PhysH308

Central forces!

Ted Brzinski, Oct 31, 2024

Let's talk about 8.9

Why is Taylor asking to do this?

- The overall goal in mechanics, generally, is to **write the minimal set of *independent, solvable* equations** to describe the motion of the system.
- Lab frame: Lagrangian is a function of x_1, y_1, x_2, y_2, t
- **The promise of CM-frame: Lagrangian will depend only on $\overrightarrow{R}, r, \phi, t$**

The system:

Two particles of equal masses attached to each other by a light straight spring and free to slide over a frictionless horizontal table.

- Lab frame: https://trinket.io/glowscript/af356f7f73a8
- CM frame: https://trinket.io/glowscript/900e2bd00945
- (~circular orbit for m=1, k=1, L0=0.1, v=0.1)

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  $\vec{R}, \phi$ **are boring!**

  (Cons of momentum)

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Overall goal:

$$\mathscr{L} = \mathscr{L}(r, \dot{r}, t)$$

- **Part (a):** "Write down the Lagrangian in terms of the coordinates r_1 and r_2, and rewrite it in terms of the CM and relative positions, R and r, using polar coordinates *(r, O)* for r."
- ***Goal:*** *Write $\mathscr{L}$ in a form that includes the things we want to eliminate $(\vec{R}, \phi)$*
- **Part (b):** Write down and solve the Lagrange equations for the CM coordinates X, Y.
- ***Goal:*** *Show that conservation of momentum allows us to eliminate $\vec{R}$ by replacing $\dot{\vec{R}}$ with $\vec{v}_0^{cm}$.*

- **Part (c):** Write down the Lagrange equations for r and ϕ. Solve these for the two special cases that r remains constant and that ϕ remains constant. Describe the corresponding motions.
- ***Goal:*** *Use conservation of angular momentum ℓ_z to get an eqn of motion in terms of only r. Show that your answer matches the two special cases $\ell_z \to 0$ and $r \to r_0$.*

Kepplerian motion

Chaps 8.5-8

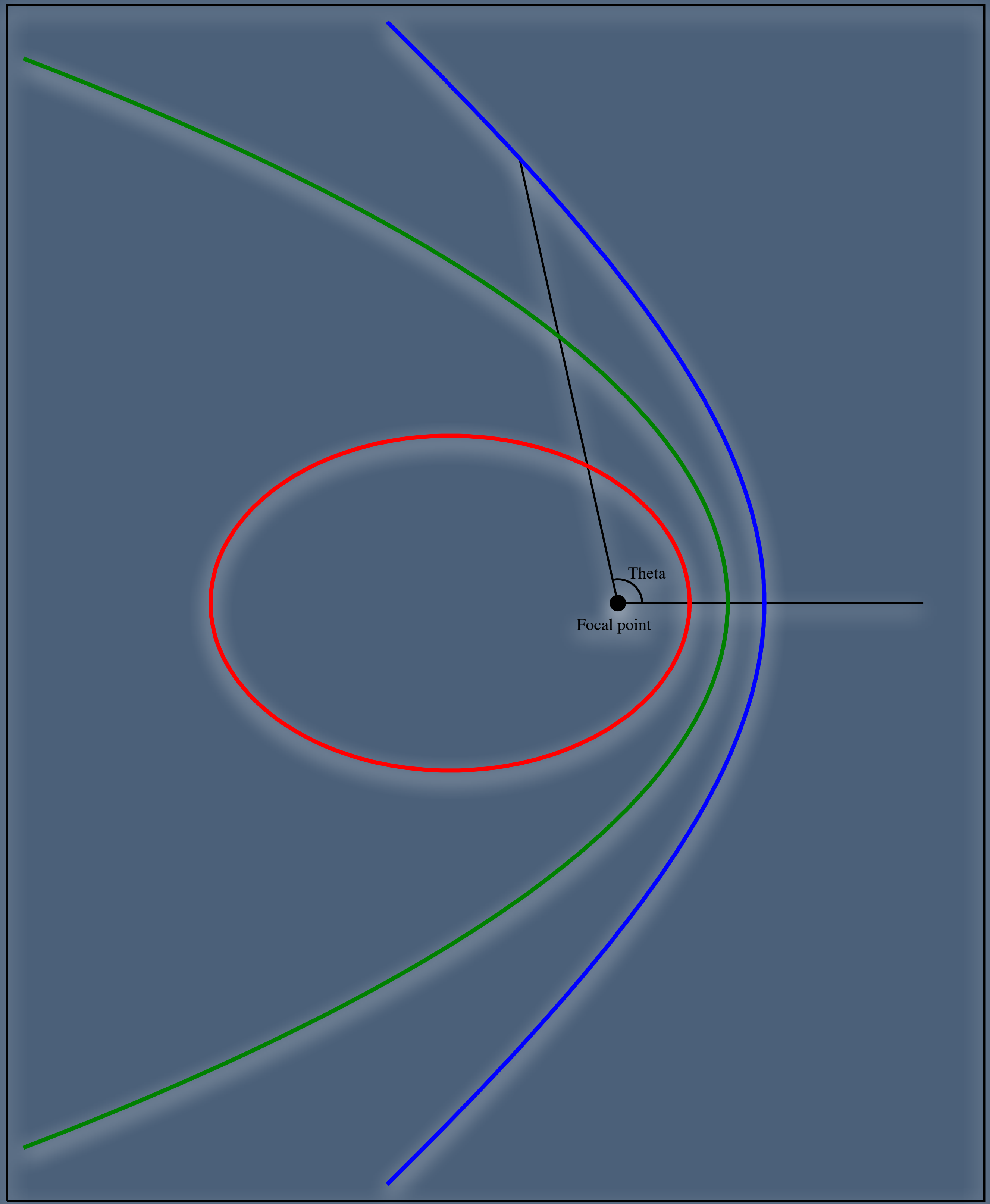
- From Newton's 2nd (pg 306):

$$u'' = -u - \frac{\mu}{\ell^2 u^2} F \text{ where}$$

$$u = u(\phi) = \frac{1}{r(\phi)} \text{ and}$$

F is any central force.

