PhysH308

Energy

Ted Brzinski, Sept 16, 2024

Recall: linear momentum

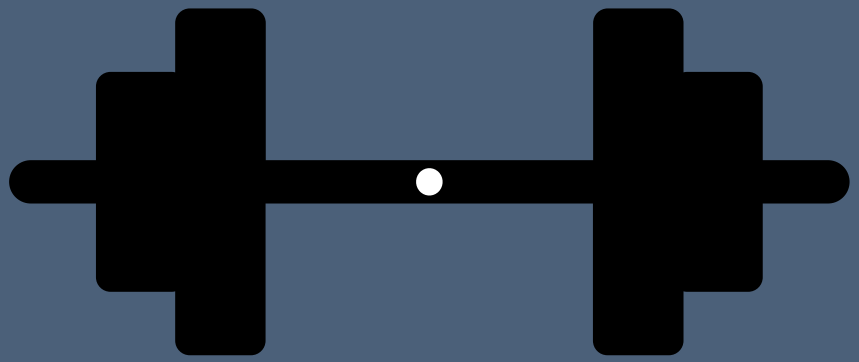
- Momentum: $\vec{p} = m\dot{\vec{x}}$
- Momentum is a conserved quantity:

  To change the momentum of a system, an external force must be applied: $\overrightarrow{F} = \dot{\vec{p}}$

  Without external forcing, momentum is constant.

Angular momentum

- Momentum can also be stored in internal degrees of freedom (i.e. rotation)



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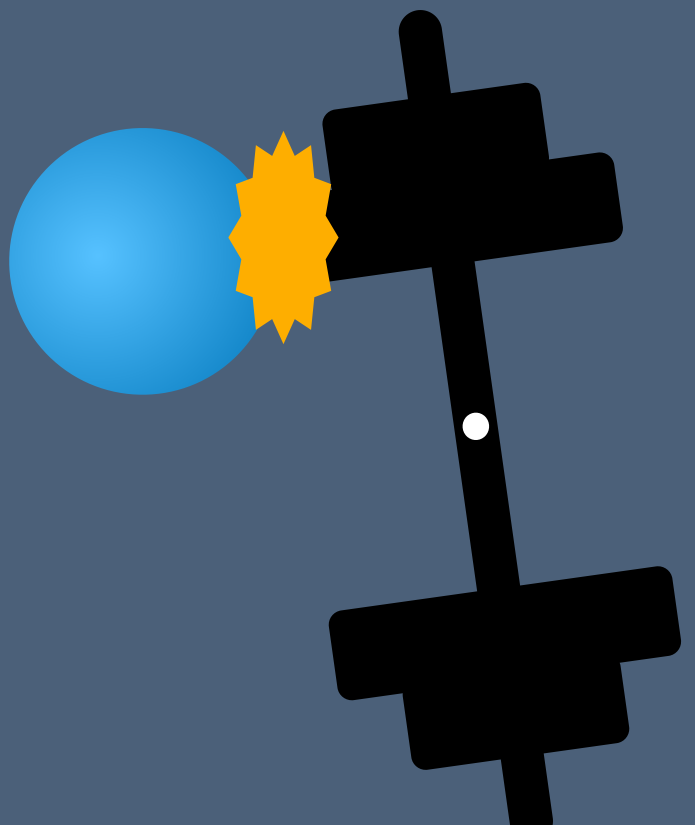

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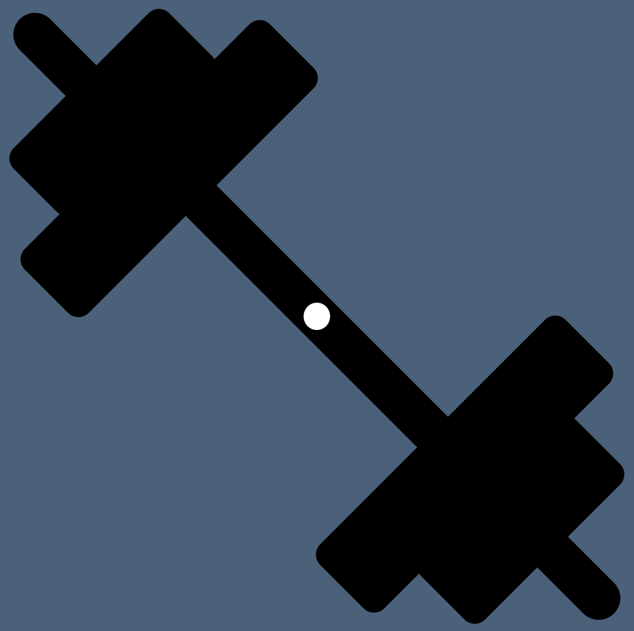

