PhysH308

Fluid Mechanics!

Ted Brzinski, Dec. 10, 2024

Course logistics

- Monday: Exam posted along with final grade projection, learning standard “punch lists” b/w 2:30 and 3:15.
- Exam week office hours:
    - T: 1-3 PM (office+zoom)
    - W: 1-3 PM (office+zoom)
    - Th/F - by appointment only, might be zoom only.
- Assessment plan

Exam format

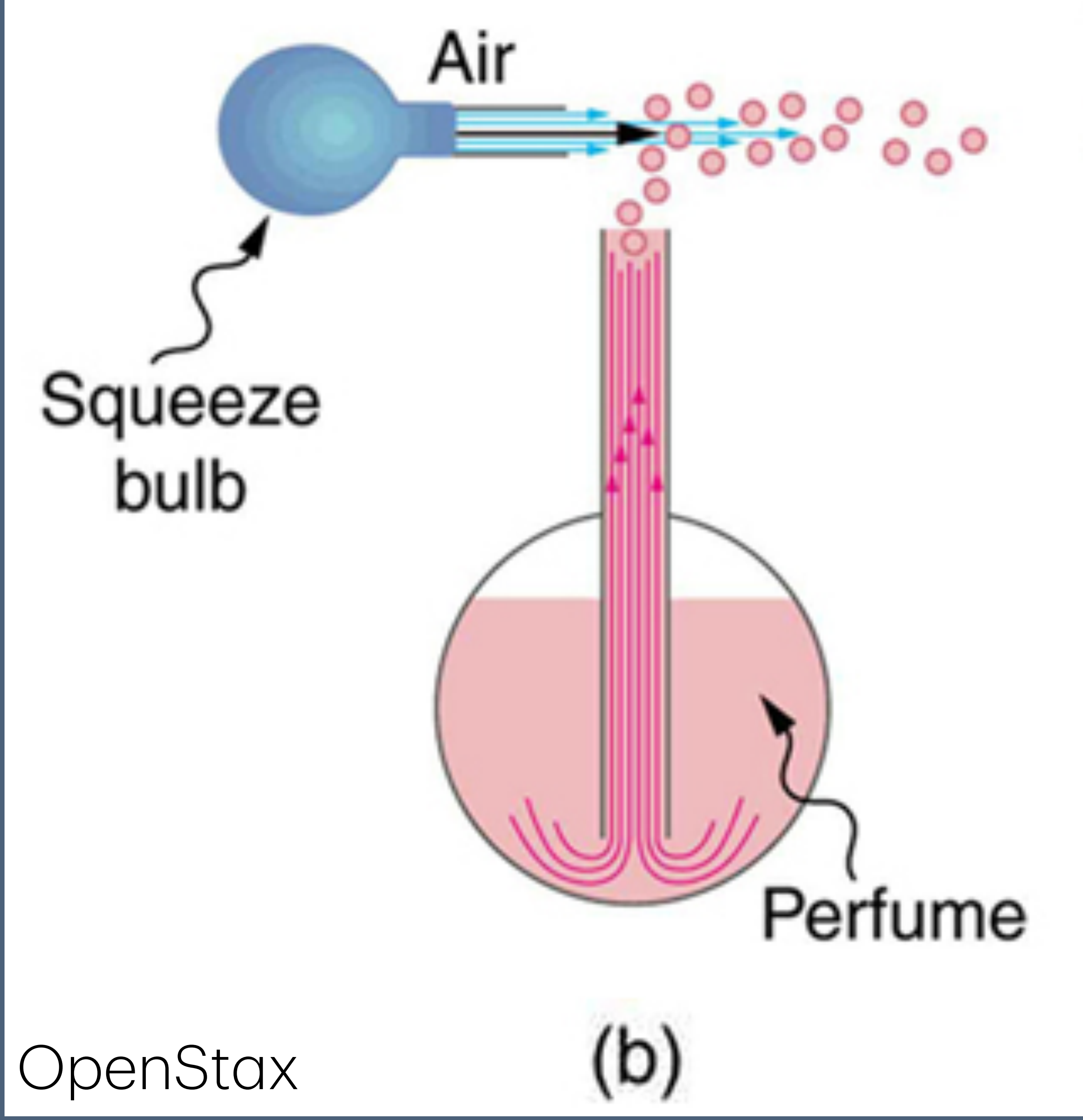
- Exam format:
  - 3 new, timed questions on fluids:
    - C7: Apply continuity to predict the behaviour of fluid flows.
    - Q7: Apply Benoulli's Theorem to understand a fluid flow and the related pressure change.
    - R7: Interpret details of a fluid flow from a plot of streamlines.
  - New, timed questions on 8 graded/resubmitted standards:
    C1-2, Q1-3, R1-2
  - Revise and resubmit graded C3-6, Q4-6, R3-6

Last time

More ideal, incompressible flow

- Leonardo's Law $\left(\dot{Q} = 0\right)$
- Bernoulli's Principle:

$$H = \frac{1}{2}v^2 + \Phi + \frac{P}{\rho} = \text{const}$$



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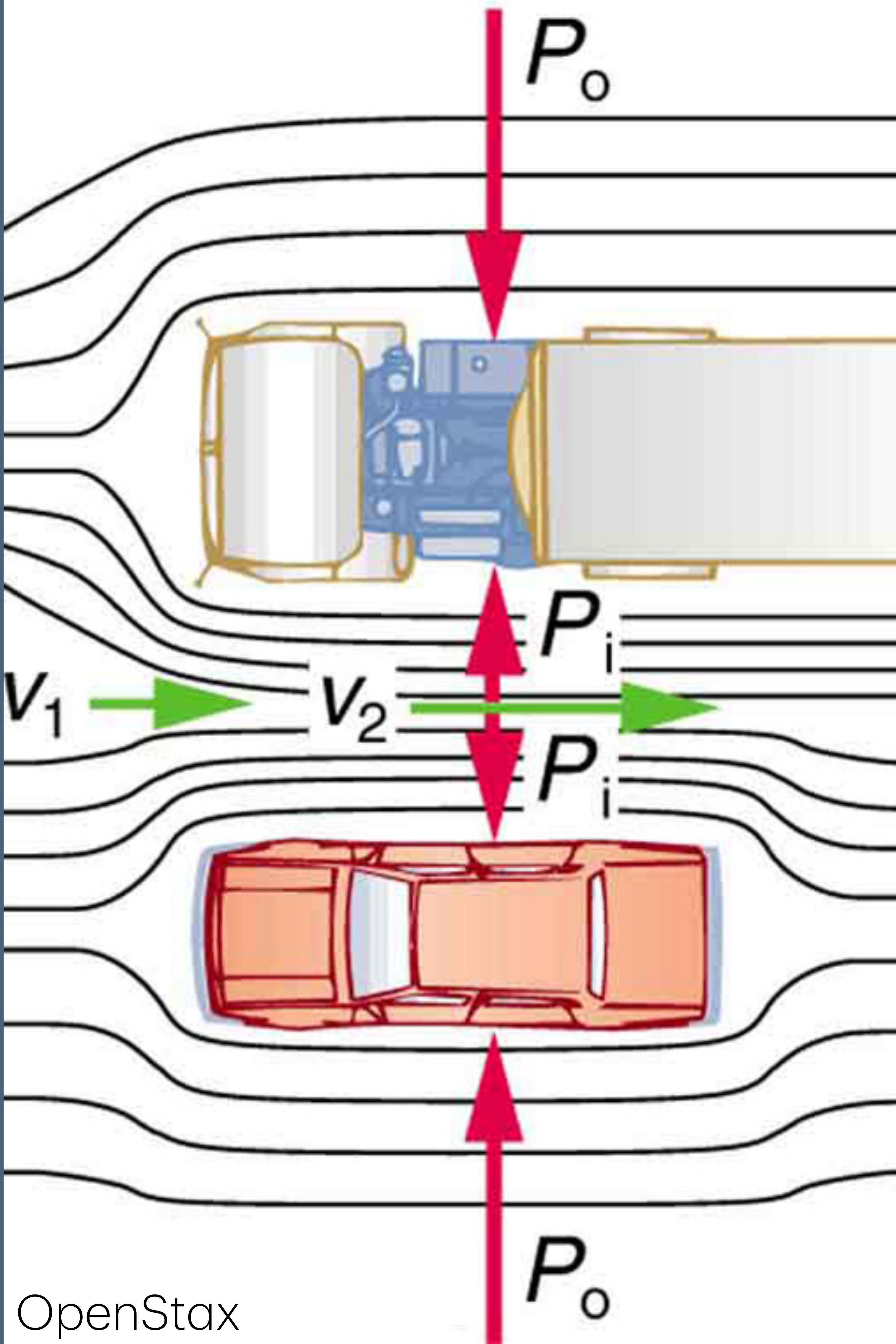

OpenStax

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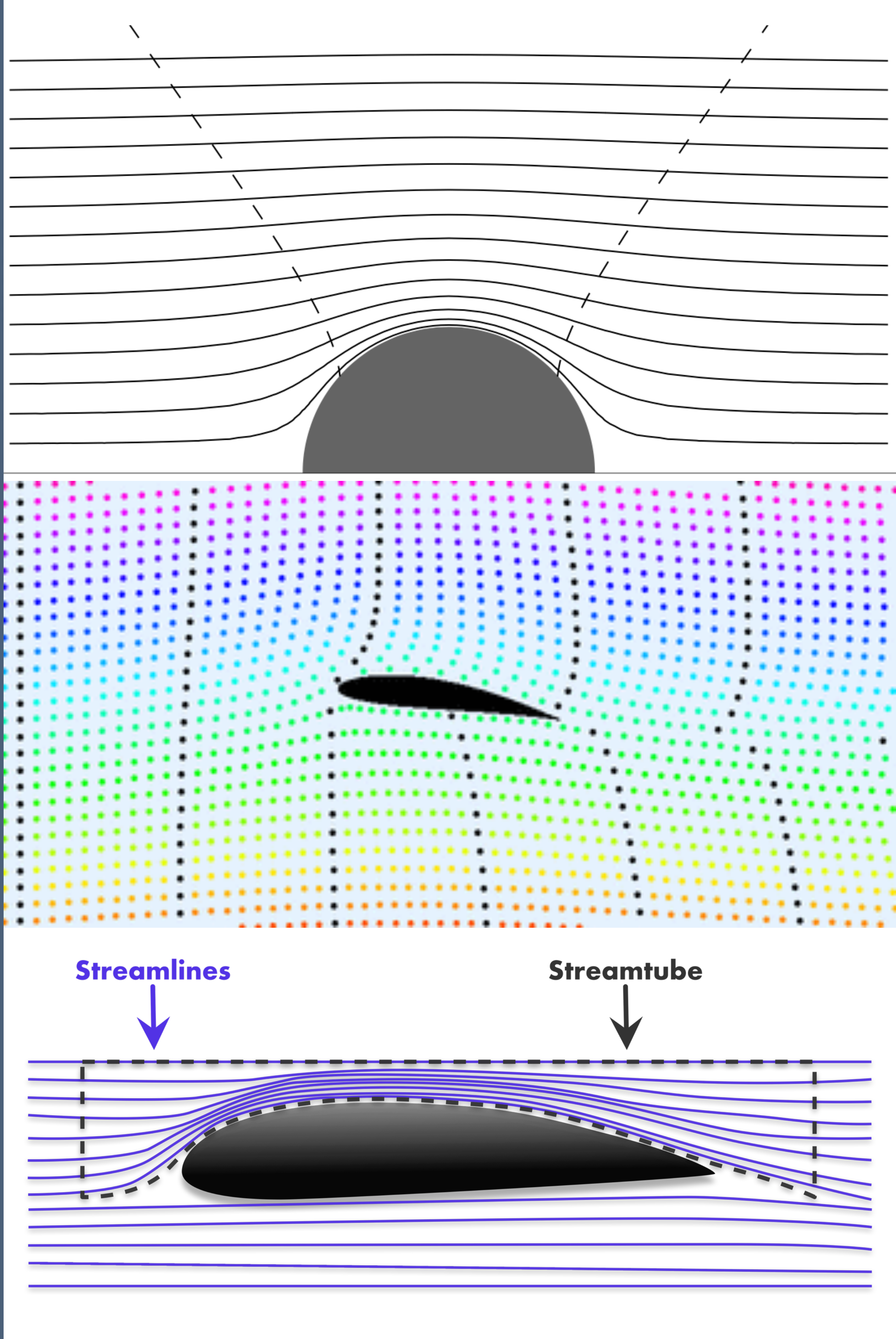
- Fun with wing shapes: LINK



