PhysH308

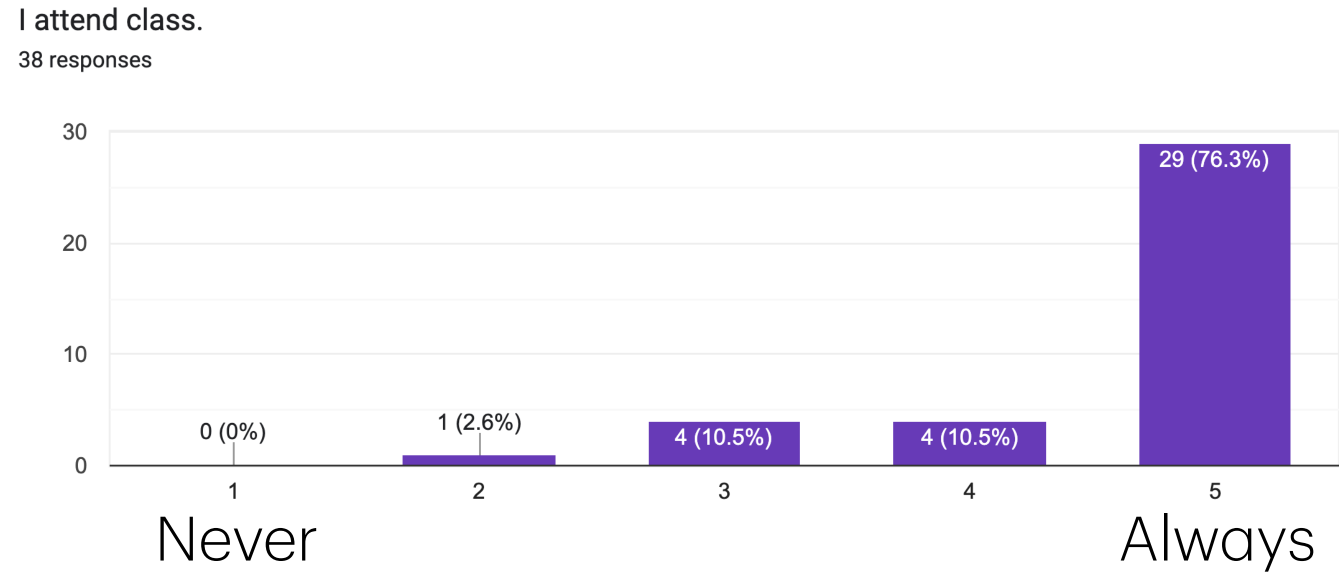
Spinning stuff!

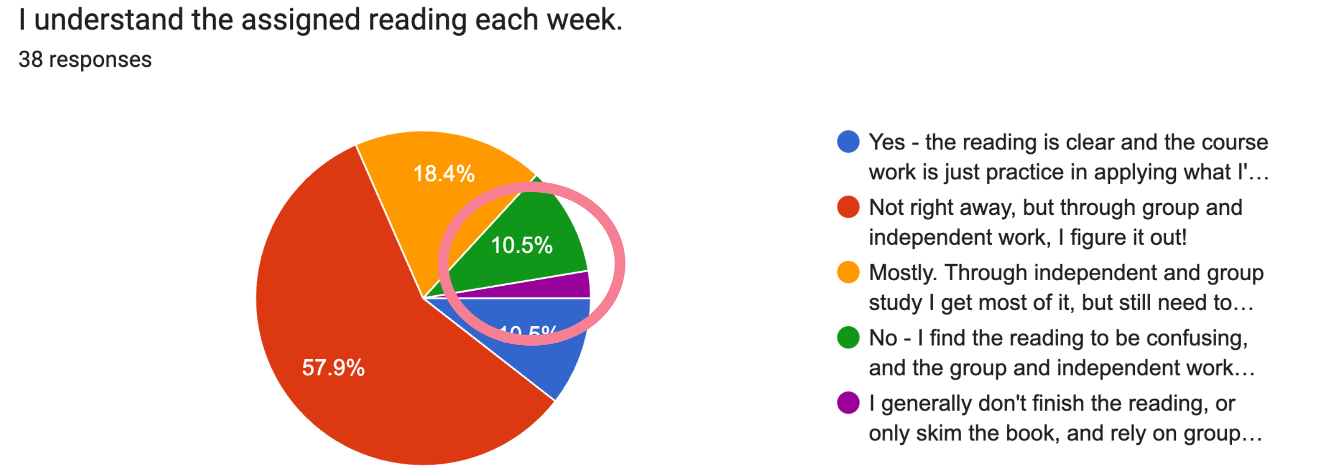
Ted Brzinski, Nov. 5, 2024

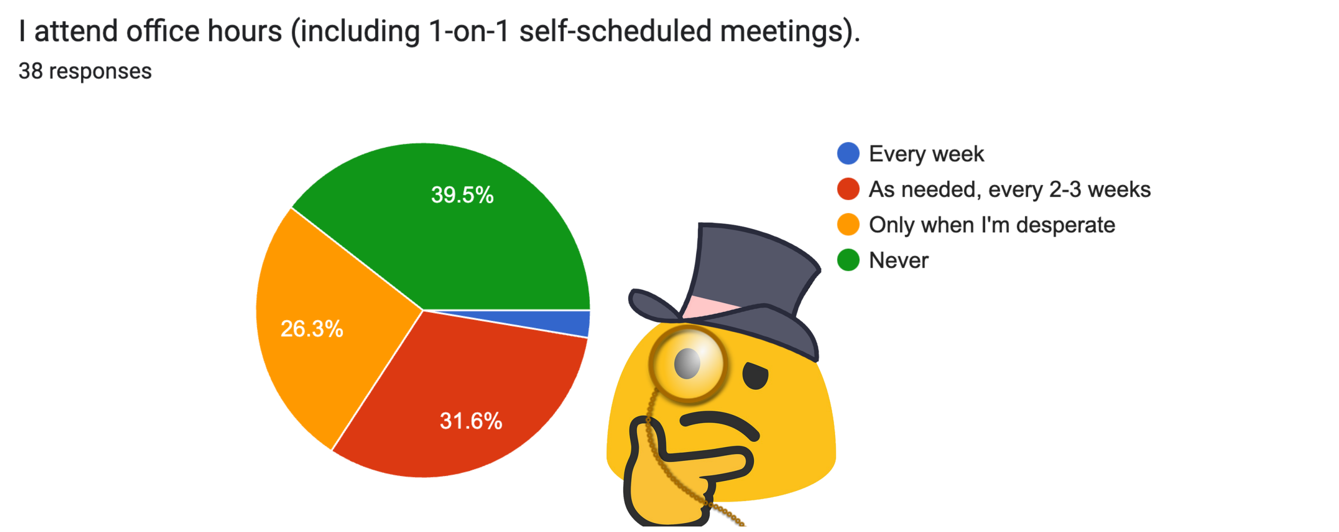
Feedback

Takeaways - engagement

- Most helpful assignment - two-stage HW
  - **Chance to change partners - survey today/tomorrow**
  - HW content: repeats, derivation vs problem solving, etc.
- Quizzes are not helpful — **no more quizzes!**
- Reading notes also frequently mentioned as positive. Deadline structure is mixed.





