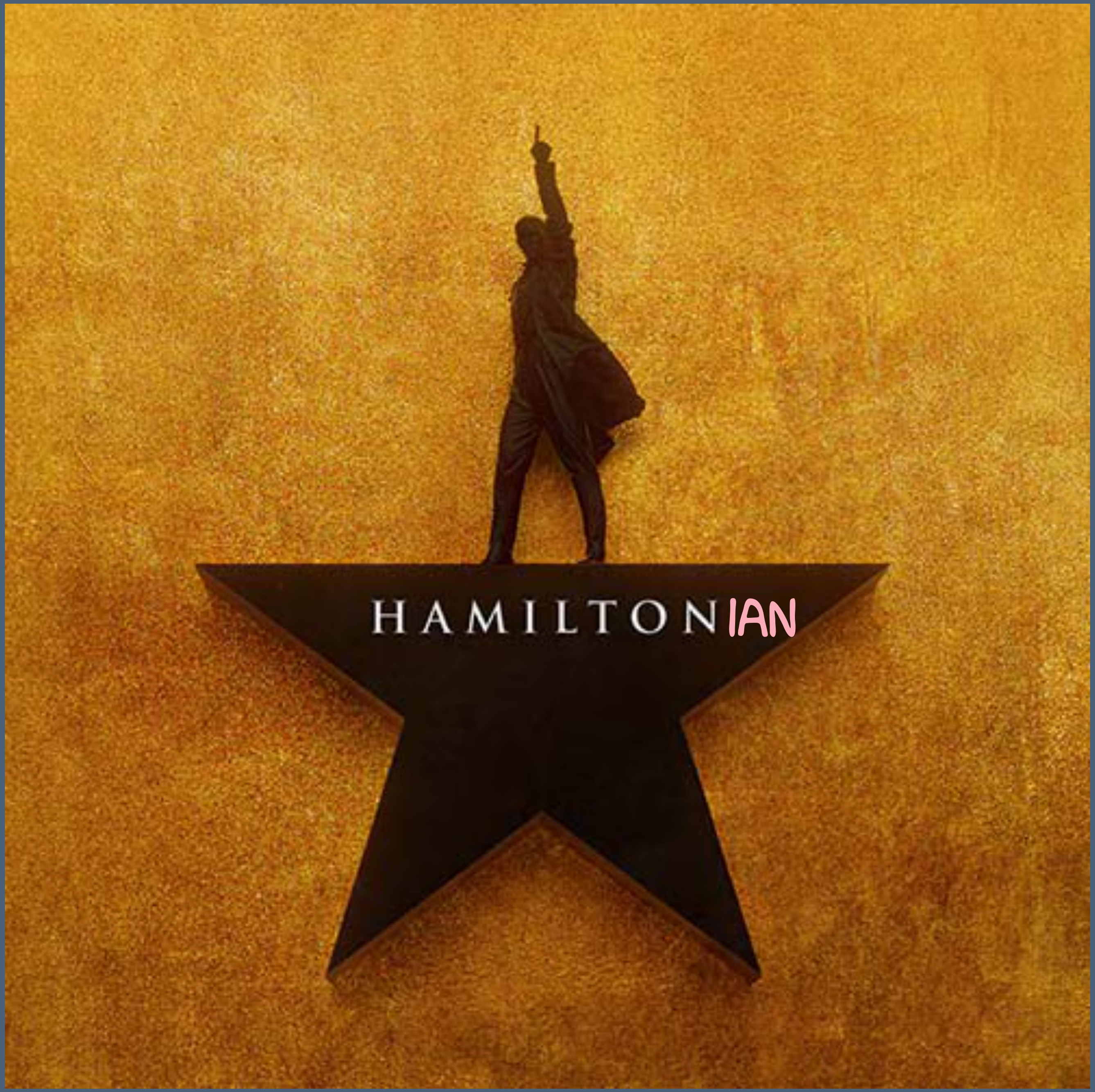
PhysH308

Hamiltonian Mechanics!



