2025-2-28 与自旋压缩态相关的一些背景

压缩系数

$$\xi_S^2 = rac{\min \Delta J_{ec{n}_\perp}^2}{j/2} = rac{4 \min \Delta J_{ec{n}_\perp}^2}{N}$$

定义平均自旋方向 $ec{n}_0 = rac{\langle ec{J}
angle}{|\langle ec{J}
angle|}$

与平均自旋垂直的方向 $\vec{n}_1, \vec{n}_2, \vec{n}_\perp = \vec{n}_1 \cos \phi + \vec{n}_2 \sin \phi$

垂直方向 $ec{n}_{\perp}$ 的自旋算符 $J_{ec{n}_{\perp}}=J_{ec{n}_{1}}\cos\phi+J_{ec{n}_{2}}\sin\phi$

垂直方向 $ec{n}_\perp$ 的自旋涨落 $(\Delta J_{ec{n}_\perp})^2 = \langle J_{ec{n}_\perp}^2
angle = ec{n}_\perp \Gamma ec{n}_\perp^T$

协方差矩阵Γ

$$\Gamma = egin{bmatrix} \langle J_{ec{n}_1}^2
angle & \operatorname{Cov}(J_{ec{n}_1},J_{ec{n}_2}) \ \operatorname{Cov}(J_{ec{n}_1},J_{ec{n}_2}) & \langle J_{ec{n}_2}^2
angle \end{bmatrix}, \quad \operatorname{Cov}(J_{ec{n}_1},J_{ec{n}_2}) = rac{1}{2} \langle [J_{ec{n}_1},J_{ec{n}_2}]_+
angle - \langle J_{ec{n}_1}
angle \, \langle J_{ec{n}_2}
angle = rac{1}{2} \langle [J_{ec{n}_1},J_{ec{n}_2}]_+
angle$$

协方差矩阵的最小本征值即为垂直方向前上的最小自旋涨落

$$\min(\Delta J_{ec{n}_{\perp}})^2 = \lambda_-$$

$$\xi_S^2 = rac{4}{N} = rac{2}{N} \left[\langle J_{ec{n}_1}^2 + J_{ec{n}_2}^2
angle - \sqrt{(\langle J_{ec{n}_1}^2 - J_{ec{n}_2}^2
angle)^2 + 4 ext{Cov}(J_{ec{n}_1}, J_{ec{n}_2})}
ight]$$

对于自旋相干态CSS, $\xi_S^2 = 1$

$$\xi_R^2 = rac{(\Delta\phi)^2}{(\Delta\phi)_{CSS}^2} = rac{N(\Delta J_{ec{n}_\perp})^2}{|\langleec{J}
angle|^2}$$

与平均自旋垂直的方向 $\vec{n}_1, \vec{n}_2, \quad \vec{n}_1 \times \vec{n}_2 = \vec{n}_0$

 $绕\vec{n}_1$ 方向转动 ϕ 角度,沿 \vec{n}_2 方向的自旋算符

$$J_{ec{n}_2}^{out} = \exp(i\phi J_{ec{n}_1}) J_{ec{n}_2} \exp(-i\phi J_{ec{n}_1}) = \cos\phi J_{ec{n}_2} - \sin\phi J_{ec{n}_0}$$

$$\langle J_{ec{n}_0}^{out}
angle = -\sin\phi\,\langle J_{ec{n}_0}
angle$$

$$(\Delta J_{ec{n}_{0}}^{out})^{2}=\cos^{2}\phi(\Delta J_{ec{n}_{0}})+\sin^{2}\phi(\Delta J_{ec{n}_{0}})^{2}-rac{1}{2}\sin(2\phi)\,\langle[J_{ec{n}_{0}},J_{ec{n}_{0}}]_{+}
angle$$