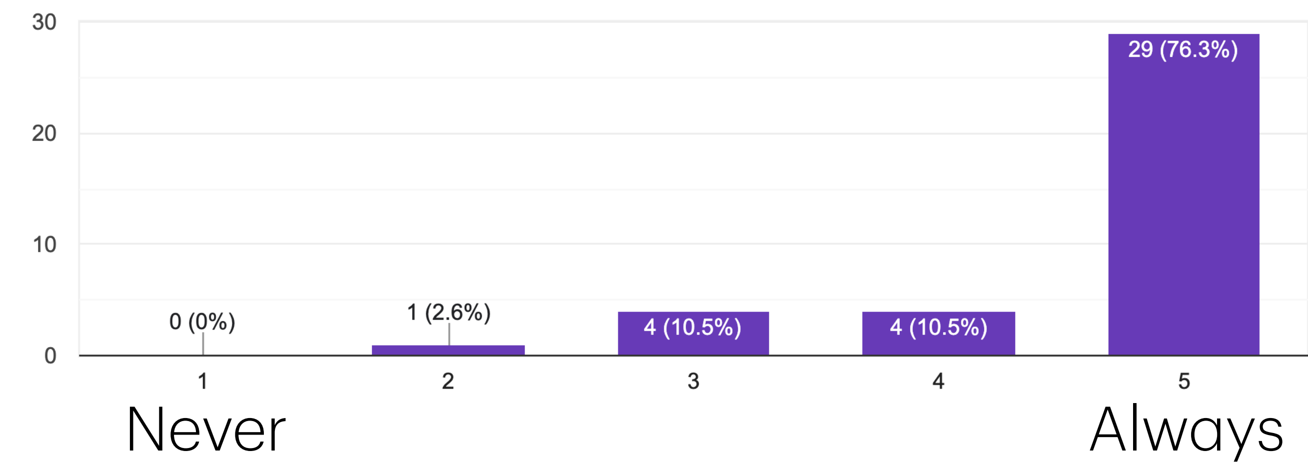


Feedback

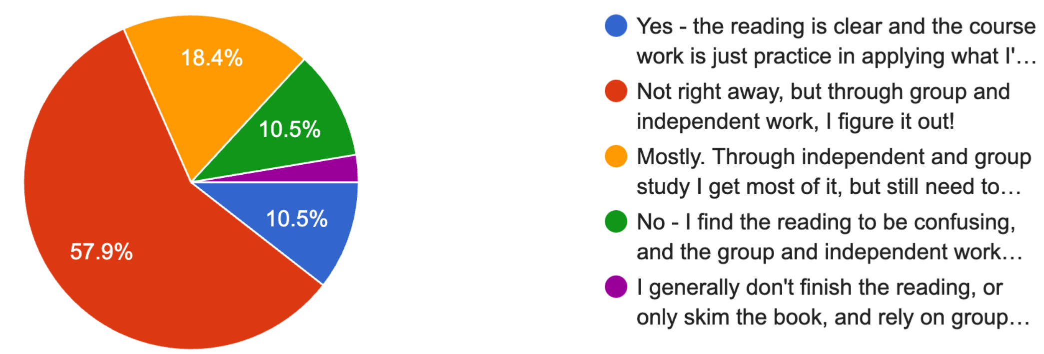
Takeaways - engagement

- *What steps could you take to improve your learning in this course?*
  - Attend office hours (more).
  - Engage with the reading more deeply/promptly.
- Changes to office hours?
  - **M/F 1:30-4 (H106)**
  - Replace T/W? Self-schedule?

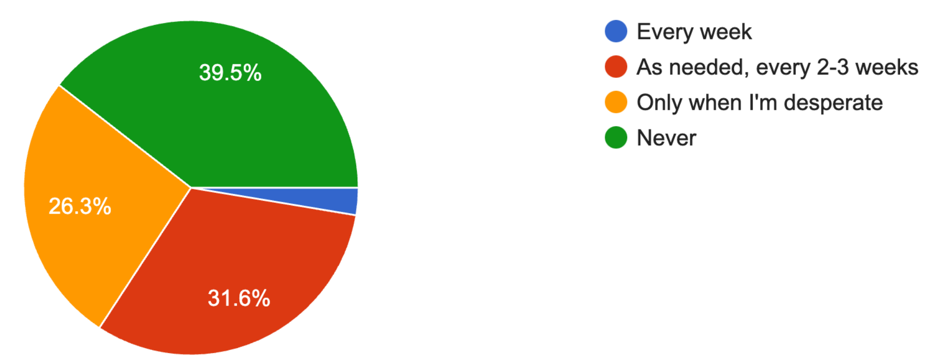
I attend class.
38 responses



I understand the assigned reading each week.
38 responses



I attend office hours (including 1-on-1 self-scheduled meetings).
38 responses



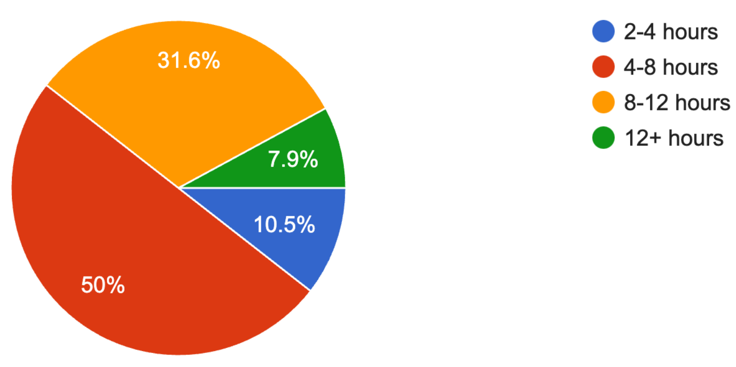
Feedback

Student experience

- Many people like the emphasis on teamwork and collaboration.
- Some people wish that they had more opportunity to engage in class-wide discussion. **Let's try Q&A@5-10min?**
- The math and geometry are intimidating to many students **(Note that you aren't alone! Good use of office hours: seeing that I get intimidated sometimes too!)**
- The constant deadlines stress out some, help others (this is a tough tension to navigate in any class!)