Ted Brzinski, Nov. 7, 2024

Last time…

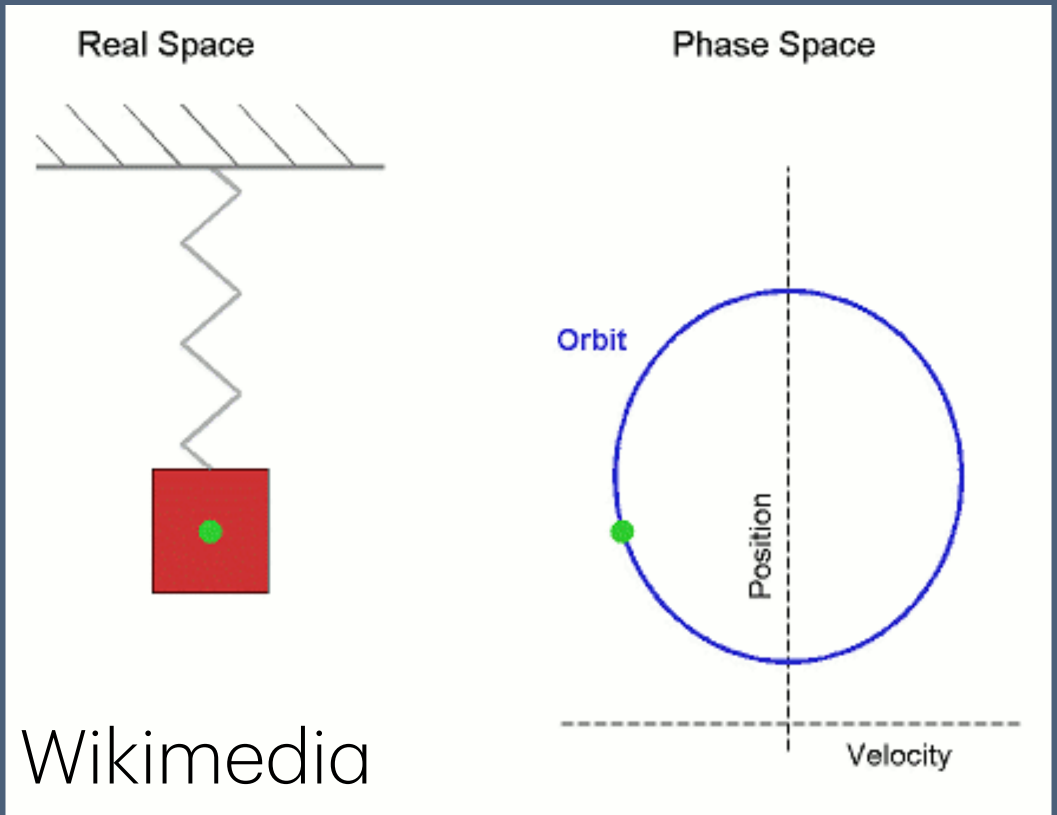
Hamiltonian and Hamilton's equations

- $\mathscr{H} \equiv p\dot{x} - \mathscr{L}$, where $\dfrac{\partial \mathscr{L}}{\partial \dot{x}} = p$
- $\dfrac{\partial \mathscr{H}}{\partial x} = -\dot{p}$
- $\dfrac{\partial \mathscr{H}}{\partial p} = \dot{x}$