Angular momentum

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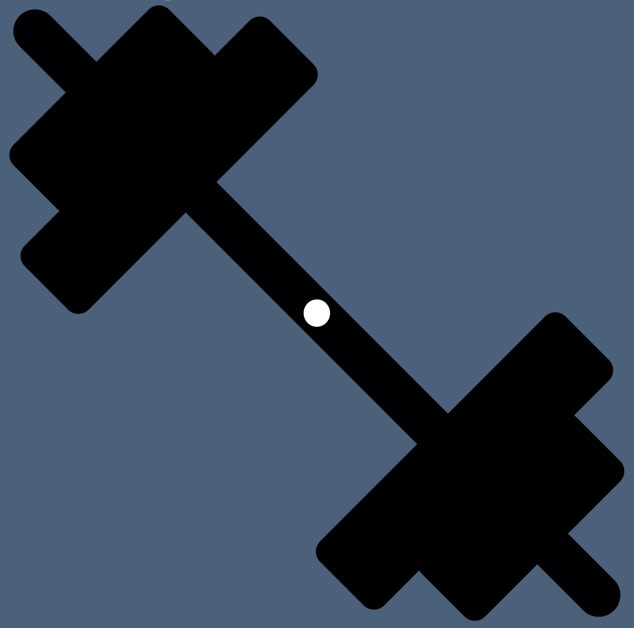
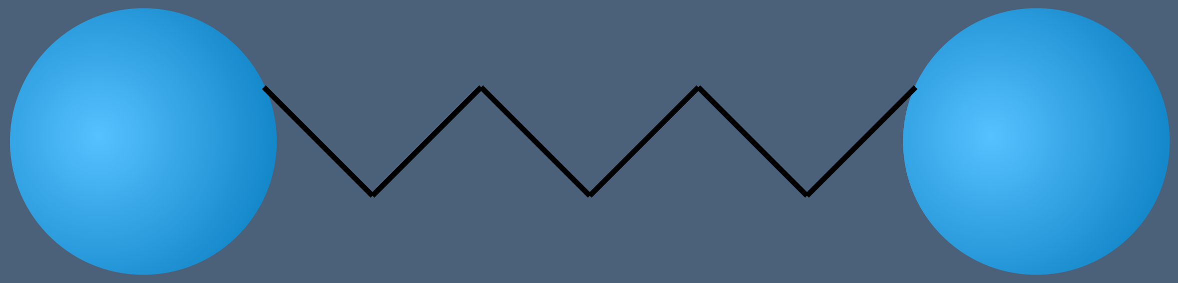
- Angular momentum:
$\overrightarrow{\ell} = \vec{r} \times \vec{p}$ or $\overrightarrow{L} = I\overrightarrow{\omega}$
- An external torque is required to change the angular momentum:
$\vec{\tau} = \dot{\overrightarrow{L}}$



Angular momentum

Rotation is weird!

- Consider a pair of balls connected by a spring, and floating in empty space.