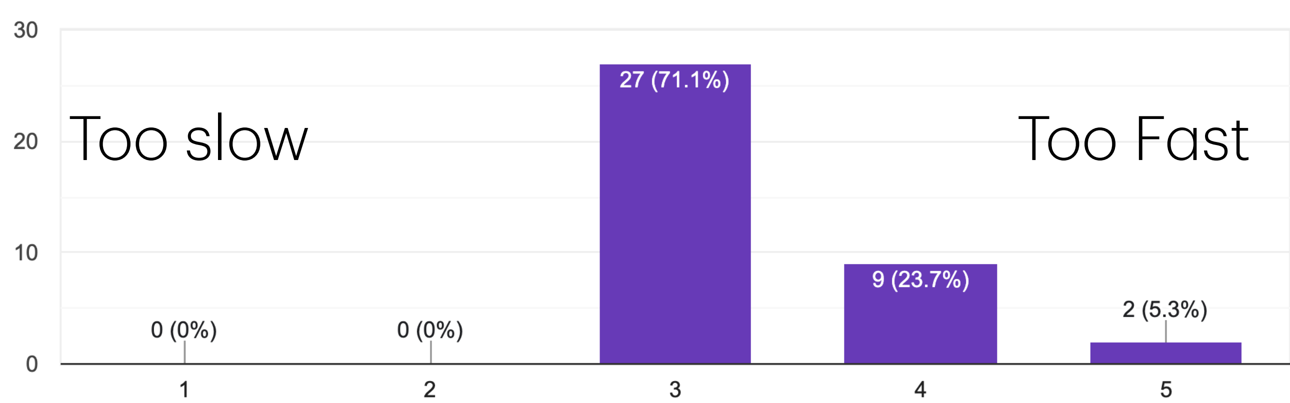
On average, how many hours a week do you spend on coursework outside of class time?

38 responses



The pace of the class is...

38 responses



Feedback

Student learning

- A strong majority (>75%) of you shared that you feel you are:
  - developing new skills
  - understand course expectations and instructions
  - being provided enough opportunities to practice and demonstrate your learning

- A weak majority (50-75%) of you shared that you are comfortable applying your learning in new/unfamiliar contexts, and that the course structure works well to support learning goals.

- There is a throughline of concern about conceptual learning vs problem solving skills.

- Y'all love and hate Lagrangian Mech! It's your "most important" and "most challenging" topic so far! (Bad news - we're playing with tops next!)

Final notes

Student comments/suggestions

- Changes to exams were appreciated. But…
  - Frequency and scalability are still major stressors
  - Several suggestions:
    - Fewer exams
    - Replace future exams with weekly independent problems (more "exams"?)
- Comments on feedback (lack thereof)
  - HW 7 is being graded now, and we will work on the most recent HW first moving forward.
  - Exams will receive more substantive feedback (in whatever form they take moving forward, including Exam II) and ~ a one-week turnaround on marks.

Something *I* appreciated: all assessment-related comments were about learning, and improving learning outcomes! Thank you!

Speaking of exams…

Exam II update

- Currently ~50% graded, on-track to finish tomorrow
- Next “exam” is tentatively released by Friday…
- Deadline creep → fewer exams → need to revise assessment and/or format

My proposal:

- 2 short *independent* problems assigned/due Friday (each on 1-2 learning standards)
- Able to resubmit once after grading
- One chance to make up all *missed* learning standards during Finals week.
- (Fewer opportunities in favor of revision, better feedback, no time limit)
- For the first 6 learning standards, I’ll try to pepper in bonus Qs.

Speaking of exams…

Exam II update

My proposal:

- 2 short *independent* problems assigned/due Friday (each on 1-2 learning standards)
- Able to resubmit once after grading
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Clarifications:

I will grade these within a week (not graders).

Format less like exam (except final), slightly longer HW with separate independent problems.

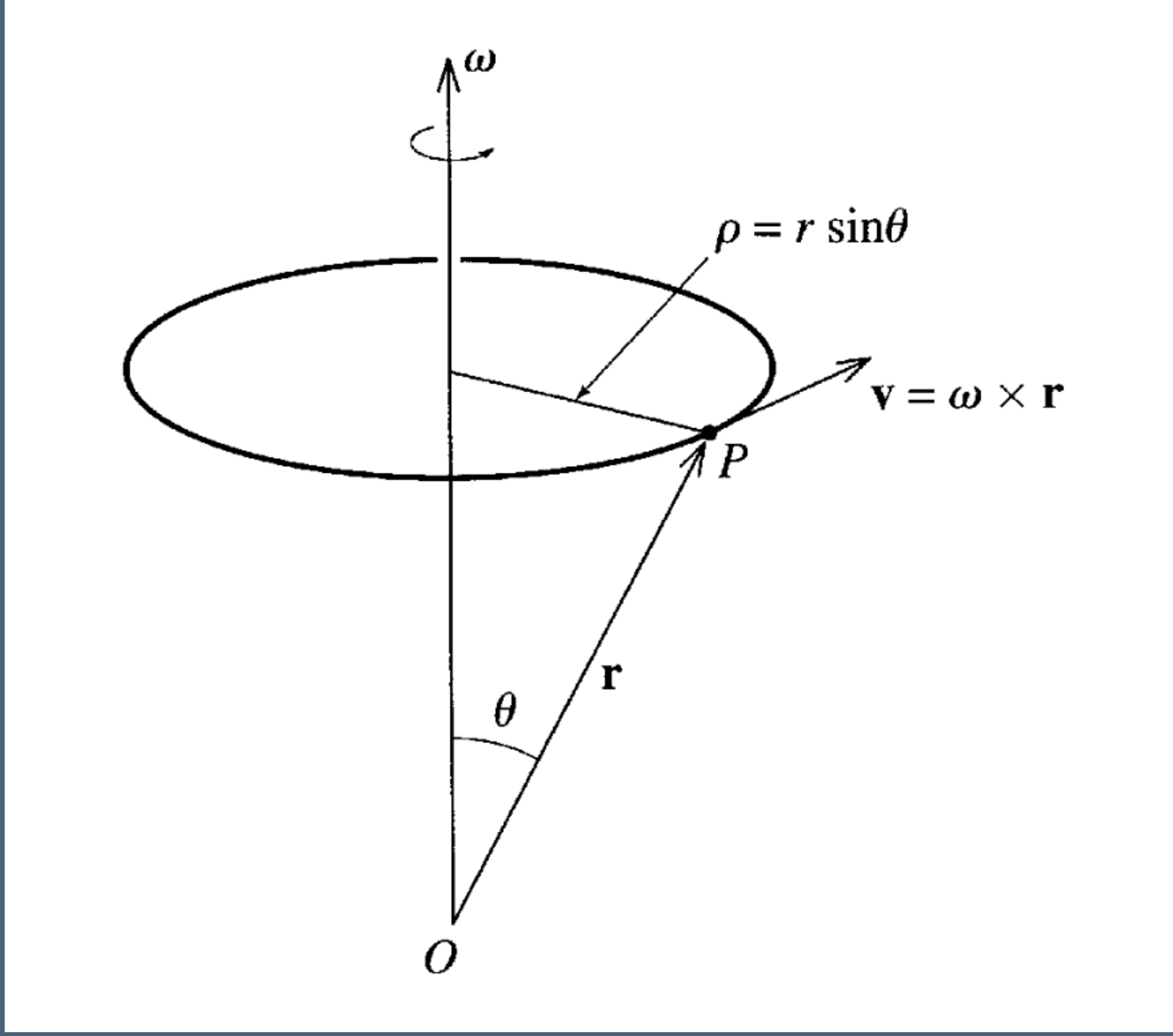
18-19 learning standards rather than the planned 22

The last week's 2 learning standards are still only on the final, with no revision on the penultimate 2.