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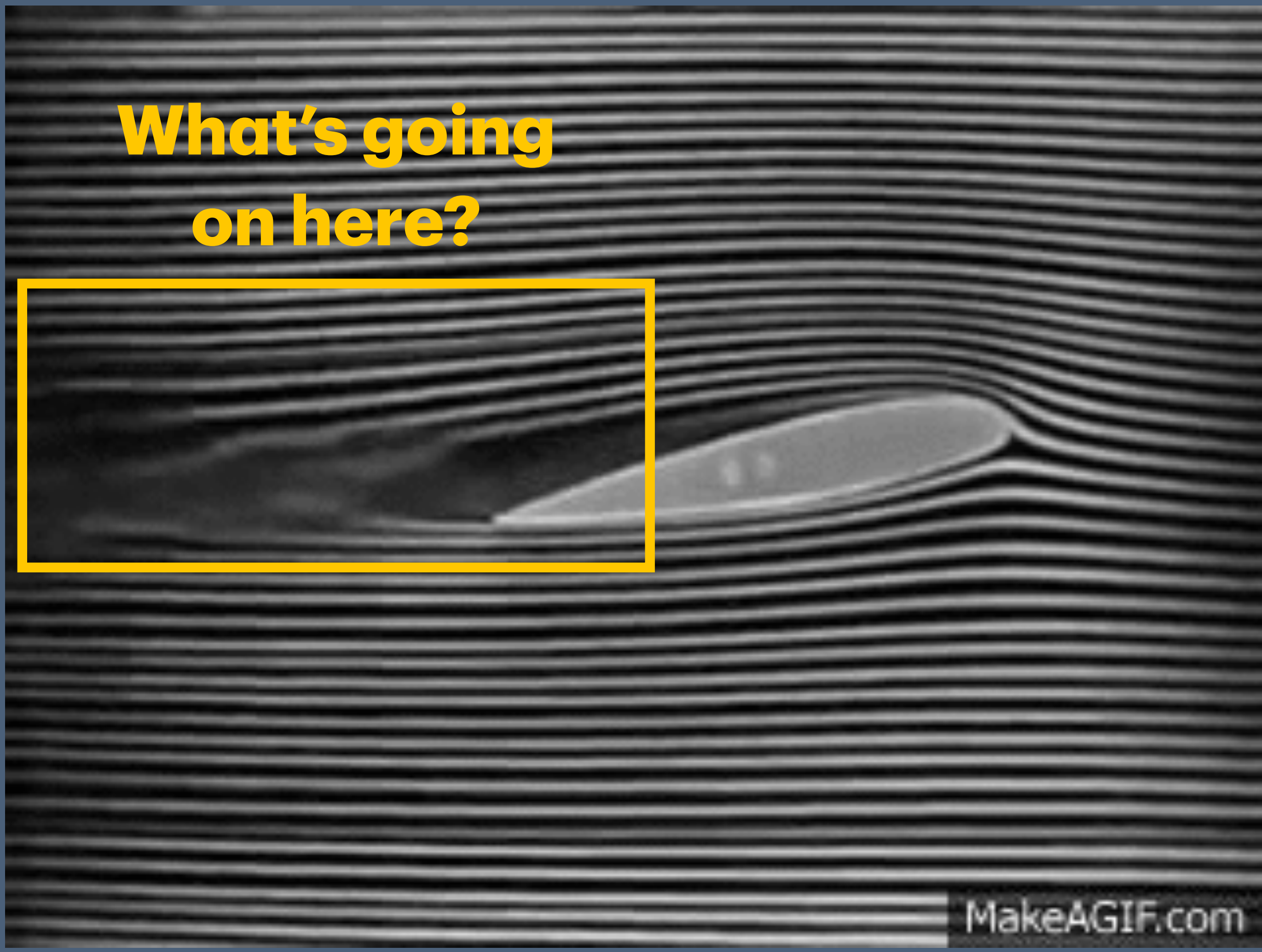
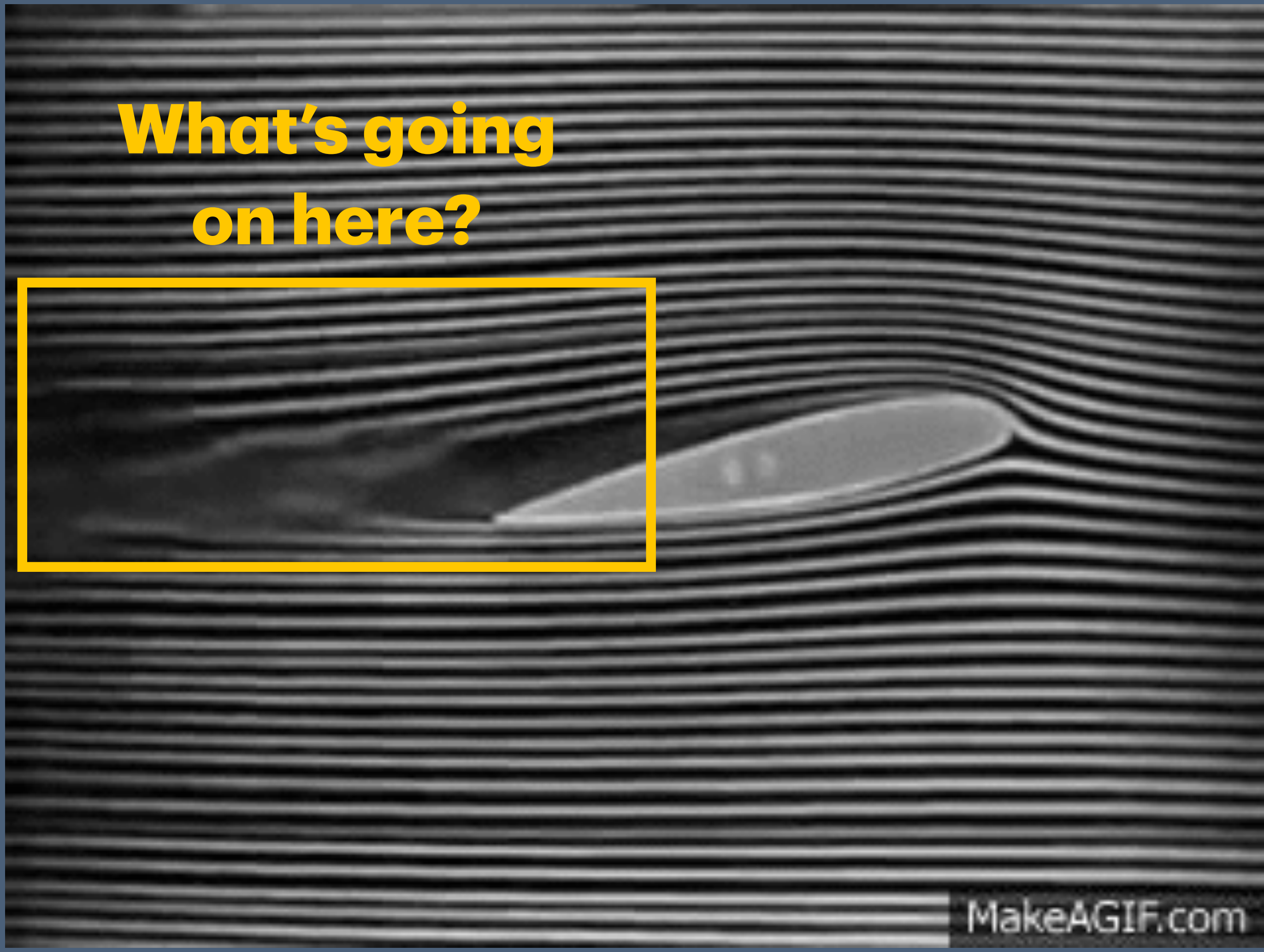