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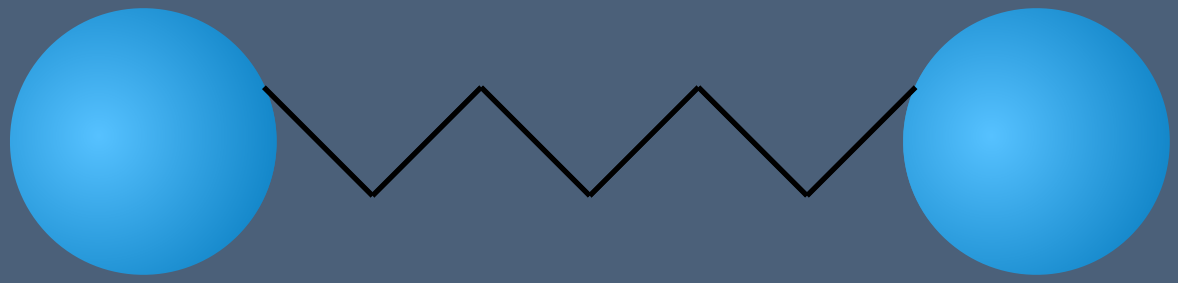

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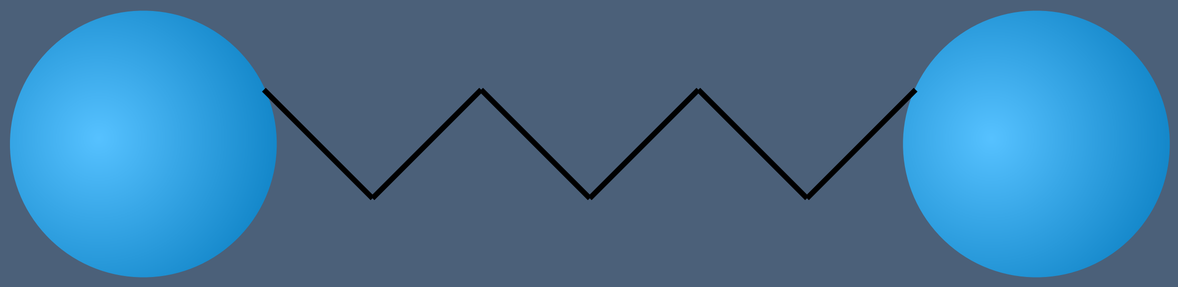

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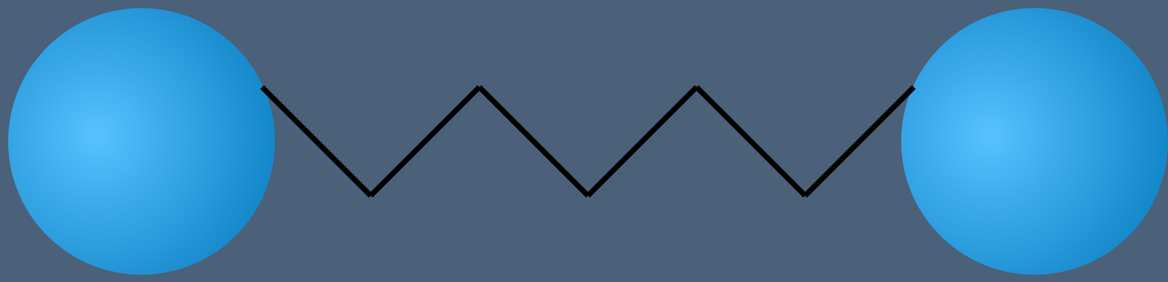

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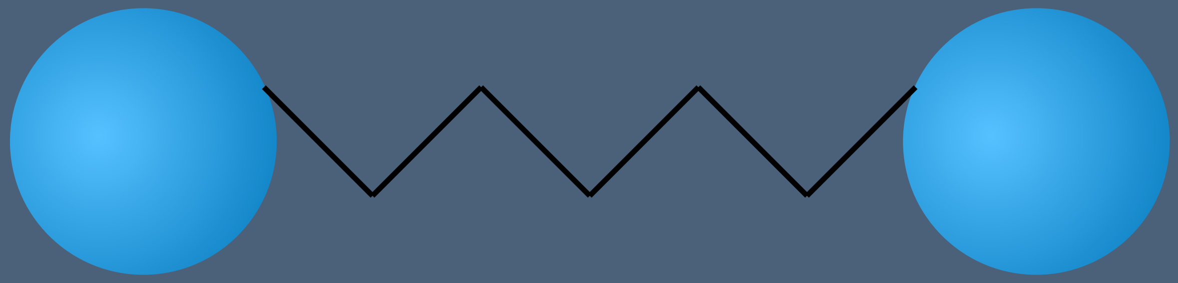