Motion of a rotating body

Angular velocity vector

- $\overrightarrow{\omega} = \omega\hat{u}$ where $\hat{u}$ is the axis of rotation
- $\dot{\vec{r}} = \overrightarrow{\omega} \times \vec{r}$ (note this doesn't include $\dot{\overrightarrow{R}}_{cm}$)
- Notation convention:
  - $\overrightarrow{\omega} = \omega\hat{u}$ for spinning objects
  - $\overrightarrow{\Omega} = \Omega\hat{u}$ for rotating frame $\mathcal{S}$ relative to an inertial frame $\mathcal{S}_0$



Motion in a rotating frame

Velocity vector in a rotating frame

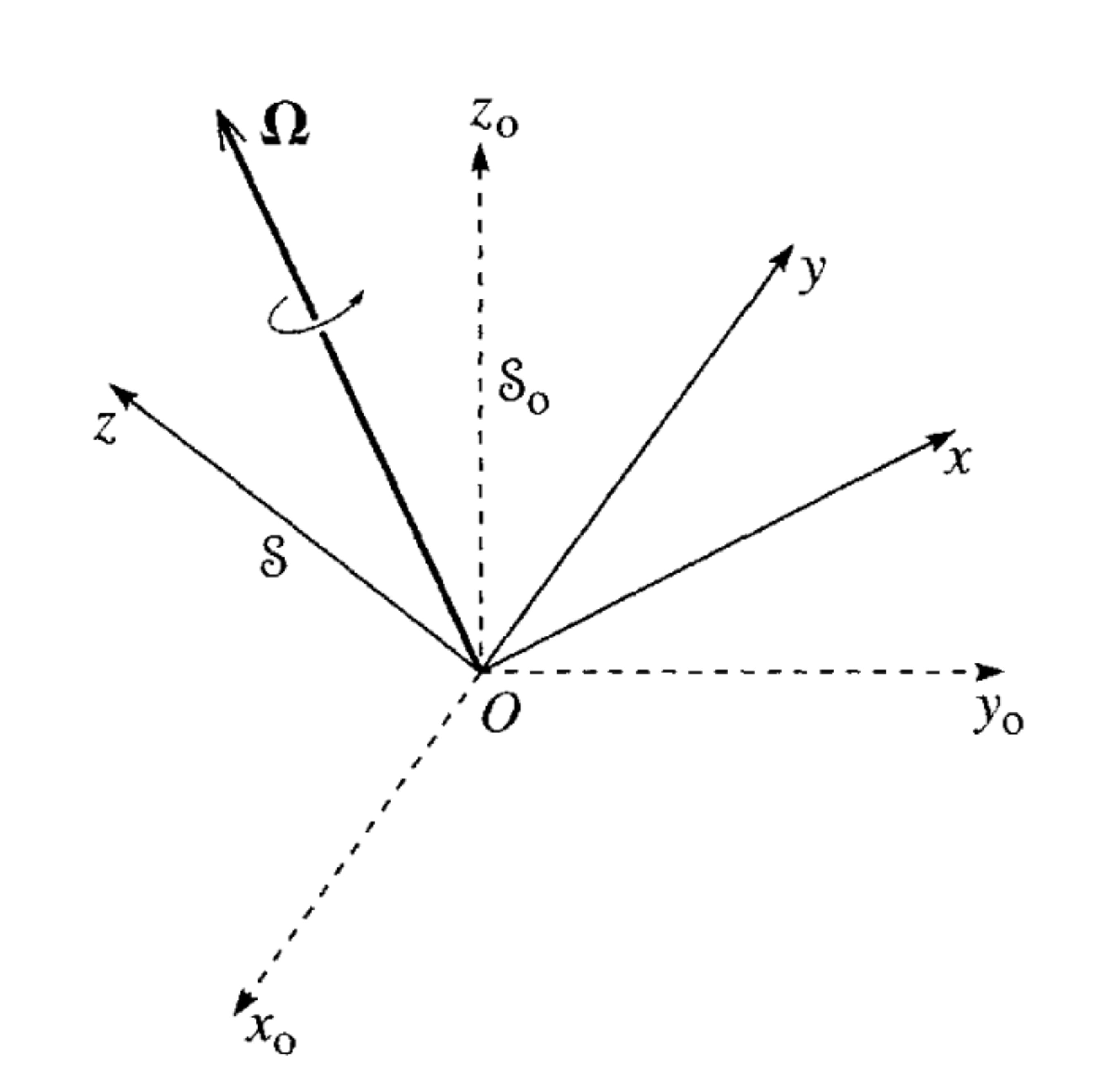
- $\overrightarrow{\Omega} = \Omega\hat{u}$ for rotating frame $\mathcal{S}$ relative to an inertial frame $\mathcal{S}_0$

- Thus, a particle at rest in $\mathcal{S}$ will have a velocity in $\mathcal{S}_0$:

$$\left(\dot{\vec{r}}\right)_{\mathcal{S}_0} = \overrightarrow{\Omega} \times \vec{r}$$

- More generally:

$$\left(\dot{\vec{r}}\right)_{\mathcal{S}_0} = \left(\dot{\vec{r}}\right)_{\mathcal{S}} + \overrightarrow{\Omega} \times \vec{r}$$

