APPENDIX A

5-Point Solutions:

$$(x_0 + x_1\mu + x_2\mu^2 + x_3\mu^3 + x_4\mu^4)(y_0 + y_4\mu + y_3\mu^2 + y_2\mu^3 + y_1\mu^4)$$
$$mod(\mu^4 + \mu^3 + \mu^2 + \mu + 1) = \widetilde{A}_3\mu^3 + \widetilde{A}_2\mu^2 + \widetilde{A}_1\mu + \widetilde{A}_0$$

$\widetilde{A}_0$=**m0+m1**+m2+m3
$\widetilde{A}_1$=**m0+m1**+m4+m5
$\widetilde{A}_2$=**m0+m1**+m6+m7
$\widetilde{A}_3$=**m0+m1**+m8+m9

5-1
m0=(x2-x4)*(y0-y3) m1=(x1-x0)*(y1-y2)
m2=(y3-y4)*(x3-x4) m3=(y0-y2)*(x0-x2)
m4=(y0-y1)*(x1-x2) m5=(y4-y2)*(x0-x3)
m6=(y4-y1)*(x1-x3) m7=(y3-y2)*(x0-x4)
m8=(y4-y0)*(x2-x3) m9=(y3-y1)*(x1-x4)

5-2
m0=(x3-x1)*(y2-y4) m1=(x4-x0)*(y1-y0)
m2=(y2-y3)*(x2-x3) m3=(y1-y4)*(x1-x4)
m4=(y1-y3)*(x2-x4) m5=(y4-y0)*(x0-x1)
m6=(y1-y2)*(x3-x4) m7=(y3-y0)*(x0-x2)
m8=(y3-y4)*(x1-x2) m9=(y2-y0)*(x0-x3)

5-3
m0=(x3-x0)*(y1-y4) m1=(x2-x1)*(y2-y3)
m2=(y1-y3)*(x1-x3) m3=(y0-y4)*(x0-x4)
m4=(y1-y2)*(x2-x3) m5=(y0-y3)*(x1-x4)
m6=(y0-y2)*(x2-x4) m7=(y3-y4)*(x0-x1)
m8=(y0-y1)*(x3-x4) m9=(y2-y4)*(x0-x2)

5-4
m0=(x3-x4)*(y0-y4) m1=(x2-x0)*(y1-y3)
m2=(y1-y2)*(x1-x2) m3=(y0-y3)*(x0-x3)
m4=(y0-y2)*(x1-x3) m5=(y4-y3)*(x0-x4)
m6=(y0-y1)*(x2-x3) m7=(y4-y2)*(x1-x4)
m8=(y4-y1)*(x2-x4) m9=(y2-y3)*(x0-x1)

5-5
m0=(x3-x2)*(y3-y4) m1=(x4-x1)*(y2-y0)
m2=(y2-y4)*(x2-x4) m3=(y0-y1)*(x0-x1)
m4=(y2-y3)*(x3-x4) m5=(y4-y1)*(x0-x2)
m6=(y4-y0)*(x1-x2) m7=(y3-y1)*(x0-x3)
m8=(y3-y0)*(x1-x3) m9=(y2-y1)*(x0-x4)

7-Point Solutions:

$$(x_0 + x_1\mu + x_2\mu^2 + \cdots + x_6\mu^6)(y_0 + y_6\mu + y_5\mu^2 + \cdots + y_1\mu^6)$$
$$mod(\mu^6 + \mu^5 + \cdots + 1) = \widetilde{A}_5\mu^5 + \cdots + \widetilde{A}_1\mu + \widetilde{A}_0$$

$\widetilde{A}_0$=**m0+m1+m2**+m3+m4+m5
$\widetilde{A}_1$=**m0+m1+m2**+m6+m7+m8
$\widetilde{A}_2$=**m0+m1+m2**+m9+m10+m11
$\widetilde{A}_3$=**m0+m1+m2**+m12+m13+m14
$\widetilde{A}_4$=**m0+m1+m2**+m15+m16+m17
$\widetilde{A}_5$=**m0+m1+m2**+m18+m19+m20

7-1
m0=(x4-x6)*(y0-y5) m1=(x3-x0)*(y1-y4) m2=(x2-x1)*(y2-y3)
m3=(y5-y6)*(x5-x6) m4=(y1-y3)*(x1-x3) m5=(y0-y4)*(x0-x4)
m6=(y1-y2)*(x2-x3) m7=(y0-y3)*(x1-x4) m8=(y6-y4)*(x0-x5)
m9=(y0-y2)*(x2-x4) m10=(y6-y3)*(x1-x5) m11=(y5-y4)*(x0-x6)
m12=(y0-y1)*(x3-x4) m13=(y6-y2)*(x2-x5) m14=(y5-y3)*(x1-x6)
m15=(y6-y1)*(x3-x5) m16=(y5-y2)*(x2-x6) m17=(y3-y4)*(x0-x1)
m18=(y6-y0)*(x4-x5) m19=(y5-y1)*(x3-x6) m20=(y2-y4)*(x0-x2)

