Lecture 1b: Program Equivalences and Fully Abstract Compilation

CS350

Marco Patrignani

What is a Compiler

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https://en.wikipedia.org/wiki/Compiler

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In this course:

- only care about the code generation phase
- takes programs written in a source language S
- output programs written in a **target language T**
- it is a function from S to **T**: $[\![\cdot]\!]^{\mathsf{S}}_{\mathbf{T}}$

What is a Compiler

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In this course:

Gross simplification:

- PL perspective on this subject (will remain for the whole course)
- Voice your concerns when this view does not match yours

Example: Insecure Compilation

```
public class Account
  private int balance = 0;

  public void deposit( int amount )
    this.balance += amount;
```

Java

source

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No access to `balance` from outside `Account`

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Java
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No access to `balance` from outside `Account`

enforced by the language

Example: Insecure Compilation

```java
public class Account
  private int balance = 0;

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    this.balance += amount;
```

Java

source

```c
typedef struct account_t {
  int balance = 0;
  void ( *deposit ) ( struct Account*, int ) =
      deposit_f;
} Account;

void deposit_f( Account* a, int amount ) {
  a→balance += amount;
  return;
}
```

C

target

Example: Insecure Compilation

Pointer arithmetic in `C` leads to security violation: undesired access to `balance`

Security is not preserved.

```
publi
  priv

  publ
    th
```

```
typedef struct account_t {
  int balance = 0;
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Secure Compilation

- **Q:** what does it mean to preserve security properties across compilation?

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- **Q:** what does it mean to preserve security properties across compilation?

- long standing question
- many anwers have been given, we focus on the formal ones
- conceptually:

  “take what was secure in the source and make it as secure in the target”

Secure Compilation

- **Q:** what does it mean to preserve security properties across compilation?

Even more questions!

- how do we identify (or specify) what is secure in the source?
- how do we preserve the meaning of a security property?

Secure Compilation

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- how do we preserve the meaning of a security property?

anwers provided in this course

Example: Confidentiality

Confidential: adjective

> *spoken, written, acted on, etc., in strict privacy or secrecy; secret:*

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```
private secret : Int = 0;

public setSecret( ) : Int {
  secret = 0;
  return 0;
}
```

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Example: Confidentiality

Confidential: adjective

- **Q:** how do we know that `secret` is confidential?

```
priva

publi
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Example: Confidentiality

Confidential: adjective

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- **Q:** how do we know that `secret` is confidential?

- Type annotations
- Program verification
- …
- Behaviour analysis
- Program equivalences

Going Forward

- Program equivalences
  - contextual equivalence, trace equivalence

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    - contextual equivalence, trace equivalence
    - fully abstract compilation, trace-preserving compilation

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- Behaviour analysis through traces
    - security properties definition

Going Forward

- Program equivalences
  - contextual equivalence, trace equivalence
  - fully abstract compilation, trace-preserving compilation
- Behaviour analysis through traces
  - security properties definition
  - robust criteria

Program Equivalence

- a possible way to know what is secure in a program

Program Equivalence

- a possible way to know what is secure in a program
- useful tool to answer many questions posed about programming languages

Quiz: Are these Equivalent Programs?

```
public Bool getTrue( x : Bool )
  return true;
```

```
public Bool getTrue( x : Bool )
  return x or true;
```

```
public Bool getTrue( x : Bool )
  return x and false;
```

```
public Bool getTrue( x : Bool )
  return false;
```

```
public Bool getFalse( x : Bool )
  return x and true;
```

- P1
- P2
- P3
- P4
- P5

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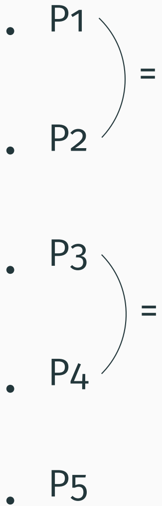

- P1
- P2

=

- P3
- P4

=

- P5

Quiz: Are these Equivalent Programs?