转动角度趋于0时的相位敏感度
$$\Delta \phi = rac{\Delta J_{ec{n}_2}}{|\partial \langle J_{ec{n}_2}^{out}
angle / \partial \phi|} = rac{\Delta J_{ec{n}_2}}{\langle J_{ec{n}_0}
angle} \left(= rac{\Delta J_{ec{n}_\perp}}{|\langle ec{J}
angle|}
ight)$$

自旋相干态的相位敏感度 $(\Delta\phi)_{CSS}=rac{1}{\sqrt{N}}$

当垂直方向 $ec{n}_{ot}$ 选为最小自旋涨落方向时 $\xi_R^2=\left(rac{j}{\langleec{J}
angle}
ight)^2\xi_S^2\geq \xi_S^2$

QFI (Quantum Fisher Information)

定义与性质

条件概率分布

待测参数为 θ , 算符 $E(\epsilon)$ 对应的测量值为 ϵ 的条件概率分布 $P(\epsilon|\theta) = \mathrm{Tr}(\rho(\theta)E(\epsilon))$ m个独立子系统测量结果 $\epsilon = (\epsilon_1, \dots, \epsilon_m)$

对数似然函数

$$L(\epsilon| heta) = \ln P(\epsilon| heta) = \sum_i \ln P(\epsilon_i| heta)$$

估计子 $\Theta(\epsilon)$

从测量结果 ϵ 得到 θ 的一个估计值 Θ ,相应也有一个分布 $P(\Theta|\theta)$ 估计子均值 $\langle\Theta\rangle=\sum_{\epsilon}\Theta(\epsilon)P(\epsilon|\theta)$ 估计子涨落 $(\Delta\Theta)^2=\sum_{\epsilon}P(\epsilon|\theta)(\Theta(\epsilon)-\langle\Theta\rangle)^2$

无偏估计

$$egin{array}{l} \langle\Theta
angle= heta,orall heta \ rac{\partial\langle\Theta
angle}{\partial heta}=1 \end{array}$$

极大似然估计子

$$\Theta_{MLE}(heta) = ext{arg}[ext{max}_{ heta} P(\epsilon| heta)]$$

Fisher信息

$$\begin{split} F(\theta) &= \langle \left(\frac{\partial L(\theta)}{\partial \theta}\right)^2 \rangle = \sum_{\epsilon} \frac{1}{P(\epsilon|\theta)} \left(\frac{\partial P(\epsilon|\theta)}{\partial \theta}\right)^2 \\ & \text{可加性} \\ F(\theta) &= \langle \sum_{i} \frac{1}{P(\epsilon_{i})} \frac{\partial P(\epsilon_{i}|\theta)}{\partial \theta} \sum_{j} \frac{1}{P(\epsilon_{j})} \frac{\partial P(\epsilon_{j}|\theta)}{\partial \theta} \rangle \\ &= \sum_{i} \sum_{\epsilon_{i}} \frac{1}{P(\epsilon_{i}|\theta)} \left(\frac{\partial P(\epsilon_{i}|\theta)}{\partial \theta}\right)^2 + \sum_{i} \sum_{\epsilon_{i}} \frac{\partial P(\epsilon_{i}|\theta)}{\partial \theta} \sum_{j} \sum_{\epsilon_{j}} \frac{\partial P(\epsilon_{j}|\theta)}{\partial \theta} \\ &= \sum_{i} \sum_{\epsilon_{i}} \frac{1}{P(\epsilon_{i}|\theta)} \left(\frac{\partial P(\epsilon_{i}|\theta)}{\partial \theta}\right)^2 + 0 = \sum_{i} F_{i}(\theta) \\ m \\ \uparrow \mathbf{2} \end{substituted}$$

Cramer-Rao 下界

$$(\Delta\Theta)^2 \geq (\Delta\Theta_{CR})^2 = rac{(rac{\partial(\Theta)}{\partial heta})^2}{F(heta)}$$