7-2
m0=(x5-x3)*(y4-y6) m1=(x6-x2)*(y3-y0) m2=(x1-x0)*(y1-y2)
m3=(y4-y5)*(x4-x5) m4=(y3-y6)*(x3-x6) m5=(y0-y2)*(x0-x2)
m6=(y3-y5)*(x4-x6) m7=(y0-y1)*(x1-x2) m8=(y6-y2)*(x0-x3)
m9=(y3-y4)*(x5-x6) m10=(y6-y1)*(x1-x3) m11=(y5-y2)*(x0-x4)
m12=(y6-y0)*(x2-x3) m13=(y5-y1)*(x1-x4) m14=(y4-y2)*(x0-x5)
m15=(y5-y0)*(x2-x4) m16=(y4-y1)*(x1-x5) m17=(y3-y2)*(x0-x6)
m18=(y5-y6)*(x3-x4) m19=(y4-y0)*(x2-x5) m20=(y3-y1)*(x1-x6)

7-3
m0=(x5-x1)*(y2-y6) m1=(x4-x2)*(y3-y5) m2=(x6-x0)*(y1-y0)
m3=(y3-y4)*(x3-x4) m4=(y2-y5)*(x2-x5) m5=(y1-y6)*(x1-x6)
m6=(y2-y4)*(x3-x5) m7=(y1-y5)*(x2-x6) m8=(y6-y0)*(x0-x1)
m9=(y2-y3)*(x4-x5) m10=(y1-y4)*(x3-x6) m11=(y5-y0)*(x0-x2)
m12=(y1-y3)*(x4-x6) m13=(y5-y6)*(x1-x2) m14=(y4-y0)*(x0-x3)
m15=(y1-y2)*(x5-x6) m16=(y4-y6)*(x1-x3) m17=(y3-y0)*(x0-x4)
m18=(y4-y5)*(x2-x3) m19=(y3-y6)*(x1-x4) m20=(y2-y0)*(x0-x5)

7-4
m0=(x5-x0)*(y1-y6) m1=(x4-x1)*(y2-y5) m2=(x3-x2)*(y3-y4)
m3=(y2-y4)*(x2-x4) m4=(y1-y5)*(x1-x5) m5=(y0-y6)*(x0-x6)
m6=(y2-y3)*(x3-x4) m7=(y1-y4)*(x2-x5) m8=(y0-y5)*(x1-x6)
m9=(y1-y3)*(x3-x5) m10=(y0-y4)*(x2-x6) m11=(y5-y6)*(x0-x1)
m12=(y1-y2)*(x4-x5) m13=(y0-y3)*(x3-x6) m14=(y4-y6)*(x0-x2)
m15=(y0-y2)*(x4-x6) m16=(y4-y5)*(x1-x2) m17=(y3-y6)*(x0-x3)
m18=(y0-y1)*(x5-x6) m19=(y3-y5)*(x1-x3) m20=(y2-y6)*(x0-x4)

7-5
m0=(x5-x4)*(y5-y6) m1=(x3-x6)*(y0-y4) m2=(x2-x0)*(y1-y3)
m3=(y4-y6)*(x4-x6) m4=(y1-y2)*(x1-x2) m5=(y0-y3)*(x0-x3)
m6=(y4-y5)*(x5-x6) m7=(y0-y2)*(x1-x3) m8=(y6-y3)*(x0-x4)
m9=(y0-y1)*(x2-x3) m10=(y6-y2)*(x1-x4) m11=(y5-y3)*(x0-x5)
m12=(y6-y1)*(x2-x4) m13=(y5-y2)*(x1-x5) m14=(y4-y3)*(x0-x6)
m15=(y6-y0)*(x3-x4) m16=(y5-y1)*(x2-x5) m17=(y4-y2)*(x1-x6)
m18=(y5-y0)*(x3-x5) m19=(y4-y1)*(x2-x6) m20=(y2-y3)*(x0-x1)

7-6
m0=(x5-x6)*(y0-y6) m1=(x4-x0)*(y1-y5) m2=(x3-x1)*(y2-y4)
m3=(y2-y3)*(x2-x3) m4=(y1-y4)*(x1-x4) m5=(y0-y5)*(x0-x5)
m6=(y1-y3)*(x2-x4) m7=(y0-y4)*(x1-x5) m8=(y6-y5)*(x0-x6)
m9=(y1-y2)*(x3-x4) m10=(y0-y3)*(x2-x5) m11=(y6-y4)*(x1-x6)
m12=(y0-y2)*(x3-x5) m13=(y6-y3)*(x2-x6) m14=(y4-y5)*(x0-x1)
m15=(y0-y1)*(x4-x5) m16=(y6-y2)*(x3-x6) m17=(y3-y5)*(x0-x2)
m18=(y6-y1)*(x4-x6) m19=(y3-y4)*(x1-x2) m20=(y2-y5)*(x0-x3)

