Lab Assignment 2

Approximate solutions to systems of first order initial value problems

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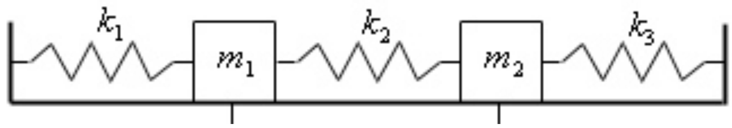
QUESTION I

a. Convert the pair of second order equations , into a system of four first-order equations for u_1 , u_2 and the velocities of the two masses, v_1 and v_2 .



$$m_1 \frac{\partial^2 u_1}{\partial t^2} = -(k_1 + k_2)u_1 + k_2 u_2 \qquad m_2 \frac{\partial^2 u_2}{\partial t^2} = k_2 u_1 - (k_2 + k_3)u_2$$

$$v_1(t) = \frac{\partial u_1}{\partial t} \qquad v_2(t) = \frac{\partial u_2}{\partial t}$$

$$\frac{\partial v_1}{\partial t} = \frac{\partial^2 u_1}{\partial t^2} \qquad \frac{\partial v_2}{\partial t} = \frac{\partial^2 u_2}{\partial t^2}$$

$$\frac{\partial u_1}{\partial t} = v_1$$
$$\frac{\partial v_1}{\partial t} = -1/m_1(k_1 + k_2)u_1 + 1/m_1 k_2 u_2$$
$$\frac{\partial u_2}{\partial t} = v_2$$
$$\frac{\partial v_2}{\partial t} = 1/m_2 k_2 u_1 - 1/m_2(k_2 + k_3)u_2$$

b. Express the system you found in part a in the form $\overline{u}' = A\overline{u}$, where $\overline{u}$ is the vector of functions u_1, v_1, u_2, v_2 , and A is a constant matrix.

$$\begin{pmatrix} u_1' \\ v_1' \\ u_2' \\ v_2' \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1/m_1(k_1+k_2) & 0 & 1/m_1 k_2 & 0 \\ 0 & 0 & 0 & 1 \\ 1/m_2 k_2 & 0 & 1/m_2(k_2+k_3) & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{pmatrix}$$

c. Use MATLAB to find the eigenvalues and eigenvectors of A in the case that:

- $m_1 = 1$
- $m_2 = 2$
- $k_1 = 1$
- $k_2 = 2$
- $k_3 = 3$

***See appendix for code*

MATLAB printout for constant matrix, eigenvalues & eigenvectors

```
A =

        0    1.0000         0         0
  -3.0000         0    2.0000         0
        0         0         0    1.0000
   1.0000         0   -2.5000         0

v =

 -0.0000 + 0.3777i  -0.0000 - 0.3777i   0.0000 - 0.5026i   0.0000 + 0.5026i
 -0.7728            -0.7728             0.5761             0.5761
  0.0000 - 0.2240i   0.0000 + 0.2240i  -0.0000 - 0.4237i  -0.0000 + 0.4237i
  0.4583 - 0.0000i   0.4583 + 0.0000i   0.4857 + 0.0000i   0.4857 - 0.0000i

r =

 -0.0000 + 2.0460i        0                  0                  0
       0            -0.0000 - 2.0460i        0                  0
       0                  0            -0.0000 + 1.1462i        0
       0                  0                  0            -0.0000 - 1.1462i
```

d. Using the eigenvalues and eigenvectors you found in part c, find the general real-valued solution of the system.

