PhysH308

Spinning *with* things!

Ted Brzinski, Nov. 7, 2024

Pre-registration ends this week!

There are a lot of upper-level options available this Spring!

Physics courses:

1. PHYSH302: Advanced Quantum, with Walter Smith
2. Bryn Mawr is offering PHYSB309 Advanced E&M
3. PHYSH304: Computational Physics with our new faculty member Vijay Singh (whose research is in computational biological physics).
4. PHYSH353: Topics in Soft Matter Physics, a special topics course with visiting faculty member Vianney Gimenez-Pinto.
5. Bryn Mawr is offering PHYSB331: Advanced Experimental Physics

Not listed as physics, but physics, Clyde Daly in Chemistry is teaching CHEMH350: Topics in Computational Chemistry (time TBD), which can use PHYSH214 as a prereq.

Astrophysics/astronomy courses:

6. ASTR344: Topics in Astrophysics: Gravitational Waves with Andrea Lommen.

Problem 10.37

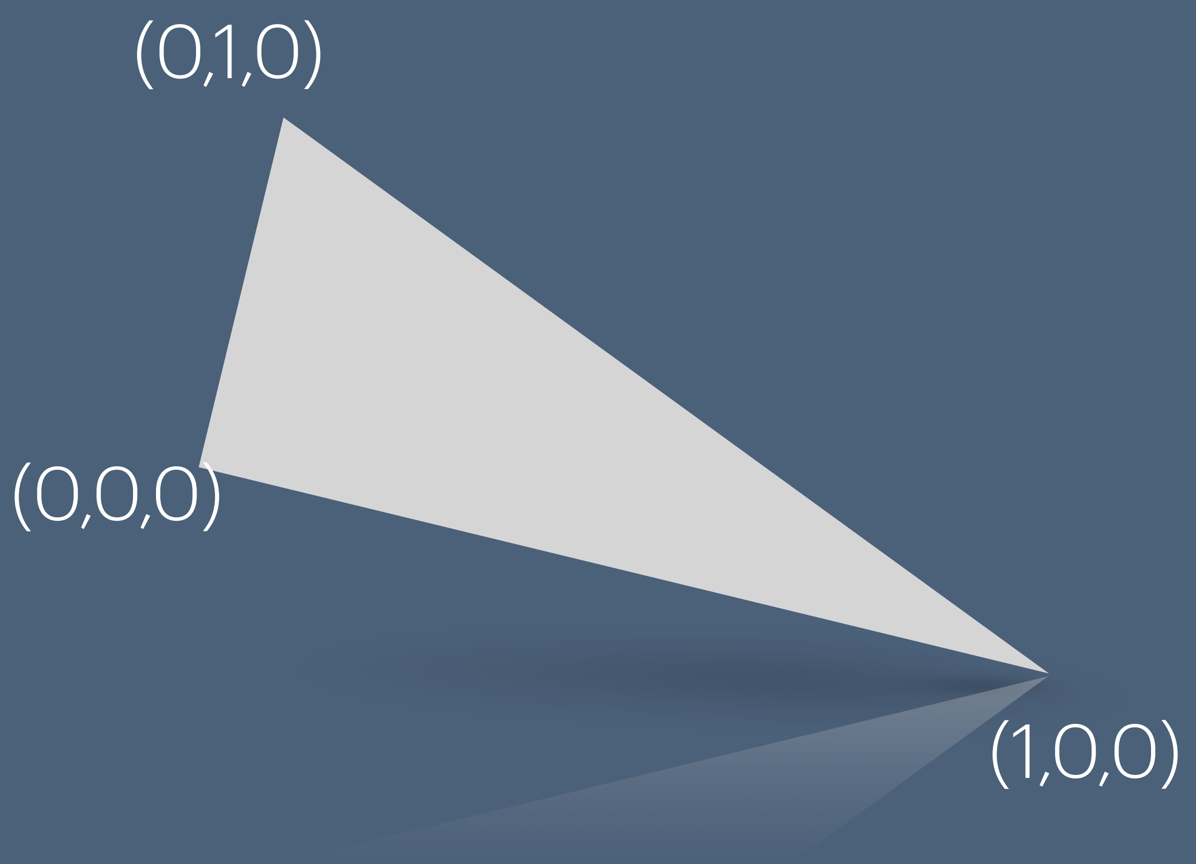
(From last week)

- Given a shape, find $\mathbf{I}$, diagonalize it, and find the principal axes

  See that from $\mathbf{I}$ in any coordinates you can find the principal axes

$$I_{xx} = m \sum \left(y^2 + z^2\right)$$

$$I_{xy} = I_{yx} = -m \sum xy$$



Why find principle axes?

- Diagonal matrices are easier to deal with!

- For rotation about these axes, $\overrightarrow{L} \parallel \overrightarrow{\omega}$ (no torque required to maintain $\hat{\omega}$) - this tells us about the (rotational translation) symmetries of the object!