- For either gravity or Coulomb: $F = -\gamma u^2$



Kepplerian motion

Chaps 8.5-8

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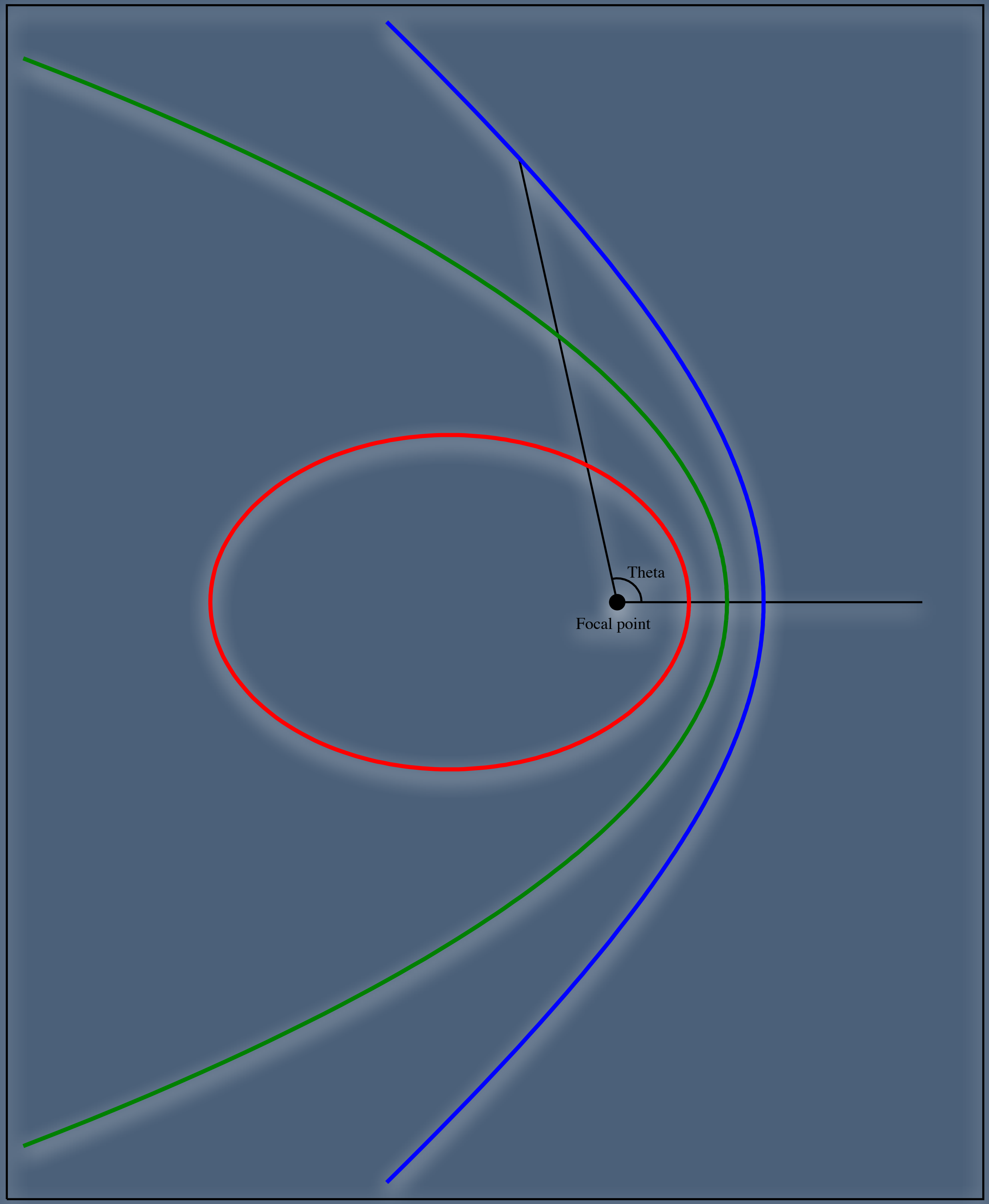
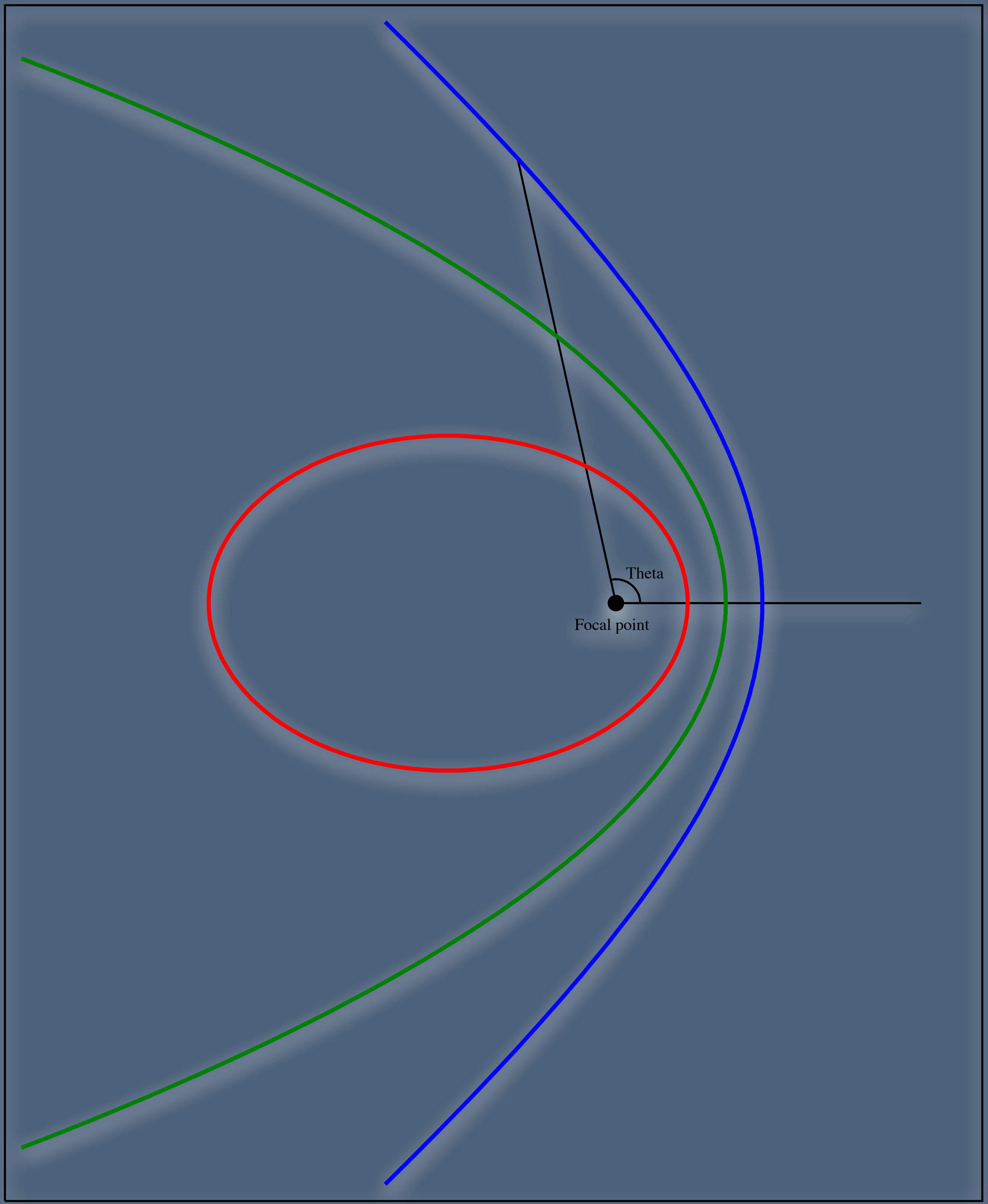


