

**\*MAT\_ORTHOTROPIC\_VISCOELASTIC**

This is Material Type 86. It allows for the definition of an orthotropic material with a viscoelastic part. This model applies to shell elements.

**NOTE:** This material does not support specification of a material angle,  $\beta_i$ , for each through-thickness integration point of a shell.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	EA	EB	EC	VF	K	
Type	A	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	G0	GINF	BETA	PRBA	PRCA	PRCB		
Type	F	F	F	F	F	F		

Card 3	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	AOPT	MANGLE			
Type	F	F	F	F	F			

Card 4	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

**\*MAT\_086****\*MAT\_ORTHOTROPIC\_VISCOELASTIC**

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
EA	Young's Modulus $E_a$
EB	Young's Modulus $E_b$
EC	Young's Modulus $E_c$
VF	Volume fraction of viscoelastic material
K	Elastic bulk modulus
G0	$G_0$ , short-time shear modulus
GINF	$G_\infty$ , long-time shear modulus
BETA	$\beta$ , decay constant
PRBA	Poisson's ratio, $\nu_{ba}$
PRCA	Poisson's ratio, $\nu_{ca}$
PRCB	Poisson's ratio, $\nu_{cb}$
GAB	Shear modulus, $G_{ab}$
GBC	Shear modulus, $G_{bc}$
GCA	Shear modulus, $G_{ca}$
AOPT	Material axes option (see <a href="#">*MAT_OPTIONTROPIC_ELASTIC</a> for a more complete description): EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with <a href="#">*DEFINE_COORDI-</a>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	NATE_NODES, and then rotated about the shell element normal by an angle MANGLE
EQ.2.0:	Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_ECTOR
EQ.3.0:	Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, MANGLE, from a line in the plane of the element defined by the cross product of the vector v with the element normal
LT.0.0:	The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
MANGLE	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card; see *ELEMENT_SHELL_BETA.
A1 A2 A3	Define components of vector <b>a</b> for AOPT = 2
V1 V2 V3	Define components of vector <b>v</b> for AOPT = 3
D1 D2 D3	Define components of vector <b>d</b> for AOPT = 2

**Remarks:**

See material types [2](#) and [24](#) for the orthotropic definition.

## \*MAT\_087

## \*MAT\_CELLULAR\_RUBBER

### \*MAT\_CELLULAR\_RUBBER

This is Material Type 87. This material model provides a cellular rubber model with confined air pressure combined with linear viscoelasticity as outlined by Christensen [1980]. See [Figure M87-1](#).

#### Card Summary:

**Card 1.** This card is required.

MID	RO	PR	N				
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**Card 2a.** This card is included if and only if  $N > 0$ .

SGL	SW	ST	LCID				
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**Card 2b.** This card is included if and only if  $N = 0$ .

C10	C01	C11	C20	C02			
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**Card 3.** This card is required.

P0	PHI	IVS	G	BETA			
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#### Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	PR	N				
Type	A	F	F	I				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
PR	Poisson's ratio; typical values are between 0.0 to 0.2. Due to the large compressibility of air, large values of Poisson's ratio generate physically meaningless results.

VARIABLE	DESCRIPTION
N	Order of fit for material model (currently < 3). If N > 0, then a least square fit is computed with uniaxial data. The parameters given on Card 2a should be specified. Also see *MAT_MOONEY_RIVLIN_RUBBER (material model 27). A Poisson's ratio of .5 is assumed for the void free rubber during the fit. The Poisson's ratio defined on Card 1 is for the cellular rubber. A void fraction formulation is used.

**Material Least Squares Fit Card.** Card 2 if N > 0, a least squares fit is computed from uniaxial data

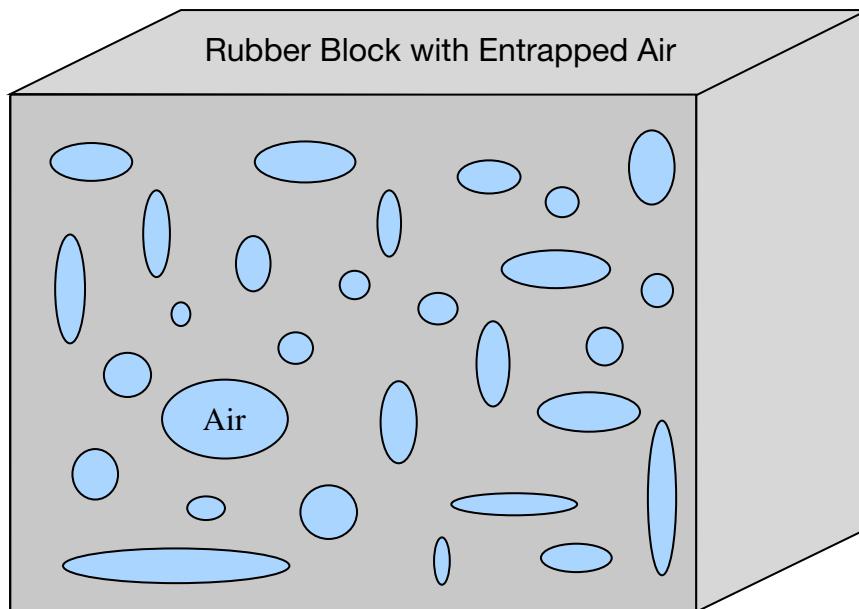
Card 2a	1	2	3	4	5	6	7	8
Variable	SGL	SW	ST	LCID				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
SGL	Specimen gauge length, $l_0$
SW	Specimen width
ST	Specimen thickness
LCID	Load curve ID giving the force as a function of actual change in the gauge length, $\Delta L$ . If SGL, SW, and ST are set to unity (1.0), then curve LCID is also engineering stress as a function of engineering strain.

**Material Constants Card.** Card 2 if N = 0, define the following constants

Card 2b	1	2	3	4	5	6	7	8
Variable	C10	C01	C11	C20	C02			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION
C10	Coefficient, $C_{10}$



**Figure M87-1.** Cellular rubber with entrapped air. By setting the initial air pressure to zero, an open cell, cellular rubber can be simulated.

VARIABLE	DESCRIPTION							
C01	Coefficient, $C_{01}$							
C11	Coefficient, $C_{11}$							
C20	Coefficient, $C_{20}$							
C02	Coefficient, $C_{02}$							

Card 3	1	2	3	4	5	6	7	8
Variable	P0	PHI	IVS	G	BETA			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION							
P0	Initial air pressure, $p_0$							
PHI	Ratio of cellular rubber to rubber density, $\phi$							
IVS	Initial volumetric strain, $\gamma_0$							
G	Optional shear relaxation modulus, $G$ , for rate effects (viscosity)							

VARIABLE	DESCRIPTION
BETA	Optional decay constant, $\beta_1$

**Remarks:**

Rubber is generally considered to be fully incompressible since the bulk modulus greatly exceeds the shear modulus in magnitude. To model the rubber as an unconstrained material a hydrostatic work term,  $W_H(J)$ , is included in the strain energy functional which is function of the relative volume,  $J$ , [Ogden 1984]:

$$W(J_1, J_2, J) = \sum_{p,q=0}^n C_{pq} (J_1 - 3)^p (J_2 - 3)^q + W_H(J)$$

$$J_1 = I_1 I_3^{-1/3}$$

$$J_2 = I_2 I_3^{-2/3}$$

In order to prevent volumetric work from contributing to the hydrostatic work the first and second invariants are modified as shown. This procedure is described in more detail by Sussman and Bathe [1987].

The effects of confined air pressure in its overall response characteristics is included by augmenting the stress state within the element by the air pressure, that is,

$$\sigma_{ij} = \sigma_{ij}^{sk} - \delta_{ij} \sigma^{air} ,$$

where  $\sigma_{ij}^{sk}$  is the bulk skeletal stress and  $\sigma^{air}$  is the air pressure.  $\sigma^{air}$  is computed from:

$$\sigma^{air} = -\frac{p_0 \gamma}{1 + \gamma - \phi} ,$$

where  $p_0$  is the initial foam pressure usually taken as the atmospheric pressure and  $\gamma$  defines the volumetric strain. The volumetric is found with

$$\gamma = V - 1 + \gamma_0 ,$$

where  $V$  is the relative volume of the voids and  $\gamma_0$  is the initial volumetric strain which is typically zero. The rubber skeletal material is assumed to be incompressible.

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau ,$$

or in terms of the second Piola-Kirchhoff stress,  $S_{ij}$ , and Green's strain tensor,  $E_{ij}$ ,

$$S_{ij} = \int_0^t G_{ijkl}(t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau ,$$

where  $g_{ijkl}(t - \tau)$  and  $G_{ijkl}(t - \tau)$  are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

Since we wish to include only simple rate effects, the relaxation function is represented by one term from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^N \alpha_m e^{-\beta_m t}$$

given by,

$$g(t) = E_d e^{-\beta_1 t}.$$

This model is effectively a Maxwell fluid which consists of a damper and spring in series. We characterize this in the input by a shear modulus,  $G$ , and decay constant,  $\beta_1$ .

The Mooney-Rivlin rubber model (model 27) is obtained by specifying  $N = 1$  without air pressure and viscosity. In spite of the differences in formulations with Model 27, we find that the results obtained with this model are nearly identical with those of material type 27 as long as large values of Poisson's ratio are used.

**\*MAT\_MTS**

This is Material Type 88. The MTS model is due to Mauldin, Davidson, and Henninger [1990] and is available for applications involving large strains, high pressures and strain rates. As described in the foregoing reference, this model is based on dislocation mechanics and provides a better understanding of the plastic deformation process for ductile materials by using an internal state variable call the mechanical threshold stress. This kinematic quantity tracks the evolution of the material's microstructure along some arbitrary strain, strain rate, and temperature-dependent path using a differential form that balances dislocation generation and recovery processes. Given a value for the mechanical threshold stress, the flow stress is determined using either a thermal-activation-controlled or a drag-controlled kinetics relationship. An equation-of-state is required for solid elements and a bulk modulus must be defined below for shell elements.

**Card Summary:**

**Card 1.** This card is required.

MID	R0	SIGA	SIGI	SIGS	SIG0	BULK	
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**Card 2.** This card is required.

HF0	HF1	HF2	SIGSO	EDOTSO	BURG	CAPA	BOLTZ
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**Card 3.** This card is required.

SM0	SM1	SM2	EDOTO	GO	PINV	QINV	EDOTI
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**Card 4.** This card is required.

GOI	PINVI	QINVI	EDOTS	GOS	PINVS	QINVS	
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**Card 5.** This card is required.

RHOCPR	TEMPPRF	ALPHA	EPS0				
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	SIGA	SIGI	SIGS	SIG0	BULK	
Type	A	F	F	F	F	F	F	

**\*MAT\_088****\*MAT\_MTS**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
SIGA	$\hat{\sigma}_a$ , dislocation interactions with long-range barriers (force/area)
SIGI	$\hat{\sigma}_i$ , dislocation interactions with interstitial atoms (force/area)
SIGS	$\hat{\sigma}_s$ , dislocation interactions with solute atoms (force/area)
SIG0	$\hat{\sigma}_0$ , initial value of $\hat{\sigma}$ at zero plastic strain (force/area) NOT USED.
BULK	Bulk modulus defined for shell elements only. Do not input for solid elements.

Card 2	1	2	3	4	5	6	7	8
Variable	HF0	HF1	HF2	SIGS0	EDOTS0	BURG	CAPA	BOLTZ
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
HF0	$a_0$ , dislocation generation material constant (force/area)
HF1	$a_1$ , dislocation generation material constant (force/area)
HF2	$a_2$ , dislocation generation material constant (force/area)
SIGS0	$\hat{\sigma}_{eso}$ , saturation threshold stress at 0° K (force/area)
EDOTS0	$\dot{\varepsilon}_{eso}$ , reference strain-rate (time <sup>-1</sup> ).
BURG	Magnitude of Burgers vector (interatomic slip distance)
CAPA	Material constant, $A$
BOLTZ	Boltzmann's constant, $k$ (energy/degree).

Card 3	1	2	3	4	5	6	7	8
Variable	SM0	SM1	SM2	EDOTO	G0	PINV	QINV	EDOTI
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SM0	$G_0$ , shear modulus at zero degrees Kelvin (force/area)
SM1	$b_1$ , shear modulus constant (force/area)
SM2	$b_2$ , shear modulus constant (degree)
EDOTO	$\dot{\epsilon}_o$ , reference strain-rate (time <sup>-1</sup> )
G0	$g_0$ , normalized activation energy for a dislocation/dislocation interaction
PINV	$1/p$ , material constant
QINV	$1/q$ , material constant
EDOTI	$\dot{\epsilon}_{o,i}$ , reference strain-rate (time <sup>-1</sup> )

Card 4	1	2	3	4	5	6	7	8
Variable	G0I	PINVI	QINVI	EDOTS	GOS	PINVS	QINVS	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
G0I	$g_{0,i}$ , normalized activation energy for a dislocation/interstitial interaction
PINVI	$1/p_i$ , material constant
QINVI	$1/q_i$ , material constant
EDOTS	$\dot{\epsilon}_{o,s}$ , reference strain-rate (time <sup>-1</sup> )

**\*MAT\_088****\*MAT\_MTS**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
G0S	$g_{0,s}$ , normalized activation energy for a dislocation/solute interaction
PINVS	$1/p_s$ , material constant
QINVS	$1/q_s$ , material constant

Card 5	1	2	3	4	5	6	7	8
Variable	RHOCPR	TEMPPRF	ALPHA	EPS0				
Type	F	F	F	F				

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RHOCPR	$\rho c_p$ , product of density and specific heat
TEMPPRF	$T_{ref}$ , initial element temperature in degrees K
ALPHA	$\alpha$ , material constant (typical value is between 0 and 2)
EPS0	$\varepsilon_o$ , factor to normalize strain rate in the calculation of $\Theta_o$ (time <sup>-1</sup> )

**Remarks:**

The flow stress  $\sigma$  is given by:

$$\sigma = \hat{\sigma}_a + \frac{G}{G_0} [s_{th} \hat{\sigma} + s_{th,i} \hat{\sigma}_i + s_{th,s} \hat{\sigma}_s] .$$

The first product in the equation for  $\sigma$  contains a micro-structure evolution variable,  $\hat{\sigma}$ , which is multiplied by a constant-structure deformation variable  $s_{th}$ :  $s_{th}$ .  $\hat{\sigma}$  is the *Mechanical Threshold Stress* (MTS) and is a function of absolute temperature,  $T$ , and the plastic strain-rates,  $\dot{\varepsilon}^P$ . The evolution equation for  $\hat{\sigma}$  is a differential hardening law representing dislocation-dislocation interactions:

$$\frac{\partial}{\partial \dot{\varepsilon}^P} \equiv \Theta_o \left[ 1 - \frac{\tanh \left( \alpha \frac{\hat{\sigma}}{\hat{\sigma}_{es}} \right)}{\tanh(\alpha)} \right] .$$

The term  $\frac{\partial \hat{\sigma}}{\partial \dot{\varepsilon}^p}$  represents the hardening due to dislocation generation while the stress ratio,  $\frac{\hat{\sigma}}{\hat{\sigma}_{es}}$ , represents softening due to dislocation recovery. The threshold stress at zero strain-hardening,  $\hat{\sigma}_{es}$ , is called the saturation threshold stress.  $\Theta_o$  is given as:

$$\Theta_o = a_o + a_1 \ln \left( \frac{\dot{\varepsilon}^p}{\varepsilon_0} \right) + a_2 \sqrt{\frac{\dot{\varepsilon}^p}{\varepsilon_0}}$$

which contains the material constants,  $a_o$ ,  $a_1$ , and  $a_2$ . The constant,  $\hat{\sigma}_{es}$ , is given as:

$$\hat{\sigma}_{es} = \hat{\sigma}_{eso} \left( \frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_{eso}} \right)^{kT/Gb^3A}$$

which contains the input constants:  $\hat{\sigma}_{eso}$ ,  $\dot{\varepsilon}_{eso}$ ,  $b$ ,  $A$ , and  $k$ . The shear modulus,  $G$ , appearing in these equations is assumed to be a function of temperature and is given by the correlation.

$$G = G_0 - b_1 / (e^{b_2/T} - 1)$$

which contains the constants:  $G_0$ ,  $b_1$ , and  $b_2$ . For thermal-activation controlled deformation  $s_{th}$  is evaluated using an Arrhenius rate equation of the form:

$$s_{th} = \left\{ 1 - \left[ \frac{kT \ln \left( \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}^p} \right)}{Gb^3 g_0} \right]^{\frac{1}{q}} \right\}^{\frac{1}{p}} .$$

The absolute temperature is given as:

$$T = T_{ref} + \frac{E}{\rho c_p} ,$$

where  $E$  is the internal energy density per unit initial volume.

**\*MAT\_089****\*MAT\_PLASTICITY\_POLYMER****\*MAT\_PLASTICITY\_POLYMER**

This is Material Type 89. An elasto-plastic material with an arbitrary stress as a function of strain curve and arbitrary strain rate dependency can be defined. It is intended for applications where the elastic and plastic sections of the response are not as clearly distinguishable as they are for metals. Rate dependency of failure strain is included. Many polymers show a more brittle response at high rates of strain. This material is supported for the commonly used solid, shell, and thick shell elements. 2D plane strain stress, plane strain, and axisymmetric elements are *not* supported.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR				
Type	A	F	F	F				
Default	none	none	none	none				

Card 2	1	2	3	4	5	6	7	8
Variable	C	P	LCSS	LCSR				
Type	F	F	I	I				
Default	0	0	0	0				

Card 3	1	2	3	4	5	6	7	8
Variable	EFTX	DAMP	RFAC	LCFAIL	NUMINT			
Type	F	F	F	I	F			
Default	0	0	0	0	0			

**VARIABLE****DESCRIPTION**

MID

Material identification. A unique number or label must be specified (see \*PART).

VARIABLE	DESCRIPTION
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
C	Strain rate parameter, $C$ , (Cowper Symonds)
P	Strain rate parameter, $P$ , (Cowper Symonds)
LCSS	<p>Load curve ID or Table ID</p> <p><b>Load Curve.</b> When LCSS is a load curve ID, it is taken as defining effective stress as a function of total effective strain.</p> <p><b>Tabular Data.</b> The table ID defines for each strain rate value a load curve ID giving the stress as a function effective strain for that rate.</p> <p><b>Logarithmically Defined Tables.</b> If the <i>first</i> value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate when the lowest strain rate and highest strain rate differ by several orders of magnitude.</p>
LCSR	<p>Load curve ID defining strain rate scaling effect on yield stress. If LCSR is negative, the load curve is evaluated using a binary search for the correct interval for the strain rate. The binary search is slower than the default incremental search, but in cases where large changes in the strain rate may occur over a single time step, it is more robust.</p>
EFTX	<p>Failure flag:</p> <ul style="list-style-type: none"> <li>EQ.0.0: Failure determined by maximum tensile strain (default).</li> <li>EQ.1.0: Failure determined only by tensile strain in local <math>x</math> direction.</li> <li>EQ.2.0: Failure determined only by tensile strain in local <math>y</math> direction.</li> </ul>
DAMP	Viscous damping factor in the units of [stress $\times$ time]. Typical values are $10^{-3}$ Ns/mm <sup>2</sup> or $10^{-4}$ Ns/mm <sup>2</sup> . If set too high, instabilities can result.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RFAC	Filtering factor for strain rate effects. Must be between 0 (no filtering) and 1 (infinite filtering). The filter is a simple low pass filter to remove high frequency oscillation from the strain rates before they are used in rate effect calculations. The cut off frequency of the filter is $[(1 - RFAC) / \text{timestep}] \text{ rad/sec}$ .
LCFAIL	Load curve ID giving variation of failure strain with strain rate. The points on the $x$ -axis should be natural log of strain rate, while the $y$ -axis should be the true strain to failure. Typically, this is measured by a uniaxial tensile test, and the strain values are converted to true strain.
NUMINT	Number of integration points which must fail before the element is deleted. This option is available for shells only.  LT.0.0: $ \text{NUMINT} $ is percentage of integration points/layers which must fail before shell element fails.

**Remarks:**

1. **\*MAT\_089 compared to \*MAT\_024.** \*MAT\_089 is the same as \*MAT\_024 except for the following points:
  - Load curve lookup for yield stress is based on equivalent uniaxial strain, not plastic strain (see [Remarks 2](#) and [3](#)).
  - Elastic stiffness is initially equal to  $E$  but will be increased according to the slope of the stress-strain curve (see [Remark 7](#)).
  - Special strain calculation is used for failure and damage (see [Remark 2](#)).
  - Failure strain depends on strain rate (see [Remark 4](#)).
2. **Strain calculation for failure and damage.** The strain used for failure and damage calculation,  $\varepsilon_{\text{pm}}$ , is based on an approximation of the greatest value of maximum principal strain encountered during the analysis:

$$\varepsilon_{\text{pm}} = \max_{i \leq n} (\varepsilon_H^i + \varepsilon_{\text{vm}}^i) ,$$

where

$n$  = current time step index

$\max_{i \leq n} (\dots)$  = maximum value attained by the argument during the calculation

$$\varepsilon_H = \frac{\varepsilon_x + \varepsilon_y + \varepsilon_z}{3}$$

$\varepsilon_x, \varepsilon_y, \varepsilon_z$  = cumulative strain in the local x, y, or z direction

$\varepsilon_{\text{vm}} = \sqrt{\frac{2}{3} \text{tr}(\boldsymbol{\varepsilon}'^T \boldsymbol{\varepsilon}')}$ , the usual definition of equivalent uniaxial strain  
 $\boldsymbol{\varepsilon}'$  = deviatoric strain tensor, where each  $\varepsilon_x$ ,  $\varepsilon_y$ , and  $\varepsilon_z$  is cumulative

3. **Yield stress load curves.** When looking up yield stress from the load curve LCSS, the  $x$ -axis value is  $\varepsilon_{\text{vm}}$ .

4. **Failure strain load curves.**

$$\varepsilon_{\text{sr}} = \frac{d\varepsilon_{\text{pm}}}{dt} = \text{strain rate for failure and damage calculation}$$

$$\varepsilon_F = \text{LCFAIL}(\varepsilon_{\text{sr}}) \\ = \text{Instantaneous true strain to failure from look-up on the curve LCFAIL}$$

5. **Damage.** A damage approach is used to avoid sudden shocks when the failure strain is reached. Damage begins when the "strain ratio,"  $R$ , reaches 1.0, where

$$R = \int \frac{d\varepsilon_{\text{pm}}}{\varepsilon_F}.$$

Damage is complete, and the element fails and is deleted, when  $R = 1.1$ . The damage,

$$D = \begin{cases} 1.0 & R < 1.0 \\ 10(1.1 - R) & 1.0 < R < 1.1 \end{cases},$$

is a reduction factor applied to all stresses. For example, when  $R = 1.05$ , then  $D = 0.5$ .

6. **Strain definitions.** Unlike other LS-DYNA material models, both the input stress-strain curve and the strain to failure are defined as total true strain, not plastic strain. The input can be defined from uniaxial tensile tests; nominal stress and nominal strain from the tests must be converted to true stress and true strain. The elastic component of strain must not be subtracted out.

7. **Elastic stiffness scaling.** The stress-strain curve is permitted to have sections steeper (i.e. stiffer) than the elastic modulus. When these are encountered the elastic modulus is increased to prevent spurious energy generation. The elastic stiffness is scaled by a factor  $f_e$ , which is calculated as follows:

$$f_e = \max \left( 1.0, \frac{s_{\text{max}}}{3G} \right)$$

where

$G$  = initial shear modulus

$s_{\text{max}}$  = maximum slope of stress-strain curve encountered during the analysis

8. **Precision.** Double precision is recommended when using this material model, especially if the strains become high.

9. **Shell numbering.** Invariant shell numbering is recommended when using this material model. See \*CONTROL\_ACCURACY.

**\*MAT\_ACOUSTIC**

This is Material Type 90. It specifies the material properties of a linear acoustic fluid. This fluid is assumed to be compressible, irrotational, inviscid, and subject to small displacements. This material is intended for general acoustic applications in either the time domain or frequency domain. See Appendix W for a description of applications. Depending on the application, it can be used with the implicit or explicit solvers.

This model is appropriate for tracking low pressure stress waves in an acoustic media, such as air or water, and can be used only with the acoustic pressure element formulation. The acoustic pressure element requires only one unknown per node. This element is very cost effective. Optionally, cavitation can be allowed.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	C	BETA	CF	ATMOS	GRAV	
Type	A	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	XN	YN	ZN		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
C	Sound speed
BETA	Damping factor. Recommended values are between 0.1 and 1.0.
CF	Cavitation flag: EQ.0.0: Off EQ.1.0: On
ATMOS	Atmospheric pressure (optional)

**\*MAT\_090****\*MAT\_ACOUSTIC**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
GRAV	Gravitational acceleration constant (optional)
XP	$x$ -coordinate of free surface point
YP	$y$ -coordinate of free surface point
ZP	$z$ -coordinate of free surface point
XN	$x$ -direction cosine of free surface normal vector
YN	$y$ -direction cosine of free surface normal vector
ZN	$z$ -direction cosine of free surface normal vector

**\*MAT\_ACOUSTIC\_COMPLEX****\*MAT\_090\_COMPLEX****\*MAT\_ACOUSTIC\_COMPLEX**

This is Material Type 90\_COMPLEX. It specifies the material properties of a linear acoustic fluid. This fluid is assumed to be compressible, irrotational, inviscid, and subject to small displacements. This material only works with acoustic elements. It is intended for direct, steady state vibration simulations with real and imaginary material properties. The model should be used with \*CONTROL\_IMPLICIT\_SSD\_DIRECT and thus only works with the implicit solver. See Appendix W for a description of this application.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RHOR	BULKR	RHOI	BULKI			
Type	A	F	F	F	F			
Default	none	none	none	none	none			

Card 2	1	2	3	4	5	6	7	8
Variable	LCIDRR	LCIDKR	LCIDRI	LCIDKI				
Type	I	I	I	I				
Default	0	0	0	0				
Remarks	2	2	2	2				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RHOR	Real part of the density, $\rho_r$
BULKR	Real part of the bulk modulus, $K_r$
RHOI	Imaginary part of the density, $\rho_i$
BULKI	Imaginary part of the bulk modulus, $K_i$
LCIDRR	Load curve ID for specifying frequency variation of $\rho_r$ .

**\*MAT\_090\_COMPLEX****\*MAT\_ACOUSTIC\_COMPLEX**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCIDKR	Load curve ID for specifying frequency variation of $K_r$ .
LCIDRI	Load curve ID for specifying frequency variation of $\rho_i$ .
LCIDKI	Load curve ID for specifying frequency variation of $K_i$ .

**Remarks:**

1. **Mass and Stiffness.** The contributions of elements using this material model are

$$\begin{aligned} [\bar{M}_f] &= \frac{-K_r}{(K_r^2 + K_i^2)} \int_V N_f^T N_f dV + \frac{iK_i}{(K_r^2 + K_i^2)} \int_V N_f^T N_f dV \\ [\bar{K}_f] &= \frac{-\rho_r}{(\rho_r^2 + \rho_i^2)} \int_V \nabla N_f^T \nabla N_f dV + \frac{i\rho_i}{(\rho_r^2 + \rho_i^2)} \int_V \nabla N_f^T \nabla N_f dV \end{aligned}$$

2. **Frequency Dependence.** If the load curve specifying the frequency variation is undefined, then the property is constant with frequency.

**\*MAT\_ACOUSTIC\_DAMP****\*MAT\_090\_DAMP****\*MAT\_ACOUSTIC\_DAMP**

This is Material Type 90\_DAMP. It specifies the material properties of a linear acoustic fluid. This fluid is assumed to be compressible, irrotational, inviscid, and subject to small displacements. This model can only be used with acoustic elements. This material works for explicit transient and direct, steady state vibration applications. See Appendix W for a description of applications. Depending on the application, it can be used with the implicit or explicit solvers.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	CEE	BETA				
Type	A	F	F	F				
Default	none	none	none	0.0				

Card 2	1	2	3	4	5	6	7	8
Variable							VDC	BETA2
Type							F	F
Default							0.0	0.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
CEE	Sound speed, $c$
BETA	Linear bulk viscosity coefficient, $\beta$
VDC	Volumetric drag coefficient, $r$
BETA2	Quadratic bulk viscosity coefficient, $\beta_2$

**Remarks:**

1. **Usage in Direct Steady State Vibration.** The bulk viscosity parameters, BETA and BETA2, are ignored in steady, state vibration simulations invoked with \*CONTROL\_IMPLICIT\_SSD\_DIRECT. The volumetric drag coefficient,  $r$ , contributes to the fluid damping matrix:

$$[W_f] = \frac{-r}{\rho^2 c^2} \int_V N_f^T N_f dV .$$

$r$  has dimensions of force / volume / velocity.

2. **Usage in Explicit Transient Analysis.** For spectral analyses (see \*CONTROL\_ACOUSTIC\_SPECTRAL), the bulk viscosity parameters, BETA and BETA2, contribute an artificial pressure:

$$\Delta p = \beta \Delta t \dot{p} + \beta_2 \frac{\Delta t^2}{\rho c^2} \dot{p} \max(\dot{p}, 0) .$$

Nonzero values of BETA and BETA2 will adversely affect the time step.

**\*MAT\_ACOUSTIC\_POROUS\_DB****\*MAT\_090\_POROUS\_DB****\*MAT\_ACOUSTIC\_POROUS\_DB**

This is Material Type 90\_POROUS\_DB. It specifies the material properties of a linear acoustic fluid. This fluid is assumed to be compressible, irrotational, inviscid, and subject to small displacements. This material works with acoustic elements. It is intended for direct, steady state forced vibration of porous materials having a rigid frame, such as glass wool. It should be used with \*CONTROL\_IMPLICIT\_SSD\_DIRECT and thus can only be used with the implicit solver. See Appendix W for a description of applications.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RHO0	CEE0	SIGMA				
Type	A	F	F	F				
Default	none	none	none	none				

Card 2	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6	C7	C8
Type	F	F	F	F	F	F	F	F
Default	none							

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RHO0	Mass density in air, $\rho_0$
CEE0	Sound speed in air, $c_0$
SIGMA	Flow resistivity, $\sigma$
$C_i$	Constants of the material model. See <a href="#">Remark 2</a> .

**Remarks:**

1. **Characteristic Impedance and Propagation Constant.** The characteristic impedance is

$$Z = \rho_o c_o (1 + c_1 X^{c_2} - i c_3 X^{c_4}) ,$$

and the propagation constant is

$$\Gamma = \frac{2\pi f}{c_o} (c_5 X^{c_6} + i(1 + c_7 X^{c_8})) ,$$

where

$$f = \frac{\omega}{2\pi} , \quad X = \frac{\rho_o f}{\sigma} .$$

2. **Delany-Bazley, Miki and Allard-Champoux Models.** C1 to C8 of various regression models for the impedance and propagation constant, including those of Delany-Bazley, Miki, and Allard-Champoux, are listed in the journal Applied Acoustics, Sound absorption of porous materials – Accuracy of prediction methods, Oliva and Hongisto, 74 (2013) 1473-1479.

**\*MAT\_SOFT\_TISSUE\_{OPTION}**

Available options include:

<BLANK>

VISCO

This is Material Type 91 (OPTION = <BLANK>) or Material Type 92 (OPTION = VISCO). This material is a transversely isotropic hyperelastic model for representing biological soft tissues, such as ligaments, tendons, and fascia. The representation provides an isotropic Mooney-Rivlin matrix reinforced by fibers having a strain energy contribution with the qualitative material behavior of collagen. The model has a viscoelasticity option which activates a six-term Prony series kernel for the relaxation function. In this case, the hyperelastic strain energy represents the elastic (long-time) response. See Weiss et al. [1996] and Puso and Weiss [1998] for additional details.

**NOTE:** This material does not support specification of a material angle,  $\beta_i$ , for each through-thickness integration point of a shell.

**Card Summary:**

**Card 1.** This card is required.

MID	R0	C1	C2	C3	C4	C5	
-----	----	----	----	----	----	----	--

**Card 2.** This card is required.

XK	XLAM	FANG	XLAM0	FAILSF	FAILSM	FAILSHR	
----	------	------	-------	--------	--------	---------	--

**Card 3.** This card is required.

AOPT	AX	AY	AZ	BX	BY	BZ	
------	----	----	----	----	----	----	--

**Card 4.** This card is required. For shells, this input does not apply, so it may be included as a blank line.

LA1	LA2	LA3	MACF				
-----	-----	-----	------	--	--	--	--

**Card 5.** This card is included for the VISCO keyword option.

S1	S2	S3	S4	S5	S6		
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**Card 6.** This card is included for the VISCO keyword option.

T1	T2	T3	T4	T5	T6		
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Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	C1	C2	C3	C4	C5	
Type	A	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
C1 - C5	Hyperelastic coefficients (see equations in Material Formulation section below)

Card 2	1	2	3	4	5	6	7	8
Variable	XK	XLAM	FANG	XLAM0	FAILSF	FAILSM	FAILSHR	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
XK	Bulk modulus
XLAM	Stretch ratio at which fibers are straightened
FANG	Angle in degrees of a material rotation about the <i>c</i> -axis, available for AOPT = 0 (shells only) and AOPT = 3 (all element types). This angle may be overridden on the element card; see *ELEMENT_SHELL_BETA and *ELEMENT_SOLID_ORTHO. See Remark 1.
XLAM0	Initial fiber stretch (optional). See Remark 2.
FAILSF	Stretch ratio for ligament fibers at failure (applies to shell elements only). If zero, failure is not considered.

VARIABLE	DESCRIPTION
FAILSM	Stretch ratio for surrounding matrix material at failure (applies to shell elements only). If zero, failure is not considered.
FAILSHR	Shear strain at failure at a material point (applies to shell elements only). If zero, failure is not considered. This failure value is independent of FAILSF and FAILSM.

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT	AX	AY	AZ	BX	BY	BZ	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
AOPT	<p>Material axes option (see <a href="#">MAT_OPTIONTROPIC_ELASTIC</a>, particularly the <a href="#">Material Directions</a> section, for details). The fiber direction depends on this coordinate system (see <a href="#">Remark 1</a>).</p> <p>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle FANG on this keyword or BETA on the *ELEMENT_SHELL_{OPTION} input.</p> <p>EQ.1.0: Locally orthotropic with material axes determined by a point, <math>P</math>, in space and the global location of the element center; this is the <math>\mathbf{a}</math>-direction. This option is for solid elements only.</p> <p>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR</p> <p>EQ.3.0: Locally orthotropic material axes determined by a vector <math>\mathbf{v}</math> and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. <math>\mathbf{a}</math> is determined by taking the cross product of <math>\mathbf{v}</math> with the normal vector, <math>\mathbf{b}</math> is determined by taking</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	the cross product of the normal vector with <b>a</b> , and <b>c</b> is the normal vector. Then <b>a</b> and <b>b</b> are rotated about <b>c</b> by an angle BETA. BETA may be set in the keyword input for the element or with FANG on this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying the angle rotation depending on the value of MACF.
	EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector <b>v</b> , and an originating point, <i>P</i> , which define the centerline axis. This option is for solid elements only.
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_OPTION).
AX, AY, AZ	<p>Vector components that depend on the value of AOPT:</p> <p>AOPT.LT.0.0: Ignored</p> <p>AOPT.EQ.1.0: Components of point <i>p</i> (XP, YP, ZP)</p> <p>AOPT.EQ.2.0: Components of vector <b>a</b> (A1, A2, A3)</p> <p>AOPT.GT.2.0: Components of vector <b>v</b> (V1, V2, V3)</p>
BX, BY, BZ	<p>Vector components that depend on the value of AOPT:</p> <p>AOPT.LE.1.0: Ignored</p> <p>AOPT.EQ.2.0: Components of vector <b>d</b> (D1, D2, D3)</p> <p>AOPT.EQ.3.0: Ignored</p> <p>AOPT.EQ.4.0: Components of point <i>p</i> (XP, YP, ZP)</p>

Card 4	1	2	3	4	5	6	7	8
Variable	LA1	LA2	LA3	MACF				
Type	F	F	F	I				

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LAX, LAY, LAZ	Local fiber orientation vector (solids only)

VARIABLE	DESCRIPTION
MACF	<p>Material axes change flag for solid elements:</p> <p>EQ.-4: Switch material axes <i>b</i> and <i>c</i> before BETA or FANG rotation</p> <p>EQ.-3: Switch material axes <i>a</i> and <i>c</i> before BETA or FANG rotation</p> <p>EQ.-2: Switch material axes <i>a</i> and <i>b</i> before BETA or FANG rotation</p> <p>EQ.1: No change, default</p> <p>EQ.2: Switch material axes <i>a</i> and <i>b</i> after BETA or FANG rotation</p> <p>EQ.3: Switch material axes <i>a</i> and <i>c</i> after BETA or FANG rotation</p> <p>EQ.4: Switch material axes <i>b</i> and <i>c</i> after BETA or FANG rotation</p>

[Figure M2-2](#) indicates when LS-DYNA applies MACF during the process to obtain the final material axes. The BETA on \*ELEMENT\_SOLID\_{OPTION} if defined is used for the rotation for all AOPT options. If BETA is not used for the element, then a rotation only occurs for AOPT = 3 where FANG is applied.

**Prony Series Card 1.** Additional card for VISCO keyword option.

Card 5	1	2	3	4	5	6	7	8
Variable	S1	S2	S3	S4	S5	S6		
Type	F	F	F	F	F	F		

**Prony Series Card 2.** Additional card for VISCO keyword option.

Card 6	1	2	3	4	5	6	7	8
Variable	T1	T2	T3	T4	T5	T6		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
S1 – S6	Factors in the Prony series (see <a href="#">Material Formulation</a> and <a href="#">Remark 3</a> )
T1 - T6	Characteristic times for Prony series relaxation kernel (see <a href="#">Material Formulation</a> and <a href="#">Remark 3</a> )

**Material Formulation:**

The overall strain energy,  $W$ , is "uncoupled" and includes two isotropic deviatoric matrix terms, a fiber term,  $F$ , and a bulk term:

$$W = C_1(\tilde{I}_1 - 3) + C_2(\tilde{I}_2 - 3) + F(\lambda) + \frac{1}{2}K[\ln(J)]^2$$

Here,  $\tilde{I}_1$  and  $\tilde{I}_2$  are the deviatoric invariants of the right Cauchy deformation tensor,  $\lambda$  is the deviatoric part of the stretch along the current fiber direction, and  $J = \det F$  is the volume ratio. The material coefficients  $C_1$  and  $C_2$  are the Mooney-Rivlin coefficients, while  $K$  is the effective bulk modulus of the material (input parameter XK).

The derivatives of the fiber term  $F$  are defined to capture the behavior of crimped collagen. The fibers are assumed to be unable to resist compressive loading - thus the model is isotropic when  $\lambda < 1$ . An exponential function describes the straightening of the fibers, while a linear function describes the behavior of the fibers once they are straightened past a critical fiber stretch level  $\lambda \geq \lambda^*$  (input parameter XLAM):

$$\frac{\partial F}{\partial \lambda} = \begin{cases} 0 & \lambda < 1 \\ \frac{C_3}{\lambda}[\exp(C_4(\lambda - 1)) - 1] & \lambda < \lambda^* \\ \frac{1}{\lambda}(C_5\lambda + C_6) & \lambda \geq \lambda^* \end{cases}$$

Coefficients  $C_3$ ,  $C_4$ , and  $C_5$  must be defined by the user.  $C_6$  is determined by LS-DYNA to ensure stress continuity at  $\lambda = \lambda^*$ . Sample values for the material coefficients  $C_1 - C_5$  and  $\lambda^*$  for ligament tissue can be found in Quapp and Weiss [1998]. The bulk modulus  $K$  should be at least 3 orders of magnitude larger than  $C_1$  to ensure near-incompressible material behavior.

Viscoelasticity is included through a convolution integral representation for the time-dependent second Piola-Kirchoff stress  $\mathbf{S}(\mathbf{C}, t)$ :

$$\mathbf{S}(\mathbf{C}, t) = \mathbf{S}^e(\mathbf{C}) + \int_0^t 2G(t-s) \frac{\partial W}{\partial \mathbf{C}(s)} ds$$

Here,  $\mathbf{S}^e$  is the elastic part of the second Piola-Kirchoff stress as derived from the strain energy, and  $G(t-s)$  is the reduced relaxation function, represented by a Prony series:

$$G(t) = \sum_{i=1}^6 S_i \exp\left(\frac{t}{T_i}\right) .$$

Puso and Weiss [1998] describe a graphical method to fit the Prony series coefficients to relaxation data that approximates the behavior of the continuous relaxation function proposed by Y-C. Fung, as quasilinear viscoelasticity.

### Remarks:

1. **Fiber direction.** For shell elements, the fiber direction lies in the plane of the element. The fiber direction is along the  $a$ -axis material direction. This direction depends on the value of AOPT.

For solids elements, the local coordinate system depends on the value of AOPT. The fiber direction is oriented in the local system using input parameters LAX, LAY, and LAZ. By default, (LAX,LAY,LAZ) = (1,0,0), and the fiber is aligned with the local  $a$ -direction.

2. **Initial fiber stretch.** An optional initial fiber stretch can be specified using XLAM0. The initial stretch is applied during the first time step. This creates preload in the model as soft tissue contacts, and equilibrium is established. For example, a ligament tissue "uncrimping strain" of 3% can be represented with initial stretch value of 1.03.
3. **Prony series input.** If the VISCO keyword option is included, at least one Prony series term ( $S_1, T_1$ ) must be defined.

**\*MAT\_093****\*MAT\_ELASTIC\_6DOF\_SPRING\_DISCRETE\_BEAM****\*MAT\_ELASTIC\_6DOF\_SPRING\_DISCRETE\_BEAM**

This is Material Type 93. This material model is defined for simulating the effects of nonlinear elastic and nonlinear viscous beams by using six springs each acting about one of the six local degrees-of-freedom. The input consists of part IDs that reference material type, \*MAT\_ELASTIC\_SPRING\_DISCRETE\_BEAM (type 74 above). Generally, these referenced parts are used only for the definition of this material model and are not referenced by any elements. The two nodes defining a beam may be coincident to give a zero length beam or offset to give a finite length beam. For finite length discrete beams the absolute value of the variable SCOR in the \*SECTION\_BEAM input should be set to a value of 2.0, which causes the local *r*-axis to be aligned along the two nodes of the beam to give physically correct behavior. The distance between the nodes of a beam should not affect the behavior of this material model. A triad is used to orient the beam for the directional springs.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	TPIDR	TPIDS	TPIDT	RPIDR	RPIDS	RPIDT
Type	A	F	I	I	I	I	I	I

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density; see also volume in *SECTION_BEAM definition.
TPIDR	Translational motion in the local <i>r</i> -direction is governed by part ID TPIDR. If zero, no force is computed in this direction.
TPIDS	Translational motion in the local <i>s</i> -direction is governed by part ID TPIDS. If zero, no force is computed in this direction.
TPIDT	Translational motion in the local <i>t</i> -direction is governed by part ID TPIDT. If zero, no force is computed in this direction.
RPIDR	Rotational motion about the local <i>r</i> -axis is governed by part ID RPIDR. If zero, no moment is computed about this axis.
RPIDS	Rotational motion about the local <i>s</i> -axis is governed by part ID RPIDS. If zero, no moment is computed about this axis.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RPIDT	Rotational motion about the local $t$ -axis is governed by part ID RPIDT. If zero, no moment is computed about this axis.

**Remarks:**

Rotational displacement is measured in radians.

**\*MAT\_094****\*MAT\_INELASTIC\_SPRING\_DISCRETE\_BEAM****\*MAT\_INELASTIC\_SPRING\_DISCRETE\_BEAM**

This is Material Type 94. This model permits elastoplastic springs with damping to be represented with a discrete beam element type 6. A yield force as a function deflection curve is used which can vary in tension and compression.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	K	F0	D	CDF	TDF	
Type	A	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	FLCID	HLCID	C1	C2	DLE	GLCID		
Type	F	F	F	F	F	I		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density; see also volume in *SECTION_BEAM definition.
K	Elastic loading/unloading stiffness. This is required input.
F0	Optional initial force. This option is inactive if this material is referenced by a part referenced by material type *MAT_INELASTIC_-6DOF_SPRING.
D	Optional viscous damping coefficient.
CDF	Compressive displacement at failure. Input as a positive number. After failure, no forces are carried. This option does not apply to zero length springs. EQ.0.0: inactive.
TDF	Tensile displacement at failure. After failure, no forces are carried. EQ.0.0: inactive.

VARIABLE	DESCRIPTION
FLCID	Load curve ID (see *DEFINE_CURVE) defining the yield force as a function of plastic deflection. If the origin of the curve is at (0,0), the force magnitude is identical in tension and compression, that is, only the sign changes. If not, the yield stress in the compression is used when the spring force is negative. The plastic displacement increases monotonically in this implementation. The load curve is required input.
HLCID	Load curve ID (see *DEFINE_CURVE) defining force as a function of relative velocity (optional). If the origin of the curve is at (0,0), the force magnitude is identical for a given magnitude of the relative velocity, that is, only the sign changes.
C1	Damping coefficient
C2	Damping coefficient
DLE	Factor to scale time units
GLCID	Optional load curve ID (see *DEFINE_CURVE) defining a scale factor as a function of deflection for load curve ID, HLCID. If zero, a scale factor of unity is assumed.

**Remarks:**

1. **Force.** To determine the force, a trial force is first computed as:

$$F^T = F^n + K \times \Delta\dot{L}(\Delta t)$$

The yield force is taken from the load curve:

$$F^Y = F_y(\Delta L^{\text{plastic}}) ,$$

where  $L^{\text{plastic}}$  is the plastic deflection, given by

$$\Delta L^{\text{plastic}} = \frac{F^T - F^Y}{S + K^{\max}} .$$

Here  $S$  is the slope of FLCID and  $K^{\max}$  is the maximum elastic stiffness:

$$K^{\max} = \max(K, 2 \times S^{\max}) .$$

The trial force is, then, checked against the yield force to determine  $F$ :

$$F = \begin{cases} F^Y & \text{if } F^T > F^Y \\ F^T & \text{if } F^T \leq F^Y \end{cases}$$

The final force, which includes rate effects and damping, is given by:

$$F^{n+1} = F \times \left[ 1 + C1 \times \Delta\dot{L} + C2 \times \text{sgn}(\Delta\dot{L}) \ln \left( \max \left\{ 1., \frac{|\Delta\dot{L}|}{DLE} \right\} \right) \right] + D \times \Delta\dot{L}$$
$$+ g(\Delta L)h(\Delta\dot{L}) .$$

2. **Yield Force Curve.** Unless the origin of the curve starts at (0,0), the negative part of the curve is used when the spring force is negative where the negative of the plastic displacement is used to interpolate,  $F_y$ . The positive part of the curve is used whenever the force is positive. In these equations,  $\Delta L$  is the change in length

$$\Delta L = \text{current length} - \text{initial length} .$$

3. **Cross-Sectional Area.** The cross-sectional area is defined on the section card for the discrete beam elements, See \*SECTION\_BEAM. The square root of this area is used as the contact thickness offset if these elements are included in the contact treatment.

**\*MAT\_INELASTIC\_6DOF\_SPRING\_DISCRETE\_BEAM**

This is Material Type 95. This material model is defined for simulating the effects of non-linear inelastic and nonlinear viscous beams by using six springs each acting about one of the six local degrees-of-freedom. The input consists of part IDs that reference material type \*MAT\_INELASTIC\_SPRING\_DISCRETE\_BEAM above (type 94). Generally, these referenced parts are used only for the definition of this material model and are not referenced by any elements. The two nodes defining a beam may be coincident to give a zero length beam, or offset to give a finite length beam. For finite length discrete beams, the absolute value of the variable SCOOR in the \*SECTION\_BEAM input should be set to a value of 2.0, which causes the local *r*-axis to be aligned along the two nodes of the beam, to give physically correct behavior. The distance between the nodes of a beam should not affect the behavior of this material model. A triad must be used to orient the beam for zero length beams.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	TPIDR	TPIDS	TPIDT	RPIDR	RPIDS	RPIDT
Type	A	F	I	I	I	I	I	I

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density, see also volume in *SECTION_BEAM definition.
TPIDR	Translational motion in the local <i>r</i> -direction is governed by part ID TPIDR. If zero, no force is computed in this direction.
TPIDS	Translational motion in the local <i>s</i> -direction is governed by part ID TPIDS. If zero, no force is computed in this direction.
TPIDT	Translational motion in the local <i>t</i> -direction is governed by part ID TPIDT. If zero, no force is computed in this direction.
RPIDR	Rotational motion about the local <i>r</i> -axis is governed by part ID RPIDR. If zero, no moment is computed about this axis.
RPIDS	Rotational motion about the local <i>s</i> -axis is governed by part ID RPIDS. If zero, no moment is computed about this axis.

**\*MAT\_095****\*MAT\_INELASTIC\_6DOF\_SPRING\_DISCRETE\_BEAM**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RPIDT	Rotational motion about the local <i>t</i> -axis is governed by part ID RPIDT. If zero, no moment is computed about this axis.

**Remarks:**

Rotational displacement is measured in radians.

**\*MAT\_BRITTLE\_DAMAGE**

This is Material Type 96. It is an anisotropic brittle damage model designed primarily for concrete though it can be applied to a wide variety of brittle materials.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	TLIMIT	SLIMIT	FTOUGH	SRETEN
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	VISC	FRA_RF	E_RF	YS_RF	EH_RF	FS_RF	SIGY	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus, $E$
PR	Poisson's ratio, $\nu$
TLIMIT	Tensile limit, $f_n$
SLIMIT	Shear limit, $f_s$
FTOUGH	Fracture toughness, $g_c$
SRETEN	Shear retention, $\beta$
VISC	Viscosity, $\eta$
FRA_RF	Fraction of reinforcement in section
E_RF	Young's modulus of reinforcement
YS_RF	Yield stress of reinforcement

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EH_RF	Hardening modulus of reinforcement
FS_RF	Failure strain (true) of reinforcement
SIGY	Compressive yield stress, $\sigma_y$
	EQ.0: No compressive yield

**Remarks:**

A full description of the tensile and shear damage parts of this material model is given in Govindjee, Kay and Simo [1994,1995]. This model admits progressive degradation of tensile and shear strengths across smeared cracks that are initiated under tensile loadings. Compressive failure is governed by a simplistic J2 flow correction that can be disabled if not desired. Damage is handled by treating the rank 4 elastic stiffness tensor as an evolving internal variable for the material. Softening induced mesh dependencies are handled by a characteristic length method [Oliver 1989].

Description of properties:

1.  $E$  is the Young's modulus of the undamaged material also known as the virgin modulus.
2.  $\nu$  is the Poisson's ratio of the undamaged material also known as the virgin Poisson's ratio.
3.  $f_n$  is the initial principal tensile strength (stress) of the material. Once this stress has been reached at a point in the body a smeared crack is initiated there with a normal that is co-linear with the 1<sup>st</sup> principal direction. Once initiated, the crack is fixed at that location, though it will convect with the motion of the body. As the loading progresses the allowed tensile traction normal to the crack plane is progressively degraded to a small machine dependent constant.

The degradation is implemented by reducing the material's modulus normal to the smeared crack plane according to a maximum dissipation law that incorporates exponential softening. The restriction on the normal tractions is given by

$$\phi_t = (\mathbf{n} \otimes \mathbf{n}) : \boldsymbol{\sigma} - f_n + (1 - \varepsilon)f_n(1 - \exp[-H\alpha]) \leq 0$$

where  $\mathbf{n}$  is the smeared crack normal,  $\varepsilon$  is the small constant,  $H$  is the softening modulus, and  $\alpha$  is an internal variable.  $H$  is set automatically by the program; see  $g_c$  below.  $\alpha$  measures the crack field intensity and is output in the equivalent plastic strain field,  $\bar{\epsilon}^p$ , in a normalized fashion.

The evolution of  $\alpha$  is governed by a maximum dissipation argument. When the normalized value reaches unity, the material's strength has been reduced to 2% of its original value in the normal and parallel directions to the smeared crack. Note that for plotting purposes it is never output greater than 5.

4.  $f_s$  is the initial shear traction that may be transmitted across a smeared crack plane. The shear traction is limited to be less than or equal to  $f_s(1 - \beta)(1 - \exp[-H\alpha])$  through the use of two orthogonal shear damage surfaces. Note that the shear degradation is coupled to the tensile degradation through the internal variable  $\alpha$  which measures the intensity of the crack field.  $\beta$  is the shear retention factor defined below. The shear degradation is taken care of by reducing the material's shear stiffness parallel to the smeared crack plane.
5.  $g_c$  is the fracture toughness of the material. It should be entered as fracture energy per unit area crack advance. Once entered the softening modulus is automatically calculated based on element and crack geometries.
6.  $\beta$  is the shear retention factor. As the damage progresses the shear tractions allowed across the smeared crack plane asymptote to the product  $\beta f_s$ .
7.  $\eta$  represents the viscosity of the material. Viscous behavior is implemented as a simple Perzyna regularization method which allows for the inclusion of first order rate effects. The use of some viscosity is recommended as it serves as regularizing parameter that increases the stability of calculations.
8.  $\sigma_y$  is a uniaxial compressive yield stress. A check on compressive stresses is made using the J2 yield function

$$\mathbf{s} : \mathbf{s} - \sqrt{\frac{2}{3}} \sigma_y \leq 0 ,$$

where  $\mathbf{s}$  is the stress deviator. If violated, a J2 return mapping correction is executed. This check is executed (1) when no damage has taken place at an integration point yet; (2) when damage has taken place at a point, but the crack is currently closed; and (3) during active damage after the damage integration (i.e. as an operator split). Note that if the crack is open the plasticity correction is done in the plane-stress subspace of the crack plane.

A variety of experimental data has been replicated using this model from quasi-static to explosive situations. Reasonable properties for a standard grade concrete would be:

Property	Value
$E$	$3.15 \times 10^6$ psi

Property	Value
$f_n$	450 psi
$f_s$	2100 psi
$\nu$	0.2
$g_c$	0.8 lbs/in
$\beta$	0.03
$\eta$	0.0 psi-sec
$\sigma_y$	4200 psi

For stability, values of  $\eta$  between 104 to 106 psi/sec are recommended. Our limited experience thus far has shown that many problems require nonzero values of  $\eta$  to run to avoid error terminations.

Various other internal variables such as crack orientations and degraded stiffness tensors are internally calculated but currently not available for output.

**\*MAT\_GENERAL\_JOINT\_DISCRETE\_BEAM**

This is Material Type 97. This model is used to define a general joint constraining any combination of degrees of freedom between two nodes. The nodes may belong to rigid or deformable bodies. In most applications the end nodes of the beam are coincident and the local coordinate system ( $r, s, t$  axes) is defined by CID (see \*SECTION\_BEAM).

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	TR	TS	TT	RR	RS	RT
Type	A	F	I	I	I	I	I	I
Remarks	1							

Card 2	1	2	3	4	5	6	7	8
Variable	RPST	RPSR						
Type	F	F						
Remarks	2							

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density; see also volume in *SECTION_BEAM definition.
TR	Translational constraint code along the $r$ -axis: EQ.0: Free EQ.1: Constrained
TS	Translational constraint code along the $s$ -axis: EQ.0: Free EQ.1: Constrained

<b>VARIABLE</b>	<b>DESCRIPTION</b>
TT	Translational constraint code along the <i>t</i> -axis: EQ.0: Free EQ.1: Constrained
RR	Rotational constraint code about the <i>r</i> -axis: EQ.0: Free EQ.1: Constrained
RS	Rotational constraint code about the <i>s</i> -axis: EQ.0: Free EQ.1: Constrained
RT	Rotational constraint code about the <i>t</i> -axis: EQ.0: Free EQ.1: Constrained
RPST	Penalty stiffness scale factor for translational constraints
RPSR	Penalty stiffness scale factor for rotational constraints

**Remarks:**

- Inertia and Stability.** For explicit calculations, the additional stiffness due to this joint may require addition mass and inertia for stability. Mass and rotary inertia for this beam element is based on the defined mass density, the volume, and the mass moment of inertia defined in the \*SECTION\_BEAM input.
- Penalty Stiffness.** The penalty stiffness applies to explicit calculations. For implicit calculations, constraint equations are generated and imposed on the system equations; therefore, these constants, RPST and RPSR, are not used.

**\*MAT\_SIMPLIFIED\_JOHNSON\_COOK\_{OPTION}**

Available options include:

<BLANK>

STOCHASTIC

This is Material Type 98 implementing Johnson/Cook strain sensitive plasticity. It is used for problems where the strain rates vary over a large range. In contrast to the full Johnson/Cook model (material type 15) this model introduces the following simplifications:

1. thermal effects and damage are ignored,
2. and the maximum stress is directly limited since thermal softening which is very significant in reducing the yield stress under adiabatic loading is not available.

An iterative plane stress update is used for the shell elements, but due to the simplifications related to thermal softening and damage, this model is 50% faster than the full Johnson/Cook implementation. To compensate for the lack of thermal softening, limiting stress values are introduced to keep the stresses within reasonable limits.

A resultant formulation for the Belytschko-Tsay, the C0 Triangle, and the fully integrated type 16 shell elements is available and can be activated by specifying either zero or one through thickness integration point on the \*SECTION\_SHELL card. While less accurate than through thickness integration, this formulation runs somewhat faster. Since the stresses are not computed in the resultant formulation, the stresses written to the databases for the resultant elements are set to zero.

This model is also available for the Hughes-Liu beam, the Belytschko-Schwer beam, and for the truss element. For the resultant beam formulation, the rate effects are approximated by the axial rate, since the thickness of the beam about its bending axes is unknown. Because this model is primarily used for structural analysis, the pressure is determined using the linear bulk modulus.

**\*MAT\_098****\*MAT\_SIMPLIFIED\_JOHNSON\_COOK**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	VP			
Type	A	F	F	F	F			
Default	none	none	none	none	0.0			

Card 2	1	2	3	4	5	6	7	8
Variable	A	B	N	C	PSFAIL	SIGMAX	SIGSAT	EPS0
Type	F	F	F	F	F	F	F	F
Default	none	0.0	0.0	0.0	$10^{17}$	SIGSAT	$10^{28}$	1.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
VP	Formulation for rate effects: EQ.0.0: scale yield stress (default) EQ.1.0: viscoplastic formulation  This option applies only to the 4-node shell and 8-node thick shell if and only if through thickness integration is used.
A	See <a href="#">Remark 1</a> .
B	See <a href="#">Remark 1</a> .
N	See <a href="#">Remark 1</a> .
C	See <a href="#">Remark 1</a> .

VARIABLE	DESCRIPTION
PSFAIL	Effective plastic strain at failure. If zero, failure is not considered.
SIGMAX	Maximum stress obtainable from work hardening before rate effects are added (optional). This option is ignored if VP = 1.0
SIGSAT	Saturation stress which limits the maximum value of effective stress which can develop after rate effects are added (optional).
EPS0	Quasi-static threshold strain rate. See description under *MAT_015.

**Remarks:**

1. **Flow Stress.** Johnson and Cook express the flow stress as

$$\sigma_y = \left( A + B\bar{\varepsilon}^p \right) (1 + C \ln \dot{\varepsilon}^*)$$

where  $A$ ,  $B$ , and  $C$  are input constants and  $\bar{\varepsilon}^p$  is the effective plastic strain.  $\dot{\varepsilon}^*$  is the normalized effective strain rate:

$$\dot{\varepsilon}^* = \frac{\dot{\varepsilon}}{\text{EPS0}} .$$

The maximum stress is limited by SIGMAX and SIGSAT by:

$$\sigma_y = \min \left\{ \min \left[ A + B\bar{\varepsilon}^p, \text{SIGMAX} \right] (1 + c \ln \dot{\varepsilon}^*), \text{SIGSAT} \right\} .$$

Failure occurs when the effective plastic strain exceeds PSFAIL.

2. **Viscoplastic.** If the viscoplastic option is active (VP = 1.0), the parameters SIGMAX and SIGSAT are ignored since these parameters make convergence of the plastic strain iteration loop difficult to achieve. The viscoplastic option replaces the effective strain rate in the forgoing equations by the effective plastic strain rate. Numerical noise is substantially reduced by the viscoplastic formulation.
3. **STOCHASTIC.** The STOCHASTIC option allows spatially varying yield and failure behavior. See \*DEFINE\_STOCHASTIC\_VARIATION for additional information.

**\*MAT\_099 \*MAT\_SIMPLIFIED\_JOHNSON\_COOK\_ORTHOTROPIC\_DAMAGE****\*MAT\_SIMPLIFIED\_JOHNSON\_COOK\_ORTHOTROPIC\_DAMAGE**

This is Material Type 99. This model, which is implemented with multiple through thickness integration points, is an extension of model 98 to include orthotropic damage as a means of treating failure in aluminum panels. Directional damage begins after a defined failure strain is reached in tension and continues to evolve until a tensile rupture strain is reached in either one of the two orthogonal directions. After rupture is detected at NUMINT integration points, the element is deleted.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	VP	EPPFR	LCDM	NUMINT
Type	A	F	F	F	F	F	I	I
Default	none	none	none	none	0.0	$10^{16}$	optional	{all}

Card 2	1	2	3	4	5	6	7	8
Variable	A	B	N	C	PSFAIL	SIGMAX	SIGSAT	EPSO
Type	F	F	F	F	F	F	F	F
Default	none	0.0	0.0	0.0	$10^{17}$	SIGSAT	$10^{28}$	1.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default) EQ.1.0: Viscoplastic formulation

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	This option applies only to the 4-node shell and 8-node thick shell if and only if through thickness integration is used.
EPPFR	Plastic strain at which material ruptures (logarithmic)
LCDM	Load curve ID defining nonlinear damage curve. See <a href="#">Figure M81-2</a> .
NUMINT	Number of through thickness integration points which must fail before the element is deleted. If zero, all points must fail. The default of all integration points is not recommended since elements undergoing large strain are often not deleted due to nodal fiber rotations which limit 0 strains at active integration points after most points have failed. Better results are obtained if NUMINT is set to 1 or a number less than one half of the number of through thickness points. For example, if four through thickness points are used, NUMINT should not exceed 2, even for fully integrated shells which have 16 integration points.
A	See <a href="#">Remark 1</a> in *MAT_098.
B	See <a href="#">Remark 1</a> in *MAT_098.
N	See <a href="#">Remark 1</a> in *MAT_098.
C	See <a href="#">Remark 1</a> in *MAT_098.
PSFAIL	Principal plastic strain at failure. If zero, failure is not considered.
SIGMAX	Maximum stress obtainable from work hardening before rate effects are added (optional). This option is ignored if VP = 1.0.
SIGSAT	Saturation stress which limits the maximum value of effective stress which can develop after rate effects are added (optional).
EPS0	Quasi-static threshold strain rate. See description under *MAT_015.

**Remarks:**

See the description for the SIMPLIFIED\_JOHNSON\_COOK model above.

**\*MAT\_SPOTWELD\_{OPTION1}\_{OPTION2}**

This is Material Type 100. The material model applies to beam element type 9 and to solid element type 1. The failure models apply to both beam and solid elements.

In the case of solid elements, if hourglass type 4 is specified then hourglass type 4 will be used; otherwise, hourglass type 6 will be automatically assigned. Hourglass type 6 is preferred.

The beam elements, based on the Hughes-Liu beam formulation, may be placed between any two deformable shell surfaces and tied with constraint contact, \*CONTACT\_SPOTWELD, which eliminates the need to have adjacent nodes at spot weld locations. Beam spot welds may be placed between rigid bodies and rigid/deformable bodies by making the node on one end of the spot weld a rigid body node which can be an extra node for the rigid body; see \*CONSTRAINED\_EXTRA\_NODES\_OPTION. In the same way rigid bodies may also be tied together with this spot weld option. This weld option should not be used with rigid body switching. The foregoing advice is valid if solid element spot welds are used; however, since the solid elements have just three degrees-of-freedom at each node, \*CONTACT\_TIED\_SURFACE\_TO\_SURFACE must be used instead of \*CONTACT\_SPOTWELD.

In flat topologies the shell elements have an unconstrained drilling degree-of-freedom which prevents torsional forces from being transmitted. If the torsional forces are deemed to be important, brick elements should be used to model the spot welds.

Beam and solid element force resultants for MAT\_SPOTWELD are written to the spot weld force file, **swforc**, and the file for element stresses and resultants for designated elements, **elout**.

*It is advisable to include all spot welds, which provide the tracked nodes, and spot welded materials, which define the reference segments, within a single \*CONTACT\_SPOTWELD interface for beam element spot welds or a \*CONTACT\_TIED\_SURFACE\_TO\_SURFACE interface for solid element spot welds. As a constraint method these interfaces are treated independently which can lead to significant problems if such interfaces share common nodal points. An added benefit is that memory usage can be substantially less with a single interface.*

Available options for OPTION1 include:

<BLANK>

DAMAGE-FAILURE

The DAMAGE-FAILURE option causes one additional line to be read with the damage parameter and a flag that determines how failure is computed from the resultants. On this line the parameter, RS, if nonzero, invokes damage mechanics combined with the plasticity model to achieve a smooth drop off of the resultant forces prior to the removal

of the spot weld. The parameter OPT determines the method used in computing resultant based failure, which is unrelated to damage.

Available options for *OPTION2* include:

<BLANK>

UNIAXIAL

The UNIAXIAL keyword option causes the transverse stresses and transverse strains to be zero for solid spot welds. The older uniaxial method, invoked with E < 0.0 on Card 1, assumed only the transverse stresses are zero. Compared to the older method, the UNIAXIAL keyword option increases the stability of the solver. See [Remark 2](#) for more details.

### **Card Summary:**

**Card 1.** This card is required.

MID	R0	E	PR	SIGY	EH	DT	TFAIL
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**Card 2a.** This card is included if no keyword option is used (<BLANK>) for *OPTION1*.

EFAIL	NRR	NRS	NRT	MRR	MSS	MTT	NF
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**Card 2b.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = -2, -1 or 0 on Card 3.

EFAIL	NRR	NRS	NRT	MRR	MSS	MTT	NF
-------	-----	-----	-----	-----	-----	-----	----

**Card 2c.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 1 on Card 3.

EFAIL	SIGAX	SIGTAU					NF
-------	-------	--------	--	--	--	--	----

**Card 2d.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 2, 12, or 22 on Card 3.

EFAIL	USRV1	USRV2	USRV3	USRV4	USRV5	USRV6	NF
-------	-------	-------	-------	-------	-------	-------	----

**Card 2e.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 3 or 4 on Card 3.

EFAIL	ZD	ZT	ZALP1	ZALP2	ZALP3	ZRRAD	NF
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## \*MAT\_100

## \*MAT\_SPOTWELD

**Card 2f.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 5 on Card 3.

EFAIL	ZD	ZT	ZT2				
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**Card 2g.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 6, 7, 9, -9 or 10 on Card 3.

EFAIL							NF
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**Card 2h.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 11 on Card 3.

EFAIL	LCT	LCC					NF
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**Card 3.** Include this card if the DAMAGE-FAILURE keyword option is used.

RS	OPT	FVAL	TRUE_T	ASFF	BETA		DMGOPT
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**Card 3.1.** Include this card if the DAMAGE-FAILURE keyword option is used and DMGOPT = -1 on Card 3.

DMGOPT	FMODE	FFCAP	TTOPT				
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**Card 4.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 12 or 22 on Card 3.

USRV7	USRV8	USRV9	USRV10	USRV11	USRV12	USRV13	USRV14
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**Card 5.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 12 or 22 on Card 3.

USRV15	USRV16	USRV17	USRV18	USRV19	USRV20	USRV21	USRV22
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### Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	EH	DT	TFAIL
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus.  LT.0.0: $ E $ is the Young's modulus. $E < 0$ invokes uniaxial stress for solid spot welds with the transverse stresses assumed to be zero. See <a href="#">Remark 2</a> . This is for when OPTION2 is unset (<BLANK>) only.
PR	Poisson's ratio
SIGY	Yield Stress:  GT.0: Initial yield stress EQ.0: Default to 1% of E LT.0: A yield curve or table is assigned by  SIGY ; see <a href="#">Remark 5</a> .
EH	Plastic hardening modulus, $E_h$
DT	Time step size for mass scaling, $\Delta t$
TFAIL	Failure time if nonzero. If zero, this option is ignored.

**Card 2 for No Failure.** Additional card when no keyword option is used (<BLANK>).

Card 2a	1	2	3	4	5	6	7	8
Variable	EFAIL	NRR	NRS	NRT	MRR	MSS	MTT	NF
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
EFAIL	Effective plastic strain in weld material at failure. The spot weld element is deleted when the plastic strain at each integration point exceeds EFAIL. If zero, failure due to effective plastic strain is not considered.
NRR	Axial force resultant $N_{rr_F}$ at failure. If zero, failure due to this component is not considered.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	GT.0: Constant value LT.0:  NRR  is a load curve ID, defining the axial force resultant $N_{rr_F}$ at failure as a function of the effective strain rate.
NRS	Force resultant $N_{rs_F}$ at failure. If zero, failure due to this component is not considered. GT.0: Constant value LT.0:  NRS  is a load curve ID, defining the force resultant $N_{rs_F}$ at failure as a function of the effective strain rate.
NRT	Force resultant $N_{rt_F}$ at failure. If zero, failure due to this component is not considered. GT.0: Constant value LT.0:  NRT  is a load curve ID, defining the force resultant $N_{rt_F}$ at failure as a function of the effective strain rate.
MRR	Torsional moment resultant $M_{rr_F}$ at failure. If zero, failure due to this component is not considered. GT.0: Constant value LT.0:  MRR  is a load curve ID, defining the torsional moment resultant $M_{rr_F}$ at failure as a function of the effective strain rate.
MSS	Moment resultant $M_{ss_F}$ at failure. If zero, failure due to this component is not considered. GT.0: Constant value LT.0:  MSS  is a load curve ID, defining the moment resultant $M_{ss_F}$ at failure as a function of the effective strain rate.
MTT	Moment resultant $M_{tt_F}$ at failure. If zero, failure due to this component is not considered. GT.0: Constant value LT.0:  MTT  is a load curve ID, defining the moment resultant $M_{tt_F}$ at failure as a function of the effective strain rate.
NF	Number of force vectors stored for filtering

**Card 2 for Resultant Based Failure.** Additional card for DAMAGE-FAILURE keyword option with OPT = -2, -1 or 0.

Card 2b	1	2	3	4	5	6	7	8
Variable	EFAIL	NRR	NRS	NRT	MRR	MSS	MTT	NF
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
EFAIL	Effective plastic strain in weld material at failure. The plastic strain must exceed the rupture strain (RS) at each integration point before deletion occurs. See Card 3.
NRR	Axial force resultant $N_{rr_F}$ at failure. If zero, failure due to this component is not considered.  GT.0: Constant value  LT.0:  NRR  is a load curve ID, defining the axial force resultant $N_{rr_F}$ at failure as a function of the effective strain rate.
NRS	Force resultant $N_{rs_F}$ at failure. If zero, failure due to this component is not considered.  GT.0: Constant value  LT.0:  NRS  is a load curve ID, defining the force resultant $N_{rs_F}$ at failure as a function of the effective strain rate.
NRT	Force resultant $N_{rt_F}$ at failure. If zero, failure due to this component is not considered.  GT.0: Constant value  LT.0:  NRT  is a load curve ID, defining the force resultant $N_{rt_F}$ at failure as a function of the effective strain rate.
MRR	Torsional moment resultant $M_{rr_F}$ at failure. If zero, failure due to this component is not considered.  GT.0: Constant value  LT.0:  MRR  is a load curve ID, defining the torsional moment resultant $M_{rr_F}$ at failure as a function of the effective strain rate.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MSS	Moment resultant $M_{ss_F}$ at failure. If zero, failure due to this component is not considered.  GT.0: Constant value  LT.0:  MSS  is a load curve ID, defining the moment resultant $M_{ss_F}$ at failure as a function of the effective strain rate.
MTT	Moment resultant $M_{tt_F}$ at failure. If zero, failure due to this component is not considered.  GT.0: Constant value  LT.0:  MTT  is a load curve ID, defining the moment resultant $M_{tt_F}$ at failure as a function of the effective strain rate.
NF	Number of force vectors stored for filtering

**Card 2 for Stress Based Failure.** Additional card for DAMAGE-FAILURE keyword option with OPT = 1.

Card 2c	1	2	3	4	5	6	7	8
Variable	EFAIL	SIGAX	SIGTAU					NF
Type	F	F	F					F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EFAIL	Effective plastic strain in weld material at failure. The plastic strain must exceed the rupture strain (RS) at each integration point before deletion occurs. See Card 3.
SIGAX	Maximum axial stress $\sigma_{rr}^F$ at failure.  GT.0.0: Constant maximum axial stress at failure  EQ.0.0: Failure due to this component is not considered.  LT.0.0:  SIGAX  is a load curve ID defining the maximum axial stress at failure as a function of the effective strain rate.
SIGTAU	Maximum shear stress $\tau^F$ at failure.  GT.0.0: Constant maximum shear stress at failure

VARIABLE	DESCRIPTION
	EQ.0.0: Failure due to this component is not considered.
	LT.0.0:  SIGTAU  is a load curve ID defining the maximum shear stress at failure as a function of the effective strain rate.

NF      Number of force vectors stored for filtering

**Card 2 for User Subroutine Based Failure.** Additional card for DAMAGE-FAILURE keyword option with OPT = 2, 12, or 22.

Card 2d	1	2	3	4	5	6	7	8
Variable	EFAIL	USRV1	USRV2	USRV3	USRV4	USRV5	USRV6	NF
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
EFAIL	Effective plastic strain in weld material at failure. The plastic strain must exceed the rupture strain (RS) at each integration point before deletion occurs. See Card 3.
USRV $n$	Failure constants for user failure subroutine, $n = 1, 2, \dots, 6$
NF	Number of force vectors stored for filtering

**Card 2 for Notch Stress Failure.** Additional card for DAMAGE-FAILURE keyword option with OPT = 3 or 4.

Card 2e	1	2	3	4	5	6	7	8
Variable	EFAIL	ZD	ZT	ZALP1	ZALP2	ZALP3	ZRRAD	NF
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
EFAIL	Effective plastic strain in weld material at failure. The plastic strain must exceed the rupture strain (RS) at each integration point before deletion occurs. See Card 3.
ZD	Notch diameter

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
ZT	Sheet thickness
ZALP1	Correction factor $\alpha_1$
ZALP2	Correction factor $\alpha_2$
ZALP3	Correction factor $\alpha_3$
ZRRAD	Notch root radius (OPT = 3 only)
NF	Number of force vectors stored for filtering

**Card 2 for Structural Stress Failure.** Additional card for DAMAGE-FAILURE keyword option with OPT = 5.

Card 2f	1	2	3	4	5	6	7	8
Variable	EFAIL	ZD	ZT	ZT2				
Type	F	F	F	F				

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EFAIL	Effective plastic strain in weld material at failure. The plastic strain must exceed the rupture strain (RS) at each integration point before deletion occurs. See Card 3.
ZD	Notch diameter
ZT	Sheet thickness
ZT2	Second sheet thickness

**Card 2 for Stress Based Failure from Resultants/Rate Effects.** Additional card for DAMAGE-FAILURE keyword option with OPT = 6, 7, 9, -9 or 10.

Card 2g	1	2	3	4	5	6	7	8
Variable	EFAIL							NF
Type	F							F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EFAIL	Effective plastic strain in weld material at failure. The plastic strain must exceed the rupture strain (RS) at each integration point before deletion occurs. See Card 3.
NF	Number of force vectors stored for filtering

**Card 2 for Resultant Based Failure for Beams depending on Loading Direction.**  
Additional card for DAMAGE-FAILURE keyword option with OPT =11.

Card 2h	1	2	3	4	5	6	7	8
Variable	EFAIL	LCT	LCC					NF
Type	F	F	F					F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EFAIL	Effective plastic strain in weld material at failure. The plastic strain must exceed the rupture strain (RS) at each integration point before deletion occurs. See Card 3.
LCT	Load curve or Table ID. Load curve defines resultant failure force under tension as a function of loading direction (in degree range 0 to 90). Table defines these curves as functions of strain rates. See remarks.
LCC	Load curve or Table ID. Load curve defines resultant failure force under compression as a function of loading direction (in degree range 0 to 90). Table defines these curves as functions of strain rates. See remarks.
NF	Number of force vectors stored for filtering

Additional card for the DAMAGE-FAILURE option.

Card 3	1	2	3	4	5	6	7	8
Variable	RS	OPT	FVAL	TRUE_T	ASFF	BETA		DMGOPT
Type	F	F	F	F	I	F		F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RS	Rupture strain (or rupture time if DMGOPT = 2 or 12). Define if and only if damage is active.
OPT	Failure option:  EQ.-9: OPT = 9 failure is evaluated and written to the <b>swforc</b> file, but element failure is suppressed.  EQ.-2: Same as option -1 but in addition, the peak value of the failure criteria and the time it occurs is stored and is written into the <b>swforc</b> database. This information may be necessary since the instantaneous values written at specified time intervals may miss the peaks. Additional storage is allocated to store this information.  EQ.-1: OPT = 0 failure is evaluated and written into the <b>swforc</b> file, but element failure is suppressed.  EQ.0: Resultant based failure EQ.1: Stress based failure computed from resultants (Toyota) EQ.2: User subroutine <b>uweldfail</b> to determine failure EQ.3: Notch stress-based failure (beam and hex assembly welds only) EQ.4: Stress intensity factor at failure (beam and hex assembly welds only) EQ.5: Structural stress at failure (beam and hex assembly welds only) EQ.6: Stress based failure computed from resultants (Toyota). In this option a shell strain rate dependent failure model is used (beam and hex assembly welds only). The static failure stresses are defined by part ID using the keyword <b>*DEFINE_SPOTWELD_RUPTURE_STRESS</b> . EQ.7: Stress based failure for solid elements (Toyota) with peak stresses computed from resultants, and strength values input for pairs of parts; see <b>*DEFINE_SPOTWELD_FAILURE_RESULTANTS</b> . Strain rate effects are optional. EQ.8: Not used EQ.9: Stress based failure from resultants (Toyota). In this option a shell strain rate dependent failure model is used (beam welds only). The static failure stresses are defined

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	by part ID using the keyword *DEFINE_SPOTWELD_-RUPTURE_PARAMETER.
	EQ.10: Stress based failure with rate effects. Failure data is defined by material using the keyword *DEFINE_SPOW-ELD_FAILURE.
	EQ.11: Resultant based failure (beams only). In this option load curves or tables LCT (tension) and LCC (compression) can be defined as resultant failure force as a function loading direction (curve) or resultant failure force as a function of loading direction for each strain rate (table).
	EQ.12: User subroutine uweldfail12 with 22 material constants to determine damage and failure
	EQ.22: user subroutine uweldfail22 with 22 material constants to determine failure
FVAL	<p>Failure parameter.</p> <p>OPT.EQ.-2: Not used</p> <p>OPT.EQ.-1: Not used</p> <p>OPT.EQ.0: Function ID (*DEFINE_FUNCTION) to define alternative Weld Failure. If this is set, the values given for NRR, NRS, NRT, MRR, MSS and MTT in Card 2 are ignored. See description of Weld Failure for OPT = 0.</p> <p>OPT.EQ.1: Not used</p> <p>OPT.EQ.2: Not used</p> <p>OPT.EQ.3: Notch stress value at failure (<math>\sigma_{KF}</math>)</p> <p>OPT.EQ.4: Stress intensity factor value at failure (<math>K_{eqF}</math>)</p> <p>OPT.EQ.5: Structural stress value at failure (<math>\sigma_{sF}</math>)</p> <p>OPT.EQ.6: Number of cycles that failure condition must be met to trigger beam deletion.</p> <p>OPT.EQ.7: Not used</p> <p>OPT.EQ.9: Number of cycles that failure condition must be met to trigger beam deletion.</p> <p>OPT.EQ.10: ID of data defined by *DEFINE_SPOTWELD_FAILURE.</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	OPT.EQ.12: Number of history variables available in user defined failure subroutine, uweldfaill2.
TRUE_T	True weld thickness. This optional value is available for solid element failure and is used to reduce the moment contribution to the failure calculation from artificially thick weld elements under shear loading, so shear failure can be modeled more accurately. Note that the behavior of TRUE_T depends on TTOPT. See <a href="#">Remark 8</a> .
	<div style="border: 1px solid black; padding: 10px;"><b>NOTE:</b> We do not recommend using TRUE_T. Instead, we recommend using TTOPT = 2 and leaving TRUE_T = 0.0. In many cases, TTOPT = 2 does a better job of removing the spurious moments. See <a href="#">Remark 9</a>.</div>
ASFF	Weld assembly simultaneous failure flag: EQ.0: Damaged elements fail individually. EQ.1: Damaged elements fail when first reaches failure criterion.
BETA	Damage model decay rate
DMGOPT	Damage option flag: EQ.-1: Flag to include Card 3.1 for additional damage fields. DMGOPT will be set on Card 3.1. EQ.0: Plastic strain based damage EQ.1: Plastic strain based damage with post damage stress limit EQ.2: Time based damage with post damage stress limit EQ.10: Like DMGOPT = 0, but failure option will initiate damage EQ.11: Like DMGOPT = 1, but failure option will initiate damage EQ.12: Like DMGOPT = 2, but failure option will initiate damage

**Damage Option Card.** Optional additional card for the DAMAGE-FAILURE option; read only if DMGOPT = -1 on Card 3.

Card 3.1	1	2	3	4	5	6	7	8
Variable	DMGOPT	FMODE	FFCAP	TTOPT				
Type	F	F	F	I				

VARIABLE	DESCRIPTION
DMGOPT	<p>Damage option flag:</p> <ul style="list-style-type: none"> <li>EQ.0: Plastic strain based damage</li> <li>EQ.1: Plastic strain based damage with post damage stress limit</li> <li>EQ.2: Time based damage with post damage stress limit</li> <li>EQ.10: Like DMGOPT = 0, but failure option will initiate damage</li> <li>EQ.11: Like DMGOPT = 1, but failure option will initiate damage</li> <li>EQ.12: Like DMGOPT = 2, but failure option will initiate damage</li> </ul>
FMODE	<p>Failure surface ratio for damage or failure, for DMGOPT = 10, 11, or 12</p> <ul style="list-style-type: none"> <li>EQ.0: Damage initiates</li> <li>GT.0: Damage or failure (see <a href="#">Remark 6</a>)</li> </ul>
FFCAP	<p>Failure function limit for OPT = 0 or -1 and DMGOPT = 10, 11, or 12</p> <ul style="list-style-type: none"> <li>EQ.0: Damage initiates</li> <li>GT.0: Damage or failure (see <a href="#">Remark 6</a>)</li> </ul>
TTOPT	<p>Options for TRUE_T / weld failure behavior:</p> <ul style="list-style-type: none"> <li>EQ.0: TRUE_T behavior of version R9 and later (see <a href="#">Remark 8</a>)</li> <li>EQ.1: TRUE_T behavior of version R8 and earlier (see <a href="#">Remark 8</a>)</li> <li>EQ.2: Weld failure is invariant with respect to the node numbering of weld elements. For this case, there is no need for the TRUE_T correction. We recommend using this option with TRUE_T set to 0.0. See <a href="#">Remark 9</a>.</li> </ul>

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**Failure Constants Card.** Additional card for OPT = 12 or 22.

Card 4	1	2	3	4	5	6	7	8
Variable	USRV7	USRV8	USRV9	USRV10	USRV11	USRV12	USRV13	USRV14
Type	F	F	F	F	F	F	F	F

**Failure Constants Card.** Additional card for OPT = 12 or 22.

Card 5	1	2	3	4	5	6	7	8
Variable	USRV15	USRV16	USRV17	USRV18	USRV19	USRV20	USRV21	USRV22
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
USRV $n$	Failure constants for OPT = 12 or 22 user defined failure, $n = 7, 8, \dots, 22$

**Remarks:**

1. **Failure Model Overview.** Spot weld material is modeled with isotropic hardening plasticity coupled to failure models. EFAIL specifies a failure strain which fails each integration point in the spot weld independently. The OPT parameter is used to specify a failure criterion that fails the entire weld element when the criterion is met. Alternatively, EFAIL and OPT may be used to initiate damage when the DAMAGE-FAILURE option is active using RS, BETA, and DMGOPT as described below.

Beam spot weld elements can use any OPT value except 7. Brick spot weld elements can use any OPT value except 3, 4, 5, 6, 9, and -9. Hex assembly spot welds can use any OPT value except 9 and -9.

For all OPT failure criteria, if a zero is input for a failure parameter on Card 2, the corresponding term will be omitted from the equation. For example, if for OPT = 0, only  $N_{rr_F}$  is nonzero, the failure surface is reduced to  $|N_{rr}| = N_{rr_F}$  (see below).

Similarly, if the failure strain EFAIL is set to zero, the failure strain model is not used. Both EFAIL and OPT failure may be active at the same time.

2. **Loading Solid Welds Uniaxially.** We have implemented two methods of loading solid and solid weld assemblies uniaxially. The older method is invoked by defining the elastic modulus,  $E$ , as a negative number where the absolute value of  $E$  is the desired value for  $E$ . This uniaxial option causes the two transverse stress terms to be assumed to be zero throughout the calculation. This assumption eliminates parasitic transverse stress that causes slow growth of plastic strain-based damage.

The other method is invoked by setting **OPTION2** to **UNIAXIAL**. This method is preferred. It causes the two transverse stress terms and the two transverse strains terms to be set to zero. It was added because we found that the older method sometimes induced spurious oscillations in the axial force, leading to premature failure.

The motivation for the uniaxial options can be explained with a weld loaded in tension. Due to Poisson's effect, an element in tension would be expected to contract in the transverse directions. However, because the weld nodes are constrained to the mid-plane of shell elements, such contraction is only possible to the degree that the shell element contracts. In other words, the uniaxial stress state cannot be represented by the weld. For plastic strain-based damage, this effect can be particularly apparent as it causes tensile stress to continue to grow very large as the stress state becomes very nearly triaxial tension. In this stress state, plastic strain grows very slowly so it appears that damage calculation is failing to knock down the stress. By simply assuming that the transverse stresses are zero, the plastic strain grows as expected and damage is much more effective.

3. **NF.** NF specifies the number of terms used to filter the stresses or resultants used in the OPT failure criterion. NF cannot exceed 30. The default value is set to zero which is generally recommended unless oscillatory resultant forces are observed in the time history databases. Although welds should not oscillate significantly, this option was added for consistency with the other spot weld options. NF affects the storage since it is necessary to store the resultant forces as history variables. The NF parameter is available only for beam element welds.
4. **Time Scaling.** The inertias of the spot welds are scaled during the first time step so that their stable time step size is  $\Delta t$ . A strong compressive load on the spot weld at a later time may reduce the length of the spot weld so that stable time step size drops below  $\Delta t$ . If the value of  $\Delta t$  is zero, mass scaling is not performed, and the spot welds will probably limit the time step size. Under most circumstances, the inertias of the spot welds are small enough that scaling them will have a negligible effect on the structural response and the use of this option is encouraged.

5. **Yield Curve or Table for SIGY.** When using a yield curve or table for SIGY, a simplified plasticity algorithm is used, assuming a linear behavior within one time increment. Thus, no iterative return mapping has to be performed.
6. **Damage.** When the DAMAGE-FAILURE option is invoked, the constitutive properties for the damaged material are obtained from the undamaged material properties. The amount of damage evolved is represented by the constant,  $\omega$ , which varies from zero if no damage has occurred to unity for complete rupture. For uniaxial loading, the nominal stress in the damaged material is given by

$$\sigma_{\text{nominal}} = \frac{P}{A}$$

where  $P$  is the applied load and  $A$  is the surface area. The true stress is given by:

$$\sigma_{\text{true}} = \frac{P}{A - A_{\text{loss}}}$$

where  $A_{\text{loss}}$  is the void area. The damage variable can then be defined:

$$\omega = \frac{A_{\text{loss}}}{A} ,$$

where

$$0 \leq \omega \leq 1 .$$

In this model, damage is initiated when the effective plastic strain in the weld exceeds the failure strain, EFAIL. If DMGOPT = 10, 11, or 12, damage will initiate when the effective plastic strain exceeds EFAIL, or when the failure criterion is met, whichever occurs first. The failure criterion is specified by the OPT parameter. If the inputted value of EFAIL = 0 and DMGOPT = 10, 11, or 12, then damage will only be initiated if the failure criterion is met. After damage initiates, the damage variable is evaluated by one of two ways:

- a) For DMGOPT = 0, 1, 10, or 11, the damage variable is a function of effective plastic strain in the weld:

$$\varepsilon_{\text{failure}}^p \leq \varepsilon_{\text{eff}}^p \leq \varepsilon_{\text{rupture}}^p \Rightarrow \omega = \frac{\varepsilon_{\text{eff}}^p - \varepsilon_{\text{failure}}^p}{\varepsilon_{\text{rupture}}^p - \varepsilon_{\text{failure}}^p} ,$$

where  $\varepsilon_{\text{failure}}^p$  = EFAIL and  $\varepsilon_{\text{rupture}}^p$  = RS. If DMGOPT = 10 or 11, and damage initiates by the failure criterion, then  $\varepsilon_{\text{failure}}^p$  is set equal to the effective plastic strain in the weld at the time of damage initiation.

- b) For DMGOPT = 2 or 12, the damage variable is a function of time:

$$t_{\text{failure}} \leq t \leq t_{\text{rupture}} \Rightarrow \omega = \frac{t - t_{\text{failure}}}{t_{\text{rupture}}}$$

where  $t_{\text{failure}}$  is the time at which damage initiates, and  $t_{\text{rupture}} = \text{RS}$ . For DMGOPT = 2,  $t_{\text{failure}}$  is set equal to the time at which  $\epsilon_{\text{eff}}^p$  exceeds EFAIL. For DMGOPT = 12,  $t_{\text{failure}}$  is set equal to either the time when  $\epsilon_{\text{eff}}^p$  exceeds EFAIL or the time when the failure criterion is met, whichever occurs first.

If DMGOPT = 0, 1, or 2, inputting EFAIL = 0 will cause damage to initiate as soon as the weld stress reaches the yield surface. Prior to version 9.1, inputting EFAIL = 0 for DMGOPT = 10, 11, 12 would similarly cause damage to initiate when the stress state reaches the yield surface, but version 9.1 and later will ignore EFAIL = 0 and only initiate damage when the failure criterion is met. If the effective plastic strain is zero when damage initiates by the failure criterion, then the yield stress of the weld is reduced to the current effective stress so that the stress state is on the yield surface and plastic strain can start to grow.

For DMGOPT = 1, the damage behavior is the same as for DMGOPT = 0, but an additional damage variable is calculated to prevent stress growth during softening. Similarly, DMGOPT = 11 behaves like DMGOPT = 10 except for the additional damage variable. This additional function is also used with DMGOPT = 2 and 12. The effect of this additional damage function is noticed only in brick and brick assembly welds in tension loading where it prevents growth of the tensile force in the weld after damage initiates.

For DMGOPT = 10, 11, or 12 an optional FMODE parameter determines whether a weld that reaches the failure surface will fail immediately or initiate damage. The failure surface calculation has shear terms, which may include the torsional moment as well as normal and bending terms. If FMODE is input with a value between 0 and 1, then when the failure surface is reached, the sum of the square of the shear terms is divided by the sum of the square of all terms. If this ratio exceeds FMODE, then the weld will fail immediately. If the ratio is less than or equal to FMODE, then damage will initiate. The FMODE option is available only for brick and brick assembly welds.

For resultant based failure (OPT = -1 or 0) and DMGOPT = 10, 11, or 12 an optional FFCAP parameter determines whether a weld that reaches the failure surface will fail immediately. After damage initiation, the failure function can reach values above 1.0. This can now be limited by the FFCAP value (should be larger than 1.0):

$$\left( \left[ \frac{\max(N_{rr}, 0)}{N_{rr_F}} \right]^2 + \left[ \frac{N_{rs}}{N_{rs_F}} \right]^2 + \left[ \frac{N_{rt}}{N_{rt_F}} \right]^2 + \left[ \frac{M_{rr}}{M_{rr_F}} \right]^2 + \left[ \frac{M_{ss}}{M_{ss_F}} \right]^2 + \left[ \frac{M_{tt}}{M_{tt_F}} \right]^2 \right)^{\frac{1}{2}} < \text{FFCAP}$$

7. **BETA.** If BETA is specified, the stress is multiplied by an exponential using  $\omega$  defined in the equations define in [Remark 6](#),

$$\sigma_d = \sigma \exp(-\beta \omega).$$

For weld elements in an assembly (see RPBHX on \*CONTROL\_SPOTWELD\_BEAM or \*DEFINE\_HEX\_SPOTWELD\_ASSEMBLY), the failure criterion is evaluated using the assembly cross section. If damage is not active, all elements will be deleted when the failure criterion is met. If damage is active, then damage is calculated independently in each element of the assembly. By default, elements of the assembly are deleted as damage in each element is complete. If ASFF = 1, then failure and deletion of all elements in the assembly will occur simultaneously when damage is complete in any one of the elements.

8. **TRUE\_T and TTOPT.** Solid weld elements and weld assemblies are tied to the mid-plane of shell materials and so typically have a thickness that is half the sum of the thicknesses of the welded shell sections. As a result, a weld under shear loading can be subject to an artificially large moment which will be balanced by normal forces transferred through the tied contact. These normal forces will cause the out-of-plane bending moment used in the failure calculation to be artificially high.

TRUE\_T was our original implementation to fix this issue. Inputting a TRUE\_T value that is smaller than the modeled thickness, for example, 10%-30% of true thickness will scale down the moment or stress that results from the balancing moment and provide more realistic failure calculations for solid elements and weld assemblies. TRUE\_T effects only the failure calculation, not the weld element behavior. If TRUE\_T = 0.0 or data is omitted, the modeled weld element thickness is used. Our preferred solution to this problem is keeping TRUE\_T = 0.0 and setting TTOPT = 2 which is discussed in [Remark 9](#).

The behavior of TRUE\_T depends on TTOPT. In LS-DYNA version R9, a modification to the TRUE\_T behavior was made to address a condition of weld assemblies that are tied to shell elements of significantly different stiffness. This change had unintended effects on the behavior of weld failure, so TTOPT was added to revert the behavior of TRUE\_T to that of R8 and earlier versions. The default behavior of TTOPT is to perform the R9 method but setting TTOPT = 1 will cause the earlier method to be used. TTOPT = 1 also invokes a second correction. With TTOPT = 0, weld assemblies use TRUE\_T as if it was a scale factor, but single element welds use it as a thickness value. Setting TTOPT = 1 corrects this so that both weld assemblies and single welds use TRUE\_T as a thickness value.

For OPT = 0 (see below), the two out-of-plane moments,  $M_{ss}$  and  $M_{tt}$  are replaced by modified terms  $\widehat{M}_{ss}$  and  $\widehat{M}_{tt}$ :

$$\left[ \frac{\max(N_{rr}, 0)}{N_{rr_F}} \right]^2 + \left[ \frac{N_{rs}}{N_{rs_F}} \right]^2 + \left[ \frac{N_{rt}}{N_{rt_F}} \right]^2 + \left[ \frac{M_{rr}}{M_{rr_F}} \right]^2 + \left[ \frac{\widehat{M}_{ss}}{M_{ss_F}} \right]^2 + \left[ \frac{\widehat{M}_{tt}}{M_{tt_F}} \right]^2 - 1 = 0$$

$$\begin{aligned}\widehat{M}_{ss} &= \begin{cases} M_{ss} - N_{rt}t(1 - t_{\text{true}}) & \text{for solid weld assemblies with TTOPT = 0} \\ M_{ss} - N_{rt}(t - t_{\text{true}}) & \text{otherwise} \end{cases} \\ \widehat{M}_{tt} &= \begin{cases} M_{tt} - N_{rs}t(1 - t_{\text{true}}) & \text{for solid weld assemblies with TTOPT = 0} \\ M_{tt} - N_{rs}(t - t_{\text{true}}) & \text{otherwise} \end{cases}\end{aligned}$$

In the above,  $t$  is the element thickness and  $t_{\text{true}}$  is the TRUE\_T parameter. For OPT = 1 (see below), the same modification is done to the moments that contribute to the normal stress, as shown below:

$$\sigma_{rr} = \frac{N_{rr}}{A} + \frac{\sqrt{\widehat{M}_{ss}^2 + \widehat{M}_{tt}^2}}{Z}.$$

9. **TTOPT = 2.** By default, failure is calculated using forces on the bottom surface of the weld as defined by nodes 1 to 4 of each element. Setting TTOPT = 2 causes the average of the forces on the bottom and top to be used so that failure is invariant. When TTOPT = 2 is used, the averaging causes the spurious moments or peak normal stress to cancel, so there is no need for a TRUE\_T correction. Therefore, the best practice is to use TTOPT = 2 and TRUE\_T = 0.0.
10. **History Data Output Files.** Spot weld force history data is written into the swforc ASCII file. In this database the resultant moments are not available, but they are in the binary time history database and in the ASCII elout file.
11. **Material Histories.** The probability of failure in solid or beam spotwelds can be estimated by retrieving the corresponding material histories for output to the d3plot database.

<i>*DEFINE_MATERIAL_HISTORIES Properties</i>						
<b>Label</b>	<b>Attributes</b>					<b>Description</b>
Instability	-	-	-	-	-	A measure between 0 and 1 related to how close the spotweld element is to fail
Damage	-	-	-	-	-	Damage in the spotweld element between 0 and 1

These two labels are supported for all options (OPT and DMGOPT, including assemblies and beams), except for user defined failure. The instability measure is the maximum over time; namely, it gives the maximum value for a given element throughout the simulation. If a damage option is invoked, then damage will initiate and increment when the instability reaches unity, and elements are not deleted until the damage value reaches unity. If no damage option is invoked, then the damage output is always zero and elements will be deleted at the point when the instability measure reaches unity

**OPT = -1 or 0**

OPT = 0 and OPT = -1 invoke a resultant-based failure criterion that fails the weld if the resultants are outside of the failure surface defined by:

$$\left[ \frac{\max(N_{rr}, 0)}{N_{rr_F}} \right]^2 + \left[ \frac{N_{rs}}{N_{rs_F}} \right]^2 + \left[ \frac{N_{rt}}{N_{rt_F}} \right]^2 + \left[ \frac{M_{rr}}{M_{rr_F}} \right]^2 + \left[ \frac{M_{ss}}{M_{ss_F}} \right]^2 + \left[ \frac{M_{tt}}{M_{tt_F}} \right]^2 - 1 = 0$$

where the *numerators* in the equation are the resultants calculated in the local coordinates of the cross section, and the *denominators* are the values specified in the input. If OPT = -1, the failure surface equation is evaluated, but element failure is suppressed. This allows easy identification of vulnerable spot welds when post-processing. Failure is likely to occur if FC > 1.0.

Alternatively, a \*DEFINE\_FUNCTION could be used to define the Weld Failure for OPT = 0. Then set FVAL = function ID. Such a function could look like this:

```
*DEFINE_FUNCTION
 100
 func(nrr,nrs,nrt,mrr,mss,mtt)= (nrr/5.0)*(nrr/5.0)
```

The six arguments for this function (nrr, ..., mtt) are the force and moment resultants during the computation.

**OPT = 1:**

OPT = 1 invokes a stress based failure model, which was developed by *Toyota Motor Corporation* and is based on the peak axial and transverse shear stresses. The weld fails if the stresses are outside of the failure surface defined by

$$\left( \frac{\sigma_{rr}}{\sigma_{rr}^F} \right)^2 + \left( \frac{\tau}{\tau^F} \right)^2 - 1 = 0$$

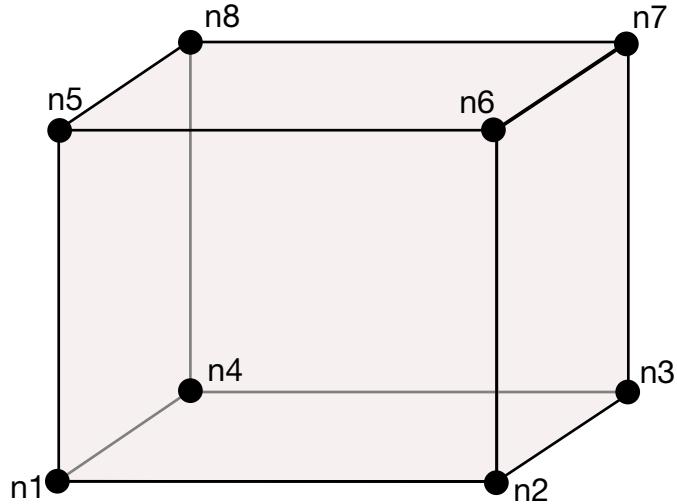
If strain rates are considered, then the failure criteria becomes:

$$\left[ \frac{\sigma_{rr}}{\sigma_{rr}^F(\dot{\epsilon}_{eff})} \right]^2 + \left[ \frac{\tau}{\tau^F(\dot{\epsilon}_{eff})} \right]^2 - 1 = 0$$

where  $\sigma_{rr}^F(\dot{\epsilon}_{eff})$  and  $\tau^F(\dot{\epsilon}_{eff})$  are defined by load curves (SIGAX and SIGTAU are less than zero). The peak stresses are calculated from the resultants using simple beam theory:

$$\sigma_{rr} = \frac{N_{rr}}{A} + \frac{\sqrt{M_{ss}^2 + M_{tt}^2}}{Z}$$

$$\tau = \frac{M_{rr}}{2Z} + \frac{\sqrt{N_{rs}^2 + N_{rt}^2}}{A}$$



**Figure M100-1.** A solid element used as spot weld is shown. When resultant based failure is used orientation is very important. Nodes n1 - n4 attach to the lower shell mid-surface and nodes n5 - n8 attach to the upper shell mid-surface. The resultant forces and moments are computed based on the assumption that the brick element is properly oriented.

where the area and section modulus are given by:

$$A = \pi \frac{d^2}{4}$$

$$Z = \pi \frac{d^3}{32}$$

In the above equations,  $d$  is the equivalent diameter of the beam element or solid element used as a spot weld.

## OPT = 2

OPT = 2 invokes a user-written subroutine `uweldfail`, documented in Appendix Q.

## OPT = 12 or 22

OPT = 12 and OPT = 22 invoke similar user-written subroutines, `uweldfail12` and `uweldfail122`, respectively. Both allow up to 22 failure parameters to be used rather than the 6 allowed with OPT = 2. OPT = 12 also allows user control of weld damage.

## OPT = 3

OPT = 3 invokes a failure based on notch stress, see Zhang [1999]. Failure occurs when the failure criterion:

$$\sigma_k - \sigma_{kF} \geq 0$$

is satisfied. The notch stress is given by the equation:

$$\sigma_k = \alpha_1 \frac{4F}{\pi dt} \left( 1 + \frac{\sqrt{3} + \sqrt{19}}{8\sqrt{\pi}} \sqrt{\frac{t}{\rho}} \right) + \alpha_2 \frac{6M}{\pi dt^2} \left( 1 + \frac{2}{\sqrt{3\pi}} \sqrt{\frac{t}{\rho}} \right) + \alpha_3 \frac{4F_{rr}}{\pi d^2} \left( 1 + \frac{5}{3\sqrt{2\pi}} \frac{d}{t} \sqrt{\frac{t}{\rho}} \right)$$

Here,

$$F = \sqrt{F_{rs}^2 + F_{rt}^2}$$

$$M = \sqrt{M_{ss}^2 + M_{tt}^2}$$

and the  $\alpha_i$  ( $i = 1, 2, 3$ ) are input corrections factors with default values of unity. If spot welds are between sheets of unequal thickness, the minimum thickness of the spot welded sheets may be introduced as a crude approximation.

#### **OPT = 4**

OPT = 4 invokes failure based on structural stress intensity, see Zhang [1999]. Failure occurs when the failure criterion:

$$K_{eq} - K_{eqF} \geq 0$$

is satisfied where

$$K_{eq} = \sqrt{K_I^2 + K_{II}^2}$$

and

$$K_I = \alpha_1 \frac{\sqrt{3}F}{2\pi d \sqrt{t}} + \alpha_2 \frac{2\sqrt{3}M}{\pi dt \sqrt{t}} + \alpha_3 \frac{5\sqrt{2}F_{rr}}{3\pi d \sqrt{t}}$$

$$K_{II} = \alpha_1 \frac{2F}{\pi d \sqrt{t}}$$

Here,  $F$  and  $M$  are as defined above for the notch stress formulas and again,  $\alpha_i$  ( $i = 1, 2, 3$ ) are input corrections factors with default values of unity. If spot welds are between sheets of unequal thickness, the minimum thickness of the spot welded sheets may be used as a crude approximation.

The maximum structural stress at the spot weld was utilized successfully for predicting the fatigue failure of spot welds, see Rupp, et. al. [1994] and Sheppard [1993]. The corresponding results from] Rupp, et. al. are listed below where it is assumed that they may be suitable for crash conditions.

#### **OPT = 5**

OPT = 5 invokes failure by

$$\max(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}) - \sigma_{sF} = 0 ,$$

where  $\sigma_{sF}$  is the critical value of structural stress at failure. It is noted that the forces and moments in the equations below refer to the beam node 1, beam node 2, and the midpoint, respectively. The three stress values,  $\sigma_{v1}, \sigma_{v2}, \sigma_{v3}$ , are defined by:

$$\sigma_{v1}(\zeta) = \frac{F_{rs1}}{\pi dt_1} \cos\zeta + \frac{F_{rt1}}{\pi dt_1} \sin\zeta - \frac{1.046\beta_1 F_{rr1}}{t_1 \sqrt{t_1}} - \frac{1.123M_{ss1}}{dt_1 \sqrt{t_1}} \sin\zeta + \frac{1.123M_{tt1}}{dt_1 \sqrt{t_1}} \cos\zeta$$

with

$$\beta_1 = \begin{cases} 0 & F_{rr1} \leq 0 \\ 1 & F_{rr1} > 0 \end{cases}$$

$$\sigma_{v2}(\zeta) = \frac{F_{rs2}}{\pi dt_2} \cos\zeta + \frac{F_{rt2}}{\pi dt_2} \sin\zeta - \frac{1.046\beta_1 F_{rr2}}{t_2 \sqrt{t_2}} + \frac{1.123M_{ss2}}{dt_2 \sqrt{t_2}} \sin\zeta - \frac{1.123M_{tt2}}{dt_2 \sqrt{t_2}} \cos\zeta$$

with

$$\beta_2 = \begin{cases} 0 & F_{rr2} \leq 0 \\ 1 & F_{rr2} > 0 \end{cases}$$

$$\sigma_{v3}(\zeta) = 0.5\sigma(\zeta) + 0.5\sigma(\zeta)\cos(2\alpha) + 0.5\tau(\zeta)\sin(2\alpha)$$

where

$$\begin{aligned} \sigma(\zeta) &= \frac{4\beta_3 F_{rr}}{\pi d^2} + \frac{32M_{ss}}{\pi d^3} \sin\zeta - \frac{32M_{tt}}{\pi d^3} \cos\zeta \\ \tau(\zeta) &= \frac{16F_{rs}}{3\pi d^2} \sin^2\zeta + \frac{16F_{rt}}{3\pi d^2} \cos^2\zeta \\ \alpha &= \frac{1}{2} \tan^{-1} \frac{2\tau(\zeta)}{\sigma(\zeta)} \\ \beta_3 &= \begin{cases} 0 & F_{rr} \leq 0 \\ 1 & F_{rr} > 0 \end{cases} \end{aligned}$$

The stresses are calculated for all directions,  $0^\circ \leq \zeta \leq 90^\circ$ , in order to find the maximum.

## **OPT = 10**

OPT = 10 invokes the failure criterion developed by Lee and Balur (2011). It is available for welds modeled by beam elements, solid elements, or solid assemblies. A detailed discussion of the criterion is given in the user's manual section for \*DEFINE\_SPOTWELD\_FAILURE.

