Homework 1 Due Tuesday, September 5th by midnight

1. (Griffiths A.1) Consider the ordinary vectors in three dimensions $a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$, with complex components.
   (a) Does the subset of all vectors with $a_z = 0$ constitute a vector space? If so, what is its dimension; if not, why not?
   (b) What about the subset of all vectors whose z component is 1? Hint: Would the sum of two such vectors be in the subset? How about the null vector?
   (c) What about the subset of vectors whose components are all equal?

2. (Griffiths A.8) Given the following two matrices:

$$A = \begin{pmatrix} -1 & 1 & i \\ 2 & 0 & 3 \\ 2i & -2i & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & 0 & -i \\ 0 & 1 & 0 \\ i & 3 & 2 \end{pmatrix}$$

   compute the following:
   (a) $A + B$
   (b) AB
   (c) $[A, B]$
   (d) A^{T}. (Note: This stands for the transpose. Griffiths writes this as $\tilde{A}$).
   (e) A^*
   (f) $A^\dagger$
   (g) $\det(B)$
   (h) B^{-1}. Check that $BB^{-1} = I$. Does A have an inverse?

3. (Griffiths A.9) Using the square matrices in Problem 2, and the column matrices

$$a = \begin{pmatrix} i \\ 2i \\ 2 \end{pmatrix}, \qquad b = \begin{pmatrix} 2 \\ (1-i) \\ 0 \end{pmatrix},$$

   find:
   (a) Aa
   (b) $a^\dagger b$
   (c) $a^{\mathsf{T}} A b$
   (d) $ab^\dagger$

4. (Griffiths A.13) Noting that $\det(A) = \det(A^{\mathsf{T}})$, show that:
   (a) the determinant of a hermitian matrix is real
   (b) the determinant of a unitary matrix has modulus 1 (hence the name)
   (c) the determinant of an orthogonal matrix (footnote 13) is either 1 or -1.

5. (Griffiths A.19) Find the eigenvalues and eigenvectors of the following matrix:

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

   Can this matrix be diagonalized?

6. (Griffiths A.27) A hermitian linear transformation must satisfy $\langle\alpha|T\beta\rangle = \langle T\alpha|\beta\rangle$ for all vectors $|\alpha\rangle$ and $|\beta\rangle$. Prove that it is (surprisingly) sufficient that $\langle\gamma|T\gamma\rangle = \langle T\gamma|\gamma\rangle$ for all vectors $|\gamma\rangle$. Hint: First let $|\gamma\rangle = |\alpha\rangle + |\beta\rangle$, and then let $|\gamma\rangle = |\alpha\rangle + i\,|\beta\rangle$.