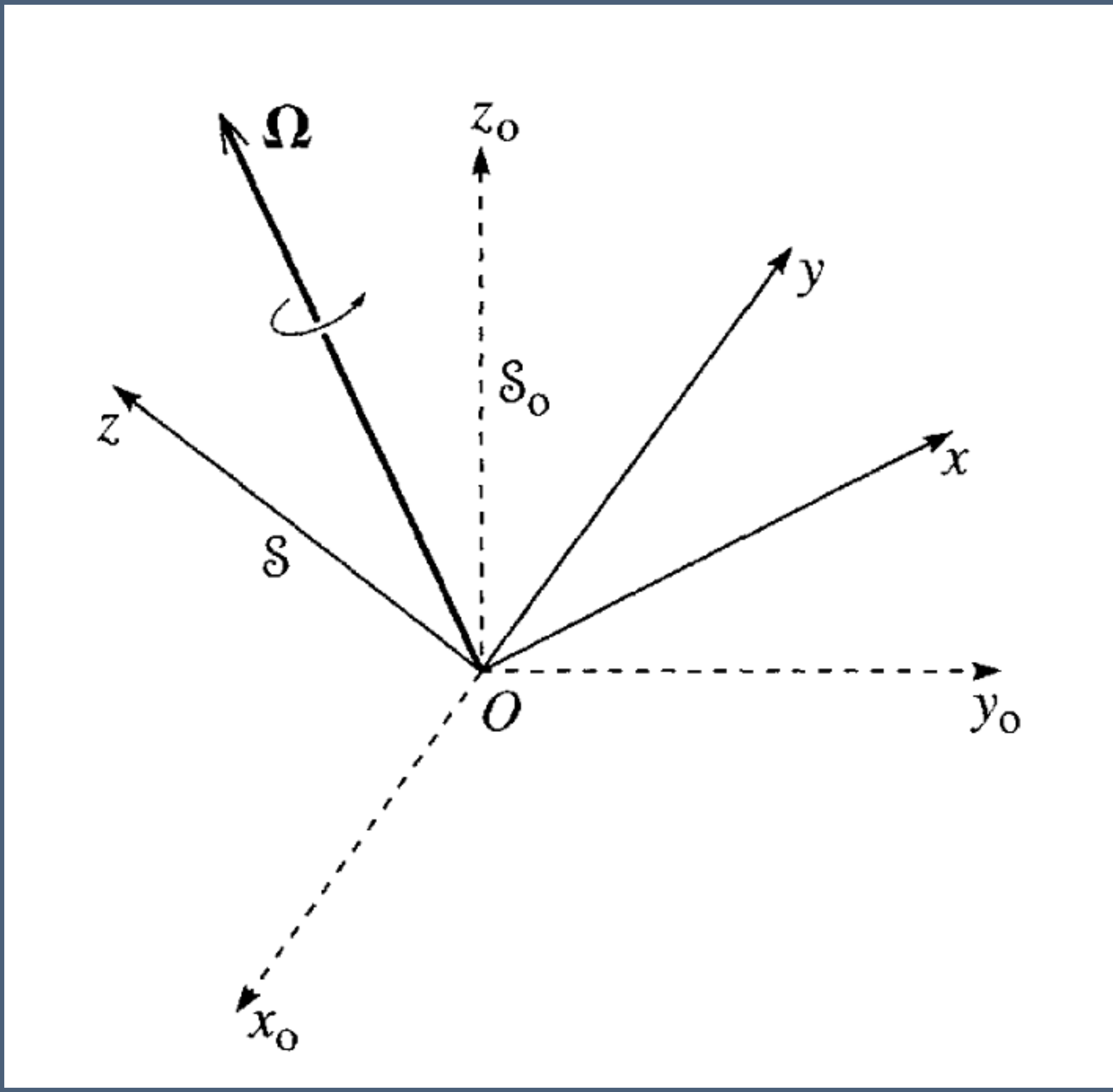


Motion in a rotating frame

Newton's 2nd in a rotating frame

- $\left(\dot{\vec{r}}\right)_{\mathcal{S}_0} = \left(\dot{\vec{r}}\right)_{\mathcal{S}} + \overrightarrow{\Omega} \times \vec{r}$
- $\left(\ddot{\vec{r}}\right)_{\mathcal{S}_0} = \left(\frac{d}{dt}\right)_{\mathcal{S}_0} \left(\dot{\vec{r}}_{\mathcal{S}} + \overrightarrow{\Omega} \times \vec{r}\right)$
- $= \left(\frac{d}{dt}\right)_{\mathcal{S}} \left(\dot{\vec{r}}_{\mathcal{S}} + \overrightarrow{\Omega} \times \vec{r}\right)$

$$+\overrightarrow{\Omega} \times \left(\dot{\vec{r}}_{\mathcal{S}} + \overrightarrow{\Omega} \times \vec{r}\right)$$



Motion in a rotating frame

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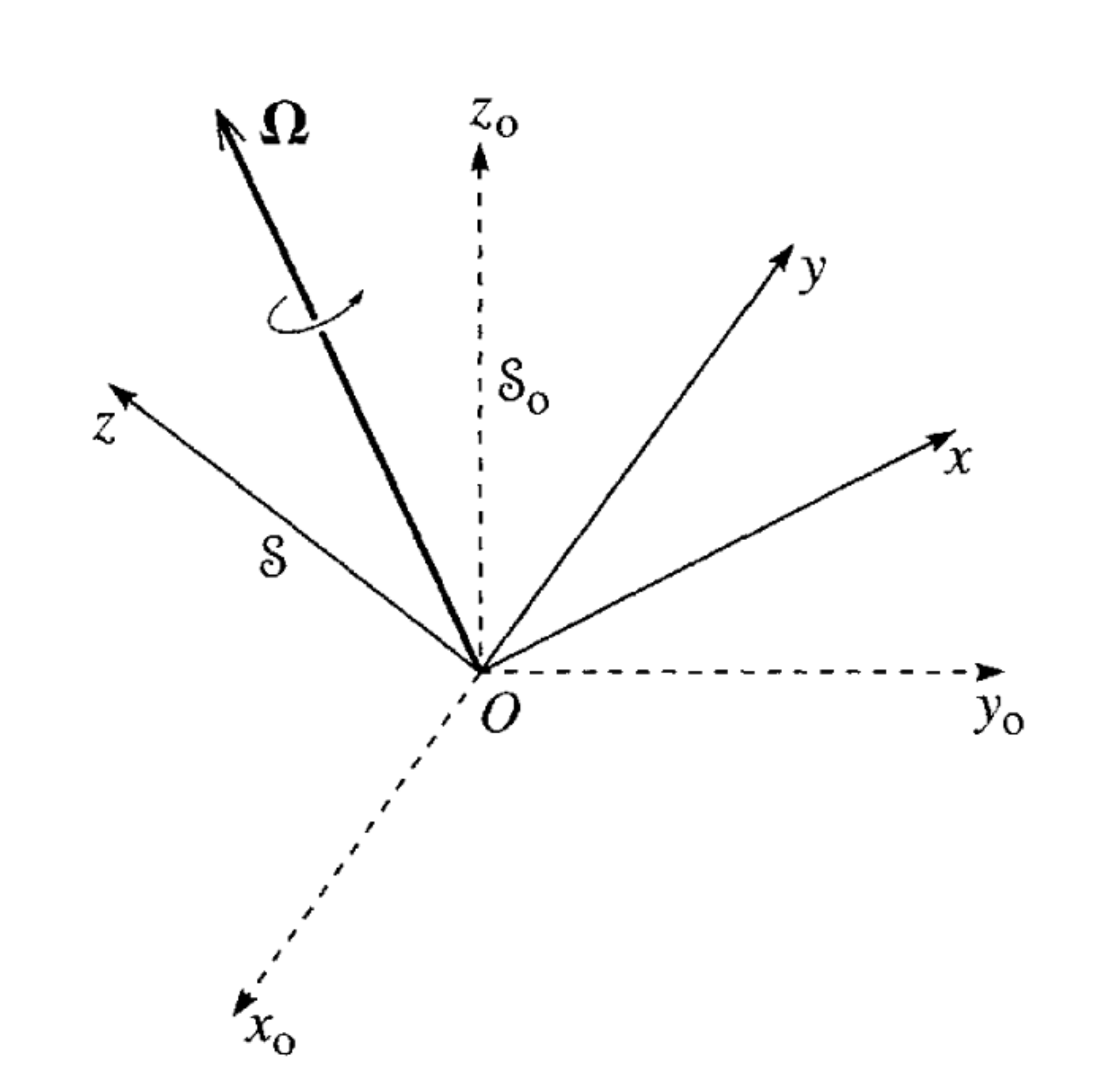
- $\left(\ddot{\vec{r}}\right)_{\mathcal{S}_0} = \left(\frac{d}{dt}\right)_{\mathcal{S}} \left(\dot{\vec{r}}_{\mathcal{S}} + \vec{\Omega} \times \vec{r}\right)$