## **OPT = 11**

OPT = 11 invokes a resultant force based failure criterion for beams. With corresponding load curves or tables LCT and LCC, resultant force at failure  $F_{fail}$  can be defined as function of loading direction  $\gamma$  (curve) or loading direction  $\gamma$  and effective strain rate  $\dot{\epsilon}$  (table):

$$F_{fail} = f(\gamma) \quad \text{or} \quad F_{fail} = f(\gamma, \dot{\epsilon})$$

with the following definitions for loading direction (in degree) and effective strain rate:

$$\gamma = \tan^{-1} \left( \left| \frac{F_{\text{shear}}}{F_{\text{axial}}} \right| \right), \quad \dot{\varepsilon} = \left[ \frac{2}{3} (\dot{\varepsilon}_{\text{axial}}^2 + \dot{\varepsilon}_{\text{shear}}^2) \right]^{1/2}$$

It depends on the sign of the axial beam force, if LCT or LCC are used for failure condition:

$$F_{\text{axial}} > 0: \quad [F_{\text{axial}}^2 + F_{\text{shear}}^2]^{1/2} > F_{\text{fail,LCT}} \rightarrow \text{failure}$$

$$F_{\text{axial}} < 0: \quad [F_{\text{axial}}^2 + F_{\text{shear}}^2]^{1/2} > F_{\text{fail,LCC}} \rightarrow \text{failure}$$

**\*MAT\_SPOTWELD\_DAIMLERCHRYSLER\_{OPTION}**

This is Material Type 100. The material model applies only to solid element type 1. If hourglass type 4 is specified, then hourglass type 4 will be used; otherwise, hourglass type 6 will be automatically assigned. Hourglass type 6 is preferred.

Spot weld elements may be placed between any two deformable shell surfaces and tied with constraint contact, \*CONTACT\_TIED\_SURFACE\_TO\_SURFACE, which eliminates the need to have adjacent nodes at spot weld locations. Spot weld failure is modeled using this card and \*DEFINE\_CONNECTION\_PROPERTIES data. Details of the failure model can be found in Seeger, Feucht, Frank, Haufe, and Keding [2005].

**NOTE:** It is advisable to include all spot welds, which provide the tracked nodes, and spot welded materials, which define the reference segments, within a single \*CONTACT\_TIED\_SURFACE\_TO\_SURFACE interface. This contact type uses constraint equations. If multiple interfaces are treated independently, significant problems can occur if such interfaces share common nodes. An added benefit is that memory usage can be substantially less with a single interface.

Available options include:

<BLANK>

UNIAXIAL

The UNIAXIAL keyword option causes the transverse stresses and transverse strains to be zero for solid spot welds. The older uniaxial method, invoked with E < 0.0 on Card 1, assumed only the transverse stresses are zero. Compared to the older method, the UNIAXIAL keyword option increases the stability of the solver. See [Remark 1](#) for more details.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR			DT	TFAIL
Type	A	F	F	F			F	F

**\*MAT\_100\_DA****\*MAT\_SPOTWELD\_DAIMLERCHRYSLER**

Card 2	1	2	3	4	5	6	7	8
Variable	EFAIL							NF
Type	F							F

Card 3	1	2	3	4	5	6	7	8
Variable	RS	ASFF		TRUE_T	CON_ID	RFILTF	JTOL	DMGOPT
Type	F	I		F	F	F	F	F

**Damage Option Card.** LS-DYNA reads this optional card only if DMGOPT = -1 on Card 3.

Card 3.1	1	2	3	4	5	6	7	8
Variable				TTOPT				
Type				I				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus.  LT.0.0: $ E $ is the Young's modulus. $E < 0$ invokes uniaxial stress for solid spot welds with the transverse stresses assumed to be zero. See <a href="#">Remark 1</a> . This is for when the keyword option is unset (<BLANK>) only.
PR	Poisson's ratio
DT	Time step size for mass scaling, $\Delta t$
TFAIL	Failure time if nonzero. If zero, this option is ignored.
EFAIL	Effective plastic strain in weld material at failure. See <a href="#">Remark 2</a> .

VARIABLE	DESCRIPTION
NF	Number of failure function evaluations stored for filtering by time averaging. The default value is set to zero which is generally recommended unless oscillatory resultant forces are observed in the time history databases. Even though these welds should not oscillate significantly, this option was added for consistency with the other spot weld options. NF affects the storage since it is necessary to store the failure terms. When NF is nonzero, the resultants in the output databases are filtered. NF cannot exceed 30.
RS	Rupture strain. See <a href="#">Remark 2</a> .
ASFF	Weld assembly simultaneous failure flag (see <a href="#">Remark 3</a> ): EQ.0: Damaged elements fail individually. EQ.1: Damaged elements fail when first reaches failure criterion.
TRUE_T	True weld thickness for single hexahedron solid weld elements. Note that the behavior of TRUE_T depends on TTOPT. See <a href="#">Remark 8</a> on *MAT_SPOTWELD.
<b>NOTE:</b> We do not recommend using TRUE_T. Instead, we recommend using TTOPT = 2 and leaving TRUE_T = 0.0. In many cases, TTOPT = 2 does a better job of removing the spurious moments. See <a href="#">Remark 9</a> on *MAT_SPOTWELD.	
CON_ID	Connection ID of *DEFINE_CONNECTION card. A negative CON_ID deactivates failure; see <a href="#">Remark 5</a> .
RFILTF	Smoothing factor on the effective strain rate (default is 0.0), potentially used in table DSIGY < 0 and in functions for PRUL.ge.2 (see *DEFINE_CONNECTION_PROPERTIES).
$\dot{\varepsilon}_n^{\text{avg}} = \text{RFILTF} \times \dot{\varepsilon}_{n-1}^{\text{avg}} + (1 - \text{RFILTF}) \times \dot{\varepsilon}_n$	
JTOL	Tolerance value for relative volume change (default: JTOL = 0.01). Solid element spot welds with a Jacobian less than JTOL will be eroded.
DMGOPT	Damage option flag: EQ.-1: Flag to include Card 3.1 for additional damage fields.

VARIABLE	DESCRIPTION
TTOPT	Options for TRUE_T / weld failure behavior:  EQ.0: TRUE_T behavior of version R9 and later (see <a href="#">Remark 8</a> on *MAT_SPOTWELD)  EQ.1: TRUE_T behavior of version R8 and earlier (see <a href="#">Remark 8</a> on *MAT_SPOTWELD)  EQ.2: Weld failure is invariant with respect to the node numbering of weld elements. For this case, there is no need for the TRUE_T correction. We recommend using this option with TRUE_T set to 0.0. See <a href="#">Remark 9</a> on *MAT_SPOTWELD.

**Remarks:**

1. **Loading solid welds uniaxially.** We have implemented two methods of loading solid and solid weld assemblies uniaxially. The older method is invoked by defining the elastic modulus,  $E$ , as a negative number where the absolute value of  $E$  is the desired value for  $E$ . This uniaxial option causes the two transverse stress terms to be assumed to be zero throughout the calculation. This assumption eliminates parasitic transverse stress that causes slow growth of plastic strain-based damage.

The other method is invoked by setting OPTION to UNIAXIAL. This method is preferred. It causes the two transverse stress terms and the two transverse strains terms to be set to zero. It was added because we found that the older method sometimes induced spurious oscillations in the axial force, leading to premature failure.

The motivation for the uniaxial options can be explained with a weld loaded in tension. Due to Poisson's effect, an element in tension would be expected to contract in the transverse directions. However, because the weld nodes are constrained to the mid-plane of shell elements, such contraction is only possible to the degree that the shell element contracts. In other words, the uniaxial stress state cannot be represented by the weld. For plastic strain-based damage, this effect can be particularly apparent as it causes tensile stress to continue to grow very large as the stress state becomes very nearly triaxial tension. In this stress state, plastic strain grows very slowly so it appears that damage calculation is failing to knock down the stress. By simply assuming that the transverse stresses are zero, the plastic strain grows as expected and damage is much more effective

2. **Connection properties.** This weld material is modeled with isotropic hardening plasticity. The yield stress and constant hardening modulus are assumed to

be those of the welded shell elements as defined in a \*DEFINE\_CONNECTION\_PROPERTIES table. \*DEFINE\_CONNECTION\_PROPERTIES data also define a failure function and the damage type. The interpretation of EFAIL and RS is determined by the choice of damage type. This is discussed in Remark 4 on \*DEFINE\_CONNECTION\_PROPERTIES.

3. **Weld assembly failure.** For weld elements in an assembly (see RPBHX on \*CONTROL\_SPOTWELD\_BEAM or \*DEFINE\_HEX\_SPOTWELD\_ASSEMBLY), the failure criterion is evaluated using the assembly cross section. If damage is not active, all elements will be deleted when the failure criterion is met. If damage is active, then damage is calculated independently in each element of the assembly. By default, elements of the assembly are deleted as damage in each element is complete. If ASFF = 1, then failure and deletion of all elements in the assembly will occur simultaneously when damage is complete in any one of the elements.
4. **Output.** Solid element force resultants for \*MAT\_SPOTWEL\_DAIMLER-CHRYSLER are written to the spot weld force file, swforc, and the file for element stresses and resultants for designated elements, ELOUT. Also, spot weld failure data is written to the file, dcfail.
5. **Deactivating weld failure.** An option to deactivate weld failure is switched on by setting CON\_ID to a negative value where the absolute value of CON\_ID becomes the connection ID. When weld failure is deactivated, the failure function is evaluated and output to swforc and dcfail, but the weld retains its full strength.

**\*MAT\_101****\*MAT\_GEPLASTIC\_SRATE\_2000a****\*MAT\_GEPLASTIC\_SRATE\_2000a**

This is Material Type 101. The GEPLASTIC\_SRATE\_2000a material model characterizes General Electric's commercially available engineering thermoplastics subjected to high strain rate events. This material model features the variation of yield stress as a function of strain rate, cavitation effects of rubber modified materials, and automatic element deletion of either ductile or brittle materials.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	RATESF	EDOTO	ALPHA	
Type	A	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	LCSS	LCFEPS	LCFSIG	LCE				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's Modulus
PR	Poisson's ratio
RATESF	Constant in plastic strain rate equation
EDOTO	Reference strain rate
ALPHA	Pressure sensitivity factor
LCSS	Load curve ID or table ID that defines the post yield material behavior. The values of this stress-strain curve are the difference of the yield stress and strain, respectively. This means the first values for both stress and strain should be zero. All subsequent values will define softening or hardening.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCFEPS	Load curve ID that defines the plastic failure strain as a function of strain rate
LCFSIG	Load curve ID that defines the maximum principal failure stress as a function of strain rate
LCE	Load curve ID that defines the unloading moduli as a function of plastic strain

**Remarks:**

The constitutive model for this approach is:

$$\dot{\varepsilon}_p = \dot{\varepsilon}_0 \exp\{A[\sigma - S(\varepsilon_p)]\} \times \exp(-p\alpha A)$$

where  $\dot{\varepsilon}_0$  and  $A$  are rate dependent yield stress parameters,  $S(\varepsilon_p)$  is the internal resistance (strain hardening), and  $\alpha$  is a pressure dependence parameter.

In this material the yield stress may vary throughout the finite element model as a function of strain rate and hydrostatic stress. Post yield stress behavior is captured in material softening and hardening values. Finally, ductile or brittle failure measured by plastic strain or maximum principal stress, respectively, is accounted for by automatic element deletion.

Although this may be applied to a variety of engineering thermoplastics, GE Plastics have constants available for use in a wide range of commercially available grades of their engineering thermoplastics.

## \*MAT\_102

## \*MAT\_INV\_HYPERBOLIC\_SIN

### \*MAT\_INV\_HYPERBOLIC\_SIN\_{OPTION}

This is Material Type 102. It allows the modeling of temperature and rate-dependent plasticity, Sheppard and Wright [1979].

Available options include:

<BLANK>

THERMAL

such that the keyword card can appear as:

\*MAT\_INV\_HYPERBOLIC\_SIN or \*MAT\_102

\*MAT\_INV\_HYPERBOLIC\_SIN\_THERMAL or \*MAT\_102\_T

### Card Summary:

**Card 1a.** This card is included if the keyword option is unset (<BLANK>).

MID	R0	E	PR	T	HC	VP	
-----	----	---	----	---	----	----	--

**Card 1b.** This card is included if the THERMAL keyword option is used.

MID	R0	ALPHA	N	A	Q	G	EPS0
-----	----	-------	---	---	---	---	------

**Card 2a.** This card is included if the keyword option is unset (<BLANK>).

ALPHA	N	A	Q	G	EPS0	LCQ	
-------	---	---	---	---	------	-----	--

**Card 2b.** This card is included if the THERMAL keyword option is used.

LCE	LCPR	LCCTE					
-----	------	-------	--	--	--	--	--

### Data Card Definitions:

Card 1 for no keyword option (<BLANK>)

Card 1a	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	T	HC	VP	
Type	A	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's Modulus
PR	Poisson's ratio
T	Initial temperature
HC	Heat generation coefficient
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default) EQ.1.0: Viscoplastic formulation

Card 1 for the THERMAL keyword option

Card 1b	1	2	3	4	5	6	7	8
Variable	MID	RO	ALPHA	N	A	Q	G	EPS0
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
ALPHA	$\alpha$ . See <a href="#">Remark 1</a> . This $\alpha$ is not the coefficient of thermal expansion.
N	See <a href="#">Remark 1</a> .
A	See <a href="#">Remark 1</a> .
Q	See <a href="#">Remark 1</a> .
G	See <a href="#">Remark 1</a> .
EPS0	Minimum strain rate considered in calculating Z

**\*MAT\_102****\*MAT\_INV\_HYPERBOLIC\_SIN**

Card 2 for no keyword option (<BLANK>)

Card 2a	1	2	3	4	5	6	7	8
Variable	ALPHA	N	A	Q	G	EPS0	LCQ	
Type	F	F	F	F	F	F	I	

VARIABLE	DESCRIPTION
ALPHA	$\alpha$ . See <a href="#">Remark 1</a> . This $\alpha$ is not the coefficient of thermal expansion.
N	See <a href="#">Remark 1</a> .
A	See <a href="#">Remark 1</a> .
Q	See <a href="#">Remark 1</a> .
G	See <a href="#">Remark 1</a> .
EPS0	Minimum strain rate considered in calculating Z.
LCQ	ID of curve specifying parameter Q: GT.0: Q as function of plastic strain. LT.0: Q as function of temperature.

Card 2 for the THERMAL keyword option

Card 2b	1	2	3	4	5	6	7	8
Variable	LCE	LCPR	LCCTE					
Type	F	F	F					

VARIABLE	DESCRIPTION
LCE	ID of curve defining the Young's modulus as a function of temperature
LCPR	ID of curve defining Poisson's ratio as a function of temperature

VARIABLE	DESCRIPTION
LCCTE	ID of curve defining the coefficient of thermal expansion as a function of temperature

**Remarks:**

1. **Material description.** Resistance to deformation is both temperature and strain rate dependent. The flow stress equation is:

$$\sigma = \frac{1}{\alpha} \sinh^{-1} \left[ \left( \frac{Z}{A} \right)^{\frac{1}{N}} \right]$$

where  $Z$ , the Zener-Holloman temperature compensated strain rate, is:

$$Z = \max(\dot{\epsilon}, \text{EPS0}) \times \exp \left( \frac{Q}{GT} \right) .$$

The units of the material constitutive constants are as follows:  $A$  (1/sec),  $N$  (dimensionless),  $\alpha$  (1/MPa), the activation energy for flow,  $Q$ (J/mol), and the universal gas constant,  $G$  (J/mol K). The value of  $G$  only varies with the unit system chosen. Typically, it is either 8.3144 J/(mol K), or 40.8825 lb in/(mol R).

The final equation necessary to complete our description of high strain rate deformation is one that enables computing the temperature change during the deformation. In the absence of a coupled thermo-mechanical finite element code, we assume adiabatic temperature change and follow the empirical assumption that 90-95% of the plastic work is dissipated as heat. Thus, the heat generation coefficient is

$$HC \approx \frac{0.9}{\rho C_v}$$

where  $\rho$  is the density of the material and  $C_v$  is the specific heat.

2. **History variables.**  $Z$  is output as history variable #11 when using the THERMAL keyword option and as history variable #8 when not using the THERMAL keyword option. See NEIPH and NEIPS on \*DATABASE\_EXTENT\_BINARY to set the number of extra history variables output to d3plot.

## \*MAT\_103

## \*MAT\_ANISOTROPIC\_VISCOPLASTIC

### \*MAT\_ANISOTROPIC\_VISCOPLASTIC

This is Material Type 103. This anisotropic-viscoplastic material model applies to shell, thick shell, solid, and SPH elements. The material constants may be fit directly or, if desired, stress as a function of strain data may be input and a least squares fit will be performed by LS-DYNA to determine the constants. Kinematic, isotropic, or a combination of kinematic and isotropic hardening may be used. A detailed description of this model can be found in the following references: Berstad, Langseth, and Hopperstad [1994]; Hopperstad and Remseth [1995]; and Berstad [1996]. Failure is based on effective plastic strain or by a user defined subroutine.

#### Card Summary:

**Card 1.** This card is required.

MID	R0	E	PR	SIGY	FLAG	LCSS	ALPHA
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**Card 2.** This card is required

QR1	CR1	QR2	CR2	QX1	CX1	QX2	CX2
-----	-----	-----	-----	-----	-----	-----	-----

**Card 3a.** Include this card for shell elements and thick shell formulations 1, 2, and 6.

VK	VM	R00	R45	R90			
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**Card 3b.** Include this card for solid elements, SPH elements, and thick shell formulations 3, 5, and 7.

VK	VM	F	G	H	L	M	N
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**Card 4.** This card is required.

AOPT	FAIL	NUMINT	MACF				
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**Card 5.** This card is required.

XP	YP	ZP	A1	A2	A3		
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**Card 6.** This card is required.

V1	V2	V3	D1	D2	D3	BETA	
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	FLAG	LCSS	ALPHA
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified.
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
SIGY	Initial yield stress
FLAG	Flag: EQ.0: Give all material parameters EQ.1: Material parameters $Q_{r1}$ , $C_{r1}$ , $Q_{r2}$ , and $C_{r2}$ for pure isotropic hardening ( $\alpha = 1$ ) are determined by a least squares fit to the curve or table specified by the variable LCSS. If $\alpha$ is input as less than 1, $Q_{r1}$ and $Q_{r2}$ are then modified by multiplying them by the factor $\alpha$ , while the factors $Q_{x1}$ and $Q_{x2}$ are taken as the product of the factor $(1 - \alpha)$ and the original parameters $Q_{r1}$ and $Q_{r2}$ , respectively, for pure isotropic hardening. $C_{x1}$ is set equal to $C_{r1}$ and $C_{x2}$ is set equal to $C_{r2}$ . $\alpha$ is input as variable ALPHA on Card 1. EQ.2: Use load curve directly, that is, no fitting is required for the parameters $Q_{r1}$ , $C_{r1}$ , $Q_{r2}$ , and $C_{r2}$ . A table is not allowed and only isotropic hardening is implemented. EQ.4: Use table definition directly. No fitting is required and the values for $Q_{r1}$ , $C_{r1}$ , $Q_{r2}$ , $C_{r2}$ , $V_k$ , and $V_m$ are ignored. Only isotropic hardening is implemented, and this option is only available for solids.
LCSS	Load curve ID or Table ID. Card 2 is ignored with this option. <b>Load Curve ID.</b> The load curve ID defines effective stress as a

**\*MAT\_103****\*MAT\_ANISOTROPIC\_VISCOPLASTIC**

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
	function of effective plastic strain. For this load curve case, viscoplasticity is modeled when the coefficients $V_k$ and $V_m$ are provided.							
	<b>Table ID.</b> The table consists of stress as a function of effective plastic strain curves indexed by strain rate. See <a href="#">Figure M24-1</a> .							
	FLAG.EQ.1: Table is used to calculate the coefficients $V_k$ and $V_m$ . FLAG.EQ.4: Table is interpolated and used directly. This option is available only for solid elements.							
ALPHA	$\alpha$ distribution of hardening used in the curve-fitting. $\alpha = 0$ is pure kinematic hardening while $\alpha = 1$ provides pure isotropic hardening.							
Card 2	1	2	3	4	5	6	7	8
Variable	QR1	CR1	QR2	CR2	QX1	CX1	QX2	CX2
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
QR1	Isotropic hardening parameter $Q_{r1}$
CR1	Isotropic hardening parameter $C_{r1}$
QR2	Isotropic hardening parameter $Q_{r2}$
CR2	Isotropic hardening parameter $C_{r2}$
QX1	Kinematic hardening parameter $Q_{\chi 1}$
CX1	Kinematic hardening parameter $C_{\chi 1}$
QX2	Kinematic hardening parameter $Q_{\chi 2}$
CX2	Kinematic hardening parameter $C_{\chi 2}$

**Shell Elements Card.** This card is included for shell elements and thick shell formulations 1, 2, and 6.

Card 3a	1	2	3	4	5	6	7	8
Variable	VK	VM	R00	R45	R90			
Type	F	F	F	F	F			

<b>VARIABLE</b>	<b>DESCRIPTION</b>
VK	Viscous material parameter $V_k$
VM	Viscous material parameter $V_m$
R00	$R_{00}$ for shell (default = 1.0)
R45	$R_{45}$ for shell (default = 1.0)
R90	$R_{90}$ for shell (default = 1.0)

**Solid Elements Card.** This card is included for solid elements, SPH elements, and thick shell formulations 3, 5, and 7.

Card 3b	1	2	3	4	5	6	7	8
Variable	VK	VM	F	G	H	L	M	N
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
VK	Viscous material parameter $V_k$
VM	Viscous material parameter $V_m$
F	$F$ in yield criteria (default = 1/2); see Remarks
G	$G$ in yield criteria (default = 1/2); see Remarks
H	$H$ in yield criteria (default = 1/2); see Remarks
L	$L$ in yield criteria (default = 3/2); see Remarks
M	$M$ in yield criteria (default = 3/2); see Remarks

**\*MAT\_103****\*MAT\_ANISOTROPIC\_VISCOPLASTIC**

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
N		N in yield criteria (default = 3/2); see Remarks						
Card 4	1	2	3	4	5	6	7	8
Variable	AOPT	FAIL	NUMINT	MACF				
Type	F	F	F	I				

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
AOPT		Material axes option (see MAT_OPTIONTROPIC_ELASTIC, particularly the <a href="#">Material Directions</a> section, for details):						
		EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.						
		EQ.1.0: Locally orthotropic with material axes determined by a point, <i>P</i> , in space and the global location of the element center; this is the <b>a</b> -direction. This option is for solid elements only.						
		EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR						
		EQ.3.0: Locally orthotropic material axes determined by a vector <b>v</b> and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. <b>a</b> is determined by taking the cross product of <b>v</b> with the normal vector, <b>b</b> is determined by taking the cross product of the normal vector with <b>a</b> , and <b>c</b> is the normal vector. Then <b>a</b> and <b>b</b> are rotated about <b>c</b> by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying						

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	BETA depending on the value of MACF.
	EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector $\mathbf{v}$ , and an originating point, $P$ , which define the centerline axis. This option is for solid elements only.
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_OPTION).
FAIL	Failure flag: LT.0.0: User defined failure subroutine is called to determine failure. This is subroutine named, MATUSR_103, in dyn21.f. EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved. GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.
NUMINT	Number of integration points which must fail before element deletion. If zero, all points must fail. This option applies to shell elements only. For the case of one point shells, NUMINT should be set to a value that is less than the number of through thickness integration points. Nonphysical stretching can sometimes appear in the results if all integration points have failed except for one point away from the midsurface because unconstrained nodal rotations will prevent strains from developing at the remaining integration point. In fully integrated shells, similar problems can occur.
MACF	Material axes change flag for solid elements: EQ.-4: Switch material axes $b$ and $c$ before BETA rotation EQ.-3: Switch material axes $a$ and $c$ before BETA rotation EQ.-2: Switch material axes $a$ and $b$ before BETA rotation EQ.1: No change, default EQ.2: Switch material axes $a$ and $b$ after BETA rotation EQ.3: Switch material axes $a$ and $c$ after BETA rotation EQ.4: Switch material axes $b$ and $c$ after BETA rotation

Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on \*ELEMENT\_-

**\*MAT\_103****\*MAT\_ANISOTROPIC\_VISCOPLASTIC**

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
SOLID_{OPTION} is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 6 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.								
Card 5	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
XP, YP, ZP		Coordinates of point $p$ for AOPT = 1 and 4						
A1, A2, A3		Components of vector $\mathbf{a}$ for AOPT = 2						

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
V1, V2, V3		Components of vector $\mathbf{v}$ for AOPT = 3 and 4						
D1, D2, D3		Components of vector $\mathbf{d}$ for AOPT = 2						
BETA		Material angle in degrees for AOPT = 0 (shells and thick shells only) and AOPT = 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, and *ELEMENT_SOLID_ORTHO.						

**Remarks:**

The uniaxial stress-strain curve is given on the following form

$$\begin{aligned}\sigma(\varepsilon_{\text{eff}}^p, \dot{\varepsilon}_{\text{eff}}^p) = & \sigma_0 + Q_{r1}[(1 - \exp(-C_{r1}\varepsilon_{\text{eff}}^p))] + Q_{r2}[1 - \exp(-C_{r2}\varepsilon_{\text{eff}}^p)] \\ & + Q_{\chi1}[(1 - \exp(-C_{\chi1}\varepsilon_{\text{eff}}^p))] + Q_{\chi2}[(1 - \exp(-C_{\chi2}\varepsilon_{\text{eff}}^p))] + V_k \dot{\varepsilon}_{\text{eff}}^p V_m\end{aligned}$$

For solids the following yield criteria is used

$$\begin{aligned}F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2 \\ = [\sigma(\varepsilon_{\text{eff}}^p, \dot{\varepsilon}_{\text{eff}}^p)]^2\end{aligned}$$

where  $\varepsilon_{\text{eff}}^p$  is the effective plastic strain and  $\dot{\varepsilon}_{\text{eff}}^p$  is the effective plastic strain rate. For shells the anisotropic behavior is given by  $R_{00}$ ,  $R_{45}$  and  $R_{90}$ . The model will work when the three first parameters in Card 3 are given values. When  $V_k = 0$ , the material will behave elasto-plastically. Default values are given by:

$$\begin{aligned}F = G = H = \frac{1}{2} \\ L = M = N = \frac{3}{2}\end{aligned}$$

$$R_{00} = R_{45} = R_{90} = 1$$

Strain rates are accounted for using the Cowper and Symonds model which scales the yield stress with the factor:

$$1 + \left(\frac{\dot{\varepsilon}_{\text{eff}}^p}{C}\right)^{1/p}$$

To convert these constants set the viscoelastic constants,  $V_k$  and  $V_m$ , to the following values:

$$\begin{aligned}V_k &= \text{SIGY} \left(\frac{1}{C}\right)^{1/p} \\ V_m &= \frac{1}{p}\end{aligned}$$

If LCSS is nonzero, substitute the initial, quasi-static yield stress for SIGY in the equation for  $V_k$  above.

This model properly treats rate effects. The viscoplastic rate formulation is an option in other plasticity models in LS-DYNA, such as \*MAT\_003 and \*MAT\_024, invoked by setting the parameter VP to 1.

## \*MAT\_103\_P

## \*MAT\_ANISOTROPIC\_PLASTIC

### \*MAT\_ANISOTROPIC\_PLASTIC

This is Material Type 103\_P. This anisotropic-plastic material model is a simplified version of the MAT\_ANISOTROPIC\_VISCOPLASTIC above. This material model applies only to shell elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	LCSS		
Type	A	F	F	F	F	F		

Card 2	1	2	3	4	5	6	7	8
Variable	QR1	CR1	QR2	CR2				
Type	F	F	F	F				

Card 3	1	2	3	4	5	6	7	8
Variable	R00	R45	R90	S11	S22	S33	S12	
Type	F	F	F	F	F	F	F	

Card 4	1	2	3	4	5	6	7	8
Variable	AOPT							
Type	F							

Card 5	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>		
MID	Material identification. A unique number or label must be specified (see *PART).		
RO	Mass density		
E	Young's modulus		
PR	Poisson's ratio		
SIGY	Initial yield stress		
LCSS	Load curve ID. The load curve ID defines effective stress as a function of effective plastic strain. Card 2 is ignored with this option.		
QR1	Isotropic hardening parameter $Q_{r1}$		
CR1	Isotropic hardening parameter $C_{r1}$		
QR2	Isotropic hardening parameter $Q_{r2}$		
CR2	Isotropic hardening parameter $C_{r2}$		
R00	$R_{00}$ for anisotropic hardening		
R45	$R_{45}$ for anisotropic hardening		
R90	$R_{90}$ for anisotropic hardening		
S11	Yield stress in local $x$ -direction. This input is ignored if $(R_{00}, R_{45}, R_{90}) > 0$ .		
S22	Yield stress in local $y$ -direction. This input is ignored if $(R_{00}, R_{45}, R_{90}) > 0$ .		
S33	Yield stress in local $z$ -direction. This input is ignored if $(R_{00}, R_{45}, R_{90}) > 0$ .		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
S12	Yield stress in local <i>xy</i> -direction. This input is ignored if $(R_{00}, R_{45}, R_{90}) > 0$ .
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description): <ul style="list-style-type: none"> <li>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA.</li> <li>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</li> <li>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal.</li> <li>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</li> </ul>
XP, YP, ZP	$x_p, y_p, z_p$ define coordinates of point <b>p</b> for AOPT = 1 and 4.
A1, A2, A3	$a_1, a_2, a_3$ define components of vector <b>a</b> for AOPT = 2.
D1, D2, D3	$d_1, d_2, d_3$ define components of vector <b>d</b> for AOPT = 2.
V1, V2, V3	$v_1, v_2, v_3$ define components of vector <b>v</b> for AOPT = 3 and 4.
BETA	Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA.

**Remarks:**

If no load curve is defined for the effective stress versus effective plastic strain, the uniaxial stress-strain curve is given on the following form

$$\sigma(\varepsilon_{\text{eff}}^p) = \sigma_0 + Q_{r1}[1 - \exp(-C_{r1}\varepsilon_{\text{eff}}^p)] + Q_{r2}[1 - \exp(-C_{r2}\varepsilon_{\text{eff}}^p)]$$

where  $\varepsilon_{\text{eff}}^p$  is the effective plastic strain. For shells the anisotropic behavior is given by  $R_{00}$ ,  $R_{45}$  and  $R_{90}$ , or the yield stress in the different direction. Default values are given by:

$$R_{00} = R_{45} = R_{90} = 1$$

if the variables R00, R45, R90, S11, S22, S33 and S12 are set to zero.

# \*MAT\_104

# \*MAT\_DAMAGE\_1

## \*MAT\_DAMAGE\_1

This is Material Type 104. This is a continuum damage mechanics (CDM) model which includes anisotropy and viscoplasticity. The CDM model applies to shell, thick shell, and solid elements. A more detailed description of this model can be found in the paper by Berstad, Hopperstad, Lademo, and Malo [1999]. This material model can also model anisotropic damage behavior by setting FLAG to -1 in Card 2.

### Card Summary:

**Card 1.** This card is required.

MID	R0	E	PR	SIGY	LCSS	LCDS	
-----	----	---	----	------	------	------	--

**Card 2a.** This card is included if FLAG = -1.

Q1	C1	Q2	C2	EPSD	EPSR		FLAG
----	----	----	----	------	------	--	------

**Card 2b.** This card is included if FLAG  $\geq 0$ .

Q1	C1	Q2	C2	EPSD	S	DC	FLAG
----	----	----	----	------	---	----	------

**Card 3a.** This card is included if the element type is shells or thick shells.

VK	VM	R00	R45	R90			
----	----	-----	-----	-----	--	--	--

**Card 3b.** This card is included if the element type is solids.

VK	VM	F	G	H	L	M	N
----	----	---	---	---	---	---	---

**Card 4.** This card is required.

AOPT		CPH	MACF	Y0	ALPHA	THETA	ETA
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**Card 5.** This card is required.

XP	YP	ZP	A1	A2	A3		
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**Card 6.** This card is required.

V1	V2	V3	D1	D2	D3	BETA	
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	LCSS	LCDS	
Type	A	F	F	F	F	I	I	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
SIGY	Initial yield stress, $\sigma_0$
LCSS	Load curve ID defining effective stress as a function of effective plastic strain. Isotropic hardening parameters on Card 2 are ignored with this option.
LCDS	Load curve ID defining nonlinear damage curve for FLAG = -1.

**Anisotropic Damage Card.** This card is included if FLAG = -1.

Card 2a	1	2	3	4	5	6	7	8
Variable	Q1	C1	Q2	C2	EPSD	EPSR		FLAG
Type	F	F	F	F	F	F		F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
Q1	Isotropic hardening parameter $Q_1$
C1	Isotropic hardening parameter $C_1$
Q2	Isotropic hardening parameter $Q_2$
C2	Isotropic hardening parameter $C_2$

**\*MAT\_104****\*MAT\_DAMAGE\_1**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EPSD	Damage threshold $\varepsilon_{\text{eff},d}^p$ . Damage effective plastic strain when material softening begins (default = 0.0).
EPSR	Effective plastic strain at which material ruptures (logarithmic).
FLAG	Damage type flag: EQ.-1: Anisotropic damage check ( <i>only for shell elements</i> ) EQ.0: Standard isotropic damage (default) EQ.1: Standard isotropic damage plus strain localization check <i>(only for shell elements)</i> EQ.10: Enhanced isotropic damage EQ.11: Enhanced isotropic damage plus strain localization check <i>(only for shell elements)</i>

**Isotropic Damage Only Card.** This card is included if FLAG  $\geq 0$ .

Card 2b	1	2	3	4	5	6	7	8
Variable	Q1	C1	Q2	C2	EPSD	S	DC	FLAG
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
Q1	Isotropic hardening parameter $Q_1$
C1	Isotropic hardening parameter $C_1$
Q2	Isotropic hardening parameter $Q_2$
C2	Isotropic hardening parameter $C_2$
EPSD	Damage threshold $\varepsilon_{\text{eff},d}^p$ . Damage effective plastic strain when material softening begins (default = 0.0).
S	Damage material constant $S$ (default = $\frac{\sigma_0}{200}$ )
DC	Critical damage value $D_C$ (default = 0.5). When the damage value, $D$ , reaches this value, the element is deleted from the calculation.

VARIABLE	DESCRIPTION
FLAG	<p>Damage type flag:</p> <p>EQ.-1: Anisotropic damage check (<i>only for shell elements</i>)</p> <p>EQ.0: Standard isotropic damage (default)</p> <p>EQ.1: Standard isotropic damage plus strain localization check (<i>only for shell elements</i>)</p> <p>EQ.10: Enhanced isotropic damage</p> <p>EQ.11: Enhanced isotropic damage plus strain localization check (<i>only for shell elements</i>)</p>

**Shell Element Material Parameters Card.** This card is included for shell or thick shell elements.

Card 3a	1	2	3	4	5	6	7	8
Variable	VK	VM	R00	R45	R90			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION
VK	Viscous material parameter, $V_k$
VM	Viscous material parameter, $V_m$
R00	$R_{00}$ for shell (default = 1.0)
R45	$R_{45}$ for shell (default = 1.0)
R90	$R_{90}$ for shell (default = 1.0)

**Brick Element Material Parameters Card.** This card is included for solid elements.

Card 3a	1	2	3	4	5	6	7	8
Variable	VK	VM	F	G	H	L	M	N
Type	F	F	F	F	F	F	F	F

**\*MAT\_104****\*MAT\_DAMAGE\_1**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
VK	Viscous material parameter, $V_k$
VM	Viscous material parameter, $V_m$
F	$F$ for solid (default = 1/2)
G	$G$ for solid (default = 1/2)
H	$H$ for solid (default = 1/2)
L	$L$ for solid (default = 3/2)
M	$M$ for solid (default = 3/2)
N	$N$ for solid (default = 3/2)

Card 4	1	2	3	4	5	6	7	8
Variable	AOPT		CPH	MACF	Y0	ALPHA	THETA	ETA
Type	F		F	I	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC, particularly the <a href="#">Material Directions</a> section, for details):  EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.  EQ.1.0: Locally orthotropic with material axes determined by a point, $P$ , in space and the global location of the element center; this is the <b>a</b> -direction. This option is for solid elements only.  EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR  EQ.3.0: Locally orthotropic material axes determined by a vector $v$ and the normal vector to the plane of the element. The plane of a solid element is the midsurface between

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. <b>a</b> is determined by taking the cross product of <b>v</b> with the normal vector, <b>b</b> is determined by taking the cross product of the normal vector with <b>a</b> , and <b>c</b> is the normal vector. Then <b>a</b> and <b>b</b> are rotated about <b>c</b> by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.
EQ.4.0:	Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector <b>v</b> , and an originating point, <i>P</i> , which define the centerline axis. This option is for solid elements only.
LT.0.0:	The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_OPTION).
CPH	Microdefects closure parameter <i>h</i> for enhanced damage (FLAG $\geq 10$ ).
MACF	Material axes change flag for solid elements: EQ.-4: Switch material axes <i>b</i> and <i>c</i> before BETA rotation EQ.-3: Switch material axes <i>a</i> and <i>c</i> before BETA rotation EQ.-2: Switch material axes <i>a</i> and <i>b</i> before BETA rotation EQ.1: No change, default EQ.2: Switch material axes <i>a</i> and <i>b</i> after BETA rotation EQ.3: Switch material axes <i>a</i> and <i>c</i> after BETA rotation EQ.4: Switch material axes <i>b</i> and <i>c</i> after BETA rotation

[Figure M2-2](#) indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on \*ELEMENT\_-SOLID\_{OPTION} is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 6 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.

**\*MAT\_104****\*MAT\_DAMAGE\_1**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
Y0	Initial damage energy release rate, $Y_0$ , for enhanced damage (FLAG $\geq 10$ ).
ALPHA	Exponent $\alpha$ for enhanced damage (FLAG $\geq 10$ )
THETA	Exponent $\theta$ for enhanced damage (FLAG $\geq 10$ )
ETA	Exponent $\eta$ for enhanced damage (FLAG $\geq 10$ )

Card 5	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
XP, YP, ZP	Coordinates of point $p$ for AOPT = 1 and 4
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3 and 4
BETA	Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, and *ELEMENT_SOLID_ORTH.

**Remarks:**

- Standard isotropic damage model (FLAG = 0 or 1).** The Continuum Damage Mechanics (CDM) model is based on an approach proposed by Lemaitre [1992]. The effective stress  $\tilde{\sigma}$ , which is the stress calculated over the section that effectively resist the forces, reads

$$\tilde{\sigma} = \frac{\sigma}{1 - D}$$

where  $D$  is the damage variable. The evolution equation for the damage variable is defined as

$$\dot{D} = \begin{cases} 0 & \text{for } \varepsilon_{\text{eff}}^p \leq \varepsilon_{\text{eff},d}^p \\ \frac{Y}{S} \dot{\varepsilon}_{\text{eff}}^p & \text{for } \varepsilon_{\text{eff}}^p > \varepsilon_{\text{eff},d}^p \text{ and } \sigma_1 > 0 \end{cases}$$

where  $\varepsilon_{\text{eff},d}^p$  is the damage threshold,  $S$  is the so-called damage energy release rate, and  $\sigma_1$  is the maximum principal stress. The damage energy density release rate is

$$Y = \frac{1}{2} \mathbf{e}_e : \mathbf{C} : \mathbf{e}_e = \frac{\sigma_{vm}^2 R_v}{2E(1 - D)^2}$$

where  $E$  is Young's modulus and  $\sigma_{vm}$  is the equivalent von Mises stress. The triaxiality function  $R_v$  is defined as

$$R_v = \frac{2}{3}(1 + \nu) + 3(1 - 2\nu) \left( \frac{\sigma_H}{\sigma_{vm}} \right)^2$$

with Poisson's ratio  $\nu$  and hydrostatic stress  $\sigma_H$ .

- Enhanced isotropic damage model (FLAG = 10 or 11).** A more sophisticated damage model that includes crack closure effects (reduced damage under compression) and more flexibility in stress state dependence and functional expressions is invoked by setting FLAG = 10 or 11. The corresponding evolution equation for the damage variable is defined as

$$\dot{D} = \left( \frac{2\tau_{\max}}{\sigma_{vm}} \right)^\eta \left( \frac{Y - Y_0}{S} \right)^\alpha (1 - D)^{1-\theta} \dot{\varepsilon}_{\text{eff}}^p$$

where  $\tau_{\max}$  is the maximum shear stress,  $Y_0$  is the initial damage energy release rate, and  $\eta$ ,  $\alpha$ , and  $\theta$  are additional material constants.  $\langle \rangle$  are the *Macaulay brackets*. The damage energy density release rate is

$$Y = \frac{1 + \nu}{2E} \left( \sum_{i=1}^3 (\langle \tilde{\sigma}_i \rangle^2 + h \langle -\tilde{\sigma}_i \rangle^2) \right) - \frac{\nu}{2E} (\langle \tilde{\sigma}_H \rangle^2 + h \langle -\tilde{\sigma}_H \rangle^2)$$

where  $\tilde{\sigma}_i$  are the principal effective stresses and  $h$  is the microdefects closure parameter that accounts for different damage behavior in tension and compression. A value of  $h \approx 0.2$  is typically observed in many experiments as stated in

Lemaitre [2000]. A parameter set of  $h = 1$ ,  $Y_0 = 0$ ,  $\alpha = 1$ ,  $\theta = 1$ , and  $\eta = 0$  should give the same results as the standard isotropic damage model (FLAG = 0 or 1) with  $\dot{\varepsilon}_{\text{eff},d}^p = 0$  as long as  $\sigma_1 > 0$ .

3. **Strain localization check (FLAG = 1 or 11).** In order to add strain localization computation to the damage models above, parameter FLAG should be set to 1 (standard damage) or 11 (enhanced damage). An acoustic tensor-based bifurcation criterion is checked and history variable no. 4 is set to 1.0 if strain localization is indicated. This is only available for shell elements.
4. **Anisotropic damage model (FLAG = -1).** At each thickness integration points, an anisotropic damage law acts on the plane stress tensor in the directions of the principal total shell strains,  $\varepsilon_1$  and  $\varepsilon_2$ , as follows:

$$\sigma_{11} = [1 - D_1(\varepsilon_1)]\sigma_{110}$$

$$\sigma_{22} = [1 - D_2(\varepsilon_2)]\sigma_{220}$$

$$\sigma_{12} = \left[1 - \frac{D_1 + D_2}{2}\right]\sigma_{120}$$

The transverse plate shear stresses in the principal strain directions are assumed to be damaged as follows:

$$\sigma_{13} = (1 - D_1/2)\sigma_{130}$$

$$\sigma_{23} = (1 - D_2/2)\sigma_{230}$$

In the anisotropic damage formulation,  $D_1(\varepsilon_1)$  and  $D_2(\varepsilon_2)$  are anisotropic damage functions for the loading directions 1 and 2, respectively. Stresses  $\sigma_{110}$ ,  $\sigma_{220}$ ,  $\sigma_{120}$ ,  $\sigma_{130}$  and  $\sigma_{230}$  are stresses in the principal shell strain directions as calculated from the undamaged elastic-plastic material behavior. The strains  $\varepsilon_1$  and  $\varepsilon_2$  are the magnitude of the principal strains calculated upon reaching the damage thresholds. Damage can only develop for tensile stresses, and the damage functions  $D_1(\varepsilon_1)$  and  $D_2(\varepsilon_2)$  are identical to zero for negative strains  $\varepsilon_1$  and  $\varepsilon_2$ . The principal strain directions are fixed within an integration point as soon as either principal strain exceeds the initial threshold strain in tension. A more detailed description of the damage evolution for this material model is given in the description of Material 81.

5. **Anisotropic viscoplasticity.** The uniaxial stress-strain curve is given in the following form

$$\sigma(r, \dot{\varepsilon}_{\text{eff}}^p) = \sigma_0 + Q_1[1 - \exp(-C_1 r)] + Q_2[1 - \exp(-C_2 r)] + V_k \dot{\varepsilon}_{\text{eff}}^p V_m ,$$

where  $r$  is the damage accumulated plastic strain, which can be calculated by

$$\dot{r} = \dot{\varepsilon}_{\text{eff}}^p (1 - D) .$$

For bricks the following yield criterion associated with the Hill criterion is used

$$F(\tilde{\sigma}_{22} - \tilde{\sigma}_{33})^2 + G(\tilde{\sigma}_{33} - \tilde{\sigma}_{11})^2 + H(\tilde{\sigma}_{11} - \tilde{\sigma}_{22})^2 + 2L\tilde{\sigma}_{23}^2 + 2M\tilde{\sigma}_{31}^2 + 2N\tilde{\sigma}_{12}^2 = \sigma(r, \dot{\epsilon}_{\text{eff}}^p)$$

where  $r$  is the damage effective viscoplastic strain and  $\dot{\epsilon}_{\text{eff}}^p$  is the effective viscoplastic strain rate. For shells the anisotropic behavior is given by the R-values:  $R_{00}$ ,  $R_{45}$ , and  $R_{90}$ . When  $V_k = 0$ , the material will behave as an elastoplastic material without rate effects. Default values for the anisotropic constants are given by:

$$F = G = H = \frac{1}{2}$$

$$L = M = N = \frac{3}{2}$$

$$R_{00} = R_{45} = R_{90} = 1$$

so that isotropic behavior is obtained.

Strain rate is accounted for using the Cowper and Symonds model which scales the yield stress with the factor:

$$1 + \left(\frac{\dot{\epsilon}}{C}\right)^{1/p}$$

To convert these constants, set the viscoelastic constants,  $V_k$  and  $V_m$ , to the following values:

$$V_k = \sigma \left(\frac{1}{C}\right)^{\frac{1}{p}}$$

$$V_m = \frac{1}{p}$$

# **\*MAT\_105**

## **\*MAT\_DAMAGE\_2**

### **\*MAT\_DAMAGE\_2**

This is Material Type 105. This is an elastic viscoplastic material model combined with continuum damage mechanics (CDM). This material model applies to shell, thick shell, and brick elements. The elastoplastic behavior is described in the description of material model 24. A more detailed description of the CDM model is given in the description of material model 104 above.

#### **Card Summary:**

**Card 1.** This card is required.

MID	R0	E	PR	SIGY	ETAN	FAIL	TDEL
-----	----	---	----	------	------	------	------

**Card 2.** This card is required.

C	P	LCSS	LCSR				
---	---	------	------	--	--	--	--

**Card 3.** This card is required.

EPSD	S	DC					
------	---	----	--	--	--	--	--

**Card 4.** This card is required.

EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
------	------	------	------	------	------	------	------

**Card 5.** This card is required.

ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
-----	-----	-----	-----	-----	-----	-----	-----

#### **Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	ETAN	FAIL	TDEL
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	$10^{20}$	0.0

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
SIGY	Yield stress
ETAN	Tangent modulus; ignored if LCSS > 0
FAIL	Failure flag: EQ.0.0: Failure due to plastic strain is not considered. GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.
TDEL	Minimum time step size for automatic element deletion

Card 2	1	2	3	4	5	6	7	8
Variable	C	P	LCSS	LCSR				
Type	F	F	I	I				
Default	0.0	0.0	0	0				

<b>VARIABLE</b>	<b>DESCRIPTION</b>
C	Strain rate parameter, $C$ ; see Remarks below.
P	Strain rate parameter, $p$ ; see Remarks below.
LCSS	Load curve ID or Table ID. Load curve ID defining effective stress as a function of effective plastic strain. If defined EPS1 - EPS8 and ES1 - ES8 are ignored. The table ID defines for each strain rate value a load curve ID giving the stress as a function of effective plastic strain for that rate, See <a href="#">Figure M24-1</a> . The stress as a function of effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value.

**\*MAT\_105****\*MAT\_DAMAGE\_2**

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
	Likewise, the stress as a function of effective plastic strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value. The strain rate parameters, C and P; the curve ID, LCSR; EPS1 - EPS8; and ES1 - ES8 are ignored if a Table ID is defined.							
LCSR	Load curve ID defining strain rate scaling effect on yield stress							

Card 3	1	2	3	4	5	6	7	8
Variable	EPSD	S	DC					
Type	F	F	F					
Default	0.0	↓	0.5					

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
EPSD	Damage threshold, $r_d$ . Damage effective plastic strain when material softening begins.							
S	Damage material constant S. Default = $\sigma_0/200$ .							
DC	Critical damage value $D_C$ . When the damage value D reaches this value, the element is deleted from the calculation.							

Card 4	1	2	3	4	5	6	7	8
Variable	EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
EPS1 - EPS8	Effective plastic strain values (optional if SIGY is defined). At least 2 points should be defined.							

Card 5	1	2	3	4	5	6	7	8
Variable	ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

**VARIABLE****DESCRIPTION**

ES1 - ES8

Corresponding yield stress values to EPS1 - EPS8.

**Remarks:**

By defining the tangent modulus (ETAN), the stress-strain behavior becomes a bilinear curve. Alternately, a curve similar to that shown in [Figure M10-1](#) is expected to be defined by (EPS1, ES1) - (EPS8, ES8); however, an effective stress as a function of effective plastic strain curve ID (LCSS) may be input instead if eight points are insufficient. The cost is roughly the same for either approach. The most general approach is to use the table definition with table ID, LCSS, discussed below.

Three options to account for strain rate effects are possible.

1. Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\varepsilon}}{C}\right)^{1/p},$$

where  $\dot{\varepsilon}$  is the strain rate,  $\dot{\varepsilon} = \sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}$ .

2. For complete generality a load curve (LCSR) to scale the yield stress may be input instead. In this curve the scale factor as a function of strain rate is defined.
3. If different stress as a function of strain curves can be provided for various strain rates, the option using the reference to a table (LCSS) can be used. Then the table input in \*DEFINE\_TABLE must be used; see [Figure M24-1](#)

A fully viscoplastic formulation is used in this model.

## \*MAT\_106

## \*MAT\_ELASTIC\_VISCOPLASTIC\_THERMAL

### \*MAT\_ELASTIC\_VISCOPLASTIC\_THERMAL

This is Material Type 106. This is an elastic viscoplastic material with thermal effects or effects from an external variable (see \*LOAD\_EXTERNAL\_VARIABLE and [Remark 4](#)).

#### Card Summary:

**Card 1.** This card is required.

MID	RO	E	PR	SIGY	ALPHA	LCSS	FAIL
-----	----	---	----	------	-------	------	------

**Card 2.** This card is required.

QR1	CR1	QR2	CR2	QX1	CX1	QX2	CX2
-----	-----	-----	-----	-----	-----	-----	-----

**Card 3.** This card is required.

C	P	LCE	LCPR	LCSIGY	LCR	LCX	LCALPH
---	---	-----	------	--------	-----	-----	--------

**Card 4.** This card is required.

LCC	LCP	TREF	LCFAIL	NUSHIS	T1PHAS	T2PHAS	TOL
-----	-----	------	--------	--------	--------	--------	-----

**Card 5.** Include this card when NUSHIS > 0.

FUSHI1	FUSHI2	FUSHI3	FUSHI4	FUSHI5	FUSHI6	FUSHI7	FUSHI8
--------	--------	--------	--------	--------	--------	--------	--------

#### Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	ALPHA	LCSS	FAIL
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus

VARIABLE	DESCRIPTION
PR	Poisson's ratio
SIGY	Initial yield stress
ALPHA	Coefficient of thermal expansion
LCSS	Load curve ID or Table ID. The load curve ID defines effective stress as a function of effective plastic strain. The table ID defines for each temperature value a load curve ID giving the stress as a function of effective plastic strain for that temperature (*DEFINE_TABLE) or it defines for each temperature value a table ID which defines for each strain rate a load curve ID giving the stress as a function of effective plastic strain (*DEFINE_TABLE_3D). The stress as a function of effective plastic strain curve for the lowest value of temperature or strain rate is used if the temperature or strain rate falls below the minimum value. Likewise, maximum values cannot be exceeded. See <a href="#">Remark 1</a> .
FAIL	Effective plastic failure strain for erosion

Card 2	1	2	3	4	5	6	7	8
Variable	QR1	CR1	QR2	CR2	QX1	CX1	QX2	CX2
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
QR1	Isotropic hardening parameter, $Q_{r1}$
CR1	Isotropic hardening parameter, $C_{r1}$
QR2	Isotropic hardening parameter, $Q_{r2}$
CR2	Isotropic hardening parameter, $C_{r2}$
QX1	Kinematic hardening parameter, $Q_{\chi1}$
CX1	Kinematic hardening parameter, $C_{\chi1}$
QX2	Kinematic hardening parameter, $Q_{\chi2}$
CX2	Kinematic hardening parameter, $C_{\chi2}$

Card 3	1	2	3	4	5	6	7	8
Variable	C	P	LCE	LCPR	LCSIGY	LCR	LCX	LCALPH
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
C	Viscous material parameter, $C$
P	Viscous material parameter, $p$
LCE	Load curve defining Young's modulus as a function of temperature or external variable (see <a href="#">Remark 4</a> ). E on Card 1 is ignored with this option.
LCPR	Load curve defining Poisson's ratio as a function of temperature or external variable (see <a href="#">Remark 4</a> ). PR on Card 1 is ignored with this option.
LCSIGY	Load curve defining the initial yield stress as a function of temperature or external variable (see <a href="#">Remark 4</a> ). SIGY on Card 1 is ignored with this option.
LCR	Load curve for scaling the isotropic hardening parameters QR1 and QR2 or the stress given by the load curve LCSS as a function of temperature or external variable (see <a href="#">Remark 4</a> )
LCX	Load curve for scaling the kinematic hardening parameters QX1 and QX2 as a function of temperature or external variable (see <a href="#">Remark 4</a> )
LCALPH	Load curve ID defining the instantaneous coefficient of thermal expansion as a function of temperature (or external variable; see <a href="#">Remark 4</a> ):

$$d\varepsilon_{ij}^{\text{thermal}} = \alpha(T) dT \delta_{ij} .$$

ALPHA on Card 1 is ignored with this option. If LCALPH is defined as the negative of the load curve ID, the curve is assumed to define the coefficient relative to a reference temperature, TREF below, such that the total thermal strain is given by

$$\varepsilon_{ij}^{\text{thermal}} = [\alpha(T)(T - T_{\text{ref}}) - \alpha(T_0)(T_0 - T_{\text{ref}})] \delta_{ij} .$$

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
		Here, temperature $T_0$ is the initial temperature.						
Card 4	1	2	3	4	5	6	7	8
Variable	LCC	LCP	TREF	LCFAIL	NUSHIS	T1PHAS	T2PHAS	TOL
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
LCC		Load curve for scaling the viscous material parameter $C$ as a function of temperature or external variable (see <a href="#">Remark 4</a> ). See <a href="#">Remark 1</a> .						
LCP		Load curve for scaling the viscous material parameter $P$ as a function of temperature or external variable (see <a href="#">Remark 4</a> )						
TREF		Reference temperature required if LCALPH is given with a negative curve ID						
LCFAIL		Load curve defining the plastic failure strain as a function of temperature or external variable (see <a href="#">Remark 4</a> ). FAIL on Card 1 is ignored with this option.						
NUHIS		Number of additional user-defined history variables, not used for EXTVAR keyword option. See <a href="#">Remarks 2</a> and <a href="#">3</a> .						
T1PHAS		Lower temperature limit for cooling rate evaluation. Cooling rate can be used as input for user defined variables.						
T2PHAS		Upper temperature limit for cooling rate evaluation. Cooling rate can be used as input for user-defined variables.						
TOL		Optional tolerance for plasticity update. The default is $10^{-6}$ for solid elements and $10^{-3}$ for shells. This parameter overrides the default tolerance for all element types.						

**User History Card.** Additional card only for NUSHIS > 0.

Card 5	1	2	3	4	5	6	7	8
Variable	FUSHI1	FUSHI2	FUSHI3	FUSHI4	FUSHI5	FUSHI6	FUSHI7	FUSHI8
Type	I	I	I	I	I	I	I	I

VARIABLE	DESCRIPTION
FUSHi	Function ID for user-defined history variables. See <a href="#">Remarks 2</a> and <a href="#">3</a> .

### Remarks:

1. **Viscous effects.** If LCSS is not given any value, the uniaxial stress-strain curve has the form:

$$\sigma(\varepsilon_{\text{eff}}^p) = \sigma_0 + Q_{r1}[1 - \exp(-C_{r1}\varepsilon_{\text{eff}}^p)] + Q_{r2}[1 - \exp(-C_{r2}\varepsilon_{\text{eff}}^p)] + Q_{\chi1}[1 - \exp(-C_{\chi1}\varepsilon_{\text{eff}}^p)] + Q_{\chi2}[1 - \exp(-C_{\chi2}\varepsilon_{\text{eff}}^p)].$$

Viscous effects are accounted for using the Cowper and Symonds model, which scales the yield stress with the factor:

$$1 + \left( \frac{\dot{\varepsilon}_{\text{eff}}^p}{C} \right)^{1/p}.$$

2. **User-defined history data.** The user can define up to eight additional history variables that are added to the list of history variables (see table in [Remark 3](#)). These values can, for example, be used to evaluate the hardness of the material.

The additional variables are to be given by respective \*DEFINE\_FUNCTION keywords as functions of the cooling rate between T2PHASE and T1PHASE, temperature, time, user-defined histories themselves, equivalent plastic strain, rate of the equivalent plastic strain, and the first six history variables (see table in [Remark 3](#)). A function declaration should, thus, look as follows:

```
*DEFINE_FUNCTION
1,user-defined history 1
uhist(trate,temp,time,usrhst1,usrhst2,...,usrhstn,epspl,
epsplrate,hist2,hist3,...,hist6) = ...
```

3. **History values.** The most important history variables of this material model are listed in the following table. To be able to post-process that data, parameters NEIPS (shells) or NEIPH (solids) must be defined on \*DATABASE\_EXTENT\_BINARY.

History Variable #	Description
1	Temperature
2	Young's modulus
3	Poisson's ratio
4	Yield stress
5	Isotropic scaling factor
6	Kinematic scaling factor
9	Effective plastic strain rate
10→ 9+NUSHIS	User-defined history variables

4. **Effect of external variables.** By default, many of the material properties can be defined as a function of the temperature field, but this material supports material definitions based on a given distribution of an external variable defined by the keyword \*LOAD\_EXTERNAL\_VARIABLE instead. Depending on the input in that loading card, one or more of the load curves LCE, LCPR, LCSIGY, LCR; LCX, LCALPH, LCC, LCP, and LCFAIL are evaluated not based on the temperature, but on the external variable. To do this, set IMP on \*LOAD\_EXTERNAL\_VARIABLE to the material property index for the desired material property. The following table lists the material property indices:

Property index	Property name	Load curve
1	Young's modulus, $E$	LCE
2	Poisson's ratio	LCPR
3	Initial yield stress	LCSIGY
4	Scale factor on the isotropic hardening parameters	LCR
5	Scale factor on the kinematic hardening parameters	LCX
6	Instantaneous coefficient of thermal expansion	LCALPH
7	Scale factor on the viscous material parameter $C$	LCC
8	Scale factor on the viscous material parameter $P$	LCP
9	Plastic failure strain	LCFAIL

## \*MAT\_107

## \*MAT\_MODIFIED\_JOHNSON\_COOK

### \*MAT\_MODIFIED\_JOHNSON\_COOK

This is Material Type 107. Adiabatic heating is included in the material formulation. Material type 107 is not intended for use in a coupled thermal-mechanical analysis or in a mechanical analysis where temperature is prescribed using \*LOAD\_THERMAL.

#### Card Summary:

**Card 1.** This card is required.

MID	R0	E	PR	BETA	XS1	CP	ALPHA
-----	----	---	----	------	-----	----	-------

**Card 2.** This card is required.

E0DOT	TR	TM	T0	FLAG1	FLAG2		
-------	----	----	----	-------	-------	--	--

**Card 3a.1.** This card is included if FLAG1 = 0.

A	B	N	C	M			
---	---	---	---	---	--	--	--

**Card 3a.2.** This card is included if FLAG1 = 0.

Q1	C1	Q2	C2				
----	----	----	----	--	--	--	--

**Card 3b.1.** This card is included if FLAG1 = 1.

SIGA	B	BETA0	BETA1				
------	---	-------	-------	--	--	--	--

**Card 3b.2.** This card is included if FLAG1 = 1.

A	N	ALPHA0	ALPHA1				
---	---	--------	--------	--	--	--	--

**Card 4a.** This card is included if FLAG2 = 0.

DC	PD	D1	D2	D3	D4	D5	
----	----	----	----	----	----	----	--

**Card 4b.** This card is included if FLAG2 = 1.

DC	WC	PHI	GAMMA				
----	----	-----	-------	--	--	--	--

**Card 5.** This card is required.

TC	TAUC						
----	------	--	--	--	--	--	--

**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	BETA	XS1	CP	ALPHA
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus, $E$
PR	Poisson's ratio, $v$
BETA	Damage coupling parameter; see <a href="#">Equation (107.3)</a> . EQ.0.0: No coupling between ductile damage and the constitutive relation EQ.1.0: Full coupling between ductile damage and the constitutive relation
XS1	Taylor-Quinney coefficient $\chi$ , see <a href="#">Equation (107.21)</a> . Gives the portion of plastic work converted into heat (normally taken to be 0.9)
CP	Specific heat $C_p$ ; see <a href="#">Equation (107.21)</a> .
ALPHA	Thermal expansion coefficient, $\alpha$

Card 2	1	2	3	4	5	6	7	8
Variable	E0DOT	TR	TM	T0	FLAG1	FLAG2		
Type	F	F	F	F	I	I		

VARIABLE	DESCRIPTION
E0DOT	Quasi-static threshold strain rate ( $\dot{\epsilon}_0 = \dot{p}_0 = \dot{r}_0$ ); see <a href="#">Equation (107.12)</a> . See description for EPS0 in *MAT_015.

**\*MAT\_107****\*MAT\_MODIFIED\_JOHNSON\_COOK**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
TR	Room temperature, see <a href="#">Equation (107.13)</a>
TM	Melt temperature, see <a href="#">Equation (107.13)</a>
T0	Initial temperature
FLAG1	Constitutive relation flag: EQ.0: Modified Johnson-Cook constitutive relation; see <a href="#">Equation (107.11)</a> . EQ.1: Zerilli-Armstrong constitutive relation, see <a href="#">Equation (107.14)</a> .
FLAG2	Fracture criterion flag: EQ.0: Modified Johnson-Cook fracture criterion; see <a href="#">Equation (107.15)</a> . EQ.1: Cockcroft-Latham fracture criterion; see <a href="#">Equation (107.19)</a> .

**Modified Johnson-Cook Constitutive Relation.** This card is included when FLAG1 = 0.

Card 3a.1	1	2	3	4	5	6	7	8
Variable	A	B	N	C	M			
Type	F	F	F	F	F			

<b>VARIABLE</b>	<b>DESCRIPTION</b>
A	Johnson-Cook yield stress $A$ ; see <a href="#">Equation (107.11)</a> .
B	Johnson-Cook hardening parameter $B$ ; see <a href="#">Equation (107.11)</a> .
N	Johnson-Cook hardening parameter $n$ ; see <a href="#">Equation (107.11)</a> .
C	Johnson-Cook strain rate sensitivity parameter $C$ ; see <a href="#">Equation (107.11)</a> .
M	Johnson-Cook thermal softening parameter $m$ ; see <a href="#">Equation (107.11)</a> .

**Modified Johnson-Cook Constitutive Relation.** This card is included when FLAG1 = 0.

Card 3a.2	1	2	3	4	5	6	7	8
Variable	Q1	C1	Q2	C2				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
Q1	Voce hardening parameter $Q_1$ (when $B = n = 0$ ); see <a href="#">Equation (107.11)</a> .
C1	Voce hardening parameter $C_1$ (when $B = n = 0$ ); see <a href="#">Equation (107.11)</a> .
Q2	Voce hardening parameter $Q_2$ (when $B = n = 0$ ); see <a href="#">Equation (107.11)</a> .
C2	Voce hardening parameter $C_2$ (when $B = n = 0$ ); see <a href="#">Equation (107.11)</a> .

**Modified Zerilli-Armstrong Constitutive Relation.** This card is included when FLAG1 = 1.

Card 3b.1	1	2	3	4	5	6	7	8
Variable	SIGA	B	BETA0	BETA1				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
SIGA	Zerilli-Armstrong parameter $\alpha_a$ ; see <a href="#">Equation (107.14)</a> .
B	Zerilli-Armstrong parameter $B$ ; see <a href="#">Equation (107.14)</a> .
BETA0	Zerilli-Armstrong parameter $\beta_0$ ; see <a href="#">Equation (107.14)</a> .
BETA1	Zerilli-Armstrong parameter $\beta_1$ ; see <a href="#">Equation (107.14)</a> .

**\*MAT\_107****\*MAT\_MODIFIED\_JOHNSON\_COOK**

**Modified Zerilli-Armstrong Constitutive Relation.** This card is included when FLAG1 = 1.

Card 3b.2	1	2	3	4	5	6	7	8
Variable	A	N	ALPHA0	ALPHA1				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
A	Zerilli-Armstrong parameter $A$ ; see <a href="#">Equation (107.14)</a> .
N	Zerilli-Armstrong parameter $n$ ; see <a href="#">Equation (107.14)</a> .
ALPHA0	Zerilli-Armstrong parameter $\alpha_0$ , see <a href="#">Equation (107.14)</a> .
ALPHA1	Zerilli-Armstrong parameter $\alpha_1$ , see <a href="#">Equation (107.14)</a> .

**Modified Johnson-Cook Fracture Criterion.** This card is included when FLAG2 = 0.

Card 4a	1	2	3	4	5	6	7	8
Variable	DC	PD	D1	D2	D3	D4	D5	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
DC	Critical damage parameter $D_c$ ; see <a href="#">Equations (107.15) and (107.22)</a> . When the damage value $D$ reaches this value, the element is eroded from the calculation.
PD	Damage threshold; see <a href="#">Equation (107.15)</a> .
D1-D5	Fracture parameters in the Johnson-Cook fracture criterion; see <a href="#">Equation (107.16)</a> .

**Cockcroft Latham Fracture Criterion.** This card is included when FLAG2 = 1.

Card 4b	1	2	3	4	5	6	7	8
Variable	DC	WC	PHI	GAMMA				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
DC	Critical damage parameter $D_c$ ; see <a href="#">Equations (107.15)</a> and <a href="#">(107.22)</a> . When the damage value $D$ reaches this value, the element is eroded from the calculation.
WC	Critical Cockcroft-Latham parameter $W_c$ , see <a href="#">Equation (107.19)</a> . When the plastic work per volume reaches this value, the element is eroded from the simulation.
PHI	Extended Cockcroft-Latham parameter $\phi$ , see <a href="#">Equation (107.20)</a> .
GAMMA	Extended Cockcroft-Latham parameter $\gamma$ , see <a href="#">Equation (107.20)</a> .

#### Additional Element Erosion Criteria Card.

Card 5	1	2	3	4	5	6	7	8
Variable	TC	TAUC						
Type	F	F						

VARIABLE	DESCRIPTION
TC	Critical temperature parameter $T_c$ ; see <a href="#">Equation (107.24)</a> . When the temperature, $T$ , reaches this value, the element is eroded from the simulation.
TAUC	Critical shear stress parameter, $\tau_c$ . When the maximum shear stress, $\tau$ , reaches this value, the element is eroded from the simulation.

#### Remarks:

An additive decomposition of the rate-of-deformation tensor  $\mathbf{d}$  is assumed, that is,

$$\mathbf{d} = \mathbf{d}^e + \mathbf{d}^p + \mathbf{d}^t \quad (107.1)$$

where  $\mathbf{d}^e$  is the elastic part,  $\mathbf{d}^p$  is the plastic part and  $\mathbf{d}^t$  is the thermal part.

The elastic rate-of-deformation  $\mathbf{d}^e$  is defined by a linear hypo-elastic relation

$$\tilde{\sigma}^{\nabla J} = \left( K - \frac{2}{3}G \right) \text{tr}(\mathbf{d}^e) \mathbf{I} + 2G\mathbf{d}^e \quad (107.2)$$

where  $\mathbf{I}$  is the unit tensor,  $K$  is the bulk modulus and  $G$  is the shear modulus. The effective stress tensor is defined by

$$\tilde{\sigma} = \frac{\sigma}{1 - \beta D} \quad (107.3)$$

where  $\sigma$  is the Cauchy-stress and  $D$  is the damage variable, while the Jaumann rate of the effective stress reads

$$\tilde{\sigma}^{\nabla J} = \tilde{\sigma} - \mathbf{W} \cdot \tilde{\sigma} - \tilde{\sigma} \cdot \mathbf{W}^T \quad (107.4)$$

Here  $\mathbf{W}$  is the spin tensor. The parameter  $\beta$  is equal to unity for coupled damage and equal to zero for uncoupled damage.

The thermal rate-of-deformation  $\mathbf{d}^T$  is defined by

$$\mathbf{d}^T = \alpha \dot{T} \mathbf{I} \quad (107.5)$$

where  $\alpha$  is the linear thermal expansion coefficient and  $T$  is the temperature.

The plastic rate-of-deformation is defined by the associated flow rule as

$$\mathbf{d}^p = \dot{r} \frac{\partial f}{\partial \sigma} = \frac{3}{2} \frac{\dot{r}}{1 - \beta D} \frac{\tilde{\sigma}'}{\tilde{\sigma}_{\text{eq}}} \quad (107.6)$$

where  $(\cdot)'$  means the deviatoric part of the tensor,  $r$  is the damage-equivalent plastic strain,  $f$  is the dynamic yield function, that is,

$$\mathbf{d}^p = \dot{r} \frac{\partial f}{\partial \sigma} = \frac{3}{2} \frac{\dot{r}}{1 - \beta D} \frac{\tilde{\sigma}'}{\tilde{\sigma}_{\text{eq}}} \quad (107.6)$$

$$f = \sqrt{\frac{3}{2} \tilde{\sigma}' : \tilde{\sigma}'} - \sigma_Y(r, \dot{r}, T) \leq 0, \quad \dot{r} \geq 0, \quad \dot{r}f = 0 \quad (107.7)$$

and  $\tilde{\sigma}_{\text{eq}}$  is the damage-equivalent stress,

$$\tilde{\sigma}_{\text{eq}} = \sqrt{\frac{3}{2} \tilde{\sigma}' : \tilde{\sigma}'} \quad (107.8)$$

The following plastic work conjugate pairs are identified

$$\dot{W}^p = \sigma : \mathbf{d}^p = \tilde{\sigma}_{\text{eq}} \dot{r} = \sigma_{\text{eq}} \dot{p} \quad (107.9)$$

where  $\dot{W}^p$  is the specific plastic work rate, and the equivalent stress  $\sigma_{eq}$  and the equivalent plastic strain  $p$  are defined as

$$\sigma_{eq} = \sqrt{\frac{3}{2} \tilde{\sigma}' : \tilde{\sigma}'} = (1 - \beta D) \tilde{\sigma}_{eq} \quad p = \sqrt{\frac{2}{3} \mathbf{d}^p : \mathbf{d}^p} = \frac{\dot{r}}{(1 - \beta D)} \quad (107.10)$$

The material strength  $\sigma_Y$  is defined by:

1. The modified Johnson-Cook constitutive relation

$$\sigma_Y = \left\{ A + Br^n + \sum_{i=1}^2 Q_i [1 - \exp(-C_i r)] \right\} (1 + \dot{r}^*)^C (1 - T^{*m}) \quad (107.11)$$

where  $A, B, C, m, n, Q_1, C_1, Q_2$ , and  $C_2$  are material parameters; the normalized damage-equivalent plastic strain rate  $\dot{r}^*$  is defined by

$$\dot{r}^* = \frac{\dot{r}}{\dot{\varepsilon}_0} \quad (107.12)$$

in which  $\dot{\varepsilon}_0$  is a user-defined reference strain rate; and the homologous temperature reads

$$T^* = \frac{T - T_r}{T_m - T_r} \quad (107.13)$$

in which  $T_r$  is the room temperature and  $T_m$  is the melting temperature.

2. The Zerilli-Armstrong constitutive relation

$$\sigma_Y = \{\sigma_a + B \exp[-(\beta_0 - \beta_1 \ln \dot{r})T] + Ar^n \exp[-(\alpha_0 - \alpha_1 \ln \dot{r})T]\} \quad (107.14)$$

where  $\sigma_a, B, \beta_0, \beta_1, A, n, \alpha_0$ , and  $\alpha_1$  are material parameters.

Damage evolution is defined by:

1. The extended Johnson-Cook damage evolution rule:

$$\Delta D = \begin{cases} 0 & p \leq p_d \\ \frac{D_c \Delta p}{p_f - p_d} & p > p_d \end{cases} \quad (107.15)$$

where the current equivalent fracture strain  $p_f = p_f(\sigma^*, \Delta p^*, T^*)$  is defined as

$$p_f = [D_1 + D_2 \exp(D_3 \sigma^*)] (1 + \Delta p^*)^{D_4} (1 + D_5 T^*) \quad (107.16)$$

Here  $D_1, D_2, D_3, D_4, D_5, D_C$ , and  $p_d$  are material parameters. The normalized equivalent plastic strain increment  $\Delta p^*$  is defined by

$$\Delta p^* = \frac{\Delta p}{\dot{\varepsilon}_0} \quad (107.17)$$

and the stress triaxiality  $\sigma^*$  reads

$$\sigma^* = \frac{\sigma_H}{\sigma_{\text{eq}}}, \quad \sigma_H = \frac{1}{3} \text{tr}(\sigma) \quad (107.18)$$

2. The Cockcroft-Latham damage evolution rule:

$$\Delta D = \frac{D_C}{W_C} \max(\sigma_1, 0) \Delta p \quad (107.19)$$

where  $D_C$  and  $W_C$  are material parameters. This assumes that the material parameters  $\phi$  or  $\gamma$  are zero. If they are not, the uncoupled extended Cockcroft-Latham damage evolution rule is used:

$$\Delta D = \frac{\sigma_{\text{eq}}}{W_C} \max \left( \phi \frac{\sigma_1}{\sigma_{\text{eq}}} + (1 - \phi) \frac{\sigma_1 - \sigma_3}{\sigma_{\text{eq}}}, 0 \right)^\gamma \Delta p \quad (107.20)$$

Adiabatic heating is calculated as

$$\dot{T} = \chi \frac{\sigma : \mathbf{d}^p}{\rho C_p} = \chi \frac{\tilde{\sigma}_{eq} \dot{r}}{\rho C_p} \quad (107.21)$$

where  $\chi$  is the Taylor-Quinney parameter,  $\rho$  is the density and  $C_p$  is the specific heat. The initial value of the temperature  $T_0$  may be specified by the user.

Element erosion occurs when one of the following several criteria are fulfilled:

1. The damage is greater than the critical value

$$D \geq D_C \quad (107.22)$$

2. The maximum shear stress is greater than a critical value

$$\tau_{\text{max}} = \frac{1}{2} \max\{|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\} \geq \tau_C \quad (107.23)$$

3. The temperature is greater than a critical value

$$T \geq T_C \quad (107.24)$$

History Variable	Description
1	Evaluation of damage $D$
2	Evaluation of stress triaxiality $\sigma^*$
3	Evaluation of damaged plastic strain $r$
4	Evaluation of temperature $T$
5	Evaluation of damaged plastic strain rate $\dot{r}$

History Variable	Description
8	Evaluation of plastic work per volume $W$
9	Evaluation of maximum shear stress $\tau_{\max}$

## \*MAT\_108

## \*MAT\_ORTHO\_ELASTIC\_PLASTIC

### \*MAT\_ORTHO\_ELASTIC\_PLASTIC

This is Material Type 108. This model combines orthotropic elastic plastic behavior with an anisotropic yield criterion. This model is implemented only for shell elements.

#### Card Summary:

**Card 1.** This card is required.

MID	R0	E11	E22	G12	PR12	PR23	PR31
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**Card 2.** This card is required.

SIGMA0	LC	QR1	CR1	QR2	CR2		
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**Card 3.** This card is required.

R11	R22	R33	R12				
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**Card 4.** This card is required.

AOPT	BETA						
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**Card 5.** This card is required.

			A1	A2	A3		
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**Card 6.** This card is required.

V1	V2	V3	D17	D2	D3		
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#### Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E11	E22	G12	PR12	PR23	PR31
Type	A	F	F	F	F	F	F	F

#### VARIABLE

#### DESCRIPTION

MID

Material identification. A unique number or label must be specified (see \*PART).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RO	Mass density
E11	Young's modulus in 11-direction
E22	Young's modulus in 22-direction
G12	Shear modulus in 12-direction
PR12	Poisson's ratio 12
PR23	Poisson's ratio 23
PR31	Poisson's ratio 31

Card 2	1	2	3	4	5	6	7	8
Variable	SIGMA0	LC	QR1	CR1	QR2	CR2		
Type	F	I	F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SIGMA0	Initial yield stress, $\sigma_0$
LC	Load curve defining effective stress as a function of effective plastic strain. If defined, QR1, CR1, QR2, and CR2 are ignored.
QR1	Isotropic hardening parameter, $Q_{R1}$
CR1	Isotropic hardening parameter, $C_{R1}$
QR2	Isotropic hardening parameter, $Q_{R2}$
CR2	Isotropic hardening parameter, $C_{R2}$

Card 3	1	2	3	4	5	6	7	8
Variable	R11	R22	R33	R12				
Type	F	F	F	F				

**\*MAT\_108****\*MAT\_ORTHO\_ELASTIC\_PLASTIC**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
R11	Yield criteria parameter, $R_{11}$
R22	Yield criteria parameter, $R_{22}$
R33	Yield criteria parameter, $R_{33}$
R12	Yield criteria parameter, $R_{12}$

Card 4	1	2	3	4	5	6	7	8
Variable	AOPT	BETA						
Type	F	F						

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):</p> <p>EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in <a href="#">Figure M2-1</a>. Nodes 1, 2 and 4 of an element are identical to the node used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA</p> <p>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR</p> <p>EQ.3.0: Locally orthotropic material axes determined by offsetting the material axes by an angle, BETA, from a line determined by taking the cross product of the vector <b>v</b> with the normal to the plane of the element</p> <p>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.</p>
BETA	Material angle in degrees for AOPT = 0 and 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA.

Card 5	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

**VARIABLE****DESCRIPTION**A1, A2, A3 Components of vector **a** for AOPT = 2

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**V1, V2, V3 Components of vector **v** for AOPT = 3D1, D2, D3 Components of vector **d** for AOPT = 2**Remarks:**

The yield function is defined as

$$f = \bar{f}(\sigma) - [\sigma_0 + R(\varepsilon^p)] ,$$

where the equivalent stress  $\sigma_{\text{eq}}$  is defined as an anisotropic yield criterion

$$\sigma_{\text{eq}} = \sqrt{F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2} .$$

Here  $F, G, H, L, M$  and  $N$  are constants obtained by testing the material in different orientations. They are defined as

$$\begin{aligned} F &= \frac{1}{2} \left( \frac{1}{R_{22}^2} + \frac{1}{R_{33}^2} - \frac{1}{R_{11}^2} \right), & L &= \frac{3}{2R_{23}^2} \\ G &= \frac{1}{2} \left( \frac{1}{R_{33}^2} + \frac{1}{R_{11}^2} - \frac{1}{R_{22}^2} \right), & M &= \frac{3}{2R_{31}^2} \\ H &= \frac{1}{2} \left( \frac{1}{R_{11}^2} + \frac{1}{R_{22}^2} - \frac{1}{R_{33}^2} \right), & N &= \frac{3}{2R_{12}^2} \end{aligned}$$

The yield stress ratios are defined as follows

$$\begin{aligned} R_{11} &= \frac{\bar{\sigma}_{11}}{\sigma_0}, & R_{12} &= \frac{\bar{\sigma}_{12}}{\tau_0} \\ R_{22} &= \frac{\bar{\sigma}_{22}}{\sigma_0}, & R_{23} &= \frac{\bar{\sigma}_{23}}{\tau_0} \\ R_{33} &= \frac{\bar{\sigma}_{33}}{\sigma_0}, & R_{31} &= \frac{\bar{\sigma}_{31}}{\tau_0} \end{aligned}$$

where  $\sigma_{ij}$  is the measured yield stress values,  $\sigma_0$  is the reference yield stress, and  $\tau_0 = \sigma_0/\sqrt{3}$ .

The strain hardening,  $R$ , is either defined by the load curve or by the extended Voce law,

$$R(\varepsilon^p) = \sum_{i=1}^2 Q_{Ri} [1 - \exp(-C_{Ri}\varepsilon^p)] ,$$

where  $\varepsilon^p$  is the effective (or accumulated) plastic strain, and  $Q_{Ri}$  and  $C_{Ri}$  are strain hardening parameters.

**\*MAT\_JOHNSON\_HOLMQUIST\_CERAMICS**

This is Material Type 110. This Johnson-Holmquist Plasticity Damage Model is useful for modeling ceramics, glass and other brittle materials. A more detailed description can be found in a paper by Johnson and Holmquist [1993].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	G	A	B	C	M	N
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	EPS0	T	SFMAX	HEL	PHEL	BETA		
Type	F	F	F	F	F	F		

Card 3	1	2	3	4	5	6	7	8
Variable	D1	D2	K1	K2	K3	FS		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Density
G	Shear modulus
A	Intact normalized strength parameter
B	Fractured normalized strength parameter
C	Strength parameter (for strain rate dependence)
M	Fractured strength parameter (pressure exponent)

<b>VARIABLE</b>	<b>DESCRIPTION</b>
N	Intact strength parameter (pressure exponent)
EPS0	Quasi-static threshold strain rate. See *MAT_015.
T	Maximum tensile pressure strength
SFMAX	Maximum normalized fractured strength (defaults to 10 <sup>20</sup> when set to 0.0).
HEL	Hugoniot elastic limit
PHEL	Pressure component at the Hugoniot elastic limit
BETA	Fraction of elastic energy loss converted to hydrostatic energy. It affects bulking pressure (history variable 1) that accompanies damage.
D1	Parameter for plastic strain to fracture
D2	Parameter for plastic strain to fracture (exponent)
K1	First pressure coefficient (equivalent to the bulk modulus)
K2	Second pressure coefficient
K3	Third pressure coefficient
FS	Element deletion criterion: LT.0.0: Fail if $p^* + t^* < 0$ (tensile failure) EQ.0.0: No failure (default) GT.0.0: Fail if the effective plastic strain > FS

**Remarks:**

The equivalent stress for a ceramic-type material is given by

$$\sigma^* = \sigma_i^* - D(\sigma_i^* - \sigma_f^*) ,$$

where

$$\sigma_i^* = a(p^* + t^*)^n(1 + c\ln\varepsilon^*)$$

represents the intact, undamaged behavior. The superscript, “\*”, indicates a normalized quantity. The stresses are normalized by the equivalent stress at the Hugoniot elastic limit, the pressures are normalized by the pressure at the Hugoniot elastic limit, and the

strain rate by the reference strain rate defined in the input. In this equation  $a$  is the intact normalized strength parameter,  $c$  is the strength parameter for strain rate dependence,  $\dot{\varepsilon}^*$  is the normalized plastic strain rate, and

$$t^* = \frac{T}{\text{PHEL}}$$

$$p^* = \frac{p}{\text{PHEL}}$$

In the above,  $T$  is the maximum tensile pressure strength, PHEL is the pressure component at the Hugoniot elastic limit, and  $p$  is the pressure.

$$D = \sum \frac{\Delta \varepsilon^p}{\varepsilon_f^p}$$

represents the accumulated damage (history variable 2) based upon the increase in plastic strain per computational cycle and the plastic strain to fracture

$$\varepsilon_f^p = d_1(p^* + t^*)^{d_2}$$

and

$$\sigma_f^* = b(p^*)^m(1 + c \ln \dot{\varepsilon}^*) \leq \text{SFMAX}$$

represents the damaged behavior. The parameter  $d_1$  controls the rate at which damage accumulates. If it is set to 0, full damage occurs in one time step, that is, instantaneously. It is also the best parameter to vary when attempting to reproduce results generated by another finite element program.

In undamaged material, the hydrostatic pressure is given by

$$P = k_1\mu + k_2\mu^2 + k_3\mu^3$$

in compression and

$$P = k_1\mu$$

in tension where  $\mu = \rho/\rho_0 - 1$ . When damage starts to occur, there is an increase in pressure. A fraction, between 0 and 1, of the elastic energy loss,  $\beta$ , is converted into hydrostatic potential energy (pressure). The details of this pressure increase are given in the reference.

Given HEL and G,  $\mu_{\text{hel}}$  can be found iteratively from

$$\text{HEL} = k_1\mu_{\text{hel}} + k_2\mu_{\text{hel}}^2 + k_3\mu_{\text{hel}}^3 + (4/3)g(\mu_{\text{hel}}/(1 + \mu_{\text{hel}}))$$

and, subsequently, for normalization purposes,

$$P_{\text{hel}} = k_1\mu_{\text{hel}} + k_2\mu_{\text{hel}}^2 + k_3\mu_{\text{hel}}^3$$

and

$$\sigma_{\text{hel}} = 1.5(\text{hel} - p_{\text{hel}})$$

**\*MAT\_110****\*MAT\_JOHNSON\_HOLMQUIST\_CERAMICS**

These are calculated automatically by LS-DYNA if  $p_{hel}$  is zero on input.

**\*MAT\_JOHNSON\_HOLMQUIST\_CONCRETE**

This is Material Type 111. This model can be used for concrete subjected to large strains, high strain rates and high pressures. The equivalent strength is expressed as a function of the pressure, strain rate, and damage. The pressure is expressed as a function of the volumetric strain and includes the effect of permanent crushing. The damage is accumulated as a function of the plastic volumetric strain, equivalent plastic strain and pressure. A more detailed description of this model can be found in the paper by Holmquist, Johnson, and Cook [1993].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	G	A	B	C	N	FC
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	T	EPS0	EFSMIN	SFMAX	PC	UC	PL	UL
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	D1	D2	K1	K2	K3	FS		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
G	Shear modulus
A	Normalized cohesive strength
B	Normalized pressure hardening

**\*MAT\_111****\*MAT\_JOHNSON\_HOLMQUIST\_CONCRETE**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
C	Strain rate coefficient
N	Pressure hardening exponent
FC	Quasi-static uniaxial compressive strength
T	Maximum tensile hydrostatic pressure
EPS0	Quasi-static threshold strain rate. See *MAT_015.
EFMIN	Amount of plastic strain before fracture
SFMAX	Normalized maximum strength
PC	Crushing pressure
UC	Crushing volumetric strain
PL	Locking pressure
UL	Locking volumetric strain
D1	Damage constant
D2	Damage constant
K1	Pressure constant
K2	Pressure constant
K3	Pressure constant
FS	Failure type: LT.0.0: Fail if damage strength < 0 EQ.0.0: Fail if $P^* + T^* \leq 0$ (tensile failure) GT.0.0: Fail if the effective plastic strain > FS

**Remarks:**

The normalized equivalent stress is defined as

$$\sigma^* = \frac{\sigma}{f'_c} ,$$

where  $\sigma$  is the actual equivalent stress, and  $f'_c$  is the quasi-static uniaxial compressive strength. The expression is defined as:

$$\sigma^* = [A(1 - D) + BP^{*N}][1 + C \ln(\dot{\varepsilon}^*)] .$$

where  $D$  is the damage parameter,  $P^* = P/f'_c$  is the normalized pressure and  $\dot{\varepsilon}^* = \dot{\varepsilon}/\dot{\varepsilon}_0$  is the dimensionless strain rate. The model incrementally accumulates damage,  $D$ , both from equivalent plastic strain and plastic volumetric strain, and is expressed as

$$D = \sum \frac{\Delta \varepsilon_p + \Delta \mu_p}{D_1(P^* + T^*)^{D_2}} .$$

Here,  $\Delta \varepsilon_p$  and  $\Delta \mu_p$  are the equivalent plastic strain and plastic volumetric strain,  $D_1$  and  $D_2$  are material constants and  $T^* = T/f'_c$  is the normalized maximum tensile hydrostatic pressure.

The damage strength, DS, is defined in compression when  $P^* > 0$  as

$$DS = f'_c \min[SFMAX, A(1 - D) + BP^{*N}][1 + C * \ln(\dot{\varepsilon}^*)]$$

or in tension if  $P^* < 0$ , as

$$DS = f'_c \max[0, A(1 - D) - A \left( \frac{P^*}{T} \right)][1 + C * \ln(\dot{\varepsilon}^*)] .$$

The pressure for fully dense material is expressed as

$$P = K_1 \bar{\mu} + K_2 \bar{\mu}^2 + K_3 \bar{\mu}^3 ,$$

where  $K_1$ ,  $K_2$  and  $K_3$  are material constants and the modified volumetric strain is defined as

$$\bar{\mu} = \frac{\mu - \mu_{lock}}{1 + \mu_{lock}} ,$$

where  $\mu_{lock}$  is the locking volumetric strain.

**\*MAT\_112****\*MATFINITEELASTICSTRAINPLASTICITY****\*MATFINITEELASTICSTRAINPLASTICITY**

This is Material Type 112. An elasto-plastic material with an arbitrary stress versus strain curve and arbitrary strain rate dependency can be defined. The elastic response of this model uses a finite strain formulation so that large elastic strains can develop before yielding occurs. This model is available for solid elements only. See Remarks below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	ETAN		
Type	A	F	F	F	F	F		
Default	none	none	none	none	none	0.0		

Card 2	1	2	3	4	5	6	7	8
Variable	C	P	LCSS	LCSR				
Type	F	F	I	I				
Default	0.0	0.0	0	0				

Card 3	1	2	3	4	5	6	7	8
Variable	EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Card 4	1	2	3	4	5	6	7	8
Variable	ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
SIGY	Yield stress
ETAN	Tangent modulus; ignored if LCSS > 0
C	Strain rate parameter, $C$ ; see Remarks below.
P	Strain rate parameter, $p$ ; see Remarks below.
LCSS	Load curve ID or table ID. <b>Load Curve ID.</b> The load curve defines effective stress as a function of effective plastic strain. If defined, EPS1 - EPS8 and ES1 - ES8 are ignored.
LCSR	<b>Table ID.</b> The table defines for each strain rate value a load curve ID giving the stress as a function of effective plastic strain for that rate; see <a href="#">Figure M24-1</a> . The stress as a function of effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress as a function of effective plastic strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value. The strain rate parameters, $C$ and $p$ ; the curve ID LCSR; EPS1 - EPS8; and ES1 - ES8 are ignored if a table ID is defined.
LCSR	Load curve ID defining strain rate scaling effect on yield stress

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EPS1 - EPS8	Effective plastic strain values (optional if SIGY is defined). At least 2 points should be defined. The first point must be zero corresponding to the initial yield stress. WARNING: If the first point is nonzero, the yield stress is extrapolated to determine the initial yield. If this option is used, SIGY and ETAN are ignored and may be input as zero.
ES1 - ES8	Corresponding yield stress values to EPS1 - EPS8.

**Remarks:**

By defining the tangent modulus ETAN, the stress strain behavior is treated using a bilinear stress strain curve. Alternately, a curve similar to that shown in [Figure M10-1](#) is expected to be defined by (EPS1, ES1) - (EPS8, ES8); however, an effective stress as a function of effective plastic strain curve (LCSS) may be input instead if eight points are insufficient. The cost is roughly the same for either approach. The most general approach is to use the table definition (LCSS) discussed below.

Three options to account for strain rate effects are possible.

1. Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\epsilon}}{C}\right)^{1/p},$$

where  $\dot{\epsilon}$  is the strain rate,  $\dot{\epsilon} = \sqrt{\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}}$ .

2. For complete generality a load curve (LCSR) to scale the yield stress may be input instead. In this curve the scale factor as a function of strain rate is defined.
3. If different stress as a function of strain curves can be provided for various strain rates, the option using the reference to a table (LCSS) can be used. Then the table input in \*DEFINE\_TABLE must be used; see [Figure M24-1](#).

**\*MAT\_TRIP****\*MAT\_113****\*MAT\_TRIP**

This is Material Type 113. This isotropic elasto-plastic material model applies to shell elements only. It features a special hardening law aimed at modelling the temperature dependent hardening behavior of austenitic stainless TRIP-steels. TRIP stands for Transformation Induced Plasticity. A detailed description of this material model can be found in Hänsel, Hora, and Reissner [1998] and Schedin, Prentzas, and Hilding [2004].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	CP	T0	TREF	TA0
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	A	B	C	D	P	Q	EOMART	VM0
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	AHS	BHS	M	N	EPS0	HMART	K1	K2
Type	F	F	F	F	F	F	F	F

**VARIABLE****DESCRIPTION**

MID           Material identification. A unique number or label must be specified (see \*PART).

RO           Mass density

E           Young's modulus

PR           Poisson's ratio

CP           Adiabatic temperature calculation option:

EQ.0.0: Adiabatic temperature calculation is disabled.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	GT.0.0: CP is the specific heat $C_p$ . Adiabatic temperature calculation is enabled.
T0	Initial temperature $T_0$ of the material if adiabatic temperature calculation is enabled.
TREF	Reference temperature for output of the yield stress as history variable 1.
TA0	Reference temperature $T_{A0}$ , the absolute zero for the used temperature scale. For example, TA0 is -273.15 if the Celsius scale is used and 0.0 if the Kelvin scale is used.
A	Martensite rate equation parameter A; see Remarks below.
B	Martensite rate equation parameter B; see Remarks below.
C	Martensite rate equation parameter C; see Remarks below.
D	Martensite rate equation parameter D; see Remarks below.
P	Martensite rate equation parameter p; see Remarks below.
Q	Martensite rate equation parameter Q; see Remarks below.
E0MART	Martensite rate equation parameter $E_{0(\text{mart})}$ ; see Remarks below.
VM0	The initial volume fraction of martensite $0.0 < V_{m0} < 1.0$ may be initialised using two different methods:  GT.0.0: $V_{m0}$ is set to VM0.  LT.0.0: Can be used only when there are initial plastic strains $\epsilon^p$ present, such as when using *INITIAL_STRESS_SHELL. The absolute value of VM0 is then the load curve ID for a function $f$ that sets $V_{m0} = f(\epsilon^p)$ . The function $f$ must be a monotonically nondecreasing function of $\epsilon^p$ .
AHS	Hardening law parameter $A_{\text{HS}}$ ; see Remarks below.
BHS	Hardening law parameter $B_{\text{HS}}$ ; see Remarks below.
M	Hardening law parameter m; see Remarks below.
N	Hardening law parameter n; see Remarks below.

VARIABLE	DESCRIPTION
EPS0	Hardening law parameter $\varepsilon_0$ ; see Remarks below.
HMART	Hardening law parameter $\Delta H_{\gamma \rightarrow \alpha'}$ ; see Remarks below.
K1	Hardening law parameter $K_1$ ; see Remarks below.
K2	Hardening law parameter $K_2$ ; see Remarks below.

**Remarks:**

Here a short description is given of the TRIP-material model. The material model uses the von Mises yield surface in combination with isotropic hardening. The hardening is temperature dependent. Therefore, this material model must be run either in a coupled thermo-mechanical solution, using prescribed temperatures or using the adiabatic temperature calculation option. Setting the parameter CP to the specific heat,  $C_p$ , of the material activates the adiabatic temperature calculation that calculates the temperature rate from the equation

$$\dot{T} = \sum_{i,j} \frac{\sigma_{ij} D_{ij}^p}{\rho C_p},$$

where  $\sigma: \mathbf{D}^p$  (the numerator) is the plastically dissipated heat. Using the Kelvin scale is recommended, even though other scales may be used without problems.

The hardening behavior is described by the following equations. The Martensite rate equation is

$$\frac{\partial V_m}{\partial \bar{\varepsilon}^p} = \begin{cases} \bar{A}^0 & \varepsilon < E_{0(\text{mart})} \\ \frac{B}{A} V_m^p \left( \frac{1 - V_m}{V_m} \right)^{\frac{B+1}{B}} \frac{[1 - \tanh(C + D \times T)]}{2} \exp\left(\frac{Q}{T - T_{A0}}\right) & \bar{\varepsilon}^p \geq E_{0(\text{mart})} \end{cases}$$

where  $\bar{\varepsilon}^p$  is the effective plastic strain and  $T$  is the temperature.

The martensite fraction is integrated from the above rate equation:

$$V_m = \int_0^\varepsilon \frac{\partial V_m}{\partial \bar{\varepsilon}^p} d\bar{\varepsilon}^p .$$

It always holds that  $0.0 < V_m < 1.0$ . The initial martensite content is  $V_{m0}$  and must be greater than zero and less than 1.0. Note that  $V_{m0}$  is not used during a restart or when initializing the  $V_{m0}$  history variable using \*INITIAL\_STRESS\_SHELL.

The yield stress is:

$$\sigma_y = \{B_{HS} - (B_{HS} - A_{HS})\exp(-m[\bar{\varepsilon}^p + \varepsilon_0]^n)\}(K_1 + K_2 T) + \Delta H_{\gamma \rightarrow \alpha'} V_m .$$

The parameters  $p$  and  $B$  should fulfill the following condition

$$\frac{1+B}{B} < p .$$

If the condition is not fulfilled, then the martensite rate will approach infinity as  $V_m$  approaches zero. Setting the parameter  $\varepsilon_0$  larger than zero (typical range 0.001 - 0.02) is recommended. Apart from the effective true strain a few additional history variables are output; see below.

### **Output History Variables:**

Variable	Description
1	Yield stress of material at temperature TREF. Useful to evaluate the strength of the material after e.g., a simulated forming operation.
2	Volume fraction martensite, $V_m$
3	CP.EQ.0.0: not used CP.GT.0.0: temperature from adiabatic temperature calculation

**\*MAT\_LAYERED\_LINEAR\_PLASTICITY**

This is Material Type 114. It is a layered elastoplastic material with an arbitrary stress as a function of strain curve. An arbitrary strain rate dependency can also be defined. This material must be used with the user defined integration rules (see \*INTEGRATION-SHELL) for modeling laminated composite and sandwich shells where each layer can be represented by elastoplastic behavior with constitutive constants that vary from layer to layer. Lamination theory is applied to correct for the assumption of a uniform constant shear strain through the thickness of the shell. Unless this correction is applied, the stiffness of the shell can be grossly incorrect leading to poor results. Generally, without the correction the results are too stiff. This model is available for shell elements only. See Remarks below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	ETAN	FAIL	TDEL
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	10 <sup>20</sup>	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	C	P	LCSS	LCSR				
Type	F	F	I	I				
Default	0.0	0.0	0	0				

Card 3	1	2	3	4	5	6	7	8
Variable	EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

**\*MAT\_114****\*MAT\_LAYERED\_LINEAR\_PLASTICITY**

Card 4	1	2	3	4	5	6	7	8
Variable	ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
SIGY	Yield stress
ETAN	Tangent modulus; ignored if LCSS > 0 is defined.
FAIL	Failure flag: LT.0.0: User defined failure subroutine, matusr_24 in dyn21.F, is called to determine failure. EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved. GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.
TDEL	Minimum time step size for automatic element deletion
C	Strain rate parameter, C; see Remarks below.
P	Strain rate parameter, p; see Remarks below.
LCSS	Load curve ID or Table ID. <b>Load Curve ID.</b> The load curve defines effective stress as a function of effective plastic strain. If defined, EPS1 - EPS8 and ES1 - ES8 are ignored.

VARIABLE	DESCRIPTION
	<b>Table ID.</b> The table defines for each strain rate value a load curve ID giving the stress as a function of effective plastic strain for that rate; see <a href="#">Figure M24-1</a> . The stress as a function of effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress as a function of effective plastic strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value. C, P, LCSR, EPS1 – EPS8, and ES1 – ES8 are ignored if a table ID is defined.
LCSR	Load curve ID defining strain rate scaling effect on yield stress
EPS1 - EPS8	Effective plastic strain values (optional if SIGY is defined). At least 2 points should be defined. The first point must be zero corresponding to the initial yield stress. WARNING: If the first point is nonzero the yield stress is extrapolated to determine the initial yield. If this option is used, SIGY and ETAN are ignored and may be input as zero.
ES1 - ES8	Corresponding yield stress values to EPS1 - EPS8.

### Remarks:

The stress strain behavior may be treated by a bilinear stress strain curve by defining the tangent modulus, ETAN. Alternately, a curve similar to that shown in [Figure M10-1](#) is expected to be defined by (EPS1, ES1) - (EPS8, ES8); however, an effective stress as a function of effective plastic strain curve (LCSS) may be input instead if eight points are insufficient. The cost is roughly the same for either approach. The most general approach is to use the table definition (LCSR) discussed below.

Three options to account for strain rate effects are possible.

1. Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\epsilon}}{C}\right)^{1/p},$$

where  $\dot{\epsilon}$  is the strain rate;  $\dot{\epsilon} = \sqrt{\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}}$ .

2. For complete generality a load curve (LCSR) to scale the yield stress may be input instead. This curve defines the scale factor as a function of strain rate.

3. If different stress as a function of strain curves can be provided for various strain rates, the option using the reference to a table (LCSS) can be used. Then the table input in \*DEFINE\_TABLE must be used; see [Figure M24-1](#).

**\*MAT\_UNIFIED\_CREEP**

This is Material Type 115. This is an elastic creep model for modeling creep behavior when plastic behavior is not considered.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	A	N	M	
Type	A	F	F	F	F	F	F	
Default	none							

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
A	Stress coefficient
N	Stress exponent
M	Time exponent

**Remarks:**

The effective creep strain,  $\bar{\epsilon}^c$ , given as:

$$\bar{\epsilon}^c = A \bar{\sigma}^n \bar{t}^m ,$$

where  $A$ ,  $n$ , and  $m$  are constants and  $\bar{t}$  is the effective time. The effective stress,  $\bar{\sigma}$ , is defined as:

$$\bar{\sigma} = \sqrt{\frac{3}{2} \sigma_{ij} \sigma_{ij}} .$$

The creep strain, therefore, is only a function of the deviatoric stresses. The volumetric behavior for this material is assumed to be elastic. By varying the time constant  $m$

primary creep ( $m < 1$ ), secondary creep ( $m = 1$ ), and tertiary creep ( $m > 1$ ) can be modeled. This model is described by Whirley and Henshall [1992].

**\*MAT\_UNIFIED\_CREEP\_ORTHO****\*MAT\_115\_O****\*MAT\_UNIFIED\_CREEP\_ORTHO**

This is Material Type 115\_O. This is an orthotropic elastic creep model for modeling creep behavior when plastic behavior is not considered. This material is available for solid elements, thick shell element formulations 3, 5, and 7, and SPH elements. It is available for both explicit and implicit dynamics.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E1	E2	E3	PR21	PR31	PR32
Type	A	F	F	F	F	F	F	F
Default	none							

Card 2	1	2	3	4	5	6	7	8
Variable	G12	G23	G13	A	N	M		
Type	F	F	F	F	F	F		
Default	none	none	none	none	none	none		

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT	MACF	XP	YP	ZP	A1	A2	A3
Type	F	F	F	F	F	F	F	F
Default	none							

**\*MAT\_115\_O****\*MAT\_UNIFIED\_CREEP\_ORTHO**

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	
Default	none							

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
Ei	Young's moduli
PR <sub>ij</sub>	Elastic Poisson's ratios
G <sub>ij</sub>	Elastic shear moduli
A	Stress coefficient
N	Stress exponent
M	Time exponent
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC, particularly the <a href="#">Material Directions</a> section, for details): <ul style="list-style-type: none"> <li>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.</li> <li>EQ.1.0: Locally orthotropic with material axes determined by a point, P, in space and the global location of the element center; this is the <b>a</b>-direction. This option is for solid elements only.</li> <li>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR</li> <li>EQ.3.0: Locally orthotropic material axes determined by a vector <b>v</b> and the normal vector to the plane of the element. The plane of a solid element is the midsurface between</li> </ul>

VARIABLE	DESCRIPTION
	the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. <b>a</b> is determined by taking the cross product of <b>v</b> with the normal vector, <b>b</b> is determined by taking the cross product of the normal vector with <b>a</b> , and <b>c</b> is the normal vector. Then <b>a</b> and <b>b</b> are rotated about <b>c</b> by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.
EQ.4.0:	Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector <b>v</b> , and an originating point, <i>P</i> , which define the centerline axis. This option is for solid elements only.
LT.0.0:	The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_OPTION).
MACF	<p>Material axes change flag for solid elements:</p> <ul style="list-style-type: none"> <li>EQ.-4: Switch material axes <i>b</i> and <i>c</i> before BETA rotation</li> <li>EQ.-3: Switch material axes <i>a</i> and <i>c</i> before BETA rotation</li> <li>EQ.-2: Switch material axes <i>a</i> and <i>b</i> before BETA rotation</li> <li>EQ.1: No change, default</li> <li>EQ.2: Switch material axes <i>a</i> and <i>b</i> after BETA rotation</li> <li>EQ.3: Switch material axes <i>a</i> and <i>c</i> after BETA rotation</li> <li>EQ.4: Switch material axes <i>b</i> and <i>c</i> after BETA rotation</li> </ul> <p><a href="#">Figure M2-2</a> indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_{OPTION} is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 4 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.</p>
XP, YP, ZP	Define coordinates of point <i>p</i> for AOPT = 1 and 4
A1, A2, A3	Define components of vector <b>a</b> for AOPT = 2

<b>VARIABLE</b>	<b>DESCRIPTION</b>
V1, V2, V3	Define components of vector <b>v</b> for AOPT = 3 and 4
D1, D2, D3	Define components of vector <b>d</b> for AOPT = 2
BETA	Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_TSHELL_BETA or *ELEMENT_SOLID_ORTHO.

**Remarks:**

The stress-strain relationship is based on an additive split of the strain,

$$\dot{\epsilon} = \dot{\epsilon}_e + \dot{\epsilon}_c .$$

Here, the multiaxial creep strain is given by

$$\dot{\epsilon}_c = \bar{\dot{\epsilon}}_c \frac{2s}{3\bar{\sigma}} ,$$

and  $\bar{\epsilon}^c$  is the effective creep strain,  $s$  the deviatoric stress

$$s = \sigma - \frac{1}{3} \text{tr}(\sigma) \mathbf{I} .$$

and  $\bar{\sigma}$  the effective stress

$$\bar{\sigma} = \sqrt{\frac{3}{2} s : s} .$$

The effective creep strain is given by

$$\dot{\epsilon}^c = A \bar{\sigma}^N t^M ,$$

where  $A$ ,  $N$ , and  $M$  are constants.

The stress increment is given by

$$\Delta\sigma = \mathbf{C} \Delta\epsilon_e = \mathbf{C} (\Delta\epsilon - \Delta\epsilon_c) ,$$

where the constitutive matrix  $\mathbf{C}$  is taken as orthotropic and can be represented in Voigt notation by its inverse as

$$\mathbf{C}^{-1} = \begin{bmatrix} \frac{1}{E_1} & -\frac{v_{21}}{E_2} & -\frac{v_{31}}{E_3} \\ -\frac{v_{12}}{E_1} & \frac{1}{E_2} & -\frac{v_{32}}{E_3} \\ -\frac{v_{13}}{E_1} & -\frac{v_{23}}{E_2} & \frac{1}{E_3} \\ & & \frac{1}{G_{12}} \\ & & \frac{1}{G_{23}} \\ & & \frac{1}{G_{13}} \end{bmatrix}.$$

## \*MAT\_116

## \*MAT\_COMPOSITE\_LAYUP

### \*MAT\_COMPOSITE\_LAYUP

This is Material Type 116. This material is for modeling the elastic responses of composite layups that have an arbitrary number of layers through the shell thickness. A pre-integration is used to compute the extensional, bending, and coupling stiffness for use with the Belytschko-Tsay resultant shell formulation. The angles of the local material axes are specified from layer to layer in the \*SECTION\_SHELL input. This material model must be used with the user defined integration rule for shells (see \*INTEGRATION\_SHELL) which allows the elastic constants to change from integration point to integration point. Since the stresses are not computed in the resultant formulation, the stresses output to the binary databases for the resultant elements are zero. Note that this shell *does not use laminated shell theory* and that storage is allocated for just one integration point (as reported in d3hsp) regardless of the layers defined in the integration rule.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	EA	EB	EC	PRBA	PRCA	PRCB
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	AOPT				
Type	F	F	F	F				

Card 3	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
EA	$E_a$ , Young's modulus in the $a$ -direction
EB	$E_b$ , Young's modulus in the $b$ -direction
EC	$E_c$ , Young's modulus in the $c$ -direction
PRBA	$\nu_{ba}$ , Poisson's ratio $ba$
PRCA	$\nu_{ca}$ , Poisson's ratio $ca$
PRCB	$\nu_{cb}$ , Poisson's ratio $cb$
GAB	$G_{ab}$ , shear modulus $ab$
GBC	$G_{bc}$ , shear modulus $bc$
GCA	$G_{ca}$ , shear modulus $ca$
AOPT	Material axes option, see <a href="#">Figure M2-1</a> : <ul style="list-style-type: none"> <li>EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in <a href="#">Figure M2-1</a>. Nodes 1, 2, and 4 of an element are identical to the nodes used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA.</li> <li>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</li> <li>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector, v, with the element normal.</li> <li>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).</li> </ul>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
XP, YP, ZP	Define coordinates of point $p$ for AOPT = 1 and 4.
A1, A2, A3	Define components of vector $\mathbf{a}$ for AOPT = 2.
V1, V2, V3	Define components of vector $\mathbf{v}$ for AOPT = 3 and 4.
D1, D2, D3	Define components of vector $\mathbf{d}$ for AOPT = 2.
BETA	Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA.

