shells and #12 for solids.

An additional, load-path independent, failure criterion can be invoked by setting FAILOPT = 1, where the current state of plastic strain is used:

$$F_2 = \frac{\varepsilon_p}{\varepsilon_{pf}}$$

Two additional parameters can be used as countermeasures against stress oscillations for this failure criterion. With NUMAVG active, plastic failure strain is averaged over NUMAVG time steps for the F_2 criterion. The value of F_2, taking into account any averaging per NUMAVG, is stored as history variable #14 for shells and #16 for solids. NUMAVG cannot exceed 30. NCYFAIL defines the number of time steps that $F_2 \geq 1$ must be met before element deletion takes place. The number of time steps that $F_2 \geq 1$ is stored as history variable #15 for shells and #19 for solids.

4. **Change in temperature.** Temperature increase is caused by plastic work

$$T = T_R + \frac{\beta}{C_p \rho} \int \sigma_y \dot{\varepsilon}_p dt$$

with room temperature T_R, dissipation factor β, specific heat C_p, and density ρ. If a coupled thermal/structural analysis is performed, temperatures from the thermal solver are used. In that case, the thermal solver receives the plastic work that is to be converted to heat as an additional volumetric heat source. If no dissipation factor is defined, the value of FWORK in *CONTROL_THERMAL_-SOLVER is used.

5. **Failure when used with *CONSTRAINED_TIED_NODES_WITH_FAILURE.** For *CONSTRAINED_TIED_NODES_WITH_FAILURE, the failure is based on the damage instead to the plastic strain.

6. **History variables.** History variables may be post-processed through additional variables. The number of additional variables for shells/solids written to the d3plot and d3thdt databases is input by the optional *DATABASE_EXTENT_-BINARY card as variable NEIPS/NEIPH. Specifically, when used with shell element type 14 or 15, history variable output will be as a solid element, not a shell element. The relevant additional variables of this material model are tabulated below:

History Variable #	Description for Shell Elements
1	Plastic strain rate
7	Plastic work
8	Ratio of plastic strain to plastic failure strain
9	Element size
10	Temperature
11	Plastic failure strain
12	Triaxiality
16	Fraction of plastic work to heat
17	LCPS: critical value

History Variable #	Description for Solid Elements
5	Plastic strain rate
8	Plastic failure strain
9	Triaxiality
10	Lode parameter
11	Plastic work
12	Ratio of plastic strain to plastic failure strain

History Variable #	Description for Solid Elements
13	Element size
14	Temperature
17	LCPS: critical value
18	Fraction of plastic work to heat

MAT_TABULATED_JOHNSON_COOK_GYS_*{OPTION}

This is Material Type 224_GYS. It is an isotropic, elastic-plastic material law with a J3-dependent yield surface. This material considers tensile/compressive asymmetry in the material response, which is essential for HCP metals. It is available for solid elements.

Available options include:

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LOG_INTERPOLATION

With LOG_INTERPOLATION, the strain rate effect in table LCK1 (Card 2) is interpolated with logarithmic interpolation.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	CP	TR	BETA	NUMINT
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	1.0	1.0

Card 2	1	2	3	4	5	6	7	8
Variable	LCK1	LCKT	LCF	LCG	LCH	LCI		
Type	I	I	I	I	I	I		
Default	0	0	0	0	0	0		

Card 3	1	2	3	4	5	6	7	8
Variable	LCCR	LCCT	LCSR	LCST	IFLAG	SFIEPM	NITER	
Type	I	I	I	I	I	F	I	
Default	0	0	0	0	0	1	100	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus: GT.0.0: Constant value is used. LT.0.0: Temperature-dependent Young's modulus given by load curve ID = -E
PR	Poisson's ratio
CP	Specific heat
TR	Room temperature
BETA	Fraction of plastic work converted into heat (superseded by FWORK in *CONTROL_THERMAL_SOLVER if a coupled thermal/structural analysis): GT.0.0: Constant value is used. LT.0.0: -BETA gives a load curve ID for strain rate dependence, a table ID for strain rate and temperature dependence, a 3-dimensional table ID for temperature (TABLE_3D), strain rate (TABLE) and plastic strain (CURVE) dependence, or a 4-dimensional table ID for triaxiality (TABLE_4D), temperature (TABLE_3D), strain rate (TABLE), and plastic strain (CURVE) dependence.
NUMINT	Number of integration points which must fail before the element is deleted. EQ.-200: Turns off erosion for solids. Not recommended unless used in conjunction with *CONSTRAINED_TIED_NODES_FAILURE.
LCK1	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) effective stress as a function of effective plastic strain for that rate.
LCKT	Table ID defining for each temperature value a load curve ID giving the (qu asi-static) effective stress as a function of effective plastic strain for that temperature.

VARIABLE	DESCRIPTION
LCF	Load curve ID or table ID. The load curve specifies plastic failure strain as a function of triaxiality. The table specifies for each Lode parameter a load curve ID giving the plastic failure strain versus triaxiality for that Lode parameter. (Table option not yet generally supported).
LCG	Load curve ID for specifying plastic failure strain as a function of plastic strain rate.
LCH	Load curve ID for specifying plastic failure strain as a function of temperature
LCI	Load curve ID, table ID, or 3D table ID. The load curve ID defines plastic failure strain as a function of element size. The table ID defines for each triaxiality a load curve ID giving the plastic failure strain versus element size for that triaxiality. If referring to a three-dimensional table ID, plastic failure strain can be a function of Lode parameter (TABLE_3D), triaxiality (TABLE), and element size (CURVE).
LCCR	Table ID. The curves in this table define compressive yield stress as a function of plastic strain or effective plastic strain (see IFLAG). The table ID defines for each plastic strain rate value or effective plastic strain rate value a load curve ID giving the (isothermal) compressive yield stress as a function of plastic strain or effective plastic strain for that rate.
LCCT	Table ID defining for each temperature value a load curve ID giving the (quasi-static) compressive yield stress as a function of strain for that temperature. The curves in this table define compressive yield stress as a function of plastic strain or effective plastic strain (see IFLAG).
LCSR	Table ID. The load curves define shear yield stress in function of plastic strain or effective plastic strain (see IFLAG). The table ID defines for each plastic strain rate value or effective plastic strain rate value a load curve ID giving the (isothermal) shear yield stress as a function of plastic strain or effective plastic strain for that rate.
LCST	Table ID defining for each temperature value a load curve ID giving the (quasi-static) shear yield stress as a function of strain for that temperature. The load curves define shear yield stress as a function of plastic strain or effective plastic strain (see IFLAG).

VARIABLE	DESCRIPTION
IFLAG	Flag to specify abscissa for LCCR, LCCT, LCSR, LCST: EQ.0: Compressive and shear yields are given as functions of plastic strain as defined in Remark 1 (default). EQ.1: Compressive and shear yields are given as functions of effective plastic strain.
SFIEPM	Scale factor on the initial estimate of the plastic multiplier
NITER	Number of secant iterations to be performed

Remarks:

1. **IFLAG.** If IFLAG = 0, the compressive and shear curves are defined as follows:

$$\sigma_c(\varepsilon_{pc}, \dot{\varepsilon}_{pc}), \quad \varepsilon_{pc} = \varepsilon_c - \frac{\sigma_c}{E}, \quad \dot{\varepsilon}_{pc} = \frac{\partial \varepsilon_{pc}}{\partial t}$$
$$\sigma_s(\gamma_{ps}, \dot{\gamma}_{ps}), \quad \gamma_{ps} = \gamma_s - \frac{\sigma_s}{G}, \quad \dot{\gamma}_{ps} = \frac{\partial \gamma_{ps}}{\partial t}$$

   Two history variables (#16 plastic strain in compression and #17 plastic strain in shear) are stored in addition to those history variables already stored for *MAT_-224.

   If IFLAG = 1, the compressive and shear curves are defined as follows:

$$\sigma_c(\dot{\lambda}, \lambda), \quad \sigma_s(\dot{\lambda}, \lambda), \quad \dot{W}_p = \sigma_{\mathrm{eff}} \dot{\lambda}$$

2. **History variables.** History variables may be post-processed through additional variables. The number of additional variables for solids written to the d3plot and d3thdt databases is input through NEIPH on optional *DATABASE_EXTENT_BINARY. The relevant additional history variables for this material model are listed below:

History Variable #	Description
5	Plastic strain rate
8	Plastic failure strain
9	Triaxiality
10	Lode parameter
11	Plastic work
12	Damage
13	Element size

History Variable #	Description
14	Temperature
16	Plastic strain in compression
17	Plastic strain in shear

*MAT_VISCOPLASTIC_MIXED_HARDENING

This is Material Type 225. An elasto-viscoplastic material with an arbitrary stress as a function of strain curve and arbitrary strain rate dependency can be defined. Kinematic, isotropic, or a combination of kinematic and isotropic hardening can be specified. Also, failure based on plastic strain can be defined.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	LCSS	BETA		
Type	A	F	F	F	I	F		
Default	none	none	none	none	none	0.0		

Card 2	1	2	3	4	5	6	7	8
Variable	FAIL							
Type	F							
Default	10^{20}							

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (*PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
LCSS	Load curve ID or Table ID. Load curve ID defining effective stress as a function of effective plastic strain The table ID defines for each strain rate value a load curve ID giving the stress as a function of effective plastic strain for that rate, See Figure M24-1. The stress as a function of effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress as a function of effective plastic strain curve for

VARIABLE	DESCRIPTION
	the highest value of strain rate is used if the strain rate exceeds the maximum value. NOTE: The strain rate values defined in the table may be given as the natural logarithm of the strain rate. If the *first* stress-strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used. Since the tables are internally discretized to equally space the points, natural logarithms are necessary, for example, if the curves correspond to rates from 10^{-4} to 10^{4}.
BETA	Hardening parameter, 0.0 < BETA < 1.0: EQ.0.0: Pure kinematic hardening EQ.1.0: Pure isotropic hardening 0.0 < BETA < 1.0: Mixed hardening
FAIL	Failure flag: LT.0.0: User-defined failure subroutine is called to determine failure EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved. GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.

MAT_KINEMATIC_HARDENING_BARLAT89_*{OPTION}

This is Material Type 226. This model combines the Yoshida & Uemori non-linear kinematic hardening rule (*MAT_125) with the 3-parameter material model of Barlat and Lian [1989] (*MAT_36) to model metal sheets under cyclic plasticity loading with anisotropy in plane stress condition. Lankford parameters are used for the definition of the anisotropy. Yoshida's theory describes the hardening rule with a "two surfaces" method: the yield surface and the bounding surface. In the forming process, the yield surface does not change in size, but its center moves with deformation; the bounding surface changes both in size and location.

Available options include:

<BLANK>

NLP

The NLP option estimates necking failure using the Formability Index (F.I.), which accounts for the non-linear strain paths seen in metal forming applications (see Remark 4). When using this option, specify IFLD in Card 3. Since the NLP option also works with a linear strain path, it is recommended to be used as the default failure criterion in metal forming. The NLP option is also available for *MAT_036, *MAT_037, and *MAT_226.

Card Summary:

Card 1. This card is required.

MID	RO	E	PR	M	R00	R45	R90

Card 2. This card is required.

CB	Y	SC	K	RSAT	SB	H	HLCID

Card 3. This card is required.

AOPT	IOPT	C1	C2	IFLD	EA	COE	

Card 4. This card is required.

			A1	A2	A3		

Card 5. This card is required.

V1	V2	V3	D1	D2	D3	BETA	

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	M	R00	R45	R90
Type	A	F	F	F	F	F	F	F
Default	none	0.0	0.0	0.0	0.0	0.0	0.0	none

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus, E. Optionally, the Young's modulus can be a function of effective plastic strain. See Remark 5. In that case this is the initial Young's modulus.
PR	Poisson's ratio, ν
M	the exponent in Barlat's yield criterion, m
R00	R_{00}, Lankford parameter in 0° direction
R45	R_{45}, Lankford parameter in 45° direction
R90	R_{90}, Lankford parameter in 90° direction

Card 2	1	2	3	4	5	6	7	8
Variable	CB	Y	SC	K	RSAT	SB	H	HLCID
Type	F	F	F	F	F	F	F	I
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	none

VARIABLE	DESCRIPTION
CB	The uppercase B defined in Yoshida's equations

VARIABLE	DESCRIPTION
Y	Hardening parameter as defined in Yoshida's equations
SC	The lowercase *c* defined in the Yoshida & Uemori's equations
K	Hardening parameter as defined in the Yoshida & Uemori's equations
RSAT	Hardening parameter as defined in the Yoshida and Uemori's equations
SB	The lowercase *b* as defined in the Yoshida & Uemori's equations
H	Anisotropic parameter associated with work-hardening stagnation, defined in the Yoshida and Uemori's equations
HLCID	Load curve ID (see *DEFINE_CURVE) giving true strain as a function of true stress. The load curve is optional and is used for error calculation only.

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT	IOPT	C1	C2	IFLD	EA	COE	
Type	F	I	F	F	I	F	F	
Default	none	none	0.0	0.0	none	0.0	0.0	

VARIABLE	DESCRIPTION
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description): EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an

VARIABLE	DESCRIPTION
	angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE__COORDINATE_VECTOR)..
IOPT	Kinematic hardening rule flag: EQ.0: Original Yoshida & Uemori formulation, EQ.1: Modified formulation; define C1, C2 as below.
C1, C2	Constants used to modify R: $R = \mathrm{RSAT} \times [(C_1 + \bar{\varepsilon}^p)^{c_2} - {C_1}^{c_2}]$
IFLD	ID of a load curve of the traditional Forming Limit Diagram (FLD) for the linear strain paths. In the load curve, abscissas represent minor strains while ordinates represent major strains. Define only when the NLP option is used.
EA	Variable controlling the change of Young's modulus, E^A. See Remark 5.
COE	Variable controlling the change of Young's modulus, ζ. See Remark 5.

Card 4	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

VARIABLE	DESCRIPTION
A1, A2, A3	Components of vector **a** for AOPT = 2

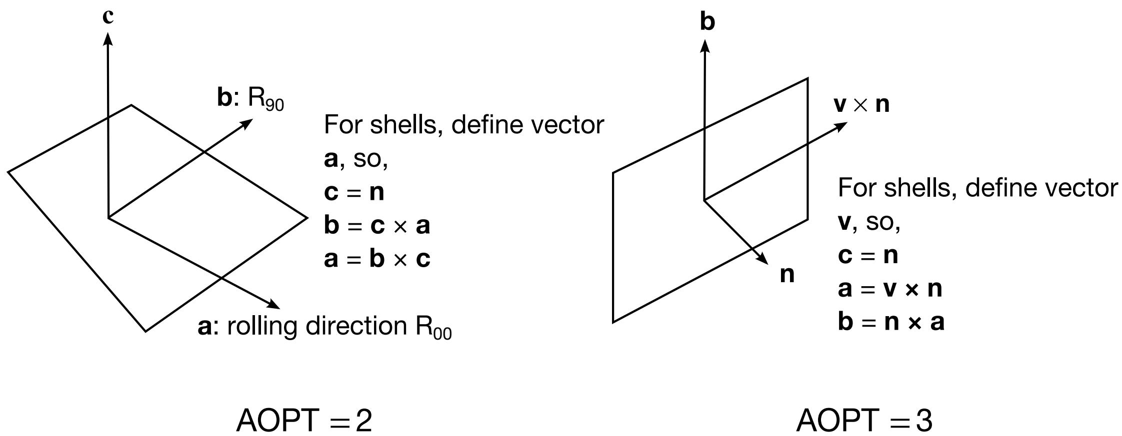

Figure M226-1. Defining sheet metal rolling direction.

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3
D1, D2, D3	Components of vector **d** for AOPT = 2
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card; see *ELEMENT_SHELL_BETA.

Remarks:

1. **Barlat and Lian's yield vriterion.** The R-values are defined as the ratio of instantaneous width change to instantaneous thickness change. That is, assume that the width W and thickness T are measured as functions of strain, then the corresponding R-value is given by:

$$R = \frac{\frac{dW}{d\varepsilon}/W}{\frac{dT}{d\varepsilon}/T} .$$

Input R00, R45 and R90 to define sheet anisotropy in the rolling, 45° and 90° direction.

Barlat and Lian's [1989] anisotropic yield criterion Φ for plane stress is defined as:

$$\Phi = a|K_1 + K_2|^m + a|K_1 - K_2|^m + c|2K_2|^m = 2\sigma_Y^m$$

for face centered cubic (FCC) materials exponent $m = 8$ is recommended and for body centered cubic (BCC) materials $m = 6$ may be used. Detailed description on the criterion can be found in the *MAT_036 manual pages.

2. **Yoshida & Uemori nonliner kinematic hardening model.** See manual pages for *MAT_125 for more details.

3. **Rolling direction of sheet metal.** The variable AOPT is used to define the rolling direction of the sheet metals. When AOPT = 2, define vector components of **a** in the direction of the rolling (R_{00}); when AOPT = 3, define vector components of **v** perpendicular to the rolling direction, as shown in Figure M226-1.

4. **A failure criterion for nonlinear strain paths (NLP).** The NLP failure criterion and corresponding post processing procedures are described in the entries for *MAT_036 and *MAT_037. The history variables for every element stored in d3plot files include:

   a) Formability Index (F.I.): #1

   b) Strain ratio β (in-plane minor strain increment/major strain increment): #2

   c) Effective strain from the planar isotropic assumption: #3

   To enable the output of these history variables to the d3plot files, NEIPS on the *DATABASE_EXTENT_BINARY card must be set to at least 3.

5. **Change in Young's modulus.** The optional change in Young's modulus is defined as a function of effective plastic strain,

$$E = E_0 - (E_0 - E_A)[1 - \exp(-\zeta\bar{\varepsilon}^p)] \ .$$

*MAT_PML_ELASTIC

This is Material Type 230. This is a perfectly-matched layer (PML) material — an absorbing layer material used to simulate wave propagation in an unbounded isotropic elastic medium — and is available only for solid 8-node bricks (element type 2). This material implements the three-dimensional version of the Basu-Chopra PML [Basu and Chopra (2003,2004), Basu (2009)].

Card	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR				
Type	A	F	F	F				
Default	none	none	none	none				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio

Remarks:

1. **Unboundedness.** A layer of this material may be placed at a boundary of a bounded domain to simulate unboundedness of the domain at that boundary: the layer absorbs and attenuates waves propagating outward from the domain, without any significant reflection of the waves back into the bounded domain. The layer cannot support any static displacement.

2. **Bounded Domain Material Properties.** It is assumed the material in the bounded domain near the layer is, or behaves like, an isotropic linear elastic material. The material properties of the layer should be set to the corresponding properties of this material.

3. **Layer Geometry.** The layer should form a cuboid box around the bounded domain, with the axes of the box aligned with the coordinate axes. Various faces

of this box may be open, as required by the geometry of the problem. For example, for a half-space problem, the “top” of the box should be open.

Internally, LS-DYNA will partition the entire PML into regions which form the “faces,” “edges” and “corners” of the above cuboid box, and generate a new material for each region. This partitioning will be visible in the d3plot file. The user may safely ignore this partitioning.

4. **Number of Elements in Layer.** The layer should have 5-10 elements through its depth. Typically, 5-6 elements are sufficient if the excitation source is reasonably distant from the layer, and 8-10 elements if it is close. The size of the elements should be similar to that of elements in the bounded domain near the layer and should be small enough to sufficiently discretize all significant wavelengths in the problem.

5. **Nodal Constraints.** The nodes on the outer boundary of the layer should be fully constrained using either *BOUNDARY_SPC or TC on *NODE. Other constraints, such as *CONSTRAINED_GLOBAL or *BOUNDARY_PRESCRIBED_MOTION with a zero-valued load curve, will not be recognized.

   Note that Ansys Mechanical uses *BOUNDARY_PRESCRIBED_MOTION with a zero-value load curve for constraining nodal degrees-of-freedom. Because of this limitation, you will have to edit the input file with a text editor.

6. **Stress and Strain.** The stress and strain values reported by this material do not have any physical significance.

*MAT_PML_ELASTIC_FLUID

This is Material Type 230_FLUID. This model is a perfectly-matched layer (PML) material with a pressure fluid constitutive law, to be used in a wave-absorbing layer adjacent to a fluid material (*MAT_ELASTIC_FLUID) in order to simulate wave propagation in an unbounded fluid medium. See the Remarks sections of *MAT_PML_ELASTIC (*MAT_230) and *MAT_ELASTIC_FLUID (*MAT_001_FLUID) for further details.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	VC				
Type	A	F	F	F				
Default	none	none	none	none				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
K	Bulk modulus
VC	Tensor viscosity coefficient

*MAT_PML_ACOUSTIC

This is Material Type 231. This is a perfectly-matched layer (PML) material — an absorbing layer material used to simulate wave propagation in an unbounded acoustic medium — and can be used only with the acoustic pressure element formulation (element type 14). This material implements the three-dimensional version of the Basu-Chopra PML for anti-plane motion [Basu and Chopra (2003,2004), Basu (2009)].

Card	1	2	3	4	5	6	7	8
Variable	MID	RO	C					
Type	A	F	F					
Default	none	none	none					

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
C	Sound speed

Remarks:

1. **Unboundedness.** A layer of this material may be placed at a boundary of a bounded domain to simulate unboundedness of the domain at that boundary: the layer absorbs and attenuates waves propagating outward from the domain, without any significant reflection of the waves back into the bounded domain. The layer cannot support any hydrostatic pressure.

2. **Material in Bounded Domain.** It is assumed the material in the bounded domain near the layer is an acoustic material. The material properties of the layer should be set to the corresponding properties of this material.

3. **Layer Geometry.** The layer should form a cuboid box around the bounded domain, with the axes of the box aligned with the coordinate axes. Various faces of this box may be open, as required by the geometry of the problem, e.g., for a half-space problem, the "top" of the box should be open.

Internally, LS-DYNA will partition the entire PML into regions which form the "faces," "edges" and "corners" of the above cuboid box, and generate a new material for each region. This partitioning will be visible in the d3plot file. The user may safely ignore this partitioning.

4. **Number of Elements in Layer.** The layer should have 5 - 10 elements through its depth. Typically, 5 - 6 elements are sufficient if the excitation source is reasonably distant from the layer, and 8 - 10 elements if it is close. The size of the elements should be similar to that of elements in the bounded domain near the layer, and should be small enough to sufficiently discretize all significant wavelengths in the problem.

5. **Nodal Constraints.** The nodes on the outer boundary of the layer should be fully constrained using either *BOUNDARY_SPC or TC on *NODE. Other constraints, such as *CONSTRAINED_GLOBAL or *BOUNDARY_PRESCRIBED_MOTION with a zero-valued load curve, will not be recognized.

   Note that Ansys Mechanical uses *BOUNDARY_PRESCRIBED_MOTION with a zero-value load curve for constraining nodal degrees-of-freedom. Because of this limitation, you will have to edit the input file with a text editor.

6. **Pressure Values.** The pressure values reported by this material do not have any physical significance.

*MAT_BIOT_HYSTERETIC

This is Material Type 232. This is a Biot linear hysteretic material, to be used for modeling the nearly-frequency-independent viscoelastic behavior of soils subjected to cyclic loading, such as in soil-structure interaction analysis [Spanos and Tsavachidis (2001), Makris and Zhang (2000), Muscolini, Palmeri and Ricciardelli (2005)]. The hysteretic damping coefficient for the model is computed from a prescribed damping ratio by calibrating with an equivalent viscous damping model for a single-degree-of-freedom system. The damping increases the stiffness of the model and thus reduces the computed time-step size.

Card	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	ZT	FD		
Type	A	F	F	F	F	F		
Default	none	none	none	none	0.0	3.25		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
ZT	Damping ratio
FD	Dominant excitation frequency in Hz

Remarks:

1. **Stress.** The stress is computed as a function of the strain rate as

$$\sigma(t) = \int_0^t C_R(t-\tau)\dot{\varepsilon}(\tau)\,d\tau$$

where

$$C_R(t) = C\left[1 + \frac{2\eta}{\pi}E_1(\beta t)\right].$$

In the above, C is the elastic isotropic constitutive tensor, η is the hysteretic damping factor, and $\beta = 2\pi f_d/10$, where f_d is the dominant excitation frequency in Hz. The function E_1 is given by

$$E_1(s) = \int_s^\infty \frac{\mathrm{e}^{-\xi}}{\xi} d\xi$$

For efficient implementation, this function is approximated by a 5-term Prony series as

$$E_1(s) \approx \sum_{k=1}^{5} b_k \mathrm{e}^{a_k s} \ ,$$

such that $b_k > 0$.

2. **Hysteretic damping factor.** The hysteretic damping factor η is obtained from the prescribed damping ratio ς as

$$\eta = \pi\varsigma/\mathrm{atan}(10) = 2.14\varsigma$$

by assuming that, for a single degree-of-freedom system, the energy dissipated per cycle by the hysteretic material is the same as that by a viscous damper, if the excitation frequency matches the natural frequency of the system.

3. **Young's modulus.** The consistent Young's modulus for this model is given by

$$E_c = E\left[1 + \frac{2\eta}{\pi} g\right] \ ,$$

where

$$g = \sum_{k=1}^{5} b_k \frac{1}{a_k \beta \Delta t_n}\left[\exp(a_k \beta \Delta t_n) - 1\right] \ .$$

Because $g > 0$, the computed element time-step size is smaller than that for the corresponding elastic element. Furthermore, the time-step size computed at any time depends on the previous time-step size. It can be demonstrated that the new computed time-step size stays within a narrow range of the previous time-step size and for a uniform mesh, converges to a constant value. For $f_d = 3.25$ Hz and $\varsigma = 0.05$, the percentage decrease in time-step size can be expected to be about 12 - 15% for initial time-step sizes of less than 0.02 secs, and about 7 - 10% for initial time-step sizes larger than 0.02 secs.

4. **Dominant frequency.** The default value of the dominant frequency is chosen to be valid for earthquake excitation.

*MAT_CAZACU_BARLAT

This is Material Type 233. This material model is for Hexagonal Closed Packet (HCP) metals and is based on the work by Cazacu et al. (2006). This model is capable of describing the yielding asymmetry between tension and compression for such materials. Moreover, a parameter fit is optional and can be used to find the material parameters that describe the experimental yield stresses. The experimental data that you should supply consists of yield stresses for tension and compression in the 00 direction, tension in the 45 and the 90 directions, and a biaxial tension test.

Available options include:

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MAGNESIUM

Including MAGNESIUM invokes a material model developed by the USAMP consortium to simulate cast Magnesium under impact loading. The model includes rate effects having a tabulated failure model including equivalent plastic strain to failure as a function of stress triaxiality and effective plastic strain rate. Element erosion will occur when the number of integration points where the damage variable has reached unity reaches some specified threshold (NUMINT). Alternatively, a Gurson type failure model can be activated, which requires less experimental data.

You must provide the evolution of the Cazacu-Barlat effective stress as a function of the energy conjugate plastic strain in the input for the hardening curve for MAT_233. With the MAGNESIUM option an alternative option for the hardening curve is available: von Mises effective stress as a function of equivalent plastic strain, which is energy conjugate to the von Mises stress.

Finally, the MAGNESIUM option allows for distortional hardening by providing hardening curves as measured in tension and compression tests. This option is however incompatible with the activation of rate effects (visco-plasticity).

With the MAGNESIUM option this material model is also available for solid elements.

> **NOTE:** Activating the MAGNESIUM options *requires* setting HR = 3 and FIT = 0.0 (also see below).

Card Summary:

Card 1. This card is required.

MID	RO	E	PR	HR	P1	P2	ITER

Card 2. This card is required.

A	C11	C22	C33	LCID	E0	K	P3

Card 3. This card is required.

AOPT				C12	C13	C23	C44

Card 4. This card is required.

XP	YP	ZP	A1	A2	A3		

Card 5. This card is required.

V1	V2	V3	D1	D2	D3	BETA	FIT

Card 6. This card is included if and only if the MAGNESIUM keyword option is used.

LC1ID	LC2ID	NUMINT	LCCID	ICFLAG	IDFLAG	LC3ID	EPSFG

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	HR	P1	P2	ITER
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be used (see *PART).
RO	Constant mass density
E	Young's modulus: GT.0.0: Constant value LT.0.0: |E| is a load curve ID that defines the Young's modulus as a function of plastic strain.
PR	Poisson's ratio
HR	Hardening rules:

VARIABLE	DESCRIPTION
	EQ.1.0: Linear hardening (default) EQ.2.0: Exponential hardening (Swift) EQ.3.0: Load curve EQ.4.0: Exponential hardening (Voce) EQ.5.0: Exponential hardening (Gosh) EQ.6.0: Exponential hardening (Hocken-Sherby) HR must be set to 3 if the MAGNESIUM keyword option is active.
P1	Material parameter: HR.EQ.1.0: Tangent modulus HR.EQ.2.0: q, coefficient for exponential hardening law (Swift) HR.EQ.4.0: a, coefficient for exponential hardening law (Voce) HR.EQ.5.0: q, coefficient for exponential hardening law (Gosh) HR.EQ.6.0: a, coefficient for exponential hardening law (Hocket-Sherby)
P2	Material parameter: HR.EQ.1.0: Yield stress for the linear hardening law HR.EQ.2.0: n, coefficient for (Swift) exponential hardening HR.EQ.4.0: c, coefficient for exponential hardening law (Voce) HR.EQ.5.0: n, coefficient for exponential hardening law (Gosh) HR.EQ.6.0: c, coefficient for exponential hardening law (Hocket-Sherby)
ITER	Iteration flag for speed: EQ.0.0: Fully iterative EQ.1.0: Fixed at three iterations. Generally, ITER = 0.0 is recommended. However, ITER = 1.0 is faster and may give acceptable results in most problems.

Card 2	1	2	3	4	5	6	7	8
Variable	A	C11	C22	C33	LCID	E0	K	P3
Type	F	F	F	F	I	F	F	F

VARIABLE	DESCRIPTION
A	Exponent in Cazacu-Barlat's orthotropic yield surface (A > 1)
C11	Material parameter (see Card 5 pos. 8): FIT.EQ.1.0.OR.EQ.2.0: Yield stress for tension in the 00 direction FIT.EQ.0.0: Material parameter c_{11}
C22	Material parameter (see Card 5 pos.8): FIT.EQ.1.0.OR.EQ.2.0: Yield stress for tension in the 45 direction FIT.EQ.0.0: Material parameter c_{22}
C33	Material parameter (see Card 5 pos.8): FIT.EQ.1.0.OR.EQ.2.0: Yield stress for tension in the 90 direction FIT.EQ.0.0: Material parameter c_{33}
LCID	Load curve ID for the hardening law (HR = 3.0), 2D Table ID for rate dependent hardening or 3D Table ID for rate-and-temperature-dependent hardening if the MAGNESIUM option is active. For the 3D table case, *MAT_ADD_THERMAL_EXPANSION could be used for thermal stress/strain effects. Note that the 3D table option is only valid for shell elements.
E0	Material parameter: HR.EQ.2.0: ε_0, initial yield strain for exponential hardening law (Swift) (default = 0.0) HR.EQ.4.0: b, coefficient for exponential hardening (Voce) HR.EQ.5.0: ε_0, initial yield strain for exponential hardening (Gosh); default = 0.0 HR.EQ.6.0: b, coefficient for exponential hardening law (Hocket-

VARIABLE	DESCRIPTION
	Sherby)
K	Material parameter (see Card 5 pos.8): FIT.EQ.1.0.OR.EQ.2.0: Yield stress for compression in the 00 direction FIT.EQ.0.0: Material parameter (-1 < k < 1)
P3	Material parameter: HR.EQ.5.0: p, coefficient for exponential hardening (Gosh) HR.EQ.6.0: n, exponent for exponential hardening law (Hocket-Sherby)

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT				C12	C13	C23	C44
Type	F				F	F	F	F

VARIABLE	DESCRIPTION
AOPT	Material axes option (see*MAT_*OPTION*TROPIC_ELASTIC for a more complete description). EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2 and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA EQ.1.0: Locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the a-direction. This option is for solid elements only. EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINED_COORDINATE_VECTOR. EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle BETA, from a line in the plane of the element defined by the cross product of the vector, **v**, with the element

VARIABLE	DESCRIPTION
	normal
	EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, p, which define the centerline axis. This option is for solid elements only.
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM, or *DEFINE_COORDINATE_VECTOR).
C12	Material parameter, c_{12}. If parameter identification (FIT = 1.0) is turned on, C12 is not used.
C13	Material parameter, c_{13}. If parameter identification (FIT = 1.0) is turned on, C13 = 0.0
C23	Material parameter. If parameter identification (FIT = 1.0) is turned on, C23 = 0.0
C44	Material parameter (see Card 5 pos.8) FIT.EQ.1.0.OR.EQ.2.0: Yield stress for the balanced biaxial tension test. FIT.EQ.0.0: Material parameter, c_{44}

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
XP - ZP	Coordinates of point p for AOPT = 1 and 4
A1 - A3	Components of vector **a** for AOPT = 2.0

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	FIT
Type	F	F	F	F	F	F	F	I

VARIABLE	DESCRIPTION
V1 - V3	Components of vector **v** for AOPT = 3.0
D1 - D3	Components of vector **d** for AOPT = 2.0
BETA	Material angle in degrees for AOPT = 0 and 3. Note that BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA
FIT	Flag for parameter identification algorithm: EQ.0.0: No parameter identification routine is used. The variables K, C11, C22, C33, C44, C12, C13 and C23 are interpreted as material parameters. FIT MUST be set to zero if MAGNESIUM option is active EQ.1.0: Parameter fit is used. The variables C11, C22, C33, C44 and K are interpreted as yield stresses in the 00 degree direction, the 45 degree direction, the 90 degree direction, the balanced biaxial tension, and the 00 degree compression, respectively. It is recommended to always check the d3hsp file to see the fitted parameters before complex jobs are submitted. EQ.2.0: Same as EQ.1.0 but also produce contour plots of the yield surface. For each material three xy-data files are created: Contour1_*n*, Contour2_*n* and Contour3_*n* where *n* equals the material number.

Magnesium Card. Additional card for MAGNESIUM keyword option.

Card 6	1	2	3	4	5	6	7	8
Variable	LC1ID	LC2ID	NUMINT	LCCID	ICFLAG	IDFLAG	LC3ID	EPSFG
Type	I	I	F	I	I	I	I	F

VARIABLE	DESCRIPTION
LC1ID	Load curve ID giving equivalent plastic strain to failure as a function of stress triaxiality or a table ID giving plastic strain to failure as a function of Lode parameter and stress triaxiality (solids)
LC2ID	Load curve ID giving equivalent plastic strain to failure as a function of equivalent plastic strain rate. The failure strain will be computed as the product of the values on LC1ID and LC2ID.
NUMINT	Number of through thickness integration points which must fail before the element is deleted (inactive for solid elements)
LCCID	Load curve ID giving effective stress as a function of plastic strain obtained from a compression stress. This load curve will activate distortional hardening. It is *not* compatible with the use of strain rate effects.
ICFLAG	Automated input conversion flag: EQ.0: Load curves provided under LCID and LCCID contain Cazacu-Barlat effective stress as a function of energy conjugate plastic strain. EQ.1: Both load curves are given in terms of von Mises stress as a function of equivalent plastic strain
IDFLAG	Damage flag: EQ.0: Failure model is of the Johnson Cook type and requires LC1ID and LC2ID as additional input. EQ.1: Failure model is of the Gurson type and requires LC3ID and EPSFG as additional input.
LC3ID	Load curve giving the critical void fraction of the Gurson model as a function of the plastic strain to failure measured in the uniaxial tensile test
EPSFG	Plastic strain to failure measured in the uniaxial tensile test. This value is used by the Gurson type failure model only.

Remarks:

This material model (*MAT_CAZACU_BARLAT) aims to model materials with strength differential and orthotropic behavior under plane stress. The yield condition includes a parameter k that describes the asymmetry between yield in tension and compression. Moreover, to include the anisotropic behavior the stress deviator, $\mathbf{S}$, undergoes a linear transformation. The principal values of the Cauchy stress deviator are substituted with the principal values of the transformed tensor, **Z**, which is represented as a vector field, defined as:

$$\mathbf{Z} = \mathbf{CS} \ . \tag{233.1}$$

Here **S** is the field comprised of the four stresses deviator components, $S_I = (s_{11}, s_{22}, s_{33}, s_{12})$,

$$\mathbf{s} = \sigma - \frac{1}{3}\mathrm{tr}(\sigma)\delta \ .$$

In the above equation, $\mathrm{tr}(\sigma)$ is the trace of the Cauchy stress tensor and δ is the Kronecker delta. For the 2D plane stress condition, the orthotropic condition gives 7 independent coefficients. The tensor **C** is represented by the 4 × 4 matrix

$$C_{IJ} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & \\ c_{12} & c_{22} & c_{23} & \\ c_{13} & c_{23} & c_{33} & \\ & & & c_{44} \end{pmatrix}.$$

The principal values of **Z** are denoted Σ_1, Σ_2, and Σ_3 and are given as the eigenvalues to the matrix composed by the components Σ_{xx}, Σ_{yy}, Σ_{zz}, and Σ_{xy}through

$$\begin{aligned} \Sigma_1 &= \frac{1}{2}\left(\Sigma_{xx} + \Sigma_{yy} + \sqrt{\left(\Sigma_{xx} - \Sigma_{yy}\right)^2 + 4\Sigma_{xy}^2}\right), \\ \Sigma_2 &= \frac{1}{2}\left(\Sigma_{xx} + \Sigma_{yy} - \sqrt{\left(\Sigma_{xx} - \Sigma_{yy}\right)^2 + 4\Sigma_{xy}^2}\right), \\ \Sigma_3 &= \Sigma_{zz} \end{aligned}$$

where

$$\begin{aligned} 3\Sigma_{xx} &= (2c_{11} - c_{12} - c_{13})\sigma_{xx} + (-c_{11} + 2c_{12} - c_{13})\sigma_{yy}, \\ 3\Sigma_{yy} &= (2c_{12} - c_{22} - c_{23})\sigma_{xx} + (-c_{12} + 2c_{22} - c_{23})\sigma_{yy}, \\ 3\Sigma_{zz} &= (2c_{13} - c_{23} - c_{33})\sigma_{xx} + (-c_{13} + 2c_{23} - c_{33})\sigma_{yy}, \\ \Sigma_{xy} &= c_{44}\sigma_{12} \end{aligned}$$

Note that the symmetry of Σ_{xy} follows from the symmetry of the Cauchy stress tensor.

The yield condition is written in the following form:

$$f\left(\Sigma, k, \varepsilon_{\mathrm{ep}}\right) = \sigma_{\mathrm{eff}}(\Sigma_1, \Sigma_2, \Sigma_3, k) - \sigma_y\left(\varepsilon_{\mathrm{ep}}\right) \le 0 \ , \tag{233.2}$$

where $\sigma_y\left(\varepsilon_{\mathrm{ep}}\right)$ is a function representing the current yield stress dependent on current effective plastic strain and k is the asymmetric parameter for yield in compression and tension. The effective stress σ_{eff} is given by

$$\sigma_{\mathrm{eff}} = \left[(|\Sigma_1| - k\Sigma_1)^a + (|\Sigma_2| - k\Sigma_2)^a + (|\Sigma_3| - k\Sigma_3)^a\right]^{1/a} \ , \tag{233.3}$$

where $k \in [-1,1]$ and $a \ge 1$. Now, let σ_{00}^T and σ_{00}^C represent the yield stress along the rolling (00 degree) direction in tension and compression, respectively. Furthermore let σ_{45}^T and σ_{90}^T represent the yield stresses in the 45 and the 90 degree directions, and last let

σ_B^T be the balanced biaxial yield stress in tension. Following Cazacu et al. (2006) the yield stresses can easily be derived.

To simplify the equations it is preferable to make the following definitions:

$$\begin{aligned} \Phi_1 &= \frac{1}{3}(2c_{11} - c_{12} - c_{13}) \\ \Phi_2 &= \frac{1}{3}(2c_{12} - c_{22} - c_{23}) \\ \Phi_3 &= \frac{1}{3}(2c_{13} - c_{23} - c_{33}) \end{aligned} \quad \text{and} \quad \begin{aligned} \Psi_1 &= \frac{1}{3}(-c_{11} + 2c_{12} - c_{13}) \\ \Psi_2 &= \frac{1}{3}(-c_{12} + 2c_{22} - c_{23}) \\ \Psi_3 &= \frac{1}{3}(-c_{13} + 2c_{23} - c_{33}) \end{aligned}$$

The yield stresses can now be written as:

1. In the 00 degree direction:

$$\begin{aligned} \sigma_{00}^T &= \left[\frac{(\sigma_{\text{eff}})^a}{(|\Phi_1| - k\Phi_1)^a + (|\Phi_2| - k\Phi_2)^a + (|\Phi_3| - k\Phi_3)^a}\right]^{1/a}, \\ \sigma_{00}^C &= \left[\frac{(\sigma_{\text{eff}})^a}{(|\Phi_1| + k\Phi_1)^a + (|\Phi_2| + k\Phi_2)^a + (|\Phi_3| + k\Phi_3)^a}\right]^{1/a} \end{aligned} \tag{233.4}$$

2. In the 45 degree direction:

$$\sigma_{45}^T = \left[\frac{(\sigma_{\text{eff}})^a}{(|\Lambda_1| - k\Lambda_1)^a + (|\Lambda_2| - k\Lambda_2)^a + (|\Lambda_3| - k\Lambda_3)^a}\right]^{1/a} \tag{233.5}$$

where

$$\begin{aligned} \Lambda_1 &= \frac{1}{4}\left[\Phi_1 + \Phi_2 + \Psi_1 + \Psi_2 + \sqrt{(\Phi_1 + \Psi_1 - \Phi_2 - \Psi_2)^2 + 4c_{44}^2}\right], \\ \Lambda_2 &= \frac{1}{4}\left[\Phi_1 + \Phi_2 + \Psi_1 + \Psi_2 - \sqrt{(\Phi_1 + \Psi_1 - \Phi_2 - \Psi_2)^2 + 4c_{44}^2}\right], \\ \Lambda_3 &= \frac{1}{2}[\Phi_3 + \Psi_3]. \end{aligned}$$

3. In the 90 degree direction:

$$\sigma_{90}^T = \left[\frac{(\sigma_{\text{eff}})^a}{(|\Psi_1| - k\Psi_1)^a + (|\Psi_2| - k\Psi_2)^a + (|\Psi_3| - k\Psi_3)^a}\right]^{1/a} \tag{233.6}$$

4. In the balanced biaxial yield occurs when both σ_{xx} and $\sigma_{yy}$are equal to:

$$\sigma_B^T = \left[\frac{(\sigma_{\text{eff}})^a}{(|\Omega_1| - k\Omega_1)^a + (|\Omega_2| - k\Omega_2)^a + (|\Omega_3| - k\Omega_3)^a}\right]^{1/a} \tag{233.7}$$

where

$$
\begin{aligned}
\Omega_1 &= \frac{1}{3}(c_{11} + c_{12} - 2c_{13}) \\
\Omega_2 &= \frac{1}{3}(c_{12} + c_{22} - 2c_{23}) \\
\Omega_3 &= \frac{1}{3}(c_{13} + c_{23} - 2c_{33})
\end{aligned}
$$

Hardening laws:

The following hardening laws are implemented:

1. Swift hardening law
2. Voce hardening law
3. Gosh hardening law
4. Hocket-Sherby hardening law
5. Loading curve, where the yield stress is given as a function of the effective plastic strain

The Swift's hardening law can be written as

$$\sigma_y(\varepsilon_{\mathrm{ep}}) = q(\varepsilon_0 + \varepsilon_{\mathrm{ep}})^n$$

where q and n are material parameters.

The Voce's equation says that the yield stress can be written in the following form

$$\sigma_y(\varepsilon_{\mathrm{ep}}) = a - be^{-c\varepsilon_{\mathrm{ep}}}$$

where a, b, and c are material parameters. The Gosh's equation is similar to Swift's equation. They only differ by a constant

$$\sigma_y(\varepsilon_{\mathrm{ep}}) = q(\varepsilon_0 + \varepsilon_{\mathrm{ep}})^n - p \ .$$

Here q, ε_0, n and p are material constants. The Hocket-Sherby equation resembles the Voce's equation, but with an additional parameter added

$$\sigma_y(\varepsilon_{\mathrm{ep}}) = a - be^{-c\varepsilon_{\mathrm{ep}}^n} \ ,$$

where a, b, c and n are material parameters.

Constitutive relation and material stiffness:

The classical elastic constitutive equation for linear deformations is the well-known Hooke's law. This relation written in a rate formulation is given by

$$\dot{\sigma} = \mathbf{D}\dot{\varepsilon}_e \ , \tag{233.8}$$

where ε_e is the elastic strain and $\mathbf{D}$ is the constitutive matrix. An over imposed dot indicates differentiation respect to time. Introducing the total strain, ε, and the plastic strain, ε_p, Eq. (233.8) is classically rewritten as

$$\dot{\sigma} = \mathbf{D}(\dot{\varepsilon} - \dot{\varepsilon}_p) \ , \tag{233.9}$$

where

$$\mathbf{D} = \frac{E}{1-\nu^2}\begin{pmatrix} 1 & v & & & \\ v & 1 & & & \\ & & \frac{1-v}{2} & & \\ & & & \frac{1-v}{2} & \\ & & & & \frac{1-v}{2} \end{pmatrix} \text{ and } (\dot{\varepsilon} - \dot{\varepsilon}_p) = \begin{pmatrix} \dot{\varepsilon}_{11} - (\dot{\varepsilon}_p)_{11} \\ \dot{\varepsilon}_{22} - (\dot{\varepsilon}_p)_{22} \\ 2\left[\dot{\varepsilon}_{12} - (\dot{\varepsilon}_p)_{12}\right] \\ 2\left[\dot{\varepsilon}_{13} - (\dot{\varepsilon}_p)_{13}\right] \\ 2\left[\dot{\varepsilon}_{23} - (\dot{\varepsilon}_p)_{23}\right] \end{pmatrix} .$$

The parameters $E$and v are the Young's modulus and Poisson's ratio, respectively.

The material stiffness $\mathbf{D}_p$ that is needed for implicit analysis can be calculated from Equation (233.9) as

$$\mathbf{D}_p = \frac{\partial \dot{\sigma}}{\partial \dot{\varepsilon}} \ .$$

The associative flow rule for the plastic strain is usually written as

$$\dot{\varepsilon}_p = \dot{\lambda}\frac{\partial f}{\partial \sigma} \ , \tag{233.10}$$

and the consistency condition reads as

$$\frac{\mathrm{d}f}{\mathrm{d}\sigma}\dot{\sigma} + \frac{\mathrm{d}f}{\mathrm{d}\varepsilon_{\mathrm{ep}}}\dot{\varepsilon}_{\mathrm{ep}} = 0 \ . \tag{233.11}$$

Note that the centralized "dot" means scalar product between two vectors. Using standard calculus one easily derives from Equations (233.9), (233.10) and (233.11) an expression for the stress rate

$$\dot{\sigma} = \left[\mathbf{D} - \frac{\left(\mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\sigma}\right)\cdot\left(\mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\sigma}\right)}{\frac{df}{d\sigma}\cdot\left(\mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\sigma}\right) - \frac{\mathrm{d}f}{\mathrm{d}\varepsilon_{ep}}}\right]\dot{\varepsilon} \tag{233.12}$$

That means that the material stiffness used for implicit analysis is given by

$$\mathbf{D}_p = \mathbf{D} - \frac{\left(\mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\sigma}\right)\cdot\left(\mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\sigma}\right)}{\frac{\mathrm{d}f}{\mathrm{d}\sigma}\cdot\left(\mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\sigma}\right) - \frac{\mathrm{d}f}{\mathrm{d}\varepsilon_{\mathrm{ep}}}}. \tag{233.13}$$

To be able to do a stress update we need to calculate the tangent stiffness and the derivative with respect to the corresponding hardening law.

When a suitable hardening law has been chosen the corresponding derivative is simple and will be left out from this document. However, the stress gradient of the yield surface is more complicated and will be outlined here.

$$\begin{aligned}\frac{\partial f}{\partial \sigma_{11}} = \frac{\partial f}{\partial \Sigma_3}\frac{1}{2}\frac{\partial f}{\partial \Sigma_1}\left[\left(1+\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Phi_1+\left(1-\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Phi_2\right] \\ +\frac{1}{2}\frac{\partial f}{\partial \Sigma_2}\left[\left(1-\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Phi_1+\left(1+\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Phi_2\right]+\Phi_3\end{aligned} \tag{233.14}$$

$$\begin{aligned}\frac{\partial f}{\partial \sigma_{22}} = \frac{1}{2}\frac{\partial f}{\partial \Sigma_1}\left[\left(1+\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Psi_1+\left(1-\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Psi_2\right] \\ +\frac{1}{2}\frac{\partial f}{\partial \Sigma_2}\left[\left(1-\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Psi_1+\left(1+\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Psi_2\right]+\frac{\partial f}{\partial \Sigma_3}\Psi_3\end{aligned} \tag{233.15}$$

and the derivative with respect to the shear stress component is

$$\frac{\partial f}{\partial \sigma_{12}} = c_{44}\frac{2\Sigma_{xy}}{\sqrt{\Sigma_T}}\left(\frac{\partial f}{\partial \Sigma_1}-\frac{\partial f}{\partial \Sigma_2}\right) \tag{233.16}$$

where

$$\Sigma_T = \left(\Sigma_{xx}-\Sigma_{yy}\right)^2+4\Sigma_{xy}^2 \tag{233.17}$$

and

$$\frac{\partial f}{\partial \Sigma_i} = f\left(\Sigma,k,\varepsilon_{ep}\right)^{\frac{1}{a}-1}(|\Sigma_i|-k\Sigma_i)^{a-1}(\mathrm{sgn}(\Sigma_i)-k)\ \text{ for }\ i=1,2,3 \tag{233.18}$$

Implementation:

Assume that the stress and strain is known at time t^n. A trial stress $\tilde{\sigma}^{n+1}$ at time t^{n+1} is calculated by assuming a pure elastic deformation, that is,

$$\tilde{\boldsymbol{\sigma}}^{n+1} = \boldsymbol{\sigma}^n+\mathbf{D}\left(\boldsymbol{\varepsilon}^{n+1}-\boldsymbol{\varepsilon}^n\right) \tag{233.19}$$

Now, if $f(\Sigma,k,\varepsilon_{ep}) \le 0$, the deformation is purely elastic, and the new stress and plastic strain are determined as

$$\begin{aligned}\boldsymbol{\sigma}^{n+1} &= \tilde{\boldsymbol{\sigma}}^{n+1} \\ \varepsilon_{\mathrm{ep}}^{n+1} &= \varepsilon_{\mathrm{ep}}^{n}\end{aligned} \tag{233.20}$$

The thickness strain increment is given by

$$\Delta\varepsilon_{33} = \varepsilon_{33}^{n+1}-\varepsilon_{33}^{n} = -\frac{v}{1-v}(\Delta\varepsilon_{11}+\Delta\varepsilon_{22}) \tag{233.21}$$

If the deformation is not purely elastic, the stress is not inside the yield surface and a plastic iterative procedure must take place as described in the following:

1. Set $m = 0$, $\boldsymbol{\sigma}_{(0)}^{n+1} = \widetilde{\boldsymbol{\sigma}}^{n+1}$, $\varepsilon_{\mathrm{ep}(0)}^{n+1} = \varepsilon_{\mathrm{ep}}^{n}$ and $\Delta\varepsilon_{11}^{p(0)} = \Delta\varepsilon_{22}^{p(0)} = 0$

2. Determine the plastic multiplier as

$$\Delta\lambda = \frac{f\left(\boldsymbol{\sigma}_{(m)}^{n+1}, \varepsilon_{\mathrm{ep}(m)}^{n+1}\right)}{\frac{\mathrm{d}f}{\mathrm{d}\boldsymbol{\sigma}}\left(\boldsymbol{\sigma}_{(m)}^{n+1}\right) \cdot \mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\boldsymbol{\sigma}}\left(\sigma_{(m)}^{n+1}\right) - \frac{\mathrm{d}f}{\mathrm{d}\varepsilon_{\mathrm{ep}}}\left(\varepsilon_{\mathrm{ep}(m)}^{n+1}\right)} \tag{233.22}$$

3. Perform a plastic corrector step: $\boldsymbol{\sigma}_{(m+1)}^{n+1} = \boldsymbol{\sigma}_{(m)}^{n+1} - \Delta\lambda\mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\sigma}\left(\boldsymbol{\sigma}_{(m)}^{n+1}\right)$ and find the increments in plastic strain according to

$$\begin{aligned}
\varepsilon_{\mathrm{ep}(m+1)}^{n+1} &= \varepsilon_{\mathrm{ep}(m)}^{n+1} + \Delta\lambda \\
\Delta\varepsilon_{11}^{p(n+1)} &= \Delta\varepsilon_{11}^{p(n)} + \Delta\lambda\frac{\partial f}{\partial\sigma_{11}}\left(\sigma_{(m)}^{n+1}\right) \\
\Delta\varepsilon_{22}^{p(n+1)} &= \Delta\varepsilon_{22}^{p(n)} + \Delta\lambda\frac{\partial f}{\partial\sigma_{22}}\left(\sigma_{(m)}^{n+1}\right)
\end{aligned} \tag{233.23}$$

4. If $\left|f\left(\boldsymbol{\sigma}_{(m+1)}^{n+1}, \varepsilon_{\mathrm{ep}}^{n}\right)\right| < \mathrm{tol}$ or $m = m_{\max}$; stop and set

$$\begin{aligned}
\boldsymbol{\sigma}^{n+1} &= \boldsymbol{\sigma}_{(m+1)}^{n+1}, \\
\varepsilon_{\mathrm{ep}}^{n+1} &= \varepsilon_{\mathrm{ep}(m+1)}^{n+1}, \\
\Delta\varepsilon_{11}^{p} &= \Delta\varepsilon_{11}^{p(m+1)}, \\
\Delta\varepsilon_{22}^{p} &= \Delta\varepsilon_{22}^{p(m+1)}.
\end{aligned} \tag{233.24}$$

Otherwise set $m = m + 1$and return to 2.

The thickness strain increment for plastic yield is calculated as

$$\Delta\varepsilon_{33} = -\frac{1}{1-v}(\Delta\varepsilon_{11} + \Delta\varepsilon_{22}) - \left(1 - \frac{v}{1-v}\right)\left(\Delta\varepsilon_{11}^{p} + \Delta\varepsilon_{22}^{p}\right) \tag{233.25}$$

History Variables for the MAGNESIUM keyword option:

The following history variables will be stored for the MAGNESIUM option:

History Variable #	Description
10	Gurson damage
11	Void fraction
12	Void fraction star

History Variable #	Description
14	Damage
15	Plastic strain to failure
17	Equivalent plastic strain (energy conjugate to von Mises stress)
19	Effective stress (Cazacu-Barlat)

*MAT_VISCOELASTIC_LOOSE_FABRIC

This is Material Type 234 developed and implemented by Tabiei et al [2004]. The model is a mechanism incorporating the crimping of the fibers as well as the trellising with re-orientation of the yarns and the locking phenomenon observed in loose fabric. The equilibrium of the mechanism allows the straightening of the fibers depending on the fiber tension. The contact force at the fiber cross over point determines the rotational friction that dissipates a part of the impact energy. The stress-strain relationship is viscoelastic based on a three-element model. The failure of the fibers is strain rate dependent. *DAMPING_PART_MASS is recommended to be used in conjunction with this material model. This material is valid for modeling the elastic and viscoelastic response of loose fabric used in body armor, blade containments, and airbags.

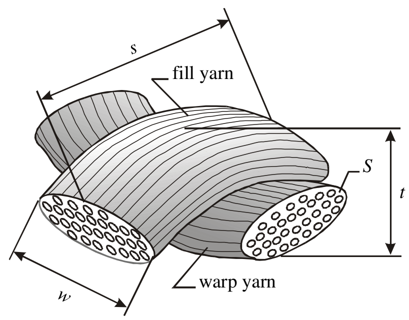

Figure M234-1. Representative Volume Cell (RVC) of the model

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E1	E2	G12	EU	THL	THI
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	TA	W	s	T	H	S	EKA	EUA
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	VMB	C	G23	EKB	AOPT			
Type	F	F	F	F	F			

Card 4	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E1	E_1, Young's modulus in the yarn axial direction
E2	E_2, Young's modulus in the yarn transverse-direction
G12	G_{12}, shear modulus of the yarns
EU	Ultimate strain at failure
THL	Yarn locking angle
THI	Initial braid angle
TA	Transition angle to locking
W	Fiber width

VARIABLE	DESCRIPTION
s	Span between the fibers
T	Real fiber thickness
H	Effective fiber thickness
S	Fiber cross-sectional area
EKA	Elastic constant of element “a”
EUA	Ultimate strain of element “a”
VMB	Damping coefficient of element “b”
C	Coefficient of friction between the fibers
G23	Transverse shear modulus
EKB	Elastic constant of element “b”
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description). EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2 and 4, as with *DEFINE_COORDINATE_NODES EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINED_COORDINATE_VECTOR EQ.3.0: Locally orthotropic material axes defined by the cross product of the vector **v** with the element normal LT.0.0: The absolute value of AOPT is a coordinate system ID (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM, or *DEFINE_COORDINATE_VECTOR).
A1 - A3	Components of vector **a** for AOPT = 2.0
V1 - V3	Components of vector **v** for AOPT = 3.0
D1 - D3	Components of vector **d** for AOPT = 2.0

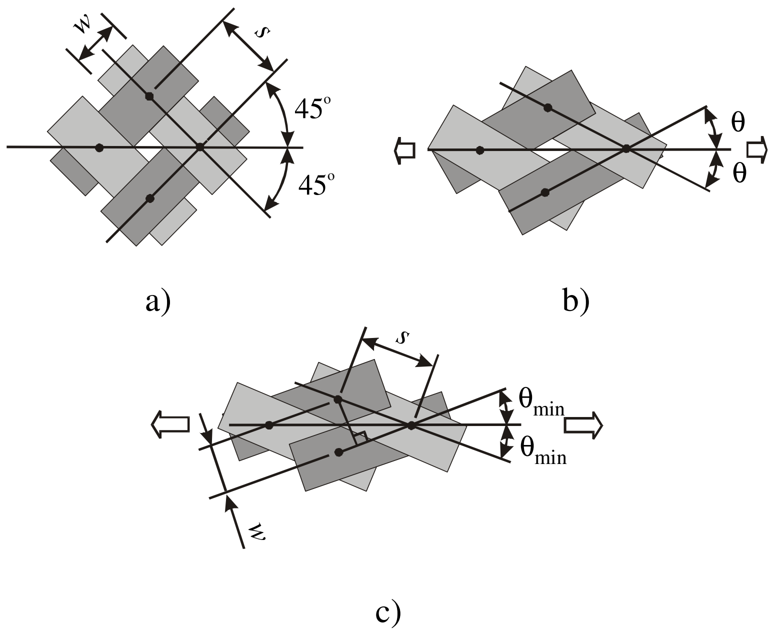

Figure M234-2. Plain woven fabric as trellis mechanism: a) initial state; b) slightly stretched in bias direction; c) stretched to locking.

Remarks:

The parameters of the Representative Volume Cell (RVC) are: the yarn span, s, the fabric thickness, t, the yarn width, w, and the yarn cross-sectional area, A. The initially orthogonal yarns (see Figure M234-2a) are free to rotate (see Figure M234-2b) up to some angle and after that the lateral contact between the yarns causes the locking of the trellis mechanism and the packing of the yarns (see Figure M234-2c).The minimum braid angle, θ_{min}, can be calculated from the geometry and the architecture of the fabric material having the yarn width, w, and the span between the yarns, s:

$$\sin(2\theta_{\min}) = \frac{w}{s} \ .$$

The range angle, θ_{lock}, and the maximum braid angle, θ_{max}, are then easily determined as:

$$\theta_{\text{lock}} = 45° - \theta_{\min} \ , \qquad \theta_{\max} = 45° + \theta_{\text{lock}}$$

The material behavior of the yarn can be simply described by a combination of one Maxwell element without the dashpot and one Kelvin-Voigt element. The 1-D model of viscoelasticity is shown in Figure M234-3. The differential equation of viscoelasticity of the yarns can be derived from the model equilibrium as in the following equation:

$$(K_a + K_b)\sigma + \mu_b\dot{\sigma} = K_a K_b \varepsilon + \mu_b K_a \dot{\varepsilon}$$

The input parameters for the viscoelasticity model of the material are only the static Young's modulus E_1, the Hookian spring coefficient (EKA) K_a, the viscosity coefficient

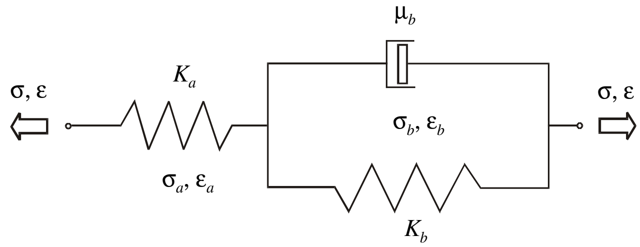

Figure M234-3. Three-element viscoelasticity model

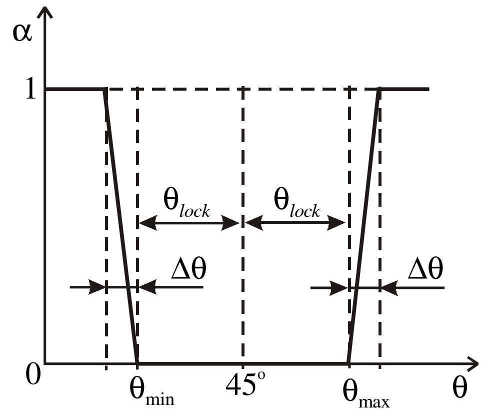

Figure M234-4. The lateral contact factor as a function of average braid angle, θ.

(VMB) μ_b, the static ultimate strain (EU) $\varepsilon_{\max}$, and the Hookian spring ultimate strain (EUA)$\varepsilon_{a,\max}$. The other parameters can be obtained as follows:

$$K_b = \frac{K_a E_1}{K_a - E_1}$$
$$\varepsilon_{b,\max} = \frac{K_a - E_1}{K_a} \varepsilon_{\max}$$

The stress in the yarns for the fill and warp is updated for the next time step as:

$$\sigma_f^{(n+1)} = \sigma_f^{(n)} + \Delta\sigma_f^{(n)}, \qquad \sigma_w^{(n+1)} = \sigma_w^{(n)} + \Delta\sigma_w^{(n)}$$

where $\Delta\sigma_f$ and $\Delta\sigma_w$ are the stress increments in the yarns. We can imagine that the RVC is smeared to the parallelepiped in order to transform the stress acting on the yarn cross-section to the stress acting on the element wall. The thickness of the membrane shell element used should be equal to the effective thickness, t_e, that can be found by dividing the areal density of the fabric by its mass density. The in-plane stress components acting on the RVC walls in the material direction of the yarns are calculated as follows for the fill and warp directions:

$$\sigma_{f11}^{(n+1)} = \frac{2\sigma_f^{(n+1)} S}{s t_e} \qquad \sigma_{w11}^{(n+1)} = \frac{2\sigma_w^{(n+1)} S}{s t_e}$$

$$\sigma_{f22}^{(n+1)} = \sigma_{f22}^{(n)} + \alpha E_2 \Delta\varepsilon_{f22}^{(n)} \qquad \sigma_{w22}^{(n+1)} = \sigma_{w22}^{(n)} + \alpha E_2 \Delta\varepsilon_{w22}^{(n)}$$

$$\sigma_{f12}^{(n+1)} = \sigma_{f12}^{(n)} + \alpha G_{12} \Delta\varepsilon_{f12}^{(n)} \qquad \sigma_{w12}^{(n+1)} = \sigma_{w12}^{(n)} + \alpha G_{12} \Delta\varepsilon_{w12}^{(n)}$$

where E_2 is the transverse Young's modulus of the yarns, G_{12} is the longitudinal shear modulus, and α is the lateral contact factor. The lateral contact factor is zero when the trellis mechanism is open and unity if the mechanism is locked with full lateral contact between the yarns. There is a transition range, $\Delta\theta \times \mathrm{TA}$, of the average braid angle, θ, in which the lateral contact factor, α, is a linear function of the average braid angle. The graph of the function $\alpha(\theta)$ is shown in Figure M234-4.

*MAT_MICROMECHANICS_DRY_FABRIC

This is Material Type 235 developed and implemented by Tabiei et al [2001]. The material model derivation includes the micro-mechanical approach and the homogenization technique usually used in composite material models. The model accounts for reorientation of the yarns and the fabric architecture. The behavior of the flexible fabric material is achieved by discounting the shear moduli of the material in free state which allows the simulation of the trellis mechanism before packing the yarns. This material is valid for modeling the elastic response of loose fabric used in inflatable structures, parachutes, body armor, blade containments, and airbags.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E1	E2	G12	G23	V12	V23
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	XT	THI	THL	BFI	BWI	DSCF	CNST	ATLR
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	VME	VMS	TRS	FFLG	AOPT			
Type	F	F	F	F	F			

Card 4	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E1	E_1, Young's modulus of the yarn in the axial-direction
E2	E_2, Young's modulus of the yarn in the transverse-direction
G12	G_{12}, shear modulus of the yarns.
G23	G_{23}, transverse shear modulus of the yarns.
V12	Poisson's ratio
V23	Transverse Poisson's ratio
XT	Stress or strain to failure (see FFLG)
THI	Initial braid angle
THL	Yarn locking angle
BFI	Initial undulation angle in fill direction
BWI	Initial undulation angle in warp direction
DSCF	Discount factor
CNST	Reorientation damping constant
ATLR	Angle tolerance for locking
VME	Viscous modulus for normal strain rate
VMS	Viscous modulus for shear strain rate
TRS	Transverse shear modulus of the fabric layer

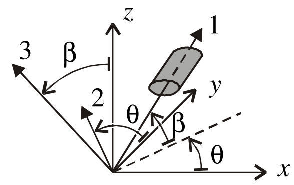

Figure M235-1. Yarn orientation schematic.

VARIABLE	DESCRIPTION
FFLG	Flag for stress-based or strain-based failure: EQ.0: XT is a stress to failure. NE.0: XT is a strain to failure.
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for more complete description). EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2 and 4, as with *DEFINE_COORDINATE_NODES EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINED_COORDINATE_VECTOR EQ.3.0: locally orthotropic material axes defined by the cross product of the vector **v**with the element normal LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM, or *DEFINE_COORDINATE_VECTOR).
A1 - A3	Components of vector **a** for AOPT = 2.0
V1 - V3	Components of vector **v** for AOPT = 3.0
D1 - D3	Components of vector **d** for AOPT = 2.0

Remarks:

The Representative Volume Cell (RVC) approach is used in the micro-mechanical model development. The direction of the yarn in each sub-cell is determined by two angles – the braid angle, θ *(the initial braid angle is 45 degrees)*, and the undulation angle of the yarn, which is different for the fill and warp-yarns, β_f and β_w (the initial undulations are normally a few degrees), respectively. The starting point for the homogenization of the material properties is the determination of the yarn stiffness matrices.

The elasticity tensor is given by

$$[C'] = [S']^{-1} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{12}}{E_1} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\mu G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\mu G_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\mu G_{12}} \end{bmatrix}^{-1}$$

where $E_1,\ E_2,\ \nu_{12},\ \nu_{23},\ G_{12}$ and G_{23} are Young's moduli, Poisson's ratios, and the shear moduli of the yarn material, respectively. μ is a discount factor, which is function of the braid angle, θ, and has value between μ_0 and 1 as shown in the next figure. Initially, in a free stress state, the discount factor is a small value (DSCF = $\mu_0 \ll 1$) and the material has very small resistance to shear deformation if any.

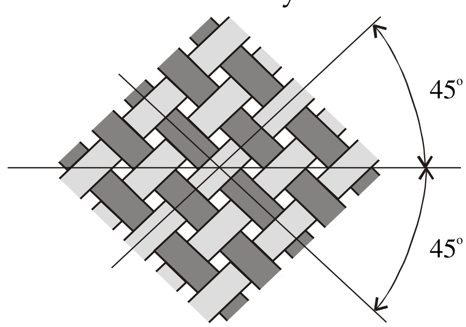

Figure M235-2. Free state of the plain woven fabric

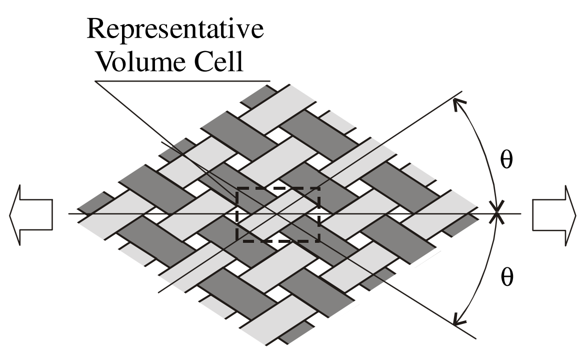

Figure M235-3. Stretched state of the plain woven fabric

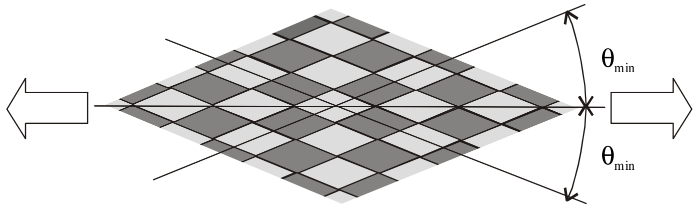

Figure M235-4. Compacted state of the plain woven fabric

When locking occurs, the fabric yarns are packed and behave like elastic media. The discount factor is unity as shown in the next figure. The micro-mechanical model is developed to account for the reorientation of the yarns up to the locking angle. The locking angle, θ_{lock}, can be obtained from the yarn width and the spacing parameter of the fabric using simple geometrical relationship. The transition range, $\Delta\theta$ (angle tolerance for locking), can be chosen to be as small as possible, but big enough to prevent high frequency oscillations during the transition to the compacted state which depends on the range to the locking angle and the dynamics of the simulated problem. The reorientation damping constant damps some of the high frequency oscillations. A simple rate effect is added by defining the viscous modulus for normal or shear strain rate (VME × $\dot{\varepsilon}_{11 \text{ or } 22}$ for normal components and VMS × $\dot{\varepsilon}_{12}$ for the shear components).

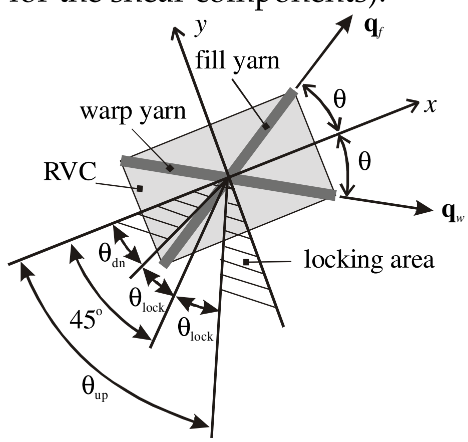

Figure M235-5. Locking angles

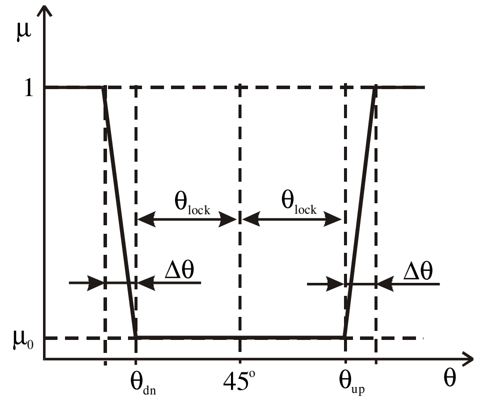

Figure M235-6. Discount factor as a function of braid angle, θ

*MAT_SCC_ON_RCC

This is Material Type 236 developed by Carney, Lee, Goldberg, and Santhanam [2007]. This model simulates silicon carbide coating on Reinforced Carbon-Carbon (RCC), a ceramic matrix. It is based upon a quasi-orthotropic, linear-elastic, plane-stress model. Additional constitutive model attributes include a simple (meaning non-damage model based) option that can model the tension crack requirement: a "stress-cutoff" in tension. This option satisfies the tension crack requirements by limiting the stress in tension but not compression and having the tensile "yielding" (that is, the stress-cutoff) be fully recoverable – not plasticity or damage based.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E0	E1	E2	E3	E4	E5
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	PR	G	G_SCL	TSL	EPS_TAN			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E0	E_0; see Remarks below.
E1	E_1; see Remarks below.
E2	E_2; see Remarks below.
E3	E_3; see Remarks below.
E4	E_4; see Remarks below.
E5	E_5, Young's modulus of the yarn in transverse-direction
PR	Poisson's ratio

VARIABLE	DESCRIPTION
G	Shear modulus
G_SCL	Shear modulus multiplier (default = 1.0)
TSL	Tensile limit stress
EPS_TAN	Strain at which E = tangent to the polynomial curve

Remarks:

This model for the silicon carbide coating on RCC is based upon a quasi-orthotropic, linear-elastic, plane-stress model, given by:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = \begin{bmatrix}\frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0\\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0\\ 0 & 0 & G_{12}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix}$$

Additional constitutive model requirements include a simple (meaning non-damage model based) option that can model the tension crack requirement: a “stress-cutoff” in tension. This option satisfies the tension crack requirements by limiting the stress in tension but not compression and having the tensile “yielding” (that is, the stress-cutoff) be fully recoverable – not plasticity or damage based.

The tension stress-cutoff separately resets the stress to a limit value when it is exceeded in each of the two principal directions. There is also a strain-based memory criterion that ensures unloading follows the same path as loading: the “memory criterion” is the tension stress assuming that no stress cutoffs were in effect. In this way, when the memory criterion exceeds the user-specified cutoff stress, the actual stress will be set to that value. When the element unloads and the memory criterion falls back below the stress cutoff, normal behavior resumes. Using this criterion is a simple way to ensure that unloading does not result in any hysteresis. The cutoff criterion cannot be based on an effective stress value because effective stress does not discriminate between tension and compression while also including shear. This means that the in plane, 1- and 2- directions must be modeled as independent to use the stress cutoff. Because the Poisson’s ratio is not zero, this assumption is not true for cracks that may arbitrarily lie along any direction. However, careful examination of damaged RCC shows that the surface cracks do, generally, tend to lie in the fabric directions, meaning that cracks tend to open in the 1- or the 2- direction independently. So the assumption of directional independence for tension cracks may be appropriate for the coating because of this observed orthotropy.

The quasi-orthotropic, linear-elastic, plane-stress model with tension stress cutoff (to simulate tension cracks) can model the as-fabricated coating properties, which do not show nonlinearities, but not the non-linear response of the flight-degraded material. Explicit finite element analysis (FEA) lends itself to *nonlinear-elastic* stress-strain relation instead of linear-elastic. Thus, instead of $\sigma = \mathbf{E}\varepsilon$, the modulus will be defined as a function of some effective strain quantity, or $\sigma = \mathbf{E}(\varepsilon_{\text{eff}})\varepsilon$, even though it is uncertain, from the available data, whether the coating response is completely nonlinear-elastic and does not include some damage mechanism.

This nonlinear-elastic model cannot be implemented into a closed form solution or into an implicit solver; however, for explicit FEA such as is used for LS-DYNA impact analysis, the modulus can be adjusted at each time step to a higher or lower value as desired. In order to model the desired S-shape response curve of flight-degraded RCC coating, a function of strain that replicates the desired response must be found. The nonlinearities in the material are assumed recoverable (elastic) and the modulus is assumed to be communicative between the 1- and 2- directions (going against the tension-crack assumption that the two directions do not interact). Sometimes stability can be a problem for this type of nonlinearity modeling; however, stability was not found to be a problem with the material constants used for the coating.

The von Mises strain is selected for the effective strain definition as it couples the 3-dimensional loading but reduces to uniaxial data, so that the desired uniaxial compressive response can be reproduced. So,

$$\varepsilon_{\text{eff}} = \frac{1}{\sqrt{2}}\frac{1}{1+\nu}\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_1-\varepsilon_3)^2+3\gamma_{12}^2}\ ,$$

where for a two-dimensional, isotropic shell element case, the z-direction strain is given by:

$$\varepsilon_3 = \frac{-\nu}{1-\nu}(\varepsilon_1+\varepsilon_2)\ .$$

The function for modulus is implemented as an arbitrary 5th order polynomial:

$$E(\varepsilon_{\text{eff}}) = A_0\varepsilon_{\text{eff}}^0 + A_1\varepsilon_{\text{eff}}^1 + \cdots + A_5\varepsilon_{\text{eff}}^5\ .$$

In the case of as-fabricated material the first coefficient, A_0, is simply the modulus E, and the other coefficients, $A_{n>0}$, are zero, reducing to a 0th order polynomial, or linear. To match the degraded stress-strain compression curve, a higher order polynomial is needed. Six conditions on stress were used (stress and its derivative at beginning, middle, and end of the curve) to obtain a 5th order polynomial, and then the derivative of that equation was taken to obtain modulus as a function of strain, yielding a 4th order polynomial that represents the degraded coating modulus as strain curve.

For values of strain which exceed the failure strain observed in the laminate compression tests, the higher order polynomial will no longer match the test data. Therefore, after a specified effective-strain, representing failure, the modulus is defined to be the tangent of the polynomial curve. As a result, the stress/strain response has a continuous derivative, which aids in avoiding numerical instabilities. The test data does not clearly define the failure strain of the coating, but in the impact test it appears that the coating has a higher compressive failure strain in bending than the laminate failure strain.

The two dominant modes of loading which cause coating loss on the impact side of the RCC (the front-side) are in-plane compression and transverse shear. The in-plane compression is measured by the peak out of plane tensile strain, ε_3. As there is no direct loading of a shell element in this direction, ε_3 is computed through Poisson's relation:

$$\varepsilon_3 = \frac{-v}{1-v}(\varepsilon_1 + \varepsilon_2) \ .$$

When ε_3 is tensile, it implies that the average of ε_1 and ε_2 is compressive. This failure mode will likely dominate when the RCC undergoes large bending, putting the front-side coating in high compressive strains. A transverse shear failure mode is expected to dominate when the debris source is very hard or very fast. By definition, the shell element cannot give a precise account of the transverse shear throughout the RCC's thickness. However, the Belytschko-Tsay shell element formulation in LS-DYNA has a first-order approximation of transverse shear that is based on the out-of-plane nodal displacements and rotations that should suffice to give a qualitative evaluation of the transverse shear. By this formulation, the transverse shear is constant through the entire shell thickness and thus violates surface-traction conditions. The constitutive model implementation records the peak value of the tensile out-of-plane strain (ε_3) and peak root-mean-sum transverse-shear: $\sqrt{\varepsilon_{13}^2 + \varepsilon_{23}^2}$.

*MAT_PML_HYSTERETIC

This is Material Type 237. This is a perfectly-matched layer (PML) material with a Biot linear hysteretic constitutive law, to be used in a wave-absorbing layer adjacent to a Biot hysteretic material (*MAT_BIOT_HYSTERETIC) in order to simulate wave propagation in an unbounded medium with material damping. This material is the visco-elastic counterpart of the elastic PML material (*MAT_PML_ELASTIC). See the Remarks sections of *MAT_PML_ELASTIC (*MAT_230) and *MAT_BIOT_HYSTERETIC (*MAT_232) for further details.

Card	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	ZT	FD		
Type	A	F	F	F	F	F		
Default	none	none	none	none	0.0	3.25		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
ZT	Damping ratio
FD	Dominant excitation frequency in Hz

*MAT_PERT_PIECEWISE_LINEAR_PLASTICITY

This is Material Type 238. It is a duplicate of Material Type 24 (*MAT_PIECEWISE_LINEAR_PLASTICITY) modified for use with *PERTURBATION_MATERIAL and solid elements in an explicit analysis. It should give exactly the same values as the original material, if used exactly the same. It exists as a separate material type because of the speed penalty (an approximately 10% increase in the overall execution time) associated with the use of a material perturbation.

See Material Type 24 (*MAT_PIECEWISE_LINEAR_PLASTICITY) for a description of the material parameters. All of the documentation for Material Type 24 applies. First creating the input deck using Material Type 24 is recommended. Additionally, the CMP variable in the *PERTURBATION_MATERIAL must be set to affect a specific variables in the MAT_238 definition as defined in the following table; for example, CMP = 5 will perturb the yield stress.

CMP Value	Material Variable
3	E
5	SIGY
6	ETAN
7	FAIL

*MAT_COHESIVE_MIXED_MODE_ELASTOPLASTIC_RATE_{OPTION}

Available options include:

<BLANK>

THERMAL

3MODES

FUNCTIONS

This is Material Type 240. This model is a rate-dependent, elastic-ideally plastic cohesive zone model. It includes a tri-linear traction-separation law with a quadratic yield and damage initiation criterion in mixed-mode loading (mode I – mode II), while the damage evolution is governed by a power-law formulation. It can be used only with cohesive element formulations; see the variable ELFORM in *SECTION_SOLID and *SECTION_-SHELL.

With the THERMAL option, some properties are defined as functions of temperature, meaning fields EMOD, GMOD, G1C_0, G2C_0, T0, S0, FG1, and FG2 must be defined as curve IDs instead of scalar values.

With the FUNCTIONS option, some properties are defined as functions of connection partner properties, meaning fields EMOD, GMOD, G1C_0, G2C_0, T0, S0, FG1, and FG2 must be defined as function IDs instead of scalar values. See remarks for details.

The keyword option 3MODES activates the possibility to include deformation/fracture mode III which could be useful for cohesive shells. Corresponding fields can be defined on optional Cards 4 and 5.

Note that 3MODES is compatible with THERMAL and FUNCTIONS, but THERMAL and FUNCTIONS cannot be used together. In other words, THERMAL_3MODES and FUNCTIONS_3MODES are allowed as keyword options, but THERMAL_FUNCTIONS is not allowed.

Card Summary:

Card 1. This card is required.

MID	RO	ROFLG	INTFAIL	EMOD	GMOD	THICK	INICRT

Card 2. This card is required.

G1C_0	G1C_INF	EDOT_G1	T0	T1	EDOT_T	FG1	LCG1C

Card 3. This card is required.

G2C_0	G2C_INF	EDOT_G2	S0	S1	EDOT_S	FG2	LCG2C

Card 4. This card is included if the 3MODES keyword option is used.

G3C_0	G3C_INF	EDOT_G3	R0	R1	EDOT_R	FG3	LCG3C

Card 5. This card is included if the 3MODES keyword option is used.

GMOD3							

Card 6. This card is optional.

RFILTF	COMPY	SMOLIM	XMU				

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ROFLG	INTFAIL	EMOD	GMOD	THICK	INICRT
Type	A	F	I	I	F/I	F/I	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
ROFLG	Flag for whether density is specified per unit area or volume: EQ.0: Specified density per unit volume (default) EQ.1: Specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero
INTFAIL	Number of integration points required for the cohesive element to be deleted. The value of INTFAIL may range from 1 to 4 with 1 the recommended value. LT.0.0: Employs a Newton-Cotes integration scheme. The element will be deleted when |INTFAIL| integration points have failed.

VARIABLE	DESCRIPTION
	EQ.0.0: Employs a Newton-Cotes integration scheme. The element will *not* be deleted even if it satisfies the failure criterion. GT.0.0: Employs a Gauss integration scheme. The element will be deleted when INTFAIL integration points have failed.
EMOD	Young's modulus of the material (Mode I). It is a curve ID if the THERMAL keyword option is used. It is a function ID if the FUNCTIONS keyword option is used.
GMOD	The shear modulus of the material (Mode II). Curve ID for THERMAL keyword option. GMOD is a function ID for the FUNCTIONS keyword option.
THICK	GT.0.0: Cohesive thickness LE.0.0: Initial thickness is calculated from nodal coordinates.
INICRT	Yield and damage initiation criterion: EQ.0.0: Quadratic nominal stress (default) EQ.1.0: Maximum nominal stress EQ.2.0: Maximum nominal stress (same as INICRT = 1.0). Additionally, it flags outputting the maximum strain as history variable #15. LT.0.0: Mixed mode with flexible exponent |INICRT|

Card 2	1	2	3	4	5	6	7	8
Variable	G1C_0	G1C_INF	EDOT_G1	T0	T1	EDOT_T	FG1	LCG1C
Type	F/I	F	F	F/I	F	F	F/I	I

VARIABLE	DESCRIPTION
G1C_0	GT.0.0: Energy release rate G_{IC} in Mode I. G1C_0 is a curve ID if the THERMAL keyword option is used. G1C_0 is a function ID if the FUNCTIONS keyword option is used. LE.0.0: Lower bound value of rate-dependent G_{IC}

VARIABLE	DESCRIPTION
G1C_INF	Upper bound value of rate-dependent G_{IC} (only considered if G1C_0 < 0)
EDOT_G1	Equivalent strain rate at yield initiation to describe the rate dependency of G_{IC} (only considered if G1C_0 < 0)
T0	GT.0.0: Yield stress in Mode I. T0 is a curve ID if the THERMAL keyword option is used. T0 is a function ID if the FUNCTIONS keyword option is used. LT.0.0: Rate-dependency is considered; see T1 and EDOT_T.
T1	Field T1, only considered if T0 < 0: GT.0.0: Quadratic logarithmic model LT.0.0: Linear logarithmic model
EDOT_T	Equivalent strain rate at yield initiation to describe the rate dependency of the yield stress in Mode I (only considered if T0 < 0)
FG1	f_{G1}, describes the tri-linear shape of the traction-separation law in Mode I. See remarks. It is a curve ID if the THERMAL keyword option is used. It is a function ID if the FUNCTIONS keyword option is used. GT.0.0: FG1 is the ratio of fracture energies, $G_{I,P}/G_{IC}$. LT.0.0: |FG1| is ratio of displacements, $(\delta_{n2} - \delta_{n1})/(\delta_{nf} - \delta_{n1})$.
LCG1C	Load curve ID which defines fracture energy GIC as a function of cohesive element thickness. G1C_0 and G1C_INF are ignored in this case.

Card 3	1	2	3	4	5	6	7	8
Variable	G2C_0	G2C_INF	EDOT_G2	S0	S1	EDOT_S	FG2	LCG2C
Type	F/I	F	F	F/I	F	F	F/I	I

VARIABLE	DESCRIPTION
G2C_0	GT.0.0: Energy release rate G_{IIC} in Mode II. If the THERMAL keyword option is used, it is a load curve ID. For the FUNCTIONS keyword option, it is a function ID. LE.0.0: Lower bound value of rate-dependent G_{IIC}
G2C_INF	Upper bound value of G_{IIC} (only considered if G2C_0 < 0)
EDOT_G2	Equivalent strain rate at yield initiation to describe the rate dependency of G_{IIC} (only considered if G2C_0 < 0)
S0	GT.0.0: Yield stress in Mode II. It is a load curve ID for the THERMAL keyword option. It is a function ID for the FUNCTIONS keyword option. LT.0.0: Rate-dependency is considered; see S1 and EDOT_S
S1	Parameter S1, only considered if S0 < 0: GT.0.0: Quadratic logarithmic model is applied. LT.0.0: Linear logarithmic model is applied.
EDOT_S	Equivalent strain rate at yield initiation to describe the rate dependency of the yield stress in Mode II (only considered if S0 < 0)
FG2	f_{G2}, describes the tri-linear shape of the traction-separation law in Mode II, see remarks. It is a load curve ID for the THERMAL keyword option. It is a function ID for the FUNCTIONS keyword option. GT.0.0: FG2 is the ratio of fracture energies, $G_{II,P}/G_{IIC}$. LT.0.0: $\lvert FG2 \rvert$ is the ratio of displacements, $(\delta_{t2} - \delta_{t1})/(\delta_{tf} - \delta_{t1})$.
LCG2C	Load curve ID which defines fracture energy GIIC as a function of cohesive element thickness. G2C_0 and G2C_INF are ignored in that case.

Additional Cards 4 and 5 for 3MODES keyword option. Properties for Mode III (out-of-plane mode in cohesive shell elements).

Card 4	1	2	3	4	5	6	7	8
Variable	G3C_0	G3C_INF	EDOT_G3	R0	R1	EDOT_R	FG3	LCG3C
Type	F/I	F	F	F/I	F	F	F/I	I

Card 5	1	2	3	4	5	6	7	8
Variable	GMOD3							
Type	F/I							

VARIABLE	DESCRIPTION
G3C_0	GT.0.0: Energy release rate G_{IIIC} in Mode III. G3C_0 is a load curve ID for the THERMAL keyword option. G3C_0 is a function ID for the FUNCTIONS keyword option. LE.0.0: Lower bound value of rate-dependent G_{IIIC}
G3C_INF	Upper bound value of rate-dependent G_{IIIC} (only considered if G3C_0 < 0)
EDOT_G3	Equivalent strain rate at yield initiation to describe the rate dependency of G_{IIIC} (only considered if G1C_0 < 0)
R0	GT.0.0: Yield stress in Mode III. R0 is a load curve ID for the THERMAL keyword option. R0 is a function ID for the FUNCTIONS keyword option. LT.0.0: Rate-dependency is considered
R1	Parameter R1, only considered if R0 < 0: GT.0.0: Quadratic logarithmic model LT.0.0: Linear logarithmic model
EDOT_R	Equivalent strain rate at yield initiation to describe the rate dependency of the yield stress in Mode III (only considered if R0 < 0)

VARIABLE	DESCRIPTION
FG3	f_{G3}, describes the tri-linear shape of the traction-separation law in Mode III; see remarks. It is a load curve ID if the THERMAL keyword option is used. It is a function ID if the FUNCTIONS keyword option is used. GT.0.0: FG3 is ratio of fracture energies, $G_{III,P}/G_{IIIC}$. LT.0.0: |FG3| is ratio of displacements, $(\delta_{s2}-\delta_{s1})/(\delta_{sf}-\delta_{s1})$.
LCG3C	Load curve ID which defines fracture energy GIIIC as a function of cohesive element thickness. G3C_0 and G3C_INF are ignored in that case.
GMOD3	Shear modulus for Mode III. GMOD3 is a load curve ID for the THERMAL keyword option. GMOD3 is a function ID for the FUNCTIONS keyword option.

This card is optional.

Card 6	1	2	3	4	5	6	7	8
Variable	RFILTF	COMPY	SMOLIM	XMU				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
RFILTF	Smoothing factor on the equivalent strain rate using an exponential moving average method: $\dot{\varepsilon}_n^{\mathrm{avg}} = \mathrm{RFILTF} \times \dot{\varepsilon}_{n-1}^{\mathrm{avg}} + (1-\mathrm{RFILTF}) \times \dot{\varepsilon}_n$ This option invokes a modified handling of strain rates (see Remarks). GT.0.0: RFILTF applied on the equivalent plastic strain rate. LT.0.0: |RFILTF| applied on the equivalent total strain rate.
COMPY	Yield under compression flag: EQ.0: Off (default) EQ.1: On

VARIABLE	DESCRIPTION
SMOLIM	Smooth treatment of asymptotic limits (such as pure shear). EQ.0: Off (default) EQ.1: On
XMU	Exponent of the mixed mode failure criterion. Default is 1.0.

Remarks:

The model is a tri-linear elastic-ideally plastic Cohesive Zone Model, developed by Marzi et al. [2009]. It looks similar to *MAT_185 but considers effects of plasticity and rate-dependency. Since the entire separation at failure is plastic, no brittle fracture behavior can be modeled with this material type.

The following description of the model is for two deformation/fracture modes (I and II) only, meaning without the option 3MODES. This is a natural choice for cohesive solid elements, where no specific distinction between in-plane and out-of-plane shear can be made. On the other hand, if this material model is used with the cohesive shell element type ±29, shear deformation can clearly be separated into in-plane shear (Mode II) and out-of-plane shear (Mode III). This can be taken into account by adding 3MODES to the keyword and defining additional Cards 4 and 5. Corresponding equations including Mode III are not explicitly given here (for the sake of brevity), but derivation of them is straightforward.

The separations, Δ_n and Δ_t, in the normal (peel) and tangential (shear) directions, respectively, are calculated from the element's separations in the integration points,

$$\Delta_n = \max(u_n, 0)$$

and

$$\Delta_t = \sqrt{u_{t1}^2 + u_{t2}^2} \ .$$

u_n is the separation in the normal direction while u_{t1} and u_{t2} is the separation in both tangential directions of the element coordinate system. The total (mixed-mode) separation Δ_m is determined by

$$\Delta_m = \sqrt{\Delta_n^2 + \Delta_t^2} \ .$$

The initial stiffnesses in both modes are calculated from the elastic Young's and shear moduli and are respectively,

$$E_n = \frac{\text{EMOD}}{\text{THICK}}$$
$$E_t = \frac{\text{GMOD}}{\text{THICK}} \ ,$$

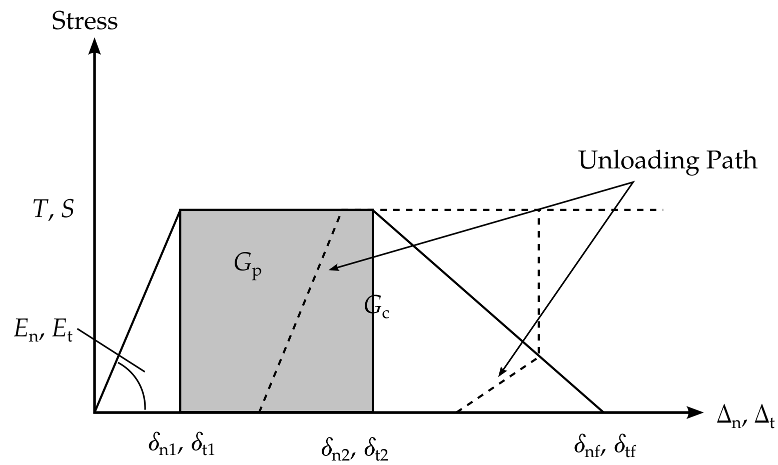

Figure M240-1. Trilinear traction separation law

where THICK, the element's thickness, is an input parameter. If THICK ≤ 0, it is calculated from the distance between the initial positions of the element's corner nodes (Nodes 1-5, 2-6, 3-7 and 4-8, respectively).

While the total energy under the traction-separation law is given by G_C, one further parameter is needed to describe the exact shape of the tri-linear material model. If the area (energy) under the constant stress (plateau) region is denoted G_P (see Figure M240-1), a parameter f_G defines the shape of the traction-separation law,

$$0 \le f_{G1} = \frac{G_{I,P}}{G_{IC}} < 1 - \frac{T^2}{2G_{IC}E_n} < 1 \quad \text{for mode I loading}$$

$$0 \le f_{G2} = \frac{G_{II,P}}{G_{IIC}} < 1 - \frac{S^2}{2G_{IIC}E_t} < 1 \quad \text{for mode II loading}$$

As a recommended alternative, the shape of the tri-linear model can be described by the following displacement ratios (triggered by negative input values for f_G):

$$0 < |f_{G1}| = \left|\frac{\delta_{n2} - \delta_{n1}}{\delta_{nf} - \delta_{n1}}\right| < 1 \quad \text{for mode I loading}$$

$$0 < |f_{G2}| = \left|\frac{\delta_{t2} - \delta_{t1}}{\delta_{tf} - \delta_{t1}}\right| < 1 \quad \text{for mode II loading}$$

While f_{G1} and f_{G2} are always constant values, T, S, G_{IC}, and G_{IIC} may be chosen as functions of an equivalent strain rate $\dot{\varepsilon}_{\mathrm{eq}}$, which is evaluated by

$$\dot{\varepsilon}_{\mathrm{eq}} = \frac{\sqrt{\dot{u}_n^2 + \dot{u}_{t1}^2 + \dot{u}_{t2}^2}}{\mathrm{THICK}} .$$

Here $\dot{u}_n$, $\dot{u}_{t1}$, and $\dot{u}_{t2}$ are the velocities corresponding to the separations u_n, u_{t1}, and u_{t2}, respectively.

For the yield stresses, two rate dependent formulations are implemented:

1. A quadratic logarithmic function:

$$T(\dot{\varepsilon}_{\mathrm{eq}}) = |\mathrm{T0}| + |\mathrm{T1}|\left[\max\left(0, \ln\frac{\dot{\varepsilon}_{\mathrm{eq}}}{\mathrm{EDOT\_T}}\right)\right]^2 \quad \text{for mode I if T0} < 0 \text{ and T1} > 0$$

$$S(\dot{\varepsilon}_{\mathrm{eq}}) = |\mathrm{S0}| + |\mathrm{S1}|\left[\max\left(0, \ln\frac{\dot{\varepsilon}_{\mathrm{eq}}}{\mathrm{EDOT\_S}}\right)\right]^2 \quad \text{for mode II if S0} < 0 \text{ and S1} > 0$$

2. A linear logarithmic function:

$$T(\dot{\varepsilon}_{\mathrm{eq}}) = |\mathrm{T0}| + |\mathrm{T1}|\max\left(0, \ln\frac{\dot{\varepsilon}_{\mathrm{eq}}}{\mathrm{EDOT\_T}}\right) \quad \text{for mode I if T0} < 0 \text{ and T1} < 0$$

$$S(\dot{\varepsilon}_{\mathrm{eq}}) = |\mathrm{S0}| + |\mathrm{S1}|\max\left(0, \ln\frac{\dot{\varepsilon}_{\mathrm{eq}}}{\mathrm{EDOT\_S}}\right) \quad \text{for mode II if S0} < 0 \text{ and S1} < 0$$

Alternatively, T and S can be set to constant values:

$$T(\dot{\varepsilon}_{\mathrm{eq}}) = \mathrm{T0} \quad \text{for mode I if T0} > 0$$
$$S(\dot{\varepsilon}_{\mathrm{eq}}) = \mathrm{S0} \quad \text{for mode II if S0} > 0$$

The rate-dependency of the fracture energies are given by:

$$G_{IC}(\dot{\varepsilon}_{\mathrm{eq}}) = |\mathrm{G1C\_0}| + (\mathrm{G1C\_INF} - |\mathrm{G1C\_0}|)\exp\left(-\frac{\mathrm{EDOT\_G1}}{\dot{\varepsilon}_{\mathrm{eq}}}\right) \quad \text{if G1C\_0} < 0$$

$$G_{IIC}(\dot{\varepsilon}_{\mathrm{eq}}) = |\mathrm{G2C\_0}| + (\mathrm{G2C\_INF} - |\mathrm{G2C\_0}|)\exp\left(-\frac{\mathrm{EDOT\_G2}}{\dot{\varepsilon}_{\mathrm{eq}}}\right) \quad \text{if G2C\_0} < 0$$

If positive values are chosen for G1C_0 or G2C_0, no rate-dependency is considered for this parameter and its value remains constant as specified by the user.

As an alternative, fracture energies GIC and GIIC can be defined as functions of cohesive element thickness by using load curves LCG1C and LCG2C, respectively. In that case, parameters G1C_0, G1C_INF, G2C_0, and G2C_INF will be ignored, and no rate dependence is considered.

Note that the equivalent strain rate $\dot{\varepsilon}_{\mathrm{eq}}$ is updated until $\Delta_m > \delta_{m1}$. Then, the model behavior depends on the equivalent strain rate at yield initiation. A modified handling of strain rates is invoked by RFILTF $\neq 0$ with which filtered strain rates are updated throughout the whole process.

Having defined the parameters describing the single modes, the mixed-mode behavior is formulated by quadratic initiation criteria for both yield stress and damage initiation, while the damage evolution follows a Power-Law. Due to reasons of readability, the following simplifications are made,

$$T = T(\dot{\varepsilon}_{\mathrm{eq}})$$
$$S = S(\dot{\varepsilon}_{\mathrm{eq}})$$
$$G_{IC} = G_{IC}(\dot{\varepsilon}_{\mathrm{eq}})$$
$$G_{IIC} = G_{IIC}(\dot{\varepsilon}_{\mathrm{eq}})$$

If the quadratic nominal stress criterion is used (INICRT = 0), the mixed-mode yield initiation displacement δ_{m1} is defined as

$$\delta_{m1} = \delta_{n1}\delta_{t1}\sqrt{\frac{1+\beta^2}{\delta_{t1}^2 + (\beta\delta_{n1})^2}}\ ,$$

where $\delta_{n1} = T/E_n$ and $\delta_{t1} = S/E_t$ are the single-mode yield initiation displacements and $\beta = \Delta_t/\Delta_n$ is the mixed-mode ratio. As an analog to the yield initiation, the damage initiation displacement δ_{m2} is defined as:

$$\delta_{m2} = \delta_{n2}\delta_{t2}\sqrt{\frac{1+\beta^2}{{\delta_{t2}}^2 + (\beta\delta_{n2})^2}}\ ,$$

where

$$\delta_{n2} = \delta_{n1} + \frac{f_{G1}G_{IC}}{T}$$
$$\delta_{t2} = \delta_{t1} + \frac{f_{G2}G_{IIC}}{S}$$

As an alternative, a maximum nominal stress criterion could be used (INICRT = 1) which results in the following expressions for yield and damage initiation displacements:

$$\delta_{m1} = \begin{cases} \delta_{n1}\sqrt{1+\beta^2} & \text{if } \beta \le \dfrac{\delta_{t1}}{\delta_{n1}} \\ \dfrac{\delta_{t1}}{\beta}\sqrt{1+\beta^2} & \text{if } \beta > \dfrac{\delta_{t1}}{\delta_{n1}} \end{cases}$$

$$\delta_{m2} = \begin{cases} \delta_{n2}\sqrt{1+\beta^2} & \text{if } \beta \le \dfrac{\delta_{t2}}{\delta_{n2}} \\ \dfrac{\delta_{t2}}{\beta}\sqrt{1+\beta^2} & \text{if } \beta > \dfrac{\delta_{t2}}{\delta_{n2}} \end{cases}$$

A third possibility is to choose INICRT < 0, which invokes a nominal stress criterion with flexible exponent:

$$\delta_{m1} = \delta_{n1}\delta_{t1}\sqrt{1+\beta^2}\left(\delta_{t1}^{|\mathrm{INICRT}|} + (\beta\delta_{n1})^{|\mathrm{INICRT}|}\right)^{-1/|\mathrm{INICRT}|}$$
$$\delta_{m2} = \delta_{n2}\delta_{t2}\sqrt{1+\beta^2}\left(\delta_{t2}^{|\mathrm{INICRT}|} + (\beta\delta_{n2})^{|\mathrm{INICRT}|}\right)^{-1/|\mathrm{INICRT}|}$$

Obviously, the special case of INICRT = -2 would lead to the same result as the quadratic criterion, INICRT = 0.

With $\gamma = \arccos\left(\frac{\langle u_n\rangle}{\Delta_m}\right)$, the ultimate (failure) displacement δ_{mf} can be written,

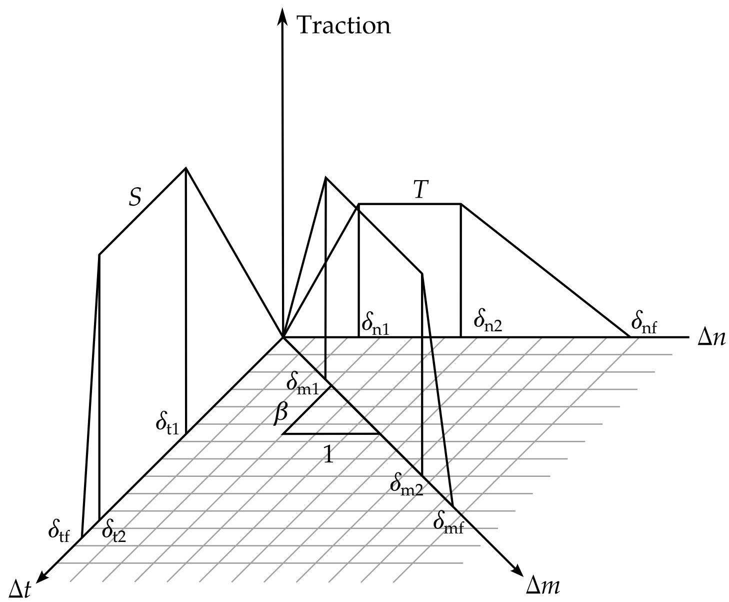

Figure M240-2. Trilinear mixed mode traction-separation law

$$\delta_{mf} = \frac{\delta_{m1}(\delta_{m1} - \delta_{m2})E_n G_{IIC}\cos^2\gamma + G_{IC}\big(2G_{IIC} + \delta_{m1}(\delta_{m1} - \delta_{m2})E_t \sin^2\gamma\big)}{\delta_{m1}(E_n G_{IIC}\cos^2\gamma + E_t G_{IC}\sin^2\gamma)} \ .$$

This formulation describes a power-law damage evolution with an exponent $\eta = 1.0$ (see *MAT_138). This is the case for XMU = 0.0 or 1.0.