Motion of a top

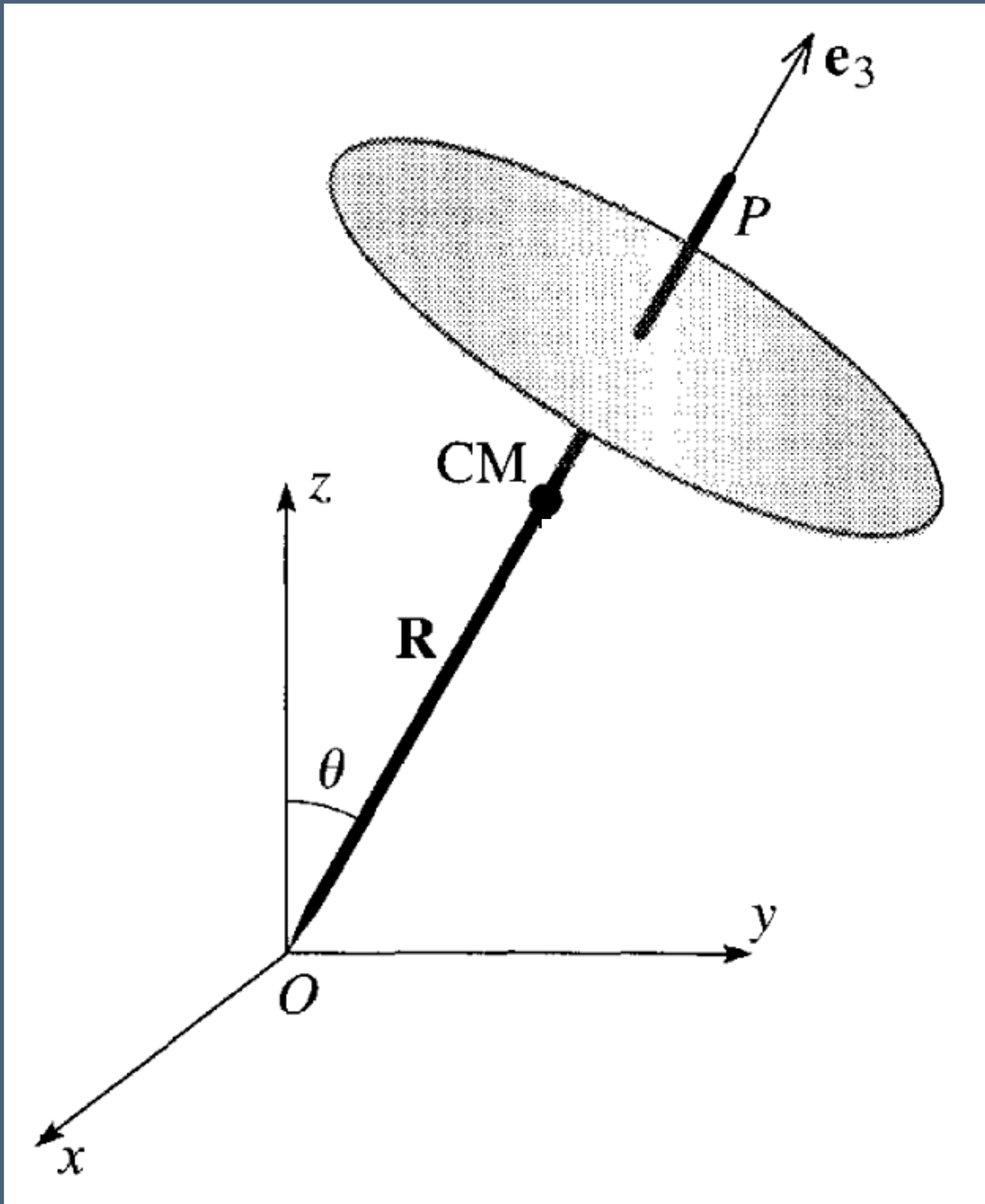
(First, without gravity)

Consider rotation of an axially symetric about an axis of symmetry $\hat{e}_3$ through the center of mass:

$$\mathbf{I} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_1 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \text{ and } \overrightarrow{\omega} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}$$

So $\overrightarrow{L} = \mathbf{I}\overrightarrow{\omega} = \lambda_3 \omega \hat{e}_3$

Now let's add gravity!



Motion of a top

Now let's add gravity!

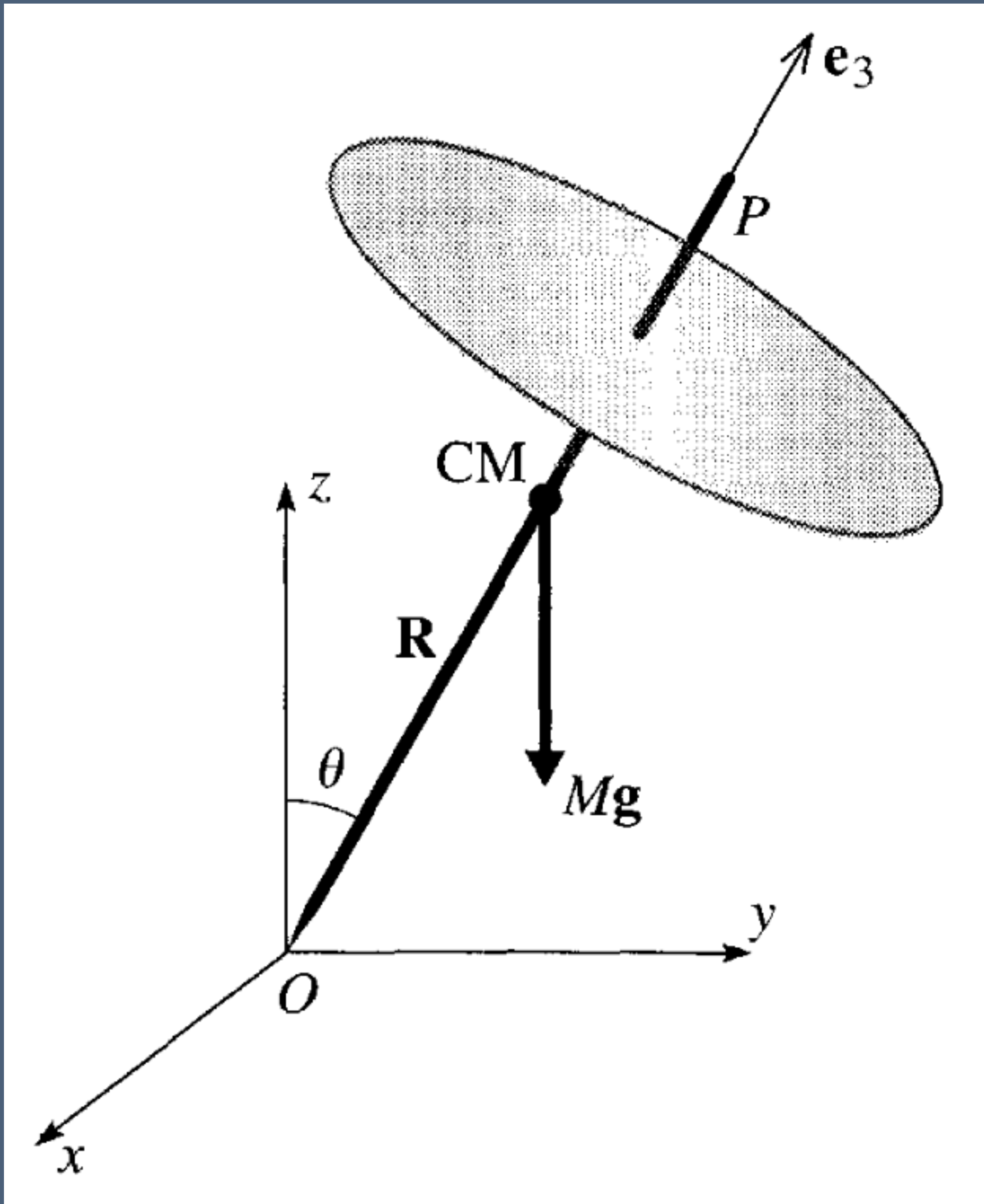
Gravity applies a force at the center of mass: $F_{cm} = -Mg\hat{z}$

The result is a torque:
$\vec{\Gamma} = \vec{F}_{cm} \times \vec{R}_{cm} = MgR_{cm}\hat{e}_3 \times \hat{z}$

Recall $\vec{\Gamma} = \dot{\vec{L}} = \lambda_3\omega\dot{\hat{e}}_3$

So $\dot{\hat{e}}_3 = \dfrac{MgR_{cm}}{\lambda_3\omega}\hat{e}_3 \times \hat{z}$

Procession about $\hat{z}$ with rate $\vec{\Omega}$!



