PhysH308

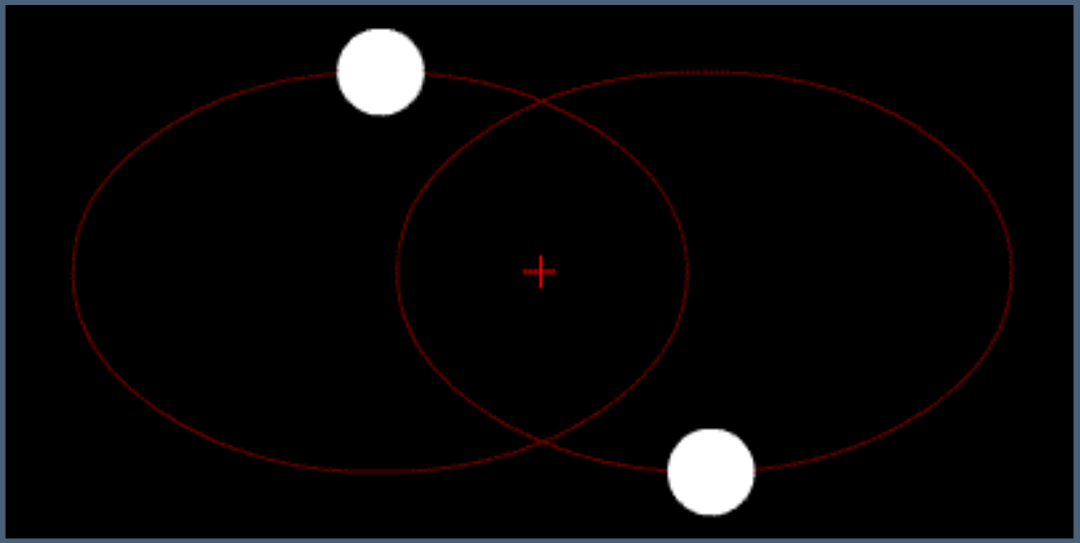
Central forces!