**Remarks:**

This material law is based on standard composite lay-up theory. The implementation, [Jones 1975], allows the calculation of the force,  $N$ , and moment,  $M$ , stress resultants from:

$$\begin{aligned} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_z^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z \end{Bmatrix} \\ \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} &= \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_z^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z \end{Bmatrix} \end{aligned}$$

where  $A_{ij}$  is the extensional stiffness,  $D_{ij}$  is the bending stiffness, and  $B_{ij}$  is the coupling stiffness which is a null matrix for symmetric lay-ups. The mid-surface strains and curvatures are denoted by  $\varepsilon_{ij}^0$  and  $\kappa_{ij}$ , respectively. Since these stiffness matrices are symmetric, 18 terms are needed per shell element in addition to the shell resultants which are integrated in time. This is considerably less storage than would typically be required with through thickness integration which requires a minimum of eight history variables per integration point. For instance, if 100 layers are used, 800 history variables would be stored. Not only is memory much less for this model, but the CPU time required is also considerably reduced.

**\*MAT\_COMPOSITE\_MATRIX**

This is Material Type 117. This material is used for modeling the elastic responses of composites where a pre-integration is used to compute the extensional, bending, and coupling stiffness coefficients for use with the Belytschko-Tsay resultant shell formulation. Since the stresses are not computed in the resultant formulation, the stresses output to the binary databases for the resultant elements are zero.

**NOTE:** This material does not support specification of a material angle,  $\beta_i$ , for each through-thickness integration point of a shell.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0						
Type	A	F						

Card 2	1	2	3	4	5	6	7	8
Variable	C11	C12	C22	C13	C23	C33	C14	C24
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	C34	C44	C15	C25	C35	C45	C55	C16
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	C26	C36	C46	C56	C66	AOPT		
Type	F	F	F	F	F	F		

**\*MAT\_117****\*MAT\_COMPOSITE\_MATRIX**

Card 5	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

MID              Material identification. A unique number or label must be specified (see \*PART).

RO              Mass density

CIJ               $C_{ij}$ , coefficients of stiffness matrix in the material coordinate system

AOPT              Material axes option (see MAT\_OPTIONTROPIC\_ELASTIC for a more complete description):

EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in [Figure M2-1](#). Nodes 1, 2, and 4 of an element are identical to the nodes used for the definition of a coordinate system as by \*DEFINE\_COORDINATE\_NODES and then rotated about the shell element normal by an angle BETA.

EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with \*DEFINE\_COORDINATE\_VECTOR

EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal

LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on \*DEFINE\_COORDINATE\_NODES, \*DEFINE\_COORDINATE\_SYSTEM or \*DEFINE\_CO-

VARIABLE	DESCRIPTION
ORDINATE_VECTOR).	
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2
BETA	Material angle in degrees for AOPT = 0 and 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA.

**Remarks:**

The calculation of the force,  $N_{ij}$ , and moment,  $M_{ij}$ , stress resultants is given in terms of the membrane strains,  $\varepsilon_i^0$ , and shell curvatures,  $\kappa_i$ , as:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_z^0 \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

where  $C_{ij} = C_{ji}$ . In this model this symmetric matrix is transformed into the element local system and the coefficients are stored as element history variables. In \*MAT\_COMPOSITE\_DIRECT, the resultants are already assumed to be given in the element local system which reduces the storage since the 21 coefficients are not stored as history variables as part of the element data.

The shell thickness is built into the coefficient matrix and, consequently, within the part ID, which references this material ID. The thickness must be uniform.

## \*MAT\_118

## \*MAT\_COMPOSITE\_DIRECT

### \*MAT\_COMPOSITE\_DIRECT

This is Material Type 118. This material is used for modeling the elastic responses of composites where a pre-integration is used to compute the extensional, bending, and coupling stiffness coefficients for use with the Belytschko-Tsay resultant shell formulation. Since the stresses are not computed in the resultant formulation, the stresses output to the binary databases for the resultant elements are zero.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0						
Type	A	F						

Card 2	1	2	3	4	5	6	7	8
Variable	C11	C12	C22	C13	C23	C33	C14	C24
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	C34	C44	C15	C25	C35	C45	C55	C16
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	C26	C36	C46	C56	C66			
Type	F	F	F	F	F			

#### VARIABLE

#### DESCRIPTION

MID              Material identification. A unique number or label must be specified (see \*PART).

RO              Mass density

VARIABLE	DESCRIPTION
CIJ	Coefficients of the stiffness matrix, $C_{ij}$

**Remarks:**

The calculation of the force,  $N_{ij}$ , and moment,  $M_{ij}$ , stress resultants is given in terms of the membrane strains,  $\varepsilon_i^0$ , and shell curvatures,  $\kappa_i$ , as:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_z^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

where  $C_{ij} = C_{ji}$ . In this model the stiffness coefficients are already assumed to be given in the element local system which reduces the storage. Great care in the element orientation and choice of the local element system, see \*CONTROL\_ACCURACY, must be observed if this model is used.

The shell thickness is built into the coefficient matrix and, consequently, within the part ID, which references this material ID, the thickness must be uniform.

**\*MAT\_119****\*MAT\_GENERAL\_NONLINEAR\_6DOF\_DISCRETE\_BEAM****\*MAT\_GENERAL\_NONLINEAR\_6DOF\_DISCRETE\_BEAM**

This is Material Type 119. It is a very general spring and damper model. This beam is based on the MAT\_SPRING\_GENERAL\_NONLINEAR option. Additional unloading options have been included. The two nodes defining the beam may be coincident to give a zero length beam or offset to give a finite length beam. For finite length discrete beams the absolute value of the variable SCOOR in the \*SECTION\_BEAM input should be set to a value of 2.0 or 3.0 to give physically correct behavior. A triad is used to orient the beam for the directional springs.

**Card Summary:**

**Card 1.** This card is required.

MID	R0	KT	KR	IUNLD	OFFSET	DAMPF	IFLAG
-----	----	----	----	-------	--------	-------	-------

**Card 2.** This card is required.

LCIDTR	LCIDTS	LCIDTT	LCIDRR	LCIDRS	LCIDRT		
--------	--------	--------	--------	--------	--------	--	--

**Card 3.** This card is required.

LCIDTUR	LCIDTUS	LCIDTUT	LCIDRUR	LCIDRUS	LCIDRUT		
---------	---------	---------	---------	---------	---------	--	--

**Card 4.** This card is required.

LCIDTDR	LCIDTDS	LCIDTDT	LCIDRDR	LCIDRDS	LCIDRDT		
---------	---------	---------	---------	---------	---------	--	--

**Card 5.** This card is required.

LCIDTER	LCIDTES	LCIDTET	LCIDRER	LCIDRES	LCIDRET		
---------	---------	---------	---------	---------	---------	--	--

**Card 6.** This card is required.

UTFAILR	UTFAILS	UTFAILT	WTFAILR	WTFAILS	WTFAILT	FCRIT	
---------	---------	---------	---------	---------	---------	-------	--

**Card 7.** This card is required.

UCFAILR	UCFAILS	UCFAILT	WCFAILR	WCFAILS	WCFAILT		
---------	---------	---------	---------	---------	---------	--	--

**Card 8.** This card is required.

IUR	IUS	IUT	IWR	IWS	IWT		
-----	-----	-----	-----	-----	-----	--	--

**Card 9.** This card is read if IFLAG = 2. It is optional, but if it is included, Cards 10 and 11 must also be included.

LM1R1S	LM1R2S	LM1R1T	LM1R2T	LM2R1S	LM2R1T		
--------	--------	--------	--------	--------	--------	--	--

**Card 10.** This card is read if IFLAG = 2. It is optional but must be included if Card 9 is included.

LUM1R1S	LUM1R2S	LUM1R1T	LUM1R2T	LUM2R1S	LUM2R1T		
---------	---------	---------	---------	---------	---------	--	--

**Card 11.** This card is read if IFLAG = 2. It is optional but must be included if Card 9 is included.

KUM1R1S	KUM1R2S	KUM1R1T	KUM1R2T	KUM2R1S	KUM2R1T	KUM2R2S	KUM2R2T
---------	---------	---------	---------	---------	---------	---------	---------

**Card 12.** This card is read if IFLAG = 2. It is optional, but if it is included Cards 13 and 14 must be included.

E1TR	E2TR	E1RR	E2RR	E1RS	E2RS	E1RT	E2RT
------	------	------	------	------	------	------	------

**Card 13.** This card is read if IFLAG = 2. It is optional but must be included if Card 12 is included.

E1M1R1S	E2M1R1S	E1M1R2S	E2M1R2S	E1M1R1T	E2M1R1T	E1M1R2T	E2M1R2T
---------	---------	---------	---------	---------	---------	---------	---------

**Card 14.** This card is read if IFLAG = 2. It is optional but must be included if Card 12 is included..

E1M2R1S	E2M2R1S	E1M2R1T	E2M2R1T				
---------	---------	---------	---------	--	--	--	--

**Card 15.** This card is read if IUNLD = 2 and IFLAG = 0 or 1. It is optional.

KTS	KTT	KRS	KRT				
-----	-----	-----	-----	--	--	--	--

### Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	KT	KR	IUNLD	OFFSET	DAMPF	IFLAG
Type	A	F	F	F	I	F	F	I

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified.
RO	Mass density; see also volume in *SECTION_BEAM definition.
KT	Translational stiffness along local <i>r</i> -axis for IUNLD = 2.0. However, if IFLAG = 2, then it is the translational stiffness for unloading along the local <i>r</i> -axis. If left blank, a value calculated by LS-DYNA will be used.
KR	Rotational stiffness along local <i>r</i> -axis for IUNLD = 2.0. However, if IFLAG = 2, then KR is the rotational stiffness for unloading along the local <i>r</i> -axis. If left blank, a value calculated by LS-DYNA will be used.
IUNLD	Unloading option (see <a href="#">Figure M119-1</a> ):  EQ.0.0: Loading and unloading follow loading curve EQ.1.0: Loading follows loading curve, unloading follows unloading curve. The unloading curve ID if undefined is taken as the loading curve. EQ.2.0: Loading follows loading curve, unloading follows unloading stiffness, KT or KR, to the unloading curve. The loading and unloading curves may only intersect at the origin of the axes. EQ.3.0: Quadratic unloading from peak displacement value to a permanent offset.
OFFSET	Offset factor between 0.0 and 1.0 to determine permanent set upon unloading if the IUNLD = 3.0. The permanent sets in compression and tension are equal to the product of this offset value and the maximum compressive and tensile displacements, respectively.
DAMPF	Damping factor for stability. Values in the neighborhood of unity are recommended. This damping factor is properly scaled to eliminate time step size dependency. Also, it is active if and only if the local stiffness is defined.
IFLAG	Formulation flag:  EQ.0: Displacement formulation which is used in all other models EQ.1: Linear strain formulation. The displacements and

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
		velocities are divided by the initial length of the beam.						
		EQ.2: A displacement formulation to simulate the buckling behavior of crushable frames						
Card 2	1	2	3	4	5	6	7	8
Variable	LCIDTR	LCIDTS	LCIDTT	LCIDRR	LCIDRS	LCIDRT		
Type	I	I	I	I	I	I		

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
LCIDTR		Load curve ID defining translational force resultant along local <i>r</i> -axis as a function of relative translational displacement. If zero, no stiffness related forces are generated for this degree of freedom. The loading curves must be defined from the most negative displacement to the most positive displacement. The force does not need to increase monotonically. The curves in this input are linearly extrapolated when the displacement range falls outside the curve definition.						
LCIDTS		Load curve ID defining translational force resultant along local <i>s</i> -axis as a function of relative translational displacement (IFLAG = 0 or 1 only).						
LCIDTT		Load curve ID defining translational force resultant along local <i>t</i> -axis as a function of relative translational displacement (IFLAG = 0 or 1 only).						
LCIDRR		Load curve for rotational moment resultant about the local <i>r</i> -axis:  IFLAG.NE.2: Load curve ID defining rotational moment resultant about local <i>r</i> -axis as a function of relative rotational displacement  IFLAG.EQ.2: Load curve ID defining rotational moment resultant about local <i>r</i> -axis as a function of relative rotational displacement at node 2						
LCIDRS		Load curve for rotational moment resultant about local <i>s</i> -axis:  IFLAG.NE.2: Load curve ID defining rotational moment resultant about local <i>s</i> -axis as a function of relative						

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	rotational displacement IFLAG.EQ.2: Load curve ID defining rotational moment resultant about local <i>s</i> -axis as a function of relative rotational displacement at node 2
LCIDRT	Load curve for rotational moment resultant about local <i>t</i> -axis: IFLAG.NE.2: Load curve ID defining rotational moment resultant about local <i>t</i> -axis as a function of relative rotational displacement IFLAG.EQ.2: Load curve ID defining rotational moment resultant about local <i>t</i> -axis as a function of relative rotational displacement at node 2

Card 3	1	2	3	4	5	6	7	8
Variable	LCIDTUR	LCIDTUS	LCIDTUT	LCIDRUR	LCIDRUS	LCIDRUT		
Type	I	I	I	I	I	I		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCIDTUR	Load curve ID defining translational force resultant along local <i>r</i> -axis as a function of relative translational displacement during unloading. The force values defined by this curve must increase monotonically from the most negative displacement to the most positive displacement. For IUNLD = 1.0, the slope of this curve must equal or exceed the loading curve for stability reasons. This is not the case for IUNLD = 2.0. For loading and unloading to follow the same path simply set LCIDTUR = LCIDTR. For options IUNLD = 0.0 or 3.0 the unloading curve is not required. For IUNLD = 2.0, if LCIDTUR is left blank or zero, the default is to use the same curve for unloading as for loading.
LCIDTUS	Load curve ID defining translational force resultant along local <i>s</i> -axis as a function of relative translational displacement during unloading (IFLAG = 0 or 1 only).
LCIDTUT	Load curve ID defining translational force resultant along local <i>t</i> -axis as a function of relative translational displacement during unloading (IFLAG = 0 or 1 only).

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
LCIDRUR	Load curve ID defining rotational moment resultant about local <i>r</i> -axis as a function of relative rotational displacement during unloading.							
LCIDRUS	Load curve for rotational moment resultant about local <i>s</i> -axis:  IFLAG.NE.2: Load curve ID defining rotational moment resultant about local <i>s</i> -axis as a function of relative rotational displacement during unloading  IFLAG.EQ.2: Load curve ID defining rotational moment resultant about local <i>s</i> -axis as a function of relative rotational displacement during unloading at node 2							
LCIDRUT	Load curve ID defining rotational moment resultant about local <i>t</i> -axis:  IFLAG.NE.2: Load curve ID defining rotational moment resultant about local <i>t</i> -axis as a function of relative rotational displacement during unloading. If zero, no viscous forces are generated for this degree of freedom  IFLAG.EQ.2: Load curve ID defining rotational moment resultant about local <i>t</i> -axis as a function of relative rotational displacement during unloading at node 2							

Card 4	1	2	3	4	5	6	7	8
Variable	LCIDTDR	LCIDTDS	LCIDTDT	LCIDRDR	LCIDRDS	LCIDRDT		
Type	I	I	I	I	I	I		

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
LCIDTDR	Load curve ID defining translational damping force resultant along local <i>r</i> -axis as a function of relative translational velocity.							
LCIDTDS	Load curve ID defining translational damping force resultant along local <i>s</i> -axis as a function relative translational velocity.							
LCIDTDT	Load curve ID defining translational damping force resultant along local <i>t</i> -axis as a function of relative translational velocity.							

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCIDRDR	Load curve ID defining rotational damping moment resultant about local <i>r</i> -axis as a function of relative rotational velocity.
LCIDRDS	Load curve ID defining rotational damping moment resultant about local <i>s</i> -axis as a function of relative rotational velocity.
LCIDRDT	Load curve ID defining rotational damping moment resultant about local <i>t</i> -axis as a function of relative rotational velocity.

Card 5	1	2	3	4	5	6	7	8
Variable	LCIDTER	LCIDTES	LCIDTET	LCIDRER	LCIDRES	LCIDRET		
Type	I	I	I	I	I	I		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCIDTER	Load curve ID defining translational damping force scale factor as a function of relative displacement in local <i>r</i> -direction.
LCIDTES	Load curve ID defining translational damping force scale factor as a function of relative displacement in local <i>s</i> -direction.
LCIDTET	Load curve ID defining translational damping force scale factor as a function of relative displacement in local <i>t</i> -direction.
LCIDRER	Load curve ID defining rotational damping moment resultant scale factor as a function of relative displacement in local <i>r</i> -rotation.
LCIDRES	Load curve ID defining rotational damping moment resultant scale factor as a function of relative displacement in local <i>s</i> -rotation.
LCIDRET	Load curve ID defining rotational damping moment resultant scale factor as a function of relative displacement in local <i>t</i> -rotation.

Card 6	1	2	3	4	5	6	7	8
Variable	UTFAILR	UTFAILS	UTFAILT	WTFAILR	WTFAILS	WTFAILT	FCRIT	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
UTFAILR	Optional, translational displacement at failure in tension. If zero, the corresponding displacement, $u_r$ , is not considered in the failure calculation.							
UTFAILS	Optional, translational displacement at failure in tension. If zero, the corresponding displacement, $u_s$ , is not considered in the failure calculation.							
UTFAILT	Optional, translational displacement at failure in tension. If zero, the corresponding displacement, $u_t$ , is not considered in the failure calculation.							
WTFAILR	Optional, rotational displacement at failure in tension. If zero, the corresponding rotation, $\theta_r$ , is not considered in the failure calculation.							
WTFAILS	Optional, rotational displacement at failure in tension. If zero, the corresponding rotation, $\theta_s$ , is not considered in the failure calculation.							
WTFAILT	Optional rotational displacement at failure in tension. If zero, the corresponding rotation, $\theta_t$ , is not considered in the failure calculation.							
FCRIT	Failure criterion (see <a href="#">Remark 1</a> ):  EQ.0.0: Two separate criteria, one for negative displacements and rotations, another for positive displacements and rotations  EQ.1.0: One criterion that considers both positive and negative displacements and rotations							

Card 7	1	2	3	4	5	6	7	8
Variable	UCFAILR	UCFAILS	UCFAILT	WCFAILR	WCFAILS	WCFAILT		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
UCFAILR	Optional, translational displacement at failure in compression. If zero, the corresponding displacement, $u_r$ , is not considered in the							

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
	failure calculation. Define as a positive number.
UCFAILS	Optional, translational displacement at failure in compression. If zero, the corresponding displacement, $u_s$ , is not considered in the failure calculation. Define as a positive number.
UCFAILT	Optional, translational displacement at failure in compression. If zero, the corresponding displacement, $u_t$ , is not considered in the failure calculation. Define as a positive number.
WCFAILR	Optional, rotational displacement at failure in compression. If zero, the corresponding rotation, $\theta_r$ , is not considered in the failure calculation. Define as a positive number.
WCFAILS	Optional, rotational displacement at failure in compression. If zero, the corresponding rotation, $\theta_s$ , is not considered in the failure calculation. Define as a positive number.
WCFAILT	Optional, rotational displacement at failure in compression. If zero, the corresponding rotation, $\theta_t$ , is not considered in the failure calculation. Define as a positive number.

Card 8	1	2	3	4	5	6	7	8
Variable	IUR	IUS	IUT	IWR	IWS	IWT		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
IUR	Initial translational displacement along local $r$ -axis.
IUS	Initial translational displacement along local $s$ -axis.
IUT	Initial translational displacement along local $t$ -axis.
IWR	Initial rotational displacement about the local $r$ -axis.
IWS	Initial rotational displacement about the local $s$ -axis.
IWT	Initial rotational displacement about the local $t$ -axis.

**Loading Rotational Moment Card.** This card is read if IFLAG = 2. It is optional. If it is included, Cards 10 and 11 must be included.

Card 9	1	2	3	4	5	6	7	8
Variable	LM1R1S	LM1R2S	LM1R1T	LM1R2T	LM2R1S	LM2R1T		
Type	I	I	I	I	I	I		

VARIABLE	DESCRIPTION
LM1R1S	Load curve ID for loading defining rotational moment resultant at node 1 about local <i>s</i> -axis as a function of relative rotational displacement at node 1.
LM1R2S	Load curve ID defining rotational moment resultant at node 1 about local <i>s</i> -axis as a function of relative rotational displacement at node 2.
LM1R1T	Load curve ID defining rotational moment resultant at node 1 about local <i>t</i> -axis as a function of relative rotational displacement at node 1.
LM1R2T	Load curve ID defining rotational moment resultant at node 1 about local <i>t</i> -axis as a function of relative rotational displacement at node 2.
LM2R1S	Load curve ID defining rotational moment resultant at node 2 about local <i>s</i> -axis as a function of relative rotational displacement at node 1.
LM2R1T	Load curve ID defining rotational moment resultant at node 2 about local <i>t</i> -axis as a function of relative rotational displacement at node 1.

**Unloading Rotational Moment Card.** This card is read if IFLAG = 2. It must be included if Card 9 is included.

Card 10	1	2	3	4	5	6	7	8
Variable	LUM1R1S	LUM1R2S	LUM1R1T	LUM1R2T	LUM2R1S	LUM2R1T		
Type	I	I	I	I	I	I		

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
LUM1R1S	Load curve ID for unloading defining rotational moment resultant at node 1 about local <i>s</i> -axis as a function of relative rotational displacement at node 1
LUM1R2S	Load curve ID for unloading defining rotational moment resultant at node 1 about local <i>s</i> -axis as a function of relative rotational displacement at node 2
LUM1R1T	Load curve ID for unloading defining rotational moment resultant at node 1 about local <i>t</i> -axis as a function of relative rotational displacement at node 1
LUM1R2T	Load curve ID for unloading defining rotational moment resultant at node 1 about local <i>t</i> -axis as a function of relative rotational displacement at node 2
LUM2R1S	Load curve ID for unloading defining rotational moment resultant at node 2 about local <i>s</i> -axis as a function of relative rotational displacement at node 1
LUM2R1T	Load curve ID for unloading defining rotational moment resultant at node 2 about local <i>t</i> -axis as a function of relative rotational displacement at node 1

**Unload Stiffness for Bending Moment Card.** This card is read if IFLAG = 2. It must be included if Card 9 is included.

Card 11	1	2	3	4	5	6	7	8
Variable	KUM1R1S	KUM1R2S	KUM1R1T	KUM1R2T	KUM2R1S	KUM2R1T	KUM2R2S	KUM2R2T
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
KUM1R1S	Optional unload stiffness for bending moment about local <i>s</i> -axis at node 1 due to relative rotation at node 1. If left blank, LS-DYNA will calculate this value.
KUM1R2S	Optional unload stiffness for bending moment about local <i>s</i> -axis at node 1 due to relative rotation at node 2. If left blank, LS-DYNA will calculate this value.

VARIABLE	DESCRIPTION
KUM1R1T	Optional unload stiffness for bending moment about local <i>t</i> -axis at node 1 due to relative rotation at node 1. If left blank, LS-DYNA will calculate this value.
KUM1R2T	Optional unload stiffness for bending moment about local <i>t</i> -axis at node 1 due to relative rotation at node 2. If left blank, LS-DYNA will calculate this value.
KUM2R1S	Optional unload stiffness for bending moment about local <i>s</i> -axis at node 2 due to relative rotation at node 1. If left blank, LS-DYNA will calculate this value.
KUM2R1T	Optional unload stiffness for bending moment about local <i>t</i> -axis at node 2 due to relative rotation at node 1. If left blank, LS-DYNA will calculate this value. .
KUM2R2S	Optional unload stiffness for bending moment about local <i>s</i> -axis at node 2 due to relative rotation at node 2. If left blank, LS-DYNA will calculate this value.
KUM2R2T	Optional unload stiffness for bending moment about local <i>t</i> -axis at node 2 due to relative rotation at node 2. If left blank, LS-DYNA will calculate this value.

**Elastic limit of loading curves.** This card is read if IFLAG = 2. It is optional. If not input, the values derived by LS-DYNA based on the related curves will be used. If it is included, Cards 13 and 14 must be included.

Card 12	1	2	3	4	5	6	7	8
Variable	E1TR	E2TR	E1RR	E2RR	E1RS	E2RS	E1RT	E2RT
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
E1TR	Negative, compressive, elastic limit of curve LCIDTR
E2TR	Positive, tensile, elastic limit of curve LCIDTR
E1RR	Negative elastic limit of curve LCIDRR
E2RR	Positive elastic limit of curve LCIDRR

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
E1RR	Negative elastic limit of curve LCIDRS
E2RR	Positive elastic limit of curve LCIDRS
E1RT	Negative elastic limit of curve LCIDRT
E2RT	Positive elastic limit of curve LCIDRT

**Elastic limit of loading curves.** This card is read if IFLAG = 2. If not input, the values derived by LS-DYNA based on the related curves will be used. It must be included if Card 12 is included.

Card 13	1	2	3	4	5	6	7	8
Variable	E1M1R1S	E2M1R1S	E1M1R2S	E2M1R2S	E1M1R1T	E2M1R1T	E1M1R2T	E2M1R2T
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
E1M1R1S	Negative, tensile, elastic limit of curve LM1R1S
E2M1R1S	Positive, tensile, elastic limit of curve LM1R1S
E1M1R2S	Negative elastic limit of curve LM1R2S
E2M1R2S	Positive elastic limit of curve LM1R2S
E1M1R1T	Negative, tensile, elastic limit of curve LM1R1T
E2M1R1T	Positive, tensile, elastic limit of curve LM1R1T
E1M1R2T	Negative elastic limit of curve LM1R2T
E2M1R2T	Positive elastic limit of curve LM1R2T

**Elastic limit of loading curves.** This card is read if IFLAG = 2. It is optional. If not input, the values derived by LS-DYNA based on the related curves will be used. It must be included if Card 12 is included.

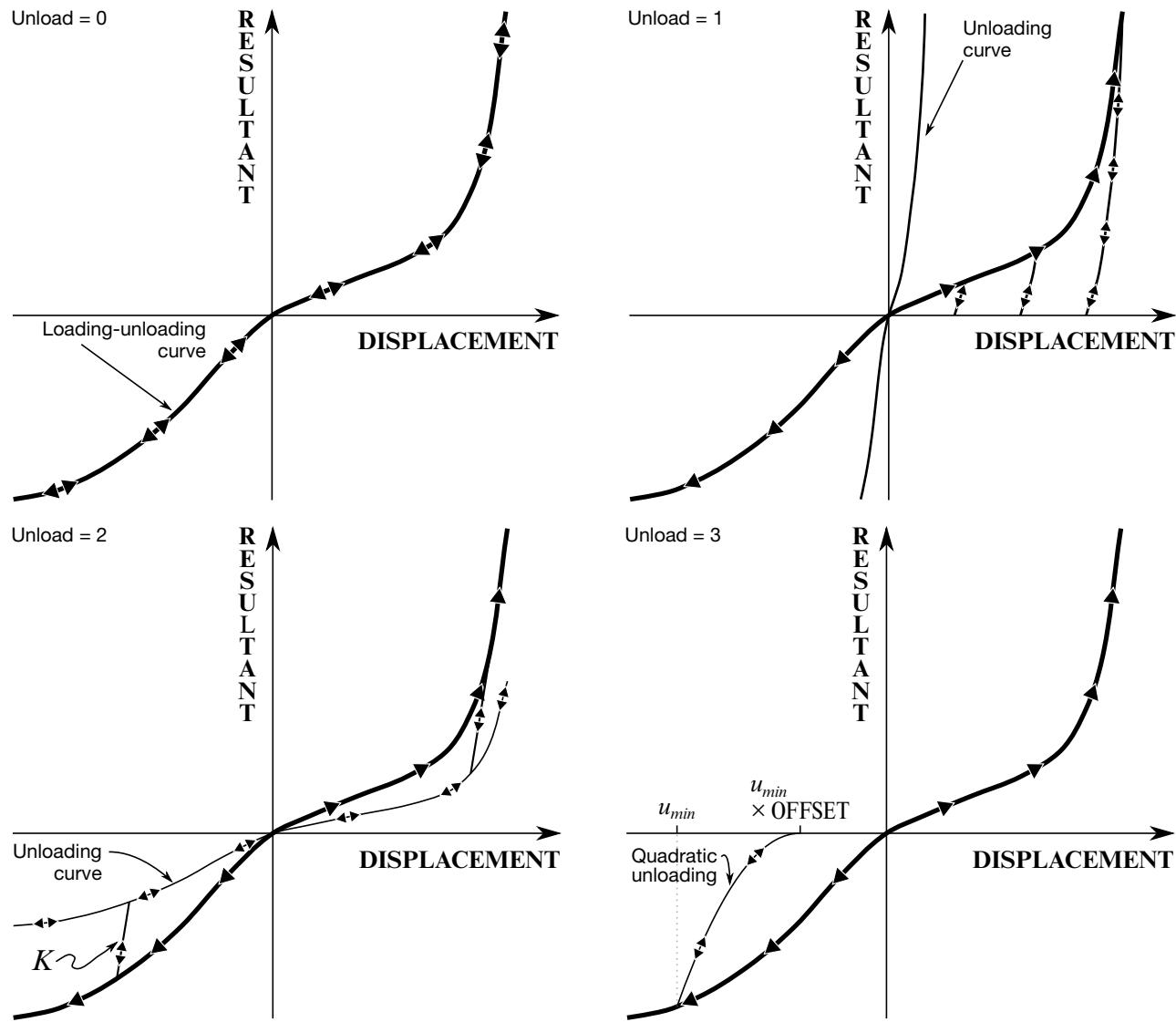
Card 14	1	2	3	4	5	6	7	8
Variable	E1M2R1S	E2M2R1S	E1M2R1T	E2M2R1T				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
E1M2R1S	Negative, tensile, elastic limit of curve LM2R1S
E2M2R1S	Positive, tensile, elastic limit of curve LM2R1S
E1M2R1T	Negative elastic limit of curve LM2R1T
E2M2R1T	Positive elastic limit of curve LM2R1T

**Unloading stiffness along local-s and local-t.** This card is read if IUNLD = 2 and IFLAG = 0 or 1. It is optional. If not input, the values along local *r*-axis, KT and KR, will be used for all axes.

Card 15	1	2	3	4	5	6	7	8
Variable	KTS	KT <sub>T</sub>	KRS	KRT				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
KTS	Translational stiffness along local <i>s</i> -axis for IUNLD = 2.0.
KT <sub>T</sub>	Translational stiffness along local <i>t</i> -axis for IUNLD = 2.0
KRS	Rotational stiffness along local <i>s</i> -axis for IUNLD = 2.0
KRT	Rotational stiffness along local <i>t</i> -axis for IUNLD = 2.0



**Figure M119-1.** Load and unloading behavior.

#### Remarks:

1. **Failure criterion.** When the catastrophic failure criterion is satisfied, the discrete element is deleted. Failure for this material depends directly on the displacement resultants. The failure criterion depends on the value of FCRIT.

If FCRIT = 0.0, failure occurs if either of the following inequalities are satisfied:

$$A^t - 1 \geq 0$$

$$A^c - 1 \geq 0$$

where

$$A^t = \left[ \frac{\max(0, u_r)}{u_r^{\text{tfail}}} \right]^2 + \left[ \frac{\max(0, u_s)}{u_s^{\text{tfail}}} \right]^2 + \left[ \frac{\max(0, u_t)}{u_t^{\text{tfail}}} \right]^2 + \left[ \frac{\max(0, \theta_r)}{\theta_r^{\text{tfail}}} \right]^2 + \left[ \frac{\max(0, \theta_s)}{\theta_s^{\text{tfail}}} \right]^2 + \left[ \frac{\max(0, \theta_t)}{\theta_t^{\text{tfail}}} \right]^2$$

$$A_c = \left[ \frac{\max(0, u_r)}{u_r^{\text{cfail}}} \right]^2 + \left[ \frac{\max(0, u_s)}{u_s^{\text{cfail}}} \right]^2 + \left[ \frac{\max(0, u_t)}{u_t^{\text{cfail}}} \right]^2 + \left[ \frac{\max(0, \theta_r)}{\theta_r^{\text{cfail}}} \right]^2 + \left[ \frac{\max(0, \theta_s)}{\theta_s^{\text{cfail}}} \right]^2 + \left[ \frac{\max(0, \theta_t)}{\theta_t^{\text{cfail}}} \right]^2$$

Positive (tension) values of displacement and rotation are considered in the first criterion and negative (compression) values in the second. Either the tension failure or the compression failure or both may be used. If any of the input failure displacements and rotations (UTFAILR etc) are left as zero, the corresponding terms will be omitted from the equations for  $A^t$  and  $A^c$  above.

If FCRIT = 1.0, then a single criterion is used:

$$A^t + A^c - 1 \geq 0$$

Thus, the combined effect of all the displacements and rotations is considered, be they positive or negative.

2. **Force.** There are two formulations for calculating the force. The first is the standard displacement formulation, where, for example, the force in a linear spring is

$$F = -K\Delta\ell$$

for a change in length of the beam of  $\Delta\ell$ . The second formulation is based on the linear strain, giving a force of

$$F = -K \frac{\Delta\ell}{\ell_0}$$

for a beam with an initial length of  $\ell_0$ . This option is useful when there are springs of different lengths but otherwise similar construction since it automatically reduces the stiffness of the spring as the length increases, allowing an entire family of springs to be modeled with a single material. Note that all the displacement and velocity components are divided by the initial length, and therefore the scaling applies to the damping and rotational stiffness.

3. **Rotational displacement.** Rotational displacement is measured in radians.

# \*MAT\_120

\*MAT\_GURSON

## \*MAT\_GURSON

This is Material Type 120. This is the Gurson dilatational-plastic model. This model is available for shell and solid elements. A detailed description of this model can be found in the following references: Gurson [1975, 1977], Chu and Needleman [1980] and Tvergaard and Needleman [1984]. The implementation in LS-DYNA is based on the implementation of Feucht [1998] and Faßnacht [1999], which was recoded at LSTC. Strain rate dependency can be defined using a table (see LCSS on Card 6).

### Card Summary:

**Card 1.** This card is required.

MID	R0	E	PR	SIGY	N	Q1	Q2
-----	----	---	----	------	---	----	----

**Card 2.** This card is required.

FC	F0	EN	SN	FN	ETAN	ATYP	FF0
----	----	----	----	----	------	------	-----

**Card 3.** This card is required.

EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
------	------	------	------	------	------	------	------

**Card 4.** This card is required.

ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
-----	-----	-----	-----	-----	-----	-----	-----

**Card 5.** This card is required.

L1	L2	L3	L4	FF1	FF2	FF3	FF4
----	----	----	----	-----	-----	-----	-----

**Card 6.** This card is required.

LCSS	LCFF	NUMINT	LCFO	LCFC	LCFN	VGTYP	DEXP
------	------	--------	------	------	------	-------	------

### Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	N	Q1	Q2
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified.
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
SIGY	Yield stress
N	Exponent for Power law (default = 0.0). This value is only used if ATYP = 1 and LCSS = 0 (see Cards 2 and 6).
Q1	Gurson flow function parameter $q_1$
Q2	Gurson flow function parameter $q_2$

Card 2	1	2	3	4	5	6	7	8
Variable	FC	F0	EN	SN	FN	ETAN	ATYP	FF0
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION
FC	Critical void volume fraction $f_c$ where voids begin to aggregate. This value is only used if LCFC = 0 (see Card 6).
F0	Initial void volume fraction, $f_0$ . This value is only used if LCF0 = 0 (see Card 6).
EN	Mean nucleation strain $\varepsilon_N$ : <ul style="list-style-type: none"> <li>GT.0.0: Constant value</li> <li>LT.0.0: Load curve ID = (-EN) which defines mean nucleation strain <math>\varepsilon_N</math> as a function of element length</li> </ul>
SN	Standard deviation $s_N$ of the normal distribution of $\varepsilon_N$ : <ul style="list-style-type: none"> <li>GT.0.0: Constant value</li> </ul>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	LT.0.0: Load curve ID = (-SN) which defines standard deviation $s_N$ of the normal distribution of $\varepsilon_N$ as a function of element length
FN	Void volume fraction of nucleating particles $f_N$ . This value is only used if LCFN = 0 (see Card 6).
ETAN	Hardening modulus. This value is only used if ATYP = 2 and LCSS = 0 (see Card 6).
ATYP	Type of hardening: EQ.0.0: Ideal plastic $\sigma_Y = \text{SIGY}$ EQ.1.0: Power law $\sigma_Y = \text{SIGY} \times \left( \frac{\varepsilon^p + \text{SIGY}/E}{\text{SIGY}/E} \right)^{1/N}$ EQ.2.0: Linear hardening $\sigma_Y = \text{SIGY} + \frac{E \times \text{ETAN}}{E - \text{ETAN}} \varepsilon^p$ EQ.3.0: 8 points curve
FF0	Failure void volume fraction $f_F$ . This value is only used if no curve is given by (L1, FF1) – (L4, FF4) and LCFF = 0 (see Cards 5 and 6).

Card 3	1	2	3	4	5	6	7	8
Variable	EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
EPS1 - EPS8	Effective plastic strain values. The first point must be zero corresponding to the initial yield stress. At least 2 points should be defined. These values are used if ATYP = 3 and LCSS = 0 (see Cards 2 and 6).							
ES1 - ES8	Corresponding yield stress values to EPS1 – EPS8. These values are used if ATYP = 3 and LCSS = 0 (see Cards 2 and 6).							

Card 5	1	2	3	4	5	6	7	8
Variable	L1	L2	L3	L4	FF1	FF2	FF3	FF4
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
L1 - L4	Element length values. These values are only used if LCFF = 0 (see Card 6).							
FF1 - FF4	Corresponding failure void volume fraction. These values are only used if LCFF = 0 (see Card 6).							

Card 6	1	2	3	4	5	6	7	8
Variable	LCSS	LCFF	NUMINT	LCFO	LCFC	LCFN	VGTYp	DEXP
Type	I	I	I	I	I	I	F	F
Default	0	0	1	0	0	0	0	3.0

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
LCSS	Load curve ID or Table ID. If defined, ATYP, EPS1 - EPS8 and ES1 - ES8 are ignored.							
	<b>Load Curve.</b> When LCSS is a load curve ID, it is taken as defining effective stress as a function of effective plastic strain.							

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	<b>Tabular Data.</b> The table ID defines for each strain rate value a load curve ID giving the effective stress as a function effective plastic strain for that rate; see <a href="#">Figure M24-1</a> and *MAT_024. When the strain rate falls below the minimum value, the stress as a function of effective plastic strain curve for the lowest value of strain rate is used.
	<b>Logarithmically Defined Tables.</b> If the <i>first</i> value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate when the lowest strain rate and highest strain rate differ by several orders of magnitude.
LCFF	Load curve ID defining failure void volume fraction, $f_F$ , as a function of element length
NUMINT	Number of integration points which must fail before the element is deleted. This option is available for shells and solids.  LT.0.0:  NUMINT  is percentage of integration points/layers which must fail before shell element fails. For fully integrated shells, a methodology is used where a layer fails if one integration point fails and then the given percentage of layers must fail before the element fails. Only available for shells.
LCF0	Load curve ID defining initial void volume fraction, $f_0$ , as a function of element length
LCFC	Load curve ID defining critical void volume fraction, $f_c$ , as a function of element length
LCFN	Load curve ID defining void volume fraction of nucleating particles, $f_N$ , as a function of element length
VGTYP	Type of void growth behavior:  EQ.0.0: Void growth in case of tension and void contraction in case of compression, but never below $f_0$ (default)  EQ.1.0: Void growth only in case of tension  EQ.2.0: Void growth in case of tension and void contraction in case of compression, even below $f_0$

VARIABLE	DESCRIPTION
DEXP	Exponent value for damage history variable 16

**Remarks:**

The Gurson flow function is defined as:

$$\Phi = \frac{\sigma_M^2}{\sigma_Y^2} + 2q_1 f^* \cosh\left(\frac{3q_2 \sigma_H}{2\sigma_Y}\right) - 1 - (q_1 f^*)^2 = 0 ,$$

where  $\sigma_M$  is the equivalent von Mises stress,  $\sigma_Y$  is the yield stress, and  $\sigma_H$  is the mean hydrostatic stress. The effective void volume fraction is defined as

$$f^*(f) = \begin{cases} f & f \leq f_c \\ f_c + \frac{1/q_1 - f_c}{f_F - f_c} (f - f_c) & f > f_c \end{cases}$$

The growth of void volume fraction is defined as

$$\dot{f} = \dot{f}_G + \dot{f}_N ,$$

where the growth of existing voids is defined as

$$\dot{f}_G = (1-f) \dot{\varepsilon}_{kk}^p$$

and nucleation of new voids is defined as

$$\dot{f}_N = A \dot{\varepsilon}_p$$

with function  $A$

$$A = \frac{f_N}{S_N \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\varepsilon_p - \varepsilon_N}{S_N}\right)^2\right] .$$

Voids are nucleated only in tension.

**History Variables:**

Shell	Solid	Description
1	1	Void volume fraction
4	2	Triaxiality variable $\sigma_H/\sigma_M$
5	3	Effective strain rate
6	4	Growth of voids
7	5	Nucleation of voids

**\*MAT\_120****\*MAT\_GURSON**

Shell	Solid	Description
11	11	Dimensionless material damage value = $\begin{cases} \frac{(f-f_0)}{(f_c-f_0)} & f \leq f_c \\ 1 + \frac{(f-f_c)}{(f_F-f_c)} & f > f_c \end{cases}$
13	13	Deviatoric part of microscopic plastic strain
14	14	Volumetric part of macroscopic plastic strain
16	16	Dimensionless material damage value = $\left(\frac{f-f_0}{f_F-f_0}\right)^{1/\text{DEXP}}$

**\*MAT\_GURSON\_JC****\*MAT\_120\_JC****\*MAT\_GURSON\_JC**

This is an enhancement of Material Type 120. This is the Gurson model with the additional Johnson-Cook failure criterion (see Card 5). This model is available for shell and solid elements. Strain rate dependency can be defined using a table (see LCSS). An extension for void growth under shear-dominated states and for Johnson-Cook damage evolution is optional.

**Card Summary:**

**Card 1.** This card is required.

MID	R0	E	PR	SIGY	N	Q1	Q2
-----	----	---	----	------	---	----	----

**Card 2.** This card is required.

FC	F0	EN	SN	FN	ETAN	ATYP	FF0
----	----	----	----	----	------	------	-----

**Card 3.** This card is required.

EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
------	------	------	------	------	------	------	------

**Card 4.** This card is required.

SIG1	SIG2	SIG3	SIG4	SIG5	SIG6	SIG7	SIG8
------	------	------	------	------	------	------	------

**Card 5.** This card is required.

LCDAM	L1	L2	D1	D2	D3	D4	LCJC
-------	----	----	----	----	----	----	------

**Card 6.** This card is required.

LCSS	LCFF	NUMINT	LCFO	LCFC	LCFN	VGTYP	DEXP
------	------	--------	------	------	------	-------	------

**Card 7.** This card is optional.

KW	BETA	M					
----	------	---	--	--	--	--	--

# \*MAT\_120\_JC

# \*MAT\_GURSON\_JC

## Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	N	Q1	Q2
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	none	none

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
SIGY	Yield stress
N	Exponent for power law. This field is only used if ATYP = 1 and LCSS = 0 (see Cards 2 and 6).
Q1	Gurson flow function parameter $q_1$
Q2	Gurson flow function parameter $q_2$

Card 2	1	2	3	4	5	6	7	8
Variable	FC	F0	EN	SN	FN	ETAN	ATYP	FF0
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION
FC	Critical void volume fraction, $f_c$ , where voids begin to aggregate

<b>VARIABLE</b>	<b>DESCRIPTION</b>
F0	Initial void volume fraction, $f_0$ . This field is only used if LCF0 = 0.
EN	Mean nucleation strain, $\varepsilon_N$ : GT.0.0: Constant value LT.0.0: Load curve ID = (-EN) which defines mean nucleation strain, $\varepsilon_N$ , as a function of element length
SN	Standard deviation, $s_N$ , of the normal distribution of $\varepsilon_N$ : GT.0.0: Constant value LT.0.0: Load curve ID = (-SN) which defines standard deviation, $s_N$ , of the normal distribution of $\varepsilon_N$ as a function of element length
FN	Void volume fraction of nucleating particles, $f_N$ . This field is only used if LCFN = 0.
ETAN	Hardening modulus. This field is only used if ATYP = 2 and LCSS = 0 (see Card 6).
ATYP	Type of hardening: EQ.0.0: Ideal plastic, $\sigma_Y = \text{SIGY}$ EQ.1.0: Power law, $\sigma_Y = \text{SIGY} \times \left( \frac{\varepsilon^p + \text{SIGY}/E}{\text{SIGY}/E} \right)^{1/N}$ EQ.2.0: Linear hardening, $\sigma_Y = \text{SIGY} + \frac{E \times \text{ETAN}}{E - \text{ETAN}} \varepsilon^p$ EQ.3.0: 8 points curve
FF0	Failure void volume fraction, $f_F$ . This field is only used if LCFF = 0 (see Card 6).

Card 3	1	2	3	4	5	6	7	8
Variable	EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
Type	F	F	F	F	F	F	F	F

**\*MAT\_120\_JC****\*MAT\_GURSON\_JC**

Card 4	1	2	3	4	5	6	7	8
Variable	SIG1	SIG2	SIG3	SIG4	SIG5	SIG6	SIG7	SIG8
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
EPS1 - EPS8	Effective plastic strain values. The first point must be zero corresponding to the initial yield stress. At least 2 points should be defined. These values are used if ATYP = 3 and LCSS = 0. See Cards 2 and 6.
ES1 - ES8	Corresponding yield stress values to EPS1 – EPS8. These values are used if ATYP = 3 and LCSS = 0. See Cards 2 and 6.

Card 5	1	2	3	4	5	6	7	8
Variable	LCDAM	L1	L2	D1	D2	D3	D4	LCJC
Type	I	F	F	F	F	F	F	I
Default	0	0.0	0.0	0.0	0.0	0.0	0.0	0

VARIABLE	DESCRIPTION
LCDAM	Load curve defining the scaling factor, $\Lambda$ , as a function of element length. It scales the Johnson-Cook failure strain (see remarks). If LCDAM = 0, no scaling is performed.
L1	Lower triaxiality factor defining failure evolution (Johnson-Cook)
L2	Upper triaxiality factor defining failure evolution (Johnson-Cook)
D1 - D4	Johnson-Cook damage parameters
LCJC	Load curve defining the scaling factor for Johnson-Cook failure as a function of triaxiality (see remarks). If LCJC > 0, parameters D1, D2 and D3 are ignored.

Card 6	1	2	3	4	5	6	7	8
Variable	LCSS	LCFF	NUMINT	LCFO	LCFC	LCFN	VGTYP	DEXP
Type	I	I	I	I	I	I	F	F
Default	0	0	1	0	0	0	0.0	3.0

VARIABLE	DESCRIPTION
LCSS	<p>Load curve ID or Table ID. If defined, ATYP, EPS1 - EPS8 and ES1 - ES8 are ignored.</p> <p><b>Load Curve.</b> When LCSS is a load curve ID, it is taken as defining effective stress as a function of effective plastic strain.</p> <p><b>Tabular Data.</b> The table ID defines for each strain rate value a load curve ID giving the effective stress as a function effective plastic strain for that rate; see <a href="#">Figure M24-1</a> and *MAT_024. When the strain rate falls below the minimum value, the stress as a function of effective plastic strain curve for the lowest value of strain rate is used.</p> <p><b>Logarithmically Defined Tables.</b> If the <i>first</i> value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate when the lowest strain rate and highest strain rate differ by several orders of magnitude.</p>
LCFF	Load curve ID defining failure void volume fraction, $f_F$ , as a function of element length
NUMINT	<p>Number of through thickness integration points which must fail before the element is deleted. This option is available for shells and solids.</p> <p><b>LT.0.0:</b> <math> NUMINT </math> is the percentage of integration points/layers which must fail before shell element fails. For fully integrated shells, a methodology is used where a layer fails if one integration point fails and then the given percentage of layers must fail before the element fails. Only available for shells.</p>

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCF0	Load curve ID defining initial void volume fraction, $f_0$ , as a function of element length
LCFC	Load curve ID defining critical void volume fraction, $f_c$ , as a function of element length
LCFN	Load curve ID defining void volume fraction of nucleating particles, $f_N$ , as a function of element length
VGTYP	Type of void growth behavior. EQ.0.0: Void growth in case of tension and void contraction in case of compression, but never below $f_0$ (default) EQ.1.0: Void growth only in case of tension EQ.2.0: Void growth in case of tension and void contraction in case of compression, even below $f_0$
DEXP	Exponent value for damage history variable 16

Optional Card (starting with version 971 release R4)

Card 7	1	2	3	4	5	6	7	8
Variable	KW	BETA	M					
Type	F	F	F					
Default	0.0	0.0	1.0					

<b>VARIABLE</b>	<b>DESCRIPTION</b>
KW	Parameter $k_\omega$ for void growth in shear-dominated states. See remarks.
BETA	Parameter $\beta$ in Lode cosine function. See remarks.
M	Parameter for generalization of Johnson-Cook damage evolution. See remarks.

**Remarks:**

The Gurson flow function is defined as:

$$\Phi = \frac{\sigma_M^2}{\sigma_Y^2} + 2q_1 f^* \cosh\left(\frac{3q_2 \sigma_H}{2\sigma_Y}\right) - 1 - (q_1 f^*)^2 = 0$$

where  $\sigma_M$  is the equivalent von Mises stress,  $\sigma_Y$  is the yield stress, and  $\sigma_H$  is the mean hydrostatic stress. The effective void volume fraction is defined as

$$f^*(f) = \begin{cases} f & f \leq f_c \\ f_c + \frac{1/q_1 - f_c}{f_F - f_c} (f - f_c) & f > f_c \end{cases}$$

The growth of void volume fraction is defined as

$$\dot{f} = \dot{f}_G + \dot{f}_N$$

where the growth of existing voids is defined as

$$\dot{f}_G = (1-f) \dot{\varepsilon}_{kk}^p + k_\omega \omega(\sigma) f (1-f) \dot{\varepsilon}_M^{pl} \frac{\sigma_Y}{\sigma_M}$$

The second term is an optional extension for shear failure proposed by Nahshon and Hutchinson [2008] with new parameter  $k_\omega$  (= 0 by default), effective plastic strain rate in the matrix  $\dot{\varepsilon}_M^{pl}$ , and Lode cosin function  $\omega(\sigma)$ :

$$\omega(\sigma) = 1 - \xi^2 - \beta \times \xi(1 - \xi), \quad \xi = \cos(3\theta) = \frac{27}{2} \frac{J_3}{\sigma_M^3}$$

with parameter  $\beta$ , Lode angle  $\theta$  and third deviatoric stress invariant  $J_3$ .

Nucleation of new voids is defined as

$$\dot{f}_N = A \dot{\varepsilon}_M^{pl}$$

with function  $A$

$$A = \frac{f_N}{S_N \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\varepsilon_M^{pl} - \varepsilon_N}{S_N}\right)^2\right].$$

Voids are nucleated only in tension.

The Johnson-Cook failure criterion is added to this material model. Based on the triaxiality ratio  $\sigma_H/\sigma_M$  failure is calculated as:

$$\sigma_H/\sigma_M > L_1 : \text{Gurson model}$$

$$L_1 \geq \sigma_H/\sigma_M \geq L_2 : \text{Gurson model and Johnson-Cook failure criteria}$$

$$L_2 < \sigma_H/\sigma_M : \text{Gurson model}$$

Johnson-Cook failure strain is defined as

$$\varepsilon_f = \left[ D_1 + D_2 \exp\left(D_3 \frac{\sigma_H}{\sigma_M}\right) \right] (1 + D_4 \ln \dot{\varepsilon}) \Lambda ,$$

where  $D_1, D_2, D_3$  and  $D_4$  are the Johnson-Cook failure parameters and  $\Lambda$  is a function for including mesh-size dependency. An alternative expression can be used, where the first term of the above equation (including  $D_1, D_2$  and  $D_3$ ) is replaced by a general function LCJC which depends on triaxiality

$$\varepsilon_f = \text{LCJC} \times \left( \frac{\sigma_H}{\sigma_M} \right) (1 + D_4 \ln \dot{\varepsilon}) \Lambda .$$

The Johnson-Cook damage parameter  $D_f$  is calculated with the following evolution equation:

$$\dot{D}_f = \frac{\dot{\varepsilon}^{pl}}{\varepsilon_f} \Rightarrow D_f = \sum \frac{\Delta \varepsilon^{pl}}{\varepsilon_f} .$$

where  $\Delta \varepsilon^{pl}$  is the increment in effective plastic strain. The material fails when  $D_f$  reaches 1.0. A more general (non-linear) damage evolution is possible if  $M > 1$  is chosen:

$$\dot{D}_f = \frac{M}{\varepsilon_f} D_f^{\left(\frac{1}{1-pM}\right)} \quad M \geq 1.0$$

### History variables:

Shell	Solid	Description
1	1	Void volume fraction
4	2	Triaxiality variable $\sigma_H/\sigma_M$
5	3	Effective strain rate
6	4	Growth of voids
7	5	Nucleation of voids
8	6	Johnson-Cook failure strain $\varepsilon_f$
9	7	Johnson-Cook damage parameter $D_f$
		Domain variable:
		EQ.0: elastic stress update
0	8	EQ.1: region (a) Gurson
		EQ.2: region (b) Gurson + Johnson-Cook
		EQ.3: region (c) Gurson

Shell	Solid	Description
11	11	Dimensionless material damage value = $\begin{cases} \frac{(f-f_0)}{(f_c-f_0)} & f \leq f_c \\ 1 + \frac{(f-f_c)}{(f_F-f_c)} & f > f_c \end{cases}$
13	13	Deviatoric part of microscopic plastic strain
14	14	Volumetric part of macroscopic plastic strain
16	16	Dimensionless material damage value = $\left(\frac{f-f_0}{f_F-f_0}\right)^{1/\text{DEXP}}$

# **\*MAT\_120\_RCDC**

**\*MAT\_GURSON\_RCDC**

## **\*MAT\_GURSON\_RCDC**

This is an enhancement of Material Type 120. This is the Gurson model with the addition of the Wilkins Rc-Dc [Wilkins et al., 1977] fracture model. This model is available for shell and solid elements. A detailed description of this model can be found in the following references: Gurson [1975, 1977], Chu and Needleman [1980], and Tvergaard and Needleman [1984].

### **Card Summary:**

**Card 1.** This card is required.

MID	R0	E	PR	SIGY	N	Q1	Q2
-----	----	---	----	------	---	----	----

**Card 2.** Description.

FC	F0	EN	SN	FN	ETAN	ATYP	FF0
----	----	----	----	----	------	------	-----

**Card 3.** This card is required.

EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
------	------	------	------	------	------	------	------

**Card 4.** This card is required.

ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
-----	-----	-----	-----	-----	-----	-----	-----

**Card 5.** This card is required.

L1	L2	L3	L4	FF1	FF2	FF3	FF4
----	----	----	----	-----	-----	-----	-----

**Card 6.** This card is required.

LCSS	LCFF	NUMINT					
------	------	--------	--	--	--	--	--

**Card 7.** This card is required.

ALPHA	BETA	GAMMA	D0	B	LAMBDA	DS	L
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	N	Q1	Q2
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	none	none

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
SIGY	Yield stress
N	Exponent for Power law. This field is only used if ATYP = 1 and LCSS = 0. See Cards 2 and 6.
Q1	Gurson flow function parameter $q_1$
Q2	Gurson flow function parameter $q_2$

Card 2	1	2	3	4	5	6	7	8
Variable	FC	F0	EN	SN	FN	ETAN	ATYP	FF0
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION
FC	Critical void volume fraction, $f_c$

**\*MAT\_120\_RCDC****\*MAT\_GURSON\_RCDC**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
F0	Initial void volume fraction, $f_0$
EN	Mean nucleation strain, $\varepsilon_N$
SN	Standard deviation, $S_N$ , of the normal distribution of $\varepsilon_N$
FN	Void volume fraction of nucleating particles
ETAN	Hardening modulus. This field is only used if ATYP = 2 and LC-SS = 0. See Card 6.
ATYP	Type of hardening: EQ.0.0: Ideal plastic, $\sigma_Y = \text{SIGY}$
	EQ.1.0: Power law, $\sigma_Y = \text{SIGY} \times \left( \frac{\varepsilon^p + \text{SIGY}/E}{\text{SIGY}/E} \right)^{1/N}$
	EQ.2.0: Linear hardening, $\sigma_Y = \text{SIGY} + \frac{E \times \text{ETAN}}{E - \text{ETAN}} \varepsilon^p$
	EQ.3.0: 8 points curve
FF0	Failure void volume fraction, $f_F$ . This field is used if no curve is given by the points (L1, FF1) – (L4, FF4) and LCFF = 0. See Cards 5 and 6.

Card 3	1	2	3	4	5	6	7	8
Variable	EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EPS1 - EPS8	Effective plastic strain values. The first point must be zero, corresponding to the initial yield stress. This option is only used if ATYP equals 3. At least 2 points should be defined. These values are used if ATYP = 3 and LCSS = 0. See Cards 2 and 6.
ES1 - ES8	Corresponding yield stress values to EPS1 - EPS8. These values are used if ATYP = 3 and LCSS = 0. See Cards 2 and 6.

Card 5	1	2	3	4	5	6	7	8
Variable	L1	L2	L3	L4	FF1	FF2	FF3	FF4
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
L1 - L4	Element length values. These fields are only used if LCFF = 0.
FF1 - FF4	Corresponding failure void volume fraction. These values are only used if LCFF = 0.

Card 6	1	2	3	4	5	6	7	8
Variable	LCSS	LCFF	NUMINT					
Type	I	I	I					
Default	0	0	1					

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCSS	Load curve ID defining effective stress as a function of effective plastic strain. ATYP is ignored with this option.
LCFF	Load curve ID defining the failure void volume fraction as a function of element length. The values L1 - L4 and FF1 - FF4 are ignored with this option.

**\*MAT\_120\_RCDC****\*MAT\_GURSON\_RCDC**

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
NUMINT		Number of through-thickness integration points that must fail before the element is deleted						
Card 7	1	2	3	4	5	6	7	8
Variable	ALPHA	BETA	GAMMA	D0	B	LAMBDA	DS	L
Type	F	F	F	F	F	F	F	F
Default	0	0	0	0	0	0	0	0

<b>VARIABLE</b>		<b>DESCRIPTION</b>
	ALPHA	Parameter $\alpha$ for the Rc-Dc model
	BETA	Parameter $\beta$ for the Rc-Dc model
	GAMMA	Parameter $\gamma$ for the Rc-Dc model
	D0	Parameter $D_0$ for the Rc-Dc model
	B	Parameter $b$ for the Rc-Dc model
	LAMBDA	Parameter $\lambda$ for the Rc-Dc model
	DS	Parameter $D_s$ for the Rc-Dc model
	L	Characteristic element length for this material

**Remarks:**

The Gurson flow function is defined as:

$$\Phi = \frac{\sigma_M^2}{\sigma_Y^2} + 2q_1 f^* \cosh \left( \frac{3q_2 \sigma_H}{2\sigma_Y} \right) - 1 - (q_1 f^*)^2 = 0 ,$$

where  $\sigma_M$  is the equivalent von Mises stress,  $\sigma_Y$  is the yield stress, and  $\sigma_H$  is the mean hydrostatic stress. The effective void volume fraction is defined as

$$f^*(f) = \begin{cases} f & f \leq f_c \\ f_c + \frac{1/q_1 - f_c}{f_F - f_c} (f - f_c) & f > f_c \end{cases}$$

The growth of the void volume fraction is defined as

$$\dot{f} = \dot{f}_G + \dot{f}_N ,$$

where the growth of existing voids is given as:

$$\dot{f}_G = (1 - f) \dot{\varepsilon}_{kk}^p ,$$

and nucleation of new voids as:

$$\dot{f}_N = A \dot{\varepsilon}_p$$

in which  $A$  is defined as

$$A = \frac{f_N}{S_N \sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\varepsilon_p - \varepsilon_N}{S_N}\right)^2\right) .$$

The Rc-Dc model is described in the following. The damage  $D$  is given by

$$D = \int \omega_1 \omega_2 d\varepsilon^p ,$$

where  $\varepsilon^p$  is the equivalent plastic strain,

$$\omega_1 = \left(\frac{1}{1 - \gamma \sigma_m}\right)^\alpha$$

is a triaxial stress weighting term, and

$$\omega_2 = (2 - A_D)^\beta$$

is an asymmetric strain weighting term. In the above  $\sigma_m$  is the mean stress and

$$A_D = \max\left(\frac{S_2}{S_3}, \frac{S_2}{S_1}\right)$$

Fracture is initiated when the accumulation of damage is

$$\frac{D}{D_c} > 1 ,$$

where  $D_c$  is the critical damage given by

$$D_c = D_0(1 + b|\nabla D|^\lambda) .$$

A fracture fraction

$$F = \frac{D - D_c}{D_s}$$

defines the degradations of the material by the Rc-Dc model.

The characteristic element length is used in the calculation of  $\nabla D$ . This factor is calculated only for elements with a smaller length than this value.

**\*MAT\_121****\*MAT\_GENERAL\_NONLINEAR\_1DOF\_DISCRETE\_BEAM****\*MAT\_GENERAL\_NONLINEAR\_1DOF\_DISCRETE\_BEAM**

This is Material Type 121. This is a very general spring and damper model. This beam is based on the MAT\_SPRING\_GENERAL\_NONLINEAR option and is a one-dimensional version of the 6DOF\_DISCRETE\_BEAM above. The forces generated by this model act along a line between the two connected nodal points. Additional unloading options have been included.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	K	IUNLD	OFFSET	DAMPF		
Type	A	F	F	I	F	F		

Card 2	1	2	3	4	5	6	7	8
Variable	LCIDT	LCIDTU	LCIDTD	LCIDTE				
Type	I	I	I	I				

Card 3	1	2	3	4	5	6	7	8
Variable	UTFAIL	UCFAIL	IU					
Type	F	F	F					

**VARIABLE****DESCRIPTION**

MID           Material identification. A unique number or label must be specified (see \*PART).

RO           Mass density; see also volume in \*SECTION\_BEAM definition.

K           Translational stiffness for unloading option 2.0

IUNLD       Unloading option (Also see [Figure M119-1](#)):

EQ.0.0: Loading and unloading follow loading curve.

EQ.1.0: Loading follows loading curve; unloading follows unloading curve. The unloading curve ID if undefined is

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	taken as the loading curve.
	EQ.2.0: Loading follows loading curve; unloading follows unloading stiffness, K, to the unloading curve. The loading and unloading curves may only intersect at the origin of the axes.
	EQ.3.0: Quadratic unloading from peak displacement value to a permanent offset.
OFFSET	Offset to determine permanent set upon unloading if the IUNLD = 3.0. The permanent sets in compression and tension are equal to the product of this offset value and the maximum compressive and tensile displacements, respectively.
DAMPF	Damping factor for stability. Values in the neighborhood of unity are recommended. This damping factor is properly scaled to eliminate time step size dependency. Also, it is active if and only if the local stiffness is defined.
LCIDT	Load curve ID defining translational force resultant along the axis as a function of relative translational displacement. If zero, no stiffness related forces are generated for this degree of freedom. The loading curves must be defined from the most negative displacement to the most positive displacement. The force does not need to increase monotonically for the loading curve. The curves are extrapolated when the displacement range falls outside the curve definition.
LCIDTU	Load curve ID defining translational force resultant along the axis as a function of relative translational displacement during unloading. The force values defined by this curve must increase monotonically from the most negative displacement to the most positive displacement. For IUNLD = 1.0, the slope of this curve must equal or exceed the loading curve for stability reasons. This is not the case for IUNLD = 2.0. For loading and unloading to follow the same path simply set LCIDTU = LCIDT.
LCIDTD	Load curve ID defining translational damping force resultant along the axis as a function of relative translational velocity.
LCIDTE	Load curve ID defining translational damping force scale factor as a function of relative displacement along the axis.

**\*MAT\_121****\*MAT\_GENERAL\_NONLINEAR\_1DOF\_DISCRETE\_BEAM**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
UTFAIL	Optional, translational displacement at failure in tension. If zero, failure in tension is not considered.
UCFAIL	Optional, translational displacement at failure in compression. If zero, failure in compression is not considered.
IU	Initial translational displacement along axis

**Remarks:**

Rotational displacement is measured in radians.

**\*MAT\_HILL\_3R**

This is Material Type 122. This is Hill's 1948 planar anisotropic material model with 3 R values.

**Card Summary:**

**Card 1.** This card is required.

MID	R0	E	PR	HR	P1	P2	
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**Card 2.** This card is required.

R00	R45	R90	LCID	E0			
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**Card 3.** This card is required.

AOPT							
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**Card 4.** This card is required.

			A1	A2	A3		
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**Card 5.** This card is required.

V1	V2	V3	D1	D2	D3	BETA	
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	HR	P1	P2	
Type	A	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus, $E$

<b>VARIABLE</b>	<b>DESCRIPTION</b>
PR	Poisson's ratio, $\nu$
HR	Hardening rule: EQ.1.0: Linear (default) EQ.2.0: Exponential EQ.3.0: Load curve
P1	Material parameter: HR.EQ.1.0: Tangent modulus HR.EQ.2.0: $k$ , strength coefficient for exponential hardening
P2	Material parameter: HR.EQ.1.0: Yield stress HR.EQ.2.0: $n$ , exponent

Card 2	1	2	3	4	5	6	7	8
Variable	R00	R45	R90	LCID	E0			
Type	F	F	F	F	F			

<b>VARIABLE</b>	<b>DESCRIPTION</b>
R00	$R_{00}$ , Lankford parameter determined from experiments
R45	$R_{45}$ , Lankford parameter determined from experiments
R90	$R_{90}$ , Lankford parameter determined from experiments
LCID	Load curve ID for the load curve hardening rule
E0	$\varepsilon_0$ for determining initial yield stress for exponential hardening. (default = 0.0)

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT							
Type	F							

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	Material axes option (see *MAT_OPTIONTROPIC_ELASTIC for a more complete description):
	EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. The material axes are then rotated about the shell element normal by an angle BETA.
	EQ.2.0: Globally orthotropic with material axes determined by the vector <b>a</b> defined below, as with *DEFINE_COORDINATE_VECTOR
	EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <b>v</b> with the element normal
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later.

Card 4	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
A1 A2 A3	Components of vector <b>a</b> for AOPT = 2

**\*MAT\_122****\*MAT\_HILL\_3R**

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
V1 V2 V3	Components of vector <b>v</b> for AOPT = 3
D1 D2 D3	Components of vector <b>d</b> for AOPT = 2
BETA	Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA.

**Remarks:**

The calculated effective stress is stored in history variable #4.

**\*MAT\_HILL\_3R\_3D****\*MAT\_122\_3D****\*MAT\_HILL\_3R\_3D**

This is Material Type 122\_3D. It combines orthotropic elastic behavior with Hill's 1948 anisotropic plasticity theory. Anisotropic plastic properties are given by 6 material parameters,  $F, G, H, L, M, N$ , which are determined by experiments. This model is implemented for solid elements.

This keyword can be written either as \*MAT\_HILL\_3R\_3D or \*MAT\_122\_3D.

**Card Summary:**

**Card 1.** This card is required.

MID	R0	EX	EY	EZ	PRXY	PRYZ	PRXZ
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**Card 2.** This card is required.

GXY	GYZ	GXZ	F	G	H	L	M
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**Card 3.** This card is required.

N	HR	P1	P2				
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**Card 4.** This card is required.

AOPT							
------	--	--	--	--	--	--	--

**Card 5.** This card is required.

XP	YP	ZP	A1	A2	A3		
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**Card 6.** This card is required.

V1	V2	V3	D1	D2	D3		
----	----	----	----	----	----	--	--

**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	EX	EY	EZ	PRXY	PRYZ	PRXZ
Type	A	F	F	F	F	F	F	F

**\*MAT\_122\_3D****\*MAT\_HILL\_3R\_3D**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
EX	$E_x$ , Young's modulus in the $x$ -direction LT.0.0:  EX  is a load curve ID defining $E_x$ as a function of temperature.
EY	$E_y$ , Young's modulus in the $y$ -direction LT.0.0:  EY  is a load curve ID defining $E_y$ as a function of temperature.
EZ	$E_z$ , Young's modulus in the $z$ -direction LT.0.0:  EZ  is a load curve ID defining $E_z$ as a function of temperature.
PRXY	$\nu_{xy}$ , Poisson's ratio $xy$ LT.0.0:  PRXY  is a load curve ID defining $\nu_{xy}$ as a function of temperature.
PRYZ	$\nu_{yz}$ , Poisson's ratio $yz$ LT.0.0:  PRYZ  is a load curve ID defining $\nu_{yz}$ as a function of temperature.
PRXZ	$\nu_{xz}$ , Poisson's ratio $xz$ LT.0.0:  PRXZ  is a load curve ID defining $\nu_{xz}$ as a function of temperature.

Card 2	1	2	3	4	5	6	7	8
Variable	GXY	GYZ	GXZ	F	G	H	L	M
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
GXY	$G_{xy}$ , shear modulus $xy$

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
	LT.0.0:  GXY  is load curve ID defining $G_{xy}$ as a function of temperature.							
GYZ	$G_{yz}$ , shear modulus $yz$ LT.0.0:  GYZ  is load curve ID defining $G_{yz}$ as a function of temperature.							
GXZ	$G_{xz}$ , shear modulus $xz$ LT.0.0:  GXZ  is load curve ID defining $G_{xz}$ as a function of temperature.							
F	Material constant in Hill's 1948 yield criterion (see <a href="#">Remark 1</a> ). LT.0.0:  F  is a load curve ID defining $F$ as a function of temperature.							
G	Material constant in Hill's 1948 yield criterion (see <a href="#">Remark 1</a> ). LT.0.0:  G  is a load curve ID defining $G$ as a function of temperature.							
H	Material constant in Hill's 1948 yield criterion (see <a href="#">Remark 1</a> ). LT.0.0:  H  is a load curve ID defining $H$ as a function of temperature.							
L	Material constant in Hill's 1948 yield criterion (see <a href="#">Remark 1</a> ). LT.0.0:  L  is a load curve ID defining $L$ as a function of temperature.							
M	Material constant in Hill's 1948 yield criterion (see <a href="#">Remark 1</a> ). LT.0.0:  M  is a load curve ID defining $M$ as a function of temperature.							

Card 3	1	2	3	4	5	6	7	8
Variable	N	HR	P1	P2				
Type	F	I	I/F	F				

**\*MAT\_122\_3D****\*MAT\_HILL\_3R\_3D**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
N	Material constant in Hill's 1948 yield criterion (see <a href="#">Remark 1</a> ). LT.0.0:  N  is a load curve ID defining N as a function of temperature.
HR	Hardening rule: EQ.1: stress-strain relationship is defined by load curve or 2D table ID, P1. P2 is ignored. EQ.2: stress-strain relationship is defined by strength coefficient k (P1) and strain hardening coefficient n (P2), as in Swift's exponential hardening equation: $\sigma_{yield} = k(\varepsilon + 0.01)^n .$
P1	Material parameter: HR.EQ.1: load curve or 2D table ID defining stress-strain curve. If P1 is a 2D table ID, the table gives stress-strain curves for different temperatures. HR.EQ.2: k, strength coefficient in $\sigma_{yield} = k(\varepsilon + 0.01)^n$
P2	Material parameter: HR.EQ.1: not used HR.EQ.2.0: n, the exponent in $\sigma_{yield} = k(\varepsilon + 0.01)^n$

Card 4	1	2	3	4	5	6	7	8
Variable	AOPT							
Type	I							

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	Material axes option (see <a href="#">MAT_OPTIONTROPIC_ELASTIC</a> for a more complete description): EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with <a href="#">*DEFINE_COORDINATE_NODES</a>

VARIABLE	DESCRIPTION							
	EQ.1.0: locally orthotropic with material axes determined by a point $p$ in space and the global location of the element center; this is the $a$ -direction.							
	EQ.2.0: globally orthotropic with material axes determined by the vectors $\mathbf{a}$ and $\mathbf{d}$ , as with *DEFINE_COORDINATE_VECTOR.							
	EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector $\mathbf{v}$ with the element normal.							
	EQ.4.0: locally orthotropic in cylindrical coordinate system with the material axes determined by a vector $\mathbf{v}$ , and an originating point, $\mathbf{p}$ , which define the centerline axis.							
Card 5	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION							
XP, YP, ZP	Coordinates of point $p$ for AOPT = 1 and 4							
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2							

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3 and 4
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2
BETA	Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SOLID_ORTHO.

**Remarks:**

1. **Hill's 1948 Yield Criterion.** Hill's yield criterion is based on the assumptions that the material is orthotropic, that hydrostatic stress does not affect yielding, and that there is no Bauschinger effect. According to Hill, when the principal axes of anisotropy are the axes of reference, the yield surface has the form

$$f = \bar{\sigma}(\sigma) - \sigma_{\text{yield}}(\varepsilon_p) = 0,$$

where the effective stress  $\bar{\sigma}$  (stored as history variable #2) is given by

$$(F + G)\bar{\sigma}^2 = F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2 ,$$

and where  $F, G, H, L, M$ , and  $N$  are material parameters of the current state of anisotropy, assuming three mutually orthogonal planes of symmetry at every point. The material  $z$ -direction is the reference direction.

Let  $X, Y, Z$  be the tensile yield stresses in the principal directions of anisotropy, then

$$\frac{\sigma_{y0}^2}{X^2} = \frac{G + H}{F + G} , \quad \frac{\sigma_{y0}^2}{Y^2} = \frac{H + F}{F + G} , \quad \frac{\sigma_{y0}^2}{Z^2} = 1 ,$$

where  $\sigma_{y0} = \sigma_{\text{yield}}(0)$ .  $F, G$ , and  $H$  are not uniquely determined, but the choice of  $F + G = 1$  gives

$$F = \frac{Z^2}{2} \left( \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2} \right)$$

$$G = \frac{Z^2}{2} \left( \frac{1}{X^2} + \frac{1}{Z^2} - \frac{1}{Y^2} \right)$$

$$H = \frac{Z^2}{2} \left( \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2} \right)$$

If  $R_{xy}$ ,  $S_{zx}$ , and  $T_{xy}$  are the yield stresses in shear with respect to the principal axes of anisotropy, then

$$L = \frac{Z^2}{2R_{xy}^2} , \quad M = \frac{Z^2}{2S_{zx}^2} , \quad N = \frac{Z^2}{2T_{xy}^2} .$$

If  $F = G = H$ , and,  $L = M = N = 3F$ , the Hill criterion reduces to the Von-Mises criterion.

The strain hardening in this model can either defined by the load curve or by Swift's exponential hardening equation:  $\sigma_{\text{yield}} = k(\varepsilon + 0.01)^n$ .

2. **Applications.** This material model is suitable for metal forming application using solid elements to account for anisotropic plasticity. NUMISHEET conferences have provided material constants of Hill's 1948 yield for many commonly used materials.

It can also be applied to multi-scale simulations of fiberglass and laminated materials, according to CYBERNET SYSTEMS CO., LTD. The elastic coefficients can be calibrated analytically by a homogenization method with tensile tests in the three orthogonal directions and three pure shear tests in the three orthogonal planes.

3. **Material Parameter Calibration.** The six material parameters required can be calibrated with nonlinear regression analysis (such as those available through LS-OPT) through a series of tensile tests in three orthogonal directions and three shear tests in three orthogonal planes.

**Revision information:**

This material model is available for explicit dynamics in both SMP and MPP starting in Revision 86100 and is available for implicit dynamics in both SMP and MPP starting in Revision 104178. It also supports temperature dependent Young's/shear modulus, Poisson ratios, and Hill parameters.

## \*MAT\_122\_TABULATED

## \*MAT\_HILL\_3R\_TABULATED

### \*MAT\_HILL\_3R\_TABULATED

This is Material Type 122. This is Hill's 1948 planar anisotropic material model with 3 R values and yield curves defined in 3 directions as well as biaxial or shear yield. It is implemented for shell elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	HR			
Type	A	F	F	F	F			

Card 2	1	2	3	4	5	6	7	8
Variable	R00	R45	R90	LC00	ICONV	LC90	LC45	LCEX
Type	F	F	F	I	I	I	I	I

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT							
Type	F							

Card 4	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus, $E$
PR	Poisson's ratio, $\nu$
HR	Hardening rule: EQ.1.0: Not applicable EQ.2.0: Not applicable EQ.3.0: Load curve
R00	$R_{00}$ , Lankford parameter determined from experiments
R45	$R_{45}$ , Lankford parameter determined from experiments
R90	$R_{90}$ , Lankford parameter determined from experiments
LC00	Load curve ID for the yield curve in the $0^\circ$ direction
ICONV	Convexity option: EQ.0.0: Convexity of the yield surface is not enforced. EQ.1.0: Convexity of the yield surface is enforced.
LC90	Load curve ID for the yield curve in the $90^\circ$ direction
LC45	Load curve ID for the yield curve in the $45^\circ$ direction
LCEX	Absolute value is load curve ID for the yield curve in shear or bi-axial: GT.0.0: Shear yield is provided. LT.0.0: Biaxial yield is provided.
AOPT	Material axes option (see <a href="#">*MAT_OPTIONTROPIC_ELASTIC</a> for a more complete description):

**\*MAT\_122\_TABULATED****\*MAT\_HILL\_3R\_TABULATED**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES that are then rotated about the shell element normal by an angle BETA
	EQ.2.0: Globally orthotropic with material axes determined by the vector <b>a</b> defined below, as with *DEFINE_COORDINATE_VECTOR
	EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <b>v</b> with the element normal
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
XP YP ZP	Coordinates of point <i>p</i> for AOPT = 1
A1 A2 A3	Components of vector <b>a</b> for AOPT = 2
V1 V2 V3	Components of vector <b>v</b> for AOPT = 3
D1 D2 D3	Components of vector <b>d</b> for AOPT = 2
BETA	Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA.

**\*MAT\_MODIFIED\_PIECEWISE\_LINEAR\_PLASTICITY\_{OPTION}**

This is Material Type 123, which is an elasto-plastic material supporting an arbitrary stress as a function of strain curve as well as arbitrary strain rate dependency. This model is available for shell and solid elements. \*MAT\_PIECEWISE\_LINEAR\_PLASTICITY is similar but lacks the enhanced failure criteria. Failure is based on effective plastic strain, thinning strain, the major principal in plane strain component, or a minimum time step size.

Available options include:

<BLANK>

LOG\_INTERPOLATION

PRESTRAIN (for shells only)

RATE

RTCL

STOCHASTIC (for shells only)

The LOG\_INTERPOLATION keyword option interpolates the strain rate effect in table LCSS with logarithmic interpolation.

The PRESTRAIN option is used to include prestrain when checking for major strain failure. The RATE option is used to account for rate dependence of thinning failure or to invoke viscoelasticity (LCEMOD). The RTCL option is used to activate RTCL damage (see Remark 1). One additional card is needed with any of these options.

The STOCHASTIC keyword option allows spatially varying yield and failure behavior. See \*DEFINE\_STOCHASTIC\_VARIATION for additional information.

**Card Summary:**

**Card 1.** This card is required.

MID	R0	E	PR	SIGY	ETAN	FAIL	TDEL
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**Card 2.** This card is required.

C	P	LCSS	LCSR	VP	EPSTHIN	EPSMAJ	NUMINT
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**Card 3.** This card is required.

EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
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**\*MAT\_123****\*MAT\_MODIFIED\_PIECEWISE\_LINEAR\_PLASTICITY**

**Card 4.** This card is required.

ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
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**Card 5.** This card included for the PRESTRAIN, RATE, and RTCL keyword options.

LCTSRF	EPS0	TRIAX	IPS	LCEMOD	BETA	RFILTF	
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	SIGY	ETAN	FAIL	TDEL
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	$10^{20}$	0.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
SIGY	Yield stress
ETAN	Tangent modulus, ignored if LCSS > 0
FAIL	Failure flag: LT.0.0: User defined failure subroutine, matusr_24 in dyn21.F, is called to determine failure. EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved. GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
TDEL		Minimum time step size for automatic element deletion						
Card 2	1	2	3	4	5	6	7	8
Variable	C	P	LCSS	LCSR	VP	EPSTHIN	EPSMAJ	NUMINT
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

<b>VARIABLE</b>		<b>DESCRIPTION</b>
	C	Strain rate parameter, C; see <a href="#">Remark 1</a> of *MAT_PIECEWISE_LINEAR_PLASTICITY.
	P	Strain rate parameter, P; see <a href="#">Remark 1</a> of *MAT_PIECEWISE_LINEAR_PLASTICITY.
	LCSS	Load curve ID or Table ID.  <b>Load Curve.</b> When LCSS is a Load curve ID, it is taken as defining effective stress versus effective plastic strain. If defined EPS1 - EPS8 and ES1 - ES8 are ignored.  <b>Tabular Data.</b> The table ID defines for each strain rate value a load curve ID giving the stress versus effective plastic strain for that rate, See <a href="#">Figure M24-1</a> . When the strain rate falls below the minimum value, the stress versus effective plastic strain curve for the lowest value of strain rate is used. Likewise, when the strain rate exceeds the maximum value the stress versus effective plastic strain curve for the highest value of strain rate is used. Fields C, P, LCSR, EPS1 - EPS8, and ES1 - ES8 are ignored if a Table ID is defined. Linear interpolation between the discrete strain rates is used by default; logarithmic interpolation is used when the LOG_INTERPOLATION option is invoked.  <b>Logarithmically Defined Tables.</b> An alternative way to invoke logarithmic interpolation between discrete strain rates is described as follows. If the <i>first</i> value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	when the lowest strain rate and highest strain rate differ by several orders of magnitude. There is some additional computational cost associated with invoking logarithmic interpolation.
LCSR	Load curve ID defining strain rate scaling effect on yield stress
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default) EQ.1.0: Viscoplastic formulation (recommended)
EPSTHIN	Thinning strain at failure. To specify thinning strain to failure as a function of plastic strain rate, see LCTSDF. GT.0.0: Total thinning strain (as in ISTUPD = 1; see *CONTROL_SHELL) LT.0.0: Plastic thinning strain  EPSTHIN  (as in ISTUPD = 4)
EPSMAJ	Major in plane strain at failure for shells (or) major principal strain at failure for solids (see <a href="#">Remark 1</a> ). LT.0: EPSMAJ =  EPSMAJ  and filtering is activated. The last twelve values of the major strain are stored at each integration point and the average value is used to determine failure.
NUMINT	Number of integration points, which must fail before the element is deleted. (If zero, all points must fail.) For fully integrated shell formulations, each of the $4 \times \text{NIP}$ integration points is counted individually in determining a total for failed integration points. NIP is the number of through-thickness integration points. As NUMINT approaches the total number of integration points (NIP for under-integrated shells, $4 \times \text{NIP}$ for fully integrated shells), the chance of instability increases. LT.0.0:  NUMINT  is the percentage of integration points/layers which must fail before the shell element fails. For fully integrated shells, a methodology is used where a layer fails if one integration point fails and then the given percentage of layers must fail before the element fails. Only available for shells.

**\*MAT\_MODIFIED\_PIECEWISE\_LINEAR\_PLASTICITY****\*MAT\_123**

Card 3	1	2	3	4	5	6	7	8
Variable	EPS1	EPS2	EPS3	EPS4	EPS5	EPS6	EPS7	EPS8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

**VARIABLE****DESCRIPTION**

EPS1 - EPS8

Effective plastic strain values. At least 2 points should be defined. The first point must be zero corresponding to the initial yield stress. If this option is used, SIGY and ETAN are ignored. WARNING: If the first point is nonzero, the yield stress is extrapolated to determine the initial yield.

Card 4	1	2	3	4	5	6	7	8
Variable	ES1	ES2	ES3	ES4	ES5	ES6	ES7	ES8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

**VARIABLE****DESCRIPTION**

ES1 - ES8

Corresponding yield stress values to EPS1 - EPS8

**RTCL/Rate Card.** Required if the PRESTRAIN, RATE, or RTCL option is active.

Card 5	1	2	3	4	5	6	7	8
Variable	LCTSRF	EPS0	TRIAX	IPS	LCEMOD	BETA	RFLTF	
Type	I	F	F	I	I	F	F	
Default	0	0.0	0.0	0	0	0.0	0.0	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCTSRF	Curve that defines the thinning strain at failure as a function of the plastic strain rate ( <i>The curve should not extrapolate to zero or failure may occur at low strain.</i> ) If LCTSRF is given, the absolute value of EPSTHIN is unimportant; however, the algebraic sign of EPSTHIN governs whether ordinate values in curve LCTSRF are interpreted as total or plastic thinning strain. For example, if plastic thinning strain should be used, then EPSTHIN should be input as a negative value.
EPS0	EPS0 parameter for RTCL damage (see <a href="#">Remark 2</a> ): EQ.0.0: RTCL damage is inactive (default). GT.0.0: RTCL damage is active.
TRIAX	RTCL damage triaxiality limit (see <a href="#">Remark 2</a> ): EQ.0.0: No limit (default) GT.0.0: Damage does not accumulate when triaxiality exceeds TRIAX.
IPS	Flag to add prestrain when checking for major strain failure (see EPSMAJ above on Card 2) for the PRESTRAIN keyword option: EQ.0: No prestrain added (default) EQ.1: Initial strain set with *INITIAL_STRAIN_SHELL will be used as a prestrain when checking for major strain failure (VP = 0 and shells only).
LCEMOD	Load curve ID defining Young's modulus as function of effective strain rate. LCEMOD ≠ 0 activates viscoelasticity. See *MAT_187L for details. The parameters BETA and RFILTF have to be defined too. (If LCEMOD ≠ 0 is used, VP = 1 should be defined and failure options EPSTHIN, EPSMAJ, NUMINT, and RTCL are currently not available. See *DEFINE_ELEMENT_EROSION to define the number of integration points for failure.)
BETA	Decay constant in viscoelastic law. BETA has the unit [1/time]. If LCEMOD > 0 is used, a non-zero value for BETA is mandatory.
RFILTF	Smoothing factor on the effective strain rate (default is 0.95). The filtered strain rate is used for the viscoelasticity (LCEMOD > 0).

$$\dot{\varepsilon}_n^{\text{avg}} = \text{RFILTF} \times \dot{\varepsilon}_{n-1}^{\text{avg}} + (1 - \text{RFILTF}) \times \dot{\varepsilon}_n$$

**Remarks:**

1. **Major principal strain failure.** The EPSMAJ parameter is compared to the major principal strain in the following senses:
  - a) For shells it is the maximum eigenvalue of the in-plane strain tensor that is incremented by the strain increments. If IPS = 1, then prestrain set with \*INITIAL\_STRAIN\_SHELL is also included in the strain measure for shells.
  - b) For solid elements it is calculated as the maximum eigenvalue to the logarithmic strain tensor

$$\varepsilon = \frac{1}{2} \ln(\mathbf{F}^T \mathbf{F}) ,$$

where  $\mathbf{F}$  is the global deformation gradient.

In sum, both element types use a natural strain measure for determining failure. The major strain calculated in this way is output as history variable #7.

2. **RTCL damage.** With the RTCL option, an RTCL damage is calculated and elements are deleted when the damage function exceeds 1.0. During each solution cycle, if the plastic strain increment is greater than zero, an increment of RTCL damage is calculated by

$$\Delta f_{\text{damage}} = \frac{1}{\varepsilon_0} f \left( \frac{\sigma_H}{\bar{\sigma}} \right)_{\text{RTCL}} d\bar{\varepsilon}^p$$

where

$$f \left( \frac{\sigma_H}{\bar{\sigma}} \right)_{\text{RTCL}} = \begin{cases} 0 & \frac{\sigma_H}{\bar{\sigma}} \leq -\frac{1}{3} \\ 2 \frac{1 + \frac{\sigma_H}{\bar{\sigma}} \sqrt{12 - 27 \left( \frac{\sigma_H}{\bar{\sigma}} \right)^2}}{3 \frac{\sigma_H}{\bar{\sigma}} + \sqrt{12 - 27 \left( \frac{\sigma_H}{\bar{\sigma}} \right)^2}} & -\frac{1}{3} < \frac{\sigma_H}{\bar{\sigma}} < \frac{1}{3} \\ \frac{1}{1.65} \exp \left( \frac{3\sigma_H}{2\bar{\sigma}} \right) & \frac{\sigma_H}{\bar{\sigma}} \geq \frac{1}{3} \end{cases}$$

and,

$\varepsilon_0$  = uniaxial fracture strain / critical damage value

$\sigma_H$  = hydrostatic stress

$\bar{\sigma}$  = effective stress

$d\bar{\varepsilon}^p$  = effective plastic strain increment

The increments are summed through time and the element is deleted when  $f_{\text{damage}} \geq 1.0$ . For  $0.0 < f_{\text{damage}} < 1.0$ , the element strength will not be degraded.

The value of  $f_{\text{damage}}$  is stored as history variable #9 and can be fringe plotted from d3plot files if the number of extra history variables requested is  $\geq 9$  on \*DATABASE\_EXTENT\_BINARY.

The optional TRIAX parameter can be used to prevent excessive RTCL damage growth and element erosion for badly shaped elements that might show unrealistically high values for the triaxiality. The triaxiality,  $\frac{\sigma_H}{\sigma}$ , is stored as history variable #11.

3. **Instability indicator.** To get an idea about the probability of failure, an indicator  $D$  is computed internally:

$$D = \max \left( \frac{\bar{\varepsilon}^p}{\text{FAIL}}, \frac{-\varepsilon_3}{\text{EPSTHIN}}, \frac{\varepsilon_I}{\text{EPSMAJ}} \right)$$

and stored as history variable #10.  $D$  ranges from 0 (intact) to 1 (failed).  $\bar{\varepsilon}^p$ ,  $-\varepsilon_3$ , and  $\varepsilon_I$  are current values of effective plastic strain, thinning strain, and major in plane strain. This instability measure, including the RTCL damage, can also be retrieved from requesting material histories

<i>*DEFINE_MATERIAL_HISTORIES Properties</i>					
<b>Label</b>	<b>Attributes</b>			<b>Description</b>	
Instability	-	-	-	-	Failure indicator $\max(D, f_{\text{damage}})$
Plastic Strain Rate	-	-	-	-	Effective plastic strain rate $\dot{\varepsilon}_{\text{eff}}^p$

4. **Implicit calculations.** For implicit calculations with this material involving severe nonlinear hardening, the radial return method may result in inaccurate stress-strain response. Setting IACC = 1 on \*CONTROL\_ACCURACY activates a fully iterative plasticity algorithm, which will remedy this. This is not to be confused with the MITER flag on \*CONTROL\_SHELL, which governs the treatment of the plane stress assumption for shell elements. If any failure model is applied with this option, incident failure will initiate damage, and the stress will continuously degrade to zero before erosion for a deformation of 1% plastic strain. For instance, if the failure strain is FAIL = 0.05, then the element is eroded when  $\bar{\varepsilon}^p = 0.06$  and the material goes from intact to completely damaged between  $\bar{\varepsilon}^p = 0.05$  and  $\bar{\varepsilon}^p = 0.06$ . The reason is to enhance implicit performance by maintaining continuity in the internal forces.

**\*MAT\_PLASTICITY\_COMPRESSION\_TENSION**

This is Material Type 124. An isotropic elastic-plastic material where unique yield stress as a function of plastic strain curves can be defined for compression and tension. Also, failure can occur based on a plastic strain or a minimum time step size. Rate effects on the yield stress are modeled either by using the Cowper-Symonds strain rate model or by using two load curves that scale the yield stress values in compression and tension, respectively. Material rate effects, which are independent of the plasticity model, are based on a 6-term Prony series Maxwell mode that generates an additional stress tensor. The viscous stress tensor is superimposed on the stress tensor generated by the plasticity.

**Card Summary:**

**Card 1.** This card is required.

MID	R0	E	PR	C	P	FAIL	TDEL
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**Card 2.** This card is required.

LCIDC	LCIDT	LCSRC	LCSRT	SRFLAG	LCFAIL	EC	RPCT
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**Card 3.** This card is required.

PC	PT	PCUTC	PCUTT	PCUTF			SRFILT
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**Card 4.** This card is required.

K							
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**Card 5.** Include up to 6 instances of this card. The next keyword ("\*") card terminates this input.

Gi	BETAi						
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	C	P	FAIL	TDEL
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	0.0	0.0	$10^{20}$	0.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
C	Strain rate parameter, C. See <a href="#">Remark 1</a> .
P	Strain rate parameter, P. See <a href="#">Remark 1</a> .
FAIL	Failure flag: LT.0.0: User defined failure subroutine, matusr_24 in dyn21.F, is called to determine failure. EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved. GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.
TDEL	Minimum time step size for automatic deletion of shell elements

Card 2	1	2	3	4	5	6	7	8
Variable	LCIDC	LCIDT	LCSRC	LCSR	SRFLAG	LCFAIL	EC	RPCT
Type	I	I	I	I	F	I	F	F
Default	0	0	0	0	0.0	0	optional	0.0

VARIABLE	DESCRIPTION
LCIDC	Load curve ID defining effective stress as a function of effective plastic strain in compression. Enter positive yield stress and plastic strain values when defining this curve.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCIDT	Load curve ID defining effective stress as a function of effective plastic strain in tension. Enter positive yield stress and plastic strain values when defining this curve.
LCSRC	Optional load curve ID defining strain rate scaling factor on yield stress as a function of strain rate when the material is in compression
LCSTRT	Optional load curve ID defining strain rate scaling factor on yield stress as a function of strain rate when the material is in tension
SRFLAG	Formulation for rate effects:  EQ.0.0: total strain rate (based on the total strain tensor) EQ.1.0: effective strain rate (based on deviatoric portion of the strain tensor) EQ.2.0: effective plastic strain rate (viscoplastic)
LCFAIL	Optional load curve ID defining effective plastic strain at failure as a function of strain rate. If LCFAIL is specified, FAIL is ignored. See <a href="#">Remark 2</a> .
EC	Optional Young's modulus for compression, $> 0$ .
RPCT	Fraction of PT and PC, used to define mean stress at which Young's modulus is E and EC, respectively. Young's modulus is E when mean stress $> RPCT \times PT$ , and EC when mean stress $< -RPCT \times PC$ . If the mean stress falls between $-RPCT \times PC$ and $RPCT \times PT$ , a linearly interpolated value is used.

Card 3	1	2	3	4	5	6	7	8
Variable	PC	PT	PCUTC	PCUTT	PCUTF			SRFILT
Type	F	F	F	F	F			F
Default	0.0	0.0	0.0	0.0	0.0			0.0

**\*MAT\_124****\*MAT\_PLASTICITY\_COMPRESSION\_TENSION**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
PC	Compressive mean stress (pressure) at which the yield stress follows load curve ID, LCIDC. If the pressure falls between PC and PT, a weighted average of the two load curves is used. Both PC and PT should be entered as positive values.
PT	Tensile mean stress at which the yield stress follows load curve ID, LCIDT
PCUTC	Pressure cut-off in compression (PCUTC must be greater than or equal to zero). PCUTC (and PCUTT) apply only to element types that use a 3D stress update, such as solids, tshell formulations 3 and 5, and SPH. When the pressure cut-off is reached the deviatoric stress tensor is set to zero and the pressure remains at its compressive value. Like the yield stress, PCUTC is scaled to account for rate effects.
PCUTT	Pressure cut-off in tension (PCUTT must be less than or equal to zero). When the pressure cut-off is reached, the deviatoric stress tensor and tensile pressure is set to zero. Like the yield stress, PCUTT is scaled to account for rate effects.
PCUTF	Pressure cut-off flag activation: EQ.0.0: Inactive EQ.1.0: Active
SRFILT	Strain rate filtering parameter in exponential moving average with admissible values ranging from 0 to 1 (available for LCSRC ≠ 0 or LCSRT ≠ 0 with SRFLAG = 0 or 1): $\dot{\epsilon}_n^{\text{avg}} = \text{SRFILT} \times \dot{\epsilon}_{n-1}^{\text{avg}} + (1 - \text{SRFILT}) \times \dot{\epsilon}_n$

Card 4	1	2	3	4	5	6	7	8
Variable	K							
Type	F							

<b>VARIABLE</b>	<b>DESCRIPTION</b>
K	Optional bulk modulus for the viscoelastic material. If nonzero, a Kelvin type behavior will be obtained. Generally, K is set to zero.

**Viscoelastic Constant Cards.** Up to 6 cards may be input. The next keyword ("\*\*") card terminates this input.

Card 5	1	2	3	4	5	6	7	8
Variable	$G_i$	BETA <i>i</i>						
Type	F	F						

VARIABLE	DESCRIPTION
$G_i$	Optional shear relaxation modulus for the <i>i</i> <sup>th</sup> term
BETA <i>i</i>	Optional shear decay constant for the <i>i</i> <sup>th</sup> term

#### Remarks:

1. **Stress-Strain Behavior.** The stress-strain behavior follows a different curve in compression than it does in tension. Tension is determined by the sign of the mean stress where a positive mean stress (meaning a negative pressure) is indicative of tension. Two curves must be defined giving the yield stress as a function of effective plastic strain for both the tension and compression regimes.

Mean stress is an invariant which can be expressed as  $(\sigma_x + \sigma_y + \sigma_z)/3$ . PC and PT define a range of mean stress values within which interpolation is done between the tensile yield surface and compressive yield surface. PC and PT are not true material properties but are just a numerical convenience so that the transition from one yield surface to the other is not abrupt as the sign of the mean stress changes. Both PC and PT are input as positive values as it is implied that PC is a compressive mean stress value and PT is tensile mean stress value.

Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor:

$$1 + \left[ \frac{\dot{\epsilon}}{C} \right]^{1/p} .$$

If SRFLAG = 0,  $\dot{\epsilon}$  is the total strain rate,

$$\dot{\epsilon} = \sqrt{\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}}.$$

2. **LCFAIL.** The LCFAIL field is only applicable when at least one of the following four conditions are met:

- a) SRFLAG = 2
- b) LCSRC is nonzero
- c) LCSRT is nonzero
- d)  $G_i$  and  $\text{BETA}_i$  values are provided.

**\*MAT\_KINEMATIC\_HARDENING\_TRANSVERSELY\_ANISOTROPIC\_{OPTION}**

This is Material Type 125. This material model combines Yoshida and Uemori's non-linear kinematic hardening rule with material type 37. Yoshida and Uemori's theory uses two surfaces to describe the hardening rule: the yield surface and the bounding surface. In the forming process, the yield surface does not change in size, but its center translates with deformation; the bounding surface changes in both size and location. In addition, the change of Young's modulus can be a function of effective plastic strain, as proposed by Yoshida and Uemori [2002]. This material type is available for shells, thick shells, and solid elements.

Available options include:

<BLANK>

NLP

The NLP option estimates necking failure using the Formability Index (F.I.), which accounts for the non-linear strain paths seen in metal forming applications (see [A Failure Criterion for Nonlinear Strain Paths \(NLP\)](#) in the remarks section). When using this option, specify **IFLD** in Card 3. Since the NLP option also works with a linear strain path, it is recommended to be used as the default failure criterion in metal forming. The NLP option is also available for \*MAT\_036, \*MAT\_037, and \*MAT\_226.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	RBAR	HLCID	OPT	
Type	A	F	F	F	F	I	I	
Default	none	none	none	none	none	0	0	

Card 2	1	2	3	4	5	6	7	8
Variable	CB	Y	SC1	K	RSAT	SB	H	SC2
Type	F	F	F	F	F	F	F	F
Default	none	0.0						

**\*MAT\_125****\*MAT\_KINEMATIC\_HARDENING\_TRANSVERSELY\_ANISOTROPIC**

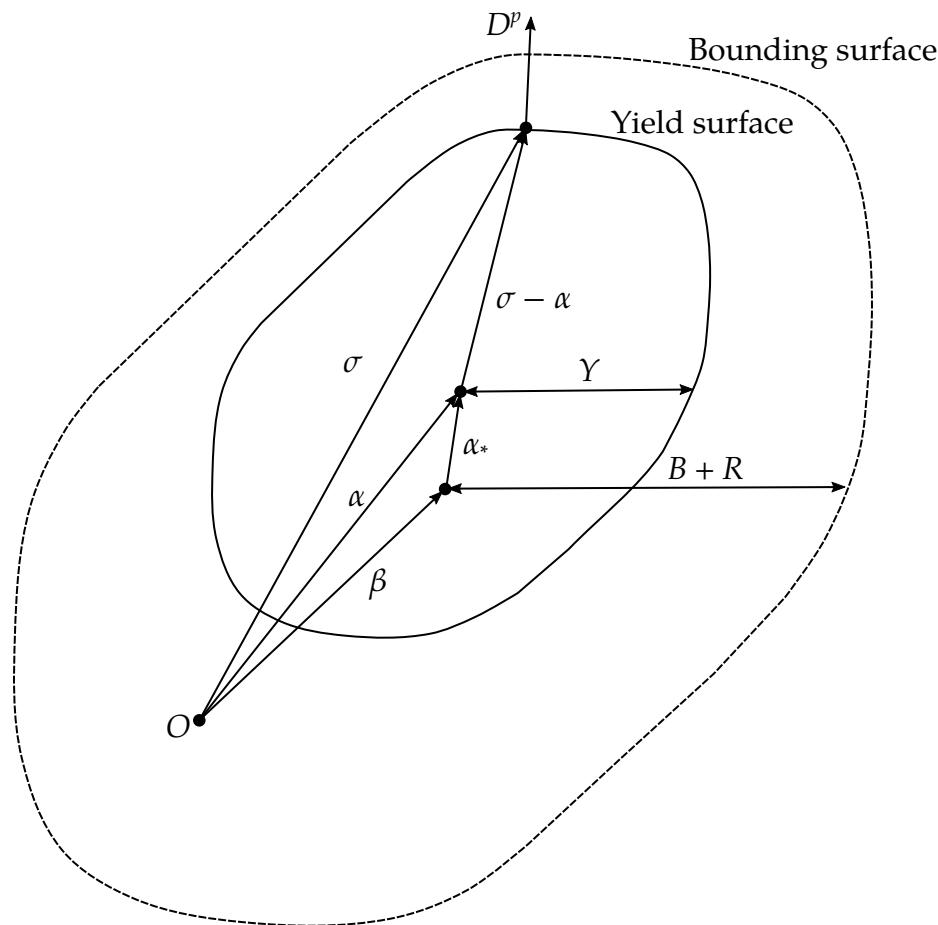
Card 3	1	2	3	4	5	6	7	8
Variable	EA	COE	IOPT	C1	C2	IFLD		
Type	F	F	I	F	F	I		
Default	none	none	0	none	none	none		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's Modulus
PR	Poisson's ratio
RBAR	Plastic anisotropic parameter $\bar{r}$ (Lankford coefficient), also commonly referred to as "r-bar value" in sheet metal forming literature. For shell elements, $\bar{r} = R_{00} = R_{45} = R_{90}$ is assumed in the plane of the shell.
HLCID	Load curve ID (see *DEFINE_CURVE) giving true strain as a function of true stress. This curve is used with OPT below and should not be referenced or used in other keywords. <i>Using this parameter is not recommended.</i>
OPT	Error calculation flag. The default value of "0" is recommended. EQ.2: LS-DYNA will perform the error calculation based on the true stress-strain curve from uniaxial tension, specified by HLCID. The corrections will be made to the cyclic load curve, both in the loading and unloading portions. Since, in some cases where loading is more complex, the accumulated plastic strain could be large (say more than 30%), the input uniaxial stress-strain curve must have enough strain range to cover the maximum expected plastic strain. Note that this variable must be set to a value of "2" if HLCID is specified and a stress-strain curve is used.
CB	The uppercase B defined in Yoshida & Uemori's equations.

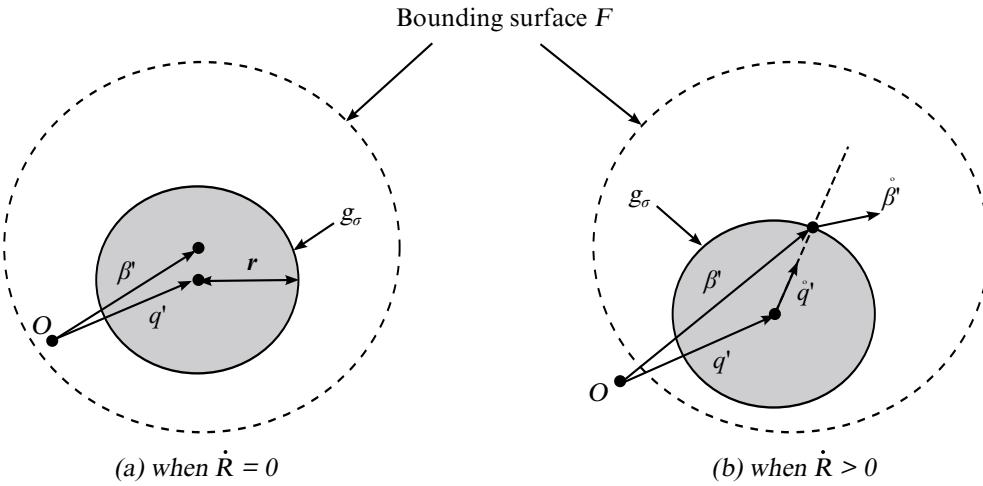
VARIABLE	DESCRIPTION
Y	Hardening parameter appearing in Yoshida & Uemori's equations.
SC1	The lowercase $c_2$ defined in Yoshida & Uemori's equations. Note the equation below from the paper:
	$c = \begin{cases} c_1 & \max(\bar{\alpha}_*) < B - Y \\ c_2 & \text{otherwise} \end{cases}$
	See more details in <a href="#">About SC1 and SC2</a> in the remarks section.
K	Hardening parameter appearing in Yoshida & Uemori's equations.
RSAT	Hardening parameter, $R_{\text{sat}}$ , appearing in Yoshida & Uemori's equations.
SB	The lowercase $b$ appearing in Yoshida & Uemori's equations.
H	Anisotropic parameter associated with work-hardening stagnation.
SC2	The lowercase $c_1$ defined in the Yoshida and Uemori's equations. Note the equation below from the paper:
	$c = \begin{cases} c_1 & \max(\bar{\alpha}_*) < B - Y \\ c_2 & \text{otherwise} \end{cases}$
	See more details in <a href="#">About SC1 and SC2</a> in the remarks section. If SC2 equals 0.0, is left blank, or equals SC1, then it turns into the basic model (the one $c$ model).
EA	Variable controlling the change of Young's modulus, $E^A$ in the following equations.
COE	Variable controlling the change of Young's modulus, $\zeta$ in the following equations.
IOPT	Modified kinematic hardening rule flag: EQ.0: Original Yoshida & Uemori formulation, EQ.1: Modified formulation. Define C1 and C2 below.
C1, C2	Constants used to modify $\dot{R}$ , so strain hardening will not saturate: $\dot{R} = \text{RSAT} \times [(C_1 + \bar{\epsilon}^p)^{c_2} - C_1^{c_2}]$

**VARIABLE****DESCRIPTION**

	Note that these variables are not the material parameter $c$ that controls the rate of the kinematic hardening in the original Yoshida & Uemori paper.
IFLD	ID of a load curve defining Forming Limit Diagram (FLD) under linear strain paths. In the load curve, abscissas represent minor strains, while ordinates represent major strains. Define only when the option NLP is used.



