

# Integer Programming

↳ Enumerative solution

# of decision possibilities

↳  $2^n$  (for binary)

↳  $\infty$  to  $n \Rightarrow$  Int.

↳ Logical constraints

↳ gates  $\overline{A} \cdot B$

$dB \leq dA$

↳ on binary variables  
↳ get in subspace

$$d_i + d_j + d_k \leq 2$$

↳ Big M problem

If only 2 from left

Total  $d \rightarrow d_0, d_1, \dots, d_i, d_j$

↳ Integer / Continuous + Binary

↳ Establish something

↳ # of items

↳ Ex. factory  $\forall N$

can be any #

$0/1$

$B_i$

$$B_i \leq 0_i \leq M \times B_i$$

↳ Constant for all  $B_i, 0_i$

$0_i$

↳ Diff. Determined

after assuming

sets Upper Bound of

list  $(0_i)$

⇒ Objective  $\rightarrow l_1 \times 50K + l_2 \times 55K + l_3 \times 65K + t_{e1} \times 200 + t_{e2} \times 200$

## Integer Programming PRACTISE :-

Foran wants to manufacture the a car :-

- ↳ 3 engine choices  $\rightarrow$  2 pseudo choices
- + 1 turbo engine choice
- + 2 break choices
- + 2 interior choices.

Decision Variables :-

$l_1, l_2, l_3, t_{e1}, t_{e2}, i_1, i_2$

↳ All are binary decisions.  $\rightarrow \in [0, 1]$

~~Constraints~~  $\rightarrow$

1. Cost associated objective

$$\Rightarrow l_1 \times 50K + l_2 \times 55K + l_3 \times 65K + t_{e1} \times 200 + t_{e2} \times 200 + i_1 \times 7K + i_2 \times 16K + i_2 \times 20K. = \text{obj}$$

↳ Minimise.

2. Constraints.

1. Horsepower constraint :-

$$l_1 \times 480 + l_2 \times 600 + l_3 \times 750 + t_{e1} \times 200 \geq 600$$

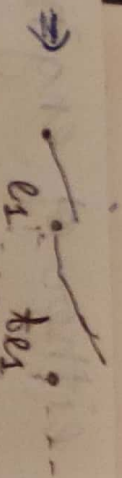
①

2. Association of engines.

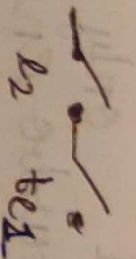
↳  $l_1$  and  $l_2$  with  $t_{e1}$

↳  $l_3$  and  $t_{e2}$  never together.

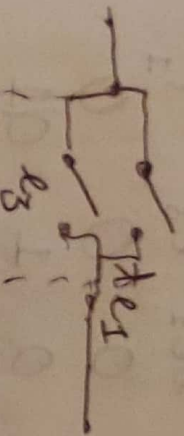




$$tel_1 \leq l_1 - (2)$$



$$tel_1 \leq l_2 - (3)$$



$$tel_1 + l_3 \leq 1 - (4)$$

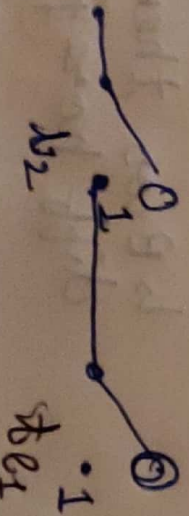
Always another line  $\therefore$  both  $tel_1$  and  $l_3 = 0$

