PhysH308

Exam and Calculus of Variations

Ted Brzinski, Oct 1, 2024

Exam

Immediate reflections

- Too many were short on time — I need to ask shorter questions
- Taking the full exam in one sitting will quickly become untenable if people are having to retake this many problems
- Many forgot to consider the learning standard as a hint (e.g. treating Q1b as a static system, solving C1 quantitatively).
- *Careful* schematics and free body diagrams correlated heavily with correct answers
- String length as a constraint condition was really challenging for many. Echoed difficulties I've observed with constrained motion in class (falling off a sphere, circular W-E problem, etc).
- Many introduced angles in their work and answered in terms of these angles rather than known quantities.
- Quantitative problems especially must be more focused - all/nothing grading is especially unforgiving in multi-part questions.
- Asking about familiar concepts in an unfamiliar context was successful in revealing understanding gaps, but students aren't going to be happy about it.

Exam

The data

- **Outcome:** The average was close to 50%.
  Goal: I want us to average 85% or more.
  Response: Retune exam difficulty (next slide), offer/suggest practice problems and study topics, adjust HW on exam weeks (amount and deadlines).

- **Outcome:** Approximately the same # completed 0, 1, 2/3 standards, just over half as many got all 3. C1 was most completed, then Q1a a close second. Q1b was distant 3rd.
  Goal: I hope to see C and R goals completed at only slightly higher rates. I want the norm to be 2-3 successful standards per exam.
  Response: Retune especially Q problems for difficulty. Aim for shorter problems (especially C).

  It seems that dynamics in Atwood machines are more unfamiliar than expected (were these covered in 101/105?) These problems will appear again in Lagrangian Mech (and constrained motion in general is an important concept), so we'll do some review of the NF solns alongside those LF problems.

- **Outcome:** Many submissions were incomplete or explicitly bemoaned the time limitations
  Goal: I want you to have plenty of time, and not to have an exam eat your week, especially as we move forward!
  Response: I am rethinking deadlines (rolling?) and rebalancing problems (next slide). Might eliminate time limits (none?) Stay tuned!

Next exam

- Aiming for Friday (probably Monday) release (we'll be 1 week behind schedule).
- Questions will not be multi-part
- I will aim for ~1/2 the time per problem.
- More questions will help students understand if they answered correctly (e.g. 'show that [xxxx] is [yyyy]" problems rather than "find [xxx]" problems).
- *I have a few further ideas I'd like your feedback on...*

Some ideas to discuss (not ready to implement)

- No due dates, but only publish follow-ups 1 week after a “not yet” submission.
- Time limit or not?
- Preview exam or not?
- Add a “revise and resubmit” grade?

The exam

Conceptual

C1 This statement is false: A ball is bounced upward off the ground. Immediately after the bounce, the ball has speed v_0, and experiences (in addition to gravity) a drag force $F_d = -m\alpha f(v)$, where $f(v)$ is a finite, positive, monotonically increasing function of v with units of velocity and no dependence on any other variables in the problem. The velocity immediately before the ball hits the ground again is described by the formula $v_f = v_0\left(\ln\left(\frac{g+\alpha v_0}{\alpha v_0}\right) - 1\right)$.

Using what you know about Newtonian physics, explain how you might have realized that the statement false. You may find the specific error in the solution or make arguments based on the physical implications of the result (limiting cases/special cases, violations of physical principles/laws, etc.).

The goal was for you to identify ways in which the final velocity is unphysical for an unknown drag force. **Possible answers:**

Limit as $\alpha \to 0, \infty$

Can $v_f \geq v_0 \Big|_{\alpha \neq 0}$?

Sign change at finite non-zero v_0?

Common mistakes:

- Drag needn't be either linear or quadratic
- Talking about time dependence
- Trying to solve the problem — you can't without knowing $f(v)$.

The exam

Quant 1 a

- Tension in a static system balances forces at either end (not the sum). For both (a) and (b), $F = T$
- You don't need an angle, but it can help. Just make sure you define it in terms of F, g, L, m.
- (d) is prompting you to check your answer against your physical intuition.

Q1a: Statics, a tense balancing act.

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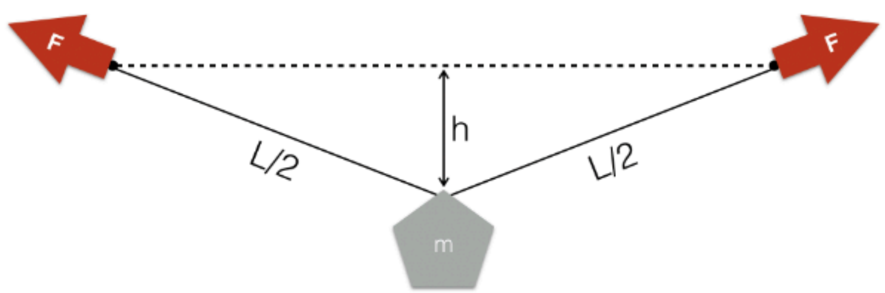
(a) Two people stand at opposite ends of a massless rope of length L. Each pulls on the rope with a force F. This scenario is depicted in the figure above. What is the tension in the rope?