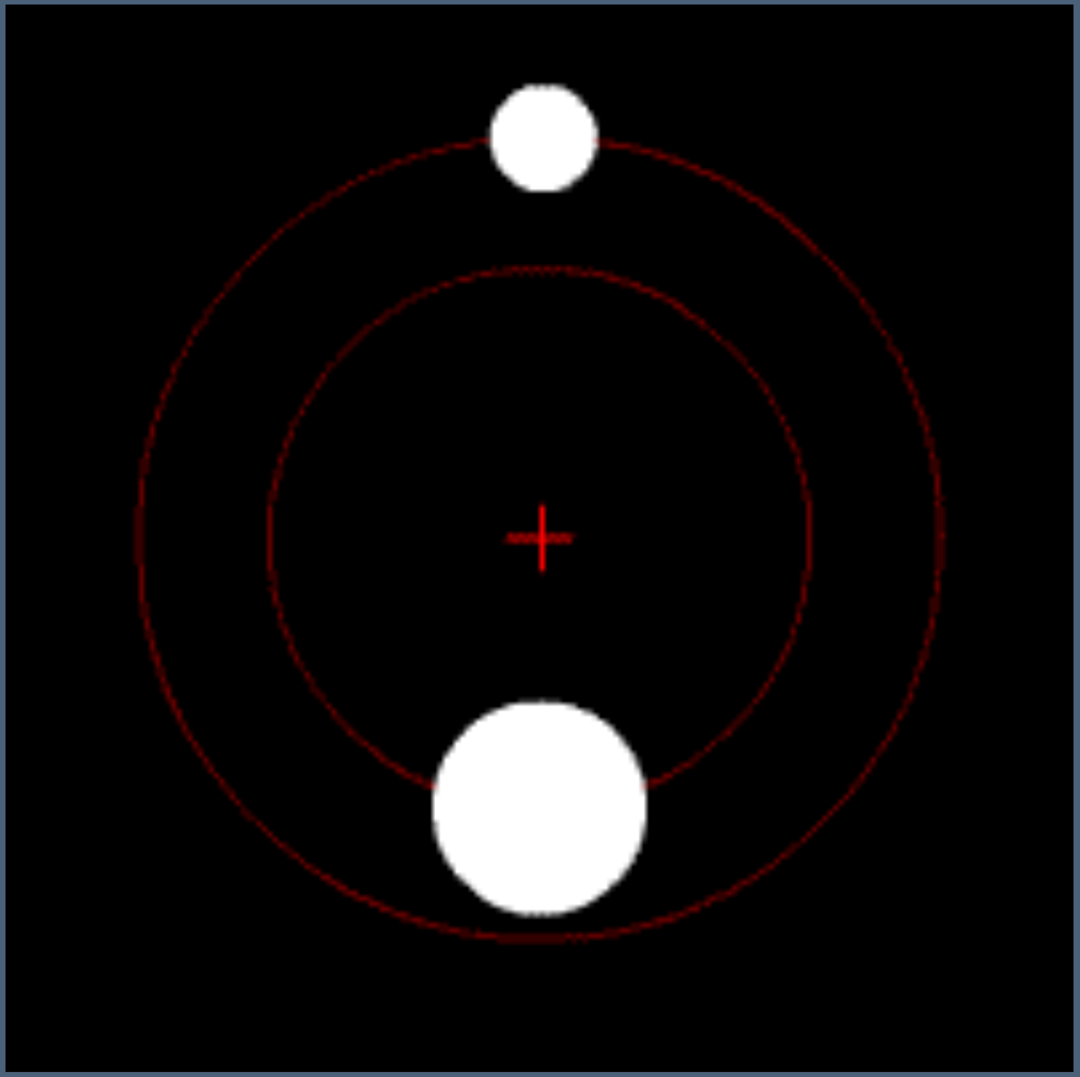
Ted Brzinski, Oct 29, 2024

The system

2 particles, conservative interaction

- Arbitrary lab frame:
  - $T = \frac{1}{2}\left(m_1 \vec{r}_1^{\,2} + m_2 \vec{r}_2^{\,2}\right)$
  - $U = U\left(\vec{r}_1 - \vec{r}_2\right)$
- CM coordinates (e.g. animations to the right)
  - $\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{M}, M = m_1 + m_2$
  - $\vec{r} = \vec{r}_1 - \vec{r}_2$





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  - $\overrightarrow{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{M}, M = m_1 + m_2$
  - $\vec{r} = \vec{r}_1 - \vec{r}_2$
- Solving for $\vec{r}_1, \vec{r}_2$
  - $\vec{r}_2 = \vec{r}_1 - \vec{r}$
  - $\overrightarrow{R} = \frac{m_1 \vec{r}_1 + m_2\left(\vec{r}_1 - \vec{r}\right)}{M}$
  - $= \vec{r}_1 \frac{m_1 + m_2}{M} - \vec{r}\frac{m_2}{M} = \vec{r}_1 - \vec{r}\frac{m_2}{M}$
  - $\vec{r}_1 = \overrightarrow{R} + \vec{r}\frac{m_2}{M}$
  - $\vec{r}_2 = \overrightarrow{R} + \vec{r}\left(\frac{m_2}{M} - 1\right) = \overrightarrow{R} - \vec{r}\frac{m_1}{M}$

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- Solving for $\vec{r}_1, \vec{r}_2$
  - $\vec{r}_2 = \vec{r}_1 - \vec{r}$
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- Kinetic Energy:
  - $T = \dfrac{1}{2}\left(m_1\vec{r}_1^{\,2} + m_2\vec{r}_2^{\,2}\right)$

    $= \dfrac{1}{2}\left(\left(m_1 + m_2\right)\overrightarrow{R} + \left(\dfrac{m_1 m_2}{M}\right)\vec{r}\right)$