	-0.3777sin (2.0460t)		0.3777sin (2.0460t)		0.5026sin (1.1462t)		-0.5026sin (1.1462t)
	-0.7728cos (2.0460t)		-0.7728cos (2.0460t)		0.5761cos (1.1462t)		0.5761cos (1.1462t)
C_1	0.2240sin (2.0460t)	C_2	-0.2240sin (2.0460t)	C_3	0.4237sin (1.1462t)	C_4	-0.4237sin (1.1462t)
	0.4583cos (2.0460t)		0.4583cos (2.0460t)		0.4857cos (1.1462t)		0.4857cos (1.1462t)

e. Find the particular solution of the system satisfying the initial conditions:

- $u_1(0) = 1$
- $u_2(0) = 2$
- $v_1(0) = 0$
- $v_2(0) = 0$

***See appendix for code*

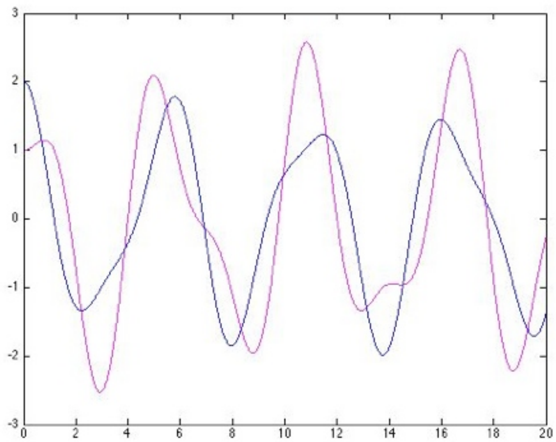
Matlab printout for RREF coefficient matrix

```
B =

         0    0.3777         0   -0.5026    1.0000
   -0.7728         0    0.5761         0         0
         0   -0.2240         0   -0.4237    2.0000
    0.4583         0    0.4587         0         0


ans =

    1.0000         0         0         0         0
         0    1.0000         0         0   -2.1331
         0         0    1.0000         0         0
         0         0         0    1.0000   -3.5926
```

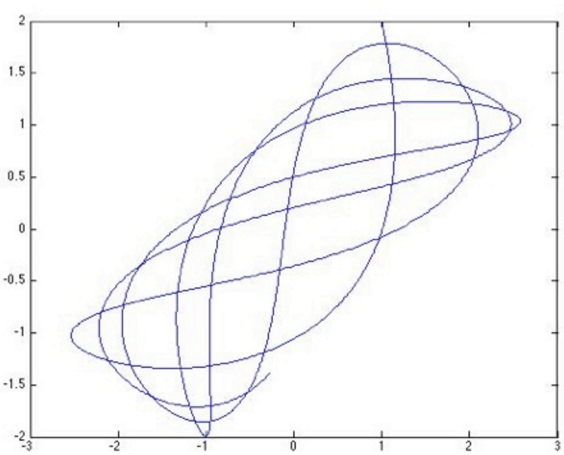
	0.3777sin(2.0460t)		-0.5026sin(1.1462t)
	-0.7728cos(2.0460t)		0.5761cos(1.1462t)
-2.1331	-0.2240sin(2.0460t)	-3.5926	-0.4237sin(1.1462t)
	0.4583cos(2.0460t)		0.4857cos(1.1462t)

f. Use MATLAB to plot the graphs of u_1 and u_2 on the interval [0,20] for the particular solution that you found, using different colors for the graphs of u_1 and u_2 .

***See appendix for code*



g. Plot the phase portrait of u_1 and u_2 .



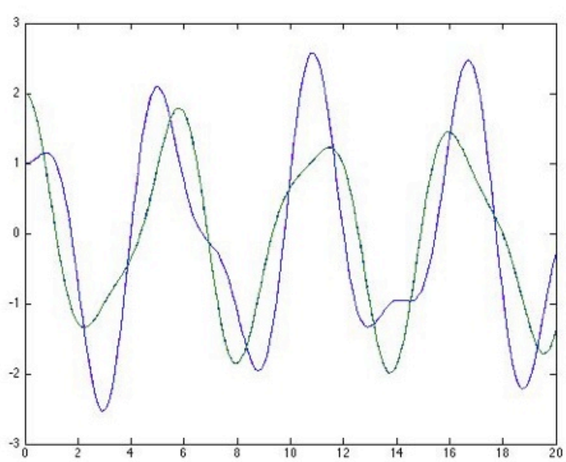
QUESTION 2

Use ode45 to solve the system in question 1, part a on the interval [0, 20], using the values m_1, m_2, k_1, k_2, k_3, from question 1, part c. Use MATLAB to sketch the graphs of u_1 and u_2 that ode45 generates in the same plot as the graphs from question 1, part f. **What do you see?**