Last time

More ideal, incompressible flow

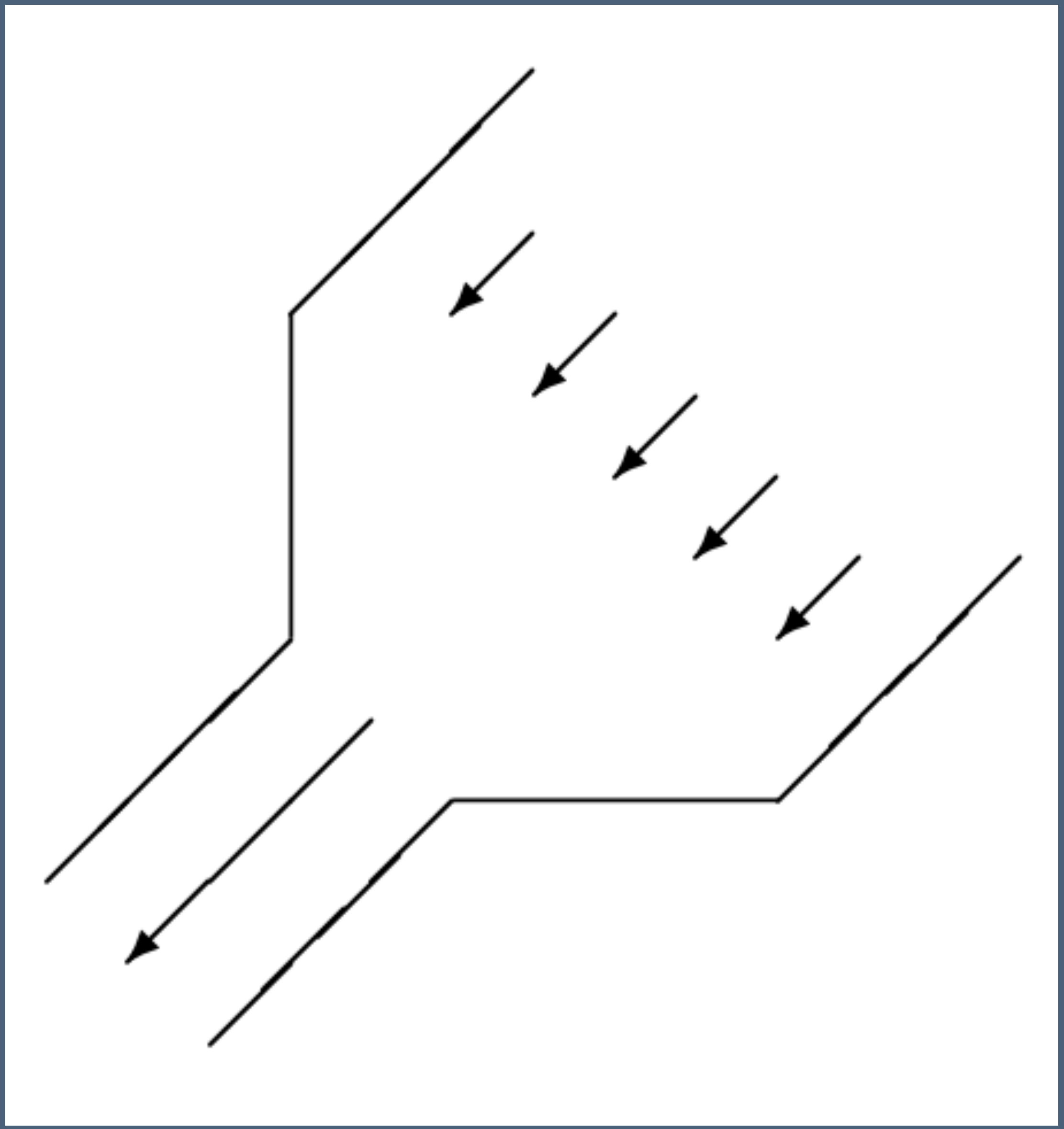
- Flow far from the wingtip is smooth, steady.
- Flow behind the wing has become unsteady, “rough”, characterized by mixing and structure at all scales
- Let’s try to understand, qualitatively, why.



