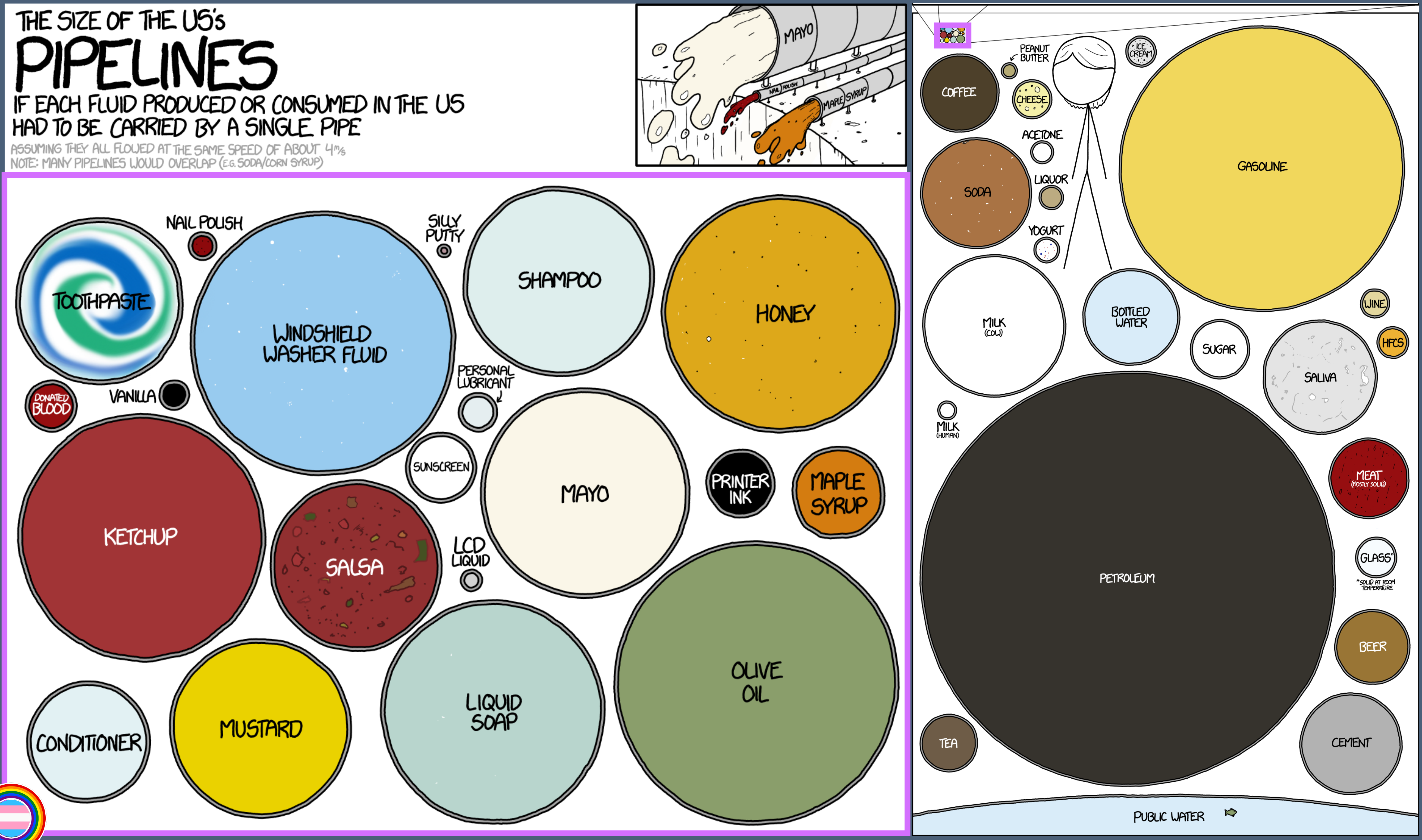
PhysH308

Fluid Mechanics!

