PhysH308

Lagrangian Mechanics

Ted Brzinski, Oct 8, 2024

Announcements

- Office hours today, tomorrow: zoom both days, only until 1:30 today
- Exam 2
- Last week's slides will go up this afternoon.

Force on a particle

By Newton's 2nd:

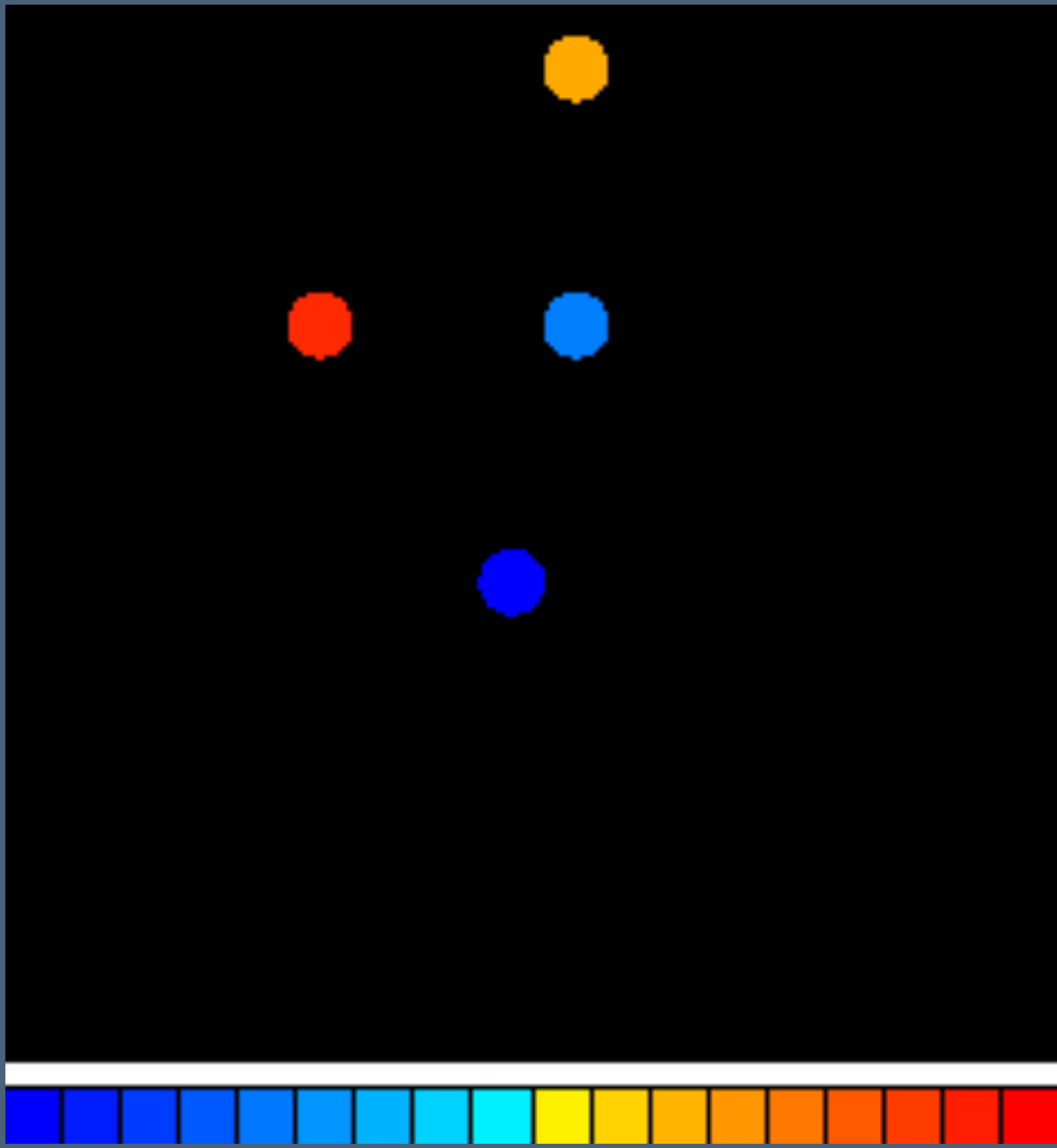
$$F = m\ddot{x} = m\dot{v} = \frac{d}{dt}mv$$

$$= \frac{d}{dt}m\frac{\partial}{\partial v}\left(\frac{v^2}{2}\right) = \frac{d}{dt}\frac{\partial T}{\partial \dot{x}}$$