With the definition of an arbitrary value for XMU, the failure displacement is given by

$$\delta_{mf} = \max\left(\delta_{m2}, \delta_{m1} - \delta_{m2} + \frac{2}{\delta_{m1}}\left[\left(\frac{E_n\cos^2\gamma}{G_{IC}}\right)^{\mathrm{XMU}} + \left(\frac{E_t\sin^2\gamma}{G_{IIC}}\right)^{\mathrm{XMU}}\right]^{-1/\mathrm{XMU}}\right)$$

After the shape of the mixed-mode traction-separation law has been determined by δ_{m1}, δ_{m2}, and δ_{mf}, the plastic separation in each element direction, $u_{n,P}$, $u_{t1,P}$, and $u_{t2,P}$ can be calculated. The plastic separation in peel direction is given by

$$u_{n,P} = \max(u_{n,P,\Delta t-1}, u_n - \delta_{m1}\cos(\gamma)\,, 0) \ .$$

In the shear direction, a shear yield separation $\delta_{t,y}$,

$$\delta_{t,y} = \sqrt{(u_{t1} - u_{t1,P,\Delta t-1})^2 + (u_{t2} - u_{t2,P,\Delta t-1})^2},$$

is defined. If $\delta_{t,y} > \delta_{m1}\sin\gamma$, the plastic shear separations in the element coordinate system are updated,

$$u_{t1,P} = u_{t1,P,\Delta t-1} + u_{t1} - u_{t1,\Delta t-1}$$
$$u_{t2,P} = u_{t2,P,\Delta t-1} + u_{t2} - u_{t2,\Delta t-1}$$

In the formulas above, $\Delta t-1$ indicates the individual value from the last time increment. In case $\Delta_m > \delta_{m2}$, the damage initiation criterion is satisfied and a damage variable D increases monotonically,

$$D = \max\left(\frac{\Delta_m - \delta_{m2}}{\delta_{mf} - \delta_{m2}}, D_{\Delta t-1}, 0\right)\ .$$

When $\Delta_m > \delta_{mf}$, complete damage ($D = 1$) is reached and the element fails in the corresponding integration point.

Finally, the peel and the shear stresses in element directions are calculated,

$$\sigma_{t1} = E_t(1-D)(u_{t1} - u_{t1,P})$$
$$\sigma_{t2} = E_t(1-D)(u_{t2} - u_{t2P})$$

In the peel direction, no damage under pressure loads is considered if $u_n - u_{n,P} > 0$

$$\sigma_n = E_n(u_n - u_{n,P})\ .$$

Otherwise,

$$\sigma_n = E_n(1-D)(u_n - u_{n,P})\ .$$

If the FUNCTIONS keyword option is used, parameters EMOD, GMOD, G1C_0, G2C_0, T0, S0, FG1, and FG2 (as well as GMOD3, G3C_0, R0, and FG3 if combined with 3MODES) should refer to *DEFINE_FUNCTION IDs. The arguments of those functions include several properties of both connection partners if corresponding solid elements are in a tied contact with shell elements.

These functions depend on:

(t1, t2) = thicknesses of both bond partners
(sy1, sy2) = initial yield stresses at plastic strain of 0.002
(sm1, sm2) = maximum engineering yield stresses (necking points)
r = strain rate
a = element area
(e1, e2) = Young's moduli

For T0 = -100 such a function could look like:

```
*DEFINE_FUNCTION
           100
     func(t1,t2,sy1,sy2,sm1,sm2,r,a,e1,e2)=0.5*(sy1+sy2)
```

Since material parameters must be identified from both bond partners during initialization, this feature is only available for a subset of material models at the moment, namely material models 24, 36, 120, 123, 124, 251, and 258.

Reference:

S. Marzi, O. Hesebeck, M. Brede and F. Kleiner (2009), A Rate-Dependent, Elasto-Plastic Cohesive Zone Mixed-Mode Model for Crash Analysis of Adhesively Bonded Joints, In Proceeding: *7th European LS-DYNA Conference, Salzburg*

*MAT_JOHNSON_HOLMQUIST_JH1

This is Material Type 241. This Johnson-Holmquist Plasticity Damage Model is useful for modeling ceramics, glass and other brittle materials. This version corresponds to the original version of the model, JH1, and Material Type 110 corresponds to JH2, the updated model.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	G	P1	S1	P2	S2	C
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	EPS0	T		ALPHA	SFMAX	BETA	DP1	
Type	F	F		F	F	F	F	

Card 3	1	2	3	4	5	6	7	8
Variable	EPFMIN	EPFMAX	K1	K2	K3	FS	FDAM	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Density
G	Shear modulus
P1	Pressure point 1 for intact material
S1	Effective stress at P1
P2	Pressure point 2 for intact material
S2	Effective stress at P2

VARIABLE	DESCRIPTION
C	Strain rate sensitivity factor
EPS0	Quasi-static threshold strain rate. See *MAT_015.
T	Maximum tensile pressure strength. This value is positive in tension.
ALPHA	Initial slope of the fractured material strength curve. See Figure M241-1.
SFMAX	Maximum strength of the fractured material
BETA	Fraction of elastic energy loss converted to hydrostatic energy (affects bulking pressure (history variable 1) that accompanies damage).
DP1	Maximum compressive pressure strength. This value is positive in compression.
EPFMIN	Plastic strain for fracture at tensile pressure T. See Figure M241-2.
EPFMAX	Plastic strain for fracture at compressive pressure DP1. See Figure M241-1.
K1	First pressure coefficient (equivalent to the bulk modulus)
K2	Second pressure coefficient
K3	Third pressure coefficient
FS	Element deletion due to hydrostatic pressure or equivalent plastic strain: LT.0.0: Delete if P < FS (tensile failure) EQ.0.0: No element deletion (default) GT.0.0: Delete element if the $\bar{\varepsilon}^p$> FS
FDAM	Failure damage value. If this damage value is reached, the element is deleted. A meaningful value would be FDAM = 1.0, for instance. EQ.0.0: No element deletion due to damage (default)

Remarks:

The equivalent stress for both intact and fractured ceramic-type materials is given by:

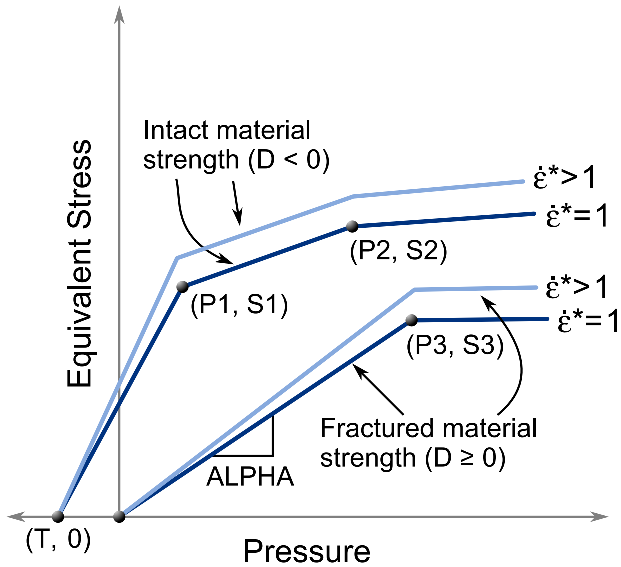

Figure M241-1. Strength: equivalent stress versus pressure.

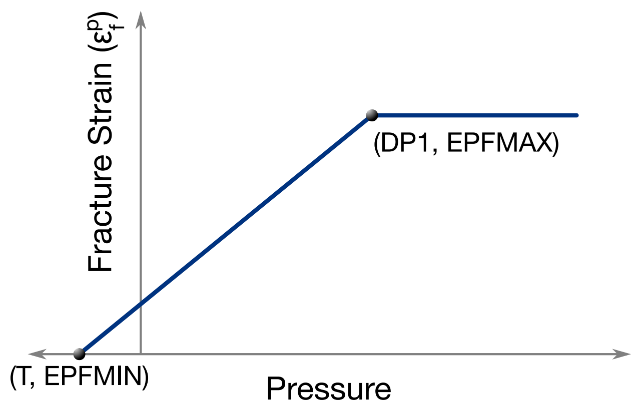

Figure M241-2. Fracture strain versus pressure.

$$\sigma_y = (1 + c \ln \dot{\varepsilon}^*)\sigma(P)$$

where $\sigma(P)$ is evaluated according to Figure M241-1.

$$D = \sum \Delta\varepsilon^p / \varepsilon_f^p (P)$$

represents the accumulated damage (history variable 2) based upon the increase in plastic strain per computational cycle and the plastic strain to fracture is evaluated according to Figure M241-2.

In undamaged material, the hydrostatic pressure is given by

$$P = k_1\mu + k_2\mu^2 + k_3\mu^3 + \Delta P$$

in compression and by

$$P = k_1\mu + \Delta P$$

in tension, where $\mu = \rho/\rho_0 - 1$. A fraction, between 0 and 1, of the elastic energy loss, β, is converted into hydrostatic potential energy (pressure). The pressure increment, ΔP, associated with the increment in the hydrostatic potential energy is calculated at fracture, where σ_y and σ_y^f are the intact and failed yield stresses, respectively. This pressure increment is applied in both compression and tension, which is not true for JH2 where the increment is added only in compression.

$$\Delta P = -k_1\mu_f + \sqrt{\left(k_1\mu_f\right)^2 + 2\beta k_1 \Delta U}$$

$$\Delta U = \frac{\sigma_y - \sigma_y^f}{6G}$$

*MAT_KINEMATIC_HARDENING_BARLAT2000

This is Material Type 242. This model combines the Yoshida non-linear kinematic hardening rule (*MAT_125) with the 8-parameter material model (*MAT_133) of Barlat et al. (2003) to model metal sheets under cyclic plasticity loading with anisotropy in plane stress conditions (see also *MAT_226). This material is available only for shell elements.

Card Summary:

Card 1. This card is required.

MID	RO	E	PR	EA	COE	M	

Card 2. This card is required.

ALPHA1	ALPHA2	ALPHA3	ALPHA4	ALPHA5	ALPHA6	ALPHA7	ALPHA8

Card 3. This card must be included as a blank card.

Card 4. This card must be included as a blank card.

Card 5. This card is required.

CB	Y	SC1	K	RSAT	SB	H	SC2

Card 6. This card is required.

AOPT		IOPT	C1	C2			

Card 7. This card is required.

			A1	A2	A3		

Card 8. This card is required.

V1	V2	V3	D1	D2	D3		

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	EA	COE	M	
Type	A	F	F	F	F	F	F	
Default	none	0.0	0.0	0.0	0.0	0.0	none	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus, E
PR	Poisson's ratio, ν
EA	E^A, parameter controlling the change of Young's modulus; see the remarks of *MAT_125. LT.0.0: |EA| is a curve ID giving the change of Young's modulus as a function of effective plastic strain.
COE	ζ, parameter controlling the change of Young's modulus; see the remarks of *MAT_125.
M	Flow potential exponent. For face centered cubic (FCC) materials $m = 8$ is recommended and for body centered cubic (BCC) materials $m = 6$ may be used. LT.0.0: |M| is a load curve ID specifying the flow potential exponent as a function of effective plastic strain.

Card 2	1	2	3	4	5	6	7	8
Variable	ALPHA1	ALPHA2	ALPHA3	ALPHA4	ALPHA5	ALPHA6	ALPHA7	ALPHA8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION
ALPHA*i*	α_i, material constants in Barlat's yield equation LT.0.0: |ALPHA*i*| is a load curve ID specifying α_i as a function of effective plastic strain.

Card 3	1	2	3	4	5	6	7	8
Variable								
Type								
Default								

Card 4	1	2	3	4	5	6	7	8
Variable								
Type								
Default								

Card 5	1	2	3	4	5	6	7	8
Variable	CB	Y	SC1	K	RSAT	SB	H	SC2
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	0.0

VARIABLE	DESCRIPTION
CB	The uppercase B defined in Yoshida's equations.
Y	Anisotropic parameter associated with work-hardening stagnation, defined in Yoshida's equations
SC1	The lowercase c_2 defined in Yoshida & Uemori's equations. Note the equation below from the paper: $$c = \begin{cases} c_1 & \max(\bar{\alpha}_*) < B - Y \\ c_2 & \text{otherwise} \end{cases}$$ See more details in About SC1 and SC2 in the remarks section of *MAT_125.
K	Hardening parameter as defined in Yoshida's equations
RSAT	Hardening parameter as defined in Yoshida's equations
SB	The lowercase b as defined in Yoshida's equations
H	Anisotropic parameter associated with work-hardening stagnation, defined in Yoshida's equations
SC2	The lowercase c_1 defined in the Yoshida and Uemori's equations. Note the equation below from the paper: $$c = \begin{cases} c_1 & \max(\bar{\alpha}_*) < B - Y \\ c_2 & \text{otherwise} \end{cases}$$ See more details in About SC1 and SC2 in the remarks section of *MAT_125. If SC2 equals 0.0, is left blank, or equals SC1, then it turns into the basic model (the one c model).

Card 6	1	2	3	4	5	6	7	8
Variable	AOPT		IOPT	C1	C2			
Type	I		I	F	F			
Default	none		none	0.0	0.0			

VARIABLE	DESCRIPTION
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description). Note AOPT may need to set to 0.0 for a simulation using the dynain file from a previous simulation. EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
IOPT	Kinematic hardening rule flag: EQ.0: Original Yoshida formulation EQ.1: Modified formulation. Define C1, C2 below.
C1, C2	Constants used to modify R: $R = \mathrm{RSAT} \times [(C_1 + \bar{\varepsilon}^p)^{c_2} - {C_1}^{c_2}]$

Card 7	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		
Default				none	none	none		

VARIABLE	DESCRIPTION
A1, A2, A3	Components of vector **a** for AOPT = 2

Card 8	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		
Default	none	none	none	none	none	none		

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3
D1, D2, D3	Components of vector **d** for AOPT = 2

Remarks:

1. **Yield Surface.** A total of eight parameters (α_1 to α_8) are needed to describe the yield surface. The parameters can be determined with tensile tests in three directions and one equal biaxial tension test. For detailed theoretical background and material parameters of some typical FCC materials, see Remarks in *MAT_133 and Barlat et al. (2003) paper.

2. **Yoshida Model.** For a more detailed description on the Yoshida model and parameters, see Remarks in *MAT_226 and *MAT_125.

3. **AOPT.** For information on the variable AOPT, see Remarks in *MAT_226.

4. **Convergence and Springback.** To improve convergence, it is recommended that *CONTROL_IMPLICIT_FORMING type '1' be used when conducting a springback simulation.

Revisions:

1. This material model is available starting in LS-DYNA R5 Revision 58432.

2. The variables EA, COE, SC1, and SC2 are available starting in Revision 133318.

*MAT_HILL_90

This is Material Type 243. This model was developed by Hill [1990] for modeling sheets with anisotropic materials under plane stress conditions. This material allows the use of the Lankford parameters for the definition of the anisotropy. All features of this model are the same as in *MAT_036, only the yield condition and associated flow rules are replaced by the Hill90 equations.

Card Summary:

Card 1. This card is required.

MID	RO	E	PR	HR	P1	P2	ITER

Card 2a. This card is included if FLAG = 0 (see Card 4).

M	R00	R45	R90	LCID	E0	SPI	P3

Card 2b. This card is included if FLAG = 1 (see Card 4).

M	AH	BH	CH	LCID	E0	SPI	P3

Card 3. This card is included if M < 0 on Card 2a/2b.

CRC1	CRA1	CRC2	CRA2	CRC3	CRA3	CRC4	CRA4

Card 4. This card is required.

AOPT	C	P	VLCID		FLAG		

Card 5. This card is required.

			A1	A2	A3		

Card 6. This card is required.

V1	V2	V3	D1	D2	D3	BETA	

Card 7. This card is optional.

USRFAIL							

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	HR	P1	P2	ITER
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus, E GT.0.0: Constant value LT.0.0: Load curve ID = (-E) which defines Young's modulus as a function of plastic strain. See Remark 1.
PR	Poisson's ratio, ν
HR	Hardening rule: EQ.1.0: Linear (default) EQ.2.0: Exponential (Swift; see Remark 3) EQ.3.0: Load curve or table with strain rate effects (see Remark 1) EQ.4.0: Exponential (Voce; see Remark 3) EQ.5.0: Exponential (Gosh; see Remark 3) EQ.6.0: Exponential (Hocket-Sherby; see Remark 3) EQ.7.0: Load curves in three directions (see Remark 1) EQ.8.0: Table with temperature dependence (see Remark 1) EQ.9.0: 3D table with temperature and strain rate dependence (see Remark 1)
P1	Material parameter: HR.EQ.1.0: Tangent modulus,

VARIABLE	DESCRIPTION
	HR.EQ.2.0: k, strength coefficient for Swift exponential hardening
	HR.EQ.4.0: a, coefficient for Voce exponential hardening
	HR.EQ.5.0: k, strength coefficient for Gosh exponential hardening
	HR.EQ.6.0: a, coefficient for Hocket-Sherby exponential hardening
	HR.EQ.7.0: Load curve ID for hardening in 45 degree direction. See Remark 1.
P2	Material parameter:
	HR.EQ.1.0: Yield stress
	HR.EQ.2.0: n, exponent for Swift exponential hardening
	HR.EQ.4.0: c, coefficient for Voce exponential hardening
	HR.EQ.5.0: n, exponent for Gosh exponential hardening
	HR.EQ.6.0: c, coefficient for Hocket-Sherby exponential hardening
	HR.EQ.7.0: Load curve ID for hardening in 90 degree direction. See Remark 1.
ITER	Iteration flag for speed:
	EQ.0.0: Fully iterative
	EQ.1.0: Fixed at three iterations
	Generally, we recommend ITER = 0.0. However, ITER = 1.0 is somewhat faster and may give acceptable results in most problems.

Lankford Parameters Card. This card is included if FLAG = 0 on Card 4.

Card 2a	1	2	3	4	5	6	7	8
Variable	M	R00	R45	R90	LCID	E0	SPI	P3
Type	F	F	F	F	I	F	F	F

VARIABLE	DESCRIPTION
M	m, exponent in Hill's yield surface. If negative, the absolute value is used. Typically, m ranges between 1 and 2 for low-r materials, such as aluminum (AA6111: $m \approx 1.5$) and is greater than 2 for high r-materials, as in steel (DP600: $m \approx 4$). See Remark 3.
R00	Lankford parameter in 0 degree direction, R_{00} (see Remark 3): GT.0.0: Constant value LT.0.0: Load curve or table ID = (-R00) which defines R_{00} as a function of plastic strain (curve) or as a function of temperature and plastic strain (table). See Remarks 1 and 2.
R45	Lankford parameter in 45 degree direction, R_{45} (see Remark 3): GT.0.0: Constant value LT.0.0: Load curve or table ID = (-R45) which defines R_{45} as a function of plastic strain (curve) or as a function of temperature and plastic strain (table). See Remarks 1 and 2.
R90	Lankford parameter in 90 degree direction, R_{90} (see Remark 3): GT.0.0: Constant value LT.0.0: Load curve or table ID = (-R90) which defines R_{90} as a function of plastic strain (curve) or as a function of temperature and plastic strain (table). See Remarks 1 and 2.
LCID	Load curve/table ID for hardening in the 0 degree direction (applies for HR = 3, 7, 8, and 9). See Remark 1.
E0	Material parameter (see Remark 3): HR.EQ.2.0: ε_0 for determining initial yield stress for Swift exponential hardening (default = 0.0) HR.EQ.4.0: b, coefficient for Voce exponential hardening HR.EQ.5.0: ε_0 for determining initial yield stress for Gosh exponential hardening (default = 0.0) HR.EQ.6.0: b, coefficient for Hocket-Sherby exponential hardening
SPI	Case I: If ε_0 is zero above and HR = 2.0 (see Remark 3). (Default = 0.0) EQ.0.0: $\varepsilon_0 = \left(E/k\right)^{1/(n-1)}$

VARIABLE	DESCRIPTION
	LE.0.02: $\varepsilon_0 = \mathrm{SPI}$ GT.0.02: $\varepsilon_0 = \left(\mathrm{SPI}/_k\right)^{1/n}$ Case II: If HR = 5.0 the strain at plastic yield is determined by an iterative procedure based on the same principles as for HR = 2.0 (see Remark 3).
P3	Material parameter (see Remark 3): HR.EQ.5.0: p, parameter for Gosh exponential hardening HR.EQ.6.0: n, exponent for Hocket-Sherby exponential hardening

Hill90 Parameters Card. This card is included for FLAG = 1.

Card 2b	1	2	3	4	5	6	7	8
Variable	M	AH	BH	CH	LCID	E0	SPI	P3
Type	F	F	F	F	I	F	F	F

VARIABLE	DESCRIPTION
M	m, exponent in Hill's yield surface. If negative, the absolute value is used. Typically, m ranges between 1 and 2 for low-R materials, such as aluminum (AA6111: $m \approx 1.5$) and is greater than 2 for high R-materials, as in steel (DP600: $m \approx 4$). See Remark 3.
AH	a, Hill90 parameter (see Remark 3)
BH	b, Hill90 parameter (see Remark 3)
CH	c, Hill90 parameter (see Remark 3)
LCID	Load curve/table ID for hardening in the 0 degree direction (applies for HR = 3, 7, 8, and 9). See Remark 1.
E0	Material parameter (see Remark 3): HR.EQ.2.0: ε_0 for determining initial yield stress for Swift exponential hardening (default = 0.0) HR.EQ.4.0: b, coefficient for Voce exponential hardening

VARIABLE	DESCRIPTION
	HR.EQ.5.0: ε_0 for determining initial yield stress for Gosh exponential hardening (default = 0.0) HR.EQ.6.0: b, coefficient for Hocket-Sherby exponential hardening
SPI	Case I: If ε_0 is zero above and HR = 2.0 (see Remark 3). (default = 0.0) EQ.0.0: $\varepsilon_0 = \left(E/_k\right)^{1/(n-1)}$ LE.0.02: $\varepsilon_0 = \mathrm{SPI}$ GT.0.02: $\varepsilon_0 = \left(\mathrm{SPI}/_k\right)^{1/n}$ Case II: If HR = 5.0 the strain at plastic yield is determined by an iterative procedure based on the same principles as for HR = 2.0 (see Remark 3).
P3	Material parameter (see Remark 3): HR.EQ.5.0: p, parameter for Gosh exponential hardening HR.EQ.6.0: n, exponent for Hocket-Sherby exponential hardening

Hardening Card. Additional Card for M < 0.

Card 3	1	2	3	4	5	6	7	8
Variable	CRC1	CRA1	CRC2	CRA2	CRC3	CRA3	CRC4	CRA4
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
CRCn	Chaboche-Rousselier hardening parameters. See Remark 4
CRAn	Chaboche-Rousselier hardening parameters. See Remark 4.

Card 4	1	2	3	4	5	6	7	8
Variable	AOPT	C	P	VLCID		FLAG		
Type	F	F	F	I		F		

VARIABLE	DESCRIPTION
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description): EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. The material axes are then rotated about the normal vector to the surface of the shell by the angle BETA. EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR. EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
C	C in Cowper-Symonds strain rate model (see Remark 3)
P	p in Cowper-Symonds strain rate model (see Remark 3). $p = 0.0$ for no strain rate effects.
VLCID	Volume correction curve ID defining the relative volume change (change in volume relative to the initial volume) as a function of the effective plastic strain. This is only used when nonzero. See Remark 1.

VARIABLE	DESCRIPTION
FLAG	Flag for interpretation of parameters. If FLAG = 1, parameters AH, BH, and CH are read instead of R00, R45, and R90. See Remark 3.

Card 5	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

VARIABLE	DESCRIPTION
A1, A2, A3	Components of vector **a** for AOPT = 2

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3
D1, D2, D3	Components of vector **d** for AOPT = 2
BETA	Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA.

This card is optional.

Card 7	1	2	3	4	5	6	7	8
Variable	USRFAIL							
Type	F							

VARIABLE	DESCRIPTION
USRFAIL	User defined failure flag:

VARIABLE	DESCRIPTION
	EQ.0: No user subroutine is called
	EQ.1: User subroutine `matusr_24` in dyn21.f is called.

Remarks:

1. **Plastic Strain in Curve Definitions.** The effective plastic strain used in this model is defined to be plastic work equivalent. A consequence of this is that for parameters defined as functions of effective plastic strain, the rolling (00) direction should be used as reference direction. For instance, the hardening curve for HR = 3 is the stress as function of strain for uniaxial tension in the rolling direction, the curve VLCID should give the relative volume change as function of strain for uniaxial tension in the rolling direction, and the load curve with ID -E should give the Young's modulus as function of strain for uniaxial tension in the rolling direction. Optionally the curve can be substituted for a table defining hardening as function of plastic strain rate (HR = 3), temperature (HR = 8), or both (HR = 9).

   Exceptions from this rule are curves defined as functions of plastic strain in the 45 and 90 directions, such as P1 and P2 for HR = 7 and negative R45 or R90. The hardening curves are here defined as measured stress as function of measured plastic strain for uniaxial tension in the direction of interest, meaning as determined from experimental testing using a standard procedure. Moreover, the curves defining the R-values are functions of the measured plastic strain for uniaxial tension in the direction of interest. These curves are transformed internally to be used with the effective stress and strain properties in the actual model. The effective plastic strain does not coincide with the plastic strain components in directions other than the rolling direction and may be somewhat confusing. Therefore, the von Mises work equivalent plastic strain is output as history variable #2 if HR = 7 or if any R-value is defined as function of the plastic strain.

2. **Determining R-Values from Curves.** The R-values in curves are defined as the ratio of instantaneous width change to instantaneous thickness change. That is, assume that the width W and thickness T are measured as a function of strain. Then the corresponding R-value is given by:

$$R = \frac{\frac{dW}{d\varepsilon}/W}{\frac{dT}{d\varepsilon}/T}$$

3. **Yield Criterion, Hill90 Parameters, and Hardening Models.** The anisotropic yield criterion Φ for plane stress is defined as:

$$\Phi = K_1^m + K_3 K_2^{(m/2)-1} + c^m K_4^{m/2} = (1 + c^m - 2a + b)\sigma_Y^m$$

where σ_Y is the yield stress. $K_i, i = 1, \ldots, 4$ are given by:

$$\begin{aligned}
K_1 &= |\sigma_x + \sigma_y| \\
K_2 &= |\sigma_x^2 + \sigma_y^2 + 2\sigma_{xy}^2| \\
K_3 &= -2a(\sigma_x^2 - \sigma_y^2) + b(\sigma_x - \sigma_y)^2 \\
K_4 &= \left|(\sigma_x - \sigma_y)^2 + 4\sigma_{xy}^2\right|
\end{aligned}$$

If FLAG = 0, the anisotropic material constants a, b, and c are obtained through R_{00}, R_{45}, and R_{90} using these 3 equations:

$$\begin{aligned}
1 + 2R_{00} &= \frac{c^m - a + \{(m+2)/2m\}b}{1 - a + \{(m-2)/2m\}b} \\
1 + 2R_{45} &= c^m \\
1 + 2R_{90} &= \frac{c^m + a + \{(m+2)/2m\}b}{1 + a + \{(m-2)/2m\}b}
\end{aligned}$$

If FLAG = 1, material parameters a (AH), b (BH), and c (CH) are used directly.

For material parameters a, b, c, and m, the following condition must be fulfilled, otherwise an error termination occurs:

$$1 + c^m - 2a + b \quad > \quad 0$$

Two even more strict conditions should be satisfied to ensure convexity of the yield surface according to Hill (1990). A warning message will be output if at least one of them is violated:

$$\begin{aligned}
&b > -2^{\left(\frac{m}{2}\right)-1}c^m \\
&b > a^2 - c^m
\end{aligned}$$

For the Swift hardening law (HR = 2), the yield strength of the material can be expressed in terms of k and n:

$$\sigma_Y = k\varepsilon^n = k(\varepsilon_0 + \bar{\varepsilon}^p)^n$$

where ε_0 is the elastic strain to yield and $\bar{\varepsilon}^p$ is the effective plastic strain (logarithmic). ε_0 can be given in the input with E0 or determined using SPI. If E0 and SPI are both set to zero, the strain to yield is found by solving for the intersection of the linearly elastic loading equation with the strain hardening equation:

$$\begin{aligned}
\sigma &= E\varepsilon \\
\sigma &= k\varepsilon^n
\end{aligned}$$

which gives the elastic strain at yield as:

$$\varepsilon_0 = \left(\frac{E}{k}\right)^{\left[\frac{1}{n-1}\right]}$$

If E0 is zero and SPI is nonzero and greater than 0.02 then:

$$\varepsilon_0 = \left(\frac{\sigma_Y}{k}\right)^{\left[\frac{1}{n}\right]}$$

The other available hardening models include the Voce equation (HR = 4) given by

$$\sigma_Y(\varepsilon_p) = a - be^{-c\varepsilon_p},$$

the Gosh equation (HR = 5) given by

$$\sigma_Y(\varepsilon_p) = k(\varepsilon_0 + \varepsilon_p)^n - p,$$

and finally, the Hocket-Sherby equation (HR = 6) given by

$$\sigma_Y(\varepsilon_p) = a - be^{-c\varepsilon_p^n}.$$

For the Gosh hardening law, the interpretation of the variable SPI is the same as for the Swift hardening law, meaning if set to zero (along with E0), the strain at yield is determined implicitly from the intersection of the strain hardening equation with the linear elastic equation.

To include strain rate effects in the model, we multiply the yield stress by a factor depending on the effective plastic strain rate. We use the Cowper-Symonds model, hence the yield stress can be written

$$\sigma_Y(\varepsilon_p, \dot{\varepsilon}_p) = \sigma_Y^s(\varepsilon_p)\left[1 + \left(\frac{\dot{\varepsilon}_p}{C}\right)^{1/p}\right] .$$

Here σ_Y^s denotes the static yield stress, C and p are material parameters, and $\dot{\varepsilon}_p$ is the effective plastic strain rate.

4. **Kinematic Hardening Model.** A kinematic hardening model is implemented following the works of Chaboche and Roussilier. A back stress $\boldsymbol{\alpha}$ is introduced such that the effective stress is computed as

$$\sigma_{\text{eff}} = \sigma_{\text{eff}}(\sigma_{11} - 2\alpha_{11} - \alpha_{22}, \sigma_{22} - 2\alpha_{22} - \alpha_{11}, \sigma_{12} - \alpha_{12})$$

The back stress is the sum of up to four terms according to

$$\alpha_{ij} = \sum_{k=1}^{4} \alpha_{ij}^k$$

and the evolution of each back stress component is as follows

$$\delta\alpha_{ij}^k = C_k\left(a_k\frac{s_{ij} - \alpha_{ij}}{\sigma_{\text{eff}}} - \alpha_{ij}^k\right)\delta\varepsilon_p$$

where C_k and a_k are material parameters,s_{ij} is the deviatoric stress tensor, σ_{eff} is the effective stress, and ε_p is the effective plastic strain.

*MAT_UHS_STEEL

This is Material Type 244. This material model is developed for both shell and solid models. It is mainly suited for hot stamping processes where phase transformations are crucial. It has five phases, and it is assumed that the blank is fully austenitized before cooling. The model also includes optional algorithms for switching between heating and cooling. The basic constitutive model is based on the work done by P. Akerstrom [2, 7].

NOTE 1: For this material "weight%" means "ppm × 10^{-4}".

NOTE 2: We include baseline values in the variable tables as possible starting values that lead to reasonable results for an alloy called 22MnB5. The values are taken from the literature.

Card Summary:

Card 1. This card is required.

MID	RO	E	PR	TUNIT	CRSH	PHASE	HEAT

Card 2. This card is required.

LCY1	LCY2	LCY3	LCY4	LCY5	KFER	KPER	B

Card 3. This card is required.

C	Co	Mo	Cr	Ni	Mn	Si	V

Card 4. This card is required.

W	Cu	P	Al	As	Ti	CWM	LCTRE

Card 5. This card is required.

THEXP1	THEXP5	LCTH1	LCTH5	TREF	LAT1	LAT5	TABTH

Card 6. This card is required.

QR2	QR3	QR4	ALPHA	GRAIN	TOFFE	TOFPE	TOFBA

Card 7. This card is required.

PLMEM2	PLMEM3	PLMEM4	PLMEM5	STRC	STRP	REACT	TEMPER

Card 8. This card is included if HEAT = 1.

AUST	FERR	PEAR	BAIN	MART	GRK	GRQR	TAU1

Card 9. This card is included if HEAT = 1.

GRA	GRB	EXPA	EXPB	GRCC	GRCM	HEATN	TAU2

Card 10. This card is included if REACT = 1.

FS	PS	BS	MS	MSIG	LCEPS23	LCEPS4	LCEPS5

Card 11. This card is included if TEMPER = 1.

LCH4	LCH5	DTCRIT	TSAMP				

Card 12. This card is included if CWM = 1.

TASTART	TAEND	TLSTART	TLEND	EGHOST	PGHOST	AGHOST	

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	TUNIT	CRSH	PHASE	HEAT
Type	A	F	F	F	F	I	I	I
Defaults	none	none	none	none	3600	0	0	0

VARIABLE	DESCRIPTION	BASELINE VALUE
MID	Material identification. A unique number or label must be specified (see *PART).	
RO	Material density	7830 Kg/m^3
E	Young's modulus: GT.0.0: Constant value LT.0.0: Temperature dependent Young's modulus given by	100 GPa [1]

VARIABLE	DESCRIPTION	BASELINE VALUE
	load curve or table ID = -E. See Remark 9 for more information about using a table to specify the Young's modulus.	
PR	Poisson's ratio: GT.0.0: Constant value LT.0.0: Temperature dependent Poisson ratio given by load curve or table ID = -PR. The table input is described in Remark 9.	0.30 [1]
TUNIT	Number of time units per hour. Default is seconds, that is, 3600 time units per hour. TUNIT is used only for hardness calculations.	3600.
CRSH	Switch to use a simple and fast material model but with the actual phases active. EQ.0: The original model where phase transitions are active and trip is used. EQ.1: A simpler and faster version. This option is mainly used when transferring the quenched blank into a crash analysis where all properties from the cooling are maintained. This option must be used with a *INTERFACE_-SPRINGBACK keyword and should be used after a quenching analysis. EQ.2: Same as 0 but trip effect is not used.	0
PHASE	Switch to include or exclude middle phases from the simulation. EQ.0: All phases active (default) EQ.1: Pearlite and bainite excluded	0

VARIABLE	DESCRIPTION	BASELINE VALUE
	EQ.2: Bainite excluded EQ.3: Ferrite and pearlite excluded EQ.4: Ferrite and bainite excluded EQ.5: Exclude middle phases (only austenite → martensite)	
HEAT	Switch to activate the heating algorithms (see Remarks 7 and 8): EQ.0: Heating is not activated which means that no transformation to austenite is possible. EQ.1: Heating is activated which means that only transformation to austenite is possible. EQ.2: Automatic switching between cooling and heating. LS-DYNA checks the temperature gradient and calls the appropriate algorithms. For example, this can be used to simulate the heat affected zone during welding. LT.0: The switch between cooling and heating is defined by a time dependent load curve with ID = |HEAT|. The ordinate should be 1.0 when heating is applied and 0.0 if cooling is preferable.	

Card 2	1	2	3	4	5	6	7	8
Variable	LCY1	LCY2	LCY3	LCY4	LCY5	KFER	KPER	B
Type	I	I	I	I	I	F	F	F
Defaults	none	none	none	none	none	0.0	0.0	0.0

VARIABLE	DESCRIPTION	BASELINE VALUE
LCY1	Load curve or table ID for austenite hardening. **Load Curve.** When LCY1 is a load curve ID, it defines input yield stress as a function of effective plastic strain. **Tabular Data (LCY1 > 0).** When LCY1 is greater than 0 and references a table ID, a 2D table references for each temperature value a hardening curve. **Tabular Data (LCY1 < 0).** When LCY1 is less than 0, |LCY1| is a 3D table ID. Each input temperature value gives a table ID which defines for each a strain rate a hardening curve.	
LCY2	Load curve ID for ferrite hardening (stress as a function of effective plastic strain)	
LCY3	Load curve or table ID for pearlite. See LCY1 for description.	
LCY4	Load curve or table ID for bainite. See LCY1 for description.	
LCY5	Load curve or table ID for martensite. See LCY1 for description.	
KFERR	Correction factor for boron in the ferrite reaction.	1.9×10^5 [2]
KPEAR	Correction factor for boron in the pearlite reaction.	3.1×10^3 [2]
B	Boron [weight %]	0.003 [2]

Card 3	1	2	3	4	5	6	7	8
Variable	C	Co	Mo	Cr	Ni	Mn	Si	V
Type	F	F	F	F	F	F	F	F
Defaults	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION	BASELINE VALUE
C	Carbon [weight %]	0.23 [2]
Co	Cobolt [weight %]	0.0 [2]
Mo	Molybdenum [weight %]	0.0 [2]
Cr	Chromium [weight %]	0.21 [2]
Ni	Nickel [weight %]	0.0 [2]
Mn	Manganese [weight %]	1.25 [2]
Si	Silicon [weight %]	0.29 [2]
V	Vanadium [weight %]	0.0 [2]

Card 4	1	2	3	4	5	6	7	8
Variable	W	Cu	P	Al	As	Ti	CWM	LCTRE
Type	F	F	F	F	F	F	I	I
Defaults	0.0	0.0	0.0	0.0	0.0	0.0	0	none

VARIABLE	DESCRIPTION	BASELINE VALUE
W	Tungsten [weight %]	0.0 [2]
Cu	Copper [weight %]	0.0 [2]
P	Phosphorous [weight %]	0.013 [2]

VARIABLE	DESCRIPTION	BASELINE VALUE
Al	Aluminum [weight %]	0.0 [2]
As	Arsenic [weight %]	0.0 [2]
Ti	Titanium [weight %]	0.0 [2]
CWM	Flag for computational welding mechanics input. One additional input card is read. EQ.1.0: Active EQ.0.0: Inactive	
LCTRE	Load curve for transformation induced strains. See Remark 14.	

Card 5	1	2	3	4	5	6	7	8
Variable	THEXP1	THEXP5	LCTH1	LCTH5	TREF	LAT1	LAT5	TABTH
Type	F	F	I	I	F	F	F	I
Defaults	0.0	0.0	none	none	273.15	0.0	0.0	none

VARIABLE	DESCRIPTION	BASELINE VALUE
THEXP1	Coefficient of thermal expansion in austenite	25.1×10^{-6} 1/K [7]
THEXP5	Coefficient of thermal expansion in martensite	11.1×10^{-6} 1/K [7]
LCTH1	Load curve for the thermal expansion coefficient for austenite: LT.0: Curve ID = -LCTH1 and TREF is used as reference temperature GT.0: Curve ID = LCTH1	0
LCTH5	Load curve for the thermal expansion coefficient for martensite:	0

VARIABLE	DESCRIPTION	BASELINE VALUE
	LT.0: Curve ID = -LCTH5 and TREF is used as reference temperature GT.0: Curve ID = LCTH5	
TREF	Reference temperature for thermal expansion. Used if LCTH1 < 0.0, LCTH5 < 0.0, or TABTH < 0.	293.15
LAT1	Latent heat for the decomposition of austenite into ferrite, pearlite and bainite. GT.0.0: Constant value LT.0.0: Load curve ID or table ID. See Remark 10 for more information.	590×10^6 J/m^3 [2]
LAT5	Latent heat for the decomposition of austenite into martensite. GT.0.0: Constant value LT.0.0: Load curve ID giving latent heat as a function of temperature Note that LAT5 is ignored if a table ID is used in LAT1.	640×10^6 J/m^3 [2]
TABTH	Table ID for thermal expansion coefficient. With this option active THEXP1, THEXP2, LCTH1 and LCTH5 are ignored. See Remark 11. GT.0: A table for instantaneous thermal expansion (TREF is ignored). LT.0: A table with thermal expansion with reference to TREF.	

Card 6	1	2	3	4	5	6	7	8
Variable	QR2	QR3	QR4	ALPHA	GRAIN	TOFFE	TOFPE	TOFBA
Type	F	F	F	F	F	F	F	F
Defaults	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION	BASELINE VALUE
QR2	Activation energy divided by the universal gas constant for the diffusion reaction of the austenite-ferrite reaction: $Q2/R$. R = 8.314472 [J/mol K].	10324 K [3] = (23000 cal/mole) × (4.184 J/cal) / (8.314 J/mole/K)
QR3	Activation energy divided by the universal gas constant for the diffusion reaction for the austenite-pearlite reaction: $Q3/R$. R = 8.314472 [J/mol K].	13432. K [3]
QR4	Activation energy divided by the universal gas constant for the diffusion reaction for the austenite-bainite reaction: $Q4/R$. R = 8.314472 [J/mol K].	15068. K [3]
ALPHA	Material constant for the martensite phase. A value of 0.011 means that 90% of the available austenite is transformed into martensite at 210 degrees below the martensite start temperature (see messag file for information), whereas a value of 0.033 means a 99.9% transformation.	0.011
GRAIN	ASTM grain size number for austenite, usually a number between 7 and 11.	6.8
TOFFE	Number of degrees that the ferrite is bleeding over into the pearlite reaction	0.0
TOFPE	Number of degrees that the pearlite is bleeding over into the bainite reaction	0.0
TOFBA	Number of degrees that the bainite is	0.0

VARIABLE	DESCRIPTION	BASELINE VALUE
	bleeding over into the martensite reaction	

Card 7	1	2	3	4	5	6	7	8
Variable	PLMEM2	PLMEM3	PLMEM4	PLMEM5	STRC	STRP	REACT	TEMPER
Type	I	F	F	F	F	F	I	I
Defaults	0.0	0.0	0.0	0.0	0.0	0.0	0	0

VARIABLE	DESCRIPTION	BASELINE VALUE
PLMEM2	Memory coefficient for the plastic strain that is carried over from the austenite. A value of 1 means that all plastic strains from austenite is transferred to the *ferrite* phase and a value of 0 means that nothing is transferred.	0.0
PLMEM3	Same as PLMEM2 but between austenite and pearlite	0.0
PLMEM4	Same as PLMEM2 but between austenite and bainite	0.0
PLMEM5	Same as PLMEM3 but between austenite and martensite	0.0
STRC	Effective strain rate parameter C. LT.0.0: load curve ID = -STRC GT.0.0: constant value EQ.0.0: strain rate NOT active	0.0
STRP	Effective strain rate parameter P. LT.0.0: load curve ID = -STRP GT.0.0: constant value EQ.0.0: strain rate NOT active	0.0

VARIABLE	DESCRIPTION	BASELINE VALUE
REACT	Flag for advanced reaction kinetics input. One additional input card is read. EQ.1.0: active EQ.0.0: inactive	0.0
TEMPER	Flag for tempering input. One additional input card is read. EQ.1.0: active EQ.0.0: inactive	0.0

Heat Card 1. Additional Card for HEAT = 1.

Card 8	1	2	3	4	5	6	7	8
Variable	AUST	FERR	PEAR	BAIN	MART	GRK	GRQR	TAU1
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.08E8

VARIABLE	DESCRIPTION	BASELINE VALUE
AUST	If a heating process is initiated at $t = 0$, this field sets the initial amount of austenite in the blank. If heating is activated at $t > 0$ during a simulation, this value is ignored. Note that, $\text{AUST} + \text{FERR} + \text{PEAR} + \text{BAIN} + \text{MART} = 1.0$	0.0
FERR	See AUST for description	0.0
PEAR	See AUST for description	0.0
BAIN	See AUST for description	0.0
MART	See AUST for description	0.0

VARIABLE	DESCRIPTION	BASELINE VALUE
GRK	Growth parameter k (μm²/sec)	10^{11} [9]
GRQR	Grain growth activation energy (J/mol) divided by the universal gas constant: Q/R where $R = 8.314472$ (J/mol K)	3×10^4 [9]
TAU1	Empirical grain growth parameter c_1 describing the function $\tau(T)$	2.08×10^8 [9]

Heat Card 2. Additional Card for HEAT =1.

Card 9	1	2	3	4	5	6	7	8
Variable	GRA	GRB	EXPA	EXPB	GRCC	GRCM	HEATN	TAU2
Type	F	F	F	F	F	F	F	F
Default	3.11	7520.	1.0	1.0	none	none	1.0	4.806

VARIABLE	DESCRIPTION	BASELINE VALUE
GRA	Grain growth parameter A	[9]
GRB	Grain growth parameter B. A table of recommended values of GRA and GRB is included in Remark 8.	[9]
EXPA	Grain growth parameter a	1.0 [9]
EXPB	Grain growth parameter b	1.0 [9]
GRCC	Grain growth parameter with the concentration of non-metals in the blank, weight% of C or N	[9]
GRCM	Grain growth parameter with the concentration of metals in the blank, lowest weight% of Cr, V, Nb, Ti, Al	[9]
HEATN	Grain growth parameter n for the austenite formation	1.0 [9]

VARIABLE	DESCRIPTION	BASELINE VALUE
TAU2	Empirical grain growth parameter c_2 describing the function $\tau(T)$	4.806 [9]

Reaction Card. Addition card for REACT = 1.

Card 10	1	2	3	4	5	6	7	8
Variable	FS	PS	BS	MS	MSIG	LCEPS23	LCEPS4	LCEPS5
Type	F	F	F	F	F	I	I	I
Default	0.0	0.0	0.0	0.0	none	none	none	none

VARIABLE	DESCRIPTION	BASELINE VALUE
FS	Manual start temperature Ferrite GT.0.0: Same temperature is used for heating and cooling. LT.0.0: Different start temperatures for cooling and heating given by load curve ID = -FS. First ordinate value is used for cooling, last ordinate value for heating.	
PS	Manual start temperature Pearlite. See FS for description.	
BS	Manual start temperature Bainite. See FS for description.	
MS	Manual start temperature Martensite. See FS for description.	
MSIG	Describes the increase of martensite start temperature for cooling due to applied stress. LT.0: Load curve ID describes MSIG as a function of triaxiality (pressure	

VARIABLE	DESCRIPTION	BASELINE VALUE
	/ effective stress). $$MS^* = MS + MSIG \times \sigma_{eff}$$	
LCEPS23	Load curve ID dependent on plastic strain that scales the activation energy QR2 and QR3. $$QRx = Qx \times LCEPS23(\varepsilon_{pl})/R$$	
LCEPS4	Load Curve ID dependent on plastic strain that scales the activation energy QR4. $$QR4 = Q4 \times LCEPS4(\varepsilon_{pl})/R$$	
LCEPS5	Load curve ID which describe the increase of the martensite start temperature for cooling as a function of plastic strain. $$MS^* = MS + MSIG \times \sigma_{eff} + LCEPS5(\varepsilon_{pl})$$	

Tempering Card. Additional card for TEMPR = 1.

Card 11	1	2	3	4	5	6	7	8
Variable	LCH4	LCH5	DTCRIT	TSAMP				
Type	I	I	F	F				
Default	0	0	0.0	0.0				

VARIABLE	DESCRIPTION	BASELINE VALUE
LCH4	Load curve ID of Vicker hardness as a function of temperature for Bainite hardness calculation	
LCH5	Load curve ID of Vicker hardness as a function of temperature for Martensite hardness calculation	

VARIABLE	DESCRIPTION	BASELINE VALUE
DTCRIT	Critical cooling rate to detect holding phase	
TSAMP	Sampling interval for temperature rate monitoring to detect the holding phase	

Computational Welding Mechanics Card. Additional card for CWM = 1.

Card 12	1	2	3	4	5	6	7	8
Variable	TASTART	TAEND	TLSTART	TLEND	EGHOST	PGHOST	AGHOST	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

VARIABLE	DESCRIPTION	BASELINE VALUE
TASTART	Annealing temperature start	
TAEND	Annealing temperature end	
TLSTART	Birth temperature start	
EGHOST	Young's modulus for ghost (quiet) material	
PGHOST	Poisson's ratio for ghost (quiet) material	
AGHOST	Thermal expansion coefficient for ghost (quiet) material	

Discussion:

The phase distribution during cooling is calculated by solving the following rate equation for each phase transition

$$\dot{X}_k = g_k(G, C, T_k, Q_k) f_k(X_k) \quad , \qquad k = 2,3,4$$

where g_k is a function, taken from Li et al., dependent on the grain number G, the chemical composition C, the temperature T, and the activation energy Q. Moreover, the function f_k is dependent on the actual phase $X_k = x_k / x_{\mathrm{eq}}$

$$f_k(X_k) = X_k^{0.4(X_k-1)}(1-X_k)^{0.4X_k} \ , \qquad k = 2,3,4$$

The true amount of martensite, that is, $k = 5$, is modelled by using the true amount of the austenite left after the bainite phase:

$$x_5 = x_1\left[1 - e^{-\alpha(\mathrm{MS}-T)}\right] \ ,$$

where x_1 is the true amount of austenite left for the reaction, α is a material dependent constant and MS is the start temperature of the martensite reaction.

The start temperatures are automatically calculated based on the composition:

1. Ferrite,

$$\begin{aligned}\mathrm{FS} = 1185 - 203 \times \sqrt{\mathrm{C}} - 15.2 \times \mathrm{Ni} + 44.7 \times \mathrm{Si} + 104 \times \mathrm{V} + 31.5 \times \mathrm{Mo} + 13.1 \times \mathrm{W} \\ - 30 \times \mathrm{Mn} - 11 \times \mathrm{Cr} - 20 \times \mathrm{Cu} + 700 \times \mathrm{P} + 400 \times \mathrm{Al} + 120 \times \mathrm{As} \\ + 400 \times \mathrm{Ti}\end{aligned}$$

2. Pearlite,

$$\mathrm{PS} = 996 - 10.7 \times \mathrm{Mn} - 16.9 \times \mathrm{Ni} + 29 \times \mathrm{Si} + 16.9 \times \mathrm{Cr} + 290 \times \mathrm{As} + 6.4 \times \mathrm{W}$$

3. Bainite,

$$\mathrm{BS} = 910 - 58 \times \mathrm{C} - 35 \times \mathrm{Mn} - 15 \times \mathrm{Ni} - 34 \times \mathrm{Cr} - 41 \times \mathrm{Mo}$$

4. Martensite,

$$\begin{aligned}\mathrm{MS} = 812 - 423 \times \mathrm{C} - 30.4 \times \mathrm{Mn} - 17.7 \times \mathrm{Ni} - 12.1 \times \mathrm{Cr} - 7.5 \times \mathrm{Mo} + 10 \times \mathrm{Co} \\ - 7.5 \times \mathrm{Si}\end{aligned}$$

where the element weight values are input on Cards 2 through 4.

The automatic start temperatures are printed to the messag file and if they are not accurate enough you can manually set them in the input deck (must be set in absolute temperature, Kelvin). If HEAT > 0, the temperature FSnc (ferrite without C) is also echoed. If the specimen exceeds that temperature, all remaining ferrite is instantaneously transformed to austenite.

Remarks:

1. **History Variables.** History variables 1 through 8 include the different phases, the Vickers hardness, the yield stress and the ASTM grain size number. Set NEIPS = 8 (shells) or NEIPH = 8 (solids) on *DATABASE_EXTENT_BINARY.

History Variable	Description
1	Amount austenite
2	Amount ferrite
3	Amount pearlite

History Variable	Description
4	Amount bainite
5	Amount martensite
6	Vickers hardness
7	Yield stress
8	ASTM grain size number (a low value means large grains and vice versa)

2. **Excluding Phases.** To exclude a phase from the simulation, set the PHASE parameter accordingly.

3. **STRC and STRP.** Note that both strain rate parameters must be set to include the effect. It is possible to use a temperature dependent load curve for both parameters simultaneously or for one parameter keeping the other constant.

4. **TUNIT.** TUNIT is time units per hour and is only used for calculating the Vicker Hardness. By default, it is assumed that the time unit is seconds. If another time unit is used, for example milliseconds, then TUNIT must be changed to TUNIT $= 3.6 \times 10^6$

5. **TSF.** The thermal speedup factor TSF of *CONTROL_THERMAL_SOLVER is used to scale reaction kinetics and hardness calculations in this material model. Strain rate dependent properties (see LCY1 to LCY5 or STRC/STRP), however, are not scaled by TSF.

6. **CRSH.** With the CRSH = 1 option it is now possible to transfer the material properties from a hot stamping simulation (CRSH = 0) into another simulation. The CRSH = 1 option reads a dynain file from a simulation with CRSH = 0 and keeps all the history variables (austenite, ferrite, pearlite, bainite, martensite, etc.) constant. This will allow steels with inhomogeneous strength to be analyzed in, for example, a crash simulation. The speed with the CRSH = 1 option is comparable with *MAT_024. Note that for keeping the speed the temperature used in the CRSH simulation should be constant and the thermal solver should be inactive.

7. **Heating and Cooling and Transformation Temperatures.** To activate the heating algorithm, set HEAT = 1 or 2. HEAT = 0 (default) activates only the cooling algorithm. Note that for HEAT = 0 you *must* check that the initial temperature of this material is above the start temperature for the ferrite transformation. The transformation temperatures are echoed in the messag and d3hsp files.

If HEAT > 0 the temperature that instantaneously transforms all ferrite back to austenite is also echoed in the messag file. If you want to heat up to 100% austenite, you must let the specimen's temperature exceed that temperature.

8. **HEAT, Grain Growth, and Re-austenization.** When HEAT is activated the re-austenitization and grain growth algorithms are also activated. The grain growth is activated when the temperature exceeds a threshold value that is given by

$$T = \frac{B}{A - \log_{10}[(\mathrm{GRCM})^a(\mathrm{GRCC})^b]} ,$$

and the rate equation for the grain growth is,

$$\dot{g} = \frac{k}{2g} e^{-\frac{Q}{RT}} .$$

The rate equation for the phase re-austenitization is given in Oddy (1996) and is here mirrored

$$\dot{x}_a = n \left[\ln \left(\frac{x_{eu}}{x_{eu} - x_a} \right) \right]^{\frac{n-1}{n}} \left[\frac{x_{eu} - x_a}{\tau(T)} \right] ,$$

where n is the parameter HEATN. The temperature dependent function $\tau(T)$ is given from Oddy as $\tau(T) = c_1(T - T_s)^{c_2}$. The empirical parameters c_1 and c_2 are calibrated in Oddy to 2.06×10^8 and 4.806 respectively. Note that τ above given in *seconds*.

Recommended values for GRA and GRB are given in the following table.

Compound	Metal	Non-metal	GRA	GRB
$Cr_{23}C_6$	Cr	C	5.90	7375
V_4C_3	V	$C_{0.75}$	5.36	8000
TiC	Ti	C	2.75	7000
NbC	Nb	$C_{0.7}$	3.11	7520
Mo_2C	Mo	C	5.0	7375
Nb(CN)	Nb	(CN)	2.26	6770
VN	V	N	3.46 + 0.12%Mn	8330
AlN	Al	N	1.03	6770
NbN	Nb	N	4.04	10230
TiN	Ti	N	0.32	8000

9. **Using the Table Capability for Temperature Dependence of Young's Modulus.** Use *DEFINE_TABLE_2D and set the abscissa value equal to 1 for the austenite YM-curve, equal to 2 for the ferrite YM-curve, equal to 3 for the pearlite YM curve, equal to 4 for the bainite YM-curve and finally equal to 5 for the martensite YM-curve. If you use the PHASE option you only need to define the curves for the included phases, but you can define all five. LS-DYNA uses the number 1 - 5 to get the right curve for the right phase. The total YM is calculated by a linear mixture law: YM = YM1 × PHASE1 + ⋯ + YM5 × PHASE5. For example:

```
*DEFINE_TABLE_2D
$ The number before curve id:s define which phase the curve
$ will be applied to.  1 = Austenite, 2 = Ferrite, 3 = Pearlite,
$ 4 = Bainite and 5 = Martensite.
      1000       0.0       0.0
                 1.0                 100
                 2.0                 200
                 3.0                 300
                 4.0                 400
                 5.0                 500
$
$ Define curves 100 - 500
*DEFINE_CURVE
$ Austenite Temp (K) - YM-Curve (MPa)
       100         0       1.0       1.0
              1300.0              50.E+3
               223.0             210.E+3
```

10. **Using the Table Capability for Latent Heat.** When using a table ID for the latent heat (LAT1) you can describe all phase transition individually. Use *DEFINE_TABLE_2D and set the abscissa values to the corresponding phase transition number, that is, 2 for austenite to ferrite, 3 for austenite to pearlite, 4 for austenite to bainite and 5 for austenite to martensite. Remark 9 demonstrates the form of a correct table definition. If a curve is missing, the corresponding latent heat for that transition will be set to zero. Also, when a table is used the LAT5 is ignored. If HEAT > 0, you also have the option to include latent heat for the transition back to Austenite. This latent heat curve is marked as 1 in the table definition of LAT1.

11. **Using the Table Capability for Thermal Expansion.** When using a table ID for the thermal expansion you can specify the expansion characteristics for each phase. That is, you can have a curve for each of the 5 phases (austenite, ferrite, pearlite, bainite, and martensite). The input is identical to the above table definitions. The table must have the abscissa values between 1 and 5 where the number correspond to phase 1 to 5. To exclude one phase from influencing the thermal expansion you simply input a curve that is zero for that phase or even easier, exclude that phase number in the table definition. For example, to exclude the bainite phase you only define the table with curves for the indices 1, 2, 3 and 5.

12. **TEMPER.** Tempering is activated by setting TEMPER to 1. When active, the default hardness calculation for bainite and martensite is altered to use an incremental update formula. The total hardness is given by $\sum_{i=1}^{5} \mathrm{HV}_i \times x_i$. When holding phases are detected, the hardness for Bainite and Martensite is updated according to

$$\mathrm{HV}_4^{n+1} = \frac{x_4^n}{x_4^{n+1}} \mathrm{HV}_4^n + \frac{\Delta x_4}{x_4^{n+1}} h_4(T), \qquad \Delta x_4 = x_4^{n+1} - x_4^n$$
$$\mathrm{HV}_5^{n+1} = \frac{x_5^n}{x_5^{n+1}} \mathrm{HV}_5^n + \frac{\Delta x_5}{x_5^{n+1}} h_5(T), \qquad \Delta x_5 = x_5^{n+1} - x_5^n$$

We detect the holding phase for Bainite and Martensite when the temperature is in the appropriate range and if average temperature rate is below DTCRIT. The average temperature rate is calculated as $T_{\mathrm{tresh}}/t_{\mathrm{tresh}}$, where $T_{\mathrm{tresh}}^{n+1} = T_{\mathrm{tresh}}^n + |\dot{T}|\Delta t$ and $t_{\mathrm{tresh}}^{n+1} = t_{\mathrm{tresh}}^n + \Delta t$. The average temperature and time are updated until $t_{\mathrm{tresh}} \geq t_{\mathrm{samp}}$.

13. **CWM (Welding).** When computational welding mechanics is activated with CWM = 1, the material can be defined to be initially in a quiet state. In this state the material (often referred to as ghost material) has thermo-mechanical properties defined by an additional card. The material is activated when the temperature reaches the birth temperature. See MAT_CWM (MAT_270) for a detailed description.

14. **LCTRE (Transformation Induced Strains).** Transformation induced strains can be included with a load curve LCTRE as a function of temperature. The load curve represents the difference between the hard phases and the austenite phase in the dilatometer curves. Therefore, positive curve values result in a negative transformation strain for austenitization and a positive transformation strain for the phase transformation from austenite to one of the hard phases.

Boron steel composition from the literature:

Element	HAZ code	Akerstrom (2)	Naderi (8)	ThyssenKrupp (5) (max amount)
B		0.003	0.003	0.005
C	0.168	0.23	0.230	0.250
Co				
Mo	0.036			0.250
Cr	0.255	0.211	0.160	0.250
Ni	0.015			
Mn	1.497	1.25	1.18	1.40
Si	0.473	0.29	0.220	0.400

Element	HAZ code	Akerstrom (2)	Naderi (8)	ThyssenKrupp (5) (max amount)
V	0.026			
W				
Cu	0.025			
P	0.012	0.013	0.015	0.025
Al	0.020			
As				
Ti			0.040	0.05
S		0.003	0.001	0.010

References:

[1] Numisheet 2008 Proceedings, The Numisheet 2008 Benchmark Study, Chapter 3, Benchmark 3, Continuous Press Hardening, Interlaken, Switzerland, Sept. 2008.

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[3] M.V Li, D.V Niebuhr, L.L Meekisho and D.G Atteridge, "A Computatinal model for the prediction of steel hardenability", Metallurgical and materials transactions B, 29B, 661-672, 1998.

[4] D.F. Watt, "An Algorithm for Modelling Microstructural Development in Weld Heat-Affected Zones (Part A) Reaction Kinetics", Acta metal. Vol. 36., No. 11, pp. 3029-3035, 1988.

[5] ThyssenKrupp Steel, "Hot Press hardening Manganese-boron Steels MBW", product information Manganese-boron Steels, Sept. 2008.

[6] Malek Naderi, "Hot Stamping of Ultra High Strength Steels", Doctor of Engineering Dissertation, Technical University Aachen, Germany, 2007.

[7] P. Akerstrom, "Numerical Implementation of a Constitutive model for Simulation of Hot Stamping", Division of Solid Mechanics, Lulea University of technology, Sweden.

[8] Malek Naderi, "A numerical and Experimental Investigation into Hot Stamping of Boron Alloyed Heat treated Steels", Steel research Int. 79 (2008) No. 2.

[9] A.S. Oddy, J.M.J. McDill and L. Karlsson, "Microstructural predictions including arbitrary thermal histories, reaustenitization and carbon segregation effects" (1996).

*MAT_PML_*OPTION*TROPIC_ELASTIC

This is Material Type 245. This is a perfectly-matched layer (PML) material for orthotropic or anisotropic media. It is to be used in a wave-absorbing layer adjacent to an orthotropic/anisotropic material (*MAT_*OPTION*TROPIC_ELASTIC) in order to simulate wave propagation in an unbounded ortho/anisotropic medium.

This material is a variant of *MAT_PML_ELASTIC (*MAT_230). It is available only for solid 8-node bricks (element type 2). The input cards exactly follow *MAT_*OPTION*-TROPIC_ELASTIC as shown below. See the variable descriptions and Remarks section of *MAT_*OPTION*TROPIC_ELASTIC (*MAT_002) for further details.

Available options include:

ORTHO

ANISO

such that the keyword cards appear:

*MAT_PML_ORTHOTROPIC_ELASTIC or MAT_245 (4 cards follow)

*MAT_PML_ANISOTROPIC_ELASTIC or MAT_245_ANISO (5 cards follow)

Card Summary:

Card 1a.1. This card is required for the ORTHO keyword option.

MID	RO	EA	EB	EC	PRBA	PRCA	PRCB

Card 1a.2. This card is required for the ORTHO keyword option.

GAB	GBC	GCA	AOPT	G	SIGF		

Card 1b.1. This card is required for the ANISO keyword option.

MID	RO	C11	C12	C22	C13	C23	C33

Card 1b.2. This card is required for the ANISO keyword option.

C14	C24	C34	C44	C15	C25	C35	C45

Card 1b.3. This card is required for the ANISO keyword option.

C55	C16	C26	C36	C46	C56	C66	AOPT

Card 2. This card is required.

XP	YP	ZP	A1	A2	A3	MACF	

Card 3.

V1	V2	V3	D1	D2	D3	BETA	REF

Data Card Definitions:

Orthotropic Card 1. Included for the ORTHO keyword option.

Card 1a.1	1	2	3	4	5	6	7	8
Variable	MID	RO	EA	EB	EC	PRBA	PRCA	PRCB
Type	A	F	F	F	F	F	F	F

Orthotropic Card 2. Included for the ORTHO keyword option.

Card 1a.2	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	AOPT	G	SIGF		
Type	F	F	F	F	F	F		

Anisotropic Card 1. Included for the ANISO keyword option.

Card 1b.1	1	2	3	4	5	6	7	8
Variable	MID	RO	C11	C12	C22	C13	C23	C33
Type	A	F	F	F	F	F	F	F

Anisotropic Card 2. Included for the ANISO keyword option.

Card 1b.2	1	2	3	4	5	6	7	8
Variable	C14	C24	C34	C44	C15	C25	C35	C45
Type	F	F	F	F	F	F	F	F

Anisotropic Card 3. Included for the ANISO keyword option.

Card 1b.3	1	2	3	4	5	6	7	8
Variable	C55	C16	C26	C36	C46	C56	C66	AOPT
Type	F	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	MACF	
Type	F	F	F	F	F	F	I	

Card 3	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	REF
Type	F	F	F	F	F	F	F	F

Remarks:

1. **Unboundedness.** A layer of this material may be placed at a boundary of a bounded domain to simulate unboundedness of the domain at that boundary. The layer absorbs and attenuates waves propagating outward from the domain, without any significant reflection of the waves back into the bounded domain. The layer cannot support any static displacement.

2. **Material Properties of Bounded Domain.** The material in the bounded domain near the layer is assumed to be, or behaves like, a linear ortho/anisotropic

material. The material properties of the layer should be set to the corresponding properties of this material.

3. **Layer Geometry.** The layer should form a cuboid box around the bounded domain, with the axes of the box aligned with the coordinate axes. Various faces of this box may be open, as required by the geometry of the problem. For instance, for a half-space problem, the "top" of the box should be open.

   Internally, LS-DYNA partitions the entire PML into regions which form the "faces", "edges" and "corners" of the above cuboid box. LS-DYNA generates a new material for each region. This partitioning will be visible in the d3plot file. You may safely ignore this partitioning.

4. **Number of Elements in the Layer.** The layer should have 5 - 10 elements through its depth. Typically, 5 - 6 elements are sufficient if the excitation source is reasonably distant from the layer, and 8 - 10 elements if it is close. The size of the elements should be similar to that of elements in the bounded domain near the layer and should be small enough to sufficiently discretize all significant wavelengths in the problem.

5. **Nodal Constraints.** The nodes on the outer boundary of the layer should be fully constrained using either *BOUNDARY_SPC or TC on *NODE. Other constraints — such as *CONSTRAINED_GLOBAL, or *BOUNDARY_PRESCRIBED_MOTION with a zero-valued load curve — will not be recognized. (Ansys Workbench uses the latter approach).

6. **Stress and Strain.** The stress and strain values reported by this material do not have any physical significance.

*MAT_PML_NULL

This is Material Type 246. This is a perfectly-matched layer (PML) material with a pressure fluid constitutive law computed using an equation of state, to be used in a wave-absorbing layer adjacent to a fluid material (*MAT_NULL with an EOS) in order to simulate wave propagation in an unbounded fluid medium. Only *EOS_LINEAR_POLYNOMIAL and *EOS_GRUNEISEN are allowed with this material. See the Remarks section of *MAT_NULL (*MAT_009) for further details. Accurate results are to be expected only for the case where the EOS presents a linear relationship between the pressure and volumetric strain.

This material is a variant of *MAT_PML_ELASTIC (*MAT_230) and is available only for solid 8-node bricks (element type 2).

Card	1	2	3	4	5	6	7	8
Variable	MID	RO	MU					
Type	A	F	F					
Default	none	none	0.0					

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
MU	Dynamic viscosity coefficient

Remarks:

1. **Unboundedness.** A layer of this material may be placed at a boundary of a bounded domain to simulate unboundedness of the domain at that boundary: the layer absorbs and attenuates waves propagating outward from the domain, without any significant reflection of the waves back into the bounded domain. The layer cannot support any static displacement.

2. **Material Properties of Bounded Domain.** It is assumed the material in the bounded domain near the layer is, or behaves like, a linear fluid material. The material properties of the layer should be set to the corresponding properties of this material.

3. **Layer Geometry.** The layer should form a cuboid box around the bounded domain, with the axes of the box aligned with the coordinate axes. Various faces of this box may be open, as required by the geometry of the problem. For example, for a half-space problem, the "top" of the box should be open.

   Internally, LS-DYNA will partition the entire PML into regions which form the "faces," "edges" and "corners" of the above cuboid box and generate a new material for each region. This partitioning will be visible in the d3plot file. The user may safely ignore this partitioning.

4. **Number of Elements in the Layer.** The layer should have 5 - 10 elements through its depth. Typically, 5 - 6 elements are sufficient if the excitation source is reasonably distant from the layer, and 8 - 10 elements if it is close. The size of the elements should be similar to that of elements in the bounded domain near the layer, and should be small enough to sufficiently discretize all significant wavelengths in the problem.

5. **Nodal Constraints.** The nodes on the outer boundary of the layer should be fully constrained using either *BOUNDARY_SPC or TC on *NODE. Other constraints, such as *CONSTRAINED_GLOBAL or *BOUNDARY_PRESCRIBED_MOTION with a zero-valued load curve, will not be recognized.

   Note that Ansys Mechanical uses *BOUNDARY_PRESCRIBED_MOTION with a zero-value load curve for fully constraining nodal degrees-of-freedom. Because of this limitation, you will have to edit the input file with a text editor.

6. **Stress and Strain.** The stress and strain values reported by this material do not have any physical significance.

*MAT_PHS_BMW

This is Material Type 248. This model is intended for hot stamping processes with phase transformation effects. It is available for shell elements *only* and is based on Material Type 244 (*MAT_UHS_STEEL). As compared with Material Type 244, Material Type 248 features:

1. a more flexible choice of evolution parameters,
2. an approach for transformation induced strains,
3. and a more accurate density calculation of individual phases.

Thus, the physical effects of the metal can be taken into account calculating the volume fractions of ferrite, pearlite, bainite and martensite for fast supercooling as well as for slow cooling conditions. Furthermore, this material model features cooling-rate dependence for several of its more crucial material parameters in order to accurately calculate the Time-Temperature-Transformation diagram dynamically. A detailed description can be found in Hippchen et al. [2013] and Hippchen [2014].

NOTE 1:	For this material "weight%" means "ppm × 10^{-4}".

NOTE 2:	For this material the phase fractions are calculated in volume percent (vol%).

NOTE 3:	The baseline values for this material are mainly taken from those for *MAT_244. They are provided to give reasonable starting results. These values may not reproduce the results from the papers by Hippchen.

Card Summary:

Card 1. This card is required.

MID	RO	E	PR	TUNIT	TRIP	PHASE	HEAT

Card 2. This card is required.

LCY1	LCY2	LCY3	LCY4	LCY5	C_F	C_P	C_B

Card 3. This card is required.

C	Co	Mo	Cr	Ni	Mn	Si	V

Card 4. This card is required.

W	Cu	P	Al	As	Ti	B	

Card 5. This card is required.

		TABRHO		TREF	LAT1	LAT5	TABTH

Card 6. This card is required.

QR2	QR3	QR4	ALPHA	GRAIN	TOFFE	TOFPE	TOFBA

Card 7. This card is required.

PLMEM2	PLMEM3	PLMEM4	PLMEM5	STRC	STRP		

Card 8. This card is required.

FS	PS	BS	MS	MSIG	LCEPS23	LCEPS4	LCEPS5

Card 9. This card is required.

LCH4	LCH5	DTCRIT	TSAMP	ISLC	IEXTRA		

Card 10. This card is required.

ALPH_M	N_M	PHI_M	PSI_M	OMG_F	PHI_F	PSI_F	CR_F

Card 11. This card is required.

OMG_P	PHI_P	PSI_P	CR_P	OMG_B	PHI_B	PSI_B	CR_B

Card 12. This card is included if HEAT ≠ 0.

AUST	FERR	PEAR	BAIN	MART	GRK	GRQR	TAU1

Card 13. This card is included if HEAT ≠ 0.

GRA	GRB	EXPA	EXPB	GRCC	GRCM	HEATN	TAU2

Card 14. This card is included if IEXTRA ≥ 1.

FUNCA	FUNCB	FUNCM	TCVUP	TCVLO	CVCRIT	TCVSL	

Card 15. This card is included if IEXTRA ≥ 2.

EPSP	EXPON						

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	TUNIT	TRIP	PHASE	HEAT
Type	A	F	F	F	F	I	I	I
Defaults	none	none	none	none	3600	0	0	0

VARIABLE	DESCRIPTION	BASELINE VALUE
MID	Material identification. A unique number or label must be specified (see *PART).	
RO	Material density at room temperature (necessary for calculating transformation induced strains)	7830 Kg/m^3
E	Youngs' modulus: GT.0.0: Constant value is used. LT.0.0: Temperature dependent Young's modulus given by load curve or table ID = -E. The table input is described in Remark 10.	100.e+09 Pa [1]
PR	Poisson's ratio: GT.0.0: Constant value is used. LT.0.0: Temperature dependent Poisson's ratio given by load curve or table ID = -PR. The table input is described in Remark 10.	0.30 [1]
TUNIT	Number of time units per hour. Default is seconds, that is, 3600 time units per hour. It is used only for hardness calculations.	3600.

VARIABLE	DESCRIPTION	BASELINE VALUE
TRIP	Flag to activate (0) or deactivate (1) trip effect calculation	0
PHASE	Switch to exclude middle phases from the simulation: EQ.0: All phases active (default) EQ.1: Pearlite and bainite active EQ.2: Bainite active EQ.3: Ferrite and pearlite active EQ.4: Ferrite and bainite active EQ.5: No active middle phases (only austenite → martensite)	0
HEAT	Heat flag as in MAT_244: EQ.0: Heating is not activated. EQ.1: Heating is activated. EQ.2: Automatic switching between cooling and heating LT.0: Switch between cooling and heating is defined by a time dependent load curve with ID |HEAT|.	

Card 2	1	2	3	4	5	6	7	8
Variable	LCY1	LCY2	LCY3	LCY4	LCY5	C_F	C_P	C_B
Type	I	I	I	I	I	F	F	F
Defaults	none	none	none	none	none	0.0	0.0	0.0

VARIABLE	DESCRIPTION	BASELINE VALUE
LCY1	Load curve or table ID for austenite hardening.	

VARIABLE	DESCRIPTION	BASELINE VALUE
	Load Curve. When LCY1 is a load curve ID, it defines input yield stress as a function of effective plastic strain. **Tabular Data (LCY1 > 0).** When LCY1 is greater than 0 and references a table ID, a 2D table references for each temperature value a hardening curve. **Tabular Data (LCY1 < 0).** When LCY1 is less than 0, |LCY1| is a 3D table ID. Each input temperature value gives a table ID which defines for each a strain rate a hardening curve.	
LCY2	Load curve or table ID for ferrite. See LCY1 for description.	
LCY3	Load curve or table ID for pearlite. See LCY1 for description.	
LCY4	Load curve or table ID for bainite. See LCY1 for description.	
LCY5	Load curve or table ID for martensite. See LCY1 for description.	
C_F	Alloy dependent factor C_f for ferrite (controls the alloying effects beside of Boron on the time-temperature-transformation start line of ferrite)	
C_P	Alloy dependent factor C_p for pearlite (see C_F for description)	
C_B	Alloy dependent factor C_b for bainite (see C_F for description)	

Card 3	1	2	3	4	5	6	7	8
Variable	C	Co	Mo	Cr	Ni	Mn	Si	V
Type	F	F	F	F	F	F	F	F
Defaults	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION	BASELINE VALUE
C	Carbon [weight %]	0.23 [2]
Co	Cobolt [weight %]	0.0 [2]
Mo	Molybdenum [weight %]	0.0 [2]
Cr	Chromium [weight %]	0.21 [2]
Ni	Nickel [weight %]	0.0 [2]
Mn	Manganese [weight %]	1.25 [2]
Si	Silicon [weight %]	0.29 [2]
V	Vanadium [weight %]	0.0 [2]

Card 4	1	2	3	4	5	6	7	8
Variable	W	Cu	P	Al	As	Ti	B	
Type	F	F	F	F	F	F	F	
Defaults	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

VARIABLE	DESCRIPTION	BASELINE VALUE
W	Tungsten [weight %]	0.0 [2]
Cu	Copper [weight %]	0.0 [2]
P	Phosphorous [weight %]	0.013 [2]

VARIABLE	DESCRIPTION	BASELINE VALUE
Al	Aluminum [weight %]	0.0 [2]
As	Arsenic [weight %]	0.0 [2]
Ti	Titanium [weight %]	0.0 [2]
B	Boron [weight %]	0.0

Card 5	1	2	3	4	5	6	7	8
Variable			TABRHO		TREF	LAT1	LAT5	TABTH
Type			I		F	F	F	I
Defaults			none		none	0.0	0.0	none

VARIABLE	DESCRIPTION	BASELINE VALUE
TABRHO	Table definition for phase and temperature dependent densities. Needed for calculation of transformation induced strains.	
TREF	Reference temperature for thermal expansion (only necessary for thermal expansion calculation with the secant method).	293.15
LAT1	Latent heat for the decomposition of austenite into ferrite, pearlite and bainite. GT.0.0: Constant value LT.0.0: Curve ID or table ID. See Remark 11 for more information.	590×10^6 J/m^3 [2]
LAT5	Latent heat for the decomposition of austenite into martensite. GT.0.0: Constant value LT.0.0: Curve ID. Note that LAT 5 is ignored if a table ID is used in LAT1.	640×10^6 J/m^3 [2]

VARIABLE	DESCRIPTION	BASELINE VALUE
TABTH	Table definition for thermal expansion coefficient. See Remark 12. GT.0: A table for instantaneous thermal expansion (TREF is ignored) LT.0: A table with thermal expansion with reference to TREF	

Card 6	1	2	3	4	5	6	7	8
Variable	QR2	QR3	QR4	ALPHA	GRAIN	TOFFE	TOFPE	TOFBA
Type	F	F	F	F	F	F	F	F
Defaults	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION	BASELINE VALUE
QR2	Activation energy divided by the universal gas constant for the diffusion reaction of the austenite-ferrite reaction: $Q2/R$. $R = 8.314472$ [J/mol K]. Load curve ID if ISLC = 2 on Card 9 (function of cooling rate).	10324 K [3] = (23000 cal/mole) × (4.184 J/cal) / (8.314 J/mole/K)
QR3	Activation energy divided by the universal gas constant for the diffusion reaction for the austenite-pearlite reaction: $Q3/R$. $R = 8.314472$ [J/mol K]. Load curve ID if ISLC = 2 on Card 9 (function of cooling rate).	13432. K [3]
QR4	Activation energy divided by the universal gas constant for the diffusion reaction for the austenite-bainite reaction: $Q4/R$. $R = 8.314472$ [J/mol K]. Load curve ID if ISLC = 2 on Card 9 (function of cooling rate).	15068. K [3]

VARIABLE	DESCRIPTION	BASELINE VALUE
ALPHA	Material constant for the martensite phase. A value of 0.011 means that 90% of the available austenite is transformed into martensite at 210 degrees below the martensite start temperature (see messag file for information), whereas a value of 0.033 means a 99.9% transformation.	0.011
GRAIN	ASTM grain size number G for austenite, usually a number between 7 and 11.	6.8
TOFFE	Number of degrees that the ferrite is bleeding over into the pearlite reaction: $T_{\mathrm{off},f}$	0.0
TOFPE	Number of degrees that the pearlite is bleeding over into the bainite reaction: $T_{\mathrm{off},p}$	0.0
TOFBA	Number of degrees that the bainite is bleeding over into the martensite reaction: $T_{\mathrm{off},b}$	0.0

Card 7	1	2	3	4	5	6	7	8
Variable	PLMEM2	PLMEM3	PLMEM4	PLMEM5	STRC	STRP		
Type	F	F	F	F	F	F		
Defaults	0.0	0.0	0.0	0.0	0.0	0.0		

VARIABLE	DESCRIPTION	BASELINE VALUE
PLMEM2	Memory coefficient for the plastic strain that is carried over from the austenite. A value of 1 means that all plastic strains from austenite is transferred to the ferrite phase and a value of 0 means that nothing is transferred.	0.0

VARIABLE	DESCRIPTION	BASELINE VALUE
PLMEM3	Same as PLMEM2 but between austenite and pearlite	0.0
PLMEM4	Same as PLMEM2 but between austenite and bainite	0.0
PLMEM5	Same as PLMEM3 but between austenite and martensite	0.0
STRC	Cowper and Symonds strain rate parameter C LT.0.0: Load curve ID = -STRC GT.0.0: Constant value EQ.0.0: Strain rate *not* active	0.0
STRP	Cowper and Symonds strain rate parameter P LT.0.0: Load curve ID = -STRP GT.0.0: Constant value EQ.0.0: Strain rate *not* active	0.0

Card 8	1	2	3	4	5	6	7	8
Variable	FS	PS	BS	MS	MSIG	LCEPS23	LCEPS4	LCEPS5
Type	F	F	F	F	F	I	I	I
Defaults	0.0	0.0	0.0	0.0	none	none	none	none

VARIABLE	DESCRIPTION	BASELINE VALUE
FS	Manual start temperature ferrite, F_S. GT.0.0: Same temperature is used for heating and cooling. LT.0.0: Different start temperatures for cooling and heating given by load curve ID = -FS. First	

VARIABLE	DESCRIPTION	BASELINE VALUE
	ordinate value is used for cooling, last ordinate value for heating.	
PS	Manual start temperature pearlite, P_S. See FS for description.	
BS	Manual start temperature bainite, B_S. See FS for description.	
MS	Manual start temperature martensite, M_S. See FS for description.	
MSIG	Describes the increase of martensite start temperature for cooling due to applied stress. LT.0: Load curve ID describes MSIG as a function of triaxiality (pressure / effective stress). $\mathrm{MS}^* = \mathrm{MS} + \mathrm{MSIG} \times \sigma_{\mathrm{eff}}$	
LCEPS23	Load curve ID dependent on plastic strain that scales the activation energy QR2 and QR3. $\mathrm{QRn} = \mathrm{Qn} \times \mathrm{LCEPS23}(\varepsilon_{\mathrm{pl}})/R$	
LCEPS4	Load curve ID dependent on plastic strain that scales the activation energy QR4. $\mathrm{QR4} = \mathrm{Q4} \times \mathrm{LCEPS4}(\varepsilon_{\mathrm{pl}})/R$	
LCEPS5	Load Curve ID which describe the increase of the martensite start temperature for cooling as a function of plastic strain. $\mathrm{MS}^* = \mathrm{MS} + \mathrm{MSIG} \times \sigma_{\mathrm{eff}} + \mathrm{LCEPS5}(\varepsilon_{\mathrm{pl}})$	

Card 9	1	2	3	4	5	6	7	8
Variable	LCH4	LCH5	DTCRIT	TSAMP	ISLC	IEXTRA		
Type	I	I	F	F	I	I		
Defaults	0	0	0.0	0.0	0	0		

VARIABLE	DESCRIPTION	BASELINE VALUE
LCH4	Load curve ID of Vickers hardness as a function of temperature for bainite hardness calculation	
LCH5	Load curve ID of Vickers hardness as a function of temperature for martensite hardness calculation	
DTCRIT	Critical cooling rate to detect holding phase	
TSAMP	Sampling interval for temperature rate monitoring to detect the holding phase	
ISLC	Flag for definition of evolution parameters on Cards 10 and 11. EQ.0.0: All 16 fields on Cards 10 and 11 are constant values. EQ.1.0: PHI_F, CR_F, PHI_P, CR_P, PHI_B, and CR_B are load curves defining values as functions of cooling rate. The remaining 10 fields on Cards 10 and 11 are constant values. EQ.2.0: QR2, QR3, and QR4 from Card 6 and all 16 fields on Cards 10 and 11 are load curves defining values as functions of cooling rate.	

VARIABLE	DESCRIPTION	BASELINE VALUE
IEXTRA	Flag to read extra cards (see Cards 14 and 15)	

Card 10	1	2	3	4	5	6	7	8
Variable	ALPH_M	N_M	PHI_M	PSI_M	OMG_F	PHI_F	PSI_F	CR_F
Type	F	F	F	F	F	F	F	F
Defaults	0.0428	0.191	0.382	2.421	0.41	0.4	0.4	0.0

VARIABLE	DESCRIPTION	BASELINE VALUE
ALPH_M	Martensite evolution parameter α_m	0.0428
N_M	Martensite evolution parameter n_m	0.191
PHI_M	Martensite evolution parameter φ_m	0.382
PSI_M	Martensite evolution exponent ψ_m. If $\psi_m < 0$, then $\psi_m = \lvert\psi_m\rvert(2 - \varsigma_a)$.	2.421
OMG_F	Ferrite grain size factor ω_f (mainly controls the alloying effect of Boron on the time-temperature-transformation start line of ferrite)	0.41
PHI_F	Ferrite evolution parameter φ_f (controls the incubation time till 1 vol% of ferrite is built)	0.4
PSI_F	Ferrite evolution parameter ψ_f (controls the time till 99 vol% of ferrite is built without effect on the incubation time)	0.4
CR_F	Ferrite evolution parameter $C_{r,f}$ (retardation coefficient to influence the kinetics of phase transformation of ferrite, should be determined at slow cooling conditions, can also be defined in dependency to the	0.0

VARIABLE	DESCRIPTION	BASELINE VALUE
	cooling rate)	

Card 11	1	2	3	4	5	6	7	8
Variable	OMG_P	PHI_P	PSI_P	CR_P	OMG_B	PHI_B	PSI_B	CR_B
Type	F	F	F	F	F	F	F	F
Defaults	0.32	0.4	0.4	0.0	0.29	0.4	0.4	0.0

VARIABLE	DESCRIPTION	BASELINE VALUE
OMG_P	Pearlite grain size factor ω_p (see OMG_F for description)	0.32
PHI_P	Pearlite evolution parameter φ_p (see PHI_F for description)	0.4
PSI_P	Pearlite evolution parameter ψ_p (see PSI_F for description)	0.4
CR_P	Pearlite evolution parameter $C_{r,p}$ (see CR_F for description)	0.0
OMG_B	Bainite grain size factor ω_b (see OMG_F for description)	0.32
PHI_B	Bainite evolution parameter φ_b (see PHI_F for description)	0.4
PSI_B	Bainite evolution parameter ψ_b (see PSI_-F for description)	0.4
CR_B	Bainite evolution parameter $C_{r,b}$ (see CR_F for description)	0.0

Heat Card 1. Additional Card for HEAT ≠ 0.

Card 12	1	2	3	4	5	6	7	8
Variable	AUST	FERR	PEAR	BAIN	MART	GRK	GRQR	TAU1
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.08E+8

VARIABLE	DESCRIPTION	BASELINE VALUE
AUST	If a heating process is initiated at $t = 0$, this field sets the initial amount of austenite in the blank. If heating is activated at $t > 0$ during a simulation, this value is ignored. Note that, $\mathrm{AUST + FERR + PEAR + BAIN + MART} = 1.0$	0.0
FERR	See AUST for description	0.0
PEAR	See AUST for description	0.0
BAIN	See AUST for description	0.0
MART	See AUST for description	0.0
GRK	Growth parameter k ($\mu m^2/sec$)	10^{11}[9]
GRQR	Grain growth activation energy (J/mol) divided by the universal gas constant: Q/R. $R = 8.314472$ (J/mol K)	3.0×10^4[9]
TAU1	Empirical grain growth parameter c_1 describing the function $\tau(T)$	2.08×10^8[9]

Heat Card 2. Additional Card for HEAT ≠ 0.

Card 13	1	2	3	4	5	6	7	8
Variable	GRA	GRB	EXPA	EXPB	GRCC	GRCM	HEATN	TAU2
Type	F	F	F	F	F	F	F	F
Default	3.11	7520.	1.0	1.0	none	none	1.0	4.806

VARIABLE	DESCRIPTION	BASELINE VALUE
GRA	Grain growth parameter A	[9]
GRB	Grain growth parameter B. A table of recommended values of GRA and GRB is included in Remark 8 of *MAT_244.	[9]
EXPA	Grain growth parameter a	1.0 [9]
EXPB	Grain growth parameter b	1.0 [9]
GRCC	Grain growth parameter with the concentration of nonmetals in the blank, weight% of C or N	[9]
GRCM	Grain growth parameter with the concentration of metals in the blank, lowest weight% of Cr, V, Nb, Ti, Al	[9]
HEATN	Grain growth parameter n for the austenite formation	1.0 [9]
TAU2	Empirical grain growth parameter c_2 describing the function $\tau(T)$	4.806 [9]

Extra Card 1. Additional Card for IEXTRA ≥ 1

Card 14	1	2	3	4	5	6	7	8
Variable	FUNCA	FUNCB	FUNCM	TCVUP	TCVLO	CVCRIT	TCVSL	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

VARIABLE	DESCRIPTION	BASELINE VALUE
FUNCA	ID of a *DEFINE_FUNCTION for saturation stress A (Hockett-Sherby approach)	
FUNCB	ID of a *DEFINE_FUNCTION for initial yield stress B (Hockett-Sherby approach)	
FUNCM	ID of a *DEFINE_FUNCTION for saturation rate M (Hockett-Sherby approach)	
TCVUP	Upper temperature for determination of average cooling velocity	
TCVLO	Lower temperature for determination of average cooling velocity	
CVCRIT	Critical cooling velocity. If the average cooling velocity is less than or equal to CVCRIT, the cooling rate at temperature TCVSL is used.	
TCVSL	Temperature for determination of cooling velocity for small cooling velocities.	

Extra Card 2. Additional Card for IEXTRA ≥ 2

Card 15	1	2	3	4	5	6	7	8
Variable	EPSP	EXPON						
Type	F	F						
Default	0.0	0.0						

VARIABLE	DESCRIPTION	BASELINE VALUE
EPSP	Plastic strain in Hockett-Sherby approach	
EXPON	Exponent in Hockett-Sherby approach	

Remarks:

1. **Start temperatures.** Start temperatures for ferrite, pearlite, bainite, and martensite can be defined manually using FS, PS, BS, and MS. Or they are initially defined using the following composition equations:

$$\begin{aligned}
F_S &= 273.15 + 912 - 203 \times \sqrt{\mathrm{C}} - 15.2 \times \mathrm{Ni} + 44.7 \times \mathrm{Si} + 104 \times \mathrm{V} + 31.5 \times \mathrm{Mo} \\
&\qquad + 13.1 \times \mathrm{W} - 30 \times \mathrm{Mn} - 11 \times \mathrm{Cr} - 20 \times \mathrm{Cu} + 700 \times \mathrm{P} + 400 \times \mathrm{Al} \\
&\qquad + 120 \times \mathrm{As} + 400 \\
P_S &= 273.15 + 723 - 10.7 \times \mathrm{Mn} - 16.9 \times \mathrm{Ni} + 29 \times \mathrm{Si} + 16.9 \times \mathrm{Cr} + 290 \times \mathrm{As} \\
&\qquad + 6.4 \times \mathrm{W} \\
B_S &= 273.15 + 637 - 58 \times \mathrm{C} - 35 \times \mathrm{Mn} - 15 \times \mathrm{Ni} - 34 \times \mathrm{Cr} - 41 \times \mathrm{Mo} \\
M_S &= 273.15 + 539 - 423 \times \mathrm{C} - 30.4 \times \mathrm{Mn} - 17.7 \times \mathrm{Ni} - 12.1 \times \mathrm{Cr} - 7.5 \times \mathrm{Mo} \\
&\qquad + 10 \times \mathrm{Co} - 7.5 \times \mathrm{Si}
\end{aligned}$$

2. **Martensite phase evolution.** Martensite phase evolution according to Lee et al. [2008, 2010] if PSI_M > 0:

$$\frac{\mathrm{d}\xi_m}{\mathrm{d}T} = \alpha_m (M_S - T)^n \xi_m^{\varphi_m} (1 - \xi_m)^{\psi_m}$$

Martensite phase evolution according to Lee et al. [2008, 2010] with extension by Hippchen et al. [2013] if PSI_M < 0:

$$\frac{\mathrm{d}\xi_m}{\mathrm{d}T} = \alpha_m (M_S - T)^n \xi_m^{\varphi_m} (1 - \xi_m)^{\psi_m (2 - \zeta_a)}$$

3. **Phase change kinetics for ferrite, pearlite and bainite.**

$$\frac{\mathrm{d}\xi_f}{\mathrm{d}t} = 2^{\omega_f G} \frac{\exp\left(-\frac{Q_f}{RT}\right)}{C_f} (F_S - T)^3 \frac{\xi^{\varphi_f(1-\xi_f)}(1-\xi_f)^{\psi_f \xi_f}}{\exp\left(C_{r,f}\xi_f^2\right)}$$
$$\text{for} \quad F_S \geq T \geq \left(P_S - T_{\text{off},f}\right)$$

$$\frac{\mathrm{d}\xi_p}{\mathrm{d}t} = 2^{\omega_p G} \frac{\exp\left(-\frac{Q_p}{RT}\right)}{C_p} (P_S - T)^3 \frac{\xi^{\varphi_p(1-\xi_p)}(1-\xi_p)^{\psi_p \xi_p}}{\exp\left(C_{r,p}\xi_p^2\right)}$$
$$\text{for} \quad P_S \geq T \geq \left(B_S - T_{\text{off},p}\right)$$

$$\frac{\mathrm{d}\xi_b}{\mathrm{d}t} = 2^{\omega_b G} \frac{\exp\left(-\frac{Q_b}{RT}\right)}{C_b} (B_S - T)^2 \frac{\xi^{\varphi_b(1-\xi_b)}(1-\xi_b)^{\psi_b \xi_b}}{\exp\left(C_{r,b}\xi_b^2\right)}$$
$$\text{for} \quad M_S \geq T \geq \left(M_S - T_{\text{off},b}\right)$$

4. **History variables.** History variables of this material model are listed in the following table. To be able to post-process that data, parameters NEIPS (shells) or NEIPH (solids) must be defined on *DATABASE_EXTENT_BINARY.

History Variable #	Description
1	Amount austenite
2	Amount ferrite
3	Amount pearlite
4	Amount bainite
5	Amount martensite
6	Vickers hardness
7	Yield stress
8	ASTM grain size number
9	Young's modulus
10	Saturation stress A (H-S approach)
11	Initial yield stress B (H-S approach)
12	Saturation rate M (H-S approach)
13	Yield stress of H-S approach $\sigma_y = A - (A - B) \times e^{-M \times \mathrm{EPSP}^{\mathrm{EXPON}}}$
17	Temperature rate
19	Current temperature

History Variable #	Description
25	Plastic strain rate
26	Effective thermal expansion coefficient

5. **Choosing/excluding phases.** To exclude a phase from the simulation, set the PHASE parameter accordingly.

6. **Strain rate effects.** Note that both strain rate parameters (STRC and STRP) must be set to include the effect. It is possible to use a temperature dependent load curve for both parameters simultaneously or for one parameter keeping the other constant.

7. **Time units.** TUNIT is time units per hour and is only used for calculating the Vicker Hardness. By default, it is assumed that the time unit is seconds. If another time unit is used, for example milliseconds, then TUNIT must be changed to TUNIT = 3.6×10^6.

8. **Thermal speedup factor.** The thermal speedup factor TSF of *CONTROL_THERMAL_SOLVER is used to scale reaction kinetics and hardness calculations in this material model. Strain rate dependent properties (see LCY1 to LCY5 or STRC/STRP), however, are not scaled by TSF.

9. **Re-austenization and grain growth with the HEAT field.** When HEAT is activated, the re-austenitization and grain growth algorithms are also activated. See Remark 8 of MAT_244 for details.

10. **Phase indexed tables.** When using a table ID to describe the Young's modulus as dependent on the temperature, use *DEFINE_TABLE_2D. Set the abscissa value equal to 1 for the austenite YM-curve, equal to 2 for the ferrite YM-curve, equal to 3 for the pearlite YM curve, equal to 4 for the bainite YM-curve and finally equal to 5 for the martensite YM-curve. When using the PHASE option only the curves for the included phases are required, but all five phases may be included. The total YM is calculated by a linear mixture law:

$$\mathrm{YM} = \mathrm{YM1} \times \mathrm{PHASE1} + \cdots + \mathrm{YM5} \times \mathrm{PHASE5}$$

For example:

```
*DEFINE_TABLE_2D
$ The number before curve id:s define which phase the curve
$ will be applied to.  1 = Austenite, 2 = Ferrite, 3 = Pearlite,
$ 4 = Bainite and 5 = Martensite.
      1000       0.0       0.0
                 1.0                 100
                 2.0                 200
                 3.0                 300
                 4.0                 400
                 5.0                 500
$
```

```
$ Define curves 100 - 500
*DEFINE_CURVE
$ Austenite Temp (K) - YM-Curve (MPa)
       100         0       1.0       1.0
            1300.0              50.E+3
             223.0             210.E+3
```

11. **Phase-indexed latent heat table.** A table ID may be specified for the Latent heat (LAT1) to describe each phase change individually. Use *DEFINE_TABLE_2D and set the abscissa values to the corresponding phase transition number. That is, 2 for the Austenite – Ferrite, 3 for the Austenite – Pearlite, 4 for the Austenite – Bainite and 5 for the Austenite – Martensite. See Remark 10 for an example of a correct table definition. If a curve is missing, the corresponding latent heat for that transition will be set to zero. Also, when a table is used, LAT2 is ignored. If HEAT > 0, the latent for the transition back to Austenite can also be included. This latent heat curve is marked as 1 in the table definition of LAT1.

12. **Phase-indexed thermal expansion table.** Tables are supported for defining different thermal expansion properties for each phase. The input is identical to the above table definitions. The table must have the abscissa values between 1 and 5 where the number correspond to phase 1 to 5. To exclude one phase from influencing the thermal expansion you simply input a curve that is zero for that phase or even easier, exclude that phase number in the table definition. For example, to exclude the bainite phase you only define the table with curves for the indices 1, 2, 3 and 5.

13. **Phase-indexed transformation induced strain properties.** Transformation induced strains can be define with a table TABRHO, where densities are defined as functions of phase (table abscissas) and temperature (load curves).

*MAT_REINFORCED_THERMOPLASTIC

This is Material Type 249. This material model describes a reinforced thermoplastic composite material. The reinforcement is defined as an anisotropic hyper-elastic material with up to three distinct fiber directions. It can be used to model unidirectional layers as well as woven and non-crimped fabrics. The matrix is modeled with a simple thermal elasto-plastic material formulation. For a composite, the overall stress is found by adding the fiber and matrix stresses.

Card Summary:

Card 1. This card is required.

MID	RO	EM	LCEM	PRM	LCPRM	LCSIGY	BETA

Card 2. This card is required.

NFIB	AOPT				A1	A2	A3

Card 3. This card is required.

V1	V2	V3	D1	D2	D3	MANGL	THICK

Card 4. This card is required.

IDF1	ALPH1	EF1	LCEF1	G23_1	G31_1		

Card 5. This card is required.

G12	LCG12	ALOC12	GLOC12	METH12			

Card 6. This card is required.

IDF2	ALPH2	EF2	LCEF2	G23_2	G31_2		

Card 7. This card is required.

G23	LCG23	ALOC23	GLOC23	METH23			

Card 8. This card is required.

IDF3	ALPH3	EF3	LCEF3	G23_3	G31_3		

Card 9. This card is optional.

POSTV	IHIS						

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EM	LCEM	PRM	LCPRM	LCSIGY	BETA
Type	A	F	F	I	F	I	I	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Density
EM	Young's modulus of matrix material
LCEM	Curve ID for Young's modulus of matrix material as a function of temperature. With this option active, EM is ignored.
PR	Poisson's ratio for matrix material
LCPR	Curve ID for Poisson's ratio of matrix material versus temperature. With this option active, PR is ignored.
LCSIGY	Load curve or table ID for strain hardening of the matrix. If a curve, then it specifies yield stress as a function of effective plastic strain. If a table, then temperatures are the table values indexing curves giving yield stress as a function of effective plastic strain (see *DEFINE_TABLE).
BETA	Parameter for mixed hardening, $0.0 \le \beta \le 1.0$. Set $\beta = 0.0$ for pure kinematic hardening and $\beta = 1.0$ for pure isotropic hardening.

Card 2	1	2	3	4	5	6	7	8
Variable	NFIB	AOPT				A1	A2	A3
Type	I	F				F	F	F

VARIABLE	DESCRIPTION
NFIB	Number of fiber families to be considered

VARIABLE	DESCRIPTION
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description): EQ.0.0: Locally orthotropic with material axes determined by element nodes, as with *DEFINE_COORDI-NATE_-NODES, and then rotated about the shell element normal by the angle MANGL. EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR. EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle. The angle may be set in the keyword input for the element or in the input for this keyword with MANGL. LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
A1, A2, A3	Components of vector **a** for AOPT = 2.

Card 3	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	MANGL	THICK
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3
D1, D2, D3	Components of vector **d** for AOPT = 2
MANGL	Material angle in degrees for AOPT = 0 and 3. It may be overwritten on the element card; see *ELEMENT_SHELL_BETA.

VARIABLE	DESCRIPTION
THICK	Balance thickness changes of the material due to the matrix description by scaling fiber stresses EQ.0: No scaling EQ.1: Scaling

Card 4	1	2	3	4	5	6	7	8
Variable	IDF1	ALPH1	EF1	LCEF1	G23_1	G31_1		
Type	I	F	F	I	F	F		

VARIABLE	DESCRIPTION
IDF1	ID for 1st fiber family for post-processing
ALPH1	Orientation angle α_1 for 1st fiber with respect to overall material direction
EF1	Young's modulus for 1st fiber family
LCEF1	Load curve for stress as a function of fiber strain of 1st fiber. With this option active, EF1 is ignored. If a curve, then it specifies input stress as a function of fiber strain. If a table, then temperatures are the table values indexing curves giving stress as function of fiber strain. The table data will be extrapolated for both strains and temperatures where necessary.
G23_1	Transverse shear modulus orthogonal to direction of 1st fiber
G31_1	Transverse shear modulus in direction of 1st fiber

Card 5	1	2	3	4	5	6	7	8
Variable	G12	LCG12	ALOC12	GLOC12	METH12			
Type	F	I	F	F	I			

VARIABLE	DESCRIPTION
G12	Linear shear modulus for shearing between fiber families 1 and 2
LCG12	Curve ID for shear stress as a function of shearing type as specified with METH12 between the 1st and 2nd fibers. See Remark 1.
ALOC12	Locking angle (in radians) for shear between fiber families 1 and 2
GLOC12	Linear shear modulus for shear angles larger than ALOC12
METH12	Option for shear response between fiber 1 and 2 (see Remark 1): EQ.0: Elastic shear response. Curve LCG12 specifies shear stress as a function of the scalar product of the fiber directions. EQ.1: Elasto-plastic shear response. Curve LCG12 specifies yield shear stress as a function of the normalized scalar product of the fiber directions. EQ.2: Elastic shear response. Curve LCG12 specifies shear stress as a function of shear angle (radians) between the fibers. EQ.3: Elasto-plastic shear response. Curve LCG12 defines yield shear stress as a function of normalized shear angle between the fibers. EQ.4: Elastic shear response. Curve LCG12 specifies shear stress as a function of shear angle (radians) between the fibers. This option is a special implementation for non-crimped fabrics, where one of the fiber families corresponds to a stitching. EQ.5: Elasto-plastic shear response. Curve LCG12 specifies yield shear stress as a function of normalized shear angle between the fibers. This option is a special implementation for non-crimped fabrics, where one of the fiber families corresponds to a stitching. EQ.10: Elastic shear response. Curve LCG12 specifies shear stress as a function of shear angle (radians) between the fibers. This option is tailored for woven fabrics and guarantees a pure shear stress response. EQ.11: Elasto-plastic shear response. Curve LCG12 specifies yield shear stress as a function of normalized shear angle. This option is tailored for woven fabrics and guarantees a pure shear stress response.

Card 6	1	2	3	4	5	6	7	8
Variable	IDF2	ALPH2	EF2	LCEF2	G23_2	G31_2		
Type	I	F	F	I	F	F		

VARIABLE	DESCRIPTION
IDF2	ID for 2nd fiber family for post-processing
ALPH2	Orientation angle α_2 for 2nd fiber with respect to overall material direction
EF2	Young's modulus for 2nd fiber family
LCEF2	Load curve for stress as a function of fiber strain of 2nd fiber. With this option active, EF2 is ignored. If a curve, then it specifies input stress as a function of fiber strain. If a table, then temperatures are the table values indexing curves giving stress as function of fiber strain. The table data will be extrapolated for both strains and temperatures where necessary.
G23_2	Transverse shear modulus orthogonal to direction of 2nd fiber
G31_2	Transverse shear modulus in direction of 2nd fiber

Card 7	1	2	3	4	5	6	7	8
Variable	G23	LCG23	ALOC23	GLOC23	METH23			
Type	F	I	F	F	I			

VARIABLE	DESCRIPTION
G23	Linear shear modulus for shearing between fiber families 2 and 3
LCG23	Curve ID for shear stress as a function of shearing type as specifies with METH23 between the 2nd and 3rd fibers. See Remark 1.
ALOC23	Locking angle (in radians) for shear between fiber families 2 and 3
GLOC23	Linear shear modulus for shear angles larger than ALOC23

VARIABLE	DESCRIPTION
METH23	Option for shear response between fibers 2 and 3 (see METH12 for input options and Remark 1).