Kepplerian motion

Chaps 8.5-8

- $u'' = -u - \frac{\mu}{\ell^2 u^2} F$
- For either gravity or Coulomb: $F = -\gamma u^2$
- Only for I.S. forces, the u's in the last term cancel, giving: $u'' = -u + \boxed{\frac{\mu\gamma}{\ell^2}}$ Constant!
- This diff EQ reduces to

$$U'' = -U = -u + \frac{\mu\gamma}{\ell^2}$$



Kepplerian motion

Chaps 8.5-8

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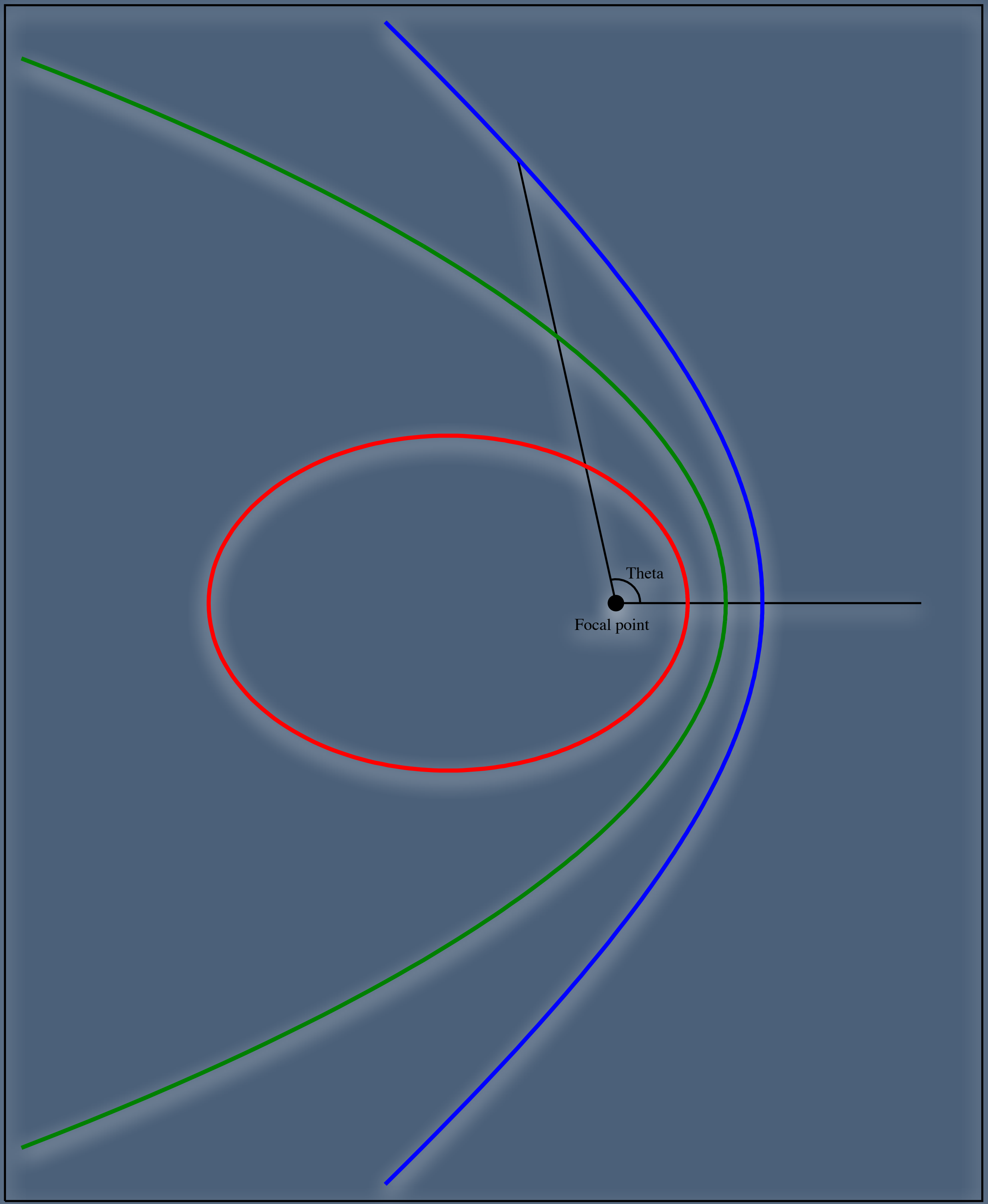