    $= \dfrac{1}{2}\left(M\overrightarrow{R}^2 + \mu\vec{r}^{\,2}\right)$

- Potential Energy:
  - $U = U\left(\vec{r}_1 - \vec{r}_2\right)$

    $= U\left(\vec{r}\right)$

The system

2 particles, conservative interaction

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  - $T = \frac{1}{2}\left(m_1 \dot{\vec{r}}_1^2 + m_2 \dot{\vec{r}}_2^2\right)$
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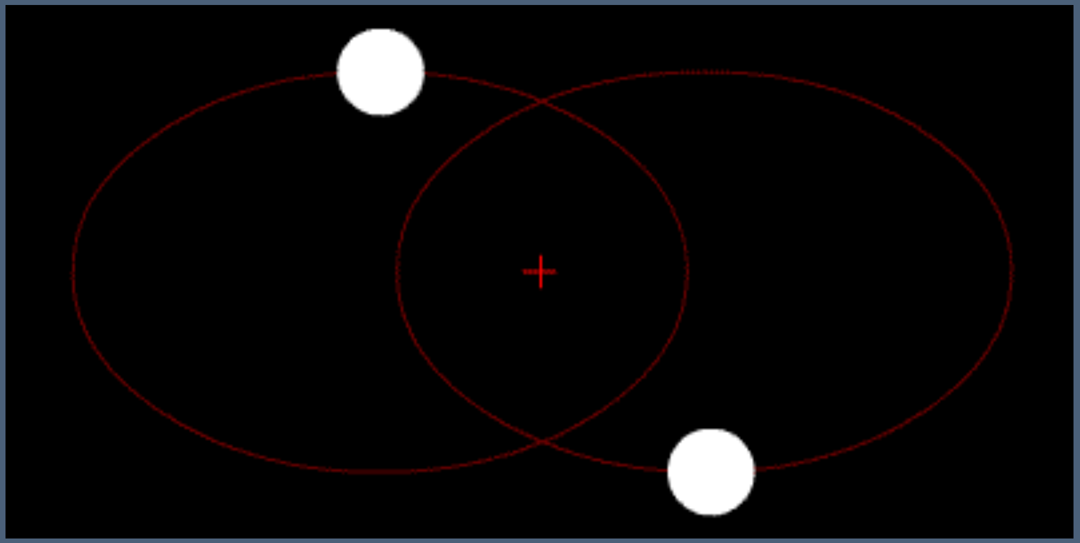
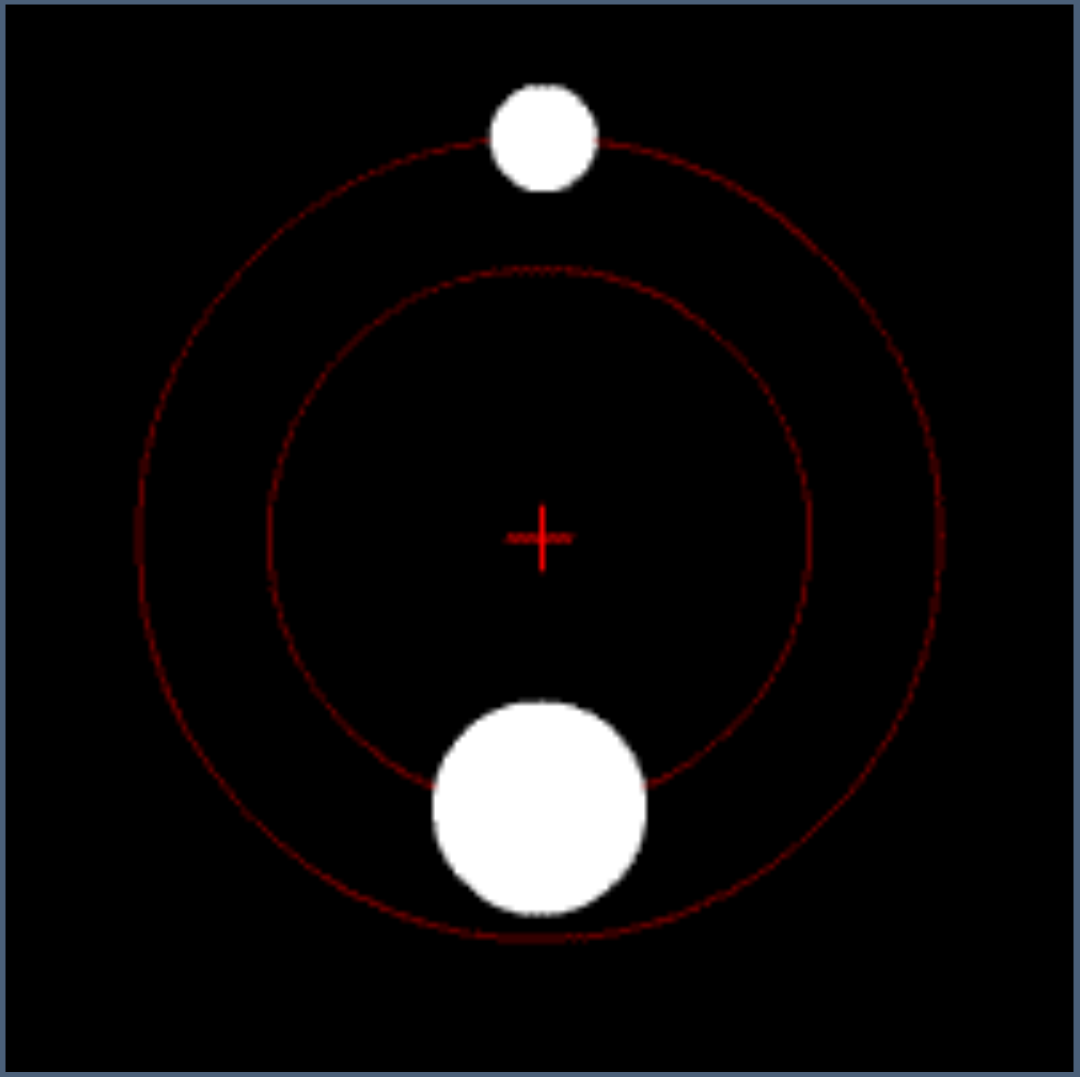
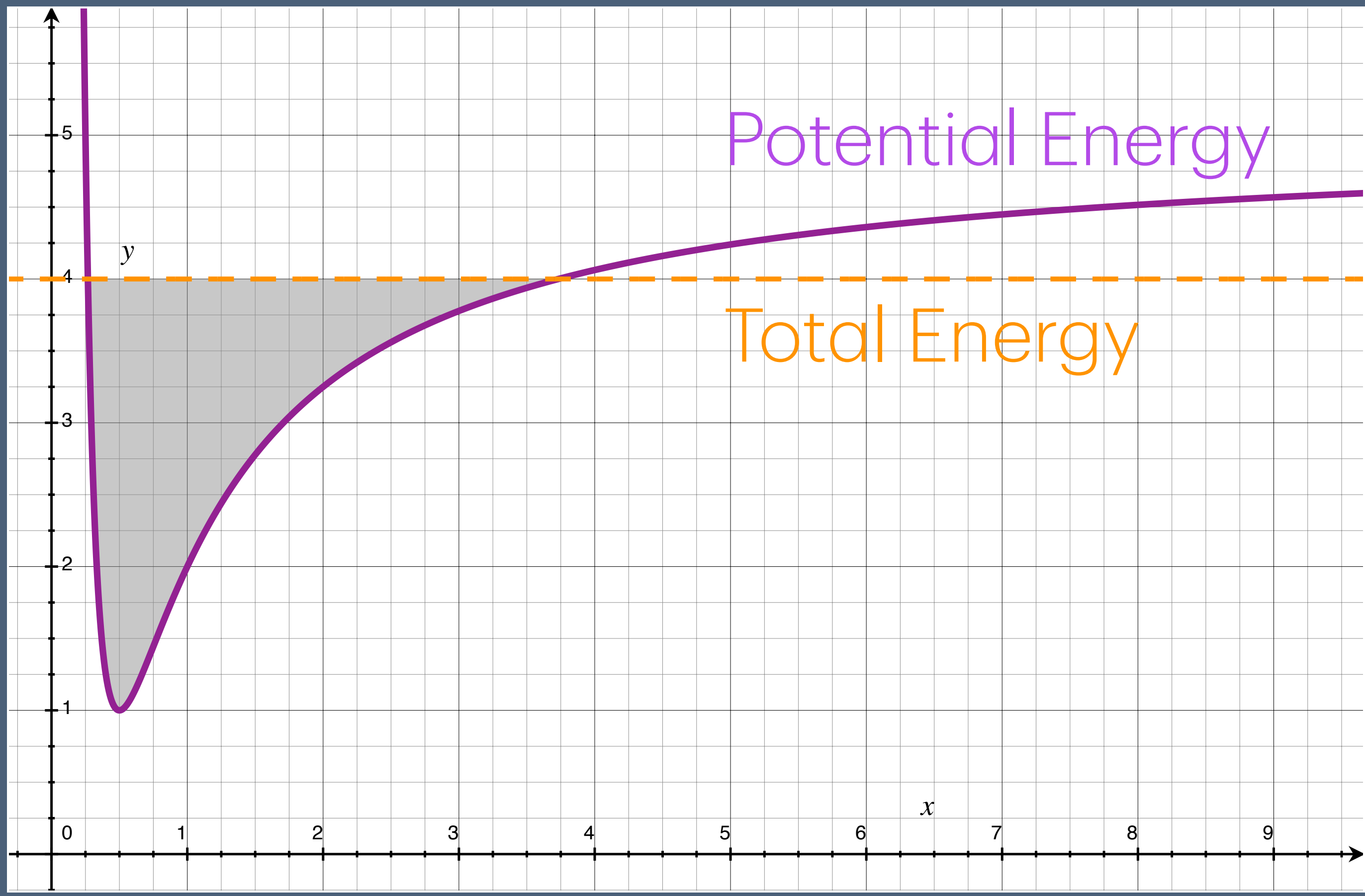