7-7
m0=(x5-x2)*(y3-y6) m1=(x4-x3)*(y4-y5) m2=(x6-x1)*(y2-y0)
m3=(y3-y5)*(x3-x5) m4=(y2-y6)*(x2-x6) m5=(y0-y1)*(x0-x1)
m6=(y3-y4)*(x4-x5) m7=(y2-y5)*(x3-x6) m8=(y6-y1)*(x0-x2)
m9=(y2-y4)*(x4-x6) m10=(y6-y0)*(x1-x2) m11=(y5-y1)*(x0-x3)
m12=(y2-y3)*(x5-x6) m13=(y5-y0)*(x1-x3) m14=(y4-y1)*(x0-x4)
m15=(y5-y6)*(x2-x3) m16=(y4-y0)*(x1-x4) m17=(y3-y1)*(x0-x5)
m18=(y4-y6)*(x2-x4) m19=(y3-y0)*(x1-x5) m20=(y2-y1)*(x0-x6)

11-Point Solutions:

$$(x_0 + x_1\mu + x_2\mu^2 + \cdots + x_{10}\mu^{10})(y_0 + y_{10}\mu + y_9\mu^2 + \cdots + y_1\mu^{10}) \quad mod(\mu^{10} + \mu^9 + \cdots + 1) = \widetilde{A}_9\mu^9 + \cdots + \widetilde{A}_1\mu + \widetilde{A}_0$$

$\widetilde{A}_0$=**m0+m1+m2+m3+m4**+m5+m6+m7+m8+m9
$\widetilde{A}_1$=**m0+m1+m2+m3+m4**+m10+m11+m12+m13+m14
$\widetilde{A}_2$=**m0+m1+m2+m3+m4**+m15+m16+m17+m18+m19
$\widetilde{A}_3$=**m0+m1+m2+m3+m4**+m20+m21+m22+m23+m24
$\widetilde{A}_4$=**m0+m1+m2+m3+m4**+m25+m26+m27+m28+m29
$\widetilde{A}_5$=**m0+m1+m2+m3+m4**+m30+m31+m32+m33+m34
$\widetilde{A}_6$=**m0+m1+m2+m3+m4**+m35+m36+m37+m38+m39
$\widetilde{A}_7$=**m0+m1+m2+m3+m4**+m40+m41+m42+m43+m44
$\widetilde{A}_8$=**m0+m1+m2+m3+m4**+m45+m46+m47+m48+m49
$\widetilde{A}_9$=**m0+m1+m2+m3+m4**+m50+m51+m52+m53+m54

11-1
m0=(x8-x10)*(y0-y9)m1=(x0-x7)*(y1-y8)m2=(x6-x1)*(y2-y7)
m3=(x5-x2)*(y3-y6)m4=(x4-x3)*(y4-y5)
m5=(y9-y10)*(x9-x10)m6=(y3-y5)*(x3-x5)m7=(y2-y6)*(x2-x6)
m8=(y1-y7)*(x1-x7)m9=(y0-y8)*(x0-x8)
m10=(y3-y4)*(x4-x5)m11=(y2-y5)*(x3-x6)m12=(y1-y6)*(x2-x7)
m13=(y0-y7)*(x1-x8)m14=(y10-y8)*(x0-x9)
m15=(y2-y4)*(x4-x6)m16=(y1-y5)*(x3-x7)m17=(y0-y6)*(x2-x8)
m18=(y10-y7)*(x1-x9)m19=(y9-y8)*(x0-x10)
m20=(y2-y3)*(x5-x6)m21=(y1-y4)*(x4-x7)m22=(y0-y5)*(x3-x8)
m23=(y10-y6)*(x2-x9)m24=(y9-y7)*(x1-x10)
m25=(y1-y3)*(x5-x7)m26=(y0-y4)*(x4-x8)m27=(y10-y5)*(x3-x9)
m28=(y9-y6)*(x2-x10)m29=(y7-y8)*(x0-x1)
m30=(y1-y2)*(x6-x7)m31=(y0-y3)*(x5-x8)m32=(y10-y4)*(x4-x9)
m33=(y9-y5)*(x3-x10)m34=(y6-y8)*(x0-x2)
m35=(y0-y2)*(x6-x8)m36=(y10-y3)*(x5-x9)m37=(y9-y4)*(x4-x10)
m38=(y6-y7)*(x1-x2)m39=(y5-y8)*(x0-x3)
m40=(y0-y1)*(x7-x8)m41=(y10-y2)*(x6-x9)m42=(y9-y3)*(x5-x10)
m43=(y5-y7)*(x1-x3)m44=(y4-y8)*(x0-x4)
m45=(y10-y1)*(x7-x9)m46=(y9-y2)*(x6-x10)m47=(y5-y6)*(x2-x3)
m48=(y4-y7)*(x1-x4)m49=(y3-y8)*(x0-x5)
m50=(y10-y0)*(x8-x9)m51=(y9-y1)*(x7-x10)m52=(y4-y6)*(x2-x4)
m53=(y3-y7)*(x1-x5)m54=(y2-y8)*(x0-x6)