Kepplerian motion

Chaps 8.5-8

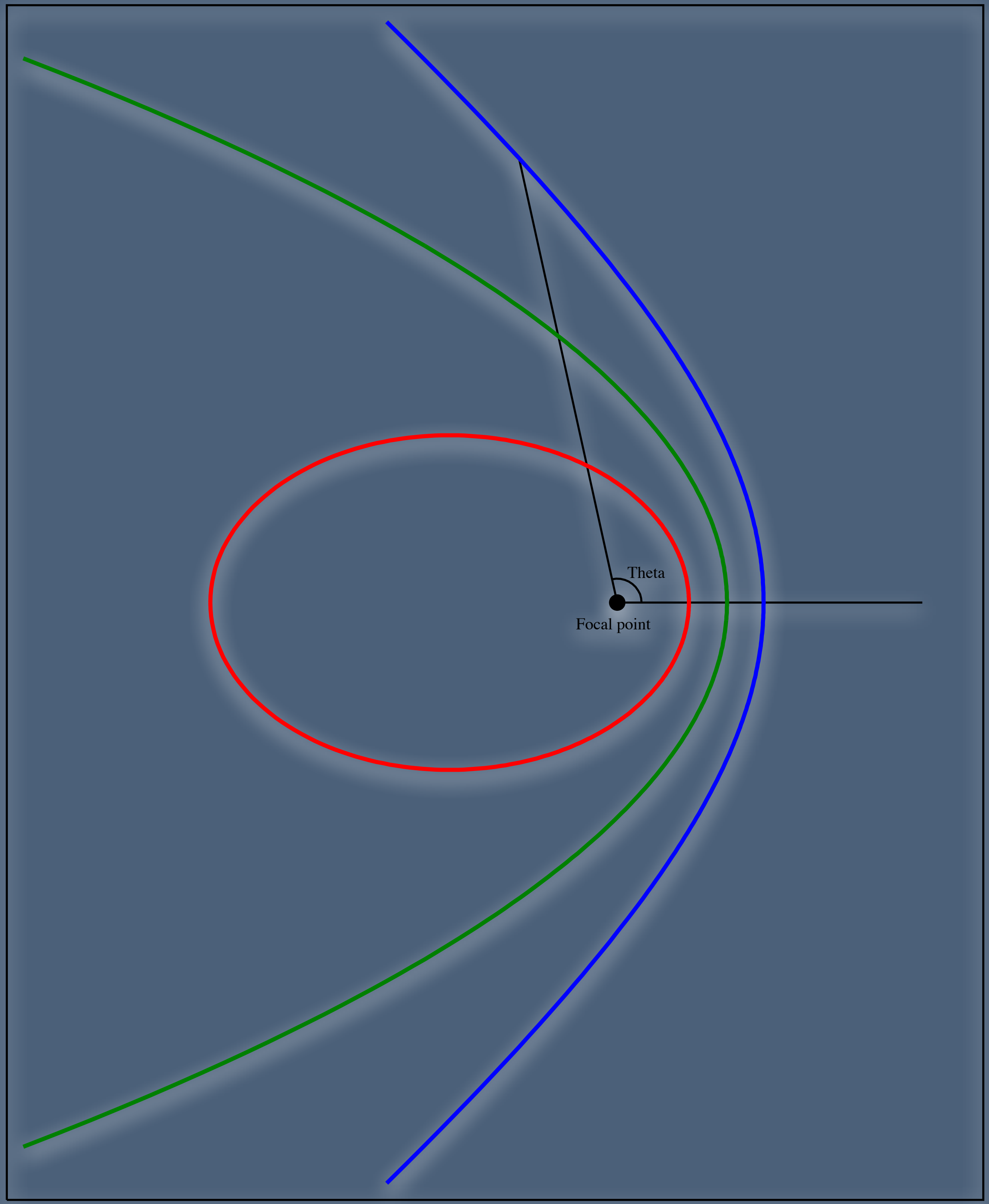
- $U'' = -U = -u + \frac{\mu\gamma}{\ell^2}$

- The general soln is $U = A\cos\left(\phi - \delta\right)$ so

$$u = A\cos\left(\phi - \delta\right) + \frac{\mu\gamma}{\ell^2}$$

- Defining our coordinates s.t. $\delta = 0$ we arrive at:

$$u = A\cos\left(\phi\right) + \frac{\mu\gamma}{\ell^2}$$



Kepplerian motion

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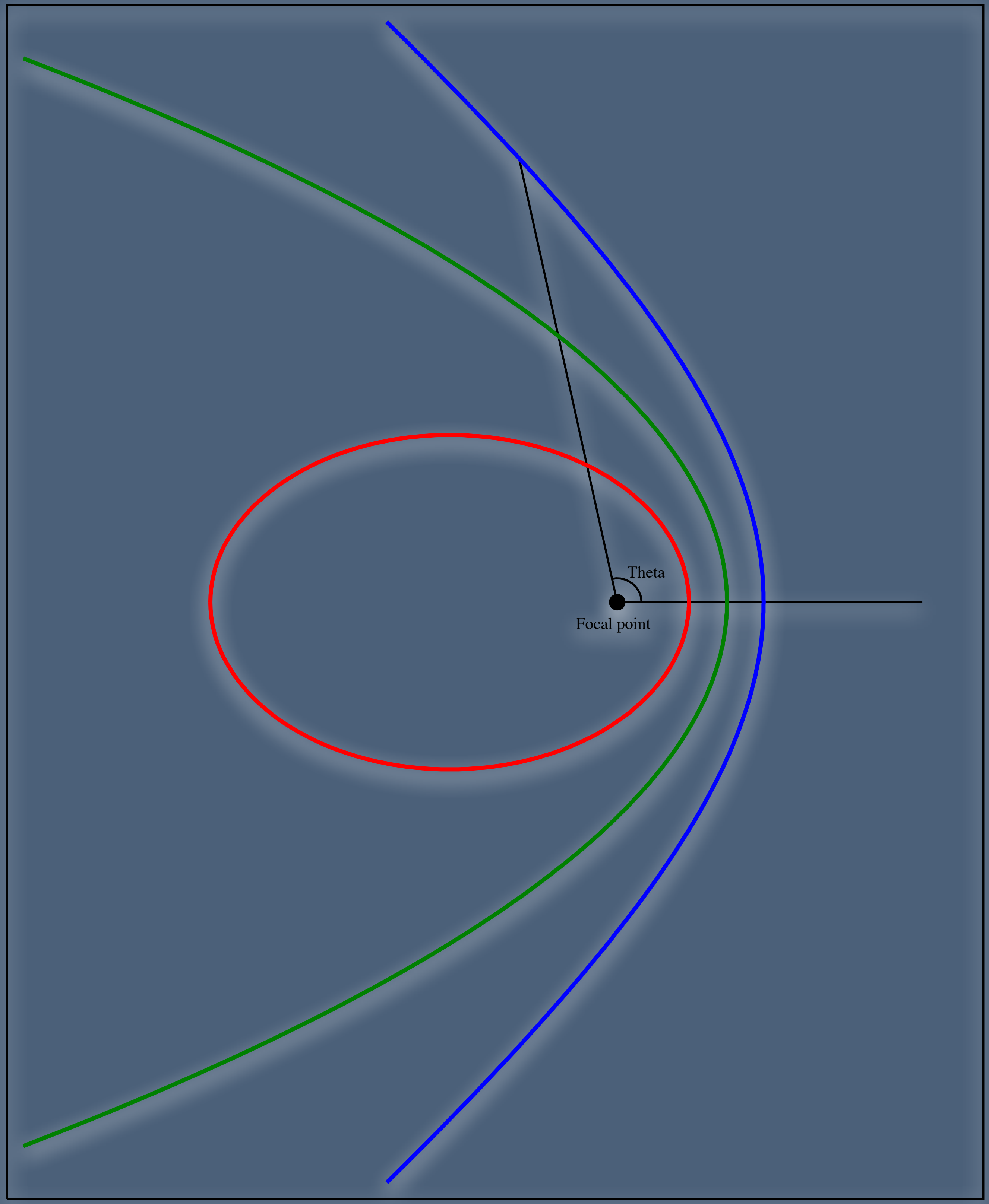
  $$u = \frac{\mu\gamma}{\ell^2}\left(1 + \epsilon\cos\phi\right)$$