This time…

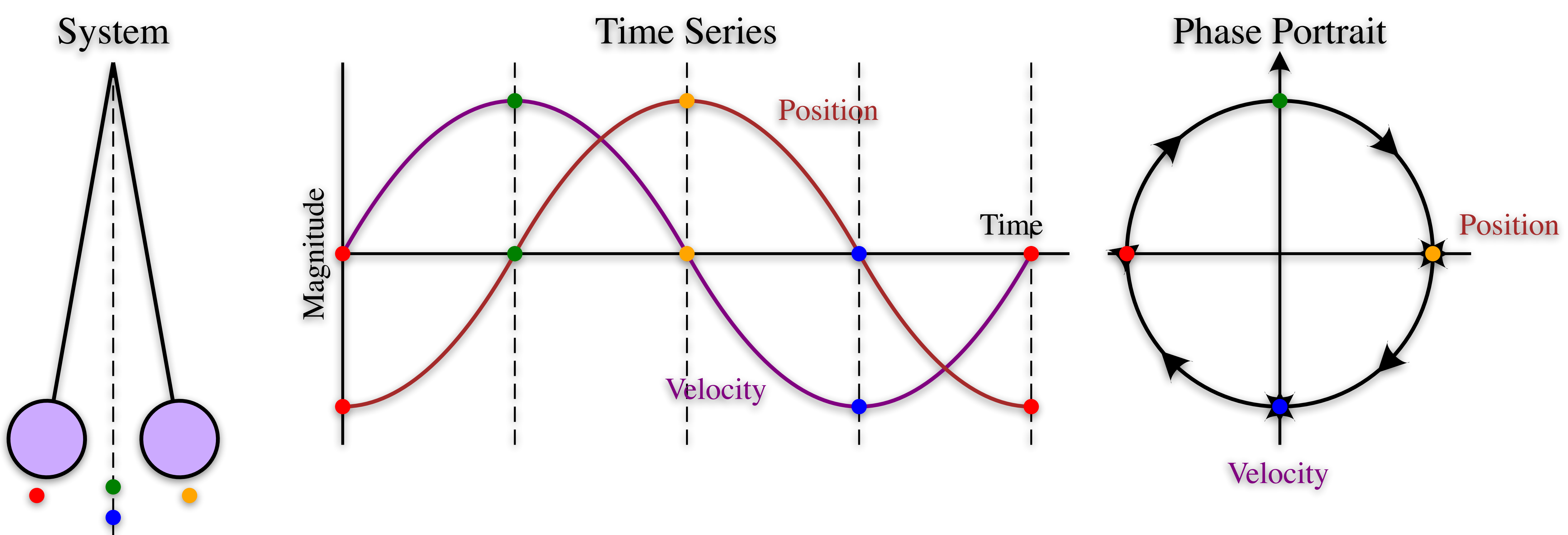
Hamiltonian and Hamilton's equations

- We can think of the Hamiltonian and the eqns of motion as a map of how momentum and position evolve in time
- Together, these can be thought of as parametric equations describing a curve in a "phase space"
- Example:



Another example:

Phase orbits and phase portraits



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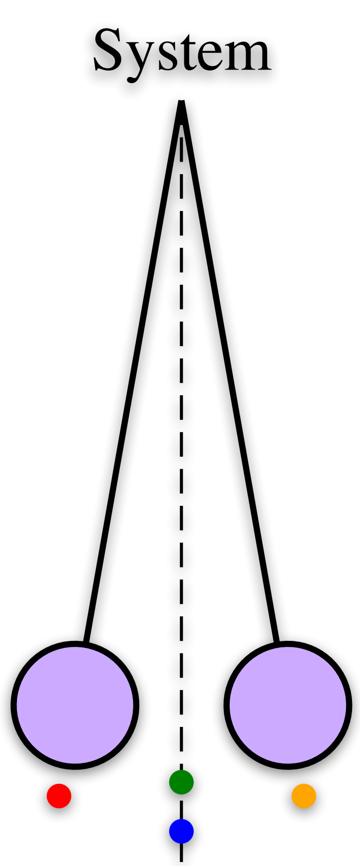
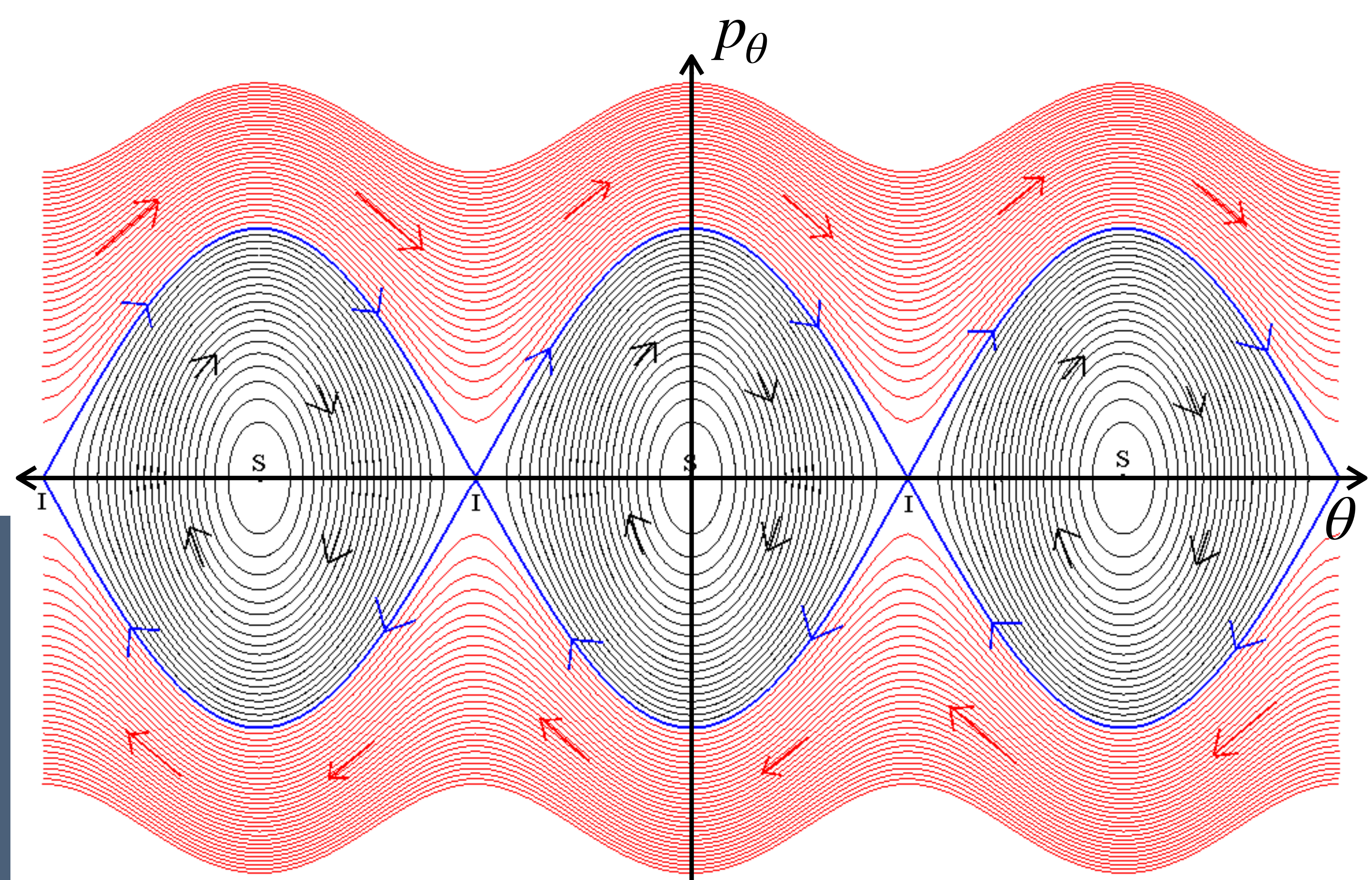
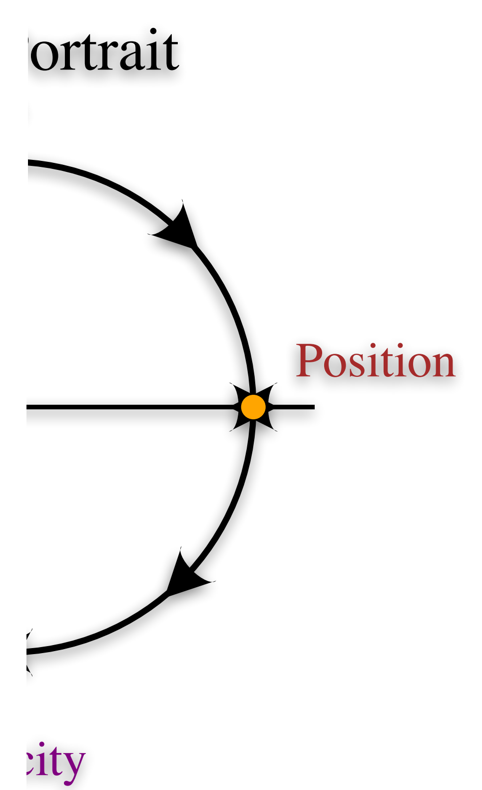
Another example:

Phase orbits and phase portraits



For a large number of initial conditions:





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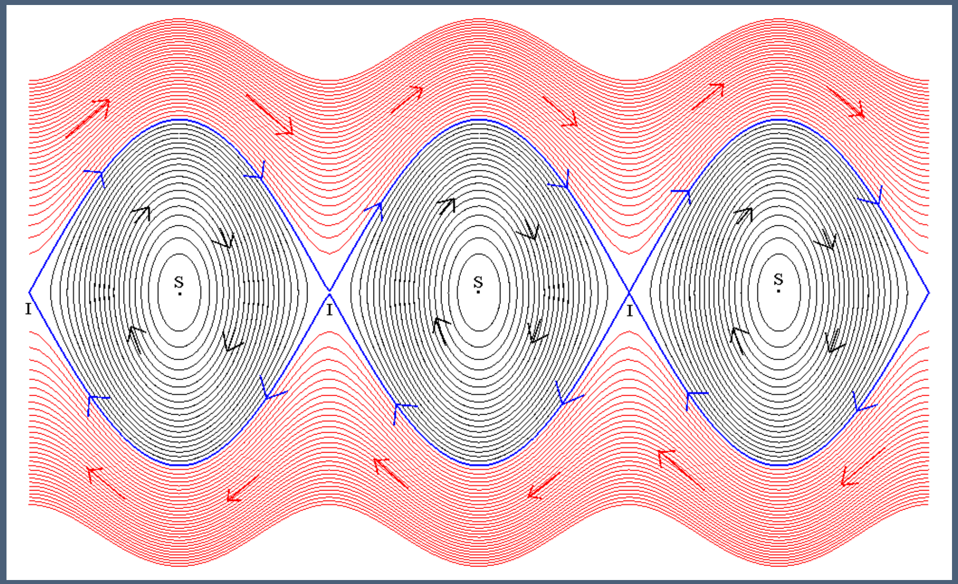
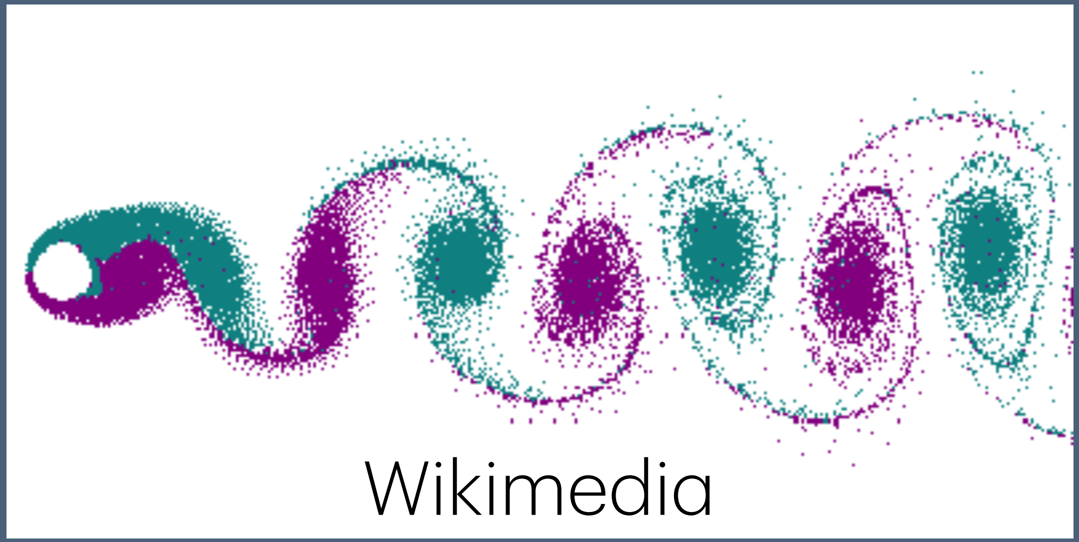
Liouville's theorem

It looks like a fluid flow for a reason!