Viscosity

Liquid friction

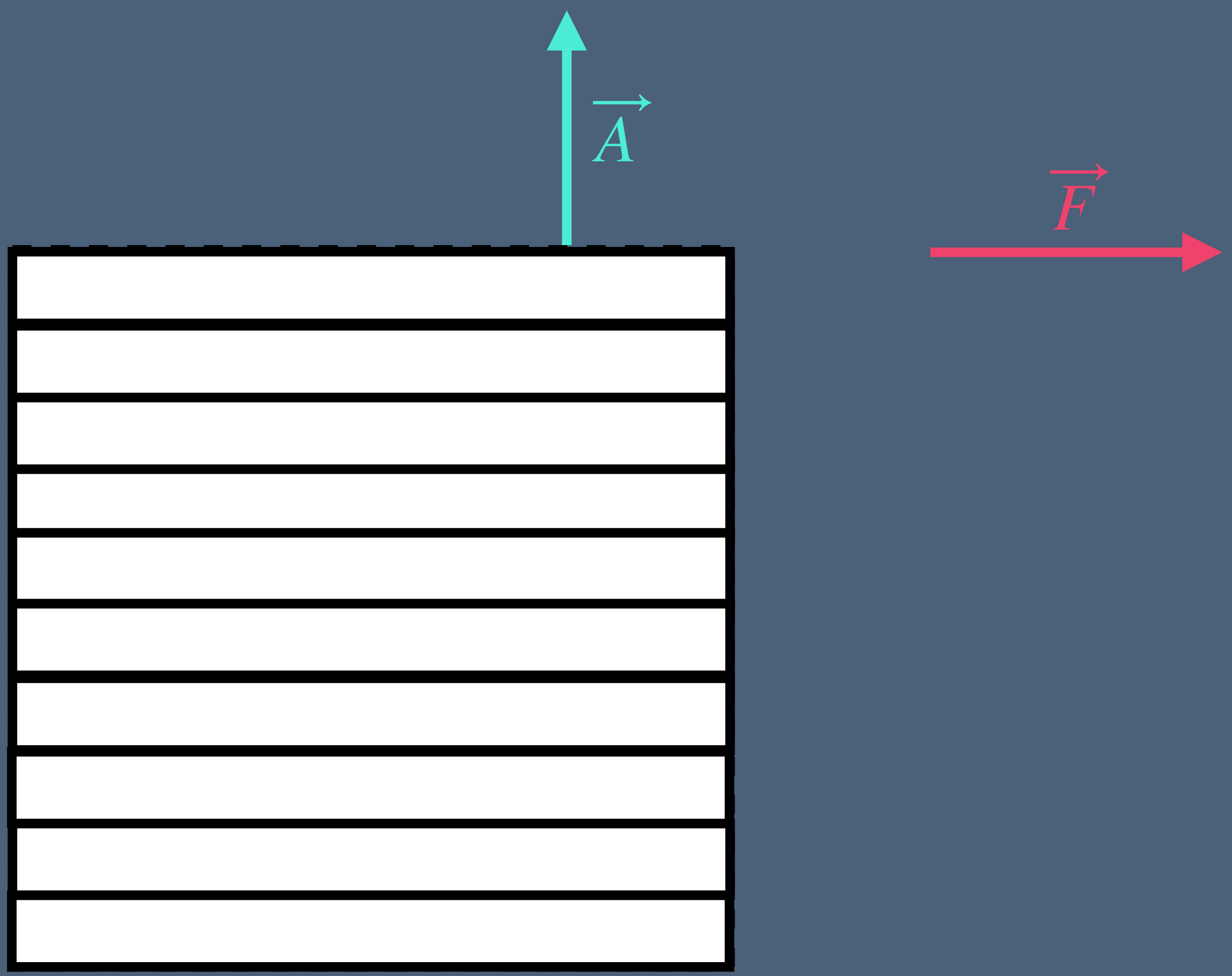
- Thus far, we've talked about ideal flows — zero dissipation, time-reversible, homogenous



Viscosity

Liquid friction

$$\tau = \frac{F}{A}$$



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- What if our fluid particles had friction?

Viscosity

Liquid friction

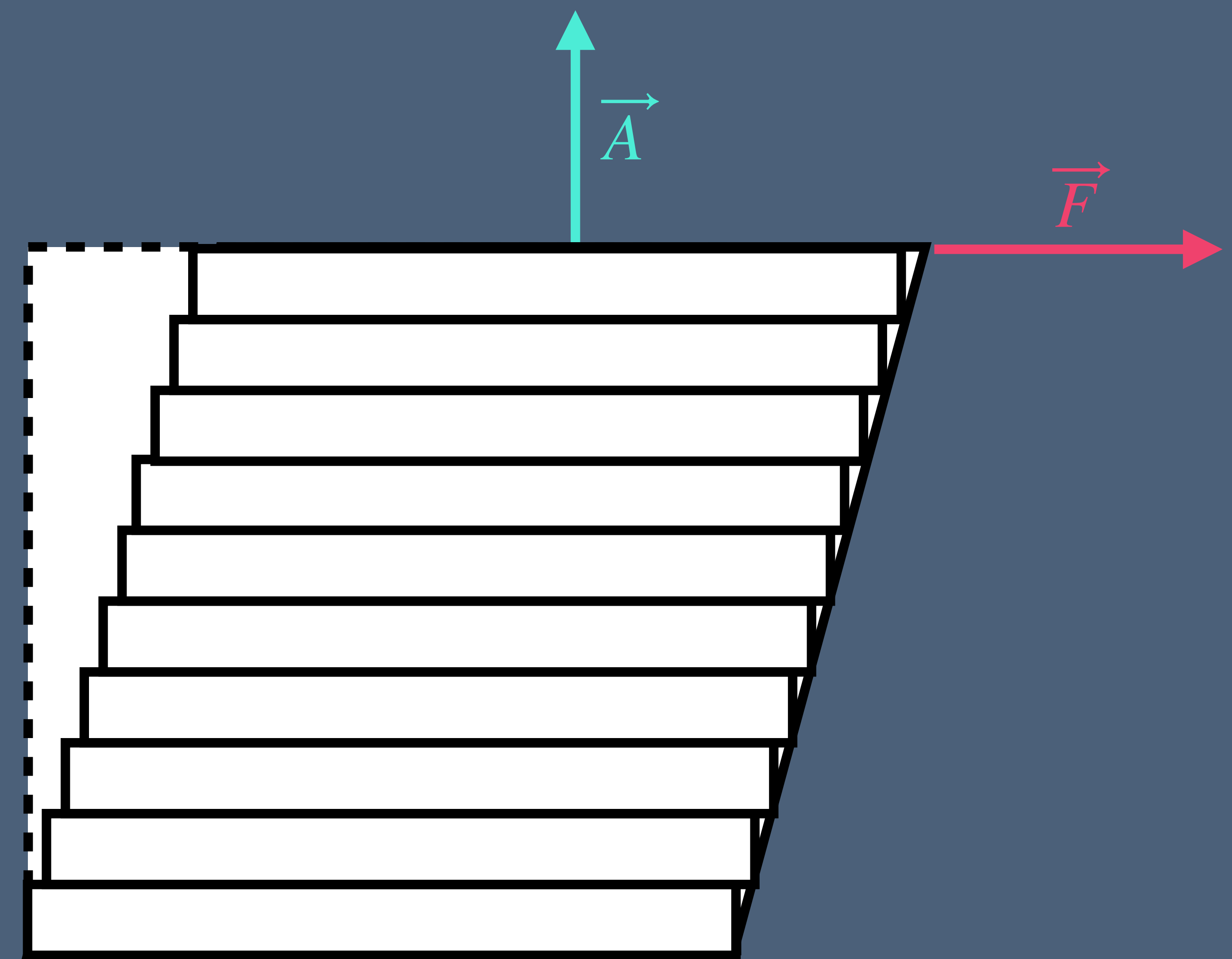
- Thus far, we've talked about ideal flows — zero dissipation, time-reversible, homogenous