3. Only 1 <sup>primary</sup> engine allowed  $\therefore$   
 $\hookrightarrow l_1 + l_2 + l_3 = 1 - (5)$

4. Only 1 break allowed  $\therefore$   
 $\hookrightarrow l_1 + l_2 = 1 - (6)$

5. Break and engine combination  $\therefore$  अपेक्षित  
 If two engine then break  $l_2$  (ये ही चुका दीना चाहिए).

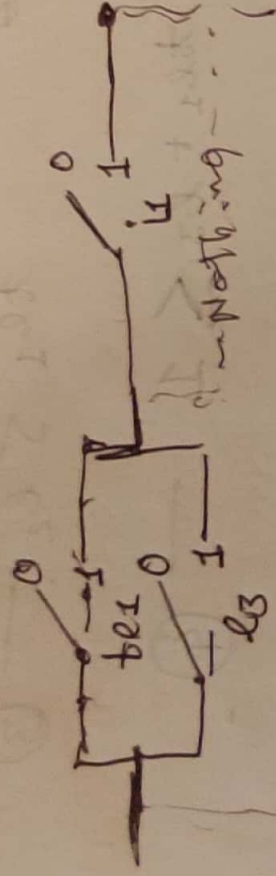
for this we require  $l_2 \rightarrow ON$ .



$$tel_1 \leq l_2 - (7)$$

6. Exactly 1 interior  
 $l_1 + l_2 = 1 - (8)$

7. Interior 1 only when either of engine .3 or turbo engine.



te1	e3	i1
0	0	0
0	1	0/1
1	0	0/1
1	1	0/1

and = Multiplication  
or = Addition - ✓

te1 + e3	i1
0	0
1	0/1
1	0/1
2	0/1

$$i_1 \leq te_1 + e_3 \text{ --- (9)}$$

Mistakes :-

1. There are two dec. vars for turbo eng.  
↳ Even if we have got 1 entity, we can have two d vars

Pseudo 2 turbo engines.  $te_1, te_2$

↳ To use it with  $e_1$   
+ To use it with  $e_2$

↳ Bcos they use diff horse power.

~~Fig~~  $A[0,1] [2,5]$

↳  $A[0,2] A[1,5]$

$A[1, [2,8]]$  → Broadcasted

↳  $A[1,2], A[1,5]$

np. ix =  $[1,2,3], [2,5]$

↳  $\begin{bmatrix} 1,2,15 \\ 2,2,25 \\ 3,2,35 \end{bmatrix}$

But  $A[1,2,3] [2,5]$   
X - Fails.



$$\Rightarrow \text{Objective} \rightarrow l_1 \times 50K + l_2 \times 55K + l_3 \times 68K + \\ t l_1 \times 8K + t l_2 \times 8K + l_1 \times 5K, l_2 \times 7K + \\ i_1 \times 25K + i_2 \times 20K.$$

$$C.1 \rightarrow l_1 \times 480 + l_2 \times 600 + l_3 \times 750 + t l_1 \times \\ 200 + t l_2 \times 150 \geq 600$$

0 1 2 3 4 5 6 7 8

↳ For a dict. if required,

$$C.2 \rightarrow t l_1 \leq l_1.$$

$$C.3 \rightarrow t l_2 \leq l_2.$$

$$C.4 \rightarrow t l_1 + t l_2 + l_3 \leq 1.$$

$$C.5 \rightarrow l_1 + l_2 + l_3 = 1.$$

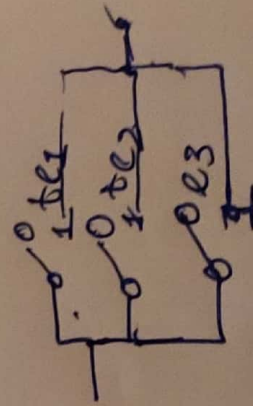
$$C.6 \rightarrow l_1 + l_2 = 1.$$

$$C.7 \rightarrow t l_1 \leq l_2 + t l_2 \leq l_2$$

$$\hookrightarrow t l_1 + t l_2 \leq 2 l_2$$

$$C.8 \rightarrow i_1 + i_2 = 1.$$

$$C.9 \rightarrow i_1 \leq t l_1 + t l_2 + l_3.$$



$$\text{f-vars} \rightarrow l_1 \quad l_2 \quad l_3$$

$$t l_1 \quad t l_2$$

$$l_1 \quad l_2$$

$$i_1 \quad i_2.$$