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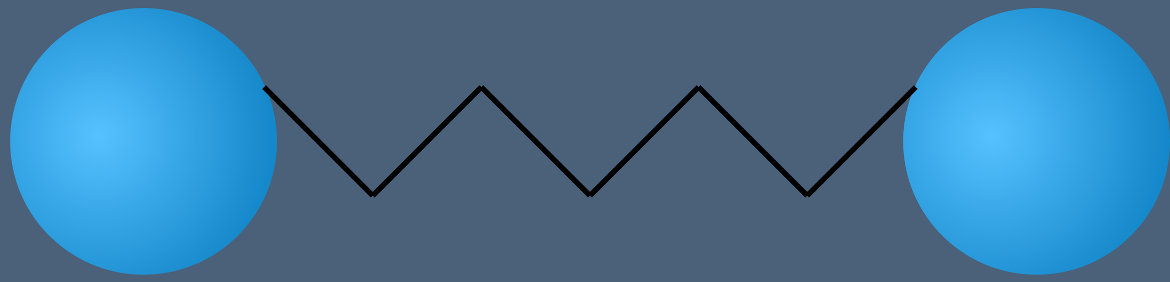
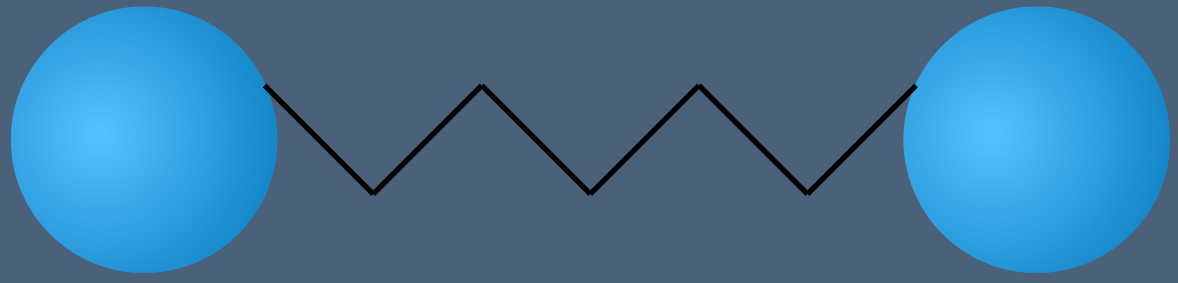


- That’s not true of the angular momentum, which can be read out of the extension of the spring!



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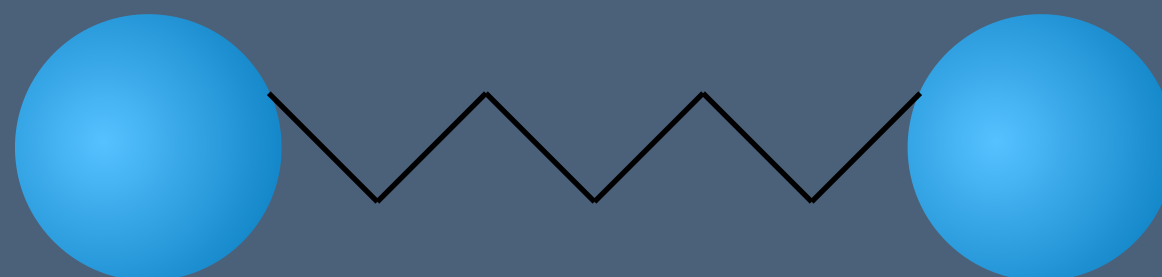

