2025-2-28 与自旋压缩态相关的一些背景

压缩系数

$$\xi_S^2=\frac{\min\Delta J_{\vec{n}_\perp}^2}{j/2}=\frac{4\min\Delta J_{\vec{n}_\perp}^2}{N}$$

定义平均自旋方向 $\vec{n}_0=\frac{\langle\vec{J}\rangle}{|\langle\vec{J}\rangle|}$
与平均自旋垂直的方向 $\vec{n}_1,\vec{n}_2,\vec{n}_\perp=\vec{n}_1\cos\phi+\vec{n}_2\sin\phi$
垂直方向$\vec{n}_\perp$的自旋算符 $J_{\vec{n}_\perp}=J_{\vec{n}_1}\cos\phi+J_{\vec{n}_2}\sin\phi$
垂直方向$\vec{n}_\perp$的自旋涨落 $(\Delta J_{\vec{n}_\perp})^2=\langle J_{\vec{n}_\perp}^2\rangle=\vec{n}_\perp\Gamma\vec{n}_\perp^T$
协方差矩阵Γ

$$\Gamma=\begin{bmatrix}\langle J_{\vec{n}_1}^2\rangle & \mathrm{Cov}(J_{\vec{n}_1},J_{\vec{n}_2})\\ \mathrm{Cov}(J_{\vec{n}_1},J_{\vec{n}_2}) & \langle J_{\vec{n}_2}^2\rangle\end{bmatrix},\quad \mathrm{Cov}(J_{\vec{n}_1},J_{\vec{n}_2})=\frac{1}{2}\langle[J_{\vec{n}_1},J_{\vec{n}_2}]_+\rangle-\langle J_{\vec{n}_1}\rangle\langle J_{\vec{n}_2}\rangle=\frac{1}{2}\langle[J_{\vec{n}_1},J_{\vec{n}_2}]_+\rangle$$

协方差矩阵的最小本征值即为垂直方向$\vec{n}_\perp$的最小自旋涨落
$\min(\Delta J_{\vec{n}_\perp})^2=\lambda_-$
$\xi_S^2=\frac{4}{N}=\frac{2}{N}\left[\langle J_{\vec{n}_1}^2+J_{\vec{n}_2}^2\rangle-\sqrt{(\langle J_{\vec{n}_1}^2-J_{\vec{n}_2}^2\rangle)^2+4\mathrm{Cov}(J_{\vec{n}_1},J_{\vec{n}_2})}\right]$
对于自旋相干态CSS，$\xi_S^2=1$

$$\xi_R^2=\frac{(\Delta\phi)^2}{(\Delta\phi)_{CSS}^2}=\frac{N(\Delta J_{\vec{n}_\perp})^2}{|\langle\vec{J}\rangle|^2}$$

与平均自旋垂直的方向 $\vec{n}_1,\vec{n}_2,\quad \vec{n}_1\times\vec{n}_2=\vec{n}_0$
绕$\vec{n}_1$方向转动ϕ角度，沿$\vec{n}_2$方向的自旋算符
$J_{\vec{n}_2}^{out}=\exp(i\phi J_{\vec{n}_1})J_{\vec{n}_2}\exp(-i\phi J_{\vec{n}_1})=\cos\phi J_{\vec{n}_2}-\sin\phi J_{\vec{n}_0}$
$\langle J_{\vec{n}_2}^{out}\rangle=-\sin\phi\langle J_{\vec{n}_0}\rangle$
$(\Delta J_{\vec{n}_2}^{out})^2=\cos^2\phi(\Delta J_{\vec{n}_2})+\sin^2\phi(\Delta J_{\vec{n}_0})^2-\frac{1}{2}\sin(2\phi)\langle[J_{\vec{n}_2},J_{\vec{n}_0}]_+\rangle$
转动角度趋于0时的相位敏感度 $\Delta\phi=\frac{\Delta J_{\vec{n}_2}}{|\partial\langle J_{\vec{n}_2}^{out}\rangle/\partial\phi|}=\frac{\Delta J_{\vec{n}_2}}{\langle J_{\vec{n}_0}\rangle}\left(=\frac{\Delta J_{\vec{n}_\perp}}{|\langle\vec{J}\rangle|}\right)$
自旋相干态的相位敏感度 $(\Delta\phi)_{CSS}=\frac{1}{\sqrt{N}}$
当垂直方向$\vec{n}_\perp$选为最小自旋涨落方向时 $\xi_R^2=\left(\frac{j}{\langle\vec{J}\rangle}\right)^2\xi_S^2\geq\xi_S^2$