**Figure M125-1.** Schematic illustration of the two-surface model.  $O$  is the original center of the yield surface;  $\alpha$  is the current center for the yield surface;  $\beta$  is the center of the bounding surface; and  $\alpha_*$  represents the relative position of the centers of the two surfaces.  $\gamma$  is the size of the yield surface and is constant throughout the deformation process.  $B + R$  represents the size of the bounding surface, with  $R$  being associated with isotropic hardening. *Reproduced from Yoshida and Uemori's original paper.*



**Figure M125-2.** Change in bounding surface (reproduced from Yoshida and Uemori's original paper).

### The Yoshida & Uemori Kinematic Hardening Model:

The following equations give the two-surface model from Yoshida and Uemori [2]:

$$\begin{aligned}\alpha_* &= \alpha - \beta \\ \alpha_* &= c \left[ \left( \frac{a}{Y} \right) (\sigma - \alpha) - \sqrt{\frac{a}{\alpha_*}} \alpha_* \right] \bar{\varepsilon}^p \\ a &= B + R - Y\end{aligned}$$

Figure M125-1 illustrates these variables. The anisotropic hardening parameter,  $\dot{R}$ , depends on IOPT. The original Yoshida and Uemori model includes saturation in strain hardening (IOPT = 0) [1, 2]. A modified version includes continuous hardening (IOPT = 1).  $\dot{R}$  changes as follows:

$$\begin{aligned}\dot{R} &= k(R_{\text{sat}} - R)\dot{\varepsilon}^p && \text{if IOPT} = 0 \\ \dot{R} &= R_{\text{sat}} \times [(C_1 + \bar{\varepsilon}^p)^{c_2} - C_1^{c_2}] && \text{if IOPT} = 1\end{aligned}$$

The following equations define the change of size and location for the bounding surface, with variable descriptions shown in Figure M125-2,

$$\begin{aligned}\beta' &= k \left( \frac{2}{3} b D - \beta' \dot{\varepsilon}^p \right) \\ \sigma_{\text{bound}} &= B + R + \beta\end{aligned}$$

The unloading process, which follows, includes work-hardening stagnation:

$$\begin{aligned}g_\sigma(\sigma', q', r') &= \frac{3}{2}(\sigma' - q') : (\sigma' - q') - r^2 \\ q' &= \mu(\beta' - q')\end{aligned}$$

$$r = h\Gamma$$

$$\Gamma = \frac{3(\beta' - q') : \overset{\circ}{\beta'}}{2r}$$

The change in Young's modulus is defined as a function of effective plastic strain,

$$E = E_0 - (E_0 - E_A)[1 - \exp(-\zeta \bar{\varepsilon}^p)] .$$

### About SC1 and SC2:

Yoshida and Uemori's paper includes a modification for the material parameter  $c$ , which controls the rate of the kinematic hardening, to describe more accurately the forward and reverse deformations of the cyclic plasticity curve in the vicinity of the initial yield. The paper gives modification of the parameter  $c$  as:

$$c = \begin{cases} c_1 & \max(\bar{\alpha}_*) < B - Y \\ c_2 & \text{otherwise} \end{cases}$$

which corresponds to:

$$c = \begin{cases} \text{SC2} & \max(\bar{\alpha}_*) < B - Y \\ \text{SC1} & \text{otherwise} \end{cases}$$

Here  $\alpha_*$  is the backstress evolution,  $\max(\bar{\alpha}_*)$  is the maximum value of  $\bar{\alpha}_*$ , and

$$\bar{\alpha}_* = \sqrt{\frac{3}{2} \alpha_* : \alpha_*} .$$

### A Failure Criterion for Nonlinear Strain Paths (NLP):

The manual pages for [\\*MAT\\_036](#) and [\\*MAT\\_037](#) describe the NLP failure criterion and corresponding post-processing procedures. The history variables for every element stored in d3plot files include:

1. Formability Index (F.I.): #1 (#24 after Revision 113708)
2. Strain ratio  $\beta$  (in-plane minor strain increment/major strain increment): #2 (#25 after Revision 113708)
3. Effective strain from the planar isotropic assumption: #3 (#26 after Revision 113708)

To enable the output of these history variables to the d3plot files, NEIPS on the [\\*DATABASE\\_EXTENT\\_BINARY](#) card must be set to at least 3.

**References:**

- [1] Shi, M.F., Zhu, X.H., Xia, Z.C. & Stoughton, T.B. (2008). Determination of nonlinear isotropic/kinematic hardening constitutive parameters for AHSS using tension and compression tests. NUMISH-EET. 2008. 137-142.
- [2] Yoshida, Fusahito & Uemori, Takeshi. (2002). A model of large-strain cyclic plasticity describing the Bauschinger effect and workhardening stagnation. International Journal of Plasticity. 18. 661-686. 10.1016/S0749-6419(01)00050-X.

## **\*MAT\_126**

## **\*MAT\_MODIFIED\_HONEYCOMB**

### **\*MAT\_MODIFIED\_HONEYCOMB**

This is Material Type 126. This material model is usually used for aluminum honeycomb crushable foam materials with anisotropic behavior. Three yield surfaces are available. The first yield surface defines the nonlinear elastoplastic material behavior separately for all normal and shear stresses, which are considered fully uncoupled. The second yield surface considers the effects of off-axis loading. It is transversely isotropic. However, because of this definition of the second yield surface, the material can collapse in a shear mode due to low shear resistance. There was no obvious way of increasing the shear resistance without changing the behavior in purely uniaxial compression. Therefore, with the third yield surface, the model has been modified so that the material's shear and hydrostatic resistance can be prescribed without affecting the uniaxial behavior. The sign of the first load curve ID, LCA, flags the choice of the second yield surface. The third yield surface is flagged by the sign of ECCU, which becomes the initial stress yield limit in simple shear. A description is given below.

The development of the second and third yield surfaces is based on experimental test results of aluminum honeycomb specimens at Toyota Motor Corporation.

The default element for this material is solid type 0, a nonlinear spring-type solid element. The recommended hourglass control is the type 2 viscous formulation for one-point integrated solid elements. When used with this constitutive model, the hourglass control's stiffness form can lead to nonphysical results since it can inhibit strain localization in the shear modes.

This material is available for solid elements and thick shell formulations 3, 5, and 7.

#### **Card Summary:**

**Card 1.** This card is required.

MID	R0	E	PR	SIGY	VF	MU	BULK
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**Card 2.** This card is required.

LCA	LCB	LCC	LCS	LCAB	LCBC	LCCA	LCR
-----	-----	-----	-----	------	------	------	-----

**Card 3.** This card is required.

EAAU	EBBU	ECCU	GABU	GBCU	GCAU	AOPT	MACF
------	------	------	------	------	------	------	------

**Card 4.** This card is required.

XP	YP	ZP	A1	A2	A3	RFAC	PRU
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**Card 5.** This card is required.

D1	D2	D3	TSEF	SSEF	VREF	TREF	SHDFLG
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**Card 6.** Include this card if AOPT = 3 or 4 (see Card 3).

V1	V2	V3					
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**Card 7.** Include this card if LCSR = -1 (see Card 2).

LCSRA	LCSR B	LCSR C	LCSR AB	LCSR BC	LCSR CA	LCSR A	LCSR B
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**Card 8.** Include this card if PRU = 2.

PRUAB	PRUAC	PRUBC	PRUBA	PRUCA	PRUCB		
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### Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	VF	MU	BULK
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	none	.05	0.0

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus for compacted honeycomb material
PR	Poisson's ratio for compacted honeycomb material
SIGY	Yield stress for fully compacted honeycomb
VF	Relative volume at which the honeycomb is fully compacted. This field is ignored for corotational solid elements, types 0 and 9.

**\*MAT\_126****\*MAT\_MODIFIED\_HONEYCOMB**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MU	Material viscosity coefficient, $\mu$ . The default value of 0.05 is recommended.
BULK	Bulk viscosity flag: EQ.0.0: Bulk viscosity is not used. This is recommended. EQ.1.0: Bulk viscosity is active and $\mu = 0$ . This will give results identical to previous versions of LS-DYNA.

Card 2	1	2	3	4	5	6	7	8
Variable	LCA	LCB	LCC	LCS	LCAB	LCBC	LCCA	LCSR
Type	I	I	I	I	I	I	I	F
Default	none	LCA	LCA	LCA	LCS	LCS	LCS	optional

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCA	Load curve ID (see *DEFINE_CURVE): LT.0: Yield stress as a function of the angle off the material axis in degrees GT.0: $\sigma_{aa}$ as a function of normal strain component aa, $\epsilon_{aa}$ . Normal strain rate effect can be considered when LCA is defined as a table, see LCSS of MAT_024 for details. Both compressive normal strain and rate are considered positive when defining the curve or table. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid and thick shell element formulations a logarithmic strain is expected. See <a href="#">Remarks 1</a> and <a href="#">3</a> . Note that LCA < 0 flags using the second yield surface (the transversely isotropic surface) and determines the definition for LCB, LCC, LCS, LCAB, LCBC, and LCCA.
LCB	Load curve ID (see *DEFINE_CURVE): LCA.LT.0: Strong axis hardening stress as a function of the volumetric strain LCA.GT.0: $\sigma_{bb}$ as a function of normal strain component bb, $\epsilon_{bb}$ .

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	Normal strain rate effect can be considered when LCB is defined as a table, see LCSS of MAT_024 for details. Both compressive normal strain and rate are considered positive when defining the curve or table. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid and thick shell element formulations a logarithmic strain is expected. See <a href="#">Remarks 1</a> and <a href="#">3</a> .
LCC	<p>Load curve ID (see *DEFINE_CURVE):</p> <p>LCA.LT.0: Weak axis hardening stress as a function of the volumetric strain</p> <p>LCA.GT.0: <math>\sigma_{cc}</math> as a function of normal strain component cc, <math>\varepsilon_{cc}</math>. Normal strain rate effect can be considered when LCC is defined as a table, see LCSS of MAT_024 for details. Both compressive normal strain and rate are considered positive when defining the curve or table. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid and thick shell element formulations a logarithmic strain is expected. See <a href="#">Remarks 1</a> and <a href="#">3</a>.</p>
LCS	<p>Load curve ID (see *DEFINE_CURVE):</p> <p>LCA.LT.0: Damage curve giving the shear stress multiplier as a function of the shear strain component. This curve definition is optional and may be used if damage is desired. IF SHDFLG = 0 (the default), the damage value multiplies the stress every time step and the stress is updated incrementally. The damage curve should be set to unity until failure begins. After failure the value should drop to 0.999 or 0.99 or any number between zero and one depending on how many steps are needed to zero the stress. Alternatively, if SHDFLG = 1, the damage value is treated as a factor that scales the shear stress compared to the undamaged value.</p> <p>LCA.GT.0: Shear stress as a function of shear strain. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid and thick shell element formulations a shear strain based on the deformed configuration is used. Each</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	component of shear stress may have its own load curve. See <a href="#">Remarks 1</a> and <a href="#">3</a> .
LCAB	Load curve ID (see *DEFINE_CURVE):  LCA.LT.0: Damage curve giving shear <i>ab</i> -stress multiplier as a function of the <i>ab</i> -shear strain component. This curve definition is optional and may be used if damage is desired. See LCS above.  LCA.GT.0: $\sigma_{ab}$ as a function of the absolute value of shear strain- <i>ab</i> , $\epsilon_{ab}$ . Shear strain rate effect can be considered when LCAB is defined as a table, see LCSS of MAT_-024 for details. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid and thick shell element formulations a shear strain based on the deformed configuration is used. See <a href="#">Remarks 1</a> and <a href="#">3</a> .
LCBC	Load curve ID (see *DEFINE_CURVE):  LCA.LT.0: Damage curve giving <i>bc</i> -shear stress multiplier as a function of the <i>ab</i> -shear strain component. This curve definition is optional and may be used if damage is desired. See LCS above.  LCA.GT.0: $\sigma_{bc}$ as a function of the absolute value of shear strain- <i>bc</i> , $\epsilon_{bc}$ . Shear strain rate effect can be considered when LCBC is defined as a table, see LCSS of MAT_-024 for details. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid and thick shell element formulations a shear strain based on the deformed configuration is used. See <a href="#">Remarks 1</a> and <a href="#">3</a> .
LCCA	Load curve ID (see *DEFINE_CURVE):  LCA.LT.0: Damage curve giving <i>ca</i> -shear stress multiplier as a function of the <i>ca</i> -shear strain component. This curve definition is optional and may be used if damage is desired. See LCS above.  LCA.GT.0: $\sigma_{ca}$ as a function of the absolute value of shear strain- <i>ca</i> , $\epsilon_{ca}$ . Shear strain rate effect can be considered when LCCA is defined as a table, see LCSS of *MAT_024 for details. For the corotational solid elements, types 0 and 9, engineering strain is

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
	expected, but for all other solid and thick shell element formulations a shear strain based on the deformed configuration is used. See <a href="#">Remarks 1</a> and <a href="#">3</a> .							
LCSR	Load curve ID (see <a href="#">*DEFINE_CURVE</a> ) for strain-rate effects defining the scale factor as a function of effective strain rate $\dot{\varepsilon} = \sqrt{\frac{2}{3}(\dot{\varepsilon}'_{ij}\dot{\varepsilon}'_{ij})}$ . This is optional. The curves defined above are scaled using this curve. Set LCSR = -1 to define a scale factor in each direction using Card 7.							

Card 3	1	2	3	4	5	6	7	8
Variable	EAAU	EBBU	ECCU	GABU	GBCU	GCAU	AOPT	MACF
Type	F	F	F	F	F	F	F	I

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
EAAU	Elastic modulus $E_{aa}$ in uncompressed configuration. LCA.LT.0: Strong axis elastic modulus in uncompressed configuration							
EBBU	Elastic modulus $E_{bb}$ in uncompressed configuration. LCA.LT.0: Weak axis elastic modulus in uncompressed configuration							
ECCU	Elastic modulus $E_{cc}$ in uncompressed configuration. LT.0.0: $ ECCU $ is the initial stress limit (yield) in simple shear, $\sigma_d^Y$ . $ECCU < 0$ activates the third yield surface if LCA < 0.							
GABU	Shear modulus $G_{ab}$ in uncompressed configuration. LCA.LT.0: Strong-weak shear modulus in uncompressed configuration							
GBCU	Shear modulus $G_{bc}$ in uncompressed configuration. LCA.LT.0: Weak-weak shear modulus in uncompressed configuration							

<b>VARIABLE</b>	<b>DESCRIPTION</b>
GCAU	Shear modulus $G_{cau}$ in uncompressed configuration. ECCU.LT.0.0: GCAU is the initial stress limit (yield) in hydrostatic compression, $\sigma_p^Y$ .
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC, particularly the <a href="#">Material Directions</a> section, for details):  EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.  EQ.1.0: Locally orthotropic with material axes determined by a point, $P$ , in space and the global location of the element center; this is the <b>a</b> -direction. This option is for solid elements only.  EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR  EQ.3.0: Locally orthotropic material axes determined by a vector <b>v</b> and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. <b>a</b> is determined by taking the cross product of <b>v</b> with the normal vector, <b>b</b> is determined by taking the cross product of the normal vector with <b>a</b> , and <b>c</b> is the normal vector. Then <b>a</b> and <b>b</b> are rotated about <b>c</b> by an angle BETA. BETA may be set in the keyword input for the element. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.  EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector <b>v</b> , and an originating point, $P$ , which define the centerline axis. This option is for solid elements only.  LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_OPTION).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MACF	<p>Material axes change flag for solid elements:</p> <p>EQ.-4: Switch material axes <math>b</math> and <math>c</math> before BETA rotation      EQ.-3: Switch material axes <math>a</math> and <math>c</math> before BETA rotation      EQ.-2: Switch material axes <math>a</math> and <math>b</math> before BETA rotation      EQ.1: No change, default      EQ.2: Switch material axes <math>a</math> and <math>b</math> after BETA rotation      EQ.3: Switch material axes <math>a</math> and <math>c</math> after BETA rotation      EQ.4: Switch material axes <math>b</math> and <math>c</math> after BETA rotation</p>

[Figure M2-2](#) indicates when LS-DYNA applies MACF to obtain the final material axes. BETA on \*ELEMENT\_SOLID\_{OPTION} is used to rotate the axes. The BETA rotation is optional.

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	RFAC	PRU
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
XP YP ZP	Coordinates of point $p$ for AOPT = 1 and 4
A1 A2 A3	Components of vector $\mathbf{a}$ for AOPT = 2
RFAC	Filtering factor for strain rate effects, see MAT_089 for details.
PRU	<p>Poisson effect option for the uncompacted status:</p> <p>EQ.0: No Poisson's effect.</p> <p>EQ.1: The Poisson's ratio ramps from 0., when an element is in its un-deformed state, to PR when it is fully compacted.</p> <p>EQ.2: Poisson's ratios are input on Card 8.</p>

**\*MAT\_126****\*MAT\_MODIFIED\_HONEYCOMB**

Card 5	1	2	3	4	5	6	7	8
Variable	D1	D2	D3	TSEF	SSEF	VREF	TREF	SHDFLG
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
D1 D2 D3	Components of vector $\mathbf{d}$ for AOPT = 2.
TSEF	Tensile strain at element failure (element will erode)  GT.0.0: Constant value  LT.0.0: $ TSEF $ is a load curve ID for the curve that defines tensile failure strain as a function of the ratio of compressive to tensile strain. See Sahraei et al. [2016] for details.
SSEF	Shear strain at element failure (element will erode)  GT.0.0: Constant value  LT.0.0: $ SSEF $ is a load curve ID for the curve that defines shear failure strain as a function of the ratio of compressive to tensile strain.
VREF	This is an optional input parameter for solid element types 1, 2, 3, 4, and 10 and thick shell formulations 3, 5, and 7. Relative volume at which the reference geometry is stored. At this time, the element behaves like a nonlinear spring. If TREF, below, is reached first, VREF has no effect.
TREF	This is an optional input parameter for solid element types 1, 2, 3, 4, and 10 and thick shell formulations 3, 5, and 7. Element time step size at which the reference geometry is stored. When this time step size is reached, the element behaves like a nonlinear spring. If VREF, above, is reached first, TREF has no effect.
SHDFLG	Flag defining treatment of damage from curves LCS, LCAB, LCBC, and LCCA (relevant only when LCA < 0):  EQ.0.0: Damage reduces shear stress every time step,  EQ.1.0: Damage = (shear stress)/(undamaged shear stress)

Additional card for AOPT = 3 or AOPT = 4.

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3					
Type	F	F	F					

**VARIABLE****DESCRIPTION**

V1 V2 V3

Components of vector v for AOPT = 3 and 4

Additional card for LCSR = -1.0

Card 7	1	2	3	4	5	6	7	8
Variable	LCSRA	LCSRAB	LCSRBC	LCSRAC	LCSRBC	LCSCA		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

LCSRA

Optional load curve ID if LCSR = -1 (see \*DEFINE\_CURVE) for strain rate effects defining the scale factor for the yield stress in the *a*-direction as a function of the *natural logarithm* of the absolute value of deviatoric strain rate in the *a*-direction. The scale factor for the lowest value of strain rate defined by the curve is used if the strain rate is zero. The scale factor for the highest value of strain rate defined by the curve also defines the upper limit of the scale factor.

LCSRAB

Optional load curve ID if LCSR = -1 (see \*DEFINE\_CURVE) for strain rate effects defining the scale factor for the yield stress in the *b*-direction as a function of the *natural logarithm* of the absolute value of deviatoric strain rate in the *b*-direction. The scale factor for the lowest value of strain rate defined by the curve is used if the strain rate is zero. The scale factor for the highest value of strain rate defined by the curve also defines the upper limit of the scale factor.

LCSRBC

Optional load curve ID if LCSR = -1 (see \*DEFINE\_CURVE) for strain rate effects defining the scale factor for the yield stress in the *c*-direction as a function of the *natural logarithm* of the absolute

**\*MAT\_126****\*MAT\_MODIFIED\_HONEYCOMB**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	value of deviatoric strain rate in the <i>c</i> -direction. The scale factor for the lowest value of strain rate defined by the curve is used if the strain rate is zero. The scale factor for the highest value of strain rate defined by the curve also defines the upper limit of the scale factor.
LCSRAB	Optional load curve ID if LCSR = -1 (see *DEFINE_CURVE) for strain rate effects defining the scale factor for the yield stress in the <i>ab</i> -direction as a function of the <i>natural logarithm</i> of the absolute value of strain rate in the <i>ab</i> -direction. The scale factor for the lowest value of strain rate defined by the curve is used if the strain rate is zero. The scale factor for the highest value of strain rate defined by the curve also defines the upper limit of the scale factor.
LCRBC	Optional load curve ID if LCSR = -1 (see *DEFINE_CURVE) for strain rate effects defining the scale factor for the yield stress in the <i>bc</i> -direction as a function of the <i>natural logarithm</i> of the absolute value of strain rate in the <i>bc</i> -direction. The scale factor for the lowest value of strain rate defined by the curve is used if the strain rate is zero. The scale factor for the highest value of strain rate defined by the curve also defines the upper limit of the scale factor.
LCRCA	Optional load curve ID if LCSR = -1 (see *DEFINE_CURVE) for strain rate effects defining the scale factor for the yield stress in the <i>ca</i> -direction as a function of the <i>natural logarithm</i> of the absolute value of strain rate in the <i>ca</i> -direction. The scale factor for the lowest value of strain rate defined by the curve is used if the strain rate is zero. The scale factor for the highest value of strain rate defined by the curve also defines the upper limit of the scale factor.

Additional card for PRU = 2.0

Card 8	1	2	3	4	5	6	7	8
Variable	PRUAB	PRUAC	PRUBC	PRUBA	PRUCA	PRUCB		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
PRU <i>ij</i>	Poisson's ratios on the <i>i-j</i> plane during uncompacted status. The <i>j</i> -direction is the direction of transverse strain when the element is stressed in the <i>i</i> -direction.

**Remarks:**

1. **Load curves and efficiency.** For efficiency, the load curves, LCA, LCB, LCC, LCS, LCAB, LCBC, and LCCA, are strongly recommended to contain exactly the same number of points with corresponding strain values on the abscissa. If this recommendation is followed, the cost of the table lookup is insignificant. Conversely, the cost increases significantly if the abscissa strain values are inconsistent between load curves.
2. **Elastic moduli.** For solid element formulations 1 and 2 and thick shell formulations 3, 5, and 7, the behavior before compaction is orthotropic, where the components of the stress tensor are uncoupled, meaning a component of strain will generate resistance in the local  $a$ -direction with no coupling to the local  $b$  and  $c$  directions. The elastic moduli vary from their initial values to the fully compacted values linearly with the relative volume:

$$E_{aa} = E_{aau} + \beta(E - E_{aau})$$

$$E_{bb} = E_{bbu} + \beta(E - E_{bbu})$$

$$E_{cc} = E_{ccu} + \beta(E - E_{ccu})$$

$$G_{ab} = G_{abu} + \beta(G - G_{abu})$$

$$G_{bc} = G_{bcu} + \beta(G - G_{bcu})$$

$$G_{ca} = G_{cau} + \beta(G - G_{cau})$$

where

$$\beta = \max \left[ \min \left( \frac{1 - V}{1 - V_f}, 1 \right), 0 \right]$$

and  $G$  is the elastic shear modulus for the fully compacted honeycomb material

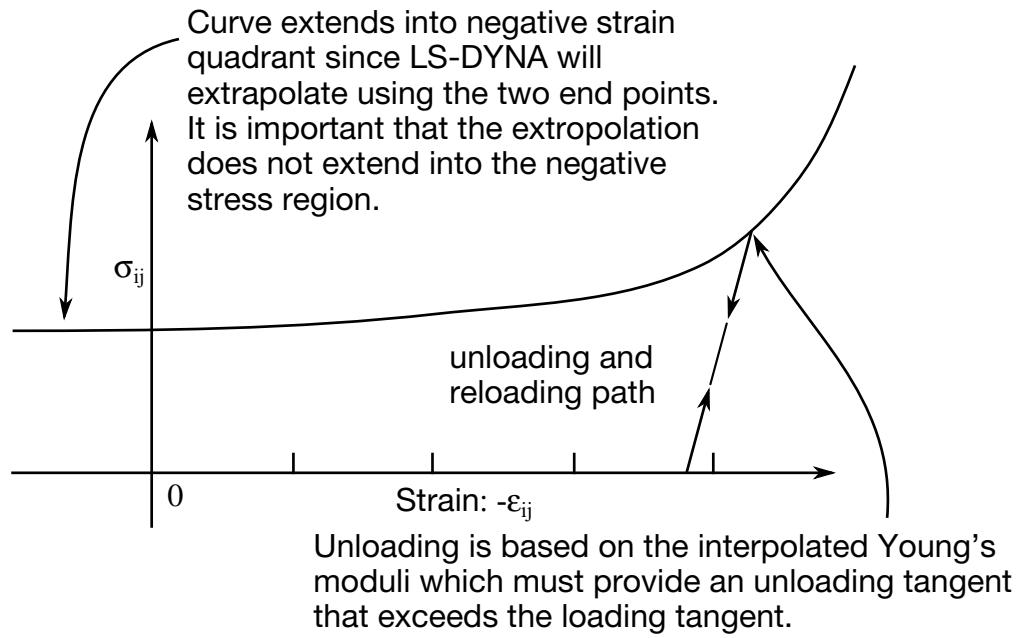
$$G = \frac{E}{2(1 + v)} .$$

The relative volume,  $V$ , is defined as the ratio of the current volume over the initial volume, and typically,  $V = 1$  at the beginning of a calculation.

For corotational solid elements, types 0 and 9, the components of the stress tensor remain uncoupled, and the uncompressed elastic moduli are used; that is, the fully compacted elastic moduli are ignored. However, calculating the element time step size still requires the Young's modulus and Poisson's ratio input on Card 1.

3. **Stress update for uncompacted material.** The load curves define the magnitude of the stress as the material undergoes deformation. The first value in the curve should be less than or equal to zero, corresponding to tension, and increase to full compaction. *Care should be taken when defining the curves so the extrapolated values do not lead to negative yield stresses.*

At the beginning of the stress update, we transform each element's stresses and strain rates into the local element coordinate system. After completing the stress update, we transform the stresses back to the global configuration. For the



**Figure M126-1.** Stress as a function of strain. Note that the “yield stress” at a strain of zero is nonzero. In the load curve definition the “time” value is the directional strain and the “function” value is the yield stress. Note that for element types 0 and 9 engineering strains are used, but for all other element types the rates are integrated in time.

uncompacted material, the trial stress components are updated using the elastic interpolated moduli according to:

$$\sigma_{aa}^{n+1\text{trial}} = \sigma_{aa}^n + E_{aa}\Delta\varepsilon_{aa}$$

$$\sigma_{ab}^{n+1\text{trial}} = \sigma_{ab}^n + 2G_{ab}\Delta\varepsilon_{ab}$$

$$\sigma_{cc}^{n+1\text{trial}} = \sigma_{cc}^n + E_{cc}\Delta\varepsilon_{cc}$$

$$\sigma_{bc}^{n+1\text{trial}} = \sigma_{bc}^n + 2G_{bc}\Delta\varepsilon_{bc}$$

$$\sigma_{bb}^{n+1\text{trial}} = \sigma_{bb}^n + E_{bb}\Delta\varepsilon_{bb}$$

$$\sigma_{ca}^{n+1\text{trial}} = \sigma_{ca}^n + 2G_{ca}\Delta\varepsilon_{ca}$$

If LCA > 0, each component of the updated stress tensor is checked to ensure that it does not exceed the permissible value determined from the load curves; for example, if

$$|\sigma_{ij}^{n+1\text{trial}}| > \lambda\sigma_{ij}(\varepsilon_{ij}) ,$$

then

$$\sigma_{ij}^{n+1} = \sigma_{ij}(\varepsilon_{ij}) \frac{\lambda\sigma_{ij}^{n+1\text{trial}}}{|\sigma_{ij}^{n+1\text{trial}}|}$$

On Card 3  $\sigma_{ij}(\varepsilon_{ij})$  is defined in the load curve specified in columns 31-40 for the aa stress component, 41-50 for the bb component, 51-60 for the cc component, and 61-70 for the ab, bc, cb shear stress components. The parameter  $\lambda$  is either unity or a value taken from the load curve number, LCSR, that defines  $\lambda$  as a

function of strain rate. Strain rate is defined here as the Euclidean norm of the deviatoric strain-rate tensor.

If  $LCA < 0$ , a transversely isotropic yield surface is obtained where the uniaxial limit stress,  $\sigma^y(\varphi, \varepsilon^{\text{vol}})$ , can be defined as a function of angle  $\varphi$  with the strong axis and volumetric strain,  $\varepsilon^{\text{vol}}$ . To facilitate the input of data to such a limit stress surface, the limit stress is written as:

$$\sigma^y(\varphi, \varepsilon^{\text{vol}}) = \sigma^b(\varphi) + (\cos\varphi)^2 \sigma^s(\varepsilon^{\text{vol}}) + (\sin\varphi)^2 \sigma^w(\varepsilon^{\text{vol}})$$

where the functions  $\sigma^b$ ,  $\sigma^s$ , and  $\sigma^w$  are represented by load curves LCA, LCB, LCC, respectively. The latter two curves can be used to include the stiffening effects that are observed as the foam material crushes to the point where it begins to lock up. To ensure that the limit stress decreases with respect to the off-angle, the curves should be defined such that the following equations hold:

$$\frac{\partial \sigma^b(\varphi)}{\partial \varphi} \leq 0$$

and

$$\sigma^s(\varepsilon^{\text{vol}}) - \sigma^w(\varepsilon^{\text{vol}}) \geq 0.$$

A drawback of this implementation was that the material often collapsed in shear mode due to low shear resistance. There was no way of increasing the shear resistance without changing the behavior in pure uniaxial compression. We have, therefore, modified the model so that the user can optionally prescribe the shear and hydrostatic resistance in the material without affecting the uniaxial behavior. We introduce the parameters  $\sigma_p^Y(\varepsilon^{\text{vol}})$  and  $\sigma_d^Y(\varepsilon^{\text{vol}})$  as the *hydrostatic* and *shear limit stresses*, respectively. These are functions of the volumetric strain and are assumed to be given by

$$\begin{aligned}\sigma_p^Y(\varepsilon^{\text{vol}}) &= \sigma_p^Y + \sigma^s(\varepsilon^{\text{vol}}) \\ \sigma_d^Y(\varepsilon^{\text{vol}}) &= \sigma_d^Y + \sigma^s(\varepsilon^{\text{vol}})\end{aligned}'$$

where we have reused the densification function  $\sigma^s$ . The new parameters are the initial hydrostatic and shear limit stress values,  $\sigma_p^Y$  and  $\sigma_d^Y$ , and are provided by the user as GCAU and |ECCU|, respectively. The negative sign of ECCU flags the third yield surface option whenever  $LCA < 0$ . The effect of the third formulation is that (i) for a uniaxial stress the stress limit is given by  $\sigma^Y(\varphi, \varepsilon^{\text{vol}})$ , (ii) for a pressure the stress limit is given by  $\sigma_p^Y(\varepsilon^{\text{vol}})$ , and (iii) for a simple shear the stress limit is given by  $\sigma_d^Y(\varepsilon^{\text{vol}})$ . Experiments have shown that the model may give noisy responses and inhomogeneous deformation modes if parameters are not carefully chosen. We, therefore, recommend (i) avoiding large slopes in the function  $\sigma^P$ , (ii) letting the functions  $\sigma^s$  and  $\sigma^w$  be slightly increasing, and (iii) avoiding large differences between the stress limit values  $\sigma^y(\varphi, \varepsilon^{\text{vol}})$ ,  $\sigma_p^Y(\varepsilon^{\text{vol}})$ , and  $\sigma_d^Y(\varepsilon^{\text{vol}})$ . These guidelines are likely to contradict how one would interpret

test data, and it is up to the user to find a reasonable trade-off between matching experimental results and avoiding the mentioned numerical side effects.

4. **Stress update for fully compacted material.** As in the uncompacted case, we transform each element's stresses and strain rates into the local element coordinate system. For fully compacted material (element formulations 1 and 2), we assume that the material behavior is elastic-perfectly plastic and updated the stress components according to:

$$s_{ij}^{\text{trial}} = s_{ij}^n + 2G\Delta\varepsilon_{ij}^{dev^{n+1/2}}$$

where the deviatoric strain increment is defined as

$$\Delta\varepsilon_{ij}^{dev} = \Delta\varepsilon_{ij} - \frac{1}{3}\Delta\varepsilon_{kk}\delta_{ij}.$$

We now check to see if the yield stress for the fully compacted material is exceeded by comparing

$$s_{\text{eff}}^{\text{trial}} = \left(\frac{3}{2}s_{ij}^{\text{trial}}s_{ij}^{\text{trial}}\right)^{1/2}$$

the effective trial stress to the yield stress,  $\sigma_y$  (Card 1, field 41-50). If the effective trial stress exceeds the yield stress, we scale back the stress components to the yield surface

$$s_{ij}^{n+1} = \frac{\sigma_y}{s_{\text{eff}}^{\text{trial}}}s_{ij}^{\text{trial}}.$$

We can now update the pressure using the elastic bulk modulus,  $K$

$$p^{n+1} = p^n - K\Delta\varepsilon_{kk}^{n+1/2}$$

$$K = \frac{E}{3(1-2\nu)}$$

and obtain the final value for the Cauchy stress

$$\sigma_{ij}^{n+1} = s_{ij}^{n+1} - p^{n+1}\delta_{ij}$$

After completing the stress update, we transform the stresses back to the global configuration.

5. **Failure.** For \*CONSTRAINED\_TIED\_NODES\_WITH\_FAILURE, the failure is based on the volumetric strain instead of the plastic strain.

**\*MAT\_ARRUDA\_BOYCE\_RUBBER****\*MAT\_127****\*MAT\_ARRUDA\_BOYCE\_RUBBER**

This is Material Type 127. This material model provides a hyperelastic rubber model (see [Arruda and Boyce 1993]) combined optionally with linear viscoelasticity as outlined by [Christensen 1980].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	K	G	N			
Type	A	F	F	F	F			

Card 2	1	2	3	4	5	6	7	8
Variable	LCID	TRAMP	NT					
Type	F	F	F					

**Viscoelastic Constant Cards.** Up to 6 cards may be input. The next keyword ("\*") card terminates this input.

Card 3	1	2	3	4	5	6	7	8
Variable	$G_i$	BETA $i$						
Type	F	F						

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
K	Bulk modulus, $K$
G	Shear modulus, $G$
N	Number of statistical links, $N$

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCID	Optional load curve ID of relaxation curve if constants $G_i$ and $\beta_i$ are determined using a least squares fit. This relaxation curve is shown in <a href="#">Figure M76-1</a> . This model ignores the constant stress.
TRAMP	Optional ramp time for loading
NT	Number of Prony series terms in optional fit. If zero, the default is 6. Currently, the maximum number is 6. Values less than 6, possibly 3-5 are recommended, since each term used adds significantly to the cost. Caution should be exercised when taking the results from the fit. Always check the results of the fit in the output file. Preferably, all generated coefficients should be positive. Negative values may lead to unstable results. Once a satisfactory fit has been achieved, it is recommended that the coefficients which are written into the output file be input in future runs.
$G_i$	Optional shear relaxation modulus for the $i^{\text{th}}$ term.
BETA <i>i</i>	Optional decay constant if $i^{\text{th}}$ term.

**Remarks:**

Rubber is generally considered to be fully incompressible since the bulk modulus greatly exceeds the shear modulus in magnitude. To model the rubber as an unconstrained material, a hydrostatic work term,  $W_H(J)$ , is included in the strain energy functional which is function of the relative volume,  $J$ , [Ogden 1984]:

$$W(J_1, J) = G \left[ \frac{1}{2} (J_1 - 3) + \frac{1}{20N} (J_1^2 - 9) + \frac{11}{1050N^2} (J_1^3 - 27) \right] \\ + G \left[ \frac{19}{7000N^3} (J_1^4 - 81) + \frac{519}{673750N^4} (J_1^5 - 243) \right] + W_H(J) ,$$

where the hydrostatic work term is in terms of the bulk modulus,  $K$ , and  $J$  as:

$$W_H(J) = \frac{K}{2} (J - 1)^2$$

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \epsilon_{kl}}{\partial \tau} d\tau ,$$

or in terms of the second Piola-Kirchhoff stress,  $S_{ij}$ , and Green's strain tensor,  $E_{ij}$ ,

$$S_{ij} = \int_0^t G_{ijkl} (t - \tau) \frac{\partial E_{kl}}{\partial \tau} d\tau ,$$

where  $g_{ijkl}(t - \tau)$  and  $G_{ijkl}(t - \tau)$  are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by up to six terms from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^N \alpha_m e^{-\beta_m t}$$

given by,

$$g(t) = \sum_{i=1}^n G_i e^{-\beta_i t} .$$

This model is effectively a Maxwell fluid which consists of a dampers and springs in series. We characterize this in the input by shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . The viscoelastic behavior is optional and an arbitrary number of terms may be used.

**\*MAT\_128****\*MAT\_HEART\_TISSUE****\*MAT\_HEART\_TISSUE**

This is Material Type 128. This material model provides a heart tissue model described in the paper by Walker *et al* [2005] as interpreted by Kay Sun. It is backward compatible with an earlier heart tissue model described in the paper by Guccione, McCulloch, and Waldman [1991]. Both models are transversely isotropic.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	C	B1	B2	B3	P	B
Type	A	F	F	F	F	F	F	F

Skip to Card 3 to activate older Guccione, McCulloch, and Waldman [1991] model.

Card 2	1	2	3	4	5	6	7	8
Variable	L0	CA0MAX	LR	M	BB	CA0	TMAX	TACT
Type	F	I	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT	MACF						
Type	F	I						

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
C	Diastolic material coefficient
B1	$b_1$ , diastolic material coefficient
B2	$b_2$ , diastolic material coefficient
B3	$b_3$ , diastolic material coefficient
P	Pressure in the muscle tissue
B	Systolic material coefficient. Omit for the earlier model.
L0	$l_0$ , sarcomere length at which no active tension develops. Omit for the earlier model.
CA0MAX	$(Ca_0)_{\max}$ , maximum peak intracellular calcium concentrate. Omit for the earlier model.
LR	$l_R$ , Stress-free sarcomere length. Omit for the earlier model.
M	Systolic material coefficient. Omit for the earlier model.
BB	Systolic material coefficient. Omit for the earlier model.
CA0	$Ca_0$ , peak intracellular calcium concentration. Omit for the earlier model.
TMAX	$T_{\max}$ , maximum isometric tension achieved at the longest sarcomere length. Omit for the earlier model.
TACT	$t_{act}$ , time at which active contraction initiates. Omit for the earlier model

VARIABLE	DESCRIPTION
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC, particularly the <a href="#">Material Directions</a> section, for details):</p> <p>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.</p> <p>EQ.1.0: Locally orthotropic with material axes determined by a point, <math>P</math>, in space and the global location of the element center; this is the <math>\mathbf{a}</math>-direction. This option is for solid elements only.</p> <p>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR</p> <p>EQ.3.0: Locally orthotropic material axes determined by a vector <math>\mathbf{v}</math> and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. <math>\mathbf{a}</math> is determined by taking the cross product of <math>\mathbf{v}</math> with the normal vector, <math>\mathbf{b}</math> is determined by taking the cross product of the normal vector with <math>\mathbf{a}</math>, and <math>\mathbf{c}</math> is the normal vector. Then <math>\mathbf{a}</math> and <math>\mathbf{b}</math> are rotated about <math>\mathbf{c}</math> by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.</p> <p>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector <math>\mathbf{v}</math>, and an originating point, <math>P</math>, which define the centerline axis. This option is for solid elements only.</p> <p>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_OPTION).</p>
MACF	<p>Material axes change flag for solid elements:</p> <p>EQ.-4: Switch material axes <math>b</math> and <math>c</math> before BETA rotation</p> <p>EQ.-3: Switch material axes <math>a</math> and <math>c</math> before BETA rotation</p> <p>EQ.-2: Switch material axes <math>a</math> and <math>b</math> before BETA rotation</p>

VARIABLE	DESCRIPTION
	EQ.1: No change, default
	EQ.2: Switch material axes <i>a</i> and <i>b</i> after BETA rotation
	EQ.3: Switch material axes <i>a</i> and <i>c</i> after BETA rotation
	EQ.4: Switch material axes <i>b</i> and <i>c</i> after BETA rotation
	<p>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_{OPTION} is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 5 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.</p>
XP, YP, ZP	Coordinates of point <i>p</i> for AOPT = 1 and 4
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3 and 4
BETA	Material angle in degrees for AOPT = 3. BETA may be overridden on the element card; see *ELEMENT_SOLID_ORTHO.

**Remarks:**

1. **Tissue Model.** The tissue model is described in terms of the energy functional that is transversely isotropic with respect to the local fiber direction,

$$W = \frac{C}{2} (e^Q - 1)$$

$$Q = b_f E_{11}^2 + b_t (E_{22}^2 + E_{33}^2 + E_{23}^2 + E_{32}^2) + b_{fs} (E_{12}^2 + E_{21}^2 + E_{13}^2 + E_{31}^2)$$

Here *C*, *b<sub>f</sub>*, *b<sub>t</sub>*, and *b<sub>fs</sub>* are material parameters and **E** is the Lagrange-Green strain.

The systolic contraction is modeled as the sum of the passive stress derived from the strain energy function and an active fiber directional component, *T<sub>0</sub>*, which is a function of time, *t*,

$$\mathbf{S} = \frac{\partial W}{\partial \mathbf{E}} - p J \mathbf{C}^{-1} + T_0(t, C a_0, l)$$

$$\boldsymbol{\sigma} = \frac{1}{J} \mathbf{F} \mathbf{S} \mathbf{F}^T$$

with  $\mathbf{S}$ , the second Piola-Kirchoff stress tensor;  $\mathbf{C}$ , the right Cauchy-Green deformation tensor;  $J$ , the Jacobian of the deformation gradient tensor  $\mathbf{F}$ ; and  $\sigma$ , the Cauchy stress tensor.

The active fiber directional stress component is defined by a time-varying elasticity model, which at end-systole, is reduced to

$$T_0 = T_{\max} \frac{Ca_0^2}{Ca_0^2 + ECa_{50}^2} C_t$$

Here,  $T_{\max}$  is the maximum isometric tension achieved at the longest sarcomere length and maximum peak intracellular calcium concentration. The length-dependent calcium sensitivity and internal variable is given by,

$$\begin{aligned} ECa_{50} &= \frac{(Ca_0)_{\max}}{\sqrt{\exp[B(l - l_0)] - 1}} \\ C_t &= 1/2(1 - \cos w) \\ l &= l_R \sqrt{2E_{11} + 1} \\ w &= \pi \frac{0.25 + t_r}{t_r} \\ t_r &= ml + bb \end{aligned}$$

A cross-fiber, in-plane stress equivalent to 40% of that along the myocardial fiber direction is added.

2. **Older Tissue Model.** The earlier tissue model is described in terms of the energy functional in terms of the Green strain components,  $E_{ij}$ ,

$$\begin{aligned} W(E) &= \frac{C}{2}(e^Q - 1) + \frac{1}{2}P(I_3 - 1) \\ Q &= b_1 E_{11}^2 + b_2(E_{22}^2 + E_{33}^2 + E_{23}^2 + E_{32}^2) + b_3(E_{12}^2 + E_{21}^2 + E_{13}^2 + E_{31}^2) \end{aligned}$$

The Green components are modified to eliminate any effects of volumetric work following the procedures of Ogden. See the paper by Guccione *et al* [1991] for more detail.

**\*MAT\_LUNG\_TISSUE****\*MAT\_129****\*MAT\_LUNG\_TISSUE**

This is Material Type 129. This material model provides a hyperelastic model for heart tissue, see [Vawter 1980] combined optionally with linear viscoelasticity as outlined by [Christensen 1980].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	K	C	DELTA	ALPHA	BETA	
Type	A	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	C1	C2	LCID	TRAMP	NT			
Type	F	F	F	F	F			

**Viscoelastic Constant Cards.** Up to 6 cards may be input. A keyword card (with a “\*” in column 1) terminates this input if less than 6 cards are used.

Card 3	1	2	3	4	5	6	7	8
Variable	GI	BETAI						
Type	F	F						

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
K	Bulk modulus
C	Material coefficient.
DELTA	$\Delta$ , material coefficient.
ALPHA	$\alpha$ , material coefficient.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BETA	$\beta$ , material coefficient.
C1	Material coefficient.
C2	Material coefficient.
LCID	Optional load curve ID of relaxation curve If constants $G_i$ and $\beta_i$ are determined via a least squares fit. This relaxation curve is shown in <a href="#">Figure M76-1</a> . This model ignores the constant stress.
TRAMP	Optional ramp time for loading.
NT	Number of Prony series terms in optional fit. If zero, the default is 6. Currently, the maximum number is 6. Values less than 6, possibly 3 - 5 are recommended, since each term used adds significantly to the cost. Caution should be exercised when taking the results from the fit. Always check the results of the fit in the output file. Preferably, all generated coefficients should be positive. Negative values may lead to unstable results. Once a satisfactory fit has been achieved it is recommended that the coefficients which are written into the output file be input in future runs.
$G_i$	Optional shear relaxation modulus for the $i^{\text{th}}$ term
BETAI $_i$	Optional decay constant if $i^{\text{th}}$ term

**Remarks:**

The material is described by a strain energy functional expressed in terms of the invariants of the Green Strain:

$$W(I_1, I_2) = \frac{C}{2\Delta} e^{(\alpha I_1^2 + \beta I_2)} + \frac{12C_1}{\Delta(1 + C_2)} [A^{(1+C_2)} - 1]$$

$$A^2 = \frac{4}{3}(I_1 + I_2) - 1$$

where the hydrostatic work term is in terms of the bulk modulus,  $K$ , and the third invariant,  $J$ , as:

$$W_H(J) = \frac{K}{2}(J - 1)^2$$

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl} (t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau$$

or in terms of the second Piola-Kirchhoff stress,  $S_{ij}$ , and Green's strain tensor,  $E_{ij}$ ,

$$S_{ij} = \int_0^t G_{ijkl} (t - \tau) \frac{\partial E_{kl}}{\partial \tau} d\tau$$

where  $g_{ijkl}(t - \tau)$  and  $G_{ijkl}(t - \tau)$  are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^N \alpha_m e^{-\beta_m t}$$

given by,

$$g(t) = \sum_{i=1}^n G_i e^{-\beta_i t}$$

This model is effectively a Maxwell fluid which consists of a dampers and springs in series. We characterize this in the input by shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . The viscoelastic behavior is optional and an arbitrary number of terms may be used.

## \*MAT\_130

## \*MAT\_SPECIAL\_ORTHOTROPIC

### \*MAT\_SPECIAL\_ORTHOTROPIC

This is Material Type 130. This model is available for Belytschko-Tsay and C0 triangular shell elements. It is based on a resultant stress formulation. In-plane behavior is treated separately from bending in order to model perforated materials, such as television shadow masks. If other shell formulations are specified, the formulation will be automatically switched to Belytschko-Tsay. As implemented, this material model cannot be used with user defined integration rules.

**NOTE:** This material does not support specification of a material angle,  $\beta_i$ , for each through-thickness integration point of a shell.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	YS	EP				
Type	A	F	F	F				

Card 2	1	2	3	4	5	6	7	8
Variable	E11P	E22P	V12P	V21P	G12P	G23P	G31P	
Type	F	F	F	F	F	F	F	

Card 3	1	2	3	4	5	6	7	8
Variable	E11B	E22B	V12B	V21B	G12B	AOPT		
Type	F	F	F	F	F	F		

Card 4	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
YS	Yield stress. This parameter is optional and approximates the yield condition. Set to zero if the behavior is elastic.
EP	Plastic hardening modulus
E11P	$E_{11p}$ , for in plane behavior
E22P	$E_{22p}$ , for in plane behavior
V12P	$\nu_{12p}$ , for in plane behavior.
V11P	$\nu_{11p}$ , for in plane behavior
G12P	$G_{12p}$ , for in plane behavior
G23P	$G_{23p}$ , for in plane behavior
G31P	$G_{31p}$ , for in plane behavior
E11B	$E_{11b}$ , for bending behavior
E22B	$E_{22b}$ , for bending behavior
V12B	$\nu_{12b}$ , for bending behavior
V21B	$\nu_{21b}$ , for bending behavior
G12B	$G_{12b}$ , for bending behavior
AOPT	Material axes option (see MAT_{OPTION}TROPIC_ELASTIC for a more complete description): EQ.0.0: Locally orthotropic with material axes determined by

**\*MAT\_130****\*MAT\_SPECIAL\_ORTHOTROPIC**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA
EQ.2.0:	Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR
EQ.3.0:	Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <b>v</b> with the element normal
LT.0.0:	The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2
V1 ,V2, V3	Components of vector <b>v</b> for AOPT = 3
BETA	Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA.

**Remarks:**

The in-plane elastic matrix for in-plane, plane stress behavior is given by:

$$\mathbf{C}_{\text{in plane}} = \begin{bmatrix} Q_{11p} & Q_{12p} & 0 & 0 & 0 \\ Q_{12p} & Q_{22p} & 0 & 0 & 0 \\ 0 & 0 & Q_{44p} & 0 & 0 \\ 0 & 0 & 0 & Q_{55p} & 0 \\ 0 & 0 & 0 & 0 & Q_{66p} \end{bmatrix}$$

The terms  $Q_{ijp}$  are defined as:

$$Q_{11p} = \frac{E_{11p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{22p} = \frac{E_{22p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{12p} = \frac{\nu_{21p}E_{11p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{44p} = G_{12p}$$

$$Q_{55p} = G_{23p}$$

$$Q_{66p} = G_{31p}$$

The elastic matrix for bending behavior is given by:

$$\mathbf{C}_{\text{bending}} = \begin{bmatrix} Q_{11b} & Q_{12b} & 0 \\ Q_{12b} & Q_{22b} & 0 \\ 0 & 0 & Q_{44b} \end{bmatrix}$$

The terms  $Q_{ijb}$  are similarly defined.

Because this is a resultant formulation, nothing is written to the six stress slots of d3plot. Resultant forces and moments may be written to elout and to dynain in place of the six stresses. The first two extra history variables may be used to complete output of the eight resultants to elout and dynain.

## \*MAT\_131

## \*MAT\_ISOTROPIC\_SMEARED\_CRACK

### \*MAT\_ISOTROPIC\_SMEARED\_CRACK

This is Material Type 131. This model was developed by Lemmen and Meijer [2001] as a smeared crack model for isotropic materials. This model is available of solid elements only and is restricted to cracks in the  $xy$ -plane. Users should choose other models unless they have the report by Lemmen and Meijer [2001].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	ISPL	SIGF	GK	SR
Type	A	F	F	F	I	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
ISPL	Failure option: EQ.0: Maximum principal stress criterion EQ.5: Smeared crack model EQ.6: Damage model based on modified von Mises strain
SIGF	Peak stress
GK	Critical energy release rate
SR	Strength ratio

#### Remarks:

The following documentation is taken nearly verbatim from the documentation of Lemmen and Meijer [2001].

Three methods are offered to model progressive failure. The maximum principal stress criterion detects failure if the maximum (most tensile) principal stress exceeds  $\sigma_{\max}$ . Upon failure, the material can no longer carry stress.

The second failure model is the smeared crack model with linear softening stress-strain using equivalent uniaxial strains. Failure is assumed to be perpendicular to the principal strain directions. A rotational crack concept is employed in which the crack directions are related to the current directions of principal strain. Therefore, crack directions may rotate in time. Principal stresses are expressed as

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix} = \begin{bmatrix} \bar{E}_1 & 0 & 0 \\ 0 & \bar{E}_2 & 0 \\ 0 & 0 & \bar{E}_3 \end{bmatrix} \begin{pmatrix} \tilde{\varepsilon}_1 \\ \tilde{\varepsilon}_2 \\ \tilde{\varepsilon}_3 \end{pmatrix} = \begin{pmatrix} \bar{E}_1 \tilde{\varepsilon}_1 \\ \bar{E}_2 \tilde{\varepsilon}_2 \\ \bar{E}_3 \tilde{\varepsilon}_3 \end{pmatrix} \quad (131.1)$$

with  $\bar{E}_1$ ,  $\bar{E}_2$  and  $\bar{E}_3$  as secant stiffness in the terms that depend on internal variables.

In the model developed for DYCOSS it has been assumed that there is no interaction between the three directions in which case stresses simply follow from

$$\sigma_j(\tilde{\varepsilon}_j) = \begin{cases} E\tilde{\varepsilon}_j & \text{if } 0 \leq \tilde{\varepsilon}_j \leq \tilde{\varepsilon}_{j,\text{ini}} \\ \bar{\sigma} \left( 1 - \frac{\tilde{\varepsilon}_j - \tilde{\varepsilon}_{j,\text{ini}}}{\tilde{\varepsilon}_{j,\text{ult}} - \tilde{\varepsilon}_{j,\text{ini}}} \right) & \text{if } \tilde{\varepsilon}_{j,\text{ini}} < \tilde{\varepsilon}_j \leq \tilde{\varepsilon}_{j,\text{ult}} \\ 0 & \text{if } \tilde{\varepsilon}_j > \tilde{\varepsilon}_{j,\text{ult}} \end{cases} \quad (131.2)$$

with  $\bar{\sigma}$  the ultimate stress,  $\tilde{\varepsilon}_{j,\text{ini}}$  the damage threshold, and  $\tilde{\varepsilon}_{j,\text{ult}}$  the ultimate strain in  $j$ -direction. The damage threshold is defined as

$$\tilde{\varepsilon}_{j,\text{ini}} = \frac{\bar{\sigma}}{E} . \quad (131.3)$$

The ultimate strain is obtained by relating the crack growth energy and the dissipated energy

$$\int \int \bar{\sigma} d\tilde{\varepsilon}_{j,\text{ult}} dV = GA \quad (131.4)$$

with  $G$  as the energy release rate,  $V$  as the element volume and  $A$  as the area perpendicular to the principal strain direction. The one point elements in LS-DYNA have a single integration point and the integral over the volume may be replaced by the volume. For linear softening it follows

$$\tilde{\varepsilon}_{j,\text{ult}} = \frac{2GA}{V\bar{\sigma}} . \quad (131.5)$$

The above formulation may be regarded as a damage equivalent to the maximum principle stress criterion.

The third model is a damage model represented by Brekelmans et. al [1991]. Here the Cauchy stress tensor,  $\sigma$ , is expressed as

$$\sigma = (1 - D)E\varepsilon \quad (131.6)$$

where  $D$  represents the current damage and the factor  $1 - D$  is the reduction factor caused by damage. The scalar damage variable is expressed as function of a so-called damage equivalent strain  $\varepsilon_d$

$$D = D(\varepsilon_d) = 1 - \frac{\varepsilon_{\text{ini}}(\varepsilon_{\text{ult}} - \varepsilon_d)}{\varepsilon_d(\varepsilon_{\text{ult}} - \varepsilon_{\text{ini}})} , \quad (131.7)$$

where

$$\varepsilon_d = \frac{k-1}{2k(1-2v)} J_1 + \frac{1}{2k} \sqrt{\left(\frac{k-1}{1-2v} J_1\right)^2 + \frac{6k}{(1+v)^2} J_2} . \quad (131.8)$$

Here the constant  $k$  represents the ratio of the strength in tension over the strength in compression

$$k = \frac{\sigma_{\text{ult, tension}}}{\sigma_{\text{ult, compression}}} , \quad (131.9)$$

$J_1$  and  $J_2$  are the first and second invariant of the strain tensor representing the volumetric and the deviatoric straining, respectively

$$\begin{aligned} J_1 &= \text{tr}(\varepsilon) \\ J_2 &= \text{tr}(\varepsilon \cdot \varepsilon) - \frac{1}{3} [\text{tr}(\varepsilon)]^2 \end{aligned} \quad (131.10)$$

If the compression and tension strength are equal, the dependency on the volumetric strain vanishes in (131.8) and failure is shear dominated. If the compressive strength is much larger than the strength in tension,  $k$  becomes small and the  $J_1$  terms in (131.8) dominate the behavior.

**\*MAT\_ORTHOTROPIC\_SMEARED\_CRACK****\*MAT\_132****\*MAT\_ORTHOTROPIC\_SMEARED\_CRACK**

This is Material Type 132. This material is a smeared crack model for orthotropic materials. It is available for solid elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	EA	EB	EC	PRBA	PRCA	PRCB
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	UINS	UISS	CERRMI	CERRMII	IND	ISD		
Type	F	F	F	F	I	I		

Card 3	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	AOPT				
Type	F	F	F	F				

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	MACF	
Type	F	F	F	F	F	F	I	

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	REF
Type	F	F	F	F	F	F	F	F

**\*MAT\_132****\*MAT\_ORTHOTROPIC\_SMEARED\_CRACK**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density.
EA	Young's modulus in <i>a</i> -direction, $E_a$
EB	Young's modulus in <i>b</i> -direction, $E_b$
EC	Young's modulus in <i>c</i> -direction, $E_c$
PRBA	Poisson's ratio $ba$ , $\nu_{ba}$
PRCA	Poisson's ratio $ca$ , $\nu_{ca}$
PRCB	Poisson's ratio $cb$ , $\nu_{cb}$
UINS	Ultimate interlaminar normal stress
UISS	Ultimate interlaminar shear stress
CERRMI	Critical energy release rate mode I
CERRMII	Critical energy release rate mode II
IND	Interlaminar normal direction: EQ.1.0: Along local <i>a</i> -axis EQ.2.0: Along local <i>b</i> -axis EQ.3.0: Along local <i>c</i> -axis
ISD	Interlaminar shear direction : EQ.4.0: Along local <i>ab</i> -axis EQ.5.0: Along local <i>bc</i> -axis EQ.6.0: Along local <i>ca</i> -axis
GAB	Shear modulus $ab$ , $G_{ab}$
GBC	Shear modulus $bc$ , $G_{bc}$
GCA	Shear modulus $ca$ , $G_{ca}$
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC, particularly the <a href="#">Material Directions</a> section, for details):

<u>VARIABLE</u>	<u>DESCRIPTION</u>
	EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.
	EQ.1.0: Locally orthotropic with material axes determined by a point, $P$ , in space and the global location of the element center; this is the $\mathbf{a}$ -direction.
	EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR
	EQ.3.0: Locally orthotropic material axes determined by a vector $\mathbf{v}$ and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, AOPT = 3 is only available for hexahedrons. $\mathbf{a}$ is determined by taking the cross product of $\mathbf{v}$ with the normal vector, $\mathbf{b}$ is determined by taking the cross product of the normal vector with $\mathbf{a}$ , and $\mathbf{c}$ is the normal vector. Then $\mathbf{a}$ and $\mathbf{b}$ are rotated about $\mathbf{c}$ by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.
	EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector $\mathbf{v}$ , and an originating point, $P$ , which define the centerline axis.
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_OPTION).
XP YP ZP	Define coordinates of point $P$ for AOPT = 1 and 4.
A1 A2 A3	Define components of vector $\mathbf{a}$ for AOPT = 2.
MACF	Material axes change flag for solid elements: EQ.-4: Switch material axes $b$ and $c$ before BETA rotation EQ.-3: Switch material axes $a$ and $c$ before BETA rotation EQ.-2: Switch material axes $a$ and $b$ before BETA rotation

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	EQ.1: No change, default
	EQ.2: Switch material axes <i>a</i> and <i>b</i> after BETA rotation
	EQ.3: Switch material axes <i>a</i> and <i>c</i> after BETA rotation
	EQ.4: Switch material axes <i>b</i> and <i>c</i> after BETA rotation
	<p><a href="#">Figure M2-2</a> indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_{OPTION} is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 5 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.</p>
V1 V2 V3	Define components of vector <b>v</b> for AOPT = 3 and 4.
D1 D2 D3	Define components of vector <b>d</b> for AOPT = 2:
BETA	Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SOLID_ORTHO.
REF	Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword *INITIAL_FOAM_REFERENCE_GEOMETRY.  EQ.0.0: Off EQ.1.0: On

**Remarks:**

This is an orthotropic material with optional delamination failure for brittle composites. The elastic formulation is identical to the DYNA3D model that uses total strain formulation. The constitutive matrix  $C$  that relates to global components of stress to the global components of strain is defined as:

$$C = T^T C_L T$$

where  $T$  is the transformation matrix between the local material coordinate system and the global system and  $C_L$  is the constitutive matrix defined in terms of the material constants of the local orthogonal material axes *a*, *b*, and *c* (see DYNA3D use manual).

Failure is described using linear softening stress strain curves for the interlaminar normal and interlaminar shear directions. The current implementation for failure is essentially two-dimensional. Damage can occur in the interlaminar normal direction and a single

interlaminar shear direction. The orientation of these directions with respect to the principal material directions must be specified by the user.

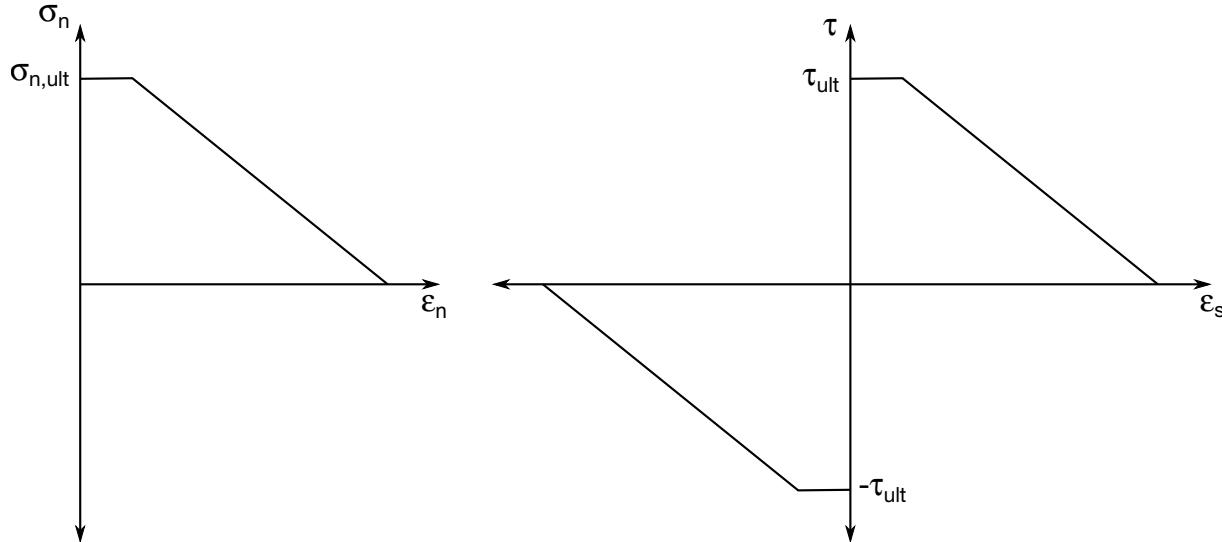
Based on specified values for the ultimate stress and the critical energy release rate bounding surfaces are defined as

$$\begin{aligned} f_n &= \sigma_n - \bar{\sigma}_n(\varepsilon_n) \\ f_s &= \sigma_s - \bar{\sigma}_s(\varepsilon_s) \end{aligned}$$

where the subscripts  $n$  and  $s$  refer to the normal and shear component. If stresses exceed the bounding surfaces, inelastic straining occurs. The ultimate strain is obtained by relating the crack growth energy and the dissipated energy. For solid elements with a single integration point it can be derived to obtain

$$\varepsilon_{i,\text{ult}} = \frac{2G_i A}{V\sigma_{i,\text{ult}}}$$

with  $G_i$  as the critical energy release rate,  $V$  as the element volume,  $A$  as the area perpendicular to the active normal direction and  $\sigma_{i,\text{ult}}$  as the ultimate stress. For the normal component failure can only occur under tensile loading. For the shear component the behavior is symmetric around zero. The resulting stress bounds are depicted in [Figure M132-1](#). Unloading is modeled with a Secant stiffness.



**Figure M132-1.** Shows stress bounds for the active normal component (left) and the archive shear component (right).

## \*MAT\_133

## \*MAT\_BARLAT\_YLD2000

### \*MAT\_BARLAT\_YLD2000

This is Material Type 133. This model was developed by Barlat et al. [2003] to overcome some shortcomings of the six parameter Barlat model implemented as material 33 (MAT\_BARLAT\_YLD96) in LS-DYNA. This model is available for shell, thick shell, and solid elements. Support for solid elements started with R12 but only for explicit analysis. The model for solid elements is based on the approach by Dunand et al. [2012].

#### Card Summary:

**Card 1.** This card is required.

MID	R0	E	PR	FIT	BETA	ITER	ISCALE
-----	----	---	----	-----	------	------	--------

**Card 2.** This card is required.

K	E0	N	C	P	HARD	A	
---	----	---	---	---	------	---	--

**Card 2.1.** This card is included if  $A < 0$ .

CRC1	CRA1	CRC2	CRA2	CRC3	CRA3	CRC4	CRA4
------	------	------	------	------	------	------	------

**Card 3a.** This card is included if FIT = 0.

ALPHA1	ALPHA2	ALPHA3	ALPHA4	ALPHA5	ALPHA6	ALPHA7	ALPHA8
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**Card 3b.1.** This card is included if FIT = 1.

SIG00	SIG45	SIG90	R00	R45	R90		
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**Card 3b.2.** This card is included if FIT = 1.

SIGXX	SIGYY	SIGXY	DXX	DYY	DXY		
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**Card 4.1.** This card is included if HARD = 3.

CP	T0	TREF	TA0				
----	----	------	-----	--	--	--	--

**Card 4.2.** This card is included if HARD = 3.

A	B	C	D	P	Q	EOMART	VMO
---	---	---	---	---	---	--------	-----

**Card 4.3.** This card is included if HARD = 3.

AHS	BHS	M	N	EPS0	HMART	K1	K2
-----	-----	---	---	------	-------	----	----

**Card 5.** This card is required.

AOPT	OFFANG	P4	HTFLAG	HTA	HTB	HTC	HTD
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**Card 6.** This card is required.

			A1	A2	A3		
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**Card 7.** This card is required.

V1	V2	V3	D1	D2	D3	USRFAIL	
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### Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	FIT	BETA	ITER	ISCALE
Type	A	F	F	F	F	F	F	F

<u>VARIABLE</u>	<u>DESCRIPTION</u>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus LE.0: -E is either a load curve ID for Young's modulus as a function of plastic strain or a table ID for Young's modulus as a function of plastic strain and temperature.
PR	Poisson's ratio LE.0: -PR is a load curve ID for Poisson's ratio as a function of temperature.
FIT	Material parameter fit flag: EQ.0.0: Material parameters are used directly on Card 3a. EQ.1.0: Material parameters are determined from test data on Cards 3b.1 and 3b.2.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BETA	Hardening parameter. Any value ranging from 0 (isotropic hardening) to 1 (kinematic hardening) may be input. This field is ignored if the flow potential exponent A is input as a negative number.
ITER	<p>Plastic iteration flag:</p> <p>EQ.0.0: Plane stress algorithm for stress return</p> <p>EQ.1.0: Secant iteration algorithm for stress return</p> <p>ITER provides an option of using three secant iterations for determining the thickness strain increment as experiments have shown that this leads to a more accurate prediction of shell thickness changes for rapid processes. A significant increase in computation time is incurred with this option so it should be used only for applications associated with high rates of loading and/or for implicit analysis.</p>
ISCALE	<p>Yield locus scaling flag:</p> <p>EQ.0.0: Scaling on - reference direction is the rolling direction (default)</p> <p>EQ.1.0: Scaling off – reference direction arbitrary</p>

Card 2	1	2	3	4	5	6	7	8
Variable	K	E0	N	C	P	HARD	A	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
K	<p>Material parameter:</p> <p>HARD.EQ.1.0: <math>k</math>, strength coefficient for exponential hardening</p> <p>HARD.EQ.2.0: <math>a</math> in Voce hardening law</p> <p>HARD.EQ.4.0: <math>k</math>, strength coefficient for Gosh hardening</p> <p>HARD.EQ.5.0: <math>a</math> in Hocket-Sherby hardening law</p>
E0	<p>Material parameter:</p> <p>HARD.EQ.1.0: <math>e_0</math>, strain at yield for exponential hardening</p>

<u>VARIABLE</u>	<u>DESCRIPTION</u>
	HARD.EQ.2.0: $b$ in Voce hardening law HARD.EQ.4.0: $\epsilon_0$ , strain at yield for Gosh hardening HARD.EQ.5.0: $b$ in Hocket-Sherby hardening law
N	Material parameter: HARD.EQ.1.0: $n$ , exponent for exponential hardening HARD.EQ.2.0: $c$ in Voce hardening law HARD.EQ.4.0: $n$ , exponent for Gosh hardening HARD.EQ.5.0: $c$ in Hocket-Sherby hardening law
C	Cowper-Symonds strain rate parameter, $C$ ; see <a href="#">Remark 1</a> .
P	Cowper-Symonds strain rate parameter, $p$ ; see <a href="#">Remark 1</a> . $\sigma_y^v(\epsilon_p, \dot{\epsilon}_p) = \sigma_y(\epsilon_p) \left( 1 + \left[ \frac{\dot{\epsilon}_p}{C} \right]^{1/p} \right)$
HARD	Hardening law: EQ.1.0: Exponential hardening: $\sigma_y = k(\epsilon_0 + \epsilon_p)^n$ EQ.2.0: Voce hardening: $\sigma_y = a - b e^{-c\epsilon_p}$ EQ.3.0: Hansel hardening (see <a href="#">Remark 4</a> ) EQ.4.0: Gosh hardening: $\sigma_y = k(\epsilon_0 + \epsilon_p)^n - p$ EQ.5.0: Hocket-Sherby hardening: $\sigma_y = a - b e^{-c\epsilon_p^q}$ LT.0.0: Absolute value defines load curve ID, table ID or 3D table ID. If it is a load curve, then yield stress is a function of plastic strain. If it is a table, then yield stress is a function of either plastic strain and plastic strain rate in case of a 2D table, or, a function of plastic strain, plastic strain rate, and temperature in case of a 3D table.
A	Flow potential exponent. For face centered cubic (FCC) materials A = 8 is recommended and for body centered cubic (BCC) materials A = 6 may be used. If the input is negative, then an extra card for Chaboche-Rousselier kinematic hardening is read, the flow potential exponent is taken as the absolute value of what is input, and BETA above is ignored.

**\*MAT\_133****\*MAT\_BARLAT\_YLD2000**

**Chaboche-Rousselier Card.** Additional Card for A < 0.

Card 2.1	1	2	3	4	5	6	7	8
Variable	CRC1	CRA1	CRC2	CRA2	CRC3	CRA3	CRC4	CRA4
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
CRC $n$	Chaboche-Rousselier kinematic hardening parameters; see <a href="#">Remark 3</a> .
CRA $n$	Chaboche-Rousselier kinematic hardening parameters; see <a href="#">Remark 3</a> .

**Direct Material Parameter Card.** Additional card for FIT = 0.

Card 3a	1	2	3	4	5	6	7	8
Variable	ALPHA1	ALPHA2	ALPHA3	ALPHA4	ALPHA5	ALPHA6	ALPHA7	ALPHA8
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
ALPHA $i$	$\alpha_i$ , see <a href="#">Remark 2</a> . If ALPHA1 is input as a negative number, then the absolute value is the ID of a load curve giving $\alpha_1$ as a function of temperature. With this choice, <i>all</i> ALPHA $i$ must be negative and given by curves.

**Test Data Card 1.** Additional card for FIT = 1.

Card 3b.1	1	2	3	4	5	6	7	8
Variable	SIG00	SIG45	SIG90	R00	R45	R90		
Type	F	F	F	F	F	F		

**Test Data Card 2.** Additional Card for FIT = 1.

Card 3b.2	1	2	3	4	5	6	7	8
Variable	SIGXX	SIGYY	SIGXY	DXX	DYY	DXY		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
SIG00	Yield stress in 00 direction  LT.0.0: -SIG00 is load curve ID, defining this stress as a function of temperature.
SIG45	Yield stress in 45 direction  LT.0.0: -SIG45 is load curve ID, defining this stress as a function of temperature.
SIG90	Yield stress in 90 direction  LT.0.0: -SIG90 is load curve ID, defining this stress as a function of temperature.
R00	R-value in 00 direction  LT.0.0: -R00 is load curve ID, defining this value as a function of temperature.
R45	R-value in 45 direction  LT.0.0: -R45 is load curve ID, defining this value as a function of temperature.
R90	R-value in 90 direction  LT.0.0: -R90 is load curve ID, defining this value as a function of temperature.
SIGXX	<i>xx</i> -component of stress on the yield surface (see <a href="#">Remark 2</a> ).
SIGYY	<i>yy</i> -component of stress on the yield surface (see <a href="#">Remark 2</a> ).
SIGXY	<i>xy</i> -component of stress on the yield surface (see <a href="#">Remark 2</a> ).
DXX	<i>xx</i> -component of tangent to the yield surface (see <a href="#">Remark 2</a> ).

**\*MAT\_133****\*MAT\_BARLAT\_YLD2000**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
DYY	$yy$ -component of tangent to the yield surface (see <a href="#">Remark 2</a> ).
DXY	$xy$ -component of tangent to the yield surface (see <a href="#">Remark 2</a> ).

**Hansel Hardening Card 1.** Additional card for HARD = 3.

Card 4.1	1	2	3	4	5	6	7	8
Variable	CP	T0	TREF	TA0				
Type	F	F	F	F				

<b>VARIABLE</b>	<b>DESCRIPTION</b>
CP	Adiabatic temperature calculation option: EQ.0.0: Adiabatic temperature calculation is disabled. GT.0.0: CP is the specific heat $C_p$ . Adiabatic temperature calculation is enabled.
T0	Initial temperature $T_0$ of the material if adiabatic temperature calculation is enabled
TREF	Reference temperature for output of the yield stress as history variable
TA0	Reference temperature $T_{A0}$ , the absolute zero for the used temperature scale, such as -273.15 if the Celsius scale is used and 0.0 if the Kelvin scale is used.

**Hansel Hardening Card 2.** Additional card for HARD = 3.

Card 4.2	1	2	3	4	5	6	7	8
Variable	A	B	C	D	P	Q	E0MART	VM0
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
A	Martensite rate equation parameter A, see <a href="#">Remark 4</a> .

VARIABLE	DESCRIPTION
B	Martensite rate equation parameter $B$ , see <a href="#">Remark 4</a> .
C	Martensite rate equation parameter $C$ , see <a href="#">Remark 4</a> .
D	Martensite rate equation parameter $D$ , see <a href="#">Remark 4</a> .
P	Martensite rate equation parameter $p$ , see <a href="#">Remark 4</a> .
Q	Martensite rate equation parameter $Q$ , see <a href="#">Remark 4</a> .
E0MART	Martensite rate equation parameter $E_{0(\text{mart})}$ , see <a href="#">Remark 4</a> .
VM0	The initial volume fraction of martensite $0.0 < V_{m0} < 1.0$ may be initialized using two different methods: GT.0.0: $V_{m0}$ is set to VM0. LT.0.0: Can be used only when there are initial plastic strains $\varepsilon^p$ present, such as when using *INITIAL_STRESS_SHELL. The absolute value of VM0 is then the load curve ID for a function, $f$ , that sets $V_{m0} = f(\varepsilon^p)$ . The function $f$ must be a monotonically nondecreasing function of $\varepsilon^p$ .

**Hansel Hardening Card 3.** Additional card for HARD = 3.

Card 4.3	1	2	3	4	5	6	7	8
Variable	AHS	BHS	M	N	EPS0	HMART	K1	K2
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
AHS	Hardening law parameter $A_{\text{HS}}$ , see <a href="#">Remark 4</a> .
BHS	Hardening law parameter $B_{\text{HS}}$ , see <a href="#">Remark 4</a> .
M	Hardening law parameter $m$ , see <a href="#">Remark 4</a> .
N	Hardening law parameter $n$ , see <a href="#">Remark 4</a> .
EPS0	Hardening law parameter $\varepsilon_0$ , see <a href="#">Remark 4</a> .
HMART	Hardening law parameter $\Delta H_{\gamma \rightarrow \alpha'}$ , see <a href="#">Remark 4</a> .

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
K1		Hardening law parameter $K_1$ , see <a href="#">Remark 4</a> .						
K2		Hardening law parameter $K_2$ , see <a href="#">Remark 4</a> .						
Card 5	1	2	3	4	5	6	7	8
Variable	AOPT	OFFANG	P4	HTFLAG	HTA	HTB	HTC	HTD
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
AOPT		Material axes option (see <a href="#">*MAT_OPTIONTROPIC_ELASTIC</a> for more details):						
		EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in Figure M133-1. Nodes 1, 2, and 4 of an element are identical to the nodes used for the definition of a coordinate system as by <a href="#">*DEFINE_COORDINATE_NODES</a> .						
		EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with <a href="#">*DEFINE_COORDINATE_VECTOR</a>						
		EQ.3.0: Locally orthotropic material axes determined by offsetting the material axes by an angle, OFFANG, from a line determined by taking the cross product of the vector $v$ with the normal to the plane of the element.						
		LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on <a href="#">*DEFINE_COORDINATE_NODES</a> , <a href="#">*DEFINE_COORDINATE_SYSTEM</a> or <a href="#">*DEFINE_COORDINATE_VECTOR</a> ).						
OFFANG		Offset angle for AOPT = 3						
P4		Material parameter:						
		HARD.EQ.4.0: $p$ in Gosh hardening law						
		HARD.EQ.5.0: $q$ in Hocket-Sherby hardening law						
HTFLAG		Heat treatment flag (see <a href="#">Remark 5</a> ):						

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	EQ.0: Preforming stage
	EQ.1: Heat treatment stage
	EQ.2: Postforming stage
HTA	Load curve or table ID for postforming parameter <i>a</i>
HTB	Load curve or table ID for postforming parameter <i>b</i>
HTC	Load curve or table ID for postforming parameter <i>c</i>
HTD	Load curve or table ID for postforming parameter <i>d</i>

Card 6	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2

Card 7	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	USRFAIL	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2
USRFAIL	User defined failure flag: EQ.0: No user subroutine is called. EQ.1: User subroutine <code>matusr_24</code> in <code>dyn21.f</code> is called.

**Remarks:**

1. **Cowper – Symonds strain rate.** Strain rate is accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left( \frac{\dot{\epsilon}_p}{C} \right)^{1/p},$$

where  $\dot{\epsilon}_p$  is the plastic strain rate. To ignore strain rate effects set both C and P to zero.

2. **Yield condition.** The yield condition for this material can be written as

$$f(\sigma, \alpha, \epsilon_p) = \sigma_{\text{eff}}(\sigma_{xx} - 2\alpha_{xx} - \alpha_{yy}, \sigma_{yy} - 2\alpha_{yy} - \alpha_{xx}, \sigma_{xy} - \alpha_{xy}) - \sigma_Y^t(\epsilon_p, \dot{\epsilon}_p, \beta) \leq 0$$

where

$$\begin{aligned} \sigma_{\text{eff}}(s_{xx}, s_{yy}, s_{xy}) &= \left[ \frac{1}{2} (\varphi' + \varphi'') \right]^{1/a} \\ \varphi' &= |X'_1 - X'_2|^a \\ \varphi'' &= |2X''_1 + X''_2|^a + |X''_1 + 2X''_2|^a \end{aligned}$$

The  $X'_i$  and  $X''_i$  are eigenvalues of  $X'_{ij}$  and  $X''_{ij}$  and are given by

$$\begin{aligned} X'_1 &= \frac{1}{2} \left( X'_{11} + X'_{22} + \sqrt{(X'_{11} - X'_{22})^2 + 4X'_{12}^2} \right) \\ X'_2 &= \frac{1}{2} \left( X'_{11} + X'_{22} - \sqrt{(X'_{11} - X'_{22})^2 + 4X'_{12}^2} \right) \end{aligned}$$

and

$$\begin{aligned} X''_1 &= \frac{1}{2} \left( X''_{11} + X''_{22} + \sqrt{(X''_{11} - X''_{22})^2 + 4X''_{12}^2} \right) \\ X''_2 &= \frac{1}{2} \left( X''_{11} + X''_{22} - \sqrt{(X''_{11} - X''_{22})^2 + 4X''_{12}^2} \right) \end{aligned}$$

respectively. The  $X'_{ij}$  and  $X''_{ij}$  are given by

$$\begin{aligned} \begin{pmatrix} X'_{11} \\ X'_{22} \\ X'_{12} \end{pmatrix} &= \begin{pmatrix} L'_{11} & L'_{12} & 0 \\ L'_{21} & L'_{22} & 0 \\ 0 & 0 & L'_{33} \end{pmatrix} \begin{pmatrix} s_{xx} \\ s_{yy} \\ s_{xy} \end{pmatrix} \\ \begin{pmatrix} X''_{11} \\ X''_{22} \\ X''_{12} \end{pmatrix} &= \begin{pmatrix} L''_{11} & L''_{12} & 0 \\ L''_{21} & L''_{22} & 0 \\ 0 & 0 & L''_{33} \end{pmatrix} \begin{pmatrix} s_{xx} \\ s_{yy} \\ s_{xy} \end{pmatrix} \end{aligned}$$

where,

$$\begin{pmatrix} L'_{11} \\ L'_{12} \\ L'_{21} \\ L'_{22} \\ L'_{33} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_7 \end{pmatrix}$$

$$\begin{pmatrix} L''_{11} \\ L''_{12} \\ L''_{21} \\ L''_{22} \\ L''_{33} \end{pmatrix} = \frac{1}{9} \begin{pmatrix} -2 & 2 & 8 & -2 & 0 \\ 1 & -4 & -4 & 4 & 0 \\ 4 & -4 & -4 & 1 & 0 \\ -2 & 8 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 9 \end{pmatrix} \begin{pmatrix} \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_8 \end{pmatrix}$$

The parameters  $\alpha_1$  to  $\alpha_8$  determine the shape of the yield surface.  $s_{xx}$ ,  $s_{yy}$ , and  $s_{xy}$  do not denote the deviatoric stress components, but the arguments are used in the  $\sigma_{\text{eff}}$  function.

Three uniaxial tests and a more general test facilitate determining the material parameters. The yield stress and R-values come from the uniaxial tests. The general test provides an arbitrary point on the yield surface, given by the stress components in the material system as

$$\sigma = \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix},$$

and a tangent to the yield surface at that particular point. For the latter, the tangential direction should be determined so that

$$d_{xx}\dot{\epsilon}_{xx}^p + d_{yy}\dot{\epsilon}_{yy}^p + 2d_{xy}\dot{\epsilon}_{xy}^p = 0 .$$

The data for the general test can be set to zero in the input deck for LS-DYNA to just fit the uniaxial data.

The effective stress (excluding back stress) can be output to the d3plot database through \*DEFINE\_MATERIAL\_HISTORIES.

<i>*DEFINE_MATERIAL_HISTORIES Properties</i>			
<b>Label</b>	<b>Attributes</b>	<b>Description</b>	
Effective Stress	- - - -	Effective stress	$\sigma_{\text{eff}}(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$ , see above

3. **Kinematic hardening model.** A kinematic hardening model is implemented following the works of Chaboche and Roussilier. A back stress,  $\alpha$ , is introduced such that the effective stress is computed as

$$\sigma_{\text{eff}} = \sigma_{\text{eff}}(\sigma_{11} - 2\alpha_{11} - \alpha_{22}, \sigma_{22} - 2\alpha_{22} - \alpha_{11}, \sigma_{12} - \alpha_{12}) .$$

The back stress is the sum of up to four terms according to

$$\alpha_{ij} = \sum_{k=1}^4 \alpha_{ij}^k ,$$

and the evolution of each back stress component is as follows

$$\delta\alpha_{ij}^k = C_k \left( a_k \frac{s_{ij} - \alpha_{ij}}{\sigma_{\text{eff}}} - \alpha_{ij}^k \right) \delta\varepsilon_p ,$$

where  $C_k$  and  $a_k$  are material parameters,  $s_{ij}$  is the deviatoric stress tensor,  $\sigma_{\text{eff}}$  is the effective stress and  $\varepsilon_p$  is the effective plastic strain. The yield condition for this case is modified according to

$$f(\sigma, \alpha, \varepsilon_p) = \sigma_{\text{eff}} (\sigma_{xx} - 2\alpha_{xx} - \alpha_{yy}, \sigma_{yy} - 2\alpha_{yy} - \alpha_{xx}, \sigma_{xy} - \alpha_{xy}) \\ - \left\{ \sigma_Y^t(\varepsilon_p, \dot{\varepsilon}_p, 0) - \sum_{k=1}^4 a_k [1 - \exp(-C_k \varepsilon_p)] \right\} \leq 0$$

in order to get the expected stress-strain response for uniaxial stress.

4. **Hansel hardening law.** The Hansel hardening law is the same as in material 113 but is repeated here for the sake of convenience.

The hardening is temperature dependent, and therefore, this material model must be run either in a coupled thermo-mechanical solution, using prescribed temperatures, or using the adiabatic temperature calculation option. Setting the parameter CP to the specific heat  $C_p$  of the material activates the adiabatic temperature calculation that calculates the temperature rate from the equation

$$\dot{T} = \sum_{i,j} \frac{\sigma_{ij} D_{ij}^p}{\rho C_p} ,$$

where  $\sigma: \mathbf{D}^p$  (the numerator) is the plastically dissipated heat. Using the Kelvin scale is recommended, even though other scales may be used without problems.

The hardening behaviour is described by the following equations. The martensite rate equation is

$$\frac{\partial V_m}{\partial \bar{\varepsilon}^p} \\ = \begin{cases} 0 & \varepsilon < E_{0(\text{mart})} \\ \frac{B}{A} V_m^p \left( \frac{1 - V_m}{V_m} \right)^{\frac{B+1}{B}} \frac{[1 - \tanh(C + D \times T)]}{2} \exp\left(\frac{Q}{T - T_{A0}}\right) & \bar{\varepsilon}^p \geq E_{0(\text{mart})} \end{cases}$$

where  $\bar{\varepsilon}^p$  is the effective plastic strain and  $T$  is the temperature. The martensite fraction is integrated from the above rate equation:

$$V_m = \int_0^\varepsilon \frac{\partial V_m}{\partial \bar{\varepsilon}^p} d\bar{\varepsilon}^p .$$

It always holds that  $0.0 < V_m < 1.0$ . The initial martensite content is  $V_{m0}$  and must be greater than zero and less than 1.0. Note that  $V_{m0}$  is not used during a restart or when initializing the  $V_m$  history variable using \*INITIAL\_STRESS\_SHELL.

The yield stress  $\sigma_y$  is

$$\sigma_y = \{B_{HS} - (B_{HS} - A_{HS})\exp(-m[\bar{\varepsilon}^p + \varepsilon_0]^n)\}(K_1 + K_2 T) + \Delta H_{\gamma \rightarrow \alpha'} V_m.$$

The parameters P and B should fulfill the following condition

$$\frac{1+B}{B} < p .$$

If not fulfilled, the martensite rate will approach infinity as  $V_m$  approaches zero. A value between 0.001 and 0.02 is recommended for  $\varepsilon_0$ .

Apart from the effective true strain, a few additional history variables are output as described in the table below.

History Variable #	Description
26	Yield stress of material at temperature TREF. This variable is useful when evaluating the strength of the material after, for example, a simulated forming operation.
27	Volume fraction martensite, $V_m$
28	If CP = 0.0, it is not used. If CP > 0.0, then it is the temperature from the adiabatic temperature calculation.

5. **Heat treatment.** Heat treatment for increasing the formability of prestrained aluminum sheets can be simulated through the use of HTFLAG, where the intention is to run a forming simulation in steps involving preforming, springback, heat treatment, and postforming. In each step the history is transferred to the next using a dynain file (see \*INTERFACE\_SPRINGBACK). The first two steps are performed with HTFLAG = 0 according to standard procedures, resulting in a plastic strain field  $\varepsilon_p^0$  corresponding to the prestrain. The heat treatment step is performed using HTFLAG = 1 in a coupled thermomechanical simulation, where the blank is heated. The coupling between thermal and mechanical processes is only through the maximum temperature  $T^0$  being stored as a history variable in the material model, corresponding to the heat treatment temperature. Here it is important to export all history variables to the dynain file for the postforming step. In the final postforming step, HTFLAG = 2, the yield stress is then augmented by the Hocket-Sherby like term

$$\Delta\sigma = b - (b - a)\exp\left[-c(\varepsilon_p - \varepsilon_p^0)^d\right],$$

where  $a, b, c$ , and  $d$  are given as tables as functions of the heat treatment temperature  $T^0$  and prestrain  $\varepsilon_p^0$ . That is, in the table definitions each load curve corresponds to a given prestrain and the load curve value is with respect to the heat treatment temperature,

$$a = a(T^0, \varepsilon_p^0) , \quad b = b(T^0, \varepsilon_p^0) , \quad c = c(T^0, \varepsilon_p^0) , \quad d = d(T^0, \varepsilon_p^0)$$

The effect of heat treatment is that the material strength decreases but hardening increases, thus typically,

$$a \leq 0 , \quad b \geq a , \quad c > 0 , \quad d > 0 .$$

**\*MAT\_VISCOELASTIC\_FABRIC**

This is Material Type 134. The viscoelastic fabric model is a variation on the general viscoelastic model of material 76. This model is valid for 3 and 4 node membrane elements only and is strongly recommended for modeling isotropic viscoelastic fabrics where wrinkling may be a problem. For thin fabrics, buckling can result in an inability to support compressive stresses; thus, a flag is included for this option. If bending stresses are important use a shell formulation with model 76.

**Card Summary:**

**Card 1.** This card is required.

MID	RO	BULK				CSE	
-----	----	------	--	--	--	-----	--

**Card 2.** If fitting is done from a relaxation curve, specify fitting parameters on this card, otherwise if constants are set on Card 3, LEAVE THIS CARD BLANK.

LCID	NT	BSTART	TRAMP	LCIDK	NTK	BSTARTK	TRAMPK
------	----	--------	-------	-------	-----	---------	--------

**Card 3.** This card is not needed if Card 2 is defined (not blank). Up to 6 of this card may be input. If fewer than 6 cards are used, then the next keyword ("\*\*") card terminates this input.

GI	BETAI	KI	BETAKI				
----	-------	----	--------	--	--	--	--

**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	BULK				CSE	
Type	I	F	F				F	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number must be specified.
RO	Mass density.
BULK	Elastic constant bulk modulus. If the bulk behavior is viscoelastic, then this modulus is used in determining the contact interface stiffness only.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
CSE	Compressive stress flag (default = 0.0): EQ.0.0: Don't eliminate compressive stresses. EQ.1.0: Eliminate compressive stresses.

**Relaxation Curve Card.** If fitting is done from a relaxation curve, specify fitting parameters on card 2, *otherwise* if constants are set on Viscoelastic Constant Cards *LEAVE THIS CARD BLANK*.

Card 2	1	2	3	4	5	6	7	8
Variable	LCID	NT	BSTART	TRAMP	LCIDK	NTK	BSTARTK	TRAMPK
Type	F	I	F	F	F	I	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCID	Load curve ID if constants, $G_i$ , and $\beta_i$ are determined using a least squares fit. See <a href="#">Figure M134-1</a> .
NT	Number of terms in shear fit. If zero, the default is 6. Currently, the maximum number is set to 6.
BSTART	In the fit, $\beta_1$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$ , $\beta_4$ is 10 times $\beta_3$ , and so on. If zero, BSTART = 0.01.
TRAMP	Optional ramp time for loading.
LCIDK	Load curve ID for bulk behavior if constants, $K_i$ and $\beta_{\kappa_i}$ are determined using a least squares fit. See <a href="#">Figure M134-1</a> .
NTK	Number of terms desired in bulk fit. If zero, the default is 6. Currently, the maximum number is set to 6.
BSTARTK	In the fit, $\beta_{\kappa_1}$ is set to zero, $\beta_{\kappa_2}$ is set to BSTARTK, $\beta_{\kappa_3}$ is 10 times $\beta_{\kappa_2}$ , $\beta_{\kappa_4}$ is 10 times $\beta_{\kappa_3}$ , and so on. If zero, BSTARTK = 0.01.
TRAMPK	Optional ramp time for bulk loading

**Viscoelastic Constant Cards.** Up to 6 cards may be input. A keyword (“\*”) card terminates this input if fewer than 6 cards are used. These cards are not needed if relaxation data is defined. The number of terms for the shear behavior may differ from that for the bulk behavior: simply insert zero if a term is not included.

Card 3	1	2	3	4	5	6	7	8
Variable	Gi	BETAI <i>i</i>	Ki	BETAK <i>i</i>				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
GI	Optional shear relaxation modulus for the $i^{\text{th}}$ term
BETAI	Optional shear decay constant for the $i^{\text{th}}$ term
KI	Optional bulk relaxation modulus for the $i^{\text{th}}$ term
BETAKI	Optional bulk decay constant for the $i^{\text{th}}$ term

### Remarks:

Rate effects are taken into account through linear viscoelasticity through a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t - \tau) \frac{\partial \epsilon_{kl}}{\partial \tau} d\tau ,$$

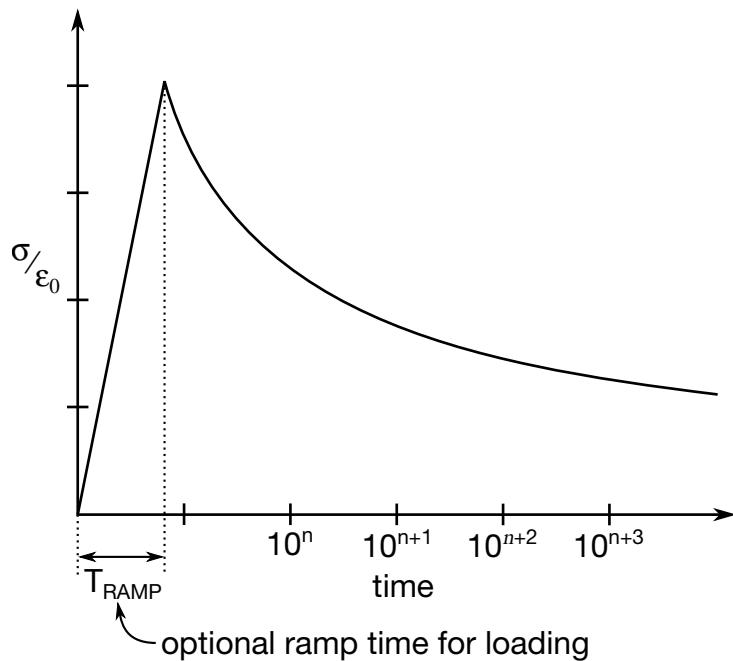
where  $g_{ijkl}(t - \tau)$  is the relaxation function. If we wish to include only simple rate effects for the deviatoric stresses, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \sum_{m=1}^N G_m e^{-\beta_m t} .$$

We characterize this function by the input shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . An arbitrary number of terms, up to 6, may be used when applying the viscoelastic model.

For volumetric relaxation, the relaxation function is also represented by the Prony series in terms of bulk moduli:

$$k(t) = \sum_{m=1}^N K_m e^{-\beta_{\kappa_m} t} .$$



**Figure M134-1.** Stress Relaxation Curve

For an example of a stress relaxation curve see [Figure M134-1](#). This curve defines stress as a function of time where time is defined on a logarithmic scale. For best results, the points defined in the load curve should be equally spaced on the logarithmic scale. Furthermore, the load curve should be smooth and defined in the positive quadrant. If non-physical values are determined by least squares fit, LS-DYNA will terminate with an error message after the initialization phase is completed. If the ramp time for loading is included, then the relaxation which occurs during the loading phase is taken into account. This effect may or may not be important.

**\*MAT\_WTM\_STM**

This is Material Type 135. This anisotropic-viscoplastic material model adopts two yield criteria for metals with orthotropic anisotropy proposed by Barlat and Lian [1989] (Weak Texture Model) and Aretz [2004] (Strong Texture Model).

**Card Summary:**

**Card 1.** This card is required.

MID	R0	E	PR	NUMFI	EPSC	WC	TAUC
-----	----	---	----	-------	------	----	------

**Card 2.** This card is required.

SIGMA0	QR1	CR1	QR2	CR2	K	LC	FLG
--------	-----	-----	-----	-----	---	----	-----

**Card 3a.** This card is included if and only if FLG = 0.

A1	A2	A3	A4	A5	A6	A7	A8
----	----	----	----	----	----	----	----

**Card 3b.** This card is included if and only if FLG = 1.

S00	S45	S90	SBB	R00	R45	R90	RBB
-----	-----	-----	-----	-----	-----	-----	-----

**Card 3c.** This card is included if and only if FLG = 2.

A	C	H	P				
---	---	---	---	--	--	--	--

**Card 4.** This card is required.

QX1	CX1	QX2	CX2	EDOT	M	EMIN	S100
-----	-----	-----	-----	------	---	------	------

**Card 5.** This card is required.

AOPT	BETA						
------	------	--	--	--	--	--	--

**Card 6.** This card is required.

			A1	A2	A3		
--	--	--	----	----	----	--	--

**Card 7.** This card is required.

V1	V2	V3	D1	D2	D3		
----	----	----	----	----	----	--	--

**\*MAT\_135****\*MAT\_WTM\_STM****Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	NUMFI	EPSC	WC	TAUC
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
NUMFI	Number of through thickness integration points that must fail before the element is deleted (remember to change this number if switching between full and reduced integration type of elements).
EPSC	Critical value $\varepsilon_{tC}$ of the plastic thickness strain (used in the CTS fracture criterion).
WC	Critical value $W_c$ for the Cockcroft-Latham fracture criterion
TAUC	Critical value $\tau_c$ for the Bressan-Williams shear fracture criterion

Card 2	1	2	3	4	5	6	7	8
Variable	SIGMA0	QR1	CR1	QR2	CR2	K	LC	FLG
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
SIGMA0	Initial mean value of yield stress $\sigma_0$ :
	GT.0.0: Constant value
	LT.0.0: Load curve ID = -SIGMA0 which defines yield stress as a function of plastic strain. Hardening parameters QR1,

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	CR1, QR2, and CR2 are ignored in that case.
QR1	Isotropic hardening parameter $Q_{R1}$
CR1	Isotropic hardening parameter $C_{R1}$
QR2	Isotropic hardening parameter $Q_{R2}$
CR2	Isotropic hardening parameter $C_{R2}$
K	$k$ , equals half YLD2003 exponent $m$ . Recommended value for FCC materials is $m = 8$ , that is, $k = 4$ .
LC	Load curve ID giving the relation between the pre-strain and the yield stress $\sigma_0$ . Similar curves for $Q_{R1}, C_{R1}, Q_{R2}, C_{R2}$ , and $W_c$ must follow consecutively from this number.
FLG	Flag to determine the card for defining yield: EQ.0: Use Card 3a for YLD2003 (STM). EQ.1: Use Card 3b for yield surface (STM – alternative input). EQ.2: Use Card 3c for YLD89 (WTM).

**YLD2003 Card.** This card 3 format is used when FLG = 0.

Card 3a	1	2	3	4	5	6	7	8
Variable	A1	A2	A3	A4	A5	A6	A7	A8
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
A1	YLD2003 parameter $a_1$
A2	YLD2003 parameter $a_2$
A3	YLD2003 parameter $a_3$
A4	YLD2003 parameter $a_4$
A5	YLD2003 parameter $a_5$
A6	YLD2003 parameter $a_6$

**\*MAT\_135****\*MAT\_WTM\_STM**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
A7	YLD2003 parameter $a_7$
A8	YLD2003 parameter $a_8$

**Yield Surface Card.** This card 3 format is used when FLG = 1.

Card 3b	1	2	3	4	5	6	7	8
Variable	S00	S45	S90	SBB	R00	R45	R90	RBB
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
S00	Yield stress in 0° direction
S45	Yield stress in 45° direction
S90	Yield stress in 90° direction
SBB	Balanced biaxial flow stress
R00	R-ratio in 0° direction
R45	R-ratio in 45° direction
R90	R-ratio in 90° direction
RBB	Balance biaxial flow ratio

**YLD89 Card.** This card 3 format used when FLG = 2.

Card 3c	1	2	3	4	5	6	7	8
Variable	A	C	H	P				
Type	F	F	F	F				

<b>VARIABLE</b>	<b>DESCRIPTION</b>
A	YLD89 parameter $a$

VARIABLE	DESCRIPTION
C	YLD89 parameter $c$
H	YLD89 parameter $h$
P	YLD89 parameter $p$

Card 4	1	2	3	4	5	6	7	8
Variable	QX1	CX1	QX2	CX2	EDOT	M	EMIN	S100
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
QX1	Kinematic hardening parameter $Q_{x1}$
CX1	Kinematic hardening parameter $C_{x1}$
QX2	Kinematic hardening parameter $Q_{x2}$
CX2	Kinematic hardening parameter $C_{x2}$
EDOT	Strain rate parameter $\dot{\epsilon}_0$
M	Strain rate parameter $m$
EMIN	Lower limit of the isotropic hardening rate $\frac{dR}{d\bar{\epsilon}}$ . This feature is included to model a non-zero and linear/exponential isotropic work hardening rate at large values of effective plastic strain. If the isotropic work hardening rate predicted by the utilized Voce-type work hardening rule falls below the specified value it is substituted by the prescribed value or switched to a power-law hardening if $S100 \neq 0$ . This option should be considered for problems involving extensive plastic deformations. If process dependent material characteristics are prescribed, that is, if $LC > 0$ the same minimum tangent modulus is assumed for all the prescribed work hardening curves. If instead $EMIN < 0$ then $-EMIN$ defines the plastic strain value at which the linear or power-law hardening approximation commences.
S100	Yield stress at 100% strain for using a power-law approximation beyond the strain defined by EMIN.

**\*MAT\_135****\*MAT\_WTM\_STM**

Card 5	1	2	3	4	5	6	7	8
Variable	AOPT	BETA						
Type	F	F						

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description): <ul style="list-style-type: none"> <li>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA</li> <li>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR</li> <li>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal</li> <li>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).</li> </ul>
BETA	Material angle in degrees for AOPT = 0 or 3. It may be overwritten on the element card; see *ELEMENT_SHELL_BETA.

Card 6	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
A1 A2 A3	Components of vector a for AOPT = 2

Card 7	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
V1 V2 V3	Components of vector $\mathbf{v}$ for AOPT = 3
D1 D2 D3	Components of vector $\mathbf{d}$ for AOPT = 2

**Remarks:**

1. **Material model.** The yield condition for this material can be written

$$t(\sigma, \alpha, \varepsilon^p, \dot{\varepsilon}^p) = \sigma_{\text{eff}}(\sigma, \alpha) - \sigma_Y(\varepsilon^p, \dot{\varepsilon}^p) .$$

The yield stress is defined as

$$\sigma_Y = [\sigma_0 + R(\varepsilon^p)] \left(1 + \frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_0}\right)^C ,$$

where the isotropic hardening reads

$$R(\dot{\varepsilon}^p) = Q_{R1}[1 - \exp(-C_{R1}\varepsilon^p)] + Q_{R2}[1 - \exp(-C_{R2}\varepsilon^p)] .$$

For the Weak Texture Model the yield function is defined as

$$\sigma_{\text{eff}} = \left[ \frac{1}{2} \{a(k_1 + k_2)^m + a(k_1 - k_2)^m + C(2k_2)^m\} \right]^{1/m}$$

where

$$k_1 = \frac{\sigma_x + h\sigma_y}{2}$$

$$k_2 = \sqrt{\left(\frac{\sigma_x + h\sigma_y}{2}\right)^2 + (p\sigma_{xy})^2}$$

For the Strong Texture Model the yield function is defined as

$$\sigma_{\text{eff}} = \left\{ \frac{1}{2} [(\sigma'_+)^m + (\sigma'_-)^m + (\sigma''_+ - \sigma''_-)^m] \right\}^{\frac{1}{m}} ,$$

where

$$\sigma'_{\pm} = \frac{a_8\sigma_x + a_1\sigma_y}{2} \pm \sqrt{\left(\frac{a_2\sigma_x - a_3\sigma_y}{2}\right)^2 + a_4^2\sigma_{xy}^2}$$

$$\sigma''_{\pm} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{a_5\sigma_x - a_6\sigma_y}{2}\right)^2 + a_7^2\sigma_{xy}}$$

Kinematic hardening can be included by

$$\alpha = \sum_{R=1}^2 \alpha_R ,$$

where each of the kinematic hardening variables  $\alpha_R$  is independent and obeys a nonlinear evolutionary equation in the form

$$\dot{\alpha}_R = C_{\alpha i} \left( Q_{\alpha i} \frac{\tau}{\sigma} - \alpha_R \right) \dot{\epsilon}^p .$$

The effective stress  $\bar{\sigma}$  is defined as

$$\bar{\sigma} = \sigma_{\text{eff}}(\tau) ,$$

where

$$\tau = \sigma - \alpha .$$

Critical thickness strain failure in a layer is assumed to occur when

$$\varepsilon_t \leq \varepsilon_{tc} ,$$

where  $\varepsilon_{tc}$  is a material parameter. It should be noted that  $\varepsilon_{tc}$  is a negative number (meaning failure is assumed to occur only in the case of thinning).

Cockcraft and Latham fracture is assumed to occur when

$$W = \int \max(\sigma_1, 0) d\varepsilon^p \geq W_C ,$$

where  $\sigma_1$  is the maximum principal stress and  $W_C$  is a material parameter.

2. **Yield surface parameters.** If FLG = 1, that is, if the yield surface parameters  $a_1$  through  $a_8$  are identified on the basis of prescribed material data internally in the material routine, files with point data for plotting of the identified yield surface, along with the predicted directional variation of the yield stress and plastic flow are generated in the directory where the LS-DYNA analysis is run. Four different files are generated for each specified material.

These files are named according to the scheme:

- a) Contour\_1#
- b) Contour\_2#
- c) Contour\_3#

## d) R\_and\_S#

where # is a value starting at 1.

The first three files contain contour data for plotting of the yield surface as shown in [Figure M135-2](#). To generate these plots a suitable plotting program should be adopted and for each file/plot, column A should be plotted as a function of column B. [Figure M135-3](#) further shows a plot generated from the final file named R\_and\_S# showing the directional dependency of the normalized yield stress (column A vs. B) and plastic strain ratio (column B vs. C).

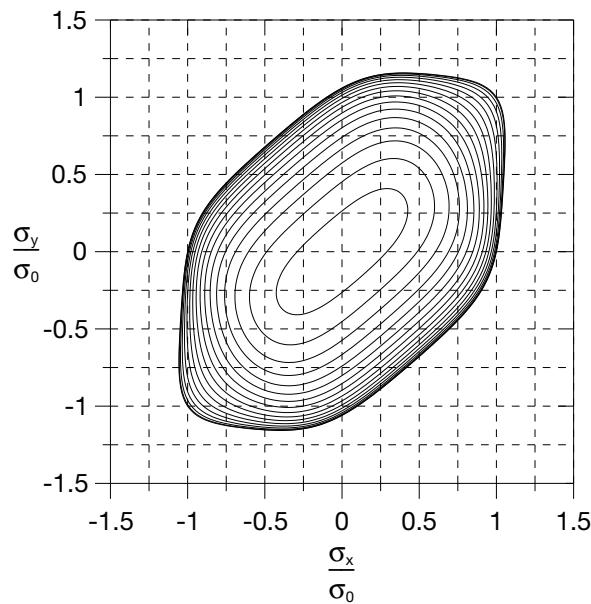
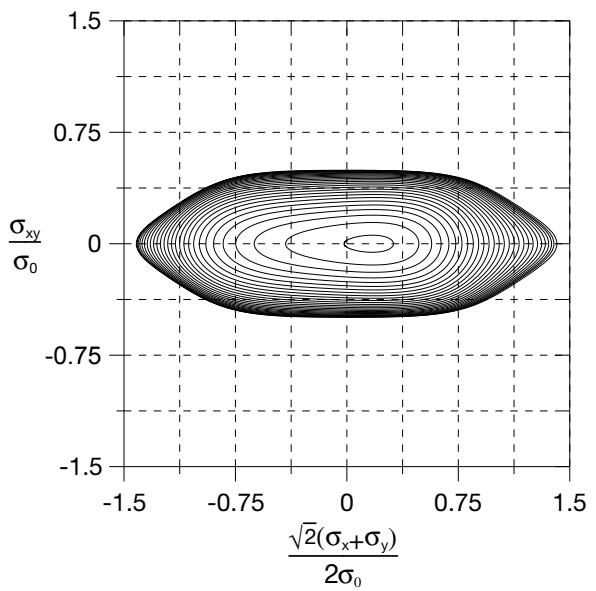
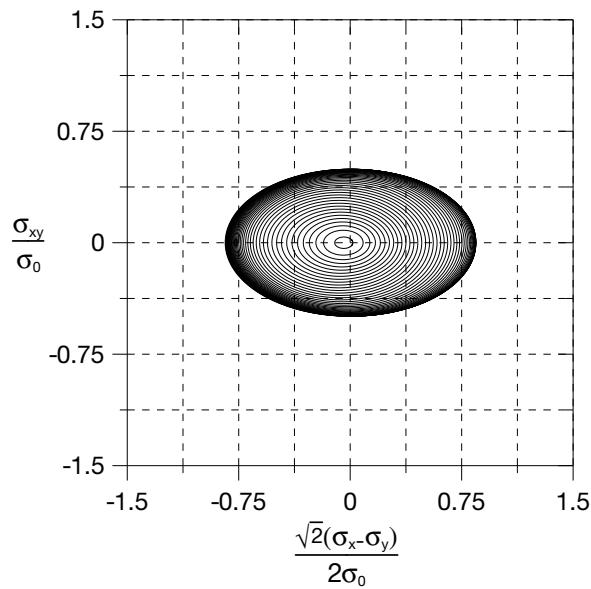
3. **History variables.** The following additional history variables can be included in the output d3plot file.