$$+\vec{\Omega} \times \left(\dot{\vec{r}}_{\mathcal{S}} + \vec{\Omega} \times \vec{r}\right)$$

- Dropping the $\mathcal{S}$ subscript and expanding the last term:

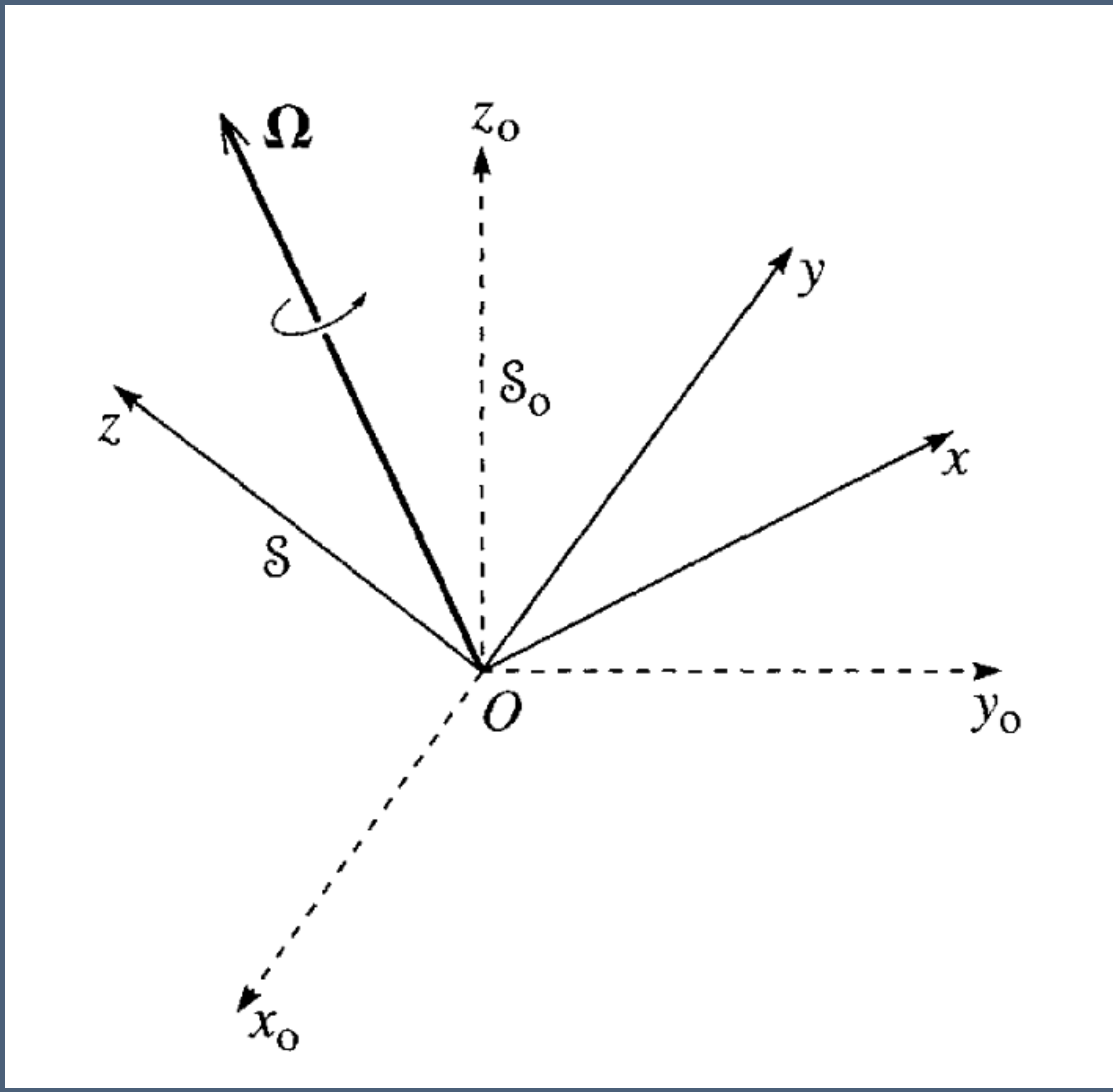
$$\left(\ddot{\vec{r}}\right)_{\mathcal{S}_0} = \ddot{\vec{r}} + 2\vec{\Omega} \times \dot{\vec{r}} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right)$$



Motion in a rotating frame

Newton's 2nd in a rotating frame

- $\left(\ddot{\vec{r}}\right)_{\mathcal{S}_0} = \ddot{\vec{r}} + 2\vec{\Omega}\times\dot{\vec{r}} + \vec{\Omega}\times\left(\vec{\Omega}\times\vec{r}\right)$