Ted Brzinski, Nov. 7, 2024

Fluids

Why we might care



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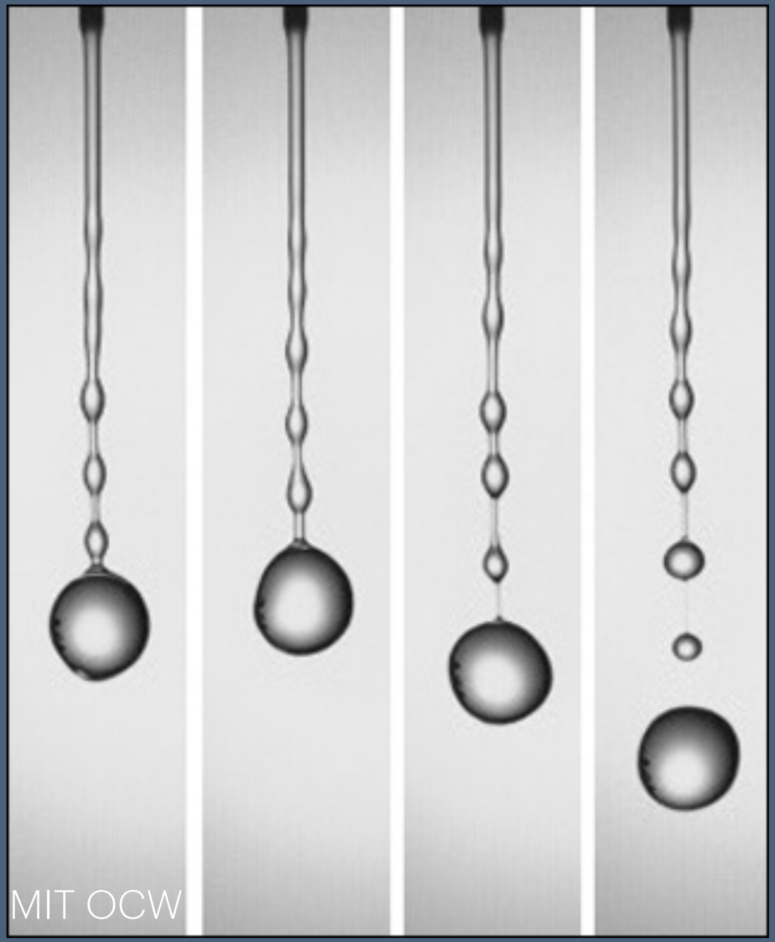


Continuum vs discrete mechanics

(Applies to continuum solids too!)

Discrete:

- Particles
- Force
  - Normal vs tangential
- Displacement
- Velocity

Continuum:

- "Material particles"
- Stress
  - Pressure vs shear
- Strain (deformation) + flow
- Velocity field (inc. strain rate)