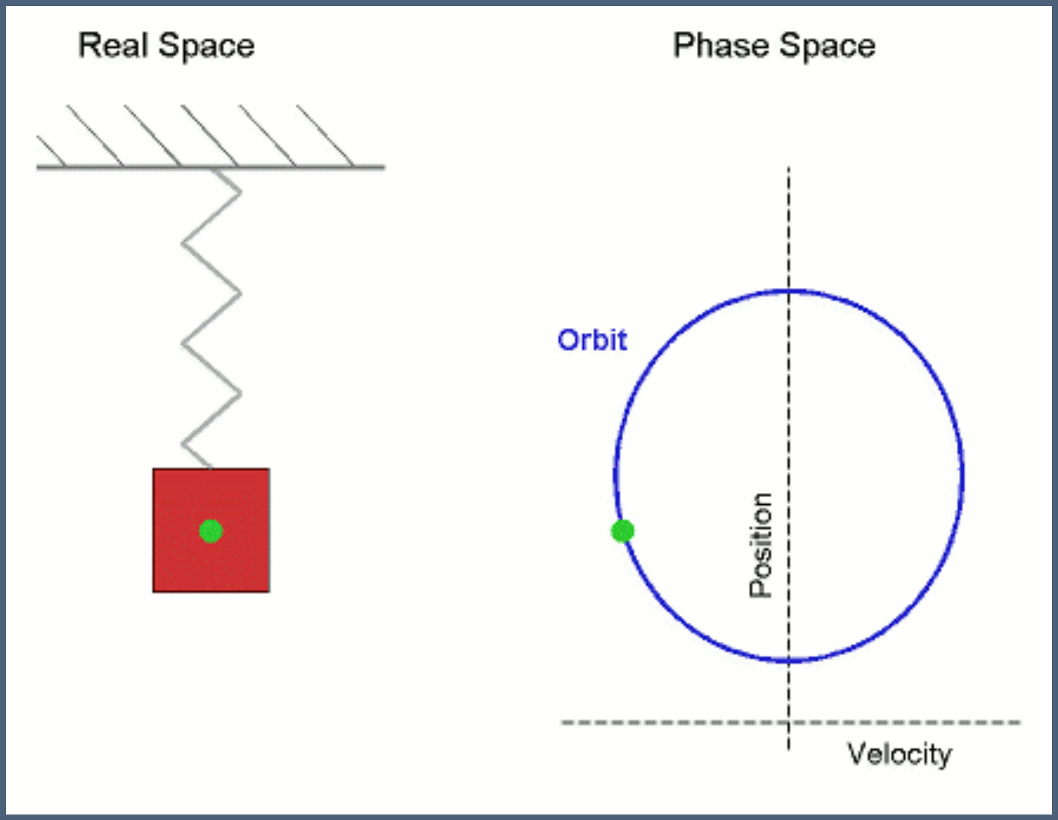


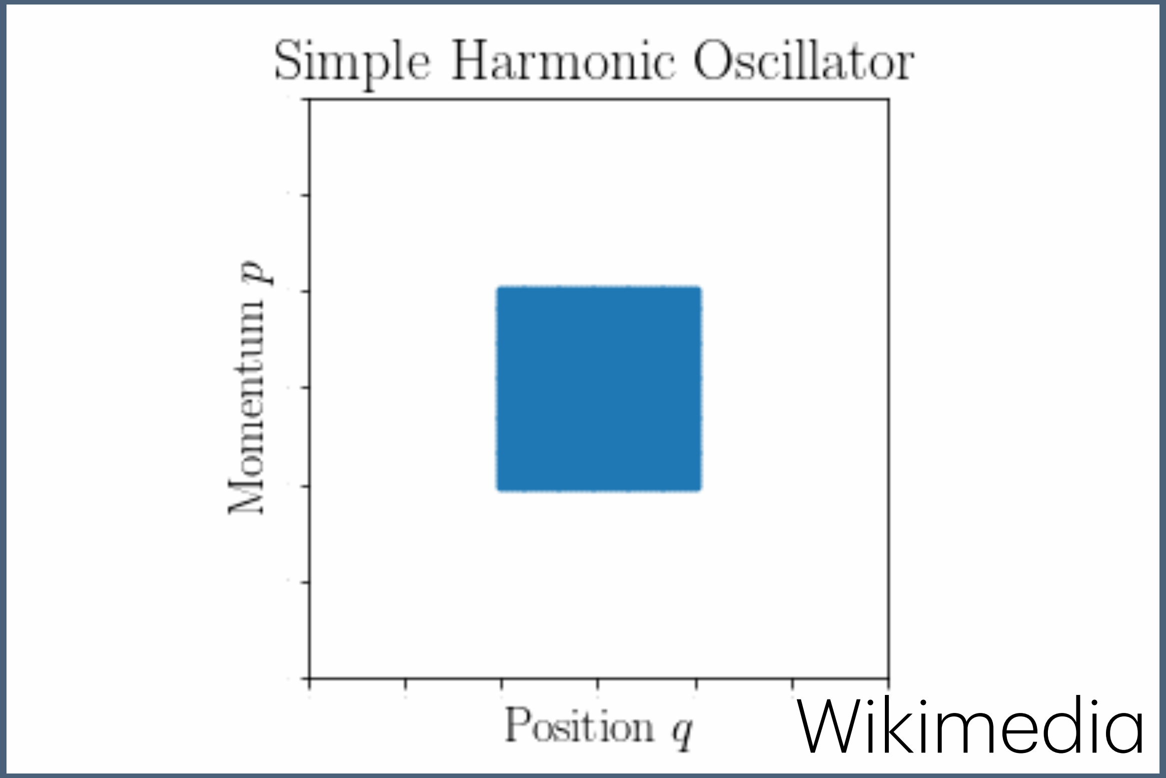


- For a ***non-dissapative*** system, the “density” of phase states traveling through phase-space is constant with time
- i.e., the flow of states in phase space maps onto the flow of an incompressible fluid in space!

Liouville's theorem

It looks like a fluid flow for a reason!