Card 8	1	2	3	4	5	6	7	8
Variable	IDF3	ALPH3	EF3	LCEF3	G23_3	G31_3		
Type	I	F	F	I	F	F		

VARIABLE	DESCRIPTION
IDF3	ID for 3rd fiber family for post-processing
ALPH3	Orientation angle α_3 for 3rd fiber with respect to overall material direction
EF3	Young's modulus for 3rd fiber family
LCEF3	Load curve for stress versus fiber strain of 3rd fiber. With this option active, EF3 is ignored. If a curve, then it specifies input stress as a function of fiber strain. If a table, then temperatures are the table values indexing curves giving stress as function of fiber strain. The table data will be extrapolated for both strains and temperatures where necessary.
G23_3	Transverse shear modulus orthogonal to direction of 3rd fiber
G31_3	Transverse shear modulus in direction of 3rd fiber

This card is optional.

Card 9	1	2	3	4	5	6	7	8
Variable	POSTV	IHIS						
Type	I							

VARIABLE	DESCRIPTION
POSTV	Defines additional history variables that might be useful for post-processing. See Remark 2.

VARIABLE	DESCRIPTION
IHIS	Flag for material properties initialization: EQ.0: Material properties defined in Cards 1 - 8 are used. GE.1: Use *INITIAL_STRESS_SHELL to initialize some material properties on an element-by-element basis (see Remark 3 below).

Remarks:

1. **Stress Calculation.** This material features an additive split of the matrix and reinforcement contributions to the stress. Therefore, the combined stress response, σ, equals the sum $\sigma^m + \sigma^f$.

   The matrix model uses an elastic-plastic material formulation with a von-Mises yield criterion. Material parameters, such as Young's modulus, Poisson's ratio and yield stress, can be given as functions of temperature. This material supports a mixed hardening approach.

   We formulated the contribution of the fiber reinforcement as a hyperelastic material. Based on the orientation angle, α_i, of the i^{th} fiber family, LS-DYNA computes an initial fiber direction in the element coordinate system $\mathbf{m}_i^0$. By using the deformation gradient, $\mathbf{F}$, the current fiber configuration is given as $\mathbf{m}_\mathrm{i} = \mathbf{F}\,\mathbf{m}_i^0$, containing all necessary information on fiber strain and reorientation. Here, this vector is always orthogonal to the shell normal and can, thus, be represented by the two in-plane vector components.

   Following standard textbook mechanics for anisotropic and hyperelastic materials, the elastic stresses within the fibers due to tension or compression are given as

$$\sigma_T^f = \sum_{i=1}^{n} \sigma_{T,i}^f(\lambda_i) = \sum_{i=1}^{n} \frac{1}{J} f_i(\lambda_i)(\mathbf{m}_\mathrm{i} \otimes \mathbf{m}_\mathrm{i}) \ ,$$

   where the function f_i of the fiber strain λ_i corresponds to the load curve LCEFi. n is the number of fiber families.

   The shear behavior of the reinforcement can be controlled by METHij. For values less than 10, the behavior is again standard textbook mechanics:

$$\sigma_S^f = \sum_{i=1}^{n-1} \sigma_{S,i,i+1}^f = \sum_{i=1}^{n-1} \frac{1}{J} g_{i,i+1}(\kappa_{i,i+1})(\mathbf{m}_\mathrm{i} \otimes \mathbf{m}_{\mathrm{i}+1}) \ .$$

   Here $\kappa_{i,i+1}$ represents the employed shear measure (scalar product or shear angle in radians). In general, the dyadic product $\mathbf{m}_\mathrm{i} \otimes \mathbf{m}_{\mathrm{i}+1}$ does not define a shear stress tensor. This formulation might result in unphysical shear behavior in the case of woven fabrics. Therefore, we devised METH*ij* = 10 and 11 to always give a pure shear stress tensor, σ_S^f.

For even values of METH*ij*, an elastic shear response is assumed. If defined, the load curve LCG*ij* corresponds to function $g_{i,j}$. In this case the values of G*ij*, ALOC*ij* and GLOC*ij* are ignored.

For odd values of METH*ij* on the other hand, an elasto-plastic shear behavior is assumed and the load curve LCG*ij* defines the yield stress value as function of a normalized shear parameter. This implies that the load curve needs to be defined for abscissa values between 0.0 and 1.0. A first elastic regime, which is controlled by the linear shear stiffness G*ij*, is assumed until the yield stress given in the load curve for normalized shear value 0.0 is reached. A second linear elastic regime is defined for shear angles (ζ_{ij})/ fiber angles (η_{ij}) larger than the locking angle ALOC*ij*. The corresponding stiffness in that regime is GLOC*ij*. At the transition point to the second elastic regime, the shear stress corresponds to the load curve value for a normalized shear of 1.0.

2. **History Data.** This material formulation outputs to d3plot additional data for post-processing to the set of history variables if requested. The parameter POSTV specifies the data to be written. Its value is calculated as

$$\text{POSTV} = a_1 + 2\,a_2 + 4\,a_3 + 8\,a_4 + 16\,a_5 + 32\,a_6 + 64\,a_7.$$

Each flag a_i is a binary number (can be either 1 or 0) and corresponds to one particular type of post-processing variable according to the following table.

Flag	Description	Variables	# History Var
a_1	Fiber angle	η_{12}, η_{23}	2
a_2	Fiber ID	IDF1, IDF2, IDF3	3
a_3	Fiber strain	$\lambda_1, \lambda_2, \lambda_3$	3
a_4	Fiber direction (in global coordinates)	$\mathbf{m}_1^g, \mathbf{m}_2^g, \mathbf{m}_3^g$	9
a_5	Individual fiber stresses	$f_1(\lambda_1), f_2(\lambda_2), f_3(\lambda_3)$	3
a_6	Fiber stress tensor	$\sigma_{11}^f, \sigma_{22}^f, \sigma_{33}^f, \sigma_{12}^f, \sigma_{23}^f, \sigma_{31}^f$	6
a_7	Fiber direction (in material coordinates)	$\mathbf{m}_1^m, \mathbf{m}_2^m, \mathbf{m}_3^m$	6

The above table also shows the order of output as well as the number of extra history variables associated with the particular flag. In total NXH extra variables are required depending on the choice of parameter POSTV. For example, the maximum number of additional variables is NXH = 32 for POSTV = 127.

As mentioned in Remark 1 fiber orientation is represented in the material subroutine as vector $\mathbf{m}_\mathrm{i}$ defined in the element coordinate system. Prior to writing to the list of histories the vector is transformed into the global coordinate system with three vector components for $a_4 = 1$ and/or into the overall material coordinate system with two vector components for $a_7 = 1$.

A more complete list of potentially helpful history variables is given in the following table. The variable NEIPS in *DATABASE_EXTENT_BINARY must be set to output these history variables.

History Variable #	Description
3	Number of fibers
4	NXH
5 → NXH + 4	Variables as described in preceding table
NXH + 5	POSTV
NXH + 6, NXH + 7	Shear angles $\xi_{12}$and ξ_{23}
NXH + 8	Matrix damage parameter d^m
NXH + 9 → NXH + 11	Fiber tensile damage parameter $d_i^{f,t}$
NXH + 12 → NXH + 14	Fiber compressive damage param. $d_i^{f,c}$
NXH + 15 → NXH + 20	Matrix stress tensor in element coordinate system
NXH+21 → NXH + 26	Deformation gradient

3. **Description of IHIS.** Some material data can be initialized on an element-by-element basis through history variables defined with *INITIAL_STRESS_SHELL starting at position HISV5.

How the data is interpreted depends on the parameter IHIS. Following the same concept as for parameter POSTV, the value of IHIS is computed by the following expression:

$$\mathrm{IHIS} = a_1 + 2\,a_2$$

Each flag a_i is a binary number (can be either 1 or 0) and corresponds to one particular type of material variable. So far, the only material variables implemented are fiber orientation in two different coordinate systems, global and material. Thus, at most one of the flags a_1 and a_2 should be set to 1.

Flag	Description	Variables	# History Var
a_1	Fiber direction (in global coordinates)	$\mathbf{m}_1^g, \mathbf{m}_2^g, \mathbf{m}_3^g$	9
a_2	Fiber direction (in material coordinates)	$\mathbf{m}_1^m, \mathbf{m}_2^m, \mathbf{m}_3^m$	6

*MAT_REINFORCED_THERMOPLASTIC_CRASH

This is Material Type 249. This material model describes a reinforced thermoplastic composite material with its damage and failure behavior. The reinforcement is modeled as an anisotropic hyper-elastic material with up to three distinguished fiber directions. It can be used to model unidirectional layers as well as woven and non-crimped fabrics. The matrix is modeled with a simple elastic plastic material formulation. For a composite, the overall stress is found by adding the fiber and matrix stresses.

Card Summary:

Card 1. This card is required.

MID	RO	EM	PRM	LCSIGY	BETA	PFL	VISC

Card 2. This card is required.

NFIB	AOPT				A1	A2	A3

Card 3. This card is required.

V1	V2	V3	D1	D2	D3	MANGL	THICK

Card 4. This card is included if VISC > 0.

VG1	VB1	VG2	VB2	VG3	VB3	VG4	VB4

Card 5. This card is required.

IDF1	ALPH1	EF1	LCEF1	G23_1	G31_1	DAF1	DAM1

Card 6. This card is required.

G12	LCG12	ALOC12	GLOC12	METH12	DAM12		

Card 7. This card is required.

IDF2	ALPH2	EF2	LCEF2	G23_2	G31_2	DAF2	DAM2

Card 8. This card is required.

G23	LCG23	ALOC23	GLOC23	METH23	DAM23		

Card 9. This card is required.

IDF3	ALPH3	EF3	LCEF3	G23_3	G31_3	DAF3	DAM3

Card 10. This card is optional.

POSTV	VISCS	IHIS					

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EM	PRM	LCSIGY	BETA	PFL	VISC
Type	A	F	F	F	I	F	F	I

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density, ρ
N	Number of phases
EM	Young's modulus of matrix material
PRM	Poisson's ratio for matrix material
LCSIGY	Load curve or table ID for strain hardening of the matrix. If a curve, then it specifies yield stress as a function of effective plastic strain. If a table, then temperatures are the table values indexing curves giving yield stress as a function of effective plastic strain (see *DEFINE_TABLE).
BETA	Parameter for mixed hardening, $0.0 \le \beta \le 1.0$. Set $\beta = 0.0$ for pure kinematic hardening and $\beta = 1.0$ for pure isotropic hardening.
PFL	Percentage of layers that must fail to initiate failure of the element (default is 100)
VISC	Viscous formulation for fibers: EQ.0: Elastic behavior GE.1: Viscoelastic behavior modeled with a Prony series. See Remark 3.

Card 2	1	2	3	4	5	6	7	8
Variable	NFIB	AOPT				A1	A2	A3
Type	I	F				F	F	F

VARIABLE	DESCRIPTION
NFIB	Number of fiber families to be considered (up to 3)
AOPT	Material axes option (see *MAT_*OPTION*TROPIC_ELASTIC for a more complete description): EQ.0.0: Locally orthotropic with material axes determined by element nodes, as with *DEFINE_COORDI-NATE_-NODES, and then rotated about the shell element normal by the angle MANGL. EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR. EQ.3.0: Locally orthotropic material axes determined by rotatingthe material axes about the element normal by an angle, MANGL, from a line in the plane of the element defined by the cross product of the vector v with the element normal LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
A1, A2, A3	Components of vector **a** for AOPT = 2

Card 3	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	MANGL	THICK
Type	F	F	F	F	F	F	F	I

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3

VARIABLE	DESCRIPTION
D1, D2, D3	Components of vector **d** for AOPT = 2
MANGL	Material angle in degrees for AOPT = 0 and 3. It may be overwritten on the element card; see *ELEMENT_SHELL_BETA.
THICK	Balance thickness changes of the material due to the matrix response when calculating the fiber stresses. Stresses can be scaled to account for the fact that fiber cross-sectional usually does not change. EQ.0: No scaling EQ.1: Scaling

Fiber Viscosity Card. Additional card for VISC > 0 only. See Remark 3.

Card 4	1	2	3	4	5	6	7	8
Variable	VG1	VB1	VG2	VB2	VG3	VB3	VG4	VB4
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
VGk	Relaxation modulus G_k for the k^{th} term of the Prony series for viscoelastic fibers
VBk	Decay constant β_k for the k^{th} term of the Prony series for viscoelastic fibers

Card 5	1	2	3	4	5	6	7	8
Variable	IDF1	ALPH1	EF1	LCEF1	G23_1	G31_1	DAF1	DAM1
Type	I	F	F	I	F	F	I	I

VARIABLE	DESCRIPTION
IDF1	ID for 1st fiber family for post-processing
ALPH1	Orientation angle α_1 for 1st fiber with respect to overall material direction

VARIABLE	DESCRIPTION
EF1	Young's modulus for 1st fiber family
LCEF1	Load curve for stress as a function of fiber strain of 1st fiber. With this option active, EF1 is ignored.
G23_1	Transverse shear modulus orthogonal to direction of 1st fiber
G31_1	Transverse shear modulus in direction of 1st fiber
DAF1	Load curve or table ID for damage parameter d_1^f for 1st fiber (see Remark 2). If a curve, DAF1 specifies damage as a function of fiber strain (for compression and elongation). If DAF1 refers to a table, then two different damage functions for tensile and compressive stresses are input. The values in the table are arbitrary and exist only to index the two curves. The first indexed curve is assumed to specify tensile damage as a function of fiber strains while second curve specifies compressive damage as a function of fiber strains. input different damage functions for tensile and compressive stresses. Any other curves input with the table definition are ignored. The damager parameter d_1^f ranges from 0.0 for an undamaged fiber to 1.0 for a failed fiber family. If all families have failed, material failure at the integration point is initiated.
DAM1	Load curve or table ID for damage parameter d_1^m for matrix material based on the current deformation status of the 1st fiber (see Remark 2). If a curve, it specifies damage as a function of fiber strain (for compression and elongation). If a table, then the values are fiber strain rates which index damage as a function of fiber strain curves. The damager parameter d_1^m ranges from 0.0 to 1.5. A value of 0.0 indicates an undamaged matrix, whereas 1.0 refers to a completely damaged matrix. To initiate failure of the composite at the integration point, a matrix damage d_1^m of 1.5 must be reached. Naturally, the mechanical behavior of the matrix does not change for damage values between 1.0 and 1.5.

Card 6	1	2	3	4	5	6	7	8
Variable	G12	LCG12	ALOC12	GLOC12	METH12	DAM12		
Type	F	I	F	F	I	I		

VARIABLE	DESCRIPTION
G12	Linear shear modulus for shearing between fiber families 1 and 2
LCG12	Curve ID for shear stress as a function of shearing type as specified with METH12 between the 1st and 2nd fibers. See Remark 1.
ALOC12	Locking angle (in radians) for shear between fiber families 1 and 2
GLOC12	Linear shear modulus for shear angles larger than ALOC12
METH12	Option for shear response between fiber 1 and 2 (see Remark 1): EQ.0: Elastic shear response. Curve LCG12 specifies shear stress as a function of the scalar product of the fiber directions. EQ.1: Elasto-plastic shear response. Curve LCG12 specifies yield shear stress as a function of the normalized scalar product of the fiber directions. EQ.2: Elastic shear response. Curve LCG12 specifies shear stress as a function of shear angle (radians) between the fibers. EQ.3: Elasto-plastic shear response. Curve LCG12 defines yield shear stress as a function of normalized shear angle between the fibers. EQ.4: Elastic shear response. Curve LCG12 specifies shear stress as a function of shear angle (radians) between the fibers. This option is a special implementation for non-crimped fabrics, where one of the fiber families corresponds to a stitching. EQ.5: Elasto-plastic shear response. Curve LCG12 specifies yield shear stress as a function of normalized shear angle between the fibers. This option is a special implementation for non-crimped fabrics, where one of the fiber families corresponds to a stitching. EQ.10: Elastic shear response. Curve LCG12 specifies shear

VARIABLE	DESCRIPTION

stress as a function of shear angle (radians) between the fibers. This option is tailored for woven fabrics and guarantees a pure shear stress response.

EQ.11: Elasto-plastic shear response. Curve LCG12 specifies yield shear stress as a function of normalized shear angle. This option is tailored for woven fabrics and guarantees a pure shear stress response

DAM12 Load curve ID defining the damage parameter d_{12}^m for the matrix as function of shear angle (radians) between the 1st and 2nd fiber (see Remark 2). The damage parameter d_{12}^m ranges from 0.0 to 1.5. A value of 0.0 indicates an undamaged matrix, whereas 1.0 refers to a completely damaged matrix. To initiate failure of the composite at the integration point, a matrix damage d_{12}^m of 1.5 must be reached. Naturally, the mechanical behavior of the matrix does not change for damage values between 1.0 and 1.5.

Card 7	1	2	3	4	5	6	7	8
Variable	IDF2	ALPH2	EF2	LCEF2	G23_2	G31_2	DAF2	DAM2
Type	I	F	F	I	F	F	I	I

VARIABLE	DESCRIPTION
IDF2	ID for 2nd fiber family for post-processing
ALPH2	Orientation angle α_2 for 2nd fiber with respect to overall material direction
EF2	Young's modulus for 2nd fiber family
LCEF2	Load curve for stress as a function of fiber strain of 2nd fiber. With this option active, EF2 is ignored.
G23_2	Transverse shear modulus orthogonal to direction of 2nd fiber
G31_2	Transverse shear modulus in direction of 2nd fiber
DAF2	Load curve or table ID for damage parameter d_2^f for 2nd fiber (see Remark 2). If a curve, DAF2 specifies damage as a function of fiber strain (for compression and elongation). If DAF2 refers to a table,

VARIABLE	DESCRIPTION
	then two different damage functions for tensile and compressive stresses are input. The values in the table are arbitrary and exist only to index the two curves. The first indexed curve is assumed to specify tensile damage as a function of fiber strains while second curve specifies compressive damage as a function of fiber strains. input different damage functions for tensile and compressive stresses. Any other curves input with the table definition are ignored. The damager parameter d_2^f ranges from 0.0 for an undamaged fiber to 1.0 for a failed fiber family. If all families have failed, material failure at the integration point is initiated.
DAM2	Load curve or table ID for damage parameter d_2^m for matrix material based on the current deformation status of the 2nd fiber (see Remark 2). If a curve, it specifies damage as a function of fiber strain (for compression and elongation). If a table, then the values are fiber strain rates which index damage as a function of fiber strain curves. The damager parameter d_2^m ranges from 0.0 to 1.5. A value of 0.0 indicates an undamaged matrix, whereas 1.0 refers to a completely damaged matrix. To initiate failure of the composite at the integration point, a matrix damage d_2^m of 1.5 must be reached. Naturally, the mechanical behavior of the matrix does not change for damage values between 1.0 and 1.5.

Card 8	1	2	3	4	5	6	7	8
Variable	G23	LCG23	ALOC23	GLOC23	METH23	DAM23		
Type	F	I	F	F	I	F		

VARIABLE	DESCRIPTION
G23	Linear shear modulus for shearing between fiber families 2 and 3
LCG23	Curve ID for shear stress as a function of shearing type as specifies with METH23 between the 2nd and 3rd fibers. See Remark 1.
ALOC23	Locking angle (in radians) for shear between fiber families 2 and 3
GLOC23	Linear shear modulus for shear angles larger than ALOC23

VARIABLE	DESCRIPTION
METH23	Option for shear response between fibers 2 and 3 (see METH12 for input options and Remark 1).
DAM23	Load curve ID defining the damage parameter d_{23}^m for the matrix as function of shear angle (in radians) between the 1st and 2nd fiber (see Remark 2). The damager parameter d_{23}^m ranges from 0.0 to 1.5. A value of 0.0 indicates an undamaged matrix, whereas 1.0 refers to a completely damaged matrix. To initiate failure of the composite at the integration point, a matrix damage d_{23}^m of 1.5 must be reached. Naturally, the mechanical behavior of the matrix does not change for damage values between 1.0 and 1.5.

Card 9	1	2	3	4	5	6	7	8
Variable	IDF3	ALPH3	EF3	LCEF3	G23_3	G31_3	DAF3	DAM3
Type	I	F	F	I	F	F	I	I

VARIABLE	DESCRIPTION
IDF3	ID for 3rd fiber family for post-processing
ALPH3	Orientation angle α_3 for 3rd fiber with respect to overall material direction
EF3	Young's modulus for 3rd fiber family
LCEF3	Load curve for stress versus fiber strain of 3rd fiber. With this option active, EF3 is ignored.
G23_3	Transverse shear modulus orthogonal to direction of 3rd fiber
G31_3	Transverse shear modulus in direction of 3rd fiber
DAF3	Load curve or table ID for damage parameter d_3^f for 3rd fiber (see Remark 2). If a curve, DAF3 specifies damage as a function of fiber strain (for compression and elongation). If DAF3 refers to a table, then two different damage functions for tensile and compressive stresses are input. The values in the table are arbitrary and exist only to index the two curves. The first indexed curve is assumed to specify tensile damage as a function of fiber strains while second curve specifies compressive damage as a function of fiber strains.

VARIABLE	DESCRIPTION
	input different damage functions for tensile and compressive stresses. Any other curves input with the table definition are ignored. The damager parameter d_3^f ranges from 0.0 for an undamaged fiber to 1.0 for a failed fiber family. If all families have failed, material failure at the integration point is initiated.
DAM3	Load curve or table ID for damage parameter d_3^m for matrix material based on the current deformation status of the 3rd fiber (see Remark 2). If a curve, it specifies damage as a function of fiber strain (for compression and elongation). If a table, then the values are fiber strain rates which index damage as a function of fiber strain curves. The damager parameter d_3^m ranges from 0.0 to 1.5. A value of 0.0 indicates an undamaged matrix, whereas 1.0 refers to a completely damaged matrix. To initiate failure of the composite at the integration point, a matrix damage d_3^m of 1.5 must be reached. Naturally, the mechanical behavior of the matrix does not change for damage values between 1.0 and 1.5.

The following card is optional.

Card 10	1	2	3	4	5	6	7	8
Variable	POSTV	VISCS	IHIS					
Type	I	F	I					

VARIABLE	DESCRIPTION
POSTV	Parameter for outputting additional history variables that might be useful for post-processing. See Remark 4.
VISCS	Portion of viscous relaxation moduli VGk that is accounted for in the time step size calculation
IHIS	Flag for material properties initialization: EQ.0: Material properties defined in Cards 1 - 9 are used GE.1: Use *INITIAL_STRESS_SHELL to initialize some material properties on an element-by-element basis (see Remark 5 below).

Remarks:

1. **Stress calculation.** This material features an additive split of the matrix and reinforcement contributions to the stress. Therefore, the combined stress response, σ, equals the sum $\sigma^m + \sigma^f$.

   The matrix uses an elastic-plastic material formulation with a von-Mises yield criterion. This material supports a mixed hardening approach.

   We formulated the contribution of the fiber reinforcement as a hyperelastic material. Based on the orientation angle, α_i, of the i^{th} fiber family, LS-DYNA computes an initial fiber direction in the element coordinate system $\mathbf{m}_i^0$. By using the deformation gradient, $\mathbf{F}$, the current fiber configuration is given as $\mathbf{m}_\mathrm{i} = \mathbf{F}\,\mathbf{m}_i^0$, containing all necessary information on fiber strain and reorientation. Here, this vector is always orthogonal to the shell normal and can, thus, be represented by the two in-plane vector components.

   Following standard textbook mechanics for anisotropic and hyperelastic materials, the elastic stresses within the fibers due to tension or compression are given as

$$\sigma_T^f = \sum_{i=1}^{n} \sigma_{T,i}^f(\lambda_i) = \sum_{i=1}^{n} \frac{1}{J} f_i(\lambda_i)(\mathbf{m}_\mathrm{i} \otimes \mathbf{m}_\mathrm{i}) \ ,$$

   where the function f_i of the fiber strain λ_i corresponds to the load curve LCEF*i*. n is the number of fiber families.

   The shear behavior of the reinforcement can be controlled by METH*ij*. For values less than 10, the behavior is again standard textbook mechanics:

$$\sigma_S^f = \sum_{i=1}^{n-1} \sigma_{S,i,i+1}^f = \sum_{i=1}^{n-1} \frac{1}{J} g_{i,i+1}(\kappa_{i,i+1})(\mathbf{m}_\mathrm{i} \otimes \mathbf{m}_{\mathrm{i}+1}) \ .$$

   Here $\kappa_{i,i+1}$ represents the employed shear measure (scalar product or shear angle in radians). In general, the dyadic product $\mathbf{m}_\mathrm{i} \otimes \mathbf{m}_{\mathrm{i}+1}$ does not define a shear stress tensor. This formulation might result in unphysical shear behavior in the case of woven fabrics. Therefore, we devised METH*ij* = 10 and 11 to always give a pure shear stress tensor, σ_S^f.

   For even values of METH*ij*, an elastic shear response is assumed. If defined, the load curve LCG*ij* corresponds to function $g_{i,j}$. In this case the values of G*ij*, ALOC*ij* and GLOC*ij* are ignored.

   For odd values of METH*ij* on the other hand, an elasto-plastic shear behavior is assumed and the load curve LCG*ij* defines the yield stress value as function of a normalized shear parameter. This implies that the load curve needs to be defined for abscissa values between 0.0 and 1.0. A first elastic regime, which is controlled by the linear shear stiffness G*ij*, is assumed until the yield stress given in the load curve for normalized shear value 0.0 is reached. A second linear elastic regime is defined for shear angles (ξ_{ij})/ fiber angles (η_{ij}) larger than the locking angle ALOC*ij*. The corresponding stiffness in that regime is GLOC*ij*. At the transition point to the second elastic regime, the shear stress corresponds to the load curve value for a normalized shear of 1.0.

2. **Damage and failure.** This material features a phenomenological description of damage and failure. User-defined load curves specify several damage variables as functions of the fiber strain values λ_i or shear $\kappa_{i,i+1}$. Here, damage parameters are always accumulated and cannot decrease during the simulation.

   If input parameter DAF*i* refers to a load curve, it specifies the damage parameter d_i^f as function of the fiber strain λ_i. If DAF*i* refers to a table, the material distinguishes between tensile and compressive damage. In that case, two parameters $d_i^{f,t}$ and $d_i^{f,c}$ are introduced as functions of the fiber strain λ_i (given by two load curves referred to by the table definition) and are both evaluated in every time step. The effective damager parameter d_i^f is then defined as

$$d_i^f(\lambda_i) = \begin{cases} d_i^{f,c}(\lambda_i), & \lambda_i < 0 \\ d_i^{f,t}(\lambda_i), & \lambda_i \geq 0 \end{cases}$$

   The damage parameter d_i^f degrades the fiber stress contribution $\sigma_{T,i}^f$:

$$\widehat{\sigma}_T^f = \sum_{i=1}^{n} \left(\mathbf{1} - d_i^f(\lambda_i)\right) \sigma_{T,i}^f(\lambda_i) \quad .$$

   Failure of the composite material at the integration point is initiated as soon as all fibers have failed: $\min_i d_i^f = 1.0$.

   We assume matrix damage to result from the fiber straining and reorientating. Consequently, the input includes load curves DAM*i* and DAM*ij* to specify damage parameters $d_i^m(\lambda_i)$ and $d_{i,i+1}^m(\kappa_{i,i+1})$, respectively. The overall matrix damage parameter d^m is given by

$$d^m = \max\left(\max_{i \leq n} d_i^m(\lambda_i), \max_{i<n} d_{i,i+1}^m(\kappa_{i,i+1})\right) \quad .$$

   Matrix failure ($d^m = 1.0$) does not necessarily initiate failure of the composite material. In this implementation, matrix damage parameters d_i^m and $d_{i,i+1}^m$ that exceed a value of 1.0 are admissible. Failure of the composite is initiated as soon as the damage parameter reaches 1.5. To account for this delayed failure, the degradation of the matrix stresses is given by:

$$\widehat{\sigma}^m = \left(1 - \min(1.0, d^m)\right)\sigma^m \quad .$$

3. **Fiber viscosity.** Input parameter VISC activates fiber viscosity. This feature adds numerical damping to the post-damage behavior of the material. Damping

might be necessary since brittle fiber failure tends to induce shockwaves through the material, resulting in oscillations or even unphysical damage propagation.

If activated, an additional viscous stress term is added to the fiber contribution:

$$\sigma_{T,v}^{f}=\sum_{i=1}^{n}\frac{1}{J}\left(\int_{0}^{t}f_v(t-\tau)\frac{\partial\lambda_i(\tau)}{\partial\tau}d\tau\right)(\mathbf{m}_\mathrm{i}\otimes\mathbf{m}_\mathrm{i}) \quad .$$

The relaxation function, f_v, is represented by up to four terms of the Prony series expansions and thus reads

$$f_v(t)=\sum_k G_k e^{-\beta_k t}$$

with relaxation moduli G_k and decay constants β_k.

4. **History data.** This material formulation outputs to d3plot additional data for post-processing to the set of history variables if requested. The parameter POSTV specifies the data to be written. Its value is calculated as

$$\text{POSTV}=a_1+2\,a_2+4\,a_3+8\,a_4+16\,a_5+32\,a_6+64\,a_7.$$

Each flag a_i is a binary number (can be either 1 or 0) and corresponds to one particular type of post-processing variable according to the following table.

Flag	Description	Variables	# of History Variables
a_1	Fiber angle	η_{12}, η_{23}	2
a_2	Fiber ID	IDF1, IDF2, IDF3	3
a_3	Fiber strain	$\lambda_1, \lambda_2, \lambda_3$	3
a_4	Fiber direction (in global coordinates)	$\mathbf{m}_1^\mathrm{g}, \mathbf{m}_2^\mathrm{g}, \mathbf{m}_3^\mathrm{g}$	9
a_5	Individual fiber stresses	$f_1(\lambda_1), f_2(\lambda_2), f_3(\lambda_3)$	3
a_6	Fiber stress tensor	$\sigma_{11}^f, \sigma_{22}^f, \sigma_{33}^f, \sigma_{12}^f, \sigma_{23}^f, \sigma_{31}^f$	6
a_7	Fiber direction (in material coordinates)	$\mathbf{m}_1^\mathrm{m}, \mathbf{m}_2^\mathrm{m}, \mathbf{m}_3^\mathrm{m}$	6

The above table also shows the order of output as well as the number of extra history variables associated with the particular flag. In total NXH extra variables are required depending on the choice of parameter POSTV. For example, the maximum number of additional variables is NXH = 32 for POSTV = 127.

As mentioned in Remark 1 fiber orientation is represented in the material subroutine as vector $\mathbf{m}_\mathrm{i}$ defined in the element coordinate system. Prior to writing to the list of histories the vector is transformed into the global coordinate system with three vector components for $a_4 = 1$ and/or into the overall material coordinate system with two vector components for $a_7 = 1$.

A more complete list of potentially helpful history variables is given in the following table. The variable NEIPS in *DATABASE_EXTENT_BINARY must be set to output these history variables.

History Variable #	Description
3	Number of fibers
4	NXH
5 → NXH + 4	Variables as described in preceding table
NXH + 5	POSTV
NXH + 6, NXH + 7	Shear angles $\xi_{12}$and ξ_{23}
NXH + 8	Matrix damage parameter d^m
NXH + 9 → NXH + 11	Fiber tensile damage parameter $d_i^{f,t}$
NXH + 12 → NXH + 14	Fiber compressive damage param. $d_i^{f,c}$
NXH + 15 → NXH + 20	Matrix stress tensor in element coordinate system
NXH+21 → NXH + 26	Deformation gradient

5. **Description of IHIS.** Some material data can be initialized on an element-by-element basis through history variables defined with *INITIAL_STRESS_SHELL starting at position HISV5.

How the data is interpreted depends on the parameter IHIS. Following the same concept as for parameter POSTV, the value of IHIS is computed by the following expression:

$$\text{IHIS} = a_1 + 2\,a_2$$

Each flag a_i is a binary number (can be either 1 or 0) and corresponds to one particular type of material variable. So far, the only material variables implemented are fiber orientation in two different coordinate systems, global and material. Thus, at most one of the flags a_1 and a_2 should be set to 1.

Flag	Description	Variables	# of History Variables
a_1	Fiber direction (in global coordinates)	$\mathbf{m}_1^\mathrm{g}, \mathbf{m}_2^\mathrm{g}, \mathbf{m}_3^\mathrm{g}$	9
a_2	Fiber direction (in material coordinates)	$\mathbf{m}_1^\mathrm{m}, \mathbf{m}_2^\mathrm{m}, \mathbf{m}_3^\mathrm{m}$	6

*MAT_REINFORCED_THERMOPLASTIC_UDFIBER

This is Material Type 249. It describes a material with unidirectional fiber reinforcements and considers up to three distinct fiber directions. Each fiber family is described by a spatially transversely isotropic neo-Hookean constitutive law. The implementation is based on an adapted version of the material described by Bonet and Burton (1998). The material is only available for thin shell elements and in explicit simulations.

Card Summary:

Card 1. This card is required.

MID	RO	EM	PRM	G	EZDEF		

Card 2. This card is required.

NFIB	AOPT	XP	YP	ZP	A1	A2	A3

Card 3. This card is required.

V1	V2	V3	D1	D2	D3	MANGL	

Card 4. This card is required.

IDF1	ALPH1	EF1	KAP1				

Card 5. This card is required.

IDF2	ALPH2	EF2	KAP2				

Card 6. This card is required.

IDF3	ALPH3	EF3	KAP3				

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EM	PRM	G	EZDEF		
Type	A	F	F	F	F	F		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label be specified (see *PART).
RO	Density
EM	Isotropic Young's modulus, E_{iso}
PR	Poisson's ratio, v
G	Linear shear modulus, G_{fib}
EZDEF	Algorithmic parameter. If set to 1, last row of deformation gradient is not updated during the calculation.

Card 2	1	2	3	4	5	6	7	8
Variable	NFIB	AOPT	XP	YP	ZP	A1	A2	A3
Type	I	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
NFIB	Number of fiber families to be considered (maximum of 3)
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description): EQ.0.0: Locally orthotropic with material axes determined by element nodes, as with *DEFINE_COORDINATE_-NODES, and then rotated about the shell element normal by the angle MANGL. EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR. EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then, **a** and **b** are rotated about **c** by an angle MANGL. MANGL may be set in the keyword input for the

VARIABLE	DESCRIPTION
	element or in the input for this keyword.
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
XP, YP, ZP	Coordinates of point *p* for AOPT = 1
A1, A2, A3	Components of vector **a** for AOPT = 2

Card 3	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	MANGL	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3
D1, D2, D3	Components of vector **d** for AOPT = 2
MANGL	Material angle in degrees for AOPT = 0 and. It may be overwritten on the element card; see *ELEMENT_SHELL_BETA.

Card 4	1	2	3	4	5	6	7	8
Variable	IDF1	ALPH1	EF1	KAP1				
Type	I	F	F	F				

Card 5	1	2	3	4	5	6	7	8
Variable	IDF2	ALPH2	EF2	KAP2				
Type	I	F	F	F				

Card 6	1	2	3	4	5	6	7	8
Variable	IDF3	ALPH3	EF3	KAP3				
Type	I	F	F	F				

VARIABLE	DESCRIPTION
IDF*i*	ID for i^{th} fiber family for post-processing
ALPH*i*	Orientation angle α_i for i^{th} fiber with respect to overall material direction
EF*i*	Young's modulus E_i for i^{th} fiber family
KAP*i*	Fiber volume ratio κ_i of i^{th} fiber family

Stress Calculation:

In this model up to three distinct fiber families are considered. We assume that there is no interaction between the families. Thus, the resulting stress tensor is the sum of the single fiber responses. Each fiber response is the sum of an isotropic and a spatially transversely isotropic neo-Hookean stress contribution, $\boldsymbol{\sigma}_i^{\text{iso}}$ and $\boldsymbol{\sigma}_i^{\text{tr}}$, respectively. The implementation is based on the work of Bonet and Burton (1998), adapted by BMW for simulation of unidirectional fabrics (see references below).

The isotropic stress tensor, $\boldsymbol{\sigma}_i^{\text{iso}}$, depends on an isotropic shear modulus, μ, and an isotropic bulk modulus, λ_i where:

$$\mu = \frac{E_{\text{iso}}}{2(1+\nu)} \text{ and } \lambda_i = \frac{E_{\text{iso}}(\nu + n_i\nu^2)}{2(1+\nu)}.$$

Here, the variable n_i denotes the ratio between stiffness orthogonally to the fibers and in fiber direction, that is, $n_i = E_{\text{iso}}/E_i$. E_{iso}, ν, and E_i are input parameters. Using the left Cauchy-Green tensor, **b,** the isotropic neo-Hookean model reads:

$$\boldsymbol{\sigma}_i^{\text{iso}} = \frac{\mu}{J}(\mathbf{b} - \mathbf{I}) + \lambda_i(J-1)\mathbf{I}\ .$$

Based on the orientation angel α_i of the i^{th} fiber family, an initial fiber direction $\mathbf{m}_i^0$ is computed. The deformation gradient, **F**, transforms the initial fiber configuration to the current fiber configuration as $\mathbf{m}_i = \mathbf{F}\mathbf{m}_i^0$. This vector contains all necessary information on fiber elongation and reorientation.

The spatially transversely isotropic neo-Hookean formulation is given by:

$$J\sigma_i^{\mathrm{tr}} = 2\beta_i(I_4 - 1)\mathbf{I} + 2\big(\alpha + 2\beta_i \ln J + 2\gamma_i(I_4 - 1)\big)\mathbf{m}_i \otimes \mathbf{m}_i - \alpha(\mathbf{b}\mathbf{m}_i \otimes \mathbf{m}_i + \mathbf{m}_i \otimes \mathbf{b}\mathbf{m}_i)$$

with material parameters

$$\alpha = \mu - G_{\mathrm{fib}}, \qquad \beta_i = \frac{E_{\mathrm{iso}}\nu^2(1-n_i)}{4m_i(1+\nu)}, \qquad m_i = 1 - \nu - 2n_i\nu^2,$$

$$\gamma_i = \frac{E_i\kappa_i(1-\nu)}{8m} - \frac{\lambda_i + 2\mu}{8} + \frac{\alpha}{2} - \beta_i.$$

The parameter EZDEF activates a modification of the model. Instead of the standard deformation gradient, $\mathbf{F}$, a modified tensor $\tilde{\mathbf{F}}$ is employed to calculate current fiber directions $\mathbf{m}_i$ and left Cauchy-Green tensor $\mathbf{b}$. For tensor $\tilde{\mathbf{F}}$ only the first two rows of the deformation gradient are updated based on the deformation of the element. This simplification can in some cases increase the stability of the model, especially if the structure undergoes large deformations.

References:

[1] Bonet, J., and A. J. Burton. "A simple orthotropic, transversely isotropic hyperelastic constitutive equation for large strain computations." *Computer methods in applied mechanics and engineering* 162.1 (1998): 151-164.

[2] Senner, T., et al. "A modular modeling approach for describing the in-plane forming behavior of unidirectional non-crimp-fabrics." *Production Engineering* 8.5 (2014): 635-643.

[3] Senner, T., et al. "Bending of unidirectional non-crimp-fabrics: experimental characterization, constitutive modeling and application in finite element simulation." *Production Engineering* 9.1 (2015): 1-10.

History Data:

History Variable #	Description
3	ID of 1st fiber
4	ID of 2nd fiber
5	ID of 3rd fiber
6 → 8	Current direction of 1st fiber
9 → 11	Current direction of 2nd fiber
12 → 14	Current direction of 3rd fiber

History Variable #	Description
15	Number of fibers
16	Projected orthogonal fiber strain (1st fiber)
17	Projected parallel fiber strain (1st fiber)
18	Shear angle (1st fiber) in rad
19	Euler-Almansi strain (1st fiber)
20	Porosity (1st fiber)
21	Fiber volume ratio (1st fiber)
22	Projected orthogonal fiber strain (2nd fiber)
23	Projected parallel fiber strain (2nd fiber)
24	Shear angle (2nd fiber) in rad
25	Euler-Almansi strain (2nd fiber)
26	Porosity (2nd fiber)
27	Fiber volume ratio (2nd fiber)
28	Projected orthogonal fiber strain (3rd fiber)
29	Projected parallel fiber strain (3rd fiber)
30	Shear angle (3rd fiber) in rad
31	Euler-Almansi strain (3rd fiber)
32	Porosity (3rd fiber)
33	Fiber volume ratio (3rd fiber)

*MAT_TAILORED_PROPERTIES

This is Material Type 251. It is similar to *MAT_PIECEWISE_LINEAR_PLASTICITY or (*MAT_024). Unlike *MAT_024, it has a 3D table option that uses a history variable (that could be hardness, pre-strain, or some other quantity) from a previous calculation to evaluate the plastic behavior as a function of 1) history variable, 2) strain rate, and 3) plastic strain. Starting with release R12, it is also possible to use a 4D table option with two history variables, that is, the plastic behavior would be a function of 1) history variable HISVN + 1, 2) history variable HISVN, 3) strain rate, and 4) plastic strain. Starting with release R15, the Young's modulus can be scaled with a factor given on history variable #8. Beginning with R16, external variables (see *LOAD_EXTERNAL_VARIABLE and Remark 5) can be used instead of history variables for evaluating the plastic and scaling the Young's modulus. This material is available for shell and solid elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR			FAIL	TDEL
Type	A	F	F	F			F	F
Default	none	none	none	none			10^{20}	0

Card 2	1	2	3	4	5	6	7	8
Variable			LCSS		VP	HISVN	PHASE	
Type			F		F	I	F	
Default			0		0	0	0	

Card 3	1	2	3	4	5	6	7	8
Variable	EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

Card 4	1	2	3	4	5	6	7	8
Variable	ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus. Spatial variation is possible using history variable #8 or an external variable. See Remarks 1 and 5.
PR	Poisson's ratio
FAIL	Failure flag: LT.0.0: Call user-defined failure subroutine, `matusr_24` in dyn21.F, to determine failure EQ.0.0: Do not consider failure. This option is recommended if failure is not of interest since many calculations will be saved. GT.0.0: Effective plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.
TDEL	Minimum time step size for automatic element deletion
LCSS	Load curve ID or table ID **Load Curve.** When LCSS is a load curve ID, it is taken as defining stress as a function of effective plastic strain. If defined, EPS1 - EPS8 and ES1 - ES8 are ignored. **Tabular Data.** The table ID defines for each strain rate value a load curve ID giving the stress as a function effective plastic strain for that rate; see Figure M24-1. When the strain rate falls below the minimum value, the stress as a function of effective plastic strain curve for the lowest value of strain rate is used. Likewise, when the strain rate exceeds the maximum value the stress as a function

VARIABLE	DESCRIPTION
	of effective plastic strain curve for the highest value of strain rate is used. EPS1 - EPS8 and ES1 - ES8 are ignored if a table ID is defined. Linear interpolation between the discrete strain rates is used by default. **Logarithmically Defined Tables.** Logarithmic interpolation between discrete strain rates is assumed if the *first* value in the table is negative, in which case LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate when the lowest strain rate and highest strain rate differ by several orders of magnitude. There is some additional computational cost associated with invoking logarithmic interpolation. **Multi-Dimensional Tables.** Stress values can also depend on history variables (or external variables). The 3D table gives stress versus plastic strain as a function of strain rates as a function of one history variable (see HISVN) or one external variable. The 4D table gives stress versus plastic strain as a function of strain rates as a function of two history variable values (see HISVN) or two external variables. See Remarks 2 and 5.
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default) EQ.1.0: Viscoplastic formulation
HISVN	Location of the history variable in the history array of *INITIAL_-STRESS_SHELL/SOLID that is used to evaluate the 3D table LCSS. If a 4D table is used, then HISVN is the location of the history variable for the *TABLE_3D value, and HISVN + 1 is the location of the history variable for the *TABLE_4D values. See Remark 4.
PHASE	Constant value to evaluate the 3D table LCSS. PHASE is only used if HISVN = 0.
EPS1 - EPS8	Effective plastic strain values (optional). At least 2 points should be defined. The first point must be zero corresponding to the initial yield stress.
ES1 - ES8	Corresponding yield stress values to EPS1 - EPS8

Remarks:

1. **Scaling the Young's modulus.** The Young's modulus can be scaled by a factor given on history variable HISV8 of *INITIAL_STRESS_SHELL/SOLID. A value of 1.0 means no scaling (default). Alternatively, it can be scaled by providing an external variable and *LOAD_EXTERNAL_VARIABLE. See Remark 5.

2. **LCSS as multi-dimensional table.** If using a 3D or 4D for LCSS, interpolation is used to find the corresponding stress value for the current plastic strain, strain rate, and history variable(s). In addition, extrapolation is used for the history variable evaluation, which means that some upper and lower "limit curves" have to be used, if extrapolation is not desired.

3. **Location of material history variables in dynain.** If using *INTERFACE_-SPRINGBACK_LSDYNA to write material history to the dynain file, the history variables of *MAT_251 (for example, hardness and temperature) are written to positions HISV6 and HISV7 of *INITIAL_STRESS_SHELL/SOLID.

4. **HISVN.** We recommend setting HISVN = 6 and putting the history variables on position HISV6 (and HISV7 if TABLE_4D is used) if using *MAT_251 in combination with *MAT_ADD_...

5. **Effect of external variables.** Instead of using history variables, it is also possible to define the spatial variation of material properties with external variables. Depending on the input in *LOAD_EXTERNAL_VARIABLE (see IMP), the Young's modulus can be scaled by the current value of an external variable (material property index IMP = 1) and/or the external variable can be used when evaluating the multi-dimensional table LCSS (indices IMP = 2 and IMP = 3 replace the first and second history variables, respectively). Note that history variables and external variables *cannot* be used at the same time in a single *MAT_-251 material. For instance, if an external variable is used for a 4D LCSS, both history variables must be replaced by external variables, and either an external variable scales the Young's modulus, or the Young's is not scaled. The following table summarizes the meaning of the indices that can be set in IMP of *LOAD_-EXTERNAL_VARIABLE:

Property index	Property name	Table
1	Young's modulus, E	-
2	Yield stress	LCSS (3D and 4D)
3	Yield stress	LCSS (4D)

*MAT_TOUGHENED_ADHESIVE_POLYMER

This is Material Type 252, the Toughened Adhesive Polymer model (TAPO). It is based on non-associated I_1 - J_2 plasticity constitutive equations and was specifically developed to represent the mechanical behaviour of crash optimized high-strength adhesives under combined shear and tensile loading. This model includes material softening due to damage, rate-dependency, and a constitutive description for the mechanical behaviour of bonded connections under compression.

A detailed description of this material can be found in Matzenmiller and Burbulla [2013]. This material model can be used with solid elements or with cohesive elements in combination with *MAT_ADD_COHESIVE.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	FLG	JCFL	DOPT	
Type	A	F	F	F	I	I	I	

Card 2	1	2	3	4	5	6	7	8
Variable	LCSS	TAU0	Q	B	H	C	GAM0	GAMM
Type	I	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	A10	A20	A1H	A2H	A2S	POW		SRFILT
Type	F	F	F	F	F	F		F

Card 4	1	2	3	4	5	6	7	8
Variable	IHIS		D1	D2	D3	D4	D1C	D2C
Type	F		F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density, ρ
E	Young's modulus, E
PR	Poisson's ratio, ν
FLG	Flag to choose between yield functions f and $\hat{f}$ (see Material Model): EQ.0: Cap in tension and nonlinear Drucker & Prager in compression EQ.2: Cap in tension and von Mises in compression
JCFL	Johnson & Cook constitutive failure criterion flag (see Material Model): EQ.0: use triaxiality factor only in tension, EQ.1: use triaxiality factor in tension and compression.
DOPT	Damage criterion flag $\widehat{D}$ or $\breve{D}$ (see Material Model): EQ.0: damage model uses damage plastic strain r, EQ.1: damage model uses plastic arc length γ_v.
LCSS	Curve ID or Table ID **Load Curve.** The curve specifies yield stress τ_Y as a function of plastic strain r. **Table Data.** If a 2D table is defined, for each strain rate value the table specifies a curve ID giving the yield stress as a function of plastic strain for that strain rate (see *DEFINE_TABLE). If a 3D table is defined, for each temperature value, a table ID is specified which, in turn, maps strain rates to curves giving the yield stress as a function of plastic strain (see DEFINE_TABLE_3D). The yield stress as a function of plastic strain curve for the lowest value of strain rate or temperature is used when the strain rate or temperature falls below the minimum value. Likewise, maximum values cannot be exceeded. Hardening variables are ignored with this option (TAU0, Q, B, H, C, GAM0, and GAMM).
TAU0	Initial shear yield stress, τ_0

VARIABLE	DESCRIPTION
Q	Isotropic nonlinear hardening modulus, q
B	Isotropic exponential decay parameter, b
H	Isotropic linear hardening modulus, H
C	Strain rate coefficient C.
GAM0	Quasi-static threshold strain rate, γ_0
GAMM	Maximum threshold strain rate, γ_m
A10	Yield function parameter: initial value a_{10} of $a_1 = \hat{a}_1(r)$
A20	Yield function parameter: initial value a_{20} of $a_2 = \hat{a}_2(r)$
A1H	Yield function parameter a_1^H for formative hardening (ignored if FLG = 2)
A2H	Yield function parameter a_2^H for formative hardening (ignored if FLG = 2)
A2S	Plastic potential parameter a_2^* for hydrostatic stress term
POW	Exponent n of the phenomenological damage model
SRFILT	Strain rate filtering parameter in exponential moving average with admissible values ranging from 0 to 1: $$\dot{\varepsilon}_n^{avg} = \mathrm{SRFILT} \times \dot{\varepsilon}_{n-1}^{avg} + (1 - \mathrm{SRFILT}) \times \dot{\varepsilon}_n$$
IHIS	Flag for additional material properties initialization based on a prior process simulation: EQ.0: No additional initialization GE.1: Use *INITIAL_STRESS_SOLID to initialize additional material properties on an element-by-element basis (see Remark 1).
D1	Johnson & Cook failure parameter d_1
D2	Johnson & Cook failure parameter d_2
D3	Johnson & Cook failure parameter d_3
D4	Johnson & Cook rate dependent failure parameter d_4

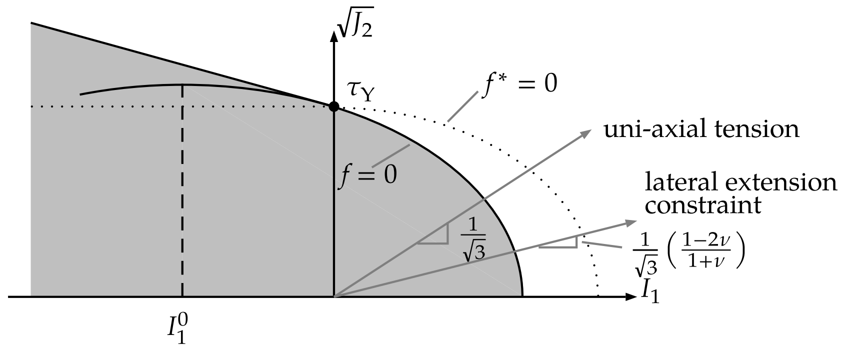


Figure M252-1. Yield function f and plastic flow potential f^*

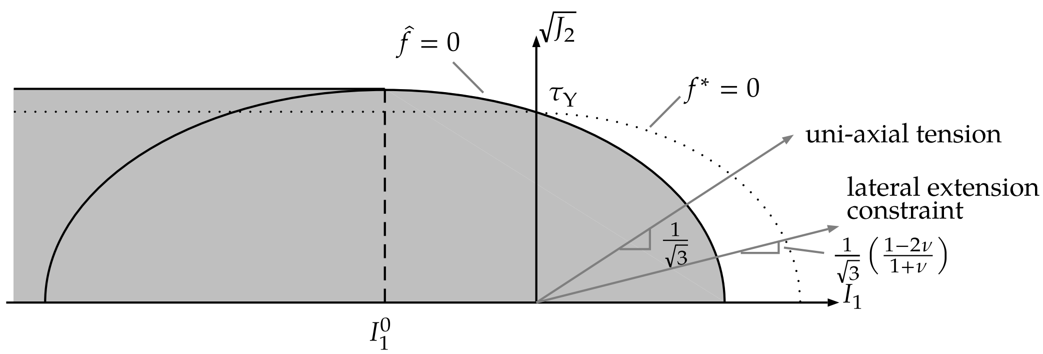


Figure M252-2. Yield function $\hat{f}$ and plastic flow potential f^*

VARIABLE	DESCRIPTION
D1C	Johnson & Cook damage threshold parameter d_{1c}
D2C	Johnson & Cook damage threshold parameter d_{2c}

Material Model:

Two different I_1-J_2 yield criteria for isotropic plasticity can be defined by parameter FLG:

1. FLG = 0 is used for the yield criterion f which is changed at the case of hydrostatic pressure $I_1 = 0$ into a nonlinear *Drucker & Prager* model (DP)

$$f := \frac{J_2}{(1-D)^2} + \frac{1}{\sqrt{3}} a_1 \tau_0 \frac{I_1}{1-D} + \frac{a_2}{3}\left\langle \frac{I_1}{1-D} \right\rangle^2 - \tau_Y^2 = 0$$

with the *Macauley* bracket $\langle \bullet \rangle$, the first invariant of the stress tensor $I_1 = \mathrm{tr}\,\boldsymbol{\sigma}$, and the second invariant of the stress deviator $J_2 = (1/2)\mathrm{tr}(\mathbf{s})^2$ (see Figure M252-1).

2. FLG = 2 is used for the yield criterion $\hat{f}$ which is changed at the vertex into the deviatoric *von Mises* yield function (see Figure M252-2) and is used for

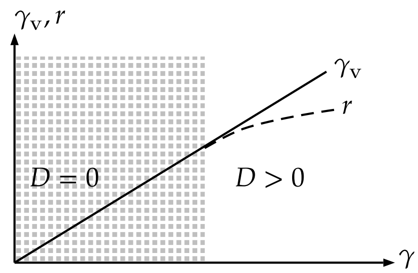


Figure M252-3. Accumulated plastic strain γ_v and damage plastic strain r as a function of strain γ

conservative calculation in case of missing uniaxial compression or combined compression and shear experiments:

$$\hat{f} := \frac{J_2}{(1-D)^2} + \frac{a_2}{3}\left\langle \frac{I_1}{1-D} + \frac{\sqrt{3}a_1\tau_0}{2a_2} \right\rangle^2 - \left(\tau_Y^2 + \frac{a_1^2\tau_0^2}{4a_2}\right) = 0 \ .$$

The yield functions f and $\hat{f}$ are formulated in terms of the effective stress tensor

$$\widetilde{\boldsymbol{\sigma}} = \boldsymbol{\sigma}/(1-D)$$

and the isotropic material damage D according to the continuum damage mechanics in Lemaitre [1992]. The stress tensor $\boldsymbol{\sigma}$ is defined in terms of the elastic strain $\boldsymbol{\varepsilon}^e$ and the isotropic damage D:

$$\boldsymbol{\sigma} = (1-D)\mathbb{C}\boldsymbol{\varepsilon}^e \ .$$

The continuity $(1-D)$ in the elastic constitutive equation above degrades the fourth order elastic stiffness tensor $\mathbb{C}$,

$$\mathbb{C} = 2G\left(\mathbb{I} - \frac{1}{3}\mathbf{1}\otimes\mathbf{1}\right) + K\,\mathbf{1}\otimes\mathbf{1}$$

with shear modulus G, bulk modulus K, fourth order identity tensor $\mathbb{I}$, and second order identity tensor $\mathbf{1}$. The plastic strain rate $\dot{\boldsymbol{\varepsilon}}^p$ is given by the non-associated flow rule

$$\dot{\boldsymbol{\varepsilon}}^p = \lambda\frac{\partial f^*}{\partial\boldsymbol{\sigma}} = \frac{\lambda}{(1-D)^2}\left(\mathbf{s} + \frac{2}{3}a_2^*\langle I_1\rangle\mathbf{1}\right)$$

with the potential f^* and an additional parameter $a_2^* < a_2$ to reduce plastic dilatancy.

$$f^* := \frac{J_2}{(1-D)^2} + \frac{a_2^*}{3}\left\langle\frac{I_1}{1-D}\right\rangle^2 - \tau_Y^2$$

The plastic arc length $\dot{\gamma}_v$ characterizes the inelastic response of the material and is defined by the Euclidean norm:

$$\dot{\gamma}_v := \sqrt{2\,\mathrm{tr}(\dot{\boldsymbol{\varepsilon}}^p)^2} = \frac{2\lambda}{(1-D)^2}\sqrt{J_2 + \frac{2}{3}(a_2^*\langle I_1\rangle)^2} \ .$$

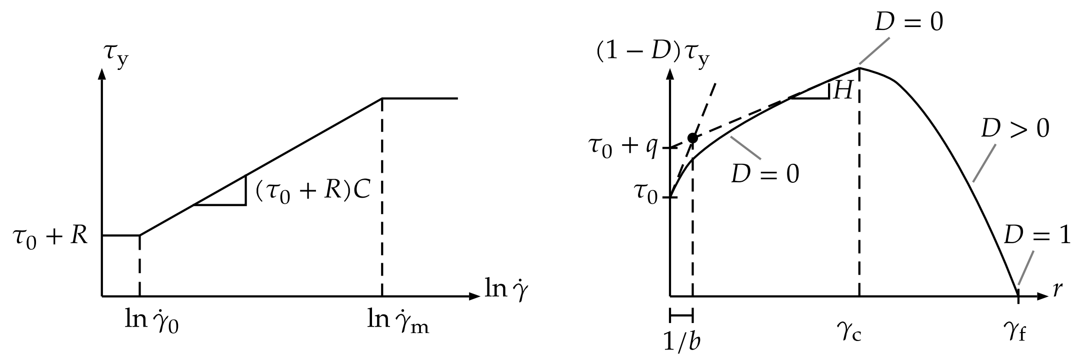


Figure M252-4. Rate-dependent tensile strength τ_Y as a function of effective strain rate $\dot{\gamma}$ (left) and effective damage plastic strain r (right)

In addition, the arc length of the damage plastic strain rate $\dot{r}$ is introduced by means of the arc length $\dot{\gamma}_v$ and the continuity $(1-D)$ as in Lemaitre [1992], where $\tilde{I}_1 = I_1/(1-D)$ and $\tilde{J}_2 = J_2/(1-D)^2$ are the effective stress invariants (see Figure M252-3).

$$\dot{r} := (1-\mathrm{D})\dot{\gamma}_v = 2\lambda\sqrt{\tilde{J}_2 + \frac{2}{3}\left(a_2^*\langle\tilde{I}_1\rangle\right)^2}$$

The rate-dependent yield strength for shear τ_Y can be defined by two alternative expressions. The first representation is an analytic expression for τ_Y:

$$\tau_Y = (\tau_0 + R)\left[1 + C\left(\left\langle\ln\frac{\dot{\gamma}}{\dot{\gamma}_0}\right\rangle - \left\langle\ln\frac{\dot{\gamma}}{\dot{\gamma}_m}\right\rangle\right)\right],\ \text{with}\ \dot{\gamma} = \sqrt{2\,\mathrm{tr}(\dot{\varepsilon})^2}\ ,$$

where the first factor $(\tau_0 + R)$ in τ_Y is given by the static yield strength with the initial yield τ_0 and the non-linear hardening contribution

$$R = q[1 - \exp(-br)] + Hr\ .$$

The second factor $[\dots]$ in τ_Y describes the rate dependency of the yield strength by a modified Johnson & Cook approach with the reference strain rates $\dot{\gamma}_0$ and $\dot{\gamma}_m$ which limit the shear strength τ_Y (see Figure M252-4).

The second representation of the yield strength τ_Y is the table definition LCSS, where hardening can be defined as a function of plastic strain, strain rate, and temperature.

Toughened structural adhesives show distortional hardening under plastic flow, that is, the yield surface changes its shape. This formative hardening can be phenomenological described by simple evolution equations of parameters $a_1 = \hat{a}_1(r)\ \wedge\ a_2 = \hat{a}_2(r)$ in the yield criterions f with the initial values a_{10} and a_{20}:

$$a_1 = \hat{a}_1(r) \wedge \dot{a}_1 = a_1^{\mathrm{H}}\dot{r}$$
$$a_2 = \hat{a}_2(r) \wedge a_2 \geq 0 \wedge \dot{a}_2 = a_2^{\mathrm{H}}\dot{r}$$

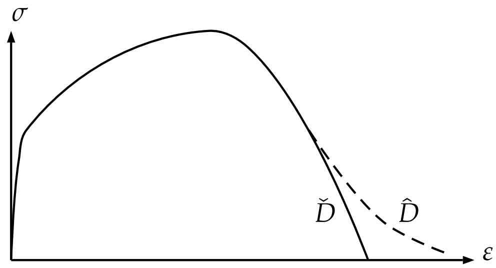


Figure M252-5. Influence of DOPT on damage softening

The parameters a_1^{H} and a_2^{H} can take positive or negative values as long as the inequality $a_2 \geq 0$ is satisfied. The criterion $a_2 \geq 0$ ensures an elliptic yield surface. The yield criterion $\hat{f}$ uses only the initial values $a_1 = a_{10}$ and $a_2 = a_{20}$ without the distortional hardening.

The empirical isotropic damage model D is based on the approach in Lemaitre [1985]. Two different evolution equations $\dot{\hat{D}}(r, \dot{r})$ and $\dot{\check{D}}(\gamma_{\mathrm{v}}, \dot{\gamma}_{\mathrm{v}})$ are available (see Figure M252-5). The damage variable D is formulated in terms of the damage plastic strain rate $\dot{r}$ (DOPT = 0)

$$\dot{D} = \dot{\hat{D}}(r, \dot{r}) = n \left\langle \frac{r - \gamma_{\mathrm{c}}}{\gamma_{\mathrm{f}} - \gamma_{\mathrm{c}}} \right\rangle^{n-1} \frac{\dot{r}}{\gamma_{\mathrm{f}} - \gamma_{\mathrm{c}}}$$

or of the plastic arc length $\dot{\gamma}_{\mathrm{v}}$ (DOPT = 1)

$$\dot{D} = \dot{\check{D}}(\gamma_{\mathrm{v}}, \dot{\gamma}_{\mathrm{v}}) = n \left\langle \frac{\gamma_{\mathrm{v}} - \gamma_{\mathrm{c}}}{\gamma_{\mathrm{f}} - \gamma_{\mathrm{c}}} \right\rangle^{n-1} \frac{\dot{\gamma}_{\mathrm{v}}}{\gamma_{\mathrm{f}} - \gamma_{\mathrm{c}}} \; ,$$

where r in contrast to γ_{v} increases non-proportionally slowly (see Figure M252-5). The strains at the thresholds γ_{c} and γ_{f} for damage initiation and rupture are functions of the triaxiality $T = \sigma_{\mathrm{m}} / \sigma_{\mathrm{eq}}$ with the hydrostatic stress $\sigma_{\mathrm{m}} = I_1/3$ and the von Mises equivalent stress $\sigma_{\mathrm{eq}} = \sqrt{3J_2}$ as in Johnson and Cook [1985].

$$\gamma_{\mathrm{c}} = [d_{1\mathrm{c}} + d_{2\mathrm{c}} \exp(-d_3 \langle T \rangle)] \left(1 + d_4 \left(\left\langle \ln \frac{\dot{\gamma}}{\dot{\gamma}_0} \right\rangle - \left\langle \ln \frac{\dot{\gamma}}{\dot{\gamma}_{\mathrm{m}}} \right\rangle \right) \right)$$

$$\gamma_f = [d_1 + d_2 \exp(-d_3 \langle T \rangle)] \left(1 + d_4 \left(\left\langle \ln \frac{\dot{\gamma}}{\dot{\gamma}_0} \right\rangle - \left\langle \ln \frac{\dot{\gamma}}{\dot{\gamma}_{\mathrm{m}}} \right\rangle \right) \right)$$

The option JCFL controls the influence of triaxiality $T = \sigma_{\mathrm{m}} / \sigma_{\mathrm{eq}}$ in the pressure range for the thresholds γ_{c} and γ_{f}. JCFL = 0 makes use of the Macauley bracket $\langle T \rangle$ for the triaxiality $T = \sigma_{\mathrm{m}} / \sigma_{\mathrm{eq}}$ while JCFL = 1 omits the Macauley bracket $\langle T \rangle$.

Remarks:

1. **Description of IHIS.** To account for results from *a prior process simulation*, it is possible to define additional material parameters on an element-by-element basis. The parameters influence stiffness, plasticity and damage behavior of the

material. LS-DYNA reads the data from the *INITIAL_STRESS_SOLID keyword beginning with history position HISV4. IHIS governs the number of read history values and their interpretation. It is defined as:

$$\text{IHIS} = a_0 + 2\,a_1 + 4\,a_2 + 8\,a_3 .$$

Here, each a_i is a binary flag set to either 1 or 0, activating or deactivating the input of particular properties as summarized in the following table, which also indicates the order in which the additional data is read.

Flag	Description	Variables	#
a_0	Scaling factors for elastic properties	α_E, α_ν	2
a_1	Scaling factors for initial yield stress and hardening	χ_c, ϕ_c	2
a_2	Scaling factors for damage strain thresholds	β, δ	2
a_3	Structural pre-damage	D_2	1

If defined by an appropriate value of IHIS ($a_0 = 1$), α_E and α_ν are scaling factors for Young's modulus E and Poissons's ratio ν, respectively. If $a_1 = 1$, then the plastic behavior is changed: the initial shear yield stress τ_0 is multiplied by factor χ_c, and the hardening modulus R is scaled by ϕ_c. Choosing $a_2 = 1$ allows locally modifying the strain thresholds γ_c and γ_f by multiplying them by β and δ, respectively. Finally, setting $a_3 = 1$ causing accounting for a pre-damaged D_2. The two damage mechanisms, represented by D and D_2, are applied multiplicatively, such that the effective stress is given by

$$\widetilde{\boldsymbol{\sigma}} = \boldsymbol{\sigma}/\big((1-D)(1-D_2)\big).$$

Note that parameter NHISV of *INITIAL_STRESS_SOLID has to be consistent with the choice of IHIS:

$$\text{NHISV} = 3 + 2\,a_0 + 2\,a_1 + 2\,a_2 + a_3$$

2. **History Variables.** The following additional history variables are available for this keyword:

History Variable #	Description
1	Damage variable, D
2	Plastic arc length, γ_v
3	Effective strain rate
4	Temperature
5	Yield stress

History Variable #	Description
6	Damaged yield stress
7	Triaxiality
8	threshold, γ_c
9	threshold, γ_f

*MAT_GENERALIZED_PHASE_CHANGE

This is Material Type 254. It is designed to model phase transformations in materials and the implied changes in the material properties. It is applicable to hot stamping, heat treatment, and welding processes for a wide range of materials. It accounts for up to 24 phases and provides a list of generic phase change mechanisms for each possible phase change. The parameters for the phase transformation laws are to be given in tabulated form.

Given the current microstructure composition, the formulation implements a temperature and strain-rate-dependent elastic-plastic material with non-linear hardening behavior. Above a certain temperature, the model shows an ideal elastic-plastic behavior with no evolution of plastic strains.

The material has been implemented for solid and shell elements and is suitable for explicit and implicit analysis.

Card Summary:

Card 1. This card is required.

MID	RO	N	E	PR	MIX	MIXR	

Card 2. This card is required.

TASTART	TAEND	CTE			EPSINI	DTEMP	

Card 2.1. Include this card if TASTART > 0 and TAEND = 0.

XASTR	XAEND	XAIPA1	XAIPA2	XAIPA3	XAFPA	CTEANN	

Card 3. This card is required.

PTLAW	PTSTR	PTEND	PTX1	PTX2	PTX3	PTX4	PTX5

Card 4. This card is required.

PTTAB1	PTTAB2	PTTAB3	PTTAB4	PTTAB5	PTTAB6	PTTAB7	

Card 5. This card is required.

PTEPS	PTRIP	PTLAT	POSTV	NUSHIS	GRAIN	T1PHAS	T2PHAS

Card 5.1. Include this card if NUSHIS > 0.

FUSHI1	FUSHI2	FUSHI3	FUSHI4	FUSHI5	FUSHI6	FUSHI7	FUSHI8

Card 6. For each of the N phases, one parameter SIGYi must be specified. A maximum of 10 instantiations of this card may be included. The next keyword ("*") card terminates this input.

SIGY1	SIGY2	SIGY3	SIGY4	SIGY5	SIGY6	SIGY7	SIGY8

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	N	E	PR	MIX	MIXR	
Type	A	F	I	F	F	I	I	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density, ρ
N	Number of phases
E	Young's modulus: GT.0.0: Constant value is used. LT.0.0: Temperature dependent Young's modulus given by load curve or table ID = -E. Tables are used to describe a temperature dependent modulus for each phase individually.
PR	Poisson's ratio: GT.0.0: Constant value is used. LT.0.0: Temperature dependent Poisson's ratio given by load curve or table ID = -E. Tables are used to describe a temperature dependent Poisson's ratio for each phase individually.
MIX	Load curve ID with initial phase concentrations
MIXR	Load curve or table ID for mixture rule. Tables are used to define temperature dependency.

Card 2	1	2	3	4	5	6	7	8
Variable	TASTART	TAEND	CTE			EPSINI	DTEMP	
Type	F	F	I			F	F	

VARIABLE	DESCRIPTION
TASTART	Temperature start for simple linear annealing (see Remark 5). If TASTART > 0 and TAEND = 0, an enhanced annealing algorithm is used (see Remark 6). In that case, TASTART is interpreted as an anneal option, and Card 2.1 is required. Possible values for the extended anneal option are: EQ.1: Linear annealing EQ.2: JMAK
TAEND	Temperature end for simple linear annealing. See Remark 5. If TASTART > 0 and TAEND = 0, an enhanced annealing algorithm is used. See Remark 6.
CTE	Coefficient of thermal expansion: GT.0.0: Constant value is used. LT.0.0: Temperature dependent CTE given by load curve or table ID = -CTE. Tables give CTE as a function of temperature for each phase individually.
EPSINI	Initial plastic strains, uniformly distributed within the part
DTEMP	Maximum temperature variation within a time step. If exceeded during the analysis, a local sub-cycling is used for the calculation of the phase transformations.

Enhanced Annealing Card. Additional card for TASTART > 0 and TAEND = 0 only. See Remark 6 for details.

Card 2.1	1	2	3	4	5	6	7	8
Variable	XASTR	XAEND	XAIPA1	XAIPA2	XAIPA3	XAFPA	CTEANN	
Type	F	F	I	I	I	F	F	

VARIABLE	DESCRIPTION
XASTR	Annealing start temperature
XAEND	Annealing end temperature
XAIPA*i*	Load curve or table ID defining the i^{th} parameter of the enhanced annealing option. Interpretation of the parameter depends on TASTART.
XAFPA	Scalar parameter of the enhanced annealing option if applicable. Interpretation of the parameter depends on TASTART.
CTEAN	Annealing option for thermal expansion: LT.0: |CTEAN| defines the upper temperature limit (cut-off temperature) for evaluation of thermal strains. EQ.0: No modification of thermal strains EQ.1: XAEND defines the upper temperature limit (cut-off temperature) for evaluation of thermal strains.

Card 3	1	2	3	4	5	6	7	8
Variable	PTLAW	PTSTR	PTEND	PTX1	PTX2	PTX3	PTX4	PTX5
Type	I	I	I	I	I	I	I	I

VARIABLE	DESCRIPTION
PTLAW	Table ID to define the phase transformation model as a function of source phase and target phase. The values in *DEFINE_TABLE are the phase numbers before transformation. The curves referenced by the table specify transformation model (ordinate) as a function of phase number after transformation. LT.0: transformation model used in heating EQ.0: no transformation GT.0: transformation model is used in cooling A variety of transformation models can be specified as ordinate values of the curves: EQ.1: Koinstinen-Marburger

VARIABLE	DESCRIPTION
	EQ.2: Johnson-Mehl-Avrami-Kolmogorov (JMAK) EQ.3: Akerstrom (only for cooling) EQ.4: Oddy (only for heating) EQ.5: Phase Recovery I (only for heating) EQ.6: Phase Recovery II (only for heating) EQ.7: Parabolic Dissolution I (only for heating) EQ.8: Parabolic Dissolution II (only for heating) EQ.9: extended Koinstinen-Marburger (only for cooling) EQ.12: JMAK for both cooling and heating See Remarks 1 and 2 for further details.
PTSTR	Table ID to define start temperatures for the transformations as a function of source phase and target phase. The values in *DEFINE_TABLE are the phase numbers before transformation. The curves referenced by the table specify start temperature (ordinate) as a function of phase number after transformation (abscissa).
PTEND	Table ID to define end temperatures for the transformations as a function of source phase and target phase. The values in *DEFINE_TABLE are the phase numbers before transformation. The curves referenced by the table specify end temperature (ordinate) as a function of phase number after transformation (abscissa).
PTXi	Table ID defining the i^{th} scalar-valued phase transformation parameter as function of source phase and target phase (see Remark 2 and Table M254-1 to determine which parameters apply). The values in *DEFINE_TABLE are the phase numbers before transformation. The curves referenced by the table specify scalar parameter (ordinate) as a function of phase number after transformation (abscissa).

Card 4	1	2	3	4	5	6	7	8
Variable	PTTAB1	PTTAB2	PTTAB3	PTTAB4	PTTAB5	PTTAB6	PTTAB7	
Type	I	I	I	I	I	I	I	

VARIABLE	DESCRIPTION
PTTAB*i*	Table ID for a 3D table defining the i^{th} tabulated phase transformation parameter as a function of source phase and target phase (see Remark 2 and Table M254-1 to determine which parameters apply). The values in *DEFINE_TABLE_3D are the phase numbers before transformation. The values in the 2D tables referenced by *DEFINE_TABLE_3D are the phase numbers after transformation. The curves referenced by the 2D tables specify a tabulated parameter (ordinate) as a function of either temperature or temperature rate (abscissa).

Card 5	1	2	3	4	5	6	7	8
Variable	PTEPS	PTRIP	PTLAT	POSTV	NUSHIS	GRAIN	T1PHAS	T2PHAS
Type	I	F	I	I	I	F	F	F

VARIABLE	DESCRIPTION
PTEPS	Table ID defining transformation induced strains. <u>If ID of 2D table</u> The values in *DEFINE_TABLE are the phase numbers before transformation. The curves referenced by the table specify strains (ordinate) as a function of phase number after transformation (abscissa). <u>If ID of 3D table</u> The values in *DEFINE_TABLE_3D are the phase numbers before transformation. The values in the 2D tables referenced by *DEFINE_TABLE_3D are the phase number after transformation. The curves referenced by the 2D tables specify induced strains as a function of temperature.
PTRIP	Flag for transformation induced plasticity (TRIP). Algorithm active for positive values of PTRIP.
PTLAT	Table ID defining transformation induced heat generation (latent heat). <u>If ID of 2D table</u>

VARIABLE	DESCRIPTION
	The values in *DEFINE_TABLE are the phase numbers before transformation. The curves referenced by the table specify heat values (ordinate) versus phase number after transformation (abscissa).
	If ID of 3D table
	The values in *DEFINE_TABLE_3D are the phase numbers before transformation. The values in the 2D tables referenced by *DEFINE_TABLE_3D are the phase number after transformation. The curves referenced by the 2D tables specify induced heat as function of temperature.
POSTV	Define additional pre-defined history variables that might be useful for post-processing. See Remark 4.
NUHIS	Number of additional user defined history variables. For details see Remarks 3 and 4.
GRAIN	Initial grain size
T1PHAS	Lower temperature limit for cooling rate evaluation. Cooling rate can be used as input for user defined variables.
T2PHAS	Upper temperature limit for cooling rate evaluation. Cooling rate can be used as input for user defined variables.

User History Card. Additional card for NUSHIS > 0 only.

Card 5.1	1	2	3	4	5	6	7	8
Variable	FUSHI1	FUSHI2	FUSHI3	FUSHI4	FUSHI5	FUSHI6	FUSHI7	FUSHI8
Type	I	I	I	I	I	I	I	I

VARIABLE	DESCRIPTION
FUSHI*i*	Function ID for user defined history variables. See Remarks 3 and 4.

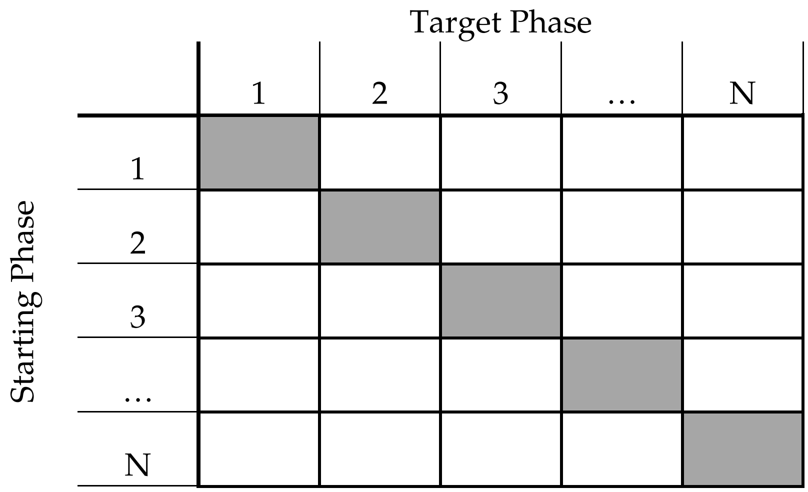
Phase Yield Stress Cards. For each of the N phases, one parameter SIGY*i* must be specified. A maximum of 10 of this card may be included. The next keyword ("*") card terminates this input.

Card 6	1	2	3	4	5	6	7	8
Variable	SIGY1	SIGY2	SIGY3	SIGY4	SIGY5	SIGY6	SIGY7	SIGY8
Type	I	I	I	I	I	I	I	I

VARIABLE	DESCRIPTION
SIGY*i*	Load curve or table ID for hardening of phase *i*. If load curve ID Input yield stress as a function of effective plastic strain. If table ID of 2D table Input temperatures as table values and hardening curves (yield stress as a function of effective plastic strain) as targets for those temperatures. If table ID of 3D table Input temperatures as main table values and strain rates as values for the sub-tables. Hardening curves (yield stress as a function of effective plastic strain) are targets for those strain rates.

Remarks:

1. **Phase transformation matrix.** All data defining the microstructure evolution is expected to be given in a tabular form. The input is structured as a two-dimensional matrix containing one row for any starting phase and one row for any target phase. The basic structure is depicted in the following table:



For the input in Card 3, the entry at position *ij* of this matrix is interpreted as scalar data used for the transformation from phase *i* to phase *j*. This could, for example, be the transformation law or the start time. In LS-DYNA, such a matrix is defined by the keyword *DEFINE_TABLE(_2D). The abscissa values are the starting phase IDs. Each load curve (*DEFINE_CURVE) that is referenced consequently defines one row of the matrix.

Some of the implemented transformation models require input data that is a function of temperature, temperature rate, equivalent plastic strain, or other values. The input of this data has the same basic input structure as the scalar values, but the matrix entries are now load curve IDs. Therefore, the input is a three-dimensional table (*DEFINE_TABLE_3D), and each row of the matrix is represented by a two-dimensional table itself defined by *DEFINE_TABLE(_2D).

2. **Phase transformation models.** This material features temperature and phase-composition-dependent elastic-plastic behavior. The phase composition is determined using a list of generic phase transformation mechanisms you can choose from for each of the possible phase transformations. So far, eight different transformation models have been implemented to describe the transition from source phase concentration, x_a, to target phase concentration, x_b. Table M254-1 at the end of this remark summarizes the input parameters necessary for the individual models.

a) *Koistinen-Marburger (KM), law 1.*

The KM formulation is tailored for non-diffusive transformations. In the most basic and commonly used version, the temperature-dependent amount of the target phase is computed as

$$x_b = (x_a + x_b)\left(1.0 - e^{-\alpha_{\text{KM}}(T_{\text{start}} - T)}\right) \ .$$

PTX1 defines the so-called Koistinen-Marburger factor, α_{KM}.

b) *Generalized Johnson-Mehl-Avrami-Kolmogorov (JMAK), law 2/12.*

This is a widely used model for diffusive phase transformation. In literature, often the incremental form of the JMAK equation is given for an isothermal, incomplete transformation:

$$x_b = x_{\text{eq}}(T)(x_a + x_b)\left(1 - e^{-\left(\frac{t}{\tau(T)}\right)^{n(T)}}\right) \ .$$

In the previous equation, exponent, n; equilibrium concentration, x_{eq}; and relaxation time, τ, are functions of the temperature.

In this material model, the differential form of the JMAK equation is employed which makes the model readily applicable for non-isothermal processes:

$$\frac{dx_b}{dt} = n(T)(k_{ab}x_a - k'_{ab}x_b)\left(\ln\left(\frac{k_{ab}(x_a + x_b)}{k_{ab}x_a - k'_{ab}x_b}\right)\right)^{\frac{n(T)-1.0}{n(T)}} \quad ,$$

In this evolution equation, the following factors are defined:

$$k_{ab} = \frac{x_{eq}(T)}{\tau(T) \times \alpha(\varepsilon^p)} f(\dot{T})$$
$$k'_{ab} = \frac{1.0 - x_{eq}(T)}{\tau(T) \times \alpha(\varepsilon^p)} f'(\dot{T})$$

As user input, load curve data for the exponent, $n(T)$, is defined in PTTAB1, the equilibrium concentration, $x_{\text{eq}}(T)$, in PTTAB2, and the relaxation time, $\tau(T)$, in PTTAB3. This model is a generalized JMAK approach that features additional parameters, such as the temperature rate correction factors, $f(\dot{T})$ and $f'(\dot{T})$, given in PTTAB4 and PTTAB5, respectively. As an optional parameter an accelerator $\alpha(\varepsilon^p)$ for the transformation can be defined as a function of equivalent plastic strain or of an external variable (see Remark 7) in PTTAB6. If not defined, a constant value of 1.0 is assumed.

Like the Koistinen-Marburger case, a temperature-dependent equilibrium concentration, $x_{\text{eq},a}$, of the source phase can optionally be defined. If defined in PTTAB7, the transformation is only active if the source phase fraction exceeds the equilibrium, meaning $x_a > x_{\mathrm{eq},a}$.

Note that the JMAK evolution can not only be activated by a choice of -2 (heating) and 2 (cooling), but also by choosing the law to be 12. In that case, the sign of the temperature rate is not checked, and the model is always active if the temperature is in the temperature window defined by the start and end temperature of the transformation.

c) *Kirkaldy, law 3.*

d) Similar to the implementation of *MAT_244, the transformation for cooling phases can be computed by the evolution equation:

$$\frac{dX_b}{dt} = 2^{0.5(G-1)} f(C)(T_{\mathrm{start}} - T)^{n_T} D(T) \frac{X_b^{n_1(1.0-X_b)}(1.0 - X_b)^{n_2 X_b}}{Y(X_b)} \quad ,$$

formulated in the normalized phase concentration

$$X_b = \frac{x_b}{x_{\mathrm{eq}}(T)} \quad .$$

In contrast to *MAT_244, the parameters for the evolution equation are not determined from the chemical composition of the material but defined directly as user input. The scalar data in PTX1 to PTX4 are interpreted as $f(C)$, n_T, n_1, and n_2. Tabulated data for $D(T)$, $Y(X_b)$, and $x_{\mathrm{eq}}(T)$ are given in PTTAB1 to PTTAB3.

e) *Oddy, law 4.*

For phase transformation in heating, the equation of Oddy can be used, which can be interpreted as a simplified JMAK relation and reads as

$$\frac{dx_b}{dt} = n \frac{x_a}{c_1 (T - T_{\mathrm{start}})^{-c_2}} \left(\ln \left(\frac{(x_a + x_b)}{x_a} \right) \right)^{\frac{n-1.0}{n}} \quad .$$

Its application requires the input of three scalar parameters, n, c_1, and c_2, that are read from the respective positions in the tables in PTX1 to PTX3.

f) *Phase recovery I, law 5.*

This phase transformation law is motivated by the recovery of the β-phase and α-phase from martensitic α-phase in titanium alloys and is a generalization of the algorithms described in the literature for this process.

The transformation takes place if and only if the amount of the target phase is below a user-defined, temperature dependent threshold, x_b^{tre}. This threshold can be defined in PTTAB3.

For $x_b < x_b^{\mathrm{tre}}$, the transformation scheme comprises three steps. First, a temperature dependent equilibrium fraction for the starting phase is calculated based on an incomplete KM equation:

$$x_{\mathrm{eq},a} = (x_a + x_b - x_{\mathrm{inc}})\left(1.0 - e^{-\alpha_{\mathrm{KM}}(T_{\mathrm{KM},s}-T)}\right) \ .$$

The KM-parameter α_{KM} and the start temperature $T_{\mathrm{KM},s}$ must be given in PTX1 and PTX2, respectively. The incompleteness parameter x_{inc} is a function of temperature defined in PTTAB4.

Second, if the current fraction of the starting phase x_a exceeds the calculated equilibrium concentration $x_{\mathrm{eq},a}$, a diffusional process follows. It is described by a JMAK approach. Its incremental form for an isothermal process is given by

$$x_a = x_{\mathrm{eq},a} + \left(x_a + x_b - x_{\mathrm{eq},a}\right)e^{-\left(\frac{t}{\tau(T)}\right)^{n(T)}} \ .$$

Naturally, a differential form of this equation is used in the model in order to be applicable to non-isothermal situations. The final calculated change Δx_a in identified with the formation of a recovery phase $x_a^r = -\Delta x_a$. The parameters for the JMAK equation are given in PTTAB1 (n) and PTTAB2 (τ).

Third, some of the recovery phase is partially transformed into the target phase:

$$\Delta x_b = \gamma(T)x_a^r \ .$$

The quotient $\gamma(T)$ can be defined in PTTAB5.

g) *Phase recovery II, law 6.*

This is the second part of the recovery and can only be defined if the previous transformation law (law 5) has also been defined with the same starting phase. This second step aims to transform the remaining fraction of the virtual, recovery phase x_a^r into the physical phases defined in the material.

In order to allow for the definition of more than two target phases for one recovery process, an optional parameter $\eta(T)$ can be defined as the only input of this transformation in PTTAB1. It is used to control the transformation by

$$\Delta x_b = \eta(T)x_a^r \ .$$

Note that x_a^r here refers to the complete fraction of the recovery phase as calculated by the JMAK approach. If the parameter is not defined, then the remainder of the virtual phase fraction is completely transformed.

h) *Parabolic growth I, law 7.*

The transformation laws 7 and 8 model the subsequent dissolution of a group of phases into one common target phase. The remaining fraction of the group after dissolution within a time step is denoted by x_{diss}. The groups are identified by a group ID that is here defined in PTX1.

You can define a dissolution function, f_{diss}, and a critical time, t_{crit}. These values are expected to be functions of temperature and are defined in PTTAB3 and PTTAB4, respectively. Based on those and the temperature dependent equilibrium concentration $x_{\mathrm{eq},b}$ (PTTAB2), a characteristic dissolution time, t_{diss}, can be calculated as

$$t_{\mathrm{diss}} = \left(\frac{x_b(T)}{x_b^{\mathrm{eq}}(T)}\right)^2 t_{\mathrm{crit}}(T) \quad .$$

Depending on the relative size of the step increment, Δt, with respect to the critical and characteristic dissolution time, the remaining group fraction x_{diss} is calculated as

$$x_{\mathrm{diss}} = \begin{cases} 1 - x_b^{\mathrm{eq}}(T) f(T) \sqrt{\Delta t + t_{\mathrm{diss}}(T)}, & \text{for } \Delta t + t_{\mathrm{diss}} < t_{\mathrm{crit}} \\ 1 - x_b^{\mathrm{eq}}(T), & \text{otherwise} \end{cases}$$

Now, the fraction x_a (the transformation of which is defined by law 7) is always assumed to be the first member of the group to by dissolved. It is algorithmically assured that there cannot be an increase in fraction x_a.

i) *Parabolic growth II, law 8.*

This law cannot be defined separately, but simulates the dissolution of the further members of the group already defined for a transformation with law 7. Naturally, the group ID must also be referenced here, and it is again given in PTX1. Furthermore, in the case of three or more members within a group the order in which the fractions are to be dissolved must be defined. For that purpose, the position in the group is defined in PTX2.

j) *Extenden Koistinen-Marburger, law 9.*

This extension to the standard Koistinen-Marburger (law 1) is motivated by the application of the material formulation to titanium and is only available in cooling.

An equilibrium concentration $x_{\mathrm{eq},a}$ of the source phase can be defined as function of the current temperature in parameter PTTAB1. The transformation is only active if the source phase fraction exceeds the equilibrium, meaning $x_a > x_{\mathrm{eq},a}$.

Furthermore, an incomplete transformation is possible in case of relatively slow cooling rates. For this purpose, you can define two rate limits $\dot{T}_{\mathrm{lim},1}$ and $\dot{T}_{\mathrm{lim},2}$ in PTX2 and PTX3, respectively, and an incompleteness parameter $x_{\mathrm{inc}}(T)$ as a function of temperature in PTTAB2. The corresponding equation for the transformation then is given by:

$$x_b = \begin{cases} (x_a + x_b)\left(1.0 - e^{-\alpha_{\mathrm{KM}}(T_{\mathrm{start}} - T)}\right) & ,\text{for } \dot{T} < \dot{T}_{\mathrm{lim},1} \\ (x_a + x_b - x_{\mathrm{inc}})\left(1.0 - e^{-\alpha_{\mathrm{KM}}(T_{\mathrm{start}} - T)}\right) & ,\text{for } \dot{T}_{\mathrm{lim},1} < \dot{T} < \dot{T}_{\mathrm{lim},2} \end{cases}$$

A summary of input parameters for the different material laws is given in the following table. If not stated otherwise, the parameters in the tabular data PTTAB*i* are expected to be functions of the current temperature, T.

Table M254-1. Summary of input parameters for the various laws

	PTLAW #								
Parameters	1	2/12	3	4	5	6	7	8	9
PTX1	α_{KM}		$f(C)$	n	α_{KM}		GID	GID	α_{KM}
PTX2			n_T	c_1	$T_{\mathrm{KM},s}$			POS	$\dot{T}_{\mathrm{lim},1}$
PTX3			n_1	c_2					$\dot{T}_{\mathrm{lim},2}$
PTX4			n_2						
PTTAB1		n	D		n	η	x_b^{tre}		$x_{\mathrm{eq},a}$
PTTAB2		x_{eq}	$Y(X_b)$		τ		$x_{\mathrm{eq},b}$		x_{inc}
PTTAB3		τ	x_{eq}		x_b^{tre}		f_{diss}		
PTTAB4		$f(\dot{T})$			x_{inc}		t_{crit}		
PTTAB5		$f'(\dot{T})$			γ				
PTTAB6		$\alpha(\varepsilon^{\mathrm{pl}})$							
PTTAB7		$x_{\mathrm{eq},a}$							

3. **User-defined history data.** You can define up to eight additional history variables that are added to the list of history variables starting at position 31 (see Remark 4). These values can, for example, be used to evaluate the hardness of the material based on different formulas given in the literature.

   The additional variables are to be given by respective *DEFINE_FUNCTION keywords in the input as functions of the current time, the user-defined histories themselves, the current phase concentrations, the current temperature, the peak temperature, the average temperature rate between T2PHASE and T1PHASE, the current yield stress, the stress tensor, and the current values for the equivalent plastic strain of the individual phases.

For example, if all 24 phases are used (N = 24) and eight additional history variables (NUSHIS = 8) are defined, a function declaration could look as follows:

```
*DEFINE_FUNCTION
1,user defined history 1
uhist(time,usrhst1,usrhst2,…,usrhst8,phase1,
phase2,…,phase24,T,Tpeak,Trate,sigy,
sigma1,sigma2,…,sigma6,
epspl1,epspl2,…,epspl24)= ...
```

In contrast, for four considered phases (N = 4) and two additional histories (NUSHIS = 2) the keyword input could be

```
*DEFINE_FUNCTION
2,user defined history 1
uhist(time,usrhst1,usrhst2,phase1,phase2,phase3,phase4,
T,Tpeak,Trate,sigy,sigma1,sigma2,…,sigma6,epspl1,epspl2,
epspl3,epspl4)= ...
```

4. **History values.** To be able to post-process values of history variables, fields NEIPS (shells) or NEIPH (solids) must be defined on *DATABASE_EXTENT_-BINARY.

   Aside from the user-defined history variables discussed in Remark 3, this material formulation can output additional pre-defined history values for post-processing. The input value of field POSTV defines the data to be written. Its value is calculated as

$$\mathrm{POSTV} = \mathrm{a}_1 + 2\,a_2 + 4\,a_3 + 8\,a_4$$

   Each flag a_i is a binary number (can be either 1 or 0) and corresponds to one particular post-processing variable according to the following table. This table also shows the order of output as well as the number of extra history variables associated with the particular flag. The values of these user-defined histories are reset when the temperature is in the annealing range.

Flag	Description	Variables	# Hist
a_1	Accumulated thermal strain	ε_T	1
a_2	Accumulated strain tensor	$\boldsymbol{\varepsilon}$	6
a_3	Plastic strain tensor	$\boldsymbol{\varepsilon}_p$	6
a_4	Equivalent strain	$\varepsilon_{\mathrm{VM}}$	1

   In total NXH extra variables are required depending on the choice of parameter POSTV. For example, the maximum number of additional variables is NXH = 14 for POSTV = 15.

A complete list of history variables for the material is given in the following table. "Position" refers to the history variable number as listed by LS-PrePost when post-processing the d3plot database. The value of N indicates the number of phases accounted for in the model.

History Variable #	Description
1 → N	Phase concentration
N + 1	Maximum temperature
N + 2	Cooling rate between T2PHAS and T1PHAS
N + 3	Yield stress
N + 4	Young's modulus
N + 5	Indicator of plastic behavior
N + 6 → N + 5 + NUSHIS	User-defined history variables
N + 6 +NUSHIS	Current temperature
N + 7 + NUHIS → N + 6 + NUHIS + NXH	Post-process history data as described in the previous table
N + 7 + NUHIS + NXH → 2 × N + 6 + NUHIS + NXH	Effective plastic strain for each phase in the microstructure

5. **Simple annealing.** When the temperature reaches the start annealing temperature TASTART, the material starts assuming its virgin properties. Beyond the start annealing temperature, it behaves as an ideal elastic-plastic material but with no evolution of plastic strains.

   For non-zero values of both TASTART and TAEND a simple annealing strategy is used. The resetting of effective plastic properties in the annealing temperature interval is done by modifying the effective plastic strain for each phase as

$$\varepsilon_p^n = \varepsilon_{p,\mathrm{start}}^n \frac{T_a^{\mathrm{end}} - T}{T_a^{\mathrm{end}} - T_a^{\mathrm{start}}} ,$$

   where $\varepsilon_{p,\mathrm{start}}^n$ is the plastic strain for phase n at the beginning of the annealing process.

6. **Enhanced annealing.** For a positive value of TASTART and TAEND = 0, an enhanced annealing strategy is employed. It requires the input of an additional keyword card.

   Above the annealing start temperature T_a^{start}, defined by XASTR, the material behaves as an ideal-plastic material, but instead of an evolution of the plastic strains, the equivalent plastic strain $\varepsilon^p$is reduced by a scale factor $\alpha(T,t)$ within the annealing temperature window

$$\varepsilon_p^n = \varepsilon_{p,\mathrm{start}}^n(\alpha(T,t)) \ .$$

The base value $\varepsilon_{p,\mathrm{start}}^n$ refers to the equivalent plastic strain found in the phase, n, when the temperature reaches the annealing start temperature for the first time. The algorithm used to determine the value of α depends on the annealing option TASTART.

a) *Linear annealing.* For TASTART = 1 a linear relation between temperature and the annealing effect is assumed, similar to the simple annealing option discussed above. But in this case an incomplete reset of the equivalent plastic strain data is possible. The scale factor, α, is a function of temperature and is given by

$$\alpha = \frac{T_a^{\mathrm{end}} - T}{T_a^{\mathrm{end}} - T_a^{\mathrm{start}}} + \alpha_{\mathrm{eq}} \frac{T - T_a^{\mathrm{start}}}{T_a^{\mathrm{end}} - T_a^{\mathrm{start}}}$$

Here, the end temperature, T_a^{end}, is defined by XAEND and the newly introduced incompleteness factor, α_{eq}, as scalar input data in XAFPA.

b) *Johnson-Mehl-Avrami-Kolmogorov (JMAK).* For TASTART = 2, the evolution of the scale factor follows a JMAK-type approach. For isothermal situations and assuming a start time for the process of 0.0, an incremental form can be explicitly stated a

$$\alpha = \alpha_{\mathrm{eq}}(T) + \Big(1 - \alpha_{\mathrm{eq}}(T)\Big) e^{-\left(\frac{t}{\tau(T)}\right)^{n(T)}} .$$

In the last equation, $n(T)$ denotes the exponent for the differential equation, $\tau(T)$ the relaxation time and $\alpha_{eq}(T)$ denotes the limit value for the scale factor for infinitely long processes. All of those are functions of temperature and, thus, require the input of load curve IDs in XAIPA1 ($n(T)$), XAIPA2 ($\alpha_{\mathrm{eq}}(T)$) and XAIPA3 ($\tau(T)$).

In the material implementation a differential form of the JMAK approach is invoked, which makes the formulation applicable to non-isothermal processes as well as independent of the start time of annealing.

7. **Effect of external variables on phase transformation.** As discussed in some detail in Remark 2b, the JMAK transformation model offers the possibility to modify the transformation speed by means of an accelerator α. α is a function of the equivalent plastic strain or an external variable input as a load curve in PTAB6. By default, the load curve is assumed to depend on equivalent plastic strain. If IMP on *LOAD_EXTERNAL_VARIABLE references the accelerator property index, which is 1, the load curve depends on the external variable instead.

*MAT_PIECEWISE_LINEAR_PLASTIC_THERMAL

This is Material Type 255, an isotropic elastoplastic material with thermal properties. It can be used for both explicit and implicit analyses. Young's modulus and Poisson's ratio can depend on the temperature by defining two load curves. Moreover, the yield stress in tension and compression are given as load curves for different temperatures by using two tables. The thermal coefficient of expansion can be given as a constant ALPHA or as a load curve LALPHA. A positive curve ID for LALPHA models the instantaneous thermal coefficient, whereas a negatives curve ID models the thermal coefficient relative to a reference temperature, TREF. The strain rate effects are modelled with the Cowper-Symonds rate model with the parameters C and P on Card 1. Failure can be based on effective plastic strain or using the *MAT_ADD_EROSION keyword.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	C	P	FAIL	TDEL
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	TABIDC	TABIDT	LALPHA		VP			
Type	I	I	I		F			

Card 3	1	2	3	4	5	6	7	8
Variable	ALPHA	TREF						
Type	F	F						

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density

VARIABLE	DESCRIPTION
E	Young's modulus: LT.0.0: |E| is a load curve ID where E is given as a function of temperature, T. The curve consists of (T, E) data pairs. GT.0.0: Constant
PR	Poisson's ratio. LT.0.0: |PR| is a load curve ID for Poisson's ratio as a function of temperature. GT.0.0: Constant
C	Strain rate parameter. See Remark 1.
P	Strain rate parameter. See Remark 1.
FAIL	Effective plastic strain when the material fails. User defined failure subroutine, matusr_24 in dyn21.F, is called to determine failure when FAIL < 0. Note that for solids the *MAT_ADD_EROSION can be used for additional failure criteria.
TDEL	A time step less than TDEL is not allowed. A step size less than TDEL will trigger automatic element deletion. This option is ignored for implicit analyses.
TABIDC	Table ID for yield stress in compression; see Remark 2.
TABIDT	Table ID for yield stress in tension; see Remark 2.
LALPHA	Load curve ID for thermal expansion coefficient as a function of temperature: GT.0: The instantaneous thermal expansion coefficient based on the following formula: $$d\varepsilon_{ij}^{\text{thermal}} = \alpha(T)dT\delta_{ij}$$ LT.0: The thermal coefficient is defined relative a reference temperature TREF, such that the total thermal strain is given by: $$\varepsilon_{ij}^{\text{thermal}} = \alpha(T)(T - T_{\text{ref}})\delta_{ij}$$ With this option active, ALPHA is ignored.
VP	Formulation for rate effects; see Remarks 1 and 2. EQ.0.0: effective total strain rate (default)

VARIABLE	DESCRIPTION
	NE.0.0: effective plastic strain rate
ALPHA	Coefficient of thermal expansion
TREF	Reference temperature, which is required if and only if LALPHA is given with a negative load curve ID

Remarks:

1. **Strain rate effects.** The strain rate effect is modelled by using the Cowper and Symonds model which scales the yield stress according to the factor

$$1 + \left(\frac{\dot{\varepsilon}_{\mathrm{eff}}}{C}\right)^{1/P}$$

where $\dot{\varepsilon}_{\mathrm{eff}} = \sqrt{\mathrm{tr}(\dot{\boldsymbol{\varepsilon}}\dot{\boldsymbol{\varepsilon}}^{\mathrm{T}})}$ is the Euclidean norm of the total strain rate tensor if VP = 0 (default), otherwise $\dot{\varepsilon}_{\mathrm{eff}} = \dot{\varepsilon}^{p}_{\mathrm{eff}}$.

2. **Yield stress tables.** The dependence of the yield stresses on the effective plastic strains is given in two tables.

   a) TABIDC gives the behaviour of the yield stresses in compression

   b) TABIDT gives the behaviour of the yield stresses in tension.

   The table indices consist of temperatures, and at each temperature a yield stress curve must be defined.

   Both TABIDC and TABIDT can be 3D tables, in which temperatures indexes the main table and strain rates are defined as values for the sub tables with hardening curves as targets for those strain rates. If the same yield stress should be used in both tension and compression, only one table needs to be defined and the same TABID is put in position 1 and 2 on Card 2. If VP = 0, effective total strain rates are used in the 3D tables, otherwise plastic strain rates.

3. **History variables.** Two history variables are added to the d3plot file, the Young's modulus and the Poisson's ratio, respectively. They can be requested through the *DATABASE_EXTENT_BINARY keyword.

4. **Nodal temperatures.** Nodal temperatures must be defined by using a coupled analysis or some other way to define the temperatures, such as *LOAD_THERMAL_VARIABLE or *LOAD_THERMAL_LOAD_CURVE.

*MAT_AMORPHOUS_SOLIDS_FINITE_STRAIN

This is Material Type 256, an isotropic elastic-viscoplastic material model intended to describe the behaviour of amorphous solids such as polymeric glasses. The model accurately captures the hardening-softening-hardening sequence and the Bauschinger effect experimentally observed at tensile loading and unloading respectively. The formulation is based on hyperelasticity and uses the multiplicative split of the deformation gradient F which makes it naturally suitable for both large rotations and large strains. Stress computations are performed in an intermediate configuration and are therefore preceded by a pull-back and followed by a push-forward. The model was originally developed by Anand and Gurtin [2003] and implemented for solid elements by Bonnaud and Faleskog [2019].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	G	MR	LL	NU0	M
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	ALPHA	H0	SCV	B	ECV	G0	S0	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
K	Bulk modulus
G	Shear modulus
MR	Kinematic hardening parameter, μ_R (see Remark 1)
LL	Kinematic hardening parameter, λ_L (see Remark 1)
NU0	Creep parameter, ν_0 (see Remark 2)
M	Creep parameter, m (see Remark 2)

VARIABLE	DESCRIPTION
ALPHA	Creep parameter, α (see Remark 2)
H0	Isotropic hardening parameter, h_0 (see Remark 3)
SCV	Isotropic hardening parameter, s_{cv} (see Remark 3)
B	Isotropic hardening parameter, b (see Remark 3)
ECV	Isotropic hardening parameter, η_{cv} (see Remark 3)
G0	Isotropic hardening parameter, g_0 (see Remark 3)
S0	Isotropic hardening parameter, s_0 (see Remark 3)

Remarks:

1. **Kinematic Hardening.** Kinematic hardening gives rise to the second hardening occurrence in the hardening-softening-hardening sequence. The constants μ_R and λ_L enter the back stress, μB (where B is the left Cauchy-Green deformation tensor), through the function μ according to:

$$\mu = \mu_R \left(\frac{\lambda_L}{3\lambda^p}\right) L^{-1} \left(\frac{\lambda^p}{\lambda_L}\right) \ , \tag{256.1}$$

where $\lambda^p = \frac{1}{\sqrt{3}} \sqrt{tr(B^p)}$ and B^p is the plastic part of the left Cauchy-Green deformation tensor and L is the Langevin function defined by:

$$L(X) = \coth(X) - X^{-1} \ .$$

2. **Creep.** This material model assumes plastic incompressibility. Nevertheless in order to account for the different behaviours in tension and compression a Drucker-Prager law is included in the creep law according to:

$$\nu^p = \nu_0 \left(\frac{\bar{\tau}}{s + \alpha\pi}\right)^{1/m} \ , \tag{256.2}$$

where ν^p is the equivalent plastic shear strain rate, $\bar{\tau}$ is the equivalent shear stress, s is the internal variable defined below and $-\pi$ is the hydrostatic stress.