测量样本数册趋于无穷的极大似然估计

$$P(\Theta_{MLE}|\theta) = \sqrt{rac{mF(heta)}{2\pi}}e^{-rac{mF(heta)}{2}(heta-\Theta_{MLE})^2}, \quad m o \infty$$
 渐近无偏 $rac{\partial \langle \Theta_{MLE}
angle}{\partial heta} = 1, \quad m o \infty$ 方差可以达到CR下界 $(\Delta\Theta_{MLE})^2 = rac{1}{mF(heta)}, \quad m o \infty$

量子Fisher信息

改变测量算符 $E(\epsilon)$ 所得的Fisher信息最大值

$$F_Q[
ho(heta)] = \max_{E(\epsilon)} F[
ho(heta), \{E(\epsilon)\}] = \mathrm{Tr}(
ho(heta) L_ heta^2)$$
可加性 $F_Q[
ho(heta)] = \sum_i F_Q[
ho_i(heta)]$

对称对数导数 $L(\theta)$

$$rac{\partial
ho}{\partial heta} = rac{1}{2} (
ho(heta) L_{ heta} + L_{ heta}
ho(heta))$$

显式形式

$$egin{aligned}
ho(heta) &= \sum_k p_k |k
angle \langle k|, \quad \langle k|k'
angle &= \delta_{k'k} \ L_ heta &= \sum_k rac{\partial_ heta p_k}{p_k} |k
angle \langle k| + 2\sum_{k,k'} rac{p_k - p_{k'}}{p_k + p_{k'}} |k
angle \langle \partial_ heta k |k'
angle \langle k'| \ F_Q[
ho(heta)] &= \sum_k rac{(\partial_ heta p_k)^2}{p_k} + 2\sum_{k,k'} rac{(p_k - p_{k'})^2}{p_k + p_{k'}} |\langle \partial_ heta k |k'
angle|^2 \end{aligned}$$

幺正演化下的显式形式

$$egin{aligned}
ho(heta) &= e^{-iH heta}
ho_0 e^{iH heta} \ L_ heta &= 2i\sum_{k,k'} rac{p_k-p_{k'}}{p_k+p_{k'}}|k
angle\,\langle k|H|k'
angle\langle k'| \ F_Q[
ho(heta)] &= 2\sum_{k,k'} rac{(p_k-p_{k'})^2}{p_k+p_{k'}}|\,\langle k|H|k'
angle|^2 \$$
 纯态情形 $L_ heta &= 2i|\psi(heta)
angle\langle \psi(heta)|H - 2iH|\psi(heta)
angle\langle \psi(heta)| \ F_Q[
ho(heta)] &= 4(\Delta H)^2 \end{aligned}$

与压缩系数的关系

结合压缩系数 ξ_R^2 的定义, $H=J_{ec{n}_1}$ $F_Q[
ho(heta)]=4(\Delta J_{ec{n}_1})^2$ 由不确定关系 $(\Delta J_{ec{n}_1})^2(\Delta J_{ec{n}_2})^2\geq \frac{1}{4}\langle J_{ec{n}_0}\rangle^2$ $\frac{N}{F_Q[
ho(heta)]}=\frac{N}{4(\Delta J_{ec{n}_1})^2}\leq \frac{N(\Delta J_{ec{n}_2})^2}{\langle J_{ec{n}_0}\rangle^2}=\xi_R^2$

自旋演化半经典数值方法

TWA (truncated Wigner approximation)

truncated Wigner approximation

位置动量空间

相点算符

$$\langle x'|\hat{A}(x,q)|x''
angle = \delta\left(x-rac{x'+x''}{2}
ight)e^{ip\cdot(x'-x'')/\hbar}, \quad \langle p'|\hat{A}(x,p)|p''
angle = \delta\left(p-rac{p'+p''}{2}
ight)e^{-ix\cdot(p'-p'')/\hbar}$$

外尔符号

$$O_W(x,p)=\mathrm{Tr}[\hat{O}\hat{A}(x,p)]=\int d^D\xi\,\langle x-rac{\xi}{2}|\hat{O}(\hat{x},\hat{p})|x+rac{\xi}{2}
angle\exp\left[rac{i}{\hbar}p\cdot\xi
ight] \ X_W(x,p)=x, P_W(x,p)=p$$
 简单形式的哈密顿量 $\hat{H}(\hat{X},\hat{P})=T(\hat{P})+V(\hat{X}),\quad H_W(x,p)=T(p)+X(x)$ 为经典哈密顿量