By definition:

$$F = -\frac{\partial}{\partial x}U$$

Thus: $-\dfrac{\partial}{\partial x}U = \dfrac{d}{dt}\dfrac{\partial T}{\partial \dot{x}}$



Force on a particle

$$-\frac{\partial}{\partial x}U = \frac{d}{dt}\frac{\partial T}{\partial \dot{x}}$$

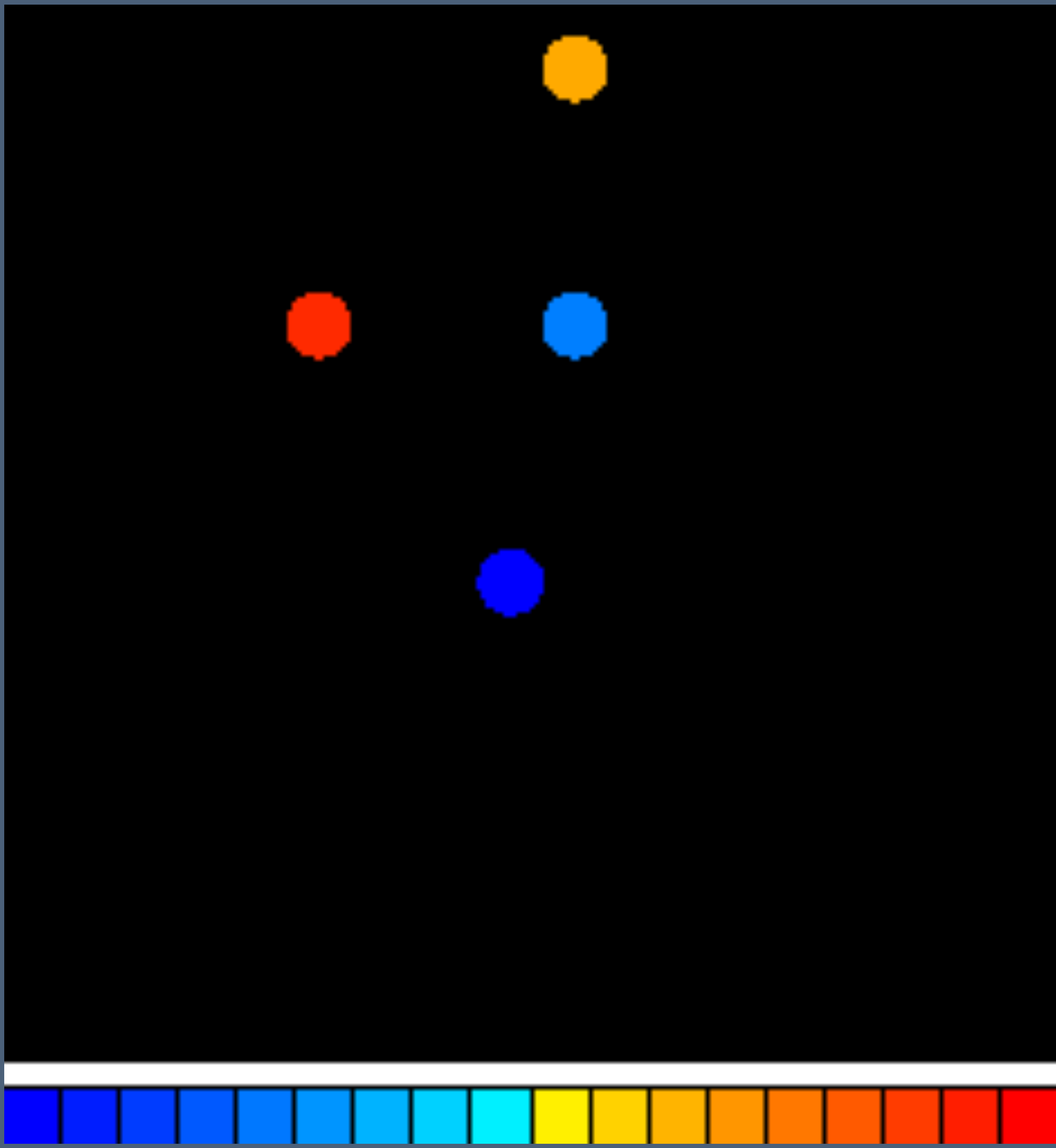
Also note that, in general,

$$\frac{\partial}{\partial x}T = 0 \text{ and } \frac{\partial U}{\partial \dot{x}} = 0$$

So $\frac{\partial}{\partial x}\mathscr{L} = \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{x}}$ where

$$\mathscr{L} = T - U$$

So what?



Force on a particle

$$\frac{\partial}{\partial x}\mathscr{L} = \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{x}} \text{ where}$$

$$\mathscr{L} = T - U$$

So what?