3. **Isotropic Hardening.** Isotropic hardening gives rise to the first hardening occurrence in the hardening-softening-hardening sequence. Two coupled internal variables are defined: the resistance to plastic flow, s, and the local free volume, η. Their evolution equations are:

$$\dot{s} = h_0 \left[1 - \frac{s}{\tilde{s}(\eta)}\right] \nu^p \tag{256.3}$$

$$\dot{\eta} = g_0 \left(\frac{s}{s_{cv}} - 1 \right) \nu^p \tag{256.4}$$

$$\tilde{s}(\eta) = s_{cv}[1 + b(\eta_{cv} - \eta)] \tag{256.5}$$

4. **Typical Material Parameters.** Typical material parameters values are given in [1] for Polycarbonate:

Variable	Value
K	2.24 GPa
G	0.857 GPA
MR	11.0 MPa
LL	1.45
NUO	0.0017 s^{-1}
M	0.011
ALPHA	0.08
H0	2.75 GPa
SCV	24.0 MPa
B	825
ECV	0.001
G0	0.006
S0	20.0 MPa

References:

[1] Anand, L., Gurtin, M.E., 2003, "A theory of amorphous solids undergoing large deformations, with application to polymeric glasses," *International Journal of Solids and Structures*, 40, pp. 1465-1487.

[2] Bonnaud, E.L., Faleskog, J., 2019, "Explicit, fully implicit and forward gradient numerical integration of a hyperelasto-viscoplastic constitutive model for amorphous polymers undergoing finite deformation," *Computational Mechanics*, 64, pp.1389–1401.

*MAT_NON_QUADRATIC_FAILURE

This is Material Type 258. This is an elastic-(visco)plastic material with a non-quadratic yield surface where isotropic work hardening is included. A ductile failure model is included in the form of a damage indicator model. The extended Cockcroft-Latham criterion is used to represent the dependence of the failure strain on stress state; see Gruben et. al. [2012]. Mesh dependency of the failure strain is damped out using a regularization scheme based on the deformation mode of the shell element. A more detailed description of this model can be found in the paper by Costas et al. [2018]. The material is available for shell elements only.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	A	KSI	
Type	A	F	F	F	F	F	F	
Default	none	none	none	none	none	none	none	

Card 2	1	2	3	4	5	6	7	8
Variable	THETA1	Q1	THETA2	Q2	THETA3	Q3		
Type	F	F	F	F	F	F		
Default	none	none	none	none	none	none		

Card 3	1	2	3	4	5	6	7	8
Variable	CS	PDOTS						
Type	F	F						
Default	none	none						

Card 4	1	2	3	4	5	6	7	8
Variable	DCRIT	WCB	WCL	WCS	CC	PHI	GAMMA	THICK
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young’s modulus
PR	Poisson’s ratio
SIGY	Initial yield stress
A	Exponent of Hershey yield criterion
KSI	Coefficient governing critical strain increment for substepping
THETA*i*	Initial hardening modulus of R_i
Q*i*	Saturation value of R_i
CS	Rate sensitivity of flow stress
PDOTS	Reference strain rate
DCRIT	Critical damage
WCB	Constant defining the damage evolution
WCL	Constant defining the damage evolution
WCS	Constant defining the damage evolution
CC	Constant defining the damage evolution
PHI	Constant defining the damage evolution

VARIABLE	DESCRIPTION
GAMMA	Constant defining the damage evolution
THICK	Element thickness if using shell formulation 16. Since releases R12.1 and R13.0, setting THICK to zero causes the thickness to be taken from *SECTION_SHELL or *ELEMENT_SHELL_THICKNESS.

Remarks:

The yield function is defined on the form

$$f = \varphi(\boldsymbol{\sigma}) - (\sigma_0 + R)$$

where $\sigma_{eq} \equiv \varphi(\boldsymbol{\sigma})$ is the equivalent stress, σ_0 is the initial yield stress, and R is the isotropic hardening variable, which is a function of the equivalent plastic strain p. The equivalent stress is defined as

$$\varphi(\boldsymbol{\sigma}) = \left[\frac{1}{2}\{|\sigma_1 - \sigma_2|^a + |\sigma_2 - \sigma_3|^a + |\sigma_3 - \sigma_1|^a\}\right]^{\frac{1}{a}}$$

where $\sigma_1 \geq \sigma_2 \geq \sigma_3$ are the ordered principal stresses. The isotropic hardening variable is expressed as

$$R(p) = \sum_{i=1}^{3} R_i\,(p) = \sum_{i=1}^{3} Q_i \left(1 - \exp\left(-\frac{\theta_i}{Q_i} p\right)\right)$$

where Q_i and θ_i are in turn the saturation value and initial hardening modulus of the hardening variable R_i. As $p \to \infty$, R attains its saturation value R_{sat}, given by

$$R_{\mathrm{sat}} = \sum_{i=1}^{3} Q_i \ .$$

Note that you can provide nonzero initial values o fplastic strain with *INITIAL_-STRESS_SHELL which initializes the simulation with nonzero $R_i(p)$ at $t = 0$.

Rate-dependent plasticity is described by a fully viscoplastic formulation. If rate dependence is invoked, the equivalent stress σ_{eq} is constrained by the viscoplastic relation

$$\sigma_{\mathrm{eq}} = (\sigma_0 + R(p))\left(1 + \frac{\dot{p}}{\dot{p}_\sigma}\right)^{C_\sigma} \quad \text{for } f > 0$$

in the plastic domain. The parameters C_σ and $\dot{p}_\sigma$ govern the rate dependence of the material, where $\dot{p}_\sigma$ is a reference strain rate.

The uncoupled version of the Extended Cockcroft-Latham (ECL) criterion is applied here to define damage evolution

$$\dot{D} = \frac{\varphi(\boldsymbol{\sigma})}{W_c}\left\langle \phi \frac{\sigma_1}{\varphi(\boldsymbol{\sigma})} + (1-\phi)\frac{\sigma_1 - \sigma_3}{\varphi(\boldsymbol{\sigma})}\right\rangle^{\gamma} \dot{p}$$

where D is the damage variable and W_c, ϕ, and γ are parameters governing the damage evolution and its dependence of the stress triaxiality and the Lode parameter. By setting $\phi = \gamma = 1$, we get the Cockcroft-Latham criterion:

$$\dot{D} = \frac{\langle \sigma_1 \rangle}{W_c}\dot{p}$$

where W_c is the Cockcroft-Latham (CL) fracture parameter.

In simulations with shell elements, the CL fracture parameter W_c is defined by

$$W_c = \Omega W_c^b + (1-\Omega)W_c^m$$

where W_c^b is the CL parameter in pure bending, W_c^m is a mesh-dependent CL parameter in membrane loading, and Ω is a bending indicator given as

$$\Omega = \frac{1}{2}\frac{\left|\dot{\varepsilon}_{3p}^{+} - \dot{\varepsilon}_{3p}^{-}\right|}{\max\left(\left|\dot{\varepsilon}_{3p}^{+}\right|, \left|\dot{\varepsilon}_{3p}^{-}\right|\right)}$$

where $\dot{\varepsilon}_{3p}^{+}$ and $\dot{\varepsilon}_{3p}^{-}$ are the plastic thickness strain rates on the two sides of the shell element. Thus, the bending indicator is $\Omega = 1$ for pure bending ($\dot{\varepsilon}_{3p}^{-} = -\dot{\varepsilon}_{3p}^{+}$) and $\Omega = 0$ ($\dot{\varepsilon}_{3p}^{-} = \dot{\varepsilon}_{3p}^{+}$) for pure membrane loading. The mesh-dependent CL parameter for membrane loading is defined by

$$W_c^m = W_c^l + \left(W_c^s - W_c^l\right)\exp\left(-c\left(\frac{l_e}{t_e} - 1\right)\right)$$

where W_c^l, W_c^s, and c are parameters, l_e is the characteristic size of the shell element, and t_e is the thickness of the shell element.

MAT_STOUGHTON_NON_ASSOCIATED_FLOW_*{OPTION}

This is Material Type 260A. This material model is implemented based on non-associated flow rule models (Stoughton 2002 and 2004). Strain rate sensitivity can be included using a load curve. This model applies to both shell and solid elements. It is available for explicit in both MPP and SMP.

Available options include:

<BLANK>

XUE

The option XUE is available for solid elements only.

Card Summary:

Card 1. This card is required.

MID	RO	E	PR	R00	R45	R90	SIG00

Card 2. This card is required.

SIG45	SIG90	SIG_B	LCIDS	LCIDV	SCALE		

Card 3. This card is included for the XUE keyword option.

EF0	PLIM	Q	GAMA	M	BETA		

Card 4. This card is required.

AOPT							

Card 5. This card is required.

XP	YP	ZP	A1	A2	A3		

Card 6. This card is required.

V1	V2	V3	D1	D2	D3		

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	R00	R45	R90	SIG00
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	1.0	R00	R00	Rem 1

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's Modulus
PR	Poisson's ratio
R00, R45, R90	Lankford parameters in rolling (0°), diagonal (45°) and transverse (90°) directions, respectively; determined from experiments. Note if R00, R45, and R90 are not defined or are set to 0.0, then R00 = R45 = R90 = 1.0, which degenerates to the Von-Mises yield.
SIG00	Initial yield stress from uniaxial tension tests in rolling (0°) direction

Card 2	1	2	3	4	5	6	7	8
Variable	SIG45	SIG90	SIG_B	LCIDS	LCIDV	SCALE		
Type	F	F	F	I	I	F		
Default	Rem 1	Rem 1	Rem 1	none	none	1.0		

VARIABLE	DESCRIPTION
SIG45	Initial yield stress from uniaxial tension tests in diagonal (45°) direction

VARIABLE	DESCRIPTION
SIG90	Initial yield stress from uniaxial tension tests in transverse (90°) directions
SIG_B	Initial yield stress from equi-biaxial stretching tests
LCIDS	ID of load curve giving stress as a function of strain hardening behavior from a uniaxial tension test along the rolling direction
LCIDV	ID of a load curve defining stress scale factors as a function strain rates, determined from experiments. An example of the curve can be found in Figure M260A-2. To know which values are used, the strain rates and strain rate scale factors are stored in the d3plot file as history variables #5 and #6, respectively.
SCALE	Parameter for speeding up the simulation while equalizing the strain rate effect. It is useful in cases where the pulling speed or punch speed is slow. See Remark 2.

XUE Card. This card is included for the XUE keyword option.

Card 3	1	2	3	4	5	6	7	8
Variable	EF0	PLIM	Q	GAMA	M	BETA		
Type	F	F	F	F	F	F		
Default	none	none	none	none	none	0.0		

VARIABLE	DESCRIPTION
EF0, PLIM, Q, GAMA, M, BETA	Material parameters for the option XUE. The parameter k in the original paper is assumed to be 1.0. Note the default BETA value of 0.0 means no progressive weakening damage. For details, refer to Xue, L., Wierzbicki, T.'s 2009 paper "*Numerical simulation of fracture mode transition in ductile plates*" in the *International Journal of Solids and Structures*. See Remark 3.

Card 4	1	2	3	4	5	6	7	8
Variable	AOPT							
Type	I							
Default	none							

VARIABLE	DESCRIPTION
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):

EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.

EQ.1.0: Locally orthotropic with material axes determined by a point, P, in space and the global location of the element center; this is the **a**-direction. This option is for solid elements only.

EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR

EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element.

EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, P, which define the centerline axis. This option is for solid elements only.

VARIABLE	DESCRIPTION
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*).

Card 5	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
XP, YP, ZP	Coordinates of point *p* for AOPT = 1 and 4
A1, A2, A3	Components of vector **a** for AOPT = 2

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3 and 4
D1, D2, D3	Components of vector **d** for AOPT = 2

The Stoughton Non-Associated Flow Rule:

In a non-associated flow rule, the material yield function is not equal to the plastic flow potential. According to Thomas B. Stoughton's paper titled "*A non-associated flow rule for sheet metal forming*" in 2002 International Journal of Plasticity 18, 687-714, and "*A pressure-sensitive yield criterion under a non-associated flow rule for sheet metal forming*" in 2004 International Journal of Plasticity 20, 705-731, the plastic potential is given by:

$$\overline{\sigma}_p = \sqrt{\sigma_{11}^2 + \lambda_p \sigma_{22}^2 - 2\nu_p \sigma_{11}\sigma_{22} + 2\rho_p \sigma_{12}^2}$$

where σ_{ij} is the stress tensor component. Here,

$$\lambda_p = \frac{1+\frac{1}{r_{90}}}{1+\frac{1}{r_0}}\ ,$$

$$\nu_p = \frac{r_0}{1+r_0}\ ,$$

$$\rho_p = \frac{\frac{1}{r_0}+\frac{1}{r_{90}}}{1+\frac{1}{r_0}}\left(\frac{1}{2}+r_{45}\right)\ .$$

r_0, r_{45}, and r_{90} are Lankford parameters in the rolling (0°), the diagonal (45°) and the transverse (90°) directions, respectively.

The yield function is given by:

$$\bar{\sigma}_y = \sqrt{\sigma_{11}^2 + \lambda_y \sigma_{22}^2 - 2\nu_y \sigma_{11}\sigma_{22} + 2\rho_y \sigma_{12}^2}\ ,$$

where

$$\lambda_y = \left(\frac{\sigma_0}{\sigma_{90}}\right)^2,$$

$$\nu_y = \frac{1}{2}\left[1 + \lambda_y - \left(\frac{\sigma_0}{\sigma_b}\right)^2\right]\ ,$$

$$\rho_y = \frac{1}{2}\left[\left(\frac{2\sigma_0}{\sigma_{45}}\right)^2 - \left(\frac{\sigma_0}{\sigma_b}\right)^2\right]\ .$$

Here σ_0, σ_{45}, σ_{90} are the initial yield stresses from uniaxial tension tests in the rolling, diagonal, and transverse directions, respectively. σ_b is the initial yield stress from an equi-biaxial stretching test.

Remarks:

1. **Defaults for SIG00, SIG45, SIG90, and SIG_B.** If not specified, SIG00, SIG45, SIG90, and SIG_B default to the first stress value in LCIDS. Note that if all four values are not specified, the non-associated flow rule degenerates to the associated flow rule.

2. **SCALE.** The variable SCALE is very useful in speeding up the simulation while equalizing the strain rate effect. For example, if the pulling speed is 15 mm/s but running the simulation at this speed will take a long time, you can increase the pulling speed to 500 mm/s while setting SCALE to 0.03. The latter settings will give the same results with the benefit of greatly reduced computational time (see Figures M260A-3 and M260A-4). Note that the increased absolute value (within a reasonable range) of mass scaling, $-1.0 \times$ dt2ms, frequently used in forming simulation does not affect the strain rates, as shown in the Figure M260A-5. See examples in Verification.

3. **XUE Parameters.** The following table lists variable names used in this material model with the corresponding symbols in Xue et al [2009] for the option XUE:

EF0	PLIM	Q	GAMA	M
ε_{f0}	P_{lim}	q	γ	m

4. **History Variables.** The history variables output to d3plot for this material depend on whether the XUE keyword option is used and whether this material is used with an EOS. When the XUE option is used, damage accumulation is output to d3plot. It is history variable #1 without an EOS and history variable #5 with an EOS. The value ranges from 0.0 to 1.0. When XUE is not used, history variable #5 is strain rates and history variable #6 is strain rate scale factors.

Verification:

Uniaxial tension tests were done on a single shell element as shown in Figure M260A-1. Strain rate effect LCIDV is input as shown in Figure M260A-2. In Figure M260A-3, pulling stress as a function of strain from various test conditions are compared with input stress-strain curve A. In summary, using the parameter SCALE, the element can be pulled much faster (500 mm/s vs. 15 mm/s) but achieve the same stress vs. strain results, the same strain rates (history variable #5), and the same strain rate scale factor (history variable #6 in Figure M260A-4). Simulation speed can be improved further with increased mass scaling $(-1.0 \times \text{dt2ms})$ without affecting the results; see Figure M260A-5.

A partial keyword input is provided below, for the case with pulling speed of 500 mm/s, strain hardening curve ID of 100, LCIDV curve ID of 105, and strain rate scale factor of 0.03.

```
*KEYWORD
*parameter_expression
R endtime     0.012
R v           500.0
*CONTROL_TERMINATION
$   ENDTIM    ENDCYC     DTMIN    ENDNEG    ENDMAS
&endtime
*MAT_STOUGHTON_NON_ASSOCIATED_FLOW
$#     mid        Ro         E        PR       R00       R45       R90     SIG00
         1 7.8000E-9   2.10E05  0.300000       1.1       1.2       1.3     311.0
$    SIG45     SIG90     SIG_B     LCIDS     LCIDV     SCALE
     305.4     321.1     290.3       100       105      0.03
$     AOPT
         3
$       XP        YP        ZP        A1        A2        A3

$       V1        V2        V3        D1        D2        D3      BETA
                 1.0
*DEFINE_CURVE
       100
         0.00000E+00         0.30130E+03
         0.10000E-01         0.42295E+03
         0.20000E-01         0.47991E+03
```

```
         0.30000E-01          0.52022E+03
         0.40000E-01          0.55126E+03
         0.50000E-01          0.57615E+03
                                           ⋮
*DEFINE_CURVE
       105
         0.00000E+00          0.10000E+01
         0.10000E+00          0.10608E+01
         0.50000E+00          0.10828E+01
         0.10000E+01          0.10923E+01
                                           ⋮
*END
```

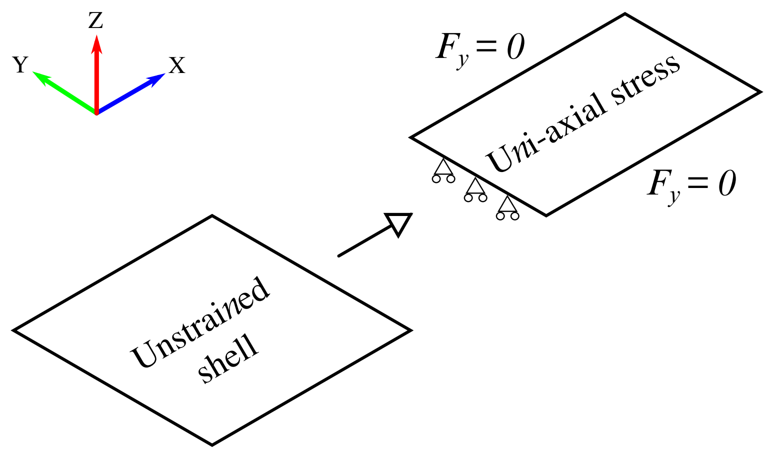


Figure M260A-1. Uniaxial tension tests on a single shell element.

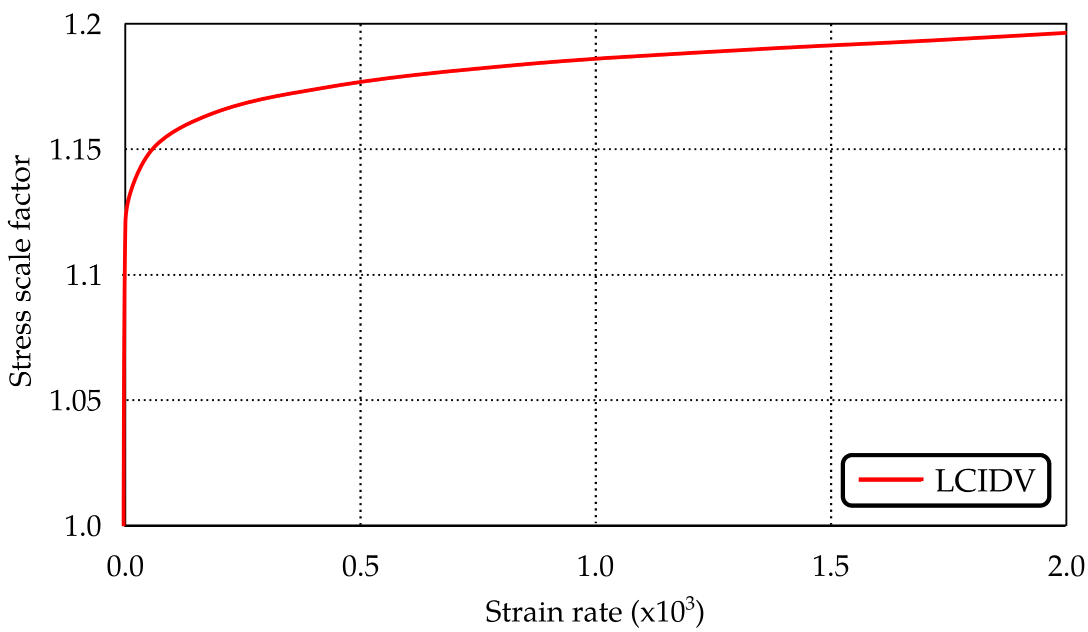
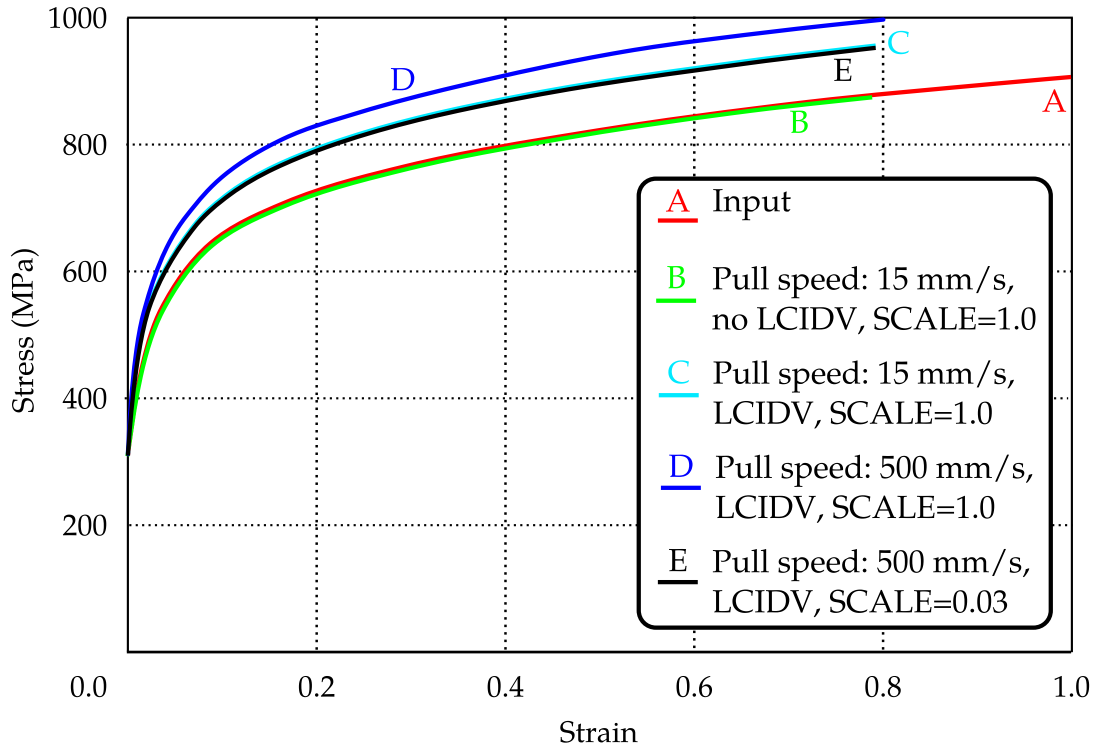


Figure M260A-2. Input LCIDV



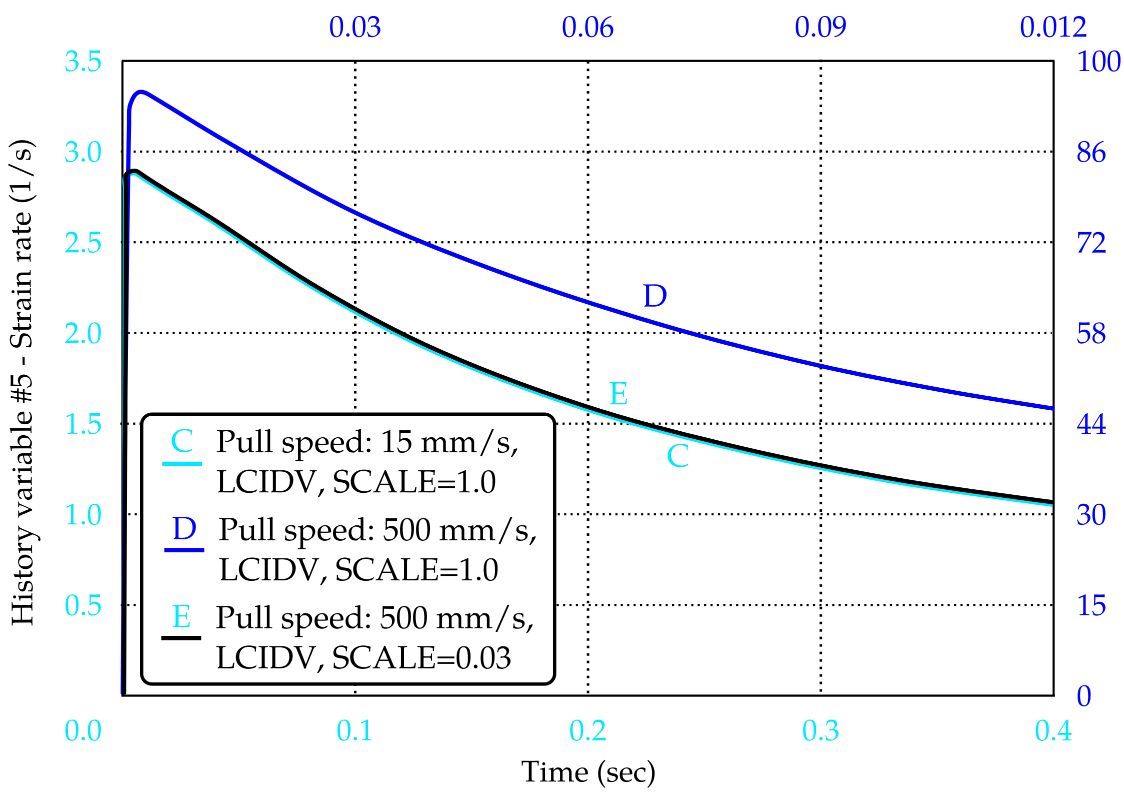


Figure M260A-3. Recovered stress-strain curve (top) and strain rates (bottom) under various conditions shown.

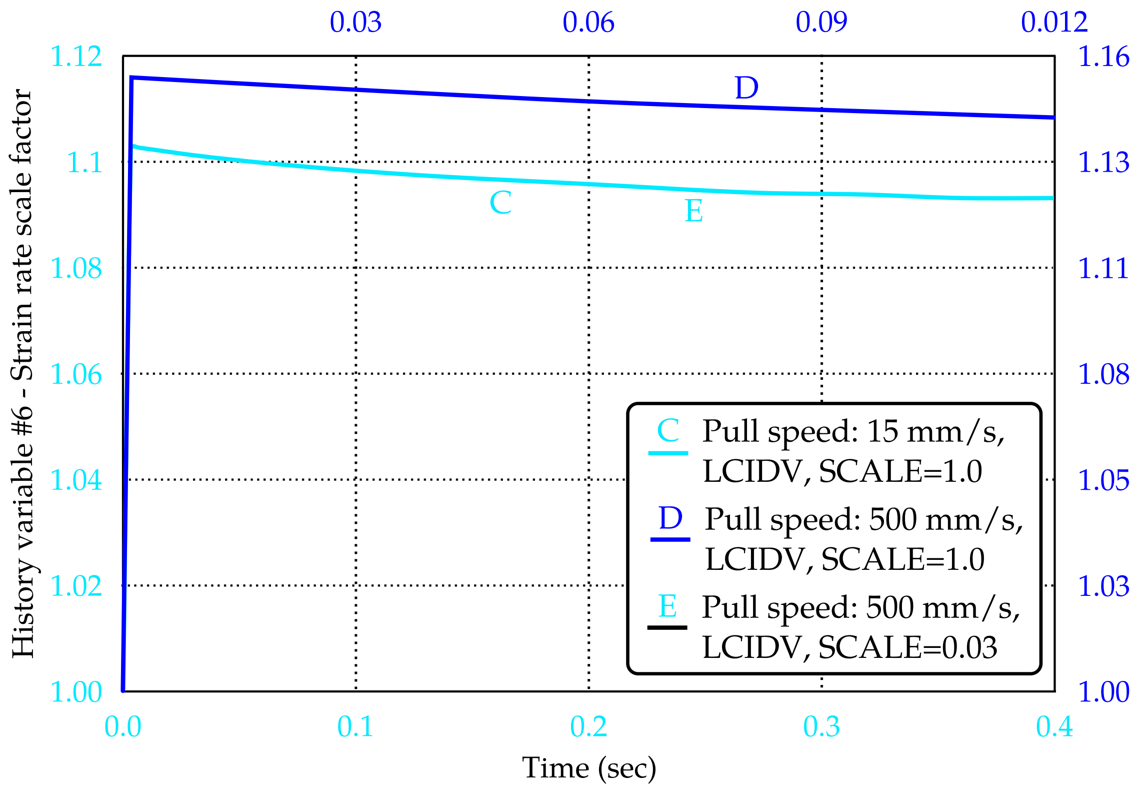


Figure M260A-4. Recovered strain rate scale factors under various conditions shown.

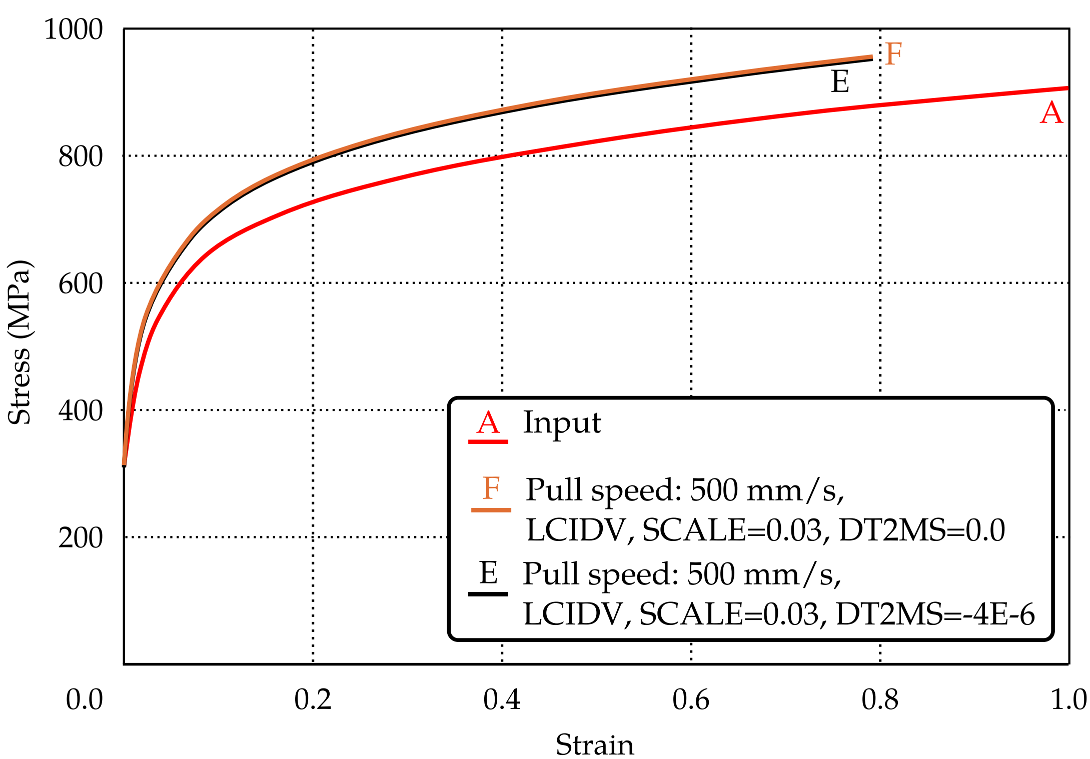


Figure M260A-5. Effect of mass scaling (-1.0*dt2ms).

MAT_MOHR_NON_ASSOCIATED_FLOW_*{OPTION}

This is Material Type 260B. This material model is implemented based on the papers by Mohr, D., et al. (2010) and Roth, C.C. and Mohr, D. (2014) [1, 2]. The Johnson-Cook plasticity model which includes strain hardening, strain rate hardening, and temperature softening is modified with a mixed Swift-Voce strain hardening function coupled with a non-associated flow rule. For certain Advanced High Strength Steels (AHSS), the non-associated flow rule accounts for the difference between directional dependency of the *r*-values (planar anisotropic) and the planar isotropic material response. A ductile fracture model is included based on Hosford-Coulomb fracture initiation model. This model applies to shell elements only.

Available options include:

<BLANK>

XUE

Card Summary:

Card 1. This card is required.

MID	RO	E	PR	P12	P22	P33	G12

Card 2. This card is required.

G22	G33	LCIDS	LCIDV	LCIDT	LFLD	LFRAC	W0

Card 3. This card is required.

A	B0	GAMMA	C	N	SCALE	SIZE0	

Card 4. This card is required.

TREF	TMELT	M	ETA	CP	TINI	DEPSO	DEPSAD

Card 5. This card is included if the XUE keyword option is used.

EF0	PLIM	Q	GAMA	M			

Card 6. This card is required.

AOPT							

Card 7. This card is required.

			A1	A2	A3		

Card 8. This card is required.

V1	V2	V3					

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	P12	P22	P33	G12
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	-0.5	1.0	3.0	-0.5

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's Modulus
PR	Poisson's ratio
P12, P22, P33	Yield function parameters, defined by Lankford parameters in rolling (0°), diagonal (45°) and transverse (90°) directions, respectively; see Remark 1.
G12	Plastic flow potential parameters, defined by Lankford parameters in rolling (0°), diagonal (45°) and transverse (90°) directions; see Remark 1.

Card 2	1	2	3	4	5	6	7	8
Variable	G22	G33	LCIDS	LCIDV	LCIDT	LFLD	LFRAC	W0
Type	F	F	I	I	I	I	I	F
Default	1.0	3.0	none	none	none	0	none	none

VARIABLE	DESCRIPTION
G22, G33	Plastic flow potential parameters, defined by Lankford parameters in rolling (0°), diagonal (45°) and transverse (90°) directions; see Remark 1.
LCIDS	Load curve ID defining stress as a function of strain hardening from a uniaxial tension test; it must be along the rolling direction. Also see Remark 2.
LCIDV	Load curve ID defining stress scale factors as a function of strain rates (see Figure M260B-1 middle) as determined from experiments. Strain rates are stored in history variable #5. Strain rate scale factors are stored in history variable #6. To output these history variables to d3plot, set NEIPS to at least "6" in *DATABASE_EXTENT_BINARY. Also see Remark 2
LCIDT	Load curve ID defining stress scale factors as a function of temperature in Kelvin (see Figure M260B-1 bottom) as determined from experiments. Temperatures are stored in history variable #4. Temperature scale factors are stored in history variable #7. To output these history variables to d3plot, set NEIPS to at least "7" in *DATABASE_EXTENT_BINARY. Also see Remark 2.
LFLD	Load curve ID defining a traditional Forming Limit Diagram for linear strain paths
LFRAC	Load curve ID defining a fracture limit curve. Leave this field empty if fields A, B0, GAMMA, C, and N are defined. However, if this field is defined, fields A, B0, GAMMA, C, and N will be ignored even if they are defined.
W0	Neck (FLD failure) width which typically is the blank thickness

Card 3	1	2	3	4	5	6	7	8
Variable	A	B0	GAMMA	C	N	SCALE	SIZE0	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	1.0	none	

VARIABLE	DESCRIPTION
A, B0, GAMMA, C, N	Material parameters (a, b_0, γ, c, n) for the rate-dependent Hosford-Coulomb fracture initiation model; see Remark 3. Ignored if LFRAC is defined.
SCALE	This field can be used to speed up the simulation while equalizing the strain rate effect, which is useful especially in cases where the pulling speed or punch speed is slow. For example, if the pulling speed is 15 mm/s but running the simulation at this speed will take a long time, the pulling speed can be increased to 500 mm/s while "SCALE" can be set to 0.03, giving the same results as those from 15 mm/s with greatly reduced computational time; see examples and Figures in *MAT_260A for details. Furthermore, the increased absolute value (within a reasonable range) of mass scaling -1.0 × dt2ms frequently used in forming simulation does not affect the strain rates, as shown in the examples and Figures in *MAT_260A.
SIZE0	Fracture gauge length used in an experimental measurement, typically between 0.2~0.5 mm

Card 4	1	2	3	4	5	6	7	8
Variable	TREF	TMELT	M	ETA	CP	TINI	DEPSO	DEPSAD
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

VARIABLE	DESCRIPTION
TREF	Material parameters for strain softening effect due to temperature. TINI is the initial temperature. See Remark 2 for other parameters' definitions.
TMELT	Reference temperature for modified Johnson-Cook Plasticity Model; see Remark 2.
M	Exponent coefficient, m, for modified Johnson-Cook Plasticity Model; see Remark 2.
ETA	Taylor-Quinney coefficient, η_k; see Remark 2.

VARIABLE	DESCRIPTION
CP	Heat capacity, C_p; see Remark 2.
TINI	Initial temperature; see Remark 2.
DEPS0	$\dot{\varepsilon}_{it}/\dot{\varepsilon}_0$; see Remark 2.
DEPSAD	$\dot{\varepsilon}_a$; see Remark 2.

XUE Card. This card is included if the XUE keyword option is used.

Card 5	1	2	3	4	5	6	7	8
Variable	EF0	PLIM	Q	GAMA	M			
Type	F	F	F	F	F			
Default	none	none	none	none	none			

VARIABLE	DESCRIPTION
EF0, PLIM, Q, GAMA, M	Material parameters for the option XUE. The parameter k in the original paper is assumed to be 1.0. For details, refer to Xue, L. and Wierzbicki, T.'s 2009 paper "Numerical simulation of fracture mode transition in ductile plates" in the *International Journal of Solids and Structures* [4].

Card 6	1	2	3	4	5	6	7	8
Variable	AOPT							
Type	F							
Default	none							

Card 7	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		
Default				none	none	none		

Card 8	1	2	3	4	5	6	7	8
Variable	V1	V2	V3					
Type	F	F	F					
Default	none	none	none					

VARIABLE	DESCRIPTION
AOPT	Material axes option (see *MAT_*OPTION* TROPIC_ELASTIC for a more complete description): EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA. EQ.2.0: globally orthotropic with material axes determined by the vector **a** for shells, as with *DEFINE_COORDINATE_VECTOR. EQ.3.0: locally orthotropic material axes determined by a line in the plane of the element defined by the cross product of the vector **v** with the element normal LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE__COORDINATE_VECTOR).
A1, A2, A3	Components of vector **a** for AOPT = 2
V1, V2, V3	Components of vector **v** for AOPT = 3

Remarks:

1. **Non-associated Flow Rule.** Referring to [1] and [2], Hill's 1948 quadratic yield function is written as:

$$f(\boldsymbol{\sigma}, k) = \bar{\sigma} - k = 0 \ ,$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor and $\bar{\sigma}$ is the equivalent stress, defined by:

$$\bar{\sigma} = \sqrt{(\mathbf{P}\boldsymbol{\sigma}) \bullet \boldsymbol{\sigma}} \ .$$

P is a symmetric positive-definite matrix defined through three independent parameters, P_{12}, P_{22}, and P_{33}:

$$\mathbf{P} = \begin{bmatrix} 1 & \mathrm{P}_{12} & 0 \\ \mathrm{P}_{12} & \mathrm{P}_{22} & 0 \\ 0 & 0 & \mathrm{P}_{33} \end{bmatrix} \ .$$

The flow rule, which defines the incremental plastic strain tensor, is written as follows:

$$d\boldsymbol{\varepsilon}_p = d\delta \frac{\partial g(\boldsymbol{\sigma})}{\partial \boldsymbol{\sigma}} \ ,$$

where $d\delta$ is a scalar plastic multiplier. The plastic potential function $g(\boldsymbol{\sigma})$ can be defined as a quadratic function in stress space:

$$g(\boldsymbol{\sigma}) = \sqrt{(\mathbf{G}\boldsymbol{\sigma}) \bullet \boldsymbol{\sigma}}$$

with,

$$\mathbf{G} = \begin{bmatrix} 1 & G_{12} & 0 \\ \mathrm{G}_{12} & \mathrm{G}_{22} & 0 \\ 0 & 0 & \mathrm{G}_{33} \end{bmatrix} \ .$$

When **P**≠**G,** the flow rule is non-associated. The associated flow rule is recovered if $\mathbf{P} = \mathbf{G}$. For example, **P** can represent an isotropic von-Mises yield surface by setting $P_{11} = P_{22} = 1.0$, $P_{12} = -0.5$, and $P_{33} = 3.0$. **G** can represent an orthotropic plastic flow potential by setting:

$$\begin{aligned} G_{12} &= -\frac{r_0}{1 + r_0} \ , \\ G_{22} &= \frac{r_0(1 + r_{90})}{r_{90}(1 + r_0)} \ , \\ G_{33} &= \frac{(1 + 2r_{45})(r_0 + r_{90})}{r_{90}(1 + r_0)} \ . \end{aligned}$$

Here r_0, r_{45}, and r_{90} are the Lankford coefficients in the rolling, diagonal and transverse directions, respectively. Experiments have shown on the stress level, some AHSS (Advanced High Strength Steel), e.g., DP590, and TRIP780 show strong directional dependency for the r-values, while nearly the same stress-strain curves have been measured in all directions. The directional dependency of r-values suggests planar anisotropy while the material response for the stress is planar isotropic, which is the main reason to employ the non-associated flow rule.

2. **A Modified Johnson-Cook Plasticity Model with Mixed Swift-Voce Hardening.** The Johnson-Cook plasticity model (1983) multiplicatively decomposes the deformation resistance into three functions representing the effects of strain hardening, strain rate, and temperature. The Johnson-Cook model is modified to include hardening saturation with a mixed Swift-Voce hardening law (Sung et al, 2010 [3]), which gives a better description of the hardening at large strain levels, thus improving the prediction of the necking and post-necking response of metal sheet:

$$\sigma_y = \left(\alpha\left(A\left(\bar{\varepsilon}_{pl} + \varepsilon_0\right)^n\right) + (1-\alpha)\left(k_0 + Q\left(1 - e^{-\beta\bar{\varepsilon}_{pl}}\right)\right)\right)\left(1 + C\ln\left(\frac{\dot{\bar{\varepsilon}}_{pl}}{\dot{\varepsilon}_0}\right)\right)\left(1 - \left(\frac{T - T_r}{T_m - T_r}\right)^m\right)$$

where $\bar{\varepsilon}_{pl}$ and $\dot{\bar{\varepsilon}}_{pl}$ are effective plastic strain and strain rate, respectively; T_m (TMELT), T_r (TREF) and T are the melting temperature, reference temperature (ambient temperature 293 K) and current temperature, respectively; and m (M) is an exponent coefficient. For other symbols' definitions refer to the aforementioned paper.

To make this material model more general and flexible, three load curves are used to define the three components of the deformation resistance. A load curve (LCIDS) is used to describe the strain hardening:

$$\alpha\left(A\left(\bar{\varepsilon}_{pl} + \varepsilon_0\right)^n\right) + (1-\alpha)\left(k_0 + Q\left(1 - e^{-\beta\bar{\varepsilon}_{pl}}\right)\right)$$

Strain rate is described by a load curve LCIDV (stress scale factor vs. strain rates, Figure M260B-1 middle), which scales the stresses based on the strain rates during a simulation:

$$1 + Cln\left(\frac{\dot{\bar{\varepsilon}}_{pl}}{\dot{\varepsilon}_0}\right)$$

The temperature softening effect is defined by another load curve LCIDT (stress scale factor as a function of temperature, Figure M260B-1 bottom), which scales the stresses based on the temperatures during the simulation:

$$1 - \left(\frac{T - T_r}{T_m - T_r}\right)^m$$

The temperature effect is a self-contained model, meaning it does not require thermal exchange with the environment. It calculates temperatures based on plastic strain and strain rate.

The temperature evolution is determined with:

$$dT = \omega[\dot{\bar{\varepsilon}}_{pl}]\frac{\eta_k}{\rho C_p}\bar{\sigma}d\bar{\varepsilon}_{pl} \ .$$

where η_k (ETA) is the Taylor-Quinney coefficient; ρ (R0) is the mass density; C_p (CP) is the heat capacity; and

$$\omega[\dot{\bar{\varepsilon}}_{pl}] = \begin{cases} 0 & for\ \dot{\bar{\varepsilon}}_{pl} < \dot{\varepsilon}_{it} \\ \dfrac{(\dot{\bar{\varepsilon}}_{pl} - \dot{\varepsilon}_{it})^2(3\dot{\varepsilon}_a - 2\dot{\bar{\varepsilon}}_{pl} - \dot{\varepsilon}_{it})}{(\dot{\varepsilon}_a - \dot{\varepsilon}_{it})^3} & for\ \dot{\varepsilon}_{it} \leq \dot{\bar{\varepsilon}}_{pl} \leq \dot{\varepsilon}_a \\ 1 & for\ \dot{\varepsilon}_a < \dot{\bar{\varepsilon}}_{pl} \end{cases}$$

Here $\dot{\varepsilon}_{it} > 0$ and $\dot{\varepsilon}_a > \dot{\varepsilon}_{it}$ define the limits of the respective domains of isothermal and adiabatic conditions ($\dot{\varepsilon}_a$ = DEPSAD). For simplicity, $\dot{\varepsilon}_{it} = \dot{\varepsilon}_0 \times$ DEPS0.

As shown in a single shell element undergoing uniaxial stretching (see Figure M260B-1), the general effect of LCIDV is to elevate the strain hardening behavior as the strain rate increases (curve "D" in Figure M260B-2 top), while the effect of LCIDT is strain softening as the temperature rises (curve "C" in Figure M260B-2 top). Initially, due to the combined effect of both LCIDV and LCIDT strain hardening may occur before the temperature rises enough to cause strain softening in the model (curve "E" in Figure M260B-2 top). The temperature and strain rates calculated for each element can be viewed with history variables #4 and #5 (curves "C" and "D" in Figure M260B-2 bottom), respectively, while the strain rate scale factors and temperature scale factors can be viewed with history variable #6 and #7, respectively.

3. **Rate-dependent Hosford-Coulomb Fracture Initiation Model.** An extension of the Hosford-Coulomb fracture initiation model is used to account for the effect of strain rate on ductile fracture. The damage accumulation is calculated through history variable #3. When this history variable reaches 1.0, fracture occurs at an equivalent plastic strain, $\bar{\varepsilon}_f$, that is,

$$\int_0^{\varepsilon_f} \frac{d\bar{\varepsilon}_{pl}}{\bar{\varepsilon}_f^{pr}[\eta, \bar{\theta}]} = 1 \ .$$

Here $\bar{\varepsilon}_f^{pr}$, η, and $\bar{\theta}$ are strain to fracture, stress triaxiality, and the Lode parameter, respectively.

The fracture parameters, A, B0, GAMMA, C, and N, are used in the following equations as (a, b_0, γ, c, n), respectively. Strain to fracture for a proportional load is given as:

$$\bar{\varepsilon}_f^{pr}[\eta,\bar{\theta}] = b(1+c)^{\frac{1}{n}}\left(\left\{\frac{1}{2}((f_1-f_2)^a+(f_2-f_3)^a+(f_1-f_3)^a)\right\}^{\frac{1}{a}} + c(2\eta+f_1+f_3)\right)^{-\frac{1}{n}}$$

where a is the Hosford exponent, c is the friction coefficient controlling the effect of triaxiality, and n is the stress state sensitivity. The Lode angle parameter dependent trigonometric functions are given as:

$$f_1[\bar{\theta}] = \frac{2}{3}\cos\left[\frac{\pi}{6}(1-\bar{\theta})\right]$$

$$f_2[\bar{\theta}] = \frac{2}{3}\cos\left[\frac{\pi}{6}(3+\bar{\theta})\right]$$

$$f_3[\bar{\theta}] = -\frac{2}{3}\cos\left[\frac{\pi}{6}(1+\bar{\theta})\right]$$

The coefficient b (strain to fracture for uniaxial or equi-biaxial stretching) is:

$$b = \begin{cases} b_0 & for\ \dot{\varepsilon}_p < \dot{\varepsilon}_0 \\ b_0\left(1+\gamma ln\left[\frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0}\right]\right) & for\ \dot{\varepsilon}_p > \dot{\varepsilon}_0 \end{cases}$$

where γ is the strain rate sensitivity.

4. **Corresponding Parameters Summary.** The following table lists variable names used in this material model and corresponding symbols employed in [1], [2], and [3]:

Variable	P12	P22	P33	G12	G22	G33	A	B0
Symbol	P_{12}	P_{22}	P_{33}	G_{12}	G_{22}	G_{33}	a	b_0
Variable	GAMMA	C	N	TREF	TMELT	M	ETA	CP
Symbol	γ	c	N	T_r	T_m	m	η_k	C_p
Variable	DEPSO	DEPSAD	RO					
Symbol	$\dot{\varepsilon}_{it}/\dot{\varepsilon}_0$	$\dot{\varepsilon}_a$	ρ					

The following table lists variable names used in this material model and corresponding symbols in Xue's 2009 paper [4], for the option XUE:

Variable	EF0	P22	P33	G12	G22			
Symbol	ε_{f0}	P_{lim}	q	γ	m			

5. **Additional History Variables.** The table below lists the extra history variables associated with this material. See NEIPS on the manual page for *DATABASE_-EXTENT_BINARY.

History Variable #	Description
3	Damage accumulation. Elements will be deleted if this variable reaches 1.0 for more than half of the through-thickness integration points.
4	Temperatures
5	Strain rates
6	Strain rate scale factors
7	Temperature scale factor

Keyword Example Input:

A sample material input card can be found below, with parameters from Mohr, D., et al. (2010) and Roth, C.C. and Mohr, D. (2014).

```
*MAT_MOHR_NON_ASSOCIATED_FLOW
$#     mid        R0         E        PR       P12       P22       P33       G12
         1 7.8000E-9  2.10E05  0.300000      -0.5       1.0       3.0   -0.4946
$      G22       G33     LCIDS     LCIDV     LCIDT      LFLD     LFRAC        W0
    0.9318    2.4653       100       105       102
$        A        B0     GAMMA         C         N     SCALE
      1.97      0.82     0.025      0.00     0.199 3.132E-3
$     TREF     TMELT         M       ETA        CP      TINI     DEPSO    DEPSAD
     293.0   1673.70     0.921       0.9     420.0     293.0  0.001164     1.379
$     AOPT
         3
$       XP        YP        ZP        A1        A2        A3

$       V1        V2        V3        D1        D2        D3      BETA
       1.0
*DEFINE_CURVE
100
         0.00000E+00         0.30130E+03
```

```
         0.10000E-01          0.42295E+03
         0.20000E-01          0.47991E+03
         0.30000E-01          0.52022E+03
         0.40000E-01          0.55126E+03
                                            ⋮
*DEFINE_CURVE
105
         0.00000E+00          0.10000E+01
         0.10000E+00          0.10608E+01
         0.50000E+00          0.10828E+01
         0.10000E+01          0.10923E+01
                                            ⋮
*DEFINE_CURVE
102
         0.29300E+03          0.10000E+01
         0.33300E+03          0.96168E+00
         0.37300E+03          0.92744E+00
         0.41300E+03          0.89459E+00
         0.45300E+03          0.86261E+00
                                            ⋮
```

References:

[1] D. Mohr, M. Dunand, K. Kim, "Evaluation of associated and non-associated quadratic plasticity models for advanced high strength steel sheets under multi-axial loading," *International Journal of Plasticity*, Vol 26, Issue 7, https://doi.org/10.1016/j.ijplas.2009.11.006, July 2010.

[2] C.C. Roth and D. Mohr, "Effect of strain rate on ductile fracture initiation in advanced high strength steel sheets: Experiments and modeling," *International Journal of Plasticity*, Vol 56, https://doi.org/10.1016/j.ijplas.2014.01.003, May 2014.

[3] J.H. Sung, J.H. Kim, R.H. Wagoner, "A plastic constitutive equation incorporating strain, strain-rate, and temperature," *International Journal of Plasticity*, Vol 26, Issue 12, https://doi.org/10.1016/j.ijplas.2010.02.005, December 2010.

[4] L. Xue and T. Wierzbicki, "Numerical simulation of fracture mode transition in ductile plates," *International Journal of Solids and Structures*, Vol 46, Issue 6, https://doi.org/10.1016/j.ijsolstr.2008.11.009, March 2009.

Revision Information::

This material model is available in SMP starting in Revision 102375. Revision history is listed below:

- Element deletion feature based on damage accumulation: Revision 109792.
- The option XUE is available starting on Revision 111531.
- Set default values for P12, P22, P33, G12, G22 and G33: Revision 116262.

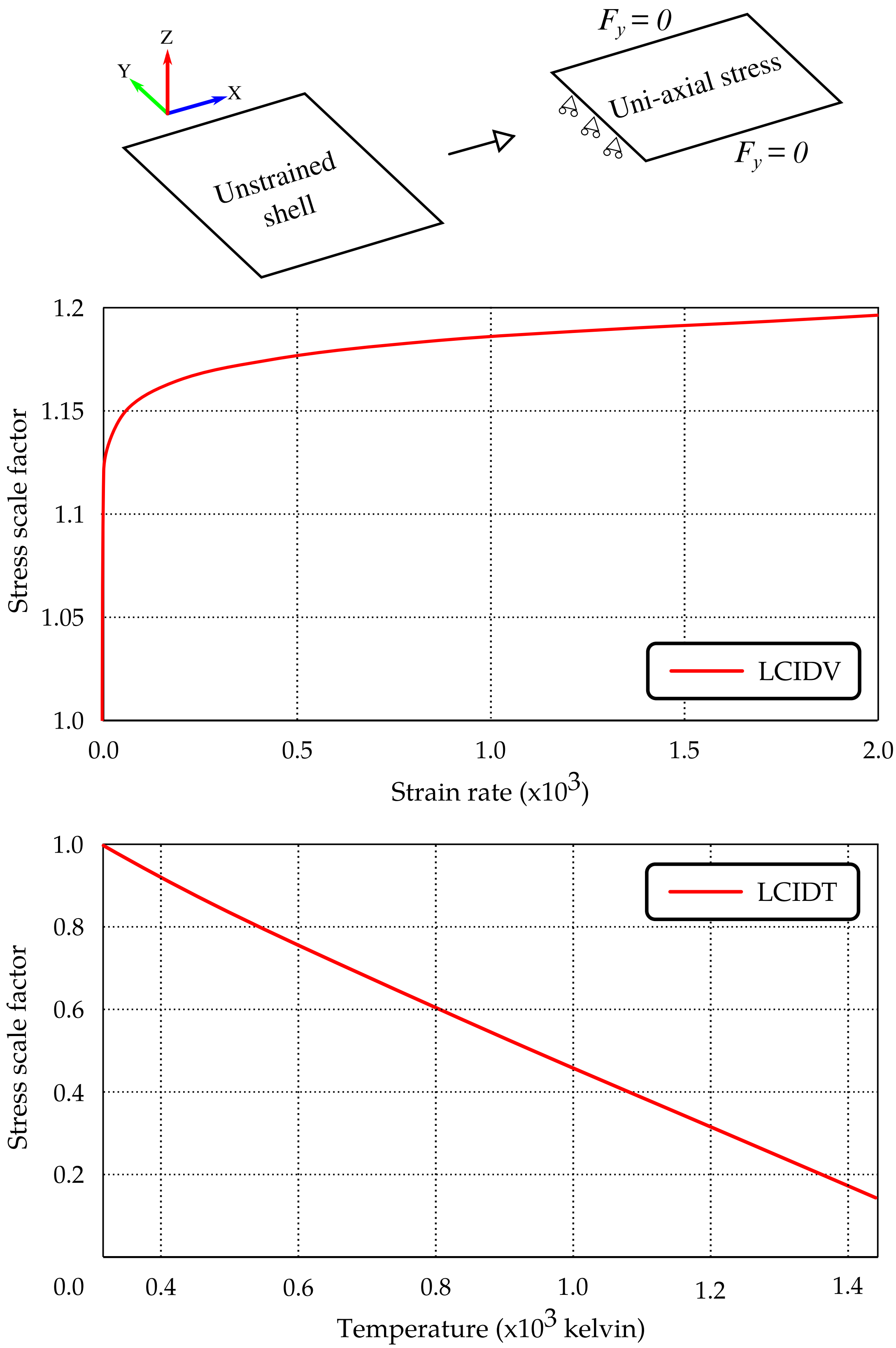
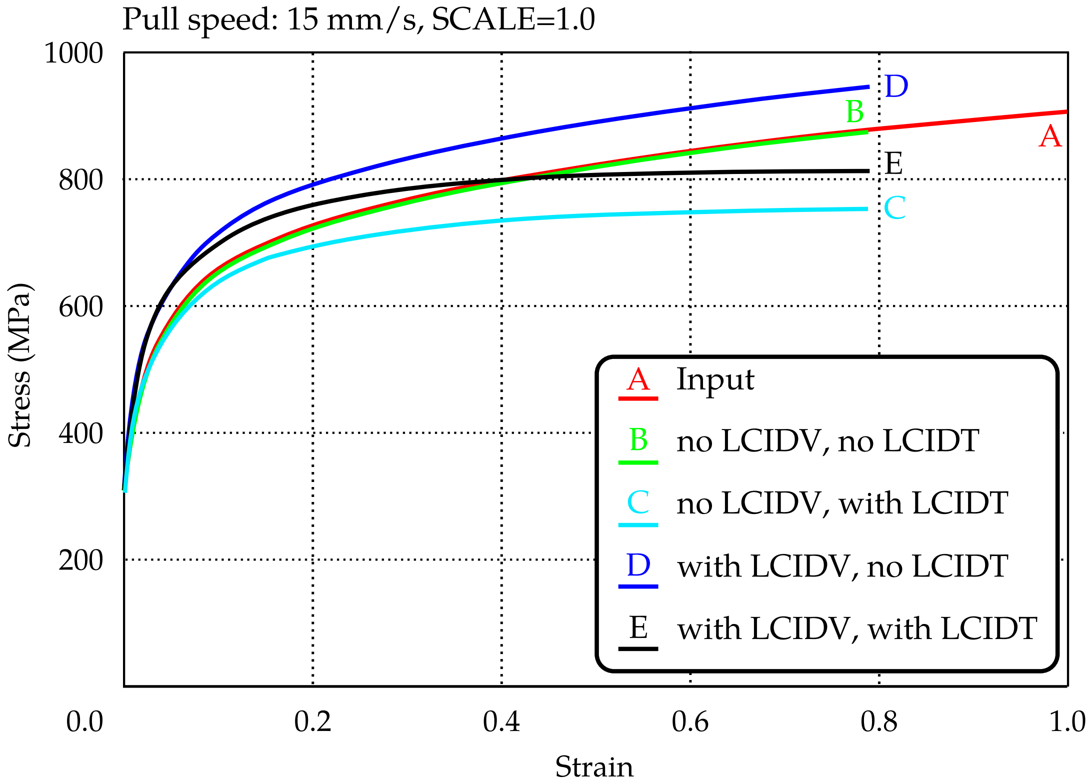


Figure M260B-1. Uniaxial stretching on a single shell element; Input curves LCIDV and LCIDT.



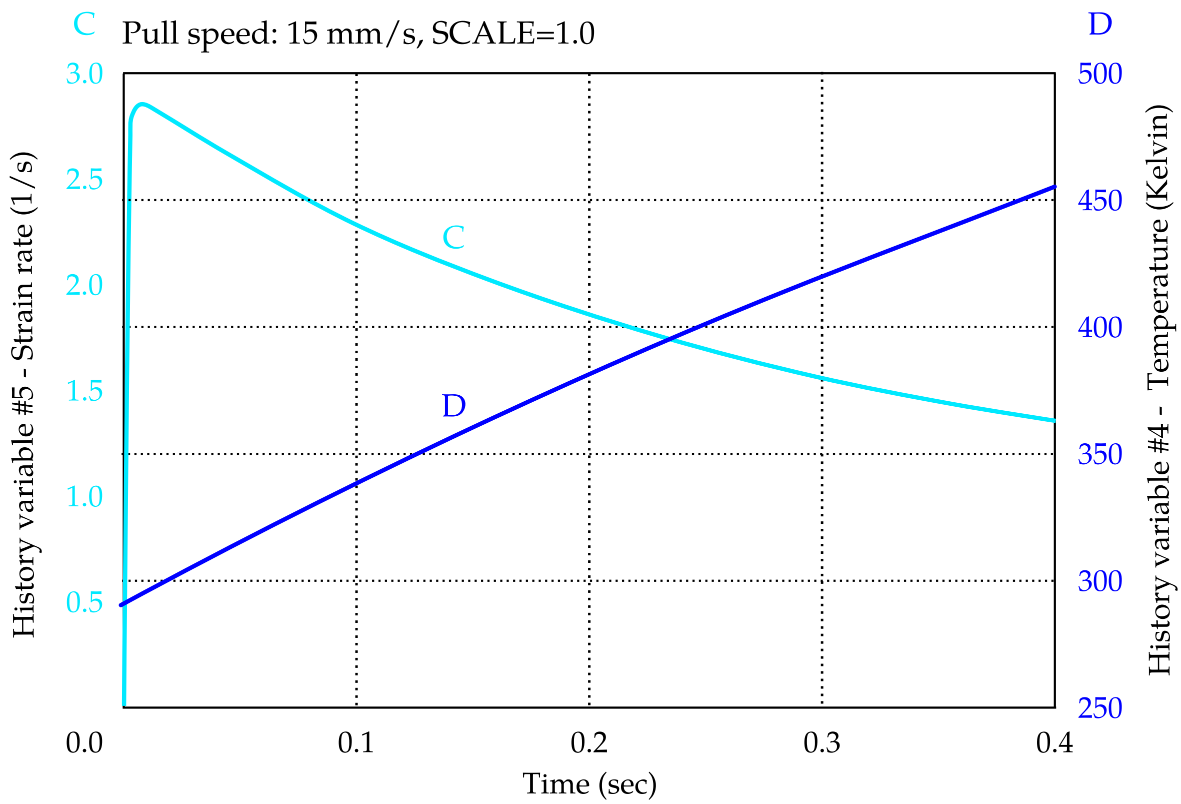


Figure M260B-2. Results of a single element uniaxial stretching - stress-strain curves (top), strain rates and temperature history under various conditions.

*MAT_LAMINATED_FRACTURE_DAIMLER_PINHO

This is Material Type 261 which is an orthotropic continuum damage model for laminated fiber-reinforced composites. See Pinho, Iannucci and Robinson [2006]. It is based on a physical model for each failure mode and considers non-linear in-plane shear behavior.

This model is implemented for shell, thick shell and solid elements.

NOTE: To apply laminated shell theory, set LAMSHT ≥ 3 in *CONTROL_SHELL.

Card Summary:

Card 1. This card is required.

MID	RO	EA	EB	EC	PRBA	PRCA	PRCB

Card 2. This card is required.

GAB	GBC	GCA	AOPT	DAF	DKF	DMF	EFS

Card 3. This card is required.

XP	YP	ZP	A1	A2	A3		

Card 4. This card is required.

V1	V2	V3	D1	D2	D3	MANGLE	

Card 5. This card is required.

ENKINK	ENA	ENB	ENT	ENL			

Card 6. This card is required.

XC	XT	YC	YT	SL			

Card 7. This card is required.

FIO	SIGY	LCSS	BETA	PFL	PUCK	SOFT	DT

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EA	EB	EC	PRBA	PRCA	PRCB
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
EA	E_a, Young's modulus in *a*-direction (longitudinal)
EB	E_b, Young's modulus in *b*-direction (transverse)
EC	E_c, Young's modulus in *c*-direction
PRBA	ν_{ba}, Poisson's ratio *ba*
PRCA	ν_{ca}, Poisson's ratio *ca*
PRCB	ν_{cb}, Poisson's ratio *cb*

Card 2	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	AOPT	DAF	DKF	DMF	EFS
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
GAB	G_{ab}, shear modulus *ab*
GBC	G_{bc}, shear modulus *bc*
GCA	G_{ca}, shear modulus *ca*
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):

VARIABLE	DESCRIPTION
	EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by an angle (see MANGLE on Card 4).
	EQ.1.0: Locally orthotropic with material axes determined by a point, P, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.
	EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR
	EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle. This angle may be set in the keyword input for the element or in the input for this keyword (see MANGLE on Card 4).
	EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, P, which define the centerline axis. This option is for solid elements only.
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*).
DAF	Flag to control failure of an integration point based on longitudinal (fiber) tensile failure (see Remarks 1 and 2):
	LT.-0.01: Integration point fails if any damage variable, that has a corresponding flag (DAF, DKF, or DMF) set ≤ 0.0, reaches 1.0. |DAF| limits the stress reduction due to damage from longitudinal tensile failure.
	EQ.0.00: Integration point fails if any damage variable, that has a corresponding flag (DAF, DKF, or DMF) set ≤ 0.0,

VARIABLE	DESCRIPTION
	reaches 1.0. DAF does not limit the stress reduction due to damage from longitudinal tensile failure.
	GT.0.01: No failure of integration point due to fiber tensile failure. This condition corresponds to history variable "da(i)" reaching 1.0. The value of DAF limits the stress reduction due to damage from longitudinal tensile failure.
DKF	Flag to control failure of an integration point based on longitudinal (fiber) compressive failure (see Remarks 1 and 2):
	LT.-0.01: Integration point fails if any damage variable, that has a corresponding flag (DAF, DKF, or DMF) set ≤ 0.0, reaches 1.0. |DKF| limits the stress reduction due to damage from longitudinal compressive failure.
	EQ.0.00: Integration point fails if any damage variable, that has a corresponding flag (DAF, DKF, or DMF) set ≤ 0.0, reaches 1.0. DKF does not limit the stress reduction due to damage from longitudinal tensile failure.
	GT.0.01: No failure of integration point due to fiber compressive failure. This condition corresponds to history variable "dkink(i)" reaching 1.0. The value of DKF limits the stress reduction due to damage from longitudinal compressive failure.
DMF	Flag to control failure of an integration point based on transverse (matrix) failure (see Remarks 1 and 2):
	LT.-0.01: Integration point fails if any damage variable, that has a corresponding flag (DAF, DKF, or DMF) set ≤ 0.0, reaches 1.0. |DMF| limits the stress reduction due to the damage from the matrix failure.
	EQ.0.00: Integration point fails if any damage variable, that has a corresponding flag (DAF, DKF, or DMF) set ≤ 0.0, reaches 1.0. DMF does not limit the stress reduction due to damage from longitudinal tensile failure.
	GT.0.01: No failure of integration point due to matrix failure. This condition corresponds to history variable "dmat(i)" reaching 1.0. The value of DMF limits the stress reduction due to damage from matrix failure.
EFS	Maximum effective strain for element layer failure. A value of

VARIABLE	DESCRIPTION
	unity would equal 100% strain. GT.0.0: Fails when effective strain calculated assuming material is volume preserving exceeds EFS LT.0.0: Fails when effective strain calculated from the full strain tensor exceeds |EFS|

Card 3	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
XP, YP, ZP	Coordinates of point *P* for AOPT = 1 and 4
A1, A2, A3	Components of vector **a** for AOPT = 2

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	MANGLE	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3
D1, D2, D3	Components of vector **d** for AOPT = 2
MANGLE	Material angle in degrees for AOPT = 0 (shells and tshells only) and AOPT = 3. MANGLE may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, and *ELEMENT_SOLID_ORTHO.

Card 5	1	2	3	4	5	6	7	8
Variable	ENKINK	ENA	ENB	ENT	ENL			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION
ENKINK	Fracture toughness for longitudinal (fiber) compressive failure mode. GT.0.0: The given value will be regularized with the characteristic element length. LT.0.0: Load curve or table ID = (-ENKINK). The load curve defines the fracture toughness for fiber compressive failure mode as a function of characteristic element length. The table defines for each strain rate value a load curve that defines the fracture toughness for fiber compressive failure mode as a function of characteristic element length. Neither case includes further regularization. The table must use the absolute value of the strain rate unless providing a logarithmically defined table. See Remark 5.
ENA	Fracture toughness for longitudinal (fiber) tensile failure mode. GT.0.0: The given value will be regularized with the characteristic element length. LT.0.0: Load curve or table ID = (-ENA) The load curve defines the fracture toughness for fiber tensile failure mode as a function of characteristic element length. The table defines for each strain rate value a load curve that defines the fracture toughness for fiber tensile failure mode as a function of characteristic element length. Neither case includes further regularization. The table definition must use the absolute value of the strain rate unless providing a logarithmically defined table. See Remark 5.
ENB	Fracture toughness for intralaminar matrix tensile failure. GT.0.0: The given value will be regularized with the characteristic element length. LT.0.0: Load curve or table ID = (-ENB). The load curve defines

VARIABLE	DESCRIPTION
	the fracture toughness for intralaminar matrix tensile failure as a function of characteristic element length. The table defines for each strain rate value a load curve that defines the fracture toughness for intralaminar matrix tensile failure as a function of characteristic element length. Neither case includes further regularization. The table definition must use the absolute value of the strain rate unless providing a logarithmically defined table. See Remark 5.
ENT	Fracture toughness for intralaminar matrix transverse shear failure. GT.0.0: The given value will be regularized with the characteristic element length. LT.0.0: Load curve or table ID = (-ENT). The load curve defines the fracture toughness for intralaminar matrix transverse shear failure as a function of characteristic element length. The table defines for each strain rate value a load curve that defines the fracture toughness for intralaminar matrix transverse shear failure as a function of characteristic element length.Neither case includes further regularization. The table must use the absolute value of the strain rate unless providing a logarithmically defined table. See Remark 5
ENL	Fracture toughness for intralaminar matrix longitudinal shear failure. GT.0.0: The given value will be regularized with the characteristic element length. LT.0.0: Load curve or table ID = (-ENL). The load curve defines the fracture toughness for intralaminar matrix longitudinal shear failure as a function of characteristic element length. The table defines for each strain rate value a load curve that defines the fracture toughness for intralaminar matrix longitudinal shear failure as a function of characteristic element length. Neither case includes further regularization. The table must use the absolute value of the strain rate unless providing a logarithmically defined table. See Remark 5.

Card 6	1	2	3	4	5	6	7	8
Variable	XC	XT	YC	YT	SL			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION
XC	Longitudinal compressive strength, a-axis (positive value). GT.0.0: Constant value LT.0.0: Load curve ID = (-XC) which defines the longitudinal compressive strength as a function of the absolute value of the longitudinal strain rate ($\dot{\epsilon}_{aa}$), except in the case of logarithmically defined curves. See Remark 5 for a discussion of logarithmically defined curves.
XT	Longitudinal tensile strength, a-axis. GT.0.0: Constant value LT.0.0: Load curve ID = (-XT) which defines the longitudinal tensile strength as a function of the absolute value of the longitudinal strain rate ($\dot{\epsilon}_{aa}$), except in the case of logarithmically defined curves. See Remark 5 for a discussion of logarithmically defined curves.
YC	Transverse compressive strength, b-axis (positive value). GT.0.0: Constant value LT.0.0: Load curve ID = (-YC) which defines the transverse compressive strength as a function of the absolute value of the transverse strain rate ($\dot{\epsilon}_{bb}$), except in the case of logarithmically defined curves. See Remark 5 for a discussion of logarithmically defined curves.
YT	Transverse tensile strength, b-axis. GT.0.0: Constant value LT.0.0: Load curve ID = (-YT) which defines the transverse tensile strength as a function of the absolute value of the transverse strain rate ($\dot{\epsilon}_{bb}$), except in the case of logarithmically defined curves. See Remark 5 for a discussion of logarithmically defined curves.

VARIABLE	DESCRIPTION
SL	Longitudinal shear strength. GT.0.0: Constant value LT.0.0: Load curve ID = (-SL) which defines the longitudinal shear strength as a function of the absolute value of the in-plane shear strain rate ($\dot{\epsilon}_{ab}$), except in the case of logarithmically defined curves. See Remark 5 for a discussion of logarithmically defined curves.

Card 7	1	2	3	4	5	6	7	8
Variable	FIO	SIGY	LCSS	BETA	PFL	PUCK	SOFT	DT
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
FIO	Fracture angle in pure transverse compression (default = 53.0°)
SIGY	In-plane shear yield stress (only used when BETA < 1.0)
LCSS	Load curve ID or Table ID. **Load Curve.** When LCSS is a load curve ID, it defines the nonlinear in-plane shear stress as a function of the absolute value of in-plane shear strain (γ_{ab}), except in the case of logarithmically defined curves. See Remark 5 for a discussion of logarithmically defined curves. **Tabular Data.** The table maps in-plane strain rate values ($\dot{\gamma}_{ab}$) to a load curve giving the in-plane shear stress as a function of the absolute value of in-plane shear strain. Note that except for in the case of logarithmically definied tables, the strain rates and shear strains must be the absolute values (see Remark 5). For strain rates below the minimum value, the curve for the lowest defined value of strain rate is used. Likewise, when the strain rate exceeds the maximum value, the curve for the highest defined value of strain rate is used.
BETA	Hardening parameter for in-plane shear plasticity ($0.0 \le \text{BETA} \le 1.0$). EQ.0.0: Pure kinematic hardening

VARIABLE	DESCRIPTION
	EQ.1.0: Pure isotropic hardening
	0.0 < BETA < 1.0: Mixed hardening
PFL	Percentage of shell or tshell layers which must fail until crashfront is initiated. For example, if |PFL| = 80.0, then 80% of layers must fail until strengths are reduced in neighboring elements. Default: all layers must fail. A single layer fails if 1 in-plane integration point fails (PFL > 0) or if 4 in-plane integration points fail (PFL < 0).
PUCK	Flag for evaluation and post-processing of Puck's inter-fiber-failure criterion (IFF); see Puck, Kopp and Knops [2002]. EQ.0.0: No evaluation of Puck's IFF-criterion. EQ.1.0: Puck's IFF-criterion will be evaluated.
SOFT	Softening reduction factor for material strength in crashfront elements (default = 1.0)
DT	Strain rate averaging option: EQ.0.0: Strain rate is evaluated using a running average. LT.0.0: Strain rate is evaluated using an average of the last 11 time steps. GT.0.0: Strain rate is averaged over the last DT time units.

Remarks:

1. **Failure surfaces.** We assemble *four* sub-surfaces, representing different failure mechanisms, to obtain the failure surface to limit the elastic domain. See Figure M261-1 for a definition of angles. They are defined as follows:

   a) longitudinal (fiber) tension,

$$f_a = \frac{\sigma_a}{X_T} = 1$$

   b) longitudinal (fiber) compression (3D-kinking model) – (transformation to fracture plane),

$$f_{\text{kink}} = \begin{cases} \left(\dfrac{\tau_T}{S_T - \mu_T \sigma_n}\right)^2 + \left(\dfrac{\tau_L}{S_L - \mu_L \sigma_n}\right)^2 = 1 & \text{if} \quad \sigma_{b^m} \le 0 \\ \left(\dfrac{\sigma_n}{Y_T}\right)^2 + \left(\dfrac{\tau_T}{S_T}\right)^2 + \left(\dfrac{\tau_L}{S_L}\right)^2 = 1 & \text{if} \quad \sigma_{b^m} > 0 \end{cases}$$

   with

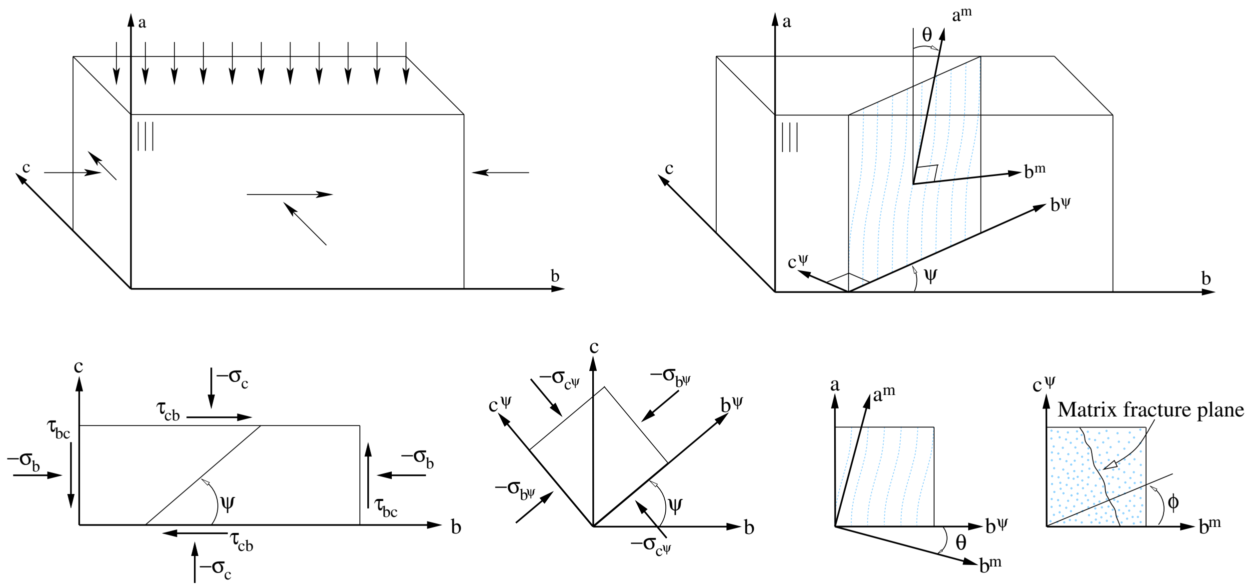


Figure M261-1. Definition of angles and stresses in fracture plane

$$
\begin{aligned}
S_T &= \frac{Y_C}{2\tan(\phi_0)} \\
\mu_T &= -\frac{1}{\tan(2\phi_0)} \\
\mu_L &= S_L \frac{\mu_T}{S_T} \\
\sigma_n &= \frac{\sigma_{b^m} + \sigma_{c^\psi}}{2} + \frac{\sigma_{b^m} - \sigma_{c^\psi}}{2}\cos(2\phi) + \tau_{b^m c^\psi}\sin(2\phi) \\
\tau_T &= -\frac{\sigma_{b^m} - \sigma_{c^\psi}}{2}\sin(2\phi) + \tau_{b^m c^\psi}\cos(2\phi) \\
\tau_L &= \tau_{a^m b^m}\cos(\phi) + \tau_{c^\psi a^m}\sin(\phi)
\end{aligned}
$$

c) transverse (matrix) failure: transverse tension,

$$f_{\text{mat}} = \left(\frac{\sigma_n}{Y_T}\right)^2 + \left(\frac{\tau_T}{S_T}\right)^2 + \left(\frac{\tau_L}{S_L}\right)^2 = 1 \quad \text{if} \quad \sigma_n \geq 0$$

with

$$
\begin{aligned}
\sigma_n &= \frac{\sigma_b + \sigma_c}{2} + \frac{\sigma_b - \sigma_c}{2}\cos(2\phi) + \tau_{bc}\sin(2\phi) \\
\tau_T &= -\frac{\sigma_b - \sigma_c}{2}\sin(2\phi) + \tau_{bc}\cos(2\phi) \\
\tau_L &= \tau_{ab}\cos\phi + \tau_{ca}\sin\phi
\end{aligned}
$$

d) transverse (matrix) failure: transverse compression/shear,

$$f_{\text{mat}} = \left(\frac{\tau_T}{S_T - \mu_T\sigma_n}\right)^2 + \left(\frac{\tau_L}{S_L - \mu_L\sigma_n}\right)^2 = 1 \quad \text{if} \quad \sigma_n < 0$$

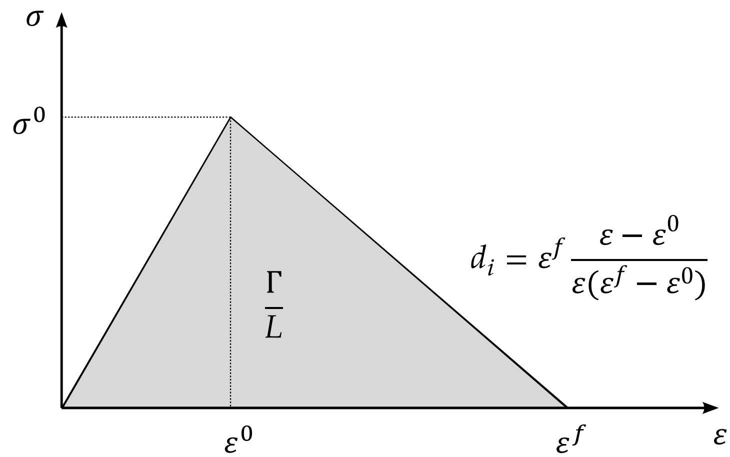


Figure M261-2. Damage evolution law

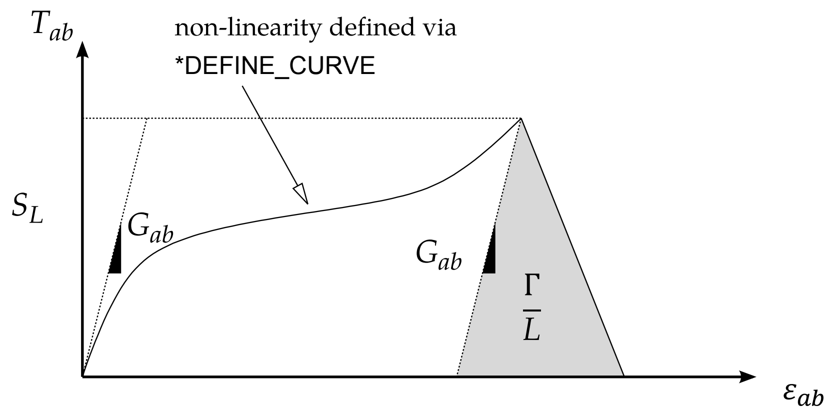


Figure M261-3. Definition of nonlinear in-plane shear behavior

2. **Damage evolution.** As long as the stress state is located within the failure surface, the model behaves in an orthotropic elastic manner. When reaching the failure criteria, the effective (undamaged) stresses will be reduced by a factor of $(1 - d)$. Here, the damage variable d represents one of the damage variables defined for the different failure mechanisms or a limit to the stress reduction set with DAF, DKF, and DMF due to the damage variable. In other words, d corresponds to one of the following values, depending on the failure mechanism: $\min(d_{\mathrm{da}}, |\mathrm{DAF}|)$ for logitudinal tension damage, $\min(d_{\mathrm{kink}}, |\mathrm{DKF}|)$ for logitudinal compression damage, or $\min(d_{\mathrm{mat}}, |\mathrm{DMF}|)$ for matrix damage. Note that DiF = 0, where i is A, K, or M, means no limit to the stress reduction due to the corresponding damage variable. Note that once the integration point fails that the stress goes to zero. A linear damage evolution law based on fracture toughnesses ($\Gamma \rightarrow$ ENKINK, ENA, ENB, ENT, ENL) and a characteristic internal element length, L, to account for objectivity drive the growth of the damage variables (d_{da}, d_{kink}, and d_{mat}). See Figure M261-2.

3. **Nonlinear in-plane shear.** To account for the characteristic nonlinear in-plane shear behavior of laminated fiber-reinforced composites, we couple a 1D elasto-

plastic formulation to a linear damage behavior upon reaching the maximum allowable stress state for shear failure. To introduce nonlinearity in the shear behavior, use *DEFINE_CURVE to define an explicit shear stress as a function of engineering shear strain curve (LCSS). See Figure M261-3 (in which ϵ_{ab} designates engineering shear strain rather than tensorial shear strain).

4. **Element deletion.** When failure has occurred in all the composite layers (through-thickness integration points), the element is deleted. Elements that share nodes with the deleted element become "crashfront" elements and can have their strengths reduced by using the SOFT parameter. An earlier initiation of crashfront elements is possible by using the parameter PFL.

5. **Logarithmically defined tables and curves.** An alternative way to invoke logarithmic interpolation between discrete strain rates or strains is described as follows. If the *first* value in the table or curve is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate or all the curve abscissa values represent the natural logarithm of a strain rate or strain. Since the tables and curves are internally discretized to equally space the table and curve values, it makes good sense from an accuracy standpoint that the table and curve values represent the natural log of strain rate or strain when the lowest strain rate or strain and highest strain rate or strain differ by several orders of magnitude. Invoking logarithmic interpolation incurs some additional computational cost.

6. **History variables.** The number of additional integration point variables written to the LS-DYNA database is input by the *DATABASE_EXTENT_BINARY definition with the variable NEIPS (shells) and NEIPH (solids). These additional variables are tabulated below (i = integration point).

   When intending to initialize the stress state using *INTIAL_STRESS_*OPTION*, the stress values SIGXX, SIGYY, etc. in *INITIAL_STRESS_*OPTION* are not used, rather stresses are determined from the total strain history variables 31 to 36.

History Variable	Description	Value	History Variable #
fa(i)	fiber tensile mode	$0 \rightarrow 1$: elastic 1: failure criterion reached	1
fkink(i)	fiber compressive mode	$0 \rightarrow 1$: elastic 1: failure criterion reached	2
fmat(i)	matrix mode	$0 \rightarrow 1$: elastic 1: failure criterion reached	3

History Variable	Description	Value	History Variable #
da(*i*)	damage fiber tension	0: elastic 1: fully damaged	5
dkink(*i*)	damage fiber compression	0: elastic 1: fully damaged	6
dmat(*i*)	damage transverse	0: elastic 1: fully damaged	7
dam(*i*)	crashfront	-1: element intact 10^{-8}: element in crashfront +1: element failed	9
fmt_p(*i*)	tensile matrix mode (Puck criteria)	$0 \rightarrow 1$: elastic 1: failure criterion reached	10
fmc_p(*i*)	compressive matrix mode (Puck criteria)	$0 \rightarrow 1$: elastic 1: failure criterion reached	11
theta_p(*i*)	angle of fracture plane (radians, Puck criteria)		12
d1tot(*i*)	Total strain in material 11-direction		31
d2tot(*i*)	Total strain in material 22-direction		32
d3tot(*i*)	Total strain in material 33-direction		33
d4tot(*i*)	Total strain in material 12-direction		34
d5tot(*i*)	Total strain in material 23-direction		35
d6tot(*i*)	Total strain in material 31-direction		36
theta	Angle θ in Figure M261-1		49
psi	Angle ψ in Figure M261-1		50
e1dot(*i*)	Averaged strain rate in longitudinal direction		54
e2dot(*i*)	Averaged strain rate in transverse direction		55

History Variable	Description	Value	History Variable #
e4dot(*i*)	Averaged engineering shear strain rate in in-plane direction		56

References:

More detailed information about this material model can be found in Pinho, Iannucci and Robinson [2006].

*MAT_LAMINATED_FRACTURE_DAIMLER_CAMANHO

This is Material Type 262 which is an orthotropic continuum damage model for laminated fiber-reinforced composites. See Maimí, Camanho, Mayugo and Dávila [2007]. It is based on a physical model for each failure mode and considers a simplified non-linear in-plane shear behavior. This model is implemented for shell, thick shell and solid elements.

Applus+ IDIADA and Toyota Motor Corporation/TOYOTA GAZOO Racing Europe developed enhancements to improve the accuracy of in-plane shear mechanisms and the predictability of material behavior under crushing-bending loads. See Alameda, Dominguez, Martin-Santos and Miura [2024]. These features have been implemented in material 262 in collaboration with DYNAmore and are specified in Cards 9 and 10.

> **NOTE:** Laminated shell theory can be applied by setting LAMSHT $\geq$ 3 in *CONTROL_SHELL.

Card Summary:

Card 1. This card is required.

MID	RO	EA	EB	EC	PRBA	PRCA	PRCB

Card 2. This card is required.

GAB	GBC	GCA	AOPT	DAF	DKF	DMF	EFS

Card 3. This card is required.

XP	YP	ZP	A1	A2	A3	DSF	

Card 4. This card is required.

V1	V2	V3	D1	D2	D3	MANGLE	MSG

Card 5. This card is required.

GXC	GXT	GYC	GYT	GSL	GXCO	GXTO	

Card 6. This card is required.

XC	XT	YC	YT	SL	XCO	XTO	

Card 7. This card is required.

FIO	SIGY	ETAN	BETA	PFL	PUCK	SOFT	DT

Card 8. This card is optional.

EPSF23	EPSR23	TSMD23	EPSF31	EPSR31	TSMD31		

Card 9. This card is optional.

EF_11T	EF_11C	EF_22T	EF_22C	EF_12	EF_23	EF_31	LCSS

Card 10. This card is optional.

CF12	CF13	CF23	SOFTC				

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EA	EB	EC	PRBA	PRCA	PRCB
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
EA	GT.0.0: E_a, Young's modulus - longitudinal direction LT.0.0: Load curve or table ID = (-EA). It is available for shells only. **Load Curve.** When -EA refers to a load curve ID, it is taken as defining the uniaxial elastic stress as a function of strain behavior in the longitudinal direction. Negative data points correspond to compression and positive values to tension. **Tabular Data.** When -EA refers to a table ID, it defines a load curve for each strain rate value. The load curves give the uniaxial elastic stress as a function of strain behavior in the longitudinal direction. **Logarithmically Defined Tables.** If the first uniaxial elastic stress as a function of strain curve in the table

VARIABLE	DESCRIPTION
	corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for *all* stress-strain curves.
EB	GT.0.0: E_b, Young's modulus - transverse direction LT.0.0: Load Curve ID or Table ID = (-EB). (shells only). **Load Curve.** When -EB refers to a load curve ID, it is taken as defining the uniaxial elastic stress as a function of strain behavior in the transverse direction. Negative data points correspond to compression, and positive values to tension. **Tabular Data.** When -EB corresponds to a table ID, it specifies a load curve for each strain rate value. The load curves give the uniaxial elastic stress as a function of strain behavior in the transverse direction. **Logarithmically Defined Tables.** If the first uniaxial elastic stress as a function of strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for *all* stress-strain curves.
EC	E_c, Young's modulus in c-direction
PRBA	ν_{ba}, Poisson's ratio *ba*
PRCA	ν_{ca}, Poisson's ratio *ca*
PRCB	ν_{cb}, Poisson's ratio *cb*

Card 2	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	AOPT	DAF	DKF	DMF	EFS
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
GAB	GT.0.0: G_{ab}, shear modulus in the *ab*-direction LT.0.0: Load Curve ID or Table ID = (-GAB)

VARIABLE	DESCRIPTION
	Load Curve. When -GAB refers to a load curve ID, it is taken as defining the elastic shear stress as a function of shear strain behavior in the *ab*-direction. **Tabular Data.** When -GAB corresponds to a table ID, it defines a load curve for each strain rate value. The load curves give the elastic shear stress as a function of shear strain behavior in the *ab*-direction. **Logarithmically Defined Tables.** If the *first* elastic shear stress as a function of shear strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for *all* shear stress-shear strain curves.
GBC	G_{bc}, shear modulus *bc*
GCA	G_{ca}, shear modulus *ca*
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description): EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle MANGLE EQ.1.0: Locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the *a*-direction. This option is for solid elements only. EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle (MANGLE) from a line in the plane of the element defined by the cross product of the vector **v** with the element normal EQ.4.0: Locally orthotropic in cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *p*, which define the centerline axis. This option is for solid elements only. LT.0.0: The absolute value of AOPT is a coordinate system ID

VARIABLE	DESCRIPTION
	number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
DAF	Flag to control failure of an integration point based on longitudinal (fiber) tensile failure: EQ.0.0: Integration point fails if any damage variable reaches 1.0. EQ.1.0: No failure of integration point due to fiber tensile failure, $da(i) = 1.0$
DKF	Flag to control failure of an integration point based on longitudinal (fiber) compressive failure: EQ.0.0: Integration point fails if any damage variable reaches 1.0. EQ.1.0: No failure of integration point due to fiber compressive failure, $\mathrm{dkink}(i) = 1.0$.
DMF	Flag to control failure of an integration point based on transverse (matrix) failure: EQ.0.0: Integration point fails if any damage variable reaches 1.0. EQ.1.0: No failure of integration point due to matrix failure, $\mathrm{dmat}(i) = 1.0$
EFS	Maximum effective strain for element layer failure. A value of unity would equal 100% strain. GT.0.0: Fails when effective strain calculated assuming material is volume preserving exceeds EFS LT.0.0: Fails when effective strain calculated from the full strain tensor exceeds |EFS|

Card 3	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	DSF	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
XP YP ZP	Coordinates of point p for AOPT = 1 and 4
A1 A2 A3	Components of vector **a** for AOPT = 2
DSF	Flag to control failure of an integration point based on in-plane shear failure: EQ.0.0: Integration point fails if any damage variable reaches 1.0. EQ.1.0: No failure of integration point due to in-plane shear failure, $\mathrm{dsl}(i) = 1.0$

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	MANGLE	MSG
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
V1 V2 V3	Components of vector **v** for AOPT = 3
D1 D2 D3	Components of vector **d** for AOPT = 2
MANGLE	Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. MANGLE may be overridden on the element card; see *ELEMENT_-SHELL_BETA and *ELEMENT_SOLID_ORTHO.
MSG	Flag to control the output of warning messages: EQ.0: Only one warning message will be written per part. GT.0: All warnings are written. LT.0: No warnings are written.

Card 5	1	2	3	4	5	6	7	8
Variable	GXC	GXT	GYC	GYT	GSL	GXCO	GXTO	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
GXC	Fracture toughness for longitudinal (fiber) compressive failure mode. GT.0.0: The given value is regularized with the characteristic element length. LT.0.0: Load curve or table ID = -GXC. If referring to a load curve, the load curve gives the fracture toughness for fiber compressive failure mode as a function of characteristic element length. If referring to a table, each strain rate value indexes a load curve that defines the fracture toughness for fiber compressive failure mode as a function of characteristic element length. In either case, no further regularization occurs.
GXT	Fracture toughness for longitudinal (fiber) tensile failure mode: GT.0.0: The given value is regularized with the characteristic element length. LT.0.0: Load curve or table ID = -GXT. If referring to a load curve, the load curve gives the fracture toughness for fiber tensile failure mode as a function of characteristic element length. If referring to a table, each strain rate value indexes a load curve that defines the fracture toughness for fiber tensile failure mode as a function of characteristic element length. In either case, no further regularization occurs.
GYC	Fracture toughness for transverse compressive failure mode. GT.0.0: The given value is regularized with the characteristic element length. LT.0.0: Load curve or table ID = -GYC. If referring to a load curve, the load curve gives the fracture toughness for transverse compressive failure mode as a function of characteristic element length. If referring to a table, each strain rate value indexes a load curve that defines the fracture toughness for transverse compressive failure mode as a function of characteristic element length. In either case, no further regularization occurs.
GYT	Fracture toughness for transverse tensile failure mode. GT.0.0: The given value is regularized with the characteristic element length.

VARIABLE	DESCRIPTION
	LT.0.0: Load curve or table ID = -GYT. If referring to a load curve,t eh load curve defines the fracture toughness for transverse tensile failure mode as a function of characteristic element length. If referring to a table, each strain rate value indexes a load curve that defines the fracture toughness for transverse tensile failure mode as a function of characteristic element length. In either case, no further regularization occurs.
GSL	Fracture toughness for in-plane shear failure mode. GT.0.0: The given value is regularized with the characteristic element length. LT.0.0: Load curve or table ID = -GSL. If referring to a load curve, the load curve gives the fracture toughness for in-plane shear failure mode as a function of characteristic element length. If referring to a table, each strain rate value indexes a load curve that defines the fracture toughness for in-plane shear failure mode as a function of characteristic element length. In either case, no further regularization occurs.
GXCO	Fracture toughness for longitudinal (fiber) compressive failure mode to define bilinear damage evolution. GT.0.0: The given value is regularized with the characteristic element length. LT.0.0: Load curve or table ID = -GXCO. If referring to a load curve, the load curve gives the fracture toughness for fiber compressive failure mode to define bilinear damage evolution as a function of characteristic element length. If referring to a table, each strain rate value indexes a load curve that defines the fracture toughness for fiber compressive failure mode to define bilinear damage evolution as a function of characteristic element length. In either case, no further regularization occurs.
GXTO	Fracture toughness for longitudinal (fiber) tensile failure mode to define bilinear damage evolution. GT.0.0: The given value is regularized with the characteristic element length. LT.0.0: Load curve or table ID = -GXTO. If referring to a load curve, the load curve defines the fracture toughness for

VARIABLE	DESCRIPTION
	fiber tensile failure mode to define bilinear damage evolution as a function of characteristic element length. If a table, each strain rate value indexes a load curve that defines the fracture toughness for fiber tensile failure mode to define bilinear damage evolution as a function of characteristic element length. In either case, no further regularization occurs.

Card 6	1	2	3	4	5	6	7	8
Variable	XC	XT	YC	YT	SL	XCO	XTO	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
XC	Longitudinal compressive strength, *a*-axis (positive value): GT.0.0: Constant value LT.0.0: Load curve ID = (-XC) which defines the longitudinal compressive strength as a function of longitudinal strain rate ($\dot{\varepsilon}_{aa}$)
XT	Longitudinal tensile strength, *a*-axis: GT.0.0: Constant value LT.0.0: Load curve ID = (-XT) which defines the longitudinal tensile strength as a function of longitudinal strain rate ($\dot{\varepsilon}_{aa}$)
YC	Transverse compressive strength, *b*-axis (positive value): GT.0.0: Constant value LT.0.0: Load curve ID = (-YC) which defines the transverse compressive strength as a function of transverse strain rate ($\dot{\varepsilon}_{bb}$)
YT	Transverse tensile strength, *b*-axis: GT.0.0: Constant value LT.0.0: Load curve ID = (-YT) which defines the transverse

VARIABLE	DESCRIPTION
	tensile strength as a function of transverse strain rate ($\dot{\varepsilon}_{bb}$)
SL	Shear strength, *ab* plane: GT.0.0: Constant value LT.0.0: Load curve ID = (-SL) which defines the longitudinal shear strength as a function of in-plane shear strain rate ($\dot{\varepsilon}_{ab}$)
XCO	Longitudinal compressive strength at inflection point (positive value): GT.0.0: Constant value LT.0.0: Load curve ID = (-XCO) which defines the longitudinal compressive strength at inflection point as a function of longitudinal strain rate ($\dot{\varepsilon}_{aa}$).
XTO	Longitudinal tensile strength at inflection point: GT.0.0: Constant value LT.0.0: Load curve ID = (-XTO) which defines the longitudinal tensile strength at inflection point as a function of longitudinal strain rate ($\dot{\varepsilon}_{aa}$)

Card 7	1	2	3	4	5	6	7	8
Variable	FIO	SIGY	ETAN	BETA	PFL	PUCK	SOFT	DT
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
FIO	Fracture angle in pure transverse compression (in degrees, default = 53.0)
SIGY	In-plane shear yield stress: GT.0.0: Constant value LT.0.0: Load curve ID = (-SIGY) which defines the in-plane shear yield stress as a function of in-plane shear strain rate ($\dot{\varepsilon}_{ab}$)

VARIABLE	DESCRIPTION
ETAN	Tangent modulus for in-plane shear plasticity: GT.0.0: Constant value LT.0.0: Load curve ID = (-ETAN) which defines the tangent modulus for in-plane shear plasticity as a function of in-plane shear strain rate ($\dot{\varepsilon}_{ab}$)
BETA	Hardening parameter for in-plane shear plasticity (0.0 ≤ BETA ≤ 1.0): EQ.0.0: Pure kinematic hardening EQ.1.0: Pure isotropic hardening 0.0 < BETA < 1.0: Mixed hardening
PFL	Percentage of layers which must fail before crashfront is initiated. For example, if |PFL| = 80.0, then 80% of the layers must fail before strengths are reduced in neighboring elements. By default, all layers must fail. A single layer fails if 1 in-plane IP fails (PFL > 0) or if 4 in-plane IPs fail (PFL < 0).
PUCK	Flag for evaluation and post-processing of Puck's inter-fiber-failure criterion (IFF, see Puck, Kopp and Knops [2002]). EQ.0.0: No evaluation of Puck's IFF-criterion EQ.1.0: Puck's IFF-criterion will be evaluated.
SOFT	Softening reduction factor for material strength in crashfront elements (default = 1.0). If SOFTC is defined as well, SOFTC is used to reduce the longitudinal compressive strength XC.
DT	Strain rate averaging option: EQ.0.0: Strain rate is evaluated using a running average. LT.0.0: Strain rate is evaluated using average of last 11 time steps. GT.0.0: Strain rate is averaged over the last DT time units.

Optional Transverse Shear Failure Card. This card is optional.

Card 8	1	2	3	4	5	6	7	8
Variable	EPSF23	EPSR23	TSMD23	EPSF31	EPSR31	TSMD31		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
EPSF23	Damage initiation transverse shear strain (23-plane)
EPSR23	Final rupture transverse shear strain (23-plane)
TSMD23	Transverse shear maximum damage; default = 0.90 (23-plane).
EPSF31	Damage initiation transverse shear strain (31-plane)
EPSR31	Final rupture transverse shear strain (31-plane)
TSMD31	Transverse shear maximum damage; default = 0.90 (31-plane).

Optional Card. This card is optional. It only applies to shell elements.

Card 9	1	2	3	4	5	6	7	8
Variable	EF_11T	EF_11C	EF_22T	EF_22C	EF_12	EF_23	EF_31	LCSS
Type	F	F	F	F	F			F

VARIABLE	DESCRIPTION
EF_11T	Tensile failure strain in longitudinal *a*-direction
EF_11C	Compressive failure strain in longitudinal *a*-direction
EF_22T	Tensile failure strain in transverse *b*-direction
EF_22C	Compressive failure strain in transverse *b*-direction
EF_12	In-plane shear failure strain in *ab*-plane
EF_23	Out-of-plane shear failure strain in *bc*-plane
EF_31	Out-of-plane shear failure strain in *ca*-plane

VARIABLE	DESCRIPTION
LCSS	Load curve ID or table ID. If this is defined, SIGY and ETAN will be ignored. **Load Curve.** When LCSS is a load curve ID, it defines the nonlinear in-plane shear stress as a function of in-plane shear strain (γ_{ab}). **Tabular Data.** The table maps in-plane strain rate values ($\dot{\gamma}_{ab}$) to a load curve giving the in-plane shear stress as a function of in-plane shear strain. For strain rates below the minimum value, the curve for the lowest defined value of strain rate is used. Likewise, when the strain rate exceeds the maximum value, the curve for the highest defined value of strain rate is used. **Logarithmically Defined Table.** An alternative way to invoke logarithmic interpolation between discrete strain rates is described as follows. If the *first* value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate when the lowest strain rate and highest strain rate differ by several orders of magnitude. There is some additional computational cost associated with invoking logarithmic interpolation.

Optional Card. This card is optional. It only applies to shell elements.