***See appendix for code*

MATLAB printout for ode45 solutions

Graph of both



Analysis:

QUESTION 3

a. Convert this second order equation to a pair of first order equations.

$$\frac{\partial^2\theta}{\partial t^2}+\frac{g}{L}\sin(\theta)=0$$

$$y_1(t)=\theta \qquad y_2(t)=\frac{d\theta}{dt}$$

$$\frac{dy_1}{dt}=y_2(t)$$
$$\frac{dy_2}{dt}=-\frac{g}{L}\sin(y_1(t))$$

b. Assuming that $L=g$, use ode45 to solve the system you found in part a on the interval [0, 10] for each of the three initial states.

- $\theta_1(0)=1$ and $\theta'_1(0)=0$
- $\theta_2(0)=0.5$ and $\theta'_2(0)=0$
- $\theta_3(0)=0.1$ and $\theta'_3(0)=0$

***See appendix for code*

MATLAB printout for ode45 solutions

c. Once again assuming that $L = g$, solve the linear differential equation above explicitly, with the same three initial sates as in part b.

$$\frac{\partial^2\theta}{\partial t^2} + \theta = 0 \text{ for } g = L, \theta = 0.$$

$\theta(t) = e^{rt}$ substitute into equation

$$ce^{rt}(r^2+1) = 0, r_1 r_3 = \pm i$$

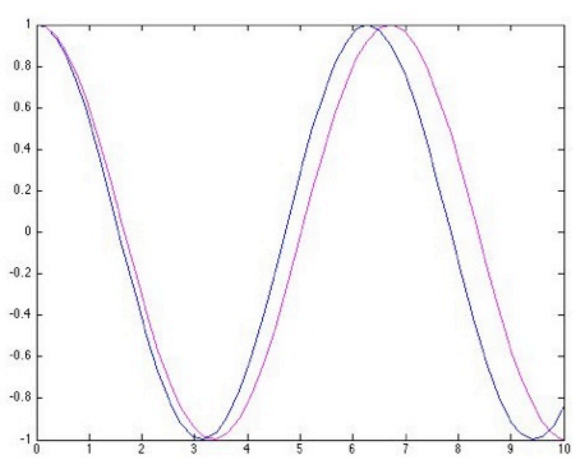
$$\theta(t) = c_1 e^{it} + c_2 e^{-it} \qquad \theta(t) = d_1\cos(t) + d_2\sin(t)$$

- $\theta_1(t) = \cos(t)$
- $\theta_2(t) = 0.5\cos(t)$
- $\theta_3(t) = 0.1\cos(t)$

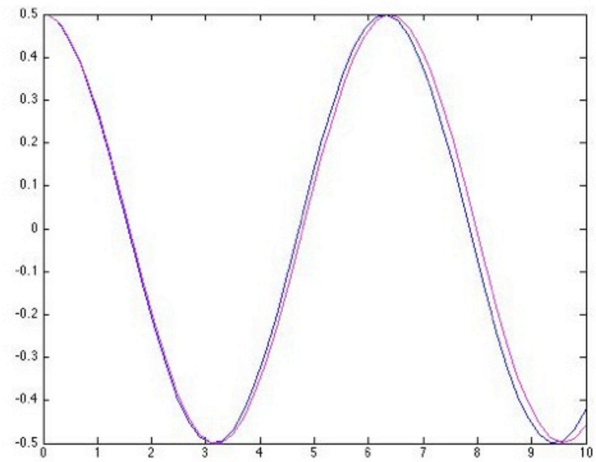
d. For each of the three initial states in part b, plot the graphs of the ode45 solution you found in part b and the solution you found in part c of the corresponding linear equation on the interval [0, 10]. **What do you see?**