APPENDIX B

5-Point DFT Solutions:

Solution set 1-10:

$\tilde{A}_3: \quad m_1 + m_2 + m_3 + m_4$

$\tilde{A}_2: \quad m_1 + m_2 + m_5 + m_6$

$\tilde{A}_1: \quad m_1 + m_2 + m_7 + m_8$

Solution set 1, 2:

$\tilde{A}_0: \quad m_1 + m_4 + m_6 + m_8 + m_9 /$
$\quad m_1 - m_4 - m_6 + m_8 + m_9$

$m_1 = (x_0 - x_4)(y_0 - y_1);$
$m_2 = (x_1 - x_3)(y_4 - y_2);$
$m_3 = (x_0 - x_3)(y_2 - y_0);$
$m_4 = (x_1 - x_2)(y_3 - y_4);$
$m_5 = (x_0 - x_2)(y_3 - y_0);$
$m_6 = (x_3 - x_4)(y_1 - y_2);$
$m_7 = (x_0 - x_1)(y_4 - y_0);$
$m_8 = (x_2 - x_4)(y_1 - y_3);$
$m_9 = (x_1 - x_2 - x_3 + x_4)(y_1 - y_2 - y_3 + y_4);/$
$\quad (x_1 - x_2 + x_3 - x_4)(y_1 - y_2 + y_3 - y_4);$

Solution set 3, 4:

$\tilde{A}_0: \quad m_1 + m_4 + m_5 + m_7 + m_9 /$
$\quad m_1 - m_4 - m_5 + m_7 + m_9$

$m_1 = (x_0 - x_1)(y_2 - y_1);$
$m_2 = (x_2 - x_4)(y_0 - y_3);$
$m_3 = (x_1 - x_4)(y_3 - y_1);$
$m_4 = (x_2 - x_3)(y_4 - y_0);$
$m_5 = (x_0 - x_4)(y_3 - y_2);$
$m_6 = (x_1 - x_3)(y_4 - y_1);$
$m_7 = (x_0 - x_3)(y_4 - y_2);$
$m_8 = (x_1 - x_2)(y_0 - y_1);$
$m_9 = (x_0 + x_2 - x_3 - x_4)(y_0 + y_2 - y_3 - y_4);/$
$\quad (x_0 - x_2 + x_3 - x_4)(y_0 - y_2 + y_3 - y_4);$

Solution set 5, 6:

$\tilde{A}_0: \quad m_2 + m_4 + m_5 + m_7 + m_9 /$
$\quad m_2 - m_4 - m_5 + m_7 + m_9$

$m_1 = (x_0 - x_3)(y_4 - y_1);$
$m_2 = (x_1 - x_2)(y_3 - y_2);$
$m_3 = (x_0 - x_2)(y_2 - y_4);$
$m_4 = (x_3 - x_4)(y_0 - y_1);$
$m_5 = (x_0 - x_1)(y_3 - y_4);$
$m_6 = (x_2 - x_4)(y_0 - y_2);$
$m_7 = (x_1 - x_4)(y_0 - y_3);$
$m_8 = (x_2 - x_3)(y_1 - y_2);$
$m_9 = (x_0 - x_1 - x_3 + x_4)(y_0 - y_1 - y_3 + y_4);/$
$\quad (x_0 - x_1 + x_3 - x_4)(y_0 - y_1 + y_3 - y_4);$

Solution set 7, 8:

$\tilde{A}_0: \quad m_2 + m_3 + m_6 + m_7 + m_9 /$
$\quad m_2 - m_3 - m_6 + m_7 + m_9$

$m_1 = (x_1 - x_4)(y_0 - y_2);$
$m_2 = (x_2 - x_3)(y_4 - y_3);$
$m_3 = (x_0 - x_4)(y_2 - y_1);$
$m_4 = (x_1 - x_3)(y_3 - y_0);$
$m_5 = (x_0 - x_3)(y_3 - y_1);$
$m_6 = (x_1 - x_2)(y_4 - y_0);$
$m_7 = (x_0 - x_2)(y_4 - y_1);$
$m_8 = (x_3 - x_4)(y_2 - y_3);$
$m_9 = (x_0 + x_1 - x_2 - x_4)(y_0 + y_1 - y_2 - y_4);/$
$(x_0 - x_1 + x_2 - x_4)(y_0 - y_1 + y_2 - y_4);$

