PhysH308

Spinning stuff!

Ted Brzinski, Nov. 7, 2024

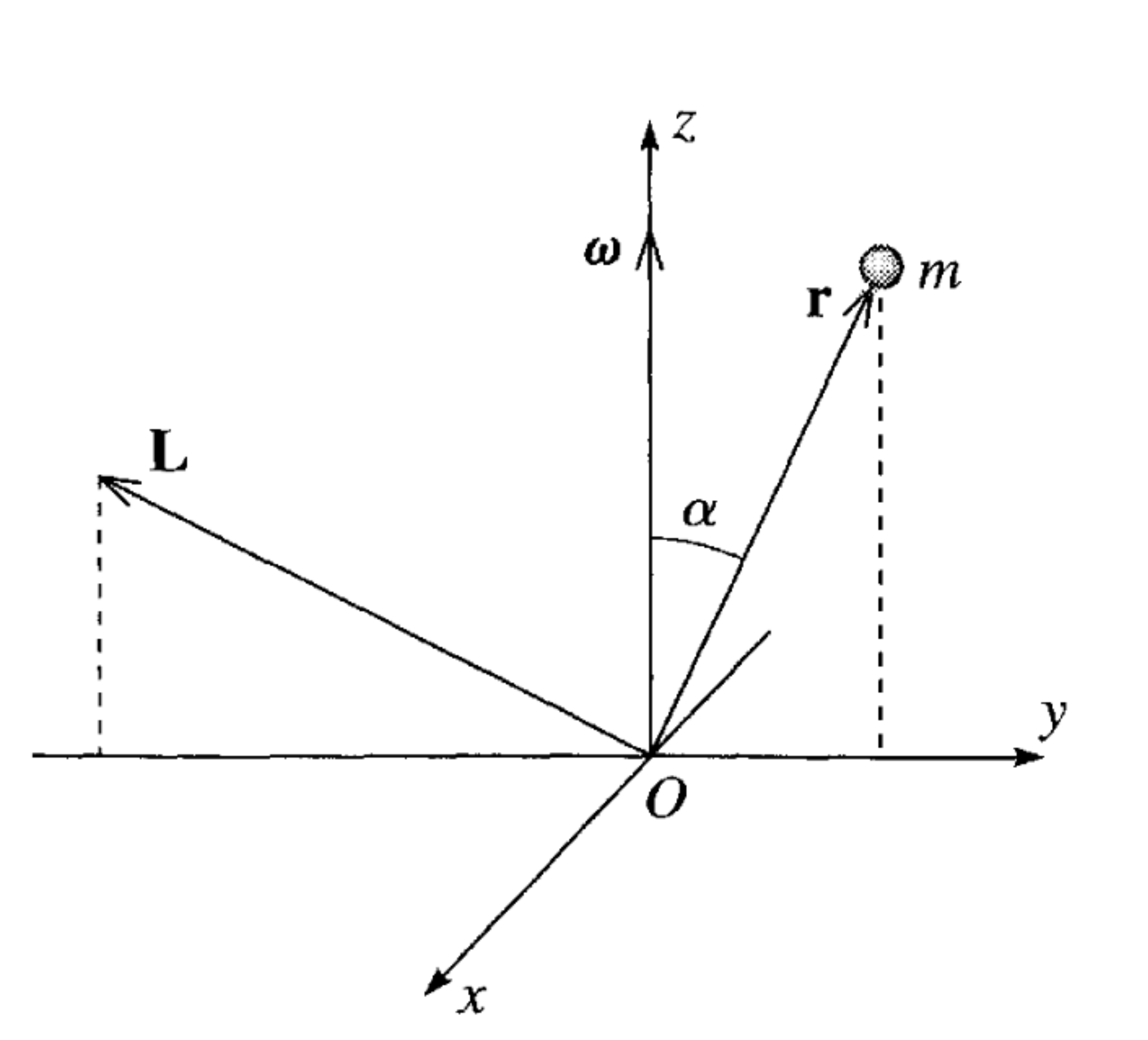
Problem 10.14

Flywheel propulsion I

- Given a big, spherical space station, how long must you spin a flywheel to rotate the space station by a constant amount?
- *Use conservation of angular momentum! You don't need* $\vec{L} = \mathbf{I}\vec{\omega}$ *yet, only* $L = I\omega$.
- *Find the energy used for the rotation.*
  (Total $\Delta E = \Delta E_{station} + \Delta E_{fw} \approx \Delta E_{fw}$ *— why?)*
  (note: torque was applied to both masses - how do we know?)

Inertia tensor

- In general: $\overrightarrow{L} = \mathbf{I}\overrightarrow{\omega}$ where $\mathbf{I}$ is the *Inertia Tensor!*
  - $I_{xx} = m\sum\left(y^2 + z^2\right)$
  - $I_{xy} = I_{yx} = -m\sum xy$
  - *(Note this works for any choice of $\overrightarrow{\omega}$ and $\mathcal{O}$)*



Inertia tensor

- In general: $\overrightarrow{L} = \mathbf{I}\overrightarrow{\omega}$ where $\mathbf{I}$ is the *Inertia Tensor!*
  - $I_{xx} = m\sum\left(y^2 + z^2\right)$
  - $I_{xy} = I_{yx} = -m\sum xy$
  - *(Note this works for any choice of $\overrightarrow{\omega}$ and $\mathcal{O}$)*