Motion of a top

Now let's add gravity!

Now let's add gravity!

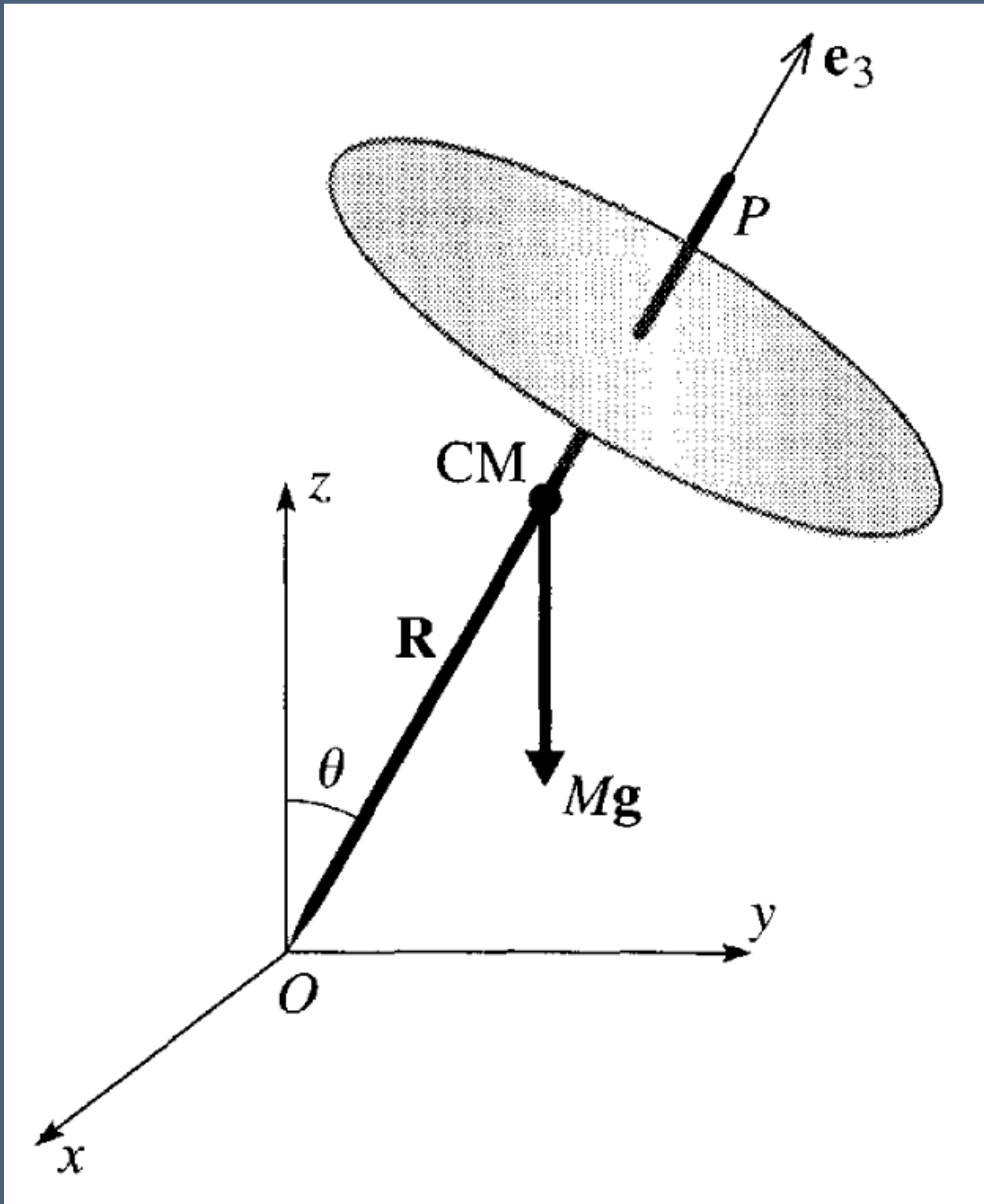
Gravity applies a force at the center of mass: $F_{cm} = -Mg\hat{z}$

The result is a torque:
$\overrightarrow{\Gamma} = \overrightarrow{F}_{cm} \times \overrightarrow{R}_{cm} = MgR_{cm}\hat{e}_3 \times \hat{z}$

Recall $\overrightarrow{\Gamma} = \dot{\overrightarrow{L}} = \lambda_3 \omega \dot{\hat{e}}_3$

So $\dot{\hat{e}}_3 = \dfrac{MgR_{cm}}{\lambda_3 \omega}\hat{e}_3 \times \hat{z} = \hat{e}_3 \times \overrightarrow{\Omega}$

Procession about $\hat{z}$ with rate $\overrightarrow{\Omega}$!



Motion of a top

Now let's add gravity!