Kepplerian motion

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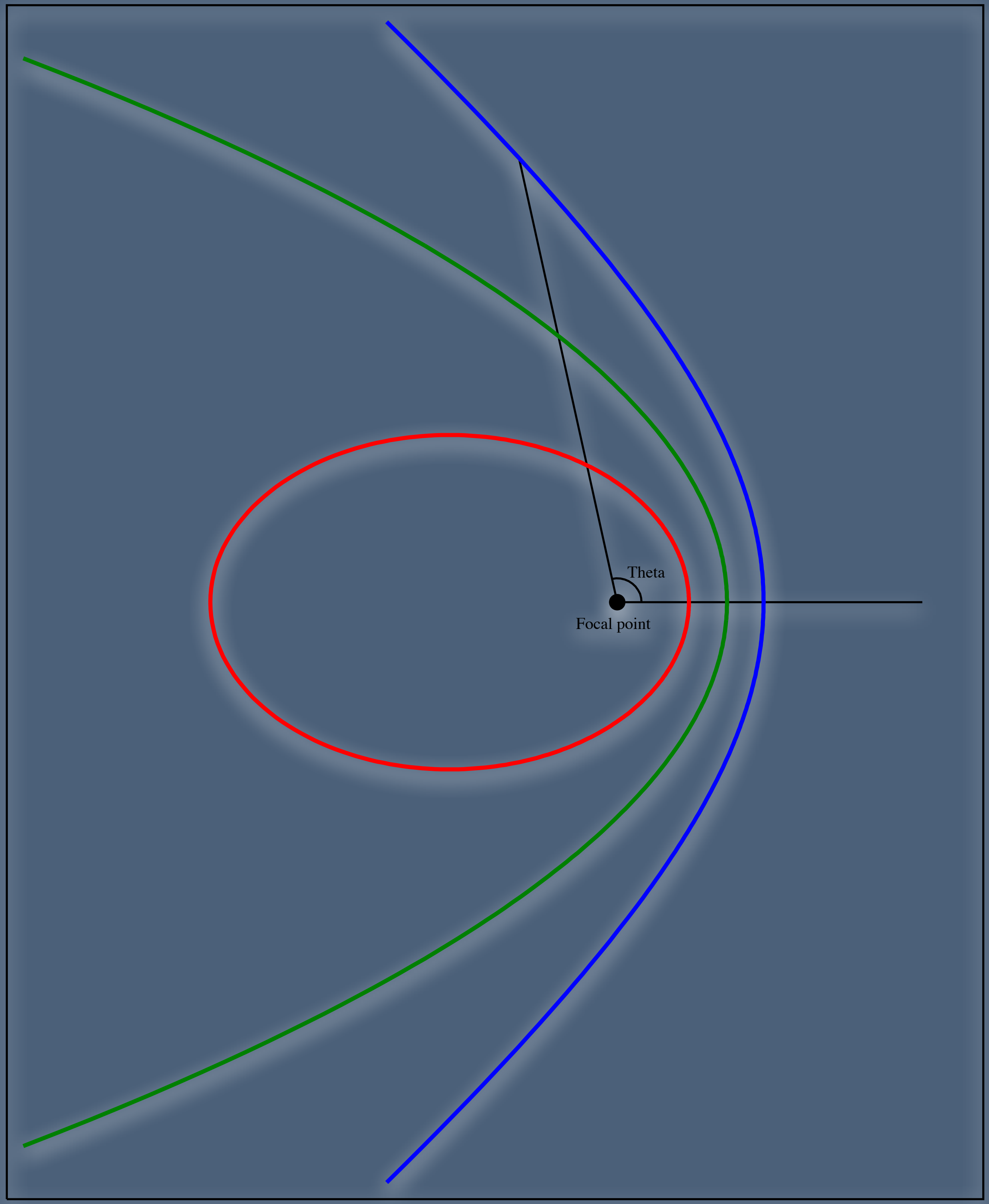
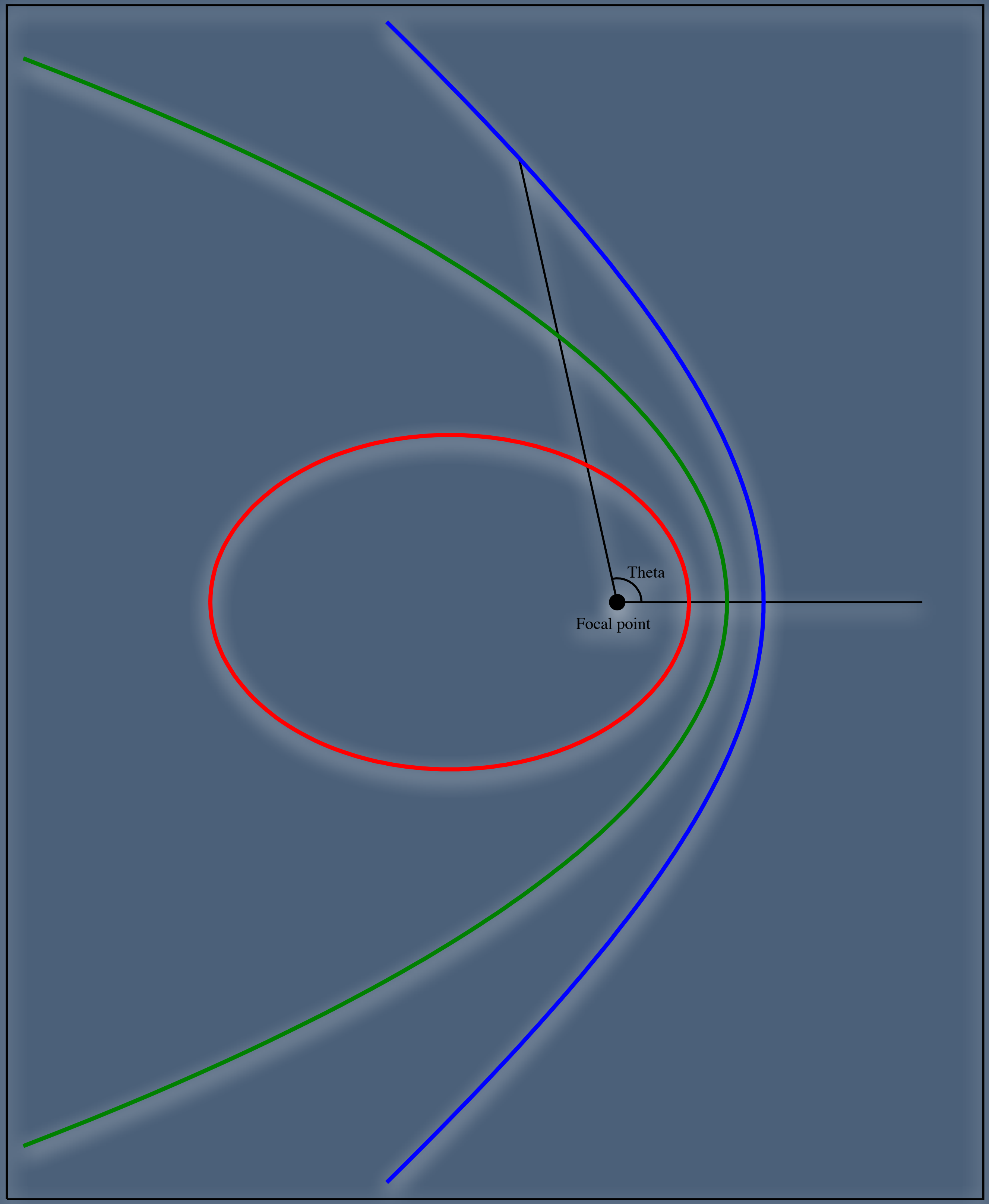


