Lecture14: Proximal Policy Optimization (PPO)

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Lecture14: Proximal Policy Optimization (PPO)
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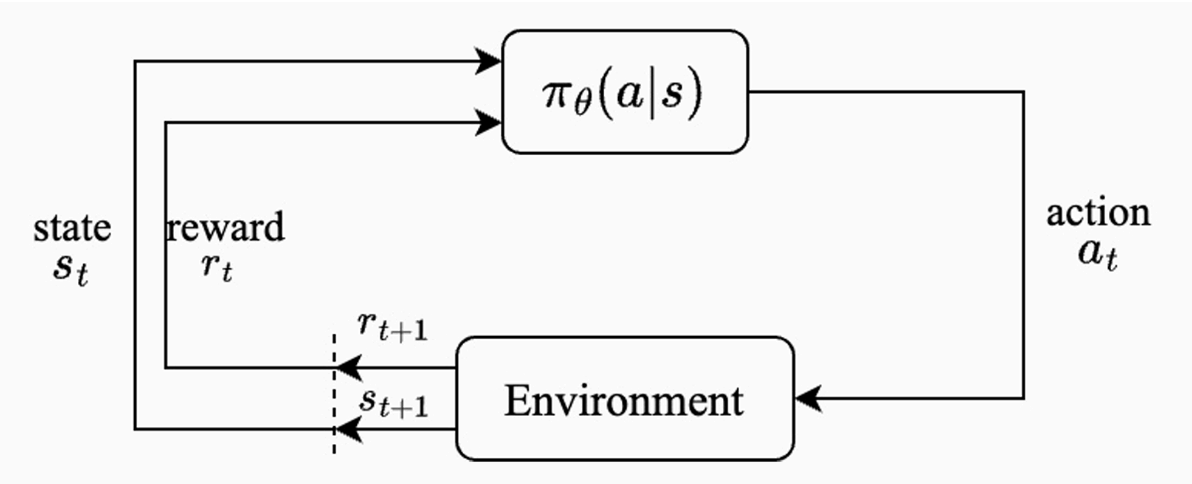
Review: Policy Optimization



- $\pi(a|s)$: Probability of action a in state s
- Objective: Maximize expected cumulative reward
  $\max_\theta \mathbf{E}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{T}\gamma^t r_t\right]$

Review: Vanilla Policy Gradient (VPG)

Input: Initial θ_0, V_ϕ

Repeat:

1. **Collect trajectories**: $\tau \sim \pi_\theta$
2. **For each timestep** t:
   - $G_t = \sum_{t'=t}^{T-1}\gamma^{t'-t} r_{t'}$ (return)
   - $A_t = G_t - V_\phi(s_t)$ (advantage function)
3. **Update** V_ϕ:

$$V_\phi = \text{argmin}_\phi \sum \|V_\phi(s_t) - G_t\|^2$$

4. **Policy Update**:

$$\theta = \theta + \alpha \sum \nabla_\theta \log \pi_\theta(a_t|s_t) A_t$$

Importance Sampling and Surrogate Loss

- Policy Gradient

$$J(\theta) = \mathbf{E}[\log \pi_\theta(a_t|s_t) \cdot A(s_t, a_t)]$$

$$\nabla_\theta J(\theta) = \mathbf{E}[\nabla_\theta \log \pi_\theta(a_t|s_t) \cdot A(s_t, a_t)]$$

- Surrogate Loss Function (替代损失函数):

$$L_{\text{surr}} = \frac{\pi_\theta(a_t|s_t)}{\pi_{\text{old}}(a_t|s_t)} A(s_t, a_t)$$

Here we prove that L_{surr} can replace policy gradient to update the policy

Prove that $\nabla_\theta \mathbf{E}(L_{\text{surr}}) = \nabla_\theta J(\theta)$ when $\theta = \theta_{\text{old}}$

$$\begin{aligned}
\nabla_\theta \mathbf{E}(L_{\text{surr}}) &= \nabla_\theta \mathbf{E}\left[\frac{\pi_\theta(a_t|s_t)}{\pi_{\text{old}}(a_t|s_t)} \cdot A(s_t, a_t)\right] \\
&= \mathbf{E}\left[\nabla_\theta \pi_\theta(a_t|s_t) \cdot \frac{1}{\pi_{\text{old}}(a_t|s_t)} \cdot A(s_t, a_t)\right] \\
&= \mathbf{E}\left[\frac{\pi_\theta(a_t|s_t)}{\pi_{\text{old}}(a_t|s_t)} \cdot \nabla_\theta \log \pi_\theta(a_t|s_t) \cdot A(s_t, a_t)\right] \\
&\approx \mathbf{E}\left[\nabla_\theta \log \pi_\theta(a_t|s_t) \cdot A(s_t, a_t)\right] \\
&= \nabla_\theta J(\theta)
\end{aligned}$$

- It update policy via:

$$\theta_{k+1} = \operatorname{argmax}_\theta \mathop{\mathbf{E}}_{s,a\sim\pi_{\theta_k}} [L(s, a, \theta_k, \theta)]$$

- Typically taking multiple steps of SGD to maximize the objective.