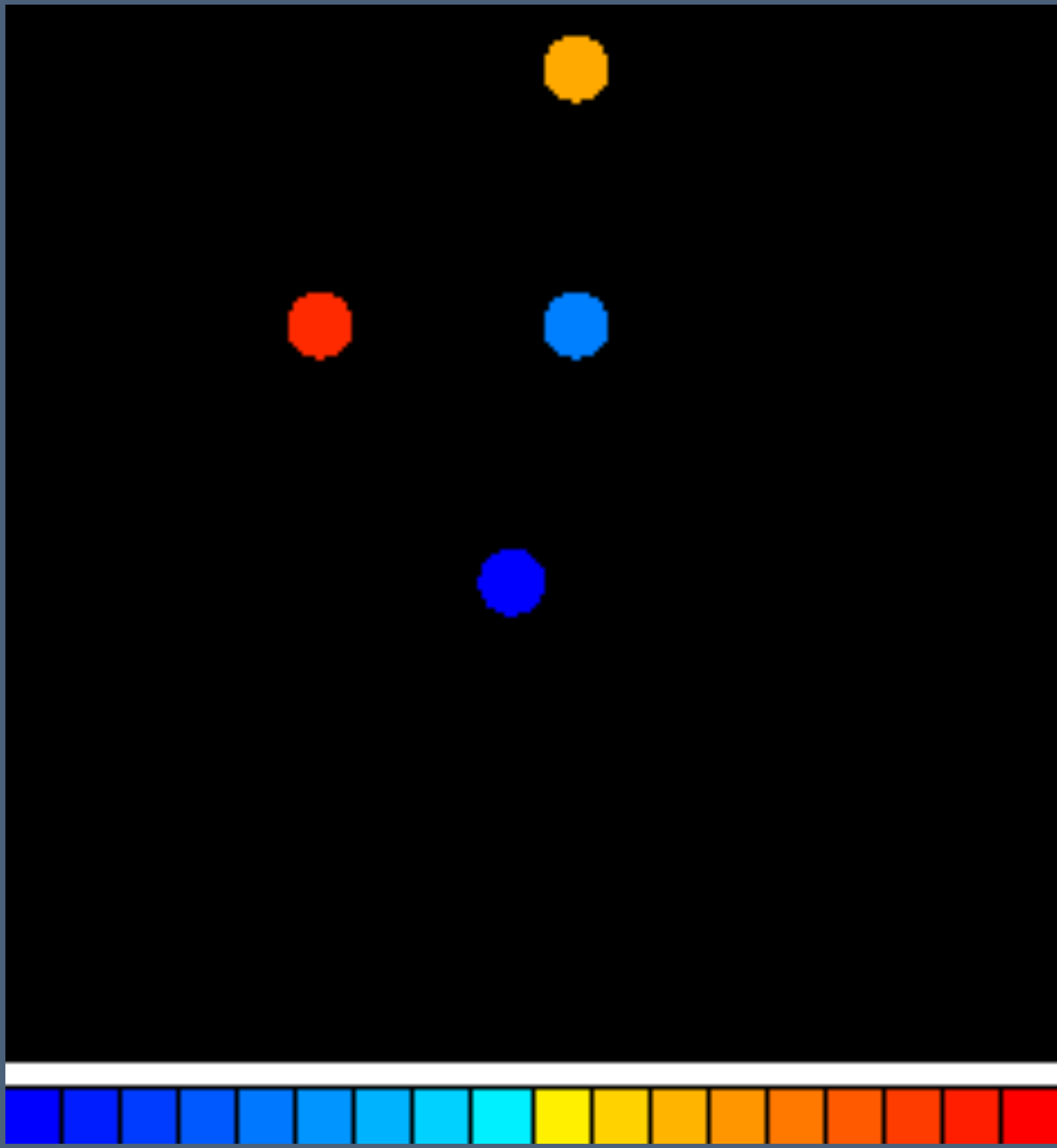
Well, that's E-L for...

$$S = \int_{t_1}^{t} \mathscr{L}\, dt$$

The "Action!!!"

What is "action"?

The thing minimized by paths described by Newton's 2nd law.