Potential minima

Stable equilibria

- Consider a given potential with a minimum (illustrated to the right)
- If the highlighted region is finite/closed, the system is bound about a stable equilibrium at the minimum.
- The Kinetic energy is given by the difference between the total and potential



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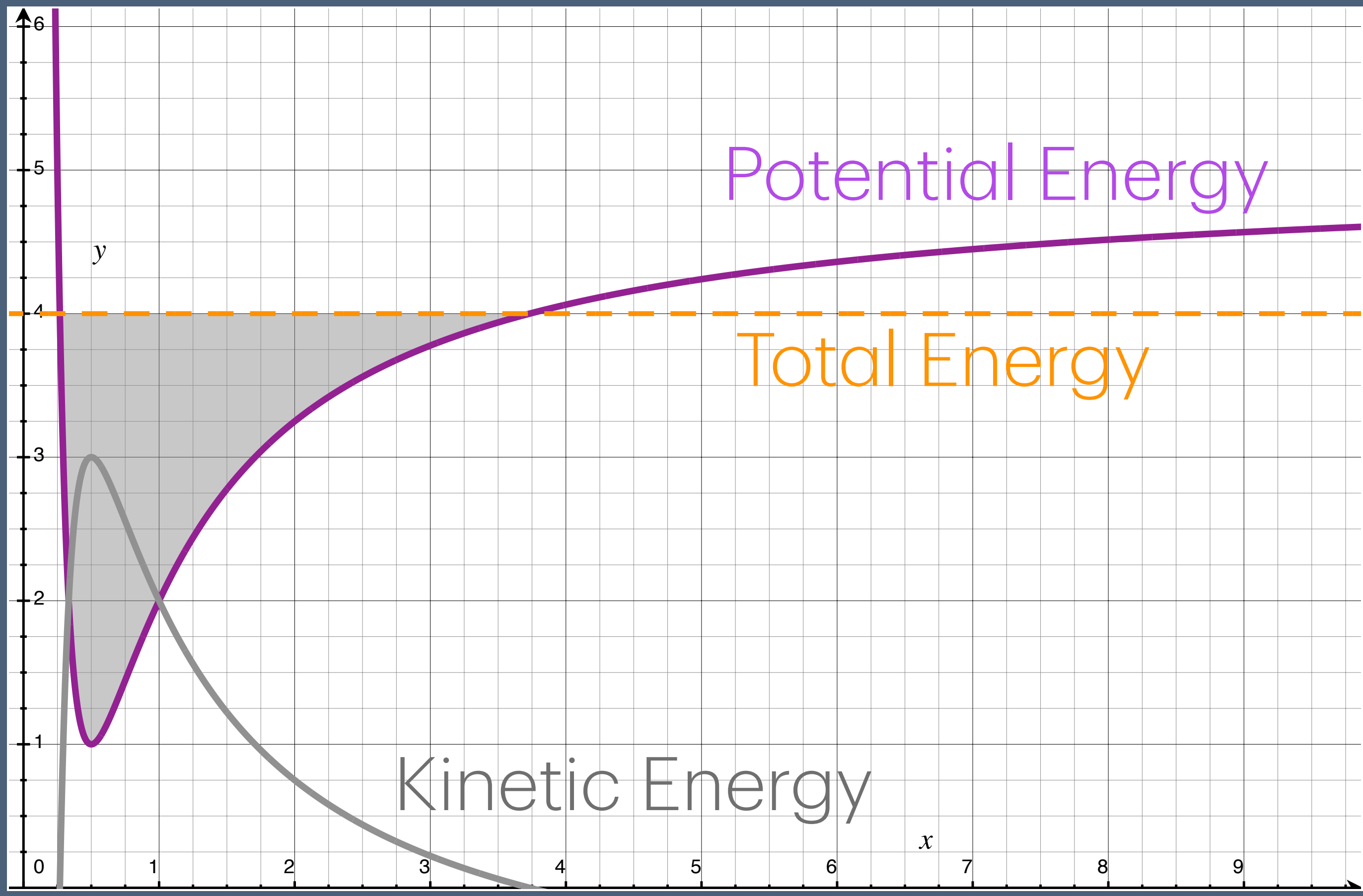