Wigner函数

$$W(x,p)=\mathrm{Tr}[\hat{
ho}\hat{A}(x,p)]=
ho_W(x,p)=\int d^D\xi\,\langle x-rac{\xi}{2}|\hat{
ho}|x+rac{\xi}{2}
angle\expigl[rac{i}{\hbar}p\cdot\xiigr],\quad\int\intrac{d^Dxd^Dp}{(2\pi\hbar)^D}W(x,p)=1$$

算符期望

满足
$$\langle \hat{O}(\hat{x},\hat{p})
angle = \int \int rac{d^D x d^D p}{(2\pi\hbar)^D} W(x,p) O_W(x,p)$$

Wigner函数的演化

辛算符
$$\Lambda = \frac{\overleftarrow{\partial}}{\partial p} \frac{\overrightarrow{\partial}}{\partial x} - \frac{\overleftarrow{\partial}}{\partial x} \frac{\overrightarrow{\partial}}{\partial p}$$

 泊松括号 $\{A,B\}_{PB} = -A\Lambda B$
 Moyal括号 $\{A,B\}_{MB} = -\frac{2}{\hbar}A\sin\left[\frac{\hbar}{2}\Lambda\right]B = -A\Lambda B + O(\hbar^2) = \{A,B\}_{PB} + O(\hbar^2)$
 算符乘积的外尔符号 $(\hat{A}\hat{B})_W(x,p) = A_W(x,p)\exp\left[-\frac{i\hbar}{2}\Lambda\right]B_W(x,p)$
 对易子的外尔符号 $([\hat{A},\hat{B}])_W = -2iA_W\sin\left[\frac{\hbar}{2}\Lambda\right]B_W = i\hbar\{A_W,B_W\}_{MB}$
 密度算符演化方程 $\dot{\hat{\rho}}(t) = \frac{1}{i\hbar}[\hat{H},\hat{\rho}(t)]$
 Wigner函数的演化方程 $\frac{\partial}{\partial t}W(x,p,t) = \{H_W(x,p),W(x,p,t)\}_{PB} + O(\hbar^2)$

t时刻的期望

对于经典哈密顿量 $H_W(x,p)$ 决定的经典轨迹 $x_{cl}(t),p_{cl}(t)$,维格纳函数近似满足刘维尔定理 $\frac{d}{dt}W(x_{cl}(t),p_{cl}(t),t)=O(\hbar^2)$ t时刻算符期望

$$\langle \hat{O}(\hat{x},\hat{p})
angle(t)=\int\intrac{d^Dx_{cl}(t)d^Dp_{cl}(t)}{(2\pi\hbar)^D}W(x_{cl}(t),p_{cl}(t))O_W(x_{cl}(t),p_{cl}(t))pprox \int\intrac{d^Dx_0d^Dp_0}{(2\pi\hbar)^D}W(x_0,p_0)O_W(x_{cl}(t),p_{cl}(t))$$

相干态空间

外尔符号

$$O_W(\psi,\psi^*) = \int \int rac{d\eta^* d\eta}{(2\pi)^M} \langle \psi - rac{\eta}{2} | \hat{O}(\hat{\psi},\hat{\psi}^\dagger) | \psi + rac{\eta}{2}
angle \exp[rac{1}{2}(\eta^*\psi - \eta\psi^*)]$$
在变换 $x = \sqrt{rac{\hbar}{2}}(\psi + \psi^*), p = i\sqrt{rac{\hbar}{2}}(\psi^* - \psi)$ 下可以回到位置动量空间的形式