Motion in a rotating frame

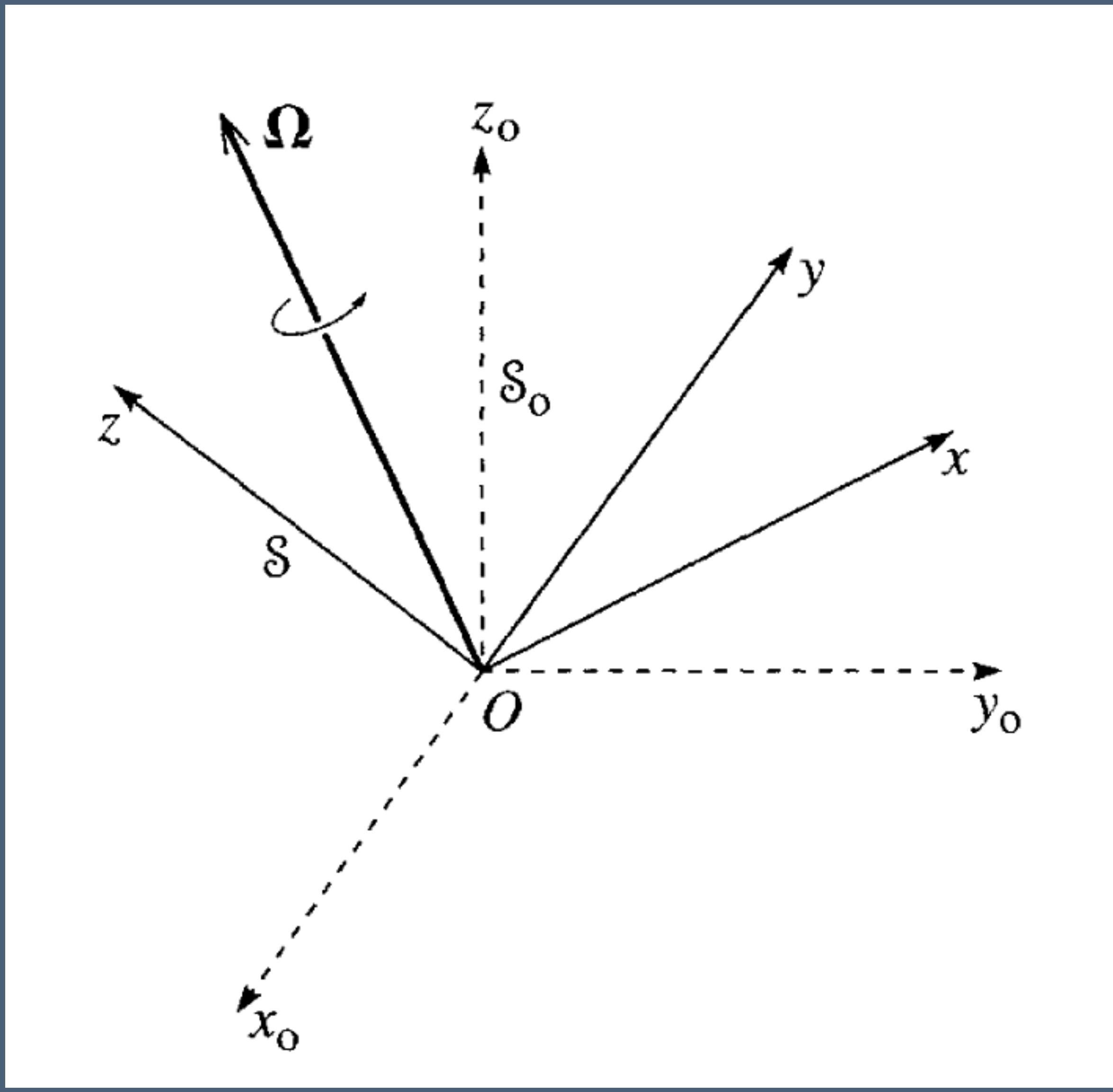
Newton's 2nd in a rotating frame

- $$\left(\ddot{\vec{r}}\right)_{\mathcal{S}_0} = \ddot{\vec{r}} + 2\vec{\Omega}\times\dot{\vec{r}} + \vec{\Omega}\times\left(\vec{\Omega}\times\vec{r}\right)$$
- $$m\ddot{\vec{r}} = \vec{F} + \boxed{2m\dot{\vec{r}}\times\vec{\Omega}} + \boxed{m\left(\vec{\Omega}\times\vec{r}\right)\times\vec{\Omega}}$$

where $\vec{F} = m\left(\ddot{\vec{r}}\right)_{\mathcal{S}_0}$

- We see familiar terms! Centrifugal and Coriolis forces!

- This notation will reappear when we discuss Euler's Angles and solve really weird rotating motion, like this...

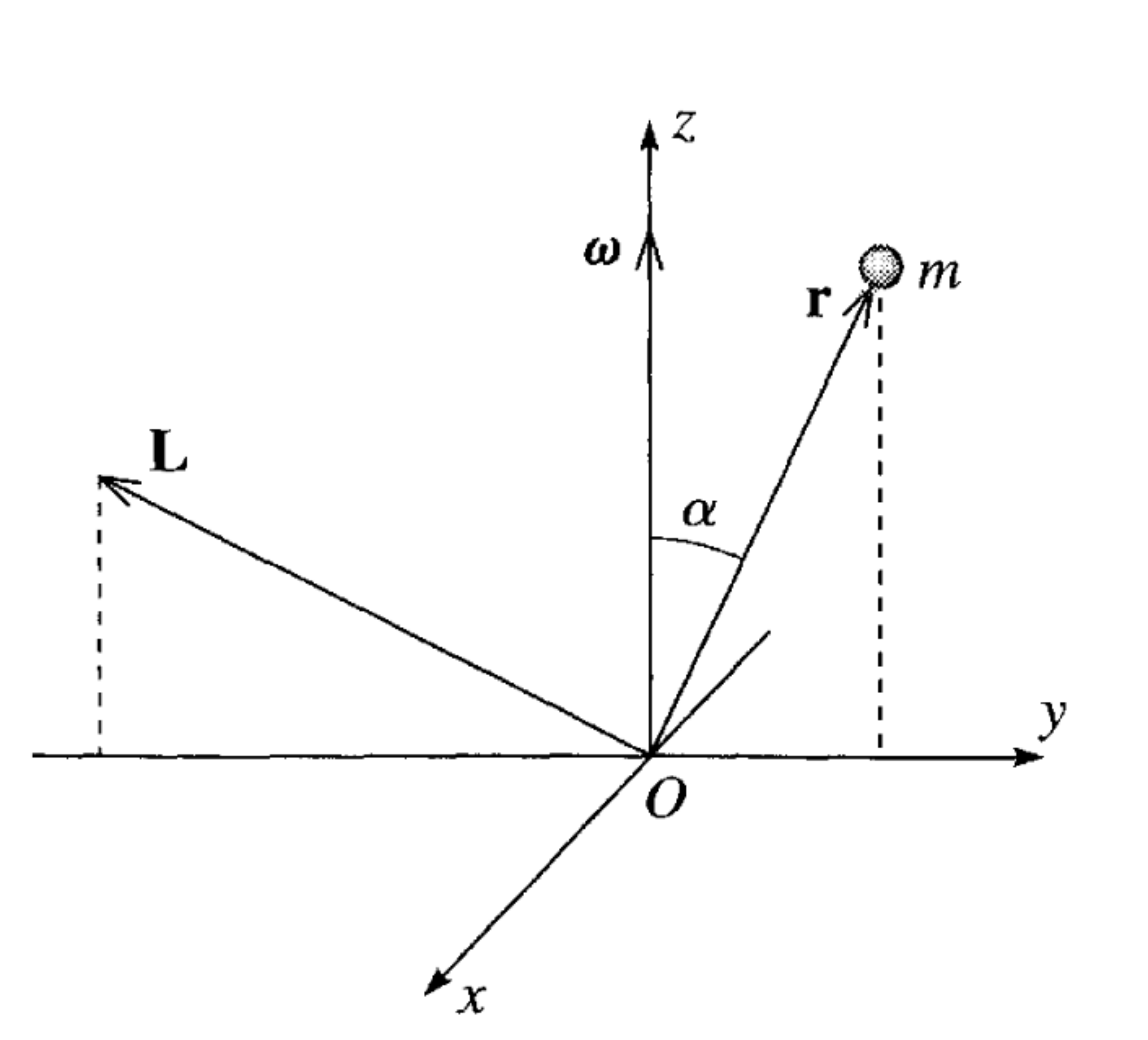


Questions?