Card 10	1	2	3	4	5	6	7	8
Variable	CF12	CF13	CF23	SOFTC				
Type	F	F	F	F				
Default	1.0	1.0	1.0	1.0				

VARIABLE	DESCRIPTION
CF12	Coupling factor for in-plane shear (*ab*-plane) damage with the fiber damage in tension: $$d_6 = 1 - [1 - d_6^*(r_{2+})](1 - d_{1+}\mathrm{CF12})$$ Here, d_6 is the in-plane shear damage, d_6^* is an intermediate damage variable needed for finding the in-plane shear damage that is a function of r_{2+}, r_{2+} is internal value of the constituative law representing an elastic domain threshold, and d_{1+} is the fiber damage in

VARIABLE	DESCRIPTION
	tension. See Maimí, Camanho, Mayugo, and Dávila [2007] for more details.
CF13	Scaling factor on the fiber damage that is used when determining the reduced transverse shear (*ca*-plane) resulting from the fiber damage: $$c_{66} = (1 - d_1\mathrm{CF13})G_{ca}$$ Here, c_{66} is reduced transverse shear modulus in the *ca*-plane, G_{ca} is the *ca* shear modulus, and $d_1$is the fiber damage. See Maimí, Camanho, Mayugo, and Dávila [2007] for more details.
CF23	Scaling factor on the in-plane shear damage that is used when determining the reduced transverse shear (*bc*-plane) resulting from the in-plane shear damage: $$c_{55} = (1 - d_6\mathrm{CF23})G_{bc}$$ Here, c_{55} is the reduced transverse shear in the *bc*-plane, G_{bc} is the *bc* shear modulus, and d_6 is the in-plane shear damage. See Maimí, Camanho, Mayugo, and Dávila [2007] for more details.
SOFTC	Softening reduction factor for XC material strength in crashfront elements. If this is not defined, XC reduces according to SOFT.

Remarks:

The failure surface to limit the elastic domain is assembled by *four* sub-surfaces, representing different failure mechanisms. They are defined as follows:

1. longitudinal (fiber) tension,

$$\phi_{1+} = \frac{\sigma_{11} - \upsilon_{12}\sigma_{22}}{X_T} = 1$$

2. longitudinal (fiber) compression – (transformation to fracture plane),

$$\phi_{1-} = \frac{\langle |\sigma_{12}^m| + \mu_L\sigma_{22}^m \rangle}{S_L} = 1$$

with

$$\begin{aligned}
\mu_L &= -\frac{S_L\cos(2\phi_0)}{Y_C\cos^2(\phi_0)} \\
\sigma_{22}^m &= \sigma_{11}\sin^2(\varphi^c) + \sigma_{22}\cos^2(\varphi^c) - 2|\sigma_{12}|\sin(\varphi^c)\cos(\varphi^c) \\
\sigma_{12}^m &= (\sigma_{22} - \sigma_{11})\sin(\varphi^c)\cos(\varphi^c) + |\sigma_{12}|\Big(\cos^2(\varphi^c) - \sin^2(\varphi^c)\Big)
\end{aligned}$$

and

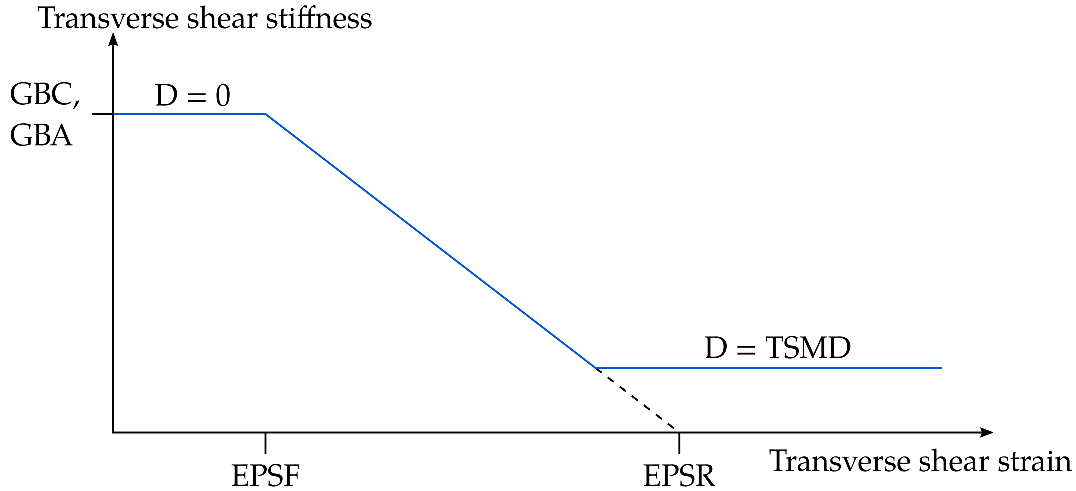


Figure M262-1. Linear Damage for Transverse Shear Behavior

$$\varphi^c = \arctan\left[\frac{1 - \sqrt{1 - 4\left(\frac{S_L}{X_C} + \mu_L\right)\frac{S_L}{X_C}}}{2\left(\frac{S_L}{X_C} + \mu_L\right)}\right]$$

3. transverse (matrix) failure: perpendicular to the laminate mid-plane,

$$\phi_{2+} = \begin{cases} \sqrt{(1-g)\frac{\sigma_{22}}{Y_T} + g\left(\frac{\sigma_{22}}{Y_T}\right)^2 + \left(\frac{\sigma_{12}}{S_L}\right)^2} = 1 & \sigma_{22} \geq 0 \\ \frac{\langle |\sigma_{12}| + \mu_L \sigma_{22} \rangle}{S_L} = 1 & \sigma_{22} < 0 \end{cases}$$

4. transverse (matrix) failure: transverse compression/shear,

$$\phi_{2-} = \sqrt{\left(\frac{\tau_T}{S_T}\right)^2 + \left(\frac{\tau_L}{S_L}\right)^2} = 1 \quad \mathit{if} \quad \sigma_{22} < 0$$

with

$$\mu_T = -\frac{1}{\tan(2\phi_0)}$$

$$S_T = Y_C \cos(\phi_0)\left[\sin(\phi_0) + \frac{\cos(\phi_0)}{\tan(2\phi_0)}\right]$$

$$\theta = \arctan\left(\frac{-|\sigma_{12}|}{\sigma_{22}\sin(\phi_0)}\right)$$

$$\tau_T = \langle -\sigma_{22}\cos(\phi_0)\left[\sin(\phi_0) - \mu_T \cos(\phi_0)\cos(\theta)\right]\rangle$$

$$\tau_L = \langle \cos(\phi_0)\left[|\sigma_{12}| + \mu_L \sigma_{22}\cos(\phi_0)\sin(\theta)\right]\rangle$$

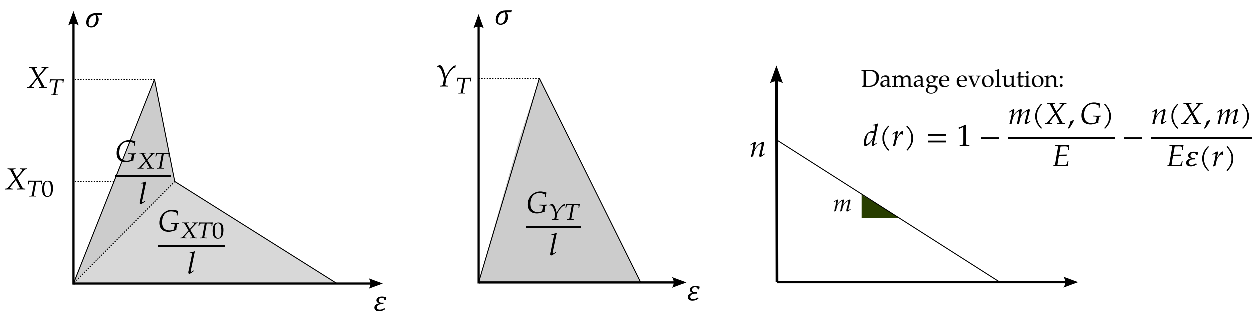


Figure M262-2. Damage evolution law

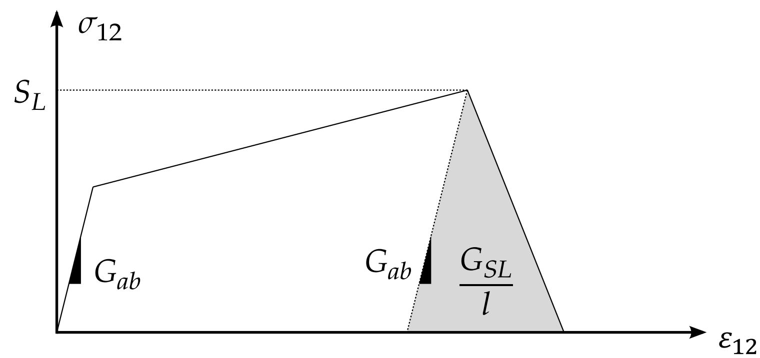


Figure M262-3. In-plane shear behavior

So long as the stress state is located within the failure surface the model behaves orthotropic elastic. The constitutive law is derived on basis of a proper definition for the ply complementary free energy density G, whose second derivative with respect to the stress tensor leads to the compliance tensor $\mathbf{H}$

$$\mathbf{H} = \frac{\partial^2 G}{\partial \sigma^2} = \begin{bmatrix} \frac{1}{(1-d_1)E_1} & -\frac{v_{21}}{E_2} & 0 \\ -\frac{v_{12}}{E_1} & \frac{1}{(1-d_2)E_2} & 0 \\ 0 & 0 & \frac{1}{(1-d_6)G_{12}} \end{bmatrix}, \qquad \begin{aligned} d_1 &= d_{1+}\frac{\langle\sigma_{11}\rangle}{|\sigma_{11}|} + d_{1-}\frac{\langle-\sigma_{11}\rangle}{|\sigma_{11}|} \\ d_2 &= d_{2+}\frac{\langle\sigma_{22}\rangle}{|\sigma_{22}|} + d_{2-}\frac{\langle-\sigma_{22}\rangle}{|\sigma_{22}|} \end{aligned}$$

Once the stress state reaches the failure criterion, a set of scalar damage variables (d_{1-}, d_{1+}, d_{2-}, d_{2+}, d_6) is introduced associated with the different failure mechanisms. A bilinear (longitudinal direction) and a linear (transverse direction) damage evolution law is used to define the development of the damage variables driven by the fracture toughness and a characteristic internal element length to account for objectivity. See Figure M262-2.

To account for the characteristic non-linear in-plane shear behavior of laminated fiber-reinforced composites a 1D elasto-plastic formulation with linear hardening is coupled to a linear damage behavior once the maximum allowable stress state for shear failure is reached. See Figure M262-3.

More detailed information about this material model can be found in Maimí, Camanho, Mayugo and Dávila [2007].

When failure has occurred in all the composite layers (through-thickness integration points), the element is deleted. Elements which share nodes with the deleted element become "crashfront" elements and can have their strengths reduced by using the SOFT parameter. An earlier initiation of crashfront elements is possible by using the parameter PFL.

The number of additional integration point variables written to the LS-DYNA database is input by the *DATABASE_EXTENT_BINARY definition with the variable NEIPS (shells) and NEIPH (solids). These additional variables are tabulated below (i = integration point).

When intending to initialize the stress state using *INTIAL_STRESS_*OPTION*, the stress values SIGXX, SIGYY, etc. in *INITIAL_STRESS_*OPTION* are not used, rather stresses are determined from the total strain history variables 31 to 36.

History Variable #	Description	Value
1	Fiber tensile mode, $\phi_{1+}(i)$	$0 \rightarrow 1$: elastic 1: failure criterion reached
2	Fiber compressive mode, $\phi_{1-}(i)$	$0 \rightarrow 1$: elastic 1: failure criterion reached
3	Tensile matrix mode, $\phi_{2+}(i)$	$0 \rightarrow 1$: elastic 1: failure criterion reached
4	Compressive matrix mode, $\phi_{2-}(i)$	$0 \rightarrow 1$: elastic 1: failure criterion reached
5	Fiber damage in tension, $d_{1+}(i)$	0: elastic 1: fully damaged
6	Fiber damage in compression, $d_{1-}(i)$	0: elastic 1: fully damaged
7	Transverse damage, $d_2(i)$	0: elastic 1: fully damaged
8	In-plane shear damage, $d_6(i)$	0: elastic 1: fully damaged

History Variable #	Description	Value
9	Crashfront	-1: element intact 10^{-8}: element in crashfront +1: element failed
10	Tensile matrix mode (Puck criteria)	$0 \rightarrow 1$: elastic 1: failure criterion reached
11	Compressive matrix mode (Puck criteria)	$0 \rightarrow 1$: elastic 1: failure criterion reached
12	Angle of fracture plane in radians (Puck criteria)	
16	Longitudinal damage, $d_1(i)$	0: elastic 1: fully damaged
17	Transverse damage in tension, $d_{2+}(i)$	0: elastic 1: fully damaged
18	Transverse damage in compression, $d_{2-}(i)$	0: elastic 1: fully damaged
31	Total strain in material 11-direction	
32	Total strain in material 22-direction	
33	Total strain in material 33-direction	
34	Total strain in material 12-direction	
35	Total strain in material 23-direction	
36	Total strain in material 31-direction	
55	Averaged strain rate in longitudinal direction	
56	Averaged strain rate in transverse direction	
57	Averaged engineering shear strain rate in in-plane direction	

*MAT_LOU-YOON_ANISOTROPIC_PLASTICITY

This is Material Type 263. It is based on the anisotropic yield function proposed by Lou and Yoon (Lou and Yoon, 2017). This yield function extends the original Drucker function into an anisotropic form using a fourth order linear transformation tensor. The non-associated flow rule (non-AFR) can be applied to accurately describe both the directional yield stresses and R-values. The anisotropic flexibility of this model can be further improved by summing up components of the anisotropic Drucker function. See the section Constitutive relations below for more details. This model is supported for shell and solid elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	HR	P1	P2	ITER
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	AFR	NFUNC	AOPT		LCID	E0	LCF	P3
Type	I	I	F		I	F	I	
Default	none	1	none		none	none	none	none

Card 3	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		
Default				none	none	none		

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		
Default	none	none	none	none	none	none		

Card 5	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6	CC	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	none	none	

Card 6	1	2	3	4	5	6	7	8
Variable	PC1	PC2	PC3	PC4	PC5	PC6	PCC	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	none	none	

Optional card that only needs to be included if LCF < 0.

Card 7	1	2	3	4	5	6	7	8
Variable	VF1	VF2	VF3	VF4	VF5			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).

VARIABLE	DESCRIPTION
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
HR	Hardening rules (see section Hardening laws below): EQ.1.0: Linear hardening (default) EQ.2.0: Exponential hardening (Swift) EQ.3.0: Load curve EQ.4.0: Exponential hardening (Voce) EQ.5.0: Exponential hardening (Gosh) EQ.6.0: Exponential hardening (Hocken-Sherby)
P1	Material parameter: HR.EQ.1.0: Tangent modulus HR.EQ.2.0: q, coefficient for exponential hardening law (Swift) HR.EQ.4.0: a, coefficient for exponential hardening law (Voce) HR.EQ.5.0: q, coefficient for exponential hardening law (Gosh) HR.EQ.6.0: a, coefficient for exponential hardening law (Hocket-Sherby)
P2	Material parameter: HR.EQ.1.0: Yield stress for the linear hardening law HR.EQ.2.0: n, coefficient for (Swift) exponential hardening HR.EQ.4.0: c, coefficient for exponential hardening law (Voce) HR.EQ.5.0: n, coefficient for exponential hardening law (Gosh) HR.EQ.6.0: c, coefficient for exponential hardening law (Hocket-Sherby)
ITER	Iteration flag for speed: EQ.0.0: Fully iterative EQ.1.0: Fixed at three iterations. Generally, ITER = 0.0 is recommended. However, ITER = 1.0 is faster and may give acceptable results in most problems.

VARIABLE	DESCRIPTION
AFR	Flag to use associated flow rule (AFR): EQ.0: Use non-AFR. EQ.1: Use AFR.
NFUNC	Number of Drucker function components. Currently NFUNC is always set to 1.
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for more complete description). EQ.0.0: Locally orthotropic with material axes determined by element nodes. The shells only the material axes are rotated about the normal vector to the surface of the shell by the angle BETA. EQ.2.0: Globally orthotropic with material axes determined by vectors defined **a** and **d** defined below, as with *DEFINED_COORDINATE_VECTOR. EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. The material directions are determined as follows: **a** is the cross product of **v** with the normal vector, **b** is the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element. LT.0.0: The absolute value of AOPT is a coordinate system ID (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM, or *DEFINE_COORDINATE_VECTOR).
LCID	Load curve ID giving the hardening law for HR = 3
E0	Material parameter: HR.EQ.2.0: ε_0, initial yield strain for exponential hardening law (Swift) (default = 0.0) HR.EQ.4.0: b, coefficient for exponential hardening (Voce)

VARIABLE	DESCRIPTION
	HR.EQ.5.0: ε_0, initial yield strain for exponential hardening (Gosh), Default = 0.0 HR.EQ.6.0: b, coefficient for exponential hardening law (Hocket-Sherby)
LCF	Fracture curve: EQ.0: No fracture curves (default) GT.0: Load curve or table ID of customized fracture curve/surface. If referring to a load curve ID, the fracture curve is defined as effective plastic strain as a function of triaxiality. If referring to a table ID, for each load parameter, an effective plastic strain as a function of. triaxiality curve can be defined (only applicable to solids EQ.-1: Drucker ductile fracture criterion. Optional Card 7 is needed in this case. VF1, VF2 and VF3 in Card 7 will be used as a, b and c in the Drucker ductile fracture criterion. See section Fracture criteria for more details. EQ.-2: DF2016 fracture criterion. Optional card 7 is needed in this case. VF1, VF2, VF3, VF4 and VF5 in Card 7 will be used as C1, C2, C3 and C in DF2016 criterion. See section Fracture criteria for more details.
P3	Material parameter: HR.EQ.5.0: p, coefficient for exponential hardening (Gosh) HR.EQ.6.0: n, exponent for exponential hardening law (Hocket-Sherby)
A1, A2, A3	Components of vector **a** for AOPT = 2.0
V1, V2, V3	Components of vector **v** for AOPT = 3.0
D1, D2, D3	Components of vector **d** for AOPT = 2.0
Ci	Anisotropic parameters c'_1 through c'_6 that defines the fourth order linear transformation tensor L'
CC	Material constant c in Drucker yield function. c is recommended to be 1.226 for BCC metals and 2 for FCC metals.
PCi	Anisotropic parameters $\hat{c}_1$ through $\hat{c}_6$ defining the fourth-order linear transformation tensor $\hat{L}$ for the plastic potential in the non-AFR

VARIABLE	DESCRIPTION
	case (see field AFR, which is input on Card 2).
PCC	Material constant $\hat{c}$ in Drucker function for the plastic potential. $\hat{c}$ is recommended to be 1.226 for BCC metals and 2 for FCC metals unless calibrated otherwise.
VF*i*	Components of the fracture criterion included for LCF < 0. See LCF (input on Card 2) for a description.

Hardening laws:

The implemented hardening laws are the following:

1. The Swift hardening law
2. The Voce hardening law
3. The Gosh hardening law
4. The Hocket-Sherby hardening law
5. A loading curve, where the yield stress is given as a function of the effective plastic strain

The Swift hardening law can be written as:

$$\sigma_y(\varepsilon_{\mathrm{ep}}) = q(\varepsilon_0 + \varepsilon_{\mathrm{ep}})^n \ ,$$

where q and n are material parameters.

The Voce equation says that the yield stress can be written in the following form:

$$\sigma_y(\varepsilon_{\mathrm{ep}}) = a - be^{-c\varepsilon_{\mathrm{ep}}} \ ,$$

where $a, b$and c are material parameters. The Gosh equation is similar to the Swift equation. They only differ by a constant

$$\sigma_y(\varepsilon_{\mathrm{ep}}) = q(\varepsilon_0 + \varepsilon_{\mathrm{ep}})^n - p \ ,$$

where q, ε_0, n and p are material constants. The Hocket-Sherby equation resemblance the Voce equation, but with an additional parameter added

$$\sigma_y(\varepsilon_{\mathrm{ep}}) = a - be^{-c\varepsilon_{\mathrm{ep}}^n} \ .$$

where a, b, c and n are material parameters.

Constitutive relations:

Drucker proposed a yield function that includes the effect of the third stress invariant in the classic Von Mises yield function. Lou and Yoon (2017) extended this yield function to an anisotropic form as shown below:

$$\bar{\sigma}_y(\sigma_{ij}) = (J_2'^3 - cJ_3'^2)^{1/6} \ .$$

Here J_2' and J_3' are the second and third invariants of the linear transformed deviatoric stress tensor $\mathbf{s}'$:

$$\mathbf{s}' = \mathbf{L}'\sigma \ .$$

The fourth order linear transformation tensor $\mathbf{L}'$ is given by:

$$\mathbf{L}' = \begin{bmatrix} (c_2' + c_3')/3 & -c_3'/3 & -c_2'/3 & 0 & 0 & 0 \\ -c_3'/3 & (c_1' + c_3')/3 & -c_1'/3 & 0 & 0 & 0 \\ -c_2'/3 & -c_1'/3 & (c_2' + c_1')/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_4' & 0 & 0 \\ 0 & 0 & 0 & 0 & c_5' & 0 \\ 0 & 0 & 0 & 0 & 0 & c_6' \end{bmatrix} .$$

c_1', c_2', c_3' and c_6' can be calibrated from uniaxial tensile yield stress along different directions and the balanced biaxial yield stress. c_4' and c_5', which are related to the through-thickness material properties, are very difficult to obtain experimentally and therefore assumed to be identical with c_6'.

With the non-associated flow rule (non-AFR), the plastic flow is not necessarily aligned with the yield surface normal and the R-values are modeled by a different plastic potential:

$$\bar{\sigma}_p(\sigma_{ij}) = (\hat{J}_2^3 - c\hat{J}_3^2)^{1/6} \ .$$

Here $\hat{J}_2$ and $\hat{J}_3$ are the second and third invariants of the linear transformed deviatoric stress tensor:

$$\hat{\mathbf{s}} = \hat{\mathbf{L}}\sigma \ .$$

And $\hat{\boldsymbol{L}}$ is defined as:

$$\hat{\mathbf{L}} = \begin{bmatrix} (\hat{c}_2 + \hat{c}_3)/3 & -\hat{c}_3/3 & -\hat{c}_2/3 & 0 & 0 & 0 \\ -\hat{c}_3/3 & (\hat{c}_1 + \hat{c}_3)/3 & -\hat{c}_1/3 & 0 & 0 & 0 \\ -\hat{c}_2/3 & -\hat{c}_1/3 & (\hat{c}_2 + \hat{c}_1)/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \hat{c}_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \hat{c}_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \hat{c}_6 \end{bmatrix} .$$

The anisotropic parameters $\hat{c}_1$, $\hat{c}_2$, $\hat{c}_3$ and $\hat{c}_6$ can be calibrated with experimentally measured R-values along different directions.

Another approach to improve the flexibility of the yield function is to sum up n components of the anisotropic Drucker functions as follows:

$$\bar{\sigma}_y(\sigma_{ij}) = \frac{1}{n}\sum_{m=1}^{n}\{[(J_2'^{(m)})^3 - c(J_3'^{(m)})^2]^{1/6}\} \ .$$

where n is an integer with $n \geq 1$. The same idea can be applied to the plastic potential in the non-AFR approach, as shown in equation:

$$\bar{\sigma}_p(\sigma_{ij}) = \frac{1}{n}\sum_{m=1}^{n}\{[(\hat{J}_2^{(m)})^3 - c(\hat{J}_3^{(m)})^2]^{1/6}\} \ .$$

Fracture criteria:

The Drucker ductile fracture criterion is given by:

$$\bar{\sigma}_f(\sigma_{ij}) = \mathrm{a}\left(bI_1 + \left(J_2^3 - cJ_3^2\right)^{1/6}\right) = 1$$

The DF2016 fracture criterion is given by:

$$\left(\frac{\sigma_1 - \sigma_3}{\bar{\sigma}_{VM}}\right)^{C_1}\left(\left\langle\frac{f(\eta, L, C)}{f(1/3\,, -1, C)}\right\rangle\right)^{C_2}\bar{\varepsilon}_f^p \ = C_3$$

Here

$$\langle x\rangle = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

and

$$f(\eta, L, C) = \eta + C_4\frac{(3 - L)}{3\sqrt{L^2 + 3}} + C \ .$$

*MAT_TABULATED_JOHNSON_COOK_ORTHO_PLASTICITY

This is Material Type 264. This is an orthotropic, elastic-plastic material law with a J3-dependent yield surface. This material considers tensile/compressive asymmetry in the material response, which is essential for HCP metals. It is available for solid elements and thick shell elements type 3, 5, and 7.

Card Summary:

Card 1. This card is required.

MID	RO	E	PR	CP	TR	BETA	NUMINT

Card 2. This card is required.

LCT00R	LCT00T	LCF	LCG	LCH	LCI		

Card 3. This card is required.

LCC00R	LCC00T	LCS45R	LCS45T	IFLAG	SFIEPM	NITER	AOPT

Card 4. This card is required.

LCT90R	LCT45R	LCTTHR	LCC90R	LCC45R	LCCTHR		

Card 5. This card is required.

LCT90T	LCT45T	LCTTHT	LCC90T	LCC45T	LCCTHT		

Card 6. This card is required.

XP	YP	ZP	A1	A2	A3	MACF	

Card 7. This card is required.

V1	V2	V3	D1	D2	D3	MANGLE	

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	CP	TR	BETA	NUMINT
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	1.0	1.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus: GT.0.0: Constant value LT.0.0: Temperature-dependent Young's modulus given by load curve ID = -E
PR	Poisson's ratio
CP	Specific heat
TR	Room temperature
BETA	Fraction of plastic work converted into heat
NUMINT	Number of failed integration points before element deletion. EQ.-200: Turns off erosion for solids. Not recommended unless used with *CONSTRAINED_TIED_NODES_FAILURE.

Card 2	1	2	3	4	5	6	7	8
Variable	LCT00R	LCT00T	LCF	LCG	LCH	LCI		
Type	I	I	I	I	I	I		
Default	0	0	0	0	0	0		

VARIABLE	DESCRIPTION
LCT00R	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) tensile yield stress as a function of plastic strain for that rate in the 00-degree direction
LCT00T	Table ID defining for each temperature value a load curve ID giving the (quasi-static) tensile yield stress as a function of plastic strain for that temperature in the 00-degree direction
LCF	Load curve or table ID. The load curve ID defines plastic failure strain as a function of triaxiality. The table ID specifies a load curve ID for each Lode parameter, giving the plastic failure strain as a function of triaxiality for that Lode parameter. (Table option yet to be generally supported.)
LCG	Load curve ID defining plastic failure strain as a function of plastic strain rate
LCH	Load curve ID defining plastic failure strain as a function of temperature
LCI	Load curve ID, table ID, or 3D table ID. The load curve gives plastic failure strain as a function of element size. The table defines a load curve ID for each triaxiality, giving the plastic failure strain as a function of element size for that triaxiality. If referring to a three-dimensional table ID, plastic failure strain can be a function of the Lode parameter (TABLE_3D), triaxiality (TABLE), and element size (CURVE).

Card 3	1	2	3	4	5	6	7	8
Variable	LCC00R	LCC00T	LCS45R	LCS45T	IFLAG	SFIEPM	NITER	AOPT
Type	I	I	I	I	I	I	I	F
Default	0	0	0	0	0	1	100	none

VARIABLE	DESCRIPTION
LCC00R	Table ID. The curves in this table define compressive yield stress as a function of plastic strain. The table specifies a load curve ID for each plastic strain rate value, giving the (isothermal) compressive yield stress as a function of plastic strain for that rate in the 00-degree direction.
LCC00T	Table ID defining for each temperature value a load curve ID giving the (quasi-static) compressive yield stress as a function of strain for that temperature. The curves in this table define compressive yield stress as a function of plastic strain in the 00-degree direction.
LCS45R	Table ID. The table defines a load curve ID for each plastic strain rate value, giving the (isothermal) shear yield stress as a function of plastic strain for that rate in the 45-degree direction.
LCS45T	Table ID. The table defines a load curve ID for each temperature value, giving the (quasi-static) shear yield stress versus strain for that temperature. The load curves define shear yield stress as a function of plastic strain or effective plastic strain (see IFLAG) in the 45-degree direction.
IFLAG	Flag to specify abscissa for LCT00R, LCC00R, LCS45R, LCT90R, LCT45R, LCTTHR, LCC90R, LCC45R, LCCTHR: EQ.0: Compressive and shear yields are given as functions of plastic strain as defined in the remarks (default). EQ.1: Compressive and shear yields are given as functions of effective plastic strain.
SFIEPM	Scale factor on the initial estimate of the plastic multiplier
NITER	Maximum number of iterations for the plasticity algorithm

VARIABLE	DESCRIPTION
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details): EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. EQ.1.0: Locally orthotropic with material axes determined by a point, P, in space and the global location of the element center; this is the **a-**direction. EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR. EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle. Either the element's input or this keyword's input (see MANGLE) sets the angle. Note that the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying the angle, depending on the value of MACF. EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, P, which define the centerline axis. LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*).

Card 4	1	2	3	4	5	6	7	8
Variable	LCT90R	LCT45R	LCTTHR	LCC90R	LCC45R	LCCTHR		
Type	I	I	I	I	I	I		
Default	0	0	0	0	0	0		

VARIABLE	DESCRIPTION
LCT90R	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) tensile yield stress as a function of plastic strain for that rate in the 90-degree direction
LCT45R	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) tensile yield stress as a function of plastic strain for that rate in the 45-degree direction
LCTTHR	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) tensile yield stress as a function of plastic strain for that rate in the thickness degree direction
LCC90R	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) compressive yield stress as a function of plastic strain for that rate in the 90-degree direction
LCC45R	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) compressive yield stress as a function of plastic strain for that rate in the 45-degree direction
LCCTHR	Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) compressive yield stress as a function of plastic strain for that rate in the thickness degree direction

Card 5	1	2	3	4	5	6	7	8
Variable	LCT90T	LCT45T	LCTTHT	LCC90T	LCC45T	LCCTHT		
Type	I	I	I	I	I	I		
Default	0	0	0	0	0	0		

VARIABLE	DESCRIPTION
LCT90T	Table ID defining for each temperature value a load curve ID giving the (quasistatic) tensile yield stress as a function of plastic strain for that rate in the 90-degree direction
LCT45T	Table ID defining for each temperature value a load curve ID giving the (quasistatic) tensile yield stress as a function of plastic strain for that rate in the 45-degree direction
LCTTHT	Table ID defining for each temperature value a load curve ID giving the (quasistatic) tensile yield stress as a function of plastic strain for that rate in the thickness degree direction
LCC90T	Table ID defining for each temperature value a load curve ID giving the (quasistatic) compressive yield stress as a function of plastic strain for that rate in the 90-degree direction
LCC45T	Table ID defining for each temperature value a load curve ID giving the (quasistatic) compressive yield stress as a function of plastic strain for that rate in the 45-degree direction
LCCTHT	Table ID defining for each temperature value a load curve ID giving the (quasistatic) compressive yield stress as a function of plastic strain for that rate in the thickness degree direction

Card 6	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	MACF	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
XP, YP, ZP	Coordinates of point *P* for AOPT = 1 and 4
A1, A2, A3	Components of vector **a** for AOPT = 2
MACF	Material axes change flag for solid elements: EQ.-4: Switch material axes *b* and *c* before BETA or MANGLE rotation EQ.-3: Switch material axes *a* and *c* before BETA or MANGLE

VARIABLE	DESCRIPTION
	rotation
	EQ.-2: Switch material axes a and b before BETA or MANGLE rotation
	EQ.1: No change, default
	EQ.2: Switch material axes a and b after BETA or MANGLE rotation
	EQ.3: Switch material axes a and c after BETA or MANGLE rotation
	EQ.4: Switch material axes b and c after BETA or MANGLE rotation
	Figure M2-2 indicates when LS-DYNA applies MACF during the procedure to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then BETA is used for the rotation for all AOPT options. Otherwise, for AOPT = 3, MANGLE input on Card 7 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no rotation will be performed.

Card 7	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	MANGLE	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3
D1, D2, D3	Components of vector **d** for AOPT = 2
MANGLE	Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SOLID_ORTHO.

Remarks:

If IFLAG = 0, the compressive and shear curves are defined as follows:

$$\sigma_{\mathrm{comp}}\left(\varepsilon_{p,\,\mathrm{comp}}, \dot{\varepsilon}_{p,\,\mathrm{comp}}\right), \quad \varepsilon_{p,\mathrm{comp}} = \varepsilon_{\mathrm{comp}} - \frac{\sigma_{\mathrm{comp}}}{E}, \quad \dot{\varepsilon}_{p,\,\mathrm{comp}} = \frac{\partial \varepsilon_{p,\mathrm{comp}}}{\partial t}$$

where comp is one of the tension (0°,45°,90°), compression (0°,45°,90°), tension and compression thickness, or shear components.

If IFLAG = 1, the compressive and shear curves are defined as follows:

$$\sigma_{\text{comp}}(\dot{\lambda},\lambda) \quad \dot{W}_p = \sigma_{\text{eff}}\dot{\lambda}$$

History variables may be post-processed through additional variables. NEIPH on *DATABASE_EXTENT_BINARY sets the number of additional variables for solids written to the d3plot and d3thdt databases. The following table lists the relevant additional variables of this material model:

History Variable #	Description
5	Strain Rate
6	Plastic failure strain
7	Triaxiality
8	Lode parameter
9	Plastic work
10	Damage
11	Element size
12	Temperature
13	Compressive plastic strain
14	Shear plastic strain

MAT_CONSTRAINED_*OPTION

This is Material Type 265. This special model defines material data for *CONSTRAINED_SPR2 or *CONSTRAINED_INTERPOLATION_SPOTWELD (aka SPR3) instead of in the input for the constraint. This material model is not available for standard elements. See the Sample Input below.

Available options include:

SPR2

SPR3

The input depends on the option used. SPR2 requires two cards, and SPR3 needs up to four cards.

Card Summary:

Card 1. Include this card if the keyword option SPR2 is used.

MID	RO	FN	FT	DN	DT	XIN	XIT

Card 2. Include this card if the keyword option SPR2 is used.

ALPHA1	ALPHA2	ALPHA3	EXPN	EXPT			

Card 3. Include this card if the keyword option SPR3 is used.

MID	RO	MODEL					

Card 4. Include this card if the keyword option SPR3 is used.

STIFF	RN	RS	ALPHA1	BETA1	LCF	LCUPF	LCUPR

Card 5a.1. Include this card if the keyword option SPR3 is used and MODEL = 1, 11, 21, 31, or 41.

STIFF2	STIFF3	STIFF4	LCDEXP	GAMMA	SROPT		

Card 5a.2. This card is optional. It is read if the keyword option SPR3 is used and MODEL = 1, 11, 21, 31, or 41.

FFN	FFB	FFS	EXFC	STIFP	MFSFC	DEFC	NPFC

Card 5b.1. Include this card if the keyword option SPR3 is used and MODEL = 2, 12, or 22.

UPFN	UPFS	ALPHA2	BETA2	UPRN	UPRS	ALPHA3	BETA3

Card 5b.2. Include this card if the keyword option SPR3 is used and MODEL = 2, 12, or 22.

MRN	MRS						

Data Card Definitions:

SPR2 Cards. Include this card if the SPR2 keyword option is used.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	FN	FT	DN	DT	XIN	XIT
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
FN	Rivet strength in tension (pull-out)
FT	Rivet strength in pure shear
DN	Failure displacement in normal direction
DT	Failure displacement in tangential direction
XIN	Fraction of failure displacement at maximum normal force
XIT	Fraction of failure displacement at maximum tangential force

SPR2 Cards. Include this card if the SPR2 keyword option is used.

Card 2	1	2	3	4	5	6	7	8
Variable	ALPHA1	ALPHA2	ALPHA3	EXPN	EXPT			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION
ALPHA1	Dimensionless parameter scaling the effective displacement
ALPHA2	Dimensionless parameter scaling the effective displacement
ALPHA3	Dimensionless parameter scaling the effective displacement. The sign of ALPHA3 can be used to choose the normal update procedure: GT.0: Incremental update (default) LT.0: Total update (recommended)
EXPN	Exponent value for load function in the normal direction
EXPT	Exponent value for load function in the tangential direction

SPR3 Cards. Include this card if the SPR3 keyword option is used.

Card 3	1	2	3	4	5	6	7	8
Variable	MID	RO	MODEL					
Type	A	F	F					

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
MODEL	Material behavior and damage model (see Remarks of *CONSTRAINED_INTERPOLATION_SPOTWELD). EQ.1: SPR3 (default)

VARIABLE	DESCRIPTION

EQ.2: SPR4

EQ.11: Same as 1 with selected material parameters as functions

EQ.12: Same as 2 with selected material parameters as functions

EQ.21: Same as 11 with slight modification

EQ.22: Same as 12 with slight modification

EQ.31: Same as 11 but with 12 more material parameters as functions

EQ.41: Same as 31 with slight modification

SPR3 cards. Include this card if the SPR3 keyword option is used.

Card 4	1	2	3	4	5	6	7	8
Variable	STIFF	RN	RS	ALPHA1	BETA1	LCF	LCUPF	LCUPR
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
STIFF	Elastic stiffness. Function ID if MODEL > 10 .
RN	Tensile strength factor, R_n. GT.0.0: Constant value unless MODEL > 10. Function ID if MODEL > 10 (see Remarks section for *CONSTRAINED_INTERPOLATION_SPOTWELD). LT.0.0: Load curve with ID |RN| giving R_n as a function of peel ratio (see Remarks section for *CONSTRAINED_INTERPOLATION_SPOTWELD)
RS	Shear strength factor, R_s. Function ID if MODEL > 10.
ALPHA1	Scaling factor α_1. Function ID if MODEL > 10.
BETA1	Exponent for plastic potential β_1. Function ID if MODEL > 10.
LCF	Load curve or table ID. Load curve ID describing force as a function of plastic displacement, that is, $F^0(\bar{u}^{\mathrm{pl}})$. Table ID describing force as a function of mode mixity (table values) and plastic displacement (curves), that is, $F^0(\bar{u}^{\mathrm{pl}}, \kappa)$.

VARIABLE	DESCRIPTION
LCUPF	Load curve ID describing plastic initiation displacement as a function of mode mixity, that is, $\bar{u}_0^{\mathrm{pl}}(\kappa)$. Only for MODEL = 1, 11, or 21. For MODEL = 1, LCUPF can also be a table ID giving plastic initiation displacement as a function of peel ratio (table values) and mode mixity (curves). See Remarks section for *CONSTRAINED_-INTERPOLATION_SPOTWELD.
LCUPR	Load curve ID describing plastic rupture displacement as a function of mode mixity, that is, $\bar{u}_f^{\mathrm{pl}}(\kappa)$. Only for MODEL = 1, 11, or 21. For MODEL = 1, LCUPF can also be a table ID giving plastic initiation displacement as a function of peel ratio (table values) and mode mixity (curves). See Remarks section for *CONSTRAINED_-INTERPOLATION_SPOTWELD.

SPR3 Cards. Include this card if the keyword option SPR3 is used and MODEL = 1, 11, 21, 31, or 41.

Card 5a.1	1	2	3	4	5	6	7	8
Variable	STIFF2	STIFF3	STIFF4	LCDEXP	GAMMA	SROPT		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
STIFF2	Elastic shear stiffness. It is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details.
STIFF3	Elastic bending stiffness. It is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details.
STIFF4	Elastic torsional stiffness. It is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details.
LCDEXP	Load curve for damage exponent as a function of mode mixity
GAMMA	Scaling factor, γ_1. It is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details.
SROPT	Shear rotation option that defines local kinematics system: EQ.0: Pure shear does not create a normal component (default).

VARIABLE	DESCRIPTION
	EQ.1: Pure shear creates a normal component.

SPR3 Cards. This card is optional. It is read if keyword option SPR3 is used and MODEL = 1, 11, 21, 31, or 41.

Card 5a.2	1	2	3	4	5	6	7	8
Variable	FFN	FFB	FFS	EXFC	STIFP	MFSFC	DEFC	NPFC
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
FFN	Resultant normal force at failure. FFN is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details.
FFB	Resultant bending force at failure. FFB is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details.
FFS	Resultant shear force at failure. FFS is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details.
EXFC	Failure function exponent. EXFC is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details.
STIFP	Plastic stiffness. If greater than zero, this replaces LCF by a simple linear hardening law: $$F^0\left(\bar{u}^{\mathrm{pl}}\right) = 1.0 + \mathrm{STIFP} \times \bar{u}^{\mathrm{pl}} \ .$$ STIFP is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details.
MFSFC	Scaling factor for torsion term in resultant shear force. MFSC is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details.
DEFC	Fading energy for damage. DEFC is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details.

VARIABLE	DESCRIPTION
NPFC	Plastic displacement offset for damage initiation. NPFC is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details.

SPR3 Cards. Include this card if the keyword option SPR3 is used and MODEL = 2, 12, or 22.

Card 5b.1	1	2	3	4	5	6	7	8
Variable	UPFN	UPFS	ALPHA2	BETA2	UPRN	UPRS	ALPHA3	BETA3
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
UPFN	Plastic initiation displacement in normal direction, $\bar{u}_{0,\mathrm{ref}}^{\mathrm{pl},n}$
UPFS	Plastic initiation displacement in shear direction, $\bar{u}_{0,\mathrm{ref}}^{\mathrm{pl},s}$
ALPHA2	Plastic initiation displacement scaling factor, α_2
BETA2	Exponent for plastic initiation displacement, β_2
UPRN	Plastic rupture displacement in normal direction, $\bar{u}_{f,\mathrm{ref}}^{\mathrm{pl},n}$
UPRS	Plastic rupture displacement in shear direction, $\bar{u}_{f,\mathrm{ref}}^{\mathrm{pl},s}$
ALPHA3	Plastic rupture displacement scaling factor, α_3
BETA3	Exponent for plastic rupture displacement, β_3

SPR3 Cards. Include this card if the keyword option SPR3 is used and MODEL = 2, 12, or 22.

Card 5b.2	1	2	3	4	5	6	7	8
Variable	MRN	MRS						
Type	F	F						

VARIABLE	DESCRIPTION
MRN	Proportionality factor for dependency, m_{R_n}
MRS	Proportionality factor for dependency, m_{R_s}

Sample Input:

With this material model it is possible to replace the following input example

```
*CONSTRAINED_SPR2
$     UPID      LPID      NSID     THICK         D        FN        FT        DN
         5         8       123       5.0       8.0      2.53       4.8       4.0
$       DT       XIN       XIT    ALPHA1    ALPHA2    ALPHA3      DENS      INTP
       7.5       0.6       0.5       0.2       0.7       1.9    7.8e-6         1
$     EXPN      EXPT     PIDVB
       8.0       8.0       999
$    XPID1     XPID2     XPID3     XPID4
        20
```

with this “split” one

```
*CONSTRAINED_SPR2
$     UPID      LPID      NSID     THICK         D        FN        FT        DN
         5         8       123       5.0       8.0      -555
$       DT       XIN       XIT    ALPHA1    ALPHA2    ALPHA3      DENS      INTP
                                                                             1
$     EXPN      EXPT     PIDVB
                           999
$    XPID1     XPID2     XPID3     XPID4
        20
*MAT_CONSTRAINED_SPR2
$      MID        RO        FN        FT        DN        DT       XIN       XIT
       555    7.8e-6      2.53       4.8       4.0       7.5       0.6       0.5
$   ALPHA1    ALPHA2    ALPHA3      EXPN      EXPT
       0.2       0.7       1.9       8.0       8.0
```

and still get the same result. Note that only the non-material data (UPID, LPID, NSID, THICK, D, INTP, PIDVB, XPID*i*) remains with the *CONSTRAINED keyword. Variables in grey are optional.

*MAT_TISSUE_DISPERSED

This is Material Type 266. This material is an invariant formulation for dispersed orthotropy in soft tissues, e.g., heart valves, arterial walls or other tissues where one or two collagen fibers are used. The passive contribution is composed of an isotropic and two anisotropic parts. The isotropic part is a simple neo-Hookean model. The first anisotropic part is passive, with two collagen fibers to choose from: (1) a simple exponential model and (2) a more advanced crimped fiber model from Freed et al. [2005]. The second anisotropic part is active described in Guccione et al. [1993] and is used for active contraction.

NOTE: This material model is obsolete. Please use MAT_ANISOTROPIC_HYPERELASTIC which contains most of the features of MAT_TISSUE_DISPERSED. For missing or additional features, please consult LST directly.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	F	SIGMA	MU	KAPPA	ACT	INIT
Type	A	F	F	F	F	F	I	I

Card 2	1	2	3	4	5	6	7	8
Variable	FID	ORTH	C1	C2	C3	THETA	NHMOD	
Type	I	I	F	F	F	F	F	

Card 3	1	2	3	4	5	6	7	8
Variable	ACT1	ACT2	ACT3	ACT4	ACT5	ACT6	ACT7	ACT8
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	ACT9	ACT10						
Type	F	F						

Card 5	1	2	3	4	5	6	7	8
Variable	AOPT	BETA	XP	YP	ZP	A1	A2	A3
Type	I	F	F	F	F	F	F	F

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density.
F	Fiber dispersion parameter governs the extent to which the fiber dispersion extends to the third dimension. F = 0 and F = 1 apply to 2D splay with the normal to the membrane being in the β and the γ-directions, respectively (see Figure M266-1). F = 0.5 applies to 3D splay with transverse isotropy. Splay will be orthotropic wheneverF $\neq$ 0.5. This parameter is ignored if INIT = 1.
SIGMA	The parameter SIGMA governs the extent of dispersion, such that as SIGMA goes to zero, the material symmetry reduces to pure transverse isotropy. Conversely, as SIGMA becomes large, the material symmetry becomes isotropic in the plane. This parameter is ignored if INIT = 1.
MU	MU is the isotropic shear modulus that models elastin. MU should be chosen such that the following relation is satisfied:

VARIABLE	DESCRIPTION
	$0.5\,(3\text{KAPPA} - 2\text{MU})/(3\text{KAPPA} + \text{MU}) < 0.5.$ Instability can occur for implicit simulations if this quotient is close to 0.5. A modest approach is a quotient between 0.495 and 0.497.
KAPPA	Bulk modulus for the hydrostatic pressure.
ACT	ACT = 1 indicates that an active model will be used that acts in the mean fiber-direction. The active model, like the passive model, will be dispersed by SIGMA and F, or if INIT = 1, with the *INITIAL_FIELD_SOLID keyword.
INIT	INIT = 1 indicates that the anisotropy eigenvalues will be given by *INITIAL_FIELD_SOLID variables in the global coordinate system (see Remark 1).
FID	The passive fiber model number. There are two passive models available: FID = 1 or FID = 2. They are described in Remark 2.
ORTH	ORTH specifies the number (1 or 2) of fibers used. When ORTH = 2 two fiber families are used and arranges symmetrically THETA degrees from the mean fiber direction and lying in the tissue plane.
C1-C3	Passive fiber model parameters.
THETA	The angle between the mean fiber direction and the fiber families. The parameter is active only if ORTH = 2 and is particularly important in vascular tissues (e.g. arteries)
NHMOD	Neo-Hooke model flag EQ.0.0: Original implementation (modified Neo-Hooke) EQ.1.0: Standard Neo-Hooke model (as in umat45 of dyn21.f)
ACT1 - ACT10	Active fiber model parameters. Note that ACT10 is an input for a time dependent load curve that overrides some of the ACTx values. See section 2 below.
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description): EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. EQ.1.0: Locally orthotropic with material axes determined by a point in space and the global location of the element

VARIABLE	DESCRIPTION
	center; this is the a-direction.
	EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.
	EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal.
	EQ.4.0: Locally orthotropic in cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, **p**, which define the centerline axis.
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
BETA	Material angle in degrees for AOPT = 3, may be overridden on the element card *ELEMANT_SOLID_ORTHO.
XP - ZP	XP, YP and ZP define the coordinates of point P for AOPT = 1 and AOPT = 4.
A1 - A3	A1, A2 and A3 define the components of vector A for AOPT = 2.
D1 - D3	D1, D2 and D3 define components of vector D for AOPT = 2.
V1 - V3	V1, V2 and V3 define components of vector V for AOPT = 3 and AOPT = 4.

Material Formulation:

Details of the passive model can be found in Freed et al. (2005) and Einstein et al. (2005). The stress in the reference configuration consists of a deviatoric matrix term, a hydrostatic pressure term, and either one (ORTHO = 1) or two (ORTH = 2) fiber terms:

$$\mathbf{S} = \kappa J(J-1)\mathbf{C}^{-1} + \mu J^{-2/3}\mathbf{DEV}\left[\frac{1}{4}\left(\mathbf{I} - \bar{\mathbf{C}}^{-2}\right)\right] + J^{-2/3}\sum_{i=1}^{n}[\sigma_i(\lambda_i) + \varepsilon_i(\lambda_i)]\mathbf{DEV}[\mathbf{K}_i]$$

where **S** is the second Piola-Kirchhoff stress tensor, J is the Jacobian of the deformation gradient, κ is the bulk modulus, σ_i is the passive fiber stress model used, and ε_i is the corresponding active fiber model used. The operator **DEV** is the deviatoric projection:

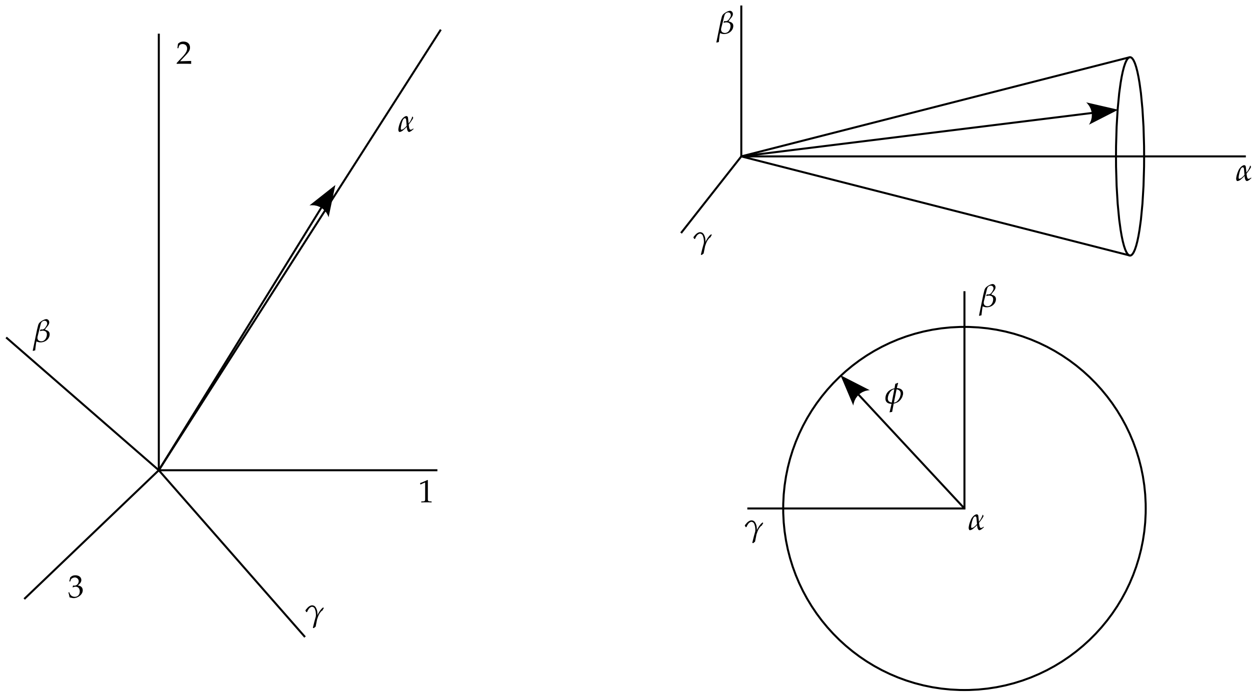


Figure M266-1. The plot on the left relates the global coordinates (1, 2, 3) to the local coordinates (α, β, γ), selected so the mean fiber direction in the reference configuration is align with the α–axis. The plots on the right show how the unit vector for a specific fiber within the fiber distribution of a 3D tissue is oriented with respect to the mean fiber direction via angles θ and ϕ.

$$\mathbf{DEV}[\bullet] = (\bullet) - \frac{1}{3}\mathrm{tr}[(\bullet)\mathbf{C}]\mathbf{C}^{-1}$$

where $\mathbf{C}$ is the right Cauchy-Green deformation tensor. The dispersed fourth invariant is $\lambda = \sqrt{\mathrm{tr}[\mathbf{K}\bar{\mathbf{C}}]}$, where $\bar{\mathbf{C}}$is the isochoric part of the Cauchy-Green deformation. Note that λ is not a stretch in the classical way, since $\mathbf{K}$ embeds the concept of dispersion. $\mathbf{K}$ is called the dispersion tensor or anisotropy tensor and is given in global coordinates. The passive and active fiber models are defined in the fiber coordinate system. In effect the dispersion tensor rotates and weights these one dimensional models, such that they are both three-dimensional and in the Cartesian framework.

In the case where, the splay parameters SIGMA and F are specified, $\mathbf{K}$ is given by:

$$\mathbf{K}_i = \frac{1}{2}\mathbf{Q}_i \begin{bmatrix} 1 + e^{-2\mathrm{SIGMA}^2} & 0 & 0 \\ 0 & \mathrm{F}\left(1 - e^{-2\mathrm{SIGMA}^2}\right) & 0 \\ 0 & 0 & (1-\mathrm{F})\left(1 - e^{-2\mathrm{SIGMA}^2}\right) \end{bmatrix} \mathbf{Q}_i^T$$

where $\mathbf{Q}$ is the transformation tensor that rotates from the local to the global Cartesian system. In the case when INIT = 1, the dispersion tensor is given by

$$\mathbf{K}_i = \mathbf{Q}_i \begin{pmatrix} \chi_i^1 & 0 & 0 \\ 0 & \chi_i^2 & 0 \\ 0 & 0 & \chi_i^3 \end{pmatrix} \mathbf{Q}_i^T$$

where the χ:s are given on the *INITIAL_FIELD_SOLID card. For the values to be physically meaningful $\chi_i^1 + \chi_i^2 + \chi_i^3 = 1$. It is the responsibility of the user to assure that this condition is met, no internal checking for this is done. These values typically come from diffusion tensor data taken from the myocardium.

Remarks:

1. Passive fiber models. Currently there are two models available.

   a) If FID = 1 a crimped fiber model is used. It is solely developed for collagen fibers. Given H_0 and R_0 compute:

   $$L_0 = \sqrt{(2\pi)^2 + (H_0)^2}, \Lambda = \frac{L_0}{H_0}$$

   and

   $$E_s = \frac{E_f H_0}{H_0 + \left(1 + \frac{37}{6\pi^2} + 2\frac{L_0^2}{\pi^2}\right)(L_0 - H_0)}.$$

   Now if the fiber stretches $\lambda < \Lambda$ the fiber stress is given by:

   $$\sigma = \xi E_s(\lambda - 1)$$

   where

   $$\xi = \frac{6\pi^2(\Lambda^2 + (4\pi^2 - 1)\lambda^2)\lambda}{\Lambda\big(3H_0^2(\Lambda^2 - \lambda^2)(3\Lambda^2 + (8\pi^2 - 3)\lambda^2) + 8\pi^2(10\Lambda^2 + (3\pi^2 - 10)\lambda^2)\big)}$$

   and if $\lambda > \Lambda$ the fiber stress equals:

   $$\sigma = E_s(\lambda - 1) + E_f(\lambda - \Lambda).$$

   In Figure M266-1 the fiber stress is rendered with $H0 = 27.5, R0 = 2$ and the transition point becomes $\Lambda = 1.1$.

   b) The second fiber model available (FID = 2) is a simpler but more useful model for the general fiber reinforced rubber. The fiber stress is simply given by:

   $$\sigma = C_1\left[e^{\frac{C_2}{2}(\lambda^2 - 1)} - 1\right].$$

The difference between the two fiber models is given in Figure M266-2.

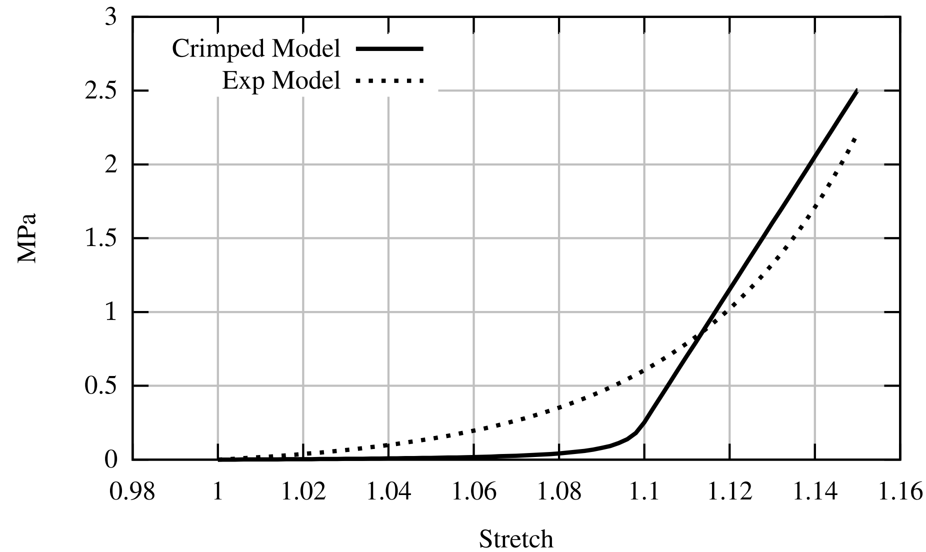


Figure M266-2. Visualization ofthe Crimped and the Exponential fiber models. Here $\Lambda = 1.1$ is the transition point in the crimped model.

The active model for myofibers (ACT = 1) is defined in Guccione et al. (1993) and is given by:

$$\sigma = T_{\max} \frac{Ca_0^2}{Ca_0^2 + ECa_{50}^2} C(t)$$

where

$$ECa_{50}^2 = \frac{(Ca_0)_{\max}}{\sqrt{e^{B\left(l_r\sqrt{2(\lambda-1)+1}-l_0\right)-1}}}$$

and B is a constant, $(Ca_0)_{\max}$ is the maximum peak intracellular calcium concentration, l_0 is the sarcomere length at which no active tension develops and l_r is the stress free sarcomere length. The function $C(t)$ is defined in one of two ways. First it can be given as:

$$C(t) = \frac{1}{2}(1 - \cos\omega(t))$$

where

$$\omega = \begin{cases} \pi \dfrac{t}{t_0} & 0 \le t < t_0 \\ \pi \dfrac{t - t_0 + t_r}{t_r} & t_0 \le t < t_0 + t_r \\ 0 & t_0 + t_r \le t \end{cases}$$

and $t_r = ml_R\lambda + b$. Secondly, it can also be given as a load curve. If a load curve should be used its index must be given in ACT10. Note that all variables that correspond to ω are neglected if a load curve is used. The active parameters on Card 3 and 4 are interpreted as:

ACT1	ACT2	ACT3	ACT4	ACT5	ACT6	ACT7	ACT8	ACT9	ACT10
$T_{\max}$	Ca_0	$(Ca_0)_{\max}$	B	l_0	t_0	m	b	l_R	LCID

References:

1. Freed AD., Einstein DR. and Vesely I., Invariant formulation for dispersed transverse isotropy in aortic heart valves – An efficient means for modeling fiber splay, Biomechan model Mechanobiol, 4, 100-117, 2005.

2. Guccione JM., Waldman LK., McCulloch AD., Mechanics of Active Contraction in Cardiac Muscle: Part II – Cylindrical Models of the Systolic Left Ventricle, J. Bio Mech, 115, 82-90, 1993.

*MAT_EIGHT_CHAIN_RUBBER

This is Material Type 267. This is an advanced rubber-like model that is tailored for glassy polymers and similar materials. It is based on Arruda's eight chain model but enhanced with non-elastic properties. This material is available for solid and SPH elements.

Card Summary:

Card 1. This card is required.

MID	RO	K	MU	N	MULL	VISPL	VISEL

Card 2. This card is required.

YLD0	FP	GP	HP	LP	MP	NP	PMU

Card 3. This card is required.

M1	M2	M3	M4	M5	TIME	VCON	

Card 4. This card is required.

Q1	B1	Q2	B2	Q3	B3	Q4	B4

Card 5. This card is required.

K1	S1	K2	S2	K3	S3		

Card 6. This card is required.

AOPT	MACF	XP	YP	ZP	A1	A2	A3

Card 7. This card is required.

V1	V2	V3	D1	D2	D3	BETA	

Card 8a. This card is included if VISEL = 1. Include up to 6 of this card. This next keyword ("*") card terminates this input.

TAU*i*	BETA*i*						

Card 8b. This card is included if VISEL = 2. Include up to 6 of this card. The next keyword ("*") card terminates this input.

TAU*i*	GAMMA*i*						

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	MU	N	MULL	VISPL	VISEL
Type	A	F	F	F	I	I	I	I
Default	none	none	0.0	0.0	0	none	0	0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
K	Bulk modulus. To get almost incompressible behavior set K one or two orders of magnitude higher than MU. Note that the poisons ratio should be kept at a realistic value. $$v = \frac{3K - 2\mathrm{MU}}{2(3K + \mathrm{MU})}.$$
MU	Shear modulus. MU is the product of the number of molecular chains per unit volume (n), Boltzmann's constant (k) and the absolute temperature (T). Thus $\mathrm{MU} = nkT$.
N	Number of rigid links between crosslinks of the soft domain region. See Remark 1.
MULL	Parameter describing which softening algorithm that shall be used (see Remarks 1 and 2). EQ.1: Strain based Mullins effect from Qi and Boyce EQ.2: Energy based Mullins, a modified version of Roxburgh and Ogden model. M1, M2, and M3 must be set.
VISPL	Parameter describing which viscoplastic formulation that should be used; see the theory section for details (see Remark 4). EQ.0: No viscoplasticity EQ.1: 2 parameter standard model; K1 and S1 must be set.

VARIABLE	DESCRIPTION
	EQ.2: 6 parameter G'Sells model; K1, K2, K3, S1, S2 and S3 must be set.
	EQ.3: 4 parameter strain hardening model; K1, K2, S1, and S2 must be set.
VISEL	Option for viscoelastic behavior; see the theory section for details. EQ.0: No viscoelasticity EQ.1: Free energy formulation based on Holzapfel and Ogden EQ.2: Formulation based on stiffness ratios from Simo et al.

Card 2	1	2	3	4	5	6	7	8
Variable	YLD0	FP	GP	HP	LP	MP	NP	PMU
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION
YLD0	Initial yield stress (see Remark 4). EQ.0.0: No plasticity GT.0.0: Initial yield stress; hardening is defined separately. LT.0.0: -YLD0 is taken as the load curve ID for the yield stress as a function of effective plastic strain.
FP-NP	Parameters for Hill's general yield surface. For Von Mises yield criteria set FP = GP = HP = 0.5 and LP = MP = NP = 1.5. See Remark 4.
PMU	Kinematic hardening parameter. It usually equals MU. See Remark 5.

Card 3	1	2	3	4	5	6	7	8
Variable	M1	M2	M3	M4	M5	TIME	VCON	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	0.0	9.0	

VARIABLE	DESCRIPTION
M1	Mullins constant (see Remarks 1 and 2): MULL.EQ.1: Constant A. For the case of a dilute solution the Mullins parameter A should be equal to 3.5. See Qi and Boyce [2004]. MULL.EQ.2: Constant M1 in the Mullins equations. M1 > 0.0 must be set.
M2	Mullins constant (see Remarks 1 and 2): MULL.EQ.1: Constant B. For a system with well dispersed particles B should somewhere around 18. See Qi and Boyce [2004]. MULL.EQ.2: Constant M2 in the Mullins equations. M2 > 0.0 must be set.
M3	Mullins constant (see Remarks 1 and): MULL.EQ.1: Constant Z. Qi recommends 0.7. MULL.EQ.2: Constant M3 in the Mullins equations. M3 > 0.0 must be set.
M4	Mullins parameter (see Remarks 1 and 2): MULL.EQ.1: Initial value of v_s. v_s must be between 0 and 1 and must be less than v_{ss} (see M5). MULL.EQ.2: Not used
M5	Mullins parameter (see Remarks 1 and 2): MULL.EQ.1: v_{ss}, saturation value of v_s. v_{ss} must be between 0 and 1 and must be greater than v_s (see M4). MULL.EQ.2: Not used

VARIABLE	DESCRIPTION
TIME	A time filter is used to smooth out the time derivative of the strain invariant over a TIME interval. Default is no smoothening but a value 100 × TIMESTEP is recommended.
VCON	A material constant for the volumetric part of the strain energy. The default is 9.0 but any value can be used to tailor the volumetric response.

Card 4	1	2	3	4	5	6	7	8
Variable	Q1	B1	Q2	B2	Q3	B3	Q4	B4
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION
Q1 - B4	Voce hardening parameters. See Remark 4.

Card 5	1	2	3	4	5	6	7	8
Variable	K1	S1	K2	S2	K3	S3		
Type	F	F	F	F	F	F		
Default	0.0	0.0	0.0	0.0	0.0	0.0		

VARIABLE	DESCRIPTION
K1 - S3	Viscoplastic parameters (see Remark 4). VISPL.EQ.1: K1 and S1 are used. VISPL.EQ.2: K1, S1, K2, S2, K3 and S3 are used. VISPL.EQ.3: K1, S1 and K2 are used.

Card 6	1	2	3	4	5	6	7	8
Variable	AOPT	MACF	XP	YP	ZP	A1	A2	A3
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details): EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. EQ.1.0: Locally orthotropic with material axes determined by a point, P, in space and the global location of the element center; this is the **a**-direction. This option is for solid elements only. EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.

VARIABLE	DESCRIPTION
	EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only.
MACF	Material axes change flag for solid elements: EQ.-4: Switch material axes *b* and *c* before BETA rotation EQ.-3: Switch material axes *a* and *c* before BETA rotation EQ.-2: Switch material axes *a* and *b* before BETA rotation EQ.1: No change, default EQ.2: Switch material axes *a* and *b* after BETA rotation EQ.3: Switch material axes *a* and *c* after BETA rotation EQ.4: Switch material axes *b* and *c* after BETA rotation Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_-SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 7 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.
XP, YP, ZP	Coordinates for point *p* for AOPT = 1 and 4
A1, A2, A3	Components of vector **a** for AOPT = 2

Card 7	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

VARIABLE	DESCRIPTION
D1, D2, D3	Components of vector **d** for AOPT = 2
V1, V2, V3	Components of vector **v** for AOPT = 3 and 4

VARIABLE	DESCRIPTION
BETA	Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SOLID_ORTHO.

Card 8a	1	2	3	4	5	6	7	8
Variable	TAU*i*	BETA*i*						
Type	F	F						
Default	0.0	0.0						

VARIABLE	DESCRIPTION
TAU*i*	Relaxation time. See Remark 3.
BETA*i*	Dissipating energy factors (see Holzapfel). See Remark 3.

Card 8b	1	2	3	4	5	6	7	8
Variable	TAU*i*	GAMMA*i*						
Type	F	F						
Default	0.0	0.0						

VARIABLE	DESCRIPTION
TAU*i*	Relaxation time. A maximum of 6 values can be used. See Remark 3.
GAMMA*i*	Gamma factors (see Simo). See Remark 3.

Remarks:

1. **Basic Theory.** This model is based on the work done by Arruda and Boyce [1993], in particular Arruda's thesis [1992]. The eight chain rubber model is based on hyper-elasticity. It is formulated with elastic strain invariants. Strain

softening is accounted for by the parameter from v_s following the work done by Qi and Boyce [2004].

The strain energy is defined in terms of the elastic deformation gradient $\mathbf{F}_e$ through the right Cauchy-Green tensor $\mathbf{C}_e = \mathbf{F}_e^T\mathbf{F}_e$ and its determinant $J_e = \det\mathbf{F}_e$ as $\Psi = \Psi_1 + \Psi_2$. Ψ_1 and Ψ_2 are the deviatoric and volumetric contributions, respectively:

$$\Psi_1 = v_s\mu\left[\sqrt{N}\Lambda_c\beta + N\ln\left(\frac{\beta}{\sinh\beta}\right)\right]$$

$$\Psi_2 = \frac{\kappa}{\nu_{\mathrm{con}}^2}\left(\nu_{\mathrm{con}}\ln J_e + \frac{1}{J_e^{\nu_{\mathrm{con}}}} - 1\right)$$

Here

$$\beta = \mathcal{L}^{-1}\left(\frac{\Lambda_c}{\sqrt{N}}\right)$$

with $\mathcal{L}^{-1}$ denoting the inverse of the Langevin function, $\mathcal{L}(x) = \coth x - 1/x$, and the amplified chain stretch is given by

$$\Lambda_c = \sqrt{X(v_s)\left(\bar{\lambda}^2 - 1\right) + 1},$$

where $\bar{\lambda}^2 = \mathrm{Tr}(\bar{\mathbf{C}}_e)/3$ and $\bar{\mathbf{C}}_e = J_e^{-2/3}\mathbf{C}_e$.

Among the constant parameters, μ is the initial modulus of the soft domain, κ is the bulk modulus, ν_{con} is a pressure influential exponent and N is the number of rigid links between crosslinks of the soft domain region. X is a general polynomial describing the interaction between the soft and hard phases (Qi and Boyce [2004] and Tobin and Mullins [1957]). It is given by

$$X(v_s) = 1 + A(1 - v_s) + B(1 - v_s)^2$$

where A and B are constants. Without the Mullins effect, $\nu_s = 1$. Otherwise, its evolution depends on the Mullins effect. See Remark 2.

The Cauchy stress is then computed as

$$\sigma = \frac{2}{J_e}\mathbf{F}_e\frac{\partial\Psi}{\partial\mathbf{C}_e}\mathbf{F}_e{}^T = v_s\mu\frac{X(v_s)}{3J_e^{5/3}}\frac{\sqrt{N}}{\Lambda_c}\beta\left(\mathbf{B}_e - \frac{\mathrm{tr}(\mathbf{B}_e)}{3}\mathbf{I}\right) + \frac{\kappa}{\nu_{\mathrm{con}}J_e}\left(1 - \frac{1}{J_e^{\nu_{\mathrm{con}}}}\right)\mathbf{I},$$

where $\mathbf{B}_e = \mathbf{F}_e\mathbf{F}_e^T$ is the left elastic Cauchy-Green tensor.

2. **Mullins Effect.** Two models for the Mullins effect are implemented.

   a) *MULL = 1*. The strain softening is developed by the evolution law taken from Boyce 2004:

$$\dot{v}_s = Z(v_{ss} - v_s)\frac{\sqrt{N} - 1}{\left(\sqrt{N} - \Lambda_c^{\max}\right)^2}\dot{\Lambda}_c^{\max} \quad ,$$

where Z is a parameter that characterizes the evolution in v_s with increasing $\dot{\Lambda}_c^{\max}$. The parameter v_{ss} is the saturation value of v_s. Note that $\dot{\Lambda}_c^{\max}$ is the maximum of Λ_c from the past:

$$\dot{\Lambda}_c^{\max} = \begin{cases} 0 & \Lambda_c < \Lambda_c^{\max} \\ \dot{\Lambda}_c & \Lambda_c > \Lambda_c^{\max} \end{cases}.$$

The structure now evolves with the deformation. The dissipation inequality requires that the evolution of the structure is irreversible $\dot{v}_s \geq 0$. See Qi and Boyce [2004].

b) *MULL* = *2*. The energy driven model is based on Ogden and Roxburgh. When activated the strain energy is automatically transformed to a standard eight chain model, meaning the variables Z, v_s and X are automatically set to 0, 1, and 1, respectively. The stress is multiplicative split of the true stress and the softening factor η.

$$\bar{\sigma} = \eta\sigma, \quad \eta = 1 - \frac{1}{\mathrm{M1}}\operatorname{erf}\left(\frac{\Psi_1^{\max} - \Psi_1}{\mathrm{M3} - \mathrm{M2}\Psi_1^{\max}}\right) \quad .$$

3. **Viscoelasticity.** Two models for viscoelasticity are implemented.

a) *VISEL* = *1*. The viscoelasticity is based on work done by Holzapfel (2004)

$$\dot{\mathbf{Q}}_\alpha + \frac{\mathbf{Q}_\alpha}{\tau_\alpha} = 2\beta_\alpha \frac{d}{dt}\frac{\partial \Psi_1}{\partial \mathbf{C}_e} = \beta_\alpha \dot{\mathbf{S}}_\mathbf{1}$$

where α is the number of viscoelastic terms (1, …, 6).

b) *VISEL* = *2*. With this option the evolution is based on work done by Simo and Hughes (2000).

$$\dot{\mathbf{Q}}_\alpha + \frac{\mathbf{Q}_\alpha}{\tau_\alpha} = 2\frac{\gamma_a}{\tau_a}\frac{d}{dt}\frac{\partial \Psi_1}{\partial \mathbf{C}_e} = \frac{\gamma_a}{\tau_a}\mathbf{S}_\mathbf{1}$$

The number of Prony terms is restricted to a maximum of 6. Also τ_α and γ_α must be greater than 0. The Cauchy stress is obtained by a push forward operation on the total second Piola-Kirchhoff stress.

$$\sigma = \frac{1}{J}\mathbf{F}_e\mathbf{S}\mathbf{F}_e{}^T.$$

4. **Viscoplasticity.** Plasticity is based on the general Hills' yield surface

$$\sigma_{eff}^2 = F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 \\ +2L\sigma_{12}^2 + 2M\sigma_{23}^2 + 2N\sigma_{13}^2$$

The hardening is either based on a load curve ID (-YLD0) or an extended Voce hardening

$$\sigma_{\text{yld}} = \sigma_{\text{yld0}} + Q_1\left(1 - e^{B_1\bar{\varepsilon}}\right) + Q_2\left(1 - e^{B_2\bar{\varepsilon}}\right) + Q_3\left(1 - e^{B_3\bar{\varepsilon}}\right) + Q_4\left(1 - e^{B_4\bar{\varepsilon}}\right) \ .$$

The evolution of the elastic deformation gradient $\mathbf{F}_e$ is written as

$$\dot{\mathbf{F}}_e = (\mathbf{L} - \mathbf{L}_p)\mathbf{F}_e$$

where $\mathbf{L}$ is the spatial velocity gradient and $\mathbf{L}_\text{p}$ is the spatial (Eulerian) plastic velocity gradient which is given by the associative flow rule

$$\mathbf{L}_p = \dot{\bar{\varepsilon}}\frac{\partial f}{\partial \sigma}$$

with f being the rate independent yield surface

$$f = \sigma_{\text{eff}} - \sigma_{\text{yld}}.$$

For rate independent plasticity (VISPL = 0) the evolution of plastic strain $\bar{\varepsilon}$ follows from the consistency conditions $f \leq 0$, $\dot{\bar{\varepsilon}} \geq 0$ and $f\dot{\bar{\varepsilon}} = 0$. For viscoplasticity these conditions are abandoned, but we instead invoke a constitutive equation for the effective plastic strain rate. This is the Perzyna (1966) overstress model, and three different formulations are available.

a) *VISPL = 1*. The evolution equation is

$$\dot{\bar{\varepsilon}} = \left(\frac{\max(f,0)}{K_1}\right)^{S_1} \ ,$$

where K_1 and S_1 are viscoplastic material parameters.

b) *VISPL = 2*. The evolution equation is

$$\dot{\bar{\varepsilon}} = \left[\frac{\max(f,0)}{K_1\left(1 - e^{-S_1(\bar{\varepsilon}+K_2)}\right)e^{S_2\bar{\varepsilon}^{K_3}}}\right]^{S_3} \ ,$$

where K_1, K_2, K_3, S_1, S_2, and S_3 are viscoplastic parameters.

c) *VISPL = 3*. The evolution equation is

$$\dot{\bar{\varepsilon}} = \left(\frac{\max(f,0)}{K_1}\right)^{S_1} (\bar{\varepsilon} + K_2)^{S_2}$$

where K_1, K_2, S_1, and S_2 are viscoplastic parameters.

5. **Kinematic Hardening.** The back stress is calculated similar to the Cauchy stress above but without the softening factors:

$$\beta = \frac{\mu_p}{3J}\frac{\sqrt{N}}{\Lambda_c}L^{-1}\left(\frac{\Lambda_c}{\sqrt{N}}\right)\left(\mathbf{I} - \frac{1}{3}I_p\mathbf{C}_p^{-1}\right) \ .$$

μ_p is a hardening material parameter (PMU). The total Piola-Kirchhoff stress is now given by $\mathbf{S}^* = \mathbf{S} - \beta$ and the total stress is given by a standard push forward operation with the elastic deformation gradient.

References:

Qi HJ., Boyce MC., Constitutive model for stretch-induced softening of stress-stretch behavior of elastomeric materials, Journal of the Mechanics and Physics of Solids, 52, 2187-2205, 2004.

Arrude EM., Characterization of the strain hardening response of amorphous polymers, PhD Thesis, MIT, 1992.

Mullins L., Tobin NR., Theoretical model for the elastic behavior of filler reinforced vulcanized rubber, Rubber Chem. Technol., 30, 555-571, 1957.

Ogden RW. Roxburgh DG., A pseudo-elastic model for the Mullins effect in Filled rubber., Proc. R. Soc. Lond. A., 455, 2861-2877, 1999.

Simo JC., Hughes TJR., Computational Inelasticity, Springer, New York, 2000.

Holzapfel GA., Nonlinear Solid Mechanics, Wiley, New-York, 2000.

*MAT_BERGSTROM_BOYCE_RUBBER

This is Material Type 269. This is a rubber model based on the Arruda and Boyce (1993) chain model accompanied with a viscoelastic contribution according to Bergström and Boyce (1998). The viscoelastic treatment is based on the physical response of a single entangled chain in an embedded polymer gel matrix, and the implementation is based on Dal and Kaliske (2009). This model is only available for solid elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	G	GV	N	NV	
Type	A	F	F	F	F	F	F	
Default	none	none	none	none	none	none	none	

Card 2	1	2	3	4	5	6	7	8
Variable	C	M	GAM0	TAUH				
Type	F	F	F	F				
Default	none	none	none	none				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
K	Elastic bulk modulus, K
G	Elastic shear modulus, G
GV	Viscoelastic shear modulus, G_v
N	Elastic segment number, N
NV	Viscoelastic segment number, N_v

VARIABLE	DESCRIPTION
C	Inelastic strain exponent, c. It should be less than zero.
M	Inelastic stress exponent, m
GAM0	Reference strain rate, $\dot{\gamma}_0$
TAUH	Reference Kirchhoff stress, $\hat{\tau}$

Remarks:

The deviatoric Kirchhoff stress for this model is the sum of an elastic and viscoelastic part according to

$$\bar{\tau} = \tau_e + \tau_v$$

The elastic part is governed by the Arruda-Boyce strain energy potential resulting in the following expression (after a Pade approximation of the Langevin function)

$$\tau_e = \frac{G}{3}\frac{3-\lambda_r^2}{1-\lambda_r^2}\left(\bar{\mathbf{b}} - \frac{\mathrm{tr}(\bar{\mathbf{b}})}{3}\mathbf{I}\right)$$

Here G is the elastic shear modulus, $\bar{\mathbf{b}}$ is the unimodular left Cauchy-Green tensor given by:

$$\bar{\mathbf{b}} = J^{-2/3}\mathbf{F}\mathbf{F}^T$$
$$J = \det \mathbf{F}$$

and λ_r is the relative network stretch given by:

$$\lambda_r^2 = \frac{tr(\bar{\mathbf{b}})}{3N}$$

The viscoelastic stress is based on a multiplicative split of the unimodular deformation gradient into unimodular elastic and inelastic parts, respectively,

$$J^{-1/3}\mathbf{F} = \mathbf{F}_e\mathbf{F}_i$$

We define

$$\mathbf{b}_e = \mathbf{F}_e\mathbf{F}_e^T$$

to be the elastic left Cauchy-Green tensor. The viscoelastic stress is given as

$$\tau_v = \frac{G_v}{3}\frac{3-\lambda_v^2}{1-\lambda_v^2}\left(\mathbf{b}_e - \frac{\mathrm{tr}(\mathbf{b}_e)}{3}\mathbf{I}\right)$$

where

$$\lambda_v^2 = \frac{\mathrm{tr}(\mathbf{b}_e)}{3N_v}$$

is the relative network stretch for the viscoelastic part. The evolution of the elastic left Cauchy-Green tensor can be written

$$\dot{\mathbf{b}}_e = \bar{\mathbf{L}}\mathbf{b}_e + \mathbf{b}_e\bar{\mathbf{L}}^T - 2\mathbf{D}_i\mathbf{b}_e$$

where the inelastic rate-of-deformation tensor is given as

$$\mathbf{D}_i = \dot{\gamma}_0(\lambda_i - 0.999)^c \left(\frac{\|\tau_v\|}{\hat{\tau}\sqrt{2}}\right)^m \frac{\tau_v}{\|\tau_v\|}$$

and

$$\bar{\mathbf{L}} = \mathbf{L} - \frac{\mathrm{tr}(\mathbf{L})}{3}\mathbf{I}$$

is the deviatoric velocity gradient. The stretch of a single chain relaxing in a polymer is linked to the inelastic right Cauchy-Green tensor as

$$\lambda_i^2 = \frac{\mathrm{tr}\left(\mathbf{F}_i^T\mathbf{F}_i\right)}{3} \geq 1 \ \ .$$

This stretch is available as the plastic strain variable in the post-processing of this material. The volumetric part is elastic and governed by the bulk modulus, the pressure for this model is given as

$$p = K\left(J^{-1} - 1\right) \ \ .$$

*MAT_CWM

This is Material Type 270. It is a thermo-elastic-plastic model with kinematic hardening that allows for material creation and annealing triggered by temperature. The acronym CWM stands for Computational Welding Mechanics, Lindström (2013, 2015). The model is intended to be used for simulating multistage weld processes. This model is available for solid and shell elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	LCEM	LCPR	LCSY	LCHR	LCAT	BETA
Type	A	F	I	I	I	I	I	F
Default	none	none	none	none	none	none	none	none

Card 2	1	2	3	4	5	6	7	8
Variable	TASTART	TAEND	TLSTART	TLEND	EGHOST	PGHOST	AGHOST	EPSINI
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	0.0

Optional Phase Change Card.

Card 3	1	2	3	4	5	6	7	8
Variable	T2PHASE	T1PHASE	ANOPT	POSTV	DTEMP	DOSPOT		
Type	F	F	F	I	F	I		
Default	optional	optional	0.0	0	0.0	0		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).

VARIABLE	DESCRIPTION
RO	Material density
LCEM	Load curve ID giving Young's modulus as a function of temperature
LCPR	Load curve ID giving Poisson's ratio as a function of temperature
LCSY	Load curve or table for yield stress. GT.0: Load curve ID giving yield stress as a function of temperature. LT.0: |LCSY| is a table ID giving yield curves for different temperatures. Each yield curve is a function of plastic strain.
LCHR	Load curve ID giving the hardening modulus as a function of temperature. LCHR is not used for LCSY < 0. The hardening modulus is then calculated from the yield curve's slope.
LCAT	Load curve (or table) ID giving the thermal expansion coefficient as a function of temperature (and maximum temperature up to the current time). In the case of a table, load curves are listed according to their maximum temperature. See Remark 1.
BETA	Fraction of isotropic hardening between 0 and 1: EQ.0.0: Kinematic hardening EQ.1.0: Isotropic hardening
TASTART	Annealing temperature start, T_a^{start}. See Remark 3.
TAEND	Annealing temperature end, T_a^{end}. See Remark 3.
TLSTART	Birth temperature start, T_l^{start}. See Remark 1.
TLEND	Birth temperature end, T_l^{end}. See Remark 1.
EGHOST	Young's modulus for ghost (quiet) material. See Remark 1.
PGHOST	Poisson's ratio for ghost (quiet) material. See Remark 1.
AGHOST	Thermal expansion coefficient for ghost (quiet) material. See Remark 1.
EPSINI	Initial plastic strains, uniformly distributed within the part.

VARIABLE	DESCRIPTION
T2PHASE	Temperature at which phase change commences. See Remark 4.
T1PHASE	Temperature at which phase change ends. See Remark 4.
ANOPT	Annealing option for thermal expansion (see Remark 3): EQ.0: No modification for thermal expansion. EQ.1: TAEND defines the upper limit (cut-off temperature) for evaluation of thermal expansion. LT.0: |ANOPT| defines the upper limit (cut-off temperature) for evaluation of thermal expansion.
POSTV	Define additional history variables that might be useful for post-processing. See Remark 5.
DTEMP	Maximum temperature variation within a time step. If exceeded during the analysis at a certain integration, a local (only for the respective integration points) sub-cycling is used for the calculation of the phase transformations. EQ.0.0: Not active (default) GT.0.0: Active
DOSPOT	Activate thinning of tied shell elements when SPOTHIN > 0 on *CONTROL_CONTACT. EQ.0: Spot weld thinning is inactive for shells tied to solids that use this material (default). EQ.1: Spot weld thinning is active for shells tied to solids that use this material.

Remarks:

1. **Material birth.** This material is initially in a quiet state, sometimes referred to as a ghost material. In this state, the material has thermo-elastic properties defined by the quiet Young's modulus, quiet Poisson's ratio, and quiet thermal expansion coefficient. These properties should represent void, meaning the Young's modulus should be small enough not to influence the surroundings but large enough to avoid numerical problems. A quiet material stress should never reach the yield point. When the temperature reaches the birth temperature, a history variable representing the indicator of the welding material is incremented. This variable follows

$$\gamma(t) = \min\left(1, \max\left[0, \frac{T_{\max} - T_l^{\text{start}}}{T_l^{\text{end}} - T_l^{\text{start}}}\right]\right)$$

where $T_{\max} = \max_{s \le t} T(s)$. This parameter is available as history variable 9 in the output database. The effective thermo-elastic material properties are interpolated as

$$\begin{aligned} E &= E(T)\gamma + E_{\text{quiet}}(1-\gamma) \\ \nu &= \nu(T)\gamma + \nu_{\text{quiet}}(1-\gamma) \\ \alpha &= \alpha(T, T_{\max})\gamma + \alpha_{\text{quiet}}(1-\gamma) \end{aligned}$$

where E, ν, and α are the Young's modulus, Poisson's ratio and thermal expansion coefficient, respectively. Here, the thermal expansion coefficient is either a temperature-dependent curve or a collection of temperature-dependent curves ordered in a table according to maximum temperature, $T_{\max}$.

2. **Stress update.** The stress update follows a classical isotropic associative thermo-elastic-plastic approach with kinematic hardening that is summarized in the following. The explicit temperature dependence is sometimes dropped for the sake of clarity.

The stress evolution is given as

$$\dot{\sigma} = \mathbf{C}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_p - \dot{\boldsymbol{\varepsilon}}_T\right)$$

where $\mathbf{C}$ is the effective elastic constitutive tensor and

$$\begin{aligned} \dot{\boldsymbol{\varepsilon}}_T &= \alpha \dot{T}\mathbf{I} \\ \dot{\boldsymbol{\varepsilon}}_p &= \dot{\varepsilon}_p \frac{3}{2}\frac{\mathbf{s} - \boldsymbol{\kappa}}{\bar{\sigma}} \end{aligned}$$

are the thermal and plastic strain rates, respectively. The latter expression includes the deviatoric stress

$$\mathbf{s} = \boldsymbol{\sigma} - \frac{1}{3}\mathrm{Tr}(\boldsymbol{\sigma})\mathbf{I},$$

the back stress $\boldsymbol{\kappa}$ and the effective stress

$$\bar{\sigma} = \sqrt{\frac{3}{2}(\mathbf{s} - \boldsymbol{\kappa}) : (\mathbf{s} - \boldsymbol{\kappa})}$$

that are involved in the plastic equations. To this end, the effective yield stress is given as

$$\sigma_Y = \sigma_Y(T) + \beta H(T)\varepsilon_p$$

and plastic strains evolve when the effective stress exceeds this value. The back stress evolves as

$$\dot{\boldsymbol{\kappa}} = (1-\beta)H(T)\dot{\varepsilon}_p \frac{\mathbf{s} - \boldsymbol{\kappa}}{\bar{\sigma}}$$

where $\dot{\varepsilon}_p$ is the rate of effective plastic strain rate that follows from consistency equations.

3. **Annealing.** When the temperature reaches the start annealing temperature, the material begins assuming its virgin properties. Beyond the start annealing temperature, it behaves as an ideal elastic-plastic material but with no evolution of plastic strains. The resetting of effective plastic properties in the annealing temperature interval is done by modifying the effective plastic strain and back stress before the stress update as

$$\varepsilon_p^{n+1} = \varepsilon_p^n \max\left[0, \min\left(1, \frac{T - T_a^{\mathrm{end}}}{T_a^{\mathrm{start}} - T_a^{\mathrm{end}}}\right)\right]$$
$$\kappa^{n+1} = \kappa^n \max\left[0, \min\left(1, \frac{T - T_a^{\mathrm{end}}}{T_a^{\mathrm{start}} - T_a^{\mathrm{end}}}\right)\right]$$

Depending on the choice for parameter ANOPT, annealing may also affect the thermal expansion of the structure. A cut-off temperature can be defined for the evaluation of the thermal expansion. Above this temperature limit, further expansion is suppressed. The cut-off temperature does not necessarily coincide with the annealing temperature.

4. **Average temperature rate.** Optional Card 3 is used to set history variable 11, which is the average temperature rate by which the temperature has gone from T2PHASE to T1PHASE. To fringe this variable, the range should be set to positive values. During the simulation it is temporarily used to store the time when the material has reached temperature T2PHASE which is stored as a negative value. A strictly positive value means that the material has reached temperature T2PHASE and gone down to T1PHASE and the history variable is (T2PHASE − T1PHASE)/(T1 − T2), where T2 is the time when temperature T2PHASE is reached and T1 is the time when temperature T1PHASE is reached. Note that T2PHASE > T1PHASE and T1 > T2. A value of zero means that the element has not yet reached temperature T2PHASE. A strictly negative value means that the element has reached temperature T2PHASE but not yet T1PHASE.

5. **History variables.** This material formulation outputs additional data for post-processing to the set of history variables if requested. The parameter POSTV defines the data to be written. Its value is calculated as

$$\mathrm{POSTV} = a_1 + 2\,a_2 + 4\,a_3 + 8\,a_4$$

Each flag a_i is a binary number (can be either 1 or 0) and corresponds to one particular post-processing variable according to the following table:

Flag	Description	Variables	# of History Variables
a_1	Accumulated thermal strain	ε_T	1

Flag	Description	Variables	# of History Variables
a_2	Accumulated strain tensor	$\boldsymbol{\varepsilon}$	6
a_3	Plastic strain tensor	$\boldsymbol{\varepsilon}_p$	6
a_4	Equivalent strain	$\varepsilon_{\mathrm{VM}}$	1

The above table also shows the order of output as well as the number of extra history variables associated with the particular flag. The values of these user-defined histories are reset when the temperature is in the annealing range.

In total NXH extra variables are required depending on the choice of parameter POSTV. For example, the maximum number of additional variables is NXH = 14 for POSTV = 15.

A complete list of history variables for the material is given in the following table. “Position” refers to the history variable number as listed by LS-PrePost when post-processing the d3plot database. The variable NEIPS in *DATABASE_EXTENT_BINARY must to set to output these history variables.

Position	Description
1 - 6	Back stress
7	Temperature at last time step
8	Yield indicator: 1 if yielding, else 0
9	Welding material indicator: 0 for ghost material, else 1
10	Maximum temperature reached
11	Average temperature rate going from T2PHASE to T1PHASE
12 $\rightarrow$ 11+NXH	User-defined history data as described in the preceding table

*MAT_POWDER

This is Material Type 271. This model is used to analyze the compaction and sintering of cemented carbides and the model is based on the works of Brandt (1998). This material is only available for solid elements.

Card Summary:

Card 1. This card is required.

MID	RO	P11	P22	P33	P12	P23	P13

Card 2. This card is required.

E0	LCK	PR	LCX	LCY	LCC	L	R

Card 3. This card is required.

CA	CD	CV	P	LCH	LCFI	SINT	TZRO

Card 3.1. Include this card if SINT = 1.

LCFK	LCFS2	DV1	DV2	DS1	DS2	OMEGA	RGAS

Card 3.2. Include this card if SINT = 1.

LCPR	LCFS3	LCTAU	ALPHA	LCFS1	GAMMA	L0	LCFKS

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	P11	P22	P33	P12	P23	P13
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).

VARIABLE	DESCRIPTION
RO	Mass density
PIJ	Initial compactness tensor P_{ij}

Card 2	1	2	3	4	5	6	7	8
Variable	E0	LCK	PR	LCX	LCY	LCC	L	R
Type	F	I	F	I	I	I	F	F
Default	none	none	none	none	none	none	none	none

VARIABLE	DESCRIPTION
E0	Initial anisotropy variable e (value between 1 and 2)
LCK	Load curve ID for bulk modulus K as function of relative density d
PR	Poisson's ratio, ν
LCX	Load curve ID for hydrostatic compressive yield X as function of relative density d
LCY	Load curve for uniaxial compressive yield Y as function of relative density d
LCC	Load curve ID for shear yield C_0 as function of relative density d
L	Yield surface parameter L relating hydrostatic compressive yield to point on hydrostatic axis with maximum strength
R	Yield surface parameter R governing the shape of the yield surface

Card 3	1	2	3	4	5	6	7	8
Variable	CA	CD	CV	P	LCH	LCFI	SINT	TZRO
Type	F	F	F	F	I	I	F	F
Default	none	none	none	none	none	none	0.0	none

VARIABLE	DESCRIPTION
CA	Hardening parameter c_a
CD	Hardening parameter c_d
CV	Hardening parameter c_v
P	Hardening exponent p
LCH	Load curve ID giving back stress parameter, H, as function of hardening parameter e
LCFI	Load curve ID giving plastic strain evolution angle, ϕ, as function of relative volumetric stress
SINT	Activate sintering: EQ.0.0: sintering off EQ.1.0: sintering on
TZRO	Absolute zero temperature, T_0

Sintering Card 1. Additional card for SINT = 1.

Card 3.1	1	2	3	4	5	6	7	8
Variable	LCFK	LCFS2	DV1	DV2	DS1	DS2	OMEGA	RGAS
Type	I	I	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

VARIABLE	DESCRIPTION
LCFK	Load curve ID for viscous compliance, f_K, as function of relative density, d
LCFS2	Load curve ID for viscous compliance, f_{S2}, as function of temperature, T
DV1	Volume diffusion coefficient d_{V1}
DV2	Volume diffusion coefficient d_{V2}
DS1	Surface diffusion coefficient d_{S1}
DS2	Surface diffusion coefficient d_{S2}
OMEGA	Blending parameter ω
RGAS	Universal gas constant, R_{gas}

Sintering Card 2. Additional card for SINT = 1.

Card 3.2	1	2	3	4	5	6	7	8
Variable	LCPR	LCFS3	LCTAU	ALPHA	LCFS1	GAMMA	L0	LCFKS
Type	I	I	I	F	I	F	F	I
Default	none	none	none	none	none	none	none	none

VARIABLE	DESCRIPTION
LCPR	Load curve ID for viscous Poisson's ratio, ν^v, as a function of relative density, d
LCFS3	Load curve ID for evolution of mobility factor, f_{S3}, as function of temperature, T
LCTAU	Load curve for relaxation time, τ, as function of temperature, T
ALPHA	Thermal expansion coefficient, α
LCFS1	Load curve ID for sintering stress scaling, f_{S1}, as function of relative density, d

VARIABLE	DESCRIPTION
GAMMA	Surface energy density, γ, which affects sintering stress
L0	Grain size, l_0, which affects sintering stress
LCFKS	Load curve ID scaling bulk modulus, f_{KS}, as function of temperature T

Remarks:

This model is intended to be used in two stages. During the first step, the compaction of a powder specimen is simulated, after which the results are dumped to file, and in a subsequent step, the model is restarted to simulate the sintering of the compacted specimen. In the following, an overview of the two different models is given; for a detailed description, we refer to Brandt (1998). The progressive stiffening in the material during compaction makes it more or less necessary to run double precision, and with constraint contacts to avoid instabilities; unfortunately, this currently limits the use of this material to the SMP version of LS-DYNA.

The powder compaction model makes use of a multiplicative split of the deformation gradient into a plastic and elastic part according to

$$\mathbf{F} = \mathbf{F}_e\mathbf{F}_p \ ,$$

where the plastic deformation gradient maps the initial reference configuration to an intermediate relaxed configuration

$$\delta\tilde{\mathbf{x}} = \mathbf{F}_p\delta\mathbf{X}$$

and subsequently the elastic part maps this onto the current loaded configuration

$$\delta\mathbf{x} = \mathbf{F}_e\delta\tilde{\mathbf{x}} \ .$$

The compactness tensor, $\mathbf{P}$, then maps the intermediate configuration onto a virtual fully compacted configuration

$$\delta\bar{\mathbf{x}} = \mathbf{P}\delta\tilde{\mathbf{x}}$$

and we define the relative density as

$$d = \det\mathbf{P} = \frac{\rho}{\bar{\rho}} \ ,$$

where ρ and $\bar{\rho}$ denote the current and fully compacted density, respectively. The elastic properties depend highly on the relative density through the bulk modulus $K(d)$, but the Poisson's ratio is assumed constant.

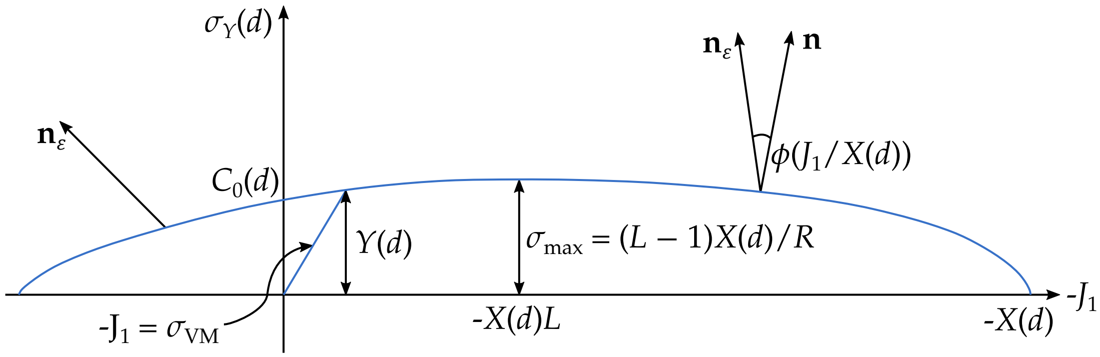


Figure M271-1. Yield surface

The yield surface is represented by two functions in the Rendulic plane according to

$$\sigma_Y(d) = \begin{cases} \frac{}{2} C_0(d) - C_1(d)J_1 - C_2(d)J_1^2 & J_1 \geq LX(d) \\ \frac{\sqrt{[(L-1)X(d)]^2 - [J_1 - LX(d)]^2}}{R} & J_1 < LX(d) \end{cases}$$

and is in this way capped in both compression and tension. Here

$$J_1 = 3\sigma^m = \mathrm{Tr}(\sigma) \ .$$

The polynomial coefficients in the expression above are chosen to give continuity at $J_1 = LX(d)$ and to give the uniaxial compressive strength $Y(d)$. Yielding is assumed to occur when the equivalent stress (note the definition) equals the yield stress

$$\sigma_{eq} = \frac{\sigma_{VM}}{\sqrt{3}} = \sqrt{\frac{1}{2}\mathbf{s}:\mathbf{s}} \leq \sigma_Y(d) \ ,$$

where

$$\mathbf{s} = \underbrace{\sigma - \sigma^m \mathbf{I}}_{\sigma^d} - \boldsymbol{\kappa}$$

in which the last term is the back stress to be described below. The yield surface does not depend on the third stress invariant. The plastic flow is non-associated, and its direction is given by

$$\mathbf{n}_\varepsilon = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \mathbf{n} \ ,$$

where

$$\mathbf{n} = \frac{\sigma_Y(d)}{\sigma_{\max}} \begin{pmatrix} \frac{\partial \sigma_Y}{\partial J_1} \\ 1 \end{pmatrix}$$

is the normal to the yield surface as depicted in the Rendulic plane above (note the sign of J_1). The angle ϕ is a function of and defined only for positive values of the relative volumetric stress $J_1/X(d) > 0$; for negative values ϕ is determined internally to achieve smoothness in the plastic flow direction and to avoid numerical problems at the tensile cap point. The above equations are for illustrative purposes; from now on, the plastic flow direction is generalized to a second-order tensor. The plastic flow rule is then

$$\dot{\boldsymbol{\varepsilon}}_p = \dot{\lambda}\mathbf{n}_\varepsilon, \qquad \dot{\varepsilon}_p^m = \frac{1}{3}Tr(\dot{\boldsymbol{\varepsilon}}_p), \qquad \dot{\boldsymbol{\varepsilon}}_p^d = \dot{\boldsymbol{\varepsilon}}_p - \dot{\varepsilon}_p^m\mathbf{I} \ .$$

The evolution of the compactness tensor is directly related to the evolution of plastic strain as

$$\dot{\mathbf{P}} = -\frac{1}{2}\left(\dot{\boldsymbol{\varepsilon}}_p\mathbf{P} + \mathbf{P}\dot{\boldsymbol{\varepsilon}}_p\right) \ ,$$

and thus, the relative density is given by

$$\dot{d} = -3\dot{\varepsilon}_p^m d \ .$$

The back stress is assumed coaxial with the deviatoric part of the compactness tensor and given by

$$\boldsymbol{\kappa} = J_1 H(e)\left(\mathbf{P} - \frac{\mathrm{Tr}(\mathbf{P})}{3}\mathbf{I}\right),$$

where e is a measure of intensity of anisotropy. This takes a value between 1 and 2 and evolves with plastic strain and plastic work according to

$$\dot{e} = c_a\sqrt{\frac{1}{2}\dot{\boldsymbol{\varepsilon}}_p^d : \dot{\boldsymbol{\varepsilon}}_p^d} - c_v J_1 \dot{\varepsilon}_p^m W(d, J_1) + c_d \dot{\boldsymbol{\varepsilon}}_p^d : \boldsymbol{\sigma} W(d, J_1) \ ,$$

where

$$W(d, J_1) = -\left[\frac{J_1}{X(d)}\right]^p \int_{d_0}^{d} \frac{X(\xi)}{3\xi} d\xi$$

and d_0 is the density in the initial uncompressed configuration. The stress update is completed by the rate equation of stress

$$\dot{\sigma} = \mathbf{C}(d) : \left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_p\right) \ ,$$

where $\mathbf{C}(d)$ is the elastic constitutive matrix.

The sintering model is a thermo and viscoelastic model where the evolution of the mean and deviatoric stress can be written as

$$\begin{aligned} \dot{\sigma}^m &= 3K^s\left(\dot{\varepsilon}^m - \dot{\varepsilon}_T - \dot{\varepsilon}_p^m\right) \\ \dot{\boldsymbol{\sigma}}^d &= 2G^s\left(\dot{\boldsymbol{\varepsilon}}^d - \dot{\boldsymbol{\varepsilon}}_p^d\right) \end{aligned}$$

The thermal strain rate is given by the thermal expansion coefficient as

$$\dot{\varepsilon}_T = \alpha\dot{T} \ ,$$

and the bulk and shear modulus are the same as for the compaction model with the exception that they are scaled by a temperature curve

$$K^s = f_{KS}(T)K(d)$$
$$G^s = \frac{3(1-2\nu)}{2(1+\nu)}K^s$$

The inelastic strain rates are different from the compaction model and is here given by

$$\dot{\boldsymbol{\varepsilon}}_p = \frac{\boldsymbol{\sigma}^d}{2G^v} + \frac{\sigma^m - \sigma^s}{3K^v}\mathbf{I}$$

which results in a viscoelastic behavior depending on the viscous compliance and sintering stress. The viscous bulk compliance can be written

$$\frac{1}{K^v} = 3f_K(d)\left\{d_{V1}\exp\left[-\frac{d_{V2}}{R_{gas}(T-T_0)}\right] + \omega d_{S1}\exp\left[-\frac{d_{S2}}{R_{gas}(T-T_0)}\right]\right\}[1+f_{S2}(T)\xi]$$

from which the viscous shear compliance is modified with aid of the viscous Poisson's ratio

$$\frac{1}{G^v} = \frac{2[1+\nu^v(d)]}{3[1-2\nu^v(d)]}\frac{1}{K^v}\ .$$

The mobility factor ξ evolves with temperature according to

$$\dot{\xi} = \frac{f_{S3}(T)\dot{T} - \xi}{\tau(T)}\ ,$$

and the sintering stress is given as

$$\sigma^s = f_{S1}(d)\frac{\gamma}{l_0}.$$

All this is accompanied with, again, the evolution of relative density given as

$$\dot{d} = -3\dot{\varepsilon}_p^m d\ .$$

*MAT_RHT

This is Material Type 272. This model is used to analyze concrete structures subjected to impulsive loadings; see Riedel et.al. (1999) and Riedel (2004).

