PhysH308

Lagrangian Mechanics

Ted Brzinski, Oct 10, 2024

Example of motion from $\mathscr{L}$

Recall this horrible problem:

We showed in class that the constraint is $\Delta y = -2\Delta x$

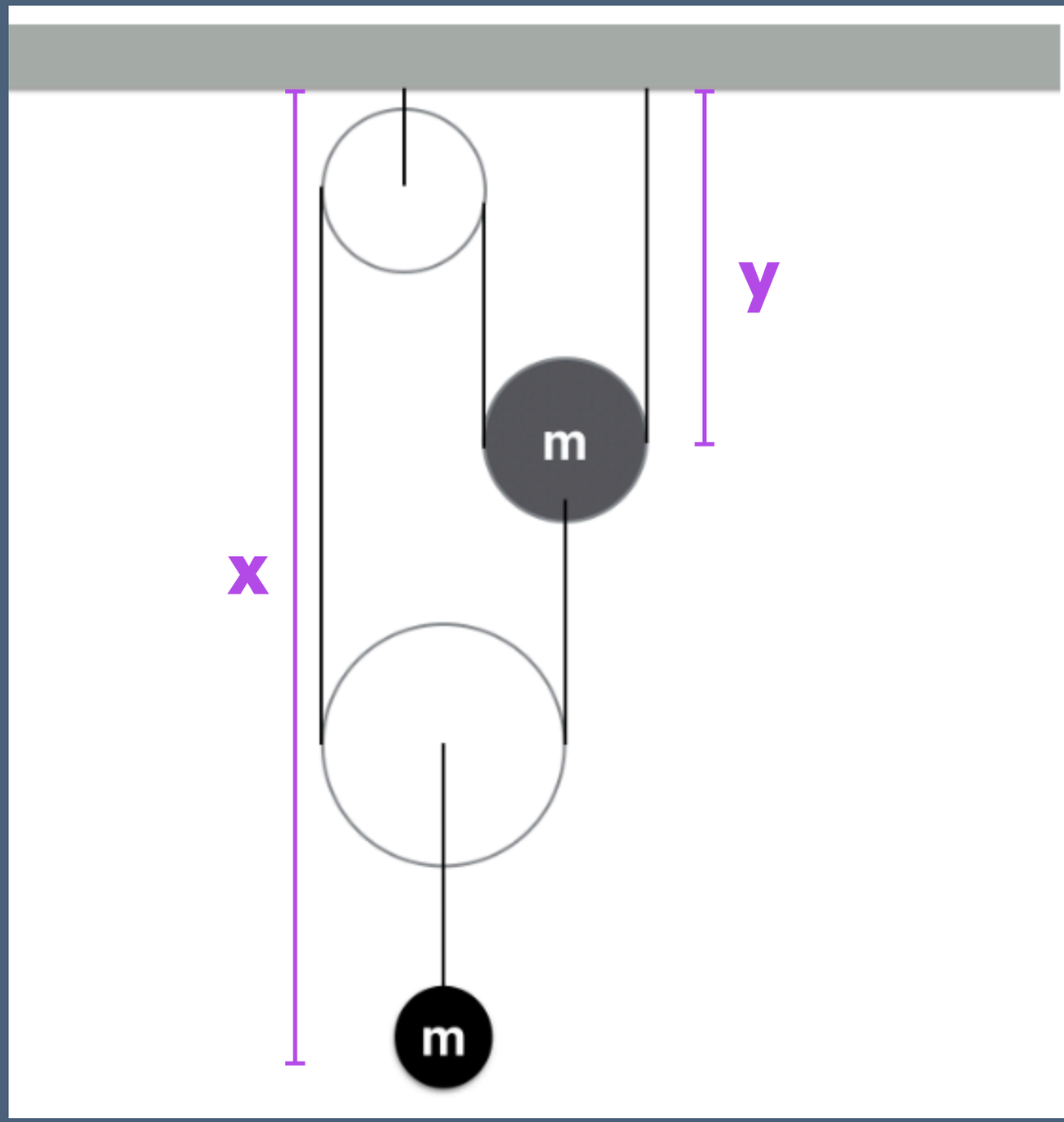
With $F = ma$:

$m\ddot{x} = mg - 2T$

AND

$m\ddot{y} = mg - T$

Solve the system of equations for the motion



Example of motion from $\mathscr{L}$

Recall this horrible problem:

We showed in class that the constraint is

$$\Delta y = -2\Delta x$$

With $\mathscr{L} = T - U$:

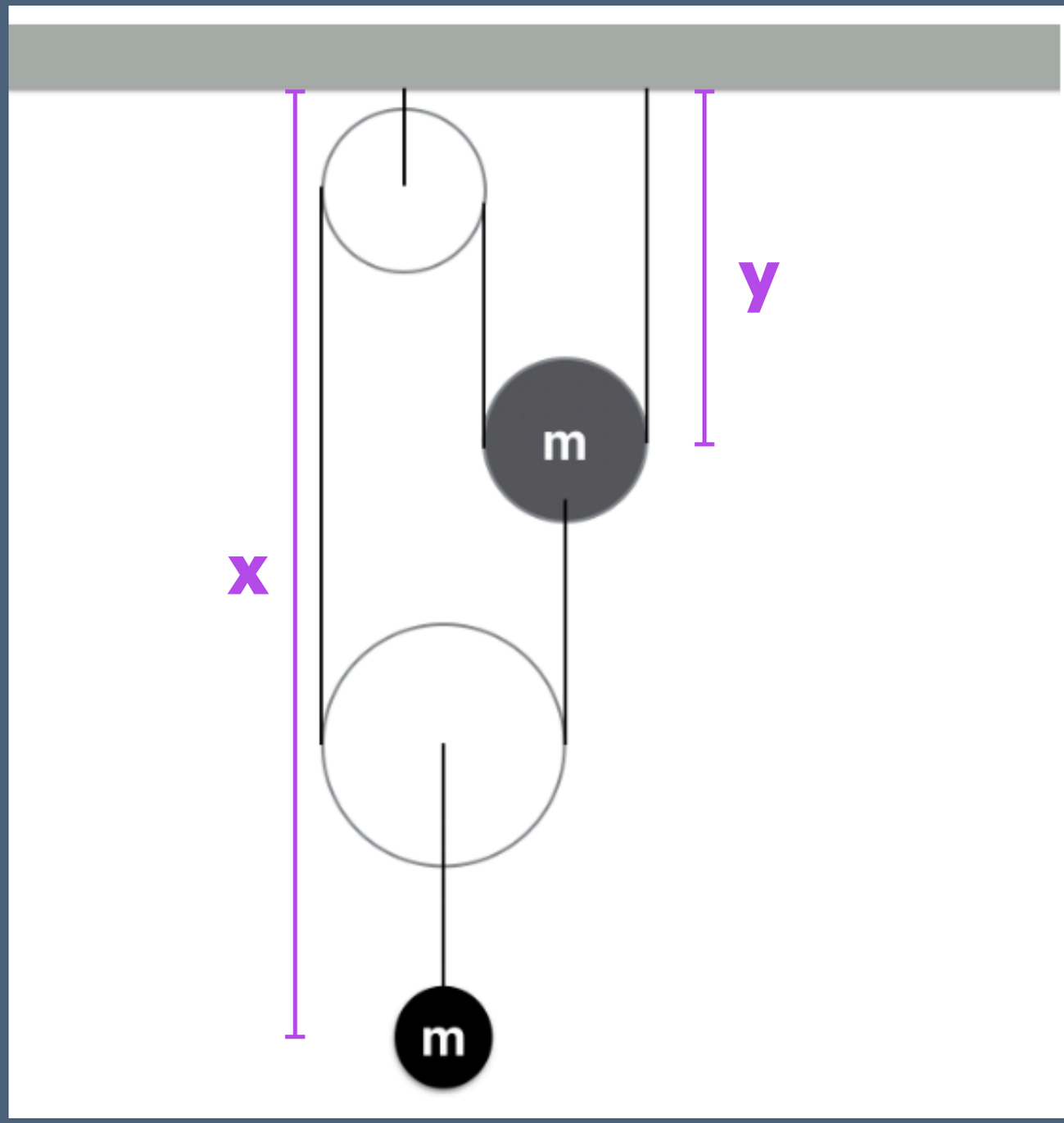
$$T = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) = \frac{5}{2}m\dot{x}^2$$