Solution set 9, 10:
$\tilde{A}_0 : \quad m_1 + m_3 + m_5 + m_7 + m_9 /$
$m_1 - m_3 - m_5 + m_7 + m_9$
$m_1 = (x_3 - x_4)(y_0 - y_4);$
$m_2 = (x_0 - x_2)(y_3 - y_1);$
$m_3 = (x_0 - x_1)(y_2 - y_3);$
$m_4 = (x_2 - x_4)(y_4 - y_1);$
$m_5 = (x_2 - x_3)(y_0 - y_1);$
$m_6 = (x_1 - x_4)(y_4 - y_2);$
$m_7 = (x_1 - x_3)(y_0 - y_2);$
$m_8 = (x_0 - x_4)(y_4 - y_3);$
$m_9 = (x_0 - x_1 - x_2 + x_3)(y_0 - y_1 - y_2 + y_3);/$
$(x_0 - x_1 + x_2 - x_3)(y_0 - y_1 + y_2 - y_3);$

Solution set 11-20:
$\tilde{A}_3 : \quad m_1 + m_2 + m_3 + m_4$
$\tilde{A}_2 : \quad m_1 + m_2 + m_5 + m_6$
$\tilde{A}_0 : \quad m_1 + m_2 + m_7 + m_8$

Solution set 11, 12:
$\tilde{A}_1 : \quad m_2 + m_4 + m_5 + m_7 + m_9 /$
$m_2 + m_4 - m_5 - m_7 + m_9$

$m_1 = (x_0 - x_4)(y_0 - y_1);$
$m_2 = (x_1 - x_3)(y_4 - y_2);$
$m_3 = (x_0 - x_3)(y_2 - y_0);$
$m_4 = (x_1 - x_2)(y_3 - y_4);$
$m_5 = (x_0 - x_2)(y_3 - y_0);$
$m_6 = (x_3 - x_4)(y_1 - y_2);$
$m_7 = (x_1 - x_4)(y_1 - y_4);$
$m_8 = (x_2 - x_3)(y_2 - y_3);$
$m_9 = (x_0 + x_1 - x_2 - x_4)(y_0 - y_1 - y_3 + y_4);/$
$(x_0 - x_1 - x_2 + x_4)(-y_0 - y_1 + y_3 + y_4);$

Solution set 13, 14:
$\tilde{A}_1 : \quad m_2 + m_4 + m_6 + m_7 + m_9 /$
$m_2 + m_4 - m_6 - m_7 + m_9$

$m_1 = (x_0 - x_1)(y_2 - y_1);$
$m_2 = (x_2 - x_4)(y_0 - y_3);$
$m_3 = (x_1 - x_4)(y_3 - y_1);$
$m_4 = (x_2 - x_3)(y_4 - y_0);$
$m_5 = (x_0 - x_4)(y_3 - y_2);$
$m_6 = (x_1 - x_3)(y_4 - y_1);$
$m_7 = (x_0 - x_2)(y_0 - y_2);$
$m_8 = (x_3 - x_4)(y_3 - y_4);$
$m_9 = (x_0 - x_1 - x_2 + x_3)(-y_0 - y_1 + y_2 + y_4);/$
$(x_0 + x_1 - x_2 - x_3)(y_0 - y_1 - y_2 + y_4);$

Solution set 15, 16:
$\tilde{A}_1 : \quad m_1 + m_4 + m_6 + m_8 + m_9 /$
$m_1 + m_4 - m_6 - m_8 + m_9$
$m_1 = (x_0 - x_3)(y_4 - y_1);$
$m_2 = (x_1 - x_2)(y_3 - y_2);$
$m_3 = (x_0 - x_2)(y_2 - y_4);$
$m_4 = (x_3 - x_4)(y_0 - y_1);$
$m_5 = (x_0 - x_1)(y_3 - y_4);$
$m_6 = (x_2 - x_4)(y_0 - y_2);$
$m_7 = (x_0 - x_4)(y_0 - y_4);$
$m_8 = (x_1 - x_3)(y_1 - y_3);$
$m_9 = (x_1 - x_2 - x_3 + x_4)(y_0 - y_1 - y_2 + y_3);/$
$(x_1 + x_2 - x_3 - x_4)(y_0 + y_1 - y_2 - y_3);$

Solution set 17, 18:
$\tilde{A}_1 : \quad m_1 + m_3 + m_5 + m_8 + m_9 /$
$m_1 + m_3 - m_5 - m_8 + m_9$
$m_1 = (x_1 - x_4)(y_0 - y_2);$
$m_2 = (x_2 - x_3)(y_4 - y_3);$
$m_3 = (x_0 - x_4)(y_2 - y_1);$
$m_4 = (x_1 - x_3)(y_3 - y_0);$
$m_5 = (x_0 - x_3)(y_3 - y_1);$
$m_6 = (x_1 - x_2)(y_4 - y_0);$
$m_7 = (x_0 - x_1)(y_0 - y_1);$
$m_8 = (x_2 - x_4)(y_2 - y_4);$
$m_9 = (x_0 + x_2 - x_3 - x_4)(y_1 - y_2 - y_3 + y_4);/$
$(x_0 - x_2 - x_3 + x_4)(-y_1 - y_2 + y_3 + y_4);$

