PhysH308

Fluid Mechanics!

Ted Brzinski, Dec. 10, 2024

Course logistics

- Next class: end-of-semester feedback, Turbulent vs Laminar flows, exam week office hours announced
- Exam week:
  - Monday: Exam posted along with final grade projection, learning standard "punch list"
  - Friday:
    - Exam due (via gradescope) by noon, **no exceptions**
    - All other coursework must be submitted, as discussed with Ted/your dean, by noon, **no exceptions**

Last time

Ideal, incompressible flow

- **12.4:** Leonardo's Law: $Q = Av = \text{constant}$
- **2nd problem (conceptual):** A pipe reduces diameter by 50% at a conical constriction. (Hint, recall the holey bottle demo from Tuesday)
    - How does the velocity change across the constriction?
    - On which side of the constriction is the pressure higher? Why?

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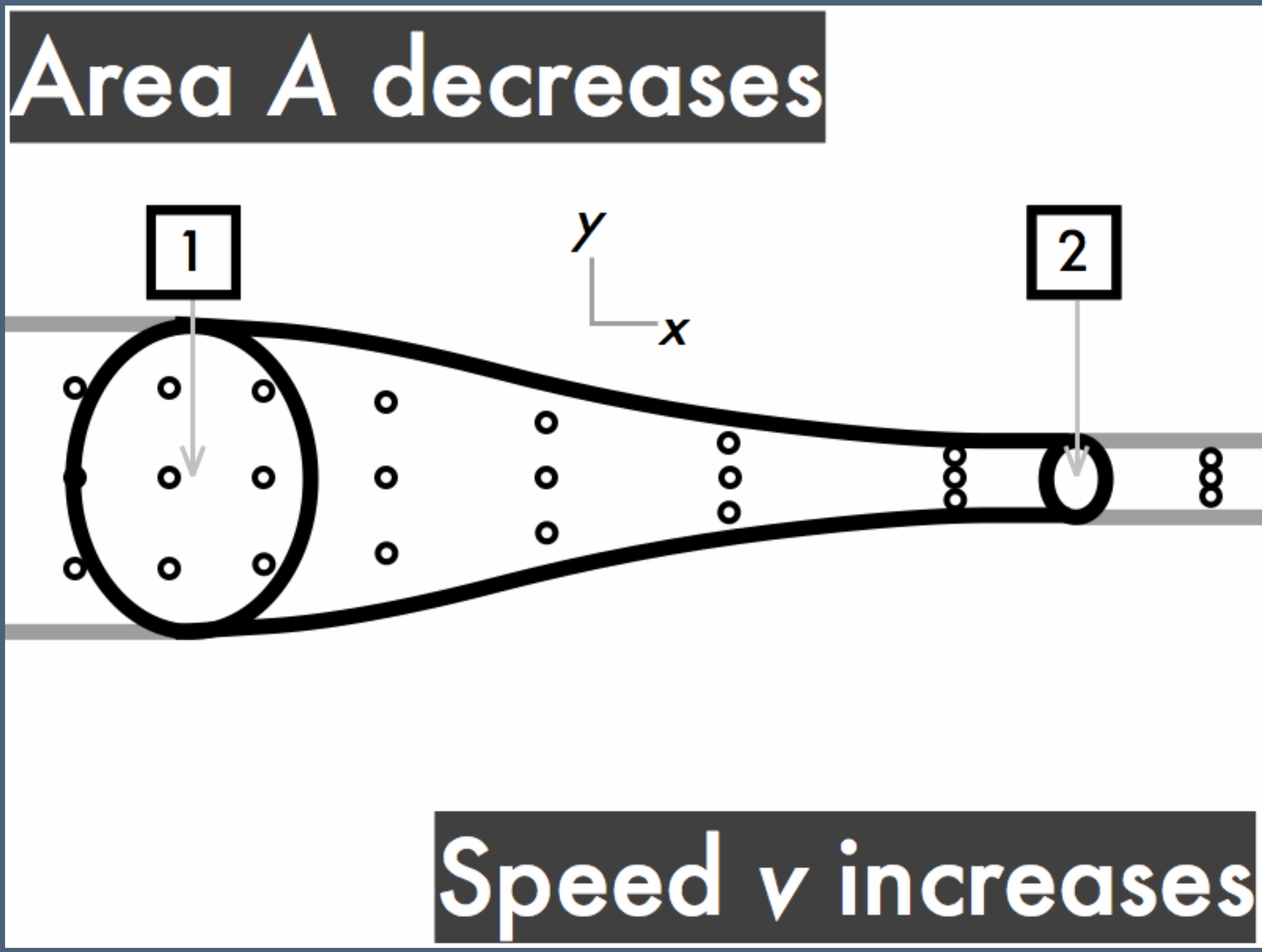