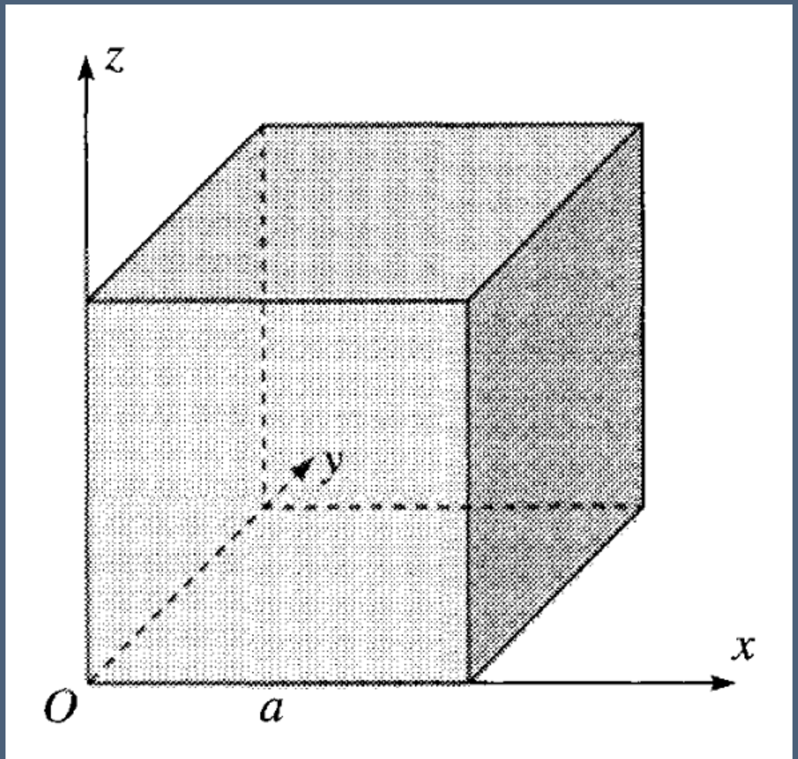
Principal axes:

- $\mathbf{I}$ can be diagonalized by choosing the right $\left(\hat{e}_x, \hat{e}_y, \hat{e}_z\right) = \left(\hat{x}, \hat{y}, \hat{z}\right)$
- In this case, $I_{ii} = \lambda_i$ where λ_i are the roots of the equation $\det\left(\mathbf{I} - \mathbf{1}\lambda\right) = 0$
- *Solving* $\left(\mathbf{I} - \lambda_i\mathbf{1}\right)\hat{e}_i = 0$ *gives* $\hat{e}_i$.

Problem 10.22

Moment of inertia practice

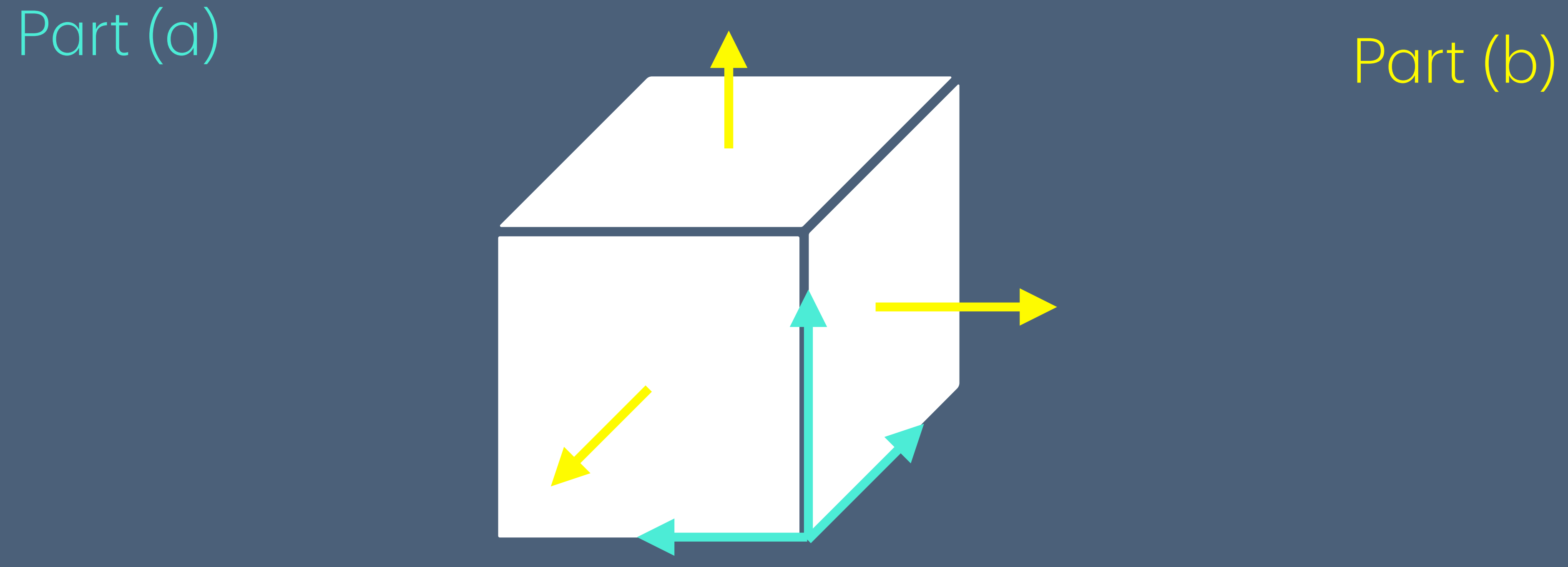
- Given a simple array of masses, practice calculating $\vec{L} = \mathbf{I}\vec{\omega}$



Problem 10.22

Moment of inertia practice

- Given a simple array of masses, practice calculating $\overrightarrow{L} = \mathbf{I}\overrightarrow{\omega}$



Problem 10.37

After 10.22

- Given a shape, find $\mathbf{I}$, diagonalize it, and find the principal axes