- What if our fluid particles had friction?

- Fluid friction is "viscosity":

$$\tau = 2\eta\dot{\gamma} = 2\eta\frac{dv_x}{dy}$$

$$\tau = \frac{F}{A}$$

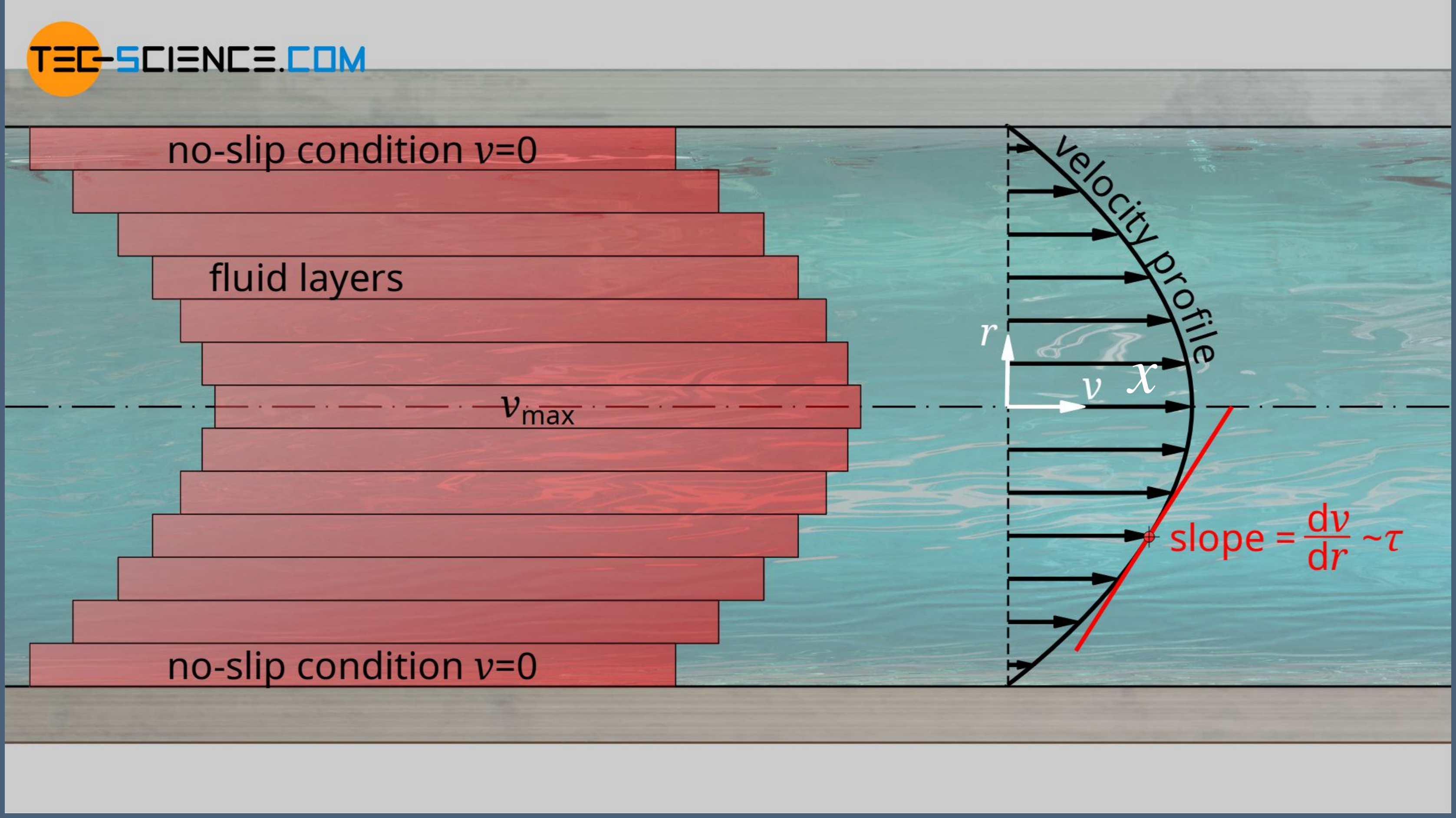


Viscous pipe flow

No-slip conditions at boundaries

$$\text{Steady flow}: F_\eta = F_P$$
$$\tau \cdot dA_\perp = A dP_\parallel$$
$$\eta \frac{dv}{dr} 2\pi r\, dx = -\pi r^2 dP$$

$$\frac{dv}{dr} = -\frac{r}{2\eta}\frac{dP}{dx}$$
$$v = -\frac{r^2}{4\eta}\frac{dP}{dx}$$