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A mass m is fixed to the rope halfway between the people. Assume both people continue holding their ropes at the same height, and continue to pull with force F. (Note, the people always pull the rope in a direction parallel to the tension, i.e. F is the only force they apply to the rope). This scenario is depicted in the figure above.

(b) What is the tension in the rope after the mass is added?

(c) At what separation ℓ must the people stand in order to hold the mass static and at What distance h below the original height of the rope does the mass hang?

(d) Explain the behavior of your answers to 2.3 in the limits of:

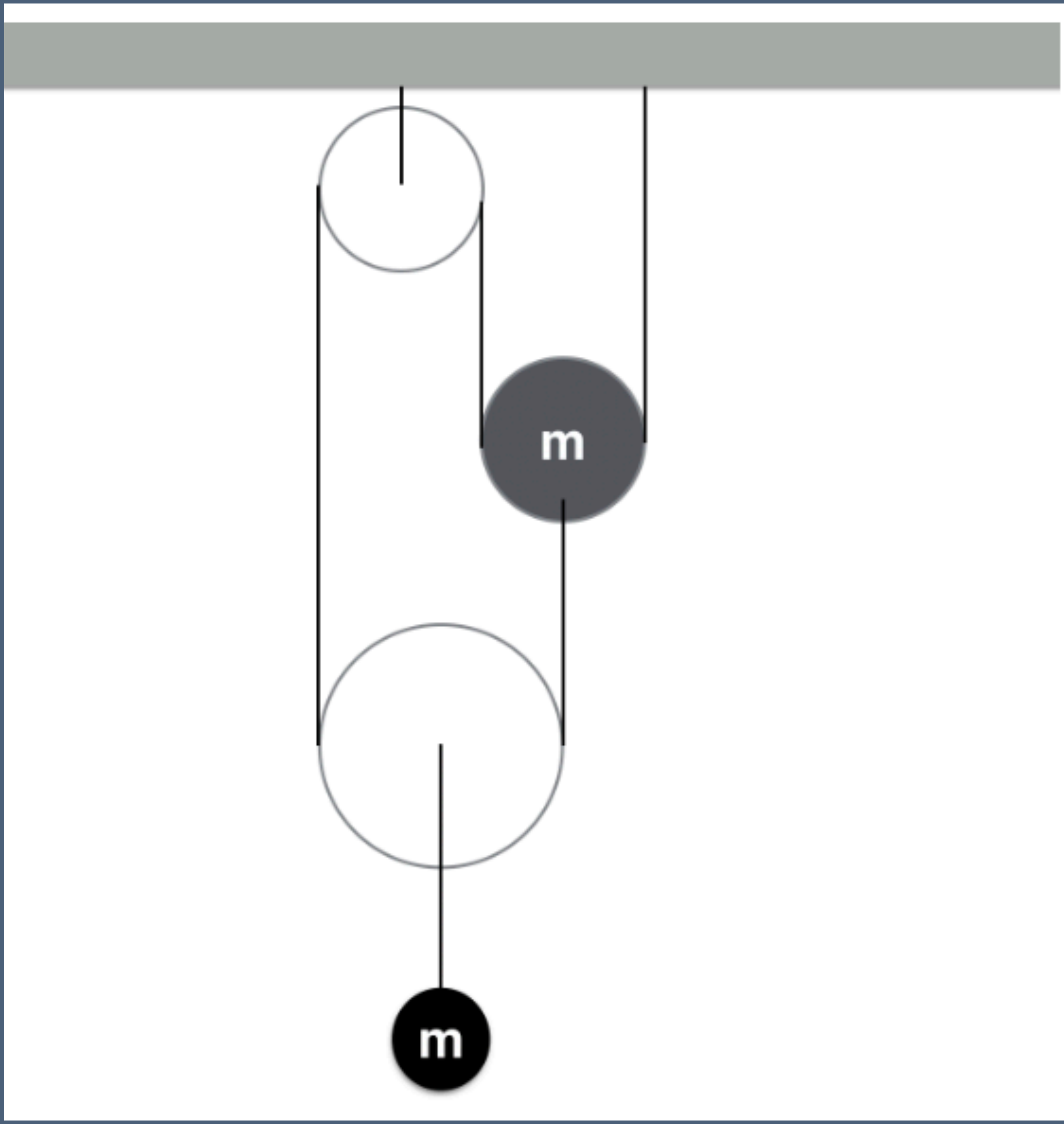
i. $\frac{mg}{2F} \to 0$

ii. $\frac{mg}{2F} \to 1$

The exam

Quant 1b

- Apply a constraint
- Identify body forces → find eqs of motion
- Solve the system of equations!