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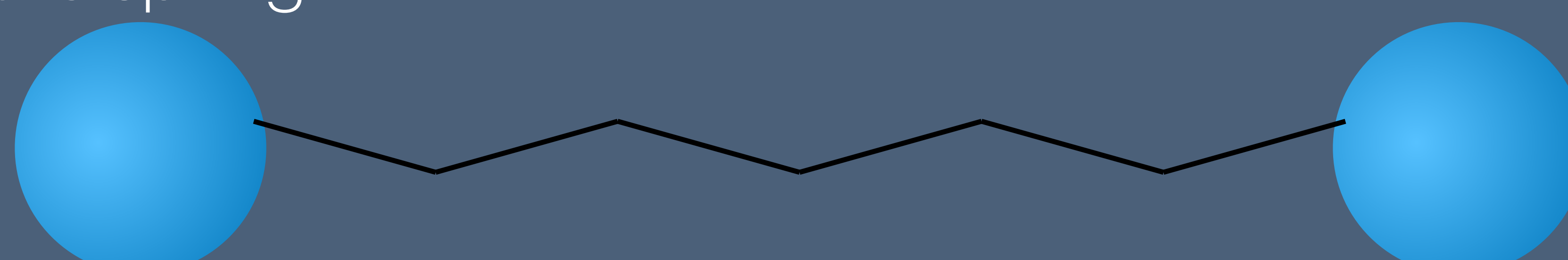
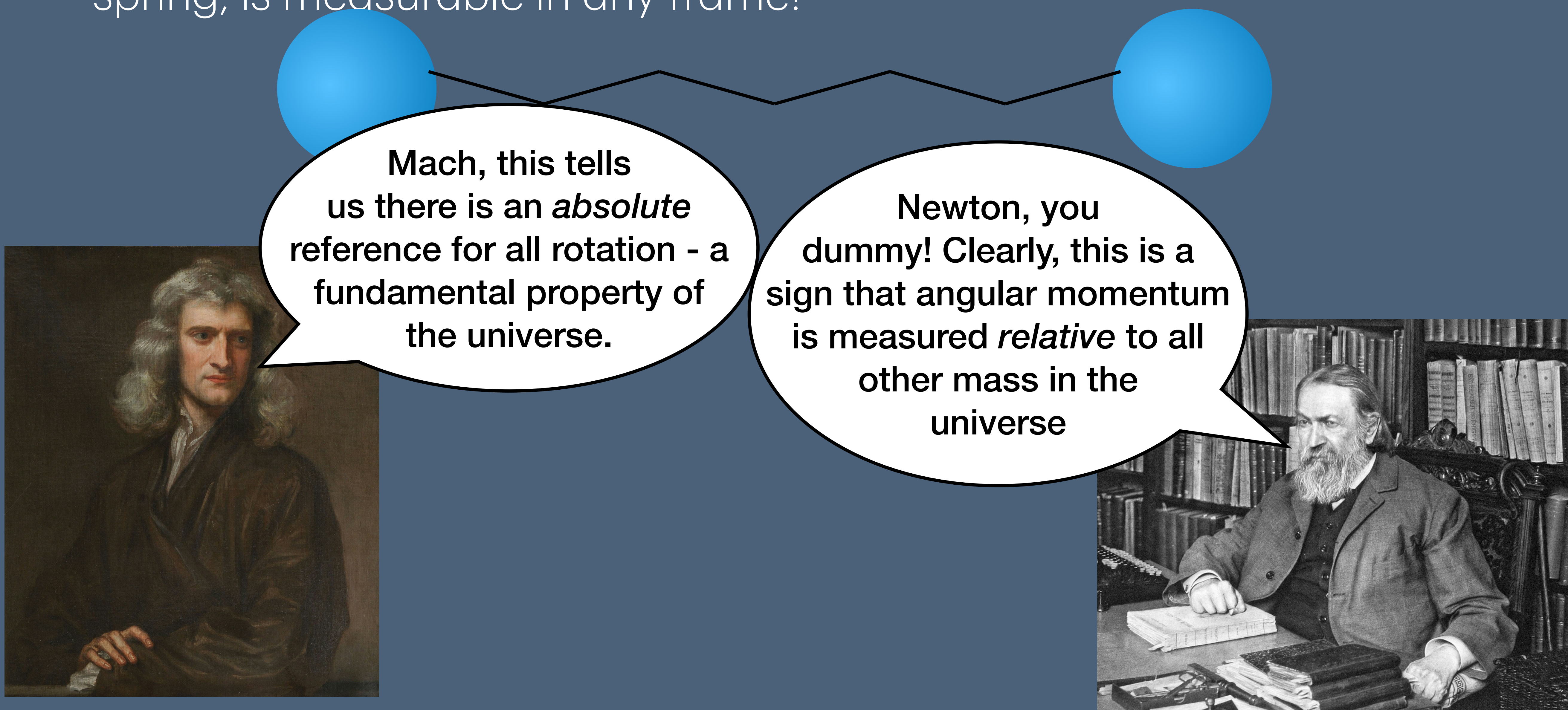


Why is rotation weird?