Deformation

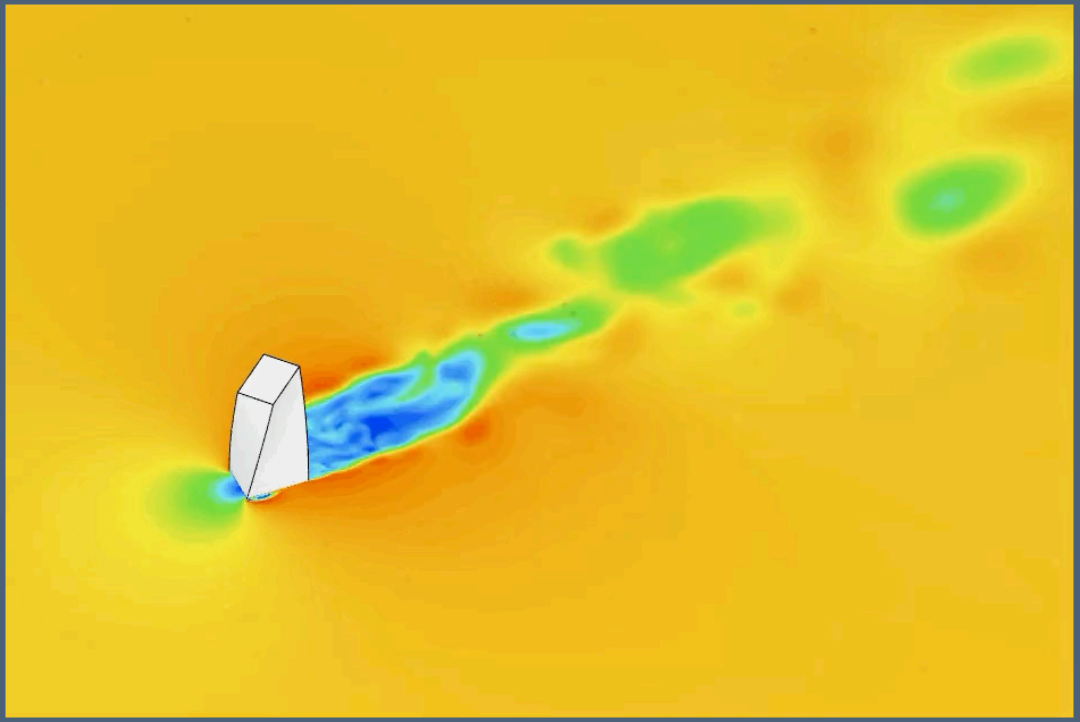
Compressible vs incompressible flows

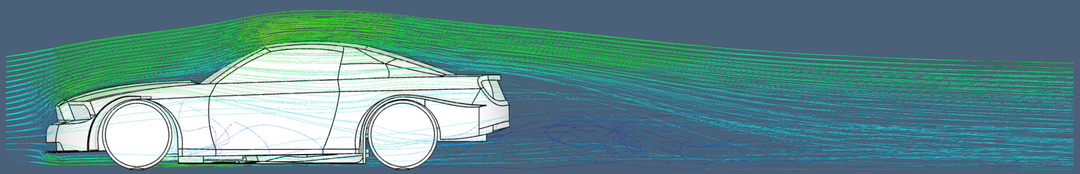
- Mach number: $M = \dfrac{\vec{v}}{c}$

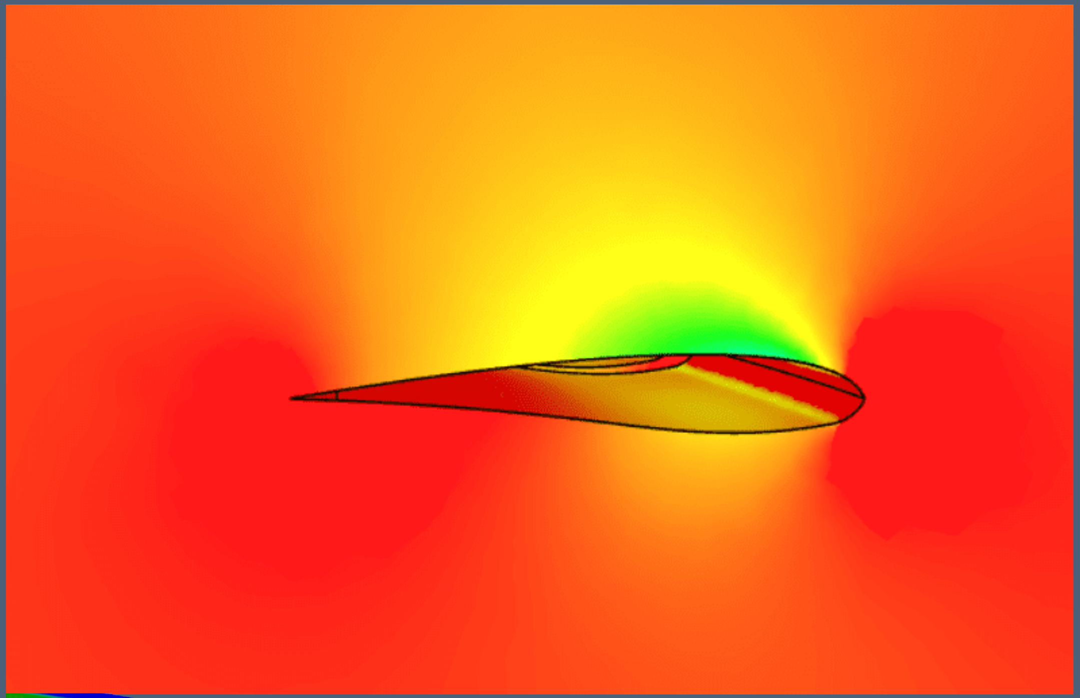
- $M \lessapprox 0.3$: Incompressible flows: volume is a conserved quantity, density is a material constant