Wigner函数

$$W(\psi,\psi^*)=\int\intrac{d\eta^*d\eta}{(2\pi)^M}\langle\psi-rac{\eta}{2}|\hat{
ho}|\psi+rac{\eta}{2}
angle\exp[rac{1}{2}(\eta^*\psi-\eta\psi^*)]$$

算符期望

$$\langle \hat{O}(\hat{\psi},\hat{\psi}^{\dagger})
angle = \int\intrac{d\psi d\psi^*}{ au^M}W(\psi,\psi^*)O_W(\psi,\psi^*)$$

Wigner函数的演化

辛算符 $\Lambda_c = \frac{\overleftarrow{\partial}}{\partial \psi} \frac{\overrightarrow{\partial}}{\partial \psi^*} - \frac{\overleftarrow{\partial}}{\partial \psi^*} \frac{\overrightarrow{\partial}}{\partial \psi}$ 泊松括号 $\{A,B\}_{PB} = A\Lambda_c B$ Moyal括号 $\{A,B\}_{MB} = 2A \sinh\left[\frac{\Lambda_c}{2}\right] B = A\Lambda_c B + \cdots = \{A,B\}_{PB} + \cdots$ 算符乘积的外尔符号 $(\hat{A}\hat{B})_W(\psi,\psi^*) = A_W(\psi,\psi^*) \exp\left[\frac{\Lambda_c}{2}\right] B_W(\psi,\psi^*)$ 对易子的外尔符号 $([\hat{A},\hat{B}])_W = 2A_W \sinh\left[\frac{\Lambda_c}{2}\right] B_W = \{A_W,B_W\}_{MB}$ 密度算符演化方程 $\dot{\rho}(t) = \frac{1}{i\hbar}[\hat{H},\hat{\rho}(t)]$ Wigner函数的演化方程 $\frac{\partial}{\partial t}W(\psi,\psi^*,t) = \{H_W(\psi,\psi^*),W(\psi,\psi^*,t)\}_{PB} + \cdots$ ψ,ψ^* 很大时,忽略泊松括号后面的项,因为 ψ,ψ^* 次数低

t时刻的期望

$$\langle \hat{O}(\hat{\psi},\hat{\psi}^\dagger)
angle(t)=\int\intrac{d\psi_0d\psi_0^*}{\pi^M}W(\psi_0,\psi_0^*)O_W(\psi_{cl}(t),\psi_{cl}^*(t))$$

自旋

Schwinger表示

$$\begin{array}{ll} \hat{s}^z = \frac{\hat{\alpha}^\dagger \hat{\alpha} - \hat{\beta}^\dagger \hat{\beta}}{2}, & \hat{s}^+ = \hat{\alpha}^\dagger \hat{\beta}, & \hat{s}^- = \hat{\beta}^\dagger \hat{\alpha} \\ \hline{ 利用相干态空间的外尔符号即可得到自旋算符的外尔符号} \\ s_W^z = \frac{|\alpha|^2 - |\beta|^2}{2}, & s_W^+ = \alpha^*\beta, & s_W^- = \beta^*\alpha \\ s_W^x = \frac{\alpha^*\beta + \beta^*\alpha}{2}, & s_W^y = \frac{\alpha^*\beta - \beta^*\alpha}{2i}, & s_W^\perp = \sqrt{(s_W^x)^2 + (s_W^y)^2} = |\alpha||\beta| \\ s_W \stackrel{\mathrm{def}}{=} \sqrt{(s_W^z)^2 + (s_W^\perp)^2} = \frac{|\alpha|^2 + |\beta|^2}{2} \\ s_W^z + s_W = |\alpha|^2 \end{array}$$

|S,S angle的Wigner函数

$$|S,S
angle = rac{(lpha^\dagger)^{2S}}{\sqrt{(2S)!}}|0
angle$$

Wigner函数

$$W(lpha, lpha^*, eta, eta^*) = 2e^{-2|lpha|^2 - 2|eta|^2} L_{2S+1}(4|lpha|^2)$$

将变量选为 s_W^z, s_W^{\perp}

$$W(s_W^z, s_W^\perp) = rac{2}{\pi s_W} e^{-4s_W} L_{2S+1}(4[s_W^z + s_W])$$

(分母中的 s_W 应该来源于积分变量代换 $\left|rac{\partial (s_W^z, s_W^{\perp})}{\partial (|lpha|^2, |eta|^2)}
ight|=s_W$)

$$\int_{-\infty}^{\infty}ds_W^z\int_0^{\infty}ds_W^{\perp}\int_0^{2\pi}d\phi W(s_W^z,s_W^{\perp})=1$$