Solution set 19, 20:
$\tilde{A}_1 : \quad m_1 + m_3 + m_6 + m_8 + m_9 /$
$m_1 + m_3 - m_6 - m_8 + m_9$

$m_1 = (x_3 - x_4)(y_0 - y_4);$
$m_2 = (x_0 - x_2)(y_3 - y_1);$
$m_3 = (x_0 - x_1)(y_2 - y_3);$
$m_4 = (x_2 - x_4)(y_4 - y_1);$
$m_5 = (x_2 - x_3)(y_0 - y_1);$
$m_6 = (x_1 - x_4)(y_4 - y_2);$
$m_7 = (x_1 - x_2)(y_1 - y_2);$
$m_8 = (x_0 - x_3)(y_0 - y_3);$
$m_9 = (x_0 - x_1 - x_3 + x_4)(-y_0 - y_2 + y_3 + y_4);/$
$(x_0 + x_1 - x_3 - x_4)(y_0 - y_2 - y_3 + y_4);$

Solution set 21-30:
$\tilde{A}_3 : \quad m_1 + m_2 + m_3 + m_4$
$\tilde{A}_1 : \quad m_1 + m_2 + m_5 + m_6$
$\tilde{A}_0 : \quad m_1 + m_2 + m_7 + m_8$

Solution set 21, 22:
$\tilde{A}_2 : \quad m_2 + m_3 + m_6 + m_8 + m_9 /$
$m_2 - m_3 - m_6 + m_8 + m_9$
$m_1 = (x_0 - x_4)(y_0 - y_1);$
$m_2 = (x_1 - x_3)(y_4 - y_2);$
$m_3 = (x_0 - x_3)(y_2 - y_0);$
$m_4 = (x_1 - x_2)(y_3 - y_4);$
$m_5 = (x_0 - x_1)(y_4 - y_0);$
$m_6 = (x_2 - x_4)(y_1 - y_3);$
$m_7 = (x_1 - x_4)(y_1 - y_4);$
$m_8 = (x_2 - x_3)(y_2 - y_3);$
$m_9 = (x_0 + x_2 - x_3 - x_4)(y_0 - y_1 - y_2 + y_3);/$
$(x_0 - x_2 - x_3 + x_4)(-y_0 - y_1 + y_2 + y_3);$

Solution set 23, 24:
$\tilde{A}_2 : \quad m_2 + m_3 + m_5 + m_8 + m_9 /$
$m_2 - m_3 - m_5 + m_8 + m_9$
$m_1 = (x_0 - x_1)(y_2 - y_1);$
$m_2 = (x_2 - x_4)(y_0 - y_3);$
$m_3 = (x_1 - x_4)(y_3 - y_1);$
$m_4 = (x_2 - x_3)(y_4 - y_0);$
$m_5 = (x_0 - x_3)(y_4 - y_2);$
$m_6 = (x_1 - x_2)(y_0 - y_1);$
$m_7 = (x_0 - x_2)(y_0 - y_2);$
$m_8 = (x_3 - x_4)(y_3 - y_4);$
$m_9 = (x_0 - x_1 - x_3 + x_4)(-y_1 + y_2 + y_3 - y_4);/$
$(x_0 + x_1 - x_3 - x_4)(-y_1 - y_2 + y_3 + y_4);$

Solution set 25, 26:
$\tilde{A}_2 : \quad m_1 + m_3 + m_5 + m_7 + m_9 /$
$m_1 - m_3 - m_5 + m_7 + m_9$
$m_1 = (x_0 - x_3)(y_4 - y_1);$
$m_2 = (x_1 - x_2)(y_3 - y_2);$
$m_3 = (x_0 - x_2)(y_2 - y_4);$
$m_4 = (x_3 - x_4)(y_0 - y_1);$
$m_5 = (x_1 - x_4)(y_0 - y_3);$
$m_6 = (x_2 - x_3)(y_1 - y_2);$
$m_7 = (x_0 - x_4)(y_0 - y_4);$
$m_8 = (x_1 - x_3)(y_1 - y_3);$
$m_9 = (x_0 + x_1 - x_2 - x_4)(-y_0 - y_2 + y_3 + y_4);/$
$(x_0 - x_1 - x_2 + x_4)(-y_0 + y_2 + y_3 - y_4);$