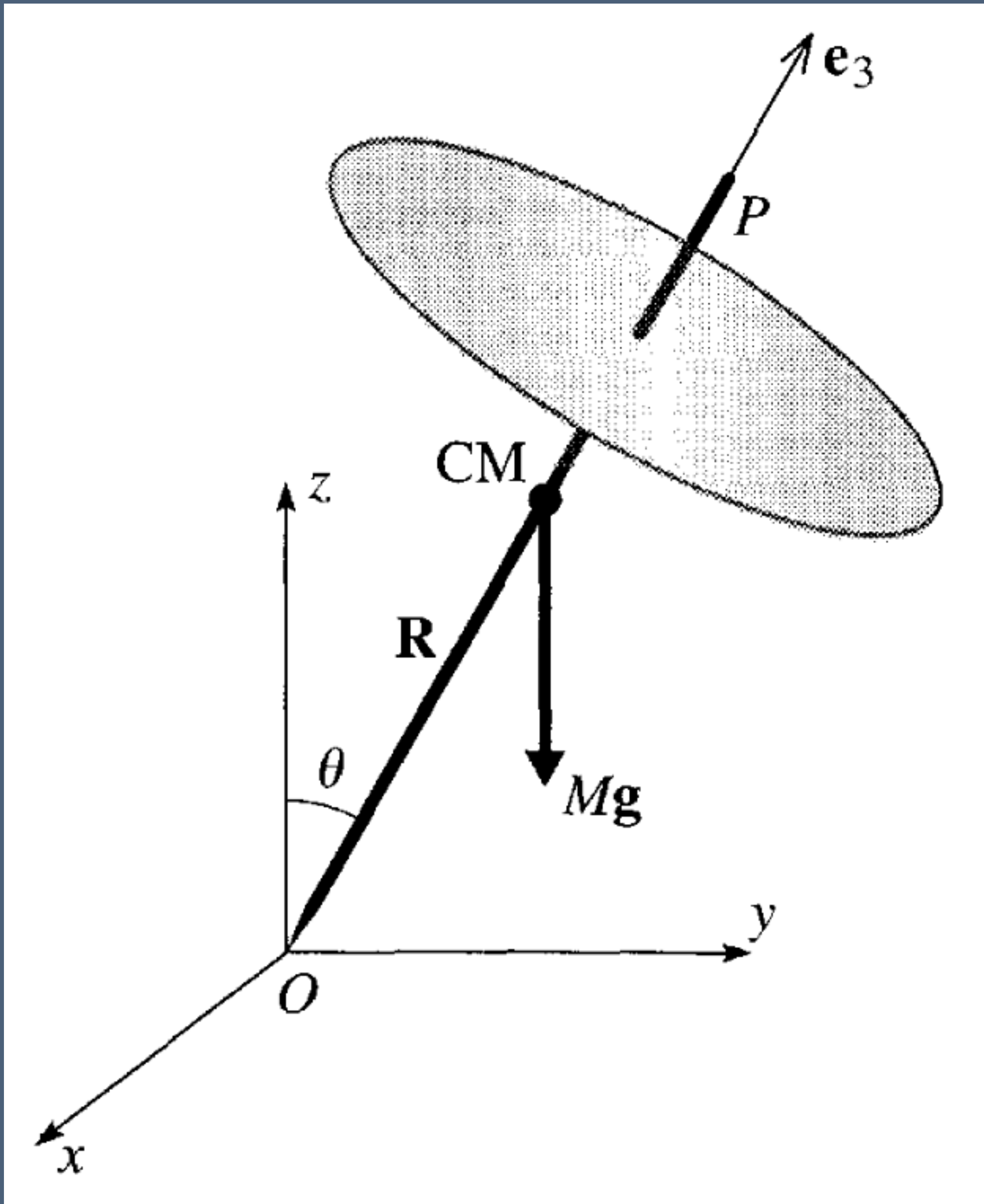
$$\dot{\hat{e}}_3 = \frac{MgR_{cm}}{\lambda_3 \omega} \hat{e}_3 \times \hat{z} = \hat{e}_3 \times \vec{\Omega}$$

Procession about $\hat{z}$ with rate $\vec{\Omega}$!

You also can get small oscillations in $\hat{\theta}$ *(nutation)* which we'll explore Thursday

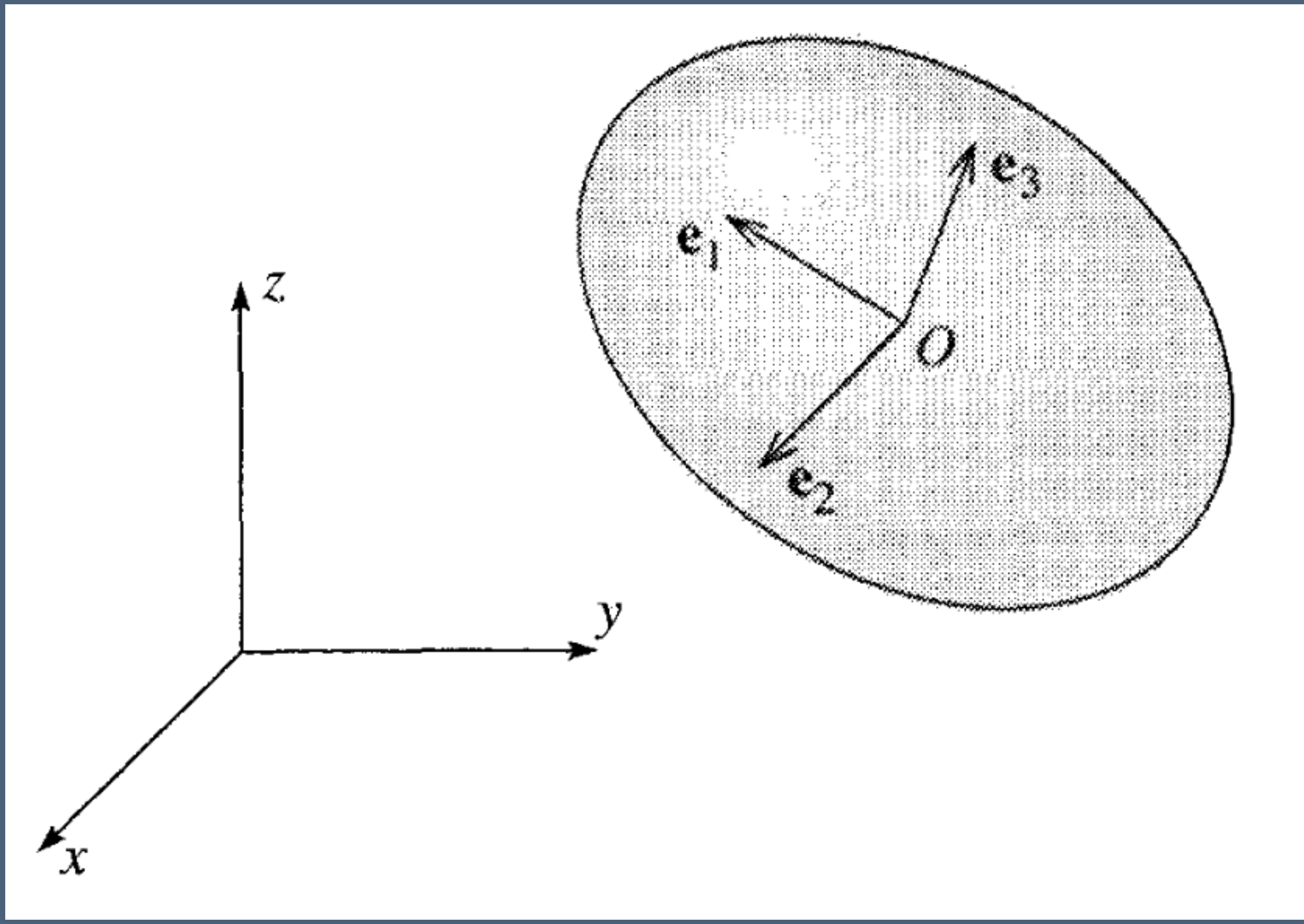
[Spintop demo!]