QFI (Quantum Fisher Information)

定义与性质

条件概率分布

待测参数为θ， 算符$E(\epsilon)$对应的测量值为ϵ的条件概率分布$P(\epsilon|\theta)=\mathrm{Tr}(\rho(\theta)E(\epsilon))$
m个独立子系统测量结果$\epsilon=(\epsilon_1,\cdots,\epsilon_m)$

对数似然函数

$L(\epsilon|\theta)=\ln P(\epsilon|\theta)=\sum_i \ln P(\epsilon_i|\theta)$

估计子$\Theta(\epsilon)$

从测量结果ϵ得到θ的一个估计值Θ，相应也有一个分布$P(\Theta|\theta)$
估计子均值$\langle\Theta\rangle=\sum_\epsilon \Theta(\epsilon)P(\epsilon|\theta)$
估计子涨落$(\Delta\Theta)^2=\sum_\epsilon P(\epsilon|\theta)(\Theta(\epsilon)-\langle\Theta\rangle)^2$

无偏估计

$\langle\Theta\rangle=\theta,\forall\theta$
$\frac{\partial\langle\Theta\rangle}{\partial\theta}=1$

极大似然估计子

$\Theta_{MLE}(\theta)=\arg[\max_\theta P(\epsilon|\theta)]$

Fisher信息

$F(\theta)=\langle\left(\frac{\partial L(\theta)}{\partial\theta}\right)^2\rangle=\sum_\epsilon\frac{1}{P(\epsilon|\theta)}(\frac{\partial P(\epsilon|\theta)}{\partial\theta})^2$
可加性
$F(\theta)=\langle\sum_i\frac{1}{P(\epsilon_i)}\frac{\partial P(\epsilon_i|\theta)}{\partial\theta}\sum_j\frac{1}{P(\epsilon_j)}\frac{\partial P(\epsilon_j|\theta)}{\partial\theta}\rangle$
$=\sum_i\sum_{\epsilon_i}\frac{1}{P(\epsilon_i|\theta)}(\frac{\partial P(\epsilon_i|\theta)}{\partial\theta})^2+\sum_i\sum_{\epsilon_i}\frac{\partial P(\epsilon_i|\theta)}{\partial\theta}\sum_j\sum_{\epsilon_j}\frac{\partial P(\epsilon_j|\theta)}{\partial\theta}$
$=\sum_i\sum_{\epsilon_i}\frac{1}{P(\epsilon_i|\theta)}(\frac{\partial P(\epsilon_i|\theta)}{\partial\theta})^2+0=\sum_i F_i(\theta)$
m个全同子系统 $F(\theta)=mF_{single}(\theta)$

Cramer-Rao 下界

$(\Delta\Theta)^2\geq(\Delta\Theta_{CR})^2=\frac{(\frac{\partial\langle\Theta\rangle}{\partial\theta})^2}{F(\theta)}$

测量样本数m趋于无穷的极大似然估计

$P(\Theta_{MLE}|\theta)=\sqrt{\frac{mF(\theta)}{2\pi}}e^{-\frac{mF(\theta)}{2}(\theta-\Theta_{MLE})^2},\quad m\to\infty$
渐近无偏 $\frac{\partial\langle\Theta_{MLE}\rangle}{\partial\theta}=1,\quad m\to\infty$
方差可以达到CR下界$(\Delta\Theta_{MLE})^2=\frac{1}{mF(\theta)},\quad m\to\infty$

