1 · Fair coins + 99 ~~unfa~~ biased coins

↳ Probability of even Heads.

↳

↳

Binomial = Combination of Bernoulli's

Two outcomes.

H, T

(p) $(1-p)$

The coins are assumed to be independe-
-nt of each other.

Thus. the 100 coin tosses will get multi-
-plied with eachother.

↳ This will generate one event of the binomial

↳ Lets say we assume to have 3 heads + 97 tails.

$$^{100}C_3 \times p^3 \times (1-p)^{97}$$

↳ 3 combinations from 100 throws.

⇒ ~~Total~~. Event SET ⇒ {0, 1H, 2H . . . . 100H}

$$\overset{1.}{(p+1-p)^{100}} = {}^{100}C_0\, p^0 (1-p)^{100} + {}^{100}C_1\, p^1 \times (1-p)^{99} \ldots$$

$$\ldots\ {}^{100}C_{50}\, p^{50} \times (1-p)^{50} + \ldots$$

$$\ldots\ {}^{100}C_{99}\, p^{99} \times (1-p)^1 + {}^{100}C_{100}\, p^{100} \times (1-p)^0$$

Lets narrow down to 3 tosses.
1 fair + 2 biased.

$$ {}^{3}C_0 \times p^{0} (1-p)^3 + {}^{3}C_1 \, p^1 \times (1-p)^2 + $$

$$ {}^{3}C_2 \times p^2 \times (1-p)^1 + {}^{3}C_3 \, p^3 \times (1-p)^0 $$

↳ These are the four combination of events.

EVEN Heads :-

$$ {}^{3}C_0 \, p^0 (1-p)^3 \quad + \quad {}^{3}C_2 \times p^2 (1-p)^1 . $$

0   2

$$ 1 \times 0.5 \times (1-p)^2 \quad + \quad 3 \times 0.5 \; ? $$

** This method FAILS :-

⇒ EMPERICAL Method

⇒ ⇒ 2 tosses → 1 fair + 1 biased. (H → p.)

Events space → HH, HT, TH, TT

prob ⟶ $0.5 \times p$; $0.5 \times (1-p)$; $0.5p$; $0.5(1-p)$

Even heads → 0, 2.

TT   HH.

$0.5 \times (1-p)$; $0.5 \times p$.

$= 0.5$

$2^3 \to 8$

3 Tosses → 1 fair + 2 biased

FAIR → Biased.

HHH
HHT ✓
HTH ✓
THH
HTT
THT
TTH
TTT ✓

Even heads :- 0, 2.

0 :- $\cancel{0.5 \times p^2}$ $0.5 \times (1-p)^2$ (T)

2 :- $0.5 \times p \times (1-p) + 0.5 \times (1-p) \times p + 0.5 \times p^2$ (T, T, H)

Total prob :-

$\cancel{0.5p^2 + 0.5\,p(1-p) + 0.5\,p(1-p)\,p + 0.5\,p^2}$ (H, T, T, H)

On rearranging.

$1+p^2-2p$ → 0.5 cancel. $2 \times 0.5(p - p^2)$

$\left[0.5 \times (1-p)^2\right] + \left[0.5\,p(1-p) + 0.5(1-p)p + 0.5 \times p^2\right]$ (T, T, T, H)

$= 0.5$

Catch from the interview.

$\rightarrow$ 1 odd + Odd = Even

↳ If fair is H → We only require odd number of Heads.

$\rightarrow$ 0 + ? = Even ⟹ ? = Even

↳ If fair is T → We require even number of HEADS.

↳ From remaining Tosses.

Independent Events

1 toss + Remaining tosses.

P(Even Heads) =

p(fair head) × p(odd heads from 99 bias)

+

p(fair tails) × p(even heads from 99 bias)

Complimentary events.

↳ In any number of tosses p(odd heads) = 1 - p(even heads)

∴ 0.5 × p(odd heads) + 0.5 × [1 - p(odd H)]

= 0.5 × P_O_H + 0.5 × 1 - 0.5 × P_O_H

= 0.5 (Ans).