```
public Bool getTrue( x : Bool )
  retur
```

```
public
  retur
```

```
public
  retur
```

```
public Bool getTrue( x : Bool )
  return false;
```

```
public Bool getFalse( x : Bool )
  return x and true;
```

Program equivalences (generally) are:

- reflexive
- transitive
- symmetric

aka: relations

P1

P4

P5

Program Equivalence

- **Q:** When are two programs equivalent?

Program Equivalence

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- When they behave the same

Program Equivalence

- **Q:** When are two programs equivalent?
- When they behave the same even if they are different

Program Equivalence

- **Q:** When are two programs equivalent?

- When they behave the same even if they are different
- Semantics (behaviour) VS Syntax (outlook)

Program Equivalence

Later in the course we'll see:

- reasoning about behaviours of programs
- defining the syntax and semantics of languages

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Program Equivalence

Defining a security property using program equivalence:
to find two programs that, albeit syntactically different, both behave in a way that respects the property, no matter how they are used.

Example: Confidentiality as P.Eq.

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public setSecret( ) : Int {
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```

```
private secret : Int = 0;

public setSecret( ) : Int {
  secret = 1;
  return 0;
}
```

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private secret : Int = 0;

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With a Java-like semantics, `secret` is never accessed from outside.

With a C-like semantics, `secret` can be accessed from outisde.

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The Language defines how to reason

(it’s what programmers already do!)

Example: Integrity as P.Eq.

```
public proxy( callback : Unit → Unit ) : Int {
  var secret = 0;
  callback();
  if ( secret == 0 ) {
    return 0;
  }
  retu
}
```

```
publi
  var secret = 0;
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Integrity: internal consistency or lack of corruption in data.

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Integrity: internal consistency or lack of corruption in data.
Maintenance of invariants

Example: Integrity as P.Eq.

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public proxy( callback : Unit → Unit ) : Int {
  var secret = 0;
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  if ( secret == 0 ) {
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  }
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```

```
public proxy( callback : Unit → Unit ) : Int {
  var secret = 0;
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  return 0;
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```

Example: Memory Allocation as P.Eq.

```
public newObjects( ) : Object {
  var x = new Object();
  var y = new Object();
  return x;
}
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Example: Memory Allocation as P.Eq.

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  var y = new Object();
  retu
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```

Guessing addresses in memory leads to common exploits: ROP, return to libc, violation of ASLR …

```
publi
  var
  var
  return y;
}
```

Example: Memory Size as P.Eq.

```
public kernel( n : Int, callback : Unit → Unit ) : Int {
  for (Int i = 0; i < n; i++){
    new Object();
  }
  callback();
  return 0;
}
```

Example: Memory Size as P.Eq.

```
public kernel( n : Int, callback : Unit → Unit ) : Int {
  for (Int i = 0; i < n; i++){
    new Object();
  }
  callback();
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}
```

```
public kernel( n : Int, callback : Unit → Unit ) : Int {



  callback();
  return 0;
}
```

Expressing Program Equivalence

Contextual Equivalence

Expressing Program Equivalence

Contextual Equivalence

(also, observational equivalence)

Contextual Equivalence (CEQ)

Two programs are equivalent if no matter what external observer interacts with them that observer cannot distinguish the programs.

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$$P_1 \simeq_{ctx} P_2 \overset{\mathsf{def}}{=} \forall \mathfrak{C}.\ \mathfrak{C}[P_1]\!\downarrow \iff \mathfrak{C}[P_2]\!\downarrow$$

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Contextual Equivalence (CEQ)

- the external observer $\mathfrak{C}$ is generally called context
- it is a program, written in the same language as P_1 and P_2
- it is the same program $\mathfrak{C}$ interacting with both P_1 and P_2 in two different runs
- so it cannot express out of language attacks (e.g., side channels)

Two *at*
exter
obse

Contextual Equivalence (CEQ)

Two programs are equivalent if no matter what exter

obse

- interaction means link and run together (like a library)

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Contextual Equivalence (CEQ)

- distinguishing means: terminate with different values
- the observer basically asks the question: *is this program P_1?*
- if the observer can find a way to distinguish P_1 from P_2, it will return `true`, otherwise `false`
- often we use *divergence* and *termination* as opposed to this boolean termination