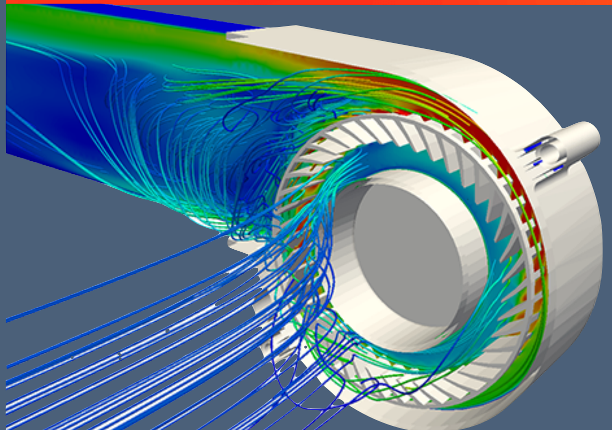
- $M \gtrapprox 0.6$: Compressible flows: volume is a function of the stresses acting on a given material particle — mass is conserved, but not volume

- $c_{air} = 343\dfrac{m}{s}$ and $c_{H_2O} = 1500\dfrac{m}{s}$









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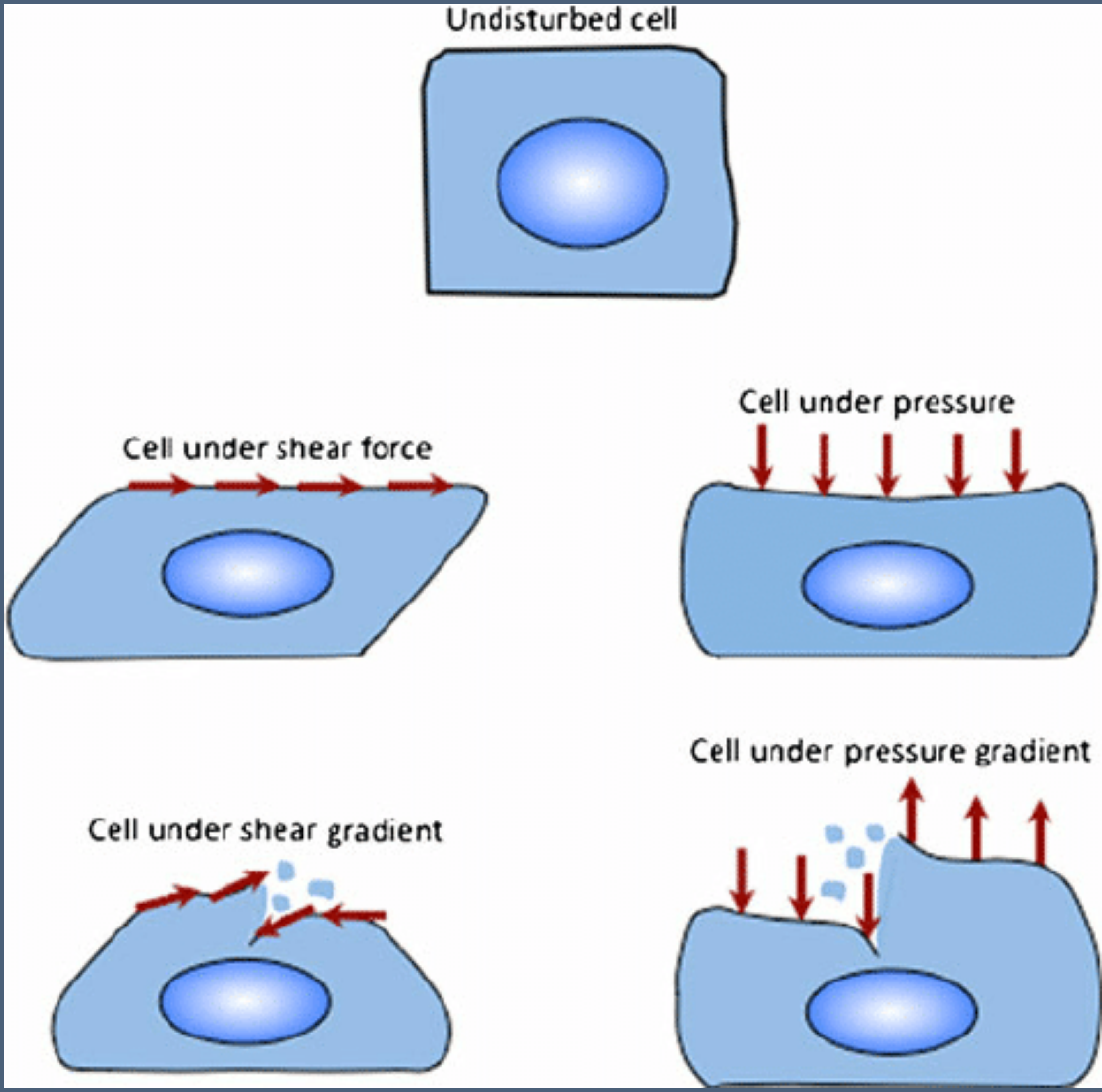
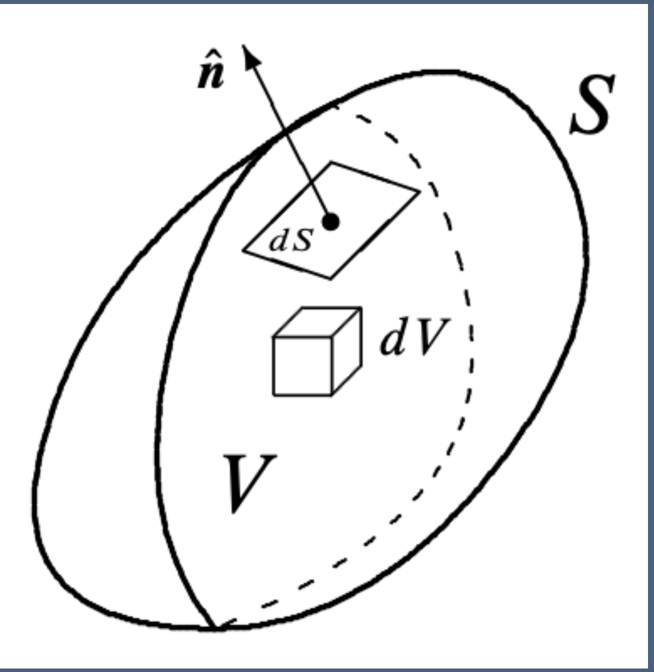
Fluid dynamics vs hydrostatics