So how can we explain the Tippe Top? [DEMO 2]



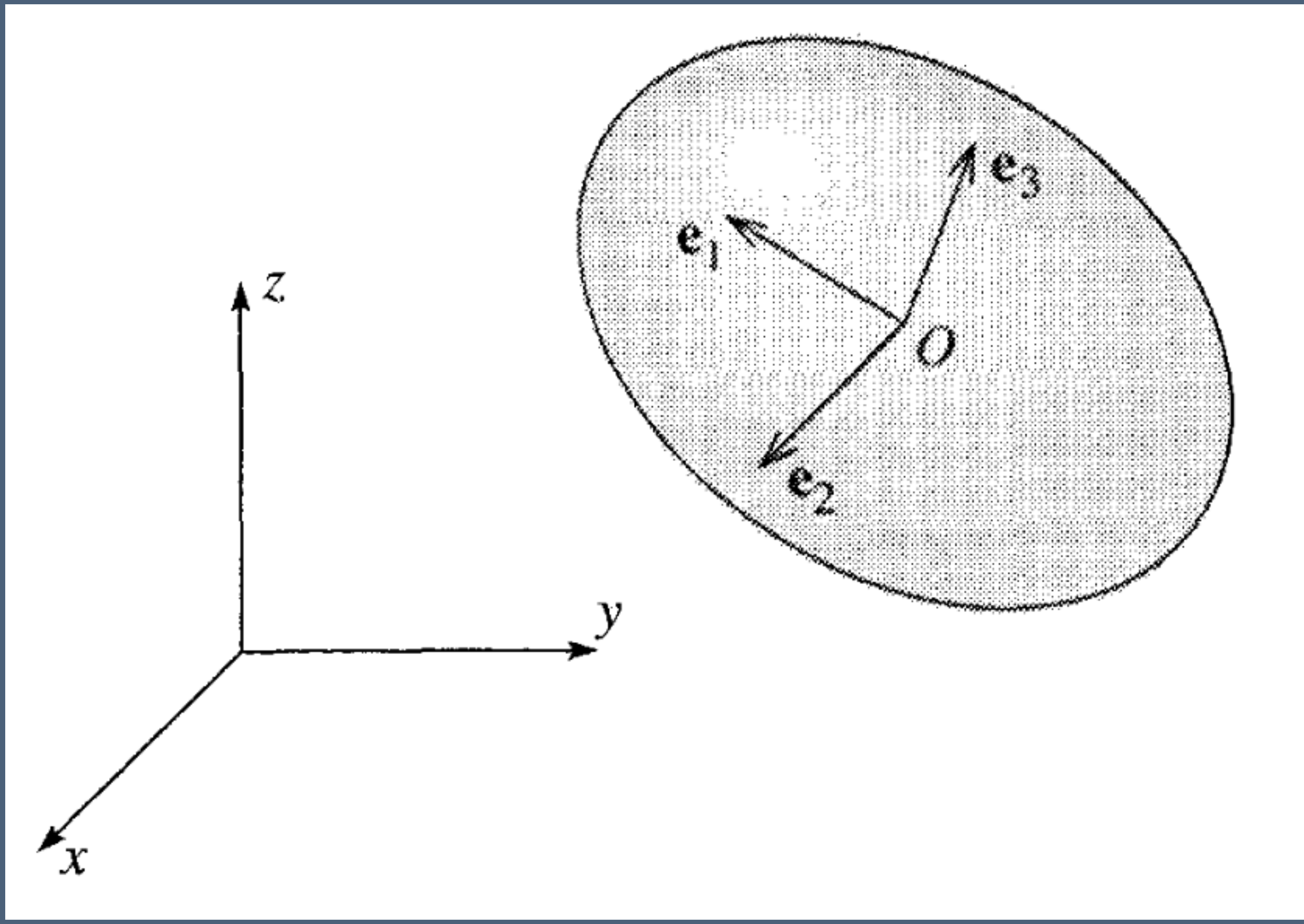
Body vs space frame

- So far we've mostly discussed behavior in the "space frame" (inertial, "lab")
- Consider a "body frame"
  - Origin fixed to the body (usually CoM), axes = principal axes
  - Co-rotating with the object, $\vec{\Omega} = \vec{\omega}$
  - $\vec{L} = \begin{pmatrix} \lambda_1 \omega_1 \\ \lambda_2 \omega_2 \\ \lambda_3 \omega_3 \end{pmatrix}$