- Angular momentum, which can be read out of the extension of the spring, is measurable in any frame!



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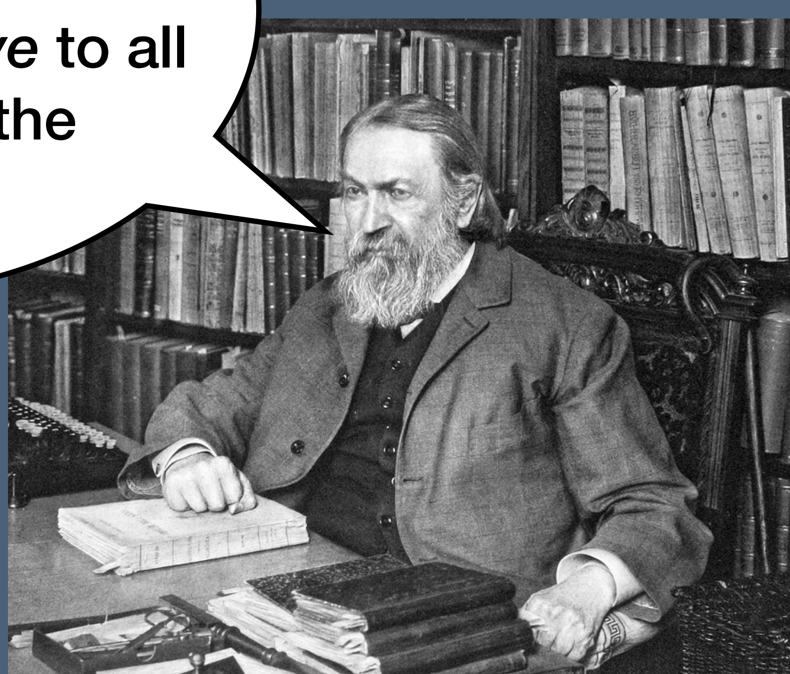
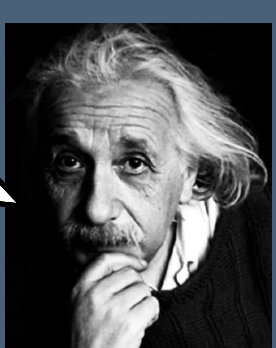