Credit: Patrick W. Len

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Ideal, incompressible flow

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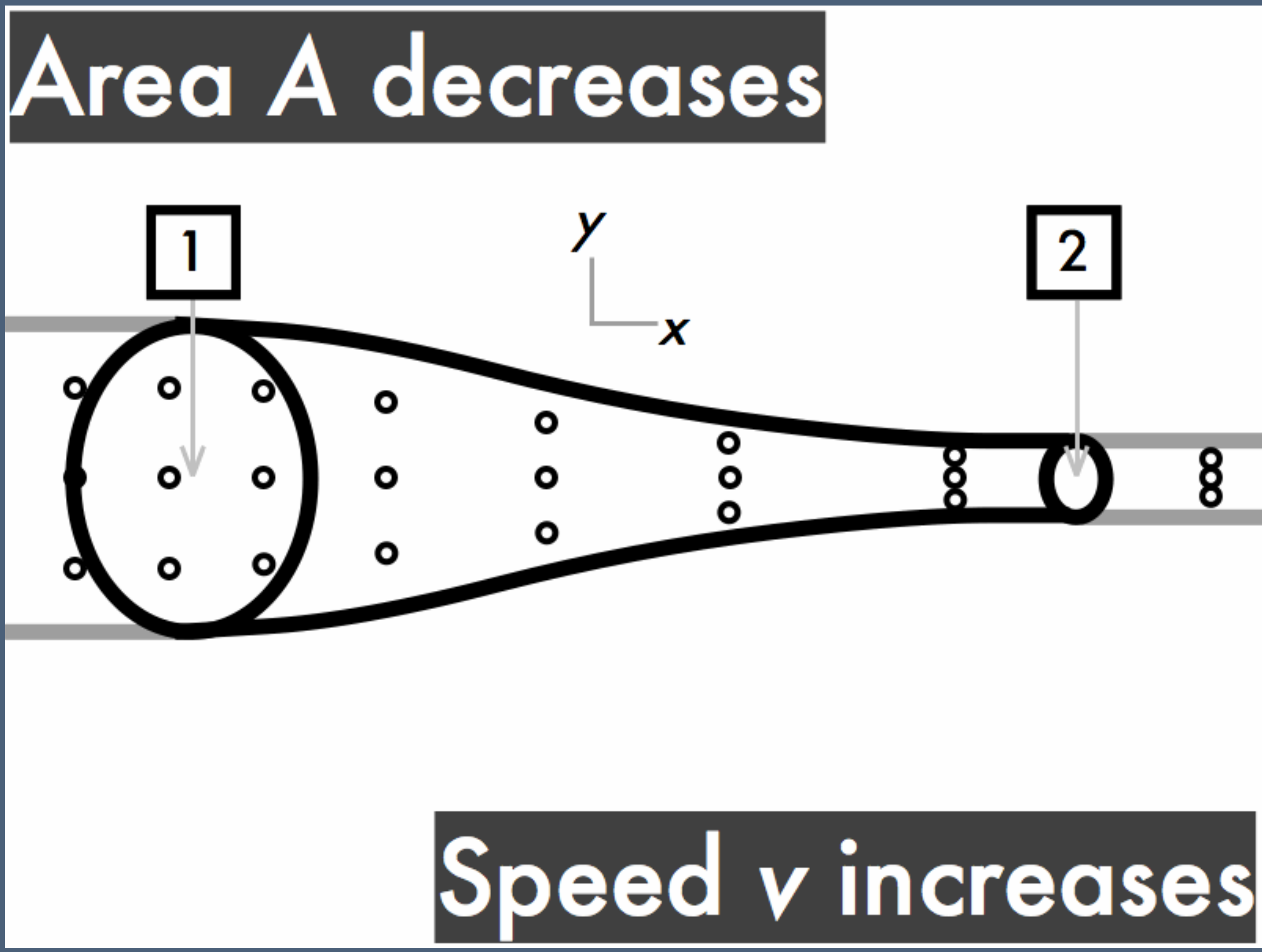

- On which side of the constriction is the pressure higher? Why?

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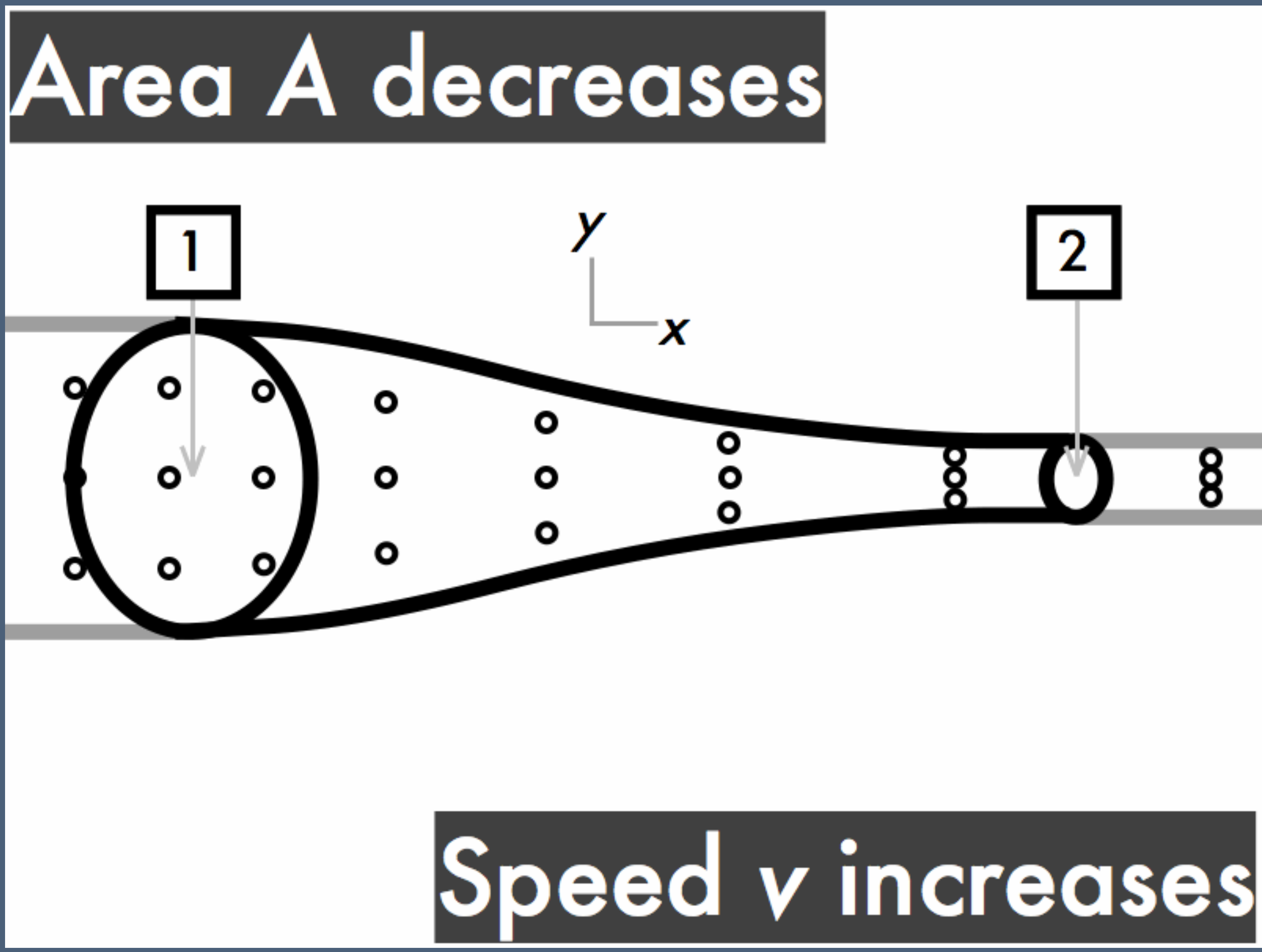
Ideal, incompressible flow