量子Fisher信息

改变测量算符$E(\epsilon)$所得的Fisher信息最大值
$F_Q[\rho(\theta)] = \max_{E(\epsilon)} F[\rho(\theta), \{E(\epsilon)\}] = \mathrm{Tr}(\rho(\theta) L_\theta^2)$
可加性$F_Q[\rho(\theta)] = \sum_i F_Q[\rho_i(\theta)]$

对称对数导数$L(\theta)$

$\frac{\partial \rho}{\partial \theta} = \frac{1}{2}(\rho(\theta) L_\theta + L_\theta \rho(\theta))$

显式形式

$\rho(\theta) = \sum_k p_k |k\rangle\langle k|, \quad \langle k|k'\rangle = \delta_{k'k}$
$L_\theta = \sum_k \frac{\partial_\theta p_k}{p_k} |k\rangle\langle k| + 2\sum_{k,k'} \frac{p_k - p_{k'}}{p_k + p_{k'}} |k\rangle \langle \partial_\theta k|k'\rangle \langle k'|$
$F_Q[\rho(\theta)] = \sum_k \frac{(\partial_\theta p_k)^2}{p_k} + 2\sum_{k,k'} \frac{(p_k - p_{k'})^2}{p_k + p_{k'}} |\langle \partial_\theta k|k'\rangle|^2$

幺正演化下的显式形式

$\rho(\theta) = e^{-iH\theta} \rho_0 e^{iH\theta}$
$L_\theta = 2i \sum_{k,k'} \frac{p_k - p_{k'}}{p_k + p_{k'}} |k\rangle \langle k|H|k'\rangle \langle k'|$
$F_Q[\rho(\theta)] = 2\sum_{k,k'} \frac{(p_k - p_{k'})^2}{p_k + p_{k'}} |\langle k|H|k'\rangle|^2$
纯态情形
$L_\theta = 2i|\psi(\theta)\rangle\langle\psi(\theta)|H - 2iH|\psi(\theta)\rangle\langle\psi(\theta)|$
$F_Q[\rho(\theta)] = 4(\Delta H)^2$

与压缩系数的关系

结合压缩系数ξ_R^2的定义，$H = J_{\vec{n}_1}$
$F_Q[\rho(\theta)] = 4(\Delta J_{\vec{n}_1})^2$
由不确定关系 $(\Delta J_{\vec{n}_1})^2 (\Delta J_{\vec{n}_2})^2 \geq \frac{1}{4} \langle J_{\vec{n}_0} \rangle^2$
$\frac{N}{F_Q[\rho(\theta)]} = \frac{N}{4(\Delta J_{\vec{n}_1})^2} \leq \frac{N(\Delta J_{\vec{n}_2})^2}{\langle J_{\vec{n}_0} \rangle^2} = \xi_R^2$

自旋演化半经典数值方法

TWA (truncated Wigner approximation)

truncated Wigner approximation

位置动量空间

相点算符

$\langle x'|\hat{A}(x,q)|x''\rangle = \delta\left(x - \frac{x'+x''}{2}\right) e^{ip\cdot(x'-x'')/\hbar}, \quad \langle p'|\hat{A}(x,p)|p''\rangle = \delta\left(p - \frac{p'+p''}{2}\right) e^{-ix\cdot(p'-p'')/\hbar}$

外尔符号

$O_W(x,p) = \mathrm{Tr}[\hat{O}\hat{A}(x,p)] = \int d^D\xi \,\langle x - \frac{\xi}{2}|\hat{O}(\hat{x},\hat{p})|x + \frac{\xi}{2}\rangle \exp\left[\frac{i}{\hbar}p\cdot\xi\right]$
$X_W(x,p) = x, P_W(x,p) = p$
简单形式的哈密顿量$\hat{H}(\hat{X},\hat{P}) = T(\hat{P}) + V(\hat{X}), \quad H_W(x,p) = T(p) + X(x)$为经典哈密顿量

Wigner函数

$W(x,p) = \mathrm{Tr}[\hat{\rho}\hat{A}(x,p)] = \rho_W(x,p) = \int d^D\xi \,\langle x - \frac{\xi}{2}|\hat{\rho}|x + \frac{\xi}{2}\rangle \exp\left[\frac{i}{\hbar}p\cdot\xi\right], \quad \int\int \frac{d^Dx d^Dp}{(2\pi\hbar)^D} W(x,p) = 1$