Kepplerian motion

Chaps 8.5-8

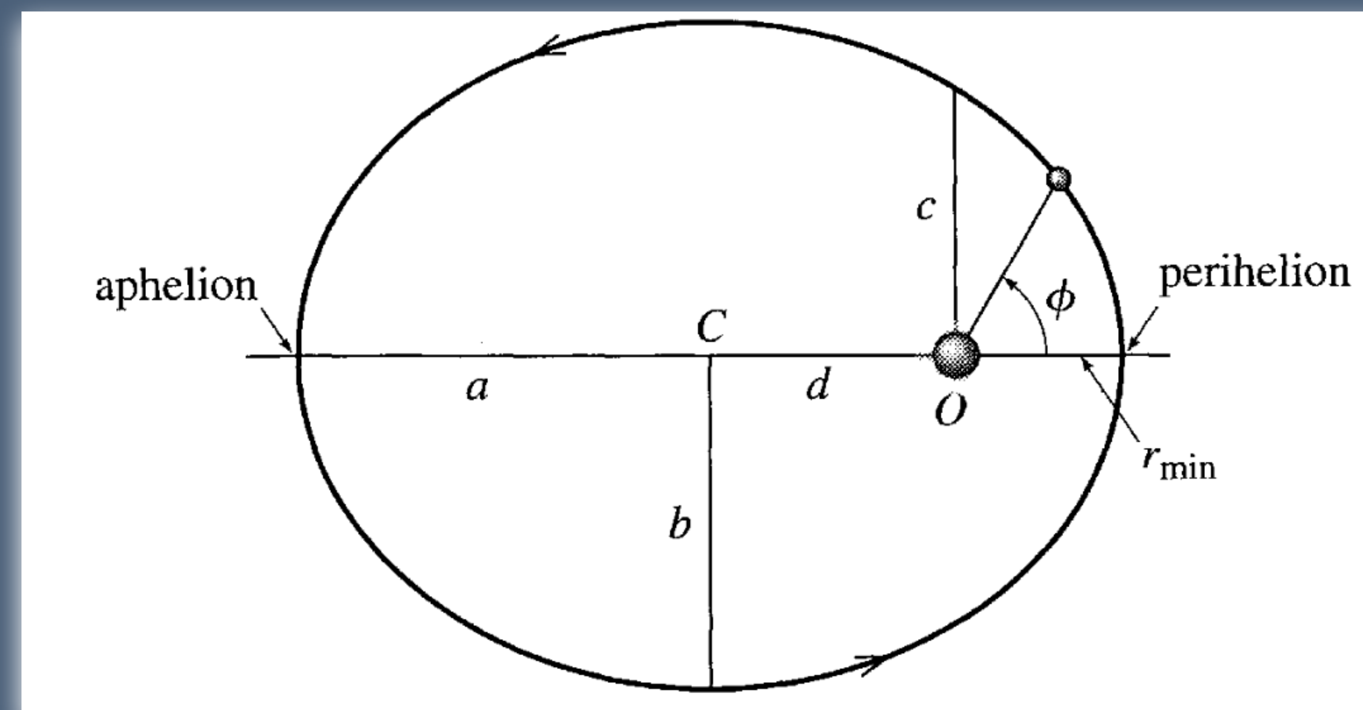
- $u = \frac{\mu\gamma}{\ell^2}\left(1 + \epsilon\cos\phi\right)$
- $r(\phi) = \frac{\ell^2}{\mu\gamma\left(1 + \epsilon\cos\phi\right)}$
- At $\epsilon = 1$, there is a single angle where $r \to \infty$. **Parabola!**
- At $\epsilon > 1$, there are a min/max angle a where $r \to \infty$. **Hyperbola!**
- $\epsilon < 1$, orbit is closed, finite. **Ellipse!**



Kepplerian motion

Chaps 8.5-6

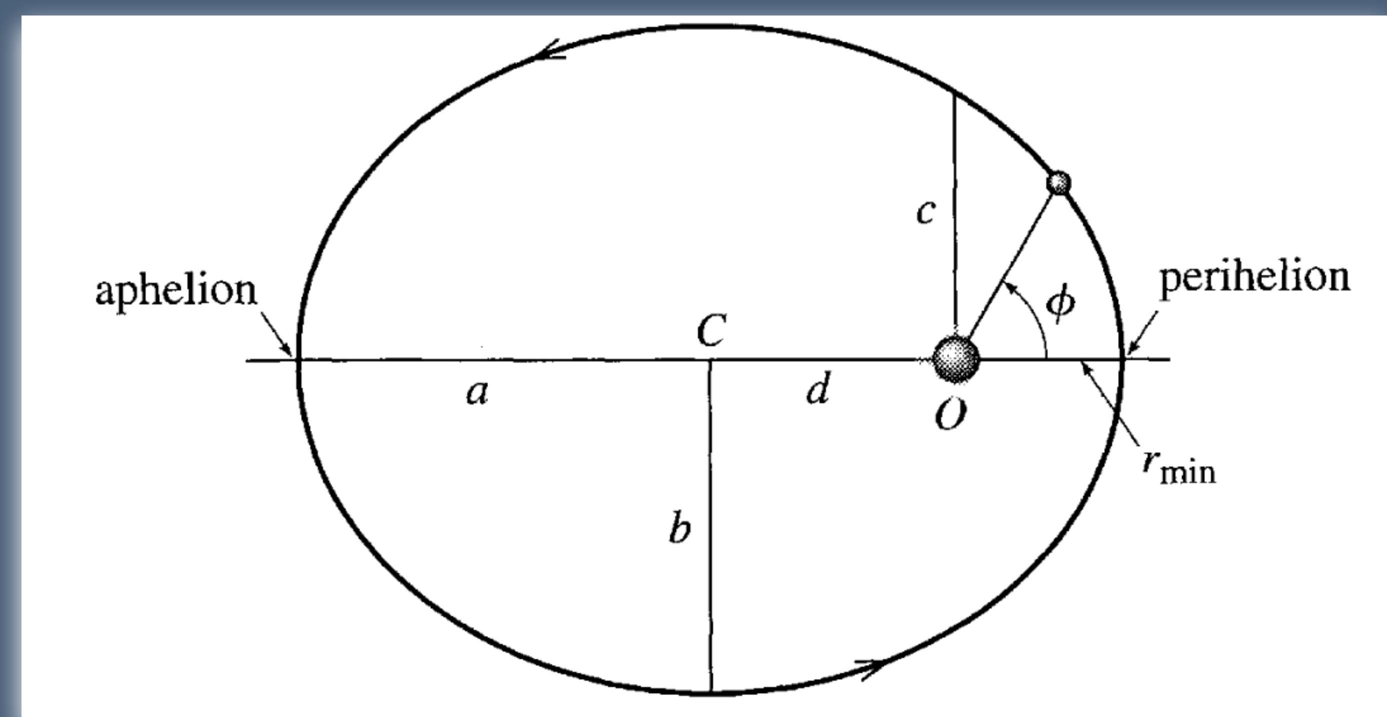
- $$r(\phi) = \frac{\ell^2}{\mu\gamma\left(1 + \epsilon\cos\phi\right)}$$
- $\epsilon < 1$, orbit is closed, finite. **Ellipse!** (K's 1st)