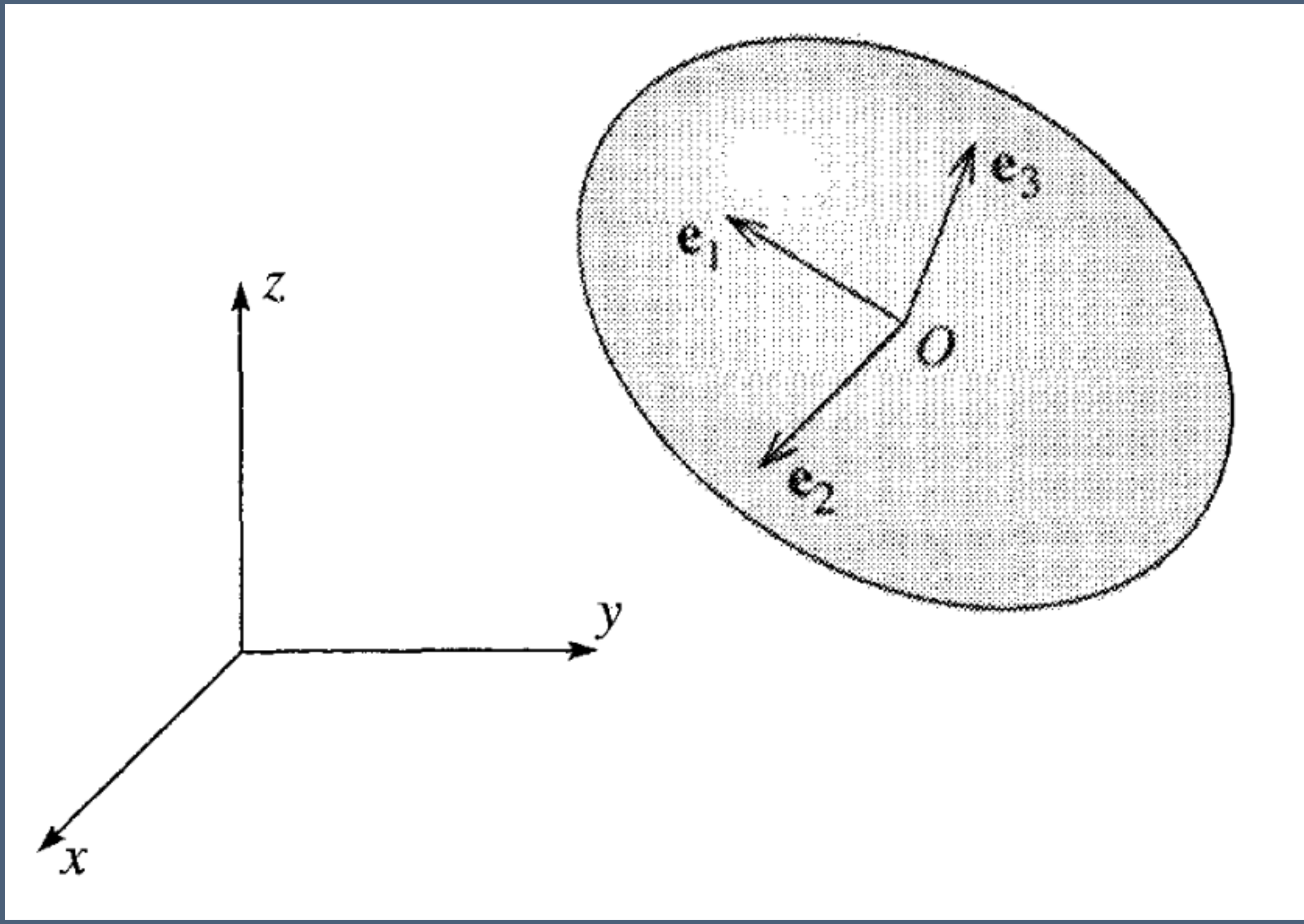
Body vs space frame

- $$\overrightarrow{L} = \begin{pmatrix} \lambda_1\omega_1 \\ \lambda_2\omega_2 \\ \lambda_3\omega_3 \end{pmatrix}$$
- *But in a rotating frame, there is an inertial torque! (Ch. 9)*
- $$\overrightarrow{\Gamma} = \dot{\overrightarrow{L}}_{space} = \dot{\overrightarrow{L}}_{body} + \omega \times \overrightarrow{L}$$
- ***Euler's Equations:***
  $$\overrightarrow{\Gamma} = \dot{\overrightarrow{L}} + \omega \times \overrightarrow{L}$$



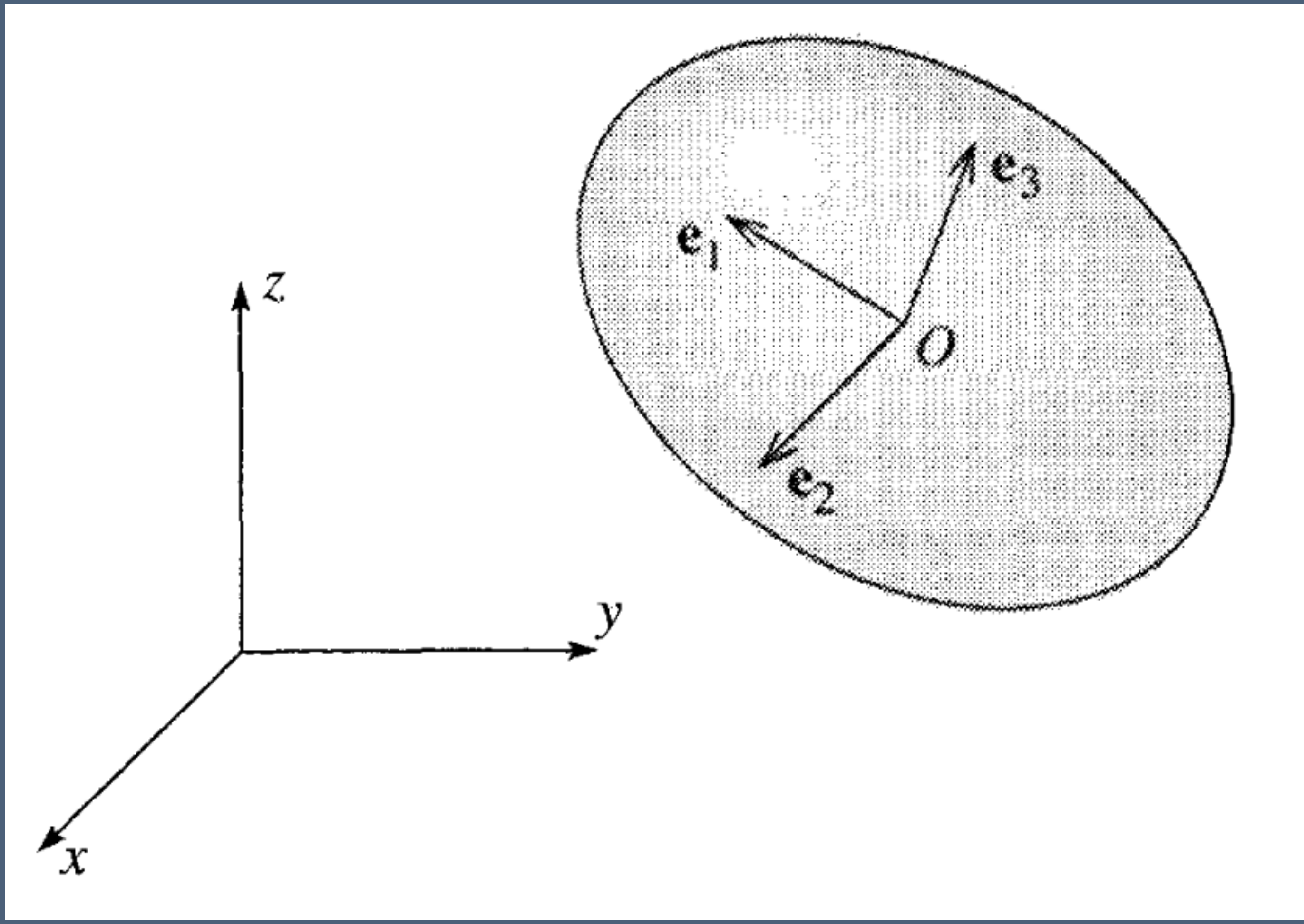
Body vs space frame

- ***Euler's Equations:***
  $\overrightarrow{\Gamma} = \dot{\overrightarrow{L}} + \omega \times \overrightarrow{L}$
- $\Gamma_i = \lambda_i \dot{\omega}_i - \left(\lambda_j - \lambda_k\right) \omega_j \omega_k$
- *With no external torque this becomes:*
  $\lambda_i \dot{\omega}_i = \left(\lambda_j - \lambda_k\right) \omega_j \omega_k$