Solution set 27, 28:
$\tilde{A}_2 : \quad m_1 + m_4 + m_5 + m_7 + m_9 /$
$m_1 - m_4 - m_5 + m_7 + m_9$
$m_1 = (x_1 - x_4)(y_0 - y_2);$
$m_2 = (x_2 - x_3)(y_4 - y_3);$
$m_3 = (x_0 - x_4)(y_2 - y_1);$
$m_4 = (x_1 - x_3)(y_3 - y_0);$
$m_5 = (x_0 - x_2)(y_4 - y_1);$
$m_6 = (x_3 - x_4)(y_2 - y_3);$
$m_7 = (x_0 - x_1)(y_0 - y_1);$
$m_8 = (x_2 - x_4)(y_2 - y_4);$
$m_9 = (x_0 - x_1 - x_2 + x_3)(-y_0 + y_1 + y_3 - y_4);/$
$(x_0 + x_1 - x_2 - x_3)(-y_0 - y_1 + y_3 + y_4);$

Solution set 29, 30:
$\tilde{A}_2 : \quad m_2 + m_4 + m_5 + m_7 + m_9 /$
$m_2 - m_4 - m_5 + m_7 + m_9$
$m_1 = (x_3 - x_4)(y_0 - y_4);$
$m_2 = (x_0 - x_2)(y_3 - y_1);$
$m_3 = (x_0 - x_1)(y_2 - y_3);$
$m_4 = (x_2 - x_4)(y_4 - y_1);$
$m_5 = (x_1 - x_3)(y_0 - y_2);$
$m_6 = (x_0 - x_4)(y_4 - y_3);$
$m_7 = (x_1 - x_2)(y_1 - y_2);$
$m_8 = (x_0 - x_3)(y_0 - y_3);$
$m_9 = (x_1 - x_2 - x_3 + x_4)(-y_0 - y_1 + y_2 + y_4);/$
$(x_1 + x_2 - x_3 - x_4)(y_0 - y_1 - y_2 + y_4);$

Solution set 31-40:
$\tilde{A}_2 : \quad m_1 + m_2 + m_3 + m_4$
$\tilde{A}_1 : \quad m_1 + m_2 + m_5 + m_6$
$\tilde{A}_0 : \quad m_1 + m_2 + m_7 + m_8$

Solution set 31, 32:
$\tilde{A}_3 : \quad m_1 + m_3 + m_5 + m_8 + m_9 /$
$m_1 + m_3 - m_5 - m_8 + m_9$

$m_1 = (x_0 - x_4)(y_0 - y_1);$
$m_2 = (x_1 - x_3)(y_4 - y_2);$
$m_3 = (x_0 - x_2)(y_3 - y_0);$
$m_4 = (x_3 - x_4)(y_1 - y_2);$
$m_5 = (x_0 - x_1)(y_4 - y_0);$
$m_6 = (x_2 - x_4)(y_1 - y_3);$
$m_7 = (x_1 - x_4)(y_1 - y_4);$
$m_8 = (x_2 - x_3)(y_2 - y_3);$
$m_9 = (x_0 - x_1 - x_2 + x_3)(y_0 + y_2 - y_3 - y_4);/$
$(x_0 - x_1 + x_2 - x_3)(-y_0 + y_2 - y_3 + y_4);$

Solution set 33, 34:
$\tilde{A}_3 : \quad m_1 + m_4 + m_6 + m_8 + m_9 /$
$m_1 + m_4 - m_6 - m_8 + m_9$
$m_1 = (x_0 - x_1)(y_2 - y_1);$
$m_2 = (x_2 - x_4)(y_0 - y_3);$
$m_3 = (x_0 - x_4)(y_3 - y_2);$
$m_4 = (x_1 - x_3)(y_4 - y_1);$
$m_5 = (x_0 - x_3)(y_4 - y_2);$
$m_6 = (x_1 - x_2)(y_0 - y_1);$
$m_7 = (x_0 - x_2)(y_0 - y_2);$
$m_8 = (x_3 - x_4)(y_3 - y_4);$
$m_9 = (x_1 - x_2 - x_3 + x_4)(-y_0 + y_1 + y_3 - y_4);/$
$(x_1 - x_2 + x_3 - x_4)(y_0 - y_1 + y_3 - y_4);$

Solution set 35, 36:
$\tilde{A}_3 : \quad m_2 + m_4 + m_6 + m_7 + m_9 /$
$m_2 + m_4 - m_6 - m_7 + m_9$
$m_1 = (x_0 - x_3)(y_4 - y_1);$
$m_2 = (x_1 - x_2)(y_3 - y_2);$
$m_3 = (x_0 - x_1)(y_3 - y_4);$
$m_4 = (x_2 - x_4)(y_0 - y_2);$
$m_5 = (x_1 - x_4)(y_0 - y_3);$
$m_6 = (x_2 - x_3)(y_1 - y_2);$
$m_7 = (x_0 - x_4)(y_0 - y_4);$
$m_8 = (x_1 - x_3)(y_1 - y_3);$
$m_9 = (x_0 + x_2 - x_3 - x_4)(-y_0 - y_1 + y_2 + y_4);/$
$(x_0 - x_2 + x_3 - x_4)(y_0 - y_1 + y_2 - y_4);$