History Variable #	Description
1	Isotropic hardening value $R_1$
2	Isotropic hardening value $R_2$
3	Increment in effective plastic strain $\Delta\bar{\epsilon}$
4	Not defined, for internal use in the material model
5	Not defined, for internal use in the material model
6	Not defined, for internal use in the material model
7	Failure in integration point EQ.0: No failure EQ.1: Failure due to EPSC, i.e. $\varepsilon_t \geq \varepsilon_{tc}$ . EQ.2: Failure due to WC, i.e. $W \geq W_c$ . EQ.3: Failure due to TAUC, i.e. $\tau \geq \tau_c$
8	Sum of incremental strain in local element $x$ -direction: $\varepsilon_{xx} = \sum \Delta\varepsilon_{xx}$
9	Sum of incremental strain in local element $y$ -direction: $\varepsilon_{yy} = \sum \Delta\varepsilon_{yy}$
10	Value of the Cockcroft-Latham failure parameter $W = \sum \sigma_1 \Delta p$
11	Plastic strain component in thickness direction $\varepsilon_t$
12	Mean value of increments in plastic strain through the thickness. (For use with the non-local instability criterion. Note that constant lamella thickness is assumed, and the instability criterion can give unrealistic results if used with a user-defined integration rule with varying lamella thickness.)

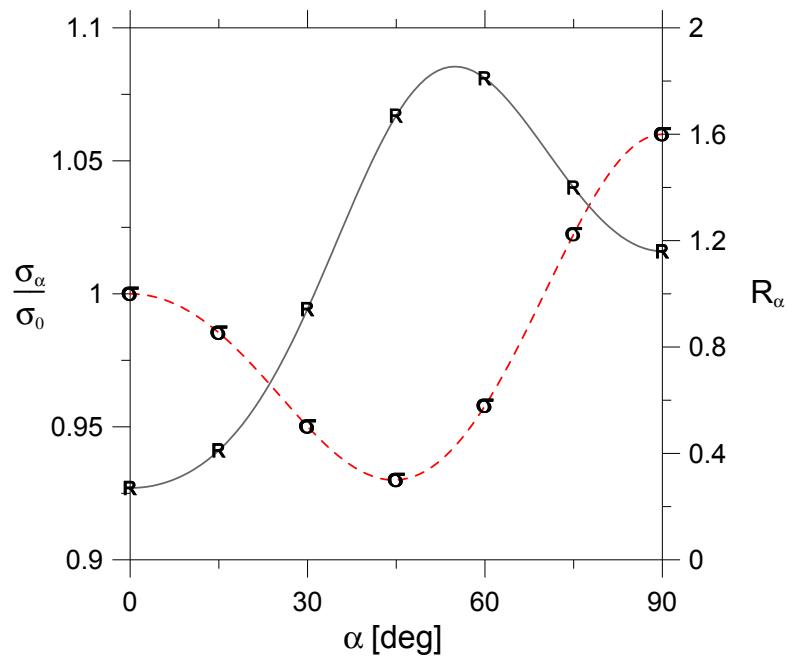
**\*MAT\_135****\*MAT\_WTM\_STM**

History Variable #	Description
13	Not defined, for internal use in the material model
14	Nonlocal value $\rho = \frac{\Delta\varepsilon_3}{\Delta\varepsilon_3^\Omega}$
17	Value of the Bressan-Williams failure parameter $\tau$

**Table M135-1.**

**(A)****(B)****(C)**

**Figure M135-2.** Contour plots of the yield surface generated from the files (a) 'Contour\_1<#>', (b) Contour\_2<#>, and (c) Contour\_3<#>.



**Figure M135-3.** Predicted directional variation of the yield stress and plastic flow generated from the file R\_and\_S<#>.

**\*MAT\_WTM\_STM\_PL****\*MAT\_135\_PL****\*MAT\_WTM\_STM\_PL**

This is Material Type 135. This anisotropic material adopts the yield criteria proposed by Aretz [2004]. The material strength is defined by McCormick's constitutive relation for materials exhibiting negative steady-state Strain Rate Sensitivity (SRS). McCormick [1998] and Zhang, McCormick and Estrin [2001].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	NUMFI	EPSC	WC	TAUC
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	SIGMA0	QR1	CR1	QR2	CR2	K		
Type	F	F	F	F	F	F		

Card 3	1	2	3	4	5	6	7	8
Variable	A1	A2	A3	A4	A5	A6	A7	A8
Type	F	F	F	F	F	F	F	F

Card 4	1	2	3	4	5	6	7	8
Variable	S	H	OMEGA	TD	ALPHA	EPS0		
Type	F	F	F	F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	AOPT	BETA						
Type	F	F						

**\*MAT\_135\_PLA****\*MAT\_WTM\_STM\_PLA**

Card 6	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 7	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
NUMFI	Number of through thickness integration points that must fail before the element is deleted (remember to change this number if switching between full and reduced integration type of elements)
EPSC	Critical value, $\varepsilon_{tC}$ , of the plastic thickness strain
WC	Critical value, $W_c$ , for the Cockcroft-Latham fracture criterion.
TAUC	Critical value, $\tau_c$ , for the shear fracture criterion.
SIGMA0	Initial yield stress, $\sigma_0$
QR1	Isotropic hardening parameter, $Q_{R1}$
CR1	Isotropic hardening parameter, $C_{R1}$
QR2	Isotropic hardening parameter, $Q_{R2}$
CR2	Isotropic hardening parameter, $C_{R2}$

VARIABLE	DESCRIPTION
K	$k$ equals half the exponent $m$ for the yield criterion
A1	Yld2003 parameter, $a_1$
A2	Yld2003 parameter, $a_2$
A3	Yld2003 parameter, $a_3$
A4	Yld2003 parameter, $a_4$
A5	Yld2003 parameter, $a_5$
A6	Yld2003 parameter, $a_6$
A7	Yld2003 parameter, $a_7$
A8	Yld2003 parameter, $a_8$
S	Dynamic strain aging parameter, $S$
H	Dynamic strain aging parameter, $H$
OMEGA	Dynamic strain aging parameter, $\Omega$
TD	Dynamic strain aging parameter, $t_d$
ALPHA	Dynamic strain aging parameter, $\alpha$
EPS0	Dynamic strain aging parameter, $\dot{\varepsilon}_0$
AOPT	Material axes option (see *MAT_OPTIONTROPIC_ELASTIC for a more complete description): <ul style="list-style-type: none"> <li>EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in <a href="#">Figure M2-1</a>, and then rotated about the shell element normal by the angle BETA. Nodes 1, 2 and 4 of an element are identical to the nodes used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES.</li> <li>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.</li> <li>EQ.3.0: Locally orthotropic material axes determined by offsetting the material axes by an angle, BETA, from a line determined by taking the cross product of the vector v</li> </ul>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	with the normal to the plane of the element.
LT.0.0:	The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
BETA	Material angle in degrees for AOPT = 0 and 3. BETA may be overwritten on the element card; see *ELEMENT_SHELL_BETA.
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2

**Remarks:**

The yield function is defined as

$$f = \sigma_{\text{eq}}(\sigma) - [\sigma_Y(t_a) + R(\varepsilon_p) + \sigma_v(\dot{\varepsilon}^p)] ,$$

where the equivalent stress  $\sigma_{\text{eq}}$  is defined as by an anisotropic yield criterion

$$\sigma_{\text{eq}} = \left[ \frac{1}{2} (|\sigma'_1|^m + |\sigma'_2|^m + |\sigma''_1 - \sigma''_2|) \right]^{\frac{1}{m}} .$$

Here

$$\begin{cases} \sigma'_1 \\ \sigma'_2 \end{cases} = \frac{a_8 \sigma_{xx} + a_1 \sigma_{yy}}{2} \pm \sqrt{\left( \frac{a_2 \sigma_{xx} - a_3 \sigma_{yy}}{2} \right)^2 + a_4^2 \sigma_{xy}^2}$$

and

$$\begin{cases} \sigma''_1 \\ \sigma''_2 \end{cases} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left( \frac{a_5 \sigma_{xx} - a_6 \sigma_{yy}}{2} \right)^2 + a_7^2 \sigma_{xy}^2} .$$

The strain hardening function,  $R$ , is defined by the extended Voce law

$$R(\varepsilon^p) = \sum_{i=1}^2 Q_{Ri} (1 - \exp(-C_{Ri} \varepsilon^p)) ,$$

where  $\varepsilon^p$  is the effective (or accumulated) plastic strain, and  $Q_{Ri}$  and  $C_{Ri}$  are strain hardening parameters.

Viscous stress,  $\sigma_v$ , is given by

$$\sigma_v = (\dot{\varepsilon}^p) = s \ln \left( 1 + \frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_0} \right) ,$$

where  $s$  represents the instantaneous strain rate sensitivity (SRS) and  $\dot{\varepsilon}_0$  is a reference strain rate. In this model the yield strength, including the contribution from dynamic strain aging (DSA) is defined as

$$\sigma_Y(t_a) = \sigma_0 + SH \left[ 1 - \exp \left\{ - \left( \frac{t_a}{t_d} \right)^\alpha \right\} \right]$$

where  $\sigma_0$  is the yield strength for vanishing average waiting time,  $t_a$  (meaning at high strain rates).  $H$ ,  $\alpha$ , and  $t_d$  are material constants linked to dynamic strain aging. It is noteworthy that  $\sigma_Y$  is an increasing function of  $t_a$ . The average waiting time is defined by the evolution equation

$$\dot{t}_a = 1 - \frac{t_a}{t_{a,ss}} ,$$

where the quasi-steady waiting time  $t_{a,ss}$  is given as

$$t_{a,ss} = \frac{\Omega}{\dot{\varepsilon}^p} .$$

Here  $\Omega$  is the strain produced by all mobile dislocations moving to the next obstacle on their path.

# \*MAT\_136

\*MAT\_VEGTER

## \*MAT\_VEGTER

This is Material Type 136 (formerly named \*MAT\_CORUS\_VEGTER), a plane stress orthotropic material model for metal forming. Yield surface construction is based on the interpolation by second-order Bezier curves, and model parameters are determined directly from a set of mechanical tests conducted for several directions. For each direction, four mechanical tests are carried out: a uniaxial, an equi-biaxial, a plane strain tensile test and a shear test. These test results are used to determine the coefficients of the Fourier directional dependency field. For a more detailed description please see Vegter and Boogaard [2006].

### Card Summary:

**Card 1.** This card is required.

MID	R0	E	PR	N	FBI	RBI0	LCID
-----	----	---	----	---	-----	------	------

**Card 2.** This card is required.

SYS	SIP	SHB	SHO	ESH	E0	ALPHA	LCID2
-----	-----	-----	-----	-----	----	-------	-------

**Card 3.** This card is required.

AOPT							
------	--	--	--	--	--	--	--

**Card 4.** This card is required.

XP	YP	ZP	A1	A2	A3		
----	----	----	----	----	----	--	--

**Card 5.** This card is required.

V1	V2	V3	D1	D2	D3	BETA	
----	----	----	----	----	----	------	--

**Card 6.** Include N+1 of this card.

FUN- <i>i</i>	RUN- <i>i</i>	FPS1- <i>i</i>	FPS2- <i>i</i>	FSH- <i>i</i>			
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR	N	FBI	RBI0	LCID
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Material density
E	Elastic Young's modulus
PR	Poisson's ratio
N	N  is the order of the Fourier series (meaning number of test groups minus one). The minimum number for  N  is 2, and the maximum is 10.  GE.0.0: Explicit cutting-plane return mapping algorithm LT.0.0: Fully implicit return mapping algorithm (more robust)
FBI	Normalized yield stress $\sigma_{bi}$ for equi-biaxial test
RBI0	Strain ratio $\sigma_{bi}(0^\circ) = \dot{\epsilon}_2(0^\circ)/\dot{\epsilon}_1(0^\circ)$ for equi-biaxial test in the rolling direction
LCID	Load curve ID or Table ID. If defined, SYS, SIP, SHB, SHO, ESH, and E0 are ignored.  <b>Load Curve.</b> When LCID is a load curve ID, it is taken as defining effective stress as a function of effective plastic strain.  <b>Tabular Data.</b> The table ID defines for each strain rate value a load curve ID giving the stress as a function effective plastic strain for that strain rate. Linear interpolation between the discrete strain rates is used by default.
	<b>Logarithmically Defined Tables.</b> A way to invoke logarithmic interpolation between discrete strain rates is described as follows. If the <i>first</i> value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate.

**\*MAT\_136****\*MAT\_VEGTER**

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
		There is some additional computational cost associated with invoking logarithmic interpolation.						
Card 2	1	2	3	4	5	6	7	8
Variable	SYS	SIP	SHB	SHO	ESH	E0	ALPHA	LCID2
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
SYS		Static yield stress, $\sigma_0$						
SIP		Stress increment parameter, $\Delta\sigma_m$						
SHB		Strain hardening parameter for large strain, $\beta$						
SHO		Strain hardening parameter for small strain, $\Omega$						
ESH		Exponent for strain hardening, $n$						
E0		Initial plastic strain, $\epsilon_0$						
ALPHA		Distribution of hardening used in the curve-fitting, $\alpha$ . $\alpha = 0$ is pure kinematic hardening while $\alpha = 1$ provides pure isotropic hardening.						
LCID2		Curve ID. The curve defines Young's modulus scaling factor with respect to the plastic strain. By default, the Young's modulus is assumed to remain constant. Effective value is between 0 and 1.						

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT							
Type	F							

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
AOPT		Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):						

**VARIABLE****DESCRIPTION**

- EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with \*DEFINE\_COORDINATE\_NODES and then rotated about the shell element normal by the angle BETA
- EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with \*DEFINE\_COORDINATE\_VECTOR
- EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector  $\mathbf{v}$  with the element normal
- LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on \*DEFINE\_COORDINATE\_NODES, \*DEFINE\_COORDINATE\_SYSTEM or \*DEFINE\_COORDINATE\_VECTOR).

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

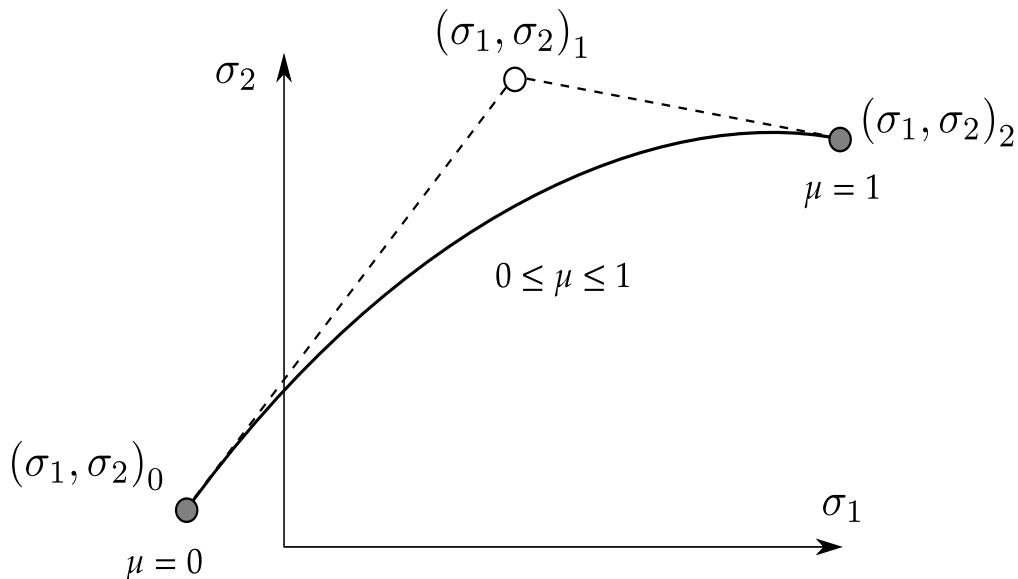
XP, YP, ZP      Coordinates of point  $p$  for AOPT = 1

A1, A2, A3      Components of vector  $\mathbf{a}$  for AOPT = 2

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

**VARIABLE****DESCRIPTION**

V1, V2, V3      Components of vector  $\mathbf{v}$  for AOPT = 3



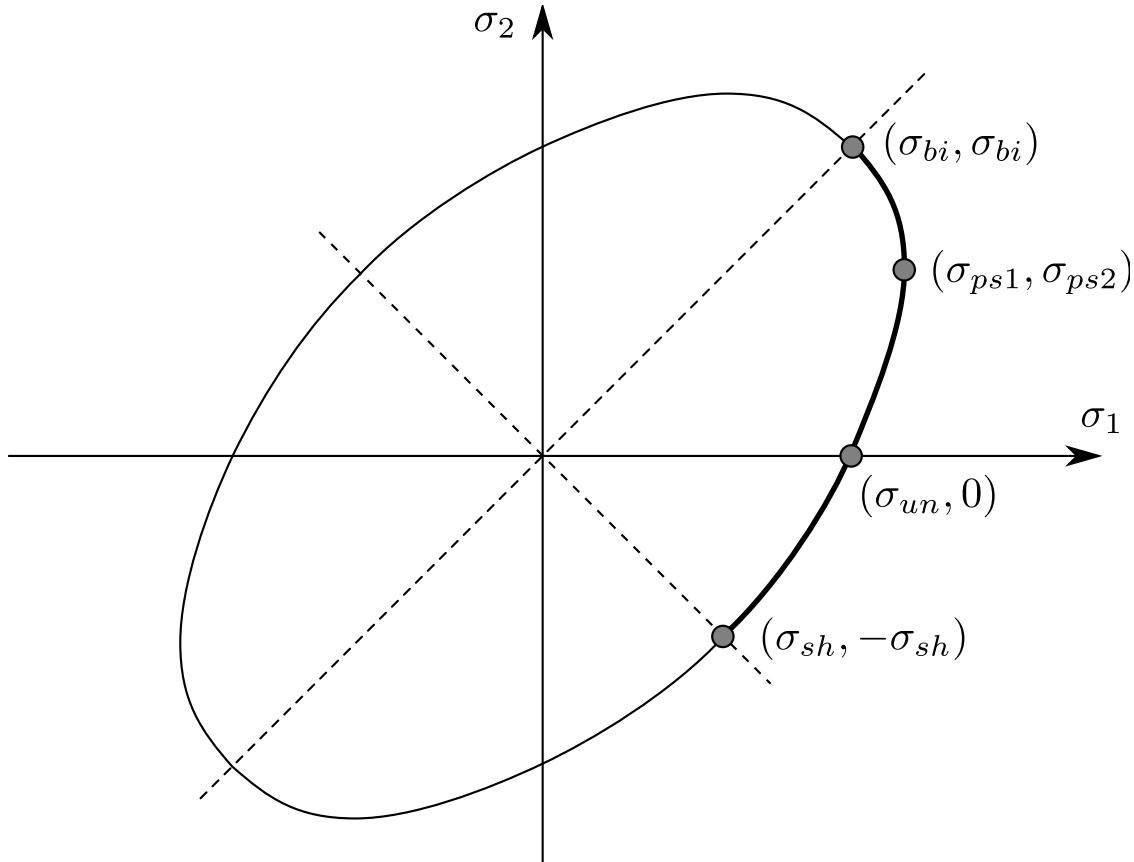
**Figure M136-1.** Bézier interpolation curve.

VARIABLE	DESCRIPTION
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2
BETA	Material angle in degrees for AOPT = 0 and 3. BETA may be overwritten on the element card; see *ELEMENT_SHELL_BETA.

**Experimental Data Cards.** The next  $N+1$  cards (see  $N$  on Card 1) contain experimental data obtained from four mechanical tests for a group of equidistantly placed directions  $\theta_i = \frac{i\pi}{2N}, i = 0, 1, 2, \dots, N$ .

Card 6	1	2	3	4	5	6	7	8
Variable	FUN- <i>i</i>	RUN- <i>i</i>	FPS1- <i>i</i>	FPS2- <i>i</i>	FSH- <i>i</i>			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION
FUN- <i>i</i>	Normalized yield stress $\sigma_{un}$ for uniaxial test for the $i^{\text{th}}$ direction
RUN- <i>i</i>	Strain ratio (R-value) for uniaxial test for the $i^{\text{th}}$ direction
FPS1- <i>i</i>	First normalized yield stress $\sigma_{ps1}$ for plain strain test for the $i^{\text{th}}$ direction



**Figure M136-2.** Vegter yield surface.

VARIABLE	DESCRIPTION
FPS2- <i>i</i>	Second normalized yield stress $\sigma_{ps2}$ for plain strain test for the $i^{\text{th}}$ direction
FSH- <i>i</i>	First normalized yield stress $\sigma_{sh}$ for pure shear test for the $i^{\text{th}}$ direction

### Remarks:

The Vegter yield locus is section-wise defined by quadratic Bézier interpolation functions. Each individual curve uses 2 reference points and a hinge point in the principal plane stress space; see [Figure M136-1](#).

The mathematical description of the Bézier interpolation is given by:

$$\left( \begin{array}{c} \sigma_1 \\ \sigma_2 \end{array} \right) = \left( \begin{array}{c} \sigma_1 \\ \sigma_2 \end{array} \right)_0 + 2\mu \left[ \left( \begin{array}{c} \sigma_1 \\ \sigma_2 \end{array} \right)_1 - \left( \begin{array}{c} \sigma_1 \\ \sigma_2 \end{array} \right)_0 \right] + \mu^2 \left[ \left( \begin{array}{c} \sigma_1 \\ \sigma_2 \end{array} \right)_2 + \left( \begin{array}{c} \sigma_1 \\ \sigma_2 \end{array} \right)_0 - 2\left( \begin{array}{c} \sigma_1 \\ \sigma_2 \end{array} \right)_1 \right],$$

where  $(\sigma_1, \sigma_2)_0$  is the first reference point,  $(\sigma_1, \sigma_2)_1$  is the hinge point, and  $(\sigma_1, \sigma_2)_2$  is the second reference point.  $\mu$  is a parameter which determines the location on the curve ( $0 \leq \mu \leq 1$ ).

Four characteristic stress states are selected as reference points: the equi-biaxial point  $(\sigma_{bi}, \sigma_{bi})$ , the plane strain point  $(\sigma_{ps1}, \sigma_{ps2})$ , the uniaxial point  $(\sigma_{un}, 0)$  and the pure shear point  $(\sigma_{sh}, -\sigma_{sh})$ ; see [Figure M136-2](#). Between the 4 stress points, 3 Bézier curves are used to interpolate the yield locus. Symmetry conditions are used to construct the complete surface. The yield locus in [Figure M136-2](#) shows the reference points of experiments for one specific direction. The reference points can also be determined for other angles to the rolling direction (planar angle  $\theta$ ). For example, if  $N = 2$  is chosen, normalized yield stresses for directions  $0^\circ$ ,  $45^\circ$ , and  $90^\circ$  should be defined. A Fourier series is used to interpolate intermediate angles between the measured points.

The Vegter yield function with isotropic hardening ( $\text{ALPHA} = 1$ ) is given as:

$$\phi = \sigma_{eq}(\sigma_1, \sigma_2, \theta) - \sigma_y(\bar{\epsilon}^p)$$

with the equivalent stress  $\sigma_{eq}$  obtained from the appropriate Bézier function related to the current stress state. The uniaxial yield stress  $\sigma_y$  can be defined as stress-strain curve LCID or alternatively as a functional expression:

$$\sigma_y = \sigma_0 + \Delta\sigma_m \left[ \beta(\bar{\epsilon}^p + \varepsilon_0) + \left(1 - e^{-\Omega(\bar{\epsilon}^p + \varepsilon_0)}\right)^n \right]$$

In case of kinematic hardening ( $\text{ALPHA} < 1$ ), the standard stress tensor is replaced by a relative stress tensor, defined as the difference between the stress tensor and a back stress tensor.

To determine the yield stress or reference points of the Vegter yield locus, four mechanical tests have to be performed for different directions. A good description about the material characterization procedure can be found in [Vegter et al. \(2003\)](#).

**\*MAT\_VEGTER\_STANDARD**

This is Material Type 136\_STD, a plane stress orthotropic material model for metal forming. Yield surface construction is based on the interpolation by second-order Bezier curves, and model parameters are determined directly from a set of mechanical tests conducted for a number of directions. For each direction, four mechanical tests are carried out: a uniaxial, an equi-biaxial, a plane strain tensile test and a shear test. The material formulation is equivalent to MAT\_VEGTER, except it requires different parameters for the plane strain tensile test and can use the Bergström-Van Liempt equation to deal with strain rate effects. These test results are used to determine the coefficients of the Fourier directional dependency field. For a more detailed description please see Vegter and Boogaard [2006].

**Card Summary:**

**Card 1.** This card is required.

MID	R0	E	PR	N	FBI	RBIO	LCID
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**Card 2.** This card is required.

SYS	SIP	SHB	SHO	ESH	E0	ALPHA	LCID2
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**Card 3.** This card is required.

AOPT		DYS	RATEN	SRNO	EXSR		
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**Card 4.** This card is required.

XP	YP	ZP	A1	A2	A3		
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**Card 5.** This card is required.

V1	V2	V3	D1	D2	D3	BETA	
----	----	----	----	----	----	------	--

**Card 6.** Include N+1 of this card.

FUN- <i>i</i>	RUN- <i>i</i>	FPS1- <i>i</i>	ALPS- <i>i</i>	FSH- <i>i</i>			
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	N	FBI	RBI0	LCID
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Material density
E	Elastic Young's modulus
PR	Poisson's ratio
N	Order of the Fourier series (meaning number of test groups minus one). The minimum number for N is 2, and the maximum is 10.
FBI	Normalized yield stress $\sigma_{bi}$ for equi-biaxial test
RBI0	Strain ratio $\sigma_{bi}(0^\circ) = \dot{\varepsilon}_2(0^\circ)/\dot{\varepsilon}_1(0^\circ)$ for equi-biaxial test in the rolling direction
LCID	Load curve ID or table ID. If defined, SYS, SIP, SHB, SHO, ESH, E0, DYS, RATEN, SRN0, and EXSR are ignored. <b>Load Curve.</b> When LCID is a load curve ID, it is taken as defining effective stress as a function of effective plastic strain. <b>Tabular Data.</b> The table ID defines for each strain rate value a load curve ID giving the stress as a function effective plastic strain for that strain rate. Linear interpolation between the discrete strain rates is used by default.
	<b>Logarithmically Defined Tables.</b> A way to invoke logarithmic interpolation between discrete strain rates is described as follows. If the <i>first</i> value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. There is some additional computational cost associated with invoking logarithmic interpolation.

**\*MAT\_VEGTER\_STANDARD****\*MAT\_136\_STD**

Card 2	1	2	3	4	5	6	7	8
Variable	SYS	SIP	SHB	SHO	ESH	E0	ALPHA	LCID2
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SYS	Static yield stress, $\sigma_0$
SIP	Stress increment parameter, $\Delta\sigma_m$
SHB	Strain hardening parameter for large strain, $\beta$
SHO	Strain hardening parameter for small strain, $\Omega$
ESH	Exponent for strain hardening, $n$
E0	Initial plastic strain, $\varepsilon_0$
ALPHA	Distribution of hardening used in the curve-fitting, $\alpha$ . $\alpha = 0$ is pure kinematic hardening while $\alpha = 1$ provides pure isotropic hardening.
LCID2	Curve ID. The curve defines Young's modulus scaling factor with respect to the plastic strain. By default, the Young's modulus is assumed to remain constant. Effective value is between 0 and 1.

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT		DYS	RATEN	SRNO	EXSR		
Type	F		F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):  EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES and then rotated about the shell element normal by the angle BETA

**\*MAT\_136\_STD****\*MAT\_VEGTER\_STANDARD**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR
	EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <b>v</b> with the element normal
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
DYS	Limit dynamic flow stress $\sigma_0^*$
RATEN	Ratio $r_{\text{enth}}$ of Boltzman constant $k$ (8.617E-5 eV/K) and maximum activation enthalpy $\Delta G_0$ (in eV): $r_{\text{enth}} = \frac{k}{\Delta G_0}$
SRN0	Limit strain rate $\dot{\varepsilon}_0$
EXSR	Exponent $m$ for strain rate behavior

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
XP, YP, ZP	Coordinates of point $p$ for AOPT = 1
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2
BETA	Material angle in degrees for AOPT = 0 and 3. BETA may be overwritten on the element card; see *ELEMENT_SHELL_BETA.

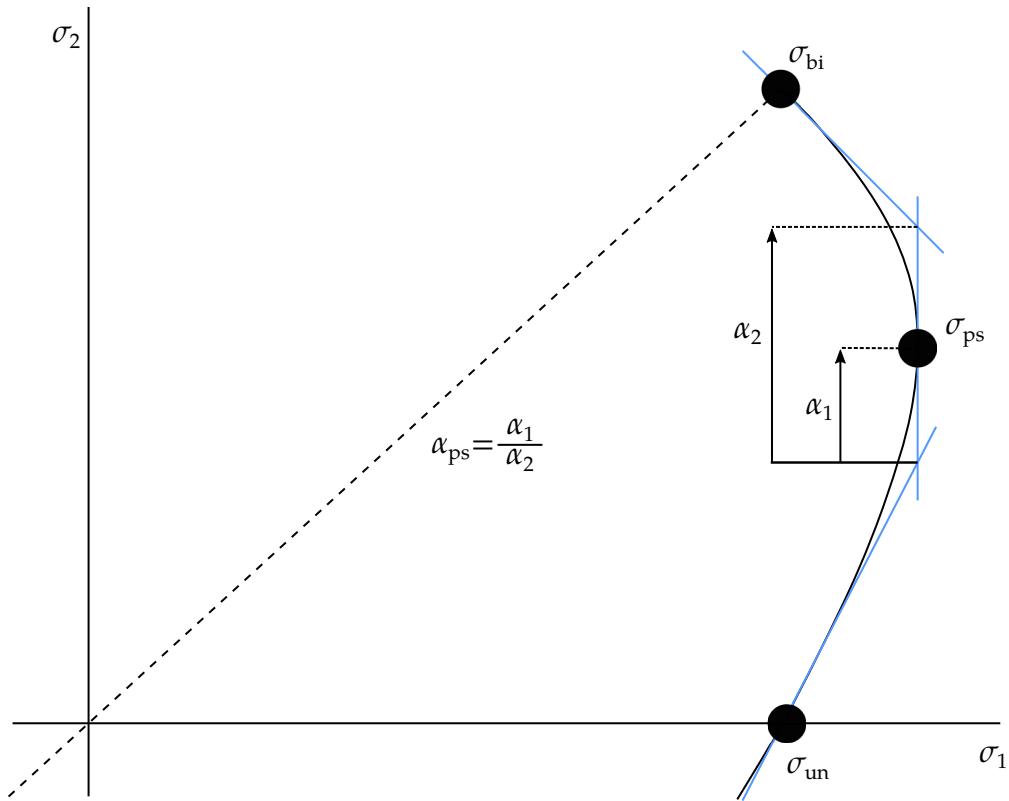
**Experimental Data Cards.** The next N+1 cards (see N on Card 1) contain experimental data obtained from four mechanical tests for a group of equidistantly placed directions  $\theta_i = i\pi/(2N)$ ,  $i = 0, 1, 2, \dots, N$ .

Card 6	1	2	3	4	5	6	7	8
Variable	FUN- <i>i</i>	RUN- <i>i</i>	FPS1- <i>i</i>	ALPS- <i>i</i>	FSH- <i>i</i>			
Type	F	F	F	F	F			

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FUN- <i>i</i>	Normalized yield stress $\sigma_{un}$ for uniaxial test for the $i^{\text{th}}$ direction
RUN- <i>i</i>	Strain ratio (R-value) for uniaxial test for the $i^{\text{th}}$ direction
FPS1- <i>i</i>	First normalized yield stress $\sigma_{ps1}$ for plain strain test for the $i^{\text{th}}$ direction
ALPS- <i>i</i>	Normalized distance $\alpha_{ps}$ of second component of plain stress point between the hinge points on both sides for the $i^{\text{th}}$ direction. See Remarks for details.
FSH- <i>i</i>	First normalized yield stress $\sigma_{sh}$ for pure shear test for the $i^{\text{th}}$ direction

### Remarks:

- Yield locus.** The yield locus description of this material is the same as for \*MAT\_VEGTER. The materials share the same Bézier interpolation for the section-wise definition of the yield locus and also use the same four characteristic stress states as reference points. They only differ in the plane-strain point definition in the input. This material MAT\_VEGTER\_STANDARD does not expect the direct input of the two components ( $\sigma_{ps1}, \sigma_{ps2}$ ), but only of the first component  $\sigma_{ps1}$ . The second component is assumed to be at a fixed distance between



**Figure M136-1.** Vegter yield surface

the hinge points on both sides. This distance is defined by factor  $\alpha_{ps} = \alpha_1/\alpha_2$ , as shown in [Figure M136-1](#). This approach is favored in most publications and has for example been discussed in the PhD-thesis of Pijlman, H. H. (2001).

To determine the yield stress or reference points of the Vegter yield locus, four mechanical tests must be performed for different directions. A good description about the material characterization procedure can be found in Vegter et al. (2003).

2. **Strain hardening.** The Vegter yield function with isotropic hardening (AL-PHA = 1) is given as:

$$\phi = \sigma_{eq}(\sigma_1, \sigma_2, \theta) - \sigma_y(\bar{\varepsilon}^p, \dot{\varepsilon}^p)$$

with the equivalent stress  $\sigma_{eq}$  obtained from the appropriate Bézier function related to the current stress state. The uniaxial yield stress  $\sigma_y$  can be defined as a yield stress curve,  $\sigma_y(\bar{\varepsilon}^p)$ , or a yield stress surface,  $\sigma_y(\bar{\varepsilon}^p, \dot{\varepsilon})$ , with LCID. In contrast to \*MAT\_VEGTER, this material also provides the temperature-dependent Bergström-Van Liempt equation as a third alternative:

$$\begin{aligned} \sigma_y(\bar{\varepsilon}^p, \dot{\varepsilon}^p, T) = & \sigma_0 + \Delta\sigma_m \left[ \beta(\bar{\varepsilon}^p + \varepsilon_0) + \left(1 - e^{-\Omega(\bar{\varepsilon}^p + \varepsilon_0)}\right)^n \right] \\ & + \sigma_0^* \left[ 1 + r_{enth} T \ln \left( \frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_0} \right) \right]^m, \end{aligned}$$

where  $T$  represents the temperature in K.

In the case of kinematic hardening ( $\text{ALPHA} < 1$ ), the standard stress tensor is replaced by a relative stress tensor, defined as the difference between the stress tensor and a back stress tensor.

## **\*MAT\_136\_2017**

**\*MAT\_VEGTER\_2017**

### **\*MAT\_VEGTER\_2017**

This is Material Type 136\_2017, a plane stress orthotropic material model for metal forming. It features the advanced Vegter yield locus based on the interpolation by second-order Bezier curves. Model parameters are determined from uniaxial test data at 0°, 45° and 90° to the rolling direction. Therefore, the same mechanical tests must be carried out as for Hill's 1948 planar anisotropic material model. For a more detailed description of the yield locus, please see Vegter and Boogaard [2006]. The relationships between the results of uniaxial testing and the advanced yield locus are introduced and discussed in Abspoel et al [2017].

#### **Card Summary:**

**Card 1.** This card is required.

MID	R0	E	PR		FBI	RBI0	LCID
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**Card 2.** This card is required.

SYS	SIP	SHB	SHO	ESH	E0	ALPHA	LCID2
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**Card 3.** This card is required.

AOPT		DYS	RATEN	SRNO	EXSR		
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**Card 4.** This card is required.

XP	YP	ZP	A1	A2	A3		
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**Card 5.** This card is required.

V1	V2	V3	D1	D2	D3	BETA	
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**Card 6.** This card is required.

RM-0	RM-45	RM-90	AG-0	AG-45	AG90		
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**Card 7.** This card is required.

R00	R45	R90					
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	E	PR		FBI	RBI0	LCID
Type	A	F	F	F		F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Material density
E	Elastic Young's modulus
PR	Poisson's ratio
FBI	Normalized yield stress $\sigma_{bi}$ for equi-biaxial test. If this value is not defined in the input, it will be approximated based on the uniaxial test result.
RBI0	Strain ratio $\sigma_{bi}(0^\circ) = \dot{\varepsilon}_2(0^\circ)/\dot{\varepsilon}_1(0^\circ)$ for equi-biaxial test in the rolling direction. If this value is not defined in the input, it will be approximated based on the uniaxial test result.
LCID	Load curve ID or Table ID. If defined, SYS, SIP, SHB, SHO, ESH, E0, DYS, RATEN, SRN0, and EXSR are ignored. <b>Load Curve.</b> When LCID is a load curve ID, it is taken as defining effective stress as a function of effective plastic strain. <b>Tabular Data.</b> The table ID defines for each strain rate value a load curve ID giving the stress as a function effective plastic strain for that strain rate. Linear interpolation between the discrete strain rates is used by default.
	<b>Logarithmically Defined Tables.</b> A way to invoke logarithmic interpolation between discrete strain rates is described as follows. If the <i>first</i> value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. There is some additional computational cost associated with invoking logarithmic interpolation.

**\*MAT\_136\_2017****\*MAT\_VEGTER\_2017**

Card 2	1	2	3	4	5	6	7	8
Variable	SYS	SIP	SHB	SHO	ESH	E0	ALPHA	LCID2
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SYS	Static yield stress, $\sigma_0$
SIP	Stress increment parameter, $\Delta\sigma_m$
SHB	Strain hardening parameter for large strain, $\beta$
SHO	Strain hardening parameter for small strain, $\Omega$
ESH	Exponent for strain hardening, $n$
E0	Initial plastic strain, $\varepsilon_0$
ALPHA	Distribution of hardening used in the curve-fitting, $\alpha$ . $\alpha = 0$ is pure kinematic hardening while $\alpha = 1$ provides pure isotropic hardening.
LCID2	Curve ID. The curve defines Young's modulus scaling factor with respect to the plastic strain. By default, the Young's modulus is assumed to remain constant. Effective value is between 0 and 1.

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT		DYS	RATEN	SRNO	EXSR		
Type	F		F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):  EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES and then rotated about the shell element normal by the angle BETA

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR
	EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <b>v</b> with the element normal
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
DYS	Limit dynamic flow stress, $\sigma_0^*$
RATEN	Ratio, $r_{\text{enth}}$ , of the Boltzmann constant, $k$ , (8.617E-5 eV/K) and maximum activation enthalpy, $\Delta G_0$ , (in eV): $r_{\text{enth}} = \frac{k}{\Delta G_0}$
SRN0	Limit strain rate, $\dot{\varepsilon}_0$
EXSR	Exponent, $m$ , for strain rate behavior

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
XP, YP, ZP	Coordinates of point $p$ for AOPT = 1
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2

**\*MAT\_136\_2017****\*MAT\_VEGTER\_2017**

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
V1, V2, V3		Components of vector <b>v</b> for AOPT = 3						
D1, D2, D3		Components of vector <b>d</b> for AOPT = 2						
BETA		Material angle in degrees for AOPT = 0 and 3. BETA may be overwritten on the element card; see *ELEMENT_SHELL_BETA.						

Card 6	1	2	3	4	5	6	7	8
Variable	RM-0	RM-45	RM-90	AG-0	AG-45	AG90		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
RM- <i>i</i>		Tensile strength for uniaxial testing at $i^\circ$ to rolling direction						
AG- <i>i</i>		Uniform elongation for uniaxial testing at $i^\circ$ to rolling direction						

Card 7	1	2	3	4	5	6	7	8
Variable	R00	R45	R90					
Type	F	F	F					

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
R00		Lankford parameter $R_{00}$						
R45		Lankford parameter $R_{45}$						
R90		Lankford parameter $R_{90}$						

**Remarks:**

1. **Yield Locus.** The yield locus description of this material is the same as for \*MAT\_VEGTER. The materials share the same Bézier interpolation for the section-wise definition of the yield locus.

The four characteristics stress states are predicted based on standard parameters from uniaxial tensile tests. This approach has been presented by Abspoel et al. (2017). Test data for 0°, 45°, and 90° to rolling direction must be given to account for anisotropic behavior of the material. The resulting formulation is then equivalent with \*MAT\_VEGTER\_STANDARD and N = 2.

2. **Strain Hardening.** The Vegter yield function with isotropic hardening (ALPHA = 1) is given as:

$$\phi = \sigma_{\text{eq}}(\sigma_1, \sigma_2, \theta) - \sigma_y(\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p, \dot{\varepsilon})$$

with the equivalent stress  $\sigma_{\text{eq}}$  obtained from the appropriate Bézier function related to the current stress state. The uniaxial yield stress  $\sigma_y$  can be defined as yield stress curve  $\sigma_y(\bar{\varepsilon}^p)$  with LCID or as  $\sigma_y(\bar{\varepsilon}^p, \dot{\varepsilon})$  with table LCID. In contrast to \*MAT\_VEGTER, this material also provides the temperature-dependent Bergström-Van Liempt equation as a third alternative:

$$\begin{aligned} \sigma_y(\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p, T) = \sigma_0 + \Delta\sigma_m & \left[ \beta(\bar{\varepsilon}^p + \varepsilon_0) + \left(1 - e^{-\Omega(\bar{\varepsilon}^p + \varepsilon_0)}\right)^n \right] \\ & + \sigma_0^* \left[ 1 + r_{\text{enth}} T \ln \left( \frac{\dot{\bar{\varepsilon}}^p}{\dot{\varepsilon}_0} \right) \right]^m, \end{aligned}$$

where  $T$  represents the temperature in K.

In the case of kinematic hardening (ALPHA < 1), the standard stress tensor is replaced by a relative stress tensor, defined as the difference between the stress tensor and a back stress tensor.

**\*MAT\_138****\*MAT\_COHESIVE\_MIXED\_MODE****\*MAT\_COHESIVE\_MIXED\_MODE**

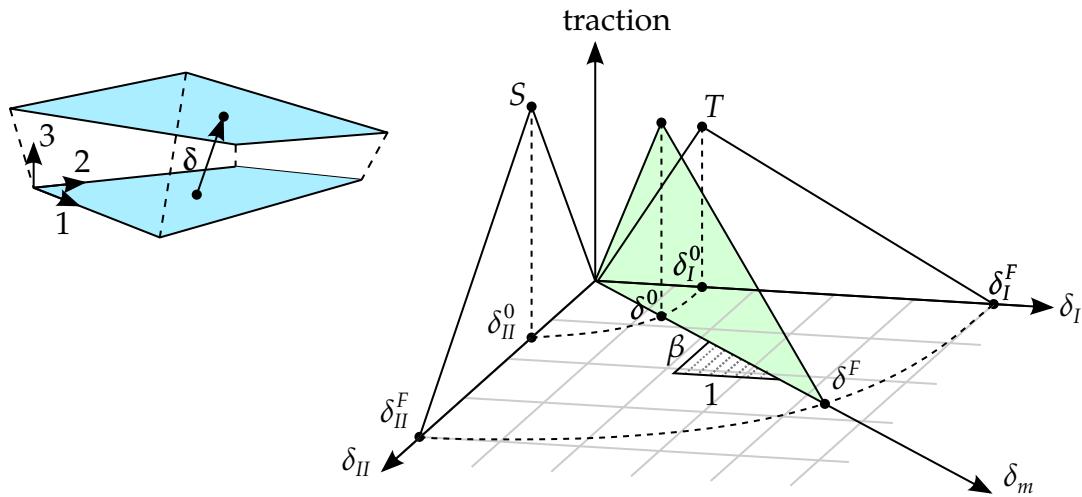
This is Material Type 138. This model is a simplification of \*MAT\_COHESIVE\_GENERAL, restricted to linear softening. It includes a bilinear traction-separation law with a quadratic mixed-mode delamination criterion and a damage formulation. This material model can only be used with cohesive element formulations; see the variable ELFORM in \*SECTION\_SOLID and \*SECTION\_SHELL.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	ROFLG	INTFAIL	EN	ET	GIC	GIIC
Type	A	F	I	F	F	F	F	F
Default	none	none	0	0.0	none	none	none	none

Card 2	1	2	3	4	5	6	7	8
Variable	XMU	T	S	UND	UTD	GAMMA		
Type	F	F	F	F	F	F		
Default	none	0.0	0.0	none	none	1.0		

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
ROFLG	Flag stating whether density is specified per unit area or volume: EQ.0: Specified density is per unit volume (default). EQ.1: Specified density is per unit area for controlling the mass of cohesive elements with an initial volume of zero.
INTFAIL	The number of integration points required for the cohesive element to be deleted. The value of INTFAIL may range from 1 to 4, with 1 being the recommended value. This field also determines the integration scheme.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	LT.0.0: Employs a Newton-Cotes integration scheme. The element will be deleted when  INTFAIL  integration points have failed.
	EQ.0.0: Employs a Newton-Cotes integration scheme. The element will <i>not</i> be deleted even if it satisfies the failure criterion.
	GT.0.0: Employs a Gauss integration scheme. The element will be deleted when INTFAIL integration points have failed.
EN	The stiffness (units of stress / length) normal to the plane of the cohesive element
ET	The stiffness (units of stress / length) in the plane of the cohesive element
GIC	Energy release rate for mode I (units of stress × length). LT.0.0: Load curve ID = (-GIC), which defines the energy release rate for mode I as a function of element size.
GIIC	Energy release rate for mode II (units of stress × length). LT.0.0: Load curve ID = (-GIIC), which defines the energy release rate for mode II as a function of element size.
XMU	Exponent of the mixed mode criteria (see <a href="#">Remark 2</a> )
T	Peak traction (stress units) in the normal direction. LT.0.0: Load curve ID = (-T), which defines peak traction in the normal direction as a function of element size. See <a href="#">Remark 4</a> . EQ.0.0: See <a href="#">Remark 1</a> . GT.0.0: Peak traction in the normal direction, T
S	Peak traction (stress units) in the tangential direction. LT.0.0: Load curve ID = (-S), which defines peak traction in the tangential direction as a function of element size. See <a href="#">Remark 4</a> . EQ.0.0: See <a href="#">Remark 1</a> . GT.0.0: Peak traction in the tangential direction, S



**Figure M138-1.** Mixed-mode traction-separation law

VARIABLE	DESCRIPTION
UND	Ultimate displacement in the normal direction
UTD	Ultimate displacement in the tangential direction
GAMMA	Additional exponent for Benzeggagh-Kenane law (default = 1.0)

#### Remarks:

1. **Ultimate Displacements.** The ultimate displacements in the normal and tangential directions are the displacements at the time when the material has entirely failed; that is, the tractions are zero. The linear stiffness for loading followed by the linear softening during the damage provides a straightforward relationship among the energy release rates, peak tractions, and ultimate displacements:

$$\text{GIC} = T \times \frac{\text{UND}}{2}$$

$$\text{GIIC} = S \times \frac{\text{UTD}}{2}$$

If the peak tractions are not specified, LS-DYNA calculates them from the ultimate displacements. See Fiolka and Matzenmiller [2005] and Gerlach, Fiolka and Matzenmiller [2005].

2. **Mixed-Mode Relative Displacement.** In this cohesive material model, the total mixed-mode relative displacement,  $\delta_m$ , is defined as  $\delta_m = \sqrt{\delta_I^2 + \delta_{II}^2}$ , where  $\delta_I = \sqrt{\delta_1^2 + \delta_2^2}$  is the separation in the normal direction (mode I) and  $\delta_{II} = \sqrt{\delta_3^2}$  is the

separation in the tangential direction (mode II). The mixed-mode damage initiation displacement  $\delta^0$  (onset of softening) is given by

$$\delta^0 = \delta_I^0 \delta_{II}^0 \sqrt{\frac{1 + \beta^2}{(\delta_{II}^0)^2 + (\beta \delta_I^0)^2}}$$

where  $\delta_I^0 = T/EN$  and  $\delta_{II}^0 = S/ET$  are the single mode damage initiation separations and  $\beta = \delta_{II}/\delta_I$  is the “mode mixity” (see [Figure M138-1](#)). The ultimate mixed-mode displacement  $\delta^F$  (total failure) for the power law ( $XMU > 0$ ) is:

$$\delta^F = \frac{2(1 + \beta^2)}{\delta^0} \left[ \left( \frac{EN}{GIC} \right)^{XMU} + \left( \frac{ET \times \beta^2}{GIIC} \right)^{XMU} \right]^{-1/XMU}$$

and, alternatively, for the Benzeggagh-Kenane law [1996] ( $XMU < 0$ ):

$$\delta^F = \frac{2}{\delta^0 \left( \frac{1}{1 + \beta^2} EN^\gamma + \frac{\beta^2}{1 + \beta^2} ET^\gamma \right)^{1/\gamma}} \left[ GIC + (GIIC - GIC) \left( \frac{\beta^2 \times ET}{EN + \beta^2 \times ET} \right)^{|XMU|} \right]$$

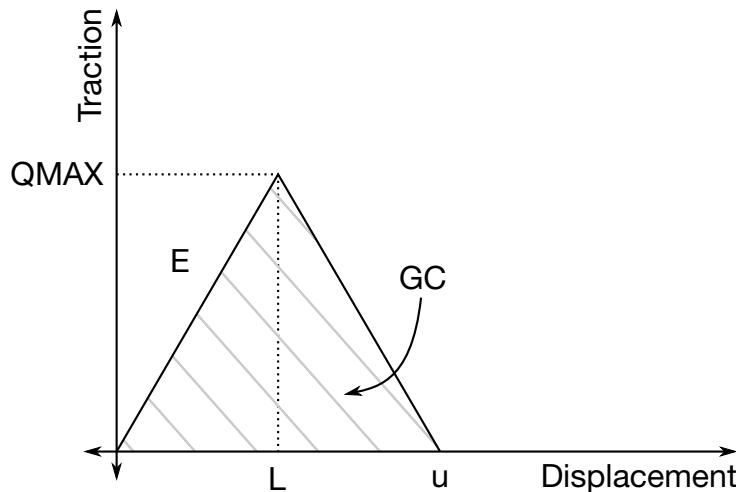
A reasonable choice for the exponent  $\gamma$  would be GAMMA = 1.0 (default) or GAMMA = 2.0.

3. **Interface Damage.** This model considers damage to the interface. The model enforces irreversible conditions with loading/unloading paths coming from/pointing to the origin.
4. **Peak Traction as Load Curves.** Peak tractions  $T$  and/or  $S$  can be defined as functions of characteristic element length (square root of mid-surface area) using a load curve. This option helps obtain the same global responses (e.g., load-displacement curve) with coarse meshes compared to the solution with a fine mesh. In general, lower peak traction values are needed for coarser meshes.
5. **Error Checks of Material Data.** We have implemented three error checks for this material model to ensure proper material data. Since the traction as a function of displacement curve is fairly simple (triangular shaped), we can check to ensure that the displacement,  $L$ , at the peak load (QMAX), is smaller than the ultimate distance for failure,  $u$ . See [Figure M138-2](#) for the used notation.

As shown in [Figure M138-2](#),

$$GC = \frac{1}{2} u \times QMAX$$

and



**Figure M138-2.** Bilinear traction-separation

$$L = \frac{Q_{MAX}}{E} .$$

To ensure the peak is not past the failure point,  $u/L$  must be larger than 1. Here,

$$u = \frac{2GC}{EL} ,$$

where GC is the energy release rate. This gives

$$\frac{u}{L} = \frac{2GC}{EL \times L} = \frac{2GC}{E \left( \frac{Q_{MAX}}{E} \right)^2} > 1 .$$

Based on this, LS-DYNA performs three error checks, one for tension, one for pure shear, and one for mixed modes:

$$\begin{aligned} \frac{u}{L} &= \frac{\delta_I^F}{\delta_I^0} = \frac{2GIC}{EN \left( \frac{T}{EN} \right)^2} > 1 \\ \frac{u}{L} &= \frac{\delta_{II}^F}{\delta_{II}^0} = \frac{2GIIC}{ET \left( \frac{S}{ET} \right)^2} > 1 \\ \frac{u}{L} &= \frac{\delta^F}{\delta^0} > 1 \end{aligned}$$

In this last equation, we did not perform the substitution as the equations are complicated and depend on the sign of XMU (see Remark 2). The value of XMU significantly affects  $\delta^F$  and should be chosen carefully. Because this check occurs during initialization, LS-DYNA computes the mode-mixity,  $\beta$ , using the displacements at failure given in input. Thus, it does not reflect any specific loading scenario.

**\*MAT\_MODIFIED\_FORCE\_LIMITED**

This is Material Type 139. This material which is for the Belytschko-Schwer resultant beam is an extension of MAT\_029. In addition to the original plastic hinge and collapse mechanisms of MAT\_029, yield moments may be defined as a function of axial force. After a hinge forms, the moment transmitted by the hinge is limited by a moment-plastic rotation relationship.

**Card Summary:**

**Card 1.** This card is required.

MID	R0	E	PR	DF	IAFLC	YTFLAG	ASOFT
-----	----	---	----	----	-------	--------	-------

**Card 2.** This card is required.

M1	M2	M3	M4	M5	M6	M7	M8
----	----	----	----	----	----	----	----

**Card 3.** This card is required.

LC1	LC2	LC3	LC4	LC5	LC6	LC7	LC8
-----	-----	-----	-----	-----	-----	-----	-----

**Card 4.** This card is required.

LPS1	SFS1	LPS2	SFS2	YMS1	YMS2		
------	------	------	------	------	------	--	--

**Card 5.** This card is required.

LPT1	SFT1	LPT2	SFT2	YMT1	YMT2		
------	------	------	------	------	------	--	--

**Card 6.** This card is required.

LPR	SFR	YMR					
-----	-----	-----	--	--	--	--	--

**Card 7.** This card is required.

LYS1	SYS1	LYS2	SYS2	LYT1	SYT1	LYT2	SYT2
------	------	------	------	------	------	------	------

**Card 8.** This card is required.

LYR	SYR						
-----	-----	--	--	--	--	--	--

**Card 9.** This card is required.

HMS1_1	HMS1_2	HMS1_3	HMS1_4	HMS1_5	HMS1_6	HMS1_7	HMS1_8
--------	--------	--------	--------	--------	--------	--------	--------

**\*MAT\_139****\*MAT\_MODIFIED\_FORCE\_LIMITED**

**Card 10.** This card is required.

LPMS1_1	LPMS1_2	LPMS1_3	LPMS1_4	LPMS1_5	LPMS1_6	LPMS1_7	LPMS1_8
---------	---------	---------	---------	---------	---------	---------	---------

**Card 11.** This card is required.

HMS2_1	HMS2_2	HMS2_3	HMS2_4	HMS2_5	HMS2_6	HMS2_7	HMS2_8
--------	--------	--------	--------	--------	--------	--------	--------

**Card 12.** This card is required.

LPMS2_1	LPMS2_2	LPMS2_3	LPMS2_4	LPMS2_5	LPMS2_6	LPMS2_7	LPMS2_8
---------	---------	---------	---------	---------	---------	---------	---------

**Card 13.** This card is required.

HMT1_1	HMT1_2	HMT1_3	HMT1_4	HMT1_5	HMT1_6	HMT1_7	HMT1_8
--------	--------	--------	--------	--------	--------	--------	--------

**Card 14.** This card is required.

LPMT1_1	LPMT1_2	LPMT1_3	LPMT1_4	LPMT1_5	LPMT1_6	LPMT1_7	LPMT1_8
---------	---------	---------	---------	---------	---------	---------	---------

**Card 15.** This card is required.

HMT2_1	HMT2_2	HMT2_3	HMT2_4	HMT2_5	HMT2_6	HMT2_7	HMT2_8
--------	--------	--------	--------	--------	--------	--------	--------

**Card 16.** This card is required.

LPMT2_1	LPMT2_2	LPMT2_3	LPMT2_4	LPMT2_5	LPMT2_6	LPMT2_7	LPMT2_8
---------	---------	---------	---------	---------	---------	---------	---------

**Card 17.** This card is required.

HMR_1	HMR_2	HMR_3	HMR_4	HMR_5	HMR_6	HMR_7	HMR_8
-------	-------	-------	-------	-------	-------	-------	-------

**Card 18.** This card is required.

LPMR_1	LPMR_2	LPMR_3	LPMR_4	LPMR_5	LPMR_6	LPMR_7	LPMR_8
--------	--------	--------	--------	--------	--------	--------	--------

**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	DF	IAFLC	YTFLAG	ASOFT
Type	A	F	F	F	F	I	F	F
Default	none	none	none	none	0.0	0	0.0	0.0

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified.
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
DF	Damping factor; see <a href="#">Remark 2</a> . <i>A proper control for the timestep must be maintained by the user.</i>
IAFLC	Axial load curve option: EQ.0: Axial load curves are force as a function of strain. EQ.1: Axial load curves are force as a function of change in length.
YTFLAG	Flag to allow beam to yield in tension: EQ.0.0: Beam does not yield in tension. EQ.1.0: Beam can yield in tension.
ASOFT	Axial elastic softening factor applied once hinge has formed. When a hinge has formed, the stiffness is reduced by this factor. If zero, this factor is ignored.

**\*MAT\_139****\*MAT\_MODIFIED\_FORCE\_LIMITED**

Card 2	1	2	3	4	5	6	7	8
Variable	M1	M2	M3	M4	M5	M6	M7	M8
Type	F	F	F	F	F	F	F	F
Default	none	0	0	0	0	0	0	0

**VARIABLE****DESCRIPTION**M1, M2,  
..., M8

Applied end moment for force as a function of strain/change in length curve. At least one moment must be defined with a maximum of 8. The values should be in ascending order.

Card 3	1	2	3	4	5	6	7	8
Variable	LC1	LC2	LC3	LC4	LC5	LC6	LC7	LC8
Type	I	I	I	I	I	I	I	I
Default	none	0	0	0	0	0	0	0

**VARIABLE****DESCRIPTION**LC1, LC2,  
..., LC8

Load curve ID (see \*DEFINE\_CURVE) defining axial force as a function of strain/change in length (see IAFLC) for the corresponding applied end moment. Define the same number as end moments. Each curve must contain the same number of points.

Card 4	1	2	3	4	5	6	7	8
Variable	LPS1	SFS1	LPS2	SFS2	YMS1	YMS2		
Type	I	F	I	F	F	F		
Default	0	1.0	LPS1	1.0	$10^{20}$	YMS1		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LPS1	Load curve ID for plastic moment as a function of rotation about the <i>s</i> -axis at node 1. If zero, this load curve is ignored.
SFS1	Scale factor for plastic moment as a function of rotation curve about the <i>s</i> -axis at node 1.
LPS2	Load curve ID for plastic moment as a function of rotation about the <i>s</i> -axis at node 2. The default is LPS1.
SFS2	Scale factor for plastic moment as a function of rotation curve about the <i>s</i> -axis at node 2. Default: SFS1.
YMS1	Yield moment about the <i>s</i> -axis at node 1 for interaction calculations (default set to $10^{20}$ to prevent interaction)
YMS2	Yield moment about the <i>s</i> -axis at node 2 for interaction calculations (default set to YMS1)

Card 5	1	2	3	4	5	6	7	8
Variable	LPT1	SFT1	LPT2	SFT2	YMT1	YMT2		
Type	I	F	I	F	F	F		
Default	0	1.0	LPT1	1.0	$10^{20}$	YMT1		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LPT1	Load curve ID for plastic moment as a function of rotation about the <i>t</i> -axis at node 1. If zero, this load curve is ignored.
SFT1	Scale factor for plastic moment as a function of rotation curve about the <i>t</i> -axis at node 1. Default = 1.0.
LPT2	Load curve ID for plastic moment as a function of rotation about the <i>t</i> -axis at node 2. Default: LPT1.
SFT2	Scale factor for plastic moment as a function of rotation curve about the <i>t</i> -axis at node 2. Default: SFT1.
YMT1	Yield moment about the <i>t</i> -axis at node 1 for interaction calculations (default set to $10^{20}$ to prevent interactions)

**\*MAT\_139****\*MAT\_MODIFIED\_FORCE\_LIMITED**

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
YMT2		Yield moment about the <i>t</i> -axis at node 2 for interaction calculations (default set to YMT1)						
Card 6	1	2	3	4	5	6	7	8
Variable	LPR	SFR	YMR					
Type	I	F	F					
Default	0	1.0	$10^{20}$					

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
LPR		Load curve ID for plastic torsional moment as a function of rotation. If zero, this load curve is ignored.						
SFR		Scale factor for plastic torsional moment as a function of rotation (default = 1.0)						
YMR		Torsional yield moment for interaction calculations (default set to $10^{20}$ to prevent interaction)						

Card 7	1	2	3	4	5	6	7	8
Variable	LYS1	SYS1	LYS2	SYS2	LYT1	SYT1	LYT2	SYT2
Type	I	F	I	F	I	F	I	F
Default	0	1.0	0	1.0	0	1.0	0	1.0

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
LYS1		ID of curve defining yield moment as a function of axial force for the <i>s</i> -axis at node 1						
SYS1		Scale factor applied to load curve LYS1						
LYS2		ID of curve defining yield moment as a function of axial force for the <i>s</i> -axis at node 2						

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SYS2	Scale factor applied to load curve LYS2
LYT1	ID of curve defining yield moment as a function of axial force for the <i>t</i> -axis at node 1
SYT1	Scale factor applied to load curve LYT1
LYT2	ID of curve defining yield moment as a function of axial force for the <i>t</i> -axis at node 2
SYT2	Scale factor applied to load curve LYT2

Card 8	1	2	3	4	5	6	7	8
Variable	LYR	SYR						
Type	I	F						
Default	0	1.0						

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LYR	ID of curve defining yield moment as a function of axial force for the torsional axis.
SYR	Scale factor applied to load curve LYR.

Card 9	1	2	3	4	5	6	7	8
Variable	HMS1_1	HMS1_2	HMS1_3	HMS1_4	HMS1_5	HMS1_6	HMS1_7	HMS1_8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

<b>VARIABLE</b>	<b>DESCRIPTION</b>
HMS1_1	Hinge moment for the <i>s</i> -axis at node 1

**\*MAT\_139****\*MAT\_MODIFIED\_FORCE\_LIMITED**

Card 10	1	2	3	4	5	6	7	8
Variable	LPMS1_1	LPMS1_2	LPMS1_3	LPMS1_4	LPMS1_5	LPMS1_6	LPMS1_7	LPMS1_8
Type	I	I	I	I	I	I	I	I
Default	0	0	0	0	0	0	0	0

**VARIABLE****DESCRIPTION**

LPMS1\_n

ID of curve defining plastic moment as a function of plastic rotation  
for the s-axis at node 1 for hinge moment HMS1\_n

Card 11	1	2	3	4	5	6	7	8
Variable	HMS2_1	HMS2_2	HMS2_3	HMS2_4	HMS2_5	HMS2_6	HMS2_7	HMS2_8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

**VARIABLE****DESCRIPTION**

HMS2\_n

Hinge moment for the s-axis at node 2

Card 12	1	2	3	4	5	6	7	8
Variable	LPMS2_1	LPMS2_2	LPMS2_3	LPMS2_4	LPMS2_5	LPMS2_6	LPMS2_7	LPMS2_8
Type	I	I	I	I	I	I	I	I
Default	0	0	0	0	0	0	0	0

**VARIABLE****DESCRIPTION**

LPMS2\_n

ID of curve defining plastic moment as a function of plastic rotation  
for the s-axis at node 2 for hinge moment HMS2\_n

**\*MAT\_MODIFIED\_FORCE\_LIMITED****\*MAT\_139**

Card 13	1	2	3	4	5	6	7	8
Variable	HMT1_1	HMT1_2	HMT1_3	HMT1_4	HMT1_5	HMT1_6	HMT1_7	HMT1_8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

**VARIABLE****DESCRIPTION**HMT1\_ *n* Hinge moment for the *t*-axis at node 1

Card 14	1	2	3	4	5	6	7	8
Variable	LPMT1_1	LPMT1_2	LPMT1_3	LPMT1_4	LPMT1_5	LPMT1_6	LPMT1_7	LPMT1_8
Type	I	I	I	I	I	I	I	I
Default	0	0	0	0	0	0	0	0

**VARIABLE****DESCRIPTION**LPMT1\_ *n* ID of curve defining plastic moment as a function of plastic rotation for the *t*-axis at node 1 for hinge moment HMT1\_ *n*

Card 15	1	2	3	4	5	6	7	8
Variable	HMT2_1	HMT2_2	HMT2_3	HMT2_4	HMT2_5	HMT2_6	HMT2_7	HMT2_8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

**VARIABLE****DESCRIPTION**HMT2\_ *n* Hinge moment for the *t*-axis at node 2

**\*MAT\_139****\*MAT\_MODIFIED\_FORCE\_LIMITED**

Card 16	1	2	3	4	5	6	7	8
Variable	LPMT2_1	LPMT2_2	LPMT2_3	LPMT2_4	LPMT2_5	LPMT2_6	LPMT2_7	LPMT2_8
Type	I	I	I	I	I	I	I	I
Default	0	0	0	0	0	0	0	0

**VARIABLE****DESCRIPTION**LPMT2\_*n*ID of curve defining plastic moment as a function of plastic rotation  
for the *t*-axis at node 2 for hinge moment HMT2\_*n*

Card 17	1	2	3	4	5	6	7	8
Variable	HMR_1	HMR_2	HMR_3	HMR_4	HMR_5	HMR_6	HMR_7	HMR_8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

**VARIABLE****DESCRIPTION**HMR\_*n*

Hinge moment for the torsional axis

Card 18	1	2	3	4	5	6	7	8
Variable	LPMR_1	LPMR_2	LPMR_3	LPMR_4	LPMR_5	LPMR_6	LPMR_7	LPMR_8
Type	I	I	I	I	I	I	I	I
Default	0	0	0	0	0	0	0	0

**VARIABLE****DESCRIPTION**LPMR\_*n*ID of curve defining plastic moment as a function of plastic rotation  
for the torsional axis for hinge moment HMR\_*n*

**Remarks:**

1. **Load Curves.** This material model is available for the Belytschko resultant beam element only. Plastic hinges form at the ends of the beam when the moment reaches the plastic moment. The plastic moment as a function of rotation relationship is specified by the user in the form of a load curve and scale factor. The points of the load curve are (plastic rotation in radians, plastic moment). Both quantities should be positive for all points, with the first point being (zero, initial plastic moment). Within this constraint any form of characteristic may be used, including flat or falling curves. Different load curves and scale factors may be specified at each node and about each of the local *s* and *t* axes.

Axial collapse occurs when the compressive axial load reaches the collapse load. Collapse load as a function of collapse deflection is specified in the form of a load curve. The points of the load curve are either (true strain, collapse force) or (change in length, collapse force). Both quantities should be entered as positive for all points and will be interpreted as compressive. The first point should be (zero, initial collapse load).

The collapse load may vary with end moment as well as with deflections. In this case several load-deflection curves are defined, each corresponding to a different end moment. Each load curve should have the same number of points and the same deflection values. The end moment is defined as the average of the absolute moments at each end of the beam and is always positive.

2. **Damping.** Stiffness-proportional damping may be added using the damping factor  $\lambda$ . This is defined as follows:

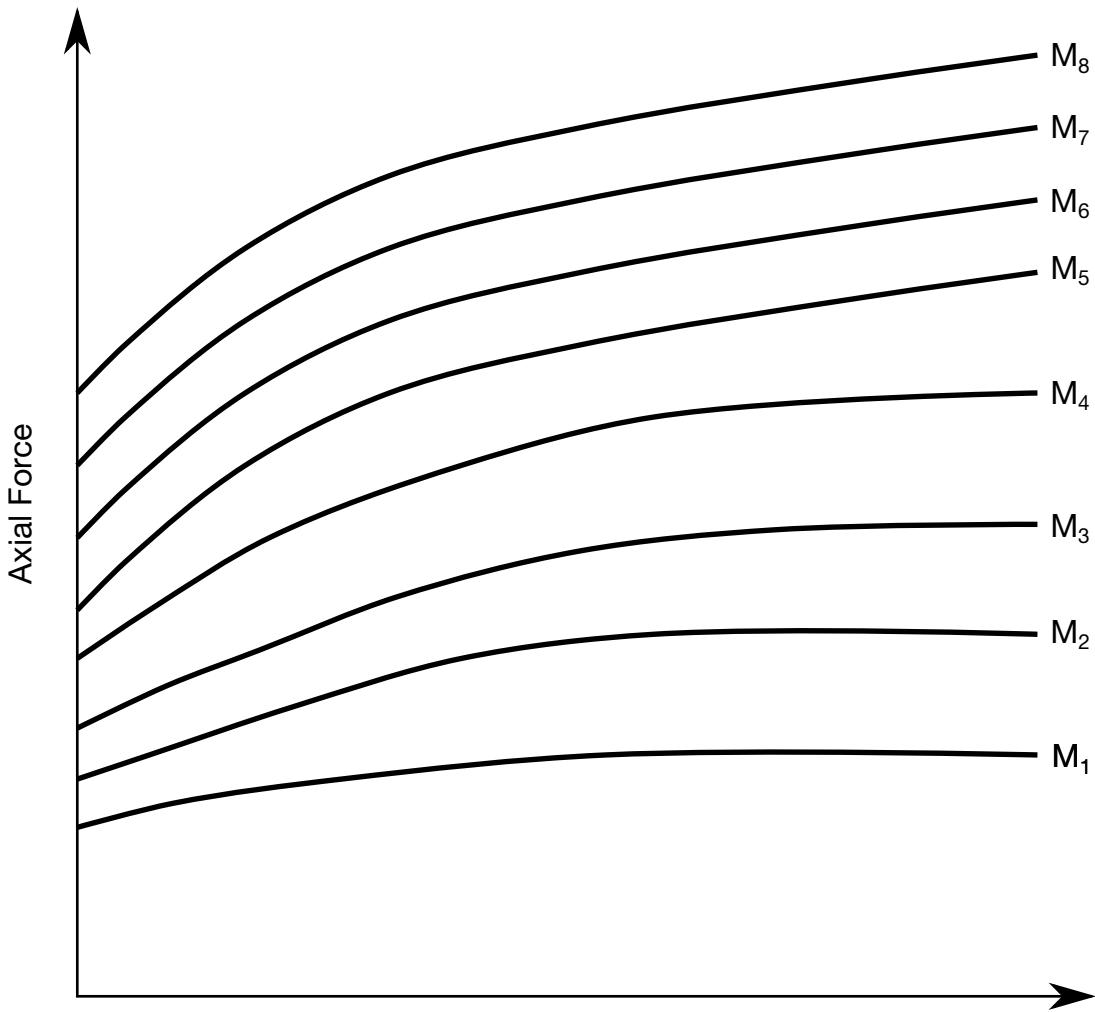
$$\lambda = \frac{2 \times \xi}{\omega}$$

where  $\xi$  is the damping factor at the reference frequency  $\omega$  (in radians per second). For example, if 1% damping at 2Hz is required

$$\lambda = \frac{2 \times 0.01}{2\pi \times 2} = 0.001592$$

If damping is used, a small time step may be required. LS-DYNA does not check this so to avoid instability it may be necessary to control the time step using a load curve. As a guide, the time step required for any given element is multiplied by  $0.3L/c\lambda$  when damping is present (*L* = element length, *c* = sound speed).

3. **Moment Interaction.** Plastic hinges can form due to the combined action of moments about the three axes. This facility is activated only when yield moments are defined in the material input. A hinge forms when the following condition is first satisfied.



Strain (or change in length; see IAFLC)

**Figure M139-1.** The force magnitude is limited by the applied end moment. For an intermediate value of the end moment LS-DYNA interpolates between the curves to determine the allowable force value.

$$\left(\frac{M_r}{M_{r\text{yield}}}\right)^2 + \left(\frac{M_s}{M_{s\text{yield}}}\right)^2 + \left(\frac{M_t}{M_{t\text{yield}}}\right)^2 \geq 1 ,$$

where

$M_r, M_s, M_t$  = current moment

$M_{r\text{yield}}, M_{s\text{yield}}, M_{t\text{yield}}$  = yield moment

Note that scale factors for hinge behavior defined in the input will also be applied to the yield moments: for example,  $M_{s\text{yield}}$  in the above formula is given by the input yield moment about the local axis times the input scale factor for the local s-axis. For strain-softening characteristics, the yield moment should generally be set equal to the initial peak of the moment-rotation load curve.

On forming a hinge, upper limit moments are set. These are given by

$$M_{r_{upper}} = \max \left( M_r, \frac{M_{r_{yield}}}{2} \right)$$

with similar conditions holding for  $M_{s_{upper}}$  and  $M_{t_{upper}}$ . Thereafter the plastic moments will be given by

$$M_{r_p} = \min(M_{r_{upper}}, M_{r_{curve}}) ,$$

where  $M_{r_p}$  is the current plastic moment and  $M_{r_{curve}}$  is the moment from the load curve at the current rotation scaled by the scale factor.  $M_{s_p}$  and  $M_{t_p}$  satisfy similar conditions.  $M_{s_p}$  and  $M_{t_p}$  satisfy similar conditions.

This provides an upper limit to the moment that can be generated; it represents the softening effect of local buckling at a hinge site. Thus, if a member is bent about the local *s*-axis, it will then be weaker in torsion and about its local *t*-axis. For moments-softening curves, the effect is to trim off the initial peak (although if the curves subsequently harden, the final hardening will also be trimmed off).

It is not possible to make the plastic moment vary with the current axial load, but it is possible to make hinge formation a function of axial load and subsequent plastic moment a function of the moment at the time the hinge formed. This is discussed in [Remark 4](#).

4. **Independent Plastic Hinge Formation.** In addition to the moment interaction equation, Cards 7 through 18 allow plastic hinges to form independently for the *s*-axis and *t*-axis at each end of the beam as well as for the torsional axis. A plastic hinge is assumed to form if any component of the current moment exceeds the yield moment as defined by the yield moment as a function axial force curves input on cards 7 and 8. If any of the 5 curves is omitted, a hinge will not form for that component. The curves can be defined for both compressive and tensile axial forces. If the axial force falls outside the range of the curve, the first or last point in the curve will be used. A hinge forming for one component of moment does not affect the other components.

Upon forming a hinge, the magnitude of that component of moment will not be permitted to exceed the current plastic moment. The current plastic moment is obtained by interpolating between the plastic moment as a function of plastic rotation curves input on cards 10, 12, 14, 16, or 18. Curves may be input for up to 8 hinge moments, where the hinge moment is defined as the yield moment at the time that the hinge formed. Curves must be input in order of increasing hinge moment and each curve should have the same plastic rotation values. The first or last curve will be used if the hinge moment falls outside the range of the curves. If no curves are defined, the plastic moment is obtained from the curves

on cards 4 through 6. The plastic moment is scaled by the scale factors on lines 4 to 6.

A hinge will form if either the independent yield moment is exceeded or if the moment interaction equation is satisfied. If both are true, the plastic moment will be set to the minimum of the interpolated value and  $M_{r_p}$ .

**\*MAT\_VACUUM**

This is Material Type 140. This model is a dummy material representing a vacuum in a multi-material Euler/ALE model. Instead of using ELFORM = 12 (under \*SECTION\_-SOLID), it is better to use ELFORM = 11 with the void material defined as the vacuum material.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RHO						
Type	A	F						

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RHO	Estimated material density. This is used only as a stability check.

**Remarks:**

The vacuum density is estimated. It should be small relative compared to air in the model (possibly at least order of magnitude  $10^3$  to  $10^6$  lighter than air).

**\*MAT\_141****\*MAT\_RATE\_SENSITIVE\_POLYMER****\*MAT\_RATE\_SENSITIVE\_POLYMER**

This is Material Type 141. This model, called the modified Ramaswamy-Stouffer model, is for the simulation of an isotropic ductile polymer with strain rate effects. See references; Stouffer and Dame [1996] and Goldberg and Stouffer [1999]. Uniaxial test data is used to fit the material parameters. This material model was implemented by Professor Ala Tabiei.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	Do	N	Z0	Q
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	OMEGA							
Type	F							

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Elastic modulus
PR	Poisson's ratio
Do	Reference strain rate ( $= 1000 \times$ max strain rate used in the test)
N	Exponent (see inelastic strain rate equation below)
Z0	Initial hardness of material, $Z_0$
Q	Material constant, $q$ (see equations below)
OMEGA	Maximum internal stress, $\Omega_m$

**Remarks:**

The inelastic strain rate is defined as:

$$\dot{\epsilon}_{ij}^I = D_o \exp \left[ -0.5 \left( \frac{Z_o^2}{3K_2} \right)^N \right] \left( \frac{S_{ij} - \Omega_{ij}}{\sqrt{K_2}} \right)$$

where the  $K_2$  term is given as:

$$K_2 = 0.5(S_{ij} - \Omega_{ij})(S_{ij} - \Omega_{ij})$$

and represents the second invariant of the overstress tensor. The elastic components of the strain are added to the inelastic strain to obtain the total strain. The following relationship defines the back stress variable rate:

$$\Omega_{ij} = \frac{2}{3}q\Omega_m \dot{\epsilon}_{ij}^I - q\Omega_{ij} \dot{\epsilon}_e^I$$

where  $q$  is a material constant,  $\Omega_m$  is a material constant that represents the maximum value of the internal stress, and  $\dot{\epsilon}_e^I$  is the effective inelastic strain rate.

**\*MAT\_142****\*MAT\_TRANSVERSELY\_ISOTROPIC\_CRUSHABLE\_FOAM****\*MAT\_TRANSVERSELY\_ISOTROPIC\_CRUSHABLE\_FOAM**

This is Material Type 142. This model is for an extruded foam material that is transversely isotropic, crushable, and of low density with no significant Poisson effect. This material is used in energy-absorbing structures to enhance automotive safety in low velocity (bumper impact) and medium high velocity (interior head impact and pedestrian safety) applications. The formulation of this foam is due to Hirth, Du Bois, and Weimar and is documented by Du Bois [2001].

This material is not wholly isotropic since the extrusion direction is preferred. The properties in directions orthogonal to the extrusion direction are, however, the same. In other words, the material is isotropic in all transversal directions to extrusion.

This material is available for solid elements and thick shell formulations 3, 5 and 7.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E11	E22	E12	E23	G	K
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	I11	I22	I12	I23	IAA	NSYM	ANG	MU
Type	I	I	I	I	I	I	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT	ISCL	BETA	MACF				
Type	F	I	F	I				

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

Card 5	1	2	3	4	5	6	7	8
Variable	D1	D2	D3	V1	V2	V3		
Type	F	F	F	F	F	F		

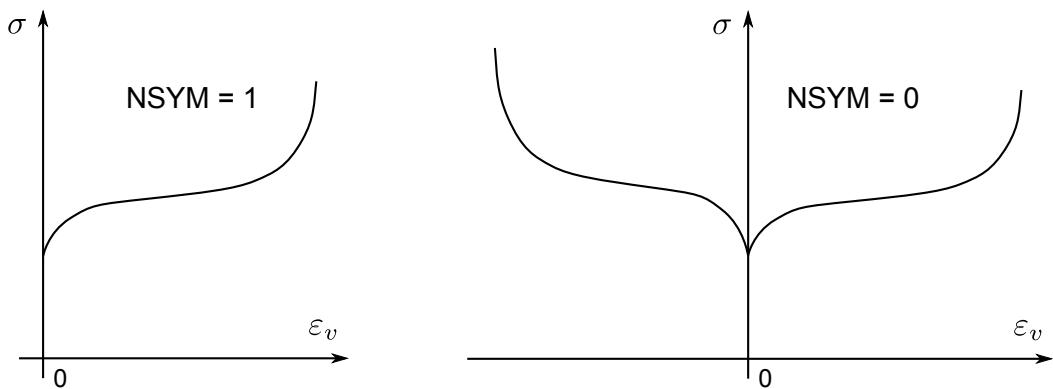
<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E11	Elastic modulus in axial direction
E22	Elastic modulus in transverse direction (E22 = E33)
E12	Elastic shear modulus (E12 = E31)
E23	Elastic shear modulus in transverse plane
G	Shear modulus
K	Bulk modulus for contact stiffness
I11	Load curve for nominal axial stress as a function of volumetric strain
I22	Load curve ID for nominal transverse stresses as a function of volumetric strain (I22 = I33)
I12	Load curve ID for shear stress component 12 and 31 as a function of volumetric strain (I12 = I31)
I23	Load curve ID for shear stress component 23 as a function of volumetric strain
IAA	Load curve ID (optional) for nominal stress as a function of volumetric strain for load at angle, ANG, relative to the material <i>a</i> -axis
NSYM	Set to unity for a symmetric yield surface in volumetric compression and tension direction
ANG	Angle corresponding to load curve ID, IAA

VARIABLE	DESCRIPTION
MU	Damping coefficient for tensor viscosity which acts in both tension and compression. Recommended values vary between 0.05 to 0.10. If zero, bulk viscosity is used instead of tensor viscosity. Bulk viscosity creates a pressure as the element compresses that is added to the normal stresses which can have the effect of creating transverse deformations when none are expected.
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC, particularly the <a href="#">Material Directions</a> section, for details):  EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.  EQ.1.0: Locally orthotropic with material axes determined by a point, $P$ , in space and the global location of the element center; this is the $\mathbf{a}$ -direction. This option is for solid elements only.  EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR  EQ.3.0: Locally orthotropic material axes determined by a vector $\mathbf{v}$ and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. $\mathbf{a}$ is determined by taking the cross product of $\mathbf{v}$ with the normal vector, $\mathbf{b}$ is determined by taking the cross product of the normal vector with $\mathbf{a}$ , and $\mathbf{c}$ is the normal vector. Then $\mathbf{a}$ and $\mathbf{b}$ are rotated about $\mathbf{c}$ by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.  EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector $\mathbf{v}$ , and an originating point, $P$ , which define the centerline axis. This option is for solid elements only.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_OPTION).
ISCL	Load curve ID for the strain rate scale factor as a function of the volumetric strain rate. The yield stress is scaled by the value specified by the load curve.
BETA	Material angle in degrees for AOPT = 0 (tshells only) and AOPT = 3 (all element types). This angle may be overridden on the element card; see *ELEMENT_TSHELL_BETA and *ELEMENT_SOLID_ORTHO.
MACF	Material axes change flag for solid elements: EQ.-4: Switch material axes <i>b</i> and <i>c</i> before BETA rotation EQ.-3: Switch material axes <i>a</i> and <i>c</i> before BETA rotation EQ.-2: Switch material axes <i>a</i> and <i>b</i> before BETA rotation EQ.1: No change, default EQ.2: Switch material axes <i>a</i> and <i>b</i> after BETA rotation EQ.3: Switch material axes <i>a</i> and <i>c</i> after BETA rotation EQ.4: Switch material axes <i>b</i> and <i>c</i> after BETA rotation  <i>Figure M2-2</i> indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_{OPTION} is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 3 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.
XP YP ZP	Coordinates of point <i>p</i> for AOPT = 1 and 4
A1 A2 A3	Components of vector <b>a</b> for AOPT = 2
D1 D2 D3	Components of vector <b>d</b> for AOPT = 2
V1 V2 V3	Define components of vector <b>v</b> for AOPT = 3 and 4

**Remarks:**

This model behaves in a more physical way for off axis loading the material than, for example, \*MAT\_HONEYCOMB which can exhibit nonphysical stiffening for loading



**Figure M142-1.** Differences between options NSYM = 1 and NSYM = 0.

conditions that are off axis. The curves given for I11, I22, I12 and I23 are used to define a yield surface of Tsai-Wu-type that bounds the deviatoric stress tensor. Hence the elastic parameters E11, E12, E22 and E23 as well as G and K must be defined in a consistent way.

For the curve definitions volumetric strain  $\varepsilon_v = 1 - V/V_0$  is used as the abscissa parameter. If the symmetric option (NSYM = 1) is used, a curve must be provided for the first quadrant, but may also be defined in both the first and second quadrants. If NSYM = 0 is chosen, the curve definitions for I11, I22, I12 and I23 (and IAA) must be in the first and second quadrant as shown in [Figure M142-1](#).

Tensor viscosity, which is activated by a nonzero value for MU, is generally more stable than bulk viscosity. A damping coefficient less than 0.01 has little effect, and a value greater than 0.10 may cause numerical instabilities.

**\*MAT\_WOOD\_{OPTION}**

This is Material Type 143. This is a transversely isotropic material. It is available for solid elements, thick shell formulations 3, 5, and 7, and SPH elements. You have the option of inputting your own material properties (<BLANK>) or requesting default material properties for Southern yellow pine (PINE) or Douglas fir (FIR). This model was developed by Murray [2002] under a contract from the FHWA.

Available options include:

<BLANK>

PINE

FIR

**Card Summary:**

**Card 1.** This card is required.

MID	RO	NPLOT	ITERS	IRATE	GHARD	IFAIL	IVOL
-----	----	-------	-------	-------	-------	-------	------

**Card 2a.** This card is included if the keyword option is set to FIR or PINE.

MOIS	TEMP	QUAL_T	QUAL_C	UNITS	IQUAL		
------	------	--------	--------	-------	-------	--	--

**Card 2b.1.** This card is included if the keyword option is unset (<BLANK>).

EL	ET	GLT	GTR	PR			
----	----	-----	-----	----	--	--	--

**Card 2b.2.** This card is included if the keyword option is unset (<BLANK>).

XT	XC	YT	YC	SXY	SYZ		
----	----	----	----	-----	-----	--	--

**Card 2b.3.** This card is included if the keyword option is unset (<BLANK>).

GF1II	GF2II	BFIT	DMAXII	GF1 $\perp$	GF2 $\perp$	DFIT	DMAX $\perp$
-------	-------	------	--------	-------------	-------------	------	--------------

**Card 2b.4.** This card is included if the keyword option is unset (<BLANK>).

FLPAR	FLPARC	POWPAR	FLPER	FLPERC	POWPER		
-------	--------	--------	-------	--------	--------	--	--

**Card 2b.5.** This card is included if the keyword option is unset (<BLANK>).

NPAR	CPAR	NPER	CPer				
------	------	------	------	--	--	--	--

**Card 3.** This card is required.

AOPT	MACF	BETA					
------	------	------	--	--	--	--	--

**Card 4.** This card is required.

XP	YP	ZP	A1	A2	A3		
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**Card 5.** This card is required.

D1	D2	D3	V1	V2	V3		
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### Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	NPLOT	ITERS	IRATE	GHARD	IFAIL	IVOL
Type	A	F	I	I	I	F	I	I

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
NPLOT	Controls what is written as component 7 to the d3plot database. LS-PrePost always blindly labels this component as effective plastic strain.  EQ.1: Parallel damage (default) EQ.2: Perpendicular damage
ITERS	Number of plasticity algorithm iterations. The default is one iteration.  GE.0: Original plasticity iteration developed by Murray [2002] LT.0: Plasticity iteration (return mapping) with non-associated flow direction for perpendicular yielding. The absolute value of ITERS is used as number of plasticity algorithm iterations.

VARIABLE	DESCRIPTION
IRATE	<p>Rate effects option:</p> <p>EQ.0: Rate effects model turned off (default).</p> <p>EQ.1: Rate effects model turned on with the original rate dependence described by Murray [2002].</p> <p>EQ.2: Rate effects model turned on with Johnson-Cook like rate dependence of the strength parameters, as described below in <a href="#">Remark 2</a>. Only works when ITERS &lt; 0 and the keyword option is unset (&lt;BLANK&gt;).</p>
GHARD	Perfect plasticity override. Values greater than or equal to zero are allowed. Positive values model late time hardening in compression (an increase in strength with increasing strain). A zero value models perfect plasticity (no increase in strength with increasing strain). The default is zero.
IFAIL	<p>Erosion perpendicular to the grain:</p> <p>EQ.0: No (default)</p> <p>EQ.1: Yes (not recommended except for debugging)</p>
IVOL	<p>Flag to invoke erosion based on negative volume or strain increments greater than 0.01:</p> <p>EQ.0: No, do not apply erosion criteria.</p> <p>EQ.1: Yes, apply erosion criteria.</p>

This card is included for the PINE and FIR keyword options.

Card 2a	1	2	3	4	5	6	7	8
Variable	MOIS	TEMP	QUAL_T	QUAL_C	UNITS	IQUAL		
Type	F	F	F	F	I	I		

VARIABLE	DESCRIPTION
MOIS	Percent moisture content. If left blank, the moisture content defaults to saturated at 30%.
TEMP	Temperature in °C. If left blank, the temperature defaults to room temperature at 20 °C

<b>VARIABLE</b>	<b>DESCRIPTION</b>											
QUAL_T	Quality factor options (see <a href="#">Remark 1</a> ). These quality factors reduce the clear wood tension, shear, and compression strengths as a function of grade.											
	EQ.0.0: Grade 1, 1D, 2, 2D. Predefined strength reduction factors are:											
	<table border="1"> <thead> <tr> <th>Wood Type</th><th>Tension/Shear Factor</th><th>Compression Factor</th></tr> </thead> <tbody> <tr> <td>Pine</td><td>0.47</td><td>0.63</td></tr> <tr> <td>Fir</td><td>0.40</td><td>0.73</td></tr> </tbody> </table>			Wood Type	Tension/Shear Factor	Compression Factor	Pine	0.47	0.63	Fir	0.40	0.73
Wood Type	Tension/Shear Factor	Compression Factor										
Pine	0.47	0.63										
Fir	0.40	0.73										
	EQ.-1.0: DS-65 or SEL STR (pine and fir). Predefined strength reduction factors are 0.80 in tension/shear and 0.93 in compression.											
	EQ.-2.0: Clear wood. No strength reduction factors are applied, that is, the reduction factors are 1.0 in tension, shear, and compression.											
	GT.0.0: User defined quality factor in tension/shear. Values between 0 and 1 are expected. Values greater than one are allowed but may not be realistic.											
QUAL_C	User defined quality factor in compression (see <a href="#">Remark 1</a> ). This input value is used if QUAL_T > 0. Values between 0 and 1 are expected. Values greater than one are allowed but may not be realistic. If left blank, a default value of QUAL_C = QUAL_T is used.											
UNITS	Units options:											
	EQ.0: GPa, mm, msec, Kg/mm <sup>3</sup> , kN.											
	EQ.1: MPa, mm, msec, g/mm <sup>3</sup> , Nt.											
	EQ.2: MPa, mm, sec, Mg/mm <sup>3</sup> , Nt.											
	EQ.3: Psi, inch, sec, lb-s <sup>2</sup> /inch <sup>4</sup> , lb											
IQUAL	Apply quality factors perpendicular to the grain:											
	EQ.0: Yes (default)											
	EQ.1: No											

This card is included if the keyword option is unset (<BLANK>).

Card 2b.1	1	2	3	4	5	6	7	8
Variable	EL	ET	GLT	GTR	PR			
Type	F	F	F	F	F			

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EL	Parallel normal modulus
ET	Perpendicular normal modulus
GLT	Parallel shear modulus (GLT = GLR)
GTR	Perpendicular shear modulus
PR	Parallel major Poisson's ratio

This card is included if the keyword option is unset (<BLANK>).

Card 2b.2	1	2	3	4	5	6	7	8
Variable	XT	XC	YT	YC	SXY	SYZ		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
XT	Parallel tensile strength
XC	Parallel compressive strength
YT	Perpendicular tensile strength
YC	Perpendicular compressive strength
SXY	Parallel shear strength
SYZ	Perpendicular shear strength

**\*MAT\_143****\*MAT\_WOOD**

This card is included if the keyword option is unset (<BLANK>).

Card 2b.3	1	2	3	4	5	6	7	8
Variable	GF1	GF2	BFIT	DMAX	GF1⊥	GF2⊥	DFIT	DMAX⊥
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
XT	Parallel tensile strength
GF1	Parallel fracture energy in tension
GF2	Parallel fracture energy in shear
BFIT	Parallel softening parameter
DMAX	Parallel maximum damage
GF1⊥	Perpendicular fracture energy in tension
GF2⊥	Perpendicular fracture energy in shear
DFIT	Perpendicular softening parameter

This card is included if the keyword option is unset (<BLANK>).

Card 2b.4	1	2	3	4	5	6	7	8
Variable	FLPAR	FLPARC	POWPAR	FLPER	FLPERC	POWPER		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
FLPAR	Rate effects parameter: IRATE.EQ.0: Ignored IRATE.EQ.1: Parallel fluidity parameter for tension and shear IRATE.EQ.2: Dimensionless parallel strain rate parameter for tension and shear (see <a href="#">Remark 2</a> )
FLPARC	Rate effects parameter:

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	IRATE.EQ.0: Ignored IRATE.EQ.1: Parallel fluidity parameter for compression IRATE.EQ.2: Dimensionless parallel strain rate parameter for compression (see <a href="#">Remark 2</a> )
POWPAR	Rate effects parameter: IRATE.EQ.0: Ignored IRATE.EQ.1: Parallel power IRATE.EQ.2: Quasi-static threshold strain rate value in the parallel direction (see <a href="#">Remark 2</a> )
FLPER	Rate effects parameter: IRATE.EQ.0: Ignored IRATE.EQ.1: Perpendicular fluidity parameter for tension and shear IRATE.EQ.2: Dimensionless perpendicular strain rate parameter for tension and shear (see <a href="#">Remark 2</a> )
FLPERC	Rate effects parameter: IRATE.EQ.0: Ignored IRATE.EQ.1: Perpendicular fluidity parameter for compression IRATE.EQ.2: Dimensionless perpendicular strain rate parameter for compression (see <a href="#">Remark 2</a> )
POWPER	Rate effects parameter: IRATE.EQ.0: Ignored IRATE.EQ.1: Perpendicular power IRATE.EQ.2: Quasi-static threshold strain rate value in the perpendicular direction (see <a href="#">Remark 2</a> )

**\*MAT\_143****\*MAT\_WOOD**

This card is included if the keyword option is unset (<BLANK>).

Card 2b.5	1	2	3	4	5	6	7	8
Variable	NPAR	CPAR	NPER	CPER				
Type	F	F	F	F				

VARIABLE	DESCRIPTION							
NPAR	Parallel hardening initiation							
CPAR	Parallel hardening rate							
NPER	Perpendicular hardening initiation							
CPER	Perpendicular hardening rate							

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT	MACF	BETA					
Type	F	I	F					

VARIABLE	DESCRIPTION							
AOPT	Material axes option (see <b>MAT_OPTIONTROPIC_ELASTIC</b> , particularly the <a href="#">Material Directions</a> section, for details):							
	EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with <b>*DEFINE_COORDINATE_NODES</b> .							
	EQ.1.0: Locally orthotropic with material axes determined by a point, $P$ , in space and the global location of the element center; this is the <b>a</b> -direction. This option is for solid elements only.							
	EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with <b>*DEFINE_COORDINATE_VECTOR</b>							
	EQ.3.0: Locally orthotropic material axes determined by a vector $v$ and the normal vector to the plane of the element. The plane of a solid element is the midsurface between							

VARIABLE	DESCRIPTION
	the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. <b>a</b> is determined by taking the cross product of <b>v</b> with the normal vector, <b>b</b> is determined by taking the cross product of the normal vector with <b>a</b> , and <b>c</b> is the normal vector. Then <b>a</b> and <b>b</b> are rotated about <b>c</b> by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.
EQ.4.0:	Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector <b>v</b> , and an originating point, <i>P</i> , which define the centerline axis. This option is for solid elements only.
LT.0.0:	The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_OPTION).
MACF	Material axes change flag for solid elements: EQ.-4: Switch material axes <i>b</i> and <i>c</i> before BETA rotation EQ.-3: Switch material axes <i>a</i> and <i>c</i> before BETA rotation EQ.-2: Switch material axes <i>a</i> and <i>b</i> before BETA rotation EQ.1: No change, default EQ.2: Switch material axes <i>a</i> and <i>b</i> after BETA rotation EQ.3: Switch material axes <i>a</i> and <i>c</i> after BETA rotation EQ.4: Switch material axes <i>b</i> and <i>c</i> after BETA rotation
BETA	Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SOLID_ORTH0 and *ELEMENT_TSHELL_BETA.

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
XP YP ZP		Coordinates of point $p$ for AOPT = 1 and 4						
A1 A2 A3		Components of vector $\mathbf{a}$ for AOPT = 2						

Card 5	1	2	3	4	5	6	7	8
Variable	D1	D2	D3	V1	V2	V3		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
D1 D2 D3		Components of vector $\mathbf{d}$ for AOPT = 2						
V1 V2 V3		Define components of vector $\mathbf{v}$ for AOPT = 3 and 4						

**Remarks:**

1. **Quality factors.** Material property data is for clear wood (small samples without defects like knots), whereas real structures are composed of graded wood. Clear wood is stronger than graded wood. Quality factors (strength reduction factors) are applied to the clear wood strengths to account for reductions in strength as a function of grade. One quality factor (QUAL\_T) is applied to the tensile and shear strengths. A second quality factor (QUAL\_C) is applied to the compressive strengths. As an option, predefined quality factors are provided based on correlations between LS-DYNA calculations and test data for pine and fir posts impacted by bogie vehicles. By default, quality factors are applied to both the parallel and perpendicular to the grain strengths. An option is available (IQUAL) to eliminate application perpendicular to the grain.

2. **Johnson-Cook-like logarithmic rate dependence.** A Johnson-Cook-like logarithmic rate dependence can be invoked by IRATE = 2 when the keyword option is unset and ITERS < 0. In that case, the strength parameters are:

$$\begin{aligned}\hat{X}_T &= X_T \left( 1 + \text{FLPAR} \times \ln \left( 1 + \frac{\dot{\varepsilon}}{\text{POWPAR}} \right) \right) \\ \hat{X}_C &= X_C \left( 1 + \text{FLPARC} \times \ln \left( 1 + \frac{\dot{\varepsilon}}{\text{POWPAR}} \right) \right) \\ \hat{Y}_T &= Y_T \left( 1 + \text{FLPER} \times \ln \left( 1 + \frac{\dot{\varepsilon}}{\text{POWPER}} \right) \right) \\ \hat{Y}_C &= Y_C \left( 1 + \text{FLPERC} \times \ln \left( 1 + \frac{\dot{\varepsilon}}{\text{POWPER}} \right) \right) \\ \hat{S}_{XY} &= S_{XY} \left( 1 + \text{FLPAR} \times \ln \left( 1 + \frac{\dot{\varepsilon}}{\text{POWPAR}} \right) \right) \\ \hat{S}_{YZ} &= S_{YZ} \left( 1 + \text{FLPER} \times \ln \left( 1 + \frac{\dot{\varepsilon}}{\text{POWPER}} \right) \right)\end{aligned}$$

The strain rate parameters, FLPAR, FLPARC, FLPER, and FLPERC, are dimensionless (factors  $\geq 0$  that quantify the strain rate dependence). POWPAR and POWPER are quasi-static threshold strain rate values in the parallel and perpendicular directions with the units of [time] $^{-1}$ . Variable  $\dot{\varepsilon}$  is an effective strain rate.

**\*MAT\_144****\*MAT\_PITZER\_CRUSHABLE\_FOAM****\*MAT\_PITZER\_CRUSHABLE\_FOAM**

This is Material Type 144. This model is for the simulation of isotropic crushable forms with strain rate effects. Uniaxial and triaxial test data have to be used. For the elastic response, the Poisson ratio is set to zero.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	K	G	PR	TY	SRTV	
Type	A	F	F	F	F	F	F	

Card 2	1	2	3	4	5	6	7	8
Variable	LCPY	LCUYS	LCSR	VC	DFLG			
Type	I	I	I	F	F			

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
K	Bulk modulus
G	Shear modulus
PR	Poisson's ratio
TY	Tension yield
SRTV	Young's modulus ( $E$ )
LCPY	Load curve ID giving pressure as a function of volumetric strain; see <a href="#">Figure M75-1</a> .
LCUYS	Load curve ID giving uniaxial stress as a function of volumetric strain; see <a href="#">Figure M75-1</a> .
LCSR	Load curve ID giving strain rate scale factor as a function of volumetric strain rate

<b>VARIABLE</b>	<b>DESCRIPTION</b>
VC	Viscous damping coefficient (.05 < recommended value < .50)
DFLG	Density flag: EQ.0.0: Use initial density. EQ.1.0: Use current density (larger step size with less mass scaling).

**Remarks:**

The logarithmic volumetric strain is defined in terms of the relative volume,  $V$ , as:

$$\gamma = -\ln(V) .$$

When defining the curves, the stress and strain pairs should be positive values starting with a volumetric strain value of zero.

## \*MAT\_145

## \*MAT\_SCHWER\_MURRAY\_CAP\_MODEL

### \*MAT\_SCHWER\_MURRAY\_CAP\_MODEL

This is Material Type 145. \*MAT\_145 is a Continuous Surface Cap Model and is a three invariant extension of \*MAT\_GEOLOGIC\_CAP\_MODEL (\*MAT\_025) that includes viscoplasticity for rate effects and damage mechanics to model strain softening. The primary references for the model are Schwer and Murray [1994], Schwer [1994], and Murray and Lewis [1994]. \*MAT\_145 was developed for geomaterials including soils, concrete, and rocks. We recommend using an updated version of a Continuous Surface Cap Model, \*MAT\_CSCM (\*MAT\_159), rather than this model, \*MAT\_SCHWER\_MURRAY\_CAP\_MODEL (\*MAT\_145).

**WARNING:** No default input parameter values are assumed, but recommendations for the more obscure parameters are provided in the descriptions that follow.

#### Card Summary:

**Card 1.** This card is required.

MID	R0	SHEAR	BULK	GRUN	SHOCK	PORE	
-----	----	-------	------	------	-------	------	--

**Card 2.** This card is required.

ALPHA	THETA	GAMMA	BETA	EFIT	FFIT	ALPHAN	CALPHA
-------	-------	-------	------	------	------	--------	--------

**Card 3.** This card is required.

R0	X0	IROCK	SECP	AFIT	BFIT	RDAMO	
----	----	-------	------	------	------	-------	--

**Card 4.** This card is required.

W	D1	D2	NPLOT	EPSMAX	CFIT	DFIT	TFAIL
---	----	----	-------	--------	------	------	-------

**Card 5.** This card is required.

FAILFL	DBETA	DDELTA	VPTAU				
--------	-------	--------	-------	--	--	--	--

**Card 6.** This card is required.

ALPHA1	THETA1	GAMMA1	BETA1	ALPHA2	THETA2	GAMMA2	BETA2
--------	--------	--------	-------	--------	--------	--------	-------

**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	SHEAR	BULK	GRUN	SHOCK	PORE	
Type	A	F	F	F	F	F	F	

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
SHEAR	Shear modulus, $G$
BULK	Bulk modulus, $K$
GRUN	Gruneisen ratio (typically = 0), $\Gamma$
SHOCK	Shock velocity parameter (typically 0), $S_l$
PORE	Flag for pore collapse EQ.0.0: Pore collapse EQ.1.0: Constant bulk modulus (typical)

Card 2	1	2	3	4	5	6	7	8
Variable	ALPHA	THETA	GAMMA	BETA	EFIT	FFIT	ALPHAN	CALPHA
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
ALPHA	Shear failure parameter, $\alpha$
THETA	Shear failure parameter, $\theta$
GAMMA	Shear failure parameter, $\gamma$

**\*MAT\_145****\*MAT\_SCHWER\_MURRAY\_CAP\_MODEL**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BETA	Shear failure parameter, $\beta$ $\sqrt{J'_2} = F_e(J_1) = \alpha - \gamma \exp(-\beta J_1) + \theta J_1$
EFIT	Dilation damage mechanics parameter (no damage = 1)
FFIT	Dilation damage mechanics parameter (no damage = 0)
ALPHAN	Kinematic strain hardening parameter, $N^\alpha$
CALPHAN	Kinematic strain hardening parameter, $c^\alpha$

Card 3	1	2	3	4	5	6	7	8
Variable	R0	X0	IROCK	SECP	AFIT	BFIT	RDAM0	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
R0	Initial cap surface ellipticity, $R$
X0	Initial cap surface $J_1$ (mean stress) axis intercept, $X(\kappa_0)$
IROCK	Material flag: EQ.0: Soils (cap can contract) EQ.1: Rock/concrete
SECP	Shear enhanced compaction
AFIT	Ductile damage mechanics parameter (=1 no damage)
BFIT	Ductile damage mechanics parameter (=0 no damage)
RDAM0	Ductile damage mechanics parameter

**\*MAT\_SCHWER\_MURRAY\_CAP\_MODEL****\*MAT\_145**

Card 4	1	2	3	4	5	6	7	8
Variable	W	D1	D2	NPLOT	EPSMAX	CFIT	DFIT	TFAIL
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
W	Plastic Volume Strain parameter, $W$
D1	Plastic Volume Strain parameter, $D_1$
D2	Plastic Volume Strain parameter, $D_2$ $\varepsilon_V^P = W \left\{ 1 - \exp \left\{ -D_1 [X(\kappa) - X(\kappa_0)] - D_2 [(X(\kappa) - X(\kappa_0))^2] \right\} \right\}$
NPLOT	History variable post-processed as effective plastic strain. (See <a href="#">Table M145-1</a> for history variables available for plotting.)
EPSMAX	Maximum permitted strain increment: EQ.0.0: $\Delta\varepsilon_{\max} = 0.05(\alpha - N^\alpha - \gamma)\min\left(\frac{1}{G}, \frac{R}{9K}\right)$ (calculated default)
CFIT	Brittle damage mechanics parameter ( $= 1$ no damage)
DFIT	Brittle damage mechanics parameter ( $= 0$ no damage)
TFAIL	Tensile failure stress

Card 5	1	2	3	4	5	6	7	8
Variable	FAILFL	DBETA	DDELTA	VPTAU				
Type	F	F	F	F				

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FAILFL	Flag controlling element deletion and effect of damage on stress (see <a href="#">Remark 1</a> ): EQ.1: $\sigma_{ij}$ reduces with increasing damage; element is deleted when fully damaged (default).

**\*MAT\_145****\*MAT\_SCHWER\_MURRAY\_CAP\_MODEL**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	EQ.-1: $\sigma_{ij}$ reduces with increasing damage; element is not deleted.
	EQ.2: $S_{ij}$ reduces with increasing damage; element is deleted when fully damaged.
	EQ.-2: $S_{ij}$ reduces with increasing damage; element is not deleted.
DBETA	Rounded vertices parameter, $\Delta\beta_0$
DDELTA	Rounded vertices parameter, $\delta$
VPTAU	Viscoplasticity relaxation time parameter, $\tau$

Card 6	1	2	3	4	5	6	7	8
Variable	ALPHA1	THETA1	GAMMA1	BETA1	ALPHA2	THETA2	GAMMA2	BETA2
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ALPHA1	Torsion scaling parameter, $\alpha_1$ LT.0.0: $ \text{ALPHA1} $ is the friction angle in degrees.
THETA1	Torsion scaling parameter, $\theta_1$
GAMMA1	Torsion scaling parameter, $\gamma_1$
BETA1	Torsion scaling parameter, $\beta_1$ $Q_1 = \alpha_1 - \gamma_1 \exp(-\beta_1 J_1) + \theta_1 J_1 \theta_2$
ALPHA2	Tri-axial extension scaling parameter, $\alpha_2$
THETA2	Tri-axial extension scaling parameter, $\theta_2$
GAMMA2	Tri-axial extension scaling parameter, $\gamma_2$
BETA2	Tri-axial extension scaling parameter, $\beta_2$ $Q_2 = \alpha_2 - \gamma_2 \exp(-\beta_2 J_1) + \theta_2 J_1$

**Remarks:**

- Damage Accumulation and Element Deletion.** FAILFL controls whether the damage accumulation applies to either the total stress tensor,  $\sigma_{ij}$ , or the deviatoric stress tensor,  $S_{ij}$ . When FAILFL = 2, damage does not diminish the ability of the material to support hydrostatic stress.