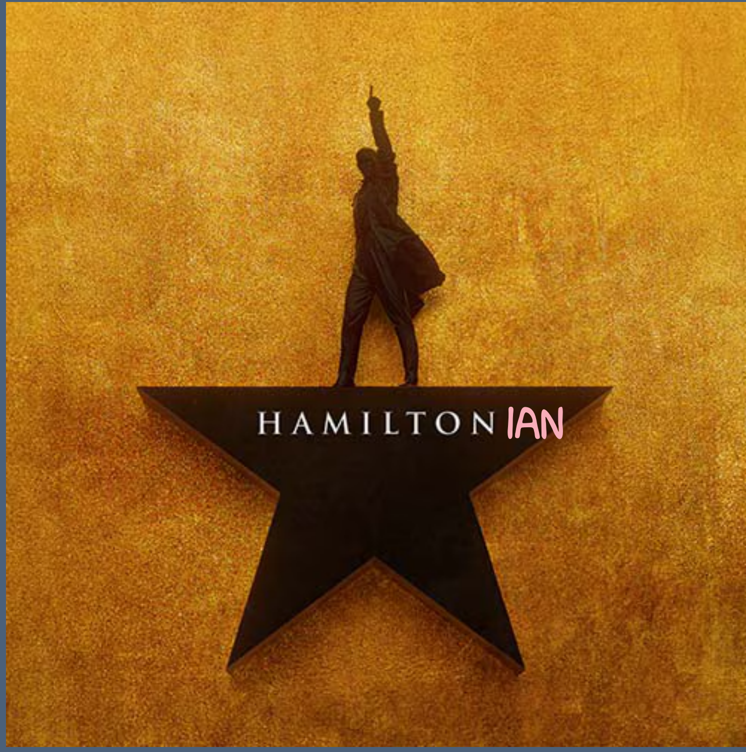




- For a ***non-dissapative*** system, the "density" of phase states traveling through phase-space is constant with time

- i.e., the flow of states in phase space maps onto the flow of an incompressible fluid in space!

Two problems



- 13.26 — Sketch some phase orbits!
  [write up]

- 13.27 — Confirm Liouville's Theorem for ballistic motion
  [in class only]