Rotating bodies

Angular momentum

- Pictured to the right, a mass on a rod is rotated around the z-axis at an angle.
- The velocity of the mass is $\vec{v} = \vec{\omega} \times \vec{r} = (-\omega y, \omega x, 0)$
- The instantaneous angular momentum is not aligned with the axis of rotation!
  - $L_z = m\vec{r} \times \vec{v} = m\left(x^2 + y^2\right)\omega = m\rho_z^2\omega$
  - $L_x = -\left|m\vec{r} \times \vec{v}\right|_x = -mz\,(x\omega)$
  - $L_y = -\left|m\vec{r} \times \vec{v}\right|_y = -mz\left(y\omega\right)$

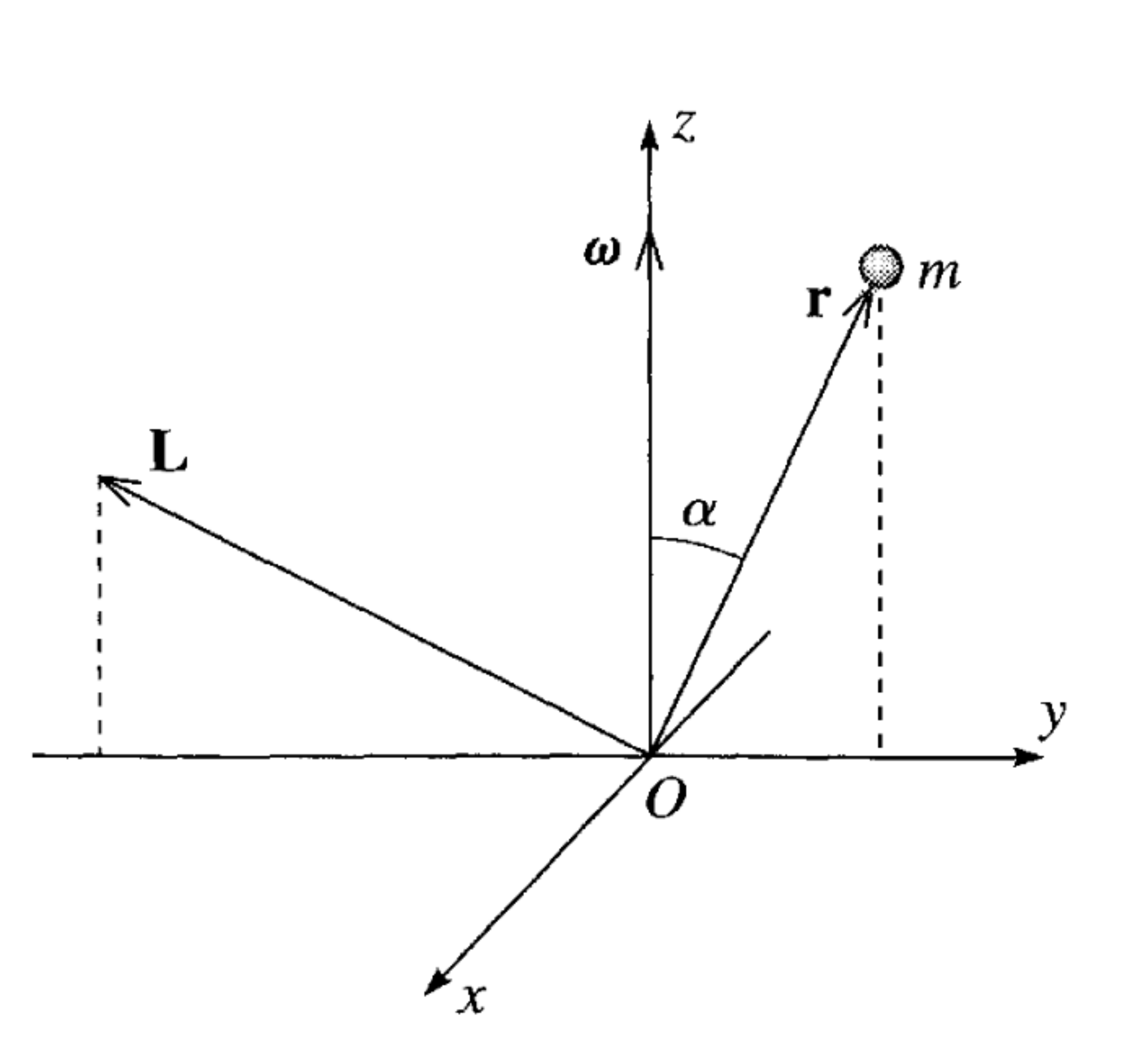


Therefore, to sustain this rotation, a torque is required!

Rotating bodies

In general

- This result differed from a case where the mass is distributed symmetrically about the rotation axis.
- In that case, we can write $L = I\omega$
- In the general case: $\overrightarrow{L} = \mathbf{I}\overrightarrow{\omega}$ where $\mathbf{I}$ is the *Inertia Tensor!*
- $I_{xx} = m\sum\left(y^2 + z^2\right)$
- $I_{xy} = I_{yx} = -m\sum xy$



Therefore, to sustain this rotation, a torque is required!

Problem 10.14

Flywheel propulsion I

- Given a big, spherical space station, how long must you spin a flywheel to rotate the space station by a constant amount? (Demo)

- You'll need

$$I_{\text{sphere}} = \rho \int r_z^2 dV = \rho \int_0^R dr \int_0^\pi d\theta \int_0^{2\pi} d\phi \left(r \sin\theta\right)^2 r^2 \sin\theta = \frac{8\pi}{15}\rho R^5 = \frac{2}{5}MR^2$$

$$\Rightarrow I_{\text{H.S}} = \rho \int r_z^2 dV = \rho \int_b^a dr \int_0^\pi d\theta \int_0^{2\pi} d\phi \left(r \sin\theta\right)^2 r^2 \sin\theta = \frac{2}{5}M\frac{a^5 - b^5}{a^3 - b^3}$$

- *And* $I_{\text{disk}} = \frac{1}{2}MR^2$

- *Use conservation of angular momentum! You don't need* $\overrightarrow{L} = \mathbf{I}\overrightarrow{\omega}$ *yet, only* $L = I\omega$.

Problem 10.22

Moment of inertia practice

- Given a simple array of masses, practice calculating $\vec{L} = \mathbf{I}\vec{\omega}$