Two *at*

exter

obse

Example: CEQ

```
private secret : Int = 0; //P1
public setSecret( ) : Int {
  secret = 0;
  return 0;
}
```

Java

```
private secret : Int = 0; //P2
public setSecret( ) : Int {
  secret = 1;
  return 0;
}
```

Java

```
// Observer P in Java
public static isItP1( ) : Bool {
  Secret.getSecret();
  ...
}
```

Java

Example: CEQ

```
typedef struct secret { // P1
  int secret = 0;
  void ( *setSec ) ( struct Secret* ) = setSec;
} Secret;
void setSec( Secret* s ) { s→secret = 0; return; }
```

C

```
typedef struct secret { // P2
  int secret = 0;
  void ( *setSec ) ( struct Secret* ) = setSec;
} Secret;
void setSec( Secret* s ) { s→secret = 1; return; }
```

C

```
// Observer P in C
int isItP1( ){
  struct Secret x;
  sec = &x + sizeof(int);
  if *sec == 0 then return true else return false
}
```

C

Inequivalences as Security Violations

- if the target programs are not equivalent ($\not\simeq_{ctx}$) then the intended security property is violated

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1. if languages have complex features

Inequivalences as Security Violations

- if the target programs are not equivalent ($\not\simeq_{ctx}$) then the intended security property is violated

When does inequivalences escape the (compiler) programmer's reasoning?

1. if languages have complex features
2. if there are more languages involved (e.g., multiple target languages)

Preserving Equivalences in Compilation

Back to our question …

- **Q:** what does it mean to preserve security properties across compilation?

Preserving Equivalences in Compilation

Back to our question …

- **Q:** what does it mean to preserve security properties across compilation?

A possible answer:

- Given source equivalent programs (which have a security property), compile them into equivalent target programs

Preserving Equivalences in Compilation

- Assumption 1: the security property is captured in the source by program equivalence

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Preserving Equivalences in Compilation

- Assumption 1: the security property is captured in the source by program equivalence
- **Crucial**: being equivalent in the target means contextual equivalence w.r.t. target observers (i.e., target programs)
- These are the attackers in the secure compilation setting

Fully Abstract Compilation

Fully Abstract Compilation

A compiler is secure if, given source equivalent programs, it compiles them into equivalent target programs

$$
\begin{aligned}
[\![\cdot]\!]^{\mathsf{S}}_{\mathbf{T}} \text{ is FAC\#1} &\overset{\text{def}}{=} \forall \mathsf{P}_1, \mathsf{P}_2\\
\text{if } & \mathsf{P}_1 \simeq_{ctx} \mathsf{P}_2\\
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Right?

Fully Abstract Compilation[1]

Wrong.

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An empty translation would fit FAC#1!

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We need the compiler also to be correct.

Roughly, turn $\Rightarrow$ into a $\iff$:

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Note: ⇐ does not mean compiler correctness in the general sense, but it's a consequence

---

Fully Abstract Compilation[1]

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We n

Roug

Criteria need to be precise and general.

$$[\![\cdot]\!]^{\mathsf{S}}_{\mathbf{T}} \text{ is FAC} \overset{\text{def}}{=} \forall \mathsf{P}_1, \mathsf{P}_2$$

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Remarks on Fully Abstract Compilation

- widely adopted since 1999

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- only preserves security property expressed as program equivalence

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- widely adopted since 1999
- only preserves security property expressed as program equivalence
- not the silver bullet: we will see shortcomings of fully abstract compilation

Conclusion

- program equivalences can be used to define security properties
- preserving (and reflecting) equivalences can be used to define a secure compiler

Further Reading

- Martin Abadi. 1999. Protection in programming-language translations.
- Andrew Kennedy. 2006. Securing the .NET Programming Model.
- Joachim Parrow. 2014. General conditions for Full Abstraction.
- Daniele Gorla and Uwe Nestman. 2014. Full Abstraction for Expressiveness: History, Myths and Facts.

- Patrignani, Ahmed, Clarke. 2019. Formal Approaches to Secure Compilation.