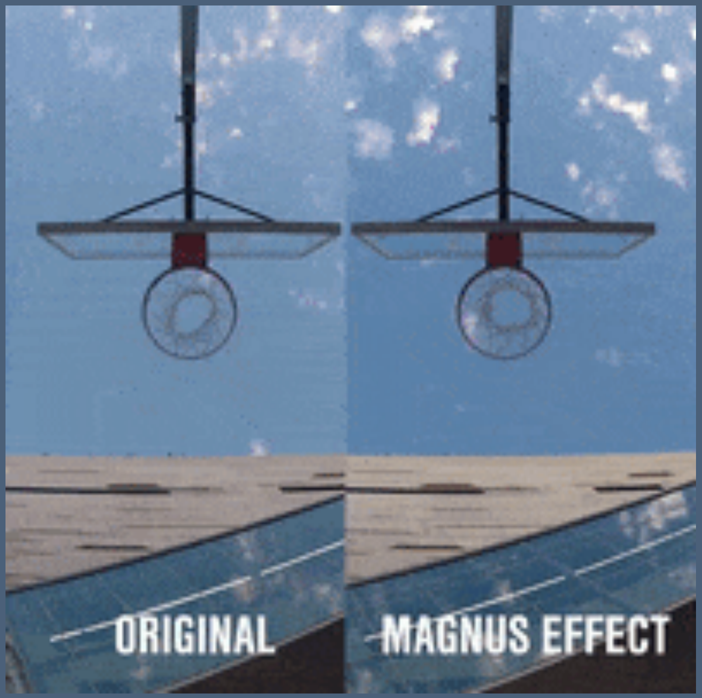


Parabolic about the center!

Magnus effect

A cool manifestation of viscous drag+Bernoulli


Veritasium



Harlem Globetrotters

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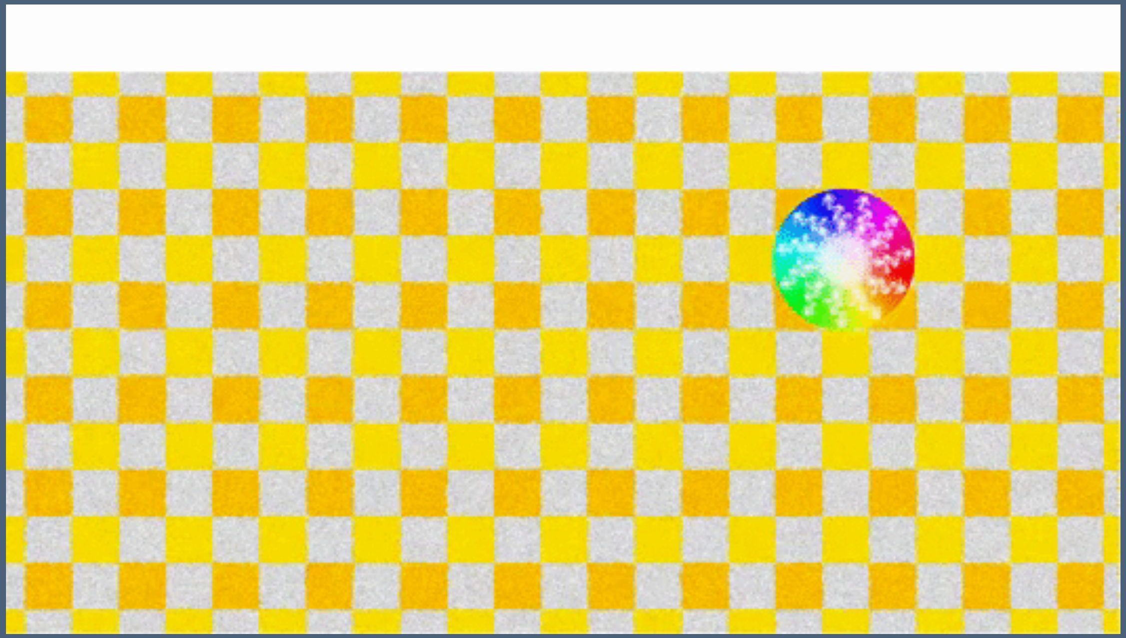
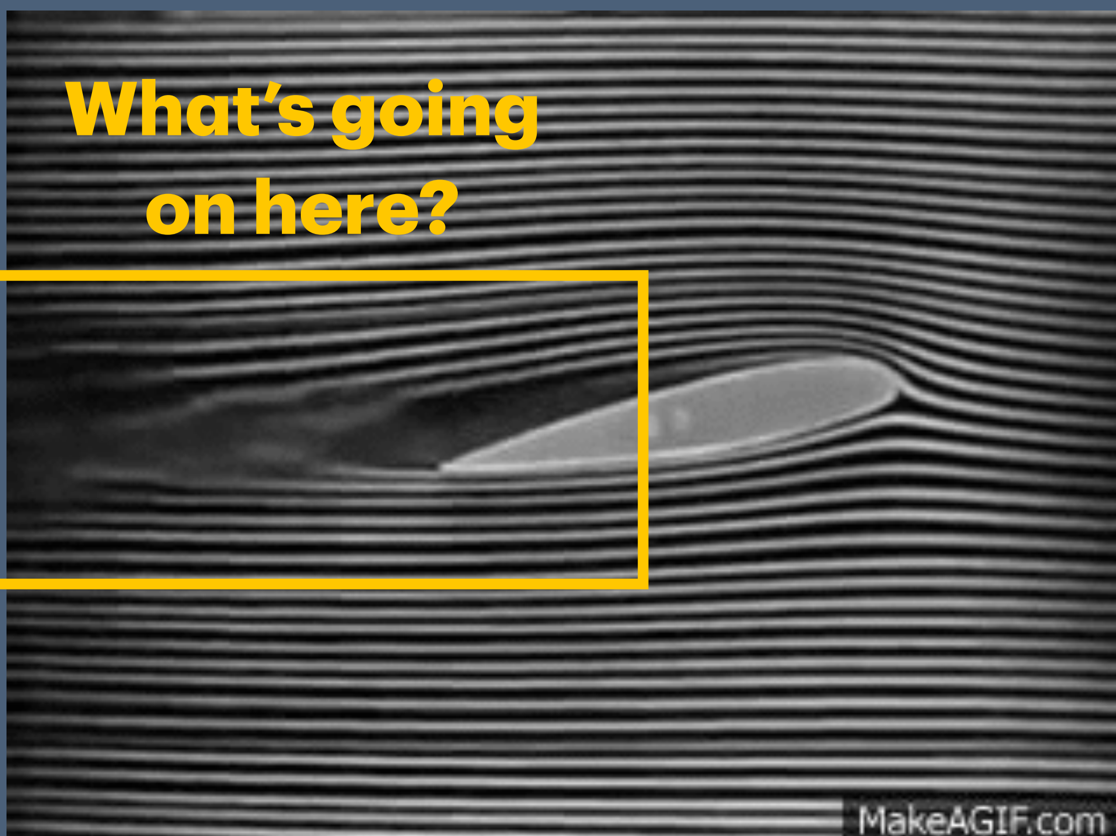
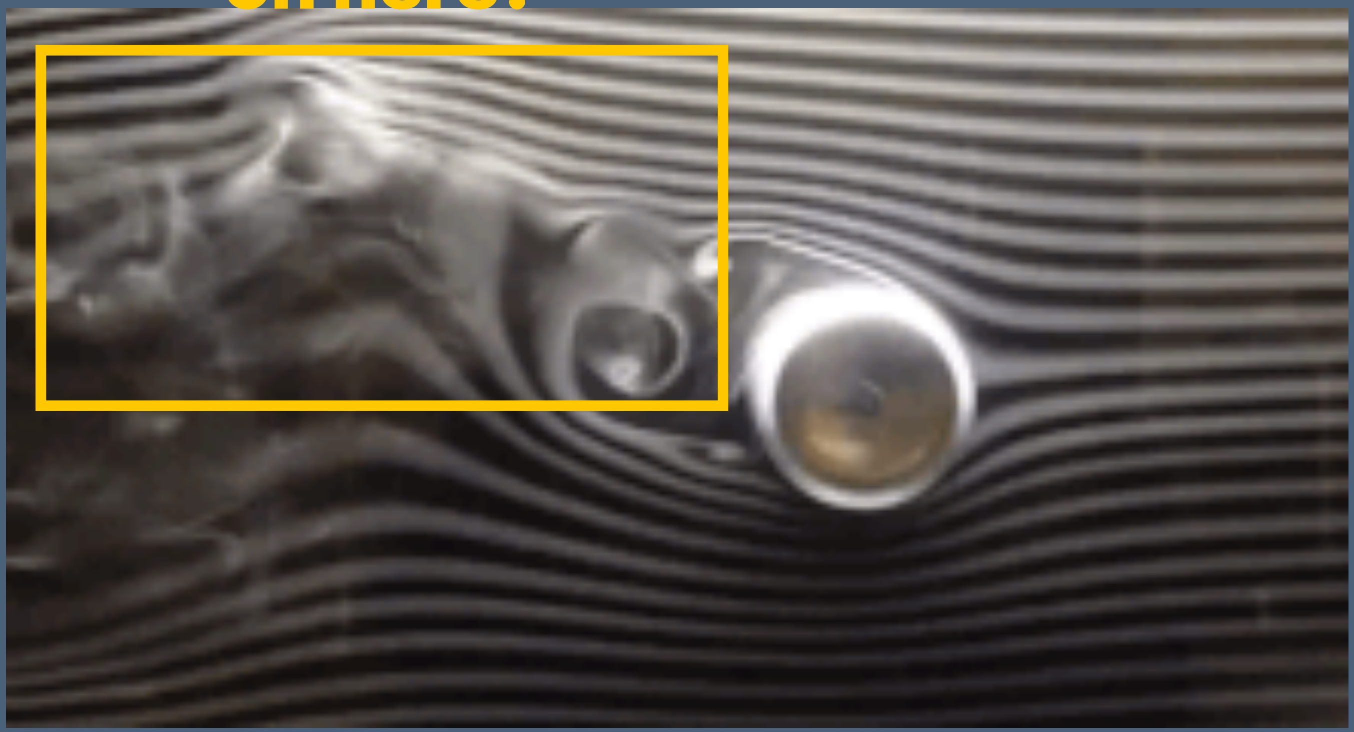