Calculus of variations

(This is weird, but awesome)

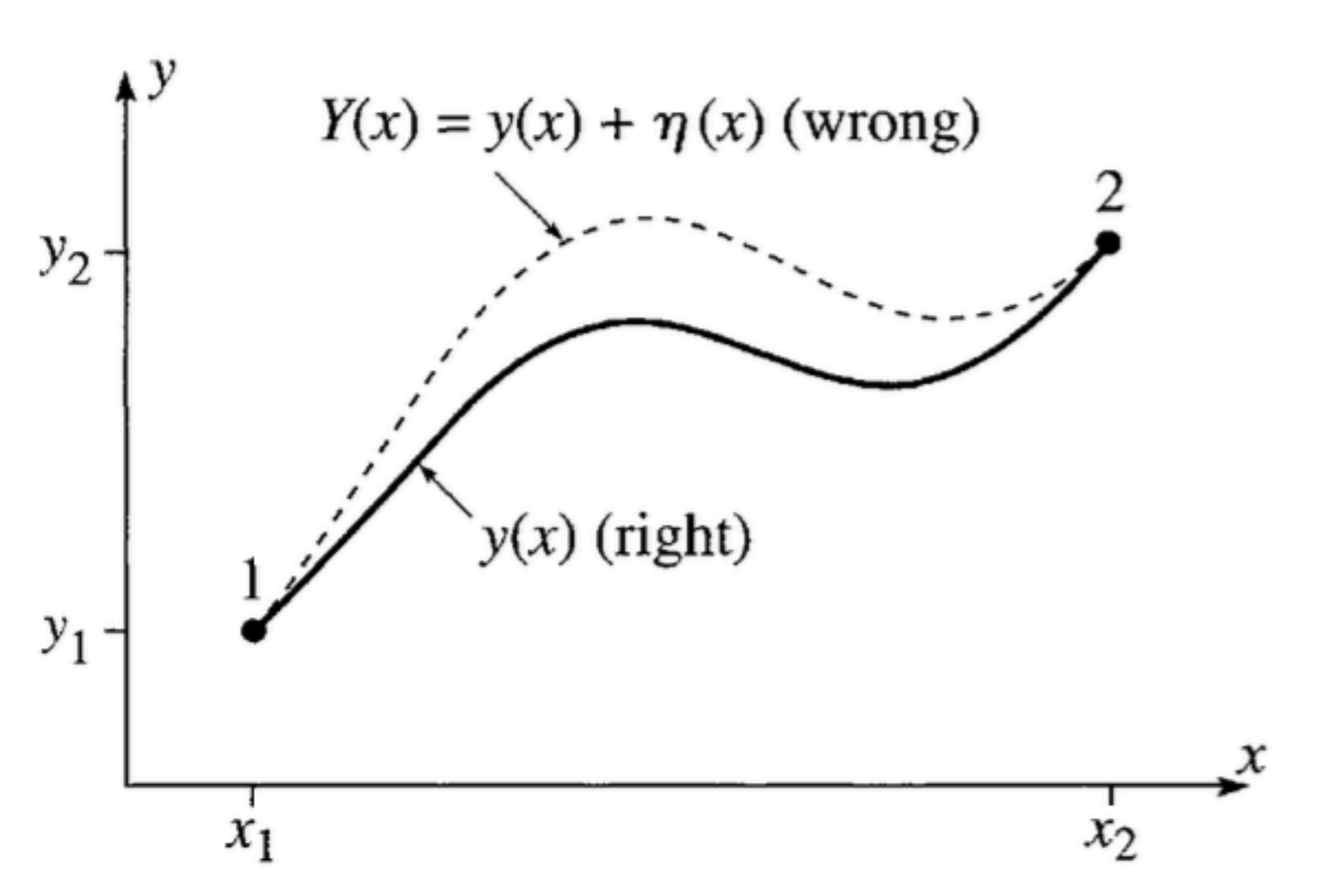
- **The goal:** find a function for a path $y\left(\vec{x}\right)$ that minimizes some quantity S (e.g., the shortest path, the quickest path, the minimum energy path, etc):
$$S = \int_{\vec{x}_1}^{\vec{x}_2} f\left(y\left(\vec{x}\right), y'\left(\vec{x}\right), \vec{x}\right) d\vec{x}$$

- **The strategy:** consider a path $Y\left(\alpha, \vec{x}\right)$ to be the sum of the ideal path $y\left(\vec{x}\right)$ and the deviation from the ideal path $\alpha\eta\left(\vec{x}\right)$.

- **The ideal solution:** $y\left(\vec{x}\right)$ is the choice of $Y\left(\alpha, \vec{x}\right)$ which minimizes S wrt α

As derived in Ch. 6.2, **this condition is equivalent to the E-L Equation:**

$$\frac{\partial f}{\partial y} = \frac{d}{d\vec{x}}\frac{\partial f}{\partial y'}$$



Problems

(If we have time!)

- In the book, the example is given of showing that a line is the shortest path between two points.
- **In 6.1**, you'll find the integral for the length along a path on a sphere
- **In 6.16**, you'll find the *shortest* path on a sphere using E-L.