大S极限下 $W(s_W^z,s_W^\perp)pprox rac{1}{\pi S}e^{-(s_W^\perp)^2/S}\delta(s_W^z-S)$

涨落为高斯分布,只能考虑集体自旋的涨落

DTWA (Discrete TWA)

离散相空间

$$\begin{split} &\alpha = (q,p) \in \{(0,0),(0,1),(1,0),(1,1)\} \\ &\vec{r}_{(0,0)} = (1,1,1), \vec{r}_{(0,1)} = (-1,-1,1), \vec{r}_{(1,0)} = (1,-1,-1), \vec{r}_{(1,1)} = (-1,1,-1) \end{split}$$

相点算符

$$egin{aligned} \hat{A}_lpha &= \hat{
ho}(ec{r}_lpha) = (1 + ec{r}_lpha \cdot \dot{ ilde{\sigma}})/2 \ \mathrm{Tr}(\hat{A}_lpha \hat{A}_eta) &= \delta_{lphaeta} \ N$$
个自旋的相点算符 $\hat{A}_{ec{lpha}} &= \prod_{lpha i=1}^N \hat{A}_{lpha i} \end{aligned}$

外尔符号

$$O_{lpha}^{W}= ext{Tr}(\hat{O}\hat{A}_{lpha})/2,\quad \hat{O}=\sum_{lpha}\hat{A}_{lpha}O_{lpha}^{W}$$

Wigner函数

$$egin{aligned} &w_{lpha}=\mathrm{Tr}(\hat{
ho}\hat{A}_{lpha})/2, \quad \hat{
ho}=\sum_{lpha}\hat{A}_{lpha}w_{lpha}, \quad \sum_{lpha}w_{lpha}=1 \ &p_1^x=w_{(0,0)}+w_{(1,0)}, \quad p_{-1}^x=w_{(0,1)}+w_{(1,1)} \ &p_1^y=w_{(0,0)}+w_{(1,1)}, \quad p_{-1}^y=w_{(0,1)}+w_{(1,0)} \ &p_1^z=w_{(0,0)}+w_{(0,1)}, \quad p_{-1}^z=w_{(1,0)}+w_{(1,1)} \ &$$
直积态的Wigner函数 $w_{ec{lpha}}=\prod_{i=1}^N w_{lpha_i} \ \end{aligned}$