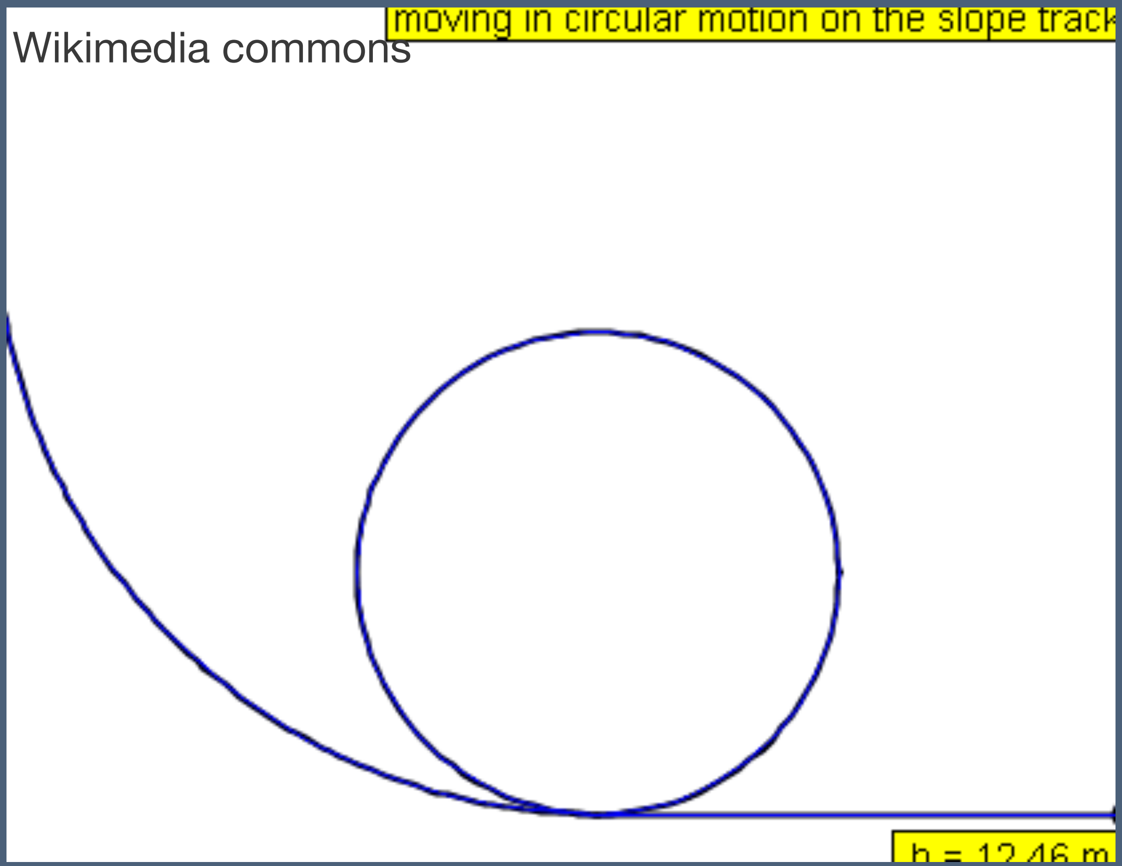
Enough momentum!

Let's talk energy!

Energy Conservation

A restatement of Newton's 2nd Law

- In general, $\vec{F}\left(\vec{r}, t\right)$ is not easily separable, and may be difficult to solve from $\vec{F} = m\dot{\vec{v}}$ directly
- $$\vec{F} = m\frac{d\vec{v}}{dt} = m\frac{d\vec{v}}{d\vec{r}}\frac{d\vec{r}}{dt} = m\vec{v}\frac{d\vec{v}}{d\vec{r}}$$
- $d\vec{r} \cdot \vec{F} = m\vec{v} \cdot d\vec{v}$
- $$\int_{\vec{r}_0}^{\vec{r}} d\vec{r} \cdot \vec{F} = \int_{\vec{v}_0}^{\vec{v}} m\vec{v} \cdot d\vec{v} = \frac{1}{2}mv^2\Big|_{v_0}^{v}$$
- Work energy theorem is N's 2nd!

