Assignment #1

Name: ______________________________ ID: ______________________________

This assignment has **5** questions, for a total of **25** marks.

Recall the following acronyms: SOS (structural operational semantics), COS (contextual operational semantics), SM (small step), BG (big step), CBV (call by value), CBN (call by name).

Question 1: **Big step-call by name** .... 4 marks

Write the operational semantics rules for a big-step, call-by-name reduction for ULC. Write the semantically correct ones only, but write them all.

Note: Use LaTeX to typeset rules, use either the `semantics` package or the macros provided on my webpage.[1] You can use the `bussproof` package to typeset whole big step reductions or typing derivations. You will likely have to split the same derivation tree into subtrees to fit all in one page. Give names to trees and refer to them in the bigger derivation for ease of reading.

[1] Here: http://theory.stanford.edu/∼mp/mp/CS358-2019_files/cmds.tex.

Question 2: **CBV and stuckness** . . . . 5 marks

Write a term t in ULC such that t in SM-CBV will get stuck (i.e., reduce to fail) but the same term t in SM-CBN will not. Show the reductions for each case.

- $t \overset{\text{def}}{=}$

1. SM-CBV

2. SM-CBN

Question 3: **Equivalence of SOS and COS** . . . . 8 marks

We saw some cases in class of the proof showing that small-step, call-by-value structured and contextual operational semantics (i.e., SOS-SM-CBV and COS-SM-CBV) are equivalent. Show the missing cases. Consider only the semantically correct rules for both semantics, i.e., BETA, OP, APP1, APP2, OP1, OP2 (TAPL page 72 plus in-class additions) for SOS-SM-CBV and rules CTX-BETA, CTX-OP, CTX for COS-SM-CBV.

1. If $t \to t'$ then $t \rightsquigarrow t'$

2. If $t \rightsquigarrow t'$ then $t \to t'$

Question 4: **Distinguish terms** .... 5 marks

Write out a term t in STLC that will reduce to two different numbers once applied to terms t_1 and t_2 below, i.e., such that $t\ t_1$ and $t\ t_2$ respectively reduce to n_1 and n_2 such that $n_1 \neq n_2$. The reduction strategy is SOS-SM-CBV, recall that if $n > m$ then $m - n = 0$. Write out the reductions too.

- $t_1 \overset{\text{def}}{=} \lambda x.\, \lambda y.\, (2 * x) - (3 * x) + ((\lambda z.\, y\ z\ x)\ 0)$
- $t_2 \overset{\text{def}}{=} \lambda x.\, \lambda y.\, (1 + x) - (3 + x) + ((\lambda z.\, y\ z\ x)\ 1)$

1. $t \overset{\text{def}}{=}$
2. $t\ t_1$ reductions.

3. $t\ t_2$ reductions.

Question 5: **Safe untypable term** . . . . 3 marks

Write out a term that is safe (i.e., it does not reduce to fail) but that cannot be typed. Show the typing derivation until it fails (i.e., no rule is applicable). Also, show how the same term would reduce according to SOS-SM-CBV semantics.