- On which side of the constriction is the pressure higher? Why?
- Recall $\dot{\vec{F}} = \dot{\vec{p}}$
- and pressure is the force per unit area!
- Thus we can interpret pressure as $P = \dfrac{dF}{dA}$
- And expect this force to be greater behind accelerating flow than in front of it!

$$Q = Av = \text{constant}$$



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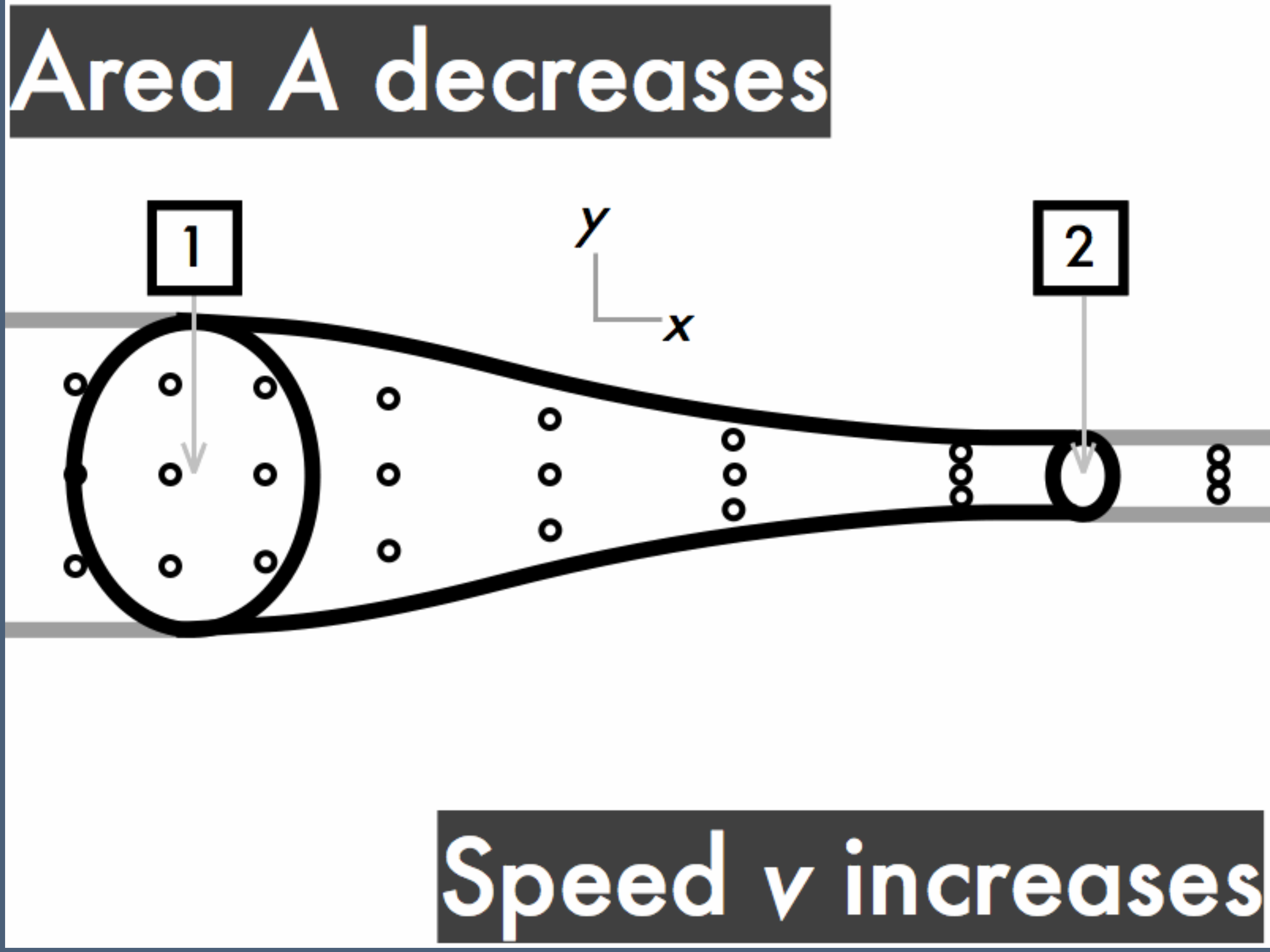