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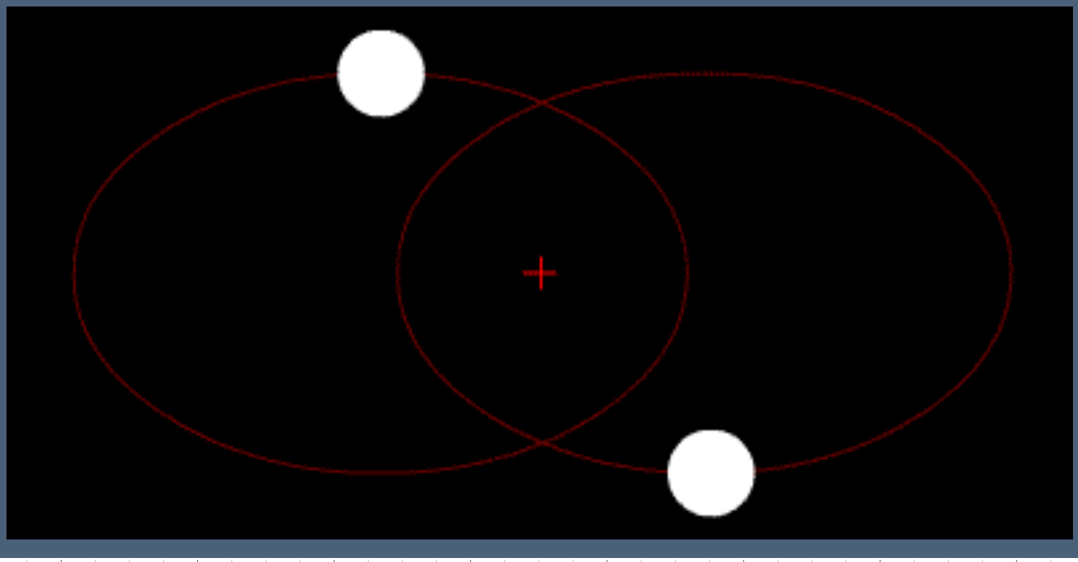
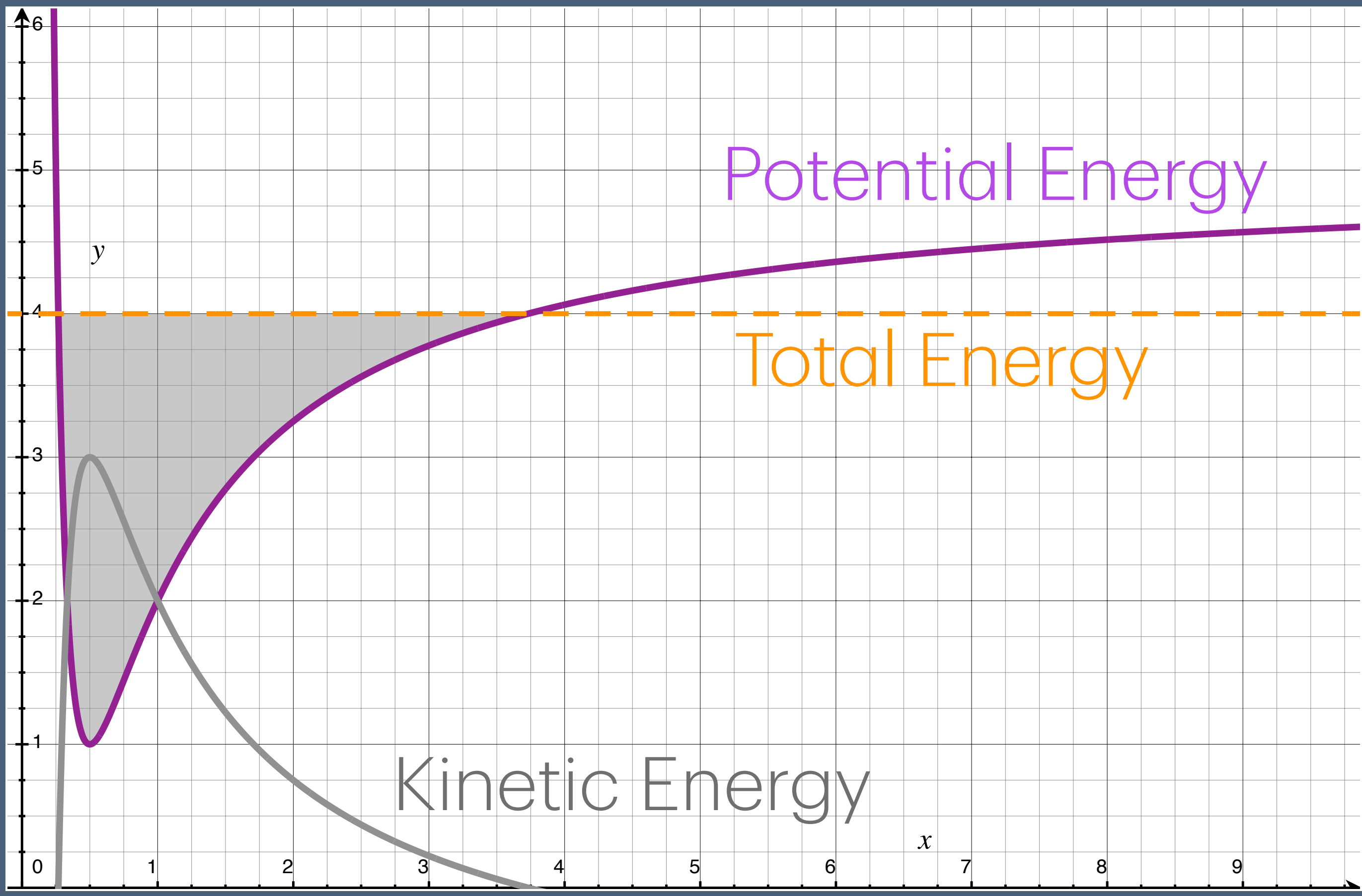
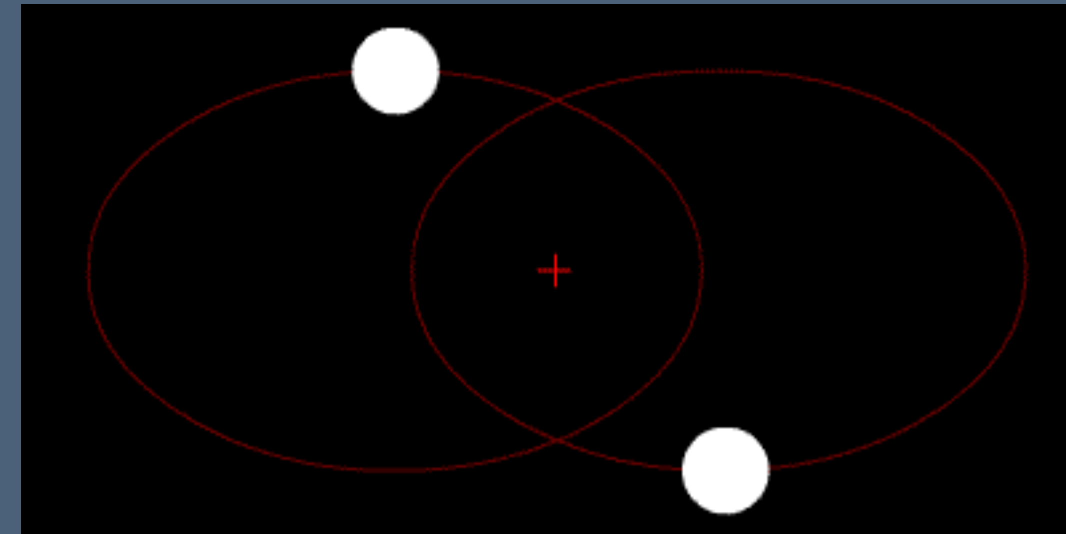