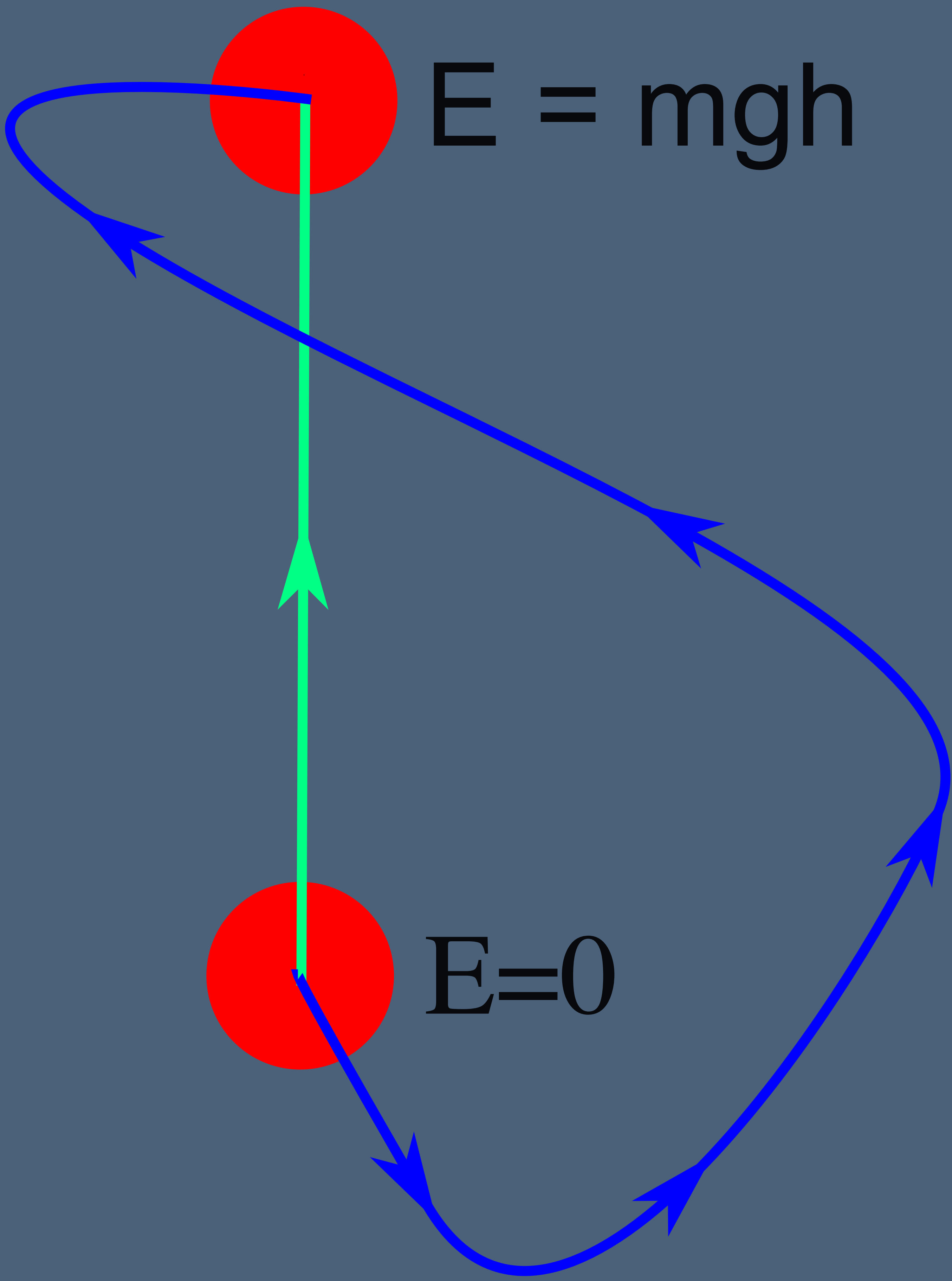


"Iron Bru Revolution" ride Pleasure Beach, Blackpool, Lancashire, England, UK.

Energy Conservation

Valid for conservative forces!

- $\int_{\vec{r}_0}^{\vec{r}} d\vec{r} \cdot \overrightarrow{F} = \frac{1}{2}mv^2 \Big|_{v_0}^{v}$
- Path integral review! When is $\overrightarrow{F}$ conservative?
  - $\nabla \times \overrightarrow{F} = 0$
  - $\int_C d\vec{r} \cdot \overrightarrow{F} = 0$
  - $\overrightarrow{F} = -\nabla V$ OR $V(\vec{r}) = -\int_0^{\vec{r}} \overrightarrow{F} \cdot d\vec{r}$
  - (Equivalent for force fields!)



Energy Conservation

A restatement of Newton's 2nd Law

- $$\Delta V\left(\vec{r}\right) = \int_{\vec{r}_0}^{\vec{r}} \overrightarrow{F} \cdot d\vec{r} = \Delta\left(\frac{1}{2}mv^2\right)_{v_0}^{v}$$

- $T = \dfrac{1}{2}mv^2$ is the energy of *motion*

- $V\left(\vec{r}\right)$ is the energy of *position*

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Today's problems

- 4.4 — Work Energy Theorem (the tough one)
- 4.8 — Potential energy (the other tough one)
- 4.12 — Gradients!