算符期望

满足 $\langle\hat{O}(\hat{x},\hat{p})\rangle = \int\int \frac{d^Dx d^Dp}{(2\pi\hbar)^D} W(x,p) O_W(x,p)$

Wigner函数的演化

辛算符 $\Lambda = \frac{\overleftarrow{\partial}}{\partial p}\frac{\overrightarrow{\partial}}{\partial x} - \frac{\overleftarrow{\partial}}{\partial x}\frac{\overrightarrow{\partial}}{\partial p}$
泊松括号 $\{A,B\}_{PB} = -A\Lambda B$
Moyal括号 $\{A,B\}_{MB} = -\frac{2}{\hbar}A\sin\left[\frac{\hbar}{2}\Lambda\right]B = -A\Lambda B + O(\hbar^2) = \{A,B\}_{PB} + O(\hbar^2)$
算符乘积的外尔符号 $(\hat{A}\hat{B})_W(x,p) = A_W(x,p)\exp\left[-\frac{i\hbar}{2}\Lambda\right]B_W(x,p)$
对易子的外尔符号 $([\hat{A},\hat{B}])_W = -2iA_W\sin\left[\frac{\hbar}{2}\Lambda\right]B_W = i\hbar\{A_W,B_W\}_{MB}$
密度算符演化方程 $\dot{\hat{\rho}}(t) = \frac{1}{i\hbar}[\hat{H},\hat{\rho}(t)]$
Wigner函数的演化方程 $\frac{\partial}{\partial t}W(x,p,t) = \{H_W(x,p), W(x,p,t)\}_{PB} + O(\hbar^2)$

t时刻的期望

对于经典哈密顿量$H_W(x,p)$决定的经典轨迹$x_{cl}(t), p_{cl}(t)$，维格纳函数近似满足刘维尔定理
$\frac{d}{dt}W(x_{cl}(t), p_{cl}(t), t) = O(\hbar^2)$
t时刻算符期望
$\langle\hat{O}(\hat{x},\hat{p})\rangle(t) = \int\int \frac{d^Dx_{cl}(t)d^Dp_{cl}(t)}{(2\pi\hbar)^D} W(x_{cl}(t),p_{cl}(t)) O_W(x_{cl}(t),p_{cl}(t)) \approx \int\int \frac{d^Dx_0 d^Dp_0}{(2\pi\hbar)^D} W(x_0,p_0) O_W(x_{cl}(t),p_{cl}(t$

相干态空间

外尔符号

$O_W(\psi,\psi^*) = \int\int \frac{d\eta^* d\eta}{(2\pi)^M}\langle\psi - \frac{\eta}{2}|\hat{O}(\hat{\psi},\hat{\psi}^\dagger)|\psi + \frac{\eta}{2}\rangle \exp[\frac{1}{2}(\eta^*\psi - \eta\psi^*)]$
在变换$x = \sqrt{\frac{\hbar}{2}}(\psi + \psi^*), p = i\sqrt{\frac{\hbar}{2}}(\psi^* - \psi)$下可以回到位置动量空间的形式

Wigner函数

$W(\psi,\psi^*) = \int\int \frac{d\eta^* d\eta}{(2\pi)^M}\langle\psi - \frac{\eta}{2}|\hat{\rho}|\psi + \frac{\eta}{2}\rangle \exp[\frac{1}{2}(\eta^*\psi - \eta\psi^*)]$

算符期望

$\langle\hat{O}(\hat{\psi},\hat{\psi}^\dagger)\rangle = \int\int \frac{d\psi d\psi^*}{\pi^M} W(\psi,\psi^*) O_W(\psi,\psi^*)$