***See appendix for code*

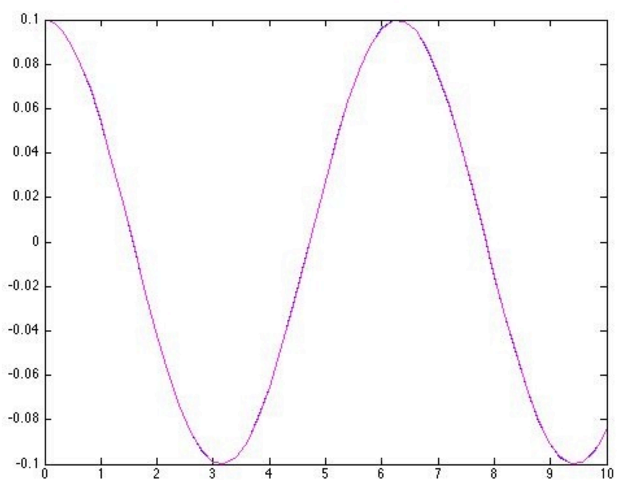
Graph $\theta_1(t) = \cos(t)$



Graph $\theta_2(t) = 0.5\cos(t)$



Graph $\theta_3(t) = 0.1\cos(t)$



Analysis:

APPENDIX

Appendix 1 - Matlab code - Main Script, Question 1

```matlab
k1=1;k2=2;k3=3;

m1=1;m2=2;

A = [0,1,0,0;(-1/m1)*(k1+k2),0,(1/m1)*k2,0;0,0,0,1;(1/m2)*k2,0,(-1/m2)*(k2+k3),0;]

[v,r] = eig(A)

B = [0,.3777,0,-.5026,1;-.7728,0,.5761,0,0;0,-.2240,0,-.4237,2;.4583,0,.4587,0,0;]

rref(B)

t = 0:.01:20;

u1 = (-2.1331)*(.3777.*cos(2.046.*t)) + (-3.5926*(-.5026.*cos(1.1462*t)));

u2 = (-2.1331)*(-.2240.*cos(2.046.*t)) + (-3.5926*(-.4237.*cos(1.1462*t)));

plot(t,u1,'--m',t,u2,'--b')

hold on;

y0 = [1;0;2;0];

[t,y] = ode45(@spring, [0,20], y0);

plot(t,y(:,1),t,y(:,3))

hold on;

plot(u1,u2)
```

Appendix 2 - Matlab code - Function Question 1

```matlab
function dy = spring(t,y)

m1=1;
m2=2;
k1=1;
k2=2;
k3=3;

dy=zeros(4,1);
dy(1) = y(2);
dy(2) = (-1/m1)*(k1+k2)*y(1)+k2*y(3);
dy(3) = y(4);
dy(4) = (1/m2)*k2*y(1)+(-1/m2)*(k2+k3)*y(3);
```

Appendix 3 - Matlab code - Main Script, Question 2

```
t=0:.01:10;

[t,y1] = ode45(@pend, [0,10], [1;0;]);

[t,y2] = ode45(@pend, [0,10], [.5;0;]);

[t,y3] = ode45(@pend, [0,10], [.1;0;]);

theta1= cos(t);

theta2= .5*cos(t);

theta3= .1*cos(t);

plot(t,y(:,1),t,y(:,2))

hold on;

plot(t,theta3,'b')

hold on;

plot(t,y3(:,1),'m')
```

Appendix 4 - Matlab code - Function Question 2

```
function dy = pend(t,y)

g=1;
L=1;

dy=zeros(2,1);
dy(1)=y(2);
dy(2)=-(g/L)*sin(y(1));
```