- The pressure is greater behind accelerating flow than in front of it!

$$P_1 > P_2$$

- This is a statement of Bernoulli's Theorem

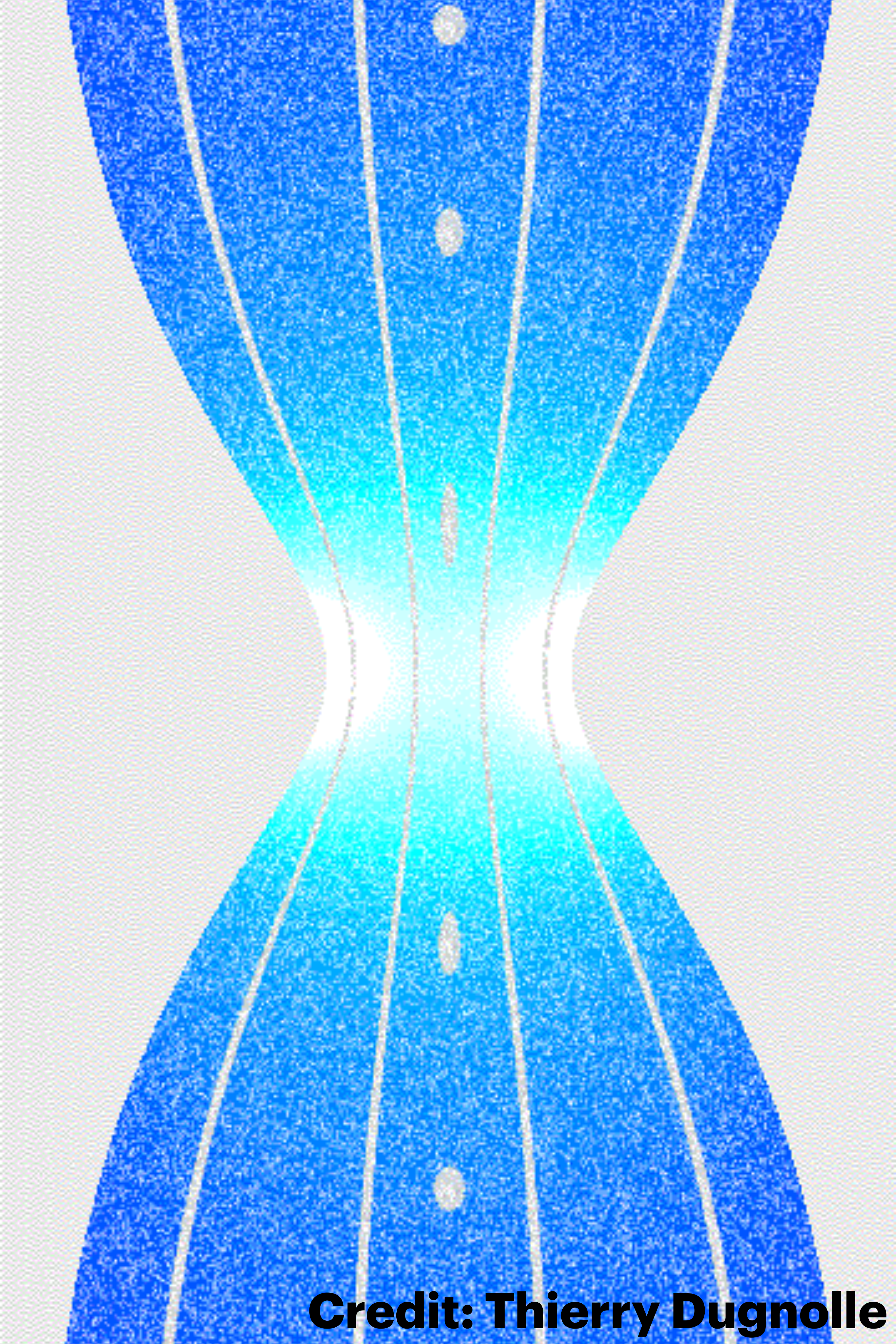
Credit: Patrick W. Len

Bernoulli's Thrm

Ideal, incompressible flow

- We can derive an analytic statement of B.T. from a conservation of energy approach…
- Kinetic energy:

$$T = \frac{1}{2}mv^2$$



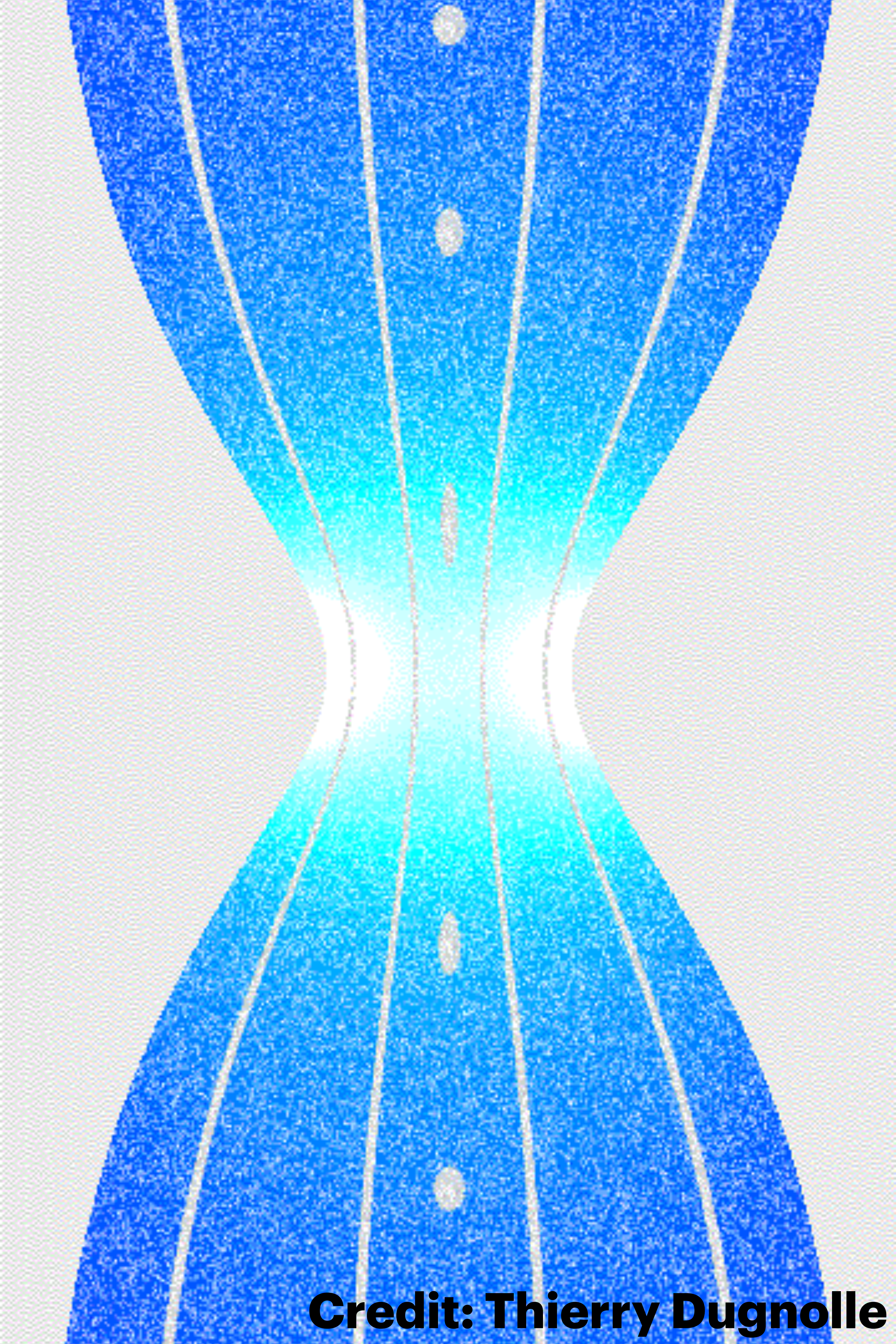
Bernoulli's Thrm

Ideal, incompressible flow

- We can derive an analytic statement of B.T. from a conservation of energy approach…

- Kinetic energy density:
$$\mathscr{T} = \frac{1}{2}\rho v^2$$

- The effective potential:
$$U = V_{\text{body}}\left(\vec{r}\right) + V_{\text{int}}\left(\delta\vec{r}\right)$$



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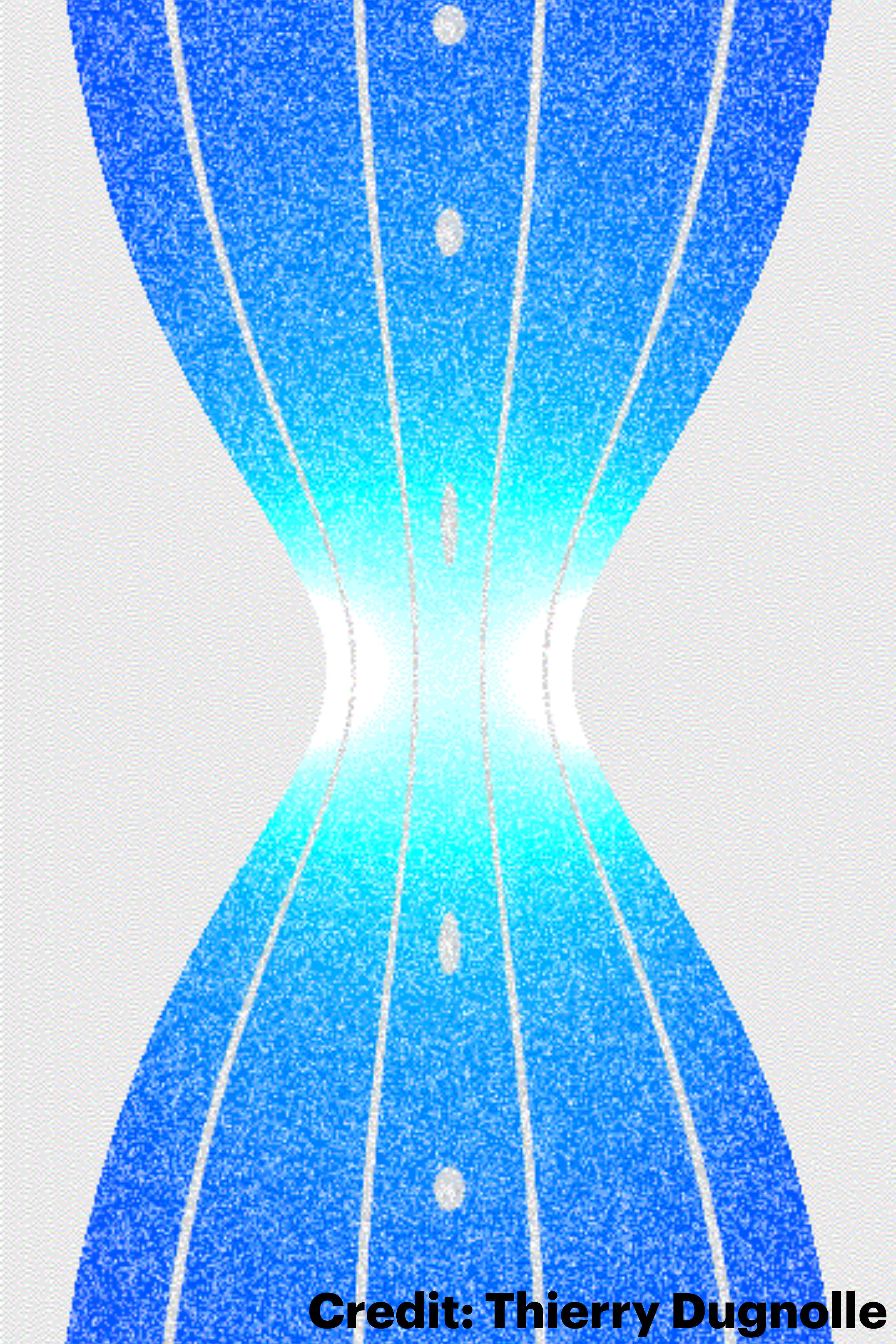
Ideal, incompressible flow