- Material particles translate and deform under stress.
- If stresses are balanced, fluid is static:
    - Shear stress sums to zero
    - Pressure sums to exactly cancel out advective forces (e.g. gravity)

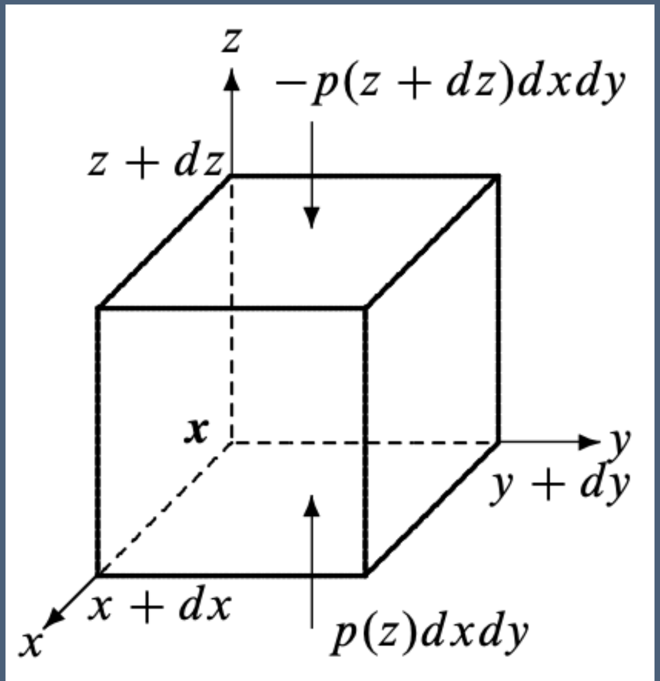


Hydrostatics

We can get pretty far with just pressure



- $[P] = \dfrac{[F]}{[A]}$
- $\mathscr{F} = -\oint_S P dS$
  (external pressure exerts a force across the material particle surface)
- $d\mathscr{F} = -\nabla P\ dV$
  (the pressure difference across a material particle gives the residual force on the particle)