Okay, but…



What's going
on here?



Navier-Stokes equation

- Recall $\dfrac{f^*}{\rho} = \dfrac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \nabla\right) \vec{v}$ is Cauchy's equation.
- $f^* = f_{ext} + f_{int} = f_{ext} - \nabla P + \eta \nabla^2 \vec{v}$ is the force density
- Combining, we get $\dfrac{1}{\rho}\left(\vec{g} - \nabla P + \eta \nabla^2 \vec{v}\right) = \dfrac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \nabla\right) \vec{v}$

Navier-Stokes equation of fluid motion!

Navier-Stokes equation

- $$\frac{1}{\rho}\left(\vec{g} - \nabla P + \eta \nabla^2 \vec{v}\right) = \frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \nabla\right)\vec{v}$$

  Navier-Stokes equation of fluid motion!

- It's useful to consider the ratio $\mathrm{Re} = \dfrac{\boxed{\rho\left(\vec{v} \cdot \nabla\right)\vec{v}}}{\boxed{\eta \nabla^2 \vec{v}}}$ **Inertia** **Viscosity**

Navier-Stokes equation

- It's useful to consider the ratio

$$\mathrm{Re} = \frac{\boxed{\rho\left(\vec{v}\cdot\nabla\right)\vec{v}}}{\boxed{\eta\,\nabla^2\vec{v}}}$$

Inertia

Viscosity

- For $\mathrm{Re} \ll 1$, flow is ***Laminar***, and is dominated by viscosity. The motion is smooth and the deformation of the fluid is time-reversible

- For $\mathrm{Re} \gtrsim 10^5$, flow is ***turbulent***, chaotic, heterogeneous, fractal — dominated by time-dependent vortices at all scales

Laminar time-reversibility demo

End of semester feedback

Also on moodle

https://forms.gle/1CsaJPwnSzWMsWEy7

Please respond by 8 AM Monday morning