FAILFL also serves as a flag to control element deletion. Fully damaged elements are deleted only if FAILFL is a positive value. When \*MAT\_145 is used with the ALE or EFG solvers, failed elements should not be eroded and so a negative value of FAILFL should be used.

- History Variables.** All the output parameters listed in [Table M145-1](#) are available for post-processing using LS-PrePost and its displayed list of history variables. The LS-DYNA input parameter NEIPH should be set to 22 on \*DATABASE\_EXTENT\_BINARY.

History Variable #	Function	Description
1	$X(\kappa)$	$J_1$ intercept of cap surface
2	$L(\kappa)$	$J_1$ value at cap-shear surface intercept
3	$R$	Cap surface ellipticity
4	$\tilde{R}$	Rubin function
5	$\varepsilon_v^p$	Plastic volume strain
6		Yield flag (= 0 elastic)
7		Number of strain sub-increments
8	$G^\alpha$	Kinematic hardening parameter
9	$J_2^\alpha$	Kinematic hardening back stress
10		Effective strain rate
11		Ductile damage
12		Ductile damage threshold
13		Strain energy

History Variable #	Function	Description
14		Brittle damage
15		Brittle damage threshold
16		Brittle energy norm
17		$J_1$ (without visco-damage/plastic)
18		$J'_2$ (without visco-damage/plastic)
19		$J'_3$ (without visco-damage/plastic)
20		$\hat{J}_3$ (without visco-damage/plastic)
21	$\beta$	Lode angle
22	$d$	Maximum damage parameter

**Table M145-1.** Output variables for post-processing using NPLOT parameter.

3. **Sample Input for Concrete.** Gran and Senseny [1996] report the axial stress as a function of strain response for twelve unconfined compression tests of concrete, used in scale-model reinforced-concrete wall tests. The Schwer & Murray Cap Model parameters provided below were used, see Schwer [2001], to model the unconfined compression test stress-strain response for the nominal 40 MPa strength concrete reported by Gran and Senseny. The basic units for the provided parameters are length in millimeters (mm), time in milliseconds (msec), and mass in grams (g). This base unit set yields units of force in Newtons (N) and pressure in Mega-Pascals (MPa).

```
*MAT_SCHWER_MURRAY_CAP_MODEL
$      MID      RO      SHEAR      BULK      GRUN      SHOCK      PORE
$      1      2.3E-3   1.048E4   1.168E4      0.0      0.0      1.0
$      ALPHA     THETA    GAMMA     BETA      EFIT      FFIT      ALPHAN    CALPHA
$      190.0     0.0      184.2    2.5E-3     0.999      0.7      2.5      2.5E3
$      R0       X0      IROCK     SECP      AFIT      BFIT      RDAM0
$      5.0       100.0     1.0      0.0      0.999      0.3      0.94
$      W        D1      D2      NPLOT     EPSMAX     CFIT      DFIT      TFAIL
$      5.0E-2    2.5E-4   3.5E-7    23.0      0.0      1.0      300.0     7.0
$      FAILFG    DBETA    DDELTA    VPTAU
$      1.0       0.0      0.0      0.0
$      ALPHA1    THETA1   GAMMA1    BETA1     ALPHA2    THETA2    GAMMA2    BETA2
$      0.747    3.3E-4     0.17     5.0E-2     0.66     4.0E-4     0.16     5.0E-2
```

4. **User Input Parameters and System of Units.** Consider the following basic units:

Length:  $L$  (e.g. millimeters - mm )

Mass: M (e.g. grams - g )

Time: T (e.g. milliseconds - ms )

The following consistent unit systems can then be derived using Newton's Law,  
 $F = Ma$ :

Force:  $F = ML/T^2$  [ g-mm/ms<sup>2</sup> = Kg-m/s<sup>2</sup> = Newton - N ]

Stress:  $\sigma = F/L^2$  [ N/mm<sup>2</sup> = 10<sup>6</sup>N/m<sup>2</sup> = 10<sup>6</sup> Pascals = MPa ]

Density:  $\rho = M/L^3$  [ g/mm<sup>3</sup> = 10<sup>6</sup> Kg/m<sup>3</sup> ]

Variable	MID	R0	SHEAR	BULK	GRUN	SHOCK	PORE	
Units		Den-sity: M/L <sup>3</sup>	Stress: F/L <sup>2</sup>	Stress: F/L <sup>2</sup>				
Variable	ALPHA	THETA	GAMMA	BETA	EFIT	FFIT	ALPHAN	CALPHA
Units	Stress: F/L <sup>2</sup>		Stress: F/L <sup>2</sup>	Stress <sup>-1</sup> : L <sup>2</sup> /F		Stress <sup>-1/2</sup> : L/F <sup>1/2</sup>	Stress: F/L <sup>2</sup>	Stress: F/L <sup>2</sup>
Variable	R0	X0	IROCK	SECP	AFIT	BFIT	RDAM0	
Units		Stress: F/L <sup>2</sup>				Stress <sup>-1/2</sup> : L/F <sup>1/2</sup>	Stress <sup>1/2</sup> : F <sup>1/2</sup> /L	
Variable	W	D1	D2	NPLOT	MAXEPS	CFIT	DFIT	TFAIL
Units		Stress <sup>-1</sup> : L <sup>2</sup> /F	Stress <sup>-2</sup> : L <sup>4</sup> /F <sup>2</sup>				Stress <sup>-1/2</sup> : L/F <sup>1/2</sup>	Stress: F/L <sup>2</sup>
Variable	FAILFG	DBETA	DDELTA	VPTAU				
Units		Angle: degrees		Time: T				
Variable	ALPHA1	THETA1	GAMMA1	BETA1	ALPHA2	THETA2	GAMMA2	BETA2
Units	Stress: F/L <sup>2</sup>		Stress: F/L <sup>2</sup>	Stress <sup>-1</sup> : L <sup>2</sup> /F	Stress: F/L <sup>2</sup>		Stress: F/L <sup>2</sup>	Stress <sup>-1</sup> : L <sup>2</sup> /F

**\*MAT\_146****\*MAT\_1DOF\_GENERALIZED\_SPRING****\*MAT\_1DOF\_GENERALIZED\_SPRING**

This is Material Type 146. This is a linear spring or damper that allows different degrees-of-freedom at two nodes to be coupled.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	K	C	SCLN1	SCLN2	DOFN1	DOFN2
Type	A	F	F	F	F	F	I	I

Card 2	1	2	3	4	5	6	7	8
Variable	CID1	CID2						
Type	I	I						

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density; see also volume in *SECTION_BEAM definition.
K	Spring stiffness
C	Damping constant
SCLN1	Scale factor on force at node 1. Default = 1.0.
SCLN2	Scale factor on force at node 2. Default = 1.0.
DOFN1	Active degree-of-freedom at node 1, a number between 1 to 6 where 1, 2 and 3 are the x, y, and z-translations and 4, 5, and 6 are the x, y, and z-rotations. If this parameter is defined in the *SECTION_BEAM definition or on the *ELEMENT_BEAM_SCALAR card, then the value here, if defined, is ignored.
DOFN2	Active degree-of-freedom at node 2, a number between 1 to 6 where 1, 2 and 3 are the x, y, and z-translations and 4, 5, and 6 are the x, y, and z-rotations. If this parameter is defined in the *SECTION_BEAM definition or on the *ELEMENT_BEAM_SCALAR card, then the value here, if defined, is ignored.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
CID1	Local coordinate system at node 1. This coordinate system can be overwritten by a local system specified on the *ELEMENT_BEAM_SCALAR or *SECTION_BEAM keyword input. If no coordinate system is specified, the global system is used.
CID2	Local coordinate system at node 2. If CID2 = 0, CID2 = CID1.

## \*MAT\_147

## \*MAT\_FHWA\_SOIL

### \*MAT\_FHWA\_SOIL

This is Material Type 147. This is an isotropic material with damage and is available for solid elements. The model has a modified Mohr-Coulomb surface to determine the pressure dependent peak shear strength. It was developed for applications involving road-base soils by Lewis [1999] for the FHWA, who extended the work of Abbo and Sloan [1995] to include excess pore water effects.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	NPLOT	SPGRAV	RHOWAT	VN	GAMMAR	ITERMX
Type	A	F	I	F	F	F	F	I
Default	none	none	1	none	1.0	0.0	0.0	1

Card 2	1	2	3	4	5	6	7	8
Variable	K	G	PHIMAX	AHYP	COH	ECCEN	AN	ET
Type	F	F	F	F	F	F		
Default	none	none	none	none	none	none	none	none

Card 3	1	2	3	4	5	6	7	8
Variable	MCONT	PWD1	PWKS	PWD2	PHIRES	DINT	VDFM	DAMLEV
Type	F	F	F	F	F	F	F	F
Default	none	none	none	none	0.0	none	none	none

Card 4	1	2	3	4	5	6	7	8
Variable	EPSMAX							
Type	F							
Default	none							

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
NPLOT	Controls what is written as component 7 to the d3plot database. LS-PrePost always blindly labels this component as effective plastic strain.
	EQ.1: Effective strain
	EQ.2: Damage criterion threshold
	EQ.3: Damage (diso)
	EQ.4: Current damage criterion
	EQ.5: Pore water pressure
	EQ.6: Current friction angle (phi)
SPGRAV	Specific gravity of soil used to get porosity.
RHOWAT	Density of water in model units - used to determine air void strain (saturation)
VN	Viscoplasticity parameter (strain-rate enhanced strength)
GAMMAR	Viscoplasticity parameter (strain-rate enhanced strength)
ITERMX	Maximum number of plasticity iterations (default 1)
K	Bulk modulus (non-zero)
G	Shear modulus (non-zero)
PHIMAX	Peak shear strength angle (friction angle in radians)

**\*MAT\_147****\*MAT\_FHWA\_SOIL**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AHYP	Coefficient A for modified Drucker-Prager Surface
COH	Cohesion $\tilde{n}$ shear strength at zero confinement (overburden)
ECCEN	Eccentricity parameter for third invariant effects
AN	Strain hardening percent of PHIMAX where non-linear effects start
ET	Strain hardening amount of non-linear effects
MCONT	Moisture content of soil. It determines the amount of air voids and should be a value between 0.0 and 1.0.
PWD1	Parameter for pore water effects on bulk modulus
PWKS1	Skeleton bulk modulus. Pore water parameter, $\tilde{n}$ , set to zero to eliminate effects.
PWD2	Parameter for pore water effects on the effective pressure (confinement)
PHIRES	The minimum internal friction angle in radians (residual shear strength)
DINT	Volumetric strain at initial damage threshold
VDFM	Void formation energy (like fracture energy)
DAMLEV	Level of damage that will cause element deletion (0.0 - 1.00)
EPSMAX	Maximum principle failure strain

**\*MAT\_FHWA\_SOIL\_NEBRASKA**

This is an option to use the default properties determined for soils used at the University of Nebraska (Lincoln). The default units used for this material are millimeter, millisecond, and kilograms. If different units are desired, the conversion factors must be input.

This is Material Type 147. This is an isotropic material with damage and is available for solid elements. The model has a modified Mohr-Coulomb surface to determine the pressure dependent peak shear strength. It was developed for applications involving road base soils.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	FCTIM	FCTMAS	FCTLEN				
Type	A	F	F	F				
Default	none	none	none	none				

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
FCTIM	Factor to multiply milliseconds by to get desired time units
FCTMAS	Factor to multiply kilograms by to get desired mass units
FCTLEN	Factor to multiply millimeters by to get desired length units

**Remarks:**

As an example, if units of seconds are desired for time, then FCTIM = 0.001.

## \*MAT\_148

## \*MAT\_GAS\_MIXTURE

### \*MAT\_GAS\_MIXTURE

This is Material Type 148. This model is for the simulation of thermally equilibrated ideal gas mixtures. This model only works with the multi-material ALE formulation (ELFORM = 11 in \*SECTION\_SOLID). This keyword must be used together with \*INITIAL\_GAS\_MIXTURE for the initialization of gas densities and temperatures. When applied in the context of ALE airbag modeling, the injection of inflator gas is done with a \*SECTION\_POINT\_SOURCE\_MIXTURE command which controls the injection process. \*MAT\_ALE\_GAS\_MIXTURE (\*MAT\_ALE\_02) is identical to this model and is another name for this material model.

#### Card Summary:

**Card 1.** This card is required.

MID	IADIAB	RUNIV	PDV				
-----	--------	-------	-----	--	--	--	--

**Card 2a.1.** Include this card if RUNIV is blank or zero.

CVMASS1	CVMASS2	CVMASS3	CVMASS4	CVMASS5	CVMASS6	CVMASS7	CVMASS8
---------	---------	---------	---------	---------	---------	---------	---------

**Card 2a.2.** Include this card if RUNIV is blank or zero.

CPMASS1	CPMASS2	CPMASS3	CPMASS4	CPMASS5	CPMASS6	CPMASS7	CPMASS8
---------	---------	---------	---------	---------	---------	---------	---------

**Card 2b.1.** Include this card if RUNIV is nonzero.

MOLWT1	MOLWT2	MOLWT3	MOLWT4	MOLWT5	MOLWT6	MOLWT7	MOLWT8
--------	--------	--------	--------	--------	--------	--------	--------

**Card 2b.2.** Include this card if RUNIV is nonzero.

CPMOLE1	CPMOLE2	CPMOLE3	CPMOLE4	CPMOLE5	CPMOLE6	CPMOLE7	CPMOLE8
---------	---------	---------	---------	---------	---------	---------	---------

**Card 2b.3.** Include this card if RUNIV is nonzero.

B1	B2	B3	B4	B5	B6	B7	B8
----	----	----	----	----	----	----	----

**Card 2b.4.** Include this card if RUNIV is nonzero.

C1	C2	C3	C4	C5	C6	C7	C8
----	----	----	----	----	----	----	----

**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	IADIAB	RUNIV	PDV				
Type	A	I	F	I				
Default	none	0	0.0	0				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
IADIAB	Flag to turn on/off adiabatic compression logic for an ideal gas. See <a href="#">Remark 5</a> .
	EQ.0: Off (default)
	EQ.1: On
RUNIV	Universal gas constant in per-mole unit (8.31447 J/(mole × K)). See <a href="#">Remark 1</a> .
PDV	Element energy update method (see <a href="#">Remark 6</a> ):
	EQ.0: Ideal gas gamma law
	EQ.1: Pressure work

**Card 2 for Per Mass Calculation.** Method (A) RUNIV = blank or 0.0.

Card 2a.1	1	2	3	4	5	6	7	8
Variable	CVMASS1	CVMASS2	CVMASS3	CVMASS4	CVMASS5	CVMASS6	CVMASS7	CVMASS8
Type	F	F	F	F	F	F	F	F

**\*MAT\_148****\*MAT\_GAS\_MIXTURE**

**Card 3 for Per Mass Calculation.** Method (A) RUNIV = blank or 0.0.

Card 2a.2	1	2	3	4	5	6	7	8
Variable	CPMASS1	CPMASS2	CPMASS3	CPMASS4	CPMASS5	CPMASS6	CPMASS7	CPMASS8
Type	F	F	F	F	F	F	F	F

**VARIABLE****DESCRIPTION**

CVMASS1 - CVMASS8 Heat capacity at constant volume for up to eight different gases in per-mass unit.

CPMASS1 - CPMAS8 Heat capacity at constant pressure for up to eight different gases in per-mass unit.

**Card 2 for Per Mole Calculation.** Method (B) RUNIV is nonzero.

Card 2b.1	1	2	3	4	5	6	7	8
Variable	MOLWT1	MOLWT2	MOLWT3	MOLWT4	MOLWT5	MOLWT6	MOLWT7	MOLWT8
Type	F	F	F	F	F	F	F	F

**Card 3 for Per Mole Calculation.** Method (B) RUNIV is nonzero.

Card 2b.2	1	2	3	4	5	6	7	8
Variable	CPMOLE1	CPMOLE2	CPMOLE3	CPMOLE4	CPMOLE5	CPMOLE6	CPMOLE7	CPMOLE8
Type	F	F	F	F	F	F	F	F

**Card 4 for Per Mole Calculation.** Method (B) RUNIV is nonzero.

Card 2b.3	1	2	3	4	5	6	7	8
Variable	B1	B2	B3	B4	B5	B6	B7	B8
Type	F	F	F	F	F	F	F	F

**Card 5 for Per Mole Calculation.** Method (B) RUNIV is nonzero.

Card 2b.4	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6	C7	C8
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MOLWT1 - MOLWT8	Molecular weight of each ideal gas in the mixture (mass-unit/mole). See <a href="#">Remark 2</a> .
CPMOLE1 - CPMOLE8	Heat capacity at constant pressure for up to eight different gases in per-mole unit. These are nominal heat capacity values typically at STP. These are denoted by the variable <i>A</i> in the equation in <a href="#">Remark 2</a> .
B1 - B8	First order coefficient for a temperature dependent heat capacity at constant pressure for up to eight different gases. These are denoted by the variable <i>B</i> in the equation in <a href="#">Remark 2</a> .
C1 - C8	Second-order coefficient for a temperature-dependent heat capacity at constant pressure for up to eight different gases. These are denoted by the variable <i>C</i> in the equation in <a href="#">Remark 2</a> .

### Remarks:

- Methods for defining gas properties.** There are 2 methods of defining the gas properties for the mixture. If RUNIV is BLANK or ZERO, Method (A) is used to define constant heat capacities where per-mass unit values of  $C_v$  and  $C_p$  are input. Only Cards 2a.1 and 2a.2 are required for this method. Method (B) is used to define constant or temperature dependent heat capacities where per-mole unit values of  $C_p$  are input. Cards 2b.1 through 2b.4 are required for this method.
- Temperature-dependent heat capacity.** The per-mass-unit, temperature-dependent, constant-pressure heat capacity is

$$C_p(T) = \frac{[CPMOLE + B \times T + C \times T^2]}{\text{MOLWT}}$$

[Table M148-1](#) shows standard SI units for these quantities.

**Table M148-1.** Standard SI units.

$C_p(T)$	CPMOLE	$B$	$C$
$\frac{\text{J}}{\text{kg K}}$	$\frac{\text{J}}{\text{mole K}}$	$\frac{\text{J}}{\text{mole K}^2}$	$\frac{\text{J}}{\text{mole K}^3}$

3. **Initial temperature and density.** \*INITIAL\_GAS\_MIXTURE specifies the initial temperature and the density of the gas species present in a mesh or part at time zero.
4. **Temperature and energy conservation.** The ideal gas mixture is assumed to be thermal equilibrium, that is, all species are at the same temperature ( $T$ ). The gases in the mixture are also assumed to follow Dalton's Partial Pressure Law,  $P = \sum_i^{\text{ngas}} P_i$ . The partial pressure of each gas is then  $P_i = \rho_i R_{\text{gas}_i} T$  where  $R_{\text{gas}_i} = \frac{R_{\text{univ}}}{M_i}$ . The individual gas species temperature equals the mixture temperature. The temperature is computed from the internal energy where the *mixture internal energy per unit volume* is used,

$$e_V = \sum_i^{\text{ngas}} \rho_i C_{V_i} T_i = \sum_i^{\text{ngas}} \rho_i C_{V_i} T$$

$$T = T_i = \frac{e_V}{\sum_i^{\text{ngas}} \rho_i C_{V_i}}$$

In general, the advection step conserves *momentum* and *internal energy*, but not *kinetic energy*. This can result in energy lost in the system and lead to a pressure drop. In \*MAT\_GAS\_MIXTURE the dissipated kinetic energy is automatically converted into heat (internal energy). Thus, in effect the total energy is conserved instead of conserving just the internal energy. This numerical scheme has been shown to improve accuracy in some cases. However, the user should always be vigilant and check the physics of the problem closely.

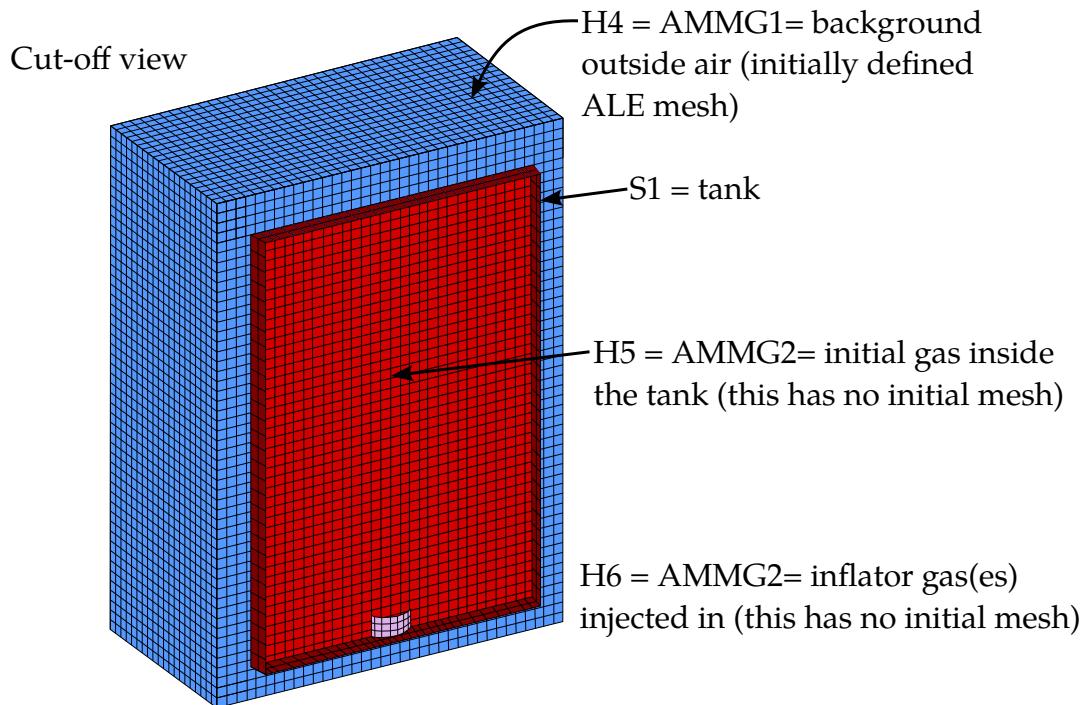
5. **IADIAB.** As an example, consider an airbag surrounded by ambient air. As the inflator gas flows into the bag, the ALE elements cut by the airbag fabric shell elements will contain some inflator gas inside and some ambient air outside. The multi-material element treatment is not perfect. Consequently the temperature of the outside air may be made artificially high after the multi-material element treatment. To prevent the outside ambient air from getting artificially high  $T$ , set IADIAB = 1 for the ambient air outside. A simple adiabatic compression equation is then assumed for the outside air. The use of this flag may be needed, but only when that air is modeled by the \*MAT\_GAS\_MIXTURE card.
6. **PDV.** In the original implementation, the ideal gas gamma law,  $e_{n+1}/e_n = (\rho_{n+1}/\rho_n)^\gamma$ , gave the element energy update. While this approach better-preserved accuracy, it violated energy balance in cases where  $\gamma$  differed across elements. For temperature-dependent cases, we recommend PDV = 1 which uses

pressure work instead of the ideal gas gamma law. PDV = 0 maintains the original behavior and is only for constant  $C_p/C_v$  cases.

**Example:**

Consider a tank test model where the Lagrangian tank (Part S1) is surrounded by an ALE air mesh (Part H4 = AMMGID 1). There are 2 ALE parts which are defined but initially have no corresponding mesh: part 5 (H5 = AMMGID 2) is the resident gas inside the tank at  $t = 0$ , and part 6 (H6 = AMMGID 2) is the inflator gas(es) which is injected into the tank when  $t > 0$ . AMMGID stands for ALE Multi-Material Group ID. Please see the figure and input below. The \*MAT\_GAS\_MIXTURE input defines the gas properties of ALE parts H5 & H6. The \*MAT\_GAS\_MIXTURE card input for both methods (A) and (B) are shown below.

The \*INITIAL\_GAS\_MIXTURE keyword input is also shown below. It basically specifies that “AMMGID 2 may be present in part or mesh H4 at  $t = 0$ , and the initial density of this gas is defined in the rho1 position which corresponds to the 1<sup>st</sup> material in the mixture (or H5, the resident gas).”

**Example Configuration:****Sample Input:**

```
$-----  
*PART
```

## \*MAT\_148

## \*MAT\_GAS\_MIXTURE

```
H5 = initial gas inside the tank
$      PID      SECID      MID      EOSID      HGID      GRAV      ADPOPT      TMID
$      5          5          5          0          5          0          0
*SECTION_SOLID
$      5          11         0
$-
$ Example 1: Constant heat capacities using per-mass unit.
$ *MAT_GAS_MIXTURE
$      MID      IADIAB      R_univ
$      5          0          0
$ Cv1_mas Cv2_mas Cv3_mas Cv4_mas Cv5_mas Cv6_mas Cv7_mas Cv8_mas
$718.7828911237.56228
$ Cp1_mas Cp2_mas Cp3_mas Cp4_mas Cp5_mas Cp6_mas Cp7_mas Cp8_mas
$1007.00058 1606.1117
$-
$ Example 2: Variable heat capacities using per-mole unit.
*MAT_GAS_MIXTURE
$      MID      IADIAB      R_univ
$      5          0      8.314470
$      MW1      MW2      MW3      MW4      MW5      MW6      MW7      MW8
$ 0.0288479 0.022256
$ Cp1_mol Cp2_mol Cp3_mol Cp4_mol Cp5_mol Cp6_mol Cp7_mol Cp8_mol
$ 29.049852 36.23388
$ B1      B2      B3      B4      B5      B6      B7      B8
$ 7.056E-3 0.132E-1
$ C1      C2      C3      C4      C5      C6      C7      C8
$ -1.225E-6 -0.190E-5
$-
$ One card is defined for each AMMG that will occupy some elements of a mesh set
*INITIAL_GAS_MIXTURE
$      SID      STYPE      MMGID      T0
$      4          1          1      298.15
$      RHO1      RHO2      RHO3      RHO4      RHO5      RHO6      RHO7      RHO8
$ 1.17913E-9
*INITIAL_GAS_MIXTURE
$      SID      STYPE      MMGID      T0
$      4          1          2      298.15
$      RHO1      RHO2      RHO3      RHO4      RHO5      RHO6      RHO7      RHO8
$ 1.17913E-9
$-
```

**\*MAT\_EMMI**

This is Material Type 151. The Evolving Microstructural Model of Inelasticity (EMMI) is a temperature and rate-dependent state variable model developed to represent the large deformation of metals under diverse loading conditions [Marin et al. 2006]. It includes various state variables to characterize effects of microstructural features, such as dislocation creation or annihilation. This model is available for 3D solid elements, 2D solid elements and thick shell forms 3 and 5.

**Card Summary:**

**Card 1.** This card is required.

MID	RHO	E	PR				
-----	-----	---	----	--	--	--	--

**Card 2.** This card is required.

RGAS	BVECT	D0	QD	CV	ADRAG	BDRAG	DMHTA
------	-------	----	----	----	-------	-------	-------

**Card 3.** This card is required.

DMPHI	DNTHTA	DNPHI	THETA0	THETAM	BETA0	BTHETA	DMR
-------	--------	-------	--------	--------	-------	--------	-----

**Card 4.** This card is required.

DNUC1	DNUC2	DNUC3	DNUC4	DM1	DM2	DM3	DM4
-------	-------	-------	-------	-----	-----	-----	-----

**Card 5.** This card is required.

DM5	Q1ND	Q2ND	Q3ND	Q4ND	CALPHA	CKAPPA	C1
-----	------	------	------	------	--------	--------	----

**Card 6.** This card is required.

C2ND	C3	C4	C5	C6	C7ND	C8ND	C9ND
------	----	----	----	----	------	------	------

**Card 7.** This card is required.

C10	A1	A2	A3	A4	A_XX	A YY	A_ZZ
-----	----	----	----	----	------	------	------

**Card 8.** This card is required.

A_XY	A_YZ	A_XZ	ALPHXX	ALPHYY	ALPHZZ	ALPHXY	ALPHYZ
------	------	------	--------	--------	--------	--------	--------

**Card 9.** This card is required.

ALPHXZ	DKAPPA	PHIO	PHICR	DLBDAG	FACTOR	RSWTCH	DMGOPT
--------	--------	------	-------	--------	--------	--------	--------

**Card 10.** This card is required.

DELASO	DIMPLO	ATOL	RTOL	DINTER			
--------	--------	------	------	--------	--	--	--

**Card 11.** This card is required. *Leave this card blank.*

--	--	--	--	--	--	--	--

### Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RHO	E	PR				
Type	A	F	F	F				

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
MID	Material identification. A unique number or label must be specified (see *PART).							
RHO	Material density							
E	Young's modulus							
PR	Poisson's ratio							

Card 2	1	2	3	4	5	6	7	8
Variable	RGAS	BVECT	D0	QD	CV	ADRAG	BDRAG	DMHTA
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
RGAS	Universal gas constant							
BVECT	Burger's vector							
D0	Pre-exponential diffusivity coefficient							
QD	Activation energy							

<b>VARIABLE</b>	<b>DESCRIPTION</b>
CV	Specific heat at constant volume
ADRAG	Drag intercept
BDRAG	Drag coefficient
DMTHTA	Shear modulus temperature coefficient

Card 3	1	2	3	4	5	6	7	8
Variable	DMPHI	DNTHTA	DNPHI	THETA0	THETAM	BETA0	BTTHETA	DMR
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
DMPHI	Shear modulus damage coefficient
DNTHTA	Bulk modulus temperature coefficient
DNPHI	Bulk modulus damage coefficient
THETA0	Reference temperature
THETAM	Melt temperature
BETA0	Coefficient of thermal expansion at reference temperature
BTTHETA	Thermal expansion temperature coefficient
DMR	Damage rate sensitivity parameter

Card 4	1	2	3	4	5	6	7	8
Variable	DNUC1	DNUC2	DNUC3	DNUC4	DM1	DM2	DM3	DM4
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
DNUC1	Nucleation coefficient 1

**\*MAT\_151****\*MAT\_EMMI**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
DNUC2	Nucleation coefficient 2
DNUC3	Nucleation coefficient 3
DNUC4	Nucleation coefficient 4
DM1	Coefficient of yield temperature dependence
DM2	Coefficient of yield temperature dependence
DM3	Coefficient of yield temperature dependence
DM4	Coefficient of yield temperature dependence

Card 5	1	2	3	4	5	6	7	8
Variable	DM5	Q1ND	Q2ND	Q3ND	Q4ND	CALPHA	CKAPPA	C1
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
DM5	Coefficient of yield temperature dependence
Q1ND	Dimensionless activation energy, $Q_1$ , for $f$
Q2ND	Dimensionless activation energy, $Q_2$ , for $r_d$
Q3ND	Dimensionless activation energy, $Q_3$ , for $R_d$
Q4ND	Dimensionless activation energy, $Q_4$ , for $R_s$
CALPHA	Coefficient for backstress, $\alpha$
CKAPPA	Coefficient for internal stress, $\kappa$
C1	Parameter, $c_1$ , for flow rule exponent, $n$

Card 6	1	2	3	4	5	6	7	8
Variable	C2ND	C3	C4	C5	C6	C7ND	C8ND	C9ND
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
C2ND	Parameter, $c_2$ , for transition rate $f$
C3	Parameter, $c_3$ , for alpha dynamic recovery, $r_d$
C4	Parameter, $c_4$ , for alpha hardening, $h$
C5	Parameter, $c_5$ , for kappa dynamic recovery, $R_d$
C6	Parameter, $c_6$ , for kappa hardening, $H$
C7ND	Parameter, $c_7$ , kappa static recovery, $R_s$
C8ND	Parameter, $c_8$ , for yield
C9ND	Parameter, $c_9$ , for temperature dependence of flow rule exponent, $n$

Card 7	1	2	3	4	5	6	7	8
Variable	C10	A1	A2	A3	A4	A_XX	A YY	A_ZZ
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
C10	Parameter, $c_{10}$ , for static recovery (set to 1)
A1	Plastic anisotropy parameter
A2	Plastic anisotropy parameter
A3	Plastic anisotropy parameter
A4	Plastic anisotropy parameter
A_XX	Initial structure tensor component

**\*MAT\_151****\*MAT\_EMMI**

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
Card 8	1	2	3	4	5	6	7	8
Variable	A_XY	A_YZ	A_XZ	ALPHXX	ALPHYY	ALPHZZ	ALPHXY	ALPHYZ
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
A_XY		Initial structure tensor component						
A_YZ		Initial structure tensor component						
A_XZ		Initial structure tensor component						
ALPHXX		Initial backstress component						
ALPHYY		Initial backstress component						
ALPHZZ		Initial backstress component						
ALPHXY		Initial backstress component						
ALPHYZ		Initial backstress component						

Card 9	1	2	3	4	5	6	7	8
Variable	ALPHXZ	DKAPPA	PHI0	PHICR	DLBDAG	FACTOR	RSWTCH	DMGOPT
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
ALPHXZ		Initial backstress component						
DKAPPA		Initial isotropic internal stress						
PHI0		Initial isotropic porosity						

<b>VARIABLE</b>	<b>DESCRIPTION</b>
PHICR	Critical cutoff porosity
DLBDAG	Slip system geometry parameter
FACTOR	Fraction of plastic work converted to heat, adiabatic
RSWTCH	Rate sensitivity switch
DMGOPT	Damage model option parameter: EQ.1.0: Pressure independent Cocks/Ashby 1980 EQ.2.0: Pressure dependent Cocks/Ashby 1980 EQ.3.0: Pressure dependent Cocks 1989

Card 10	1	2	3	4	5	6	7	8
Variable	DELASO	DIMPLO	ATOL	RTOL	DINTER			
Type	F	F	F	F	F			

<b>VARIABLE</b>	<b>DESCRIPTION</b>
DELASO	Temperature option: EQ.0.0: Driven externally EQ.1.0: Adiabatic
DIMPLO	Implementation option flag: EQ.1.0: Combined viscous drag and thermally activated dislocation motion EQ.2.0: Separate viscous drag and thermally activated dislocation motion
ATOL	Absolute error tolerance for local Newton iteration
RTOL	Relative error tolerance for local Newton iteration
DNITER	Maximum number of iterations for local Newton iteration

Leave this card blank (but include it!).

Card 11	1	2	3	4	5	6	7	8
Variable								
Type								

**Remarks:**

1. **EMMI Plasticity Model.** The following equations summarize the evolution equations and material functions for the EMMI model. See [Marin et al 2006] for more details.

$$\begin{aligned}\nabla \alpha &= h \mathbf{d}^p - r_d \dot{\varepsilon}^p \bar{\alpha} \alpha \\ \dot{\kappa} &= (H - R_d \kappa) \dot{\varepsilon}^p - R_s \kappa \sinh(Q_s \kappa) \\ \mathbf{d}^p &= \sqrt{\frac{3}{2}} \dot{\varepsilon}^p \mathbf{n}, \dot{\varepsilon}^p = f \sinh^n \left[ \left( \frac{\bar{\sigma}}{\kappa + Y} - 1 \right) \right]\end{aligned}$$

$\dot{\varepsilon}^p$ – equation	$\alpha$ – equation	$\kappa$ – equation
$f = c_2 \exp\left(\frac{Q_1}{\theta}\right)$ $n = \frac{c_9}{\theta} - c_1$ $Y = c_8 \hat{Y}(\theta)$	$r_d = c_3 \exp\left(\frac{-Q_2}{\theta}\right)$ $h = c_4 \hat{\mu}(\theta)$	$R_d = c_5 \exp\left(\frac{-Q_3}{\theta}\right)$ $H = c_6 \hat{\mu}(\theta)$ $R_s = c_7 \exp\left(\frac{-Q_4}{\theta}\right)$ $Q_s = c_{10} \exp\left(\frac{-Q_5}{\theta}\right)$

**Table M151-1.** Plasticity Material Functions of EMMI Model.

2. **Void Growth.** The following equations extend the EMMI material model for void growth. See [Marin et al 2006] for more details

$$\begin{aligned}\dot{\varphi} &= \frac{3}{\sqrt{2}} (1 - \varphi) \hat{G}(\bar{\sigma}_{eq}, \bar{p}, \varphi) \dot{\varepsilon}^p \\ \hat{G}(\bar{\sigma}_{eq}, \bar{p}_\tau, \varphi) &= \frac{3}{\sqrt{3}} \left[ \frac{1}{(1 - \varphi)m + 1} - 1 \right] \sinh \left[ \frac{2(2m - 1)}{2m + 1} \frac{\langle \bar{p} \rangle}{\bar{\sigma}_{eq}} \right]\end{aligned}$$

**\*MAT\_DAMAGE\_3**

This is Material Type 153. This model has up to 10 back stress terms for kinematic hardening combined with isotropic hardening and a damage model for modeling low cycle fatigue and failure. The model is based on Huang [2009]. It is available for solid, shell, thick shell, and beam elements. This model is supported for both explicit and implicit analysis. For beams the model is restricted to 3 back stress terms, temperature independent data and KHFLG = 0; while for solids, shells, and thick shells up to 10 back stress terms can be used, including temperature effects and parameter fit from uniaxial cyclic stress-strain tests (KHFLG > 0).

**Card Summary:**

**Card 1.** This card is required.

MID	R0	E	PR	SIGY	HARDI	BETA	LCSS
-----	----	---	----	------	-------	------	------

**Card 2.** This card is required.

HARDK1	GAMMA1	HARDK2	GAMMA2	SRC	SRP	HARDK3	GAMMA3
--------	--------	--------	--------	-----	-----	--------	--------

**Card 3.** This card is required.

IDAM	IDS	IDEF	EPSD	S	T	DC	KHFLG
------	-----	------	------	---	---	----	-------

**Card 4a.** This card is only read when KHFLG = 0. It is optional.

HARDK4	GAMMA4						
--------	--------	--	--	--	--	--	--

**Card 4b.** This card is included if KHFLG > 0.

LCKH	NKH						
------	-----	--	--	--	--	--	--

**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	HARDI	BETA	LCSS
Type	A	F	F	F	F	F	F	I

**\*MAT\_153****\*MAT\_DAMAGE\_3**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density, $\rho$
E	Young's modulus, $E$ LT.0: -E gives the curve ID for $E$ as a function of temperature.
PR	Poisson's ratio, $\nu$ LT.0: -PR gives the curve ID for $\nu$ as a function of temperature.
SIGY	Initial yield stress, $\sigma_{y0}$ (ignored if LCSS > 0)
HARDI	Isotropic hardening modulus, $H$ (ignored if LCSS > 0)
BETA	Isotropic hardening parameter, $\beta$ . Set $\beta = 0$ for linear isotropic hardening. (Ignored if LCSS > 0 or if HARDI = 0.)
LCSS	Load curve or table ID defining effective stress as a function of effective plastic strain (and temperature in the table case) for isotropic hardening. For a table each curve corresponds to a temperature. The first abscissa value (effective plastic strain) in each curve must be zero corresponding to the initial yield stress. The first ordinate value in each curve is the initial yield stress.

Card 2	1	2	3	4	5	6	7	8
Variable	HARDK1	GAMMA1	HARDK2	GAMMA2	SRC	SRP	HARDK3	GAMMA3
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
HARDK $j$	Kinematic hardening modulus, $C_j$ LT.0: -HARDK $j$ gives the curve ID for $C_j$ as a function of temperature.
GAMMA $j$	Kinematic hardening parameter, $\gamma_j$ . Set $\gamma_j = 0$ for linear kinematic hardening. Ignored if HARDK $j$ = 0.

VARIABLE	DESCRIPTION
	LT.0: -GAMMA $j$ gives the curve ID for $\gamma_j$ as a function of temperature.
SRC	Strain rate parameter, $C$ , for Cowper Symonds strain rate model; see remarks below. If zero, rate effects are not considered. LT.0: -SRC gives the curve ID for $C$ as a function of temperature.
SRP	Strain rate parameter, $p$ , for Cowper Symonds strain rate model; see remarks below. If zero, rate effects are not considered. LT.0: -SRP gives the curve ID for $p$ as a function of temperature.

Card 3	1	2	3	4	5	6	7	8
Variable	IDAM	IDS	I <sup>D</sup> E <sup>P</sup>	EPSD	S	T	DC	KHFLG
Type	I	I	I	F	F	F	F	I

VARIABLE	DESCRIPTION
IDAM	Isotropic damage flag: EQ.0: Damage is inactivated. IDS, I <sup>D</sup> E <sup>P</sup> , EPSD, S, T, and DC are ignored. EQ.1: Damage is activated.
IDS	Output stress flag: EQ.0: Undamaged stress, $\tilde{\sigma}$ , is output. EQ.1: Damaged stress, $\tilde{\sigma}(1 - D)$ , is output.
I <sup>D</sup> E <sup>P</sup>	Damaged plastic strain: EQ.0: Plastic strain is accumulated, $r = \int \dot{\varepsilon}^{pl}$ . EQ.1: Damaged plastic strain is accumulated, $r = \int (1 - D) \dot{\varepsilon}^{pl}$ .
EPSD	Damage threshold, $r_d$ . Damage accumulation begins when $r > r_d$ .
S	Damage material constant, $S$ . Default = $\sigma_{y0}/200$ .
T	Damage material constant, $t$ . Default = 1.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
DC	Critical damage value, $D_c$ . When damage value reaches the critical value, the element is deleted from calculation. Default = 0.5.
KHFLG	<p>Kinematic hardening flag:</p> <p>EQ.0: Use kinematic hardening parameters <math>\text{HARDK}_j</math> and <math>\text{GAMMA}_j</math> (default).</p> <p>EQ.1: Kinematic hardening parameters (<math>C_j, \gamma_j</math>) given by load curve or table if temperature is considered. NKH data points used (with a maximum of 10) in each curve. <math>\text{HARDK}_j</math> and <math>\text{GAMMA}_j</math> fields are ignored.</p> <p>EQ.2: Fits NKH kinematic hardening parameters (<math>C_j, \gamma_j</math>) to uniaxial stress-strain data at constant temperature for a half-cycle, meaning it fits</p> $\sigma_i = \sigma_y(\varepsilon_i^p) + \sum_{j=1}^{NKH} \frac{C_j}{\gamma_j} (1 - \exp(-\gamma_j \varepsilon_i^p))$ <p>to stress as a function of plastic strain data. The stress, <math>\sigma_i</math>, can be given in field LCKH as a function of strain, <math>\varepsilon_i^p</math>, in a load curve or as a function of strain and temperature, <math>T</math>, in a table. <math>\text{HARDK}_j</math> and <math>\text{GAMMA}_j</math> fields are ignored.</p> <p>EQ.3: Fits NKH kinematic hardening parameters (<math>C_j, \gamma_j</math>) to uniaxial stress-strain data for the tensile part of a stabilized cycle, meaning it fits</p> $\sigma_i = \frac{\sigma_1 + \sigma_N}{2} + \sum_{j=1}^{NKH} \frac{C_j}{\gamma_j} (1 - 2 \exp(-\gamma_j \varepsilon_i^p))$ <p>to <math>N</math> stress as a function of plastic strain data. This data is given by LCKH as either a load curve or table depending on if temperature is included. Here the first data point is chosen such that <math>\varepsilon_1^p = 0</math>. <math>\text{HARDK}_j</math> and <math>\text{GAMMA}_j</math> fields are ignored.</p> <p>EQ.4: Fits NKH kinematic hardening parameters (<math>C_j, \gamma_j</math>) to uniaxial stress-strain data for different stabilized cycles, that is, it fits</p> $\sigma_i = \sigma_y(\varepsilon_i^p) + \sum_{j=1}^{NKH} \frac{C_j}{\gamma_j} \tanh(\gamma_j \varepsilon_i^p),$ <p>to max stress as a function of max plastic strain data over <math>N</math> cycles. This data is given by LCKH as either a load</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
	curve or table depending on if temperature is defined. HARDK $j$ and GAMMA $j$ fields are ignored.							

Optional Card 4 (read only if KHFLG = 0)

Card 4a	1	2	3	4	5	6	7	8
Variable	HARDK4	GAMMA4						
Type	F	F						

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
HARDK4	Kinematic hardening modulus, $C_4$  LT.0: -HARDK4 gives the curve ID for $C_4$ as a function of temperature.							
GAMMA4	Kinematic hardening parameter, $\gamma_4$ . Set $\gamma_4 = 0$ for linear kinematic hardening. Ignored if HARDK4 = 0.  LT.0: -GAMMA4 gives the curve ID for $\gamma_4$ as a function of temperature.							

Card 4 (included if and only if KHFLG &gt; 0)

Card 4b	1	2	3	4	5	6	7	8
Variable	LCKH	NKH						
Type	I	I						

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
LCKH	Load curve or table ID defining kinematic hardening when KHFLG > 0. A table is used when temperature dependence is considered. Depending on KHFLG, it gives either $(C_j, \gamma_j)$ values or stress as a function of plastic strain with optional temperature dependence.							
NKH	Number of kinematic hardening parameters when KHFLG > 0. Up to 10 back stresses can be used.							

**Model Description:**

This model is based on the work of Lemaitre [1992], and Dufailly and Lemaitre [1995]. It is a pressure-independent plasticity model with the yield surface defined by the function

$$F = \bar{\sigma} - \sigma_y = 0 ,$$

where  $\sigma_y$  is uniaxial yield stress,

$$\sigma_y = \sigma_{y0} + \frac{H}{\beta} [1 - \exp(-\beta r)] .$$

By setting  $\beta = 0$ , a linear isotropic hardening is obtained

$$\sigma_y = \sigma_{y0} + Hr ,$$

where  $\sigma_{y0}$  is the initial yield stress. In the above,  $\bar{\sigma}$  is the equivalent von Mises stress, with respect to the deviatoric effective stress,

$$\mathbf{s}_e = \text{dev}[\tilde{\boldsymbol{\sigma}}] - \boldsymbol{\alpha} = \mathbf{s} - \boldsymbol{\alpha} .$$

Here  $\mathbf{s}$  is deviatoric stress and  $\boldsymbol{\alpha}$  is the back stress, which is the sum of up to four terms according to:

$$\boldsymbol{\alpha} = \sum_j \boldsymbol{\alpha}_j .$$

$\tilde{\boldsymbol{\sigma}}$  is effective stress (undamaged stress), based on Continuum Damage Mechanics model [Lemaitre 1992],

$$\tilde{\boldsymbol{\sigma}} = \frac{\boldsymbol{\sigma}}{1 - D} .$$

Here  $D$  is the isotropic damage scalar, which is bounded by 0 and 1

$$0 \leq D \leq 1 .$$

$D = 0$  represents a damage-free material RVE (representative volume element), while  $D = 1$  represents a fully broken material RVE in two parts. In fact, fracture occurs when  $D = D_c < 1$ , modeled as element removal. The evolution of the isotropic damage value related to ductile damage and fracture (the case where the plastic strain or dissipation is much larger than the elastic one, [Lemaitre 1992]) is defined as

$$\dot{D} = \begin{cases} \left(\frac{Y}{S}\right)^t \dot{\varepsilon}^{pl} & \text{when } r > r_d \text{ and } \frac{\sigma_m}{\sigma_{eq}} > -\frac{1}{3} \\ 0 & \text{otherwise} \end{cases}$$

where  $\sigma_m/\sigma_{eq}$  is the stress triaxiality,  $r_d$  is damage threshold,  $S$  is a material constant, and  $Y$  is strain energy density release rate:

$$Y = \frac{1}{2} \boldsymbol{\varepsilon}^{\text{el}} : \mathbf{D}^{\text{el}} : \boldsymbol{\varepsilon}^{\text{el}} .$$

Here  $\mathbf{D}^{\text{el}}$  represents the fourth-order elasticity tensor and  $\boldsymbol{\varepsilon}^{\text{el}}$  is elastic strain. In the above,  $t$  is a material constant, introduced by Dufailly and Lemaitre [1995], to provide an

additional degree of freedom for modeling low-cycle fatigue ( $t = 1$  in Lemaitre [1992]). Dufailly and Lemaitre [1995] also proposed a simplified method to fit experimental results and get  $S$  and  $t$ .

The equivalent Mises stress is defined as

$$\bar{\sigma}(\mathbf{s}_e) = \sqrt{\frac{3}{2} \mathbf{s}_e \cdot \mathbf{s}_e} = \sqrt{\frac{3}{2} \|\mathbf{s}_e\|} .$$

The model assumes associated plastic flow

$$\dot{\epsilon}^{pl} = \frac{\partial F}{\partial \sigma} d\lambda = \frac{3}{2} \frac{\mathbf{s}_e}{\bar{\sigma}} d\lambda ,$$

where  $d\lambda$  is the plastic consistency parameter. The evolution of the kinematic component of the model is defined as [Armstrong and Frederick 1966]:

$$\dot{\alpha}_j = \begin{cases} \frac{2}{3} C_j \dot{\epsilon}^{pl} - \gamma_j \alpha_j \dot{\epsilon}^{pl} & \text{if IDEP} = 0 \\ (1 - D) \left( \frac{2}{3} C_j \dot{\epsilon}^{pl} - \gamma_j \alpha_j \dot{\epsilon}^{pl} \right) & \text{if IDEP} = 1 \end{cases}$$

The damaged plastic strain is accumulated as

$$r = \begin{cases} \int \dot{\epsilon}^{pl} & \text{if IDEP} = 0 \\ \int (1 - D) \dot{\epsilon}^{pl} & \text{if IDEP} = 1 \end{cases}$$

where  $\dot{\epsilon}^{pl}$  is the equivalent plastic strain rate

$$\dot{\epsilon}^{pl} = \sqrt{\frac{2}{3} \dot{\epsilon}^{pl} : \dot{\epsilon}^{pl}} .$$

$\dot{\epsilon}^{pl}$  represents the rate of plastic flow.

Strain rate is accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left( \frac{\dot{\epsilon}}{C} \right)^{1/p}$$

where  $\dot{\epsilon}$  is the strain rate.

### Uniaxial cyclic tension and compression:

This material can be used to model cyclic hardening plasticity, including effects known as *plastic shakedown* and *strain ratcheting*. To understand how the plasticity parameters qualitatively influence the behavior in uniaxial tension and compression, we restrict ourselves to a discussion concerning linear isotropic hardening with initial yield  $\sigma_Y$  and hardening modulus  $H$ . We also only include two kinematic hardening terms. For the

kinematic part, we use one linear term with hardening  $C_0$  (and decay coefficient  $\gamma_0 = 0$ ) and one combined term with hardening  $C$  and decay coefficient  $\gamma$ . The elastic Young's modulus is denoted  $E$ , and we neglect any forms of temperature or rate effects.

While this is merely an attempt to explain the phenomena, estimating the parameters that reflect the actual behavior of the physical material may be difficult. Because of this, we recommend the fitting options provided by KHFLG, where even the effects of temperature can be accounted for.

### *Strain induced deformation*

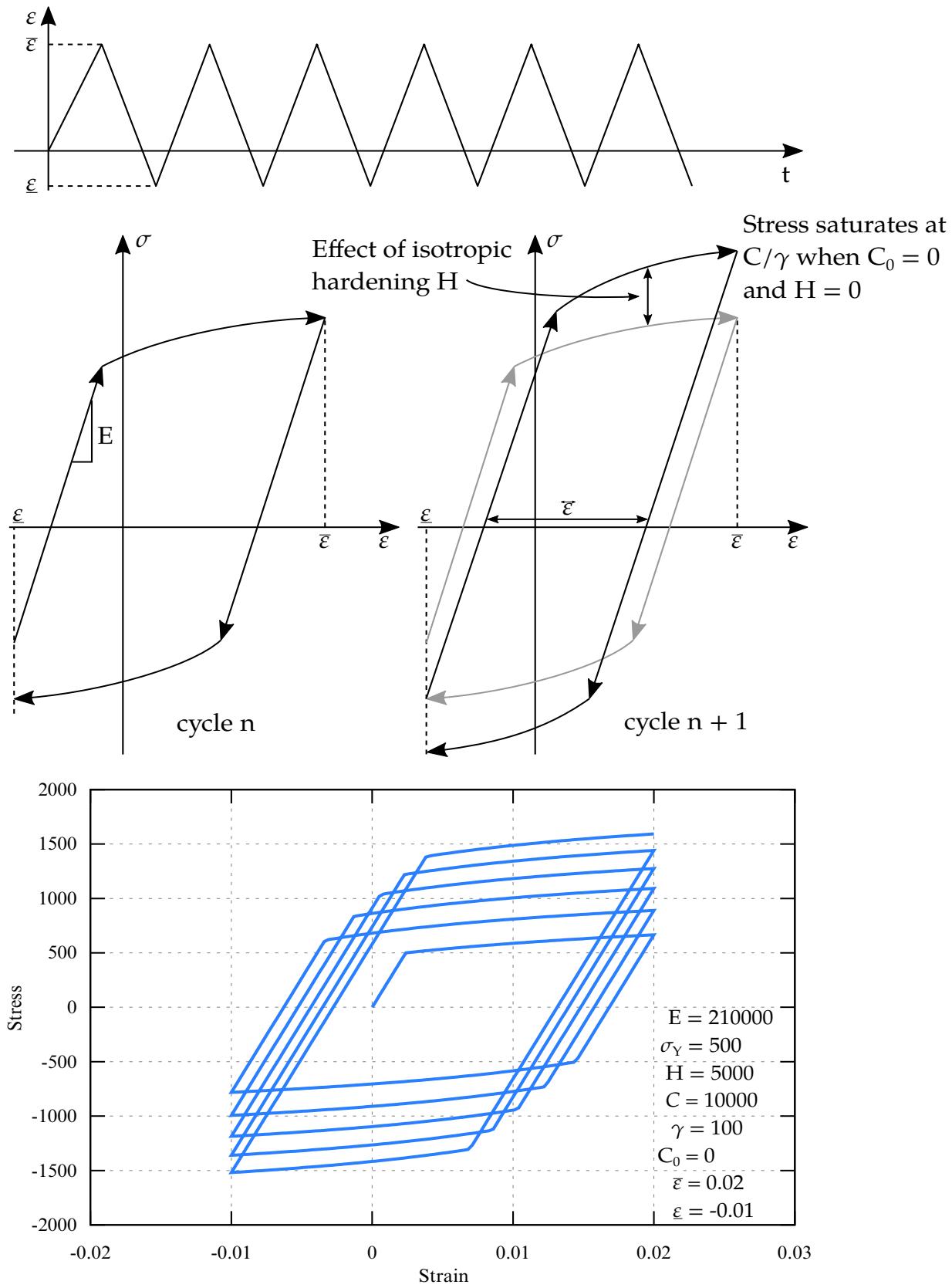
Consider the cyclic deformation depicted in [Figure M153-1](#) in which the uniaxial strain ranges between  $\underline{\varepsilon}$  and  $\bar{\varepsilon}$ . Two subsequent stress-strain cycles are shown.

If the isotropic hardening modulus  $H = 0$ , then the cycles are identical. For nonzero hardening  $H$ , the stress level increases with each cycle and the strain width indicated by  $\bar{\varepsilon}$  decreases. As the yield surface expands, the isotropic hardening effect diminishes, and we tend towards a stable cycle; this phenomenon is called *plastic shakedown*.

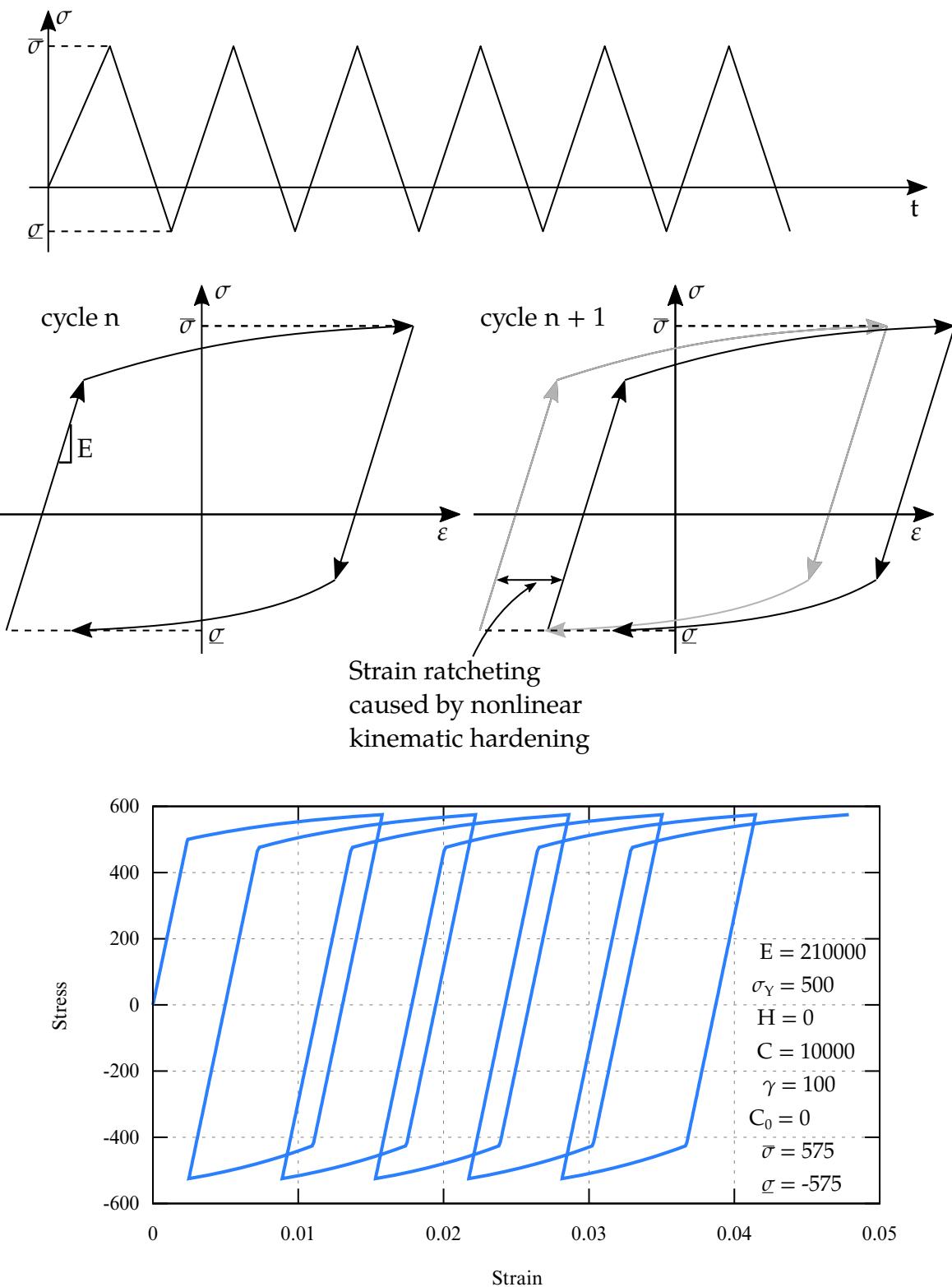
If both  $H$  and  $C_0$  are zero, then the end of each cycle tends towards ideal plastic since the presence of a nonzero  $\gamma$  saturates the level of back stress and consequently the stress  $\sigma$  itself. This physical phenomenon is unlikely. Therefore, we recommend having at least one linear kinematic hardening term present in combination with some isotropic hardening for a realistic behavior.

### *Stress induced deformation*

For stress induced deformation, we impose a cyclic stress between  $\underline{\sigma}$  and  $\bar{\sigma}$  as shown in [Figure M153-2](#) and investigate two subsequent stress-strain cycles. In this case a combination of isotropic hardening and nonlinear kinematic hardening may cause a drift in strain. This drift is referred to as *ratcheting strain* and may be considered a creep phenomenon. Even without the isotropic hardening  $H$ , a nonzero mean stress,  $(\bar{\sigma} + \underline{\sigma})/2$ , in the cycle causes a ratcheting effect. We again recommend using combinations of isotropic, linear and nonlinear kinematic hardening for accurate predictions of this creep behavior.



**Figure M153-1.** Schematic of Uniaxial Plastic Shakedown Phenomenon



**Figure M153-2.** Schematic of Uniaxial Strain Ratcheting Phenomenon

**Material Model Comparison:**

**Table M153-1** below shows the difference between MAT 153 (for KHFLG = 0) and MAT 104/105. MAT 153 is less computationally expensive than MAT 104/105. Kinematic hardening, which already exists in MAT 103, is included in MAT 153 but not in MAT 104/105.

	MAT 153	MAT 104	MAT 105
Computational cost	1.0	3.0	3.0
Isotropic hardening	One component	Two components	One component
Kinematic hardening	Four components	N/A	N/A
Output stress	$\text{IDS} = 0 \Rightarrow \tilde{\sigma}$ $\text{IDS} = 1 \Rightarrow \tilde{\sigma}(1 - D)$	$\tilde{\sigma}(1 - D)$	$\tilde{\sigma}(1 - D)$
Damaged plastic strain	$\text{IDEP} = 0 \Rightarrow r = \int \dot{\varepsilon}^{\text{pl}}$ $\text{IDEP} = 1 \Rightarrow r = \int (1 - D) \dot{\varepsilon}^{\text{pl}}$	$r = \int (1 - D) \dot{\varepsilon}^{\text{pl}}$	$r = \int (1 - D) \dot{\varepsilon}^{\text{pl}}$
Accumulation when	$\frac{\sigma_m}{\sigma_{eq}} > -\frac{1}{3}$	$\sigma_1 > 0$	$\sigma_1 > 0$
Isotropic plasticity	Yes	Yes	Yes
Anisotropic plasticity	No	Yes	No
Isotropic damage	Yes	Yes	Yes
Anisotropic damage	No	Yes	No

**Table M153-1.** Differences between MAT 153 and MAT 104/105**History Variables:**

Additional history variables, which can be written by using variables NEIPH and NEIPS in \*DATABASE\_EXTENT\_BINARY, are as follows:

History Variable #	Description
1	Damage, $D$

**\*MAT\_153****\*MAT\_DAMAGE\_3**

History Variable #	Description
2	Back stress term 1 in the 11-direction
3	Back stress term 1 in the 22-direction
4	Back stress term 1 in the 12-direction
5	Back stress term 1 in the 23-direction
6	Back stress term 1 in the 31-direction
7	Back stress term 2 in the 11-direction
8	Back stress term 2 in the 22-direction
9	Back stress term 2 in the 12-direction
10	Back stress term 2 in the 23-direction
11	Back stress term 2 in the 31-direction
12	Back stress term 3 in the 11-direction
13	Back stress term 3 in the 22-direction
14	Back stress term 3 in the 12-direction
15	Back stress term 3 in the 23-direction
16	Back stress term 3 in the 31-direction
17	Back stress term 4 in the 11-direction
18	Back stress term 4 in the 22-direction
19	Back stress term 4 in the 12-direction
20	Back stress term 4 in the 23-direction
21	Back stress term 4 in the 31-direction

**\*MAT\_DESHPANDE\_FLECK\_FOAM**

This is Material Type 154 for solid elements. This material is for modeling aluminum foam used as a filler material in aluminum extrusions to enhance the energy absorbing capability of the extrusion. Such energy absorbers are used in vehicles to dissipate energy during impact. This model was developed by Reyes, Hopperstad, Berstad, and Langseth [2002] and is based on the foam model by Deshpande and Fleck [2000].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RHO	E	PR	ALPHA	GAMMA		
Type	A	F	F	F	F	F		
Default	none	none	none	none	none	none		

Card 2	1	2	3	4	5	6	7	8
Variable	EPSD	ALPHA2	BETA	SIGP	DERFI	CFAIL	PFAIL	NUM
Type	F	F	F	F	F	F	F	I
Default	none	none	none	none	none	↓	↓	1000

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label must be specified (see *PART).
RHO	Mass density
E	Young's modulus
PR	Poisson's ratio
ALPHA	Controls shape of yield surface
GAMMA	Material parameter, $\gamma$ ; see Remarks.
EPSD	Densification strain

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ALPHA2	Material parameter, $\alpha_2$ ; see Remarks.
BETA	Material parameter, $\beta$ ; see Remarks.
SIGP	Material parameter, $\sigma_p$ ; see Remarks.
DERFI	Type of derivation used in material subroutine: EQ.0: Numerical derivation EQ.1: Analytical derivation
CFAIL	Tensile volumetric strain at failure. Default is no failure due to tensile volumetric strain.
PFAIL	Maximum principal stress at failure. Must be sustained NUM ( $> 0$ ) timesteps to fail element. Default is no failure due to maximum principal stress.
NUM	Number of timesteps at or above PFAIL to trigger element failure

**Remarks:**

The yield stress function,  $\Phi$ , is defined by:

$$\Phi = \hat{\sigma} - \sigma_y .$$

The equivalent stress,  $\hat{\sigma}$ , is given by:

$$\hat{\sigma}^2 = \frac{\sigma_{VM}^2 + \alpha^2 \sigma_m^2}{1 + \left(\frac{\alpha}{3}\right)^2} ,$$

where,  $\sigma_{VM}$ , is the von Mises effective stress:

$$\sigma_{VM} = \sqrt{\frac{2}{3} \boldsymbol{\sigma}^{\text{dev}} : \boldsymbol{\sigma}^{\text{dev}}} .$$

In this equation  $\sigma_m$  and  $\boldsymbol{\sigma}^{\text{dev}}$  are the mean and deviatoric stress:

$$\boldsymbol{\sigma}^{\text{dev}} = \boldsymbol{\sigma} - \sigma_m \mathbf{I} .$$

The yield stress,  $\sigma_y$ , can be expressed as:

$$\sigma_y = \sigma_p + \gamma \frac{\hat{\varepsilon}}{\varepsilon_D} + \alpha_2 \ln \left[ \frac{1}{1 - \left( \frac{\hat{\varepsilon}}{\varepsilon_D} \right)^\beta} \right] .$$

Here,  $\sigma_p$ ,  $\alpha_2$ ,  $\gamma$ , and  $\beta$  are material parameters. The densification strain  $\varepsilon_D$  is defined as:

$$\varepsilon_D = -\ln \left( \frac{\rho_f}{\rho_{f0}} \right) ,$$

where  $\rho_f$  is the foam density and  $\rho_{f0}$  is the density of the virgin material.

## \*MAT\_155

## \*MAT\_PLASTICITY\_COMPRESSION\_TENSIONEOS

### \*MAT\_PLASTICITY\_COMPRESSION\_TENSIONEOS

This is Material Type 155. An isotropic elastic-plastic material where unique yield stress as a function of plastic strain curves can be defined for compression and tension. Also, failure can occur based on a plastic strain or a minimum time step size. Rate effects on the yield stress are modeled either by using the Cowper-Symonds strain rate model or by using two load curves that scale the yield stress values in compression and tension, respectively. Material rate effects, which are independent of the plasticity model, are based on a 6-term Prony series Maxwell mode that generates an additional stress tensor. The viscous stress tensor is superimposed on the stress tensor generated by the plasticity. Pressure is defined by an equation of state, which is required to utilize this model. This model is applicable to solid elements and SPH.

#### Card Summary:

**Card 1.** This card is required.

MID	R0	E	PR	C	P	FAIL	TDEL
-----	----	---	----	---	---	------	------

**Card 2.** This card is required.

LCIDC	LCIDT	LCSRC	LCSRT	SRFLAG			
-------	-------	-------	-------	--------	--	--	--

**Card 3.** This card is required.

PC	PT	PCUTC	PCUTT	PCUTF	SCALEP	SCALEE	
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**Card 4.** This card is required.

K							
---	--	--	--	--	--	--	--

**Card 5.** This card is optional. Up to six cards may be input. The next keyword ("\*") card terminates this input.

Gi	BETAi						
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	C	P	FAIL	TDEL
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	0.0	0.0	$10^{20}$	0.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified.
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
C	Strain rate parameter, $C$ ; see Remarks below.
P	Strain rate parameter, $p$ ; see Remarks below.
FAIL	Failure flag: LT.0.0: User defined failure subroutine, <code>matusr_24</code> in <code>dyn21.F</code> , is called to determine failure EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved. GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation.
TDEL	Minimum time step size for automatic element deletion

**\*MAT\_155****\*MAT\_PLASTICITY\_COMPRESSION\_TENSIONEOS**

Card 2	1	2	3	4	5	6	7	8
Variable	LCIDC	LCIDT	LCSR C	LCSR T	SRFLAG			
Type	I	I	I	I	F			
Default	0	0	0	0	0.0			

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCIDC	Load curve ID defining effective yield stress as a function of effective plastic strain in compression
LCIDT	Load curve ID defining effective yield stress as a function of effective plastic strain in tension.
LCSR C	Optional load curve ID defining strain rate scaling effect on yield stress when the material is in compression (compressive yield stress scaling factor as a function of strain rate).
LCSR T	Optional load curve ID defining strain rate scaling effect on yield stress when the material is in tension (tensile yield stress scaling factor as a function of strain rate).
SRFLAG	Formulation for rate effects: EQ.0.0: Total strain rate EQ.1.0: Deviatoric strain rate

Card 3	1	2	3	4	5	6	7	8
Variable	PC	PT	PCUTC	PCUTT	PCUTF	SCALEP	SCALEE	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
PC	Compressive mean stress at which the yield stress follows load curve ID LCIDC. If the pressure falls between PC and PT, a weighted average of the two load curves is used.
PT	Tensile mean stress at which the yield stress follows load curve ID LCIDT.
PCUTC	Pressure cut-off in compression. When the pressure cut-off is reached, the deviatoric stress tensor is set to zero. The compressive pressure is not, however, limited to PCUTC. Like the yield stress, PCUTC is scaled to account for rate effects.
PCUTT	Pressure cut-off in tension. When the pressure cut-off is reached, the deviatoric stress tensor and tensile pressure is set to zero. Like the yield stress, PCUTT is scaled to account for rate effects.
PCUTF	Pressure cut-off flag: EQ.0.0: inactive EQ.1.0: active
SCALEP	Scale factor applied to the yield stress after the pressure cut-off is reached in either compression or tension. If SCALEP = 0.0 (default), the deviatoric stress is set to zero after the cut-off is reached.
SCALEE	Scale factor applied to the yield stress after the strain exceeds the failure strain set by FAIL. If SCALEE = 0.0 (default), the deviatoric strain is set to zero if the failure strain is exceeded. <i>If both SCALEP &gt; 0 and SCALEE &gt; 0 and both failure conditions are met, then the minimum scale factor is used.</i>

Card 4	1	2	3	4	5	6	7	8
Variable	K							
Type	F							

<b>VARIABLE</b>	<b>DESCRIPTION</b>
K	Optional bulk modulus for the viscoelastic material. If nonzero a Kelvin type behavior will be obtained. Generally, K is set to zero.

**\*MAT\_155****\*MAT\_PLASTICITY\_COMPRESSION\_TENSIONEOS**

**Viscoelastic Constant Cards.** Card format for viscoelastic constants. Up to 6 cards may be input. The next keyword ("\*\*") cards terminates this input.

Card 5	1	2	3	4	5	6	7	8
Variable	$G_i$	BETA <i>i</i>						
Type	F	F						

VARIABLE	DESCRIPTION
$G_i$	Optional shear relaxation modulus for the $i^{\text{th}}$ term
BETA <i>i</i>	Optional shear decay constant for the $i^{\text{th}}$ term

**Remarks:**

The effective yield stress as a function of effective plastic strain behavior follows a different curve in compression than it does in tension. Tension is determined by the sign of the mean stress where a positive mean stress (meaning a negative pressure) is indicative of tension. Two curves must be defined giving the yield stress as a function of effective plastic strain. One curve is for the tensile regime and the other curve is for the compressive regime.

Mean stress is an invariant which can be expressed as  $(\sigma_x + \sigma_y + \sigma_z)/3$ . PC and PT define a range of mean stress values within which interpolation is done between the tensile yield surface and compressive yield surface. PC and PT are not true material properties but are just a numerical convenience so that the transition from one yield surface to the other is not as abrupt as the sign of the mean stress changes. Both PC, PT, PCUTC, and PCUTT may all be input as positive values. It is implied that PC and PCUTC are compressive values and that PT and PCUTT are tensile values. The algebraic sign given these variables by the user is inconsequential.

Strain rate may be accounted for by using either two curves of yield stress scaling factor as a function of strain rate or a Cowper and Symonds model. The two curves in the former approach are used directly, that is, the curves are not rediscretized before being used by the material model. The Cowper and Symonds model scales the yield stress with the factor:

$$1 + \left(\frac{\dot{\epsilon}}{C}\right)^{1/p},$$

where  $\dot{\epsilon}$  is the strain rate,

$$\dot{\varepsilon} = \sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}} .$$

**History Variables:**

History Variable	Description
4	Tensile pressure cutoff (set to zero if tensile or compressive failure occurs)
5	The cutoff flag, initially equals 1; set to 0 if tensile or compressive failure occurs.
6	The failure mode flag EQ.0: No failure EQ.1: Compressive failure EQ.2: Tensile failure EQ.3: Failure by plastic strain
7	The current flow stress

# \*MAT\_156

# \*MAT\_MUSCLE

## \*MAT\_MUSCLE

This is Material Type 156 for truss elements. This material is a Hill-type muscle model with activation and a parallel damper. Also, see \*MAT\_SPRING\_MUSCLE (\*MAT\_S15) where a description of the theory is available.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	SNO	SRM	PIS	SSM	CER	DMP
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	ALM	SFR	SVS	SVR	SSP			
Type	F	F	F	F	F			
Default	0.0	1.0	1.0	1.0	0.0			

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Material density in the initial undeformed configuration
SNO	Initial stretch ratio, $l_0/l_{\text{orig}}$ , meaning the length as defined by the nodal points at $t = 0$ divided by the original initial length. The density for the nodal mass calculation is RO/SNO, or $\rho l_{\text{orig}}/l_0$ .
SRM	Maximum strain rate
PIS	Peak isometric stress corresponding to the dimensionless value of unity in the dimensionless stress as a function of strain; see SSP below.
SSM	Strain when the dimensionless stress as a function of strain, SSP below, reaches its maximum stress value
CER	Constant, governing the exponential rise of SSP. Required if SSP = 0.0.

VARIABLE	DESCRIPTION
DMP	Damping constant (stress $\times$ time units)
ALM	Activation level as a function of time: LT.0.0: Absolute value gives load curve ID. GE.0.0: Constant value of ALM is used.
SFR	Scale factor for strain rate maximum as a function of activation level, $a(t)$ : LT.0.0: Absolute value gives load curve ID. GE.0.0: Constant value of 1.0 is used.
SVS	Active dimensionless tensile stress as a function of the stretch ratio, $l/l_{\text{orig}}$ : LT.0.0: Absolute value gives load curve ID. GE.0.0: Constant value of 1.0 is used.
SVR	Active dimensionless tensile stress as a function of the normalized strain rate, $\dot{\varepsilon}$ : LT.0.0: Absolute value gives load curve ID. GE.0.0: Constant value of 1.0 is used.
SSP	Isometric dimensionless stress as a function of the stretch ratio, $l/l_{\text{orig}}$ , for the parallel elastic element: LT.0.0: Absolute value gives load curve ID or table ID (see Remarks). EQ.0.0: Exponential function is used (see Remarks). GT.0.0: Constant value of 0.0 is used.

**Remarks:**

The material behavior of the muscle model is adapted from \*MAT\_S15 (the spring muscle model) and treated here as a standard material. The initial length of muscle is calculated automatically. The force, relative length and shortening velocity are replaced by stress, strain, and strain rate. A new parallel damping element is added.

The strain  $\varepsilon$  and normalized strain rate  $\dot{\varepsilon}$  are defined respectively as

$$\begin{aligned}\varepsilon &= \frac{l}{l_{\text{orig}}} - 1 \\ &= \text{SNO} \times \frac{l}{l_0} - 1\end{aligned}$$

and,

$$\begin{aligned}\dot{\varepsilon} &= \frac{l}{l_{\text{orig}}} \frac{\dot{\varepsilon}}{\dot{\varepsilon}_{\text{max}}} \\ &= \text{SNO} \times \frac{l}{l_0} \times \frac{\dot{\varepsilon}}{\text{SFR} \times \text{SRM}}\end{aligned}$$

where  $\dot{\varepsilon} = \Delta\varepsilon/\Delta t$  (current strain increment divided by current time step),  $l$  is the current muscle length, and  $l_{\text{orig}}$  is the original muscle length.

From the relation above, it is known:

$$l_{\text{orig}} = \frac{l_0}{1 + \varepsilon_0}$$

where  $\varepsilon_0 = \text{SNO} - 1$  and  $l_0$  is the muscle length at  $t = 0$ .

Stress of Contractile Element is:

$$\sigma_1 = \sigma_{\text{max}} a(t) f\left(\frac{l}{l_{\text{orig}}}\right) g(\dot{\varepsilon}) ,$$

where  $\sigma_{\text{max}} = \text{PIS}$ ,  $a(t) = \text{ALM}$ ,  $f(l/l_{\text{orig}}) = \text{SVS}$ , and  $g(\dot{\varepsilon}) = \text{SVR}$ .

Stress of Passive Element is:

$$\sigma_2 = \begin{cases} \sigma_{\text{max}} h\left(\frac{l}{l_{\text{orig}}}\right) & \text{for curve} \\ \sigma_{\text{max}} h\left(\dot{\varepsilon}, \frac{l}{l_{\text{orig}}}\right) & \text{for table} \end{cases}$$

where  $h = \text{SSP}$ . For  $\text{SSP} < 0$ , the absolute value gives a load curve ID or table ID. The load curve defines isometric dimensionless stress  $h$  as a function of stretch ratio  $l/l_{\text{orig}}$ . The table defines for each normalized strain rate  $\dot{\varepsilon}$  a load curve giving the isometric dimensionless stress  $h$  as a function of stretch ratio  $l/l_{\text{orig}}$  for that rate.

For the exponential relationship ( $\text{SSP} = 0$ ):

$$h\left(\frac{1}{l_{\text{orig}}}\right) = \begin{cases} 0 & \frac{1}{l_{\text{orig}}} < 1 \\ \frac{1}{\exp(\text{CER}) - 1} \left[ \exp\left(\frac{\text{CER}}{\text{SSM}} \varepsilon\right) - 1 \right] & \frac{1}{l_{\text{orig}}} \geq 1 \quad \text{CER} \neq 0 \\ \frac{\varepsilon}{\text{SSM}} & \frac{1}{l_{\text{orig}}} \geq 1 \quad \text{CER} = 0 \end{cases}$$

Stress of Damping Element is:

$$\sigma_3 = \text{DMP} \times \frac{l}{l_{\text{orig}}} \dot{\varepsilon} .$$

Total Stress is:

$$\sigma = \sigma_1 + \sigma_2 + \sigma_3 .$$

**\*MAT\_157****\*MAT\_ANISOTROPIC\_ELASTIC\_PLASTIC****\*MAT\_ANISOTROPIC\_ELASTIC\_PLASTIC**

This is Material Type 157. This material model is a combination of the anisotropic elastic material model (\*MAT\_002) and the anisotropic plastic material model (\*MAT\_103\_P). Brittle orthotropic failure based on a phenomenological Tsai-Wu or Tsai-Hill criterion can be defined. This material is available for solid, shell, and thick shell (formulations 1, 2, and 6) elements.

**Card Summary:**

**Card 1.** This card is required.

MID	R0	SIGY	LCSS	QR1	CR1	QR2	CR2
-----	----	------	------	-----	-----	-----	-----

**Card 2.** This card is required.

C11	C12	C13	C14	C15	C16	C22	C23
-----	-----	-----	-----	-----	-----	-----	-----

**Card 3.** This card is required.

C24	C25	C26	C33	C34	C35	C36	C44
-----	-----	-----	-----	-----	-----	-----	-----

**Card 4a.** Include this card if the element type is shells or thick shells.

C45	C46	C55	C56	C66	R00	R45	R90
-----	-----	-----	-----	-----	-----	-----	-----

**Card 4b.** Include this card if the element type is solids.

C45	C46	C55	C56	C66	F	G	H
-----	-----	-----	-----	-----	---	---	---

**Card 5a.** Include this card if the element type is shells or thick shells.

S11	S22	S33	S12	AOPT	VP		
-----	-----	-----	-----	------	----	--	--

**Card 5b.** Include this card if the element type is solids.

L	M	N		AOPT	VP		MACF
---	---	---	--	------	----	--	------

**Card 6.** This card is required.

XP	YP	ZP	A1	A2	A3	ID3UPD	EXTRA
----	----	----	----	----	----	--------	-------

**Card 7.** This card is required.

V1	V2	V3	D1	D2	D3	BETA	IHIS
----	----	----	----	----	----	------	------

**Card 8.** Include this card if EXTRA > 0.

XT	XC	YT	YC	SXY	FF12		NCFAIL
----	----	----	----	-----	------	--	--------

**Card 9.** Include this card if EXTRA > 0.

ZT	ZC	SYZ	SZX	FF23	FF31		
----	----	-----	-----	------	------	--	--

### Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	SIGY	LCSS	QR1	CR1	QR2	CR2
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
SIGY	Initial yield stress
LCSS	Load curve ID or Table ID:  <b>Load Curve.</b> When LCSS is a load curve ID, it is taken as defining effective stress as a function of effective plastic strain. If defined, QR1, CR1, QR2, and CR2 are ignored.  <b>Tabular Data.</b> The table ID defines for each strain rate value a load curve ID giving the stress as a function of effective plastic strain for that rate. See <a href="#">Figure M24-1</a> . When the strain rate falls below the minimum value, the load curve for the lowest value of strain rate is used. Likewise, when the strain rate exceeds the maximum value, the load curve for the highest value of strain rate is used.

**Logarithmically Defined Tables.** An alternative way to invoke logarithmic interpolation between discrete strain rates is described as follows. If the *first* value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
	when the lowest strain rate and highest strain rate differ by several orders of magnitude. Logarithmic interpolation has some additional computational cost.
QR1	Isotropic hardening parameter
CR1	Isotropic hardening parameter
QR2	Isotropic hardening parameter
CR2	Isotropic hardening parameter

Card 2	1	2	3	4	5	6	7	8
Variable	C11	C12	C13	C14	C15	C16	C22	C23
Type	F	F	F	F	F	F	F	F

Card 3	1	2	3	4	5	6	7	8
Variable	C24	C25	C26	C33	C34	C35	C36	C44
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
$C_{ij}$	The $ij^{\text{th}}$ term in the $6 \times 6$ anisotropic constitutive matrix. Note that 1 corresponds to the $a$ material direction, 2 to the $b$ material direction, and 3 to the $c$ material direction.

**Anisotropic Constants Card for Shells.** Include this card if the element type is shells or thick shells.

Card 4a	1	2	3	4	5	6	7	8
Variable	C45	C46	C55	C56	C66	R00	R45	R90
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
$C_{ij}$	The $ij^{\text{th}}$ term in the $6 \times 6$ anisotropic constitutive matrix
R00	$R_{00}$ for shell (default = 1.0)
R45	$R_{45}$ for shell (default = 1.0)
R90	$R_{90}$ for shell (default = 1.0)

**Anisotropic Constants Card for Solids.** Include this card if the element type is solids.

Card 4b	1	2	3	4	5	6	7	8
Variable	C45	C46	C55	C56	C66	F	G	H
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
$C_{ij}$	The $ij^{\text{th}}$ term in the $6 \times 6$ anisotropic constitutive matrix
F	$F$ for solid (default = 1/2)
G	$G$ for solid (default = 1/2)
H	$H$ for solid (default = 1/2)

**Shell Yield Stress Card.** Include this card if the element type is shells or thick shells.

Card 5a	1	2	3	4	5	6	7	8
Variable	S11	S22	S33	S12	AOPT	VP		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
S11	Yield stress in local- $x$ direction (shells only). This input is ignored when R00, R45, and R90 are greater than 0.
S22	Yield stress in local- $y$ direction (shells only). This input is ignored when R00, R45, and R90 are greater than 0.

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
S33	Yield stress in local-z direction (shells only). This input is ignored when R00, R45, and R90 are greater than 0.
S12	Yield stress in local $xy$ -direction (shells only). This input is ignored when R00, R45, and R90 are greater than 0.
AOPT	Material axes option (see <b>MAT_OPTIONTROPIC_ELASTIC</b> , particularly the <a href="#">Material Directions</a> section, for details): <ul style="list-style-type: none"> <li>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with <b>*DEFINE_COORDINATE_NODES</b>. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle <b>BETA</b>.</li> <li>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with <b>*DEFINE_COORDINATE_VECTOR</b></li> <li>EQ.3.0: Locally orthotropic material axes determined by a vector <b>v</b> and the normal vector to the plane of the element. <b>a</b> is determined by taking the cross product of <b>v</b> with the normal vector, <b>b</b> is determined by taking the cross product of the normal vector with <b>a</b>, and <b>c</b> is the normal vector. Then <b>a</b> and <b>b</b> are rotated about <b>c</b> by an angle <b>BETA</b>. <b>BETA</b> may be set in the keyword input for the element or in the input for this keyword.</li> <li>LT.0.0: The absolute value of <b>AOPT</b> is a coordinate system ID number (<b>CID</b> on <b>*DEFINE_COORDINATE_OPTION</b>).</li> </ul>
VP	Formulation for rate effects: <ul style="list-style-type: none"> <li>EQ.0.0: Scale yield stress (default)</li> <li>EQ.1.0: Viscoplastic formulation</li> </ul>

**Anisotropic Constants Card for Solids.** Include this card if the element type is solids.

Card 5b	1	2	3	4	5	6	7	8
Variable	L	M	N		AOPT	VP		MACF
Type	F	F	F		F	F		F

VARIABLE	DESCRIPTION
L	$L$ for solid (default = 3/2)
M	$M$ for solid (default = 3/2)
N	$N$ for solid (default = 3/2)
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC, particularly the <a href="#">Material Directions</a> section, for details):</p> <p>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.</p> <p>EQ.1.0: Locally orthotropic with material axes determined by a point, <math>P</math>, in space and the global location of the element center; this is the <b>a</b>-direction.</p> <p>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR</p> <p>EQ.3.0: Locally orthotropic material axes determined by a vector <b>v</b> and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. <b>a</b> is determined by taking the cross product of <b>v</b> with the normal vector, <b>b</b> is determined by taking the cross product of the normal vector with <b>a</b>, and <b>c</b> is the normal vector. Then <b>a</b> and <b>b</b> are rotated about <b>c</b> by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.</p> <p>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector <b>v</b>, and an originating point, <math>P</math>, which define the centerline axis.</p> <p>LT.0.0: The absolute value of AOPT is a coordinate system ID number (*DEFINE_COORDINATE_OPTION).</p>

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<b>VARIABLE</b>	<b>DESCRIPTION</b>
VP	Formulation for rate effects: EQ.0.0: Scale yield stress (default) EQ.1.0: Viscoplastic formulation
MACF	Material axes change flag for solid elements: EQ.-4: Switch material axes <i>b</i> and <i>c</i> before BETA rotation EQ.-3: Switch material axes <i>a</i> and <i>c</i> before BETA rotation EQ.-2: Switch material axes <i>a</i> and <i>b</i> before BETA rotation EQ.1: No change, default EQ.2: Switch material axes <i>a</i> and <i>b</i> after BETA rotation EQ.3: Switch material axes <i>a</i> and <i>c</i> after BETA rotation EQ.4: Switch material axes <i>b</i> and <i>c</i> after BETA rotation

[Figure M2-2](#) indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on \*ELEMENT\_SOLID\_{OPTION} is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 7 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.

Card 6	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	ID3UPD	EXTRA
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
XP, YP, ZP	Coordinates of point <i>p</i> for AOPT = 1 and 4
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2
EXTRA	Flag to input further data to include failure with Cards 8 and 9: EQ.1.0: Tsai-Wu (stress-based) parameters. See <a href="#">Remark 3</a> . EQ.2.0: Tsai-Hill (stress-based) parameters See <a href="#">Remark 4</a> . EQ.3.0: Tsai-Wu (strain-based) parameters. See <a href="#">Remark 5</a> .

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	EQ.4.0: Tsai-Hill (strain-based) parameters. See <a href="#">Remark 6</a> .
ID3UPD	Flag for transverse through-thickness strain update (thin shells only): EQ.0.0: Reflects $R$ -values by splitting the strain tensor into elastic and plastic components EQ.1.0: Elastic update using total strain tensor

Card 7	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	IHIS
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3 and 4
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2
BETA	Material angle in degrees for AOPT = 0 (shells and tshells only) and AOPT = 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA and *ELEMENT_SOLID_ORTHO.
IHIS	Flag for material properties initialization: EQ.0: Material properties defined in Cards 1 - 5 are used GE.1: Use *INITIAL_STRESS_SOLID/SHELL to initialize material properties on an element-by-element basis for solid or shell elements, respectively (see <a href="#">Remarks 1</a> and <a href="#">2</a> below).

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Two additional cards for EXTRA > 0.

Card 8	1	2	3	4	5	6	7	8
Variable	XT	XC	YT	YC	SXY	FF12		NCFAIL
Type	F	F	F	F	F	F		I
Default	10 <sup>20</sup>	0.0		10				

VARIABLE	DESCRIPTION
XT	<p>Longitudinal tensile strength, <math>a</math>-axis, for EXTRA = 1 and 2 or longitudinal tensile strain at failure, <math>a</math>-axis, for EXTRA = 3 and 4:</p> <p>GT.0.0: Constant value</p> <p>LT.0.0: Load curve ID = (-XT) which defines either the longitudinal tensile strength (EXTRA = 1 and 2) or the longitudinal tensile strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate.</p>
XC	<p>Longitudinal compressive strength, <math>a</math>-axis, for EXTRA = 1 and 2 or longitudinal compressive strain at failure, <math>a</math>-axis, for EXTRA = 3 and 4:</p> <p>GT.0.0: Constant value</p> <p>LT.0.0: Load curve ID = (-XC) which defines either the longitudinal compressive strength (EXTRA = 1 and 2) or the longitudinal compressive strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate.</p> <p>Longitudinal compressive strengths and longitudinal compressive strains at failure should be positive.</p>
YT	<p>Transverse tensile strength, <math>b</math>-axis, for EXTRA = 1 and 2 or transverse tensile strain at failure, <math>b</math>-axis, for EXTRA = 3 and 4:</p> <p>GT.0.0: Constant value</p> <p>LT.0.0: Load curve ID = (-YT) which defines either the transverse tensile strength (EXTRA = 1 and 2) or the</p>

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	transverse tensile strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate.
YC	Transverse compressive strength, <i>b</i> -axis, for EXTRA = 1 and 2 or transverse compressive strain at failure, <i>b</i> -axis, for EXTRA = 3 and 4:  GT.0.0: Constant value  LT.0.0: Load curve ID = (-YC) which defines either the transverse compressive strength (EXTRA = 1 and 2) or the transverse compressive strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate.  Transverse compressive strengths and transverse compressive strains at failure should be positive.
SXY	Shear strength, <i>ab</i> -plane, for EXTRA = 1 and 2 or shear strain at failure, <i>ab</i> -plane, for EXTRA = 3 and 4:  GT.0.0: Constant value  LT.0.0: Load curve ID = (-SXY) which defines the shear strength (EXTRA = 1 and 2) or the shear strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate.
FF12	Scale factor between -1 and +1 for interaction term F12. See <a href="#">Remark 3</a> . It applies for EXTRA = 1 and 3.
NCFAIL	Number of time steps to reduce stresses until element deletion.

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Card 9	1	2	3	4	5	6	7	8
Variable	ZT	ZC	SYZ	SZX	FF23	FF31		
Type	F	F	F	F	F	F		
Default	$10^{20}$	$10^{20}$	$10^{20}$	$10^{20}$	0.0	0.0		

**VARIABLE****DESCRIPTION**

ZT

This field applies to *solid elements only*. Transverse tensile strength, *c*-axis, for EXTRA = 1 and 2 or transverse tensile strain at failure, *c*-axis, for EXTRA = 3 and 4:

GT.0.0: Constant value

LT.0.0: Load curve ID = (-ZT) which defines either the transverse tensile strength (EXTRA = 1 and 2) or the transverse tensile strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, all strain rate values are assumed to be given as a natural logarithm of the strain rate.

ZC

This field applies to *solid elements only*. Transverse compressive strength, *c*-axis, for EXTRA = 1 and 2 or transverse compressive strain at failure, *c*-axis, for EXTRA = 3 and 4:

GT.0.0: Constant value

LT.0.0: Load curve ID = (-ZC) which defines either the transverse compressive strength (EXTRA = 1 and 2) or the transverse compressive strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, all strain rate values are assumed to be given as a natural logarithm of the strain rate.

Transverse compressive strengths and transverse compressive strains at failure should be positive.

SYZ

This field applies to *solid elements only*. Shear strength, *bc*-plane, for EXTRA = 1 and 2 or shear strain at failure, *bc*-plane, for EXTRA = 3 and 4:

GT.0.0: Constant value

LT.0.0: Load curve ID = (-SYZ) which defines the shear strength

VARIABLE	DESCRIPTION
	(EXTRA = 1 and 2) or the shear strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate.
SZX	This field applies to <i>solid elements only</i> . Shear strength, <i>ca</i> -plane, for EXTRA = 1 and 2 or shear strain at failure, <i>ca</i> -plane, for EXTRA = 3 and 4: GT.0.0: Constant value LT.0.0: Load curve ID = (-SZX) which defines the shear strength (EXTRA = 1 and 2) or the shear strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate.
FF23	Scale factor between -1 and +1 for interaction term F23. See <a href="#">Remark 3</a> . This field applies to <i>solid elements only</i> . It applies for EXTRA = 1 and 3.
FF31	Scale factor between -1 and +1 for interaction term F31. See <a href="#">Remark 3</a> . This field applies to <i>solid elements only</i> . It applies for EXTRA = 1 and 3.

**Remarks:**

1. **Description of IHIS (Solid Elements).** Several of this material's parameters may be overwritten on an element-by-element basis through history variables using the \*INITIAL\_STRESS\_SOLID keyword. Bitwise (binary) expansion of IHIS determines which material properties are to be read:

$$\text{IHIS} = a_4 \times 16 + a_3 \times 8 + a_2 \times 4 + a_1 \times 2 + a_0,$$

where each  $a_i$  is a binary flag set to either 1 or 0. The  $a_i$  are interpreted according to the following table.

Flag	Description	Variables	#
$a_0$	Material directions	$q_{11}, q_{12}, q_{13}, q_{31}, q_{32}, q_{33}$	6
$a_1$	Anisotropic stiffness	$C_{ij}$	21
$a_2$	Anisotropic constants	F, G, H, L, M, N	6

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Flag	Description	Variables	#
$a_3$	Stress-strain curve	LCSS	1
$a_4$	Strength limits	XT, XC, YT, YC, ZT, ZC, SXY, SYZ, SZX	9

The NHISV field on \*INITIAL\_STRESS\_SOLID must be set equal to the sum of the number of variables to be read in, which depends on IHIS (and the  $a_i$ ):

$$\text{NHISV} = 6a_0 + 21a_1 + 6a_2 + a_3 + 9a_4.$$

Then, in the following order, \*INITIAL\_STRESS\_SOLID processes the history variables, HISVi, as:

- a) 6 material direction parameters when  $a_0 = 1$
- b) 21 anisotropic stiffness parameters when  $a_1 = 1$
- c) 6 anisotropic constants when  $a_2 = 1$
- d) 1 parameter when  $a_3 = 1$
- e) 9 strength parameters when  $a_4 = 1$

The  $q_{ij}$  terms are the first and third rows of a rotation matrix for the rotation from a co-rotational element's system and the  $a$ - $b$ - $c$  material directions. The  $c_{ij}$  terms are the upper triangular terms of the symmetric stiffness matrix,  $c_{11}, c_{12}, c_{13}, c_{14}, c_{15}, c_{16}, c_{22}, c_{23}, c_{24}, c_{25}, c_{26}, c_{33}, c_{34}, c_{35}, c_{36}, c_{44}, c_{45}, c_{46}, c_{55}, c_{56}$ , and  $c_{66}$ .

2. **Description of IHIS (Shell Elements).** Several of this material's parameters may be overwritten on an element-by-element basis through history variables using the \*INITIAL\_STRESS\_SHELL keyword. Bitwise (binary) expansion of IHIS determines which material properties are to be read:

$$\text{IHIS} = a_4 \times 16 + a_3 \times 8 + a_2 \times 4 + a_1 \times 2 + a_0,$$

where each  $a_i$  is a binary flag set to either 1 or 0. The  $a_i$  are interpreted according to the following table.

Flag	Description	Variables	#
$a_0$	Material directions	$q_1, q_2$	2
$a_1$	Anisotropic stiffness	$C_{ij}$	21
$a_2$	Anisotropic constants	$r_{00}, r_{45}, r_{90}$	3
$a_3$	Stress-strain curve	LCSS	1
$a_4$	Strength limits	XT, XC, YT, YC, SXY	5

The NHISV field on \*INITIAL\_STRESS\_SHELL must be set equal to the sum of the number of variables to be read in, which depends on IHIS (and the  $a_i$ ):

$$\text{NHISV} = 2a_0 + 21a_1 + 3a_2 + a_3 + 5a_4.$$

Then, in the following order, \*INITIAL\_STRESS\_SHELL processes the history variables, HISVi, as:

- a) 2 material direction parameters when  $a_0 = 1$
- b) 21 anisotropic stiffness parameters when  $a_1 = 1$
- c) 3 anisotropic constants when  $a_2 = 1$
- d) 1 parameter when  $a_3 = 1$
- e) 5 strength parameters when  $a_4 = 1$

The  $q_i$  terms are the material direction cosine and sine for the rotation from a co-rotational element's system to the a-b-c material directions. The  $c_{ij}$  terms are the upper triangular terms of the symmetric stiffness matrix,  $c_{11}, c_{12}, c_{13}, c_{14}, c_{15}, c_{16}, c_{22}, c_{23}, c_{24}, c_{25}, c_{26}, c_{33}, c_{34}, c_{35}, c_{36}, c_{44}, c_{45}, c_{46}, c_{55}, c_{56}$ , and  $c_{66}$ .

3. **Tsai-Wu failure criterion (EXTRA = 1, stress-based).** EXTRA = 1 with the definition of corresponding parameters on Cards 8 and 9 invokes brittle failure with different strengths in tension and compression in all main material directions. The model used is the phenomenological Tsai-Wu failure criterion which requires that

$$\begin{aligned} & \left( \frac{1}{XT} - \frac{1}{XC} \right) \sigma_{aa} + \left( \frac{1}{YT} - \frac{1}{YC} \right) \sigma_{bb} + \left( \frac{1}{ZT} - \frac{1}{ZC} \right) \sigma_{cc} + \frac{1}{XT \times XC} \sigma_{aa}^2 \\ & + \frac{1}{YT \times YC} \sigma_{bb}^2 + \frac{1}{ZT \times ZC} \sigma_{cc}^2 + \frac{1}{SXY^2} \sigma_{ab}^2 + \frac{1}{SYZ^2} \sigma_{bc}^2 + \frac{1}{SZX^2} \sigma_{ca}^2 \\ & + 2 \times F_{12} \times \sigma_{aa} \sigma_{bb} + 2 \times F_{23} \times \sigma_{bb} \sigma_{cc} + 2 \times F_{31} \times \sigma_{cc} \sigma_{aa} < 1 \end{aligned}$$

for the 3-dimensional case (solid elements) with three planes of symmetry with respect to the material coordinate system. The interaction terms  $F_{12}$ ,  $F_{23}$ , and  $F_{31}$  are given by

$$\begin{aligned} F_{12} &= \text{FF12} \times \sqrt{\frac{1}{XT \times XC \times YT \times YC}} \\ F_{23} &= \text{FF23} \times \sqrt{\frac{1}{YT \times YC \times ZT \times ZC}} \\ F_{31} &= \text{FF31} \times \sqrt{\frac{1}{ZT \times ZC \times XT \times XC}} \end{aligned}$$

For the 2-dimensional case of plane stress (shell elements), this expression reduces to:

$$\left( \frac{1}{XT} - \frac{1}{XC} \right) \sigma_{aa} + \left( \frac{1}{YT} - \frac{1}{YC} \right) \sigma_{bb} + \frac{1}{XT \times XC} \sigma_{aa}^2 + \frac{1}{YT \times YC} \sigma_{bb}^2 + \frac{1}{SXY^2} \sigma_{ab}^2 + 2 \times F_{12} \times \sigma_{aa} \sigma_{bb} < 1$$

If these conditions are violated, then the stress tensor reduces to zero over NCFAIL time steps, and then the element erodes. A small value for NCFAIL (< 50) is recommended to avoid unphysical behavior; the default is 10.

4. **Tsai-Hill failure criterion (EXTRA = 2, stress-based).** EXTRA = 2 with the definition of corresponding parameters on Cards 8 and 9 (FF12, FF23, and FF31 are not used in this model) invokes brittle failure with different strengths in tension and compression in all main material directions. The model is based on the HILL criterion which can be written as

$$(G + H) \sigma_{aa}^2 + (F + H) \sigma_{bb}^2 + (F + G) \sigma_{cc}^2 - 2H\sigma_{aa}\sigma_{bb} - 2F\sigma_{bb}\sigma_{cc} - 2G\sigma_{cc}\sigma_{aa} + 2N\sigma_{ab}^2 + 2L\sigma_{bc}^2 + 2M\sigma_{ca}^2 < 1$$

for the 3-dimensional case. The constants  $H, F, G, N, L$ , and  $M$  can be expressed in terms of the strength limits (which then becomes the TSAI-HILL criterion) as

$$\begin{aligned} G + H &= \frac{1}{X_i^2} & 2N &= \frac{1}{SXY^2} & H &= 0.5 \times \left( \frac{1}{X_i^2} + \frac{1}{Y_i^2} - \frac{1}{Z_i^2} \right) \\ F + H &= \frac{1}{Y_i^2} & 2L &= \frac{1}{SYZ^2} & F &= 0.5 \times \left( \frac{1}{Y_i^2} + \frac{1}{Z_i^2} - \frac{1}{X_i^2} \right) \\ F + G &= \frac{1}{Z_i^2} & 2M &= \frac{1}{SZX^2} & G &= 0.5 \times \left( \frac{1}{X_i^2} + \frac{1}{Z_i^2} - \frac{1}{Y_i^2} \right) \end{aligned}$$

where the current stress state defines whether the compressive or the tensile strength limit will enter into the equation:

$$\begin{aligned} X_i &= \begin{cases} XT & \text{if } \sigma_{aa} > 0 \\ XC & \text{if } \sigma_{aa} < 0 \end{cases} \\ Y_i &= \begin{cases} YT & \text{if } \sigma_{bb} > 0 \\ YC & \text{if } \sigma_{bb} < 0 \end{cases} \\ Z_i &= \begin{cases} ZT & \text{if } \sigma_{cc} > 0 \\ ZC & \text{if } \sigma_{cc} < 0 \end{cases} \end{aligned}$$

For the 2-dimensional case of plane stress (shell elements) the TSAI-HILL criterion reduces to:

$$(G + H) \sigma_{aa}^2 + (F + H) \sigma_{bb}^2 - 2H\sigma_{aa}\sigma_{bb} + 2N\sigma_{ab}^2 < 1$$

with

$$G + H = \frac{1}{X_i^2}$$

$$F + H = \frac{1}{Y_i^2}$$

$$H = 0.5 \times \frac{1}{X_i^2}$$

$$2N = \frac{1}{SXY^2}$$

If these conditions are violated, then the stress tensor will be reduced to zero over NCFAIL time steps and the element will be eroded. A small value for NCFAIL (< 50) is recommended to avoid unphysical behavior; the default is 10.

5. **Tsai-Wu failure criterion (EXTRA = 3, strain-based).** EXTRA = 3 invokes brittle failure with different strain limits in tension and compression in all main material directions. The failure criterion is like that of EXTRA = 1 as described in [Remark 3](#), but instead of using the stress tensor, the criterion is evaluated based on the current strain tensor. Consequently, the material parameters XT, XC, YT, ... give the limit strains at failure in the various directions.
6. **Tsai-Hill failure criterion (EXTRA = 4, strain-based).** EXTRA = 4 invokes brittle failure with different strain limits in tension and compression in all main material directions. The failure criterion is like that of EXTRA = 2 as described in [Remark 4](#), but instead of using the stress tensor, the criterion is evaluated based on the current strain tensor. Consequently, the material parameters XT, XC, YT, ... give the limit strains at failure in the various directions.

**\*MAT\_158****\*MAT\_RATE\_SENSITIVE\_COMPOSITE\_FABRIC****\*MAT\_RATE\_SENSITIVE\_COMPOSITE\_FABRIC**

This is Material Type 158. Depending on the type of failure surface, this model may be used to model rate sensitive composite materials with unidirectional layers, complete laminates, and woven fabrics. A viscous stress tensor, based on an isotropic Maxwell model with up to six terms in the Prony series expansion, is superimposed on the rate independent stress tensor of the composite fabric. The viscous stress tensor approach should work reasonably well if the stress increases due to rate affects are up to 15% of the total stress. This model is implemented for both shell and thick shell elements. The viscous stress tensor is effective at eliminating spurious stress oscillations.

**Card Summary:**

**Card 1.** This card is required.

MID	R0	EA	EB	(EC)	PRBA	TAU1	GAMMA1
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**Card 2.** This card is required.

GAB	GBC	GCA	SLIMT1	SLIMC1	SLIMT2	SLIMC2	SLIMS
-----	-----	-----	--------	--------	--------	--------	-------

**Card 3.** This card is required.

AOPT	TSIZE	ERODS	SOFT	FS			
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**Card 4.** This card is required.

XP	YP	ZP	A1	A2	A3	PRCA	PRCB
----	----	----	----	----	----	------	------

**Card 5.** This card is required.

V1	V2	V3	D1	D2	D3	BETA	
----	----	----	----	----	----	------	--

**Card 6.** This card is required.

E11C	E11T	E22C	E22T	GMS			
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**Card 7.** This card is required.

XC	XT	YC	YT	SC			
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**Card 8.** This card is required.

K							
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**Card 9.** Include up to 6 of this card. This input ends with the next keyword ("\*") card.

Gi	BETAi						
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#### Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EA	EB	(EC)	PRBA	TAU1	GAMMA1
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
EA	$E_a$ , Young's modulus - longitudinal direction
EB	$E_b$ , Young's modulus - transverse direction
(EC)	$E_c$ , Young's modulus - normal direction (not used)
PRBA	$\nu_{ba}$ , Poisson's ratio $ba$
TAU1	$\tau_1$ , stress limit of the first slightly nonlinear part of the shear stress as a function of shear strain curve. The values $\tau_1$ and $\gamma_1$ are used to define a curve of shear stress as a function of shear strain. These values are input if FS, defined in Card 3, is set to -1.
GAMMA1	$\gamma_1$ , strain limit of the first slightly nonlinear part of the shear stress as a function of shear strain curve

Card 2	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	SLIMT1	SLIMC1	SLIMT2	SLIMC2	SLIMS
Type	F	F	F	F	F	F	F	F

**\*MAT\_158****\*MAT\_RATE\_SENSITIVE\_COMPOSITE\_FABRIC**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
GAB	$G_{ab}$ , shear modulus $ab$
GBC	$G_{bc}$ , shear modulus $bc$
GCA	$G_{ca}$ , shear modulus $ca$
SLIMT1	Factor to determine the minimum stress limit after stress maximum (fiber tension)
SLIMC1	Factor to determine the minimum stress limit after stress maximum (fiber compression)
SLIMT2	Factor to determine the minimum stress limit after stress maximum (matrix tension)
SLIMC2	Factor to determine the minimum stress limit after stress maximum (matrix compression)
SLIMS	Factor to determine the minimum stress limit after stress maximum (shear)

Card 3	1	2	3	4	5	6	7	8
Variable	AOPT	TSIZE	ERODS	SOFT	FS			
Type	F	F	F	F	F			

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):  EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA  EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR  EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle (BETA) from a line in the plane of the element

VARIABLE	DESCRIPTION
	defined by the cross product of the vector <b>v</b> with the element normal
LT.0.0:	The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR.
TSIZE	Time step for automatic element deletion
ERODS	Maximum effective strain for element layer failure. A value of unity would equal 100% strain.
SOFT	Softening reduction factor for strength in the crashfront.
FS	Failure surface type:  EQ.1.0: Smooth failure surface with a quadratic criterion for both the fiber ( <i>a</i> ) and transverse ( <i>b</i> ) directions. This option can be used with complete laminates and fabrics.  EQ.0.0: Smooth failure surface in the transverse ( <i>b</i> ) direction with a limiting value in the fiber ( <i>a</i> ) direction. This model is appropriate for unidirectional (UD) layered composites only.  EQ.-1: Faceted failure surface. When the strength values are reached, then damage evolves in tension and compression for both the fiber and transverse direction. Shear behavior is also considered. This option can be used with complete laminates and fabrics.