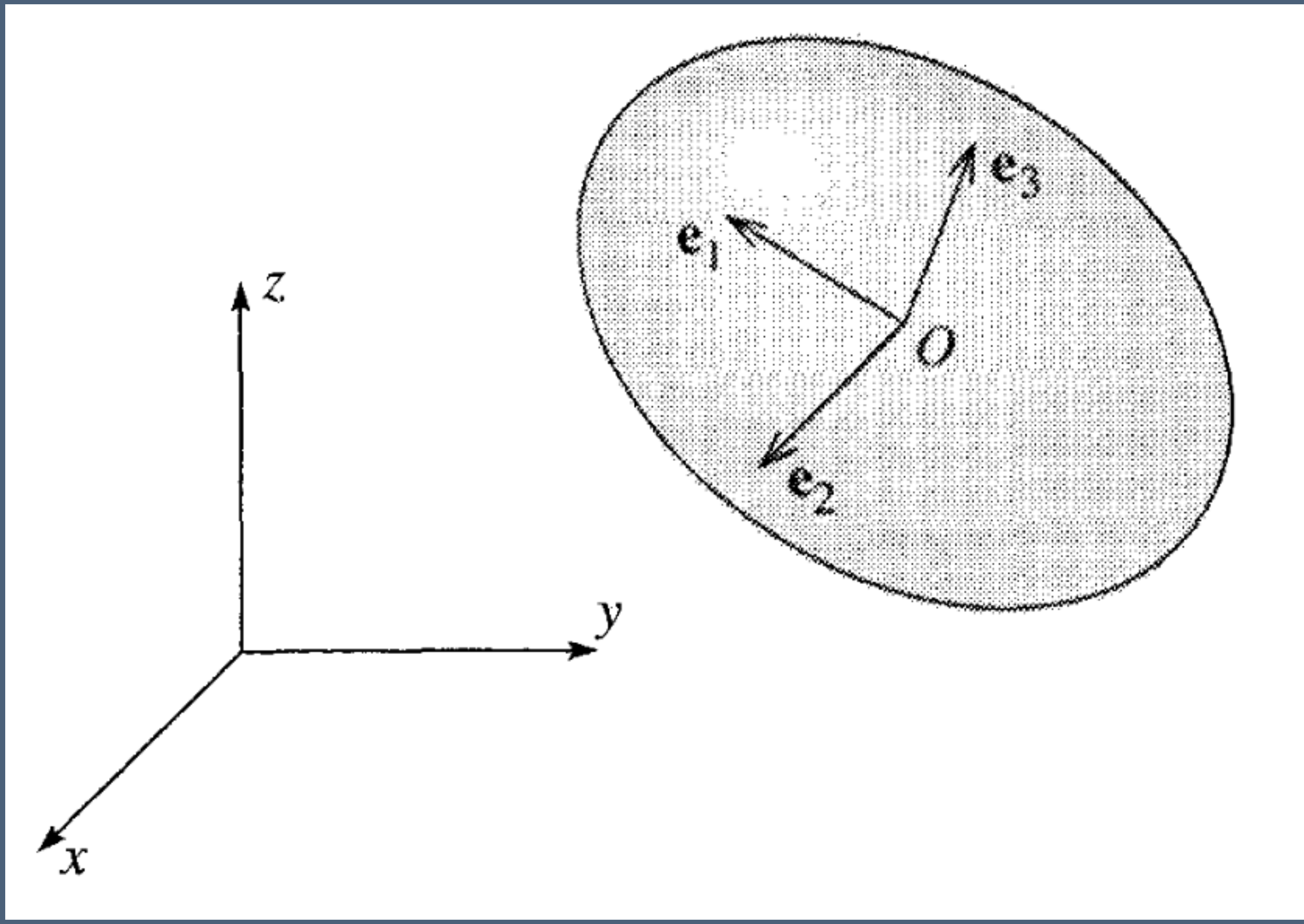
Rotation without torque

- ***Euler's Equations:***
  $\overrightarrow{\Gamma} = \dot{\overrightarrow{L}} + \omega \times \overrightarrow{L}$
- $\Gamma_i = \lambda_i \dot{\omega}_i - \left(\lambda_j - \lambda_k\right) \omega_j \omega_k$
- *With no external torque this becomes:*
  $\lambda_i \dot{\omega}_i = \left(\lambda_j - \lambda_k\right) \omega_j \omega_k$
- *Consider the case where*
  $\omega_3 \approx \text{constant} \gg \omega_1, \omega_2$