Wigner函数的演化

辛算符 $\Lambda_c = \frac{\overleftarrow{\partial}}{\partial\psi}\frac{\overrightarrow{\partial}}{\partial\psi^*} - \frac{\overleftarrow{\partial}}{\partial\psi^*}\frac{\overrightarrow{\partial}}{\partial\psi}$
泊松括号 $\{A, B\}_{PB} = A\Lambda_c B$
Moyal括号 $\{A, B\}_{MB} = 2A\sinh\left[\frac{\Lambda_c}{2}\right]B = A\Lambda_c B + \cdots = \{A, B\}_{PB} + \cdots$
算符乘积的外尔符号 $(\hat{A}\hat{B})_W(\psi, \psi^*) = A_W(\psi, \psi^*)\exp\left[\frac{\Lambda_c}{2}\right]B_W(\psi, \psi^*)$
对易子的外尔符号 $([\hat{A}, \hat{B}])_W = 2A_W\sinh\left[\frac{\Lambda_c}{2}\right]B_W = \{A_W, B_W\}_{MB}$
密度算符演化方程 $\dot{\hat{\rho}}(t) = \frac{1}{i\hbar}[\hat{H}, \hat{\rho}(t)]$
Wigner函数的演化方程 $\frac{\partial}{\partial t}W(\psi, \psi^*, t) = \{H_W(\psi, \psi^*), W(\psi, \psi^*, t)\}_{PB} + \cdots$
ψ, ψ^*很大时，忽略泊松括号后面的项，因为ψ, ψ^*次数低

t时刻的期望

$\langle\hat{O}(\hat{\psi}, \hat{\psi}^\dagger)\rangle(t) = \int\int\frac{d\psi_0 d\psi_0^*}{\pi^M}W(\psi_0, \psi_0^*)O_W(\psi_{cl}(t), \psi_{cl}^*(t))$

自旋

Schwinger表示

$\hat{s}^z = \frac{\hat{\alpha}^\dagger\hat{\alpha} - \hat{\beta}^\dagger\hat{\beta}}{2}, \quad \hat{s}^+ = \hat{\alpha}^\dagger\hat{\beta}, \quad \hat{s}^- = \hat{\beta}^\dagger\hat{\alpha}$
利用相干态空间的外尔符号即可得到自旋算符的外尔符号
$s_W^z = \frac{|\alpha|^2 - |\beta|^2}{2}, \quad s_W^+ = \alpha^*\beta, \quad s_W^- = \beta^*\alpha$
$s_W^x = \frac{\alpha^*\beta + \beta^*\alpha}{2}, \quad s_W^y = \frac{\alpha^*\beta - \beta^*\alpha}{2i}, \quad s_W^\perp \overset{\text{def}}{=} \sqrt{(s_W^x)^2 + (s_W^y)^2} = |\alpha||\beta|$
$s_W \overset{\text{def}}{=} \sqrt{(s_W^z)^2 + (s_W^\perp)^2} = \frac{|\alpha|^2 + |\beta|^2}{2}$
$s_W^z + s_W = |\alpha|^2$

$|S, S\rangle$的Wigner函数

$|S, S\rangle = \frac{(\alpha^\dagger)^{2S}}{\sqrt{(2S)!}}|0\rangle$
Wigner函数
$W(\alpha, \alpha^*, \beta, \beta^*) = 2e^{-2|\alpha|^2 - 2|\beta|^2}L_{2S+1}(4|\alpha|^2)$
将变量选为$s_W^z, s_W^\perp$
$W(s_W^z, s_W^\perp) = \frac{2}{\pi s_W}e^{-4s_W}L_{2S+1}(4[s_W^z + s_W])$
（分母中的s_W应该来源于积分变量代换$\left|\frac{\partial(s_W^z, s_W^\perp)}{\partial(|\alpha|^2, |\beta|^2)}\right| = s_W$）
$\int_{-\infty}^{\infty}ds_W^z\int_0^\infty ds_W^\perp\int_0^{2\pi}d\phi W(s_W^z, s_W^\perp) = 1$
大S极限下 $W(s_W^z, s_W^\perp) \approx \frac{1}{\pi S}e^{-(s_W^\perp)^2/S}\delta(s_W^z - S)$
涨落为高斯分布，只能考虑集体自旋的涨落

