Assignment #4

Name: _____ ID: _____

This assignment has **5** questions, for a total of **25** marks.

Question 1: **Polymorphic behaviour** ..... 5 marks

Given a closed term t of type $\forall\alpha.\,\alpha \to \alpha \to \alpha \to \alpha$, and three closed values v_1, v_2, v_3 of type τ, define and prove that t only reduces to any of its supplied inputs.

Question 2: **Compatibility lemma for pair** . . . . . 3 marks

Assuming:

- $\Gamma \vDash t_1 : \tau_1$
- $\Gamma \vDash t_2 : \tau_2$

Prove:

- $\Gamma \vDash \langle t_1, t_2 \rangle : \tau_1 \times \tau_2$

Question 3: **Compatibility lemma for projection** ..... 3 marks

Assuming:

- $\Gamma \vDash t_1 : \tau_1 \times \tau_2$

Prove:

- $\Gamma \vDash t_1.1 : \tau_1$

Question 4: **Compatibility lemma for unpack** . . . . . 7 marks

Assuming:

- $\Delta \vdash \tau'$
- $\Delta, \Gamma \vDash t : \exists \alpha.\tau$
- $\Delta; \alpha, \Gamma; x : \tau \vDash t' : \tau'$

Prove:

- $\Delta, \Gamma \vDash \mathsf{unpack}\ t\ \mathsf{as}\ \langle \alpha, x \rangle\ \mathsf{in}\ t' : \tau'$

Question 5: **Free theorems** . . . . . 7 marks

Consider type $\tau = \forall\alpha.\forall\beta.\alpha \to \beta$. Is it inhabited? If yes, show a term that inhabits τ and prove that such a term is in the term relation for τ. If not, prove that there is no term in the term relation for τ.