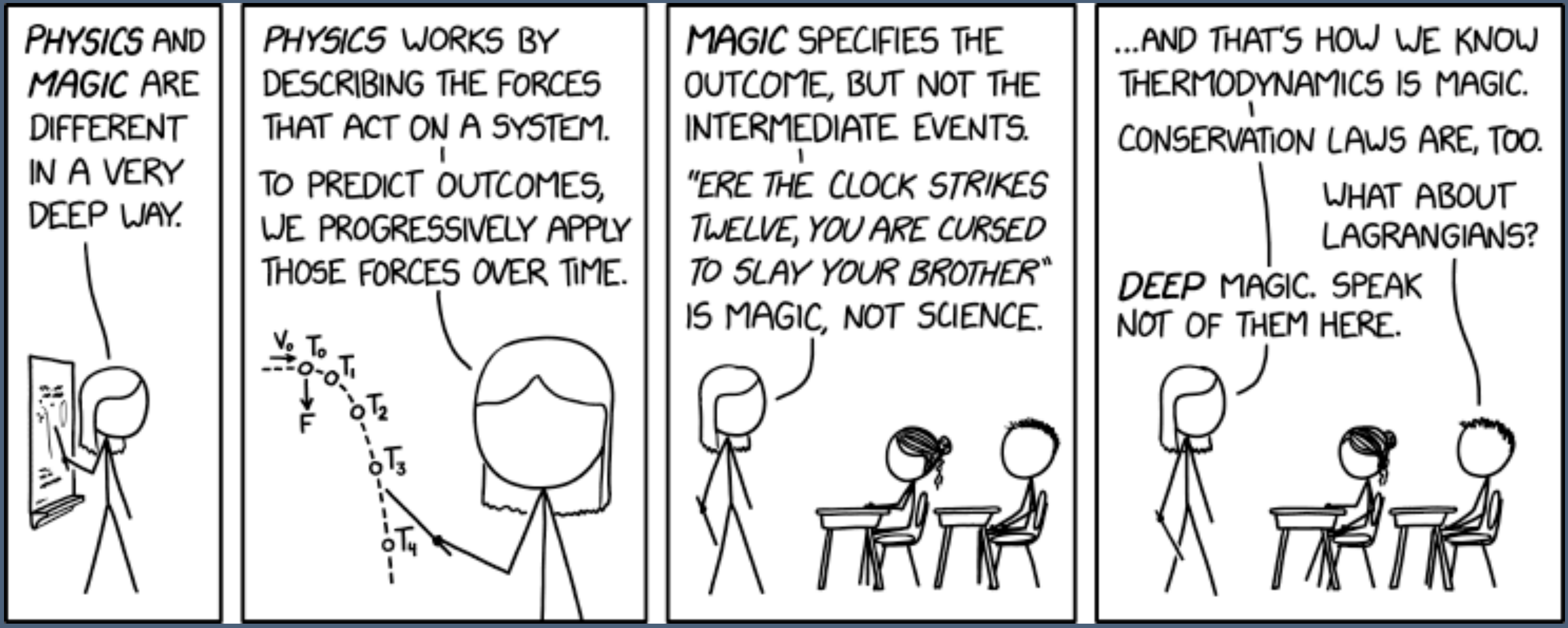


Okay Ted. Fine. But why would I do that?

(Fair question)

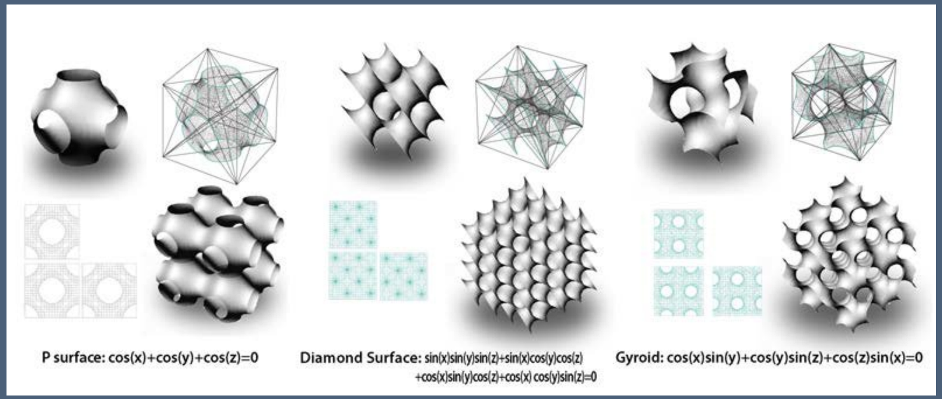
- It's often easier to solve E-L for S than to solve a bunch of $F = ma$. Especially for coupled/constrained motion (we'll see that in practice tomorrow).

- We get the equation of motion from a statement that $F = F$, weird!



Problems

- 6.19: our last problem from Chap. 6, finding a minimal surface! (Soap films, planar defects in LC, 3D printing)



- 7.2: Langrangian mech reproduces a familiar result!