Card Summary:

Card 1. This card is required.

MID	RO	SHEAR	ONEMPA	EPSF	B0	B1	T1

Card 2. This card is required.

A	N	FC	FS*	FT*	Q0	B	T2

Card 3. This card is required.

E0C	E0T	EC	ET	BETAC	BETAT	PTF	

Card 4. This card is required.

GC*	GT*	XI	D1	D2	EPM	AF	NF

Card 5. This card is required.

GAMMA	A1	A2	A3	PEL	PCO	NP	ALPHA0

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	SHEAR	ONEMPA	EPSF	B0	B1	T1
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
SHEAR	Elastic shear modulus

VARIABLE	DESCRIPTION
ONEMPA	Unit conversion factor defining 1 MPa in the pressure units used. It can also be used for automatic generation of material parameters for a given compressive strength (see remarks). EQ.0: defaults to 1.0 EQ.-1: parameters generated in m, s and kg (Pa) EQ.-2: parameters generated in mm, s and tonne (MPa) EQ.-3: parameters generated in mm, ms and kg (GPa) EQ.-4: parameters generated in in, s and dozens of slugs (psi) EQ.-5: parameters generated in mm, ms and g (MPa) EQ.-6: parameters generated in cm, μs and g (Mbar) EQ.-7: parameters generated in mm, ms and mg (kPa)
EPSF	Eroding plastic strain (default is 2.0)
B0	Parameter for polynomial EOS
B1	Parameter for polynomial EOS
T1	Parameter for polynomial EOS

Card 2	1	2	3	4	5	6	7	8
Variable	A	N	FC	FS*	FT*	Q0	B	T2
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
A	Failure surface parameter *A*
N	Failure surface parameter *N*
FC	Compressive strength
FS*	Relative shear strength
FT*	Relative tensile strength
Q0	Lode angle dependence factor

VARIABLE	DESCRIPTION
B	Lode angle dependence factor
T2	Parameter for polynomial EOS

Card 3	1	2	3	4	5	6	7	8
Variable	E0C	E0T	EC	ET	BETAC	BETAT	PTF	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
E0C	Reference compressive strain rate
E0T	Reference tensile strain rate
EC	Break compressive strain rate
ET	Break tensile strain rate
BETAC	Compressive strain rate dependence exponent (optional)
BETAT	Tensile strain rate dependence exponent (optional)
PTF	Pressure influence on plastic flow in tension (default is 0.001)

Card 4	1	2	3	4	5	6	7	8
Variable	GC*	GT*	XI	D1	D2	EPM	AF	NF
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
GC*	Compressive yield surface parameter
GT*	Tensile yield surface parameter
XI	Shear modulus reduction factor
D1	Damage parameter

VARIABLE	DESCRIPTION
D2	Damage parameter
EPM	Minimum damaged residual strain
AF	Residual surface parameter
NF	Residual surface parameter

Card 5	1	2	3	4	5	6	7	8
Variable	GAMMA	A1	A2	A3	PEL	PCO	NP	ALPHA0
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
GAMMA	Gruneisen gamma
A1	Hugoniot polynomial coefficient
A2	Hugoniot polynomial coefficient
A3	Hugoniot polynomial coefficient
PEL	Crush pressure
PCO	Compaction pressure
NP	Porosity exponent
ALPHA	Initial porosity

Remarks:

In the RHT model, the shear and pressure part is coupled in which the pressure is described by the Mie-Gruneisen form with a polynomial Hugoniot curve and a p-α compaction relation. For the compaction model, we define a history variable representing the porosity α that is initialized to $\alpha_0 > 1$. This variable represents the current fraction of density between the matrix material and the porous concrete and will decrease with increasing pressure, that is, the reference density is expressed as $\alpha\rho$. The evolution of this variable is given as

$$\alpha(t) = \max\left(1, \min\left\{\alpha_0, \min_{s\le t}\left[1 + (\alpha_0 - 1)\left(\frac{p_{\text{comp}} - p(s)}{p_{\text{comp}} - p_{\text{el}}}\right)^N\right]\right\}\right) \ ,$$

where $p(t)$ indicates the pressure at time t. This expression also involves the initial pore crush pressure p_{el}, compaction pressure p_{comp} and porosity exponent N. For later use, we define the cap pressure, or current pore crush pressure, as

$$p_c = p_{\text{comp}} - \left(p_{\text{comp}} - p_{\text{el}}\right)\left(\frac{\alpha - 1}{\alpha_0 - 1}\right)^{1/N} \quad .$$

The remainder of the pressure (EOS) model is given in terms of the porous density ρ and specific internal energy e (with respect to the porous density). Depending on user inputs, it is either governed by ($B_0 > 0$)

$$p(\rho, e) = \frac{1}{\alpha}\begin{cases}(B_0 + B_1\eta)\alpha\rho e + A_1\eta + A_2\eta^2 + A_3\eta^3 & \eta > 0 \\ B_0\alpha\rho e + T_1\eta + T_2\eta^2 & \eta < 0\end{cases}$$

or ($B_0 = 0$)

$$p(\rho, e) = \Gamma\rho e + \frac{1}{\alpha}p_H(\eta)\left[1 - \frac{1}{2}\Gamma\eta\right]$$
$$p_H(\eta) = A_1\eta + A_2\eta^2 + A_3\eta^3$$

together with

$$\eta(\rho) = \frac{\alpha\rho}{\alpha_0\rho_0} - 1 \ .$$

For the shear strength description we use

$$p^* = \frac{p}{f_c}$$

as the pressure normalized with the compressive strength parameter. We also use $\mathbf{s}$ to denote the deviatoric stress tensor and $\dot{\varepsilon}_p$ the plastic strain rate. The effective plastic strain is thus denoted ε_p and can be viewed as such in the post processor of choice.

For a given stress state and rate of loading, the elastic-plastic yield surface for the RHT model is given by

$$\sigma_y(p^*, \mathbf{s}, \dot{\varepsilon}_p, \varepsilon_p^*) = f_c\sigma_y^*(p^*, F_r(\dot{\varepsilon}_p, p^*), \varepsilon_p^*)R_3(\theta, p^*)$$

and is the composition of two functions and the compressive strength parameter f_c. The first describes the pressure dependence for principal stress conditions $\sigma_1 < \sigma_2 = \sigma_3$ and is expressed in terms of a failure surface and normalized plastic strain as

$$\sigma_y^*(p^*, F_r, \varepsilon_p^*) = \sigma_f^*\left(\frac{p^*}{\gamma}, F_r\right)\gamma$$

with

$$\gamma = \varepsilon_p^* + (1 - \varepsilon_p^*)F_eF_c \ .$$

The failure surface is given as

$$\sigma_f^*(p^*, F_r) = \begin{cases} A\left[p^* - \dfrac{F_r}{3} + \left(\dfrac{A}{F_r}\right)^{-1/n}\right]^n & 3p^* \geq F_r \\[2ex] \dfrac{F_r f_s^*}{Q_1} + 3p^*\left(1 - \dfrac{f_s^*}{Q_1}\right) & F_r > 3p^* \geq 0 \\[2ex] \dfrac{F_r f_s^*}{Q_1} - 3p^*\left(\dfrac{1}{Q_2} - \dfrac{f_s^*}{Q_1 f_t^*}\right) & 0 > 3p^* > 3p_t^* \\[2ex] 0 & 3p_t^* > 3p^* \end{cases}$$

in which p_t^* is the failure cut-off pressure

$$p_t^* = \frac{F_r Q_2 f_s^* f_t^*}{3(Q_1 f_t^* - Q_2 f_s^*)} \ ,$$

F_r is a dynamic increment factor, and

$$Q_1 = R_3\left(\frac{\pi}{6}, 0\right)$$
$$Q_2 = Q(p^*)$$

In these expressions, f_t^* and f_s^* are the tensile and shear strength of the concrete relative to the compressive strength f_c and the Q values are introduced to account for the tensile and shear meridian dependence. Further details are given in the following.

To describe reduced strength on shear and tensile meridian the factor

$$R_3(\theta, p^*) = \frac{2(1 - Q^2)\cos\theta + (2Q - 1)\sqrt{4(1 - Q^2)\cos^2\theta + 5Q^2 - 4Q}}{4(1 - Q^2)\cos^2\theta + (1 - 2Q)^2}$$

is introduced, where θ is the Lode angle given by the deviatoric stress tensor $\mathbf{s}$ as

$$\cos 3\theta = \frac{27\,\det(\mathbf{s})}{2\overline{\sigma}(\mathbf{s})^3}$$
$$\overline{\sigma}(\mathbf{s}) = \sqrt{\frac{3}{2}\mathbf{s} : \mathbf{s}}$$

The maximum reduction in strength is given as a function of relative pressure

$$Q = Q(p^*) = Q_0 + Bp^* \ .$$

Finally, the strain rate dependence is given by

$$F_r(\dot{\varepsilon}_p, p^*) = \begin{cases} F_r^c & 3p^* \geq F_r^c \\ F_r^c - \dfrac{3p^* - F_r^c}{F_r^c + F_r^t f_t^*}\left(F_r^t - F_r^c\right) & F_r^c > 3p^* \geq -F_r^t f_t^* \\ F_r^t & -F_r^t f_t^* > 3p^* \end{cases}$$

in which

$$F_r^{\frac{c}{t}}(\dot{\varepsilon}_p) = \begin{cases} \left(\dfrac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0^{c/t}}\right)^{\beta_{c/t}} & \dot{\varepsilon}_p^{c/t} \geq \dot{\varepsilon}_p \\ \gamma_{\frac{c}{t}} \sqrt[3]{\dot{\varepsilon}_p} & \dot{\varepsilon}_p > \dot{\varepsilon}_p^{c/t} \end{cases} .$$

The parameters involved in these expressions are given as (f_c is in *MPa* below)

$$\beta_c = \frac{4}{20 + 3f_c}$$

$$\beta_t = \frac{2}{20 + f_c}$$

and $\gamma_{c/t}$ is determined from continuity requirements, but it is also possible to choose the rate parameters via inputs.

The elastic strength parameter used above is given by

$$F_e(p^*) = \begin{cases} g_c^* & 3p^* \geq F_r^c g_c^* \\ g_c^* - \dfrac{3p^* - F_r^c g_c^*}{F_r^c g_c^* + F_r^t g_t^* f_t^*}\left(g_t^* - g_c^*\right) & F_r^c g_c^* > 3p^* \geq -F_r^t g_t^* f_t^* \\ g_t^* & -F_r^t g_t^* f_t^* > 3p^* \end{cases}$$

while the cap of the yield surface is represented by

$$F_c(p^*) = \begin{cases} 0 & p^* \geq p_c^* \\ \sqrt{1 - \left(\dfrac{p^* - p_u^*}{p_c^* - p_u^*}\right)^2} & p_c^* > p^* \geq p_u^* \\ 1 & p_u^* > p^* \end{cases}$$

where

$$p_c^* = \frac{p_c}{f_c}$$

$$p_u^* = \frac{F_r^c g_c^*}{3} + \frac{G^* \varepsilon_p}{f_c}$$

The hardening behavior is described linearly with respect to the plastic strain, where

$$\varepsilon_p^* = \min\left(\frac{\varepsilon_p}{\varepsilon_p^h}, 1\right)$$
$$\varepsilon_p^y = \frac{\sigma_y(p^*, \mathbf{s}, \dot{\varepsilon}_p, \varepsilon_p^*)(1 - F_e F_c)}{\gamma 3G^*}$$

here

$$G^* = \xi G$$

where G is the shear modulus of the virgin material and ξ is a reduction factor representing the hardening in the model.

When hardening states reach the ultimate strength of the concrete on the failure surface, damage is accumulated during further inelastic loading controlled by plastic strain. To this end, the plastic strain at failure is given as

$$\varepsilon_p^f = \begin{cases} D_1[p^* - (1-D)p_t^*]^{D_2} & p^* \geq (1-D)p_t^* + \left(\frac{\varepsilon_p^m}{D_1}\right)^{1/D_2} \\ \varepsilon_p^m & (1-D)p_t^* + \left(\frac{\varepsilon_p^m}{D_1}\right)^{1/D_2} > p^* \end{cases}$$

The damage parameter is accumulated with plastic strain according to

$$D = \int_{\varepsilon_p^h}^{\varepsilon_p} \frac{d\varepsilon_p}{\varepsilon_p^f}$$

and the resulting damage surface is given as

$$\sigma_d(p^*, \mathbf{s}, \dot{\varepsilon}_p) = \begin{cases} \sigma_y(p^*, \mathbf{s}, \dot{\varepsilon}_p, 1)(1-D) + Df_c\sigma_r^*(p^*) & 0 \leq p^* \\ \sigma_y(p^*, \mathbf{s}, \dot{\varepsilon}_p, 1)\left(1 - D - \frac{p^*}{p_t^*}\right) & (1-D)p_t^* \leq p^* < 0 \end{cases}$$

where

$$\sigma_r^*(p^*) = A_f\{p^*\}^{n_f} \ .$$

Plastic flow occurs in the direction of deviatoric stress, meaning

$$\dot{\varepsilon}_p \sim \mathbf{s} \ ,$$

but for tension there is an option to set the parameter PFC to a number corresponding to the influence of plastic volumetric strain. If $\lambda \leq 1$ is used to denote this parameter, then for the special case of $\lambda = 1$

$$\dot{\varepsilon}_p \sim \mathbf{s} - p\mathbf{I} \ .$$

This was introduced to reduce noise in tension that was observed on some test problems. A failure strain can be used to erode elements with severe deformation which by default is set to 200%.

For simplicity, automatic generation of material parameters is available using ONEMPA < 0; no other parameters are needed. If FC = 0 then the 35 MPa strength concrete in Riedel (2004) is generated in the units specified by the value of ONEMPA. For FC > 0 FC then specifies the actual strength of the concrete in the units specified by the value of ONEMPA. The other parameters are generated by interpolating between the 35 MPa and 140 MPa strength concretes as presented in Riedel (2004). Any automatically generated parameter may be overridden by the user; one of these parameters may be the initial porosity ALPHA0 of the concrete.

For post-processing, the following history variables may be of interest:

History Variable	Description
2	Internal energy per volume (ρe)
3	Porosity value (α)
4	Damage value (D)

or as an alternative use a material history list

*DEFINE_MATERIAL_HISTORIES Properties					
Label	**Attributes**				**Description**
Damage	-	-	-	-	Damage value D

*MAT_CONCRETE_DAMAGE_PLASTIC_MODEL

*MAT_CDPM

This is Material Type 273. CDPM is a damage plastic concrete model based on Grassl et al. (2011, 2013) and Grassl and Jirásek (2006). This model aims to simulate the failure of concrete structures subjected to dynamic loadings. It describes the characterization of the failure process subjected to multi-axial and rate-dependent loading. The model is based on effective stress plasticity and includes a damage model based on plastic and elastic strain measures. This material model is available only for solids.

This material model includes many parameters for the advanced user, but most have default values based on experimental tests. They might not be useful for all concrete and load paths, but the values provide a good starting point. If the default values are not good enough, see the remarks at the end for a discussion of these parameters.

More details on this material can be found at:
http://petergrassl.com/Research/DamagePlasticity/CDPMLSDYNA/index.html

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	ECC	QH0	FT	FC
Type	A	F	F	F	F	F	F	F
Default	none	none	none	0.2	AUTO	0.3	none	none

Card 2	1	2	3	4	5	6	7	8
Variable	HP	AH	BH	CH	DH	AS	DF	FC0
Type	F	F	F	F	F	F	F	F
Default	0.5	0.08	0.003	2.0	1.0E-6	15.0	0.85	AUTO

Card 3	1	2	3	4	5	6	7	8
Variable	TYPE	BS	WF	WF1	FT1	STRFLG	FAILFLG	EFC
Type	F	F	F	F	F	F	F	F
Default	0.0	1.0	none	0.15 × WF	0.3 × FT	0.0	0.0	1.0E-4

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus. The sign determines if an anisotropic (E positive) or an isotropic (E negative) damage formulation is used. See Remark 1. The Young's modulus is taken as the absolute value of this parameter. See Remark 3.
PR	Poisson's ratio
ECC	Eccentricity parameter. See Remark 2. EQ.0.0: ECC is calculated from Jirásek and Bazant (2002) as $\mathrm{ECC} = \frac{1+\epsilon}{2-\epsilon}, \quad \epsilon = \frac{f_t(f_{bc}^2 - f_c^2)}{f_{bc}(f_c^2 - f_t^2)}, \quad f_{bc} = 1.16 f_c$
QH0	Initial hardening defined as $\mathrm{FC}_0/\mathrm{FC}$ where FC_0 is the compressive stress at which the initial yield surface is reached.
FT	Uniaxial tensile strength (stress), f_t. See Remarks 2 and 3.
FC	Uniaxial compression strength (stress), f_c. See Remarks 2 and 3.
HP	Hardening parameter, H_p. The default, HP = 0.5, is the value used in Grassl et al. (2011) for a strain-rate-dependent material response (STRFLG = 1). For applications without a strain rate effect (STRFLG = 0), a value of HP = 0.01 is recommended, which has been used in Grassl et al. (2013). See Remark 2.
AH	Hardening ductility parameter 1, A_h. See Remark 2.

VARIABLE	DESCRIPTION
BH	Hardening ductility parameter 2, B_h. See Remark 2.
CH	Hardening ductility parameter 3, C_h. See Remark 2.
DH	Hardening ductility parameter 4, D_h. See Remark 2.
AS	Ductility parameter during damage, A_s. See Remark 3.
DF	Flow rule parameter, D_f. See Remark 2.
FC0	Rate-dependent parameter, f_{c0}. It is only needed if STRFLG = 1. The recommended value is 10 MPa, which has to be entered consistently with the system of units used. See Remark 4.
TYPE	Flag for damage type (see Remark 3): EQ.0.0: Linear damage formulation EQ.1.0: Bilinear damage formulation EQ.2.0: Exponential damage formulation EQ.3.0: No damage The best results are obtained with the bilinear formulation.
BS	Damage ductility exponent during damage, B_s. See Remark 3.
WF	Tensile threshold value for linear damage formulation, w_f. It controls the tensile softening branch of the exponential tensile damage formulation. See Remark 3.
WF1	Tensile threshold value for the second part of the bilinear damage formulation, w_{f1}. The default is 0.15 × WF. See Remark 3.
FT1	Tensile strength threshold value for the bilinear damage formulation, f_{t1}. The default is 0.3 × FT. See Remark 3.
STRFLG	Strain rate flag: EQ.1.0: Strain rate dependent (see Remark 4) EQ.0.0: Not strain rate dependent
FAILFLG	Failure flag. EQ.0.0: Not active, meaning no erosion GT.0.0: Active. Ane element erodes if ω_t and ω_c equal 1 in FAILFLG percent of the integration points. For

VARIABLE	DESCRIPTION
	example, if FAILFLG = 0.60, 60% of all integration points must fail before erosion.
EFC	Parameter controlling the compressive damage softening branch of the exponential compressive damage formulation, ε_{fc}. See Remark 3.

Remarks:

1. **Stress depending on the damage model.** The stress for the anisotropic damage plasticity model (E > 0 in the input) is defined as

$$\boldsymbol{\sigma} = (1-\omega_t)\boldsymbol{\sigma}_t + (1-\omega_c)\boldsymbol{\sigma}_c$$

where $\boldsymbol{\sigma}_t$ and $\boldsymbol{\sigma}_c$ are the positive and negative part of the effective stress, $\boldsymbol{\sigma}_{\mathrm{eff}}$, determined in the principal stress space. The scalar functions ω_t and ω_c are damage parameters.

The stress for the isotropic damage plasticity model (E < 0 in the input) is defined as

$$\boldsymbol{\sigma} = (1-\omega_t)\boldsymbol{\sigma}_{\mathrm{eff}}$$

The effective stress, $\boldsymbol{\sigma}_{\mathrm{eff}}$, is defined according to the damage mechanics convention as

$$\boldsymbol{\sigma}_{\mathrm{eff}} = \mathbf{D}_e : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_p)$$

2. **Plasticity.** The yield surface is described by the Haigh-Westergaard coordinates: the volumetric effective stress, σ_v, the norm of the deviatoric effective stress, ρ, and the Lode angle, θ. The following equation gives the yield surface:

$$f_p(\sigma_v,\rho,\theta,\kappa) = \left[[1-q_1(\kappa)]\left(\frac{\rho}{\sqrt{6}f_c}+\frac{\sigma_v}{f_c}\right)^2 + \sqrt{\frac{3}{2}}\frac{\rho}{f_c}\right]^2 + m_0 q_1(\kappa)^2 q_2(\kappa)\left[\frac{\rho}{\sqrt{6}f_c}r(\cos\theta)+\frac{\sigma_v}{f_c}\right] - q_1^2(\kappa)q_2^2(\kappa)\ .$$

The variables q_1 and q_2 depend on the hardening variable κ. The parameter f_c is the uniaxial compressive strength.

The following function controls the shape of the deviatoric part:

$$r(\cos\theta) = \frac{4(1-e^2)\cos^2\theta + (2e-1)^2}{2(1-e^2)\cos\theta + (2e-1)\sqrt{4(1-e^2)\cos^2\theta + 5e^2 - 4e}}$$

where e is the eccentricity parameter (ECC). The parameter m_0 is the friction parameter. It is defined as:

$$m_0 = \frac{3(f_c^2 - f_t^2)}{f_c f_t}\frac{e}{e+1} \ ,$$

where f_t is the tensile strength.

The flow rule is non-associative, meaning that the plastic flow's direction is not normal to the yield surface. This aspect is essential for modeling concrete because an associative flow rule gives an overestimated maximum stress. The plastic potential is given by:

$$g(\sigma_v, \rho, \kappa) = \left\{[1 - q_1(\kappa)]\left(\frac{\rho}{\sqrt{6}f_c} + \frac{\sigma_v}{f_c}\right)^2 + \sqrt{\frac{3}{2}}\frac{\rho}{f_c}\right\}^2 + q_1(\kappa)\left(\frac{m_0 \rho}{\sqrt{6}f_c} + \frac{m_g(\sigma_v, \kappa)}{f_c}\right) \ ,$$

where

$$m_g(\sigma_v, \kappa) = A_g(\kappa) B_g(\kappa) f_c e^{\frac{\sigma_v - q_2 f_t/3}{B_g f_c}}$$

and

$$A_g = \frac{3 f_t q_2(\kappa)}{f_c} + \frac{m_0}{2}, \qquad B_g = \frac{q_2(\kappa)}{3}\frac{1 + f_t/f_c}{\ln\dfrac{A_g}{3q_2 + \dfrac{m_0}{2}} + \ln\left(\dfrac{D_f + 1}{2D_f - 1}\right)}$$

The hardening laws q_1 and q_2 control the shape of the yield surface and the plastic potential. They are defined as:

$$q_1(\kappa) = \begin{cases} q_{h0} + (1 - q_{h0})(\kappa^3 - 3\kappa^2 + 3\kappa) - H_p(\kappa^3 - 3\kappa^2 + 2\kappa) & \text{if } \kappa < 1 \\ 1 & \text{if } \kappa \geq 1 \end{cases}$$
$$q_2(\kappa) = \begin{cases} 1 & \text{if } \kappa < 1 \\ 1 + H_p(\kappa - 1) & \text{if } \kappa \geq 1 \end{cases}$$

The evolution for the hardening variable is given by

$$\dot{\kappa} = \frac{4\dot{\lambda}\cos^2\theta}{x_h(\sigma_v)}\left\|\frac{dg}{d\sigma}\right\|$$

It sets the rate of the hardening variable to the norm of the plastic strain rate scaled by a ductility measure, which is defined as:

$$x_h(\sigma_v) = \begin{cases} A_h - (A_h - B_h)e^{-\frac{R_h(\sigma_v)}{C_h}} & \text{if } R_h(\sigma_v) \geq 0 \\ E_h e^{\frac{R_h(\sigma_v)}{F_h}} + D_h & \text{if } R_h(\sigma_v) < 0 \end{cases}$$

Here,

$$E_h = B_h - D_h, \qquad F_h = \frac{(B_h - D_h)C_h}{A_h - B_h}, \qquad R_h(\sigma_v) = -\frac{\sigma_v}{f_c} - \frac{1}{3}$$

3. **Damage.** Damage initializes when the equivalent strain, $\tilde{\varepsilon}$, reaches the threshold value $\varepsilon_0 = f_t/E$, where the equivalent strain is defined as

$$\tilde{\varepsilon} = \frac{\varepsilon_0 m_0}{2}\left[\frac{\rho}{\sqrt{6}f_c}r(\cos\theta) + \frac{\sigma_V}{f_c}\right] + \sqrt{\frac{\varepsilon_0^2 m_0^2}{4}\left(\frac{\rho}{\sqrt{6}f_c}r(\cos\theta) + \frac{\sigma_V}{f_c}\right)^2 + \frac{3\varepsilon_0^2\rho^2}{2f_c^2}}$$

A stress-inelastic displacement law describes tensile damage. For linear, bilinear, and exponential damage types, the stress value f_t and the displacement value w_f must be defined. Additional parameters f_{t1} and w_{f1} must be defined for the bilinear type. Figure M273-1 illustrates how the input parameters control the stress softening for the different damage models.

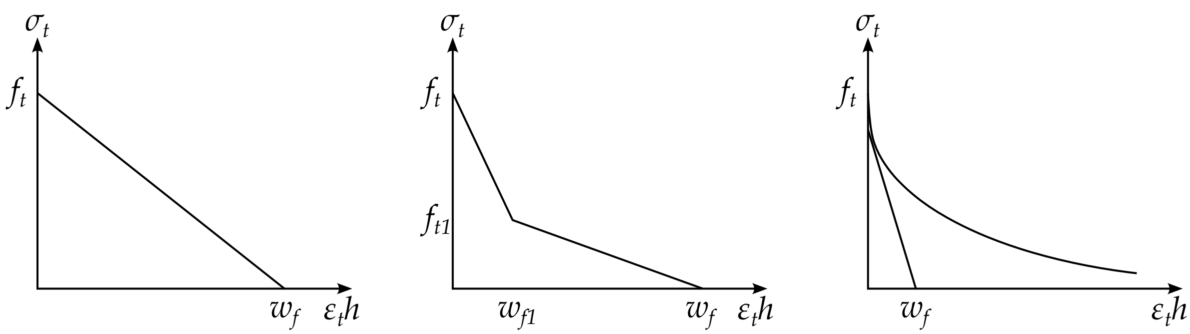


Figure M273-1. Stress softening due to damage in tension. The figures from left to right show this behavior for linear, bilinear, and exponential damage, respectively.

The variable h in Figure M273-1 is a mesh-dependent measure used to convert strains to displacements. The variable ε_t is called the inelastic tensile strain and is defined as the sum of the irreversible plastic strain ε_p and the reversible strain $w_t(\varepsilon - \varepsilon_p)$ (in compression $w_c(\varepsilon - \varepsilon_p)$).

A damage ductility measure, x_s, models the influence of multi-axial stress states on the softening:

$$x_s = 1 + (A_s - 1)R_s^{B_s}$$

Here, A_s and B_s are input parameters, and

$$R_s = \begin{cases} -\dfrac{\sqrt{6}\sigma_v}{\rho} & \text{if } \sigma_v \le 0 \\ 0 & \text{if } \sigma_v > 0 \end{cases}$$

The inelastic strain is then modified according to:

$$\varepsilon_i = \frac{\varepsilon_i}{x_s}$$

An exponential stress-inelastic strain law controls compressive damage. Stress value f_c and inelastic strain ε_{fc} need to be specified. Figure M273-2 illustrates how the input parameters affect stress softening. A small value of ε_{fc}, such as 1.0E-4 (the default), causes a brittle form of damage.

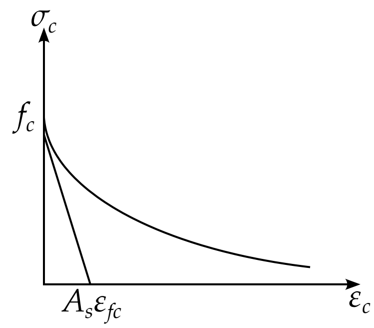


Figure M273-2. Stress softening due to damage in compression

4. **Strain rate.** Concrete is strongly rate dependent. If the loading rate increases, the tensile and compressive strengths increase and are more prominent in tension than in compression. $\alpha_r \geq 1$ models this dependency. The rate dependency is included by scaling both the equivalent strain rate and the inelastic strain. The rate parameter is defined by

$$\alpha_r = (1 - X_{\text{compression}})\alpha_{rt} + X_{\text{compression}}\alpha_{rc} \ ,$$

where $X_{\text{compression}}$ is a continuous compression measure (= 1 means only compression, = 0 means only tension). For tension:

$$\alpha_{rt} = \begin{cases} 1 & \text{if} \qquad \dot{\varepsilon}_{\text{max}} < 30 \times 10^{-6}\ s^{-1} \\ \left(\dfrac{\dot{\varepsilon}_{\text{max}}}{\dot{\varepsilon}_{t0}}\right)^{\delta_t} & \text{if } 30 \times 10^{-6} < \dot{\varepsilon}_{\text{max}} < 1\ s^{-1} \\ \beta_t \left(\dfrac{\dot{\varepsilon}_{\text{max}}}{\dot{\varepsilon}_{t0}}\right)^{\frac{1}{3}} & \text{if} \qquad \dot{\varepsilon}_{\text{max}} > 1\ s^{-1} \end{cases}$$

where $\delta_t = \frac{1}{1+8f_c/f_{c0}}$, $\beta_t = e^{6\delta_t - 2}$, and $\dot{\varepsilon}_{t0} = 1 \times 10^{-6}\ s^{-1}$. For compression, the corresponding rate factor is given by:

$$\alpha_{rc} = \begin{cases} 1 & \text{if} \qquad |\dot{\varepsilon}_{\text{min}}| < 30 \times 10^{-6}\ s^{-1} \\ \left[S \dfrac{|\dot{\varepsilon}_{\text{min}}|}{\dot{\varepsilon}_{c0}}\right]^{1.026\delta_c} & \text{if } 30 \times 10^{-6} < |\dot{\varepsilon}_{\text{min}}| < 1\ s^{-1} \\ \beta_c \left[\dfrac{|\dot{\varepsilon}_{\text{min}}|}{\dot{\varepsilon}_{c0}}\right]^{\frac{1}{3}} & \text{if} \qquad |\dot{\varepsilon}_{\text{min}}| > 30\ s^{-1} \end{cases}$$

where $\delta_c = \frac{1}{5+9f_c/f_{c0}}$, $\beta_c = e^{6.156\delta_c - 2}$, and $\dot{\varepsilon}_{c0} = 30 \times 10^{-6}\ s^{-1}$. f_{c0} is an input parameter. A recommended value is 10 MPa.

5. **History variables.** Extra history variables of interest are listed in the following table. Set NEIPH in *DATABASE_EXTENT_BINARY to request these variables.

History Variable #	Description
1	Hardening variable, κ. See Remark 2.
15	Damage in tension, ω_t. See Remark 1.
16	Damage in compression, ω_c. See Remark 1.

*MAT_PAPER

This is Material Type 274. This is an orthotropic elastoplastic model for paper materials, based on Xia (2002) and Nygards (2009). It is available for solid and shell elements. Solid elements use a hyperelastic-plastic formulation, while shell elements use a hypoelastic-plastic formulation. The material is available for explicit and implicit simulations; see Remark 5.

Card Summary:

Card 1. This card is required.

MID	RO	E1	E2	E3	PR21	PR32	PR31

Card 2. This card is required.

G12	G23	G13	E3C	CC	TWOK		ROT

Card 3. This card is required.

S01	A01	B01	C01	S02	A02	B02	C02

Card 4. This card is required.

S03	A03	B03	C03	S04	A04	B04	C04

Card 5. This card is required.

S05	A05	B05	C05	PRP1	PRP2	PRP4	PRP5

Card 6. This card is required.

ASIG	BSIG	CSIG	TAU0	ATAU	BTAU		

Card 7. This card is required.

AOPT	MACF	XP	YP	ZP	A1	A2	A3

Card 8. This card is required.

V1	V2	V3	D1	D2	D3	BETA	

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E1	E2	E3	PR21	PR32	PR31
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Material density
E*i*	Young's modulus in direction *i*, E_i
PR*ij*	Elastic Poisson's ratio ν_{ij}

Card 2	1	2	3	4	5	6	7	8
Variable	G12	G23	G13	E3C	CC	TWOK		ROT
Type	F	F	F	F	F	F		F
Default	none	none	none	none	none	none		0.0

VARIABLE	DESCRIPTION
G*ij*	Elastic shear modulus in direction, G_{ij}
E3C	Elastic compression parameter
CC	Elastic compression exponent
TWOK	Exponent in in-plane yield surface

VARIABLE	DESCRIPTION
ROT	Option for two-dimensional solids (shell element forms 13, 14, or 15): EQ.0.0: No rotation of material axes (default). Direction of material axes are solely defined by AOPT. It is only possible to rotate in shell-plane. EQ.1.0: Rotate coordinate system around material 1-axis such that 2-axis coincides with shell normal. This rotation is done in addition to AOPT. EQ.2.0: Rotate coordinate system around material 2-axis such that 1-axis coincides with shell normal. This rotation is done in addition to AOPT.

In plane Yield Surface Card 1.

Card 3	1	2	3	4	5	6	7	8
Variable	S01	A01	B01	C01	S02	A02	B02	C02
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

In plane Yield Surface Card 2.

Card 4	1	2	3	4	5	6	7	8
Variable	S03	A03	B03	C03	S04	A04	B04	C04
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

In plane Yield Surface Card 3.

Card 5	1	2	3	4	5	6	7	8
Variable	S05	A05	B05	C05	PRP1	PRP2	PRP4	PRP5
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	1/2	2/15	1/2	2/15

VARIABLE	DESCRIPTION
S0i	i^{th} in-plane plasticity yield parameter LT.0.0: |S0i| is a load curve ID; see Remark 1.
A0i	i^{th} in-plane plasticity hardening parameter
B0i	i^{th} in-plane plasticity hardening parameter
C0i	i^{th} in-plane plasticity hardening parameter
PRP1	Tensile plastic Poisson's ratio in direction 1
PRP2	Tensile plastic Poisson's ratio in direction 2
PRP4	Compressive plastic Poisson's ratio in direction 1
PRP5	Compressive plastic Poisson's ratio in direction 2

Out of Plane and Transverse Shear Yield Surface Card.

Card 6	1	2	3	4	5	6	7	8
Variable	ASIG	BSIG	CSIG	TAU0	ATAU	BTAU		
Type	F	F	F	F	F	F		
Default	none	none	none	none	none	none		

VARIABLE	DESCRIPTION
ASIG	Out-of-plane plasticity yield parameter

VARIABLE	DESCRIPTION
BSIG	Out-of-plane plasticity hardening parameter
CSIG	Out-of-plane plasticity hardening parameter
TAU0	Transverse shear plasticity yield parameter
ATAU	Transverse shear plasticity hardening parameter
BTAU	Transverse shear plasticity hardening parameter

Orthotropic Parameter Card 1.

Card 7	1	2	3	4	5	6	7	8
Variable	AOPT	MACF	XP	YP	ZP	A1	A2	A3
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

VARIABLE	DESCRIPTION

AOPT

Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):

EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.

EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a**-direction. This option is for solid elements only.

EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR

EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of

VARIABLE	DESCRIPTION
	the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.
	EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, P, which define the centerline axis. This option is for solid elements only.
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*).
MACF	Material axes change flag for solid elements:
	EQ.-4: Switch material axes b and c before BETA rotation
	EQ.-3: Switch material axes a and c before BETA rotation
	EQ.-2: Switch material axes a and b before BETA rotation
	EQ.1: No change, default
	EQ.2: Switch material axes a and b after BETA rotation
	EQ.3: Switch material axes a and c after BETA rotation
	EQ.4: Switch material axes b and c after BETA rotation
	Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_-SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 8 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.
XP, YP, ZP	Coordinates of point p for AOPT = 1 and 4
A1, A2, A3	Components of vector **a** for AOPT = 2

Orthotropic Parameter Card 2.

Card 8	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	
Default	none	none	none	none	none	none	none	

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3 and 4
D1, D2, D3	Components of vector **d** for AOPT = 2
BETA	Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA or *ELEMENT_-SOLID_ORTHO.

Remarks:

1. **Hardening function.** Each hardening function, q_i (note that $q_6 = q_3$), is given by a load curve if $S_i^0 < 0$, otherwise

$$q_i\left(\varepsilon_p^f\right) = S_i^0 + A_i^0 \tanh\left(B_i^0 \varepsilon_p^f\right) + C_i^0 \varepsilon_p^f .$$

2. **Material model for solid elements.** The stress-strain relationship for solid elements is based on a multiplicative split of the deformation gradient into an elastic and a plastic part

$$\mathbf{F} = \mathbf{F}_e \mathbf{F}_p \ .$$

The elastic Green strain is formed as

$$\mathbf{E}_e = \frac{1}{2}\left(\mathbf{F}_e^{\mathrm{T}} \mathbf{F}_e - \mathbf{I}\right) \ ,$$

and the 2nd Piola-Kirchhoff stress as

$$\mathbf{S} = \mathbf{C}\mathbf{E}_e \ ,$$

where the constitutive matrix is taken as orthotropic and can be represented in Voigt notation by its inverse as

$$\mathbf{C}^{-1}=\begin{bmatrix} \frac{1}{E_1} & -\frac{v_{21}}{E_2} & -\frac{v_{31}}{E_3} & & & \\ -\frac{v_{12}}{E_1} & \frac{1}{E_2} & -\frac{v_{32}}{E_3} & & & \\ -\frac{v_{13}}{E_1} & -\frac{v_{23}}{E_2} & \frac{1}{E_3} & & & \\ & & & \frac{1}{G_{12}} & & \\ & & & & \frac{1}{G_{23}} & \\ & & & & & \frac{1}{G_{13}} \end{bmatrix}.$$

In out-of-plane compression the stress is modified according to

$$S_{33}=C_{31}E_{11}^e+C_{32}E_{22}^e+\begin{cases} E_3E_{33}^e, & E_{33}^e\geq 0\\ E_3^c[1-\exp(-C_cE_{33}^e)], & E_{33}^e<0\end{cases}$$

Three yield surfaces are present: in-plane, out-of-plane, and transverse shear. The in-plane yield surface is given as (see Remark 1)

$$f=\sum_{i=1}^{6}\left[\frac{\max(0,S{:}N_i)}{q_i\left(\varepsilon_p^f\right)}\right]^{2k}-1\leq 0\ ,$$

with the six yield plane normals (in strain Voigt notation)

$$\begin{aligned}
N_1&=\begin{bmatrix}\frac{1}{\sqrt{1+v_{1p}^2}} & -\frac{v_{1p}}{\sqrt{1+v_{1p}^2}} & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}\ ,\\
N_2&=\begin{bmatrix}-\frac{v_{2p}}{\sqrt{1+v_{2p}^2}} & \frac{1}{\sqrt{1+v_{2p}^2}} & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}\ ,\\
N_3&=\begin{bmatrix}0 & 0 & 0 & \sqrt{2} & 0 & 0\end{bmatrix}^{\mathrm{T}}\ ,\\
N_4&=-\begin{bmatrix}\frac{1}{\sqrt{1+v_{4p}^2}} & -\frac{v_{4p}}{\sqrt{1+v_{4p}^2}} & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}\ ,\\
N_5&=-\begin{bmatrix}-\frac{v_{5p}}{\sqrt{1+v_{5p}^2}} & \frac{1}{\sqrt{1+v_{5p}^2}} & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}\ ,\\
N_6&=-N_3\ .
\end{aligned}$$

The yield planes describe the following states

Plane	Stress State
1	Tension in material direction 1
2	Tension in material direction 2
3	Positive shear in the 1, 2-direction
4	Compression in material direction 1
5	Compression in material direction 2
6	Negative shear in the 1, 2-direction

The out-of-plane surface is given as

$$g = \frac{-S_{33}}{A_\sigma + B_\sigma \exp\left(-C_\sigma \varepsilon_p^g\right)} - 1 \le 0 \ ,$$

and the transverse shear surface is

$$h = \frac{\sqrt{S_{13}^2 + S_{23}^2}}{\tau_0 + [A_\tau - \min(0, S_{33})\, B_\tau]\varepsilon_p^h} - 1 \le 0 \ .$$

The flow rule is given by the evolution of the plastic deformation gradient

$$\dot{\mathbf{F}}_p = \mathbf{L}_p \mathbf{F}_p \ ,$$

where the plastic velocity gradient is given as

$$\mathbf{L}_p = \begin{bmatrix} \dot{\varepsilon}_p^f \dfrac{\partial f}{\partial S_{11}} & \dot{\varepsilon}_p^f \dfrac{\partial f}{\partial S_{12}} & \dot{\varepsilon}_p^h \dfrac{\partial h}{\partial S_{13}} \\ \dot{\varepsilon}_p^f \dfrac{\partial f}{\partial S_{12}} & \dot{\varepsilon}_p^f \dfrac{\partial f}{\partial S_{22}} & \dot{\varepsilon}_p^h \dfrac{\partial h}{\partial S_{23}} \\ \dot{\varepsilon}_p^h \dfrac{\partial h}{\partial S_{13}} & \dot{\varepsilon}_p^h \dfrac{\partial h}{\partial S_{23}} & \dot{\varepsilon}_p^g \dfrac{\partial g}{\partial S_{33}} \end{bmatrix} \ ,$$

and where it is implicitly assumed that the involved derivatives in the expression of the velocity gradient is appropriately normalized.

3. **Material model for shell elements.** The stress-strain relationship for shell elements is based on an additive split of the rate of deformation into an elastic and a plastic part

$$\mathbf{D} = \mathbf{D}_e + \mathbf{D}_p \ ,$$

and the rate of Cauchy stress is given by

$$\dot{\sigma} = \mathbf{C}\mathbf{D}_e \ .$$

In out-of-plane compression the stress rate is modified according to

$$\dot{\sigma}_{33} = C_{31} D_{11}^e + C_{32} D_{22}^e + D_{33}^e \begin{cases} E_3, & \varepsilon_{33}^e \ge 0 \\ E_3^c C_c \exp(-C_c \varepsilon_{33}^e)\,, & \varepsilon_{33}^e < 0 \end{cases}$$

For shell elements, $D_{33}^{p}=0$, and only two yield surfaces are present: the in-plane yield surface (see Remark 1),

$$f=\sum_{i=1}^{6}\left[\frac{\max(0,\sigma:N_i)}{q_i\left(\varepsilon_\mathrm{p}^f\right)}\right]^{2k}-1\le 0\ ,$$

and the transverse-shear yield surface,

$$h=\frac{\sqrt{\sigma_{13}^2+\sigma_{23}^2}}{\tau_0+[A_\tau-\min(0,\sigma_{33})\,B_\tau]\varepsilon_\mathrm{p}^h}-1\le 0\ .$$

For this case, the plastic flow rule is given by

$$\dot{\boldsymbol{\varepsilon}}_p=\mathbf{D}_p=\mathbf{L}_p\ ,$$

where the plastic velocity gradient is given as

$$\mathbf{L}_p=\begin{bmatrix}\dot{\varepsilon}_\mathrm{p}^f\dfrac{\partial f}{\partial\sigma_{11}} & \dot{\varepsilon}_\mathrm{p}^f\dfrac{\partial f}{\partial\sigma_{12}} & \dot{\varepsilon}_\mathrm{p}^h\dfrac{\partial h}{\partial\sigma_{13}}\\ \dot{\varepsilon}_\mathrm{p}^f\dfrac{\partial f}{\partial\sigma_{12}} & \dot{\varepsilon}_\mathrm{p}^f\dfrac{\partial f}{\partial\sigma_{22}} & \dot{\varepsilon}_\mathrm{p}^h\dfrac{\partial h}{\partial\sigma_{23}}\\ \dot{\varepsilon}_\mathrm{p}^h\dfrac{\partial h}{\partial\sigma_{13}} & \dot{\varepsilon}_\mathrm{p}^h\dfrac{\partial h}{\partial\sigma_{23}} & 0\end{bmatrix}\ .$$

4. **History variables.** The *Effective Plastic Strain* is

$$\varepsilon_p=\sqrt{\left(\varepsilon_p^f\right)^2+\left(\varepsilon_p^g\right)^2+\left(\varepsilon_p^h\right)^2}\ .$$

The other history variables are listed below.

History Variable #	Solid Elements	Shell Elements
1	ε_p^f	Q_{11} in element to material rotation tensor
2	ε_p^g	Q_{12} in element to material rotation tensor
3	ε_p^h	ε_p^f
4		ε_p^h

5. **Tangent stiffness.** The shell hypoelastic-plastic formulation produces a symmetric tangent stiffness. For the solid hyperelastic-plastic formulation, the tangent stiffness is nonsymmetric. However, unless LCPACK on *CONTROL_IMPLICIT_SOLVER is set to 3, a simplified symmetric tangent will be used for solid elements. This simplified tangent is based on the assumption of small elastic strains. For some problems, using the nonsymmetric tangent significantly improves the convergence rate.

*MAT_SMOOTH_VISCOELASTIC_VISCOPLASTIC

This is Material Type 275, a smooth viscoelastic viscoplastic model based on the works of Hollenstein et.al. [2013, 2014] and Jabareen [2015]. The stress response is rheologically represented by HJR (Hollenstein-Jabareen-Rubin) elements in parallel (see Figure M275-1), where each element exhibits combinations of viscoelastic and viscoplastic characteristics. The model is based on large displacement hyper-elastoplasticity and the numerical implementation is strongly objective; this together with the smooth characteristics makes it especially suitable for implicit analysis.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K					
Type	A	F	F					

HJR Element Cards. At least 1 and optionally up to 6 cards should be input. The next keyword ("*") card terminates this input.

Card 2	1	2	3	4	5	6	7	8
Variable	A0	B0	A1	B1	M	KAPAS	KAPA0	SHEAR
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
K	Elastic bulk modulus
A0	Rate-dependent understress viscoplastic parameter
B0	Rate-independent understress plasticity parameter
A1	Rate-dependent overstress viscoplastic parameter
B1	Rate-independent overstress plasticity parameter
M	Exponential hardening parameter

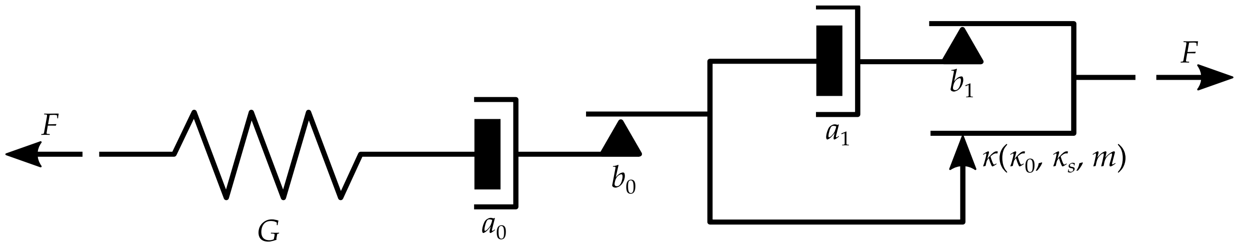


Figure M275-1. Rheological representation of an HJR element, including the associated parameters

VARIABLE	DESCRIPTION
KAPAS	Saturated yield strain
KAPA0	Initial yield strain
SHEAR	Elastic shear modulus

Remarks:

The Cauchy stress for this smooth viscoelastic viscoplastic material is given by

$$\boldsymbol{\sigma} = K(J-1)\mathbf{I} + \sum_{i=1}^{6} \mathbf{s}_i \quad ,$$

where K is the elastic bulk modulus provided on the first card and $J = \det(\mathbf{F})$ is the relative volume with $\boldsymbol{F}$ being the total deformation gradient. The deviatoric stresses, $\boldsymbol{s}_i$, are coming from the *HJR (Hollenstein-Jabareen-Rubin)* elements in parallel. Up to 6 such elements can be defined for the deviatoric response and a rheological representation of one is shown in Figure M275-1. Each element is associated with 8 material parameters that are provided on the optional cards and characterize its inelastic response. All this allows for a wide range of stress strain relationships. The critical part involves estimating parameters for a given test suite. Some elaboration on the physical interpretation of the individual parameters in the context of uniaxial stress is given following a general description of the model.

We analyze one HJR element by letting $\overline{\mathbf{B}}$ denote the associated isochoric elastic left Cauchy-Green tensor. Define

$$\widetilde{\overline{\mathbf{B}}} = \overline{\mathbf{B}} - \tfrac{1}{3}\alpha\mathbf{I}, \text{ where } \alpha = \mathrm{tr}(\overline{\mathbf{B}}) \quad .$$

The evolution of $\overline{\mathbf{B}}$ is given by

$$\dot{\overline{\mathbf{B}}} = \mathbf{L}\overline{\mathbf{B}} + \overline{\mathbf{B}}\mathbf{L}^{\mathrm{T}} - \frac{2}{3}\mathrm{tr}(\mathbf{D})\overline{\mathbf{B}} - \dot{\Gamma}\mathbf{A}, \text{ where } \mathbf{A} = \overline{\mathbf{B}} - \left[\frac{3}{\mathrm{tr}(\overline{\mathbf{B}}^{-1})}\right]\mathbf{I} \quad ,$$

where $\mathbf{D}$ is the rate-of-deformation and $\dot{\Gamma}$ governs the inelastic deformation. The functional form of $\dot{\Gamma}$ is summarized in the following set of equations

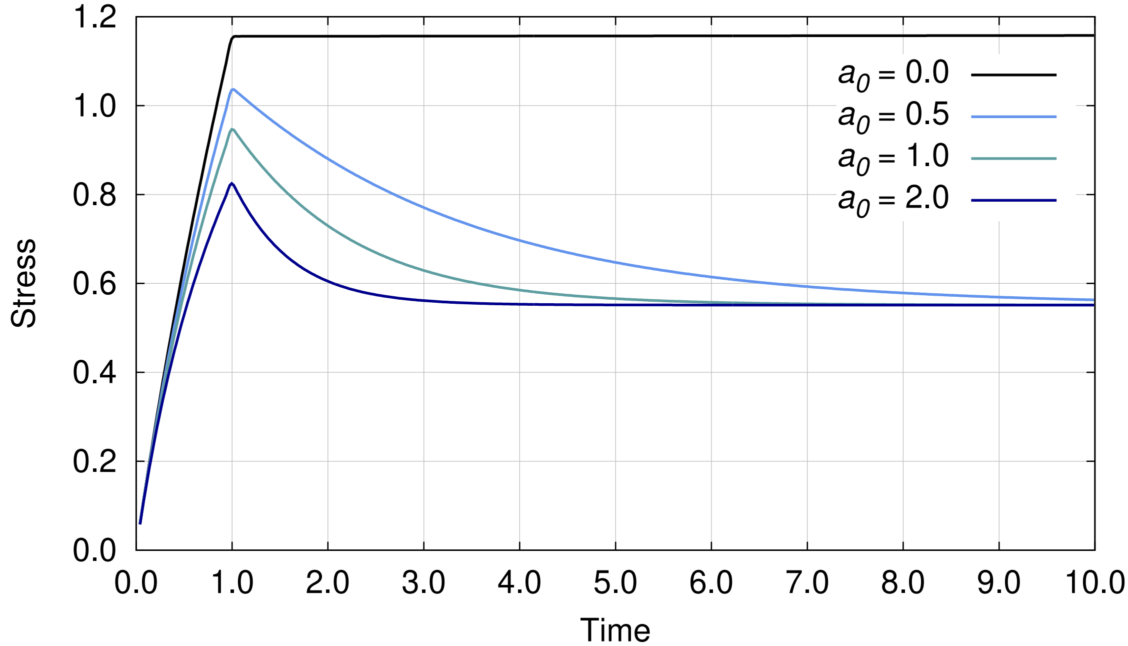


Figure M275-2. Influence of parameter a_0 on stress relaxation

$$\dot{\Gamma} = \dot{\Gamma}_0 + \langle g \rangle \dot{\Gamma}_1$$
$$\dot{\Gamma}_i = a_i + b_i \dot{\varepsilon}, \qquad i = 0,1$$
$$g = 1 - \frac{\kappa}{\tilde{\gamma}}$$

where

$$\langle g \rangle = \max(0, g)$$
$$\dot{\varepsilon} = \sqrt{\frac{2}{3} \tilde{\mathbf{D}} \colon \tilde{\mathbf{D}}}$$
$$\widetilde{D} = D - \frac{1}{3} \mathrm{tr}(\mathbf{D}) \mathbf{I}$$
$$\tilde{\gamma} = \sqrt{\frac{3}{8} \widetilde{\mathbf{B}} : \widetilde{\mathbf{B}}}$$
$$\dot{\kappa} = m \dot{\Gamma}_1 \langle g \rangle (\kappa_s - \kappa)$$

A hyperelastic law with a strain energy potential for the distortional deformation given by

$$\psi(\alpha) = \frac{G}{2}(\alpha - 3)$$

yields a contribution to the deviatoric Cauchy stress of

$$\mathbf{s} = G J^{-1} \widetilde{\mathbf{B}} \ .$$

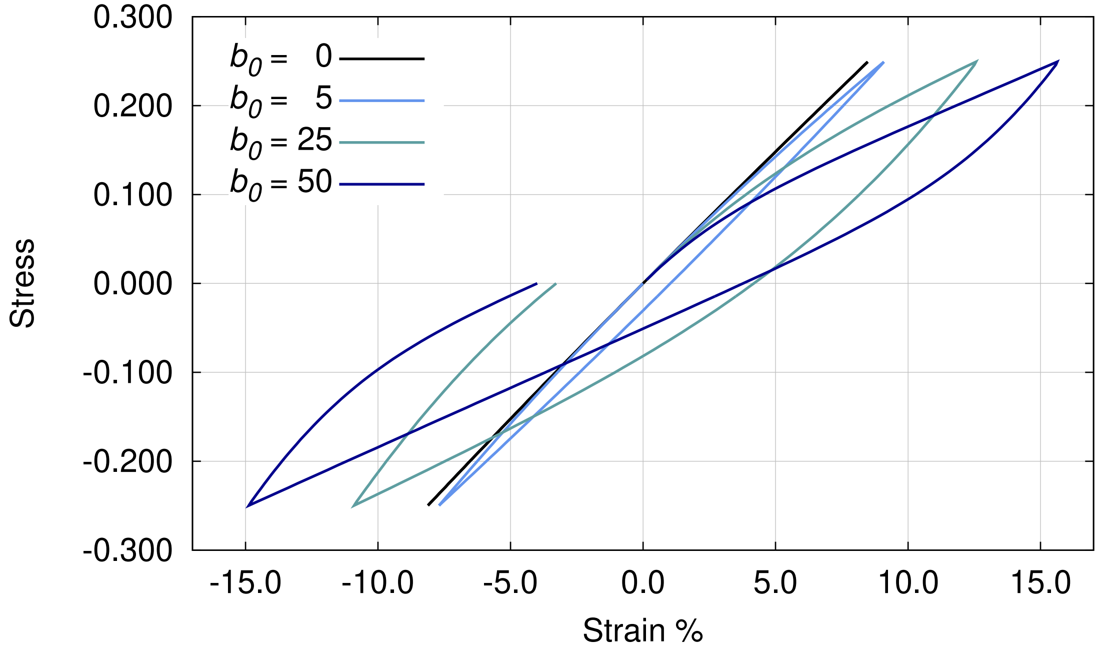


Figure M275-3. Influence of b_0 in cyclic loading

In uniaxial stress at constant total distortional rate of deformation $\pm\dot{\varepsilon}$ (tension or compression), these equations can be reduced to scalar correspondents

$$\frac{\dot{\bar{b}}}{\bar{b}} = 2\left(\pm\dot{\varepsilon} - \dot{\Gamma}\,\frac{\bar{b}\sqrt{\bar{b}} - 1}{2\bar{b}\sqrt{\bar{b}} + 1}\right)$$

$$\tau = G\left(\bar{b} - \frac{1}{\sqrt{\bar{b}}}\right) \tag{M275.1}$$

where $\bar{b}$ is the component of $\overline{\mathbf{B}}$ in the direction of deformation and τ is the uniaxial Kirchhoff stress. The evolution of Γ follows the equations above with

$$\tilde{\gamma} = \tfrac{1}{2}\left|\bar{b} - 1/\sqrt{\bar{b}}\right|.$$

Even though analytical solutions may be out of reach, this would be the basis for estimating as well as interpreting the material parameters. Obviously the shear modulus G (SHEAR) provides the elastic deviatoric stiffness, for a purely elastic material just define one such parameter and leave out all the other parameters on the same card. If several cards are used, the effective elastic shear stiffness is the sum of the contributions from each of the corresponding HJR elements. An interesting observation is that the stress in a HJR element saturates to a value given by the solution of $\bar{b}$ to

$$\begin{aligned}\bar{b}\sqrt{\bar{b}}\,\left(\pm 2 - \left\{b_0 + b_1 + \frac{a_0 + a_1}{\dot{\varepsilon}}\right\}\right) \pm 2\sqrt{\bar{b}}\kappa_s\left(b_1 + \frac{a_1}{\dot{\varepsilon}}\right) \\ + \left(\pm 1 + \left\{b_0 + b_1 + \frac{a_0 + a_1}{\dot{\varepsilon}}\right\}\right) = 0\end{aligned} \tag{M275.2}$$

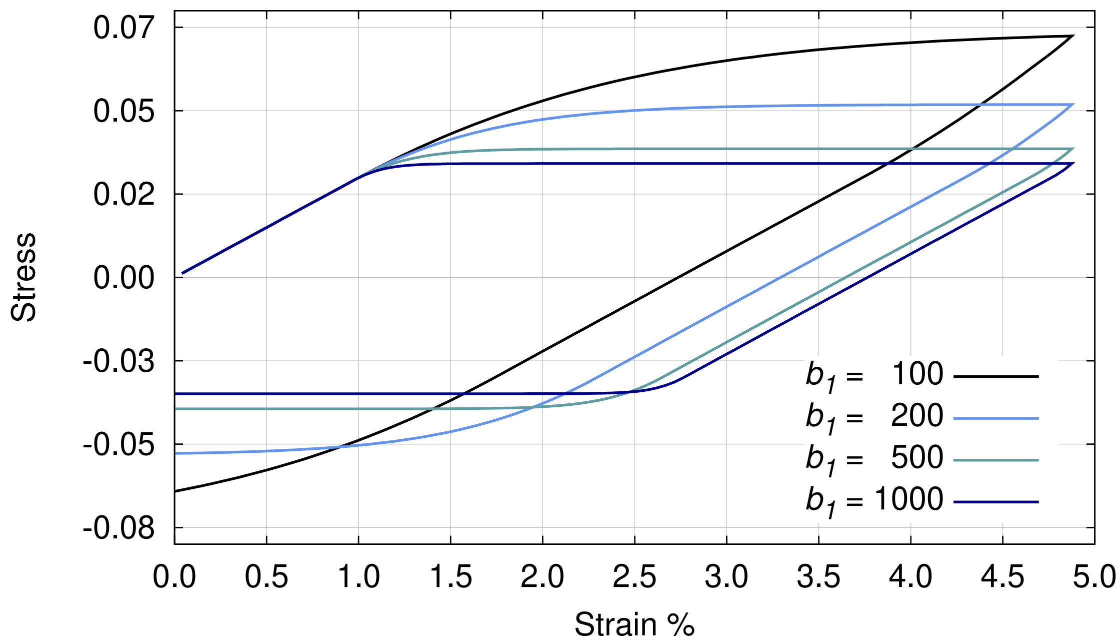


Figure M275-4. Effect of b_1 in cyclic loading

in tension (+) and compression (−), followed by application of (M275.1) above, assuming that

$$b_0 + b_1 + \frac{a_0 + a_1}{\dot{\varepsilon}} > 2$$

in tension and

$$b_0 + b_1 + \frac{a_0 + a_1}{\dot{\varepsilon}} > 1$$

in compression. This expression will be used in special cases below when examining each inelastic material parameter individually; the material parameters above are input on the HJR element cards as A0, B0, A1, B1 and KAPAS.

A Maxwell material is obtained by providing an element with a nonzero a_0 (A0) and with other parameters set to zero. This parameter should be interpreted as the viscoelastic relaxation coefficient determining the rate at which the stress relaxes to zero (see parameter BETA in *MAT_VISCOELASTIC). In Figure M275-2 a stress relaxation is shown for a strain controlled problem using two HJR elements and normalized material parameters using a bulk modulus of $K = 1$. For the first element $G = 0.5$ and for the other $G = 1$ while a_0 varies; all other parameters are zero. The engineering strain is ramped to 50% from $t = 0$ to $t = 1$ and then kept constant. The response is very similar to other viscoelastic models in LS-DYNA. Not surprisingly, a HJR element with $a_0 > 0$ (and $a_1 = b_1 = 0$) will always relax to zero stress, which follows from (M275.1) and (M275.2); thus the relaxed stress in this case comes from the purely elastic element. A general viscoelastic material can be obtained by putting several such HJR elements in parallel, in analogy to *MAT_-GENERAL_VISCOELASTIC.

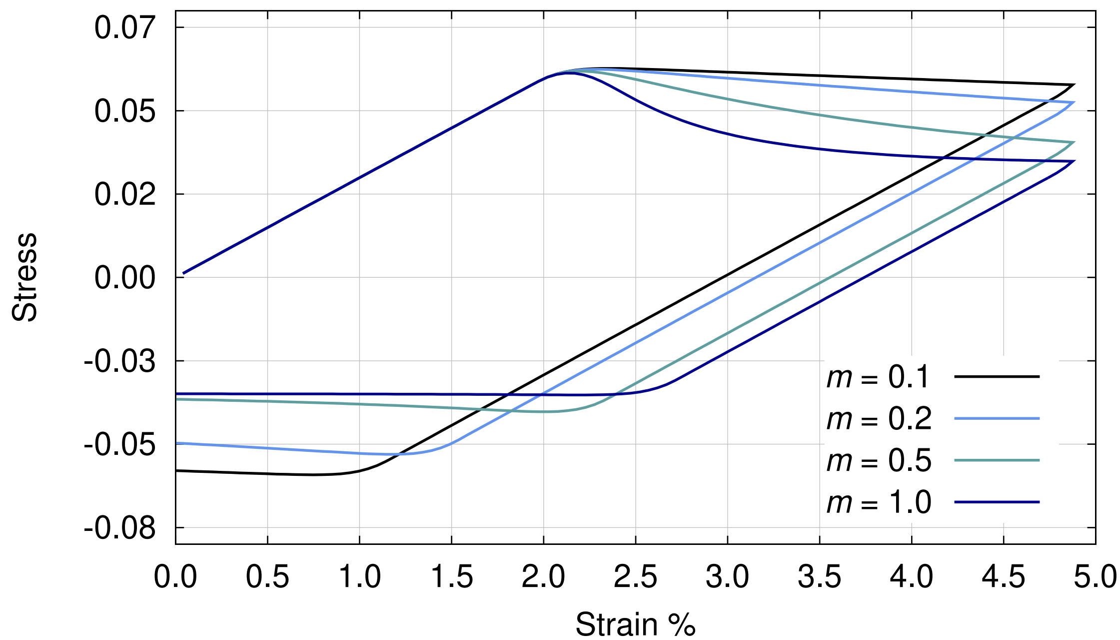


Figure M275-5. Softening response in cyclic loading for various values of m

For a nonzero b_0 (B0) with other parameters set to zero, a rate independent plastic response is obtained exhibiting zero yield stress, that is, inelastic strains develop immediately upon loading. From (M275.2) the value of b_0 determines the saturated stress value for the associated HJR element by (M275.1) and

$$\bar{b} = \left(\frac{b_0 \pm 1}{b_0 \mp 2}\right)^{2/3}$$

in tension (+) and compression (−), respectively. A smooth response is obtained that is characterized by hysteresis as shown in Figure M275-3. The same material parameters as in the previous example are used with the exception of varying b_0 instead of a_0. The deformation is controlled by a cyclic Cauchy stress between −0.25 and 0.25; for larger b_0 a hysteresis is observed. It should however be mentioned that the hysteresis vanishes as $b_0 \to \infty$ as the stress for the second element saturates quickly to a small value, so it is not trivial to quantitatively estimate the amount of hysteresis for a given parameter setting and deformation.

Rate independent plasticity with a nonzero yield stress can be obtained by a nonzero b_1 (B1) in combination with parameters κ_0 (KAPA0), κ_s (KAPAS) and m (M). The yield stress in the sense of von Mises is given by

$$\sigma_Y = 2GJ^{-1}\kappa$$

from which κ is interpreted as the current yield strain. Here b_1 determines the amount of overstress through (M275.1) and (M275.2), requiring the solution of a non-trivial

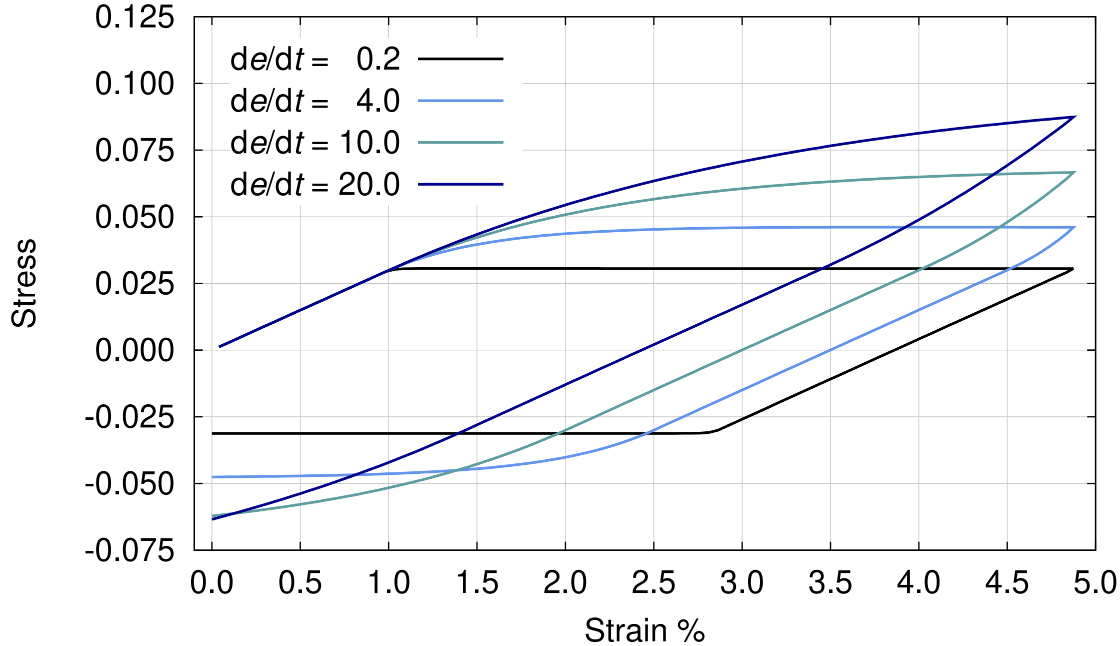


Figure M275-6. Strain rate dependence for $a_1 = 1000$ and $b_1 = 10$

polynomial equation. This is exemplified in Figure M275-4 using one HJR element with $K = 1$, $G = 1.5$, $\kappa_0 = \kappa_s = 0.01$ and $m = 0$. The engineering strain is ramped up to 5% and down to 0 and b_1 is varied with all other parameters zero; the response tends to an elastic-perfectly plastic as b_1 increases. The saturated stress value for $b_1 \to \infty$ can be calculated as

$$\bar{b} = \left[\left(\frac{1}{2} + \sqrt{\frac{1}{4} \mp \frac{8\kappa_s^3}{27}}\right)^{1/3} + \left(\frac{1}{2} - \sqrt{\frac{1}{4} \mp \frac{8\kappa_s^3}{27}}\right)^{1/3}\right]^2 \tag{M275.3}$$

employing (M275.1).

Isotropic strain hardening ($\kappa_s > \kappa_0$) or softening ($\kappa_s < \kappa_0$) is obtained with $m > 0$; κ tends exponentially towards κ_s at a rate determined by m. Using $b_1 = 1000$ (meaning very little overstress), $\kappa_0 = 0.02$, and $\kappa_s = 0.01$ while varying m, the softening response in Figure M275-5 is obtained. The rate at which the element hardens is difficult to quantitatively estimate, but presumably it depends not only on m but also on b_1. It is important to note however that for small to moderate b_1 the model appears to harden with $m = 0$, which is due to larger overstress. The hardening determined by m can be determined from a loading, unloading and reloading cycle to detect how the the yield strain κ changes; see Hollenstein et.al. [2013].

Finally, a_1 (A1) is the viscoplastic parameter determining how stress responds to change in strain rate. Its interpretation is very similar to that of a_0; stress increases with increasing loading rate and relaxes to the saturated stress value given by (M275.1) and (M275.2). In Figure M275-6 a rate dependency is illustrated for $K = 1$, $G = 1.5$, $\kappa_0 = \kappa_s = 0.01$ and

$m = 0$, where we have set $a_1 = 1000$ and $b_1 = 10$. The engineering strain rate varies from 0.2 to 20 and for small strain rates (M275.3) can be used for estimating the saturated stress, but in general (M275.2) must be used.

Putting several HJR elements in parallel can thus provide a fairly general combination of viscoelastic/viscoplastic response with isotropic hardening/softening, but this of course requires a rich test suite and a good way of estimating the material parameters. Presumably it is often sufficient to neglect some effects and work with only a subset of the material parameters.

For post-processing, the effective plastic strain in this model is defined as

$$\varepsilon_p = \sqrt{\frac{2}{3}\boldsymbol{\varepsilon}_p : \boldsymbol{\varepsilon}_p} \ ,$$

where

$$\boldsymbol{\varepsilon}_p = \boldsymbol{\varepsilon}_t - \boldsymbol{\varepsilon}_e$$

is a crude estimation of the difference between total and elastic strain. We set

$$\boldsymbol{\varepsilon}_t = \frac{1}{2J}\left[\mathbf{B} - \frac{1}{3}\mathrm{tr}(\mathbf{B})\mathbf{I}\right]$$
$$\boldsymbol{\varepsilon}_e = \frac{1}{2G}\left[\sigma - \frac{1}{3}\mathrm{tr}(\sigma)\mathbf{I}\right]$$

where

$$\mathbf{B} = J^{-2/3}\mathbf{F}\mathbf{F}^{\mathrm{T}}$$

and G here is the sum of all shear moduli defined on the HJR element cards. Note that this does not correspond to the traditional measure of effective plastic strain which should be accounted for when validating results.

*MAT_CHRONOLOGICAL_VISCOELASTIC

This is Material Type 276. This material model provides a general viscoelastic Maxwell model having up to 6 terms in the Prony series expansion and is useful for modeling dense continuum rubbers and solid explosives. It is similar to Material Type 76 but allows the incorporation of aging effects on the material properties. Either the coefficients of the Prony series expansion or a relaxation curve may be specified to define the viscoelastic deviatoric and bulk behavior.

The material model can also be used for laminated shells with either an elastic or viscoelastic layer. To activate the laminated shell, set the formulation flag on *CONTROL_-SHELL. With the laminated option, a user-defined integration rule is needed.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	BULK	PCF	EF	TREF	A	B
Type	A	F	F	F	F	F	F	F

Relaxation Curve. If fitting is done from a relaxation curve, specify fitting parameters on this card. *Otherwise*, if constants are set on Viscoelastic Constant Cards, *LEAVE THIS CARD BLANK*.

Card 2	1	2	3	4	5	6	7	8
Variable	LCID	NT	BSTART	TRAMP	LCIDK	NTK	BSTARTK	TRAMPK
Type	F	I	F	F	F	I	F	F

Viscoelastic Constant Cards. Up to 12 cards may be input. The next keyword ("*") card terminates this input. These cards are not needed if relaxation data is defined. The number of terms for the shear behavior may differ from that for the bulk behavior: simply insert zero if a term is not included. If an elastic layer is defined, only G*i* and K*i* need to be defined (note in an elastic layer only one card is needed).

Card 3	1	2	3	4	5	6	7	8
Variable	G*i*	BETA*i*	K*i*	BETAK*i*				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
BULK	Elastic bulk modulus
PCF	Tensile pressure elimination flag for solid elements only. If set to unity, tensile pressures are set to zero.
EF	Elastic flag: EQ.0: Layer is viscoelastic. EQ.1: Layer is elastic.
TREF	Reference temperature for shift function (must be greater than zero)
A	Chronological coefficient $\alpha(t_a)$. See Remarks below.
B	Chronological coefficient $\beta(t_a)$. See Remarks below.
LCID	Load curve ID for deviatoric behavior if constants, G_i and β_i, are determined using a least squares fit. See Figure M76-1 for an example relaxation curve.
NT	Number of terms in shear fit. The default is 6. Fewer than NT terms will be used if the fit produces one or more negative shear moduli. Currently, the maximum number is 6.
BSTART	In the fit, β_1 is set to zero, β_2 is set to BSTART, β_3 is 10 times β_2, β_4 is 10 times β_3 , and so on. If zero, BSTART is determined by an iterative trial and error scheme.
TRAMP	Optional ramp time for loading
LCIDK	Load curve ID for bulk behavior if constants, K_i, and β_{K_i} are determined via a least squares fit. See Figure M76-1 for an example relaxation curve.
NTK	Number of terms desired in bulk fit. The default is 6. Currently, the maximum number is 6.
BSTARTK	In the fit, β_{K_1} is set to zero, β_{K_2} is set to BSTARTK, β_{K_3} is 10 times β_{K_2}, β_{K_4} is 10 times β_{K_3}, and so on. If zero, BSTARTK is determined

VARIABLE	DESCRIPTION
	by an iterative trial and error scheme.
TRAMPK	Optional ramp time for bulk loading
G*i*	Optional shear relaxation modulus for the i^{th} term
BETA*i*	Optional shear decay constant for the i^{th} term
K*i*	Optional bulk relaxation modulus for the i^{th} term
BETAK*i*	Optional bulk decay constant for the i^{th} term

Remarks:

The Cauchy stress, σ_{ij}, is related to the strain rate by

$$\sigma_{ij}(t) = -p\delta_{ij} + \int_0^t g_{ijkl}(t-\tau)\frac{\partial\varepsilon_{kl}(\tau)}{\partial\tau}d\tau$$

For this model, it is postulated that the mathematical form is preserved in the constitutive equation for aging; however two new material functions, $g'_0(t_a)$ and $g'_1(t_a,t)$ are introduced to replace g_0 and $g_1(t)$, which is expressed in terms of a Prony series as in material model 76, *MAT_GENERAL_VISCOELASTIC. The aging time is denoted by t_a.

$$\sigma_{ij}(t_a,t) = -p\delta_{ij} + \int_0^t g'_{ijkl}(t_a,t-\tau)\frac{\partial\varepsilon_{kl}(\tau)}{\partial\tau}d\tau$$

where

$$g'_{ijkl}(t_a,t) = \alpha(t_a)g_{ijkl}[\beta(t_a)t] \;.$$

Here $\alpha(t_a)$ and $\beta(t_a)$ are two new material properties that are functions of the aging time t_a. The material properties functions $\alpha(t_a)$ and $\beta(t_a)$ will be determined using experimental results. For determination of $\alpha(t_a)$ and $\beta(t_a)$, the above equations can be written in the following form

$$\begin{aligned}\log\left(\sigma_{ij} - p\delta_{ij}\right)_{t_a,t} &= \log\alpha(t_a) + \log\left(\sigma_{ij} - p\delta_{ij}\right)_{t_a=0,t\to\xi} \\ \log\xi &= \log\beta(t_a) + \log t\end{aligned}$$

Therefore, if one plots the stress as a function of time on log-log scales, with the vertical axis being the stress and the horizontal axis being the time, then the stress-relaxation curve for any aged time history can be obtained directly from the stress-relaxation curve at $t_a = 0$ by imposing a vertical shift and a horizontal shift on the stress-relaxation curves. The vertical shift and the horizontal shift are $\log\alpha(t_a)$ and $\log\beta(t_a)$ respectively.

*MAT_ADHESIVE_CURING_VISCOELASTIC

This is Material Type 277. It is useful for modeling adhesive materials during chemical curing. This material model provides a general viscoelastic Maxwell model having up to 16 terms in the Prony series expansion. It is similar to Material Type 76, but the viscoelastic properties not only depend on the temperature but also on an internal variable representing the state of cure for the adhesive. The kinematics of the curing process depend on temperature as well as on temperature rate and follow the Kamal model.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K1	K2	C1	C2	M	N
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	CHEXP1	CHEXP2	CHEXP3	LCCHEXP	LCTHEXP	R	TREFEXP	DOCREXP
Type	F	F	F	I	I	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	WLFTREF	WLFA	WLFB	LCG0	LCK0	IDOC	INCR	QCURE
Type	F	F	F	I	I	F	I	F

Viscoelastic Constant Cards. Up to 16 cards may be input. A keyword ("*") card terminates this input if fewer than 16 cards are used. The number of terms for the shear behavior may differ from that for the bulk behavior: simply insert zero if a term is not included.

Card 4	1	2	3	4	5	6	7	8
Variable	G*i*	BETAG*i*	K*i*	BETAK*i*				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
K1	Parameter k_1 for Kamal model
K2	Parameter k_2 for Kamal model
C1	Parameter c_1 for Kamal model
C2	Parameter c_2 for Kamal model
M	Exponent m for Kamal model
N	Exponent n for Kamal model
CHEXP1	Quadratic parameter γ_2 for chemical shrinkage
CHEXP2	Linear parameter γ_1 for chemical shrinkage
CHEXP3	Constant parameter γ_0 for chemical shrinkage
LCCHEXP	|LCCHEXP| is a load curve ID defining the coefficient for chemical shrinkage $\gamma(\alpha)$ as a function of the state of cure, α. If set, parameters CHEXP1, CHEXP2 and CHEXP3 are ignored. See Remark 1 below.
LCTHEXP	|LCTHEXP| is a load curve ID or table ID defining the coefficient of thermal expansion $\beta(\alpha, T)$ as a function of cure, α, and temperature, T. If referring to a load curve, parameter $\beta(T)$ is a function of temperature, T.
R	Gas constant, R, for Kamal model
TREFEXP	Reference temperature, T_0, for secant form of thermal expansion. See Remark 1 below.
DOCREXP	Reference degree of cure, α_0, for sequential form of chemical expansion. See Remark 1 below.
WLFTREF	Reference temperature for either the Arrhenius or Williams-Landel-Ferry shift function (must be greater than zero for the shift function). Set to zero (along with WLFA and WLFB) to not apply scaling. See Remark 2.

VARIABLE	DESCRIPTION
WLFA	Coefficient for the Arrhenius and the Williams-Landel-Ferry shift functions. Set to zero (along with WLFTREF and WLFB) to not apply scaling. See Remark 2.
WLFB	Coefficient for the Williams-Landel-Ferry shift function. Set to zero for the Arrhenius shift function or to not apply scaling (to not apply scaling also set WLFTREF and WLFA to zero). See Remark 2.
LCG0	Load curve ID defining the instantaneous shear modulus, G_0, as a function of state of cure
LCK0	Load curve ID defining the instantaneous bulk modulus, K_0, as a function of state of cure
IDOC	Initial degree of cure, α_I
INCR	Switch between incremental and total stress formulation: EQ.0: Total form (default) EQ.1: Incremental form (recommended)
QCURE	Heat generation factor, relating the heat generated in one time step with the increment of the degree of cure in that step
G*i*	Shear relaxation modulus for the i^{th} term for fully cured material
BETAG*i*	Shear decay constant for the i^{th} term for fully cured material
K*i*	Bulk relaxation modulus for the i^{th} term for fully cured material.
BETAK*i*	Bulk decay constant for the i^{th} term for fully cured material

Remarks:

1. **Material Formulation.** Within this material formulation an internal variable α has been included to represent the degree of cure for the adhesive. The evolution equation for this variable is given by the Kamal model and reads

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \left(k_1 \exp\left(\frac{-c_1}{RT}\right) + k_2 \exp\left(\frac{-c_2}{RT}\right)\alpha^m\right)(1-\alpha)^n \ .$$

The chemical reaction of the curing process results in a shrinkage of the material. The coefficient of the chemical shrinkage $\gamma(\alpha)$ can either be given by a load curve or by using the quadratic expression

$$\gamma(\alpha) = \gamma_2\alpha^2 + \gamma_1\alpha + \gamma_0 \ .$$

For positive values of the parameter LCCHEXP, a differential form is used to compute the chemical strains:

$$d\varepsilon^{\mathrm{ch}} = \gamma(\alpha)d\alpha \ .$$

Otherwise a secant form defines the strains:

$$\varepsilon^{\mathrm{ch}} = \gamma(\alpha)(\alpha - \alpha_0) - \gamma(\alpha_I)(\alpha_I - \alpha_0) \ .$$

Consequently, the definition of $\gamma(\alpha)$ as quadratic expression goes along the secant formulation.

Analogously, the thermal strains are either defined in a secant or differential form, depending on the load curve parameter LCTHEXP. For positive values of that parameter, the differential form is applied, otherwise the secant form is used. For the latter the reference temperature T_0 is identified with the input parameter TREFEXP. In both cases the coefficient of thermal expansion can be given as table depending on degree of cure and temperature.

Finally, the Cauchy stress, σ_{ij}, is related to the strain rate by

$$\sigma_{ij}(t) = \int_0^t g_{ijkl}(t-\tau)\frac{\partial\varepsilon_{kl}(\tau)}{\partial\tau}d\tau \ .$$

The relaxation functions $g_{ijkl}(t-\tau)$ are represented in this material formulation by up to 16 terms (not including the instantaneous modulus G_0) of the Prony series:

$$\mathrm{g}(t,\alpha) = G_0(\alpha) - \sum_i G_i(\alpha) + \sum_i G_i(\alpha)\, e^{-\beta_i t} \ .$$

For the sake of simplicity, a constant ratio $G_i(\alpha)/G_0(\alpha)$ for all degrees of cure is assumed. Consequently, it suffices to define one term $G_0(\alpha)$ as a function of the degree of cure and further coefficients for the fully cured state of the adhesive:

$$\mathrm{g}(t,\alpha) = G_0(\alpha)\left(1 - \sum_i \frac{G_{i,\alpha=1.0}}{G_{0,\alpha=1.0}}\left(1 - e^{-\beta_i t}\right)\right) \ .$$

2. **Temperature Effect on the Stress Relaxation.** A possible temperature effect on the stress relaxation (see Remark 1) is accounted for by the Williams-Landel-Ferry (WLF) shift function or the Arrhenius shift function. For details on this function, please see material formulation 76, *MAT_GENERAL_VISCOELASTIC. If all three values (WLFTREF, WLFA, and WLFB) are nonzero, the WLF function is used; the Arrhenius function is used if WLFB is zero; and no scaling is applied if all three values are zero.

*MAT_CF_MICROMECHANICS

This is Material Type 278 developed for draping and curing analysis of pre-impregnated (prepreg) carbon fiber sheets. This material model is a mixture of *MAT_234 [Tabiei et. al.] and *MAT_277. It was developed with the collaboration of Professor Tabiei from UC. *MAT_234 provides the reorientation and locking phenomenon of fibers while *MAT_-277 provides the viscoelastic behavior of epoxy resin. Both the epoxy resin and the fiber orientation and deformation contribute to the overall stress.

Card Summary:

Card 1. This card is required.

MID	RO	E1	E2	G12	G23	EU	C

Card 2. This card is required.

EKA	EUA	VMB	EKB	THL	TA	THI1	THI2

Card 3. This card is required.

W	SPAN	THICK	H	AREA	E3	PR13	PR23

Card 4. This card is required.

AOPT	A1	A2	A3	XP	YP	ZP	

Card 5. This card is required.

V1	V2	V3	D1	D2	D3		

Card 6. This card is required.

VYARN							

Card 7. This card is required.

K1	K2	C1	C2	M	N		

Card 8. This card is required.

CHEXP1	CHEXP2	CHEXP3	LCCHEXP	LCTHEXP	R	TREF	DOCREF

Card 9. This card is required.

WLFTREF	WLFA	WLFB	LCG0	LCK0	IDOC	INCR	QCURE

Card 10. Up to 14 cards may be input. The next keyword ("*") card terminates this input.

G*i*	BETAG*i*	K*i*	BETAK*i*				

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E1	E2	G12	G23	EU	C
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E1	Young's modulus in the yarn's axial direction, E_1
E2	Young's modulus in the yarn's transverse direction, E_2
G12	Shear modulus of the yarns, G_{12}
G23	Transverse shear modulus
EU	Ultimate strain at failure
C	Coefficient of friction between the fibers

Card 2	1	2	3	4	5	6	7	8
Variable	EKA	EUA	VMB	EKB	THL	TA	THI1	THI2
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
EKA	Elastic constant of element "a"

VARIABLE	DESCRIPTION
EUA	Ultimate strain of element “a”
VMB	Damping coefficient of element “b”
EKB	Elastic constant of element “b”
THL	Yarn locking angle
TA	Transition angle of locking
THI1	Initial braid angle 1
THI2	Initial braid angle 2

Card 3	1	2	3	4	5	6	7	8
Variable	W	SPAN	THICK	H	AREA	E3	PR13	PR23
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
W	Fiber width
SPAN	Span between the fibers
THICK	Real fiber thickness
H	Effective fiber thickness
AREA	Fiber cross-sectional area
E3	Young’s modulus, E_3, in the “thickness” direction as defined by the 3rd axis of the material coordinate system (solids only)
PR13	Transverse Poisson’s ratio ν_{13} (solids only)
PR23	Transverse Poisson’s ratio ν_{23} (solids only)

Card 4	1	2	3	4	5	6	7	8
Variable	AOPT	A1	A2	A3	XP	YP	ZP	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
AOPT	Material axes option (see MAT_*OPTION* TROPIC_ELASTIC for a more complete description): EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. EQ.1.0: Locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the *a*-direction. EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR. EQ.3.0: Locally orthotropic material axes for each integration point determined by rotating the material axes about the element normal by an angle, B*i* (see *PART_COMPOSITE), from a line in the plane of the element defined by the cross product of the vector **v** with the element normal. EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, **p**, which define the centerline axis. This option is for solid elements only. LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
A1, A2, A3	Components of vector **a** for AOPT = 2.0
XP, YP, ZP	Coordinates of point **p** for AOPT = 1.0 and 4.0

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3.0 and 4.0
D1, D2, D3	Components of vector **d** for AOPT = 2.0

Card 6	1	2	3	4	5	6	7	8
Variable	VYARN							
Type	F							

VARIABLE	DESCRIPTION
VYARN	Volume fraction of yarn

Card 7	1	2	3	4	5	6	7	8
Variable	K1	K2	C1	C2	M	N		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
K1	Parameter k_1 for Kamal model
K2	Parameter k_2 for Kamal model
C1	Parameter c_1 for Kamal model
C2	Parameter c_2 for Kamal model
M	Exponent m for Kamal model
N	Exponent n for Kamal model

Card 8	1	2	3	4	5	6	7	8
Variable	CHEXP1	CHEXP2	CHEXP3	LCCHEXP	LCTHEXP	R	TREF	DOCREF
Type	F	F	F	I	I	F	F	F

VARIABLE	DESCRIPTION
CHEXP1	Quadratic parameter γ_2 for chemical shrinkage
CHEXP2	Quadratic parameter γ_1 for chemical shrinkage
CHEXP3	Quadratic parameter γ_0 for chemical shrinkage
LCCHEXP	Load curve ID to define the coefficient for chemical shrinkage $\gamma(\alpha)$ as a function of the state of cure α. If set, parameters CHEXP1, CHEXP2, and CHEXP3 are ignored.
LCTHEXP	Load curve ID or table ID defining the instantaneous coefficient of thermal expansion $\beta(\alpha, T)$ as a function of cure α and temperature T. If referring to a load curve, parameter $\beta(T)$ is a function of temperature T.
R	Gas constant R for Kamal model
TREF	Reference temperature T_0 for secant form of thermal expansion
DOCREF	Reference degree of cure α_0 for sequential form of chemical expansion

Card 9	1	2	3	4	5	6	7	8
Variable	WLFTREF	WLFA	WLFB	LCG0	LCK0	IDOC	INCR	QCURE
Type	F	F	F	I	I	F	I	F

VARIABLE	DESCRIPTION
WLFTREF	Reference temperature for WLF shift function
WLFA	Parameter A for WLF shift function

VARIABLE	DESCRIPTION
WLFB	Parameter *B* for WLF shift function
LCG0	Load curve ID defining the instantaneous shear modulus G_0 as a function of state of cure
LCK0	Load curve ID defining the instantaneous bulk modulus K_0 as a function of state of cure
IDOC	Initial degree of cure
INCR	Flag for stress formulation: EQ.0: Total formulation (default) EQ.1: Incremental formulation (recommended)
QCURE	Heat generation factor, relating the heat generated in one time step with the increment of the degree of cure in that step

Viscoelastic Constant Cards. Up to 14 cards may be input. A keyword ("*") card terminates this input if fewer than 14 cards are used. The number of terms for the shear behavior may differ from that for the bulk behavior: simply insert zero if a term is not included.

Card 10	1	2	3	4	5	6	7	8
Variable	G*i*	BETAG*i*	K*i*	BETAK*i*				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
G*i*	Shear relaxation modulus for the i^{th} term for fully cured material
BETAG*i*	Shear decay constant for the i^{th} term for fully cured material
K*i*	Bulk relaxation modulus for the i^{th} term for fully cured material
BETAK*i*	Bulk decay constant for the i^{th} term for fully cured material

*MAT_COHESIVE_PAPER

This is Material Type 279. This is a cohesive model for paper materials and can be used only with cohesive element formulations; see the variable ELFORM in *SECTION_SOLID and *SECTION_SHELL.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ROFLG	INTFAIL	EN0	ET0	EN1	ET1
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

Card 2	1	2	3	4	5	6	7	8
Variable	T0N	DN	T1N	T0T	DT	T1T	E3C	CC
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

Card 3	1	2	3	4	5	6	7	8
Variable	ASIG	BSIG	CSIG	FAILN	FAILT			
Type	F	F	F	F	F			
Default	none	none	none	none	none			

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density

VARIABLE	DESCRIPTION
ROFLG	Flag for whether density is specified per unit area or volume: EQ.0: Specifies density per unit volume (default) EQ.1: Specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero.
INTFAIL	The number of integration points required for the cohesive element to be deleted. The value of INTFAIL may range from 1 to 4 with 1 the recommended value. LT.0.0: Employs a Newton-Cotes integration scheme. The element will be deleted when |INTFAIL| integration points have failed. EQ.0.0: Employs a Newton-Cotes integration scheme. The element will *not* be deleted even if it satisfies the failure criterion. GT.0.0: Employs a Gauss integration scheme. The element will be deleted when INTFAIL integration points have failed.
EN0	The initial tensile stiffness (units of stress / length) normal to the plane of the cohesive element.
EN1	The final tensile stiffness (units of stress / length) normal to the plane of the cohesive element.
ET0	The initial stiffness (units of stress / length) tangential to the plane of the cohesive element.
ET1	The final stiffness (units of stress / length) tangential to the plane of the cohesive element.
T0N	Peak tensile traction in normal direction.
DN	Scale factor (unit of length).
T1N	Final tensile traction in normal direction.
T0T	Peak tensile traction in tangential direction. If negative, the absolute value indicates a curve with respect to the normal traction.
DT	Scale factor (unit of length). If negative, the absolute value indicates a curve with respect to the normal stress.

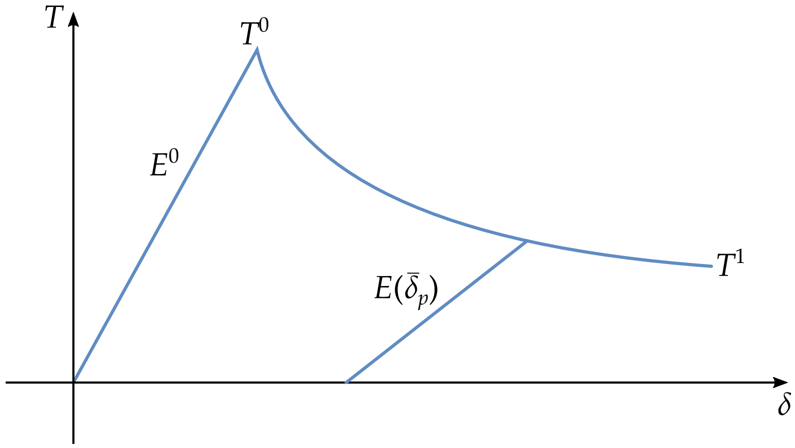


Figure M279-1. Traction-separation law

VARIABLE	DESCRIPTION
T1T	Final traction in tangential direction. If negative, the absolute value indicates a curve with respect to the normal traction.
E3C	Elastic parameter in normal compression.
CC	Elastic parameter in normal compression.
ASIG	Plasticity hardening parameter in normal compression.
BSIG	Plasticity hardening parameter in normal compression.
CSIG	Plasticity hardening parameter in normal compression.
FAILN	Maximum effective separation distance in normal direction. Beyond this distance failure occurs.
FAILT	Maximum effective separation distance in tangential direction. Beyond this distance failure occurs.

Remarks:

In this elastoplastic cohesive material, the normal and tangential directions are treated separately, but can be connected by expressing the in-plane traction parameters as functions of the normal traction. In the normal direction the material uses different models in tension and compression.

Normal tension:

Assume the total separation is an additive split of the elastic and plastic separation

$$\delta = \delta_e + \delta_p \; .$$

In normal tension ($\delta_e > 0$) the elastic traction is given by

$$T = E\delta_e = E(\delta - \delta_p) \geq 0,$$

where the tensile normal stiffness

$$E = (E_N^0 - E_N^1)\exp\left(\frac{-\bar{\delta}_p}{\delta_N}\right) + E_N^1\,,$$

depends on the effective plastic separation in the normal direction

$$\bar{\delta}_p = \int |\mathrm{d}\delta_p|\,.$$

Yield traction for tensile loads in normal direction is given by

$$T_{\mathrm{yield}} = (T_N^0 - T_N^1)\exp\left(\frac{-\bar{\delta}_p}{\delta_N}\right) + T_N^1 \geq 0,$$

and yielding occurs when $T > T_{\mathrm{yield}} \geq 0$. The above elastoplastic model gives the traction-separation law depicted in Figure M279-1.

Normal compression:

In normal compression the elastic traction is

$$T = E_3^c[1 - \exp(-C_c\delta_e)] \leq 0,$$

and the yield traction is

$$T_{\mathrm{yield}} = -\left[A_\sigma + B_\sigma \exp\left(-C_\sigma\bar{\delta}_p\right)\right] \leq 0,$$

with yielding if $T < T_{\mathrm{yield}} \leq 0$.

Tangential traction:

Assume the total separation is an additive split of the elastic and plastic separation in each in-plane direction

$$\delta_i = \delta_e^i + \delta_p^i, \qquad i = 1{,}2.$$

The elastic traction is given by

$$T_i = E\delta_e^i = E(\delta_i - \delta_p^i),$$

where the tensile normal stiffness

$$E = (E_T^0 - E_T^1)\exp\left(\frac{-\bar{\delta}_p}{\delta_T}\right) + E_T^1,$$

depends on the effective plastic separation

$$\bar{\delta}_p = \int \mathrm{d}\delta_p\,, \qquad \mathrm{d}\delta_p = \sqrt{\left(\mathrm{d}\delta_p^1\right)^2 + \left(\mathrm{d}\delta_p^2\right)^2}.$$

Yield traction is given by

$$T_{\text{yield}} = \left(T_T^0 - T_T^1\right)\exp\left(\frac{-\bar{\delta}_p}{\delta_T}\right) + T_T^1,$$

and yielding occurs when

$$T_1^2 + T_2^2 - T_{yield}^2 \geq 0.$$

The plastic flow increment follows the flow rule

$$\mathrm{d}\delta_p^i = \frac{T_i}{\sqrt{T_1^2 + T_2^2}}\mathrm{d}\delta_p.$$

The above elastoplastic model gives the traction-separation law depicted in Figure M279-1.

History variables

This material uses five history variables. Effective separation in the tangential direction is saved as Effective Plastic Strain. History variable 1 and 2 indicates the plastic separation in each tangential direction. Effective plastic separation and plastic separation in the normal direction are saved as history variable 3 and 4, respectively.

MAT_GLASS_*{OPTION}

Available options include:

<BLANK>

STOCHASTIC

SPM

This is Material Type 280. It is a smeared fixed crack model with a selection of different brittle, stress-state dependent failure criteria such as Rankine, Mohr-Coulomb, or Drucker-Prager. The model incorporates up to 2 (orthogonal) cracks per integration point, simultaneous failure over element thickness, and crack closure effects. It is available for shell elements and thick shell types 1, 2 and 6. It is only available for explicit analysis.

The STOCHASTIC keyword option allows spatially varying tensile strength behavior. See *DEFINE_STOCHASTIC_VARIATION for additional information.

The SPM keyword option invokes the additional use of the Glass Strength Prediction Model (GSPM) developed by Rudshaug et al. [2023]. See Remark 11.

Card Summary:

Card 1. This card is required.

MID	RO	E	PR			IMOD	ILAW

Card 2. This card is required.

FMOD	FT	FC	AT	BT	AC	BC	FTSCL

Card 3. This card is required.

SFSTI	SFSTR	CRIN	ECRCL	NCYCR	NIPF		

Card 4. Include this card when using the SPM keyword option.

NUMIT	FDMIN	FDMAX	KCRIT	FDENS	ACMN	ACSTD	ACMIN

Card 5. Include this card when using the SPM keyword option.

ACMAX	JUMAR	KTH	V0	N	TSCL	EXPA	NINC

Card 6. Include this card when using the SPM keyword option.

FPERC							

Card 7. This card is optional.

EPSCR	ENGCRT	RADCRT	RATENL	RFILTF	FRACEN	CTRACK	GRPFT

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR			IMOD	ILAW
Type	A	F	F	F			F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density, ρ
E	Young's modulus, E
PR	Poisson's ratio, ν
IMOD	Flag to choose degradation procedure when critical stress is reached: EQ.0.0: Softening in NCYCR load steps. Define SFSTI, SFSTR, and NCYCR (default). EQ.1.0: Damage model for softening. Define ILAW, AT, BT, AC, and BC.
ILAW	Flag to choose damage evolution law if IMOD = 1.0 (see Remark 5): EQ.0.0: Same damage evolution for tensile and compressive failure (default) EQ.1.0: Different damage evolution for tensile failure and compressive failure.

Card 2	1	2	3	4	5	6	7	8
Variable	FMOD	FT	FC	AT	BT	AC	BC	FTSCL
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
FMOD	Flag to choose between failure criteria (see Remark 1): EQ.0.0: Rankine maximum stress (default) EQ.1.0: Mohr-Coulomb EQ.2.0: Drucker-Prager EQ.10.0: Rankine with modified compressive failure EQ.11.0: Mohr-Coulomb with modified compressive failure EQ.12.0: Drucker-Prager with modified compressive failure
FT	Tensile strength, f_t. GT.0.0: Constant value LT.0.0: Load curve ID = |FT|, which gives tensile strength as a function of effective strain rate (RFILTF is recommended). If used with FTSCL > 0, |FT| specifies a curve for tensile strength vs. strain rate, and FTSCL scales the strength values from that curve as long as the material is intact. If cracked, neighbors get non-scaled values from that curve. RATENL is set to zero in that case. Logarithmic interpolation between strain rates is assumed if the first abscissa value in the curve is negative, in which case LS-DYNA assumes that all the abscissa values represent the natural logarithm of a strain rate.
FC	Compressive strength, f_c.
AT	Tensile damage evolution parameter α_t. Can be interpreted as the residual load carrying capacity ratio for tensile failure ranging from 0 to 1.
BT	Tensile damage evolution parameter, β_t. It controls the softening velocity for tensile failure.
AC	Compressive damage evolution parameter, α_c. Can be interpreted as the residual load carrying capacity ratio for compressive failure ranging from 0 to 1.

VARIABLE	DESCRIPTION
BC	Compressive damage evolution parameter β_c. It controls the softening velocity for compressive failure.
FTSCL	Scale factor for the tensile strength (default = 1.0): $$FT_{mod} = FTSCL \times FT$$ If RATENL = 0.0 (see Card 4), then the tensile strength drops to its original value, FT, as soon as the first crack happens in the associated part. In this case, FTSCL > 1.0 can be helpful in modeling high-force peaks in impact events. If RATENL ≠ 0.0, the tensile strength of an element is evaluated depending on the smoothed effective strain rate when a crack forms in a neighboring element (see Remark 7).

Card 3	1	2	3	4	5	6	7	8
Variable	SFSTI	SFSTR	CRIN	ECRCL	NCYCR	NIPF		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
SFSTI	Scale factor for stiffness after failure. For example, SFSTI = 0.001 means that stiffness is reduced to 0.1% of the elastic stiffness at failure.
SFSTR	Scale factor for stress in case of failure. For example, SFSTR = 0.01 means that stress is reduced to 1% of the failure stress at failure.
ICRIN	Flag for crack strain initialization: EQ.0.0: Initial crack strain is the strain at failure (default). EQ.1.0: Initial crack strain is zero.
ECRCL	Crack strain necessary to reactivate certain stress components after crack closure
NCYCR	Number of cycles in which the stress is reduced to SFSTR × failure stress
NIPF	Number of failed through-thickness integration points needed to fail *all* through-thickness integration points for IMOD = 0

This card is included if the SPM keyword option is used.

Card 4	1	2	3	4	5	6	7	8
Variable	NUMIT	FDMIN	FDMAX	KCRIT	FDENS	ACMN	ACSTD	ACMIN
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
NUMIT	Number of virtual flaw maps
FDMIN	Minimum flaw depth
FDMAX	Maximum flaw depth
KCRIT	Critical stress intensity
FDENS	Flaw density
ACMN	Flaw depth-to-half-length ratio, mean value
ACSTD	Flaw depth-to half-length ratio, standard deviation
ACMIN	Flaw depth-to-half-length ratio, minimum value

This card is included if the SPM keyword option is used.

Card 5	1	2	3	4	5	6	7	8
Variable	ACMAX	JUMAR	KTH	V0	N	TSCL	EXPA	NINC
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
ACMAX	Flaw depth-to-half-length ratio, maximum value
JUMAR	Area of a jumbo glass plate
KTH	Stress intensity threshold for subcritical crack growth
V0	Terminal velocity of subcritical crack growth

VARIABLE	DESCRIPTION
N	Subcritical crack growth exponent
TSCL	Time scaling factor, TSCL > 0. Scales the time step increment used to calculate the subcritical crack growth velocity: $$v = \frac{da}{dt} = V0\left(\frac{K_I}{K_{IC}}\right)^N$$ $$dt = (t_{i+1} - t_i)\times TSCL$$
EXPA	Exponent for moving exponential averaging, 0<= EXPA <= 1 $$\overline{K}_{I,i} = EXPA\times K_{I,i} + (1 - EXPA)\overline{K}_{I,i-1}$$
NINC	Defines the number of increments to run per GSPM update, NINC ≥ 1. NINC = 1 means that GSPM runs at each increment, while NINC = 10 results in one GSPM running every tenth increment.

This card is included if the SPM keyword option is used.

Card 6	1	2	3	4	5	6	7	8
Variable	FPERC							
Type	F							

VARIABLE	DESCRIPTION
FPERC	Failure percentile, 0 < FPERC ≤ 1. When the number of virtual glass plates with fracture initiation exceeds FPERC × NUMIT, fracture initiation is triggered in the simulation.

Optional card.