- We can derive an analytic statement of B.T. from a conservation of energy approach...

- Kinetic energy density:
$$\mathscr{T} = \frac{1}{2}\rho v^2$$

- The effective potential density:
$$\mathscr{U} = \rho\Phi\left(\vec{r}\right) + P$$



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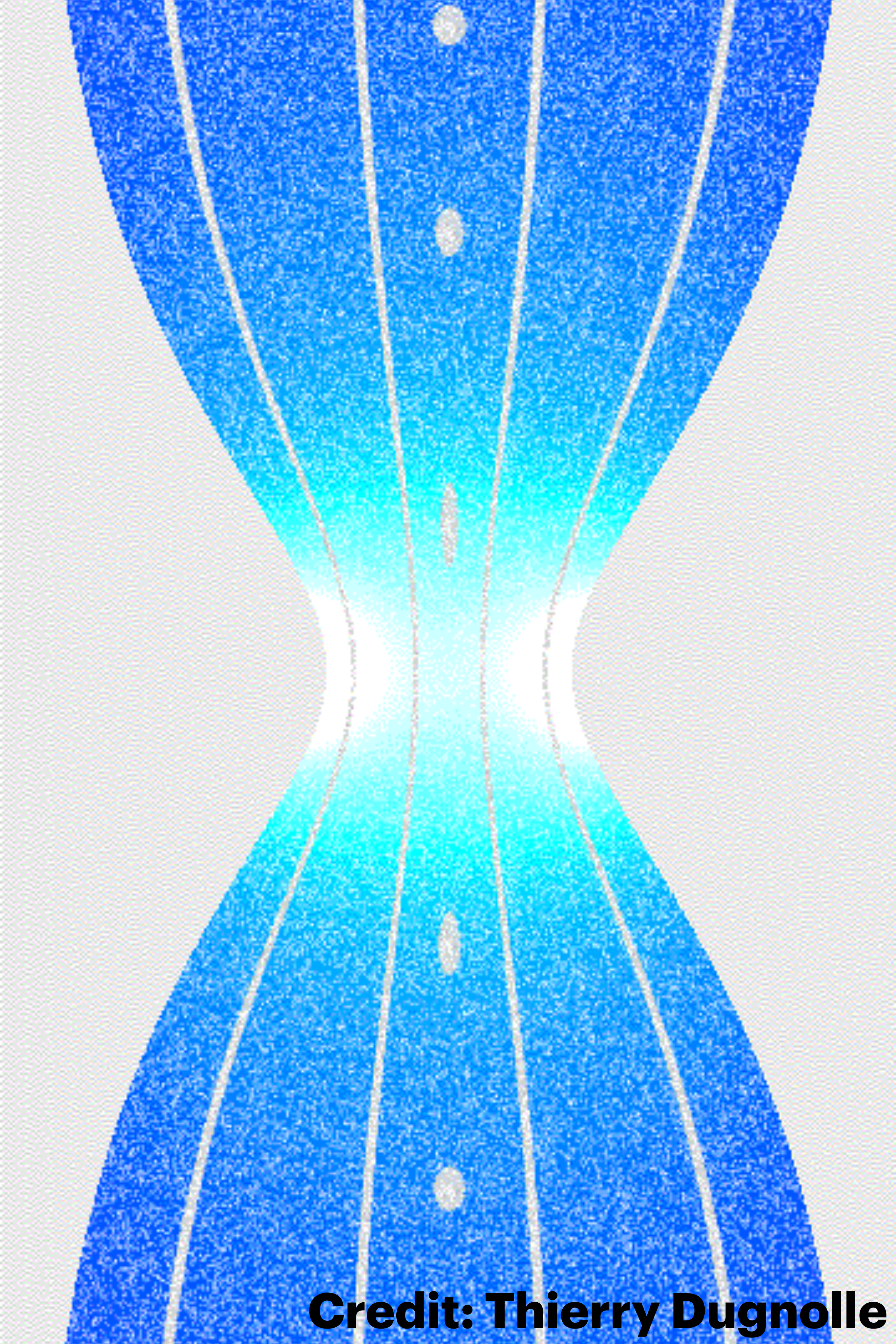
Bernoulli's Thrm

