Lecture4: Instantaneous Velocity of Moving Frames

Instantaneous Velocity of Rotating Frame

$$\frac{\mathrm{d}}{\mathrm{d}t}R_A(t) = [\omega_A(t)]R_A(t) \Rightarrow [\omega_A(t)] = \dot{R}_A(t)R_A^{-1}(t)$$

Proof:

- In world coordinate

$$\begin{align}
R_A(t) &= [\hat{x}_A \quad \hat{y}_A \quad \hat{z}_A] \tag{1}\\
\dot{\hat{x}}_A &= \omega_A \times \hat{x}_A \tag{2}\\
\dot{\hat{y}}_A &= \omega_A \times \hat{y}_A \tag{3}\\
\dot{\hat{z}}_A &= \omega_A \times \hat{z}_A \tag{4}\\
\dot{R}_A &= \left[\dot{\hat{x}}_A \quad \dot{\hat{y}}_A \quad \dot{\hat{z}}_A\right] = \omega_A \times R_A = [\omega_A]R_A \tag{5}\\
{}^o\dot{R}_A &= [{}^o\omega_A]{}^oR_A \Rightarrow [{}^o\omega_A] = {}^o\dot{R}_A{}^oR_A^{-1} \tag{6}
\end{align}$$

- In {A} frame

  Remember that $[R\omega] = R[\omega]R^T$

$$\begin{align}
{}^A\omega_A &= {}^AR_o{}^o\omega_A \tag{7}\\
[{}^o\omega_A] &= {}^o\dot{R}_A{}^oR_A^{-1} \tag{8}\\
[{}^A\omega_A] &= [{}^AR_o{}^o\omega_A] \tag{9}\\
&= {}^AR_o[{}^o\omega_A]{}^AR_o^T \tag{10}\\
&= {}^AR_o{}^o\dot{R}_A{}^oR_A^{-1}{}^oR_A \tag{11}\\
&= {}^AR_o{}^o\dot{R}_A \tag{12}\\
[{}^A\omega_A] &= {}^oR_A^{-1}{}^o\dot{R}_A \tag{13}
\end{align}$$

Instantaneous Velocity of Moving Frame

$$\frac{\mathrm{d}}{\mathrm{d}t}T_A(t) = [\mathcal{V}_A(t)]T_A(t) \Rightarrow [\mathcal{V}_A(t)] = \dot{T}_A(t)T_A^{-1}(t)$$

You can find the proof in noted lecture slide. The process is similar to rotation frame.