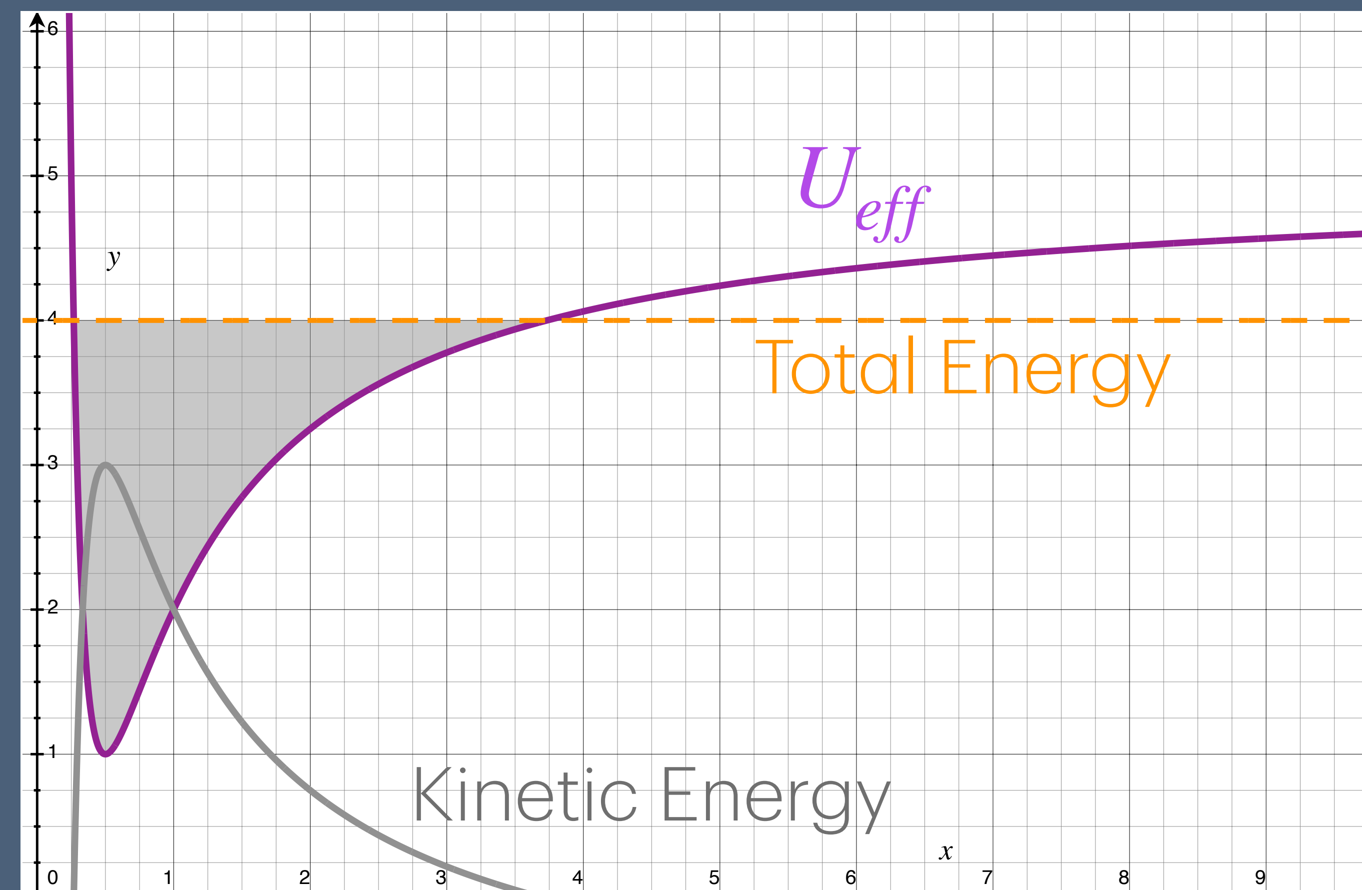


But none of our central potentials have minima!!!