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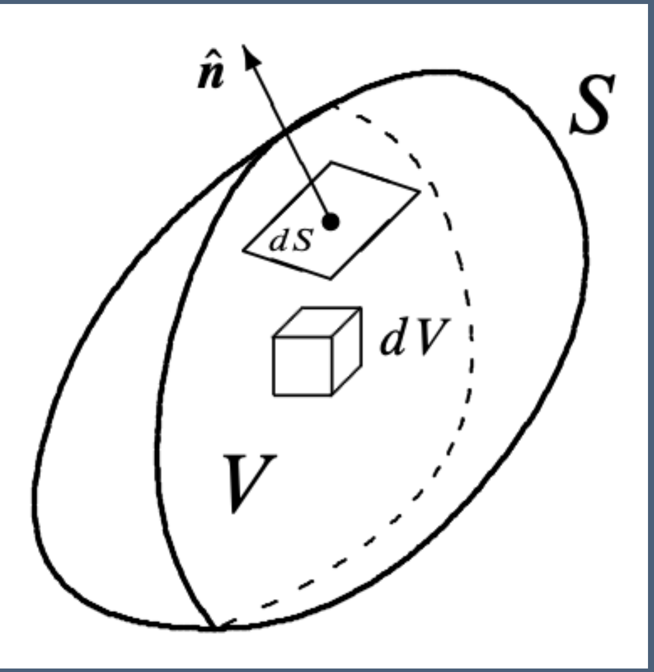
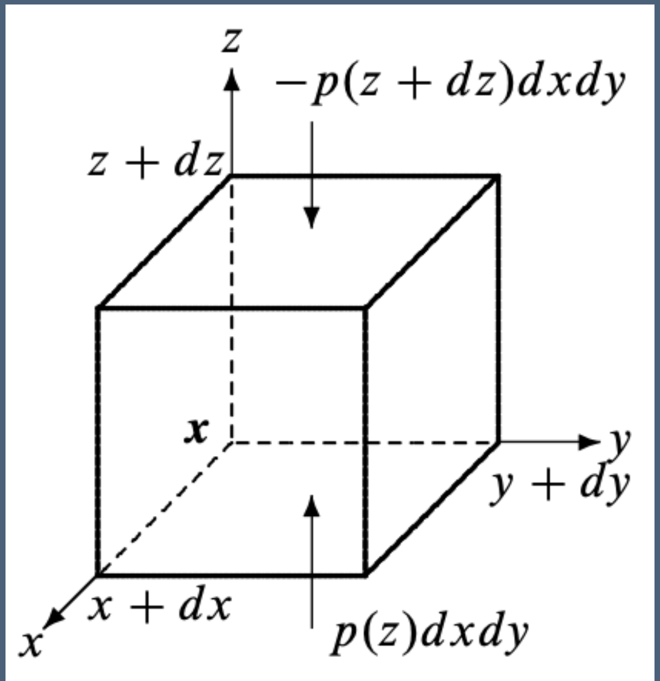




Hydrostatics

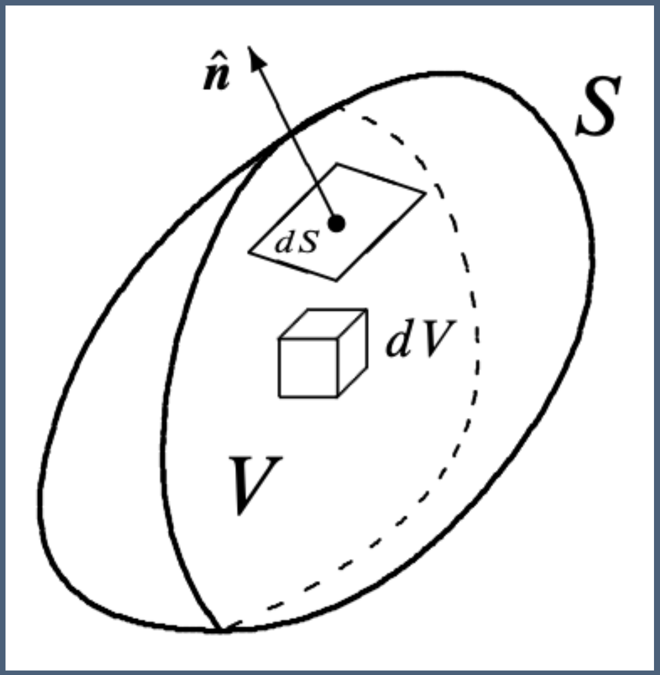
We can get pretty far with just pressure