Kepplerian motion

Chaps 8.5-6

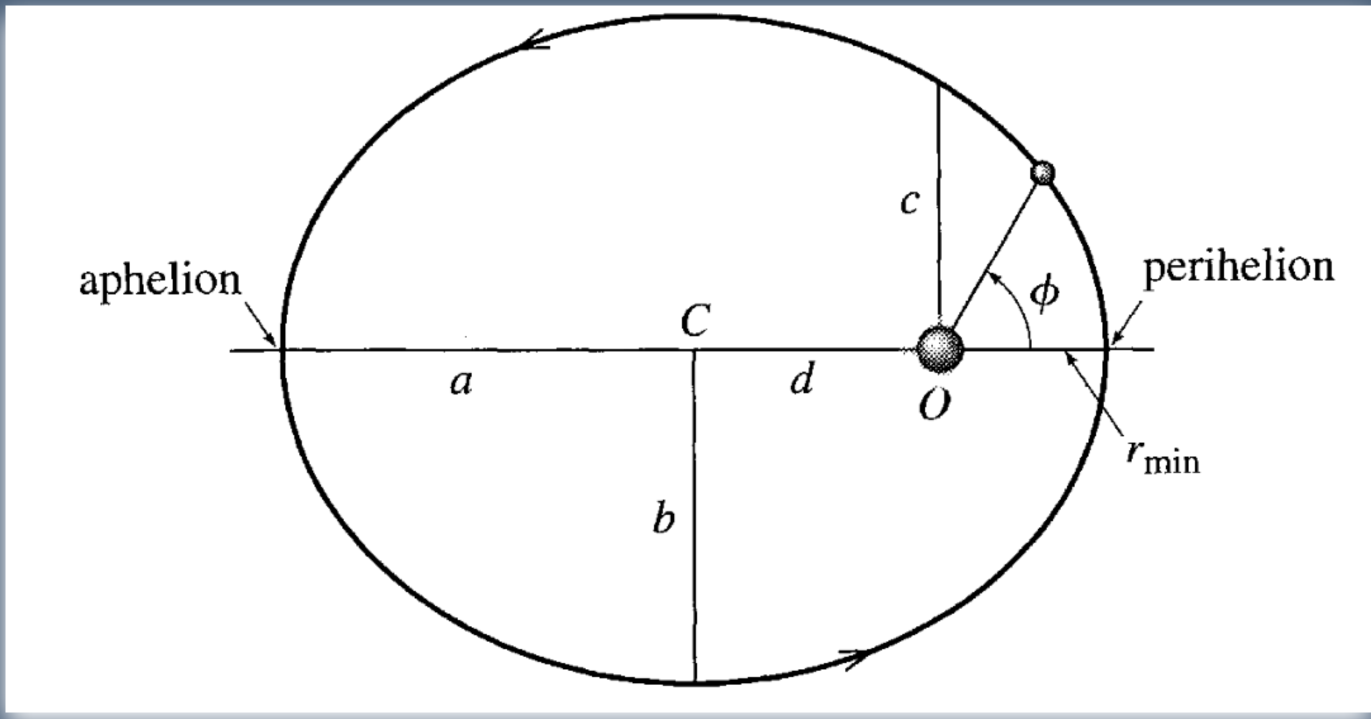
- $r(\phi) = \dfrac{c}{1 + \epsilon \cos \phi}$
- $\epsilon < 1$, orbit is closed, finite. **Ellipse!** (K's 1st)



Kepplerian motion

Chaps 8.5-6

- $r(\phi) = \dfrac{c}{1 + \epsilon \cos \phi}$
- $\epsilon < 1$, orbit is closed, finite. **Ellipse!** (K's 1st)
- Aphelion: $r = \dfrac{c}{1 - \epsilon}$
- Perihelion: $r = \dfrac{c}{1 + \epsilon}$



Kepplerian motion

Chaps 8.5-6

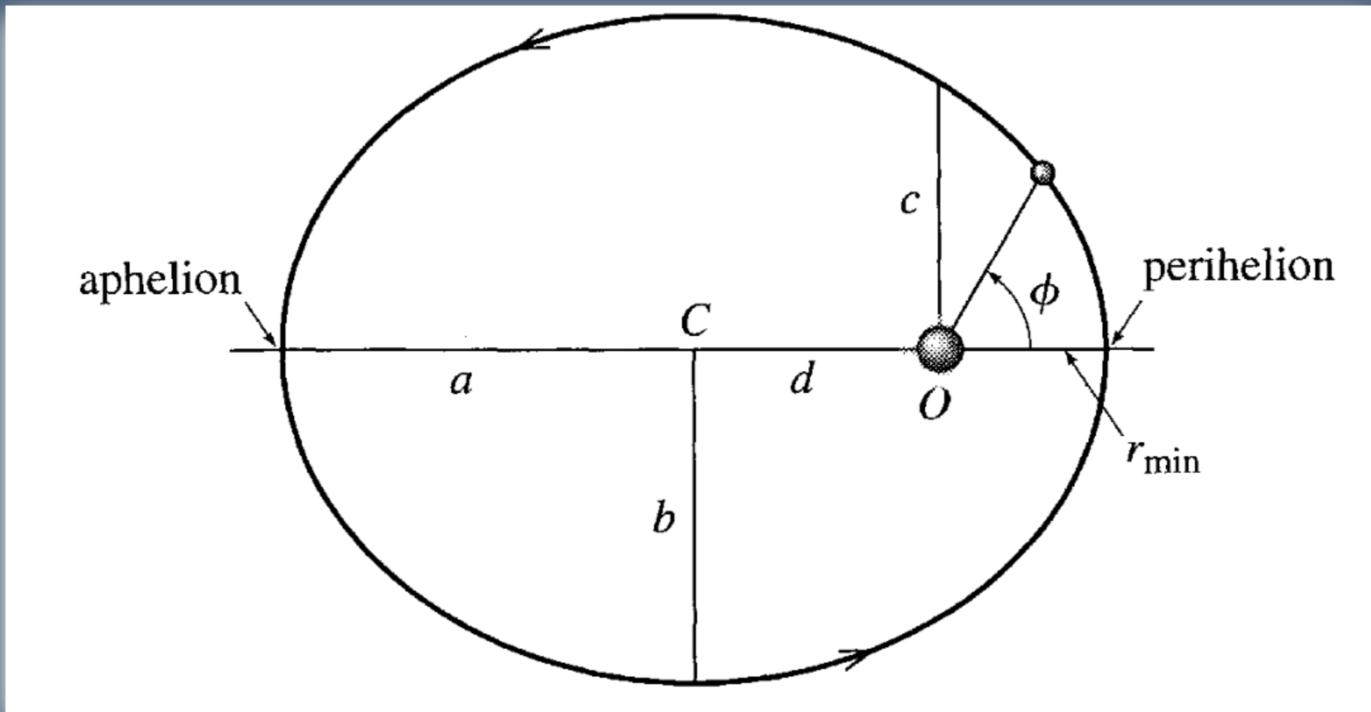
- Ellipse! (Keppler's first)