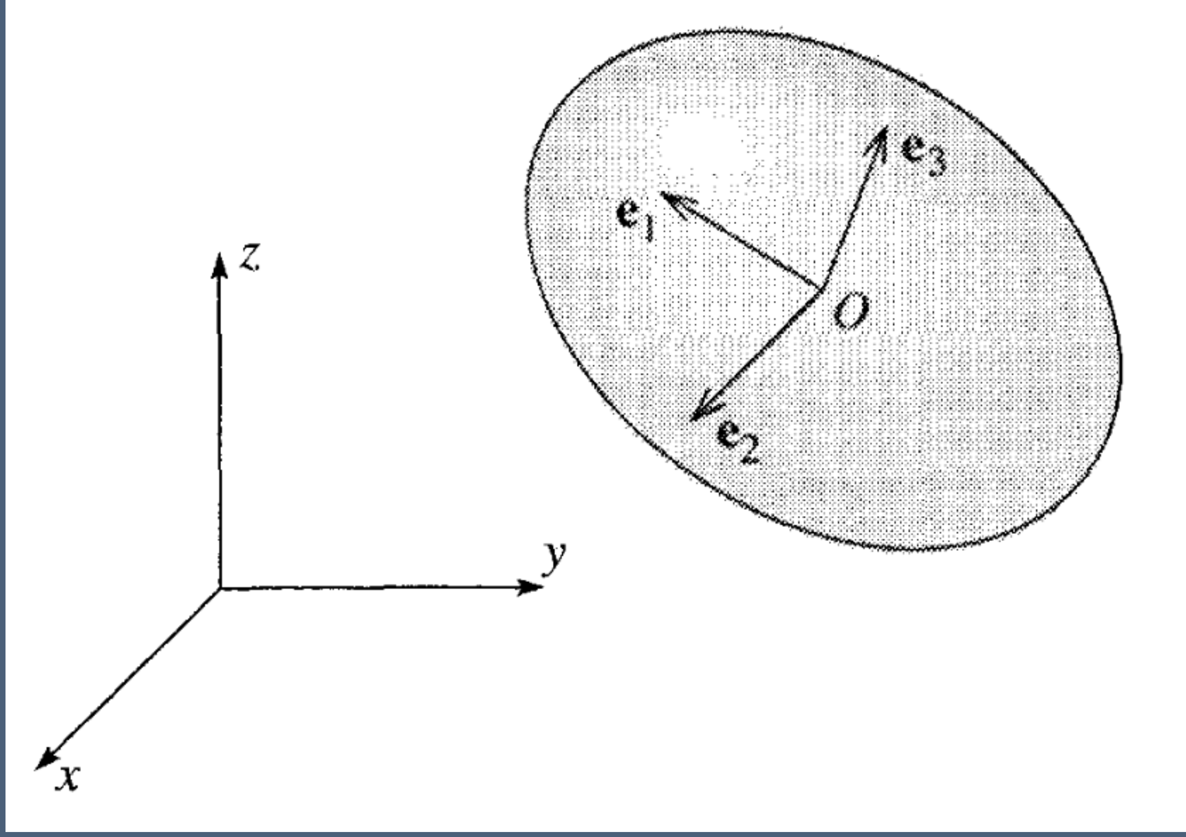
Rotation without torque

- $\lambda_i \dot{\omega}_i = \left(\lambda_j - \lambda_k\right) \omega_j \omega_k$
- *Consider the case where* $\omega_3 \approx \text{constant} \gg \omega_1, \omega_2$
- $\lambda_1 \dot{\omega}_1 = \left[\left(\lambda_2 - \lambda_3\right) \omega_3\right] \omega_2$
- $\lambda_1 \ddot{\omega}_1 = \left[\left(\lambda_2 - \lambda_3\right) \omega_3\right] \dot{\omega}_2$
- $\lambda_2 \dot{\omega}_2 = \left[\left(\lambda_3 - \lambda_1\right) \omega_3\right] \omega_1$



Rotation without torque

- $\lambda_i \dot{\omega}_i = \left(\lambda_j - \lambda_k\right) \omega_j \omega_k$
- *Consider the case where* $\omega_3 \approx \text{constant} \gg \omega_1, \omega_2$
- $\lambda_1 \dot{\omega}_1 = \left[\left(\lambda_2 - \lambda_3\right) \omega_3\right] \omega_2$
- $\ddot{\omega}_1 = -\left[\left(\lambda_3 - \lambda_2\right)\left(\lambda_3 - \lambda_1\right) \frac{\omega_3^2}{\lambda_1 \lambda_2}\right] \omega_1$

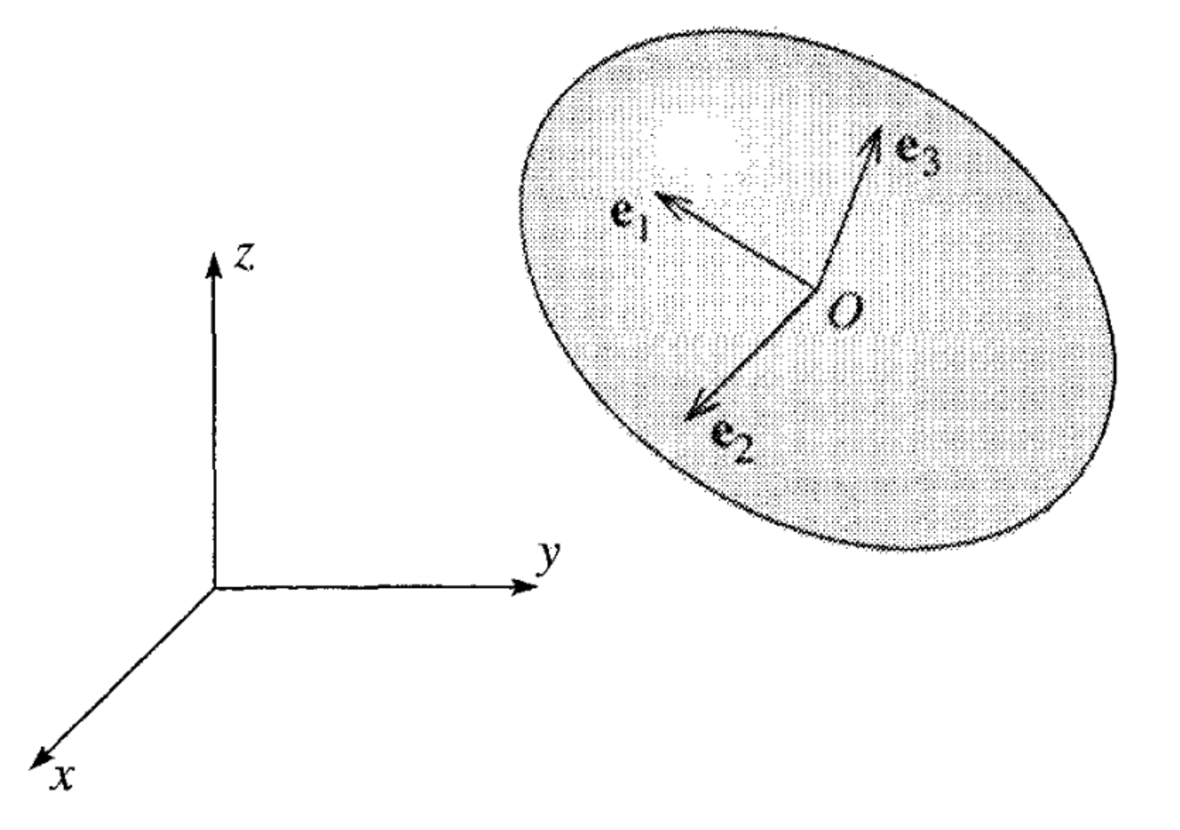


e.g. wobbly frisbee!

Intermediate Moment theorem!

AKA Tennis Racquet Thrm

- $$\ddot{\omega}_1 = -\left[\left(\lambda_3 - \lambda_2\right)\left(\lambda_3 - \lambda_1\right)\frac{\omega_3^2}{\lambda_1 \lambda_2}\right]\omega_1$$



- *When is rotation stable?*
- *Unstable?*

Problem 10.42

Wobbly book frisbee!

- How wobbly is the spinning of a book about it's shortest principal axis?
- Ex. 10.2 will be helpful, along with $\ddot{\omega}_1 = -\left[\frac{(\lambda_3 - \lambda_2)(\lambda_3 - \lambda_1)}{\lambda_1 \lambda_2}\omega_3^2\right]\omega_1$

Problem 10.37

- Given a shape, find $\mathbf{I}$, diagonalize it, and find the principal axes

$$I_{xx} = m \sum \left(y^2 + z^2\right)$$

$$I_{xy} = I_{yx} = -m \sum xy$$