Card 7	1	2	3	4	5	6	7	8
Variable	EPSCR	ENGCRT	RADCRT	RATENL	RFILTF	FRACEN	CTRACK	GRPFT
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
EPSCR	Critical value to trigger element deletion. This can be useful to get rid of highly distorted elements. GT.0.0: EPSCR is effective critical strain. LT.0.0: |EPSCR| is critical crack opening displacement.
ENGCRT	Critical energy for nonlocal failure criterion; see Remark 6.
RADCRT	Critical radius for nonlocal failure criterion; see Remark 6.
RATENL	Quasi-static strain rate threshold variable which activates a non-local, strain rate dependent tensile strength adaption; see Remark 7.
RFILTF	Smoothing factor on the effective strain rate for the evaluation of the current tensile strength if RATENL > 0.0; see Remark 7. $$\dot{\varepsilon}_n^{\mathrm{avg}} = \mathrm{RFILTF} \times \dot{\varepsilon}_{n-1}^{\mathrm{avg}} + (1 - \mathrm{RFILTF}) \times \dot{\varepsilon}_n$$
FRACEN	Fracture energy (units of stress × length). An alternative orthotropic damage model with linear softening is invoked with this option. Values smaller than $0.5 \times f_t \times \frac{f_t}{E} \times l_e$ (element size) lead to immediate failure. This is the area under the elastic stress-displacement line until f_t is reached. Only larger values result in actual residual energy after crack initiation. Variables SFSTI, SFSTR, and NCYCR are ignored with this option. You can specify a spatially varying scale factor for FRACEN by setting history variable #14 with *INITIAL_STRESS_SHELL.
CTRACK	Flag for optional crack tracking algorithm (see Remark 10): EQ.0.0: Inactive EQ.1.0: Active
GRPFT	Optional group number for strength reduction. If several parts use *MAT_280 with potentially different material parameters, giving them the same value of GRPT causes them to experience tensile strength reduction by FTSCL at the same time (RATENL = 0).

Remarks:

1. **Plane stress failure criteria.** The underlying material behavior before failure is isotropic, small strain linear elasticity with Young's modulus, E, and Poisson's

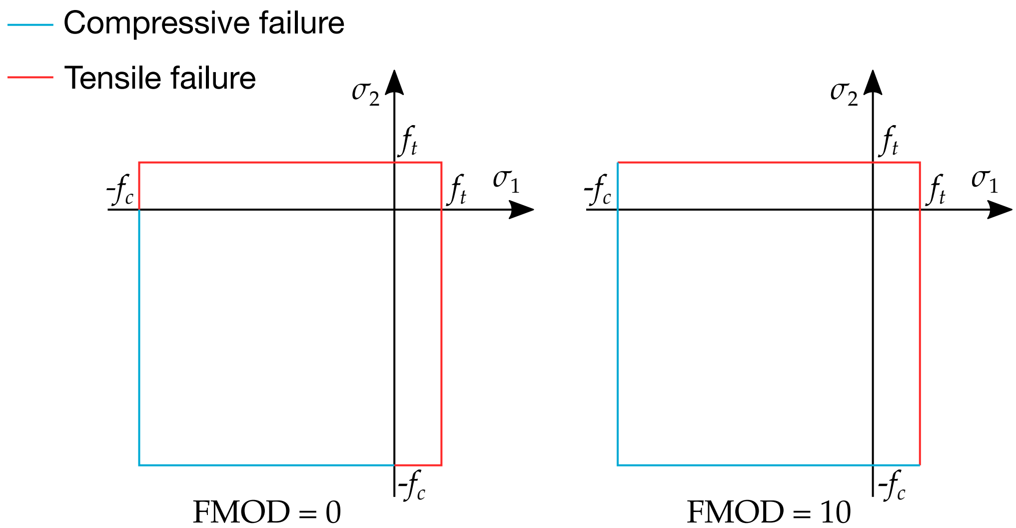


Figure M280-1. Rankine failure criterion. With FMOD = 10, the form of the failure criterion does not change but what is considered compressive failure is modified.

ratio, ν. Asymmetric (tension-compression dependent) failure happens as soon as one of the following plane stress failure criteria is violated.

a) *Rankine Maximum Stress.* For FMOD = 0, a maximum stress criterion (Rankine) is used where principal stresses, σ_1 and σ_2, are bound by tensile strength, f_t, and compressive strength, f_c, as follows:

$$-f_c < \{\sigma_1, \sigma_2\} < f_t$$

b) *Mohr-Coulomb.* With FMOD = 1, the Mohr-Coulomb criterion with expressions in four different categories is used:

$$\sigma_1 > 0 \text{ and } \sigma_2 > 0: \qquad \max\left(\frac{\sigma_1}{f_t}, \frac{\sigma_2}{f_t}\right) < 1$$

$$\sigma_1 < 0 \text{ and } \sigma_2 < 0: \qquad \max\left(-\frac{\sigma_1}{f_c}, -\frac{\sigma_2}{f_c}\right) < 1$$

$$\sigma_1 > 0 \text{ and } \sigma_2 < 0: \qquad \frac{\sigma_1}{f_t} - \frac{\sigma_2}{f_c} < 1$$

$$\sigma_1 < 0 \text{ and } \sigma_2 > 0: \qquad -\frac{\sigma_1}{f_c} + \frac{\sigma_2}{f_t} < 1$$

c) *Drucker-Prager.* For FMOD = 2, the plane stress Drucker-Prager criterion is given by:

$$\frac{1}{2f_c}\left[\left(\frac{f_c}{f_t} - 1\right)(\sigma_1 + \sigma_2) + \left(\frac{f_c}{f_t} + 1\right)\sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}\right] < 1$$

The modified versions, FMOD = 10, 11, 12, change what is considered tensile and compressive failure. The form of the failure stays the same for each type. In each case, a line with slope $-f_c/f_t$ distinguishes the difference between the two types of failure. See Figure M280-1 for an example of how the tensile and compressive failure are modified for the Rankine failure criterion.

2. **Crack formation.** As soon as failure happens in the tensile regime, a crack occurs perpendicular to the maximum principal stress direction. That means a crack coordinate system is set up and stored, defined by a relative angle with respect to the element coordinate system. Appropriate stress and stiffness tensor components (e.g. normal to the crack) are reduced according to SFSTR and SFSTI if IMOD = 0. The stress reduction takes place in a period of NCYCR time step cycles. For IMOD = 1.0 the stress and stiffness tensor are reduced by a damage model (see Remark 5). A second crack orthogonal to the first crack is possible which can open and close independently from the first one, further reducing the element stiffness.

3. **Crack closure.** To deal with crack closure, the current strain in the principal stress direction is stored as the initial crack strain (ICRIN = 0, default), or the initial crack strain is set to zero (ICRIN = 1). After failure, the crack strain is tracked, so that later crack closure will be detected. If that is the case, appropriate stress and stiffness tensor components (e.g., compressive) are reactivated so that, e.g., under pressure, a load could be carried and cause nonzero stress perpendicular to the crack.

4. **Number of failed integration points.** If the critical number of failed integration points (NIPF) in one element is reached, all integration points over the element thickness fail as well. The default value of NIPF=1 resembles the fact, that a crack in a glass plate immediately runs through the thickness.

5. **Damage model.** IMOD = 1 invokes a damage model for stress and stiffness softening. The corresponding evolution law for ILAW = 0 is given by:

$$D = \begin{cases} 0 & \text{for } \kappa \le \kappa^0 \\ 1 - \dfrac{\kappa^0}{\kappa}\left(1 - \alpha_{t,c} + \alpha_{t,c} e^{-\beta_{t,c}(\kappa-\kappa^0)}\right) & \text{otherwise} \end{cases}$$

meaning tensile and compressive failure are treated in the same fashion.

However, with ILAW = 1 the damage evolution for tensile failure is given by:

$$D = \begin{cases} 0 & \text{for } \kappa \le \kappa^0 \\ 1 - \dfrac{\kappa^0}{\kappa}\left(1 - \alpha_t + \alpha_t e^{-\beta_t(\kappa-\kappa^0)}\right) & \text{otherwise} \end{cases}$$

while damage for compressive failure evolves as (more delayed stress reduction):

$$D = \begin{cases} 0 & \text{for } \kappa \le \kappa^0 \\ 1 - \dfrac{\kappa^0}{\kappa}(1 - \alpha_c) - \alpha_c e^{-\beta_c(\kappa-\kappa^0)} & \text{otherwise} \end{cases}$$

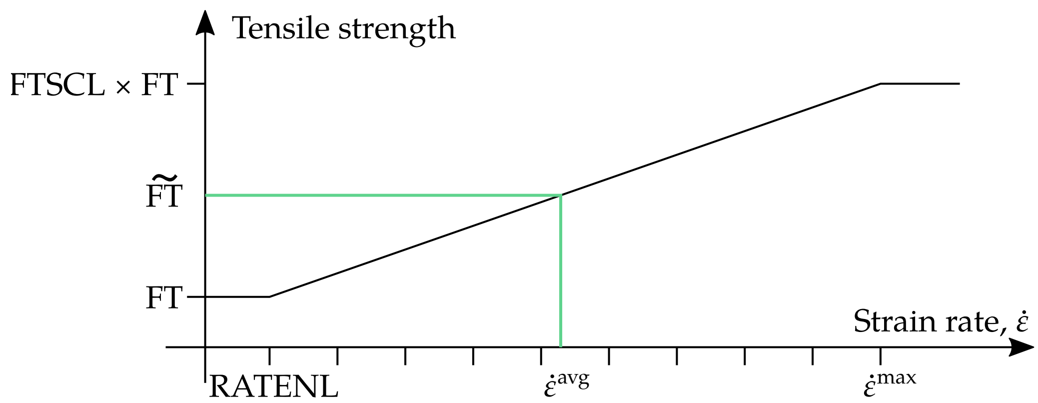


Figure M280-2. Modified tensile strength of elements neighboring an element that has at least one failed integration point as a function of strain rate when RATENL > 0.0. $\dot{\varepsilon}^{\max} = 10^9 \times \mathrm{RATENL}$.

6. **Nonlocal failure.** Use ENGCRT and RADCRT to specify a nonlocal failure criterion. This criterion is mainly intended for windshield impact. The same procedure is used as in *MAT_ADD_EROSION; see Remark 1i there.

7. **Strain-rate-dependent, nonlocal tensile strength.** If RATENL > 0.0, all elements in the appropriate part are initialized with a tensile strength of FTSCL × FT. If one integration point in an element fails, the tensile strength in the neighboring elements is set to $\widetilde{\mathrm{FT}}$ where

$$\widetilde{\mathrm{FT}} = \begin{cases} \mathrm{FT} & \text{if } \dot{\varepsilon}^{\mathrm{avg}} \le \mathrm{RATENL} \\ \mathrm{FT} + (\mathrm{FTSCL} \times \mathrm{FT} - \mathrm{FT}) \dfrac{\ln\left(\dfrac{\dot{\varepsilon}^{\mathrm{avg}}}{\mathrm{RATENL}}\right)}{\ln\left(\dfrac{\dot{\varepsilon}^{\max}}{\mathrm{RATENL}}\right)} & \text{if } \mathrm{RATENL} < \dot{\varepsilon}^{\mathrm{avg}} < \dot{\varepsilon}^{\max} \\ \mathrm{FTSCL} \times \mathrm{FT} & \text{if } \dot{\varepsilon}^{\mathrm{avg}} > \dot{\varepsilon}^{\max} \end{cases}$$

Here $\dot{\varepsilon}^{\max} = 10^9 \times \mathrm{RATENL}$. See Figure M280-2 for a plot of this tensile strength as a function of average strain rate. The average strain rate in this case is calculated as:

$$\dot{\varepsilon}_n^{\mathrm{avg}} = \mathrm{RFILTF} \times \dot{\varepsilon}_{n-1}^{\mathrm{avg}} + (1 - \mathrm{RFILTF}) \times \dot{\varepsilon}_n$$

where n is the time step.

8. **Element based (or stochastically varied) tensile strength.** You can define a spatially varying tensile strength with history variable #13 which is a scale factor on the strength. The history variable #13 value can be filled with *INITIAL_-STRESS_SHELL or can come from *DEFINE_STOCHASTIC_VARIATION.

9. **Vector plot of crack direction.** Extra history variables #15, #16, and #17 store the global coordinates of the first crack direction. These coordinates can be used to represent the crack as a vector, such as in LS-PrePost with Post→Vector→Hist. var. cosine.

10. **Crack tracking algorithm.** Cracks often follow the shell element meshing directions, e.g., it is often observed that cracks are “trapped” in element rows and cannot travel freely through the structure. This phenomenon is also called directional mesh-bias dependency. To alleviate this issue, a nonlocal crack tracking algorithm can be invoked with CTRACK = 1. Among other things, it weakens neighboring elements in the first crack direction more strongly. Currently this option must be used in conjunction with the nonlocal model discussed in Remark 7 (RATENL > 0.0) or the other nonlocal option invoked with FT < 0.0. To make this option work with MPP, the whole glass part must be put on one processor, such as by using *CONTROL_MPP_DECOMPOSITION_ARRANGE_PARTS with TYPE = 10.

11. **GSPM.** The glass strength prediction model is a Monte-Carlo-based fracture initiation predictor. It combines the theories of linear elastic fracture mechanics (LEFM) and sub-critical crack growth (SCG) to generate a representative sample of virtual glass plates that are monitored during the simulation. Each virtual glass plate has the same geometry as the respective glass part and includes a unique set of surface flaws. The flaw distribution parameters (NUMIT, FDMIN, FDMAX, FDENS, ACMN, ACSTD, ACMIN, ACMAX, and JUMAR) combined with the LEFM and SCG parameters (KCRIT, KTH, V0, and N) determine the strength distribution of the representative sample of virtual glass plates. The virtual glass plates are monitored at every NINC increment when the flaw status is updated. If the stress intensity factor is critical for a flaw, fracture initiates for the corresponding virtual glass plate. The number of virtual glass plates with fracture initiation exceeding FPERC × NUMIT triggers fracture initiation in the simulation. Selecting the failure percentile value, FPERC, enables determining the location within the generated representative sample of the virtual glass plate used to trigger fracture initiation. If time scaling is used in the simulation, the parameter TSCL can be used to scale the time increment calculated in the GSPM. EXPA is a moving average exponent that can be used to exponentially average the stress intensity factor to account for noise.

12. **History variables.** This material has the following additional history variables that can be output to the d3plot file.

History Variable #	Description
1	Crack flag: EQ.0: No crack EQ.1: One crack

History Variable #	Description
	EQ.2: Two cracks EQ.-1: Failed under compression
2	Direction of 1st principle stress as angle in radians with respect to the element direction. The shell normal defines the positive angle direction. The 1st crack direction is perpendicular to the direction of 1st principle stress.
3	Angle in radians that defines the orthogonal to the 2nd crack direction (with respect to the element direction).
4	Failure criterion value, see Remark 1
7	Current tensile strength value
8	Effective strain rate (if FT < 0 or RATENL > 0 is used)
9	Crack opening displacement (1st crack)
10	Time to failure / GSPM initial crack flag
11	Damage in 1st crack direction (only with FRACEN > 0)
12	Damage in 2nd crack direction (only with FRACEN > 0)
13	Scale factor for tensile strength
14	Scale factor for fracture energy
15	Global x-coordinate of 1st crack direction
16	Global y-coordinate of 1st crack direction
17	Global z-coordinate of 1st crack direction

*MAT_SHAPE_MEMORY_ALLOY

This is Material Type 291, a micromechanics-inspired constitutive model for shape-memory alloys that accounts for the initiation and saturation of phase. This model is based on Kelly, Stebner, and Bhattacharya (2016) and is available for solid elements only.

Card Summary:

Card 1. This card is required.

MID	RHO	EM	EA	PRM	PRA	AOPT	STYPE

Card 2. This card is required.

CPM	CPA	LH	TC	TMF	TMS	TAS	TAF

Card 3. This card is required.

A1I	A2I	BI	CI	KI	MI	KL	ML

Card 4. This card is required.

A1S	A2S	BS	CS	KS	MS		

Card 5. This card is required.

DOL	DOM						

Card 6. This card is required.

XP	YP	ZP	A1	A2	A3	MACF	

Card 7. This card is required.

V1	V2	V3	D1	D2	D3	BETA	REF

Card 8.1. Include this card if STYPE = 1.

N11	N22	N33	N44	N55	N66	N12	N23

Card 8.2. Include this card if STYPE = 1.

N34	N45	N56	N13	N24	N35	N46	N14

Card 8.3. Include this card if STYPE = 1.

N25	N36	N15	N26	N16			

Card 9. This card is optional.

KP	MP	KC	MC				

Card 10. This card is optional.

DOP	QP	NP	QL	NL	QM	NM	

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RHO	EM	EA	PRM	PRA	AOPT	STYPE
Type	A	F	F	F	F	F	I	I

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RHO	Mass density
EM	Martensite Young's modulus
EA	Austenite Young's modulus
PRM	Martensite Poisson's ratio
PRA	Austenite Poisson's ratio
AOPT	Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details): EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. EQ.1.0: Locally orthotropic with material axes determined by a point, P, in space and the global location of the element center; this is the **a**-direction. EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR

VARIABLE	DESCRIPTION
	EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.
	EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis.
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_OPTION).
STYPE	Initiation/saturation surface type (see Remark 1): EQ.0: Uses strain invariants (default) EQ.1: Uses principal strains

Card 2	1	2	3	4	5	6	7	8
Variable	CPM	CPA	LH	TC	TMF	TMS	TAS	TAF
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
CPM	Martensite volumetric heat capacity (density times specific heat capacity)
CPA	Austenite volumetric heat capacity (density times specific heat capacity)

VARIABLE	DESCRIPTION
LH	Volumetric latent heat of transformation (density times specific latent heat)
TC	Thermodynamic temperature
TMF	Martensite finish temperature, optional; see Remark 2.
TMS	Martensite start temperature, optional; see Remark 2.
TAS	Austenite start temperature, optional; see Remark 2.
TAF	Austenite finish temperature, optional; see Remark 2.

Card 3	1	2	3	4	5	6	7	8
Variable	A1I	A2I	BI	CI	KI	MI	KL	ML
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
A1I	Tension/compression asymmetry for initiation surface
A2I	Tension/compression asymmetry for initiation surface
BI	Radius for initiation surface
CI	Eccentricity of initiation surface with respect to material direction
KI	Coefficient in initiation energy
MI	Exponent in initiation energy
KL	Coefficient in volume fraction energy
ML	Exponent in volume fraction energy

Card 4	1	2	3	4	5	6	7	8
Variable	A1S	A2S	BS	CS	KS	MS		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
A1S	Tension/compression asymmetry for saturation surface
A2S	Tension/compression asymmetry for saturation surface
BS	Radius for saturation surface
CS	Eccentricity of saturation surface with respect to material direction
KS	Coefficient in saturation energy
MS	Exponent in saturation energy

Card 5	1	2	3	4	5	6	7	8
Variable	D0L	D0M						
Type	F	F						

VARIABLE	DESCRIPTION
D0L	Initial driving force for volume fraction transformation
D0M	Initial driving force for martensite strain transformation

Card 6	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	MACF	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
XP, YP, ZP	Coordinates of point P for AOPT = 1 and 4

VARIABLE	DESCRIPTION
A1, A2, A3	Components of vector **a** for AOPT = 2
MACF	Material axes change flag for solid elements: EQ.-4: Switch material axes *b* and *c* before BETA rotation EQ.-3: Switch material axes *a* and *c* before BETA rotation EQ.-2: Switch material axes *a* and *b* before BETA rotation EQ.1: No change, default EQ.2: Switch material axes *a* and *b* after BETA rotation EQ.3: Switch material axes *a* and *c* after BETA rotation EQ.4: Switch material axes *b* and *c* after BETA rotation Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_-SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, for AOPT = 3, the BETA input on Card 7 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.

Card 7	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	REF
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
V1, V2, V3	Components of vector **v** for AOPT = 3 and 4
D1, D2, D3	Components of vector **d** for AOPT = 2
BETA	Material angle in degrees for AOPT = 3. This angle may be overridden on the element card; see *ELEMENT_SOLID_ORTHO.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY. EQ.0.0: Off EQ.1.0: On

Anisotropy Parameter Cards. This card and the following two cards are included if STYPE = 1.

Card 8.1	1	2	3	4	5	6	7	8
Variable	N11	N22	N33	N44	N55	N66	N12	N23
Type	F	F	F	F	F	F	F	F

Card 8.2	1	2	3	4	5	6	7	8
Variable	N34	N45	N56	N13	N24	N35	N46	N14
Type	F	F	F	F	F	F	F	F

Card 8.3	1	2	3	4	5	6	7	8
Variable	N25	N36	N15	N26	N16			
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
N*ij*	Additional anisotropy parameters for initiation/saturation surface, relative to material axis given by AOPT. Used for STYPE = 1.

Plasticity Parameter Card 1. The following two cards are optional.

Card 9	1	2	3	4	5	6	7	8
Variable	KP	MP	KC	MC				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
KP	Coefficient in plastic energy

VARIABLE	DESCRIPTION
MP	Exponent in plastic energy
KC	Coefficient in coupling energy
MC	Exponent in coupling energy

Plasticity Parameter Card 2. This card is optional.

Card 10	1	2	3	4	5	6	7	8
Variable	D0P	QP	NP	QL	NL	QM	NM	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
D0P	Initial driving force for plastic transformation
QP	Isotropic hardening coefficient in plastic kinetic relation
NP	Isotropic hardening exponent in plastic kinetic relation
QL	Isotropic hardening coefficient in volume fraction kinetic relation
NL	Isotropic hardening exponent in volume fraction kinetic relation
QM	Isotropic hardening coefficient in martensite kinetic relation
NM	Isotropic hardening exponent in martensite kinetic relation

Remarks:

1. **Material model.** The total strain ε is composed of elastic strain ε_e, martensite strain ε_m, and plastic strain ε_p according to the additive split

$$\varepsilon = \varepsilon_e + \lambda\varepsilon_m + \varepsilon_p \ ,$$

where $0 \leq \lambda \leq 1$ is the volume fraction of martensite. Initially, the material is only composed of austenite, that is, $\lambda = 0$. The material is assumed to be isotropic elastic

$$\sigma = \mathbf{C}(\lambda)\varepsilon_e \ ,$$

and the martensite strain is assumed to be trace-free

$$\mathrm{tr}(\boldsymbol{\varepsilon}_m) = 0 \ .$$

Given the total strain $\boldsymbol{\varepsilon}$ and temperature T, this model finds λ, $\boldsymbol{\varepsilon}_m$, and $\boldsymbol{\varepsilon}_p$ that minimize the mechanical energy

$$U = W + D \ ,$$

where W is the Helmholtz free energy and D is the dissipated energy. The Helmholtz free energy here is given by

$$\begin{aligned} W\big(\boldsymbol{\varepsilon}, \lambda, \boldsymbol{\varepsilon}_m, \boldsymbol{\varepsilon}_p, T\big) = \frac{1}{2}\boldsymbol{\sigma}\colon \boldsymbol{\varepsilon}_e + \lambda\omega(T) - c(\lambda)T \ln\left(\frac{T}{\mathrm{TC}}\right) + \lambda G_I(\boldsymbol{\varepsilon}_m) \\ + G_S(\lambda\boldsymbol{\varepsilon}_m) + G_\lambda(\lambda) + G_P\big(\boldsymbol{\varepsilon}_p\big) + \lambda G_C\big(\boldsymbol{\varepsilon}_m, \boldsymbol{\varepsilon}_p\big), \end{aligned}$$

where

$$\begin{aligned} \omega(T) &= \mathrm{LH}\frac{T - \mathrm{TC}}{\mathrm{TC}} \quad , \\ c(\lambda) &= \lambda \times \mathrm{CPM} + (1 - \lambda) \times \mathrm{CPA} \ . \end{aligned}$$

The functions G_I and G_S denote initiation energy and saturation energy, respectively, and are defined as

$$G_i(\mathbf{X}) = \begin{cases} \mathrm{K}i \times \max(g_i(\mathbf{X}), 0)^{\mathrm{M}i+1} & \mathrm{K}i \geq 0 \\ -\mathrm{K}i \times \log(\max(g_i(\mathbf{X}), 1))^{\mathrm{M}i+1} & \mathrm{K}i < 0 \end{cases}, \qquad i = I, S,$$

where the functions of g_I and g_S depends on the parameter STYPE and $\mathbf{X}$ is a tensor. Negative values of $g_i$lead to no contribution to the free energy. Thus, $g_i \leq 0$ defines the set of admissible values for λ and ε_m.

For STYPE = 0, we have

$$g_i(\mathbf{X}) = -1 + \frac{1}{\mathrm{B}i}\left[\left(\frac{1}{2}\mathbf{X}\colon\mathbf{X}\right)^{\frac{3}{2}} - \mathrm{A1}i \times \det(\mathbf{X}) - \mathrm{C}i \times \left|\mathbf{n}^T \bullet \mathbf{X}\,\mathbf{n}\right|^3\right], \qquad i = I, S,$$

Where $\mathrm{B}i > 0$. The direction vector, $\mathbf{n}$, is given by the main material direction defined with AOPT.

For STYPE = 1, we have

$$g_i(\mathbf{X}) = -1 + \frac{1}{\mathrm{B}_i^{\mathrm{A2}i+1}} \sum_{n=1,2,3} \left[|\mu_n(\mathbf{N}\colon\mathbf{X})| - \mathrm{A1}i \times \mu_n(\mathbf{N}\colon\mathbf{X})\right]^{\mathrm{A2}i+1}, \qquad i = I, S,$$

for $-1 < \mathrm{A1}i < 1$, $\mathrm{A2}i > 0$, $\mathrm{B}i > 0$, and the principal values μ_n. The anisotropy tensor, $\mathbf{N}$, is relative to the main material direction, $\mathbf{n}$, defined with AOPT.

The function G_λ denotes martensite volume fraction energy and is defined as

$$G_\lambda(\lambda) = \frac{\mathrm{KL} \times \lambda^{\mathrm{ML}+1}}{\mathrm{ML} + 1}, \qquad \mathrm{KL}, \mathrm{ML} \geq 0.$$

Thus, the amount of stored energy in the system can increase with increasing volume fraction.

The function G_P denotes plastic strain energy and is defined as

$$G_P(\boldsymbol{\varepsilon}_p) = \mathrm{KP} \times \|\boldsymbol{\varepsilon}_p\|^{\mathrm{MP}+1}, \qquad \mathrm{KP}, \mathrm{MP} \geq 0,$$

and G_C denotes coupling energy and is defined as

$$G_C(\boldsymbol{\varepsilon}_m, \boldsymbol{\varepsilon}_p) = -\mathrm{KC} \times \|\boldsymbol{\varepsilon}_p\|^{\mathrm{MC}+1} \boldsymbol{\varepsilon}_m : \boldsymbol{\varepsilon}_p, \qquad \mathrm{KC}, \mathrm{MC} \geq 0.$$

The driving forces for λ, $\boldsymbol{\varepsilon}_m$ and $\boldsymbol{\varepsilon}_p$ are defined as

$$\begin{aligned}
d_\lambda &= -\frac{\partial W}{\partial \lambda} = -\frac{1}{2}\mathbf{C}'(\lambda)\boldsymbol{\varepsilon}_e : \boldsymbol{\varepsilon}_e + \boldsymbol{\sigma} : \boldsymbol{\varepsilon}_m - \omega(T) + c'(\lambda) T \ln\left(\frac{T}{\mathrm{TC}}\right) \\
&\qquad - G_I(\boldsymbol{\varepsilon}_m) - \boldsymbol{\varepsilon}_m : G_s'(\lambda \boldsymbol{\varepsilon}_m) - G_\lambda'(\lambda) - G_C(\boldsymbol{\varepsilon}_m, \boldsymbol{\varepsilon}_p), \\
\mathbf{d}_{\varepsilon_m} &= -\frac{\partial W}{\partial \boldsymbol{\varepsilon}_m} = \lambda \left(\boldsymbol{\sigma} - G_I'(\boldsymbol{\varepsilon}_m) - G_s'(\lambda \boldsymbol{\varepsilon}_m) - \frac{\partial G_C}{\partial \boldsymbol{\varepsilon}_m}(\boldsymbol{\varepsilon}_m, \boldsymbol{\varepsilon}_p) \right), \\
\mathbf{d}_{\varepsilon_p} &= -\frac{\partial W}{\partial \boldsymbol{\varepsilon}_p} = \boldsymbol{\sigma} - G_P'(\boldsymbol{\varepsilon}_p) - \lambda \frac{\partial G_C}{\partial \boldsymbol{\varepsilon}_p}(\boldsymbol{\varepsilon}_m, \boldsymbol{\varepsilon}_p),
\end{aligned}$$

and typically govern the evolution of λ, $\boldsymbol{\varepsilon}_m$, and $\boldsymbol{\varepsilon}_p$ through evolution equations

$$\begin{aligned}
\dot{\lambda} &= f_\lambda(d_\lambda), \\
\dot{\boldsymbol{\varepsilon}}_m &= \mathbf{f}_{\varepsilon_m}(\mathbf{d}_{\varepsilon_m}), \\
\dot{\boldsymbol{\varepsilon}}_p &= \mathbf{f}_{\varepsilon_p}(\mathbf{d}_{\varepsilon_p}).
\end{aligned}$$

In this model, λ, $\boldsymbol{\varepsilon}_m$, and $\boldsymbol{\varepsilon}_m$ can, however, evolve freely, and the evolution is instead postulated to satisfy the kinetic relations:

$$\begin{aligned}
\dot{\lambda} &= 0 && \text{if } |d_\lambda| < \mathrm{D0L} + \mathrm{QL}\|\boldsymbol{\varepsilon}_p\|^{\mathrm{NL}}, \\
d_\lambda \dot{\lambda} &\geq 0, \\
|d_\lambda| &\leq \mathrm{D0L} + \mathrm{QL}\|\boldsymbol{\varepsilon}_p\|^{\mathrm{NL}},
\end{aligned}$$

for volume fraction,

$$\begin{aligned}
\dot{\boldsymbol{\varepsilon}}_m &= 0 && \text{if } \|\mathbf{d}_{\varepsilon_m}\| < \lambda\left(\frac{\mathrm{D0M}}{\sqrt{1.5}} + \mathrm{QM}\|\boldsymbol{\varepsilon}_p\|^{\mathrm{NM}}\right), \\
\mathbf{d}_{\varepsilon_m} : \dot{\boldsymbol{\varepsilon}}_m &\geq 0, \\
\|\mathbf{d}_{\varepsilon_m}\| &\leq \lambda\left(\frac{\mathrm{D0M}}{\sqrt{1.5}} + \mathrm{QM}\|\boldsymbol{\varepsilon}_p\|^{\mathrm{NM}}\right) && \text{if } g_i(\boldsymbol{\varepsilon}_m) \leq 0.
\end{aligned}$$

for martensite strain, and

$$\begin{aligned}
\dot{\boldsymbol{\varepsilon}}_p &= 0 && \text{if } \|\mathbf{d}_{\varepsilon_p}\| < \frac{\mathrm{D0P}}{\sqrt{1.5}} + \mathrm{QP}\|\boldsymbol{\varepsilon}_p\|^{\mathrm{NP}}, \\
\mathbf{d}_{\varepsilon_p} : \dot{\boldsymbol{\varepsilon}}_p &\geq 0, \\
\|\mathbf{d}_{\varepsilon_p}\| &\leq \frac{\mathrm{D0P}}{\sqrt{1.5}} + \mathrm{QP}\|\boldsymbol{\varepsilon}_p\|^{\mathrm{NP}}.
\end{aligned}$$

for plastic strain. The norm is here defined as

$$\|\mathbf{d}\| = \sqrt{\mathbf{d}:\mathbf{d}},$$

and the scaling factor $\sqrt{1.5}$ implies that D0M and D0P corresponds to the von Mises stress at which the martensite and plastic strains start to develop.

The above kinetic relations correspond to the rate of dissipation

$$\dot{D} = d_\lambda \dot{\lambda} + \mathbf{d}_{\boldsymbol{\varepsilon}_m}:\dot{\boldsymbol{\varepsilon}}_m + \mathbf{d}_{\boldsymbol{\varepsilon}_p}:\dot{\boldsymbol{\varepsilon}}_p \geq 0,$$

and are incorporated in the model by minimizing the mechanical energy over one time-step

$$\begin{aligned}
\Delta U &= \int_t^{t+\Delta t} \dot{U} ds = \int_t^{t+\Delta t} (\dot{W} + \dot{D}) ds = \Delta W + \int_t^{t+\Delta t} \dot{D} ds \\
&\leq \Delta W + \int_t^{t+\Delta t} \Bigg(\left(\mathrm{D0L} + \mathrm{QL}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NL}}\right)|\dot{\lambda}| + \lambda \left(\frac{\mathrm{D0M}}{\sqrt{1.5}} + \mathrm{QM}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NM}}\right)\|\dot{\boldsymbol{\varepsilon}}_m\| \\
&\qquad + \left(\frac{\mathrm{D0P}}{\sqrt{1.5}} + \mathrm{QP}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NP}}\right)\|\dot{\boldsymbol{\varepsilon}}_p\| \Bigg) ds \\
&\approx \Delta W + \left(\mathrm{D0L} + \mathrm{QL}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NL}}\right)|\lambda_{t+\Delta t} - \lambda_t| \\
&\qquad + \lambda_t \left(\frac{\mathrm{D0M}}{\sqrt{1.5}} + \mathrm{QM}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NM}}\right)\|\boldsymbol{\varepsilon}_m^{t+\Delta t} - \boldsymbol{\varepsilon}_m^t\| \\
&\qquad + \left(\frac{\mathrm{D0P}}{\sqrt{1.5}} + \mathrm{QP}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NP}}\right)\|\boldsymbol{\varepsilon}_p^{t+\Delta t} - \boldsymbol{\varepsilon}_p^t\|,
\end{aligned}$$

with respect to $\lambda_{t+\Delta t}$, $\boldsymbol{\varepsilon}_m^{t+\Delta t}$, and $\boldsymbol{\varepsilon}_p^{t+\Delta t}$. The evolution of λ, $\boldsymbol{\varepsilon}_m$, and $\boldsymbol{\varepsilon}_p$ is thus constrained by the time step.

Minimizing the mechanical energy over one time step with respect to $\lambda_{t+\Delta t}$, $\boldsymbol{\varepsilon}_m^{t+\Delta t}$, and $\boldsymbol{\varepsilon}_p^{t+\Delta t}$ gives the optimality constraints

$$\begin{aligned}
\frac{\partial U_{t+\Delta t}}{\partial \lambda_{t+\Delta t}} &= -d_\lambda^{t+\Delta t} + \left(\mathrm{D0L} + \mathrm{QL}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NL}}\right)\mathrm{sign}(\lambda_{t+\Delta t} - \lambda_t) = 0, \\
\frac{\partial U_{t+\Delta t}}{\partial \boldsymbol{\varepsilon}_m^{t+\Delta t}} &= -\mathbf{d}_{\boldsymbol{\varepsilon}_m}^{t+\Delta t} + \lambda_t \left(\frac{\mathrm{D0M}}{\sqrt{1.5}} + \mathrm{QM}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NM}}\right)\frac{(\boldsymbol{\varepsilon}_m^{t+\Delta t} - \boldsymbol{\varepsilon}_m^t)}{\|\boldsymbol{\varepsilon}_m^{t+\Delta t} - \boldsymbol{\varepsilon}_m^t\|} = \mathbf{0}, \\
\frac{\partial U_{t+\Delta t}}{\partial \boldsymbol{\varepsilon}_p^{t+\Delta t}} &= -\mathbf{d}_{\boldsymbol{\varepsilon}_p}^{t+\Delta t} + \left(\frac{\mathrm{D0P}}{\sqrt{1.5}} + \mathrm{QP}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NP}}\right)\frac{(\boldsymbol{\varepsilon}_p^{t+\Delta t} - \boldsymbol{\varepsilon}_p^t)}{\|\boldsymbol{\varepsilon}_p^{t+\Delta t} - \boldsymbol{\varepsilon}_p^t\|} = \mathbf{0},
\end{aligned}$$

and incorporating the trace-free condition on $\boldsymbol{\varepsilon}_m$ and $\boldsymbol{\varepsilon}_p$ gives

$$\begin{gathered}
\frac{\partial U_{t+\Delta t}}{\partial \lambda_{t+\Delta t}} = 0, \\
\frac{\partial U_{t+\Delta t}}{\partial \left(\boldsymbol{\varepsilon}_q^{t+\Delta t}\right)_i} - \frac{\partial U_{t+\Delta t}}{\partial \left(\boldsymbol{\varepsilon}_q^{t+\Delta t}\right)_3} = 0, \qquad i = 1{,}2,
\end{gathered}$$

$$\frac{\partial U_{t+\Delta t}}{\partial\left(\boldsymbol{\varepsilon}_q^{t+\Delta t}\right)_i} = 0, \qquad i = 4{,}5{,}6,$$

for $q = m, p$.

2. **Transition temperatures.** The initial (zero stress) austenite and martensite start and finish temperatures, TMF ≤ TMS ≤ TAS ≤ TAF, can be given instead of the parameters *"TC"* , KL, and D0L. The temperatures are defined as

$$\begin{aligned}\mathrm{TMS} &= \mathrm{TC}\left(1 - \frac{\mathrm{D0L}}{\mathrm{LH}}\right)\\ \mathrm{TMF} &= \mathrm{TC}\left(1 - \frac{\mathrm{D0L} + \mathrm{KL}}{\mathrm{LH}}\right)\\ \mathrm{TAS} &= \mathrm{TC}\left(1 + \frac{\mathrm{D0L} - \mathrm{KL}}{\mathrm{LH}}\right)\\ \mathrm{TAF} &= \mathrm{TC}\left(1 + \frac{\mathrm{D0L}}{\mathrm{LH}}\right)\end{aligned}$$

which gives

$$\begin{aligned}\mathrm{TC} &= \frac{1}{2}(\mathrm{TMS} + \mathrm{TAF})\\ \frac{\mathrm{KL}}{\mathrm{LH}} &= 1 - \frac{\mathrm{TMF} + \mathrm{TAS}}{\mathrm{TMS} + \mathrm{TAF}}\\ \frac{\mathrm{D0L}}{\mathrm{LH}} &= \frac{\mathrm{TAF} - \mathrm{TMS}}{\mathrm{TAF} + \mathrm{TMS}}\end{aligned}$$

Thus, if 0 < TMF ≤ TMS ≤ TAS ≤ TAF are given as keyword input, then *"TC"* , KL, and D0L are calculated as above, and their keyword values are ignored.

3. **Heat generation.** The internal energy is

$$\epsilon = W + \eta T \ ,$$

and the entropy is defined as

$$\eta = -\frac{\partial W}{\partial T} = -\lambda\frac{\mathrm{LH}}{\mathrm{TC}} + c(\lambda)\left(1 + \ln\left(\frac{T}{\mathrm{TC}}\right)\right) \ .$$

From the differential of W, we have

$$\begin{aligned}\dot{W} &= \frac{\partial W}{\partial \boldsymbol{\varepsilon}}\dot{\boldsymbol{\varepsilon}} + \frac{\partial W}{\partial \lambda}\dot{\lambda} + \frac{\partial W}{\partial \boldsymbol{\varepsilon}_m}\dot{\boldsymbol{\varepsilon}}_m + \frac{\partial W}{\partial \boldsymbol{\varepsilon}_p}\dot{\boldsymbol{\varepsilon}}_p + \frac{\partial W}{\partial T}\dot{T}\\ &= \boldsymbol{\sigma}\colon\dot{\boldsymbol{\varepsilon}} - d_\lambda\dot{\lambda} - \mathbf{d}_{\boldsymbol{\varepsilon}_m}\colon\dot{\boldsymbol{\varepsilon}}_m - \mathbf{d}_{\boldsymbol{\varepsilon}_p}\colon\dot{\boldsymbol{\varepsilon}}_p - \eta\dot{T}\\ &= \boldsymbol{\sigma}\colon\dot{\boldsymbol{\varepsilon}} - \dot{D} - \eta\dot{T}\end{aligned}$$

and thus

$$\dot{\epsilon} = \dot{W} + \dot{\eta}T + \eta\dot{T} \ = \boldsymbol{\sigma}\colon\dot{\boldsymbol{\varepsilon}} - \dot{D} - \eta\dot{T} + (\dot{\eta}T + \eta\dot{T}) = \boldsymbol{\sigma}\colon\dot{\boldsymbol{\varepsilon}} - \dot{D} - \dot{\eta}T \ .$$

Combining this with the energy balance

$$\dot{e} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} + \nabla \cdot (k \nabla T) \ ,$$

we get

$$c(\lambda)\dot{T} = \nabla \cdot (k \nabla T) + Q \ ,$$

with the volumetric heat generation rate

$$Q = T \frac{\mathrm{LH}}{\mathrm{TC}} \dot{\lambda} - T c'(\lambda) \dot{\lambda} \left(1 + \ln \left(\frac{T}{\mathrm{TC}}\right)\right) + \dot{D} \ .$$

4. **History variables.** The following history variables are available:

History Variable #	Description
1-6	Strain in local coordinate system ($\boldsymbol{\varepsilon}$)
7-12	Martensite strain in local system ($\boldsymbol{\varepsilon}_m$)
13	Martensite volume fraction (λ)
14	Volumetric heat generation (Q)
15-20	Stress in local system ($\boldsymbol{\sigma}$)
21-26	Plastic strain in local system ($\boldsymbol{\varepsilon}_p$)

In a thermal analysis, *MAT_291 can be coupled with *MAT_THERMAL_ISOTROPIC_TD_LC with load curves HCLC/TCLC depending on history variable 13, and load curve TGRLC depending on history variable 14. Note that *MAT_291 uses volumetric quantities for CPA, CPM, and LH, while the thermal materials use specific quantities, meaning the volumetric quantity divided by density.

*MAT_ELASTIC_PERI

This is Material Type 292. This material is valid for modeling brittle elastic materials with peridynamics solids. Material failure is captured through a bond-based peridynamics model. See Ren et al 2017 for details about this model.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	GT	GS	PSX	PSY	PSZ
Type	A	I	F	F	F	F	F	F
Default	none	none	none	10^{20}	10^{20}	0.0	0.0	0.0

Thermal Residual Stress Analysis Card. This card is optional. It is only needed for performing a thermally-induced residual stress analysis.

Card 2	1	2	3	4	5	6	7	8
Variable	GF	ALPHAT						
Type	I	F						
Default	0	0.0						

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Material density
E	Young's modulus
GT	Fracture energy release rate
GS	Fracture energy release rate for compression
PSX	Initial strain along the x-axis
PSY	Initial strain along the y-axis

VARIABLE	DESCRIPTION
PSZ	Initial strain along the *z*-axis
GF	Flag to perform a thermal residual stress analysis during dynamic relaxation (see Remark1): EQ.0: No thermal residual stress analysis. GT.0: Perform thermal residual stress analysis. GF is a function ID for the function defining the thermal field: f(x,y,z,t). x, y, and z give the position, and t is time. LT.0: Perform thermal residual stress analysis. Read peritprofile.txt which gives the initial temperature field for the thermal residual stress analysis.
ALPHAT	Isotropic thermal expansion coefficient for residual stress analysis

Remarks:

1. **Thermal residual stress analysis.** Peridynamics supports performing a thermally-induced residual stress analysis during the dynamic relaxation phase. To perform this analysis, set GF to a nonzero value. The sign of GF determines how the temperature is provided for the analysis. If GF > 0, it refers to the ID of a function giving the thermal field as a function of position and time. GF < 0 causes reading file peritprofile.txt, which must provide the initial temperature over the region. This file must have the following format:

```
x, y, z, temp
```

where x, y, and z are position coordinates, and temp is the temperature at that position.

References:

B Ren, CT Wu, E Askari (2017) A 3D discontinuous Galerkin finite element method with the bond-based peridynamics model for dynamic brittle failure analysis, International Journal of Impact Engineering 99, 14-25.

*MAT_ELASTIC_PERI_LAMINATE

This is Material Type 292A. This material is for modeling unidirectional fiber reinforced polymer laminates with peridynamics. Each lamina is modeled as a transversely isotropic material while the matrix is assumed to be isotropic. See Ren et al 2018 for details about this model.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E1	E2	PR12	G12		
Type	I/A	F	F	F	F	F		
Default	none	none	none	none	none	none		

Card 2	1	2	3	4	5	6	7	8
Variable	FOPT	FC1	FC2	FCC1	FCC2	FCD	FCDC	
Type	I	F	F	F	F	F	F	
Default	none	none	none	none	none	none	none	

Card 3	1	2	3	4	5	6	7	8
Variable	V1	V2	V3					
Type	F	F	F					
Default	none	none	none					

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Material density

VARIABLE	DESCRIPTION
E1	Young's modulus-longitudinal direction for one lamina (1-direction)
E2	Young's modulus-transverse direction for one lamina (2-direction)
PR12	Poisson's ratio in the lamina plane
G12	Shear modulus in the 12-direction
FOPT	Failure criterion type for FC1, FC2, FCC1, FCC2, FCD, and FCDC: EQ.1: Energy release rate EQ.2: Failure stretch ratio for tension (recommended)
FC1	Tension failure criterion for longitudinal direction, 1-direction
FC2	Tension failure criterion for transverse direction, 2-direction
FCC1	Compression failure criterion for longitudinal direction, 1-direction
FCC2	Compression failure criterion for transverse direction, 2-direction
FCD	Tension delamination failure criterion
FCDC	Compression delamination failure criterion
V1, V2, V3	Components of the reference fiber direction in the global coordinate system

References:

B Ren, CT Wu, P Seleson, D Zeng, D Lyu (2018) A peridynamic failure analysis of fiber-reinforced composite laminates using finite element discontinuous Galerkin approximations, International Journal of Fracture 214 (1), 49-68.

*MAT_COMPRF

This is Material Type 293. This material models the behavior of pre-impregnated (pre-preg) composite fibers during the high-temperature preforming process. In addition to providing stress and strain, it also provides warp and weft yarn directions and stretch ratios after the forming process. The major applications of the model are for materials used in lightweight automobile parts.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ET	EC	PR	G121	G122	G123
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	G124	G125	G126	GAMMAL	VF	EF3	VF23	EM
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	VM	EPSILON	THETA	BULK	G			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Continuum equivalent mass density.
ET	Tensile modulus along the fiber yarns, corresponding to the slope of the curve in Figure M293-2 in the Stable Modulus region from a uniaxial tension test. See Remark 6.
EC	Compression modulus along the fiber yarns, reversely calculated using bending tests when all the other material properties are determined. See Remark 6.

VARIABLE	DESCRIPTION
PR	Poisson's ratio. See Remark 6.
G12*i*	Coefficients for the bias-extension angle change-engineering stress curve in Figure M293-3. G121 to G126 corresponds to the 6th-order to 1st-order factors of the loading curve. See Remark 6.
GAMMAL	Shear locking angle, in degrees. See Remark 6.
VF	Fiber volume fraction in the prepreg composite.
EF3	Transverse compression modulus of the dry fiber.
VF23	Transverse Poisson's ratio of the dry fiber
EM	Young's modulus of the cured resin.
VM	Poisson's ratio of the cured resin
EPSILON	Stretch ratio at the end of the undulation stage during the uniaxial tension test. Example shown in Figure M293-2. See Remark 6.
THETA	Initial angle offset between the fiber direction and the element direction. To reduce simulation error, when building the model, the elements should be aligned to the same direction as much as possible.
BULK	Bulk modulus of the prepreg material
G	Shear modulus of the prepreg material

Remarks:

1. **Fiber and resin properties.** The dry fiber properties, EF3 and VF23, and the cure resin properties, EM and VM, are used to calculate the through-thickness elastic modulus of the prepreg using the rule of mixture. These properties will not affect the in-plane deformation of the prepreg during the preforming simulation.

2. **Shear locking.** In most of the preforming cases, the angle between the fiber yarns will not reach the shear-locking state. This model is not designed for, and, therefore, not recommended for simulating shear locking.

3. **History variables.** History variable 1 represents the angle between warp/weft yarns. History variables 2 and 3 are the stretch ratio of fibers in the 1 and 2 directions, respectively.

4. **BULK and G.** BULK and G are used by the contact algorithm. Changing these parameters will not affect the final simulation result significantly (but it may affect the time step).

5. **Model description.** Woven composite prepregs are characterized using a non-orthogonal coordinate system having two principal directions: one aligned with the longitudinal warp yarns and the other with the transverse weft yarns. Prior to deformation the warp and weft yarns are orthogonal. The directions and the fiber stretch ratios are determined from the deformation gradient. In Figure M293-1, the angles α and β refer to the relative of the rotation of the warp yarn coordinate to the local corotational x coordinate and the angle between the warp and weft yarns, respectively [2,3,4].

   The stress from material deformation is divided into two parts: (1) stress caused by the fiber stretch, $\boldsymbol{\sigma}^f$, as shown in Figure M293-1 (a); (2) stress caused by the fiber rotation, $\boldsymbol{\sigma}^m$, as shown in Figure M293-1 (b). The total stress tensor, $\boldsymbol{\sigma}$, in the local corotational $x - y$ coordinate system is the sum where the components are given below [3]:

$$\sigma_{xx}^f = \sigma_1^f \cos^2\alpha + \sigma_2^f \cos^2(\alpha+\beta) \tag{1}$$

$$\sigma_{xy}^f = \sigma_{yx}^f = \frac{1}{2}\sigma_1^f \sin 2\alpha + \frac{1}{2}\sigma_2^f \sin 2(\alpha+\beta) \tag{2}$$

$$\sigma_{yy}^f = \sigma_1^f \sin^2\alpha + \sigma_2^f \sin^2(\alpha+\beta) \tag{3}$$

$$\sigma_{xx}^m = \frac{\sigma_1^m + \sigma_2^m}{2} + \frac{\sigma_1^m - \sigma_2^m}{2}\cos(2\alpha+\beta) \tag{4}$$

$$\sigma_{xy}^m = \sigma_{yx}^m = \frac{\sigma_1^m - \sigma_2^m}{2}\sin(2\alpha+\beta) \tag{5}$$

$$\sigma_{yy}^m = \frac{\sigma_1^m + \sigma_2^m}{2} - \frac{\sigma_1^m - \sigma_2^m}{2}\cos(2\alpha+\beta) \tag{6}$$

$$\sigma_{xx} = \sigma_{xx}^f + \sigma_{xx}^m \tag{7}$$

$$\sigma_{xy} = \sigma_{yx} = \sigma_{xy}^f + \sigma_{xy}^m \tag{8}$$

$$\sigma_{yy} = \sigma_{yy}^f + \sigma_{yy}^m \tag{9}$$

6. **Material property characterization.** The non-orthogonal stress components caused by yarn stretch and rotation at various deformation states will be characterized via a set of experiments, which are uniaxial tension, bias-extension and cantilever beam bending tests. All the tests need to be performed at the pre-forming temperature. See references [1] and [3] for more details.

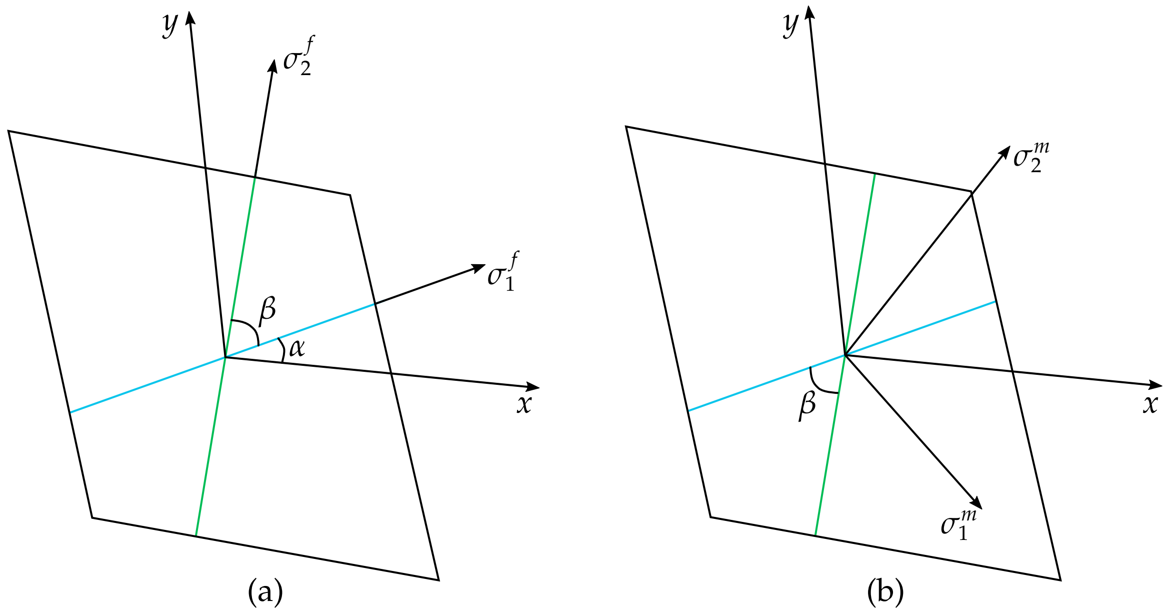


Figure M293-1. Stress components caused by (a) stretch in fiber directions and (b) rotation of the fibers [3].

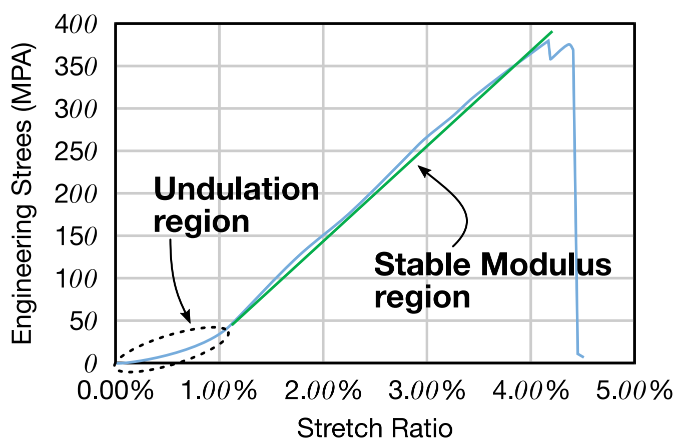


Figure M293-2. An example of the engineering stress as a function of stretch ratio from the uniaxial tension test [3].

The uniaxial tension test is used to obtain the fiber direction undulation strains and the stable tensile moduli, together with the in-plane Poisson's ratio (PR). A typical test result is shown in Figure M293-2. From the stretch ratio-engineering stress curve, the tensile modulus, ET, and the stretch ratio at the end of undulation, EPSILON, can be captured.

The bias-extension test is used to characterize the shear behavior of the composite needed for fields G12*i*. The test procedure comes from the benchmark test literature [1]. An example of the bias-extension test angle change-engineering stress curve is shown in Figure M293-3.

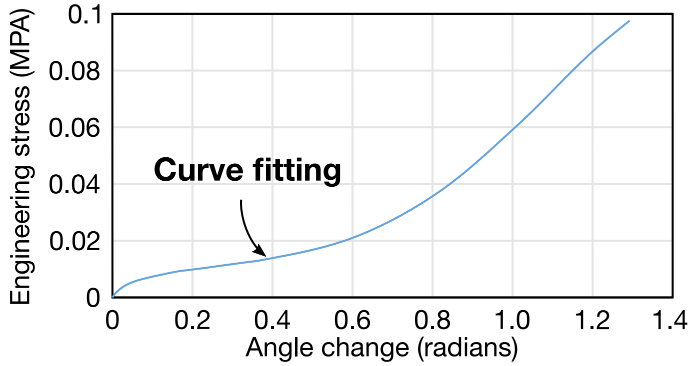


Figure M293-3. An example of the angle change-engineering stress curve from the bias-extension test. The curve fit for this example is $y = -0.29x^6 + 1.09x^5 - 1.68x^4 + 1.37x^3 - 0.56x^2 + 0.12x$. For this example curve the inputs into LS-DYNA are G121 = −0.29, G122 = 1.09, G123 = −1.68, G124 = 1.37, G125 = −0.56, and G126 = −0.12 [3].

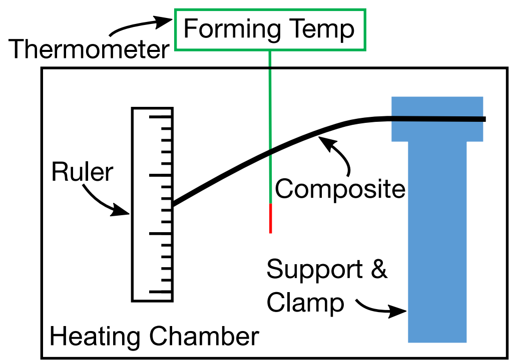


Figure M293-4. Bending test setup [3]

The angle change is calculated by using the equation [1]:

$$\gamma = \frac{\pi}{2} - 2\cos^{-1}\frac{D+d}{\sqrt{2}D}$$

where d is the cross-head displacement and D is the difference between the original height and the original width of the sample. This equation holds only before the shear locking angle, specified in field GAMMAL, which is measured directly at the end of the test, so the curve should end when the fiber yarn angle reaches the shear locking state.

The bending test should be performed to characterize the compression modulus along the yarn directions, as specified in the EC field. The test setup is shown in Figure M293-4. The composite specimen is held in a clamp and deforms under its own gravity. During the test, the composite is heated to the preforming temperature and the tip displacement is recorded. Due to the nonlinearity of the tensile modulus, the compression modulus is reversely calculated using a simulation: it is adjusted until the simulation leads to similar tip displacement to the real experiment case. The starting point for the compression modulus iteration can be set as about 100X of the shear modulus when the warp and weft yarns are perpendicular to each other.

7. **Element type.** The material model is available for shell elements with OSU = 1 and INN = 2 in the CONTROL_ACCURACY card. It is recommended to use a double-precision version of LS-DYNA.

References:

[1] J. Cao, R. Akkerman, P. Boisse, J. Chen, H.S. Cheng, E.F. de Graaf, J.L. Gorczyca, P. Harrison, G. Hivet, J. Launay, W. Lee, L. Liu, S.V. Lomov, A. Long, E. de Luycker, F. Morestin, J. Padvoiskis, X.Q. Peng, J. Sherwood, Tz. Stoilova, X.M. Tao, I. Verpoest, A. Willems, J. Wiggers, T.X. Yu, B. Zhu, Characterization of mechanical behavior of woven fabrics: Experimental methods and benchmark results, Composites Part A: Applied Science and Manufacturing, Volume 39, Issue 6, 2008, Pages 1037-1053, ISSN 1359-835X.

[2] Pu Xue, Xiongqi Peng, Jian Cao, A non-orthogonal constitutive model for characterizing woven composites, Composites Part A: Applied Science and Manufacturing, Volume 34, Issue 2, 2003, Pages 183-193, ISSN 1359-835X.

[3] Weizhao Zhang, Huaqing Ren, Biao Liang, Danielle Zeng, Xuming Su, Jeffrey Dahl, Mansour Mirdamadi, Qiangsheng Zhao, Jian Cao, A non-orthogonal material model of woven composites in the preforming process, CIRP Annals - Manufacturing Technology, Volume 66, Issue 1, 2017, Pages 257-260, ISSN 0007-8506.

[4] X.Q. Peng, J. Cao, A continuum mechanics-based non-orthogonal constitutive model for woven composite fabrics, Composites Part A: Applied Science and Manufacturing, Volume 36, Issue 6, 2005, Pages 859-874, ISSN 1359-835X.

*MAT_ANISOTROPIC_HYPERELASTIC

This is Material Type 295 which includes a collection of *(nearly-in)compressible, (an)isotropic, hyperelastic* material models primarily aimed at describing the mechanical behavior of biological soft tissues. Some of the material models may also be used to analyze a wider class of materials including fiber-reinforced elastomers and stretchable fabrics.

The constitutive laws are implemented in a modular fashion. Each module may be invoked at most once, however, the order of modules is interchangeable. Each module may comprise of different models. Consequently, one may easily change models in a module and include additional modules to account for more complex material behavior within the same keyword. Extending an existing module with a new model or even including a new module is straightforward and does not require a new material keyword.

Card Summary:

Card 1. This card is required.

MID	RHO	AOPT					

Card 2. ISOtropic module. This card and all related cards below are required.

TITLE	ITYPE	BETA	NU				

Card 2.1a. Include this card if ITYPE = ±1.

MU1	MU2	MU3	MU4	MU5	MU6	MU7	MU8

Card 2.2a. Include this card if ITYPE = ±1.

ALPHA1	ALPHA2	ALPHA3	ALPHA4	ALPHA5	ALPHA6	ALPHA7	ALPHA8

Card 2.1b. Include this card if ITYPE = -2.

C1	C2	C3					

Card 2.1c. Include this card if ITYPE = ±3.

K1	K2						

Card 3. ANISOtropic module. This card and all related cards below are optional.

TITLE	ATYPE	INTYPE	NF				

Card 3.1. Include this card if ATYPE = ±1. Include a pair of this card and one of the following two cards for each fiber family $i = 1,\ldots,$NF, that is, 2 × NF cards in total.

THETA	A	B					

Card 3.2a. Include this card if FTYPE = 1.

FTYPE	FCID	K1	K2				

Card 3.2b. Include this card if FTYPE = 2.

FTYPE	FCID	E	R0NORM	H0NORM			

Card 3.3. Include this card if INTYPE = 1.

K1	K2						

Card 4. ACTIVE module. This and all related cards below are optional and may only be used in combination with the ANISOtropic module.

TITLE	ACTYPE	ACDIR	ACID	ACTHR	SF	SS	SN

Card 4.1a. Include this card if ACTYPE = 1.

T0	CA2ION	CA2IONM	N	TAUMAX	ST	B	L0

Card 4.2a. Include this card if ACTYPE = 1.

L	DTMAX	MR	TR				

Card 4.1b. Include this card if ACTYPE = 2.

T0	CA2ION	CA2IONM	N	TAUMAX	ST	B	L0

Card 4.2b. Include this card if ACTYPE = 2.

L	ETA						

Card 4.1c. Include this card if ACTYPE = 3.

T0	CA2ION	CA2ION50	N	TAUMAX	ST	L	ETA

Card 4.1d. Include this card if ACTYPE = 4.

T0	CA2ION50	CA2IONM	N	TAUMAX	ST	CA2ION0	TCA

Card 4.2d. Include this card if ACTYPE = 4.

L	ETA						

Card 4.1e. Include this card if ACTYPE = 5.

FSEID	FLID	FVID	ALPHAID				

Card 5. This card is optional and must be used in combination with the ANISOtropic module only.

XP	YP	ZP	A1	A2	A3	MACF	

Card 6. This card is optional and must be used in combination with the ANISOtropic module only.

V1	V2	V3	D1	D2	D3	BETA	REF

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RHO	AOPT					
Type	A	F	F					

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RHO	Mass density
AOPT	Material axes option (see *MAT_002 for a more complete description):

EQ.0.0: Locally orthotropic with material axes determined by element nodes. The **a**-direction is from node 1 to node 2 of the element. The **b**-direction is orthogonal to the **a**-direction and is in the plane formed by nodes 1, 2, and 4. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.

EQ.1.0: Locally orthotropic with material axes determined by a

VARIABLE	DESCRIPTION
	point, P, in space and the global location of the element center; this is the **a**-direction. This option is for solid elements only.
	EQ.2.0: Globally orthotropic with material axes determined by vectors **a** and **d** input below, as with *DEFINE_COORDINATE_VECTOR
	EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on MACF.
	EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, P, which define the centerline axis. This option is for solid elements only.
	LT.0.0: |AOPT| is a coordinate system ID (see *DEFINE_COORDINATE_*OPTION*).

Isotropic Module Card.

Card 2	1	2	3	4	5	6	7	8
Variable	TITLE	ITYPE	BETA	NU				
Type	A10	I	F	F				
Default	none	none	0.0	none				

VARIABLE	DESCRIPTION
TITLE	Module title which must be set to ISO
ITYPE	Type of isotropic model (see Remarks 1 and 2): EQ.±1: Compressible/nearly-incompressible Ogden [12] (see Remark 4) EQ.-2: Yeoh [13] EQ.±3: Compressible/nearly-incompressible Holzapfel-Ogden [1], [7]
BETA	Volumetric response function coefficient
NU	Poisson's ratio (see Remark 3)

Ogden Model Card 1. This card is only defined if ITYPE = ±1.

Card 2.1a	1	2	3	4	5	6	7	8
Variable	MU1	MU2	MU3	MU4	MU5	MU6	MU7	MU8
Type	F	F	F	F	F	F	F	F

Ogden Model Card 2. This card is only defined if ITYPE = ±1.

Card 2.2a	1	2	3	4	5	6	7	8
Variable	ALPHA1	ALPHA2	ALPHA3	ALPHA4	ALPHA5	ALPHA6	ALPHA7	ALPHA8
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MUi	Ogden moduli, with $i = 1,\ldots,8$
ALPHAi	Ogden constants, with $i = 1,\ldots,8$

Yeoh Model Card. This card is only defined if ITYPE = -2.

Card 2.1b	1	2	3	4	5	6	7	8
Variable	C1	C2	C3					
Type	F	F	F					

VARIABLE	DESCRIPTION
C*i*	Yeoh moduli, with $i = 1,2,3$

Holzapfel-Ogden Model Card. This card is only defined if ITYPE = ±3.

Card 2.1c	1	2	3	4	5	6	7	8
Variable	K1	K2						
Type	F	F						

VARIABLE	DESCRIPTION
K1	Holzapfel-Ogden modulus
K2	Holzapfel-Ogden constant

Anisotropic Module Card.

Card 3	1	2	3	4	5	6	7	8
Variable	TITLE	ATYPE	INTYPE	NF				
Type	A10	I	I	I				
Default	none	none	none	none				

VARIABLE	DESCRIPTION
TITLE	Module title which must be set to ANISO

VARIABLE	DESCRIPTION
ATYPE	Type of anisotropic model: EQ.±1: General structure tensor-based; see Holzapfel et al. [8] (see Remark 5)
INTYPE	Type of interaction/coupling (see Remarks 6 and 7): EQ.0: None EQ.1: Holzapfel-Ogden [1], [5]
NF	Number of fiber families (see Remarks 5 and 6)

General Structure Tensor-Based Model Card A. This card is only defined if ATYPE = ±1. Include a pair of this card and one of the 2 cards following this card for each fiber family i = 1,…,NF, that is, 2 × NF cards in total.

Card 3.1	1	2	3	4	5	6	7	8
Variable	THETA	A	B					
Type	F	F	F					

VARIABLE	DESCRIPTION
THETA	Mean fiber family orientation angle with respect to the **a** material axis in the **ab** material plane in degrees
A	First structure tensor parameter
B	Second structure tensor parameter

Holzapfel-Gasser-Ogden Model Card. This card is only defined if FTYPE = 1.

Card 3.2a	1	2	3	4	5	6	7	8
Variable	FTYPE	FCID	K1	K2				
Type	I	I	F	F				

Freed-Doehring Model Card. This card is only defined if FTYPE = 2.

Card 3.2b	1	2	3	4	5	6	7	8
Variable	FTYPE	FCID	E	R0NORM	H0NORM			
Type	I	I	F	F	F			

VARIABLE	DESCRIPTION
FTYPE	Type of fiber model: EQ.1: Holzapfel-Gasser-Ogden [6] EQ.2: Freed-Doehring [2]
FCID	Curve ID defining the fiber stress as a function of fiber stretch, default if nonzero.
K1	Holzapfel-Gasser-Ogden modulus
K2	Holzapfel-Gasser-Ogden constant
E	Fiber modulus
R0NORM	Initial crimp/coil amplitude normalized with respect to the initial fiber radius (R_0/r_0)
H0NORM	Initial crimp/coil wavelength normalized with respect to the initial fiber radius (H_0/r_0)

Holzapfel-Ogden Coupling Model Card(s). These cards are only defined if INTYPE = 1.

Card 3.3	1	2	3	4	5	6	7	8
Variable	K1	K2						
Type	F	F						

VARIABLE	DESCRIPTION
K1	Coupling modulus between the fiber and sheet directions
K2	Coupling constant between the fiber and sheet directions

Active Module Card.

Card 4	1	2	3	4	5	6	7	8
Variable	TITLE	ACTYPE	ACDIR	ACID	ACTHR	SF	SS	SN
Type	A10	I	I	I	F	F	F	F
Default	none	none	0	0	0.0	none	none	none

VARIABLE	DESCRIPTION
TITLE	Module title which must be set to ACTIVE
ACTYPE	Type of active model: EQ.1: Guccione-Waldman-McCulloch [4] EQ.2: Guccione-Waldman-McCulloch [4] and Hunter-Nash-Sands [9] EQ.3: Hunter-Nash-Sands [9] EQ.4: Hunter-Nash-Sands [9] and Hunter-McCulloch-ter Keurs [10] EQ.5: Martins-Pato-Pires [14]
ACDIR	Direction of active tension: EQ.0: Active tension develops along the mean fiber orientation of all fiber families. GT.0: Active tension develops along the mean orientation of the ACDIRth fiber family.
ACID	Activation curve ID (takes priority over T0 for ACTYPE = 1, 2, 3, or 4 when defined, see Remark 8)
ACTHR	(De/re)activation threshold (see Remark 8)
SF	Active stress scaling factor in the fiber direction (see Remark 9)
SS	Active stress scaling factor in the transverse sheet direction (see Remark 9)

VARIABLE	DESCRIPTION
SN	Active stress scaling factor in the transverse normal direction (see Remark 9)

Guccione-Waldman-McCulloch Model Card 1. This card is only defined if ACTYPE = 1.

Card 4.1a	1	2	3	4	5	6	7	8
Variable	T0	CAION	CAIONM	N	TAUMAX	ST	B	L0
Type	F	F	F	F	F	F	F	F

Guccione-Waldman-McCulloch Model Card 2. This card is only defined if ACTYPE = 1.

Card 4.2a	1	2	3	4	5	6	7	8
Variable	L	DTMAX	MR	TR				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
T0	Starting time of active stress development
CA2ION	Intercellular calcium ion concentration
CA2IONM	Maximum intercellular calcium ion concentration
N	Hill coefficient
TAUMAX	Peak isometric tension under maximum activation
ST	Active fiber stress scaling factor in the transverse directions (see Remark 9)
B	Shape coefficient
L0	Sarcomere length with no active tension
L	Reference (stress-free) sarcomere length

VARIABLE	DESCRIPTION
DTMAX	Time to peak tension
MR	Slope of linear relaxation versus sarcomere length relation
TR	Time intercept of linear relaxation as a function of sarcomere length relation

Guccione-Waldman-McCulloch and Hunter-Nash-Sands Model Card 1. This card is only defined if ACTYPE = 2.

Card 4.1b	1	2	3	4	5	6	7	8
Variable	T0	CA2ION	CA2IONM	N	TAUMAX	ST	B	L0
Type	F	F	F	F	F	F	F	F

Guccione-Waldman-McCulloch and Hunter-Nash-Sands Model Card 2. This card is only defined if ACTYPE = 2.

Card 4.2b	1	2	3	4	5	6	7	8
Variable	L	ETA						
Type	F	F						

VARIABLE	DESCRIPTION
T0	Starting time of active stress development
CA2ION	Intercellular calcium ion concentration
CA2IONM	Maximum intercellular calcium ion concentration
N	Hill coefficient
TAUMAX	Peak isometric tension under maximum activation
ST	Active fiber stress scaling factor in the transverse directions (see Remark 9)
B	Shape coefficient

VARIABLE	DESCRIPTION
L0	Sarcomere length with no active tension
L	Reference (stress-free) sarcomere length
ETA	Scaling parameter

Hunter-Nash-Sands Model Card 1. This card is only defined if ACTYPE = 3.

Card 4.1c	1	2	3	4	5	6	7	8
Variable	T0	CA2ION	CA2ION50	N	TAUMAX	ST	L	ETA
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
T0	Starting time of active stress development
CA2ION	Intercellular calcium ion concentration
CA2ION50	Intercellular calcium ion concentration at half of peak isometric tension
N	Hill coefficient
TAUMAX	Peak isometric tension under maximum activation
ST	Active fiber stress scaling factor in the transverse directions (see Remark 9)
L	Reference (stress-free) sarcomere length
ETA	Scaling parameter

Hunter-Nash-Sands and Hunter-McCullogh-ter Keurs Model Card A. This card is only defined if ACTYPE = 4.

Card 4.1d	1	2	3	4	5	6	7	8
Variable	T0	CA2ION50	CA2IONM	N	TAUMAX	ST	CA2ION0	TCA
Type	F	F	F	F	F	F	F	F

Hunter-Nash-Sands and Hunter-McCullogh-ter Keurs Model Card B. This card is only defined if ACTYPE = 4.

Card 4.2d	1	2	3	4	5	6	7	8
Variable	L	ETA						
Type	F	F						

VARIABLE	DESCRIPTION
T0	Starting time of active stress development
CA2ION50	Intercellular calcium ion concentration at half of peak isometric tension
CA2IONM	Maximum intercellular calcium ion concentration
N	Hill coefficient
TAUMAX	Peak isometric tension under maximum activation
ST	Active fiber stress scaling factor in the transverse directions (see Remark 9)
CA2ION0	Intercellular calcium ion concentration at rest
TCA	Shape coefficient
L	Reference (stress-free) sarcomere length
ETA	Scaling parameter

Martins-Pato-Pires Model Card A. This card is only defined if ACTYPE = 5.

Card 4.1e	1	2	3	4	5	6	7	8
Variable	FSEID	FLID	FVID	ALPHAID				
Type	I	I	I	I				

VARIABLE	DESCRIPTION
FSEID	Serial stress function ID (see Remark 10)
FLID	Normalized force-contractile stretch curve ID
FVID	Normalized force-contractile stretch rate curve ID
ALPHAID	Activation curve ID

Local Coordinate System Card A. These cards are only defined in combination with the ANISOtropic module.

Card 5	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	MACF	
Type	F	F	F	F	F	F	I	

Local Coordinate System Card B.

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	REF
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
XP, YP, ZP	Coordinates of point *P* for AOPT = 1 and 4
A1, A2, A3	Components of vector **a** for AOPT = 2
MACF	Material axes change flag for solid elements (see *MAT_002 for more details):

VARIABLE	DESCRIPTION
	EQ.-4: Switch material axes b and c before BETA rotation EQ.-3: Switch material axes a and c before BETA rotation EQ.-2: Switch material axes a and b before BETA rotation EQ.1: No change, default EQ.2: Switch material axes a and b after BETA rotation EQ.3: Switch material axes a and c after BETA rotation EQ.4: Switch material axes b and c after BETA rotation
V1, V2, V3	Components of vector **v** for AOPT = 3 and 4
D1, D2, D3	Components of vector **d** for AOPT = 2
BETA	Material angle in degrees for AOPT = 0 (shells and thick shells only) and AOPT = 3 (all element types). This angle may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, and *ELEMENT_SOLID_ORTHO.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY. EQ.0.0: Off EQ.1.0: On

Remarks:

1. **Volumetric strain energy function.** The pure volumetric part of the strain energy function is defined as part of the ISOtropic module.

2. **compressible and nearly incompressible models.** Depending on the sign of ITYPE and ATYPE, several formulations are available. Negative model numbers indicate that the corresponding part of the strain energy function is considered isochoric. Furthermore, the sign of FTYPE and INTYPE is directly linked to ATYPE. For example, if ATYPE is negative, both fiber and fiber interaction models are in their isochoric form. Consequently, compressible and nearly incompressible anisotropy is obtained by using both INTYPE and ATYPE with or without a sign, respectively.

3. **Incompressibility limit.** While there is no strict lower bound on the Poisson's ratio (v_L), for nearly incompressible materials, LS-DYNA will issue a warning message if $v < v_L = 0.49$.

4. **Special cases of the compressible/nearly incompressible Ogden model.** The following described special cases of the OGDEN Model (ITYPE = ±1).

   a) The (nearly-in)compressible neo-Hookean model is obtained as special case of ITYPE = (-)1 with $\mu_1 > 0, \alpha_1 = 1$.

   b) The Mooney-Rivlin model is obtained as a special case of ITYPE = -1 with $\mu_i \alpha_i > 0$ for *i* = *1,2*, $\alpha_1 = 2$, and $\alpha_2 = -2$.

   c) By setting $\beta = -1$ and ITYPE = -1, one obtains an equivalent formulation with *MAT_077_O.

5. **General structure tensor.** Considering the anisotropic part of the strain energy function, one may distinguish angular integration (AI) and general structure tensor (GST) based models. Owing to their numerical efficacy, currently, all models in LS-DYNA rely on the general structure tensor [7], [8]. Parameters defining the structure tensor and fiber models need to be provided for each fiber family. Consequently, one may use different structure tensors and/or fiber models to describe the behavior of the individual fiber families.

   The model proposed by Freed *et al.* [3] is a special case of the general structure tensor-based models assuming rotational symmetric fiber dispersion, determining the parameters A and B using a normal distribution, and invoking the fiber model introduced by Freed and Doehring [2].

6. **Fiber families.** Characteristic material directions within the plane are defined by fiber families. The number of fiber families for INTYPE = 0 is currently limited to 3. If INTYPE = 1, the number of fiber families is limited to 2, representing the fiber and sheet directions, respectively. The fiber and sheet directions form an orthonormal basis.

7. **Coupling (quasi-)invariants.** To further enhance the material model, coupling (quasi-) invariants associated with pairs of directions may be included in the strain-energy function. Following the formulation in Holzapfel and Ogden [7] and Eriksson *et al.* [1], a single coupling invariant defined between the orthonormal fiber and sheet directions is included with INTYPE = 1.

8. **Onset of active stress.** The input for this model gives several different methods for triggering the activation and deactivation of active stress development.

   a) For ACTYPE = 1, 2, 3, and 4, T0 specifies the time at which active stress development is activated. This method does not include deactivation. The other methods take priority over setting T0.

   b) For all ACTYPE options, you can specify ACID and ACTHR. ACID represents the evolution of either the calcium ion concentration (ACTYPE = 1, 2, 3, or 5) or the transmembrane potential (ACTYPE = 4) over time. ACTHR is a threshold value for one of these quantities depending on ACTYPE. When the calcium ion concentration or transmembrane potential from the curve exceeds the threshold, the active stress development is activated. When it is less than the threshold, the active stress development is deactivated. With this method, the active stress development can be reactivated again when the value in the curve exceeds the threshold.

c) For all ACTYPE methods, if you set up a coupled problem with the electrophysiology solver, ACTHR again gives the threshold value for the calcium ion concentration (ACTYPE = 1, 2, 3, or 5) or transmembrane potential (ACTYPE = 4). The electrophysiology solver provides the value to compare to the threshold to activate and deactivate the active stress development. As with ACID, the active stress development is activated when the value exceeds ACTHR and deactivates when the value is less than ACTHR. With this method, the active stress development can be reactivated again when the value exceeds the threshold.

9. **Active stress development.** Active stress is developed along direction(s) defined by ACDIR and may be scaled using the scaling factors SF, SS, and SN. For ACTYPE < 5, if SS and SN are zero, they are reset internally to ST.

Depending on ACDIR active stress may develop along one or multiple fiber families. Consider a single fiber family with unit fiber orientation vector $\mathbf{e}_f$. Let τ_A be the active stress. Then, the active stress tensor in the local fiber frame is:

$$\boldsymbol{\tau}_A = \tau_A\left(\mathrm{SF}\ \mathbf{e}_f \otimes \mathbf{e}_f + \mathrm{SS}\ \mathbf{e}_s \otimes \mathbf{e}_s + \mathrm{SN}\ \mathbf{e}_n \otimes \mathbf{e}_n\right) \ .$$

Here $\mathbf{e}_s$ and $\mathbf{e}_n$ are the unit vectors in the sheet and normal directions that form a basis with $\mathbf{e}_f$.

If active tension develops along multiple fiber families, then the active stress tensor is:

$$\boldsymbol{\tau}_A = \sum_{i=1}^{\mathrm{NF}} \boldsymbol{\tau}_{A_i} \ .$$

In the above NF is the number of fiber families and $\boldsymbol{\tau}_{A_i}$ is the active stress tensor for the i^{th} fiber family.

10. **Serial stress function.** The serial stress function needs to be expressed in terms of the fiber stretch λ and contractile stretch λ^{CE}. Thus, the elastic stretch in the serial element λ^{SE} needs to be eliminated using the multiplicative decomposition of the fiber stretch, that is, $\lambda = \lambda^{\mathrm{CE}}\lambda^{\mathrm{SE}}$.

11. **History variables.** The history variables are listed in the table below. The default number of history variables depends on the used modules. If only the

mandatory ISOtropic module is used, the number of history variables is 9. Including the ANISOtropic and ACTIVE modules in a hierarchical fashion yields an additional 12 and 9 history variables, that is, making the total number of history variables 21 and 30, respectively.

History Variable #	Definition
1-9	Deformation gradient (column-wise storage)
10-15	First two rows of the rotation matrix defining the material coordinate system **a-b-c**
16-18	Fiber stretch in each fiber family
19-21	Fiber stress in each fiber family
22-24	Active fiber stress in each fiber family
25	Calcium ion concentration at t^n if ACTYPE = 1,2,3,5) Transmembrane potential at t^n if ACTYPE = 4
26	Calcium ion concentration at t^{n-1} if ACTYPE = 1,2,3,5) Transmembrane potential at t^{n-1} if ACTYPE = 4
27	Time since onset of activation
28-30	Contractile stretch in the dashpot at t^n if ACTYPE = 5

References:

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*MAT_ANAND_VISCOPLASTICITY

This is Material Type 296. This visco-plastic model by Professor Anand uses a set of evolution equations instead of a loading-unloading criterion to describe dislocation motion and the hardening or softening behavior of materials. This model can be applied to simulate solders used in electronic packaging.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	YM	PR	ALPHA	A1	RATIOQR	XI
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	M	S0	H0	A2	SBAR	N		TREF
Type	F	F	F	F	F	F		F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
YM	Young's Modulus
PR	Poisson's ratio
ALPHA	Coefficient of thermal expansion, α
A1	Pre-exponential factor, A
RATIOQR	Ratio of the activation energy, Q (J/mol), to the universal gas constant, R (J/mol/K)
XI	Multiplier of stress, ξ
M	Strain rate sensitivity, m
S0	Initial value of deformation resistance, s_0

VARIABLE	DESCRIPTION
H0	Hardening/softening constant, H_0
A2	Strain rate sensitivity of hardening or softening, a
SBAR	Coefficient of deformation resistance saturation value, $\bar{s}$
N	Strain rate sensitivity of deformation resistance saturation value, n
TREF	Reference temperature, T_{ref}

Remarks:

In the Anand model, the equivalent stress σ is proportional to the deformation resistance s and depends upon the temperature T and the equivalent elastic strain $\dot{\varepsilon}^p$ as

$$\sigma = c(T, \dot{\varepsilon}^p)s \ ,$$

where c is a material parameter defined by

$$c = \frac{1}{\xi}\sinh^{-1}\left[\left(\frac{\dot{\varepsilon}^p}{A}\exp(\frac{Q}{RT})\right)^m\right] \ .$$

The material parameter c depends on material constants defined in the variable list above and the universal gas constant R.

The above equations can be rearranged to express the equivalent inelastic strain rate $\dot{\varepsilon}^p$ in terms σ, T, and s as

$$\dot{\varepsilon}^p = A\exp\left(-\frac{Q}{RT}\right)\left[\sinh(\xi\frac{\sigma}{s})\right]^{1/m} \ .$$

This is called the flow equation.

The rate of deformation resistance $\dot{s}$ is defined as

$$\dot{s} = H\dot{\varepsilon}^p \ ,$$

where

$$H = H_0\left|1 - \frac{s}{s_s}\right|^a sign\left(1 - \frac{s}{s_s}\right) .$$

In the above equation, s_s is the deformation resistance saturation value which is defined as

$$s_s = \bar{s}\left[\frac{\dot{\varepsilon}^p}{A}\exp(\frac{Q}{RT})\right]^n \ .$$

With the equivalent inelastic strain rate $\dot{\varepsilon}^p$, the inelastic strain components can be computed based on a normality hypothesis of the Prandtl-Reuss flow law:

$$\dot{\boldsymbol{\varepsilon}}^p = \sqrt{\frac{3}{2}}\dot{\varepsilon}^p\mathbf{N} \ .$$

In the above equation, the direction of plastic flow $\mathbf{N}$ is defined as

$$\mathbf{N} = \sqrt{\frac{3}{2}}\frac{\mathbf{S}}{\sigma} \ ,$$

where $\mathbf{S}$ is the deviatoric part of the stress $\boldsymbol{\sigma}$.

The Cauchy Stress $\mathbf{T}$ for this model is

$$\mathbf{T} = J^{e-1}\mathbf{R}^e\mathbf{M}^e\mathbf{R}^{e^T} \ ,$$

where

$$\mathbf{M}^e = \mathbf{C}[\mathbf{E}^e - \alpha(T - T_0)] \ .$$

T_0 is the initial temperature. $\mathbf{C}$ is defined as

$$\mathbf{C} \stackrel{\text{def}}{=} 2G\left(\mathbb{I}^s - \frac{1}{3}\mathbf{I}\otimes\mathbf{I}\right) + K\mathbf{I}\otimes\mathbf{I} \ .$$

References:

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*MAT_DMN_COMPOSITE_FRC

This is Material Type 303. It is a machine-learning-based multiscale material model for analysis of injection-molded fiber-reinforced composites (FRC). The multiscale material model can predict the macroscopic material responses (stress, equivalent plastic strain, etc.) based on the microscopic material information. Using this material model requires providing the geometric descriptors for material microstructures (i.e., fiber orientation tensor, fiber volume fraction, etc.) and the material properties of each base material (i.e., fiber and matrix), respectively. Only certain constitutive laws described in Remarks 2 and 3 are supported for the base materials. The FIBAND (FIBer-distribution-based Anisotropic Damage) model is available for predicting the failure of injection-molded composites with arbitrary fiber distributions.

To obtain the geometrical information for the microstructures, use an injection molding simulation software, such as Moldex3D. LS-PrePost can import the injection molding results into LS-DYNA models (see Remark 5 and Workflow to import fiber data from Moldex3D).

This model is available in R14 or newer versions of MPP/SMP double precision LS-DYNA. Currently, this 3D multiscale material model supports explicit dynamic finite element analysis using eight-node hexahedron solid elements, four-node tetrahedron solid elements, and type 25 four-node shell elements.

Card Summary:

Card 1. This card is required.

MID							

Card 2. This card is required.

FVF	RO	RF	RM	FL	FD		

Card 3. This card is required.

F_E	F_PR	ISO	DAM				

Card 3.1. Include this card if ISO = 1 in Card 3.

F_EL	F_ET	F_PRTL	F_PRTT	F_GLT			

Card 4. This card is required.

M_E	M_PR	M_SY	M_H1	M_H2	M_H3	ITC	

Card 4.1. Include this card if ITC = 1 or 3 in Card 4.

M_EC	M_PRC	M_SYC	M_H1C	M_H2C	M_H3C	PT	PC

Card 5. Include this card if DAM = 1 on Card 3.

D_C				D_ERO			

Card 6. This card is required.

LCIDT	LCIDC	LCFS	LCFA	LCSRS	LCSRA		

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID							
Type	A							
Default	none							

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).

Card 2	1	2	3	4	5	6	7	8
Variable	FVF	RO	RF	RM	FL	FD		
Type	F	F	F	F	F	F		
Default	none	none	none	none	none	none		

VARIABLE	DESCRIPTION
FVF	φ^f, fiber volume fraction. If RF and RM are given, a nonzero FVF must be specified to calculate the mass density of the overall fiber-reinforced composite. This value can be overwritten by *INI-

VARIABLE	DESCRIPTION
	TIAL_STRESS_SHELL or *INITIAL_STRESS_SOLID. See Remark 4.
RO	ρ^c, mass density of the overall fiber-reinforced composite. This value will be neglected if RF and RM are given, respectively.
RF	ρ^f, mass density of the fiber phase
RM	ρ^m, mass density of the matrix phase
FL	Fiber length. Alternatively, if you want to specify the fiber aspect ratio, set FL to the fiber aspect ratio and FD to 1.0.
FD	Fiber diameter. Alternatively, if you want to directly specify the fiber aspect ratio, set FD to 1.0 and FL to the fiber aspect ratio.

Card 3	1	2	3	4	5	6	7	8
Variable	F_E	F_PR	ISO	DAM				
Type	F	F	I	I				
Default	none	none	0	0				

VARIABLE	DESCRIPTION
F_E	E^f, Young's modulus of the fiber phase if the fiber property is isotropic. See Remark 2.
F_PR	v_{tl}^f, Poisson's ratio of the fiber phase if the fiber property is isotropic.
ISO	Flag for anisotropy of the fiber phase: EQ.0: Isotropic fiber material property EQ.1: Transversely isotropic fiber material property
DAM	Flag for the composite failure model: EQ.0: Do not consider material damage. EQ.1: Use the FIBAND model: See Remark 4.

Transversely Isotropic Fiber Material Card. Include this card if ISO = 1 on Card 3.

Card 3.1	1	2	3	4	5	6	7	8
Variable	F_EL	F_ET	F_PRTL	F_PRTT	F_GLT			
Type	F	F	F	F	F			
Default	none	none	none	none	none			

VARIABLE	DESCRIPTION
F_EL	E_l^f, Young's modulus of the fiber phase along the fiber's longitudinal direction, l. See Remark 2.
F_ET	E_t^f, Young's modulus of the fiber phase along the fiber's transverse direction t. Note that the transversely isotropic model becomes isotropic by setting $E_t^f = E_l^f$, $v_{tt}^f = v_{tl}^f$, and $G_{lt}^f = E_l^f / \left[2\left(1+v_{tl}^f\right)\right]$.
F_PRTL	v_{tl}^f, Poisson's ratio of the fiber phase
F_PRTT	v_{tt}^f, Poisson's ratio of the fiber phase. Note that the transversely isotropic model becomes isotropic by setting $E_t^f = E_l^f$, $v_{tt}^f = v_{tl}^f$, and $G_{lt}^f = E_l^f / \left[2\left(1+v_{tl}^f\right)\right]$.
F_GLT	G_{lt}^f, shear modulus of the fiber phase in the lt direction. Note that the transversely isotropic model becomes isotropic by setting $E_t^f = E_l^f$, $v_{tt}^f = v_{tl}^f$, and $G_{lt}^f = E_l^f / \left[2\left(1+v_{tl}^f\right)\right]$.

Card 4	1	2	3	4	5	6	7	8
Variable	M_E	M_PR	M_S1	M_S2	M_S3	M_S4	ITC	
Type	F	F	F	F	F	F	I	
Default	none	none	none	none	none	none	0	

VARIABLE	DESCRIPTION
M_E	E^m, Young's modulus of the matrix phase. See Remark 3.
M_PR	υ^m, Poisson's ratio of the matrix phase
M_S1	s_1^m, plastic yielding parameter of the matrix phase
M_S2	s_2^m, plastic yielding parameter of the matrix phase
M_S3	s_3^m, plastic yielding parameter of the matrix phase
M_S4	h_0^m, plastic yielding parameter of the matrix phase
ITC	Option for the elastoplastic material law for the matrix phase. EQ.0: No tension-compression asymmetry in material properties EQ.1: Use tension-compression asymmetric material properties EQ.2: Use a viscoplastic formulation to account for strain rate effects, where a table can define the yield strength as a function of the equivalent plastic strain for various strain rates EQ.3: Use tension-compression asymmetric material properties in a viscoplastic formulation to account for strain rate effects

Tension-Compression Asymmetry Card. Include this card if ITC = 1 or 3.

Card 4.1	1	2	3	4	5	6	7	8
Variable	M_EC	M_PRC	M_S1C	M_S2C	M_S3C	M_S4C	PT	PC
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	0.0	0.0

VARIABLE	DESCRIPTION
M_EC	E^m, Young's modulus of the matrix phase in compression
M_PRC	υ^m, Poisson's ratio of the matrix phase in compression
M_S1C	s_1^m, plastic yielding parameter of the matrix phase in compression

VARIABLE	DESCRIPTION
M_S2C	s_2^m, plastic yielding parameter of the matrix phase in compression
M_S3C	s_3^m, plastic yielding parameter of the matrix phase in compression
M_S4C	h_0^m, plastic yielding parameter of the matrix phase in compression
PT	Absolute value of the tensile mean stress threshold beyond which the tensile material properties are adopted. If the mean stress $(\sigma_{XX} + \sigma_{YY} + \sigma_{ZZ})/3$ falls within the range $[-\mathrm{PC}, \mathrm{PT}]$, a weighted average of the tensile and compressive material properties is used for the matrix phase. See Remark 3.
PC	Absolute value of the compressive mean stress threshold beyond which compressive material properties are adopted.

Material Failure Card. Include this card if DAM = 1 on Card 3.

Card 5	1	2	3	4	5	6	7	8
Variable	D_C				D_ERO			
Type	F				F			
Default	none				optional			

VARIABLE	DESCRIPTION
D_C1	Critical damage threshold value
D_ERO	Maximum damage value beyond which the element is eroded. Element erosion does not occur if this value is left empty.

Card 6	1	2	3	4	5	6	7	8
Variable	LCIDT	LCIDC	LCFS	LCFA	LCSRS	LCSRA		
Type	I	I	I	I	I	I		
Default	none	none	none	none	0	0		

VARIABLE	DESCRIPTION
LCIDT	Load curve or table ID. The load curve is available for ITC = 0 and 1 while the table is available for ITC = 2 and 3. **Load Curve.** When LCIDT is a load curve ID, data points representing the accumulated equivalent plastic strain and the corresponding yield strength for the matrix phase are respectively given in the first column and the second column of the corresponding load curve in *DEFINE_CURVE. If ITC = 1 is specified in Card 4, this load curve describes the matrix material in tension only. See Remark 3. **Tabular Data.** If ITC = 2 or 3 is specified in Card 4, LCIDT is treated as a table ID. Data points representing different strain rates are given in one column of the corresponding table in *DEFINE_TABLE, followed by the definitions of load curves for the yield strength for the matrix phase as a function of effective plastic strain at each given strain rate value. See *DEFINE_TABLE. Linear interpolation of the yield strengths at different given strain rates is used by default. If the strain rate values fall out of range, extrapolation is not used; instead, either the first or last curve determines the yield strength as a function of effective plastic strain, which depends on whether the strain rate falls below the minimum given value or exceeds the maximum given value, respectively. If ITC = 3 is specified in Card 4, this table describes the matrix material in tension only. **Logarithmically-Defined Tables.** If ITC = 2 or 3 is specified in Card 4, LCIDT refers to a table ID. In addition, if the first value in the table is negative, all data points in the table represent the natural logarithm of strain rates, and logarithmic interpolation of the yield strengths at discrete given strain rates is used. Note that this option works only when the lowest strain rate has a value less than 1.0. For values greater than or equal to 1.0, use the LOG_INTERPOLATION option.
LCIDC	ID for a load curve or table. Similar to LCIDT, LCIDC is the ID of a load curve or table that contains the accumulated equivalent plastic strain and the yield strength for the matrix phase in compression. This parameter is available for ITC = 1 (load curve) and 3 (table). The description for the load curve and table is the same as for LCIDT but for compression. See Remark 3.
LCFS	ID of a load curve giving the equivalent failure strain, $\bar{\varepsilon}^F$, (ordinate) as a function of the stress triaxiality state (abscissa). This curve should only be defined when DAM = 1.

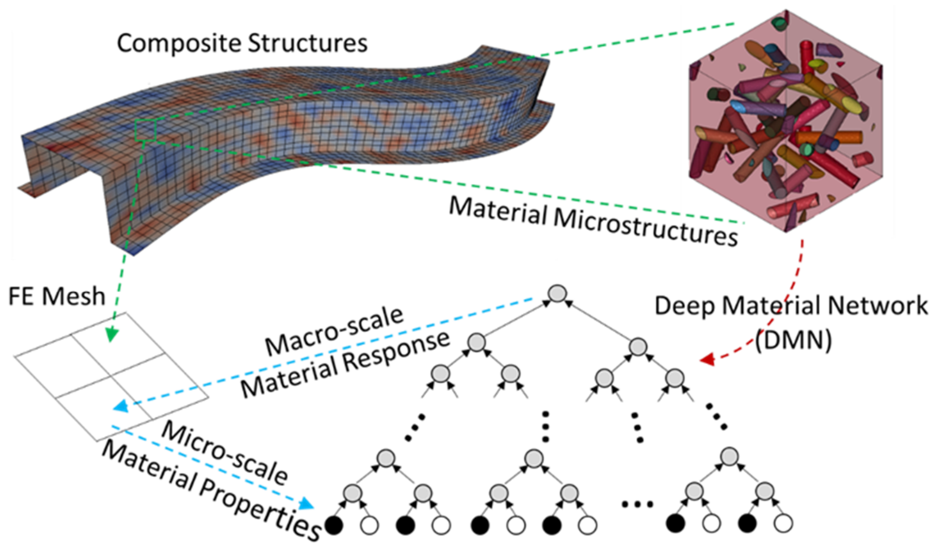

Figure M303-1. Schematic of the simulation framework for concurrent multiscale nonlinear analysis of injection-molded fiber-reinforced composite structures.

VARIABLE	DESCRIPTION
LCFA	ID of a load curve giving the failure anisotropy factor, $\bar{\beta}^F$, (ordinate) as a function of the stress triaxiality state (abscissa). $\bar{\beta}^F$ falls within the range [0,1]. This curve should only be defined when DAM = 1.
LCSRS	ID of a load curve giving scaling factor SRS (ordinate) as a a function of strain rate (abscissa). The factor SRS scales the failure strain, $\bar{\varepsilon}^F$, defined with LCFS. It is optional.
LCSRA	ID of a load curve giving scaling factor SRA (ordinate) as a a function of strain rate (abscissa). The factor SRA scales the failure anisotropy factor, $\bar{\beta}^F$, defined with LCFA. It is optional.

Remarks:

1. **Mechanistic machine learning-based multiscale simulation.** The core algorithm of this multiscale simulation is the DMN (Deep Material Network) based upon a mechanistic machine learning technique [1, 2, 3, 4, 5, 6]. As described in [4, 5, 6], the machine learning model creation requires an "offline" training process, which involves the learning of composite material physics hidden in the high-fidelity RVE simulation-based data. To cover a wide variety of material microstructures of fiber-reinforced composites, we adopted a transfer learning method [3, 4, 6] to generate different networks based on the actual material

microstructure information at each integration point of the finite element mesh. In LS-DYNA, we have implemented a trained DMN database. It can effectively predict the highly nonlinear macroscopic material behaviors. The computational cost of DMN is orders-of-magnitude lower than finite element simulation of high-fidelity 3D RVEs containing complex material microstructures. The overall concurrent multiscale simulation framework enabled by DMN is depicted in Figure M303-1.

Different from conventional material models, this machine learning-based multiscale material model is data-driven, so its simulation capability can be continuously enhanced as more high-quality training data for fiber-reinforced composites are supplied in the future. Enhanced DMN databases with new functions will be available in the future release of LS-DYNA.

2. **Constitutive laws for fiber materials.** By default, the constitutive behaviors of the fiber material are modeled with isotropic elasticity. If you set ISO to 1 on Card 3, then the fiber material is modeled with a transversely isotropic elasticity. Its symmetric compliance matrix takes the following form:

$$\begin{bmatrix} 1/E_l^f & -v_{tl}^f/E_t^f & -v_{tl}^f/E_t^f & 0 & 0 & 0 \\ -v_{tl}^f/E_t^f & 1/E_t^f & -v_{tt}^f/E_t^f & 0 & 0 & 0 \\ -v_{tl}^f/E_t^f & -v_{tt}^f/E_t^f & 1/E_t^f & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{lt}^f & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\left(1+v_{tl}^f\right)/E_l^f & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{lt}^f \end{bmatrix}$$

Here E_l^f and E_t^f are the Young's moduli along the fiber's longitudinal (l) and transverse (t) directions, respectively; $v_{tl}^f$and v_{tt}^f are the Poisson's ratios; and G_{lt}^f is the shear modulus. If the five material parameters given in Card 3.1 satisfy $E_l^f = E_t^f$, $v_{tl}^f = v_{tt}^f$, and $G_{lt}^f = E_l^f/\left[2\left(1+v_{tl}^f\right)\right]$, then the elastic fiber's material model becomes isotropic which is the same as simply providing the two material constants in Card 3.

3. **Constitutive laws for matrix materials.** The matrix materials are modeled with an associated elastoplastic constitutive model. If ITC = 0 in Card 4, then the material properties given in Card 4 and/or LCIDT in Card 6 will be used for the matrix material, and Card 4.1 should not appear in the input file. However, if ITC = 1 or 3 is specified in Card 4, the model for matrix material considers tension-compression asymmetry. In this case, Card 4 and/or LCIDT specify tensile material properties while Card 4.1 and/or LCIDC specify compressive material properties. By default, the sign of the mean stress $(\sigma_{XX} + \sigma_{YY} + \sigma_{ZZ})/3$ determines tension with a positive sign indicating that the material is in tension. Numerically, an abrupt transition from the tensile and compressive yield surfaces

may cause convergence difficulty. To avoid this numerical issue, we can assign small positive numbers (e.g., a small percentage of the yield strength) to PT and PC in Card 4.1 which define a mean stress range [−PC, PT] for which weighted averaged values of the tensile and compressive properties are used in the simulation.

Three von Mises yield functions with different hardening laws are available, including:

a) A yield function based on the following isotropic hardening law:

$$s_Y^m = s_1^m + s_2^m \bar{\varepsilon}_P^m - s_3^m \exp(-h_0^m \bar{\varepsilon}_P^m)$$

where s_Y^m denotes the current yield strength for the matrix phase, $\bar{\varepsilon}_P^m$ denotes the accumulated equivalent plastic strain of the matrix material, and plastic yielding parameters h_0^m, s_1^m, s_2^m, and s_3^m are defined in Card 4 or Card 4.1. If $s_3^m = 0$, the yield function becomes equivalent to a linear hardening law with a hardening coefficient s_2^m an initial yield strength s_1^m. Otherwise, $s_1^m - s_3^m$ represents the initial yield strength.

b) A yield function based on an input hardening curve. The curve ID is given as LCIDT or LCIDC in Card 6, where the curve data's first and second columns represent, respectively, the accumulated equivalent plastic strain, $\bar{\varepsilon}_P^m$, and the corresponding yield strength, s_Y^m.

c) A yield function based on an input hardening table. The table ID is given as LCIDT or LCIDC in Card 6, where the associated curves provide the accumulated equivalent plastic strain $\bar{\varepsilon}_P^m$ and the corresponding yield strength s_Y^m at different strain rates.

If a curve or table is provided in Card 6, the corresponding yield strength and hardening parameters defined in Card 4 or 4.1 are ignored.

Based on feedback and requests, we will develop other constitutive laws for the base materials to capture more complex behaviors of composites, such as material failures.

4. **FIBAND model for short-fiber-reinforced composites.** The FIBer-distribution-based ANisotropic Damage (FIBAND) model predicts the stress triaxiality and strain-rate-dependent anisotropic failures of injection-molded fiber-reinforced composites. DAM = 1 on Card 3 enables this model.

For unidirectionally aligned short-fiber-reinforced composites (UD-SFRC), larger values of the equivalent failure strain, $\bar{\varepsilon}^F$, correspond to a higher failure strength, and larger values of the factor, $\bar{\beta}^F$, correspond to higher degrees of anisotropy in the failure strength. These two parameters are defined as functions of the stress triaxiality state through load curves LCFS and LCFA in Card 6. Note that $\bar{\beta}^F$ falls within the range [0,1]. The factors defined by LCSRS and LCSRA can scale $\bar{\varepsilon}^F$ and $\bar{\beta}^F$, respectively. In addition, the critical damage threshold value, D_C on Card 5, falls within the range [0,1]. Higher values correspond to faster damage growth rates that are suitable for capturing more brittle failures.

For injection-molded composites with more complex fiber orientation states, FIBAND models the anisotropic failure of the composite by representing its microstructure as an aggregate of UD-SFRC unit cells with various orientations. The stress triaxiality and strain-rate-dependent damage initiation and growth are predicted based on the parameters in Cards 5 and 6 in conjunction with the local fiber distribution state. The local fiber distribution state is parametrized by the second-order fiber orientation tensor and the fiber volume fraction, which are mapped from injection molding data (see Remark 5 and Workflow to import fiber data from Moldex3D).