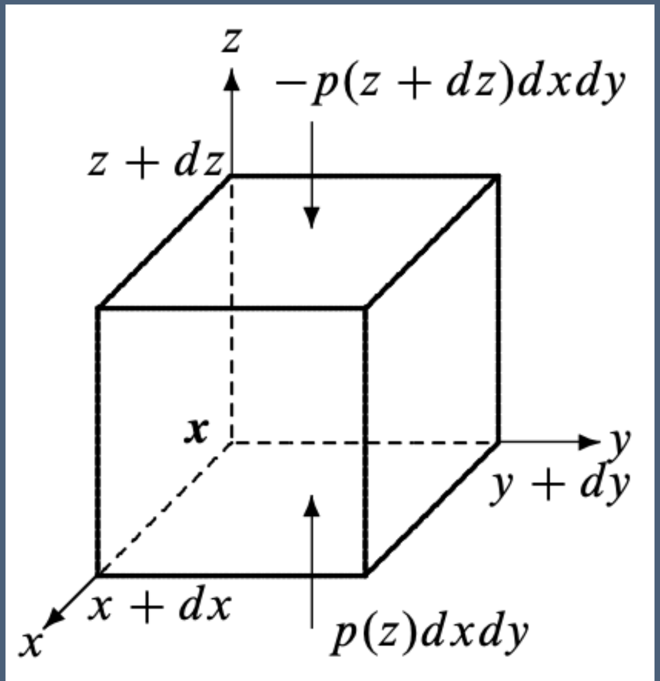
- Thus, hydrostaticity requires:

$$\oint_S P dS = \int_V \nabla P \, dV$$

- Gauss' law: The pressure across the surface of the particle (flux) must equal the internal pressure of the particle (divergence).

- Pascal's Law: pressure is isotropic (same in all directions)