DTWA (Discrete TWA)

discrete truncated Wigner approximation

离散相空间

$\alpha=(q,p)\in\{(0,0),(0,1),(1,0),(1,1)\}$
$\vec{r}_{(0,0)}=(1,1,1),\vec{r}_{(0,1)}=(-1,-1,1),\vec{r}_{(1,0)}=(1,-1,-1),\vec{r}_{(1,1)}=(-1,1,-1)$

相点算符

$\hat{A}_\alpha=\hat{\rho}(\vec{r}_\alpha)=(1+\vec{r}_\alpha\cdot\vec{\hat{\sigma}})/2$
$\mathrm{Tr}(\hat{A}_\alpha\hat{A}_\beta)=\delta_{\alpha\beta}$
N个自旋的相点算符 $\hat{A}_{\vec{\alpha}}=\prod_{\otimes i=1}^N\hat{A}_{\alpha_i}$

外尔符号

$O_\alpha^W=\mathrm{Tr}(\hat{O}\hat{A}_\alpha)/2,\quad \hat{O}=\sum_\alpha\hat{A}_\alpha O_\alpha^W$

Wigner函数

$w_\alpha=\mathrm{Tr}(\hat{\rho}\hat{A}_\alpha)/2,\quad \hat{\rho}=\sum_\alpha\hat{A}_\alpha w_\alpha,\quad \sum_\alpha w_\alpha=1$
$p_1^x=w_{(0,0)}+w_{(1,0)},\quad p_{-1}^x=w_{(0,1)}+w_{(1,1)}$
$p_1^y=w_{(0,0)}+w_{(1,1)},\quad p_{-1}^y=w_{(0,1)}+w_{(1,0)}$
$p_1^z=w_{(0,0)}+w_{(0,1)},\quad p_{-1}^z=w_{(1,0)}+w_{(1,1)}$
直积态的Wigner函数 $w_{\vec{\alpha}}=\prod_{i=1}^N w_{\alpha_i}$

算符期望

$\langle O\rangle=\sum_{\vec{\alpha}}w_{\vec{\alpha}}O_{\vec{\alpha}}^W$

t时刻的算符期望

$\langle O\rangle(t)=\sum_{\vec{\alpha}}w_{\vec{\alpha}}(0)O_{\vec{\alpha}}^W(t)$
$O_{\vec{\alpha}}^W(t)=\mathrm{Tr}(\hat{O}(t)\hat{A}_{\vec{\alpha}})/2=\mathrm{Tr}(\hat{O}(0)\hat{U}(t)\hat{A}_{\vec{\alpha}}\hat{U}^\dagger(t))/2,\quad U(t)=\exp(-i\hat{H}t/\hbar)$
$\hat{A}_{\vec{\alpha}}(t)=\hat{U}(t)\hat{A}_{\vec{\alpha}}\hat{U}^\dagger(t)$
$i\dot{\hat{A}}_{\vec{\alpha}}(t)=[\hat{H},\hat{A}_{\vec{\alpha}}]$
约化相点算符
$\hat{A}_j=\mathrm{Tr}_{\cancel{j}}(\hat{A}_{\vec{\alpha}}),\quad \hat{A}_{j,k}=\mathrm{Tr}_{\cancel{j},\cancel{k}}(\hat{A}_{\vec{\alpha}}),\quad \hat{A}_{j,k,l}=\mathrm{Tr}_{\cancel{j},\cancel{k},\cancel{l}}(\hat{A}_{\vec{\alpha}})$
包含单体作用和两体作用的哈密顿量
$\hat{H}=\sum_j\hat{H}_j+\sum_{j,l}\hat{H}_{j,l}=\frac{1}{2}\sum_{i=1}^N\vec{\Omega}_i\cdot\vec{\sigma}_i+\frac{1}{2}\sum_{i\neq j}^N\vec{\sigma}_i^T\cdot J_{ij}\cdot\vec{\sigma}_i$
BBGKY级联方程
$i\dot{\hat{A}}_j=[\hat{H}_j,\hat{A}_j]+\sum_{l=1,l\neq j}^N\mathrm{Tr}_l([\hat{H}_{j,l},\hat{A}_{j,l}])$
$i\dot{\hat{A}}_{j,k}=[\hat{H}_j+\hat{H}_k+\hat{H}_{j,k},\hat{A}_{j,k}]+\sum_{l=1,l\neq j,k}^N\mathrm{Tr}_l([\hat{H}_{j,l}+\hat{H}_{k,l},\hat{A}_{j,k,l}])$