5. **Heterogeneous distributions of fiber orientations and fiber volume fractions.** Due to the manufacturing process, fiber-reinforced composites contain heterogeneous distributions of material microstructures, such as different fiber orientations, fiber volume fractions, and thermally/chemically-induced residual stresses at different locations of the composite structure. This information can be obtained from either experimental measurements or injection molding simulation software packages. If these microstructure data are available, they can be used as initial conditions in LS-DYNA by defining the keyword *INITIAL_STRESS_SOLID or *INITIAL_STRESS_SHELL, depending on the finite element formulations adopted in the macroscale numerical model.

   a) If solid finite elements (e.g., eight-node hexahedron or four-node tetrahedron elements) are used, the *INITIAL_STRESS_SOLID keyword can be used to define the fiber information. As shown in the following example, 6 history variables initialize the components of the symmetric fiber orientation tensor, $(A_{XX})_e^p$, $(A_{YY})_e^p$, $(A_{XY})_e^p$, $(A_{YZ})_e^p$, $(A_{XZ})_e^p$, and the fiber volume fraction, $(fvf)_e^p$, at each integration point of the finite element mesh. The subscript $e = 1, 2, 3, \ldots$ denotes the finite element index, and the superscript $p = 1, 2, 3, \ldots$ denotes the integration point index. Note that, the component $(A_{ZZ})_e^p$ of the fiber orientation tensor is not provided in this keyword because it can be easily calculated based on its relationship with the $(A_{XX})_e^p$ and $(A_{YY})_e^p$ components, i.e., $(A_{ZZ})_e^p = 1.0 - (A_{YY})_e^p - (A_{XX})_e^p$. Starting with R15, *MAT_303 offers an effective method of capturing the effects of the manufacturing-process-induced residual stress field on the mechanical performance. If the residual stress effects need to be considered, *INITIAL_STRESS_SOLID can provide the six residual stress components at each integration point. See *INITIAL_STRESS_SOLID for details.

```
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$  Define Fiber Information in *INITIAL_STRESS_SOLID
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
*KEYWORD
$
$...>....1....>....2....>....3....>....4....>....5....>....6....>....7....>....8
*INITIAL_STRESS_SOLID
$      EID      NINT     NHISV     LARGE
         1         1         6         1
$.....>.......1.......>.......2.......>.......3.......>.......4.......>.......5
$          SIGXX           SIGYY           SIGZZ           SIGXY           SIGYZ
             0.0             0.0             0.0             0.0             0.0
$          SIGZX             EPS           HISV1           HISV2           HISV3
             0.0             0.0       (A_XX)_1^1      (A_YY)_1^1      (A_XY)_1^1
$          HISV4           HISV5           HISV6
      (A_YZ)_1^1      (A_XZ)_1^1       (fvf)_1^1

…

$      EID      NINT     NHISV     LARGE
         e         1         6         1
$          SIGXX           SIGYY           SIGZZ           SIGXY           SIGYZ
             0.0             0.0             0.0             0.0             0.0
$          SIGZX             EPS           HISV1           HISV2           HISV3
             0.0             0.0       (A_XX)_e^1      (A_YY)_e^1      (A_XY)_e^1
$          HISV4           HISV5           HISV6
      (A_YZ)_e^1      (A_XZ)_e^1       (fvf)_e^1
…

$
*END
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
```

b) If shell finite elements (shell formulation 25 that supports the use of 3D constitutive laws) are used, the *INITIAL_STRESS_SHELL keyword can be used to define the fiber information. If we assume that the one-point quadrature scheme is adopted in the in-plane direction while 3 integration points exist along the shell thickness direction (note: you can define the number of through-thickness integration points), then we need to define 6 history variables at each of the 3 integration points of every shell finite element. These history variables include the components of the symmetric fiber orientation tensor, $(A_{XX})_e^p$, $(A_{YY})_e^p$, $(A_{XY})_e^p$, $(A_{YZ})_e^p$, $(A_{XZ})_e^p$, and the fiber volume fraction, $(fvf)_e^p$. The subscript $e = 1, 2, 3, \ldots$ denotes the finite element index, and the superscript $p = 1, 2, 3, \ldots$ denotes the integration point index. Please refer to the following example. Starting with R15, *MAT_303 offers an effective method of capturing the effects of the manufacturing-process-induced residual stress field on the mechanical performance. If the residual stress effects need to be considered, *INITIAL_STRESS_SHELL can provide the six residual stress components at each integration point. See *INITIAL_STRESS_SHELL for details.

```
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$  Define Fiber Information in *INITIAL_STRESS_SHELL
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
*KEYWORD
$
$...>....1....>....2....>....3....>....4....>....5....>....6....>....7....>....8
*INITIAL_STRESS_SHELL
$      EID    NPLANE    NTHICK     NHISV    NTENSR     LARGE
         e         1         3         6         0         1
$.......>.......1.......>.......2.......>.......3.......>.......4.......>.......5
$              T           SIGXX           SIGYY           SIGZZ           SIGXY
        -0.77459             0.0             0.0             0.0             0.0
$          SIGYZ           SIGZX             EPS
             0.0             0.0             0.0
$          HISV1           HISV2           HISV3           HISV4           HISV5
    $(A_{XX})_e^1$  $(A_{YY})_e^1$  $(A_{XY})_e^1$  $(A_{YZ})_e^1$  $(A_{XZ})_e^1$
$          HISV6
       $(fvf)_e^1$
$              T           SIGXX           SIGYY           SIGZZ           SIGXY
         0.00000             0.0             0.0             0.0             0.0
$          SIGYZ           SIGZX             EPS
             0.0             0.0             0.0
$          HISV1           HISV2           HISV3           HISV4           HISV5
    $(A_{XX})_e^2$  $(A_{YY})_e^2$  $(A_{XY})_e^2$  $(A_{YZ})_e^2$  $(A_{XZ})_e^2$
$          HISV6
       $(fvf)_e^2$
$              T           SIGXX           SIGYY           SIGZZ           SIGXY
         0.77459             0.0             0.0             0.0             0.0
$          SIGYZ           SIGZX             EPS
             0.0             0.0             0.0
$          HISV1           HISV2           HISV3           HISV4           HISV5
    $(A_{XX})_e^3$  $(A_{YY})_e^3$  $(A_{XY})_e^3$  $(A_{YZ})_e^3$  $(A_{XZ})_e^3$
$          HISV6
       $(fvf)_e^3$
…

$
*END
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
```

Workflow to import fiber data from Moldex3D:

LS-PrePost 4.11 or newer versions support the mapping of Moldex3D injection molding simulation results (fiber orientation tensor A_{ij} and the fiber concentration c) onto LS-DYNA mechanical models. The mapped data will be exported as material history state variables in the initial stress keyword cards *INITIAL_STRESS_SHELL or *INITIAL_STRESS_SOLID (see Remark 5). In addition, if warpage data are incorporated in the mapping, a new mesh considering the initial warpage deformation will be created automatically. These mapped data will be used by LS-DYNA when the multiscale material model for fiber-reinforced composites, *MAT_DMN_COMPOSITE_FRC, is used in the finite element analysis. For this discussion, Moldex3D provides the data for the "source" parts which are mapped by LS-PrePost onto the "target" LS-DYNA parts.

LS-PrePost provides an advanced option and a basic option for data mapping. We recommend the advanced option because it is more efficient and offers more functionalities (such as considering molding-induced warpage and residual stress effects) than the basic option. Depending on which LS-PrePost mapping option is chosen, the injection molding simulation result data can be exported in different file formats from Moldex3D:

1. The basic option requires exporting a .k format file for the mesh data (element connectivity and nodal coordinates), a .o2d file for the fiber orientation tensor at each element, and a .fcd file for the fiber concentration at each node from Moldex3D as follows:

   a) Click *Results* in the top toolbar of the Moldex3D software, and select *FEA interface*.

   b) In *FEA Interface Function Option*, choose *LS-Dyna* as the stress solver, and then select *fiber concentration output* and *fiber orientation output*. Next, click *Export* to create the .k, .o2d, and .fcd files, which are located in the project folder.

   Note that the above procedure creates two .k files (LS-DYNA format mesh files) simultaneously, but the one that treats each element as a part should be discarded. The other .k file contains the finite element mesh information and will be used in LS-PrePost.

2. If the LS-PrePost advanced option is chosen, a similar procedure can be followed, but only a single .mdx format file needs to be exported. The .k, .o2d, and .fcd files are no longer required from Moldex3D.

In both cases, to map the data from these Moldex3D files to an LS-DYNA finite element model, click *Application* in the top toolbar of LS-PrePost, select *Plastics Analysis*, and click *Import Data from Moldex3D*. A Moldex3D dialog box will appear, as shown in Figure M303-2.

To map the information from these Moldex3D files (source) to an LS-DYNA finite element model (target), perform the following procedure in LS-PrePost:

1. **Import files.**

   a) Click the *Browse* button under *Target Mesh* and select the .k file for the LS-DYNA mechanical model. The .k file must contain the element connectivity and nodal coordinates for the target parts to be used in the LS-DYNA structural analysis.

   b) If using the advanced option, click the *Browse* button for *Moldex3D Data* and select the .mdx file exported from Moldex3D. The .mdx file contains the

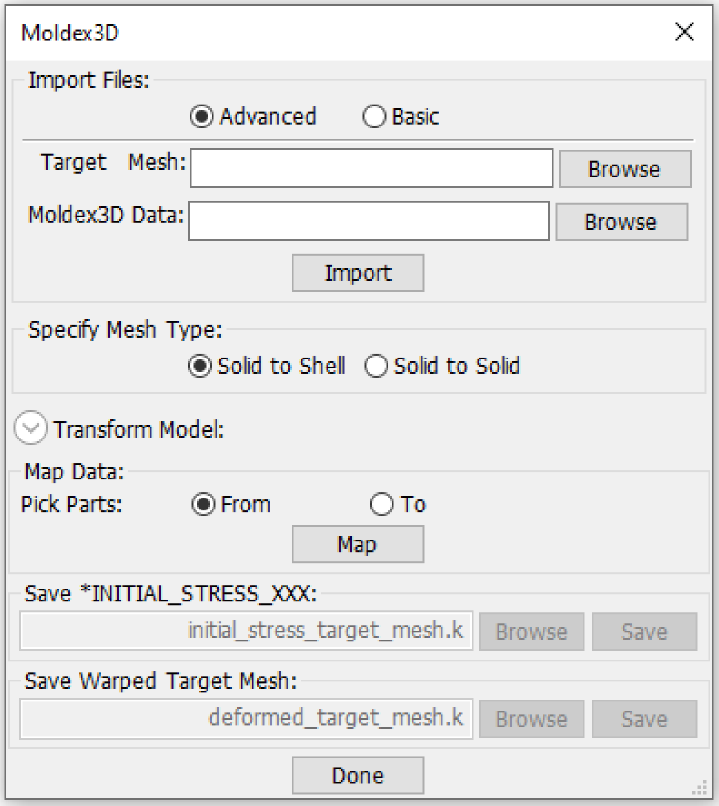

Figure M303-2. Moldex3D mapping GUI in LS-PrePost 4.11 (available in January 2024). Steps 1 through 5 below use this GUI.

mesh, fiber orientation tensor distribution, fiber concentration (i.e., fiber volume fraction × 100), manufacturing process-induced residual stress, and warpage data for the source parts that are used in the Moldex3D injection molding analysis.

If using the basic option, the following steps import the Moldex3D files:

i) Click the *Browse* button under *Moldex3D Mesh* and select the .k file exported from Moldex3D. The .k file must contain the element connectivity and nodal coordinates for the source parts that are used in the injection molding analysis in Moldex3D.

ii) Click the *Browse* button under *Moldex3D O2D* and select the .o2d file exported from Moldex3D. The .o2d file contains the fiber orientation tensor distribution in the source part.

iii) Click the *Browse* button under *Moldex3D FCD* and select the .fcd file exported from Moldex3D. The .fcd file contains the fiber concentration (i.e., fiber volume fraction × 100) distribution in the source part. If the .fcd file is not available, this step can be skipped, and the fiber volume fraction distribution is considered homogeneous in the target part based on the parameter FVF given in *MAT_-DMN_COMPOSITE_FRC.

c) Click the button *Import* so the source and target models will be imported and visualized in the same global coordinate system.

2. **Specify the mesh type.** Choose *Solid to Shell* if the target part is discretized by shell finite elements or *Solid to Solid* if the target part is discretized by solid finite elements.

   Note that, in the current LS-PrePost mapping function, only solid elements can be used in the source parts from Moldex3D.

   For *Solid to Shell*, LS-PrePost automatically identifies the number of through-thickness integration points defined in the keyword card *SECTION_SHELL for shell finite elements in the target part. If the number of through-thickness integration points is not defined for the target part, LS-PrePost will use two through-thickness integration points in the solid-to-shell mapping.

3. **Transform the model.** If the source part and the target part are oriented in the same direction in the global coordinate system, this step should be skipped. Otherwise, a rigid body rotation needs to be performed to rotate the source part into the same direction as the target part.

   To perform this transformation, click *Transform Model*, and pick three nodes (S1, S2, S3) in the source model that define a plane (plane A), and then pick three nodes (T1, T2, T3) in the target model that define a plane (plane B). LS-PrePost automatically applies a 3D rigid body rotation that transforms plane A to plane B when it maps the Moldex3D data from the source part to the target part in subsequent steps.

   After the transformation, nodes S1 and T1 will have identical global coordinates. While the coordinate of node S2 can be different from node T2 after the transformation, the line S1-S2 (i.e., the straight-line connecting node S1 and node S2) must be parallel to the straight line T1-T2 after the transformation. Similarly, the transformed coordinates of nodes S3 and T3 can be different, but the line S1-S3 must be parallel to the line T1-T3 after the transformation.

4. **Map the data.** To map the data:

   a) Click *From* in *Pick Parts* and select the source part from the Moldex3D source model.

   b) Click *To* in *Pick Parts* and select the target part from the LS-DYNA model.

   c) Click the *Map* button to map the data from the source part to the target part.

5. **Save the mapped results.**

   a) Under *Save *INITIAL_STRESS_XXX*, click *Browse* to specify the path and filename for a new keyword file, and then click *Save*. These steps create a new keyword file containing the cards *INITIAL_STRESS_SHELL or *INITIAL_STRESS_SOLID, which should be included in the LS-DYNA finite element analysis.

   b) Under *Save Warped Target Mesh*, click *Browse* to specify the path and filename for a new keyword file, and then click *Save*. These steps produce a new keyword file containing the warped target mesh. This file should be included in the LS-DYNA finite element analysis. Note that this step is only necessary when the Moldex3D .mdx file contains warpage data.

   c) Click *Done* to exit the Moldex3D mapping function.

6. **Visualization.** To visualize the data, click *Post*, choose *Fringe Component*, and select either the source or target part to view contour plots of the original or mapped data, respectively. To view residual stress and shell thickness data, use *Dynain* in the *Fringe Component* GUI. Fiber orientation and volume fraction data can be selected in *History* in the *Fringe Component* GUI. To visualize warpage displacement data, use *User* in the *Fringe Component* GUI.

References:

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[2] Liu, Z. and C.T. Wu, "Exploring the 3D architectures of deep material network in data-driven multiscale mechanics," *Journal of the Mechanics and Physics of Solids*, Vol. 127, pp. 20-46, (2019).

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[4] Liu, Z., H. Wei, T. Huang, and C.T. Wu, "Intelligent multiscale simulation based on process-guided composite database," (2020). https://arxiv.org/abs/2003.09491

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Modeling of Short Fiber–Reinforced Composites." Journal of Engineering Mechanics, 149(3), 04023003, (2023).

[7] Wei, H., Hu, W., Wu, C. T., Pavia, F., "AI-empowered LS-DYNA ICME simulation technique for multiscale predictive modeling of composites." American Society for Composites (ASC_ 39th Annual Technical Conference & US-Japan Joint Symposium, October 21-23, 2024, San Diego, California, USA.

*MAT_HOT_PLATE_ROLLING

This is Material Type 305. This model is for hot rolling of steel. It can only be used with solid elements for explicit simulation. The model contains the following features: work hardening, dynamic softening, static recovery, and static recrystallization. Input parameters are calibrated from Gleeble tests at various deformation rates and temperatures; see Schill et. al. [2021] and references therein.

Card Summary:

Card 1. This card is required.

MID	RO	E	PR	ALPHAT	BETA	VP	TOL

Card 2. This card is required.

YB	QDEF	R	A	B	MINRT	POST	ODESOL

Card 3. This card is required.

ASIG0	BSIG0	ASIGS	BSIGS	ASIGSS	BSIGSS	AEPS	BEPS

Card 4. This card is required.

THRES	M	ALPHA	NUD	U0	K	NU	BNU

Card 5. This card is required.

T50	N	A50	D	GSF	P	Q	QREX

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	ALPHAT	BETA	VP	TOL
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	0.0	1.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see*PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
ALPHAT	Thermal expansion coefficient, α_T.
BETA	Mixed hardening parameter, $0 \leq \beta \leq 1$. $\beta = 0$ for isotropic and $\beta = 1$ for kinematic hardening.
VP	Formulation for rate effects in plasticity update: EQ.0.0: No plastic strain rate dependence in yield stress (default) EQ.1.0: Plastic strain rate dependence in yield stress. Slower but more stable (recommended).
TOL	Multiplication factor (must be > 0.0) on tolerance criteria for plasticity and annealing iterations. LT.1.0: Increases accuracy at greater computational cost EQ.1.0: Default value GT.1.0: Decreases accuracy at less computational cost

Work Hardening and Dynamic Softening Parameters Card.

Card 2	1	2	3	4	5	6	7	8
Variable	YB	QDEF	R	A	B	MINRT	POST	ODESOL
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	0.0	0.0

VARIABLE	DESCRIPTION
BY	Work hardening parameter, B_y. See Work Hardening and Dynamic Softening.

VARIABLE	DESCRIPTION
QDEF	Work hardening activation energy, Q_{def}. See Work Hardening and Dynamic Softening.
R	Work hardening gas constant, R. See Work Hardening and Dynamic Softening.
A	Dynamic softening parameter, a. See Work Hardening and Dynamic Softening.
B	Dynamic softening parameter, b. See Work Hardening and Dynamic Softening.
MINRT	Work hardening minimum (plastic) strain rate, $\dot{\varepsilon}_{min}$, in Zener-Hollomon parameter. See Work Hardening and Dynamic Softening.
POST	Save additional history variables for post-processing with POST = 1
ODESOL	Solver for static recovery stress: EQ.0.0: Trapezoidal rule (default) EQ.1.0: Heun's method: Faster but less stable.

Second Work Hardening and Dynamic Softening Parameters Card.

Card 3	1	2	3	4	5	6	7	8
Variable	ASIG0	BSIG0	ASIGS	BSIGS	ASIGSS	BSIGSS	AEPS	BEPS
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

VARIABLE	DESCRIPTION
ASIG0, ASIGS	Parameters a_i, $i = 0, s, ss$, to calculate σ_0, σ_s, and σ_{ss}, respectively, from the Zener-Hollomon parameter. See Work Hardening and Dynamic Softening.
BSIG0, BSIGS	Parameters b_i, $i = 0, s, ss$, to calculate σ_0, σ_s, and σ_{ss}, respectively, from the Zener-Hollomon parameter. See Work Hardening and Dynamic Softening.

VARIABLE	DESCRIPTION
AEPS	Parameter a_{ε_s} used to calculate the saturation strain, ε_s, for dynamic relaxation from the Zener-Hollomon parameter. See Work Hardening and Dynamic Softening.
BEPS	Parameter a_{ε_s} used to calculate the saturation strain, ε_s, for dynamic relaxation from the Zener-Hollomon parameter. See Work Hardening and Dynamic Softening.

Static Recovery Parameters Card. See Static Recovery and Static Recrystallization.

Card 4	1	2	3	4	5	6	7	8
Variable	THRES	M	ALPHA	NUD	U0	K	NU	BNU
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

VARIABLE	DESCRIPTION
THRES	Static recovery strain rate threshold. THRES > 0 turns off dynamic softening, meaning sets A = 0.
M	Taylor factor, M, for static recovery stress
ALPHA	α parameter for static recovery stress
NUD	Debye frequency, ν_D, for static recovery stress
U0	Activation energy, U_0, for static recovery stress
K	Boltzmann constant, k
NU	Interaction volume, ν, for static recovery stress
BNU	Burger's vector, b_ν, for static recovery stress

Static Recrystallization Parameters Card. See Static Recovery and Static Recrystallization.

Card 5	1	2	3	4	5	6	7	8
Variable	T50	N	A50	D	GSF	P	Q	QREX
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	none	none

VARIABLE	DESCRIPTION
T50	Time required to reach 50% static recrystallization
N	Static recrystallization exponent
A50	Scale parameter for strain dependent recrystallization time: EQ.0.0: T50 parameter is used for recrystallization time. GT.0.0: T50 parameter is ignored. Recrystallization time is calculated with A50, D, GSF, P, Q, and QREX. LT.0.0: T50 parameter is used. Recrystallization factor and combined recovery stress calculated using A50, D, GSF, P, Q, QREX are additional history variables if POST = 1.
D	Length parameter for strain dependent recrystallization time
GSF	Exponent for strain dependent recrystallization time
P	Exponent for strain dependent recrystallization time
Q	Exponent for strain dependent recrystallization time
QREX	Activation energy for strain dependent recrystallization time

Material Model:

This material model uses a hypo-elastoplastic formulation

$$\dot{\sigma} = \mathbf{C}\dot{\varepsilon}_e = \mathbf{C}\left(\dot{\varepsilon} - \dot{\varepsilon}_T - \dot{\varepsilon}_p\right) \;,$$

with thermal strain rate

$$\dot{\varepsilon}_T = \alpha_T \dot{T}\mathbf{I} \;,$$

and plastic strain rate

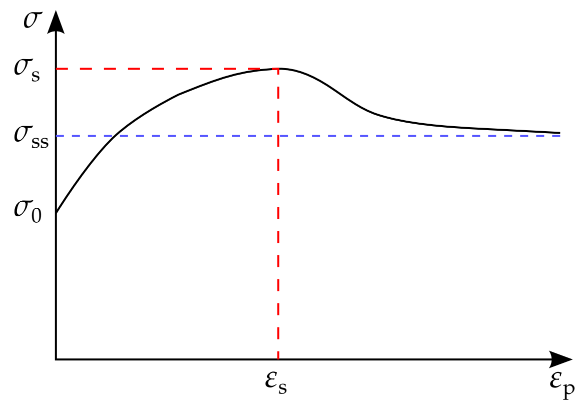

Figure M305-1. Stress-strain with work hardening and dynamic softening

$$\dot{\boldsymbol{\varepsilon}}_p = \dot{\varepsilon}_p \frac{3}{2}\frac{\mathbf{s}-\boldsymbol{\alpha}}{\sigma_{\mathrm{VM}}} \ ,$$

where $\mathbf{s}$ is the deviatoric stress, $\boldsymbol{\alpha}$ is the back stress, ε_p is the effective plastic strain, and

$$\sigma_{\mathrm{VM}} = \sqrt{\frac{3}{2}(\mathbf{s}-\boldsymbol{\alpha}):(\mathbf{s}-\boldsymbol{\alpha})} \ ,$$

is the Von Mises stress.

Using a mixed kinematic isotropic-kinematic hardening with mixing factor $\beta \in [0,1]$ and a nonlinear hardening function $h(T, \varepsilon_p, \dot{\varepsilon}_p)$, the back stress evolves according to

$$\dot{\boldsymbol{\alpha}} = \beta H \dot{\varepsilon}_p \frac{\mathbf{s}-\boldsymbol{\alpha}}{\sigma_{\mathrm{VM}}} \ ,$$

where $H = \partial h/\partial \varepsilon_p$ is the material hardening. The yield stress becomes

$$\sigma_y = h + \beta(\sigma_0 - h) \ ,$$

where σ_0 is the initial yield stress. We will discuss the nonlinear hardening function in Work Hardening and Dynamic Softening below.

We will next discuss the models for work hardening, dynamic recrystallization, static recovery, and static recrystallization.

Work Hardening and Dynamic Softening

To begin this discussion, we need to introduce the Zener-Hollomon parameter. The initial yield stress (σ_0), the saturation or peak stress (σ_s), the steady state stress (σ_{ss}), and the saturation strain (ε_s) which describe the stress-strain curve in the work hardening and dynamic softening regime (see Figure M305-1) can depend on the deformation temperature and strain rate. The Zener-Hollomon provides this dependence. The parameter is given by

$$Z = \max(\dot{\varepsilon}, \dot{\varepsilon}_{\min}) \exp\left(\frac{Q_{\mathrm{def}}}{RT}\right) \ .$$

The stresses then have the general form of:

$$\sigma_i = a_i \ln Z + b_i, \qquad i = 0, s, ss \ ,$$

and the saturation strain, similarly, is given by

$$\varepsilon_s = a_{\varepsilon_s} \ln Z + b_{\varepsilon_s} \ .$$

In the above, $\dot{\varepsilon}$ is the effective strain rate, and $\dot{\varepsilon}_{\mathrm{min}}$ is the minimum strain rate for which the parameter fit is done to prevent unphysical values of σ_i and ε_s. If VP = 1 the effective *plastic* strain rates are used here. Q_{def} is the activation energy, R is the gas constant, and T is the temperature.

The work hardening model is based on the interplay between storage and annihilation of dislocations described by the Estrin and Mecking model

$$\sigma_{\mathrm{EM}} = \sqrt{\sigma_s^2 - (\sigma_s^2 - \sigma_0^2)\exp\left(-2B_y\varepsilon_p\right)} \ ,$$

where B_y is a material parameter and ε_p is the effective plastic strain.

We include the effect of softening due to dynamic recrystallization in the prediction of the flow stress beyond a saturation strain, ε_s. To model this effect, we introduce the dynamic recrystallization fraction

$$X_{drx} = \begin{cases} 0, & \varepsilon_p < \varepsilon_s \\ 1 - \exp\left(-a\left(\varepsilon_p - \varepsilon_s\right)^b\right), & \varepsilon_p \geq \varepsilon_s \end{cases}$$

The transient flow stress due to work hardening and dynamic softening is, then, predicted using a mixture law between the steady state stress, σ_{ss}, and the Estrin Mecking stress, σ_{EM}:

$$\sigma = \sigma_{\mathrm{EM}} - X_{\mathrm{drx}}(\sigma_s - \sigma_{ss}) \ .$$

The resulting hardening function then becomes

$$h = (1 - X_{\mathrm{drx}})\sigma_{\mathrm{EM}} + X_{\mathrm{drx}}\sigma_{ss}.$$

Static Recovery and Static Recrystallization

After deformation, the material softens due to static recovery and static recrystallization. The static recovery stress, σ_{srx}, is modeled by

$$\sigma_{\mathrm{srx}} = \sigma_0 + \Delta\sigma$$

where σ_0 is the initial yield stress and $\Delta\sigma$ is the change in stress due to dislocation climb. $\Delta\sigma$ changes with time by

$$\frac{d\Delta\sigma}{dt} = -\frac{64\Delta\sigma^2}{9M^3\alpha^2E(T)}\nu_D \exp\left(-\frac{U_0}{RT}\right)\sinh\left(\frac{\Delta\sigma\nu b_\nu^3}{kT}\right) \ .$$

At the start of recovery $\Delta\sigma = \sigma_{\mathrm{VM}} - \sigma_0$. M, α, ν_D, and b_v are physical constants related to the properties of the FCC iron lattice, U_0 is the activation energy for climb, ν is the interaction volume, and R and k are the universal gas constant and Boltzmann constant, respectively.

Static recovery starts when the material is under plastic load and the plastic strain rate is lower than THRES. It stops when the plastic strain rate is higher than THRES.

Softening is assumed to be caused by recrystallized grain growth. Deformed structure with high dislocation density is replaced with new grains with a low dislocation density and constant stress σ_0. The recrystallized fraction is described by the static recrystallization fraction X_{srx} which is described via a standard JMAK expression

$$X_{\text{srx}} = 1 - \exp\left(-0.693\left(\frac{t - t_{\text{start}}}{t_{50}}\right)^n\right) ,$$

where t is the total time, t_{start} is the start time of the recovery and t_{50} is the time required to reach 50% recrystallization.

Finally, the combined recovery stress state is expressed by a law of mixtures

$$\sigma_r = X_{\text{srx}}\sigma_0 + (1 - X_{\text{srx}})\sigma_{\text{srx}}.$$

If A50 $\neq$ 0, the recrystallization time t_{50} may be calculated from

$$t_{50} = |A_{50}|(\varepsilon_p^{\text{tstart}})^p \left(\dot{\varepsilon}_p^{\text{tstart}} \exp\left(\frac{Q_{\text{def}}}{RT}\right)\right)^q d^{G_{\text{sf}}} \exp\left(\frac{Q_{\text{rex}}}{RT}\right) ,$$

where $\varepsilon_p^{\text{tstart}}$ is the plastic strain at start of recovery. For A50 > 0, the parameter T50 is ignored, and the material routine calculates t_{50} with the above expression. For A50 < 0, the material routine uses the parameter T50, but, if POST = 1, LS-DYNA additionally calculates history variables $X_{\text{srx}}^{\text{post}}$ and σ_r^{post} with t_{50} from the expression above.

During static recovery, stress and effective plastic strain is annealed, meaning stress and back stress is scaled with

$$\gamma = \frac{\sigma_r - \sigma_0}{\sigma_{\text{VM}} - \sigma_0}, \qquad 0 \leq \gamma \leq 1,$$

and the annealed plastic strain solves

$$\sigma_y(\varepsilon_p^{\text{annealed}}) = \gamma\sigma_y(\varepsilon_p).$$

History variables:

The following history variables are available by default:

History Variable #	Description
1-6	Back stress

History Variable #	Description
7	Temperature
8	Plastic strain rate
9	Plastic strain at start of recovery
10	Time since start of recovery ($t - t_{\text{start}}$). It is set to -1 when inactive.
11	Static recovery stress, σ_{srx}
12	Combined recovery stress, σ_r

The following additional variables are available if POST = 1:

History Variable #	Description
13	Initial yield stress, σ_0
14	Saturation stress, σ_s
15	Steady state stress, σ_{ss}
16	Saturation strain, ε_s
17	Dynamic recrystallization fraction, X_{drx}
18	Yield stress, σ_y
19	Static recrystallization fraction, X_{srx}

The following additional variables are available if POST = 1 and A50 < 0:

History Variable #	Description
20	Derived static recrystallization fraction, $X_{\text{srx}}^{\text{post}}$
21	Derived combined recovery stress, σ_r^{post}

References:

Schill, M., Karlsson, J., Magnusson, H., Huyan, F., Nosar, N.S., Lagergren, J., Narström, T., and Johansson, F. "Simulation of Hot Plate Rolling using LS-DYNA," *13th European LS-DYNA Conference* (2021).

*MAT_GENERALIZED_ADHESIVE_CURING

This is Material Type 307. It incorporates a modular approach for modeling adhesive materials during chemical curing. This material model provides a general viscoelastic Maxwell model defined by its Prony series expansion of up to 18 terms that considers the effects of temperature and degree of cure. It is supported for solid and cohesive solid elements.

Card Summary:

Card 1. This card is required.

MID	RO	GASC	IDOC	INCR	QCURE	TZERO	

Card 2. This card is required.

CKOPT	CK1	CK2	CK3	CK4	CK5	CK6	CK7

Card 2.1. Include this card if |CKOPT| > 3 and |CKOPT| < 11.

CK8	CK9	CK10	CK11	CK12	CK13	CK14	CK15

Card 2.2. Include this card if CKOPT = 5, 6, 9, or 10.

CK16	CK17	CK18	CK19	CK20	CK21	CK22	CK23

Card 3. Include this card if CKOPT < 0.

CDOPT	CD1	CD2	CD3				

Card 4. Include this card if CKOPT < 0.

CTGOPT	CTG1	CTG2	CTG3				

Card 5. This card is required.

CEOPT	CE1	CE2	CE3	CE4			

Card 6. This card is required.

TEOPT	TE1	TE2					

Card 7. This card is required.

THOPT	TH1	TH2	TH3	TH4	TH5	TH6	TH7

Card 8. This card is required.

TVOPT	TV1	TV2					

Card 9. This card is required.

PHOPT	PH1	PH2	PH3	PH4	PH5	PH6	

Card 10. This card is required.

PVOPT	PV1	PV2	PV3	PV4			

Card 11. This card is required.

PL1OPT	PL11	PL12	PL13	PL14	PL15		

Card 12. This card is required.

PL2OPT	PL21	PL22	PL23	PL24	PL25	PL26	

Card 13. This card is required.

DAOPT	DAEVO	DATRIA	DA1	DA2	DA3	DA4	DA5

Card 14. This card is required.

DA6	DA7	DA8	DA9	DA10	DA11	PDA1	PGEL

Card 15a. The keyword reader assumes the input deck includes this version of Card 15 if, in the first instantiation of this card, the value in the first entry is ≥ 0.0. Input up to 18 instantiations of this card. The next keyword ("*") card terminates this input if using fewer than 18 cards. The number of terms for the shear behavior may differ from that for the bulk behavior: insert zero to exclude a term.

G*i*	BETAG*i*	K*i*	BETAK*i*				

Card 15b. The keyword reader assumes the input deck includes this version of Card 15 if the value in the first entry is < 0.0

VISOPT	NUE						

Card 16a. Include this card if VISOPT = -1 on Card 15b. Input up to 13 instantiations of this card. The next keyword ("*") card terminates this input if using fewer than 13 cards.

E*i*	BETA*i*	E*j*	BETA*j*				

Card 16b. Include this card if VISOPT = -2. Include up to 13 instantiations of this card. The next keyword ("*") card terminates this input if using fewer than 13 cards.

G*i*	BETA*i*	G*j*	BETA*j*				

Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	GASC	IDOC	INCR	QCURE	TZERO	
Type	A	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
GASC	Gas constant, R
IDOC	Initial degree of cure, p_I: EQ.0.0: Uncured initial state, $p_I = 0$ GT.0.0: Uniformly distributed initial state of cure with p_I = IDOC LT.0.0: Use *INITIAL_STRESS_OPTION to define a potentially nonuniform distribution for the initial state of cure (history variable #2).
INCR	Switch between incremental and total stress formulation: EQ.1: Incremental form (default, recommended) EQ.2: Total form
QCURE	Heat generation factor, relating the heat generated in one time step with the increment of the degree of cure in that step
TZERO	Temperature value with respect to the temperature scale used in the input deck for a temperature of 0 K. See Remark 1.

Curing Kinetics Card 1

Card 2	1	2	3	4	5	6	7	8
Variable	CKOPT	CK1	CK2	CK3	CK4	CK5	CK6	CK7
Type	I	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
CKOPT	Curing kinetics option (see Remark 2 for details): EQ.0: No curing kinetics EQ.1: Generalized model, with load curves for pre-exponential factors EQ.2: Extended Kamal model EQ.-2: Extended Kamal model with diffusion control EQ.3: Kamal model EQ.-3: Kamal model with diffusion control EQ.4: Three-species reaction kinetics model EQ.-4: Three-species reaction kinetics model with diffusion control EQ.5: Five-species reaction kinetics model EQ.6: Five-species reaction kinetics model EQ.7: Three-species reaction kinetics model EQ.8: Three-species reaction kinetics model EQ.9: Four-species reaction kinetics model EQ.10: Five-species reaction kinetics model EQ.11: Model-free kinetics
CK*i*	i^{th} curing kinetics model parameter. The meaning of the parameter depends on the choice of CKOPT. For details, see Remark 2.

Curing Kinetics Card 2. Additional card for |CKOPT| > 3 and |CKOPT| < 11.

Card 2.1	1	2	3	4	5	6	7	8
Variable	CK8	CK9	CK10	CK11	CK12	CK13	CK14	CK15
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
CK*i*	i^{th} curing kinetics model parameter. The meaning of the parameter depends on the choice of CKOPT. For details, see Remark 2.

Curing Kinetics Card 3. Additional card for CKOPT = 5, 6, 9, or 10.

Card 2.2	1	2	3	4	5	6	7	8
Variable	CK16	CK17	CK18	CK19	CK20	CK21	CK22	CK23
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
CK*i*	i^{th} curing kinetics model parameter. The meaning of the parameter depends on the choice of CKOPT. For details, see Remark 2.

Curing Kinetics Diffusion Control Card 1. Additional card for CKOPT < 0.

Card 3	1	2	3	4	5	6	7	8
Variable	CDOPT	CD1	CD2	CD3				
Type	I	F	F	F				

VARIABLE	DESCRIPTION
CDOPT	Diffusion control mechanism option (see Remark 16): EQ.1: Williams-Landel-Ferry (WLF) type EQ.2: Arrhenius shift-like mechanism EQ.3: Combined Williams-Landel-Ferry (WLF) and Arrhenius

VARIABLE	DESCRIPTION
	shift-like mechanism
CD*i*	*i*th diffusion control model parameter. The meaning of the parameter depends on the choice of CDOPT. For details, see Remark 16.

Curing Kinetics Diffusion Control Card 2. Additional card for CKOPT < 0.

Card 4	1	2	3	4	5	6	7	8
Variable	CTGOPT	CTG1	CTG2	CTG3				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
CTGOPT	Glass transition temperature for diffusion control (see Remark 16): EQ.1: DiBenedetto equation EQ.2: Hesekamp equation
CTGD*i*	*i*th parameter for calculating glass transition temperature. The meaning of the parameter depends on the choice of CTOPT. For details, see Remark 16.

Card 5	1	2	3	4	5	6	7	8
Variable	CEOPT	CE1	CE2	CE3	CE4			
Type	I	F	F	F	F			

VARIABLE	DESCRIPTION
CEOPT	Chemical expansion option (see Remark 3 for details): EQ.0: No chemical expansion EQ.1: Differential form with load curve input EQ.2: Secant form with load curve input EQ.3: Secant form with a polynomial expression
CE*i*	*i*th chemical expansion model parameter. The meaning of the

VARIABLE	DESCRIPTION
	parameter depends on the choice of CEOPT. For details, see Remark 3.

Card 6	1	2	3	4	5	6	7	8
Variable	TEOPT	TE1	TE2					
Type	I	F	F					

VARIABLE	DESCRIPTION
TEOPT	Thermal expansion option (see Remark 4): EQ.0: No thermal expansion EQ.1: Differential form with load curve input EQ.2: Secant form with load curve input
TE*i*	i^{th} thermal expansion parameter. The meaning of the parameter depends on the choice of TEOPT. For details, see Remark 4.

Card 7	1	2	3	4	5	6	7	8
Variable	THOPT	TH1	TH2	TH3	TH4	TH5	TH6	TH7
Type	I	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
THOPT	Option for the horizontal temperature shift of the viscoelastic master curve as given by the Prony series expansion (see Remarks 5 and 6): EQ.0: No temperature shift EQ.1: Williams-Landel-Ferry (WLF) shift function EQ.2: Arrhenius shift function EQ.3: Combined WLF (above glass transition temperature T_g) and Arrhenius (below T_g) shift function EQ.4: As 3, but with a glass transition temperature $T_g(p)$ as a

VARIABLE	DESCRIPTION
	function of the degree of cure p EQ.5: Direct input of shift factors as a function of temperature EQ.6: Combined extended WLF (above $T_g(p)$) and exponential shift function (below $T_g(p)$)
TH*i*	i^{th} shifting parameter. The meaning of the parameter depends on the choice of THOPT. For details, see Remark 6.

Card 8	1	2	3	4	5	6	7	8
Variable	TVOPT	TV1	TV2					
Type	I	F	F					

VARIABLE	DESCRIPTION
TVOPT	Option for the vertical temperature shift of the master viscoelastic curve as given by the Prony series expansion. See Remarks 5 and 7 for details. EQ.0: No temperature shift EQ.1: Shifting of the complete master curves $G(t)$ EQ.2: Shifting of all terms G_i and K_i, but not G_∞ and K_∞
TV*i*	The meaning of the shifting parameters depends on the choice of TVOPT. For details, see Remark 7.

Card 9	1	2	3	4	5	6	7	8
Variable	PHOPT	PH1	PH2	PH3	PH4	PH5	PH6	
Type	I	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
PHOPT	Option for the horizontal shift due to curing effects of the master viscoelastic curve as given by the Prony series expansion (see Remarks 5 and 8):

VARIABLE	DESCRIPTION
	EQ.0: No shift EQ.1: Eom model EQ.2: Direct input of shift factors as function of the degree of cure
PHi	i^{th} shifting parameter. The meaning of the parameter depends on the choice of PHOPT. For details, see Remark 8.

Card 10	1	2	3	4	5	6	7	8
Variable	PVOPT	PV1	PV2	PV3	PV4			
Type	I	F	F	F	F			

VARIABLE	DESCRIPTION
PVOPT	Option for the vertical shift of the master viscoelastic curve due to curing effects as given by the Prony series expansion (see Remarks 5 and 9 for details): EQ.0: No shift EQ.1: Input of instantaneous moduli G_0 and K_0 as a function of degree of cure p. Assumption of constant ratios $G_i(p)/G_0(p)$ and $K_i(p)/K_0(p)$. EQ.2: Input of long-term moduli $G_\infty(p)$ and $K_\infty(p)$ as functions of degree of cure p and of scaling functions for other moduli G_i and K_i.
PVi	i^{th} shifting parameter. The meaning of the parameter depends on the choice of PVOPT. For details, see Remark 9.

Card 11	1	2	3	4	5	6	7	8
Variable	PL1OPT	PL11	PL12	PL13	PL14	PL15		
Type	I	F	F	F	F	F		

VARIABLE	DESCRIPTION
PL1OPT	Option for yield function description (see Remarks 10 and 11): EQ.0: No plasticity EQ.1: Version of Toughened Adhesive Polymer model (TAPO) with cap in tension and Drucker & Prager in compression with distortional hardening under plastic flow EQ.2: Version of Toughened Adhesive Polymer model (TAPO) with cap in tension and von Mises in compression EQ.3: Version of Toughened Adhesive Polymer model (TAPO) with cap in tension and Drucker & Prager in compression with distortional hardening under temperature change.
PL1*i*	i^{th} yield surface parameter. The meaning of the parameter depends on the choice of PL1OPT. For details, see Remark 11.

Card 12	1	2	3	4	5	6	7	8
Variable	PL2OPT	PL21	PL22	PL23	PL24	PL25	PL26	PL27
Type	I	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
PL2OPT	Option for yield stress description (see Remarks 10 and 12 for details): EQ.0: No plasticity EQ.1: Tabular input for yield stress as a function of curing, temperature, and plastic strains. EQ.2: Tabular input for initial yield stress as a function of curing and temperature and hardening as a function of curing, temperature, and plastic strains. EQ.3: Load curve inputs for effects of curing on initial yield stress and on hardening. Load curve input for temperature dependence of initial yield stress. Tabular input for hardening as a function of temperature and strain EQ.4: Load curve inputs for effects of curing and temperature on the parameters for the yield stress definitions in the Toughened Adhesive Polymer model (TAPO)

VARIABLE	DESCRIPTION
	EQ.5: Yield stress definitions in the Toughened Adhesive Polymer model (TAPO). No influence of temperature or curing.
PL2*i*	i^{th} yield stress parameter. The meaning of the parameter depends on the choice of PL2OPT. For details, see Remark 12.

Card 13	1	2	3	4	5	6	7	8
Variable	DAOPT	DAEVO	DATRIA	DA1	DA2	DA3	DA4	DA5
Type	I	I	I	F	F	F	F	F

VARIABLE	DESCRIPTION
DAOPT	Material damaging option (damage parameter D_1), defines the strain thresholds γ_c and γ_f for damage initiation and rupture (see Remark 13): EQ.0: No material damage EQ.1: Version of Toughened Adhesive Polymer model (TAPO): Strain threshold exponential function of triaxiality. Load curve inputs for temperature and cure dependence. EQ.2: Version of Toughened Adhesive Polymer model (TAPO). Same as 1, but without temperature and cure dependence. EQ.3: Version of Toughened Adhesive Polymer model (TAPO). Same as 1, but with additional strain rate dependency.
DAEVO	Effective strain measure used for material damage evolution (see Remark 13): EQ.0: Arc length of damage plastic strain EQ.1: Arc length of plastic strain EQ.2: Arc length of viscoelastic-plastic strain rate
DATRIAX	Triaxiality flag for calculation of strain thresholds γ_c and γ_f for damage initiation and rupture of material damage option (see Remark 13): EQ.0: Use triaxiality factor only in tension

VARIABLE	DESCRIPTION
	EQ.1: Use triaxiality factor in tension and compression
DAi	i^{th} material damage parameter. The meaning of the parameter depends on the choice of DAOPT for the evolution of damage parameter D_1. For details, see Remark 13.

Card 14	1	2	3	4	5	6	7	8
Variable	DA6	DA7	DA8	DA9	DA10	DA11	PDA1	PDA2
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
DAi	i^{th} material damage parameter. The meaning of the parameter depends on the choice of DAOPT for the evolution of damage parameter D_1. For details, see Remark 13.
PDA1	Parameter for the (pre-) damage formulation due to for example viscous fingering. It defines the damage parameter D_2 as function of the thickness strain ε_{33} and the degree of cure p. For details, see Remark 14. EQ.0: No damage GT.0: Use exponential approach LT.0: Load curve input with ID $\lvert$PDA1$\rvert$ input for $D_2(\varepsilon_{33})$
PGEL	Gelation point p_{gel} as needed to switch between evolution of damage parameters D_1 and D_2. For details, see Remark 13 and Remark 14.

Viscoelastic Constant Card. The keyword reader assumes the input deck includes this version of Card 15 if, in the first instantiation of this card, the value in the first entry is ≥ 0.0. Input up to 18 instantiations of this card. The next keyword ("*") card terminates this input if using fewer than 18 cards. The number of terms for the shear behavior may differ from that for the bulk behavior: insert zero to exclude a term.

Card 15a	1	2	3	4	5	6	7	8
Variable	G*i*	BETAG*i*	K*i*	BETAK*i*				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
G*i*	Shear relaxation modulus for the i^{th} term
BETAG*i*	Shear decay constant for the i^{th} term
K*i*	Bulk relaxation modulus for the i^{th} term
BETAK*i*	Bulk decay constant for the i^{th} term

Viscoelastic Option Card. The keyword reader assumes the input deck includes this version of Card 15 if the value in the first entry is < 0.0.

Card 15b	1	2	3	4	5	6	7	8
Variable	VISOPT	PR						
Type	F	F						

VARIABLE	DESCRIPTION
VISOPT	Viscous option determining the input of the Prony series: EQ.-1: Prony series input for Young's modulus $E(t)$. Prony series for shear modulus $G(t)$ and bulk modulus $K(t)$ are derived from it, assuming a constant Poisson's ratio. EQ.-2: Prony series input for shear modulus $G(t)$. The Prony series for the bulk modulus $K(t)$ is derived from it, assuming a constant Poisson's ratio.
PR	Constant Poisson's ratio ν

Viscoelastic Constant Cards for VISOPT = -1. Include up to 13 instantiations of this card if VISOPT = -1. See Remark 5.

Card 16a	1	2	3	4	5	6	7	8
Variable	E*i*	BETA*i*	E*j*	BETA*j*				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
E*i*	Relaxation modulus for the i^{th} term
BETA*i*	Decay constant for the i^{th} term
E*j*	Relaxation modulus for the j^{th} term
BETA*j*	Decay constant for the j^{th} term

Viscoelastic Constant Cards for VISOPT = -2. Include up to 13 instantiations of this card if VISOPT = -2. See Remark 5.

Card 16b	1	2	3	4	5	6	7	8
Variable	G*i*	BETA*i*	G*j*	BETA*j*				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
G*i*	Shear relaxation modulus for the i^{th} term
BETA*i*	Decay constant for the i^{th} term
G*j*	Shear relaxation modulus for the j^{th} term
BETA*j*	Decay constant for the j^{th} term

Remarks:

1. **Temperature scale.** This material formulation requires providing the material data with respect to a consistent temperature unit. For all the curing kinetics models described in Remark 2, except CKOPT = 1, it is necessary to define the

temperature T in Kelvin. Consequently, if considering curing, all temperature-dependent input should be given for temperature data in Kelvin.

TZERO enables including this material model in a simulation set up in the Celsius temperature scale. It defines the temperature value T_{0K} in the user system for 0 K. Thus, to run a simulation with the Celsius scale, set T_{0K} to approximately -273. For a model using the Kelvin scale, T_{0K} is 0.

For all temperature-dependent values in this material, LS-DYNA uses the modified temperature

$$T = T_{\text{user}} - T_{0K} \ ,$$

where T_{user} refers to the temperature value in the simulation.

2. **Curing kinetics.** This material formulation includes an internal variable p to represent the degree of cure for the adhesive. In all cases, it is the result of a set of chemical reactions. The number of species in the reaction, the number of reaction steps, and the reaction kinetics applied depend on the choice of the curing kinetics option CKOPT.

   All options CKOPT except 1 use the Arrhenius formula:

   $$K_i^{\text{chem}}(T) = k_i \exp\left(-\frac{Q_i}{RT}\right) \ .$$

   In the above, R is the universal gas constant. GASC in Card 1 sets R.

   For the positive values of CKOPT, $K_i^{\text{chem}}(T)$ is used as an effective parameter in the given kinetics models below, that is, $K_i(T) = K_i^{\text{chem}}(T)$. The negative options CKOPT = -2,-3,-4 feature a diffusion control mechanism, which introduces an additional term $K^{\text{diff}}(T)$ to calculate the values $K_i(T)$ as

   $$K_i(T) = \left(\frac{1}{K_i^{\text{chem}}(T)} + \frac{1}{K^{\text{diff}}(T)}\right)^{-1}$$

   The definition of the diffusion control effect K_i^{diff} depends on the input of Cards 3 and 4 and is discussed in Remark 16.

   A table at the end of this remark gives the input structure for the parameters used by the different CKOPT options.

   a) *Two-species reaction kinetics model (CKOPT = 1, 2,-2, 3 and -3)*

      We can directly give the evolution equation in terms of the degree of cure p by identifying it with the product c_2 of a chemical reaction with two chemical species. In the most general form, it reads

      $$\dot{p} = K_1(T)(1-p)^{n_1} + K_2(T)p^{m_2}(1-p)^{n_2} \ .$$

The functions $K_1(T)$ and $K_2(T)$ are the load curves for CKOPT = 1. The other options follow the equations stated above. The standard Kamal model (CKOPT = 3 and -3) introduces a simplification to the above equations with $n_1 = n_2 = n$.

b) *Three-species reaction kinetics model (CKOPT = 4 and -4)*

This option represents a system of chemical reactions involving three chemical species, A, B, and C, with two reaction steps (n^{th} order with autocatalysis). We denote the concentrations of the species with c_1, c_2, and c_3. The following gives the evolution equations for the concentrations of the reactant, c_1, and intermediate, c_2:

$$\begin{aligned}\dot{c}_1 &= -K_1(T)(1.+k_{c1}c_2)c_1^{n_1} \\ \dot{c}_2 &= \quad K_1(T)(1.+k_{c1}c_2)c_1^{n_1} - K_2(T)(1.+k_{c2}c_3)c_2^{n_2}\end{aligned}$$

with input parameters k_{ci} and n_i. The identity $c_3 = 1.-c_1 - c_2$ eliminates the concentration c_3 of product species C from the equations. Therefore, the algorithm internally only uses the concentrations c_1 and c_2. Thus, this model requires initial values $c_{1,0}$ and $c_{2,0}$.

Finally, we determine the degree of cure by a linear combination:

$$\begin{aligned}p &= F_1(1.-c_1) + (1.-F_1)(1.-c_1 - c_2) \\ &= 1.-c_1 - c_2 + F_1 c_2\end{aligned}$$

with an additional factor F_1.

c) *Five-species reaction kinetics models (CKOPT = 5 and 6)*

These options represent systems of chemical reactions with five chemical species A, B, C, D, and E with concentrations c_1, c_2, c_3, c_4, and c_5. The four reaction steps (n^{th} order with autocatalysis) of the system result in evolution equations for the reactant c_1 and intermediates c_2, c_3, and c_4 as follows

$$\begin{aligned}\dot{c}_1 &= -K_1(T)(1.+k_{c1}c_2)c_1^{n_1} \\ \dot{c}_2 &= \quad K_1(T)(1.+k_{c1}c_2)c_1^{n_1} - K_2(T)\left(1.+k_{c2}\tilde{c}_{\text{Opt}}\right)c_2^{n_2} \\ \dot{c}_3 &= \quad K_2(T)\left(1.+k_{c2}\tilde{c}_{\text{Opt}}\right)c_2^{n_2} - K_3(T)(1.+k_{c3}c_4)c_3^{n_3} \\ \dot{c}_4 &= \quad K_3(T)(1.+k_{c3}c_4)c_3^{n_3} - K_4(T)(1.+k_{c4}c_5)c_4^{n_4}\end{aligned}$$

with input parameters k_{ci} and n_i. The identity $c_5 = 1.-c_1 - c_2 - c_3 - c_4$ eliminates the concentration c_5 of the product E from the system. Consequently, the algorithm internally only uses the concentrations c_1, c_2, c_3, and c_4 internally and requires input of their initial values $c_{1,0}, c_{2,0}, c_{3,0}$ an, d $c_{4,0}$.

The options CKOPT = 5 and 6 only differ in the species used in the autocatalysis in the second reaction step. For option CKOPT = 5, we implemented an autocatalysis by D. Thus, the value of $\tilde{c}_{\text{Opt}}$ in the above equations is the concentration c_4 ($\tilde{c}_{\text{CKOPT=5}} = c_4$). We use an autocatalysis by C as the second reaction step for CKOPT = 6. Consequently, $\tilde{c}_{\text{Opt}}$ is c_3 ($\tilde{c}_{\text{CKOPT=6}} = c_3$).

Finally, we determine the degree of cure p by a linear combination of the concentrations with scaling factors F_1, F_2, and F_3:

$$p = (1. - c_1 - c_2 - c_3 - c_4) + F_1(c_2 + c_3 + c_4) + F_2(c_3 + c_4) + F_3(c_4)$$

d) *Three-species reaction kinetics model (CKOPT = 7)*

This option implements another system of chemical reactions involving three chemical species A, B, and C (concentrations denoted by c_1, c_2, and c_3). It involves three reaction steps. The first reaction step (A → B) is an n^{th} order reaction with autocatalysis by B. The reaction from the intermediate B to the product C follows the extended Kamal model. The evolution equations thus read

$$\begin{aligned}\dot{c}_1 &= -K_1(T)(1. + k_{c1}c_2)c_1^{n_1} \\ \dot{c}_2 &= K_1(T)(1. + k_{c1}c_2)c_1^{n_1} - K_2(T)c_2^{n_2}c_3^{m_2} - K_3(T)c_2^{n_3}\end{aligned}$$

with input parameters k_{c1}, m_2 and n_i. The identity $c_3 = 1. - c_1 - c_2$ removes concentration c_3 of the product C from the equations. Therefore, the algorithm internally only uses the concentrations c_1 and c_2 and requires initial values $c_{1,0}$ and $c_{2,0}$.

Finally, a linear combination with the factor F_1 determines the degree of cure:

$$p = 1. - c_1 - c_2 + F_1 c_2$$

e) *Three-species reaction kinetics model (CKOPT = 8)*

This option represents the third system of chemical reactions with three chemical species A, B, and C (concentrations denoted by c_1, c_2, c_3). It involves two reaction steps. The first reaction step (A → B) follows the Prout-Tompkins equation, the second (B→ C) is described by an n^{th} order reaction. The evolution equations read as:

$$\begin{aligned}\dot{c}_1 &= -K_1(T)c_1^{n_1}c_2^{\mathrm{m}_1} \\ \dot{c}_2 &= K_1(T)c_1^{n_1}c_2^{\mathrm{m}_1} - K_2(T)c_2^{n_2}\end{aligned}$$

with input parameters m_1 and n_i. The identity $c_3 = 1. - c_1 - c_2$ replaces the concentration c_3 of the product. Thus, the internal calculation does not use c_3. The calculation requires initial values $c_{1,0}$ and $c_{2,0}$.

Finally, a linear combination determines the degree of cure:

$$p = 1. - c_1 - c_2 + F_1 c_2$$

with an additional factor F_1.

f) *Four-species reaction kinetics model (CKOPT = 9)*

This option implements a reduced version of the system of chemical reactions defined by CKOPT = 6. This option only considers four chemical species A, B, C, and D and three reaction steps. The following gives the evolution equations for the reactant c_1 and intermediates c_2 and c_3 as:

$$\begin{aligned}\dot{c}_1 &= -K_1(T)(1.+k_{c1}c_2)c_1^{n_1}\\ \dot{c}_2 &= \quad K_1(T)(1.+k_{c1}c_2)c_1^{n_1} - K_2(T)(1.+k_{c2}c_3)c_2^{n_2}\\ \dot{c}_3 &= \quad K_2(T)(1.+k_{c2}c_3)c_2^{n_2} - K_3(T)(1.+k_{c3}c_4)c_3^{n_3}\end{aligned}$$

with input parameters k_{ci} and n_i. The identity $c_4 = 1.-c_1-c_2-c_3$ eliminates the concentration c_4 of the product D, which allows expressing the system in terms of the concentrations c_1, c_2 and c_3. Initial values $c_{1,0}, c_{2,0}$ and $c_{3,0}$ are needed to solve the system.

A linear combination determines the degree of cure p:

$$p = (1.-c_1-c_2-c_3) + F_1(c_2+c_3) + F_2(c_3) \quad .$$

g) *Five-species reaction kinetics model (CKOPT = 10)*

Option CKOPT = 10 models a system of chemical reactions with five chemical species: A, B, C, D, and E with concentrations c_1, c_2, c_3, c_4, and c_5. The first reaction involves the reactant A and the product D (n^{th} order with autocatalysis). The second reaction changes species B into species E through an intermediate species C. Modeling this reaction involves two reaction steps (n^{th} order with autocatalysis by C and an n^{th} order reaction). The evolution equations are:

$$\begin{aligned}\dot{c}_1 &= -K_1(T)(1.+k_{c1}c_4)c_1^{n_1}\\ \dot{c}_2 &= -K_2(T)(1.+k_{c2}c_3)c_2^{n_2}\\ \dot{c}_3 &= \quad K_2(T)(1.+k_{c2}c_3)c_2^{n_2} - K_3(T)c_3^{n_3}\end{aligned}$$

with input parameters k_{ci} and n_i. The equations $c_4 = 1.-c_1$ and $c_5 = 1.-c_2-c_3$ give the concentrations c_4 and c_5 of the products D and E. Consequently, these equations reduce the system to three unknown concentrations, c_1, c_2, and c_3. Therefore, solving the system requires the input of initial values $c_{1,0}, c_{2,0}$ and $c_{3,0}$.

The following equation determines the degree of cure p from the concentrations and factors F_1 and F_2:

$$p = F_1(1.-c_1) + (1.-F_1)\big(F_2(1.-c_2) + (1.-F_2)(1.-c_2-c_3)\big)$$

h) *Model-free kinetics approach (CKOPT = 11)*

This option allows for a direct, tabulated input of the evolution equation governing the curing process. This choice for CKOPT requires inputting load curves for a logarithmic scaling function $\ln(A'(p))$ and the activation energy $Q(p)$ as functions of the degree of cure p. The differential equation then reads:

$$\dot{p} = \exp\left(\ln\left(A'(p)\right)\right) \times \exp\left(-\frac{Q(p)}{RT}\right)$$

CKOPT	1	2	3	4	5	6	7	8	9	10	11
CK1	$K_1(T)$	k_1	k_1	k_1	k_1	k_1	k_1	k_1	k_1	k_1	$\ln A'(p)$
CK2	$K_2(T)$	k_2	k_2	k_2	k_2	k_2	k_2	k_2	k_2	k_2	$Q(p)$
CK3	m_2	Q_1	Q_1	Q_1	k_3	k_3	k_3	Q_1	k_3	k_3	
CK4	n_1	Q_2	Q_2	Q_2	k_4	k_4	Q_1	Q_2	Q_1	Q_1	
CK5	n_2	m_2	m_2	n_1	Q_1	Q_1	Q_2	n_1	Q_2	Q_2	
CK6		n_1	n	n_2	Q_2	Q_2	Q_3	m_1	Q_3	Q_3	
CK7		n_2		k_{c1}	Q_3	Q_3	n_1	n_2	n_1	n_1	
CK8				k_{c2}	Q_4	Q_4	n_2	F_1	n_2	n_2	
CK9				F_1	n_1	n_1	m_2	$c_{1,0}$	n_3	n_3	
CK10				$c_{1,0}$	n_2	n_2	n_3	$c_{2,0}$	k_{c1}	k_{c1}	
CK11				$c_{2,0}$	n_3	n_3	k_{c1}		k_{c2}	k_{c2}	
CK12					n_4	n_4	F_1		k_{c3}	F_1	
CK13					k_{c1}	k_{c1}	$c_{1,0}$		F_1	F_2	
CK14					k_{c2}	k_{c2}	$c_{2,0}$		F_2	$c_{1,0}$	
CK15					k_{c3}	k_{c3}			$c_{1,0}$	$c_{2,0}$	
CK16					k_{c4}	k_{c4}			$c_{2,0}$	$c_{3,0}$	
CK17					F_1	F_1			$c_{3,0}$		
CK18					F_2	F_2					
CK19					F_3	F_3					
CK20					$c_{1,0}$	$c_{1,0}$					

CKOPT	1	2	3	4	5	6	7	8	9	10	11
CK21					$c_{2,0}$	$c_{2,0}$					
CK22					$c_{3,0}$	$c_{3,0}$					
CK23					$c_{4,0}$	$c_{4,0}$					

3. **Chemical shrinkage.** The chemical reaction of the curing process results in shrinkage of the material. Three options are available to model this behavior.

   For CEOPT = 1 and 2, the coefficient of chemical shrinkage, $\gamma(p)$, is specified with a load curve. For CEOPT = 3, the coefficient is given by the following quadratic expression:

   $$\gamma(p) = \gamma_2 p^2 + \gamma_1 p + \gamma_0 \ .$$

   For CEOPT = 1, this load curve is used to compute the chemical strains by the following differential form:

   $$d\varepsilon^{\text{ch}} = \gamma(p)dp.$$

   CEOPT = 2 and 3 invoke a secant form, such that the strains are computed as:

   $$\varepsilon^{\text{ch}} = \gamma(p) \times (p - p_R) - \gamma(p_I) \times (p_I - p_R) \ ,$$

   with a reference degree of cure p_R and initial degree of cure p_I.

   The following table summarizes the input structure.

CEOPT	CE1	CE2	CE3	CE4
1	$\gamma(p)$			
2	$\gamma(p)$	p_R		
3	γ_0	γ_1	γ_2	p_R

4. **Thermal expansion.** Like the strains resulting from chemical shrinkage discussed in Remark 3, the thermal strains are either defined in a secant or differential form. In both cases the coefficient of thermal expansion $\eta(p, T)$ can be given as function of degree of cure p and temperature T and requires the input by of two-dimensional tabular data.

   Option TEOPT = 1 refers to the differential form

   $$d\varepsilon^{\text{th}} = \eta(p, T)\ dT \ \ .$$

   TEOPT = 2 invokes the secant formulation which requires the specification of an additional reference temperature T_R

$$\varepsilon^{\mathrm{th}} = \eta(p,T) \times (T - T_R) - \eta(p,T_I) \times (T_I - T_R) \ \ .$$

Coefficient $\eta(p,T)$ is specified with a 2D table (*DEFINE_TABLE_2D) whose ID is provided by parameter TE1. The values given in the table input correspond to the degree of cure and the abscissa of the referenced curve to temperature. If a load curve is reference by parameter TE1, the coefficient η is assumed to be a function of temperature.

The following table summarizes the input structure.

TEOPT	TE1	TE2
1	$\eta(p,T)$	
2	$\eta(p,T)$	T_R

5. **Stress relaxation.** The Cauchy stress, σ_{ij}, is related to the strain rate by:

$$\sigma_{ij}(t) = \int_0^t g_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}(\tau)}{\partial \tau} d\tau \ \ .$$

The relaxation functions $g_{ijkl}(t-\tau)$ are represented in this material formulation by terms of the Prony series for the shear modulus G and the bulk modulus K as functions of time t. For the shear modulus G, the series expansion is given by:

$$G(t) = G_\infty + \sum_{i=1}^{n_G} G_i e^{-\beta_i^G t} = G_0 - \sum_{i=1}^{n_G} G_i + \sum_{i=1}^{n_G} G_i e^{-\beta_i^G t},$$

with shear relaxation moduli G_i and decay constants β_i^G. The relation between the shear equilibrium modulus G_∞ and the instantaneous shear modulus G_0 is given by

$$G_\infty = G_0 - \sum_i G_i.$$

A similar Prony series definition is expected for the bulk modulus $K(t)$:

$$K(t) = K_\infty + \sum_{i=1}^{n_K} K_i e^{-\beta_i^K t} = K_0 - \sum_{i=1}^{n_K} K_i + \sum_{i=1}^{n_K} K_i e^{-\beta_i^K t},$$

with bulk relaxation moduli K_i and decay constants β_i^K.

This material model provides several options for inputting the Prony series data. If the first entry in Card 15 is non-negative, the input strategy defaults to VISOPT = 0. For this option, provide the terms for individual Prony series for $G(t)$ and $K(t)$ with up to 18 Prony series terms for each ($n_G \le 18$, $n_K \le 18$).

If the first entry in Card 15 is negative, it represents the option VISOPT. A negative value VISOPT < 0 implies the following coupling between the Prony series terms G_i and K_i:

$$K_i = \frac{2 + 2\nu}{3(1 - 2\nu)} G_i \quad \text{and} \quad \beta_i^K = \beta_i^G \,.$$

Note that Poisson's ratio ν in the above equation is constant. This approach allows accounting for up to 25 Prony series terms ($n_G = n_K \leq 25$). For VIOPT = -1, LS-DYNA interprets the input relaxation constants as terms E_i for Young's modulus $E(t)$ and internally translates them into the necessary constants G_i. For VIOPT = -2, LS-DYNA assumes a direct input of the shear relaxation moduli G_i.

In most applications the viscoelastic properties depend on temperature and degree of cure. In this material, shifting functions acting on the moduli G_i, G_0 and G_∞ (vertical shifting) and on the decay constants β_i (horizontal shifting) apply these dependencies. Note that, if not stated otherwise, LS-DYNA applies the same shifting operations to the shear and bulk moduli. Cards 7 to 10 set the shifting functions. We discuss these functions in Remarks 6, 7, 8 and 9.

6. **Horizontal temperature shift.** You can account for a possible temperature effect on the stress relaxation (see Remark 5) by a horizontal shift operation on the relaxation curve, implemented by the scaling of the decay constants β_i with a factor $a_T(T)$.

For THOPT = 1, the Williams-Landel-Ferry (WLF) shift function is used:

$$\ln\big(a_T(T)\big) = \frac{-A(T - T_R)}{B + T - T_R}$$

with constant parameters A and B and the reference temperature T_R.

THOPT = 2 invokes the Arrhenius shift function which requires the input of a reference temperature T_R and one parameter C:

$$\ln\big(a_T(T)\big) = C\left(\frac{1}{T} - \frac{1}{T_R}\right)$$

For many adhesive materials, the qualitative behavior of the temperature dependence changes with the glass transition temperature T_G from an Arrhenius- to a WLF-type description of the shifting. THOPT = 3 provides this behavior:

$$\ln\big(a_T(T)\big) = \begin{cases} C\left(\frac{1}{T} - \frac{1}{T_G}\right) & T \leq T_G \\ \frac{-A(T - T_G)}{B + T - T_G} & T > T_G \end{cases}$$

It has been proposed in literature to extend this option by a curing that depends on the glass transition temperature $T_G = T_G(p)$, such that the shifting factor reads

$$\ln\left(a_T(T,p)\right) = \begin{cases} C\left(\frac{1}{T} - \frac{1}{T_G(p)}\right) & T \le T_G(p) \\ \frac{-A(T - T_G(p))}{B + T - T_G(p)} & T > T_G(p) \end{cases}$$

This feature corresponds to THOPT = 4. The glass transition temperature must be input as a load curve.

THOPT = 6 replaces the Arrhenius shift function with an exponential approach. Above the glass transition temperature T_G, an extension of the WLF-type shift function is used:

$$a_T(T,p) = \begin{cases} \left(1 - D\left(T - T_G(p)\right)\right)^C & T \le T_G(p) \\ \exp\left(\min\left(\frac{-A_1(T - T_G(p))}{B_1 + T - T_G(p)}, \frac{-A_2(T - T_G(p))}{B_2 + T - T_G(p)}\right)\right) & T > T_G(p) \end{cases}$$

For the options discussed so far, no difference in temperature dependence is made between the shear and bulk moduli. The same scaling is applied to both Prony series expansions.

Finally, THOPT = 5 allows defining direct input for scaling factors a_T^G and a_T^K for the shear and bulk moduli, respectively. Load curve or table IDs are expected as input. The load curves (either referenced by the table or by the input) define the logarithm of the factors, that is, $\ln\left(a_T^G\right)$ and $\ln\left(a_T^G\right)$, as functions of temperature. In case of a table ID input, an additional dependence on the degree of cure can be accounted for.

The parameters input for the different options are shown in the following table.

THOPT	TH1	TH2	TH3	TH4	TH5	TH6	TH7
1	A	B	T_R				
2	C	T_R					
3	A	B	C	T_G			
4	A	B	C	$T_G(p)$			
5	$\ln(a_T^G(T))$ / $\ln(a_T^G(p,T))$	$\ln(a_T^K(T))$ / $\ln(a_T^K(p,T))$					
6	A_1	B_1	C	D	A_2	B_2	$T_G(p)$

7. **Vertical temperature shift.** To model the effect of temperature on the viscoelastic response, you can apply a vertical shift to the master relaxation curve (see Remark 5). The shear relaxation moduli (G_i and G_∞) and bulk relaxation moduli (K_i and K_∞) are scaled by temperature dependent scaling factors $b_T^G(T)$ and $b_T^K(T)$, respectively, to achieve this shift. The input parameters for the factors need to be load curves. Here, parameter TV1 refers to $b_T^G(T)$ and TV2 to $b_T^K(T)$.

   For TVOPT = 1 the entire relaxation curve is scaled. In contrast, TVOPT = 2 causes shifting of only the time dependent terms of the Prony series and, consequently, only the moduli G_i and K_i are scaled.

8. **Horizontal p-shift.** The effect of curing on the viscoelastic property of an adhesive material can be modelled by a horizontal shift of the relaxation curve (see Remark 5), meaning by scaling the decay moduli β_i. The scaling factors are denoted in this case by a_c.

   For PHOPT = 1, an analytical expression based on Eom et al is implemented

$$\log(a_c(p)) = \begin{cases} c(p - p_{\mathrm{gel}}) + a_{\mathrm{gel}} & p < p_{\mathrm{gel}} \\ a_{\mathrm{gel}} H^{(p-p_{\mathrm{gel}})} \left(\dfrac{p_f - p}{p_f - p_{\mathrm{gel}}} \right)^m & p \geq p_{\mathrm{gel}} \end{cases}$$

   with p_{gel} and a_{gel} being properties at the gelation point of the material. This shift is applied to both the shear and bulk moduli.

   PHOPT = 2 offers the possibility of a direct input of the scaling factors as functions of degree of cure. Here, load curves defining $\log(a_c^G)$ for shifting the shear curve and $\log(a_c^K)$ for shifting the bulk curve are expected.

   The set of input parameters is summarized in the following table.

PHOPT	PH1	PH2	PH3	PH4	PH5	PH6
1	p_{gel}	a_{gel}	c	H	p_f	m
2	$\log\left(a_c^G(p)\right)$	$\log\left(a_c^K(p)\right)$				

9. **Vertical p-shift.** We have implemented two different approaches to represent the effect of curing on the viscoelasticity through vertical shifting operations. The vertical shifting operations apply to the master curves $G(t)$ and $K(t)$ as defined in Remark 5.

   The first approach (PVOPT = 1) is taken from *MAT_277 and assumes a constant ratio $G_i(p)/G_0(p)$ for all degrees of cure. Consequently, it suffices to define one term $G_0(p)$ as a function of the degree of cure and further coefficients for the fully cured state of the adhesive:

$$G(t,p) = G_0(p)\left(1 - \sum_i \frac{G_{i,p=1.0}}{G_{0,\mathrm{p}=1.0}}\left(1 - e^{-\beta_i t}\right)\right) .$$

PVOPT = 2 distinguishes the effect of curing on the equilibrium moduli from its effect on the time-depending terms of the Prony series. Consequently, load curve IDs are expected to define $G_\infty(p)$ and $K_\infty(p)$ as well as scaling factors $b_c^G(p)$ and $b_c^K(p)$. The latter are applied to all G_i and K_i, respectively. This is also reflected by the input structure shown in the following table.

PVOPT	PV1	PV2	PV3	PV4
1	$G_0(p)$	$K_0(p)$		
2	$G_\infty(p)$	$K_\infty(p)$	$b_c^G(p)$	$b_c^K(p)$

10. **Plasticity.** This material features an isotropic plasticity formulation with a non-associated flow rule closely related to the TAPO model implemented in *MAT_-252. Both, the yield criterion F as well as the flow potential F^*, are defined in terms of invariants $\tilde{I}_1$ and $\tilde{J}_2$of the effective stress tensor:

$$\tilde{\boldsymbol{\sigma}} = \boldsymbol{\sigma}/(1 - D_1)(1 - D_2),$$

where the evolution of the damage parameters D_1 and D_2 is defined separately.

The general form of F and F^* in this model is given by

$$\begin{aligned} F &= f(\tilde{I}_1, \tilde{J}_2, r, T) - \tau_Y^2(p, T, r) = 0 \\ F^* &= f^*(\tilde{I}_1, \tilde{J}_2) - \tau_Y^2(p, T, r) \end{aligned}$$

The yield surface f and yield strength τ_Y are functions of the arc length of the damage plastic strain rate $\dot{r}$, which is defined by means of the arc length of the plastic strain rate $\dot{\gamma}_\mathrm{v}$ as in Lemaitre [1992]:

$$\dot{r} = (1 - D_1)\dot{\gamma}_\mathrm{v} = (1 - D_1\ )\sqrt{2\ \mathrm{tr}(\dot{\boldsymbol{\varepsilon}}^\mathrm{p})^2}.$$

The plastic strain rate $\dot{\boldsymbol{\varepsilon}}^\mathrm{p}$ is given by the non-associated flow rule

$$\dot{\boldsymbol{\varepsilon}}^\mathrm{p} = \lambda \frac{\partial F^*}{\partial \boldsymbol{\sigma}}.$$

The expressions for f and f^* or in other words the form of yield surface and flow potential are determined by the choice of parameters in Card 11. The yield strength computation is defined by Card 12. For details, see Remarks 11, 12, 13, and 14.

11. **Yield surface.** The yield surface definition is controlled by choice of parameter PL1OPT in Card 11. For the currently available options PL1OPT = 1, 2 or 3, the same flow potential is assumed:

$$f^*(\tilde{I}_1, \tilde{J}_2) = \tilde{J}_2 + \frac{a_2^*}{3}\langle \tilde{I}_1 \rangle^2 \ ,$$

where a_2^* is a user-defined material parameter.

Choosing PL1OPT = 1 results in a cap model in tension and nonlinear Drucker & Prager in compression with a distortional hardening under plastic flow. There is no temperature dependence for function f in this case, which reads:

$$f(\tilde{I}_1, \tilde{J}_2, r) = \tilde{J}_2 + \frac{1}{\sqrt{3}} a_1(r)\ \tau_0\ \tilde{I}_1 + \frac{a_2(r)}{3}\langle \tilde{I}_1 \rangle^2 \ .$$

Distortional hardening is introduced by phenomenological descriptions for parameters $a_1(r)$ and $a_2(r)$:

$$a_1(r) = a_{10} + a_1^{\mathrm{H}} r$$
$$a_2(r) = \max\left(a_{20} + a_2^{\mathrm{H}} r, 0.0\right)$$

PL1OPT = 2 does not consider distortional hardening and refers to a cap model in tension and a von Mises yield function in compression:

$$f(\tilde{I}_1, \tilde{J}_2) = \tilde{J}_2 + \frac{a_{20}}{3}\left\langle \tilde{I}_1 + \frac{\sqrt{3} a_{10} \tau_0}{2\, a_{20}} \right\rangle^2 - \frac{a_{10}^2\ \tau_0^2}{4\, a_{20}}.$$

Finally, PL1OPT = 3 refers to a temperature dependent yield surface. Equivalently to PL1OPT = 1 a cap model in tension and nonlinear Drucker & Prager in compression is used, but the distortional hardening is defined with respect to the current temperature:

$$f(\tilde{I}_1, \tilde{J}_2, T) = \tilde{J}_2 + \frac{1}{\sqrt{3}} a_{10} \tau_0 \tilde{I}_1 + \frac{a_2(T)}{3}\langle \tilde{I}_1 \rangle^2,$$

with the simple linear temperature dependence

$$a_2(T) = a_{20}\left(1 - m_{a2}\ (T - T_0)\right).$$

Input parameters for the different options can be found in the following table:

PL1OPT	PL11	PL12	PL13	PL14	PL15
1	a_{10}	a_{20}	a_2^*	a_1^H	a_2^H
2	a_{10}	a_{20}	a_2^*		
3	a_{10}	a_{20}	a_2^*	m_{a2}	T_0

12. **Yield strength.** The yield strength τ_Y is defined by the parameters in Card 12. Different options are available to define temperature and degree of cure dependent hardening behavior. The most general option (PL2OPT = 1) is a three-dimensional tabular input for $\tau_Y(p,T,r)$, employing *DEFINE_TABLE_3D. Here p is the degree of cure, T is the temperature, and r is the damage plastic strain.

    For PL2OPT = 2, initial strength τ_0 and hardening R are defined independently with tabular data. Their sum represents the current yield strength:

    $$\tau_Y(p,T,r) = \tau_0(p,T) + R(p,T,r).$$

    A two-dimensional table (*DEFINE_TABLE_2D) is required to define $\tau_0(p,T)$ as a function of degree of cure p and temperature T. The hardening part $R(p,T,r)$ naturally requires a three-dimensional table (*DEFINE_TABLE_3D).

    PL2OPT = 3 employs the same split between initial and hardening part as the second option, but it is further assumed, that the effect of curing can be modelled by different scaling operations $\chi_c(p)$ and $\phi_c(p)$:

    $$\tau_Y(p,T,r) = \tau_{0\theta}(T)\chi_c(p) + R_\theta(T,r)\phi_c(p).$$

    The input only requires a two-dimensional tabular input for the hardening $R_\theta(T,r)$ and three load curve definitions (see *DEFINE_CURVE) for $\tau_{0\theta}(T)$, $\chi_c(p)$ and $\phi_c(p)$.

    PL2OPT = 4 only differs from PL2OPT = 3 in the input of the temperature dependent hardening part $R_\theta(T,r)$. Instead of tabular data, an exponential hardening behavior is assumed. When PL2OPT = 4 is invoked, the following analytical expression for $R_\theta(T,r)$ is used:

    $$R_\theta(T,r) = H_\theta(T)r + q_\theta(T)\left(1 - e^{-b_\theta(T)r}\right)\ .$$

    Temperature dependencies for the parameters H_θ, q_θ, and b_θ requires load curve input.

    The simplest version is invoked by PL2OPT = 5, where the yield strength $\tau_Y(r)$ is a function solely of the plastic strain data. Again, an exponential hardening is assumed:

    $$\tau_Y(r) = \tau_0 + Hr + q\left(1 - e^{-br}\right).$$

    The input of the parameters is shown in the following table.

PL2OPT	PL21	PL22	PL23	PL24	PL25	PL26
1	$\tau_Y(p,T,r)$					
2	$\tau_0(p,T)$	$R(p,T,r)$				
3	$\tau_{0\theta}(T)$	$\chi_c(p)$	$R_\theta(T,r)$	$\phi_c(p)$		

PL2OPT	PL21	PL22	PL23	PL24	PL25	PL26
4	$\tau_{0\theta}(T)$	$\chi_c(p)$	$q_\theta(T)$	$b_\theta(T)$	$H_\theta(T)$	$\phi_c(p)$
5	τ_0	q	B	H		

13. **Material damage.** Material damage can occur for this material when in a solid-like state. The material becomes solid-like when the current degree of cure p reaches the gelation point p_{gel}, given by parameter PGEL. A different damage mechanism occurs in the liquid phase, as discussed in Remark 14.

    The material damage is described in terms of the damage parameter D_1. Its evolution is based on the approach in Lemaitre [1985]. For $p \geq p_{\mathrm{gel}}$, the general formulation can be defined in terms of a chosen strain measure ζ as follows:

    $$\dot{D}_1 = \dot{D}_1(\zeta, \dot{\zeta}) = n \left\langle \frac{\zeta - \gamma^{\mathrm{c}}}{\gamma^{\mathrm{f}} - \gamma^{\mathrm{c}}} \right\rangle^{n-1} \frac{\dot{\zeta}}{\gamma^{\mathrm{f}} - \gamma^{\mathrm{c}}} \ .$$

    The parameter DAEVO defines the strain measure ζ. For DAEVO = 0, the arc length of the damage plastic strain rate is used: $\dot{\zeta} = \dot{r}$. The arc length of plastic strain rate $\dot{\gamma}_{\mathrm{v}}$ governs the damage evolution for DAEVO = 1, that is, $\dot{\zeta} = \dot{\gamma}_{\mathrm{v}}$. DAEVO = 2 employs the viscoelastic-plastic strain rate $\dot{\gamma}$ as strain rate measure:

    $$\dot{\zeta} = \dot{\gamma} = \sqrt{2\mathrm{tr}((\dot{\boldsymbol{\varepsilon}}^{\mathrm{vp}})^2)}, \qquad \dot{\boldsymbol{\varepsilon}}^{\mathrm{vp}} = \dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{\mathrm{th}} - \dot{\boldsymbol{\varepsilon}}^{\mathrm{ch}} \ .$$

    The strains at the thresholds γ_{c} and γ_{f} for damage initiation and rupture depend on a function $\xi(\eta)$ of the triaxiality η. This function $\xi(\eta)$ determines if triaxiality is considered only under tensile loading or under tensile and compressive loading. Consequently, there are two choices available: For DATRIAX = 0, the Macauley bracket is used ($\xi(\eta) = \langle \eta \rangle$), whereas DATRIX = 1 reduces ξ to the identity ($\xi(\eta) = \eta$).

    The particular equations for the strain thresholds γ_{c} and γ_{f} are determined by the damage option parameter DAOPT. Choosing DAOPT = 1 allows for temperature and degree of cure dependence:

    $$\gamma^{\mathrm{c}} = \left(d_1^{\mathrm{c}} + d_2^{\mathrm{c}} \left(e^{-d_3 \xi(\eta)} \right) \right) d_\theta(T) \beta(p)$$
    $$\gamma^{\mathrm{f}} = \left(d_1 + d_2 \left(e^{-d_3 \xi(\eta)} \right) \right) d_\theta(T) \delta(p)$$

    Functions $d_\theta(T)$, $\beta(p)$, and $\delta(p)$ each require a load curve input. These functions are omitted in the simplified option DAOPT = 2, for which the strain thresholds reduce to

    $$\gamma^{\mathrm{c}} = \left(d_1^{\mathrm{c}} + d_2^{\mathrm{c}} \left(e^{-d_3 \xi(\eta)} \right) \right)$$
    $$\gamma^{\mathrm{f}} = \left(d_1 + d_2 \left(e^{-d_3 \xi(\eta)} \right) \right)$$

Strain rate effects for the definition of the thresholds are incorporated into option DAOPT = 3 following Johnson and Cook [1985], which leads to

$$\gamma^{\mathrm{c}} = \left(d_1^{\mathrm{c}} + d_2^{\mathrm{c}}\left(e^{-d_3\xi(\eta)}\right)\right) d_\theta(T)\beta(p)\left(1 + d_4\left\langle \ln \dot{\gamma}/\dot{\gamma}_0 \right\rangle\right)$$
$$\gamma^{\mathrm{f}} = \left(d_1 + d_2\left(e^{-d_3\xi(\eta)}\right)\right) d_\theta(T)\delta(p)\left(1 + d_4\left\langle \ln \dot{\gamma}/\dot{\gamma}_0 \right\rangle\right)$$

The parameter input is summarized in the following table:

DAOPT	DA1	DA2	DA3	DA4	DA5	DA6
1	n	d_1^c	d_2^c	d_1	d_2	d_3
2	n	d_1^c	d_2^c	d_1	d_2	d_3
3	n	d_1^c	d_2^c	d_1	d_2	d_3

DAOPT	DA7	DA8	DA9	DA10	DA11
1	$d_\theta(T)$	$\beta(p)$	$\delta(p)$		
2					
3	$d_\theta(T)$	$\beta(p)$	$\delta(p)$	d_4	$\dot{\gamma}_0$

14. **Viscous fingering.** Viscous fingering can occur if the connected partners (partially) separate while the connecting adhesive is still in the liquid phase, resulting in an incomplete bonding of the partners. For this material model we implemented a rather simple phenomenological approach. An additional damage parameter D_2 models the effect of viscous fingering. D_2 accounts for the reduction of the effective adhesive area. Furthermore, we assume that D_2 can be expressed as function $\delta_A(\varepsilon_{33})$ of the thickness strain ε_{33} of the element.

    In the liquid phase (meaning $p < p_{\mathrm{gel}}$), this damage mechanism is active if parameter PDA1 is nonzero. For positive values of PDA1, the parameter is interpreted as scalar input β_A for an exponential approach:

    $$D_2 = \delta_A(\varepsilon_{33}) = 1 - \exp(-\beta_A\varepsilon_{33})$$

    Alternatively, a negative input for PDA1 implies a direct input of $D_2 = \delta_A(\varepsilon_{33})$ as a load curve with ID = |PDA1|.

    As soon as the degree of cure exceeds the gelation point p_{gel}, the mechanism is stopped and the damage parameter D_2 remains constant. The value for p_{gel} is to be given as input parameter PGEL.

15. **Histories variables.** The most important history variables are listed in the following table:

History Variable #	Description
1	Temperature, T
2	Degree of cure, p
3	Chemical expansion
4	Thermal expansion
5	Initial temperature, T_0
6	Material damage parameter, D_1
7	Viscous fingering damage parameter, D_2
8	Effective strain measure, ζ, for material damage
9	Thickness strain ε_{33}
10	Current effective shear modulus, $G(t)$
11	Current effective bulk modulus, $K(t)$
12	Concentration c_1 of species A
13	Concentration c_2 of species B
14	Concentration c_3 of species C
15	Concentration c_4 of species D

16. **Diffusion control mechanism definition.** Diffusion processes in the material may affect the curing process of the material. For example, they may slow down the chemical reaction for temperatures around the glass transition temperature, T_G, which is itself a function of the degree of cure, p. Parameter CDOPT sets the model giving the diffusion effect, $K^{\mathrm{diff}}(T)$, discussed in Remark 2. CTGOPT defines the relationship between $T_G(p)$ and p needed by the CDOPT models.

We begin with a discussion of the models specified with CTGOPT because $T_G(p)$ is needed by the diffusion effect models. Currently, the material distinguishes between two different models for $T_G(p)$: DiBenedetto and Hesekamp. The DiBenedetto model (CTGOPT = 1) requires the input of the glass transition temperatures for uncured (T_{G0}) and for fully cured ($T_{G\infty}$) material. With a transition parameter λ, the DiBenedetto equations then reads:

$$T_G(p) = T_{G0} + \lambda\, p\, \frac{T_{G\infty} - T_{G0}}{1 - (1 - \lambda)p}.$$

The Hesekamp equation (CTGOPT = 2) is given in terms of the glass transition temperature for uncured material (T_{G0}) and two algorithmic parameters (g_1 and g_2):

$$T_G(p) = T_{G0} \exp\left(\frac{g_1 p}{g_2 - p}\right).$$

The following table summarizes how the necessary parameters are defined in the input of Card 4.

CTGOPT	CTG1	CTG2	CTG3
1	T_{G0}	$T_{G\infty}$	λ
2	T_{G0}	g_1	g_2

With $T_G(p)$ defined, we can describe the models for $K^{\text{diff}}(T)$ set with CDOPT. CDOPT = 1 invokes a function similar to the Williams-Landel-Ferry (WLF) shift function:

$$K^{\text{diff}}(T) = k^{\text{diff}} \exp\left(\frac{c_1(T - T_G(p))}{c_2 + T - T_G(p)}\right).$$

CDOPT = 2 models the diffusion effect with an equation closely related to the Arrhenius shift function:

$$K^{\text{diff}}(T) = k^{\text{diff}} \exp\left(\frac{-c_1 T_g(p)^2}{c_2}\left(\frac{1}{T} - \frac{1}{T_G(p)}\right)\right).$$

Finally, CDOPT = 3 combines the above approaches and switches formulations at the glass transition temperature:

$$K^{\text{diff}}(T) = \begin{cases} k^{\text{diff}} \exp\left(\dfrac{c_1(T - T_G(p))}{c_2 + T - T_G(p)}\right) & T \le T_G(p) \\ k^{\text{diff}} \exp\left(\dfrac{-c_1 T_g(p)^2}{c_2}\left(\dfrac{1}{T} - \dfrac{1}{T_G(p)}\right)\right) & T > T_G(p) \end{cases}$$

In all three of the above equations, the input parameter k^{diff} defines the diffusion constant at the glass transition temperature ($k^{\text{diff}} = K^{\text{diff}}(T_g(p))$). In addition, these models require two material parameters, c_1 and c_2. The following table shows the parameters input in Card 3:

CDOPT	CD1	CD2	CD3
1/2/3	k^{diff}	c_1	c_2

*MAT_RRR_POLYMER

This is Material Type 317. It is for analysis of isotropic polymers, such as thermoplastics. This rheological network model was developed to incorporate rate, relaxation, and recovery effects in plastics up to yield plateau. Damping and creep effects spanning from milliseconds to years can be represented. It works for both the explicit and implicit solver and uses a numerically efficient implementation. Only solid elements are supported.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0						
Type	A	F						

Card 2	1	2	3	4	5	6	7	8
Variable	ESTR1	EEND1	EELIM1	PR1				
Type	F	F	F	F				

Card 3	1	2	3	4	5	6	7	8
Variable	ESTR2	EEND2	EELIM2	PR2				
Type	F	F	F	F				

Card 4	1	2	3	4	5	6	7	8
Variable	ESTR3	EEND3	EELIM3	PR3				
Type	F	F	F	F				

Card 5	1	2	3	4	5	6	7	8
Variable	MSTR1	MEND1	ECLIM1	SGLIM1	A1	PRV1		
Type	F	F	F	F	F	F		

Card 6	1	2	3	4	5	6	7	8
Variable	MSTR2	MEND2	ECLIM2	SGLIM2	A2	PRV2		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
ESTR*i*	Starting Young's modulus, E_{s_i}, in link i, $i = 1,2,3$.
EEND*i*	Ending Young's modulus, E_{e_i}, in link i, $i = 1,2,3$.
EELIM*i*	Elastic limit, $\bar{\varepsilon}_{e_i}$, in link i, $i = 1,2,3$.
PR*i*	Poisson ratio, ν_i, in link i, $i = 1,2,3$.
MSTR*i*	Starting exponent, m_{s_i}, in link i, $i = 1,2$.
MEND*i*	Ending exponent, m_{e_i}, in link i, $i = 1,2$.
ECLIM*i*	Creep strain limit, $\bar{\varepsilon}_{c_i}$, in link i, $i = 1,2$.
SGLIM*i*	Effective stress limit, $\bar{\sigma}_i$, in link i, $i = 1,2$.
A*i*	Reference creep strain rate, α_i, in link i, $i = 1,2$.
PRV*i*	Viscous Poisson ratio, μ_i, in link i, $i = 1,2$. Default: 0.5.

Remarks:

1. **Material model.** The material model is due to M. Lindvall and is composed of two viscoelastic links and one purely elastic link. Each link is characterized by its own set of parameters resulting in a Cauchy stress, $\boldsymbol{\sigma}_i$ with $i = 1,2,3$, so the complete stress is given by:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_3.$$

Each link is a hypo-elasto-viscoelastic model with a stress rate on the form[5]

$$\dot{\boldsymbol{\sigma}} = \mathbf{C}(\varepsilon_e)(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_c).$$

Here $\mathbf{C}(\varepsilon_e)$ is the isotropic Hooke tensor which depends on the effective elastic strain:

$$\varepsilon_e = \sqrt{\frac{1}{1 + 2\mu^2}\boldsymbol{\varepsilon}_e^{\mathrm{dev}} : \boldsymbol{\varepsilon}_e^{\mathrm{dev}}}.$$

The elastic strain quantities are given by $\boldsymbol{\varepsilon}_e = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_c$, $\varepsilon_e^{\mathrm{mean}} = \frac{1}{3}\boldsymbol{\varepsilon}_e : \mathbf{I}$, and $\boldsymbol{\varepsilon}_e^{\mathrm{dev}} = \boldsymbol{\varepsilon}_e - \varepsilon_e^{\mathrm{mean}}\mathbf{I}$. μ is the viscous Poisson ratio. The exact expression for $\mathbf{C}(\varepsilon_e)$ is indirectly defined by a strain-dependent Young's modulus $E(\varepsilon_e)$ and constant Poisson ratio, ν:

$$E(\varepsilon_e) = E_s + (E_e - E_s)\tanh\left(\frac{\varepsilon_e}{\bar{\varepsilon}_e}\right).$$

$\bar{\varepsilon}_e$ is the elastic limit, and E_s and E_e are the starting and ending Young's moduli. These are all input on Cards 2 through 4, except for μ which is input on Cards 5 and 6.

The creep strain tensor evolves as[6]

$$\dot{\boldsymbol{\varepsilon}}_c = a\left(\frac{\sigma^{\mathrm{eff}}}{\overline{\sigma}}\right)^{m(\varepsilon_c)} \frac{(1 + \upmu)\boldsymbol{\sigma} + 3\upmu \mathrm{p} I}{\sigma^{\mathrm{eff}}},$$

where

$$p = -\frac{1}{3}\boldsymbol{\sigma} : \mathbf{I}$$
$$\mathbf{s} = \boldsymbol{\sigma} + p\mathbf{I}$$
$$\sigma^{\mathrm{eff}} = \sqrt{(1 + \mu)\boldsymbol{\sigma} : \boldsymbol{\sigma} - 9\upmu \mathrm{p}^2}$$

σ^{eff} is the effective stress. $\overline{\sigma}$ is the effective stress limit, μ is the viscous Poisson ratio, and a is the reference creep strain rate. These are all input on Cards 5 and 6. The exponent $m(\varepsilon_c)$ depends on the effective creep strain as

[5] For the sake of convenience, we drop the link subscripts and superscripts, and also emphasize that the rates are to be interpreted as objective.

[6] For elastic link #3, $a = 0$, meaning there is no creep strain.

$$m(\varepsilon_c) = m_s + (m_e - m_s)\tanh\left(\frac{\varepsilon_c}{\bar{\varepsilon}_c}\right),$$

where

$$\varepsilon_c = \sqrt{\frac{1}{1+2\mu^2}\boldsymbol{\varepsilon}_c : \boldsymbol{\varepsilon}_c}.$$

In the above, m_s, m_e, and $\bar{\varepsilon}_c$ are input parameters (see Cards 5 and 6). m_s and m_e are the starting and ending exponents. $\bar{\varepsilon}_c$ is the creep strain limit.

2. **History variables.** This material model outputs 10 history variables. To output the history variables, set the variable NEIPH in *DATABASE_EXTENT_BINARY.

History Variable #	Definition
1	Effective elastic strain for link 1, ε_e^1
2	Effective elastic strain for link 2, ε_e^2
3	Effective elastic strain for link 3, ε_e^3
4	Effective creep strain for link 1, ε_c^1
5	Effective creep strain for link 2, ε_c^2
6	Effective exponent for link 1, $m(\varepsilon_c^1)$
7	Effective exponent for link 2, $m(\varepsilon_c^2)$
8	Effective von Mises stress for link 1, σ_1^{eff}
9	Effective von Mises stress for link 2, σ_2^{eff}
10	Effective von Mises stress for link 3, σ_3^{eff}

References:

Borrvall, T., and Lindvall, M. “A Pragmatic Approach to the Modelling of Nonlinear Rheological Networks for Polymers,” *North American LS-DYNA User Forum 2023*.

*MAT_TNM_POLYMER

This is Material Type 318. It is for the analysis of isotropic polymers, such as thermoplastics. It works for both the explicit and implicit solvers. This keyword is supported for solid elements and some thick shell elements (ELFORM = 3, 5, and 7). However, 2D continuum elements (shell formulations 13, 14, and 15) are *not* supported for implicit.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO						
Type	A	F						

Card 2	1	2	3	4	5	6	7	8
Variable	MUA	THETAH	LAMBL	KAPPA	TAUHA	A	MA	N
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	MUBI	MUBF	BETA	TAUHB	MB	MUC	Q	ALPHA
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	THETA0	IBULK	IG	TSSTIF	GAMMA0			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label be specified (see *PART).
RO	Mass density

VARIABLE	DESCRIPTION
MUA	Shear modulus for network A, μ_A
THETAH	Temperature factor, $\hat{\theta}$
LAMBL	Locking stretch, λ_L
KAPPA	Bulk modulus, κ
TAUHA	Flow resistance of network A, $\hat{\tau}_A$
A	Pressure dependence of flow, a
MA	Stress exponential of network A, m_A
N	Temperature exponential, n
MUBI	Initial shear modulus for network B, μ_{Bi}
MUBF	Final shear modulus for network B, μ_{Bf}
BETA	Evolution rate of shear modulus for network B, β
TAUHB	Flow resistance of network B, $\hat{\tau}_B$
MB	Stress exponential of network B, m_B
MUC	Shear modulus for network C, μ_C
Q	Relative contribution of I_1 and I_2 on network C, q
ALPHA	Thermal expansion coefficient
THETA0	Reference temperature, θ_0
IBULK	Internal bulk modulus
IG	Internal shear modulus
TSSTIF	Transversal stiffness for shells
GAMMA0	Reference strain rate

Remarks:

1. **Material model.** The material model is due to J. Bergström and consists of a rheologic network of three hyperelastic springs: A, B, and C. The springs act in

parallel so that the total deformation gradient ($\mathbf{F}^{\mathrm{tot}}$), and thus the total strain, is the same for each of them. The deformation gradient is made up of both a thermal part, $\mathbf{F}^{\mathrm{th}}$, and a mechanical part, $\mathbf{F}$, in a multiplicative manner: $\mathbf{F}^{\mathrm{tot}} = \mathbf{F}\mathbf{F}^{\mathrm{th}}$. The thermal part takes the form $\mathbf{F}^{\mathrm{th}} = (1 + \alpha(\theta - \theta_0))\mathbf{I}$, with α as the thermal expansion coefficient, θ as the temperature, θ_0 as a reference temperature, and $\mathbf{I}$ as the unit tensor. The mechanical part, $\mathbf{F}$, depends on the network.

In network A, $\mathbf{F}$ is multiplicatively decomposed into elastic and viscoplastic parts

$$\mathbf{F} = \mathbf{F}_A^e \mathbf{F}_A^v \ .$$

The Cauchy stress is defined by a temperature-dependent variant of the Arruda-Boyce eight-chain model

$$\boldsymbol{\sigma}_A = \frac{\mu_A}{J_A^e \overline{\lambda_A^e}} \left(1 + \frac{\theta - \theta_0}{\hat{\theta}}\right) \frac{\mathcal{L}^{-1}\left(\dfrac{\overline{\lambda_A^e}}{\lambda_L}\right)}{\mathcal{L}^{-1}\left(\dfrac{1}{\lambda_L}\right)} \operatorname{dev}(\mathbf{b}_A^e) + \kappa(J_A^e - 1)\mathbf{I} \ ,$$

where μ_A is the (constant) shear modulus, and κ is the bulk modulus. $\mathcal{L}^{-1}$ is the inverse of the Langevin function $\mathcal{L}(x) = \coth(x) - 1/x$. In the above,

$$\begin{aligned} J_A^e &= \det(\mathbf{F}_A^e) \\ \mathbf{b}_A^e &= (J_A^e)^{-\frac{2}{3}} \mathbf{F}_A^e (\mathbf{F}_A^e)^T \\ \overline{\lambda_A^e} &= \sqrt{\frac{\operatorname{tr}(\mathbf{b}_A^e)}{3}} \end{aligned}$$

$\mathbf{b}_A^e$ is the Cauchy-Green strain tensor. $\overline{\lambda_A^e}$ is the so-called chain stretch, while λ_L is the chain-locking stretch.

Similar to network A, the Cauchy stress for network B is given by the eight-chain model

$$\boldsymbol{\sigma}_B = \frac{\mu_B}{J_B^e \overline{\lambda_B^e}} \left(1 + \frac{\theta - \theta_0}{\hat{\theta}}\right) \frac{\mathcal{L}^{-1}\left(\dfrac{\overline{\lambda_B^e}}{\lambda_L}\right)}{\mathcal{L}^{-1}\left(\dfrac{1}{\lambda_L}\right)} \operatorname{dev}(\mathbf{b}_B^e) + \kappa(J_B^e - 1)\mathbf{I} \ ,$$

where now

$$\begin{aligned} \mathbf{F} &= \mathbf{F}_B^e \mathbf{F}_B^v \\ \mathbf{b}_B^e &= (J_B^e)^{-\frac{2}{3}} \mathbf{F}_B^e (\mathbf{F}_B^e)^T \\ J_B^e &= \det(\mathbf{F}_B^e) \\ \overline{\lambda_B^e} &= \sqrt{\frac{\operatorname{tr}(\mathbf{b}_B^e)}{3}} \ . \end{aligned}$$

However, unlike in network A, the shear modulus, μ_B, in network B evolves with plastic strain from a starting value μ_{Bi} to a final value μ_{Bf} according to

$$\dot{\mu}_B = -\beta(\mu_B - \mu_{Bf})\dot{\gamma}_A \ ,$$

Here, $\dot{\gamma}_A$ is the viscoplastic flow rate in network A defined by

$$\dot{\gamma}_A = \dot{\gamma}_0 \left(\frac{\tau_A}{\hat{\tau}_A + aR(p_A)}\right)^{m_A} \left(\frac{\theta}{\theta_0}\right)^n \ ,$$

with pressure $p_A = -\mathrm{tr}(\boldsymbol{\sigma}_A)/3$ and von Mises-like stress $\tau_A = \sqrt{\mathrm{dev}(\boldsymbol{\sigma}_A)\colon\mathrm{dev}(\boldsymbol{\sigma}_A)}$. $\hat{\tau}_A$ is the flow resistance, and a, β, m_A, n, and $\dot{\gamma}_0$ are other given material parameters. $R(x) = (x + |x|)/2$ is a ramp function.

The viscoplastic deformation gradient in network A is

$$\dot{\mathbf{F}}_A^v = \dot{\gamma}_A \mathbf{F}_A^{e\,-1} \mathrm{dev}(\boldsymbol{\sigma}_A)\mathbf{F}/\tau_A \ ,$$

and a similar relation holds for $\mathbf{F}_B^v$.

In network C, the Cauchy stress is, again, defined by a variant of the eight-chain model

$$(1+q)\boldsymbol{\sigma}_C = \frac{\mu_C}{J\bar{\lambda}}\left(1 + \frac{\theta - \theta_0}{\hat{\theta}}\right)\frac{\mathcal{L}^{-1}\left(\frac{\bar{\lambda}}{\lambda_L}\right)}{\mathcal{L}^{-1}\left(\frac{1}{\lambda_L}\right)}\mathrm{dev}(\mathbf{b}) + \kappa(J_B^e - 1)\mathbf{I}$$
$$+q\frac{\mu_C}{J}\left(I_1\mathbf{b} - \frac{2I_2}{3}\mathbf{I} - \mathbf{b}^2\right) \ ,$$

where

$$\begin{aligned}\mathbf{b} &= (J)^{-\frac{2}{3}}\mathbf{F}(\mathbf{F})^T \\ J &= \det(\mathbf{F}) \\ \bar{\lambda} &= \sqrt{\frac{\mathrm{tr}(\mathbf{b})}{3}} \ .\end{aligned}$$

I_1 and I_2 are the 1st and 2nd invariants of $\mathbf{b}$. The parameter q controls the influence of these invariants. μ_C is the (constant) shear modulus.

The total stress is the sum of the stress in the network:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_A + \boldsymbol{\sigma}_B + \boldsymbol{\sigma}_C \ .$$

2. **History variables.** This material model outputs 21 history variables. To output the history variables, set the variable NEIPH in *DATABASE_EXTENT_BINARY. History variables #1-9 are the components of the viscoplastic deformation gradient, $\mathbf{F}_A^v$, in network A. Similarly, history variables #10-18 are the components of $\mathbf{F}_B^v$. History variable #19 is the shear modulus, μ_B. In addition, for implicit simulations, history variables #20 and #21 are the accumulated plastic strains γ_A and γ_B.