- 2nd law: Area swept by vector from sun to orbiting body is constant in time

$$\frac{dA}{dt} = \frac{\ell}{2\mu} \text{ (sect. 3.4)}$$

- 3rd: Equivalent to 2nd (period):

$$T = \sqrt{\frac{4\pi^2 \mu c^3}{\gamma\left(1-\epsilon^2\right)^3}} = \sqrt{\frac{4\pi^2 \mu a^3}{\gamma}}$$



Today's problem(s)

Stupidly eccentric vs falling through the Sun?

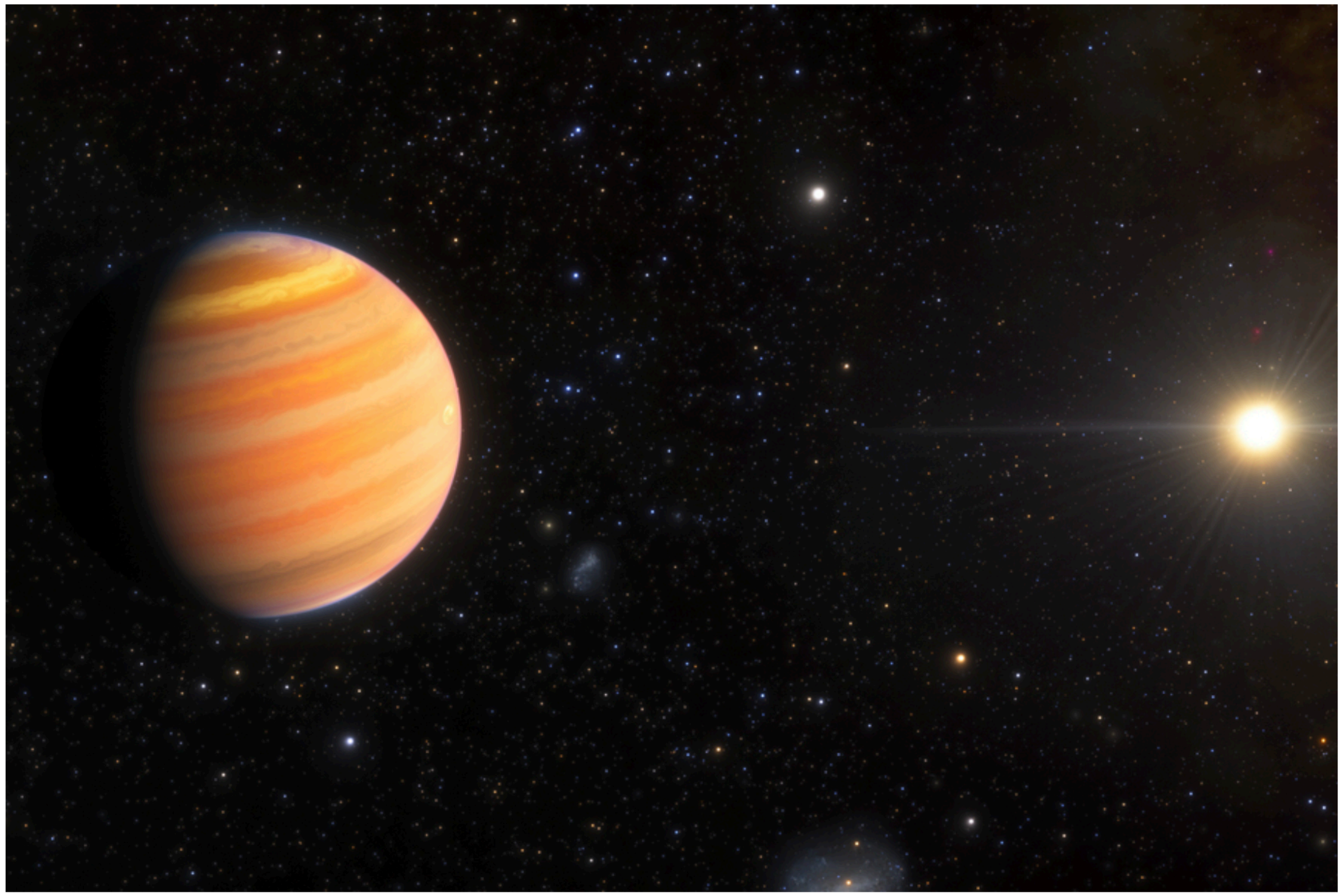
8.21

- **(a)** the limit that $\ell \to 0$, what is the shape of the orbit? (You need this to answer (b)!)
- **(b)** What is the period of the orbit for $\ell \to 0$?
- **(c-d)** What is the period for $\ell = 0$ (hint, it's different from (b))
- **(e)** Talk through why (d) and (b) are different! (Demonstrate physical reasoning)

Astronomers spot a highly "eccentric" planet on its way to becoming a hot Jupiter

The planet's wild orbit offers clues to how such large, hot planets take shape.

Jennifer Chu | MIT News
July 17, 2024



The new planet, which astronomers labeled TIC 241249530 b, orbits a star that is about 1,100 light-years from Earth. The planet circles its star in a highly "eccentric" orbit, meaning that it comes extremely close to the star before slinging far out, then doubling back, in a narrow, elliptical circuit. If the planet was part of our solar system, it would come 10 times closer to the sun than Mercury, before hurtling out, just past Earth, then back around. By the scientists' estimates, the planet's stretched-out orbit has the highest eccentricity of any planet detected to date.