TSP - optimization

Travelling Salesperson Problem.

↳ Integer programming problem.

↳ Travel each city

↳ Order of city so that distance is minimum.

→ City x City matrix with distances.

City! ways to do/perform this

↳ visit each city once.
visit all the cities.

A B C .. 3
3/2x1

↳ solution (feasible) ~~to find~~ for this problem
↳ Need to find optimal solution.

⇒ In every city, we need to choose which city to select for our next trip

*
↳ We can define city x city matrix for our decision variable.

↳ x_{ij} : Binary Travel from i to j

*↳ Can always constraint $x_{ii} = 0$ → ~~Even distance are 0~~

* OBJECTIVE :-
Minimize distances

↳ Map/Multiply ~~of~~ binary decision with distance matrix.

→ Element wise/dot product

⇒ Enter each city once ; Leaves each city once.

X_{ij} → From i to j

$$\sum_{i=1}^{N} X_{ij} = 1$$ (Come from any city, you can visit j only once)

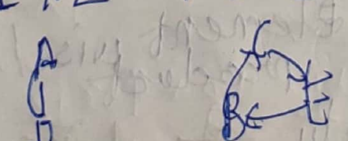
$$\sum_{j=1}^{N} X_{ij} = 1$$ (Visit any city but you can leave i only once)

↳ We will have 5 x_{ij} that are positive/1; we need to rearrange them at the end to find the road-map.

ISSUES :-

1. We need a contraint that supervises that we covered each city once.
2. There are no sub loops → for 5 cities

$1 \times 2 + 1 \times 3$ →



x_{AD} ; x_{DA} ; x_{CE}, x_{EB}, x_{BC} = 1   sum is 5 but all cities are not covered.

my first hunch → $x_{ij} + x_{ji} \leq 2$.

↳ only removes 2 city loop

→ x_i

$7 = 4 + 3$ ↳ $5 + 2$ We need to eliminate all small loops.

⇒ ~~At~~ times $d_{ij} \neq d_{ji}$ (can take different paths A–B, B–A)

N = 5

23
24 34
25 35 45

* STROKE of Brilliance.

$(n-1)^2 - (n-1)$ constraints

$u_2 - u_3 + N \times x_{23} \leq N-1$

$u_i - u_j + N \times x_{ij} \leq N-1$

$u_3 - u_2 + N \times x_{32} \leq N-1$.

→ Add these two

$N(\underbrace{x_{23} + x_{32}}_{\text{cannot be both } 1}) \leq 2N-2$.

→ Removes 2-city loops.

2 3

1, 3, 5

→ 1 possible 3 ~~too~~ node subloop

$u_1 - u_3 + \cancel{N x_{13}} \leq N-1$

$u_1 - u_5 + \cancel{N x_{15} \leq N-1}$

$\cancel{u_3 - u_5} + N x_{35} \leq \cancel{N}-1$

$\cancel{u_5 - u_3} + N \cancel{x_{53}} \leq N-1$

$u_3 - \cancel{u_1} + N \cancel{x}_{31}$

$u_5 - u_1 + N x_{51}$

cannot be 3?

$u_1 - u_3 + N x_{13} \leq N-1$

$u_3 - u_5 + N x_{35} \leq N-1$

$u_5 - u_1 + N x_{51} \leq N-1$

$N(\underbrace{x_{13} + x_{35} + x_{51}}) \leq 3N-3$

Final N sized loop not ignored because neither of i or j incorporates 1

↳ Can be some other value too.

Decision variables $u_{1 \text{ to } N}$

a_{ij} ↳ $N^2 + N$

u_1 can be skipped but for consistency.

Total constraints :-

N → 1 time visit / arrival

+ N → 1 time departure.

+ $(N-1)^2 - (N-1)$ → No sub-looping.

x

$i \to 0$ to N

A → 0 to N

0 : 5 ; 5 : 10 ; 10 : 15

I ↳ row, $i{*}N : (i+1)N$ = 1 → will be included in constraints

$i{*}N : (i+1)N$

II

↳ row, i : $N{*}N - (N-i)$ ← 5th

[[00, 01, 02, 03, 04
10, 11, 12, 13, 14
20, 21, 22, 23, 24
30, 31, 32, 33, 34
40, 41, 42, 43, 44]]

0, 5, 10, 15, 20

More intuitive way :-

0 - row, [0, 1, 2, 3, 4] + Ni ← 5

↳ row slider.

↳ 00, 01, 02, 03, 04, 10
11, - - - -

0 1 2 3 4

5 6 7 8 9

row, [0, 1, 2, 3, 4]×N + i

↳ Column slider.