Ideal, incompressible flow

- We can derive an analytic statement of B.T. from a conservation of energy approach…
- Total energy density:
$$\mathscr{H} = \rho H \equiv \frac{1}{2}\rho v^2 + \rho\Phi\left(\vec{r}\right) + P$$
- This quantity is conserved along streamlines! (Consider particle path for steady flow)



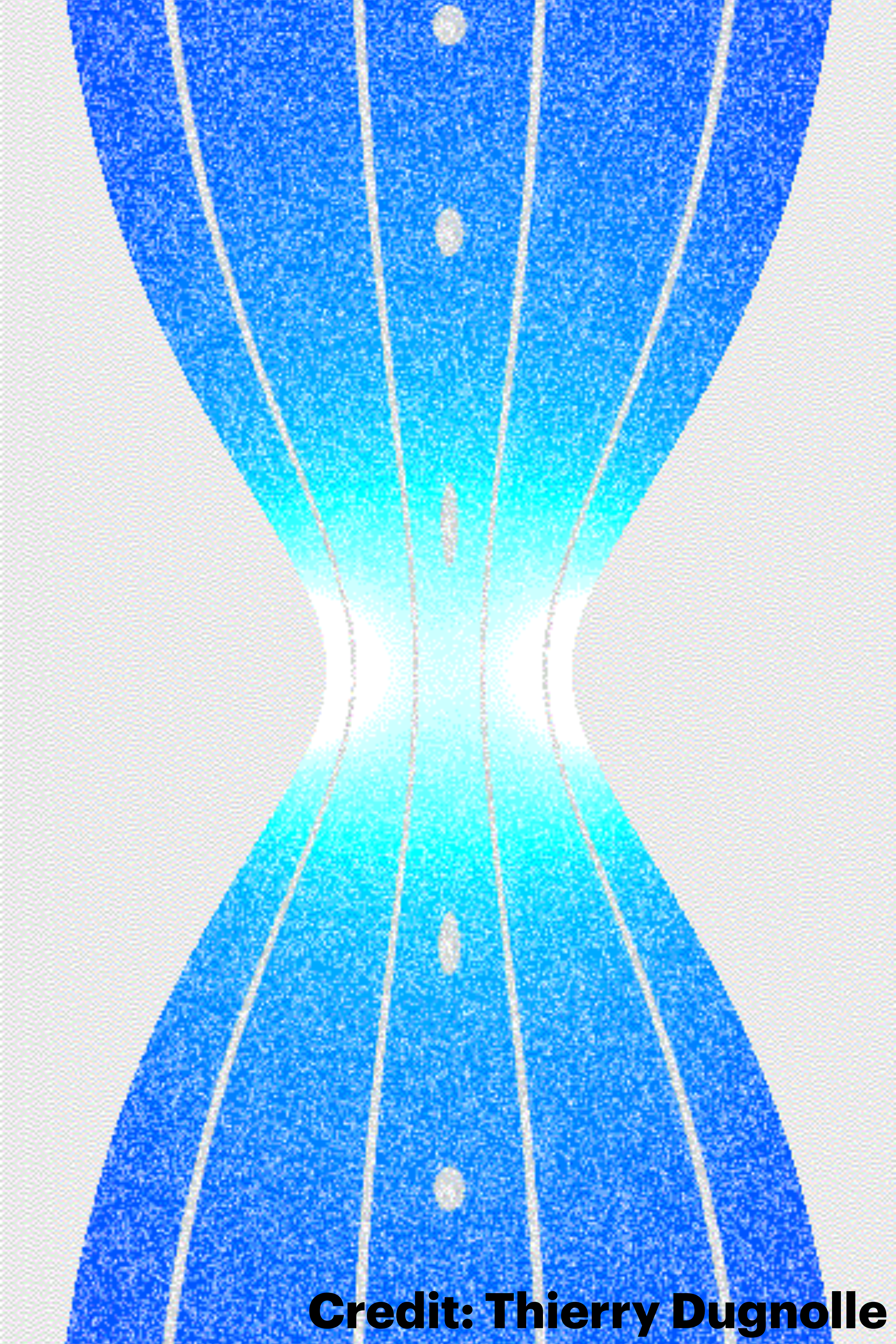
Bernoulli's Thrm

Ideal, incompressible flow

- We can derive an analytic statement of B.T. from a conservation of energy approach...
- Total energy density:

$$\underbrace{\boxed{\rho H}}_{\text{Total pressure}} \equiv \overbrace{\boxed{\frac{1}{2}\rho v^2}}^{\text{Dynamic pressure}} + \underbrace{\boxed{\rho\Phi\left(\vec{r}\right) + \overbrace{\boxed{P}}^{\text{Static pressure}}}}_{\text{Stagnation pressure}}$$

- This quantity is conserved along streamlines! (Consider particle path for steady flow)