截断关联

$\hat{A}_{j,k} \approx \hat{A}_j \hat{A}_k$

$i\dot{\hat{A}}_j = [\hat{H}_j, \hat{A}_j] + \sum_{l=1,l\neq j}^{N} \mathrm{Tr}_l([\hat{H}_{j,l}, \hat{A}_j \hat{A}_l])$

等价于让$\hat{A}_{\vec{\alpha}}$始终保持直积态得到的方程，求解规模得以缩小

对于每个相空间抽样$\vec{\alpha}$，演化方程可以写为经典自旋$\dot{r}_{\vec{\alpha}}$的演化

$\frac{d\vec{r}_i}{dt} = \vec{\Omega}^i_{eff} \times \vec{r}_i, \quad \vec{\Omega}^i_{eff} = \vec{\Omega}_i + 2\sum_{j=1,j\neq i}^{N} J_{ij} \cdot \vec{r}_i$

DDTWA (Dissipative Discrete TWA)

dissipative discrete truncated Wigner approximation

$\dot{\rho} = -i[H, \rho] + L_{deph}(\rho) + L_{decay}(\rho)$

$L_{deph}(\rho) = \frac{\Gamma_\phi}{2} \sum_{i=1}^{N} (\sigma_i^z \rho \sigma_i^z - \rho)$

$L_{decay} = \frac{\Gamma}{2} \sum_{i=1}^{N} (2\sigma_i^- \rho \sigma_i^+ - \sigma_i^+ \sigma_i^- \rho - \rho \sigma_i^+ \sigma_i^-)$

直接使用DTWA的局限

经典自旋演化过程中的模长不再守恒，相互作用项将出现偏差

目标是尽可能保持模长不变

Dephasing的处理

$d\vec{r}_i = \vec{\Omega}^i_{eff} \times \vec{r}_i dt + d\vec{r}_i|_{deph}$

$dr_i^x|_{deph} = -\Gamma_\phi r_i^x dt - \sqrt{2\Gamma_\phi} r_i^y dW_i$

$dr_i^y|_{deph} = -\Gamma_\phi r_i^y dt - \sqrt{2\Gamma_\phi} r_i^x dW_i$

$dr_i^z|_{deph} = 0$

维纳过程 $\langle dW_i \rangle = 0, \langle (dW_i)^2 \rangle = dt$，从高斯分布中抽样

模长变化$\langle d\vec{r}_i^2 \rangle = 0$

Decay的处理

$d\vec{r}_i = \vec{\Omega}^i_{eff} \times \vec{r}_i dt + d\vec{r}_i|_{decay}$

$dr_i^x|_{decay} = -\frac{\Gamma}{2} r_i^x dt - \sqrt{\Gamma} r_i^y dW_i$

$dr_i^y|_{decay} = -\frac{\Gamma}{2} r_i^y dt + \sqrt{\Gamma} r_i^x dW_i$

$dr_i^z|_{decay} = -\Gamma(r_i^z + 1)dt + \sqrt{\Gamma}(r_i^z + 1)dW_i$

维纳过程 $\langle dW_i \rangle = 0, \langle (dW_i)^2 \rangle = dt$，从高斯分布中抽样

模长变化$\langle d\vec{r}_i^2 \rangle = \Gamma(1 - \langle (r_i^z)^2 \rangle)dt$

由于$\langle d(r_i^z)^2 \rangle = -2\Gamma r_i^z (r_i^z + 1)dt + \Gamma(r_i^z + 1)^2 dt$

r_i^z小于0时，$d(r_i^z)^2$大于0，$\langle (r_i^z) \rangle$趋于1，模长变化被抑制