Card 4	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3	PRCA	PRCB
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
XP, YP, ZP	Coordinates of point <i>p</i> for AOPT = 1
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2

**\*MAT\_158****\*MAT\_RATE\_SENSITIVE\_COMPOSITE\_FABRIC**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
PRCA	$\nu_{ca}$ , Poisson's ratio $ca$ (default = PRBA)
PRCB	$\nu_{cb}$ , Poisson's ratio $cb$ (default = PRBA)

Card 5	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2
BETA	Material angle in degrees for AOPT = 0 and 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA.

Card 6	1	2	3	4	5	6	7	8
Variable	E11C	E11T	E22C	E22T	GMS			
Type	F	F	F	F	F			

<b>VARIABLE</b>	<b>DESCRIPTION</b>
E11C	Strain at longitudinal compressive strength, $a$ -axis
E11T	Strain at longitudinal tensile strength, $a$ -axis
E22C	Strain at transverse compressive strength, $b$ -axis
E22T	Strain at transverse tensile strength, $b$ -axis
GMS	Strain at shear strength, $ab$ -plane

Card 7	1	2	3	4	5	6	7	8
Variable	XC	XT	YC	YT	SC			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION
XC	Longitudinal compressive strength; see <a href="#">Remark 2</a> of *MAT_058.
XT	Longitudinal tensile strength; see <a href="#">Remark 2</a> of *MAT_058.
YC	Transverse compressive strength, <i>b</i> -axis, see <a href="#">Remark 2</a> of *MAT_-058.
YT	Transverse tensile strength, <i>b</i> -axis; see <a href="#">Remark 2</a> of *MAT_058.
SC	Shear strength, <i>ab</i> -plane; see <a href="#">Remark 2</a> of *MAT_058.

Card 8	1	2	3	4	5	6	7	8
Variable	K							
Type	F							

VARIABLE	DESCRIPTION
K	Optional bulk modulus for the viscoelastic material. If nonzero, a Kelvin type behavior will be obtained. Generally, K is set to zero.

**Viscoelastic Cards.** Up to 6 cards may be input. The next keyword ("\*") card terminates this input.

Card 9	1	2	3	4	5	6	7	8
Variable	Gi	BETAI <i>i</i>						
Type	F	F						

**\*MAT\_158****\*MAT\_RATE\_SENSITIVE\_COMPOSITE\_FABRIC**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
$G_i$	Optional shear relaxation modulus for the $i^{\text{th}}$ term
BETA <i>i</i>	Optional shear decay constant for the $i^{\text{th}}$ term

**Remarks:**

1. **Related material.** See the Remarks for material type 58, [\\*MAT\\_LAMINATED\\_COMPOSITE\\_FABRIC](#), for the treatment of the composite material.
2. **Rate effects.** Rate effects are taken into account through a Maxwell model using linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t-\tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau ,$$

where  $g_{ijkl}(t-\tau)$  is the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional. Since we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \sum_{m=1}^N G_m e^{-\beta_m t}$$

We characterize this in the input by the shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . An arbitrary number of terms, not exceeding 6, may be used when applying the viscoelastic model. The composite failure is not directly affected by the presence of the viscous stress tensor.

**\*MAT\_CSCM\_{OPTION}**

This is Material Type 159. This material model is a smooth or continuous surface cap model and is available for solid elements in LS-DYNA. The user has the option of inputting his own material properties (<BLANK> option) or requesting default material properties for normal strength concrete (CONCRETE). See [Murray 2007] for a more complete model description.

Available options include:

<BLANK>

CONCRETE

**Card Summary:**

**Card 1.** This card is required.

MID	RO	NPLOT	INCRE	IRATE	ERODE	RECOV	ITRETRC
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**Card 2.** This card is required.

PRED							
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**Card 3.** This card is included if and only if the CONCRETE keyword option is used.

FPC	DAGG	UNITS					
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**Card 4.** This card is included if and only if the keyword option is unused (<OPTION>).

G	K	ALPHA	THETA	LAMBDA	BETA	NH	CH
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**Card 5.** This card is included if and only if the keyword option is unused (<OPTION>).

ALPHA1	THETA1	LAMBDA1	BETA1	ALPHA2	THETA2	LAMBDA2	BETA2
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**Card 6.** This card is included if and only if the keyword option is unused (<OPTION>).

R	X0	W	D1	D2			
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**Card 7.** This card is included if and only if the keyword option is unused (<OPTION>).

B	GFC	D	GFT	GFS	PWRC	PWRT	PMOD
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**Card 8.** This card is included if and only if the keyword option is unused (<OPTION>).

ETAOC	NC	ETAOT	NT	OVERC	OVERT	SRATE	REPON
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	NPLOT	INCRE	IRATE	ERODE	RECOV	ITRETRC
Type	A	F	I	F	I	F	F	I

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
NPLOT	Controls what is written as component 7 to the d3plot database. LS-PrePost always labels this component as effective plastic strain: EQ.1: Maximum of brittle and ductile damage (default) EQ.2: Maximum of brittle and ductile damage, with recovery of brittle damage EQ.3: Brittle damage EQ.4: Ductile damage EQ.5: $\kappa$ (intersection of cap with shear surface) EQ.6: $X_0$ (intersection of cap with pressure axis) EQ.7: $\epsilon_v^P$ (plastic volume strain).
INCRE	Maximum strain increment for subincrementation. If left blank, a default value is set during initialization based upon the shear strength and stiffness.
IRATE	Rate effects options: EQ.0: Rate effects model turned off (default). EQ.1: Rate effects model turned on.
ERODE	Elements erode when damage exceeds 0.99 and the maximum principal strain exceeds ERODE – 1.0. For erosion that is independent of strain, set ERODE equal to 1.0. Erosion does not occur if ERODE is less than 1.0.

VARIABLE	DESCRIPTION
RECOV	<p>The modulus is recovered in compression when RECOV is equal to 0.0 (default). The modulus remains at the brittle damage level when RECOV is equal to 1.0. Partial recovery is modeled for values of RECOV between 0.0 and 1.0. Two options are available:</p> <ol style="list-style-type: none"> <li>1. If RECOV is a value between 0.0 and 1.0, then recovery is based upon the sign of the pressure invariant only.</li> <li>2. If RECOV is a value between 10.0 and 11.0, then recovery is based upon the sign of both the pressure and volumetric strain. In this case, RECOV = RECOV – 10, and a flag is set to request the volumetric strain check.</li> </ol>
IRETRC	<p>Cap retraction option:</p> <p>EQ.0: Cap does not retract (default).</p> <p>EQ.1: Cap retracts.</p>

Card 2	1	2	3	4	5	6	7	8
Variable	PRED							
Type	F							

VARIABLE	DESCRIPTION
PRED	Pre-existing damage ( $0 \leq \text{PRED} < 1$ ). If left blank, the default is zero (no pre-existing damage).

**Concrete Properties Card.** This card is included if and only if the CONCRETE keyword option is used.

Card 3	1	2	3	4	5	6	7	8
Variable	FPC	DAGG	UNITS					
Type	F	F	I					

<b>VARIABLE</b>	<b>DESCRIPTION</b>
FPC	Unconfined compression strength, $f'_c$ . Material parameters are internally fit to data for unconfined compression strengths between about 20 and 58 MPa (2,901 to 8,412 psi), with emphasis on the mid-range between 28 and 48 MPa (4,061 and 6,962 psi). If left blank, the default for FPC is 30 MPa.
DAGG	Maximum aggregate size, $D_{agg}$ . Softening is fit to data for aggregate sizes between 8 and 32 mm (0.3 and 1.3 inches). If left blank, the default for DAGG is 19 mm (3/4 inch).
UNITS	Units options: EQ.0: GPa, mm, msec, kg/mm <sup>3</sup> , kN EQ.1: MPa, mm, msec, g/mm <sup>3</sup> , N EQ.2: MPa, mm, sec, Mg/mm <sup>3</sup> , N EQ.3: Psi, inch, sec, lbf-s <sup>2</sup> /in <sup>4</sup> , lbf EQ.4: Pa, m, sec, kg/m <sup>3</sup> , N

**User Defined Properties Card.** This card is included if and only if the keyword option is left blank.

Card 4	1	2	3	4	5	6	7	8
Variable	G	K	ALPHA	THETA	LAMBDA	BETA	NH	CH
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
G	Shear modulus
K	Bulk modulus
ALPHA	Tri-axial compression surface constant term, $\alpha$
THETA	Tri-axial compression surface linear term, $\theta$
LAMBDA	Tri-axial compression surface nonlinear term, $\lambda$
BETA	Tri-axial compression surface exponent, $\beta$
NH	Hardening initiation, $N_H$

VARIABLE	DESCRIPTION
CH	Hardening rate, $C_H$

**User Defined Properties Card.** This card is included if and only if the keyword option is left blank.

Card 5	1	2	3	4	5	6	7	8
Variable	ALPHA1	THETA1	LAMBDA1	BETA1	ALPHA2	THETA2	LAMBDA2	BETA2
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
ALPHA1	Torsion surface constant term, $\alpha_1$
THETA1	Torsion surface linear term, $\theta_1$
LAMBDA1	Torsion surface nonlinear term, $\lambda_1$
BETA1	Torsion surface exponent, $\beta_1$
ALPHA2	Tri-axial extension surface constant term, $\alpha_2$
THETA2	Tri-axial extension surface linear term, $\theta_2$
LAMBDA2	Tri-axial extension surface nonlinear term, $\lambda_2$
BETA2	Tri-axial extension surface exponent, $\beta_2$

**User Defined Properties Card.** This card is included if and only if the keyword option is left blank.

Card 6	1	2	3	4	5	6	7	8
Variable	R	X0	W	D1	D2			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION
R	Cap aspect ratio, $R$

<b>VARIABLE</b>	<b>DESCRIPTION</b>
X0	Cap initial location, $X_0$
W	Maximum plastic volume compaction, $W$
D1	Linear shape parameter, $D_1$
D2	Quadratic shape parameter, $D_2$

**User Defined Properties Card.** This card is included if and only if the keyword option is left blank.

Card 7	1	2	3	4	5	6	7	8
Variable	B	GFC	D	GFT	GFS	PWRC	PWRT	PMOD
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
B	Ductile shape softening parameter, $B$
GFC	Fracture energy in uniaxial stress, $G_{fc}$
D	Brittle shape softening parameter, $D$
GFT	Fracture energy in uniaxial tension, $G_{ft}$
GFS	Fracture energy in pure shear stress, $G_{fs}$
PWRC	Shear-to-compression transition parameter
PWRT	Shear-to-tension transition parameter
PMOD	Modify moderate pressure softening parameter

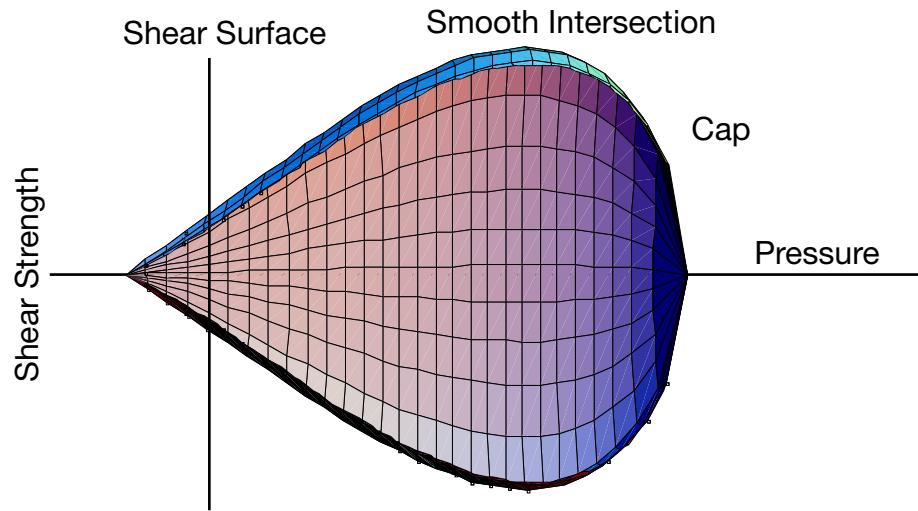
**User Defined Properties Card.** This card is included if and only if the keyword option is left blank.

Card 8	1	2	3	4	5	6	7	8
Variable	ETA0C	NC	ETA0T	NT	OVERC	OVERT	SRATE	REPOW
Type	F	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
ETA0C	Rate effects parameter for uniaxial compressive stress, $\eta_{0c}$
NC	Rate effects power for uniaxial compressive stress, $N_c$
ETA0T	Rate effects parameter for uniaxial tensile stress, $\eta_{0t}$
NT	Rate effects power for uniaxial tensile stress, $N_t$
OVERC	Maximum overstress allowed in compression
OVERT	Maximum overstress allowed in tension
SRATE	Ratio of effective shear stress to tensile stress fluidity parameters
REPOW	Power which increases fracture energy with rate effects

### Remarks:

- Model Overview.** This is a cap model with a smooth intersection between the shear yield surface and hardening cap, as shown in [Figure M159-1](#). The initial damage surface coincides with the yield surface. Rate effects are modeled with viscoplasticity. For a complete theoretical description, with references and example problems see [Murray 2007] and [Murray, Abu-Odeh and Bligh 2007].
- Stress Invariants.** The yield surface is formulated in terms of three stress invariants:  $J_1$  which is the first invariant of the stress tensor,  $J'_2$  which is the second invariant of the deviatoric stress tensor, and  $J'_3$  which is the third invariant of the deviatoric stress tensor. The invariants are defined in terms of the deviatoric stress tensor,  $S_{ij}$ , and pressure,  $P$ , as follows:



**Figure M159-1.** General shape of concrete model yield surface in two dimensions.

$$\begin{aligned} J_1 &= 3P \\ J'_2 &= \frac{1}{2}S_{ij}S_{ij} \\ J'_3 &= \frac{1}{3}S_{ij}S_{ik}S_{ki} \end{aligned}$$

3. **Plasticity Surface.** The three invariant yield function is based on these three invariants, and the cap hardening parameter,  $\kappa$ , as follows:

$$f(J_1, J'_2, J'_3, \kappa) = J'_2 - \mathfrak{R}^2 F_f^2 F_c .$$

Here  $F_f$  is the shear failure surface,  $F_c$  is the hardening cap, and  $\mathfrak{R}$  is the Rubin three-invariant reduction factor. The cap hardening parameter  $\kappa$  is the value of the pressure invariant at the intersection of the cap and shear surfaces.

Trial elastic stress invariants are temporarily updated using the trial elastic stress tensor,  $\sigma^T$ . These are denoted  $J_1^T$ ,  $J_2^{T^T}$ , and  $J_3^{T^T}$ . Elastic stress states are modeled when  $f(J_1^T, J_2^{T^T}, J_3^{T^T}, \kappa^T) \leq 0$ . Elastic-plastic stress states are modeled when  $f(J_1^T, J_2^{T^T}, J_3^{T^T}, \kappa^T) \leq 0$ . In this case, the plasticity algorithm returns the stress state to the yield surface such that  $f(J_1^P, J_2^{P^T}, J_3^{P^T}, \kappa^P) = 0$ . This is accomplished by enforcing the plastic consistency condition with associated flow.

4. **Shear Failure Surface.** The strength of concrete is modeled by the shear surface in the tensile and low confining pressure regimes:

$$F_f(J_1) = \alpha - \lambda \exp(-\beta J_1) + \theta J_1 .$$

Here the values of  $\alpha$ ,  $\beta$ ,  $\lambda$ , and  $\theta$  are selected by fitting the model surface to strength measurements from triaxial compression (TXC) tests conducted on plain concrete cylinders.

5. **Rubin Scaling Function.** Concrete fails at lower values of  $\sqrt{3J'_2}$  (principal stress difference) for triaxial extension (TXE) and torsion (TOR) tests than it does for TXC tests conducted at the same pressure. The Rubin scaling function,  $\mathfrak{R}$ , determines the strength of concrete for any state of stress relative to the strength for TXC, using  $\mathfrak{R}F_f$ . Strength in torsion is modeled as  $Q_1F_f$ . Strength in TXE is modeled as  $Q_2F_f$ , where:

$$Q_1 = \alpha_1 - \lambda_1 \exp(-\beta_1 J_1) + \theta_1 J_1$$

$$Q_2 = \alpha_2 - \lambda_2 \exp(-\beta_2 J_1) + \theta_2 J_1$$

6. **Cap Hardening Surface.** The strength of concrete is modeled by a combination of the cap and shear surfaces in the low to high confining pressure regimes. The cap is used to model plastic volume change related to pore collapse (although the pores are not explicitly modeled). The isotropic hardening cap is a two-part function that is either unity or an ellipse:

$$F_c(J_1, \kappa) = 1 - \frac{[J_1 - L(\kappa)][|J_1 - L(\kappa)| + J_1 - L(\kappa)]}{2 [X(\kappa) - L(\kappa)]^2},$$

where  $L(\kappa)$  is defined as:

$$L(\kappa) = \begin{cases} \kappa & \text{if } \kappa > \kappa_0 \\ \kappa_0 & \text{otherwise} \end{cases}$$

The equation for  $F_c$  is equal to unity for  $J_1 \leq L(\kappa)$ . It describes the ellipse for  $J_1 > L(\kappa)$ . The intersection of the shear surface and the cap is at  $J_1 = \kappa$ .  $\kappa_0$  is the value of  $J_1$  at the *initial* intersection of the cap and shear surfaces before hardening is engaged (before the cap moves). The equation for  $L(\kappa)$  restrains the cap from retracting past its initial location at  $\kappa_0$ .

The intersection of the cap with the  $J_1$  axis is at  $J_1 = X(\kappa)$ . This intersection depends upon the cap ellipticity ratio  $R$ , where  $R$  is the ratio of its major to minor axes:

$$X(\kappa) = L(\kappa) + RF_f[L(\kappa)].$$

The cap moves to simulate plastic volume change. The cap expands ( $X(\kappa)$  and  $\kappa$  increase) to simulate plastic volume compaction. The cap contracts ( $X(\kappa)$  and  $\kappa$  decrease) to simulate plastic volume expansion, called dilation. The motion (expansion and contraction) of the cap is based upon the hardening rule:

$$\varepsilon_v^p = W \left[ 1 - e^{-D_1(X-X_0)-D_2(X-X_0)^2} \right].$$

Here  $\varepsilon_v^p$  is the plastic volume strain,  $W$  is the maximum plastic volume strain, and  $D_1$  and  $D_2$  are model input parameters.  $X_0$  is the initial location of the cap when  $\kappa = \kappa_0$ .

The five input parameters ( $X_0$ ,  $W$ ,  $D_1$ ,  $D_2$ , and  $R$ ) are obtained from fits to the pressure-volumetric strain curves in isotropic compression and uniaxial strain.  $X_0$  determines the pressure at which compaction initiates in isotropic

compression,  $R$ , combined with  $X_0$ , determines the pressure at which compaction initiates in uniaxial strain.  $D_1$  and  $D_2$  determine the shape of the pressure-volumetric strain curves.  $W$  determines the maximum plastic volume compaction.

7. **Shear Hardening Surface.** In unconfined compression, the stress-strain behavior of concrete exhibits nonlinearity and dilation prior to the peak. Such behavior is modeled with an initial shear yield surface,  $N_H F_f$ , which hardens until it coincides with the ultimate shear yield surface,  $F_f$ . Two input parameters are required. One parameter,  $N_H$ , initiates hardening by setting the location of the initial yield surface. A second parameter,  $C_H$ , determines the rate of hardening (amount of nonlinearity).
8. **Damage.** Concrete exhibits softening in the tensile and low to moderate compressive regimes.

$$\sigma_{ij}^d = (1 - d)\sigma_{ij}^{vp}$$

A scalar damage parameter,  $d$ , transforms the viscoplastic stress tensor without damage, denoted  $\sigma^{vp}$ , into the stress tensor with damage, denoted  $\sigma^d$ . Damage accumulation is based upon two distinct formulations, which we call brittle damage and ductile damage. The initial damage threshold is coincident with the shear plasticity surface, so the threshold does not have to be specified by the user.

- a) *Ductile Damage.* Ductile damage accumulates when the pressure,  $P$ , is compressive and an energy-type term,  $\tau_c$ , exceeds the damage threshold,  $\tau_{0c}$ . Ductile damage accumulation depends upon the total strain components,  $\varepsilon_{ij}$ , as follows:

$$\tau_c = \sqrt{\frac{1}{2}\sigma_{ij}\varepsilon_{ij}}$$

The stress components,  $\sigma_{ij}$  are the elasto-plastic stresses (with kinematic hardening) calculated before application of damage and rate effects.

- b) *Brittle Damage.* Brittle damage accumulates when the pressure is tensile and an energy-type term,  $\tau_t$ , exceeds the damage threshold,  $\tau_{0t}$ . Brittle damage accumulation depends upon the maximum principal strain,  $\varepsilon_{max}$ , as follows:

$$\tau_t = \sqrt{E\varepsilon_{max}^2} .$$

As damage accumulates, the damage parameter,  $d$ , increases from an initial value of zero, towards a maximum value of one, using the following formulations:

$$\text{Brittle Damage: } d(\tau_t) = \frac{0.999}{D} \left[ \frac{1+D}{1+D e^{-C(\tau_t-\tau_{0t})}} - 1 \right]$$

$$\text{Ductile Damage: } d(\tau_c) = \frac{d_{\max}}{B} \left[ \frac{1+B}{1+B e^{-A(\tau_c-\tau_{0c})}} - 1 \right]$$

The damage parameter that is applied to the six stresses is equal to the current maximum of the brittle or ductile damage parameter. The parameters  $A$  and  $B$  or  $C$  and  $D$  set the shape of the softening curve plotted as stress-displacement or stress-strain. The parameter  $d_{\max}$  is the maximum damage level that can be attained. It is internally calculated and is less than one at moderate confining pressures. See [Murray 2007] for a description of how  $d_{\max}$  is calculated for different loading regimes. The compressive softening parameter,  $A$ , may also be reduced with confinement, using the input field PMOD, as follows:

$$A = A(d_{\max} + 0.001)^{\text{PMOD}}$$

9. **Regulating Mesh Size Sensitivity.** The concrete model maintains constant fracture energy, regardless of element size. The fracture energy is defined here as the area under the stress-displacement curve from peak strength to zero strength. This is done by internally formulating the softening parameters  $A$  and  $C$  (see [Remark 8](#)) in terms of the element length,  $l$  (cube root of the element volume), the fracture energy,  $G_f$ , the initial damage threshold,  $\tau_{0t}$  or  $\tau_{0c}$ , and the softening shape parameters,  $D$  or  $B$ .

The fracture energy is calculated from up to five user-specified input fields: GFC, GFS, GFT, PWRC, and PWRT. The user specifies three distinct fracture energy values. These are the fracture energy in uniaxial tensile stress, GFT; pure shear stress, GFS; and uniaxial compressive stress, GFC. The model internally selects the fracture energy from equations which interpolate between the three fracture energy values as a function of the stress state (expressed using two stress invariants). The interpolation equations depend upon the user-specified input powers PWRC and PWRT, as follows:

$$\text{Tensile Pressure: } G_f = \text{GFS} + \overbrace{\left( \frac{-J_1}{\sqrt{3J'_2}} \right)}^{k_t}^{\text{PWRT}} \quad [\text{GFT} - \text{GFS}]$$

$$\text{Compressive Pressure: } G_f = \text{GFS} + \overbrace{\left( \frac{J_1}{\sqrt{3J'_2}} \right)}^{k_c}^{\text{PWRC}} \quad [\text{GFC} - \text{GFS}]$$

The internal parameters  $k_c$  and  $k_t$  are restricted to the interval  $[0,1]$ .

10. **Element Erosion.** An element loses all strength and stiffness as  $d \rightarrow 1$ . To prevent computational difficulties with very low stiffness, element erosion is

available as a user option. An element erodes when  $d > 0.99$  and the maximum principal strain is greater than a user supplied input value, ERODE – 1.0.

11. **Viscoplastic Rate Effects.** At each time step, the viscoplastic algorithm interpolates between the elastic trial stress,  $\sigma_{ij}^T$ , and the inviscid stress (without rate effects),  $\sigma_{ij}^P$ , to set the viscoplastic stress (with rate effects),  $\sigma_{ij}^{VP}$ :

$$\sigma_{ij}^{VP} = (1 - \gamma)\sigma_{ij}^T + \gamma\sigma_{ij}^P,$$

where

$$\gamma = \frac{\Delta t / \eta}{1 + \Delta t / \eta}.$$

This interpolation depends upon the effective fluidity coefficient,  $\eta$ , and the time step,  $\Delta t$ . The effective fluidity coefficient is internally calculated from five user-supplied input parameters and interpolation equations:

$$\text{Tensile Pressure: } \eta = \eta_s + \left( \frac{-J_1}{\sqrt{3J'_2}} \right)^{\text{PWRT}} [\eta_t - \eta_s]$$

$$\text{Compressive Pressure: } \eta = \eta_s + \left( \frac{J_1}{\sqrt{3J'_2}} \right)^{\text{PWRC}} [\eta_c - \eta_s]$$

where

$$\begin{aligned} \eta_s &= \text{SRATE} \times \eta_t \\ \eta_t &= \frac{\text{ETA0T}}{\dot{\epsilon}^{\text{NT}}} \\ \eta_c &= \frac{\text{ETA0C}}{\dot{\epsilon}^{\text{NC}}} \end{aligned}$$

The input parameters are ETA0T and NT for fitting uniaxial tensile stress data, ETA0X and NC for fitting the uniaxial compressive stress data, and SRATE for fitting shear stress data. The effective strain rate is  $\dot{\epsilon}$ .

This viscoplastic model may predict substantial rate effects at high strain rates ( $\dot{\epsilon} > 100$ ). To limit rate effects at high strain rates, the user may input overstress limits in tension OVERT and compression OVERC. These input fields limit calculation of the fluidity parameter, as follows:

$$\text{if } E\dot{\epsilon}\eta > \text{OVER, then } \eta = \frac{m}{E\dot{\epsilon}}.$$

Here  $m = \text{OVERT}$  when the pressure is tensile and  $m = \text{OVERC}$  when the pressure is compressive.

The user has the option of increasing the fracture energy as a function of effective strain rate using the REPOW input parameter, as follows:

$$G_f^{\text{rate}} = G_f \left[ 1 + \frac{E\dot{\epsilon}\eta}{r^s \sqrt{E}} \right]^{\text{REPOW}}$$

Here  $G_f^{\text{rate}}$  is the fracture energy enhanced by rate effects, and  $r^s$  is an internally calculated damage threshold determined before applying viscoplasticity (see [Murray 2007] for more details). The term in brackets is only applied if it is greater than or equal to one and is the approximate ratio of the dynamic to static strength.

**\*MAT\_160****\*MAT\_ALE\_INCOMPRESSIBLE****\*MAT\_ALE\_INCOMPRESSIBLE**

This is Material Type 160. This material is for modeling incompressible flows with the ALE solver. It should be used with solid element formulations 6 or 12 (see \*SECTION\_-SOLID). A projection method enforces the incompressibility condition.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	PC	MU				
Type	A	F	F	F				
Default	none	none	0.0	0.0				

Card 2	1	2	3	4	5	6	7	8
Variable	TOL	DTOUT	NCG	METH				
Type	F	F	I	I				
Default	$10^{-8}$	$10^{10}$	50	-7				

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART)
RO	Material density
PC	Pressure cutoff ( $\leq 0.0$ )
MU	Dynamic viscosity coefficient
TOL	Tolerance for the convergence of the conjugate gradient
DTOUT	Time interval between screen outputs
NCG	Maximum number of loops in the conjugate gradient
METH	Conjugate gradient methods: EQ.-6: Solves Poisson's equation for the pressure.

**VARIABLE****DESCRIPTION**

EQ.-7: Solves Poisson's equation for the pressure increment.

## \*MAT\_161-162

## \*MAT\_COMPOSITE\_MSC

### \*MAT\_COMPOSITE\_MSC\_{OPTION}

Available options include:

<BLANK>

DMG

These are Material Types 161 and 162. These models may be used to model the progressive failure analysis for composite materials consisting of unidirectional and woven fabric layers. The progressive layer failure criteria have been established by adopting the methodology developed by Hashin [1980] with a generalization to include the effect of highly constrained pressure on composite failure. These failure models can be used to effectively simulate fiber failure, matrix damage, and delamination behavior under all conditions - opening, closing, and sliding of failure surfaces. The model with the DMG keyword option (material 162) is a generalization of the basic layer failure model of Material 161 by adopting the damage mechanics approach for characterizing the softening behavior after damage initiation. These models require an additional license from Materials Sciences Corporation, which developed and supports these models. These models are supported for solid elements.

### Card Summary:

**Card 1.** This card is required.

MID	R0	EA	EB	EC	PRBA	PRCA	PRCB
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**Card 2.** This card is required.

GAB	GBC	GCA	AOPT	MACF			
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**Card 3.** This card is required.

XP	YP	ZP	A1	A2	A3		
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**Card 4.** This card is required.

V1	V2	V3	D1	D2	D3	BETA	
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**Card 5.** This card is required.

SAT	SAC	SBT	SBC	SCT	SFC	SFS	SAB
-----	-----	-----	-----	-----	-----	-----	-----

**Card 6.** This card is required.

SBC	SCA	SFFC	AMODEL	PHIC	E_LIMT	S_DELM	
-----	-----	------	--------	------	--------	--------	--

**Card 7.** This card is required.

OMGMX	ECRSH	EEXPN	CERATE1	AM1			
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**Card 8.** This card is included if the DMG keyword option is used.

AM2	AM3	AM4	CERATE2	CERATE3	CERATE4		
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### Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	EA	EB	EC	PRBA	PRCA	PRCB
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material ID. A unique number or label must be specified (see *PART).
RO	Mass density
EA	$E_a$ , Young's modulus - longitudinal direction
EB	$E_b$ , Young's modulus - transverse direction
EC	$E_c$ , Young's modulus - through thickness direction
PRBA	$\nu_{ba}$ , Poisson's ratio $ba$
PRCA	$\nu_{ca}$ , Poisson's ratio $ca$
PRCB	$\nu_{cb}$ , Poisson's ratio $cb$

Card 2	1	2	3	4	5	6	7	8
Variable	GAB	GBC	GCA	AOPT	MACF			
Type	F	F	F	F	I			

**\*MAT\_161-162****\*MAT\_COMPOSITE\_MSC**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
GAB	$G_{ab}$ , shear modulus $ab$
GBC	$G_{bc}$ , shear modulus $bc$
GCA	$G_{ca}$ , shear modulus $ca$
AOPT	<p>Material axes option (see MAT_OPTIONTROPIC_ELASTIC, particularly the <a href="#">Material Directions</a> section, for details):</p> <p>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES..</p> <p>EQ.1.0: Locally orthotropic with material axes determined by a point, <math>P</math>, in space and the global location of the element center; this is the <b>a</b>-direction.</p> <p>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR</p> <p>EQ.3.0: Locally orthotropic material axes determined by a vector <b>v</b> and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, AOPT = 3 is only available for hexahedrons. <b>a</b> is determined by taking the cross product of <b>v</b> with the normal vector, <b>b</b> is determined by taking the cross product of the normal vector with <b>a</b>, and <b>c</b> is the normal vector. Then <b>a</b> and <b>b</b> are rotated about <b>c</b> by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.</p> <p>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector <b>v</b>, and an originating point, <math>P</math>, which define the centerline axis.</p> <p>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_OPTION).</p>

VARIABLE	DESCRIPTION
MACF	<p>Material axes change flag for solid elements:</p> <p>EQ.-4: Switch material axes <math>b</math> and <math>c</math> before BETA rotation      EQ.-3: Switch material axes <math>a</math> and <math>c</math> before BETA rotation      EQ.-2: Switch material axes <math>a</math> and <math>b</math> before BETA rotation      EQ.1: No change, default      EQ.2: Switch material axes <math>a</math> and <math>b</math> after BETA rotation      EQ.3: Switch material axes <math>a</math> and <math>c</math> after BETA rotation      EQ.4: Switch material axes <math>b</math> and <math>c</math> after BETA rotation</p>

[Figure M2-2](#) indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on \*ELEMENT\_SOLID\_{OPTION} is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 4 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed.

Card 3	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
XP, YP, ZP	Coordinates of point $p$ for AOPT = 1 and 4
A1, A2, A3	Components of vector $\mathbf{a}$ for AOPT = 2

Card 4	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

**\*MAT\_161-162****\*MAT\_COMPOSITE\_MSC**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
V1, V2, V3	Components of vector $\mathbf{v}$ for AOPT = 3 and 4
D1, D2, D3	Components of vector $\mathbf{d}$ for AOPT = 2
BETA	Layer in-plane rotational angle in degrees. It may be override

Card 5	1	2	3	4	5	6	7	8
Variable	SAT	SAC	SBT	SBC	SCT	SFC	SFS	SAB
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SAT	Longitudinal tensile strength
SAC	Longitudinal compressive strength
SBT	Transverse tensile strength
SBC	Transverse compressive strength
SCT	Through thickness tensile strength
SFC	Crush strength
SFS	Fiber mode shear strength
SAB	Matrix mode shear strength, $ab$ plane; see remarks.

Card 6	1	2	3	4	5	6	7	8
Variable	SBC	SCA	SFFC	AMODEL	PHIC	E_LIMIT	S_DELM	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SBC	Matrix mode shear strength, $bc$ plane; see remarks.
SCA	Matrix mode shear strength, $ca$ plane; see remarks.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SFFC	Scale factor for residual compressive strength
AMODEL	Material models: EQ.1.0: Unidirectional layer model EQ.2.0: Fabric layer model
PHIC	Coulomb friction angle for matrix and delamination failure, < 90
E_LIMIT	Element eroding axial strain
S_DELM	Scale factor for delamination criterion

Card 7	1	2	3	4	5	6	7	8
Variable	OMGMX	ECRSH	EEXPN	CERATE1	AM1			
Type	F	F	F	F	F			

<b>VARIABLE</b>	<b>DESCRIPTION</b>
OMGMX	Limit damage parameter for elastic modulus reduction
ECRSH	Limit compressive volume strain for element eroding
EEXPN	Limit tensile volume strain for element eroding
CERATE1	Coefficient for strain rate dependent strength properties
AM1	Coefficient for strain rate softening property for fiber damage in $\alpha$ -direction

**Failure Card.** Additional card for DMG keyword option.

Card 8	1	2	3	4	5	6	7	8
Variable	AM2	AM3	AM4	CERATE2	CERATE3	CERATE4		
Type	F	F	F	F	F	F		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
AM2	Coefficient for strain rate softening property for fiber damage in <i>b</i> -direction
AM3	Coefficient for strain rate softening property for fiber crush and punch shear damage
AM4	Coefficient for strain rate softening property for matrix and delamination damage
CERATE2	Coefficient for strain rate dependent axial moduli
CERATE3	Coefficient for strain rate dependent shear moduli
CERATE4	Coefficient for strain rate dependent transverse moduli

**Material Models:**

The unidirectional and fabric layer failure criteria and the associated property degradation models for material 161 are described as follows. All the failure criteria are expressed in terms of stress components based on ply level stresses ( $\sigma_a, \sigma_b, \sigma_c, \tau_{ab}, \tau_{bc}, \tau_{ca}$ ) and the associated elastic moduli are ( $E_a, E_b, E_c, G_{ab}, G_{bc}, G_{ca}$ ). Note that for the unidirectional model, *a*, *b* and *c* denote the fiber, in-plane transverse and out-of-plane directions, respectively, while for the fabric model, *a*, *b* and *c* denote the in-plane fill, in-plane warp and out-of-plane directions, respectively.

**Unidirectional Lamina Model:**

Three criteria are used for fiber failure, one in tension/shear, one in compression and another one in crush under pressure. They are chosen in terms of quadratic stress forms as follows:

1. Tensile/shear fiber mode:

$$f_1 = \left( \frac{\langle \sigma_a \rangle}{S_{aT}} \right)^2 + \left( \frac{\tau_{ab}^2 + \tau_{ca}^2}{S_{FS}^2} \right) - 1 = 0$$

2. Compression fiber mode:

$$f_2 = \left( \frac{\langle \sigma'_a \rangle}{S_{aC}} \right)^2 - 1 = 0, \quad \sigma'_a = -\sigma_a + \left\langle -\frac{\sigma_b + \sigma_c}{2} \right\rangle$$

3. Crush mode:

$$f_3 = \left( \frac{\langle p \rangle}{S_{FC}} \right)^2 - 1 = 0, \quad p = -\frac{\sigma_a + \sigma_b + \sigma_c}{3}$$

Here  $\langle \rangle$  are Macaulay brackets,  $S_{aT}$  and  $S_{aC}$  are the tensile and compressive strengths in the fiber direction, and  $S_{FS}$  and  $S_{FC}$  are the layer strengths associated with the fiber shear and crush failure, respectively.

Matrix mode failures must occur without fiber failure, and hence they will be on planes parallel to fibers. For simplicity, only two failure planes are considered: one is perpendicular to the planes of layering and the other one is parallel to them. The matrix failure criteria for the failure plane perpendicular and parallel to the layering planes, respectively, have the forms:

1. Perpendicular matrix mode:

$$f_4 = \left( \frac{\langle \sigma_b \rangle}{S_{bT}} \right)^2 + \left( \frac{\tau_{bc}}{S'_{bc}} \right)^2 + \left( \frac{\tau_{ab}}{S_{ab}} \right)^2 - 1 = 0$$

2. Parallel matrix mode (Delamination):

$$f_5 = S^2 \left\{ \left( \frac{\langle \sigma_c \rangle}{S_{bT}} \right)^2 + \left( \frac{\tau_{bc}}{S''_{bc}} \right)^2 + \left( \frac{\tau_{ca}}{S_{ca}} \right)^2 \right\} - 1 = 0$$

Here  $S_{bT}$  is the transverse tensile strength. Based on the Coulomb-Mohr theory, the shear strengths for the transverse shear failure and the two axial shear failure modes are assumed to be the forms,

$$\begin{aligned} S_{ab} &= S_{ab}^{(0)} + \tan(\varphi) \langle -\sigma_b \rangle \\ S'_{bc} &= S_{bc}^{(0)} + \tan(\varphi) \langle -\sigma_b \rangle \\ S_{ca} &= S_{ca}^{(0)} + \tan(\varphi) \langle -\sigma_c \rangle \\ S''_{bc} &= S_{bc}^{(0)} + \tan(\varphi) \langle -\sigma_c \rangle \end{aligned}$$

where  $\varphi$  is a material constant as  $\tan(\varphi)$  is similar to the coefficient of friction, and  $S_{ab}^{(0)}$ ,  $S_{ca}^{(0)}$  and  $S_{bc}^{(0)}$  are the shear strength values of the corresponding tensile modes.

Failure predicted by the criterion of  $f_4$  can be referred to as transverse matrix failure, while the matrix failure predicted by  $f_5$ , which is parallel to the layer, can be referred as the delamination mode when it occurs within the elements that are adjacent to the ply interface. Note that a scale factor  $S$  is introduced to provide better correlation of delamination area with experiments. The scale factor  $S$  can be determined by fitting the analytical prediction to experimental data for the delamination area.

When fiber failure in tension/shear mode is predicted in a layer by  $f_1$ , the load carrying capacity of that layer is completely eliminated. All the stress components are reduced to zero instantaneously (100 time steps to avoid numerical instability). For compressive fiber failure, the layer is assumed to carry a residual axial load, while the transverse load carrying capacity is reduced to zero. When the fiber compressive failure mode is reached due to  $f_2$ , the axial layer compressive strength stress is assumed to reduce to a residual value  $S_{RC}$  ( $= SFFC \times S_{AC}$ ). The axial stress is then assumed to remain constant, meaning

$\sigma_a = -S_{RC}$ , for continuous compressive loading, while the subsequent unloading curve follows a reduced axial modulus to zero axial stress and strain state. When the fiber crush failure occurs, the material is assumed to behave elastically for compressive pressure,  $p > 0$ , and to carry no load for tensile pressure,  $p < 0$ .

When a matrix failure (delamination) in the  $ab$ -plane is predicted, the strength values for  $S_{ca}^{(0)}$  and  $S_{bc}^{(0)}$  are set to zero. This results in reducing the stress components  $\sigma_c$ ,  $\tau_{bc}$  and  $\tau_{ca}$  to the fractured material strength surface. For tensile mode,  $\sigma_c > 0$ , these stress components are reduced to zero. For compressive mode,  $\sigma_c < 0$ , the normal stress  $\sigma_c$  is assumed to deform elastically for the closed matrix crack. Loading on the failure envelop, the shear stresses are assumed to 'slide' on the fractured strength surface (frictional shear stresses) like in an ideal plastic material, while the subsequent unloading shear stress-strain path follows reduced shear moduli to the zero shear stress and strain state for both  $\tau_{bc}$  and  $\tau_{ca}$  components.

The post failure behavior for the matrix crack in the a-c plane due to  $f_4$  is modeled in the same fashion as that in the  $ab$ -plane as described above. In this case, when failure occurs,  $S_{ab}^{(0)}$  and  $S_{bc}^{(0)}$  are reduced to zero instantaneously. The post fracture response is then governed by failure criterion of  $f_5$  with  $S_{ab}^{(0)} = 0$  and  $S_{bc}^{(0)} = 0$ . For tensile mode,  $\sigma_b > 0$ ,  $\sigma_b$ ,  $\tau_{ab}$  and  $\tau_{bc}$  are zero. For compressive mode,  $\sigma_b < 0$ ,  $\sigma_b$  is assumed to be elastic, while  $\tau_{ab}$  and  $\tau_{bc}$  'slide' on the fracture strength surface as in an ideal plastic material, and the unloading path follows reduced shear moduli to the zero shear stress and strain state. It should be noted that  $\tau_{bc}$  is governed by both the failure functions and should lie within or on each of these two strength surfaces.

### Fabric Lamina Model:

The fiber failure criteria of Hashin for a unidirectional layer are generalized to characterize the fiber damage in terms of strain components for a plain weave layer. The fill and warp fiber tensile/shear failure are given by the quadratic interaction between the associated axial and shear stresses, that is,

$$f_6 = \left(\frac{\langle\sigma_a\rangle}{S_{aT}}\right)^2 + \frac{(\tau_{ab}^2 + \tau_{ca}^2)}{S_{aFS}^2} - 1 = 0$$

$$f_7 = \left(\frac{\langle\sigma_b\rangle}{S_{bT}}\right)^2 + \frac{(\tau_{ab}^2 + \tau_{bc}^2)}{S_{bFS}^2} - 1 = 0$$

where  $S_{aT}$  and  $S_{bT}$  are the axial tensile strengths in the fill and warp directions, respectively, and  $S_{aFS}$  and  $S_{bFS}$  are the layer shear strengths due to fiber shear failure in the fill and warp directions. These failure criteria are applicable when the associated  $\sigma_a$  or  $\sigma_b$  is positive. It is assumed  $S_{aFS} = SFS$ , and

$$S_{bFS} = SFS \times \frac{S_{bT}}{S_{aT}} .$$

When  $\sigma_a$  or  $\sigma_b$  is compressive, it is assumed that the in-plane compressive failure in both the fill and warp directions are given by the maximum stress criterion, that is,

$$f_8 = \left[ \frac{\langle \sigma'_a \rangle}{S_{aC}} \right]^2 - 1 = 0, \quad \sigma'_a = -\sigma_a + \langle -\sigma_c \rangle$$

$$f_9 = \left[ \frac{\langle \sigma'_b \rangle}{S_{bC}} \right]^2 - 1 = 0, \quad \sigma'_b = -\sigma_b + \langle -\sigma_c \rangle$$

where  $S_{aC}$  and  $S_{bC}$  are the axial compressive strengths in the fill and warp directions, respectively. The crush failure under compressive pressure is

$$f_{10} = \left( \frac{\langle p \rangle}{S_{FC}} \right)^2 - 1 = 0, \quad p = -\frac{\sigma_a + \sigma_b + \sigma_c}{3}.$$

A plain weave layer can fail under in-plane shear stress without the occurrence of fiber breakage. This in-plane matrix failure mode is given by

$$f_{11} = \left( \frac{\tau_{ab}}{S_{ab}} \right)^2 - 1 = 0,$$

where  $S_{ab}$  is the layer shear strength due to matrix shear failure.

Another failure mode, which is due to the quadratic interaction between the thickness stresses, is expected to be mainly a matrix failure. This through the thickness matrix failure criterion is

$$f_{12} = S^2 \left\{ \left( \frac{\langle \sigma_c \rangle}{S_{cT}} \right)^2 + \left( \frac{\tau_{bc}}{S_{bc}} \right)^2 + \left( \frac{\tau_{ca}}{S_{ca}} \right)^2 \right\} - 1 = 0,$$

where  $S_{cT}$  is the through the thickness tensile strength, and  $S_{bc}$  and  $S_{ca}$  are the shear strengths assumed to depend on the compressive normal stress  $\sigma_c$ , meaning

$$\begin{Bmatrix} S_{ca} \\ S_{bc} \end{Bmatrix} = \begin{Bmatrix} S_{ca}^{(0)} \\ S_{bc}^{(0)} \end{Bmatrix} + \tan(\varphi) \langle -\sigma_c \rangle.$$

When failure predicted by this criterion occurs within elements that are adjacent to the ply interface, the failure plane is expected to be parallel to the layering planes, and, thus, can be referred to as the delamination mode. Note that a scale factor  $S$  is introduced to provide better correlation of delamination area with experiments. The scale factor  $S$  can be determined by fitting the analytical prediction to experimental data for the delamination area.

Similar to the unidirectional model, when fiber tensile/shear failure is predicted in a layer by  $f_6$  or  $f_7$ , the load carrying capacity of that layer in the associated direction is completely eliminated. For compressive fiber failure due to by  $f_8$  or  $f_9$ , the layer is assumed to carry a residual axial load in the failed direction, while the load carrying capacity transverse to the failed direction is assumed unchanged. When the compressive axial stress in a layer reaches the compressive axial strength  $S_{aC}$  or  $S_{bC}$ , the axial layer stress is assumed to be reduced to the residual strength  $S_{aRC}$  or  $S_{bRC}$  where  $S_{aRC} = SFFC \times S_{aC}$  and  $S_{bRC} =$

SFFC  $\times S_{bC}$ . The axial stress is assumed to remain constant, that is,  $\sigma_a = -S_{aCR}$  or  $\sigma_b = -S_{bCR}$ , for continuous compressive loading, while the subsequent unloading curve follows a reduced axial modulus. When the fiber crush failure is occurred, the material is assumed to behave elastically for compressive pressure,  $p > 0$ , and to carry no load for tensile pressure,  $p < 0$ .

When the in-plane matrix shear failure is predicted by  $f_{11}$  the axial load carrying capacity within a failed element is assumed unchanged, while the in-plane shear stress is assumed to be reduced to zero.

For through the thickness matrix (delamination) failure given by equation  $f_{12}$ , the in-plane load carrying capacity within the element is assumed to be elastic, while the strength values for the tensile mode,  $S_{ca}^{(0)}$  and  $S_{bc}^{(0)}$ , are set to zero. For tensile mode,  $\sigma_c > 0$ , the through the thickness stress components are reduced to zero. For compressive mode,  $\sigma_c < 0$ ,  $\sigma_c$  is assumed to be elastic, while  $\tau_{bc}$  and  $\tau_{ca}$  'slide' on the fracture strength surface as in an ideal plastic material, and the unloading path follows reduced shear moduli to the zero shear stress and strain state.

The effect of strain-rate on the layer strength values of the fiber failure modes is modeled by the strain-rate dependent functions for the strength values  $\{S_{RT}\}$  as

$$\{S_{RT}\} = \{S_0\} \left( 1 + C_{rate1} \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)$$

$$\{S_{RT}\} = \begin{Bmatrix} S_{aT} \\ S_{aC} \\ S_{bT} \\ S_{bC} \\ S_{FC} \\ S_{FS} \end{Bmatrix}, \quad \dot{\varepsilon} = \begin{Bmatrix} |\dot{\varepsilon}_a| \\ |\dot{\varepsilon}_a| \\ |\dot{\varepsilon}_b| \\ |\dot{\varepsilon}_b| \\ |\dot{\varepsilon}_c| \\ (\dot{\varepsilon}_{ca}^2 + \dot{\varepsilon}_{bc}^2)^{1/2} \end{Bmatrix}$$

where  $C_{rate1}$  is the strain-rate constants, and  $\{S_0\}$  are the strength values of  $\{S_{RT}\}$  at the reference strain-rate  $\dot{\varepsilon}_0$ .

### **Damage Model:**

The damage model is a generalization of the layer failure model of Material 161 by adopting the MLT damage mechanics approach, Matzenmiller et al. [1995], for characterizing the softening behavior after damage initiation. Complete model description is given in Yen [2002]. The damage functions, which are expressed in terms of ply level engineering strains, are converted from the above failure criteria of fiber and matrix failure modes by neglecting the Poisson's effect. Elastic moduli reduction is expressed in terms of the associated damage parameters  $\omega_i$ :

$$E'_i = (1 - \omega_i) E_i$$

where  $\omega_i$  is given by

$$\omega_i = 1 - \exp\left(-\frac{r_i^{m_i}}{m_i}\right), \quad r_i \geq 0, \quad i = 1, \dots, 6.$$

In the above  $E_i$  are the initial elastic moduli,  $E'_i$  are the reduced elastic moduli,  $r_i$  are the damage thresholds computed from the associated damage functions for fiber damage, matrix damage and delamination, and  $m_i$  are material damage parameters, which are currently assumed to be independent of strain-rate. The damage function is formulated to account for the overall nonlinear elastic response of a lamina including the initial 'hardening' and the subsequent softening beyond the ultimate strengths.

In the damage model (material 162), the effect of strain-rate on the nonlinear stress-strain response of a composite layer is modeled by the strain-rate dependent functions for the elastic moduli  $\{E_{RT}\}$  as

$$\{E_{RT}\} = \{E_0\} \left( 1 + \{C_{rate}\} \ln \frac{\{\dot{\varepsilon}\}}{\dot{\varepsilon}_0} \right)$$

$$\{E_{RT}\} = \begin{Bmatrix} E_a \\ E_b \\ E_c \\ G_{ab} \\ G_{bc} \\ G_{ca} \end{Bmatrix} \quad \{\dot{\varepsilon}\} = \begin{Bmatrix} |\dot{\varepsilon}_a| \\ |\dot{\varepsilon}_b| \\ |\dot{\varepsilon}_c| \\ |\dot{\varepsilon}_{ab}| \\ |\dot{\varepsilon}_{bc}| \\ |\dot{\varepsilon}_{ca}| \end{Bmatrix} \quad \{C_{rate}\} = \begin{Bmatrix} C_{rate2} \\ C_{rate2} \\ C_{rate4} \\ C_{rate3} \\ C_{rate3} \\ C_{rate3} \end{Bmatrix}$$

where  $\{C_{rate}\}$  are the strain-rate constants.  $\{E_0\}$  are the modulus values of  $\{E_{RT}\}$  at the reference strain-rate  $\dot{\varepsilon}_0$ .

### Element Erosion:

A failed element is eroded in any of three different ways:

1. If fiber tensile failure in a unidirectional layer is predicted in the element and the axial tensile strain is greater than E\_LIMT. For a fabric layer, both in-plane directions are failed and exceed E\_LIMT.
2. If compressive relative volume in a failed element is smaller than ECRSH.
3. If tensile relative volume in a failed element is greater than EEXPN.

### Damage History Parameters:

Information about the damage history variables for the associated failure modes can be plotted in LS-PrePost. These additional history variables are tabulated below:

**\*MAT\_161-162****\*MAT\_COMPOSITE\_MSC**

History Variable	Description	Value	History Variable #
efa( $I$ )	Fiber mode in $a$		7
efb( $I$ )	Fiber mode in $b$		8
efp( $I$ )	Fiber crush mode	EQ.0: elastic	9
em( $I$ )	Perpendicular matrix mode	GE.1: failed	10
ed( $I$ )	Parallel matrix/delamination mode		11
delm( $I$ )	Delamination mode		12

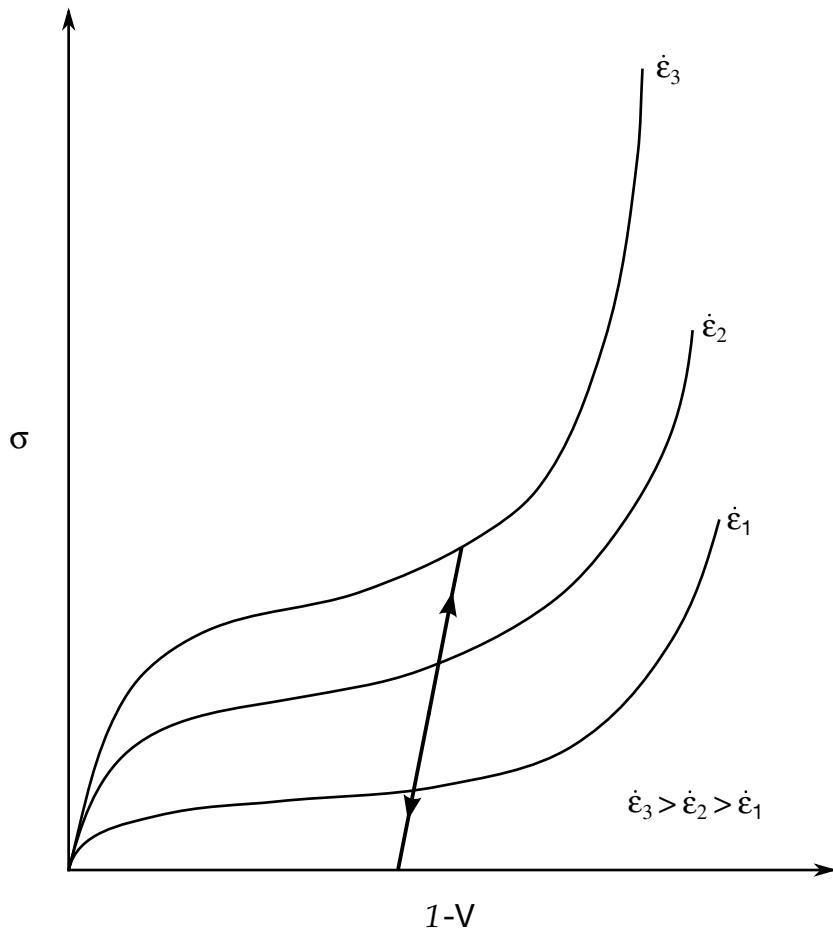
**\*MAT\_MODIFIED\_CRUSHABLE\_FOAM****\*MAT\_163****\*MAT\_MODIFIED\_CRUSHABLE\_FOAM**

This is Material Type 163 which is dedicated to modeling crushable foam with optional damping, tension cutoff, and strain rate effects. Unloading is fully elastic. Tension is treated as elastic-perfectly-plastic at the tension cut-off value.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	TID	TSC	DAMP	NCYCLE
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	0.10	12.

Card 2	1	2	3	4	5	6	7	8
Variable	SRCLMT	SFLAG						
Type	F	I						
Default	$10^{20}$	0						

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
TID	Table ID defining yield stress as a function of volumetric strain, $\gamma$ , at different strain rates.
TSC	Tensile stress cutoff. A nonzero, positive value is strongly recommended for realistic behavior.
DAMP	Rate sensitivity via damping coefficient (.05 < recommended value < .50).



**Figure M163-1.** Rate effects are defined by a family of curves giving yield stress as a function of volumetric strain where  $V$  is the relative volume.

VARIABLE	DESCRIPTION
NCYCLE	Number of cycles to determine the average volumetric strain rate.
SRCLMT	Strain rate change limit.
SFLAG	The strain rate in the table may be the true strain rate (SFLAG = 0) or the engineering strain rate (SFLAG = 1).

#### Remarks:

The volumetric strain is defined in terms of the relative volume,  $V$ , as:

$$\gamma = 1 - V$$

The relative volume is defined as the ratio of the current to the initial volume. In place of the effective plastic strain in the d3plot database, the integrated volumetric strain (natural logarithm of the relative volume) is output.

This material is an extension of material 63, \*MAT\_CRUSHABLE\_FOAM. It allows the yield stress to be a function of both volumetric strain rate and volumetric strain. Rate effects are accounted for by defining a table of curves using \*DEFINE\_TABLE. Each curve defines the yield stress as a function volumetric strain for a different strain rate. The yield stress is obtained by interpolating between the two curves that bound the strain rate.

To prevent high frequency oscillations in the strain rate from causing similar high frequency oscillations in the yield stress, a modified volumetric strain rate is used when interpolating to obtain the yield stress. The modified strain rate is obtained as follows. If NYCLE is > 1, then the modified strain rate is obtained by a time average of the actual strain rate over NCYCLE solution cycles. For SRCLMT > 0, the modified strain rate is capped so that during each cycle, the modified strain rate is not permitted to change more than SRCLMT multiplied by the solution time step.

**\*MAT\_164****\*MAT\_BRAIN\_LINEAR\_VISCOELASTIC****\*MAT\_BRAIN\_LINEAR\_VISCOELASTIC**

This is Material Type 164. This material is a Kelvin-Maxwell model for modeling brain tissue, which is valid for solid elements only. See Remarks below.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	BULK	G0	GI	DC	F0	SO
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	0.0	0.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
BULK	Bulk modulus (elastic)
G0	Short-time shear modulus, $G_0$
GI	Long-time (infinite) shear modulus, $G_\infty$
DC	Constant depending on formulation option (FO) below: FO.EQ.0.0: Maxwell decay constant, $\beta$ FO.EQ.1.0: Kelvin relaxation constant, $\tau$
FO	Formulation option: EQ.0.0: Maxwell EQ.1.0: Kelvin
SO	Strain (logarithmic) output option to control what is written as component 7 to the d3plot database. (LS-PrePost always blindly labels this component as effective plastic strain.) The maximum values are updated for each element each time step: EQ.0.0: Maximum principal strain that occurs during the calculation

VARIABLE	DESCRIPTION
	EQ.1.0: Maximum magnitude of the principal strain values that occurs during the calculation
	EQ.2.0: Maximum effective strain that occurs during the calculation

**Remarks:**

1. **Maxwell Model.** The shear relaxation behavior is described for the Maxwell model by:

$$G(t) = G + (G_0 - G_\infty)e^{-\beta t} .$$

A Jaumann rate formulation is used

$$\overset{\nabla}{\sigma}_{ij} = 2 \int_0^t G(t - \tau) D'_{ij}(\tau) dt ,$$

where the prime denotes the deviatoric part of the stress rate,  $\overset{\nabla}{\sigma}_{ij}$ , and the strain rate  $D_{ij}$ .

2. **Kelvin Model.** For the Kelvin model the stress evolution equation is defined as:

$$\dot{s}_{ij} + \frac{1}{\tau} s_{ij} = (1 + \delta_{ij}) G_0 \dot{e}_{ij} + (1 + \delta_{ij}) \frac{G_\infty}{\tau} \dot{e}_{ij} .$$

The strain data as written to the d3plot database may be used to predict damage, see [Bandak 1991].

**\*MAT\_165****\*MAT\_PLASTIC\_NONLINEAR\_KINEMATIC****\*MAT\_PLASTIC\_NONLINEAR\_KINEMATIC**

This is Material Type 165. This relatively simple model, based on a material model by Lemaitre and Chaboche [1990], is suited to model nonlinear kinematic hardening plasticity. The model accounts for the nonlinear Bauschinger effect, cyclic hardening, and ratcheting. Huang [2009] programmed this model and provided it as a user subroutine. It is a very cost effective model and is available for shell and solid elements.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY	H	C	GAMMA
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	0.0	0.0	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	FS							
Type	F							
Default	$10^{16}$							

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
SIGY	Initial yield stress, $\sigma_{y0}$
H	Isotropic plastic hardening modulus
C	Kinematic hardening modulus

<b>VARIABLE</b>	<b>DESCRIPTION</b>
GAMMA	Kinematic hardening parameter, $\gamma$
FS	Failure strain for eroding elements

**Remarks:**

If the isotropic hardening modulus,  $H$ , is nonzero, the size of the surface increases as a function of the equivalent plastic strain,  $\varepsilon^p$ :

$$\sigma_y = \sigma_{y0} + H\varepsilon^p .$$

The rate of evolution of the kinematic component is a function of the plastic strain rate:

$$\dot{\alpha} = [Cn - \gamma\alpha]\dot{\varepsilon}^p ,$$

where  $n$  is the flow direction. The term,  $\gamma\alpha\dot{\varepsilon}^p$ , introduces the nonlinearity into the evolution law, which becomes linear if the parameter,  $\gamma$ , is set to zero.

## \*MAT\_166

## \*MAT\_MOMENT\_CURVATURE\_BEAM

### \*MAT\_MOMENT\_CURVATURE\_BEAM

This is Material Type 166. This material is for performing nonlinear elastic or multi-linear plastic analysis of Belytschko-Schwer beams with user-defined axial force-strain, moment curvature and torque-twist rate curves. If strain, curvature or twist rate is located outside the curves, use extrapolation to determine the corresponding rigidity. For multi-linear plastic analysis, the user-defined curves are used as yield surfaces.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	ELAF	EPFLG	CTA	CTB	CTT
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	0.0	0.0	0.0	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	N1	N2	N3	N4	N5	N6	N7	N8
Type	F	F	F	F	F	F	F	F
Default	none	none	0.0 / none	0.0	0.0	0.0	0.0	0.0

Card 3	1	2	3	4	5	6	7	8
Variable	LCMS1	LCMS2	LCMS3	LCMS4	LCMS5	LCMS6	LCMS7	LCMS8
Type	I	I	I	I	I	I	I	I
Default	none	none	0 / none	0	0	0	0	0

**\*MAT\_MOMENT\_CURVATURE\_BEAM****\*MAT\_166**

Card 4	1	2	3	4	5	6	7	8
Variable	LCMT1	LCMT2	LCMT3	LCMT4	LCMT5	LCMT6	LCMT7	LCMT8
Type	I	I	I	I	I	I	I	I
Default	none	none	0 / none	0	0	0	0	0

Card 5	1	2	3	4	5	6	7	8
Variable	LCT1	LCT2	LCT3	LCT4	LCT5	LCT6	LCT7	LCT8
Type	I	I	I	I	I	I	I	I
Default	none	none	0 / none	0	0	0	0	0

**Multilinear Plastic Analysis Card.** Additional card for EPFLG = 1.

Card 6	1	2	3	4	5	6	7	8
Variable	CFA	CFB	CFT	HRULE	REPS	RBETA	RCAPAY	RCAPAZ
Type	F	F	F	F	F	F	F	F
Default	1.0	1.0	1.0	0.0	$10^{20}$	$10^{20}$	$10^{20}$	$10^{20}$

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus. This variable controls the time step size and must be chosen carefully. Increasing the value of E will decrease the time step size.
ELAF	Load curve ID for the axial force-strain curve

**\*MAT\_166****\*MAT\_MOMENT\_CURVATURE\_BEAM**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EPFLG	Function flag: EQ.0.0: Nonlinear elastic analysis EQ.1.0: Multi-linear plastic analysis
CTA, CTB, CTT	Type of axial force-strain, moment-curvature, and torque-twist rate curves (see Remarks): EQ.0.0: Curve is symmetric. EQ.1.0: Curve is asymmetric.
N1 - N8	Axial forces at which moment-curvature curves are given. The axial forces must be ordered monotonically increasing. At least two axial forces must be defined if the curves are symmetric. At least three axial forces must be defined if the curves are asymmetric.
LCMS1 - LCMS8	Load curve IDs for the moment-curvature curves about axis S under corresponding axial forces.
LCMT1 - LCMT8	Load curve IDs for the moment-curvature curves about axis T under corresponding axial forces.
LCT1 - LCT8	Load curve IDs for the torque-twist rate curves under corresponding axial forces.
CFA, CFB, CFT	For multi-linear plastic analysis only. Ratio of axial, bending and torsional elastic rigidities to their initial values, no less than 1.0 in value.
HRULE	Hardening rule, for multi-linear plastic analysis only. EQ.0.0: Isotropic hardening GT.0.0.AND.LT.1.0: Mixed hardening EQ.1.0: Kinematic hardening
REPS	Rupture effective plastic axial strain
RBETA	Rupture effective plastic twist rate
RCAPAY	Rupture effective plastic curvature about axis S
RCAPAZ	Rupture effective plastic curvature about axis T

**Remarks:**

For symmetric curves (see fields CTA, CTB, and CTT above), all data points must be in the first quadrant, and at least three data points need to be given, starting from the origin, followed by the yield point.

For asymmetric curves, at least five data points are needed and exactly one point must be at the origin. The two points on both sides of the origin record the positive and negative yield points.

The last data point(s) has no physical meaning: it serves only as a control point for interpolation or extrapolation.

The curves are input by the user and treated in LS-DYNA as linearly piecewise functions. The curves must be monotonically increasing while the slopes must be monotonically decreasing.

**\*MAT\_167****\*MAT\_MCCORMICK****\*MAT\_MCCORMICK**

This is Material Type 167. This material is intended for finite plastic deformations. McCormick's constitutive relation for materials exhibiting negative steady-state Strain Rate Sensitivity (SRS) defines this material's strength. See McCormick [1988] and Zhang, McCormick and Estrin [2001].

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	SIGY			
Type	A	F	F	F	F			

Card 2	1	2	3	4	5	6	7	8
Variable	Q1	C1	Q2	C2				
Type	F	F	F	F				

Card 3	1	2	3	4	5	6	7	8
Variable	S	H	OMEGA	TD	ALPHA	EPS0		
Type	F	F	F	F	F	F		

**VARIABLE****DESCRIPTION**

MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
SIGY	Initial yield stress
Q1	Isotropic hardening parameter, $Q_1$
C1	Isotropic hardening parameter, $C_1$

VARIABLE	DESCRIPTION
Q2	Isotropic hardening parameter, $Q_2$
C2	Isotropic hardening parameter, $C_2$
S	Dynamic strain aging parameter, $S$
H	Dynamic strain aging parameter, $H$
OMEGA	Dynamic strain aging parameter, $\Omega$
TD	Dynamic strain aging parameter, $t_d$
ALPHA	Dynamic strain aging parameter, $\alpha$
EPS0	Reference strain rate, $\dot{\epsilon}_0$

**Remarks:**

The uniaxial stress-strain curve is given in the following form:

$$\sigma(\varepsilon^p, \dot{\varepsilon}^p) = \sigma_Y(t_a) + R(\varepsilon^p) + \sigma_v(\dot{\varepsilon}^p) .$$

Viscous stress  $\sigma_v$  is given by

$$\sigma_v(\dot{\varepsilon}^p) = S \times \ln \left( 1 + \frac{\dot{\varepsilon}^p}{\dot{\epsilon}_0} \right) ,$$

where  $S$  represents the instantaneous strain rate sensitivity and  $\dot{\epsilon}_0$  is a reference strain rate.

In the McCormick model the yield strength includes a dynamic strain aging (DSA) contribution. The yield strength is defined as

$$\sigma_Y(t_a) = \sigma_o + S \times H \times \left[ 1 - \exp \left\{ - \left( \frac{t_a}{t_d} \right)^\alpha \right\} \right] ,$$

where  $\sigma_o$  is the yield strength for vanishing average waiting time  $t_a$ , and  $H$ ,  $\alpha$ , and  $t_d$  are material constants linked to dynamic strain aging.

The average waiting time is defined by the evolution equation

$$\dot{t}_a = 1 - \frac{t_a}{t_{a,ss}} ,$$

where the quasi-steady state waiting time  $t_{a,ss}$  is given as

$$t_{a,ss} = \frac{\Omega}{\dot{\varepsilon}^p} .$$

The strain hardening function  $R$  is defined by the extended Voce Law

$$R(\varepsilon^p) = Q_1[1 - \exp(-C_1\varepsilon^p)] + Q_2[1 - \exp(-C_2\varepsilon^p)] .$$

**\*MAT\_POLYMER**

This is Material Type 168. This model is implemented for brick elements.

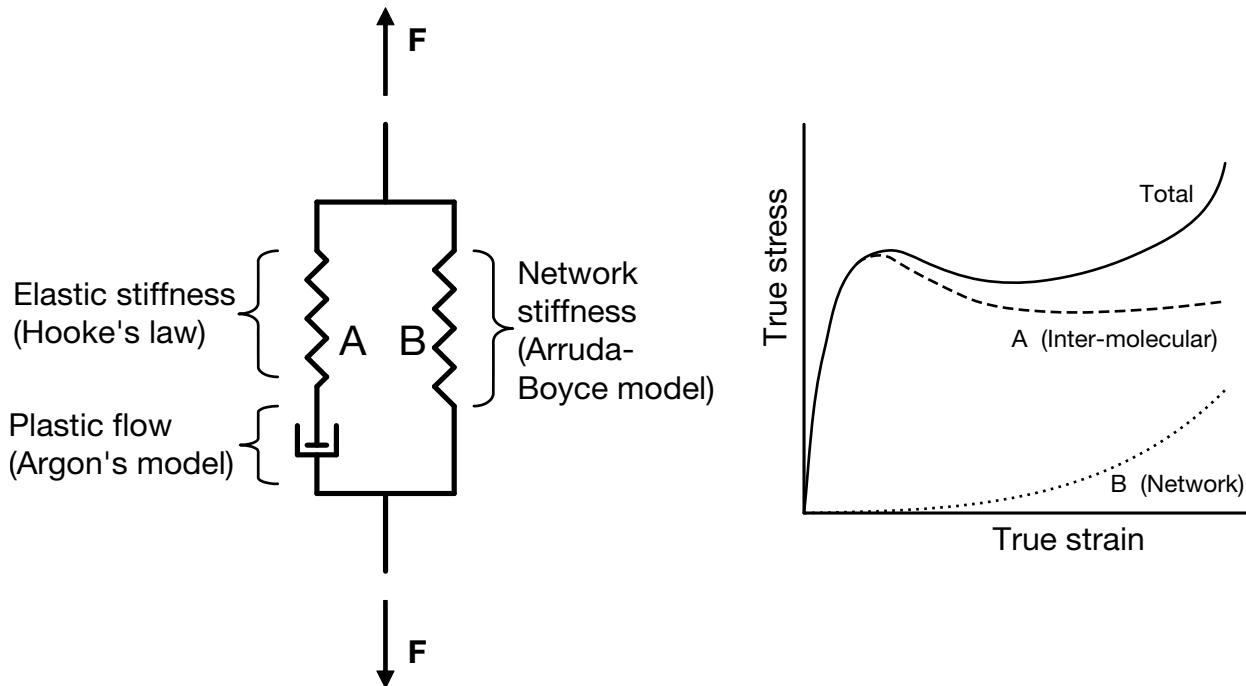
Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	GAMMA0	DG	SC	ST
Type	A	F	F	F	F	F	F	F

Card 2	1	2	3	4	5	6	7	8
Variable	TEMP	K	CR	N	C			
Type	F	F	F	F	F			

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus, $E$
PR	Poisson's ratio, $\nu$
GAMMA0	Pre-exponential factor, $\dot{\gamma}_{0A}$
DG	Energy barrier to flow, $\Delta G$
SC	Shear resistance in compression, $S_c$
ST	Shear resistance in tension, $S_t$
TEMP	Absolute temperature, $\theta$
K	Boltzmann constant, $k$
CR	Product, $C_r = nk\theta$
N	Number of "rigid links" between entanglements, $N$
C	Relaxation factor, $C$

**Remarks:**

The polymer is assumed to have two basic resistances to deformation:



**Figure M168-1.** Stress decomposition in inter-molecular and network contributions.

1. An intermolecular barrier to deformation related to relative movement between molecules.
2. An evolving anisotropic resistance related to straightening of the molecule chains.

The model which is implemented and presented in this paper is mainly based on the framework suggested by Boyce et al. [2000]. Going back to the original work by Haward and Thackray [1968], they considered the uniaxial case only. The extension to a full 3D formulation was proposed by Boyce et al. [1988]. Moreover, Boyce and co-workers have during a period of 20 years changed or further developed the parts of the original model. Haward and Thackray [1968] used an Eyring model to represent the dashpot in Fig. M168-1, while Boyce et al. [2000] employed the double-kink model of Argon [1973] instead. Part B of the model, describing the resistance associated with straightening of the molecules, contained originally a one-dimensional Langevin spring [Haward and Thackray, 1968], which was generalized to 3D with the eight-chain model by Arruda and Boyce [1993].

The main structure of the model presented by Boyce et al. [2000] is kept for this model. Recognizing the large elastic deformations occurring for polymers, a formulation based

on a Neo-Hookean material is here selected for describing the spring in resistance A in Figure M168-1.

Referring to Figure M168-1, it is assumed that the deformation gradient tensor is the same for the two resistances (Part A and B)

$$\mathbf{F} = \mathbf{F}_A = \mathbf{F}_B$$

while the Cauchy stress tensor for the system is assumed to be the sum of the Cauchy stress tensors for the two parts

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_A + \boldsymbol{\sigma}_B .$$

### Part A: Inter-Molecular Resistance:

The deformation is decomposed into elastic and plastic parts,  $\mathbf{F}_A = \mathbf{F}_A^e \mathbf{F}_A^p$ , where it is assumed that the intermediate configuration  $\bar{\Omega}_A$  defined by  $\mathbf{F}_A^p$  is invariant to rigid body rotations of the current configuration. The velocity gradient in the current configuration  $\Omega$  is defined by

$$\mathbf{L}_A = \dot{\mathbf{F}}_A \mathbf{F}_A^{-1} = \mathbf{L}_A^e + \mathbf{L}_A^p$$

Owing to the decomposition,  $\mathbf{F}_A = \mathbf{F}_A^e \mathbf{F}_A^p$ , the elastic and plastic rate-of-deformation and spin tensors are defined by

$$\begin{aligned}\mathbf{L}_A^e &= \mathbf{D}_A^e + \mathbf{W}_A^e = \dot{\mathbf{F}}_A^e (\mathbf{F}_A^e)^{-1} \\ \mathbf{L}_A^p &= \mathbf{D}_A^p + \mathbf{W}_A^p = \mathbf{F}_A^e \dot{\mathbf{F}}_A^p (\mathbf{F}_A^p)^{-1} (\mathbf{F}_A^e)^{-1} = \mathbf{F}_A^e \bar{\mathbf{L}}_A^p (\mathbf{F}_A^e)^{-1}\end{aligned}$$

where  $\bar{\mathbf{L}}_A^p = \dot{\mathbf{F}}_A^p (\mathbf{F}_A^p)^{-1}$ . The Neo-Hookean material represents an extension of Hooke's law to large elastic deformations and may be chosen for the elastic part of the deformation when the elastic behavior is assumed to be isotropic.

$$\boldsymbol{\tau}_A = \lambda_0 \ln J_A^e \mathbf{I} + \mu_0 (\mathbf{B}_A^e - \mathbf{I})$$

where  $\boldsymbol{\tau}_A = J_A \boldsymbol{\sigma}_A$  is the Kirchhoff stress tensor of Part A and  $J_A^e = \sqrt{\det \mathbf{B}_A^e} = J_A$  is the Jacobian determinant. The elastic left Cauchy-Green deformation tensor is given by  $\mathbf{B}_A^e = \mathbf{F}_A^e \mathbf{F}_A^{e T}$ .

The flow rule is defined by

$$\mathbf{L}_A^p = \dot{\gamma}_A^p \mathbf{N}_A$$

where

$$\mathbf{N}_A = \frac{1}{\sqrt{2} \tau_A} \boldsymbol{\tau}_A^{\text{dev}}, \quad \tau_A = \sqrt{\frac{1}{2} \text{tr}(\boldsymbol{\tau}_A^{\text{dev}})^2}$$

and  $\boldsymbol{\tau}_A^{\text{dev}}$  is the stress deviator. The rate of flow is taken to be a thermally activated process

$$\dot{\gamma}_A^p = \dot{\gamma}_{0A} \exp \left[ -\frac{\Delta G(1 - \tau_A/s)}{k\theta} \right]$$

where  $\dot{\gamma}_{0A}$  is a pre-exponential factor,  $\Delta G$  is the energy barrier to flow,  $s$  is the shear resistance,  $k$  is the Boltzmann constant and  $\theta$  is the absolute temperature. The shear resistance,  $s$ , is assumed to depend on the stress triaxiality,  $\sigma^*$ :

$$s = s(\sigma^*), \quad \sigma^* = \frac{\text{tr } \boldsymbol{\sigma}_A}{3\sqrt{3}\tau_A} .$$

The exact dependence is given by a user-defined load curve, which is linear between the shear resistances in compression and tension. These resistances are denoted  $s_c$  and  $s_t$ , respectively.

### Part B: Network Resistance:

The network resistance is assumed to be nonlinear elastic with deformation gradient  $\mathbf{F}_B = \mathbf{F}_B^N$ , meaning, any viscoplastic deformation of the network is neglected. The stress-stretch relation is defined by

$$\boldsymbol{\tau}_B = \frac{nk\theta}{3} \frac{\sqrt{N}}{\bar{\lambda}_N} \mathcal{L}^{-1} \left( \frac{\bar{\lambda}_N}{\sqrt{N}} \right) (\bar{\mathbf{B}}_B^N - \bar{\lambda}_N^2 \mathbf{I})$$

where  $\boldsymbol{\tau}_B = J_B \boldsymbol{\sigma}_B$  is the Kirchhoff stress for Part B,  $n$  is the chain density and  $N$  the number of “rigid links” between entanglements. In accordance with Boyce et. al [2000], the product,  $nk\theta$ , is denoted  $C_R$  herein. Moreover,  $\mathcal{L}^{-1}$  is the inverse Langevin function,  $\mathcal{L}(\beta) = \coth \beta - 1/\beta$ , and further

$$\bar{\mathbf{B}}_B^N = \bar{\mathbf{F}}_B^N \bar{\mathbf{F}}_B^{N T}, \quad \bar{\mathbf{F}}_B^N = J_B^{-1/3} \mathbf{F}_B^N, \quad J_B = \det \mathbf{F}_B^N, \quad \bar{\lambda}_N = \left[ \frac{1}{3} \text{tr } \bar{\mathbf{B}}_B^N \right]^{\frac{1}{2}}$$

The flow rule defining the rate of molecular relaxation reads

$$\mathbf{L}_B^F = \dot{\gamma}_B^F \mathbf{N}_B$$

where

$$\mathbf{N}_B = \frac{1}{\sqrt{2} \tau_B} \boldsymbol{\tau}_B^{\text{dev}}, \quad \tau_B = \sqrt{\frac{1}{2} \boldsymbol{\tau}_B^{\text{dev}} : \boldsymbol{\tau}_B^{\text{dev}}}$$

The rate of relaxation is taken equal to

$$\dot{\gamma}_B^F = C \left( \frac{1}{\bar{\lambda}_F - 1} \right) \tau_B$$

where

$$\bar{\lambda}_F = \left[ \frac{1}{3} \text{tr} \left( \mathbf{F}_B^F \{ \mathbf{F}_B^F \}^T \right) \right]^{\frac{1}{2}}$$

The model has been implemented into LS-DYNA using a semi-implicit stress-update scheme [Moran et. al 1990], and is available for the explicit solver only.

## \*MAT\_169

## \*MAT\_ARUP\_ADHESIVE

### \*MAT\_ARUP\_ADHESIVE

This is Material Type 169. This material model was created for adhesive bonding in aluminum structures. The plasticity model is not volume-conserving, so it avoids the spuriously high tensile stresses that can develop when modeling adhesive with traditional elasto-plastic material models. It is available *only* for solid elements of formulations 1, 2 and 15. Unless THKDIR = 1, the smallest dimension of the element is assumed to be the through-thickness dimension of the bond.

#### Card Summary:

**Card 1.** This card is required.

MID	R0	E	PR	TENMAX	GCTEN	SHRMAX	GCSHR
-----	----	---	----	--------	-------	--------	-------

**Card 2.** This card is required.

PWRT	PWRS	SHRP	SHT_SL	EDOTO	EDOT2	THKDIR	EXTRA
------	------	------	--------	-------	-------	--------	-------

**Card 3.** This card is included if EXTRA = 1 or 3.

TMAXE	GCTE	SMAXE	GCSE	PWRTE	PWRSE		
-------	------	-------	------	-------	-------	--	--

**Card 4.** This card is included if EXTRA = 1 or 3.

FACET	FACCT	FACES	FACCS	SOFTT	SOFTS		
-------	-------	-------	-------	-------	-------	--	--

**Card 5.** This card is included when EDOT2  $\neq$  0.

SDFAC	SGFAC	SDEFAC	SGEFAC				
-------	-------	--------	--------	--	--	--	--

**Card 6.** This card is included if EXTRA = 2 or 3.

BTHK	OUTFAIL	FSIP	FBR713	ELF2NS			
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	E	PR	TENMAX	GCTEN	SHRMAX	GCSHR
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	$10^{20}$	$10^{20}$	$10^{20}$	$10^{20}$

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
E	Young's modulus
PR	Poisson's ratio
TENMAX	Maximum through-thickness tensile stress (see <a href="#">Remark 7</a> ): GT.0.0: Constant value LT.0.0: $ TENMAX $ is a function ID.
GCTEN	Energy per unit area to fail the bond in tension (see <a href="#">Remark 7</a> ): GT.0.0: Constant value LT.0.0: $ GCTEN $ is a function ID.
SHRMAX	Maximum through-thickness shear stress (see <a href="#">Remark 7</a> ): GT.0.0: Constant value LT.0.0: $ SHRMAX $ is a function ID.
GCSHR	Energy per unit area to fail the bond in shear (see <a href="#">Remark 7</a> ): GT.0.0: Constant value LT.0.0: $ GCSHR $ is a function ID.

**\*MAT\_169****\*MAT\_ARUP\_ADHESIVE**

Card 2	1	2	3	4	5	6	7	8
Variable	PWRT	PWRS	SHRP	SHT_SL	EDOT0	EDOT2	THKDIR	EXTRA
Type	F	F	F	F	F	F	F	F
Default	2.0	2.0	0.0	0.0	1.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION
PWRT	Power law term for tension
PWRS	Power law term for shear
SHRP	Shear plateau ratio (optional): GT.0.0: Constant value LT.0.0:  SHRP  is a function ID (see <a href="#">Remark 7</a> ).
SHT_SL	Slope (non-dimensional) of yield surface at zero tension (see <a href="#">Remark 3</a> )
EDOT0	Strain rate at which the “static” properties apply
EDOT2	Strain rate at which the “dynamic” properties apply (Card 5)
THKDIR	Through-thickness direction flag (see <a href="#">Remark 1</a> ): EQ.0.0: Smallest element dimension (default) EQ.1.0: Direction from nodes 1-2-3-4 to nodes 5-6-7-8
EXTRA	Flag to input further data: EQ.1.0: Interfacial failure properties (Cards 3 and 4) EQ.2.0: Bond thickness and more (Card 6) EQ.3.0: Both of the above

**Interfacial Failure Properties Card.** Additional card for EXTRA = 1 or 3.

Card 3	1	2	3	4	5	6	7	8
Variable	TMAXE	GCTE	SMAXE	GCSE	PWRTE	PWRSE		
Type	F	F	F	F	F	F		
Default	$10^{20}$	$10^{20}$	$10^{20}$	$10^{20}$	2.0	2.0		

VARIABLE	DESCRIPTION
TMAXE	Maximum tensile force per unit length on edges of joint
GCTE	Energy per unit length to fail the edge of the bond in tension
SMAXE	Maximum shear force per unit length on edges of joint
GCSE	Energy per unit length to fail the edge of the bond in shear
PWRTE	Power law term for tension
PWRSE	Power law term for shear

**Interfacial Failure Properties Card.** Additional card for EXTRA = 1 or 3.

Card 4	1	2	3	4	5	6	7	8
Variable	FACET	FACCT	FACES	FACCS	SOFTT	SOFTS		
Type	F	F	F	F	F	F		
Default	1.0	1.0	1.0	1.0	1.0	1.0		

VARIABLE	DESCRIPTION
FACET	Stiffness scaling factor for edge elements – tension
FACCT	Stiffness scaling factor for interior elements – tension
FACES	Stiffness scaling factor for edge elements – shear
FACCS	Stiffness scaling factor for interior elements – shear

**\*MAT\_169****\*MAT\_ARUP\_ADHESIVE**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SOFTT	Tensile strength reduction factor applied when a neighbor fails
SOFTS	Shear strength reduction factor applied when a neighbor fails

**Dynamic Strain Rate Card.** Additional card for EDOT2 ≠ 0.

Card 5	1	2	3	4	5	6	7	8
Variable	SDFAC	SGFAC	SDEFAC	SGEFAC				
Type	F	F	F	F				
Default	1.0	1.0	1.0	1.0				

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SDFAC	Factor on TENMAX and SHRMAX at strain rate EDOT2: GT.0.0: Constant value LT.0.0:  SDFAC  is a function ID (see <a href="#">Remark 7</a> ).
SGFAC	Factor on GCTEN and GCSHR at strain rate EDOT2: GT.0.0: Constant value LT.0.0:  SGFAC  is a function ID (see <a href="#">Remark 7</a> ).
SDEFAC	Factor on TMAXE and SMAXE at strain rate EDOT2
SGEFAC	Factor on GCTE and GCSE at strain rate EDOT2

**Bond Thickness Card.** Additional card for EXTRA = 2 or 3.

Card 6	1	2	3	4	5	6	7	8
Variable	BTHK	OUTFAIL	FSIP	FBR713	ELF2NS			
Type	F	F	F	F	F			
Default	0.0	0.0	0.0	0.0	0.0			

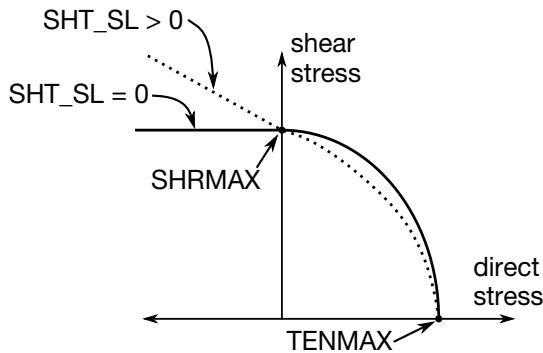
VARIABLE	DESCRIPTION
BTHK	<p>Bond thickness (overrides thickness from element dimensions; see <a href="#">Remark 1</a>):</p> <p>LT.0.0: <math> BTHK </math> is bond thickness, but critical time step remains unaffected. Helps to avoid very small time steps, but it can affect stability.</p>
OUTFAIL	<p>Flag for additional output to <b>messag</b> file which includes information about damage initiation time, failure function terms and forces:</p> <p>EQ.0.0: Off</p> <p>EQ.1.0: On</p>
FSIP	Effective in-plane strain at failure
FBR713	<p>Fallback option to get results from previous version. See <a href="#">Remark 8</a>.</p> <p>EQ.0.0: Off</p> <p>EQ.1.0: LS-DYNA release R7.1.3</p>
ELF2NS	<p>Volumetric smearing option for ELFORM = 2. See <a href="#">Remark 9</a>.</p> <p>EQ.0.0: Usual ELFORM = 2 behavior with volumetric smearing</p> <p>EQ.1.0: Volumetric smearing is turned off.</p>

**Remarks:**

1. **Through-thickness direction and bond thickness.** The through-thickness direction is identified from the smallest dimension of each element by default (THKDIR = 0.0). It is expected that this dimension will be smaller than in-plane dimensions (typically 1 - 2 mm compared with 5 - 10 mm). If this is not the case, one can set the through-thickness direction using element numbering (THKDIR = 1.0). Then the thickness direction is expected to point from lower face (nodes 1-2-3-4) to upper face (nodes 5-6-7-8). For wedge elements, these faces are the two triangular faces (nodes 1-2-5) and (nodes 3-4-6).

The bond thickness is assumed to be the element size in the thickness direction. This may be overridden using BTHK. In this case the behavior becomes independent of the element thickness. The elastic stiffness is affected by BTHK, so it is necessary to set the characteristic element length to a smaller value

$$l_e^{\text{new}} = \sqrt{BTHK \times l_e^{\text{old}}} .$$



**Figure M169-1.** Figure illustrating the yield surface

This again affects the critical time step of the element, that is, a small BTHK can decrease the element time step significantly.

2. **Bond stiffness and strength.** In-plane stresses are set to zero: it is assumed that the stiffness and strength of the substrate are large compared with that of the adhesive, given the relative thicknesses.

If the substrate is modeled with shell elements, it is expected that these will lie at the mid-surface of the substrate geometry. Therefore, the solid elements representing the adhesive will be thicker than the actual bond. If the elastic compliance of the bond is significant, this can be corrected by increasing the elastic stiffness property  $E$ .

3. **Stress and failure.** The yield and failure surfaces are treated as a power-law combination of direct tension and shear across the bond:

$$\left(\frac{\sigma}{\sigma_{\max}}\right)^{\text{PWRT}} + \left(\frac{\tau}{\tau_{\max} - \text{SHT\_SL} \times \sigma}\right)^{\text{PWRS}} = 1.0$$

At yield SHT\_SL is the slope of the yield surface at  $\sigma = 0$ . See [Figure M169-1](#).

The stress-displacement curves for tension and shear are shown in [Figure M169-2](#). In both cases, GC is the area under the curve. The displacement to failure in tension is given by

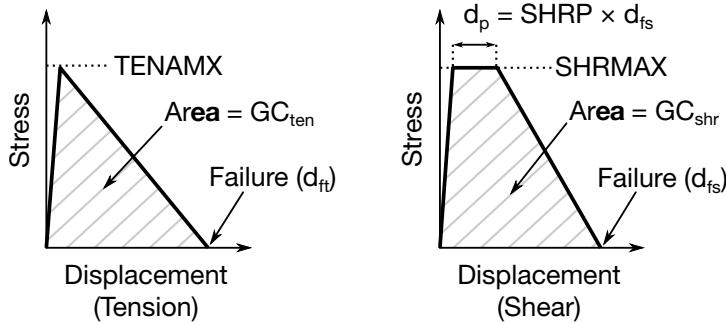
$$d_{ft} = 2 \left( \frac{GCTEN}{TENMAX} \right) ,$$

subject to a lower limit

$$d_{ft, \min} = \left( \frac{2L_0}{E'} \right) TENMAX ,$$

where  $L_0$  is the initial element thickness (or BTHK if used) and

$$E' = \frac{E(1-\nu)}{(1-2\nu)(1+\nu)} .$$



**Figure M169-2.** Stress-Displacement Curves for Tension and Shear

If GCTEN is input such that  $d_{ft} < d_{ft, min}$ , LS-DYNA will automatically increase GCTEN to make  $d_{ft} = d_{ft, min}$ . Therefore, GCTEN has a minimum value of

$$GCTEN \geq \frac{L_0}{E'} (\text{TENMAX})^2$$

Similarly, the minimum value for GCSHR is

$$GCSHR \geq \frac{L_0}{G} (\text{SHRMAX})^2$$

where  $G$  is the elastic shear modulus.

Because of the algorithm used, yielding in tension across the bond does not require strains in the plane of the bond – unlike the plasticity models, plastic flow is not treated as volume-conserving.

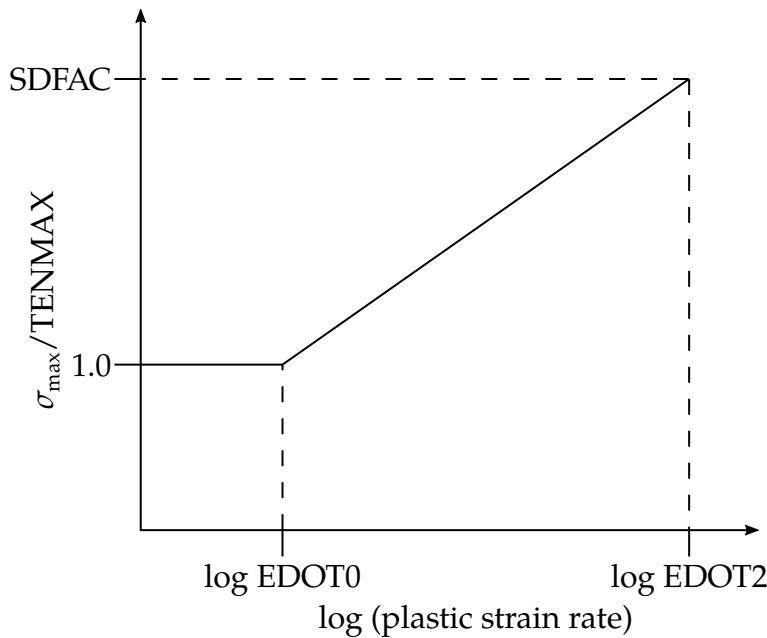
4. **Output variables.** The plastic strain output variable, PS, has a special meaning:

$0 < PS < 1$ : PS is the maximum value of the yield function experienced since time zero.

$1 < PS < 2$ : The element has yielded, and the strength is reducing towards failure – yields at PS = 1, fails at PS = 2.

Extra history variables may be requested for solid elements (NEIPH on \*DATABASE\_EXTENT\_BINARY). They are described in the following table.

History Variable #	Description
1	Damage caused by cohesive deformation on a scale of 0 at first yield to 1 at failure
2	Damage caused by interfacial deformation (see <a href="#">Remark 6</a> ) on a scale of 0 at first yield to 1 at failure
5	Current thickness dimension of the element
6	Current strain rate (relevant if EDOT0 and EDOT2 are defined, see <a href="#">Remark 5</a> )



**Figure M169-3.** Figure illustrating rate effects

History Variable #	Description
10	Direct stress in the local z-direction (bond tensile stress)
12	Through-thickness shear stress in the local yz-direction
13	Through-thickness shear stress in the local zx-direction

5. **Rate effects.** When the plastic strain rate rises above  $EDOT0$ , rate effects are assumed to scale with the logarithm of the plastic strain rate, as in the example shown in [Figure M169-3](#) for cohesive tensile strength with dynamic factor  $SDFAC$ . The same form of relationship is applied for the other dynamic factors. If  $EDOT0$  is zero or blank, no rate effects are applied. Rate effects are applied using the viscoplastic method.
6. **Interfacial failure.** Interfacial failure is assumed to arise from stress concentrations at the edges of the bond – typically the strength of the bond becomes almost independent of bond length. This type of failure is usually more brittle than cohesive failure. To simulate this, LS-DYNA identifies the free edges of the bond (made up of element faces that are not shared by other elements of material type \*MAT\_ARUP\_ADHESIVE, excluding the faces that bond to the substrate). Only these elements can fail initially. The neighbors of failed elements can then develop free edges and fail in turn.

In real adhesive bonds, the stresses at the edges can be concentrated over very small areas; in typical finite element models the elements are much too large to capture this. Therefore, the concentration of loads onto the edges of the bond is accomplished artificially by stiffening elements containing free edges (e.g.,

FACET, FACES > 1) and reducing the stiffness of interior elements (e.g., FACCT, FACCS < 1). Interior elements are allowed to yield at reduced loads (equivalent to  $TMAXE \times FACET/FACCT$  and  $SMAXE \times FACES/FACCS$ ) to prevent excessive stresses from developing before the edge elements have failed - but cannot be damaged until they become edge elements after the failure of their neighbors.

7. **Function arguments.** Parameters TENMAX, GCTEN, SHRMAX, GCSHR, SHRP, SDFAC, and SGFAC can be defined as negative values. In that case, the absolute values refer to \*DEFINE\_FUNCTION IDs. The arguments of those functions include several properties of both connection partners if corresponding solid elements are in a tied contact with shell elements.

These functions depend on:

(t1, t2) = thicknesses of both bond partners  
(sy1, sy2) = initial yield stresses at plastic strain of 0.002  
(sm1, sm2) = maximum engineering yield stresses (necking points)  
r = strain rate  
a = element area  
(e1, e2) = Young's moduli

For TENMAX = -100 such a function could look like:

```
*DEFINE_FUNCTION
  100
  func(t1,t2,sy1,sy2,sm1,sm2,r,a,e1,e2)=0.5*(sy1+sy2)
```

Since material parameters must be identified from both bond partners during initialization, this feature is only available for a subset of material models at the moment, namely material models 24, 36, 120, 123, 124, 251, and 258.

8. **Older versions.** Some corrections were made to this material model that can cause results to be different in R8 and later versions compared to R7.1.3 and earlier versions. To avoid recalibration of old material data, it is possible to recover previous results with option FBR713 = 1. The corrections were related to the post-yield stress-strain response not matching the description in the manual, with the difference being most noticeable when (a) the elastic stiffness was low, such that the elastic displacement to yield was of the same order as the element thickness; or (b) when the power-law terms PWRS, PWRT were not both equal to 2, and strain rate effects were specified (EDOT2, SDFAC).
9. **Volumetric smearing.** The element formulation given by ELFOM = 2 on \*SECTION\_SOLID smears the volumetric strain across the eight integration points. This smearing can sometimes cause an unstable dynamic response with \*MAT\_ARUP\_ADHESIVE. The smearing can be turned off by setting ELF2NS to 1. ELFNS has no effect for other element formulations or when FBR713 is nonzero.

## \*MAT\_170

## \*MAT\_RESULTANT\_ANISOTROPIC

### \*MAT\_RESULTANT\_ANISOTROPIC

This is Material Type 170. This model is available for the Belytschko-Tsay and the C0 triangular shell elements. It is based on a resultant stress formulation. In-plane behavior is treated separately from bending for modeling perforated materials, such as television shadow masks. The plastic behavior of each resultant is specified with a load curve and is completely uncoupled from the other resultants. If other shell formulations are specified, the formulation will be automatically switched to Belytschko-Tsay. As implemented, this material model cannot be used with user defined integration rules.

**NOTE:** This material does not support specification of a material angle,  $\beta_i$ , for each through-thickness integration point of a shell.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0						
Type	A	F						

Card 2	1	2	3	4	5	6	7	8
Variable	E11P	E22P	V12P	V21P	G12P	G23P	G31P	
Type	F	F	F	F	F	F	F	

Card 3	1	2	3	4	5	6	7	8
Variable	E11B	E22B	V12B	V21B	G12B	AOPT		
Type	F	F	F	F	F	F		

Card 4	1	2	3	4	5	6	7	8
Variable	LN11	LN22	LN12	LQ1	LQ2	LM11	LM22	LM12
Type	F	F	F	F	F	F	F	F

Card 5	1	2	3	4	5	6	7	8
Variable				A1	A2	A3		
Type				F	F	F		

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
MID	Material identification. A unique number or label must be specified.
RO	Mass density
E11P	$E_{11p}$ , for in-plane behavior
E22P	$E_{22p}$ , for in-plane behavior
V12P	$\nu_{12p}$ , for in-plane behavior
V11P	$\nu_{11p}$ , for in-plane behavior
G12P	$G_{12p}$ , for in-plane behavior
G23P	$G_{23p}$ , for in-plane behavior
G31P	$G_{31p}$ , for in-plane behavior
E11B	$E_{11b}$ , for bending behavior
E22B	$E_{22b}$ , for bending behavior
V12B	$\nu_{12b}$ , for bending behavior
V21B	$\nu_{21b}$ , for bending behavior
G12B	$G_{12b}$ , for bending behavior

VARIABLE	DESCRIPTION
AOPT	Material axes option (see MAT_OPTIONTROPIC_ELASTIC for a more complete description):  EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA.  EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.  EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector <b>v</b> with the element normal.  LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR).
LN11	Yield curve ID for $N_{11}$ , the in-plane force resultant
LN22	Yield curve ID for $N_{22}$ , the in-plane force resultant
LN12	Yield curve ID for $N_{12}$ , the in-plane force resultant
LQ1	Yield curve ID for $Q_1$ , the transverse shear resultant
LQ2	Yield curve ID for $Q_2$ , the transverse shear resultant
LM11	Yield curve ID for $M_{11}$ , the moment
LM22	Yield curve ID for $M_{22}$ , the moment
LM12	Yield curve ID for $M_{12}$ , the moment
A1, A2, A3	$(a_1, a_2, a_3)$ , components of vector <b>a</b> for AOPT = 2
V1, V2, V3	$(v_1, v_2, v_3)$ , components of vector <b>v</b> for AOPT = 3
D1, D2, D3	$(d_1, d_2, d_3)$ , components of vector <b>d</b> for AOPT = 2
BETA	Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA.

**Remarks:**

The in-plane elastic matrix for in-plane, plane stress behavior is given by:

$$\mathbf{C}_{\text{in plane}} = \begin{bmatrix} Q_{11p} & Q_{12p} & 0 & 0 & 0 \\ Q_{12p} & Q_{22p} & 0 & 0 & 0 \\ 0 & 0 & Q_{44p} & 0 & 0 \\ 0 & 0 & 0 & Q_{55p} & 0 \\ 0 & 0 & 0 & 0 & Q_{66p} \end{bmatrix}$$

The terms  $Q_{ijp}$  are defined as:

$$Q_{11p} = \frac{E_{11p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{22p} = \frac{E_{22p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{12p} = \frac{\nu_{12p}E_{11p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{44p} = G_{12p}$$

$$Q_{55p} = G_{23p}$$

$$Q_{66p} = G_{31p}$$

The elastic matrix for bending behavior is given by:

$$\mathbf{C}_{\text{bending}} = \begin{bmatrix} Q_{11b} & Q_{12b} & 0 \\ Q_{12b} & Q_{22b} & 0 \\ 0 & 0 & Q_{44b} \end{bmatrix}$$

The terms  $Q_{ijp}$  are similarly defined.

Because this is a resultant formulation, no stresses are output to **d3plot**, and forces and moments are reported to **elout** in place of stresses.

**\*MAT\_171****\*MAT\_STEEL\_CONCENTRIC\_BRACE****\*MAT\_STEEL\_CONCENTRIC\_BRACE**

This is Material Type 171. It represents the cyclic buckling and tensile yielding behavior of steel braces and is intended primarily for seismic analysis. Use only for beam elements with ELFORM = 2 (Belytschko-Schwer beam).

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	YM	PR	SIGY	LAMDA	FBUCK	FBUCK2
Type	A	F	F	F	F	F	F	F
Default	none	none	none	none	none	optional	optional	0.0

Card 2	1	2	3	4	5	6	7	8
Variable	CCBRF	BCUR	EPTCRIT	DAMF1	DAMF2	DAMEP1	DAMEP2	
Type	F	F	F	F	F	F	F	
Default	optional	optional	0.01	optional	optional	optional	optional	

Card 3	1	2	3	4	5	6	7	8
Variable	TS1	TS2	TS3	TS4	CS1	CS2	CS3	CS4
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	TS1	TS2	TS3	TS4

**VARIABLE****DESCRIPTION**

MID              Material identification. A unique number or label must be specified (see \*PART).

RO              Mass density

YM              Young's modulus

<b>VARIABLE</b>	<b>DESCRIPTION</b>
PR	Poisson's ratio
SIGY	Yield stress
LAMDA	Slenderness ratio, $\lambda$ (optional – see remarks)
FBUCK	Initial buckling load (optional – see remarks. If used, should be positive)
FBUCK2	Optional extra term in initial buckling load – see remarks
CCBRF	Reduction factor on initial buckling load for cyclic behavior
BCUR	Optional load curve giving compressive buckling load ( $y$ -axis) as a function of compressive strain ( $x$ -axis - both positive)
EPTCRIT	Tensile plastic strain to reduce buckling strength to cyclic value
DAMF1	FEMA threshold at which damage begins (see <a href="#">Remark 5</a> ). EQ.0: No damage or failure based on FEMA thresholds
DAMF2	FEMA threshold at which element is eroded, applicable only if DAMF1 > 0
DAMEP1	Cumulative plastic strain at which damage begins (see <a href="#">Remark 5</a> ). EQ.0: No damage or failure based on plastic strain
DAMEP2	Cumulative plastic strain at which element is eroded, applicable only if DAMEP1 > 0
TS1 - TS4	Tensile axial strain FEMA thresholds 1 to 4 (see <a href="#">Remark 3</a> )
CS1 - CS4	Compressive axial strain FEMA thresholds 1 to 4 (see <a href="#">Remark 3</a> )

**Remarks:**

1. **General.** The brace element is intended to represent the buckling, yielding and cyclic behavior of steel elements, such as tubes or I-sections, that carry only axial loads. A single beam element should be used to represent each structural element. Empirical relationships are used to determine the buckling and cyclic load-deflection behavior. Details of the axial response are given after the Remarks.

2. **Strain Definitions.** Output variables, and the damage and failure treatment, refer to the following strain definitions, all of which relate to the axial direction of the beam element:
  - a) Total strain: Change of length divided by initial length, positive in tension.
  - b) Plastic strain: Current inelastic strain, defined as total strain minus elastic strain. It is positive for tensile strains, negative for compressive strains. The term "plastic strain" is used here irrespective of whether the inelastic behavior represents yielding or buckling.
  - c) High-tide plastic strain: Maximum plastic strain that has occurred during the analysis. Separate values are recorded for tensile and compressive plastic strains.
  - d) Cumulative plastic strain: The sum of the absolute values of the plastic strain increments. The cumulative plastic strain increases whenever yielding or buckling occurs. For cyclic loading in the plastic or buckling regimes, this strain measure increases with each cycle.
3. **FEMA Thresholds.** FEMA thresholds are used in performance-based earthquake engineering to classify the response into categories such as "Elastic", "Immediate Occupancy", "Life Safe", etc., according to the level of deformation of each structural element. During the analysis, the maximum high-tide tensile and compressive plastic strains are recorded. These are checked against the user-defined limits TS1 to TS4 and CS1 to CS4. The output flag is then set to 0, 1, 2, 3, or 4 according to which limits have been passed. The value in the output files is the highest such flag from tensile or compressive strains.
4. **Output.** In addition to the six resultants written for all beam elements, this material model writes further extra history variables to the d3plot and d3thdt files, given in the table below. The data is written in the same position in these files as where integrated beams write the stresses and strains at integration points requested by BEAMIP on \*DATABASE\_EXTENT\_BINARY. Therefore, some post-processors may interpret this data as if the elements were integrated beams with 4 integration points, and in that case the data may be accessed by selecting the appropriate integration point and data component:

Int. Point	Data component in post-processor	Description
1	XX(RR) axial stress	Total axial deformation/strain
3	ZX(TR) shear stress	Internal energy

Int. Point	Data component in post-processor	Description
4	XX(RR) axial stress	Current buckling/yield force in compression
4	XY(RS) shear stress	Tensile high-tide plastic strain
4	ZX(TR) shear stress	Compressive high-tide plastic strain
4	Equivalent plastic strain	Cumulative plastic strain
4	XX(RR) axial strain	FEMA flag

5. **Damage and failure.** Optionally, damage and failure (element erosion) can be modelled. DAMF1 and DAMF2 control damage and failure based on high tide plastic strains. DAMEP1 and DAMEP2 control damage and failure based on cumulative plastic strain (fatigue-type damage). A combination of both of the above is obtained if all four input parameters are defined.

DAMF1 and DAMF2 refer to the “FEMA” output flag (see [Remark 3](#) above). Only integer values 0, 1, 2, 3 or 4 are meaningful because those are the possible values of the FEMA output flag. DAMEP1 and DAMEP2 refer to values of cumulative plastic strain.