算符期望

$$\langle O
angle = \sum_{ec{lpha}} w_{ec{lpha}} O^W_{ec{lpha}}$$

t时刻的算符期望

$$\begin{split} \langle O \rangle(t) &= \sum_{\vec{\alpha}} w_{\vec{\alpha}}(0) O_{\vec{\alpha}}^W(t) \\ O_{\vec{\alpha}}^W(t) &= \mathrm{Tr}(\hat{O}(t) \hat{A}_{\vec{\alpha}})/2 = \mathrm{Tr}(\hat{O}(0) \hat{U}(t) \hat{A}_{\vec{\alpha}} \hat{U}^\dagger(t))/2, \quad U(t) = \exp(-i \hat{H} t/\hbar) \\ \hat{A}_{\vec{\alpha}}(t) &= \hat{U}(t) \hat{A}_{\vec{\alpha}} \hat{U}^\dagger(t) \\ i \hat{A}_{\vec{\alpha}}(t) &= [\hat{H}, \hat{A}_{\vec{\alpha}}] \\ \text{约化相点算符} \\ \hat{A}_j &= \mathrm{Tr}_f(\hat{A}_{\vec{\alpha}}), \quad \hat{A}_{j,k} = \mathrm{Tr}_{f,\not{K}}(\hat{A}_{\vec{\alpha}}), \quad \hat{A}_{j,k,l} = \mathrm{Tr}_{f,\not{K},f}(\hat{A}_{\vec{\alpha}}) \\ \text{包含单体作用和两体作用的哈密顿量} \\ \hat{H} &= \sum_j \hat{H}_j + \sum_{j,l} \hat{H}_{j,l} = \frac{1}{2} \sum_{i=1}^N \vec{\Omega}_i \cdot \vec{\sigma}_i + \frac{1}{2} \sum_{i\neq j}^N \vec{\sigma}_i^T \cdot J_{ij} \cdot \vec{\sigma}_i \\ \mathrm{BBGKY级联方程} \\ i \hat{A}_j &= [\hat{H}_j, \hat{A}_j] + \sum_{l=1,l\neq j}^N \mathrm{Tr}_l([\hat{H}_{j,l}, \hat{A}_{j,l}]) \\ i \hat{A}_{j,k} &= [\hat{H}_j + \hat{H}_k + \hat{H}_{j,k}, \hat{A}_{j,k}] + \sum_{l=1,l\neq j,k}^N \mathrm{Tr}_l([\hat{H}_{j,l} + \hat{H}_{k,l}, \hat{A}_{j,k,l}]) \end{split}$$

截断关联

$$\hat{A}_{j,k}pprox\hat{A}_j\hat{A}_k$$
 $i\hat{A}_j=[\hat{H}_j,\hat{A}_j]+\sum_{l=1,l
eq j}^N\mathrm{Tr}_l([\hat{H}_{j,l},\hat{A}_j\hat{A}_l])$ 等价于让 $\hat{A}_{ec{lpha}}$ 始终保持直积态得到的方程,求解规模得以缩小对于每个相空间抽样 $ec{lpha}$,演化方程可以写为经典自旋 $\dot{r}_{ec{lpha}}$ 的演化 $rac{dec{r}_i}{dt}=ec{\Omega}_{eff}^i imesec{r}_i,\quad ec{\Omega}_{eff}^i=ec{\Omega}_i+2\sum_{j=1,j
eq i}^NJ_{ij}\cdotec{r}_i$

DDTWA (Dissipative Discrete TWA)

dissipative discrete truncated Wigner approximation

$$egin{aligned} \dot{
ho} &= -i[H,
ho] + L_{deph}(
ho) + L_{decay}(
ho) \ L_{deph}(
ho) &= rac{\Gamma_{\phi}}{2} \sum_{i=1}^{N} (\sigma_{i}^{z}
ho \sigma_{i}^{z} -
ho) \ L_{decay} &= rac{\Gamma}{2} \sum_{i=1}^{N} (2\sigma_{i}^{-}
ho \sigma_{i}^{+} - \sigma_{i}^{+} \sigma_{i}^{-}
ho -
ho \sigma_{i}^{+} \sigma_{i}^{-}) \end{aligned}$$

直接使用DTWA的局限

经典自旋演化过程中的模长不再守恒,相互作用项将出现偏差 目标是尽可能保持模长不变

Dephasing的处理

$$dec{r}_i = ec{\Omega}_{eff}^i imes ec{r}_i dt + dec{r}_i|_{deph}$$

$$dr_i^x|_{deph} = -\Gamma_\phi r_i^x dt - \sqrt{2\Gamma_\phi} r_i^y dW_i$$

$$dr_i^y|_{deph} = -\Gamma_\phi r_i^y dt - \sqrt{2\Gamma_\phi} r_i^x dW_i$$

$$dr_i^z|_{deph} = 0$$
 维纳过程 $\langle dW_i \rangle = 0, \langle (dW_i)^2 \rangle = dt$,从高斯分布中抽样 模长变化 $\langle dec{r}_i^2 \rangle = 0$

Decay的处理