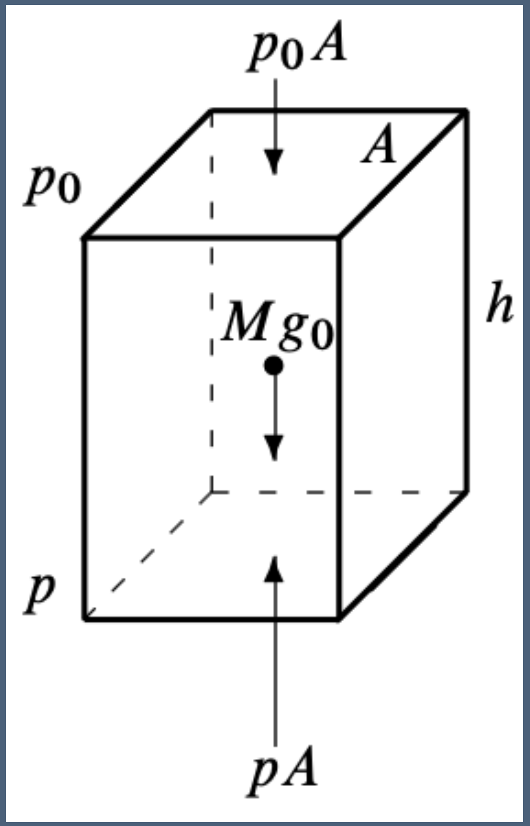


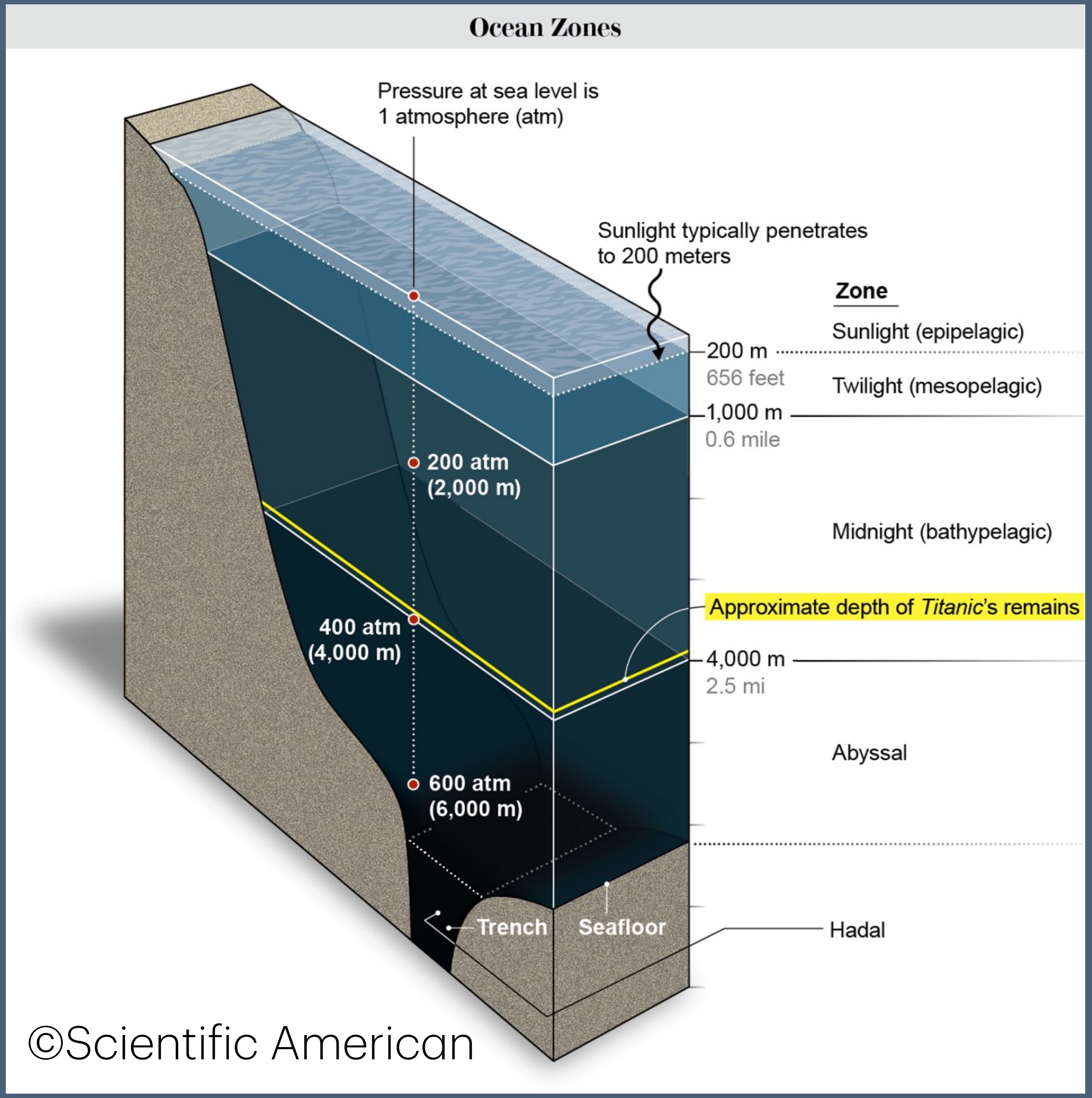


Hydrostatic pressure

The incompressible case

- $\int_S P dS = \int_V \nabla P \, dV$
- $\left[\int_S P dS \right]_z = PA - P_0 A$
- $\int_V \nabla P \, dV = Mg\hat{z} = \rho V g\hat{z} = \rho(Ah)g\hat{z}$
  (force balance)
- Setting the two equal and dividing by A yields:
  $P = P_0 + \rho g z$





Demo time!

Bottle with 3 holes

- Why, with 1 hole, was there no flow?
- Why, with 2 holes, was there flow from one hole?
- Why did the water flow as it did with 3 holes?

Demo time!

Lautrup 2.5

2.5 An open jar contains two non-mixable liquids with densities $\rho_1 > \rho_2$. The heavy layer has thickness h_1 and the light layer on top of it has thickness h_2. **(a)** An open glass tube is now lowered vertically into the liquids toward the bottom of the jar. Describe how high the liquids rise in the tube (disregarding capillary effects). **(b)** The open tube is already placed in the container with its opening close to the bottom when the heavy fluid is poured in, followed by the light. How high will the heavy fluid rise in the tube?

$$P = P_0 + \rho g z$$