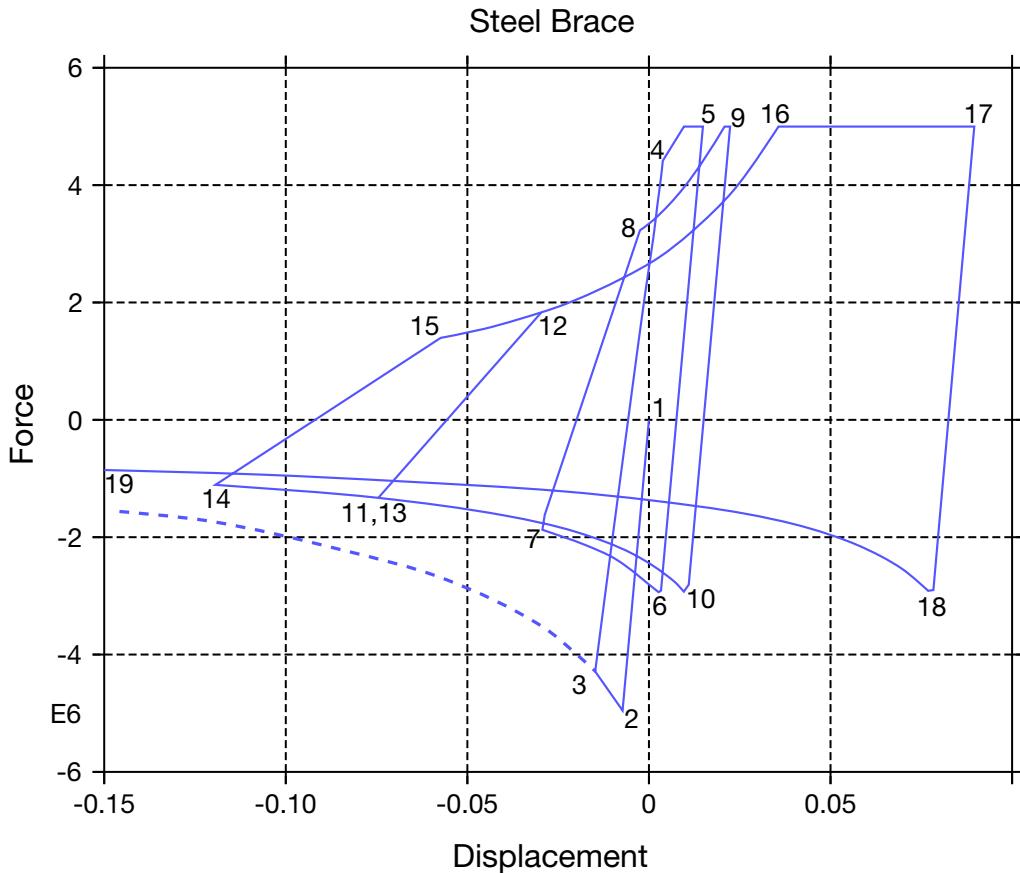
Damage is modelled with a scaling factor,  $D$ , that multiplies the stiffness and strength of the element. DAMF1 and DAMEP1 define thresholds at which damage begins. Until that point is reached, the damage algorithm has no effect and  $D = 1$ . DAMF2 and DAMEP2 define the threshold at which damage is complete. At that point,  $D = 0$ , meaning the element has no remaining stiffness or strength and is deleted. Between DAMF1 and DAMF2 and between DAMEP1 and DAMEP2,  $D$  ramps down linearly from  $D = 1.0$ , when damage begins, to  $D = 0.0$ , when damage is complete.

If both damage mechanisms are modelled (i.e., DAMF1, DAMF2, DAMEP1, DAMEP2 are all nonzero), the damage scaling factors for the two mechanisms are multiplied together. Thus, damage and failure can occur by either mechanism depending on which thresholds are reached first.

### Axial response:

The cyclic behavior is shown in [Figure M171-1](#) (compression shown as negative force and displacement). The initial buckling load (point 2) is:

$$F_{b, \text{initial}} = \text{FBUCK} + \frac{\text{FBUCK2}}{L^2},$$

**Figure M171-1.** Cyclic Behavior of a Steel Brace

where FBUCK and FBUCK2 are input parameters, and  $L$  is the length of the beam element. If neither FBUCK nor FBUCK2 is defined, the default is that the initial buckling load is

$$\text{SIGY} \times A,$$

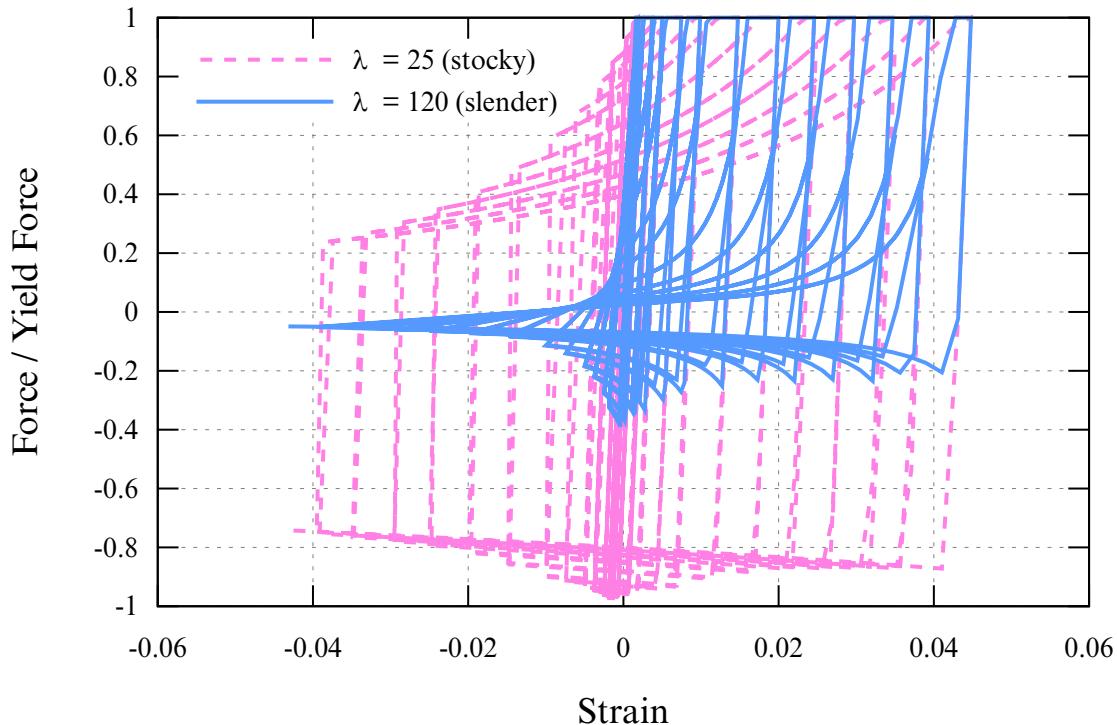
where  $A$  is the cross sectional area. The buckling curve (shown dashed) has the form:

$$F(d) = \frac{F_{b, \text{initial}}}{\sqrt{A\delta + B}}$$

where  $\delta$  is |strain/yield strain|, and  $A$  and  $B$  are internally calculated functions of slenderness ratio ( $\lambda$ ) and loading history.

The member slenderness ratio,  $\lambda$ , is defined as  $\frac{kL}{r}$ , where  $k$  depends on end conditions,  $L$  is the element length, and  $r$  is the radius of gyration such that  $Ar^2 = I$  (and  $I = \min(I_{yy}, I_{zz})$ );  $\lambda$  will by default be calculated from the section properties and element length using  $k = 1$ . Optionally, this may be overridden by input parameter LAMDA to allow for different end conditions.

Optionally, you may provide a buckling curve BCUR. The points of the curve give compressive displacement (x-axis) as a function of force (y-axis); the first point should have



**Figure M171-2.** Comparing the stress-strain response for two values of  $\lambda$

zero displacement and the initial buckling force. Displacement and force should both be positive. The initial buckling force must not be greater than the yield force.

The tensile yield force (point 5 and segment 16-17 in [Figure M171-1](#)) is defined by

$$F_y = \text{SIGY} \times A,$$

where yield stress SIGY is an input parameter and  $A$  is the cross-sectional area.

Following initial buckling and subsequent yield in tension, the member is assumed to be damaged. The initial buckling curve is then scaled by input parameter CCBRF, leading to reduced strength curves such as segments 6-7, 10-14 and 18-19. This reduction factor is typically in the range 0.6 to 1.0 (smaller values for more slender members). By default, CCBRF is calculated using SEAOC 1990:

$$\text{CCBRF} = \frac{1}{\left(1 + \frac{0.5\lambda}{\pi\sqrt{\frac{E}{0.5\sigma_y}}}\right)}$$

When tensile loading is applied after buckling, the member must first be straightened before the full tensile yield force can be developed. This is represented by a reduced unloading stiffness (such as segment 14-15) and the tensile reloading curve (segments 8-9 and 15-16). Further details can be found in Bruneau, Uang, and Whittaker [1998] and Structural Engineers Association of California [1974, 1990, 1996].

## **\*MAT\_171**

## **\*MAT\_STEEL\_CONCENTRIC\_BRACE**

The response of stocky (low  $\lambda$ ) and slender (high  $\lambda$ ) braces are compared in [Figure M171-2](#). These differences are achieved by altering the input value LAMDA (or the section properties of the beam) and FBUCK.

**\*MAT\_CONCRETE\_EC2**

This is Material Type 172. This model is available for shell, thick shell (formulations 1, 2, and 6), and Hughes-Liu beam elements. The material model can represent plain concrete only, reinforcing steel only, or a smeared combination of concrete and reinforcement. The model includes concrete cracking in tension and crushing in compression and reinforcement yield, hardening, and failure. Properties are thermally sensitive; the material model can be used for fire analysis. Material data and equations governing the behavior (including thermal properties) are taken from Eurocode 2 (EC2). See the remarks below for more details on how the standard is applied in the material model.

Although the material model offers many options, a reasonable response may be obtained by entering only RO, FC, and FT for plain concrete. If reinforcement is present, YMREINF, SUREINF, FRACRX, and FRACRY must be defined, or for an alternative way to model the reinforcement, see [\\*MAT\\_203/\\*MAT\\_HYSTERETIC\\_REINFORCEMENT](#). Note that, from release R10 onwards, the number of possible cracks has been increased from 2 (0 and 90 degrees) to 4 (see TYPEC on Card 1 and Tensile response under the [Material Behavior of Concrete](#) section).

**NOTE:** This material does not support the specification of a material angle,  $\beta_i$ , for each through-thickness integration point of a shell (ICOMP = 1 on \*SECTION\_SHELL, Bi on \*PART\_COMPOSITE or Bi on \*ELEMENT\_SHELL\_COMPOSITE).

**Card Summary:**

**Card 1.** This card is required.

MID	RO	FC	FT	TYPEC	UNITC	ECUTEN	FCC

**Card 2.** This card is required.

ESOFT	LCHAR	MU	TAUMXF	TAUMXC	ECRAGG	AGGSZ	UNITL

**Card 3.** This card is required.

YMREINF	PRREINF	SUREINF	TYPER	FRACRX	FRACRY	LCRSU	LCALPS

**Card 4.** This card is required.

AOPT	ET36	PRT36	ECUT36	LCALPC	DEGRAD	ISHCHK	UNLFAC

**\*MAT\_172****\*MAT\_CONCRETE\_EC2**

**Card 5.** Include this card if AOPT > 0.0.

XP	YP	ZP	A1	A2	A3		
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**Card 6.** Include this card if AOPT > 0.0.

V1	V2	V3	D1	D2	D3	BETA	
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**Card 7.** Include this card if ISHCHK ≠ 0.

TYPESC	P_OR_F	EFFD	GAMSC	ERODET	ERODEC	ERODER	TMPOFF
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**Card 8.** Include this card if TYPEC = 6 or 9.

EC1_6	ECSP69	GAMCE9	PHIEF9				
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**Card 9.** Include this card if FT < 0.0.

FT2	FTSHR	LCFTT	WRO_G	ZSURF	LCFIB		
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	FC	FT	TYPEC	UNITC	ECUTEN	FCC
Type	A	F	F	F	F	F	F	F
Default	none	none	none	0.0	1.0	1.0	0.0025	↓

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
FC	Compressive strength of concrete (stress units). Its meaning depends on TYPEC. TYPEC.EQ.1,2,3,4,5,7,8: FC is the actual compressive strength. TYPEC.EQ.6: FC is the unconfined compressive strength used in Mander equations.

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	TYPEC.EQ.9: FC is the characteristic compressive strength ( $f_{ck}$ in EC2 1-1). See also FCC and the remarks below.
FT	Tensile stress to cause cracking. If FT < 0.0, then Card 9 is read.
TYPEC	Concrete aggregate type for stress-strain-temperature relationships (see <a href="#">Remark 3</a> ): EQ.1.0: Siliceous (default), Draft EC2 Annex (fire engineering) EQ.2.0: Calcareous, Draft EC2 Annex (fire engineering) EQ.3.0: Non-thermally-sensitive using ET3, ECU3 EQ.4.0: Lightweight EQ.5.0: Fiber-reinforced EQ.6.0: Non-thermally-sensitive, Mander algorithm EQ.7.0: Siliceous, EC2 1-2:2004 (fire engineering) EQ.8.0: Calcareous, EC2 1-2:2004 (fire engineering) EQ.9.0: EC2 1-1:2004 (general and buildings)  To obtain the pre-R10 behavior, that is, a maximum of 2 cracks, add 100 to TYPEC. For example, 109 means two cracks and EC2 1-1:2004 (general and buildings).
UNITC	Factor to convert stress units to MPa (used in shear capacity checks and for application of EC2 formulae when TYPEC = 9). For example, if the model units are Newtons and meters, UNITC = $10^{-6}$ .
ECUTEN	Strain to fully open a crack
FCC	Relevant only if TYPEC = 6 or 9.  TYPEC.EQ.6: FCC is the compressive strength of confined concrete used in Mander equations. Default: unconfined properties are assumed (FCC = FC).  TYPEC.EQ.9: FCC is the actual compressive strength. If blank, this will be set equal to the mean compressive strength ( $f_{cm}$ in EC2 1-1) as required for serviceability calculations (8MPa greater than FC). For ultimate load calculations, you can set FCC to a factored characteristic compressive strength. See remarks below.

**\*MAT\_172****\*MAT\_CONCRETE\_EC2**

Card 2	1	2	3	4	5	6	7	8
Variable	ESOFT	LCHAR	MU	TAUMXF	TAUMXC	ECRAGG	AGGSZ	UNITL
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.4	$10^{20}$	$1.161 \times FT$	0.001	0.0	1.0

VARIABLE	DESCRIPTION
ESOFT	Tension stiffening (slope of stress-strain curve post-cracking in tension). See <a href="#">Figure M172-1</a> .
LCHAR	Characteristic length at which ESOFT applies. It is also used as crack spacing in aggregate-interlock calculations.
MU	Friction on crack planes (max shear = $\mu \times$ compressive stress)
TAUMXF	Maximum friction shear stress on crack planes (ignored if AG-GSZ > 0.0 - see remarks).
TAUMXC	Maximum through-thickness shear stress after cracking (see remarks).
ECRAGG	Strain parameter for aggregate interlock (ignored if AGGSZ > 0.0 - see remarks).
AGGSZ	Aggregate size (length units - used in NS3473 aggregate interlock formula - see remarks).
UNITL	Factor to convert length units to millimeters (used only if AG-GSZ > 0.0 - see remarks). For example, if the model unit is meters, UNITL = 1000.

Card 3	1	2	3	4	5	6	7	8
Variable	YMREINF	PRREINF	SUREINF	TYPER	FRACRX	FRACRY	LCRSU	LCALPS
Type	F	F	F	F	F	F	I	I
Default	none	0.0	0.0	2.0	0.0	0.0	0	0

VARIABLE	DESCRIPTION
YMREINF	Young's modulus of reinforcement
PRREINF	Poisson's ratio of reinforcement
SUREINF	Ultimate stress of reinforcement
TYPER	Type of reinforcement for stress-strain-temperature relationships (see <a href="#">Remark 3</a> ):  EQ.1.0: Hot rolled reinforcing steel, Draft EC2 Annex (fire) EQ.2.0: Cold worked reinforcing steel (default), Draft EC2 Annex (fire) EQ.3.0: Quenched/tempered prestressing steel, Draft EC2 Annex (fire) EQ.4.0: Cold worked prestressing steel, Draft EC2 Annex (fire) EQ.5.0: Non-thermally sensitive using load curve LCRSU EQ.7.0: Hot rolled reinforcing steel, EC2 1-2:2004 (fire) EQ.8.0: Cold worked reinforcing steel, EC2 1-2:2004 (fire)
FRACRX	Fraction of reinforcement ( $x$ -axis). For example, to obtain 1% reinforcement set FRACRX = 0.01. See <a href="#">Remark 1</a> .
FRACRY	Fraction of reinforcement ( $y$ -axis). For example, to obtain 1% reinforcement set FRACRY = 0.01. See <a href="#">Remark 1</a> .
LCRSU	Load curve for TYPER = 5 giving non-dimensional factor on SUREINF as a function of plastic strain (overrides stress-strain function from EC2).
LCALPS	Optional load curve giving thermal expansion coefficient of reinforcement as a function of temperature (overrides function from EC2).

Card 4	1	2	3	4	5	6	7	8
Variable	AOPT	ET36	PRT36	ECUT36	LCALPC	DEGRAD	ISHCHK	UNLFAC
Type	F	F	F	F	I	F	I	F
Default	0.0	0.0	0.25	↓	none	0.0	0	0.5

VARIABLE	DESCRIPTION
AOPT	Material axes option (see *MAT_002 for a more complete description):  EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.  EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.  EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal.
ET36	Young's modulus of concrete (TYPEC = 3 and 6). For other values of TYPEC, the Young's modulus is calculated internally (see remarks).
PRT36	Poisson's ratio of concrete. Applies to all values of TYPEC.
ECUT36	Strain to failure of concrete in compression (TYPEC = 3 and 6). See "Compressive response..." in the <a href="#">Material Behavior of Concrete</a> section below. Default is 0.02 for TYPEC = 3 and $1.1 \times EC1\_6$ for TYPEC = 6.
LCALPC	Optional load curve giving thermal expansion coefficient of concrete as a function of temperature – overrides relationship from EC2.
DEGRAD	If non-zero, the compressive strength of concrete parallel to an open crack will be reduced (see remarks).

<b>VARIABLE</b>	<b>DESCRIPTION</b>
ISHCHK	Set this flag to 1 to include Card 7 (shear capacity check and other optional input data).
UNLFAC	Stiffness degradation factor after crushing (0.0 to 1.0 – see <a href="#">Figure M172-4</a> ).

Additional card for AOPT &gt; 0.0.

Card 5	1	2	3	4	5	6	7	8
Variable	XP	YP	ZP	A1	A2	A3		
Type	F	F	F	F	F	F		
Default	0.0	0.0	0.0	0.0	0.0	0.0		

<b>VARIABLE</b>	<b>DESCRIPTION</b>
XP, YP, ZP	Not used
A1, A2, A3	Components of vector <b>a</b> for AOPT = 2.0

Additional card for AOPT &gt; 0.

Card 6	1	2	3	4	5	6	7	8
Variable	V1	V2	V3	D1	D2	D3	BETA	
Type	F	F	F	F	F	F	F	
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

<b>VARIABLE</b>	<b>DESCRIPTION</b>
V1, V2, V3	Components of vector <b>v</b> for AOPT = 3.0
D1, D2, D3	Components of vector <b>d</b> for AOPT = 2.0
BETA	Material angle in degrees for AOPT = 3.0. BETA may be overridden on the element card; see <b>*ELEMENT_SHELL_BETA</b>

**\*MAT\_172****\*MAT\_CONCRETE\_EC2**

Include if ISHCHK ≠ 0.

Card 7	1	2	3	4	5	6	7	8
Variable	TYPESC	P_OR_F	EFFD	GAMSC	ERODET	ERODEC	ERODER	TMPOFF
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	2.0	0.01	0.05	0.0

VARIABLE	DESCRIPTION
TYPESC	Type of shear capacity check (see <a href="#">Remark 5</a> ): EQ.1: BS 8110, no failure even if capacity is exceeded EQ.2: ACI 318-05M, no failure even if capacity is exceeded EQ.11: BS 8110, failure occurs if capacity is exceeded EQ.12: ACI 318-05M, failure occurs if capacity is exceeded
P_OR_F	If BS8110 shear check, percent reinforcement – for example, if 0.5%, input 0.5. If ACI shear check, ratio (cylinder strength/FC) - defaults to 1.
EFFD	Effective section depth (length units) used in the shear capacity check. This is usually the section depth excluding the cover concrete.
GAMSC	Load factor used in BS8110 shear capacity check
ERODET	Crack-opening strain at which element is deleted; see <a href="#">Remark 7</a> .
ERODEC	Compressive strain used in erosion criteria; see <a href="#">Remark 7</a> .
ERODER	Reinforcement plastic strain used in erosion criteria; see <a href="#">Remark 7</a> .
TMPOFF	Constant to be added to the model's temperature unit to convert into degrees Celsius. For example, if the model's temperature unit is degrees Kelvin, set TMPOFF to -273. Degrees Celsius temperatures are then used throughout the material model for LCALPC and the default thermally-sensitive properties.

Additional card for TYPEC = 6 or 9.

Card 8	1	2	3	4	5	6	7	8
Variable	EC1_6	ECSP69	GAMCE9	PHIEF9				
Type	F	F	F	F				
Default	see remarks	see remarks	0.0	0.0				

VARIABLE	DESCRIPTION
EC1_6	Strain at maximum compressive stress for Type 6 concrete
ECSP69	Spalling strain in compression for TYPEC = 6 and 9
GAMCE9	Material factor that divides the Youngs Modulus (TYPEC = 9)
PHIEF9	Effective creep ratio (TYPEC = 9)

Define this card only if FT < 0.0.

Card 9	1	2	3	4	5	6	7	8
Variable	FT2	FTSHR	LCFTT	WRO_G	ZSURF	LCFIB		
Type	F	F	F	F	F	I		
Default	FT	FT2	0.0	0.0	0.0	0		

VARIABLE	DESCRIPTION
FT2	Tensile strength used for calculating tensile response
FTSHR	Tensile strength used for calculating post-crack shear response
LCFTT	Load curve defining factor on tensile strength as a function of time
WRO_G	Density times gravity for water pressure in cracks
ZSURF	Z-coordinate of water surface (for water pressure in cracks)

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCFIB	Optional load curve defining the tensile response. It is intended for fiber-reinforced concrete. The $x$ -axis of the curve is tensile strain. The $y$ -axis of the curve is a non-dimensional scale factor on the tensile strength FT2. If defined, this curve overrides ECUTEN.

**Remarks:**

1. **Material types.** This material model can be used to represent unreinforced concrete ( $\text{FRACR} = 0.0$  where  $\text{FRACR} = \max(\text{FRACX}, \text{FRACY})$ ), reinforcing steel ( $\text{FRACR} = 1.0$ ), or a smeared combination of reinforced concrete with evenly distributed reinforcement ( $0.0 < \text{FRACR} < 1.0$ ). Concrete is modeled as an initially isotropic material with a non-rotating smeared crack approach in tension, together with a plasticity model for compressive loading. Reinforcement is treated as separate sets of bars in the local material  $x$ - and  $y$ -axes. The reinforcement is assumed not to carry through-thickness shear or in-plane shear. Therefore, this material model should not be used to model steel-only sections; that is, do not create a section in which all the integration points are of \*MAT\_172 with both FRACRX and FRACRY set to 1.0.
2. **Creating reinforced concrete sections.** Reinforced concrete sections for shell or beam elements may be created using \*PART\_COMPOSITE (for shells) or \*INTEGRATION\_BEAM (for beams). Create one material definition representing the concrete using \*MAT\_CONCRETE\_EC2 with  $\text{FRACR} = 0.0$ . Create another material definition representing the reinforcement using \*MAT\_CONCRETE\_EC2 with  $\text{FRACRX}$  and/or  $\text{FRACRY} = 1.0$ . The material ID of each integration point is then set to represent either concrete or steel. The position of each integration point within the cross-section and its cross-sectional area are chosen to represent the actual distribution of reinforcement. If desired, \*MAT\_HYSTERETIC\_REINFORCEMENT can be used for the reinforcement layers instead of \*MAT\_CONCRETE\_EC2.
3. **Eurocode 2.** Eurocode 2 (EC2) contains different sections applicable to general structural engineering versus fire engineering. The latter contains different data for different types of concrete and steel and has been revised during its history. TYPEC and TYPER control the version and section of the EC2 document from which the material data is taken and the types of concrete and steel represented. In the descriptions of TYPEC and TYPER above, "Draft EC2 Annex (fire engineering)" means data taken from the 1995 draft Eurocode 2 Part 1-2 (for fire engineering), ENV 1992-1-2:1995. These defaults are suitable for general use where elevated temperatures are not considered.

EC2 was then issued in 2004 (described above as EC2 1-2:2004 (fire)) with revised stress-strain data at elevated temperatures (TYPEC and TYPER = 7 or 8). These settings are recommended for analyses with elevated temperatures.

Meanwhile, Eurocode 2 Part 1-1 (for general structural engineering), EC2 1-1:2004, contains material data and formulae that differ from Part 1-2; these are obtained by setting TYPEC = 9. This setting is recommended where compatibility is required with the structural engineering data and assumptions of Part 1-1 of the Eurocode.

A further option for modeling concrete, TYPEC = 6, is provided for applications, such as seismic engineering, in which the different stress-strain behaviors of confined versus unconfined concrete must be captured. This option uses equations by Mander et al. and does not relate directly to Eurocode 2.

4. **Local material axes.** The local material axes define the directions of the reinforcement bars. If the reinforcement directions are inconsistent across neighboring elements, the response may be less stiff than intended – this is equivalent to the bars being bent at the element boundaries. Local material axes default to the same as the element axes, with the local  $x$ -direction pointing from Node 1 to Node 2. The local material axes can be controlled using the angle BETA on \*ELEMENT\_SHELL\_BETA or AOPT and associated input parameters in the material definition. See material type 2 for a description of the different AOPT settings.

Only the reinforcement response depends on the local material axes, not the concrete response. Therefore, it is not usually necessary to set the local material axes for material definitions that do not have reinforcement (i.e., FRACRX = 0 and FRACRY = 0). However, when a reinforced concrete section is defined using \*PART\_COMPOSITE, and either the shear capacity check is invoked (TYPESC > 0, see [Remark 5](#)) or CMPFLG is set on \*DATABASE\_EXTENT\_BINARY, all layers in the \*PART\_COMPOSITE definition need to have identical material axes. This can be achieved by using the BETA angle on \*ELEMENT\_SHELL\_BETA or inputting identical AOPT parameters for all the material definitions referenced by the \*PART\_COMPOSITE card.

5. **Through-thickness shear.** In this material model, cracks are initiated only by in-plane stresses caused by axial and bending effects. Once a crack has formed, the through-thickness shear stress is limited by considerations of aggregate interlock or friction on the crack surfaces. If the in-plane stresses are insufficient to cause cracks, the through-thickness shear strength is, by default, unlimited. Thus, failures caused primarily by through-thickness shear will not be predicted. The Shear Capacity Check option (see TYPESC) allows you to address this limitation. Two classes of behavior are available (described in a and b),

together with different options for the calculation of shear capacity (described in c and d):

- a) TYPESC < 10 represents the situation where sufficient shear reinforcement will be provided to prevent any through-thickness failure. LS-DYNA generates extra history variables, so you can compare the shear demand to shear capacity to assess the requirements for shear reinforcement (see the list of additional history variables in [Remark 8](#)). Furthermore, we assume that the shear reinforcement prevents inelastic through-thickness shear deformation. Thus, through-thickness slipping on crack planes is automatically disabled.
- b) TYPESC > 10 represents the situation where no shear reinforcement is provided. A brittle failure occurs if the shear capacity is exceeded. Note, however, that the shear capacity is calculated from equations in design codes and may be quite conservative.
- c) If TYPESC = 1 or 11, the shear capacity calculation is based on BS 8110-1:1997. These values of TYPESC require supplying the percentage reinforcement (P\_OR\_F), the effective depth of section EFD (this typically excludes the cover concrete), and the load factor GAMSC. These are used in Table 3.8 of BS 8110-1:1997 to determine the design shear stress. The “shear capacity” is this design shear stress times the total section thickness (force per unit width), modified according to Equation 6b of BS 8110 to allow axial load.
- d) If TYPESC = 2 or 12, the shear capacity calculation is based on ACI 318-05M. The shear capacity,  $\emptyset V_n$ , is calculated assuming  $\emptyset = 0.75$  and taking  $V_n$  from ACI 318-05M equations 11-4 (for compressive axial load) or 11-8 (for tensile axial load). In these equations,  $f'_c$  is taken as P\_OR\_F × FC,  $d$  as EFD, and  $b_w$  as 1 to give shear capacity as a force per unit width. Note that in LS-DYNA versions prior to R13, Equation 11-4 was incorrectly implemented, so results from the TYPESC = 2 check before R13 should not be used.

The “shear demand” (actual shear force per unit width) is compared to the shear capacity for all the above options. This process is performed separately for each element’s two local reinforcement directions. When defining sections using \*PART\_COMPOSITE or integration rules with multiple sets of material properties, each set of material properties referenced must have the same local material axes (see [Remark 4](#)). The shear demand and axial load (used in calculating the shear capacity) are summed across the integration points within the section. The extra history variables for capacity, demand, and the difference between capacity and demand relate to the whole section (not each integration point separately). Thus, the same values are written to all the integration points within an element.

6. **Thermal expansion.** By default, thermal expansion properties from EC2 are used. If no temperatures are defined in the model, properties for 20°C are used. For TYPEC = 3, 6, or 9, and TYPER = 5, there is no thermal expansion by default, and the properties do not vary with temperature. Defining curves of thermal expansion coefficient as a function of temperature (LCALPC, LCALPR) overrides the default thermal expansion behavior. These apply no matter the selected types of TYPEC and TYPER.
7. **Erosion criteria.** Elements are deleted from the calculation when all their integration points have reached the erosion criteria. Because this material model can represent plain concrete without reinforcement, pure reinforcement without concrete, or a smeared combination, the criteria depend on the modeled type (see [Remark 1](#)). There are three criteria:
  - a) *Concrete Tensile Strain Limit.* The concrete tensile (crack-opening) strain limit ERODET has been exceeded.
  - b) *Concrete Compressive Strain Limit.* The concrete compressive strain limit ERODEC +  $\varepsilon_{csp}$  has been exceeded.  $\varepsilon_{csp}$  is the strain at which the stress-strain relation falls to zero.
  - c) *Reinforcement Strain Limit.* The reinforcement strain limit ERODER +  $\varepsilon_{rsp}$  has been exceeded.  $\varepsilon_{rsp}$  is the strain at which the stress-strain relation falls to zero. However, if LCRSU > 0,  $\varepsilon_{rsp}$  is assumed to be 2.0.

The table below indicates which criteria apply to each of the variations of material type. Note that FRACR = max(FRACX, FRACY) as discussed in [Remark 1](#).

FRACR	Material Type	Erosion Criteria	Erosion Criteria (in plain English)
FRACR = 0.0	Pure concrete	(a).OR.(b)	The concrete tensile strain limit or concrete compressive strain limit conditions are satisfied.
0.0 < FRACR < 1.0	Smeared combination	((a).OR.(b)).AND.(c)	The reinforcement strain limit and either the concrete tensile strain limit or the concrete compressive strain limit are satisfied.
FRACR = 1.0	Pure steel reinforcement	(c)	The reinforcement strain limit is satisfied.

If both FRACRX and FRACRY are nonzero, the reinforcement erosion criterion is applied as follows: in LS-DYNA versions up to and including R14, the

reinforcement erosion criterion must be met in both local directions ( $X$  and  $Y$ ) before erosion occurs. In versions from R15 onwards, erosion occurs when either direction  $X$  or direction  $Y$  reaches the erosion criterion. The R14 treatment had the counterintuitive side-effect of preventing erosion of elements under large uniaxial strains because the reinforcement in the low-strain direction had not reached its erosion limit.

8. **Output.** “Plastic Strain” is the maximum of the plastic strains in the reinforcement in the two local directions.

Extra history variables may be requested for shell elements (NEIPS on \*DATABASE\_EXTENT\_BINARY). They are described in the following table.

History Variable #	Description
1	Current crack opening strain (if two cracks are present, max of two)
2	Equivalent uniaxial strain for concrete compressive behavior
3	Number of cracks (0, 1, 2, 3, or 4)
4	Temperature
5	Thermal strain
6	Current crack opening strain for the first crack to form
7	Current crack opening strain for the crack at 90 degrees to the first crack
8	Max crack opening strain for the first crack to form
9	Max crack opening strain for the crack at 90 degrees to the first crack
10	TYPESC.EQ.0: Maximum through-thickness shear stress (resultant of local $YZ$ and $ZX$ shear stresses) TYPESC.GE.1: Maximum difference (shear demand minus capacity) that has occurred so far in either of the two reinforcement directions
11	TYPESC.EQ.0: Maximum through-thickness $YZ$ shear stress (element axes) TYPESC.GE.1: Maximum difference (shear demand minus capacity) that has occurred so far in reinforcement $x$ -direction
12	TYPESC.EQ.0: Maximum through-thickness $ZX$ shear stress (element axes) TYPESC.GE.1: Maximum difference (shear demand minus capacity) that has occurred so far in reinforcement $y$ -direction

History Variable #	Description
13	TYPESC.GE.1: Current shear demand minus capacity in reinforcement $x$ -direction
14	TYPESC.GE.1: Current shear demand minus capacity in reinforcement $y$ -direction
15	TYPESC.GE.1: Current shear capacity, $V_{cx}$ , in reinforcement $x$ -direction
16	TYPESC.GE.1: Current shear capacity, $V_{cy}$ , in reinforcement $y$ -direction
17	TYPESC.GE.1: Current shear demand, $V_x$ , in reinforcement $x$ -direction
18	TYPESC.GE.1: Current shear demand, $V_y$ , in reinforcement $y$ -direction
19	TYPESC.GT.0: Maximum shear demand that has occurred so far in reinforcement $x$ -direction
20	TYPESC.GT.0: Maximum shear demand that has occurred so far in reinforcement $y$ -direction
21	Current strain in reinforcement ( $x$ -direction)
22	Current strain in reinforcement ( $y$ -direction)
23	Engineering shear strain (slip) across the first crack
24	Engineering shear strain (slip) across the second crack
25	$x$ -stress in concrete (element local axes)
26	$y$ -stress in concrete (element local axes)
27	$xy$ -stress in concrete (element local axes)
28	$yz$ -stress in concrete (element local axes)
29	$xz$ -stress in concrete (element local axes)
30	Reinforcement stress ( $a$ -direction)
31	Reinforcement stress ( $b$ -direction)
32	TYPESC.GT.0: Current shear demand $V_{max}$
33	TYPESC.GT.0: Maximum $V_{max}$ that has occurred so far
34	TYPESC.GT.0: Current shear capacity $V_{c\theta}$
35	TYPESC.GT.0: Excess shear, $V_{max} - V_{c\theta}$
36	TYPESC.GT.0: Maximum excess shear that has occurred so far
56	Max crack opening strain for the crack at 45 degrees to the first crack

History Variable #	Description
57	Max crack opening strain for the crack at -45 degrees to the first crack
58	Current crack opening strain for the crack at 45 degrees to the first crack
59	Current crack opening strain for the crack at -45 degrees to the first crack

In the above table  $V_{\max}$  is given by

$$V_{\max} = \sqrt{V_x^2 + V_y^2} ,$$

where  $V_x$  and  $V_y$  is the shear demand reinforcement in  $x$  and  $y$  directions, respectively. Additionally,

$$V_{c\theta} = \frac{V_{\max}}{\sqrt{\left(\frac{V_x}{V_{cx}}\right)^2 + \left(\frac{V_y}{V_{cy}}\right)^2}} ,$$

where  $V_{cx}$  and  $V_{cy}$  are the shear capacities in the  $x$ - and  $y$ -directions, respectively.

Note that the concrete stress history variables are stored in element local axes irrespective of AOPT; that is, local  $x$  is always the direction from node 1 to node 2. The reinforcement stresses are in the reinforcement directions; these do take account of AOPT.

### **Material Behavior of Concrete:**

#### Thermal sensitivity

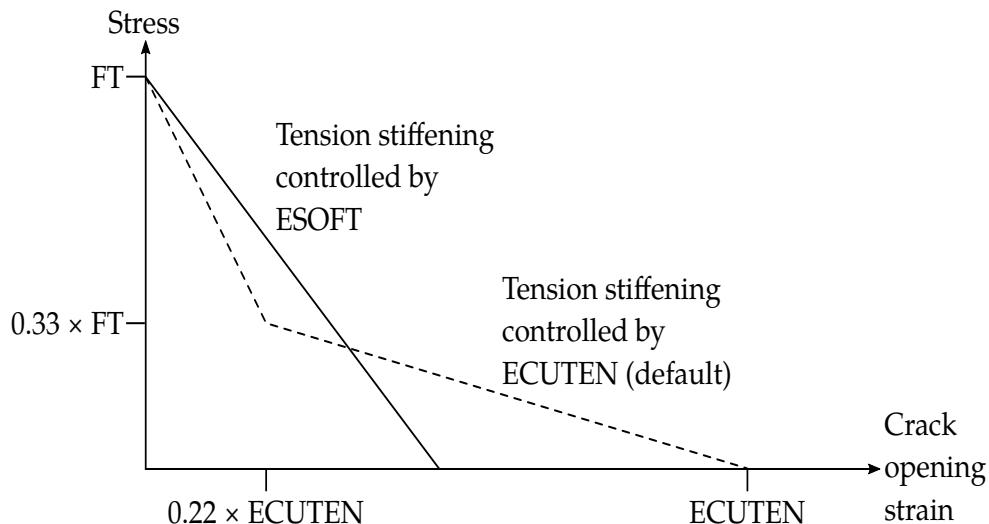
For TYPEC = 1,2,4,5,7,8, the material properties are thermally-sensitive. If no temperatures are defined in the model, it behaves as if at 20°C. Pre-programmed relationships between temperature and concrete properties are taken from the EC2 document. The thermal expansion coefficient is as defined in EC2, is nonzero by default, and is a function of temperature. This coefficient may be overridden by inputting the curve LCALPC. TYPEC = 3, 6, and 9 are not thermally sensitive and have no thermal expansion coefficient by default.

#### Tensile response

The concrete is assumed to crack in tension when the maximum in-plane principal stress reaches FT. A non-rotating smeared crack approach is used. Cracks can open and close repeatedly under hysteretic loading. When a crack is closed, it can carry compression according to the normal compressive stress-strain relationships. The direction of the

crack relative to the element coordinate system is stored when the crack first forms. The material can carry compression parallel to the crack even when the crack is open. Further cracks may then form at pre-determined angles to the first crack if the tensile stress in that direction reaches FT. In versions up to R9, the number of further cracks is limited to one, at 90 degrees to the first crack. In versions starting from R10, up to three additional cracks can form at 45, 90, and 135 degrees to the first crack. The tensile stress is limited to FT only in the available crack directions. The tensile stress in other directions is unlimited and could exceed FT. This is a limitation of the non-rotating crack approach and may lead to models being non-conservative; that is, the response is stronger than implied by the input. Increasing the possible number of cracks from two to four significantly reduces this error and may cause models to seem “weaker” in R10 than in R9 under some loading conditions. An option to revert to the previous two-crack behavior is available in R10 and later – add 100 to TYPEC.

After initial cracking, the tensile stress reduces with increasing tensile strain. A finite amount of energy must be absorbed to create a fully open crack. In practice, the reinforcement holds the concrete together, allowing it to continue to take some tension (this effect is known as tension-stiffening). The options available for the stress-strain curve are shown in [Figure M172-1](#). The piecewise curve is used by default. The simple linear curve applies only if ESOFT > 0.0 and ECUTEN = 0.0. A further option is to define the stress-strain response through a load curve; see LCFIB (intended for fiber-reinforced concrete).



**Figure M172-1.** Tensile Behavior of Concrete

LCHAR can optionally be used to maintain constant energy per unit area of crack irrespective of mesh size; that is, the crack opening displacement is fixed rather than the crack opening strain. LCHAR × ECUTEN is then the displacement to open a crack fully. For the actual elements, crack opening displacement is estimated by strain ×  $\sqrt{\text{area}}$ . Note that if LCHAR is defined, it is also used as the crack spacing in the NS 3473 aggregate interlock calculation.

For the thermally-sensitive values of TYPEC, the relationship of FT with temperature is taken from EC2 – there is no input option to change this. FT is assumed to remain at its input value at temperatures up to 100°C, then to reduce linearly with temperature to zero at 600°C. Up to 500°C, the crack opening strain ECUTEN increases with temperature such that the fracture energy to open the crack remains constant. Above 500°C, the crack opening strain does not increase further.

Some concrete design codes and standards stipulate that the tensile strength of concrete should be assumed to be zero. However, for MAT\_CONCRETE\_EC2, we do not recommend setting FT to zero because:

- Cracks will form at random orientations caused by small dynamic tensile stresses, leading to unexpected behavior when the loading increases because the crack orientations are fixed when the cracks first form;
- The shear strength of cracked concrete may also become zero in the analysis (according to the aggregate interlock formula, the post-crack shear strength is assumed proportional to FT).

These problems may be tackled by using the inputs on Card 9. Firstly, separate tensile strengths may be input for the tensile response and for calculating the shear strength of cracked concrete. Secondly, by using the load curve LCFTT, the tensile strength may be ramped gradually down to zero after the static loads have been applied, ensuring that the cracks will form in the correct orientation.

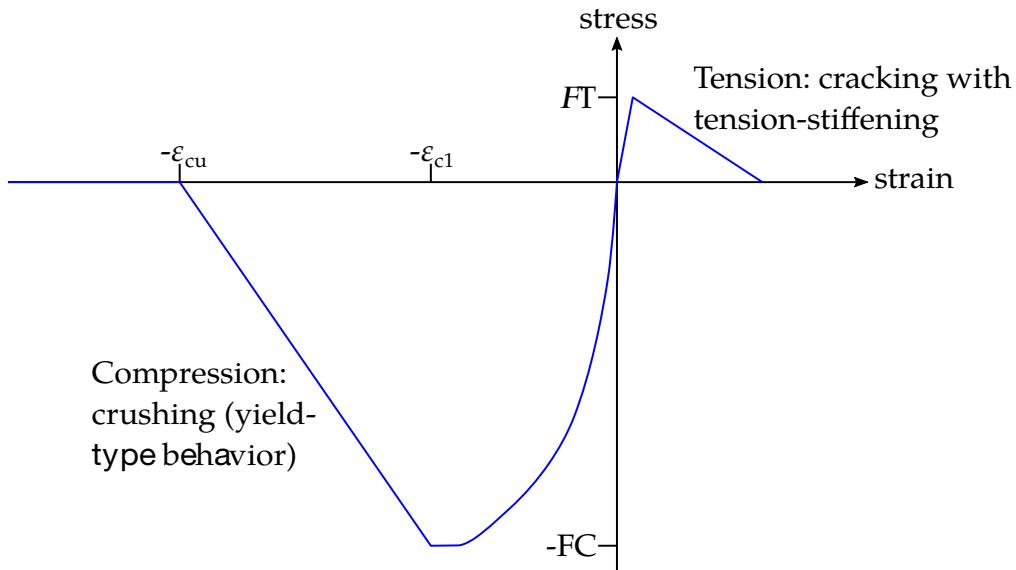
#### Compressive response for TYPEC = 1, 2, 4, 5, 7, and 8

For TYPEC = 1, 2, 4, 5, 7, and 8, the compressive behavior of the concrete initially follows a stress-strain curve defined in EC2 as:

$$\text{Stress} = \text{FC}_{\max} \times \left[ \left( \frac{\varepsilon}{\varepsilon_{c1}} \right) \times \frac{3}{2 + \left( \frac{\varepsilon}{\varepsilon_{c1}} \right)^3} \right],$$

where  $\varepsilon_{c1}$  is the strain at which the ultimate compressive strength,  $\text{FC}_{\max}$ , is reached, and  $\varepsilon$  is the current equivalent uniaxial compressive strain.

The initial elastic modulus is given by  $E = 3 \times \text{FC}_{\max} / 2\varepsilon_{c1}$ . Upon reaching  $\text{FC}_{\max}$ , the stress decreases linearly with increasing strain, reaching zero at a strain  $\varepsilon_{cu}$ . Strains  $\varepsilon_{c1}$  and  $\varepsilon_{cu}$  are by default taken from EC2 and are functions of temperature. At 20°C, they are values 0.0025 and 0.02, respectively.  $\text{FC}_{\max}$  is also a function of temperature, given by the input parameter FC (which applies at 20°C) times a temperature-dependent softening factor taken from EC2. The differences among TYPEC = 1, 2, 4, 5, 7, and 8 are limited to (a) different reductions of FC at elevated temperatures and (b) different values of  $\varepsilon_{c1}$  at elevated temperatures.



**Figure M172-2.** Concrete stress strain behavior

#### Compressive response for TYPEC = 3

For TYPEC = 3, the stress-strain behavior follows the same form described above. To override the default values of the Young's modulus and  $\varepsilon_{cu}$ , set ET36 and ECUT36, respectively. In this case, the strain,  $\varepsilon_{c1}$ , is calculated from the elastic stiffness, and there is no thermal sensitivity.

#### Compressive response for TYPEC = 6

For TYPEC = 6, the above compressive crushing behavior is replaced with the equations proposed by Mander. This algorithm can model unconfined or confined concrete; for unconfined, leave FCC blank. For confined concrete, input the confined compressive strength as FCC.

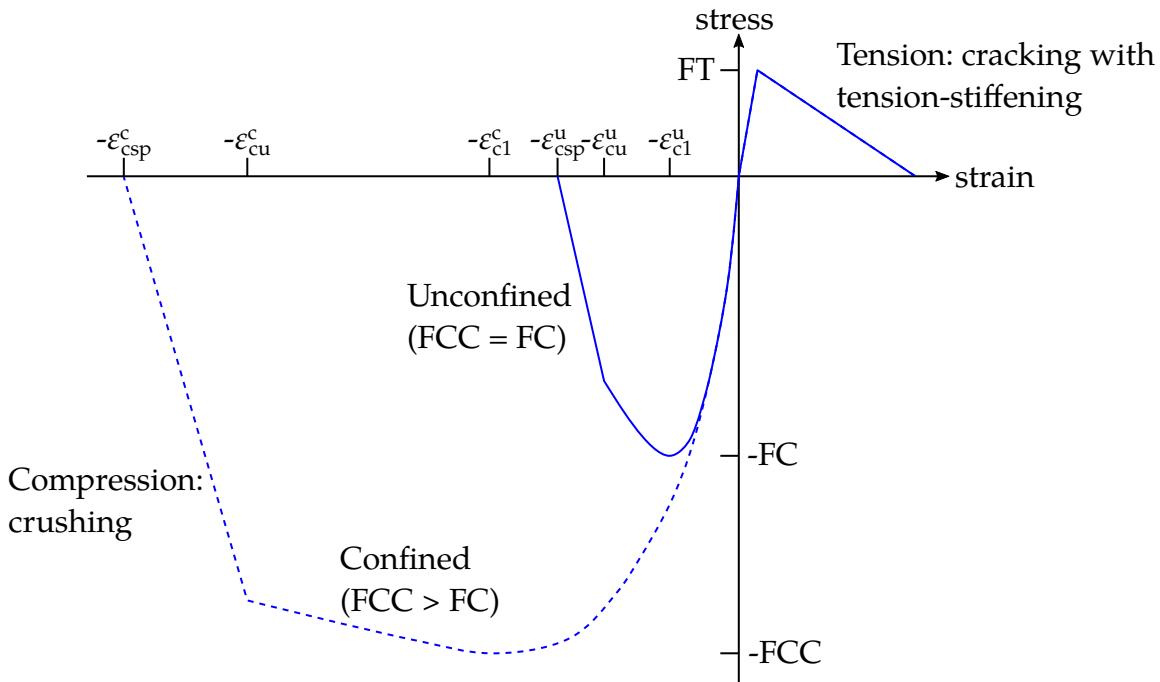
As indicated in [Figure M172-3](#),  $\varepsilon_{c1}$  is the strain at maximum compressive stress,  $\varepsilon_{cu}$  is the ultimate compressive strain, and  $\varepsilon_{csp}$  is the spalling strain. Default values for these quantities for both confined and unconfined concrete are calculated as follows:

$$\varepsilon_{c1} = 0.002 \times \left[ 1 + 5 \left( \frac{FCC}{FC} - 1 \right) \right]$$

$$\varepsilon_{cu} = 1.1 \times \varepsilon_{c1}$$

$$\varepsilon_{csp} = \varepsilon_{cu} + 2 \frac{FCC}{E}$$

Note that for unconfined concrete, FCC = FC causing  $\varepsilon_{c1}$  to default to 0.002. To override the default values  $\varepsilon_{c1}$ ,  $\varepsilon_{cu}$ , and  $\varepsilon_{csp}$ , set EC1\_6, ECUT36, and ECSP69, respectively.



**Figure M172-3.** Type 6 concrete. Values with superscripts *u* and *c* specify they are for the unconfined and confined curves, respectively.

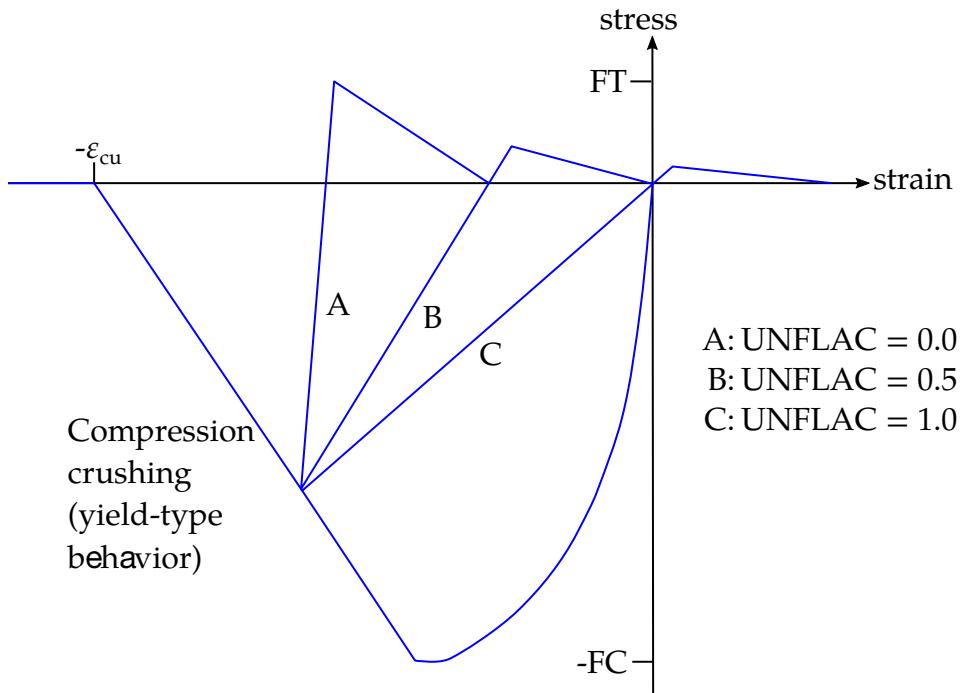
#### Compressive response: TYPEC = 9

For TYPEC = 9, the input parameter FC is the characteristic cylinder strength in the stress units of the model. FC × UNITC is assumed to be  $f_{ck}$ , the strength class in MPa units. The mean tensile strength  $f_{ctm}$ , mean Young's modulus  $E_{cm}$ , and the strains used to construct the stress-strain curve, such as  $\varepsilon_{c1}$ , are by default evaluated automatically from tabulated functions of  $f_{ck}$  given in Table 3.1 of EC2. Input parameter FCC provides the material's compressive strength of the material. It defaults to the mean compressive strength  $f_{cm}$  defined in EC2 as  $f_{ck} + 8\text{MPa}$ . Inputting FCC explicitly overrides the default compressive strength. The stress-strain curve follows this form:

$$\frac{\text{Stress}}{\text{FCC}} = \frac{k\eta - \eta^2}{1 + (k - 2)\eta},$$

where FCC is the input parameter FCC (default:  $= (f_{ck} + 8\text{MPa})/\text{UNITC}$ ),  $\eta = \text{strain}/\varepsilon_{c1}$ ,  $k = 1.05E \times \varepsilon_{c1}/\text{FCC}$ , and  $E$  is the Young's modulus.

The default parameters are intended to be appropriate for a serviceability analysis (mean properties), so default  $FT = f_{ctm}$  and default  $E = E_{cm}$ . For an ultimate load analysis, FCC should be the "design compressive strength" (normally the factored characteristic strength, including any appropriate material factors); FT should be input as the factored characteristic tensile strength; GAMCE9 may be input (a material factor that divides the Young's Modulus so  $E = E_{cm}/\text{GAMCE9}$ ); and a creep factor PHIEF9 may be input that scales  $\varepsilon_{c1}$  by  $(1 + \text{PHIEF9})$ .



**Figure M172-4.** Concrete unloading behavior

#### Unload/Reload Stiffness (All Concrete Types):

The parameter UNLFAC (default = 0.5) determines the reduction of the elastic modulus during compressive loading. See [Figure M172-4](#). UNLFAC = 0.0 means no reduction; the initial elastic modulus applies during unloading and reloading. UNLFAC = 1.0 means that unloading results in no permanent strain. Intermediate values imply a permanent strain linearly interpolated between these extremes. The same factor reduces the tensile strength and the elastic modulus.

#### Optional Compressive Strength Degradation due to Cracking:

By default, the compressive strength of cracked and uncracked elements is the same. If DEGRAD is non-zero, the formula from BS8110 reduces compressive strength during or after crack opening has occurred:

$$\text{Reduction factor} = \min \left( 1.0, \frac{1.0}{0.8 + 100\varepsilon_{t\max}} \right),$$

where  $\varepsilon_{t\max}$  is the maximum (tensile) crack-opening strain that has occurred up to the current time.

**Shear Strength on Cracking Planes:**

Before cracking, the through-thickness shear stress in the concrete is unlimited., unless TYPESC > 10 (see Remark 5). For cracked elements, shear stress on the crack plane (magnitude of shear stress including element-plane and through-thickness terms) is treated in one of two ways:

1. If AGGSZ > 0.0, the relationship from Modified Compression Field Theory is used to model the aggregate-interlock that allows cracked concrete to carry shear loading. The maximum shear stress that can be carried on the crack plane,  $\tau_{max}$ , depends on compressive stress on the crack  $\sigma_c$ (if the crack is closed) or on crack opening width  $w$  (if the crack is open):

$$\tau_{max} = 0.18\tau_{rm} + 1.64\sigma_c - 0.82 \frac{\sigma_c^2}{\tau_{rm}}$$

$$\tau_{rm} = \frac{2FTSHR}{0.31 + \frac{24w}{(D_0 + 16)}}$$

FTSHR is defined on Card 9 and defaults to FT on Card 1.

UNITL is compulsory when AGGSZ is non-zero. This is the factor that converts model length units to millimeters; that is, the aggregate size in millimeters  $D_0 = AGGSZ \times UNITL$ .

The crack width is estimated from  $w = UNITL \times \varepsilon_{cro} \times L_e$ , where  $\varepsilon_{cro}$  is the crack opening strain and  $L_e$  is the crack spacing.  $L_e$  is taken as LCHAR if non-zero or is equal to element size if LCHAR is zero.

Optionally, TAUMXC may be used to set the maximum shear stress when the crack is closed, and the normal stress is zero – by default, this works out as  $1.161 \times FT$  from the above equations. If TAUMXC is defined, the shear stress from the NS3473 formula,  $\tau_{max}$ , is scaled by TAUMXC /  $1.161 \times FT$ .

2. If AGGSZ = 0.0, the aggregate interlock is modeled by this formula:

$$\tau_{max} = \frac{TAUMXC}{1.0 + \frac{\varepsilon_{cro}}{ECRAGG}} + \min(MU \times \sigma_{comp}, TAUMXF) ,$$

where  $\tau_{max}$  is the maximum shear stress carried across a crack;  $\sigma_{comp}$ is the compressive stress across the crack (this is zero if the crack is open); and ECRAGG is the crack opening strain at which the input shear strength TAUMXC is halved. Again, TAUMXC defaults to  $1.161 \times FT$ .

Note that if a shear capacity check is specified, the above applies only to in-plane shear, while the through-thickness shear is unlimited.

**Reinforcement:**

The reinforcement is treated as separate bars providing resistance only in the local  $x$ - and  $y$ -directions – it does not carry shear in-plane or out-of-plane.

For TYPER = 1, 2, 3, 4, 7, and 8, the behavior is thermally sensitive and follows stress-strain relationships of a form defined in EC2. At 20°C (or if no thermal input is specified), the behavior is elastic-perfectly-plastic with Young's Modulus EREINF and ultimate stress SUREINF, up to the onset of failure, after which the stress reduces linearly with increasing strain until final failure. At elevated temperatures, a nonlinear transition between the elastic and the perfectly plastic phases exists, and temperature-dependent factors defined in EC2 scale down EREINF and SUREINF. The strain at which failure occurs depends on the reinforcement type (TYPER) and the temperature. For example, for hot-rolled reinforcing steel at 20°C, failure begins at 15% strain and is complete at 20%. The thermal expansion coefficient is as defined in EC2 and is a function of temperature. This may be overridden by inputting the curve LCAPLS. The differences between TYPER = 1, 2, 4, 7, 8 are limited to (a) different reductions of EREINF and SUREINF at elevated temperatures, (b) different nonlinear transitions between elastic and plastic phases and (c) the strains at which softening begins and is complete.

The default stress-strain curve for reinforcement may be overridden using TYPER = 5 and LCRSU. In this case, the reinforcement properties are not temperature-sensitive, and  $SUREINF \times f(\varepsilon_p)$  gives the yield stress, where  $f(\varepsilon_p)$  is the load curve value at the current plastic strain. To include failure of the reinforcement, the curve should reduce to zero at the desired failure strain and remain zero for higher strains. Note that by default, LS-DYNA re-interpolates the input curve to have 100 equally-spaced points; if the last point on the curve is at very high strain, then the initial part of the curve may become poorly defined.

# \*MAT\_173

# \*MAT\_MOHR\_COULOMB

## \*MAT\_MOHR\_COULOMB

This is Material Type 173. It is for solid elements, thick shells, and SPH particles only and is intended to represent cohesive or sandy soils and other granular materials. A simple soil model is obtained by defining Fields 1 through 4 of Card 1 together with PHI and/or CVAL while leaving all other fields blank. Joints (planes of weakness) may be added if required; the material then represents rock. The joint treatment is identical to that of [\\*MAT\\_JOINTED\\_ROCK](#).

### Card Summary:

**Card 1.** This card is required.

MID	RO	GMOD	RNU		PHI	CVAL	PSI
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**Card 2.** This card is required.

NOVOID	NPLANES	EXTRA	LCCPDR	LCCPT	LCCJDR	LCCJT	LCSFAC
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**Card 3.** This card is required.

GMODDP	GMODGR	LCGMEP	LCPHIEP	LCPSIEP	LCGMST	CVALGR	ANISO
--------	--------	--------	---------	---------	--------	--------	-------

**Card 4.** Include this card if EXTRA > 0.

LCGMT	LCCVT	LCPHT	EPDAM1	EPDAM2			
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**Card 5.** Include if NPLANES > 0. Repeat this card for each plane (maximum of 6 planes).

DIP	DIPANG	CPLANE	FRPLANE	TPLANE	SHRMAX	LOCAL	
-----	--------	--------	---------	--------	--------	-------	--

### Data Card Definitions:

	1	2	3	4	5	6	7	8
Variable	MID	RO	GMOD	RNU	(blank)	PHI	CVAL	PSI
Type	A	F	F	F		F	F	F
Default	none	none	none	none		none	none	0.0

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
GMOD	Elastic shear modulus
RNU	Poisson's ratio
PHI	Angle of friction (radians)
CVAL	Cohesion value (shear strength at zero normal stress)
PSI	Dilation angle (radians)

Card 2	1	2	3	4	5	6	7	8
Variable	NOVOID	NPLANES	EXTRA	LCCPDR	LCCPT	LCCJDR	LCCJT	LCSFAC
Type	1	I	I	I	I	I	I	I
Default	0	0	0	0	0	0	0	0

VARIABLE	DESCRIPTION
NOVOID	Voiding behavior flag (see <a href="#">Remarks 8</a> and <a href="#">9</a> ): EQ.0: Voiding behavior on EQ.1: Voiding behavior off
NPLANES	Number of joint planes (maximum of 6)
EXTRA	Flag to input further data. If EXTRA > 0, then Card 4 is read.
LCCPDR	Load curve for extra cohesion for base material (dynamic relaxation)
LCCPT	Load curve for extra cohesion for base material (transient)
LCCJDR	Load curve for extra cohesion for joints (dynamic relaxation)
LCCJT	Load curve for extra cohesion for joints (transient)

**\*MAT\_173****\*MAT\_MOHR\_COULOMB**

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
Card 3	1	2	3	4	5	6	7	8
Variable	GMODDP	GMODGR	LCGMEP	LCPHIEP	LCPSIEP	LCGMST	CVALGR	ANISO
Type	F	F	I	I	I	I	F	F
Default	0.0	0.0	0	0	0	0	0.0	1.0

<b>VARIABLE</b>		<b>DESCRIPTION</b>						
GMODDP		z-coordinate at which GMOD and CVAL are correct						
GMODGR		Gradient of GMOD as a function of z-coordinate (usually negative)						
LCGMEP		Load curve of GMOD as a function of plastic strain (overrides GMODGR)						
LCPHIEP		Load curve of PHI as a function of plastic strain						
LCPSIEP		Load curve of PSI as a function of plastic strain						
LCGMST		(Leave blank)						
CVALGR		Gradient of CVAL as a function of z-coordinate (usually negative)						
ANISO		Factor applied to elastic shear stiffness in global XZ and YZ planes						

**Card 4.** Define Card 4 only if EXTRA > 0.

Card 4	1	2	3	4	5	6	7	8
Variable	LCGMT	LCCVT	LCPHT	EPDAM1	EPDAM2			
Type	F	F	F	F	F			
Default	0.0	0.0	0.0	$10^{20}$	0.0			

VARIABLE	DESCRIPTION
LCGMT	Load curve of nondimensional factor on GMOD as a function of time
LCCVT	Load curve of nondimensional factor on CVAL as a function of time
LCPHT	Load curve of nondimensional factor on PHI as a function of time
EPDAM1	Plastic strain or volumetric void strain at which damage begins
EPDAM2	Plastic strain or volumetric void strain at which element is eroded

**Plane Cards.** Define if NPLANES > 0. Repeat Card 5 for each plane (maximum 6 planes).

Card 5	1	2	3	4	5	6	7	8
Variable	DIP	DIPANG	CPLANE	FRPLANE	TPLANE	SHRMAX	LOCAL	
Type	F	F	F	F	F	F	I	
Default	0.0	0.0	0.0	0.0	0.0	10 <sup>20</sup>	0	

VARIABLE	DESCRIPTION
DIP	Angle of the plane in degrees below the horizontal (see <a href="#">Remark 11</a> )
DIPANG	Plan view angle (degrees) of downhill vector drawn on the plane
CPLANE	Cohesion for shear behavior on plane
PHPLANE	Friction angle for shear behavior on plane (degrees)
TPLANE	Tensile strength across plane (generally zero or very small)
SHRMAX	Max shear stress on plane (upper limit, independent of compression)
LOCAL	Axes (see <a href="#">Remark 12</a> ): EQ.0: DIP and DIPANG are with respect to the global axes. EQ.1: DIP and DIPANG are with respect to the local element axes.

**Remarks:**

1. **Mohr-Coulomb yield surface.** This material has a Mohr-Coulomb yield surface, given by

$$\tau_{\max} = C + \sigma_n \tan(\text{PHI}) ,$$

where  $\tau_{\max}$  is the maximum shear stress on any plane,  $\sigma_n$  is the normal stress on that plane (positive in compression),  $C$  is the cohesion, and PHI is the friction angle. The plastic potential function is of the form

$$\beta\sigma_k - \sigma_i + \text{constant},$$

where  $\sigma_k$  is the maximum principal stress,  $\sigma_i$  is the minimum principal stress, and

$$\beta = \frac{1 + \sin(\text{PSI})}{1 - \sin(\text{PSI})} .$$

2. **Depth-dependent properties.** If depth-dependent properties are used (see GMODDP, GMODGR, CVALGR), the model must be oriented with the z-axis in the upward direction.
3. **Plastic Strain.** Plastic strain is defined as

$$\sqrt{\frac{2}{3}\varepsilon_{pij}\varepsilon_{pij}} ,$$

that is, the same way as for other elastoplastic material models.

4. **Plastic strain adjustments.** Friction and dilation angles PHI and PSI may vary with plastic strain (see LCPHIEP and LCPSIEP). To model heavily consolidated materials under large shear strains, as the strain increases, the dilation angle typically reduces to zero, and the friction angle reduces to a lower pre-consolidation value.

For similar reasons, the shear modulus may reduce with plastic strain (see LCG-MEP), but this option may sometimes give unstable results.

5. **Additional cohesion.** The load curves, LCCPDR, LCCPT, LCCJDR, and LCCJT, allow extra cohesion to be specified as a function of time. The cohesion is additional to that specified in the material parameters. This is intended for use during the initial stages of an analysis to allow application of gravity or other loads without cracking or yielding, and for the cracking or yielding then to be introduced in a controlled manner. This is done by specifying extra cohesion that exceeds the expected stresses initially, then declining to zero. If no curves are specified, no extra cohesion is applied.
6. **Time-dependent properties.** LCSFAC, the load curve for factor on strength, applies simultaneously to the cohesion and tan (PHI) of the base material and

all joints. This feature is intended to reduce the strength of the material gradually to explore factors of safety. If no curve is present, a constant factor of 1 is assumed. Values much greater than 1.0 may cause problems with stability. Alternatively, separate functions of time may be defined for each of the properties GMOD, CVAL, and PHI using load curves LCGMT, LCCVT, and LCPHT, respectively.

7. **ANISO.** The anisotropic factor, ANISO, applies to the elastic shear stiffness in the global XZ and YZ planes. It can be used only in a pure Mohr-Coulomb mode (NPLANES = 0).
8. **Tensile pressure limit.** For a friction angle greater than zero, the Mohr-Coulomb yield surface implies a tensile pressure limit equal to CVAL/tan(PHI). By default, voids develop in the material when this pressure limit is reached, and the pressure will never become more tensile than the tensile pressure limit. The volumetric void strain is tracked and is reversible if the strain is reversed.
9. **NOVOID.** If NOVOID = 1, then the tensile pressure limit is not applied and stress states in which the pressure is more tensile than CVAL/tan(PHI) are permitted but will be purely hydrostatic with no shear stress. NOVOID is recommended in Multi-Material ALE simulations in which the development of voids or air space is already accounted for by the Multi-Material ALE.
10. **Soil or rock.** To model soil, set NJOINT = 0. The joints allow modeling of rock and are treated identically to those of [\\*MAT\\_JOINTED\\_ROCK](#).
11. **Joint plane orientations.** The joint plane orientations are defined by the angle of a “downhill vector” drawn on the plane, meaning the vector is oriented within the plane to obtain the maximum possible downhill angle. DIP is the angle of this line below the horizontal. DIPANG is the plan-view angle of the line (pointing downhill) measured clockwise from the global Y-axis about the global Z-axis.
12. **Masonry and joint planes.** Joint planes are generally defined in the global axis system if they are taken from survey data, and the material represents rock. For this case, set LOCAL = 0. In other cases, it may be more convenient to define the joint plane angles, DIP and DIPANG, relative to the element local axis system (to do this, set LOCAL = 1). For example, this material model can be used to represent masonry with the weak planes representing the mortar joint. In this situation, these joints may be parallel to the local element axes throughout the mesh.

The choice of defining the joint angles relative to global versus local coordinates is available only for solid elements. For thick shell elements (\*ELEMENT\_TSHELL), DIP and DIPANG are always relative to the element's local axis, and the setting of LOCAL is ignored.

13. **Rigid body motion.** The joint planes rotate with the rigid body motion of the elements, irrespective of whether their initial definitions are in the global or local axis system.
14. **Extra history variables.** Extra history variables may be requested (see NEIPH on \*DATABASE\_EXTENT\_BINARY). They are described in the following table:

History Variable #	Description
1	Mobilized strength fraction for base material
2	Volumetric void strain
3	Maximum stress overshoot during plasticity calculation
4 – 9	Crack opening strain for planes 1 through 6
10 – 15	Crack accumulated engineering shear strain for planes 1 through 6
16 – 20	Current shear utilization for planes 1 through 6
21 – 27	Maximum shear utilization to date for planes 1 through 6
33	Elastic shear modulus (for checking depth-dependent input)
34	Cohesion (for checking depth-dependent input)

**\*MAT\_RC\_BEAM****\*MAT\_174****\*MAT\_RC\_BEAM**

This is Material Type 174. It is for Hughes-Liu beam elements only. The material model can represent plain concrete only, reinforcing steel only, or a smeared combination of concrete and reinforcement. The main emphasis of this material model is the cyclic behavior. It is intended primarily for seismic analysis.

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	EUNL	PR	FC	EC1	EC50	RESID
Type	A	F	F	F	F	F	F	F
Default	none	none	Rem 2	0.0	none	0.0022	Rem 2	0.2

Card 2	1	2	3	4	5	6	7	8
Variable	FT	UNITC	(blank)	(blank)	(blank)	ESOFT	LCHAR	OUTPUT
Type	F	F	F	F	F	F	F	F
Default	Rem 2	1.0	none	none	none	Rem 2	none	0

Card 3	1	2	3	4	5	6	7	8
Variable	FRACR	YMREIN	PRREIN	SYREIN	SUREIN	ESHR	EUR	RREINF
Type	F	F	F	F	F	F	F	F
Default	0.0	none	0.0	0.0	SYREIN	0.03	0.2	4.0

**VARIABLE****DESCRIPTION**

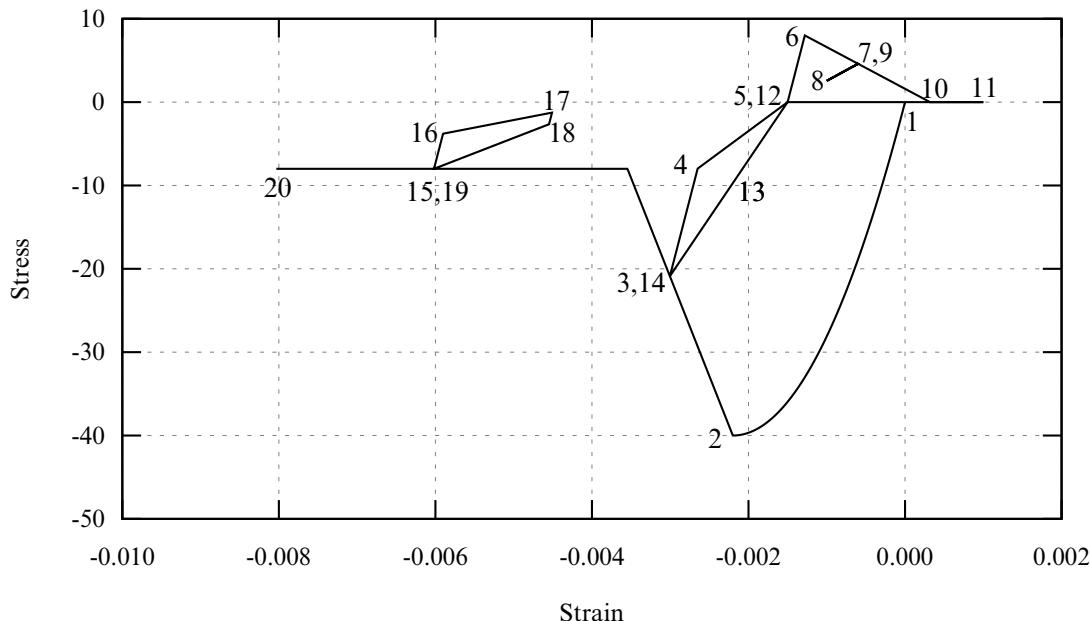
MID

Material identification. A unique number or label must be specified (see \*PART).

RO

Mass density

<b>VARIABLE</b>	<b>DESCRIPTION</b>
EUNL	Initial unloading elastic modulus (see <a href="#">Remark 2</a> )
PR	Poisson's ratio.
FC	Cylinder strength (stress units)
EC1	Strain at which stress FC is reached.
EC50	Strain at which the stress has dropped to 50% FC
RESID	Residual strength factor
FT	Maximum tensile stress
UNITC	Factor to convert stress units to MPa (see <a href="#">Remark 2</a> )
ESOFT	Slope of stress-strain curve post-cracking in tension
LCHAR	Characteristic length for strain-softening behavior
OUTPUT	Output flag controlling what is written as "plastic strain" (see <a href="#">Remark 4</a> ): EQ.0.0: Curvature EQ.1.0: "High-tide" plastic strain in reinforcement
FRACR	Fraction of reinforcement (for example, for 1% reinforcement FRACR = 0.01). See <a href="#">Remark 1</a> .
YMREIN	Young's Modulus of reinforcement
PRREIN	Poisson's Ratio of reinforcement
SYREIN	Yield stress of reinforcement
SUREIN	Ultimate stress of reinforcement
ESHR	Strain at which reinforcement begins to harden
EUR	Strain at which reinforcement reaches ultimate stress
R_REINF	Dimensionless Ramberg-Osgood parameter $r$ . If zero, a default value of 4.0 will be used. If set to -1, parameters will be calculated from Kent & Park formulae (see <a href="#">Remark 3</a> ).



**Figure M174-1.** Example response for Concrete

#### Remarks:

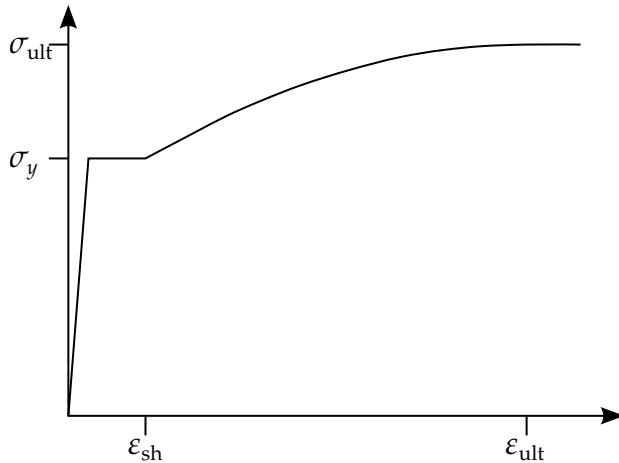
1. **Creating sections for reinforced concrete beams.** This material model can be used to represent unreinforced concrete ( $\text{FRACR} = 0$ ), steel ( $\text{FRACR} = 1$ ), or reinforced concrete with evenly distributed reinforcement ( $0 < \text{FRACR} < 1$ ).

Alternatively, you can specify the distribution in a section with \*INTEGRATION\_BEAM. In this case, the PID field for each integration point on \*INTEGRATION\_BEAM identifies the material for that integration point. You should create one part for concrete and another for steel. These parts should reference two materials of type \*MAT\_RC\_BEAM, but one with  $\text{FRACR} = 0$  and the other with  $\text{FRACR} = 1$ . Then, by assigning one or other of these part IDs to each integration point, the reinforcement can be applied to the correct locations within the section of the beam.

2. **Modeling Concrete.** In monotonic compression, we use the approach of Park and Kent, as described in Park & Paulay [1975]. The material follows a parabolic stress-strain curve up to a maximum stress equal to the cylinder strength  $\text{FC}$ . Thereafter, the strength decays linearly with strain until the residual strength is reached.

Default values for some material parameters will be calculated automatically as follows:

$$\text{EC50} = \frac{(3 + 0.29\text{FC})}{145\text{FC} - 1000}$$



**Figure M174-2.** Monotonic tensile loading of the reinforcement

where FC is in MPa as per Park and Kent test data.

$$EUNL = \text{initial tangent slope} = \frac{2FC}{EC1}$$

Input values for EUNL lower than this are not permitted, but higher values may be defined if desired.

$$FT = 1.4 \left( \frac{FC}{10} \right)^{\frac{2}{3}}$$

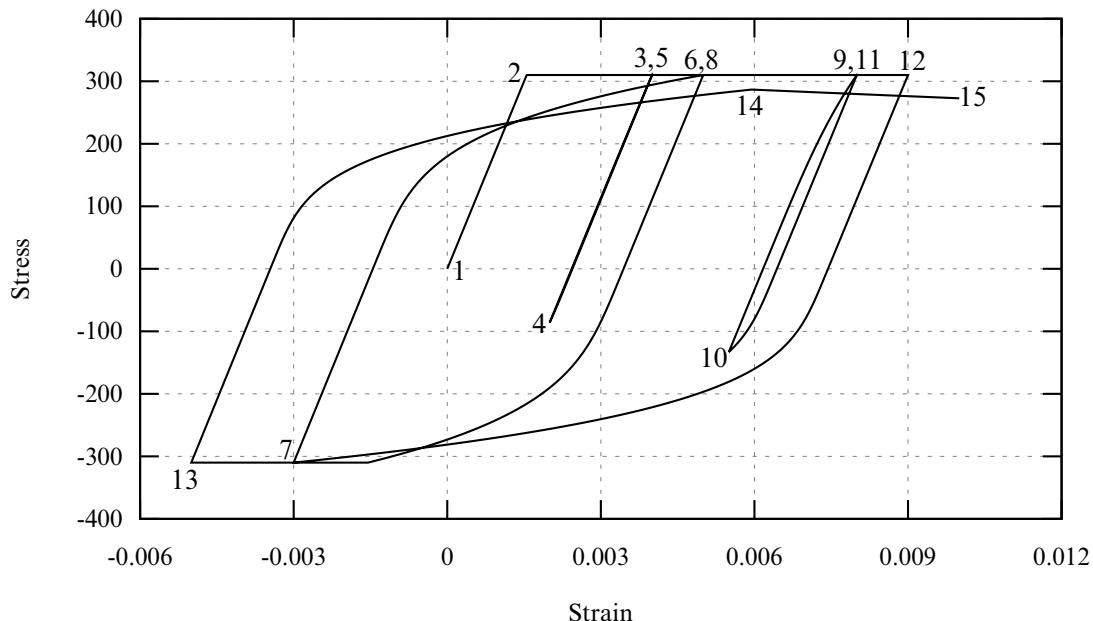
where FC is in MPa as per Park and Kent test data.

$$ESOFT = EUNL$$

Input values higher than EUNL are not permitted. UNITC is used only to calculate default values for the above parameters from FC.

Strain-softening behavior tends to lead to deformations being concentrated in one element, and hence the overall force-deflection behavior of the structure can be mesh-size-dependent if the softening is characterized by strain. To avoid this, you may define a characteristic length (LCHAR). This is the length of specimen (or element) that would exhibit the defined monotonic stress-strain relationship. LS-DYNA adjusts the stress-strain relationship after ultimate load for each element, such that all elements irrespective of their length will show the same deflection during strain softening (that is, between ultimate load and residual load). Therefore, although deformation will still be concentrated in one element, the load-deflection behavior should be the same irrespective of element size. For tensile behavior, ESOFT is similarly scaled.

Cyclic behavior is broadly suggested by Blakeley and Park [1973] as described in Park & Paulay [1975]. The stress-strain response lies within the Park-Kent envelope and is characterized by stiff initial unloading response at slope EUNL



**Figure M174-3.** Stress vs. strain hysteresis plot for the reinforcement with RRE-INF = 4.0

followed by a less stiff response if it unloads to less than half the current strength. Reloading stiffness degrades with increasing strain.

In tension, the stress rises linearly with strain until a tensile limit FT is reached. Thereafter the stiffness and strength decays with increasing strain at a rate ES-OFT. The stiffness also decays such that unloading always returns to strain at which the stress most recently changed to tensile.

3. **Modeling the Reinforcement.** Monotonic loading of the reinforcement results in the stress-strain curve shown in [Figure M174-2](#), which is parabolic between  $\varepsilon_{sh}$  and  $\varepsilon_{ult}$ . The same curve acts as an envelope on the hysteretic behavior when the  $x$ -axis is cumulative plastic strain.

Unloading from the yielded condition is elastic until the load reverses. Thereafter, the Bauschinger Effect (reduction in stiffness at stresses less than yield during cyclic deformation) is represented by following a Ramberg-Osgood relationship until the yield stress is reached:

$$\varepsilon - \varepsilon_s = \left( \frac{\sigma}{E} \right) \left\{ 1 + \left( \frac{\sigma}{\sigma_{CH}} \right)^{r-1} \right\}$$

where  $\varepsilon$  and  $\sigma$  are strain and stress,  $\varepsilon_s$  is the strain at zero stress,  $E$  is Young's Modulus, and  $r$  and  $\sigma_{CH}$  are as defined below

We have two options for calculating  $r$  and  $\sigma_{CH}$ , which is performed at each stress reversal:

- a) If RREINF is input as -1,  $r$  and  $\sigma_{CH}$  are calculated internally from formulae given in Kent and Park. Parameter  $r$  depends on the number of stress reversals. Parameter  $\sigma_{CH}$  depends on the plastic strain that occurred between the previous two stress reversals. The formulae were statistically derived from experiments but may not fit all circumstances. In particular, large differences in behavior may be caused by the presence or absence of small stress reversals such as could be caused by high frequency oscillations. Therefore, results might sometimes be unduly sensitive to small changes in the input data.
  - b) If RREINF is entered by the user or left blank,  $r$  is held constant while  $\sigma_{CH}$  is calculated on each reversal such that the Ramberg-Osgood curve meets the monotonic stress-strain curve at the point from which it last unloaded. For example, points 6 and 8 are coincident in [Figure M174-3](#). The default setting of 4.0 for RREINF gives similar hysteresis behavior to that described by Kent & Park but is unlikely to be so sensitive to small changes of input data.
4. **Output.** We recommend setting BEAMIP on \*DATABASE\_EXTENT\_BINARY to request stress and strain output at the individual integration points. Note that for \*MAT\_RC\_BEAM either element curvature or high tide plastic strain for the reinforcement is written to the output files in place of plastic strain depending on the setting of OUTPUT. In the post-processor, select “plastic strain” to display your selection of OUTPUT. For curvature, LS-DYNA compares the absolute values of the curvatures about the local  $y$  and  $z$  axes and outputs the larger value. In the post-processor, to display the total axial strain (elastic + plastic) at that integration point, select “axial strain.” This can be combined with axial stress to create hysteresis plots, such as those shown in [Figures M174-1](#) and [M174-3](#).

**\*MAT\_VISCOELASTIC\_THERMAL**

This is Material Type 175. This material model provides a general viscoelastic Maxwell model having up to 12 terms in the prony series expansion and is useful for modeling dense continuum rubbers and solid explosives. Either the coefficients of the prony series expansion or a relaxation curve may be specified to define the viscoelastic deviatoric and bulk behavior. Note that \*MAT\_GENERAL\_VISCOELASTIC (Material Type 76) has all the capability of \*MAT\_VISCOELASTIC\_THERMAL and additionally offers more terms (18) in the prony series expansion and an optional scaling of material properties with moisture content.

**Card Summary:**

**Card 1.** This card is required.

MID	RO	BULK	PCF	EF	TREF	A	B
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**Card 2.** If fitting is done from a relaxation curve, specify fitting parameters on Card 2; otherwise if constants are set on Card 3, *LEAVE THIS CARD BLANK*.

LCID	NT	BSTART	TRAMP	LCIDK	NTK	BSTARTK	TRAMPK
------	----	--------	-------	-------	-----	---------	--------

**Card 3.** These cards are not needed if data is defined using Card 2. This card can be input up to 6 times. The keyword ("\*") card terminates this input.

Gi	BETAi	Ki	BETAKi				
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	BULK	PCF	EF	TREF	A	B
Type	A	F	F	F	F	F	F	F

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
BULK	Elastic bulk modulus

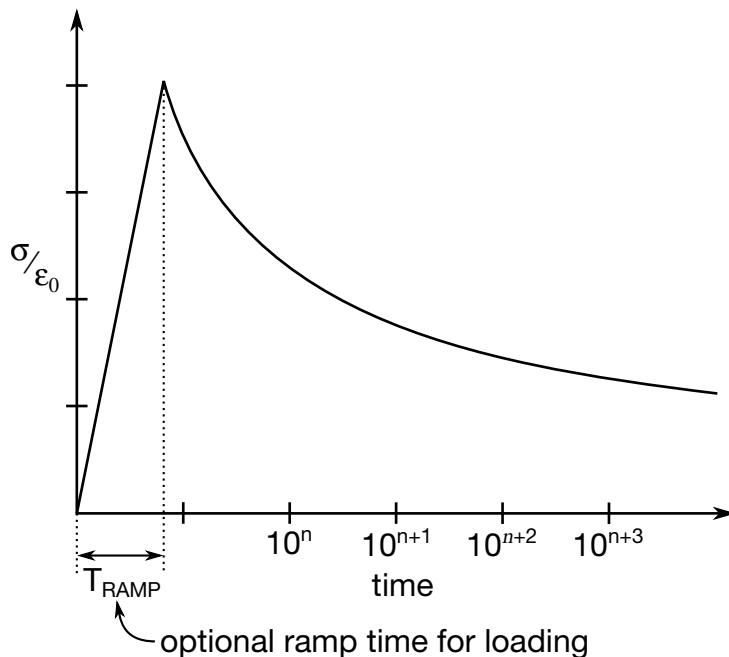
**\*MAT\_175****\*MAT\_VISCOELASTIC\_THERMAL**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
PCF	Tensile pressure elimination flag for solid elements only. If set to unity, tensile pressures are set to zero.
EF	Elastic flag: EQ.0: The layer is viscoelastic. EQ.1: The layer is elastic.
TREF	Reference temperature for shift function (must be greater than zero)
A	Coefficient for the Arrhenius and the Williams-Landel-Ferry shift functions
B	Coefficient for the Williams-Landel-Ferry shift function

**Relaxation Curve Card.** If fitting is done from a relaxation curve, specify fitting parameters on Card 2; otherwise if constants are set on Viscoelastic Constant Cards, LEAVE THIS CARD BLANK.

Card 2	1	2	3	4	5	6	7	8
Variable	LCID	NT	BSTART	TRAMP	LCIDK	NTK	BSTARTK	TRAMPK
Type	F	I	F	F	F	I	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCID	Load curve ID for deviatoric behavior if constants $G_i$ and $\beta_i$ are determined using a least squares fit. This relaxation curve is shown in <a href="#">Figure M175-1</a> .
NT	Number of terms in shear fit. If zero, 6 terms are used by default. Fewer than NT terms will be used if the fit produces one or more negative shear moduli. Currently, the maximum number is set to 6.
BSTART	In the fit, $\beta_1$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$ , $\beta_4$ is 10 times $\beta_3$ , and so on. If zero, BSTART is determined by an iterative trial and error scheme.
TRAMP	Optional ramp time for loading



**Figure M175-1.** Relaxation curve. This curve defines stress as a function of time where time is defined on a logarithmic scale. For best results, the points defined in the load curve should be equally spaced on the logarithmic scale. Furthermore, the load curve should be smooth and defined in the positive quadrant. If nonphysical values are determined by least squares fit, LS-DYNA will terminate with an error message after the initialization phase is completed. If the ramp time for loading is included, then the relaxation which occurs during the loading phase is taken into account. This effect may or may not be important.

VARIABLE	DESCRIPTION
LCIDK	Load curve ID for bulk behavior if constants, $K_i$ , and $\beta\kappa_i$ are determined using a least squares fit. This relaxation curve is shown in <a href="#">Figure M175-1</a> .
NTK	Number of terms desired in bulk fit. If zero 6 terms are used by default. Currently, the maximum number is set to 6.
BSTARTK	In the fit, $\beta\kappa_1$ is set to zero, $\beta\kappa_2$ is set to BSTARTK, $\beta\kappa_3$ is 10 times $\beta\kappa_2$ , $\beta\kappa_4$ is 10 times $\beta\kappa_3$ , and so on. If zero, BSTARTK is determined by an iterative trial and error scheme.
TRAMPK	Optional ramp time for bulk loading

**\*MAT\_175****\*MAT\_VISCOELASTIC\_THERMAL**

**Viscoelastic Constant Cards.** Up to 6 cards may be input. The next keyword ("\*\*") card terminates this input. These cards are not needed if relaxation data is defined (Card 2). The number of terms for the shear behavior may differ from that for the bulk behavior: simply insert zero if a term is not included. If an elastic layer is defined, only  $G_i$  and  $K_i$  need to be defined (note in an elastic layer only one card is needed).

Card 3	1	2	3	4	5	6	7	8
Variable	$G_i$	BETA <i>i</i>	$K_i$	BETAK <i>i</i>				
Type	F	F	F	F				

VARIABLE	DESCRIPTION
$G_i$	Optional shear relaxation modulus for the $i^{\text{th}}$ term
BETA <i>i</i>	Optional shear decay constant for the $i^{\text{th}}$ term
$K_i$	Optional bulk relaxation modulus for the $i^{\text{th}}$ term
BETAK <i>i</i>	Optional bulk decay constant for the $i^{\text{th}}$ term

**Remarks:**

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t-\tau) \frac{\partial \epsilon_{kl}}{\partial \tau} d\tau ,$$

where  $g_{ijkl}(t-\tau)$  is the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \sum_{m=1}^N G_m e^{-\beta_m t} .$$

We characterize this in the input by shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . An arbitrary number of terms, up to 6, may be used when applying the viscoelastic model.

For volumetric relaxation, the relaxation function is also represented by the Prony series in terms of bulk moduli:

$$k(t) = \sum_{m=1}^N K_m e^{-\beta_{k_m} t}$$

The Arrhenius and Williams-Landel-Ferry (WLF) shift functions account for the effects of the temperature on the stress relaxation. A scaled time,  $t'$ ,

$$t' = \int_0^t \Phi(T) dt,$$

is used in the relaxation function instead of the physical time. The Arrhenius shift function is

$$\Phi(T) = \exp \left[ -A \left( \frac{1}{T} - \frac{1}{T_{\text{REF}}} \right) \right],$$

and the Williams-Landel-Ferry shift function is

$$\Phi(T) = \exp \left( -A \frac{T - T_{\text{REF}}}{B + T - T_{\text{REF}}} \right).$$

If all three values (TREF, A, and B) are not zero, the WLF function is used; the Arrhenius function is used if B is zero; and no scaling is applied if all three values are zero.

## \*MAT\_176

## \*MAT\_QUASILINEAR\_VISCOELASTIC

### \*MAT\_QUASILINEAR\_VISCOELASTIC

This is Material Type 176. This is a quasi-linear, isotropic, viscoelastic material based on a one-dimensional model by Fung [1993], which represents biological soft tissues, such as the brain. It is implemented for solid and shell elements. As of LS-DYNA version 971, a second formulation has been implemented that allows for larger strains, but in general, will not give the same results as the previous (default) implementation.

#### Card Summary:

**Card 1.** This card is required.

MID	R0	K	LC1	LC2	N	GSTART	M
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**Card 2.** This card is required.

S0	E_MIN	E_MAX	GAMA1	GAMA2	K	EH	FORM
----	-------	-------	-------	-------	---	----	------

**Card 3.** This card is included if and only if LC1 = 0.

G1	BETA1	G2	BETA2	G3	BETA3	G4	BETA4
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**Card 4.** This card is included if and only if LC1 = 0.

G5	BETA5	G6	BETA6	G7	BETA7	G8	BETA8
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**Card 5.** This card is included if and only if LC1 = 0.

G9	BETA9	G10	BETA10	G11	BETA11	G12	BETA12
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**Card 6.** This card is included if and only if LC2 = 0.

C1	C2	C3	C4	C5	C6		
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#### Data Card Definitions:

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	K	LC1	LC2	N	GSTART	M
Type	A	F	F	I	I	F	F	F
Default	none	none	none	0	0	6	1/TMAX	6

VARIABLE	DESCRIPTION
MID	Material identification. A unique number or label must be specified (see *PART).
RO	Mass density
K	Bulk modulus
LC1	Load curve ID that defines the relaxation function in shear. This curve is used to fit the coefficients $G_i$ and $\text{BETAI}_i$ . If zero, define the coefficients directly. The latter is recommended.
LC2	Load curve ID that defines the instantaneous elastic response in compression and tension. If zero, define the coefficients directly. <i>Symmetry is not assumed if only the tension side is defined; therefore, defining the response in tension only, may lead to nonphysical behavior in compression. Also, this curve should give a softening response for increasing strain without any negative or zero slopes. A stiffening curve or one with negative slopes is generally unstable.</i>
N	Number of terms used in the Prony series which must be less than or equal to 6. This number should be equal to the number of decades of time covered by the experimental data. Define this number if LC1 is nonzero. Carefully check the fit in the d3hsp file to ensure that it is valid, since the least square fit is not always reliable.
GSTART	Starting value for least square fit. If zero, a default value is set equal to the inverse of the largest time in the experiment. Define this number if LC1 is nonzero.
M	Number of terms used to determine the instantaneous elastic response. This variable is ignored with the new formulation but is kept for compatibility with the previous input.

Card 2	1	2	3	4	5	6	7	8
Variable	S0	E_MIN	E_MAX	GAMA1	GAMA2	K	EH	FORM
Type	F	F	F	F	F	F	F	I
Default	0.0	-0.9	5.1	0.0	0.0	0.0	0.0	0

<b>VARIABLE</b>	<b>DESCRIPTION</b>
SO	<p>Strain (logarithmic) output option to control what is written as component 7 to the d3plot database. (LS-PrePost always blindly labels this component as effective plastic strain.) The maximum values are updated for each element each time step:</p> <p>EQ.0.0: Maximum principal strain that occurs during the calculation</p> <p>EQ.1.0: Maximum magnitude of the principal strain values that occurs during the calculation</p> <p>EQ.2.0: Maximum effective strain that occurs during the calculation</p>
E_MIN	Minimum strain used to generate the load curve from $C_i$ . The default range is -0.9 to 5.1. The computed solution will be more accurate if the user specifies the range used to fit the $C_i$ . Linear extrapolation is used outside the specified range.
E_MAX	Maximum strain used to generate the load curve from $C_i$ .
GAMA1	Material failure parameter (see *MAT_SIMPLIFIED_RUBBER and <a href="#">Figure M181-1</a> )
GAMA2	Material failure parameter (see *MAT_SIMPLIFIED_RUBBER)
K	Material failure parameter that controls the volume enclosed by the failure surface (see *MAT_SIMPLIFIED_RUBBER):  LE.0.0: Ignore failure criterion GT.0.0: Use actual K value for failure criteria
EH	Damage parameter (see *MAT_SIMPLIFIED_RUBBER)
FORM	Formulation of model.  EQ.0: Original model developed by Fung, which always relaxes to a zero stress state as time approaches infinity EQ.1: Alternative model, which relaxes to the quasi-static elastic response EQ.-1: Improvement on FORM = 0 where the instantaneous elastic response is used in the viscoelastic stress update, not just in the relaxation, as in FORM = 0. Consequently, the constants for the elastic response do not need to be scaled.

<b>VARIABLE</b>	<b>DESCRIPTION</b>							
	In general, formulations 1 and 2 won't give the same responses.							

**Viscoelastic Constants Card 1.** Additional card for LC1 = 0.

Card 3	1	2	3	4	5	6	7	8
Variable	G1	BETA1	G2	BETA2	G3	BETA3	G4	BETA4
Type	F	F	F	F	F	F	F	F

**Viscoelastic Constants Card 2.** Additional card for LC1 = 0.

Card 4	1	2	3	4	5	6	7	8
Variable	G5	BETA5	G6	BETA6	G7	BETA7	G8	BETA8
Type	F	F	F	F	F	F	F	F

**Viscoelastic Constants Card 3.** Additional card for LC1 = 0.

Card 5	1	2	3	4	5	6	7	8
Variable	G9	BETA9	G10	BETA10	G11	BETA11	G12	BETA12
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
$G_i$	Coefficients of the relaxation function. The number of coefficients is currently limited to 6 although 12 may be read in to maintain compatibility with the previous formulation's input. Define these coefficients if LC1 is set to zero. At least 2 coefficients must be non-zero.
$\text{BETA}_i$	Decay constants of the relaxation function. Define these coefficients if LC1 is set to zero. The number of coefficients is currently limited to 6 although 12 may be read in to maintain compatibility with the previous formulation's input.

**\*MAT\_176****\*MAT\_QUASILINEAR\_VISCOELASTIC**

**Instantaneous Elastic Reponses Card.** Additional card for LC2 = 0.

Card 6	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6		
Type	F	F	F	F	F	F		

VARIABLE	DESCRIPTION
Ci	Coefficients of the instantaneous elastic response in compression and tension. Define these coefficients only if LC2 is set to zero.

**Remarks:**

The equations for the original model (FORM = 0) are given as:

$$\begin{aligned}\sigma_V(t) &= \int_0^t G(t-\tau) \frac{\partial \sigma_\varepsilon[\varepsilon(\tau)]}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial \tau} d\tau \\ G(t) &= \sum_{i=1}^n G_i e^{-\beta t} \\ \sigma_\varepsilon(\varepsilon) &= \sum_{i=1}^k C_i \varepsilon^i\end{aligned}$$

where  $G$  is the shear modulus. Effective strain (which can be written to the d3plot database) is calculated as follows:

$$\varepsilon^{\text{eff}} = \sqrt{\frac{2}{3} \varepsilon_{ij} \varepsilon_{ij}}$$

The polynomial for instantaneous elastic response should contain only odd terms if symmetric tension-compression response is desired.

The new model (FORM = 1) is based on the hyperelastic model used in \*MAT\_SIMPLIFIED\_RUBBER assuming incompressibility. The one-dimensional expression for  $\sigma_\varepsilon$  generates the uniaxial stress-strain curve and an additional visco-elastic term is added on,

$$\begin{aligned}\sigma(\varepsilon, t) &= \sigma_{SR}(\varepsilon) + \sigma_V(t) \\ \sigma_V(t) &= \int_0^t G(t-\tau) \frac{\partial \varepsilon}{\partial \tau} d\tau\end{aligned}$$

where the first term to the right of the equals sign is the hyperelastic stress and the second is the viscoelastic stress. Unlike the previous formulation, where the stress always relaxes to zero, the current formulation relaxes to the hyperelastic stress.

**\*MAT\_HILL\_FOAM**

Purpose: This is Material Type 177. This is a highly compressible foam based on the strain-energy function proposed by Hill [1979]; also see Storakers [1986]. Poisson's ratio effects are taken into account.

**Card Summary:**

**Card 1.** This card is required.

MID	R0	K	N	MU	LCID	FITTYPE	LCSR
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**Card 2.** This card is included if LCID = 0.

C1	C2	C3	C4	C5	C6	C7	C8
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**Card 3.** This card is included if LCID = 0.

B1	B2	B3	B4	B5	B6	B7	B8
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**Card 4.** This card is optional.

R	M						
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	K	N	MU	LCID	FITTYPE	LCSR
Type	A	F	F	F	F	I	I	I
Default	none	none	none	0.	0.	0	0	0

**VARIABLE****DESCRIPTION**

MID           Material identification. A unique number or label must be specified (see \*PART).

RO           Mass density

K           Bulk modulus. This modulus is used for determining the contact interface stiffness. See [Remark 2](#).

**\*MAT\_177****\*MAT\_HILL\_FOAM**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
N	Material constant. Define if LCID = 0 below; otherwise, N is fit from the load curve data. See <a href="#">Remark 2</a> .
MU	Damping coefficient.
LCID	Load curve ID that defines the force per unit area as a function of the stretch ratio. This curve can be given for either uniaxial or bi-axial data depending on FITTYPE. See <a href="#">Remark 1</a> .
FITTYPE	Type of fit: EQ.1: Uniaxial data EQ.2: Biaxial data EQ.3: Pure shear data
LCSR	Load curve ID that defines the uniaxial or biaxial stretch ratio (see FITTYPE) as a function of the transverse stretch ratio.

**Material Constant Card 1.** Additional card for LCID = 0.

Card 2	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6	C7	C8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

<b>VARIABLE</b>	<b>DESCRIPTION</b>
$C_i$	Material constants. See equations below. Define up to 8 coefficients if LCID = 0.

**Material Constant Card 2.** Additional card for LCID = 0.

Card 3	1	2	3	4	5	6	7	8
Variable	B1	B2	B3	B4	B5	B6	B7	B8
Type	F	F	F	F	F	F	F	F
Default	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIABLE	DESCRIPTION
Bi	Material constants. See equations below. Define up to 8 coefficients if LCID = 0.

**Mullins Effect Card.** This card is optional.

Card 4	1	2	3	4	5	6	7	8
Variable	R	M						
Type	F	F						
Default	0.0	0.0						

VARIABLE	DESCRIPTION
R	Mullins effect model <i>r</i> coefficient
M	Mullins effect model <i>m</i> coefficient

#### Remarks:

- Load Curve Fit.** If load curve data is defined, the fit generated by LS-DYNA must be closely checked in the d3hsp output file. The nonlinear least squares procedure in LS-DYNA, which is used to fit the data, may be inadequate.
- Material Model.** The Hill strain energy density function for this highly compressible foam is given by:

$$W = \sum_{j=1}^m \frac{C_j}{b_j} \left[ \lambda_1^{b_j} + \lambda_2^{b_j} + \lambda_3^{b_j} - 3 + \frac{1}{n} (J^{-nb_j} - 1) \right]$$

where  $C_j$ ,  $b_j$ , and  $n$  are material constants.  $J = \lambda_1\lambda_2\lambda_3$  and represents the ratio of the deformed to the undeformed state. The constant  $m$  is internally set to 4. If the number of points in the curve is less than 8, then  $m$  is set to the number of points divided by 2. The principal Cauchy stresses are:

$$t_i = \sum_{j=1}^m \frac{C_j}{J} \left[ \lambda_i^{b_j} - J^{-nb_j} \right] \quad i = 1,2,3 .$$

From the above equations the shear modulus is:

$$\mu = \frac{1}{2} \sum_{j=1}^m C_j b_j ,$$

and the bulk modulus is:

$$K = 2\mu \left( n + \frac{1}{3} \right) .$$

LS-DYNA uses the value for  $K$  defined in the input in the calculation of contact forces and for the material time step. Generally, this value should be equal to or greater than the  $K$  given in the above equation.

**\*MAT\_VISCOELASTIC\_HILL\_FOAM**

This is Material Type 178. This material is a highly compressible foam based on the strain-energy function proposed by Hill [1979]; also see Storakers [1986]. The extension to include large strain viscoelasticity is due to Feng and Hallquist [2002].

**Card Summary:**

**Card 1.** This card is required.

MID	R0	K	N	MU	LCID	FITTYPE	LCSR
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**Card 2.** This card is required.

LCVE	NT	GSTART					
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**Card 3.** This card is defined if and only if LCID = 0.

C1	C2	C3	C4	C5	C6	C7	C8
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**Card 4.** This card is defined if and only if LCID = 0.

B1	B2	B3	B4	B5	B6	B7	B8
----	----	----	----	----	----	----	----

**Card 5.** Include up to 12 of this card. The next keyword ("\*") card terminates this input.

Gi	BETAi						
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**Data Card Definitions:**

Card 1	1	2	3	4	5	6	7	8
Variable	MID	R0	K	N	MU	LCID	FITTYPE	LCSR
Type	A	F	F	F	F	I	I	I
Default	none	none	none	0.0	0.05	0	0	0

**VARIABLE****DESCRIPTION**

MID

Material identification. A unique number or label must be specified (see \*PART).

**\*MAT\_178****\*MAT\_VISCOELASTIC\_HILL\_FOAM**

<b>VARIABLE</b>	<b>DESCRIPTION</b>
RO	Mass density
K	Bulk modulus. This modulus is used for determining the contact interface stiffness.
N	Material constant. Define if LCID = 0 below; otherwise, N is fit from the load curve data. See remarks below.
MU	Damping coefficient (0.05 < recommended value < 0.50)
LCID	Load curve ID that defines the force per unit area as a function of the stretch ratio. This curve can be given for either uniaxial or bi-axial data depending on FITTYPE. Load curve LCSR below must also be defined.
FITTYPE	Type of fit: EQ.1: Uniaxial data EQ.2: Biaxial data
LCSR	Load curve ID that defines the uniaxial or biaxial stress ratio (see FITTYPE) as a function of the transverse stretch ratio

Card 2	1	2	3	4	5	6	7	8
Variable	LCVE	NT	GSTART					
Type	I	I	F					
Default	0	6	1/TMAX					

<b>VARIABLE</b>	<b>DESCRIPTION</b>
LCVE	Optional load curve ID that defines the relaxation function in shear. This curve is used to fit the coefficients $G_i$ and $\text{BETA}_i$ (see Card 5). If zero, define the coefficients directly (recommended).
NT	Number of terms used to fit the Prony series, which must be less than or equal to 12. This number should be equal to the number of decades of time covered by the experimental data. Define this number if LCVE is nonzero. Carefully check the fit in the d3hsp file to ensure that it is valid, since the least square fit is not always

<b>VARIABLE</b>	<b>DESCRIPTION</b>
	reliable.
GSTART	Starting value for the least squares fit. If zero, a default value is set equal to the inverse of the largest time in the experiment. Define this number if LCVE is nonzero. See remarks below.

**Material Constant Card 1.** Additional card for LCID = 0

Card 3	1	2	3	4	5	6	7	8
Variable	C1	C2	C3	C4	C5	C6	C7	C8
Type	F	F	F	F	F	F	F	F

**Material Constant Card 2.** Additional card for LCID = 0

Card 4	1	2	3	4	5	6	7	8
Variable	B1	B2	B3	B4	B5	B6	B7	B8
Type	F	F	F	F	F	F	F	F

<b>VARIABLE</b>	<b>DESCRIPTION</b>
$C_i$	Material constants. See remarks below. Define up to 8 coefficients.
$B_i$	Material constants. See remarks below. Define up to 8 coefficients.

**Viscoelastic Constant Cards.** Up to 12 cards may be input. The next keyword ("\*") card terminates this input.

Card 5	1	2	3	4	5	6	7	8
Variable	$G_i$	BETAI $i$						
Type	F	F						

<b>VARIABLE</b>	<b>DESCRIPTION</b>
$G_i$	Optional shear relaxation modulus for the $i^{\text{th}}$ term

<b>VARIABLE</b>	<b>DESCRIPTION</b>
BETAI	Optional decay constant if $i^{\text{th}}$ term

**Remarks:**

If load curve data is defined, the fit generated by LS-DYNA must be closely checked in the d3hsp output file. It may occur that the nonlinear least squares procedure in LS-DYNA, which is used to fit the data, is inadequate.

The Hill strain energy density function for this highly compressible foam is given by:

$$W = \sum_{j=1}^n \frac{C_j}{b_j} \left[ \lambda_1^{b_j} + \lambda_2^{b_j} + \lambda_3^{b_j} - 3 + \frac{1}{n} (J^{-nb_j} - 1) \right] ,$$

where  $C_j$ ,  $b_j$ , and  $n$  are material constants and  $J = \lambda_1 \lambda_2 \lambda_3$  represents the ratio of the deformed to the undeformed state. The principal Cauchy stresses are

$$\tau_{ii} = \sum_{j=1}^n \frac{C_j}{J} \left[ \lambda_i^{b_j} - J^{-nb_j} \right] \quad i = 1, 2, 3$$

From the above equations the shear modulus is:

$$\mu = \frac{1}{2} \sum_{j=1}^m C_j b_j$$

and the bulk modulus is:

$$K = 2\mu \left( n + \frac{1}{3} \right)$$

The value for  $K$  defined in the input is used in the calculation of contact forces and for the material time step. Generally, this value should be equal to or greater than the  $K$  given in the above equation.

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl} (t - \tau) \frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau ,$$

or in terms of the second Piola-Kirchhoff stress,  $S_{ij}$ , and Green's strain tensor,  $E_{ij}$ ,

$$S_{ij} = \int_0^t G_{ijkl} (t - \tau) \frac{\partial E_{kl}}{\partial \tau} d\tau ,$$

where  $g_{ijkl}(t - \tau)$  and  $G_{ijkl}(t - \tau)$  are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^N \alpha_m e^{-\beta_m t}$$

given by,

$$g(t) = \sum_{i=1}^n G_i e^{-\beta_i t}$$

This model is effectively a Maxwell fluid which consists of a dampers and springs in series. We characterize this in the input by shear moduli,  $G_i$ , and decay constants,  $\beta_i$ . The viscoelastic behavior is optional and an arbitrary number of terms may be used.