Proximal Policy Optimization (PPO)

- PPO introduces a clipping mechanism to prevent large, destabilizing updates.
- Clipped objective:

$$L(s, a, \theta_k, \theta) = \min\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_k}(a|s)} A^{\pi_{\theta_k}}(s, a), \text{clip}\left(\frac{\pi_\theta(a|s)}{\pi_{\theta_k}(a|s)}, 1-\epsilon, 1+\epsilon\right) A^{\pi_{\theta_k}}(s, a)\right)$$

  - ϵ is hyperparameter which roughly says how far away the new policy is allowed to go from the old.

- Clipped objective in another form:

$$L(s, a, \theta_k, \theta) = \min\left(\frac{\pi_\theta(a \mid s)}{\pi_{\theta_k}(a \mid s)} A^{\pi_{\theta_k}}(s, a),\ g(\epsilon, A^{\pi_{\theta_k}}(s, a))\right),$$
$$\text{where } g(\epsilon, A) = \begin{cases} (1+\epsilon)A & \text{if } A \geq 0, \\ (1-\epsilon)A & \text{if } A < 0. \end{cases}$$

- When the advantage is positive:

- Action a becomes more likely but has a limit.
- Good action appears more but not too more
- When the advantage is negative:
  - Action a becomes less likely but has a limit.
  - Bad action appears less but not too less.

Pseudo code:

Algorithm 1 PPO-Clip

Require: Initial policy parameters θ_0, initial value function parameters ϕ_0

for $k = 0, 1, 2, \ldots$ **do**

Collect set of trajectories $\mathcal{D}_k = \{\tau_i\}$ by running policy $\pi_k = \pi(\theta_k)$ in the environment.

Compute rewards-to-go $\hat{G}_t$.

Compute advantage estimates, $\hat{A}_t$ (using any method of advantage estimation) based on the current value function V_{ϕ_k}.

Update the policy by maximizing the PPO-Clip objective:

$$\theta_{k+1} = \arg\max_\theta \frac{1}{|\mathcal{D}_k|T} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \min\left(\frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_k}(a_t|s_t)} A^{\pi_{\theta_k}}(s_t, a_t), g(\epsilon, A^{\pi_{\theta_k}}(s_t, a_t))\right),$$

typically via stochastic gradient ascent with Adam.

Fit value function by regression on mean-squared error:

$$\phi_{k+1} = \arg\min_\phi \frac{1}{|\mathcal{D}_k|T} \sum_{\tau \in \mathcal{D}_k} \sum_{t=0}^{T} \left(V_\phi(s_t) - \hat{G}_t\right)^2,$$

typically via some gradient descent algorithm.

end for

General Advantage Estimation (GAE)

Advantage $\hat{A}_t^{(k)}$:

$$\hat{A}_t^{(k)} = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots + \gamma^k V(s_{t+k}) - V(s)$$

GAE takes a weighted average of $\hat{A}_t^{(k)}$ to balance bias and variance.

$$\hat{A}_t^{(k)} = A_t^{\text{GAE}} = \sum_k w_k \hat{A}_t^{(k)}$$

The derivation process of a recursive form

- $w_k = \lambda^{k-1}$, $\lambda \in [0, 1]$
- $\delta_t = r_t + \gamma V(s_{t+1}) - V(s_t)$

Then:

- $k = 1$: $\hat{A}_t^{(1)} = \delta_t$
- $k = 2$: $\hat{A}_t^{(2)} = \delta_t + \gamma\delta_{t+1}$

- $k=3$: $\hat{A}_t^{(3)} = \delta_t + \gamma\delta_{t+1} + \gamma^2\delta_{t+2}$

Takes a weighted average:

$$\begin{aligned}
\hat{A}_t &= (1-\lambda)(\hat{A}_t^{(1)} + \lambda\hat{A}_t^{(2)} + \lambda^2\hat{A}_t^{(3)} + \ldots) \\
&= (1-\lambda)(\delta_t + \lambda\delta_t + \gamma\lambda\delta_{t+1} + \lambda^2\delta_t + \gamma\lambda^2\delta_{t+1} + \gamma^2\lambda^2\delta_{t+2} + \ldots) \\
&= (1-\lambda)[\delta_t\left(1+\lambda+\lambda^2+\ldots\right) + \gamma\delta_{t+1}\left(\lambda+\lambda^2+\ldots\right) + \gamma^2\delta_{t+2}\left(\lambda^2+\lambda^3+\ldots\right) + \cdots] \\
&= \delta_t + \gamma\lambda\delta_{t+1} + (\gamma\lambda)^2\delta_{t+2} + \cdots \\
&= \delta_t + \gamma\lambda\hat{A}_{t+1}
\end{aligned}$$

Special Case:

- $\lambda=0$: $\hat{A}_t = \delta_t = \hat{A}_t^{(1)}$
- $\lambda=1$: $\hat{A}_t = \delta_t + \gamma\hat{A}_{t+1} = \hat{A}_t^{(\infty)}$

Code Example

- [Code Example]