1. *Commutators and Angular Momentum*

   (Suppressing hats since we have lots of operators here.) Using the fact that $[x, p_x] = i\hbar$, $[y, p_y] = i\hbar$, and $[z, p_z] = i\hbar$, verify various claims made in class on Tuesday:

   a. $[L_x, L_y] = i\hbar L_z$
   b. $[L^2, L_z] = 0$
   c. $[L_z, L_\pm] = \pm\hbar L_\pm$
   d. $L_\pm L_\mp = L^2 - L_z^2 \pm \hbar L_z$

   I'm using the abbreviated notion where the last two problems are really four problems. I recommend that you prove them as four problems because it is much less likely you will make a sign error that way.

2. *Isotropic 3D Harmonic Oscillator*

   Solve the three-dimensional time-independent Schrödinger equation with the potential
   $$V(\vec{r}) = \frac{1}{2}\mu\omega^2 r^2$$
   using separation of variables. You may simply quote results about the 1D harmonic oscillator; you do not have to rederive them.

   a. Write down the wave function $\psi(x, y, z)$ in terms of energy eigenvalues E_x, E_y, E_z associated to each dimension.
   b. What are the possible values of E_x, E_y, and E_z?
   c. What is the degeneracy of the first, second, and third energy level? (That is, how many mutually orthogonal energy eigenstates are there that have the lowest, second lowest, and third lowest total energy?)

3. *Coordinate transformations and derivatives* Recall the standard definition of spherical polar coordinates.

   a. Write down x, y, and z as a function of r, θ, and ϕ.
   b. Write down r, θ, and ϕ as a function of x, y, and z.
   c. Using the chain rule, show that
   $$\frac{\partial}{\partial\phi} = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}$$