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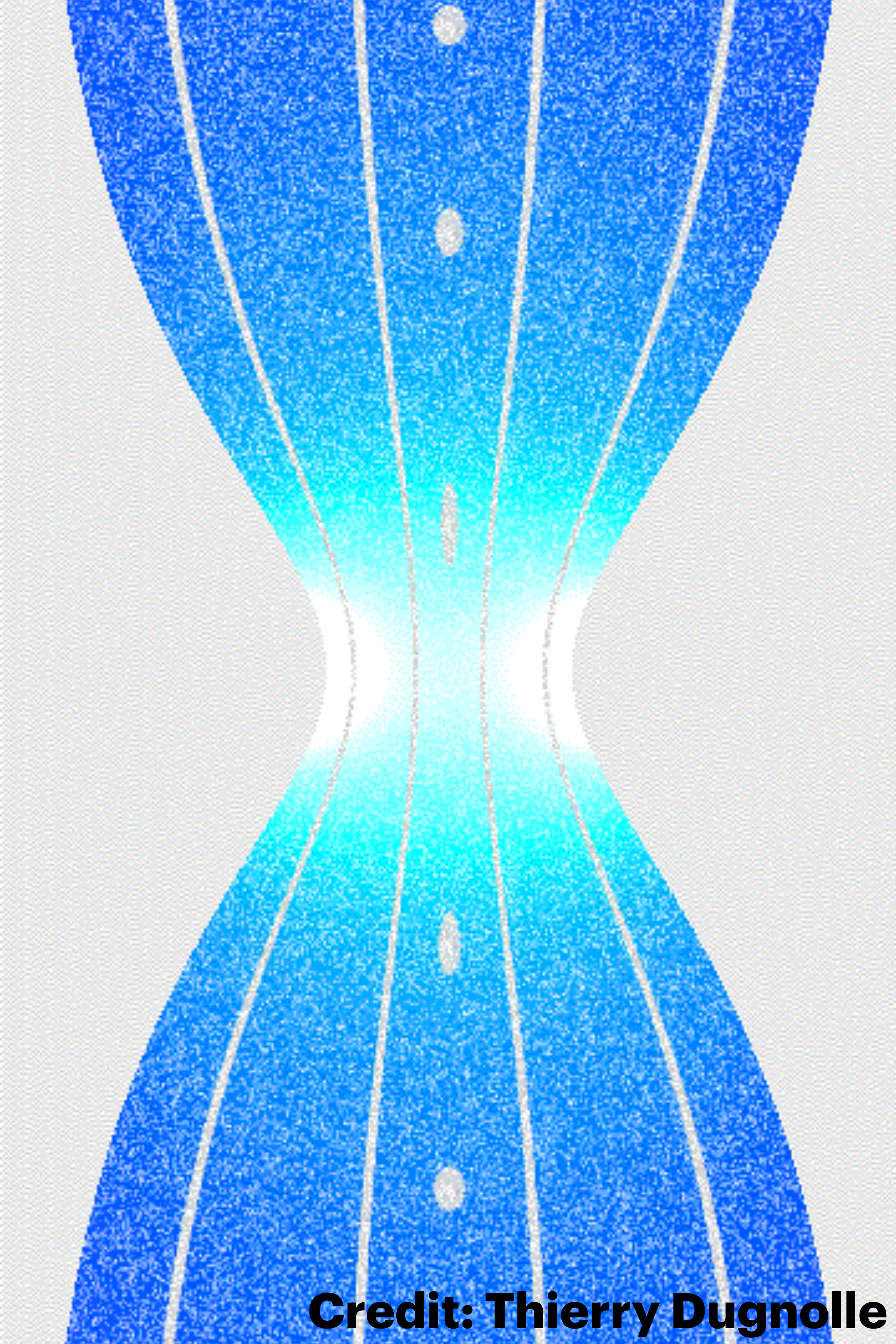
Bernoulli's Thrm

Ideal, incompressible flow

- $H \equiv \frac{1}{2}v^2 + \Phi\left(\vec{r}\right) + \frac{P}{\rho}$

  is the *Bernoulli Field* and is constant along streamlines.



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An example

Conical constriction

- Leonardo's law let's us calculate $v(x)$
- Bernoulli tells us that, along streamlines,
$$\frac{1}{2}v^2(x) + \frac{P(x)}{\rho} = H = \text{constant}$$
- $P_0 = \text{constant} - \frac{1}{2}\rho v_0^2$
- $P(x) = \text{constant} - \frac{1}{2}\rho v_0^2 \left(1 - \frac{x}{2}\right)^4$
- $\Delta P = P(x) - P_0 = -\frac{1}{32}\rho v_0^2 x^4$

$$R = \left(1 - \frac{x}{2}\right) 0.1 \ \text{meters}$$

$$A = \pi R^2 = \pi \left(1 - \frac{x}{2}\right)^2 0.01 \ \text{meters}^2$$

$$v(x) = \frac{A(x)}{A(x=0)} v_0 = v_0 \left(1 - \frac{x}{2}\right)^2$$

Let's practice using Bernoulli!

(No write-ups this week!)

$$H = \frac{1}{2}v^2 + \Phi\left(\vec{r}\right) + \frac{P}{\rho}$$

1. *Bernoulli near your car:* You're driving in your car and trying out a new barometer. Most of the windows are closed, but one is open a little. There is no wind. You find that, when the car is moving at 105 km/h, the pressure inside drops by 667 Pa. Assume a density of air of 1.21 $\mathrm{kg/m^3}$.
    - What is the speed of the air just outside the window relative to the car, expressed in km/h? (Hint: the correct answer is not 105 km/h.)
    - Explain why this is so, qualitatively.
2. *Bernoulli near your faucet:* Water flows out of a kitchen faucet of 1.25 cm diameter at the rate 0.1 L/s. The bottom of the sink is 45 cm below the faucet outlet.
    - Will the cross-sectional area of the fluid stream increase, decrease, or remain constant between the faucet outlet and the bottom of the sink? Explain qualitatively.
    - Find the cross-sectional area of the stream as a function of distance y above the sink bottom, and plot it.
    - If a plate is held directly under the faucet, find the force required to hold the plate in a horizontal position as a function of y and plot it.