Effective potential

"Potential" from fictitious forces



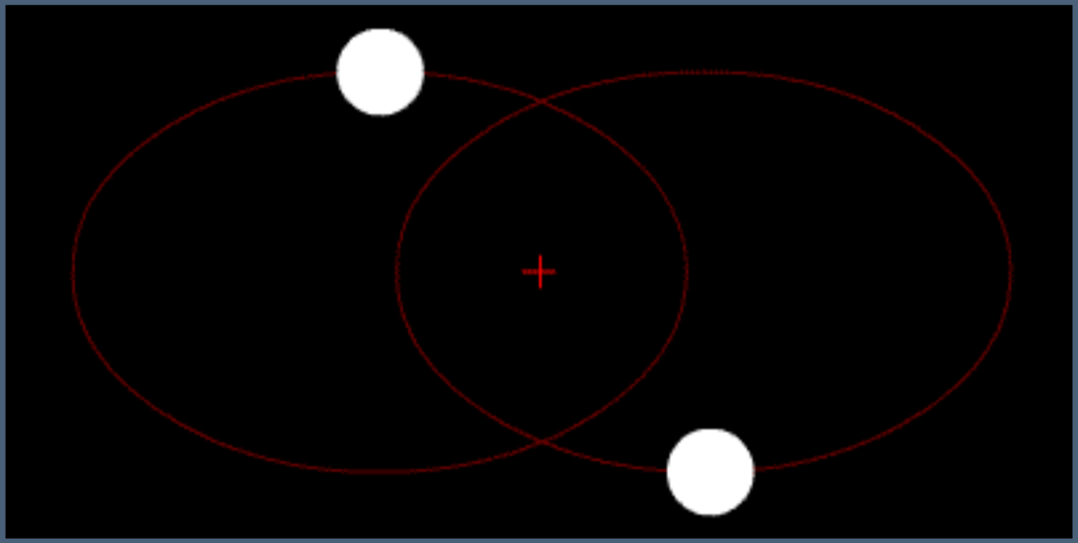


- For central forces, we've already seen it's useful to reduce $\mathscr{L}$ to 1-D by replacing $\dot{\phi}$ with ℓ_z:

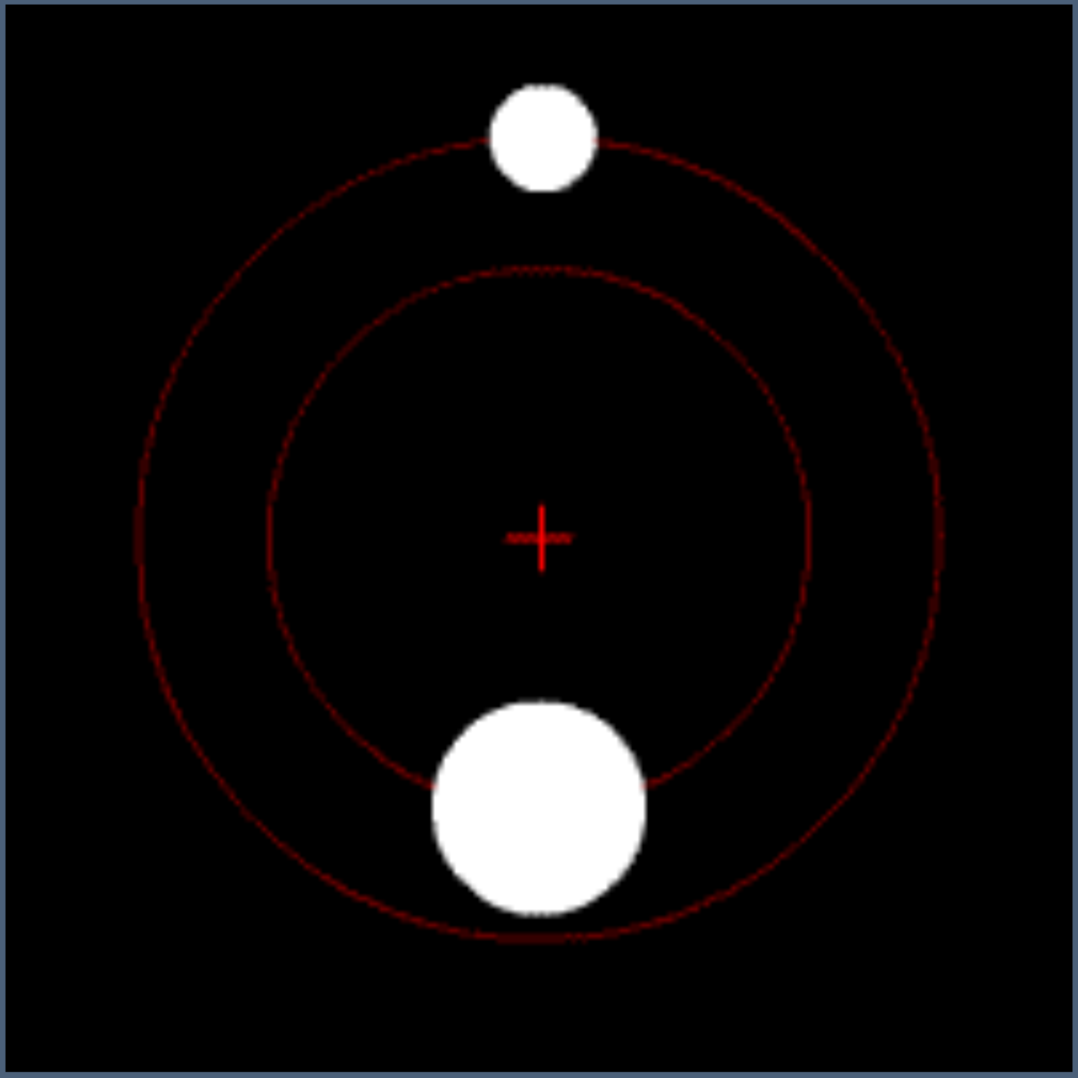
$$\mathscr{L} = \frac{1}{2}\mu\dot{\vec{r}}^2 + \frac{\ell^2}{2\mu r^2} - U(r)$$

- Gathering the r-dependent terms: $U_{eff} = U(r) - \dfrac{\ell^2}{2\mu r^2}$

Problems



- 8.9 - CM frame



- 8.12 - Effective potential
- Thursday - we'll go through derivation of the Kepler orbits, chaps 8.6-7.