Solution set 37, 38:
$\tilde{A}_3 : \quad m_2 + m_3 + m_6 + m_7 + m_9 /$
$m_2 + m_3 - m_6 - m_7 + m_9$
$m_1 = (x_1 - x_4)(y_0 - y_2);$
$m_2 = (x_2 - x_3)(y_4 - y_3);$
$m_3 = (x_0 - x_3)(y_3 - y_1);$
$m_4 = (x_1 - x_2)(y_4 - y_0);$
$m_5 = (x_0 - x_2)(y_4 - y_1);$
$m_6 = (x_3 - x_4)(y_2 - y_3);$
$m_7 = (x_0 - x_1)(y_0 - y_1);$
$m_8 = (x_2 - x_4)(y_2 - y_4);$
$m_9 = (x_0 - x_1 - x_3 + x_4)(-y_0 + y_1 + y_2 - y_3);/$
$(x_0 - x_1 + x_3 - x_4)(y_0 - y_1 + y_2 - y_3);$

Solution set 39, 40:
$\tilde{A}_3 : \quad m_1 + m_4 + m_6 + m_7 + m_9 /$
$m_1 + m_4 - m_6 - m_7 + m_9$
$m_1 = (x_3 - x_4)(y_0 - y_4);$
$m_2 = (x_0 - x_2)(y_3 - y_1);$
$m_3 = (x_2 - x_3)(y_0 - y_1);$
$m_4 = (x_1 - x_4)(y_4 - y_2);$
$m_5 = (x_1 - x_3)(y_0 - y_2);$
$m_6 = (x_0 - x_4)(y_4 - y_3);$
$m_7 = (x_1 - x_2)(y_1 - y_2);$
$m_8 = (x_0 - x_3)(y_0 - y_3);$
$m_9 = (x_0 + x_1 - x_2 - x_4)(-y_1 + y_2 + y_3 - y_4);/$
$(x_0 - x_1 + x_2 - x_4)(-y_1 + y_2 - y_3 + y_4);$

APPENDIX C

7-Point Heuristic WFTA:

$(x_0 + x_1\mu + x_2\mu^2 + \cdots + x_6\mu^6)(y_0 + y_6\mu + y_5\mu^2 + \cdots + y_1\mu^6) \mod(\mu^6 + \mu^5 + \cdots + 1) = \widetilde{A}_5\mu^5 + \cdots + \widetilde{A}_1\mu + \widetilde{A}_0$

$\widetilde{A}_5$= m3 - m1 + m6 - m7 + m9 + m13 - m15 - m18;
$\widetilde{A}_4$= m3 - m2 + m6 - (2*m7)/3 - (2*m8)/3 + (4*m9)/3 - m10/6 - m11/2+ m12 + (2*m13)/3 + (2*m14)/3 - (4*m15)/3 + m16/6 + m17/2 - 2*m18;
$\widetilde{A}_3$= m6 + m7/3 - (2*m8)/3 + m9/3 - m10/6 + m11/2 + m13/3 + m14/3 - (2*m15)/3 + m16/3 - 2*m18;
$\widetilde{A}_2$= m3 - m1 + m6 - m12 + m13/3 + m14/3 - (2*m15)/3 + m16/3 - m18;
$\widetilde{A}_1$= (4*m3)/3 - (2*m2)/3 - (2*m1)/3 - m4/6 - m5/2 + 2*m6 - m7 + m9 + m13/3 + m14/3 - (2*m15)/3 + m16/3 - m18;
$\widetilde{A}_0$= m1/3 - (2*m2)/3 + m3/3 - m4/6 + m5/2 + m6 - m8 + m9 - m13/3 +(2*m14)/3 - m15/3 + m16/6 - m17/2;

m1=(x0-x2)*(y0-y5);
m2=(x0-x1)*(y0-y6);
m3=(x1-x2)*(y6-y5)
m4=(x0+x1+x2-3x6)*(y0+y6+y5-3y1);
m5=(x0+x2-x1-x6)*(y0+y5-y1-y6);
m6=(x0-x6)*(y0-y1)

m7=(x3-x5)*(y4-y2);
m8=(x4-x3)*(y3-y4);
m9=(x4-x5)*(y3-y2)
m10=(x3+x4+x5-3x6)*(y4+y3+y2-3y1);
m11=(x3+x5-x4-x6)*(y4+y2-y3-y1);
m12=(x3-x6)*(y4-y1)

m13=(x0+x5-x3-x2)*(y0+y2-y5-y4);
m14=(x1-x4+x3-x0)*(y6-y3-y0+y4);
m15=(x1+x5-x2-x4)*(y6+y2-y5-y3);
m16=(x0+x1+x2-x3-x4-x5)*(y0+y6+y5-y3-y4-y2);
m17=(x0-x3+x4-x1+x2-x5)*(y0-y4+y3-y6+y5-y2);
m18=(x0-x3)*(y0-y4)