AND

$$U = -mg(x + y) + U_0 = mgx$$

E-L: $$\frac{\partial \mathscr{L}}{\partial x} = \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{x}}$$

$$mg = -5m\ddot{x}$$



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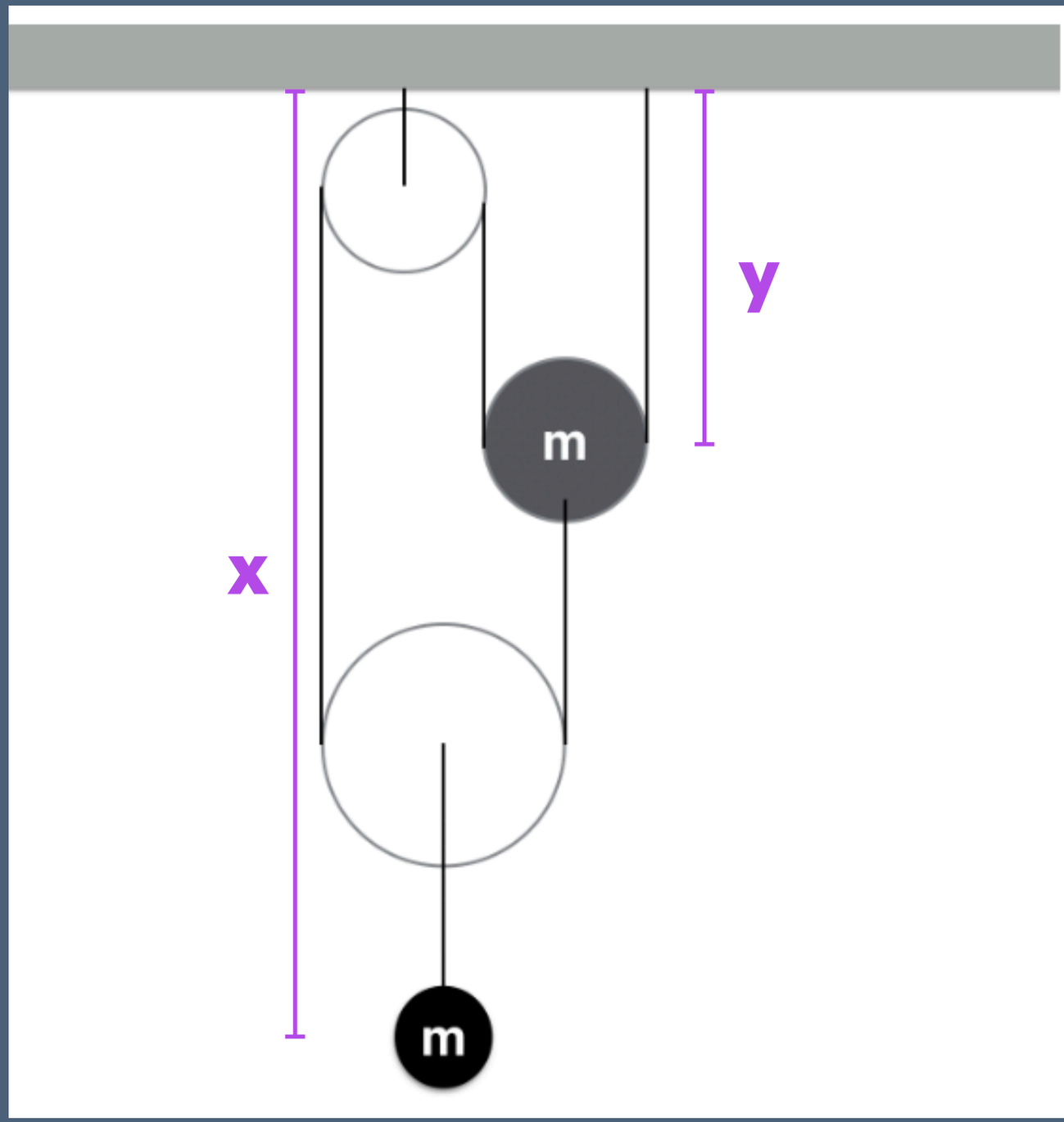
$$\ddot{x} = -\frac{g}{5}$$

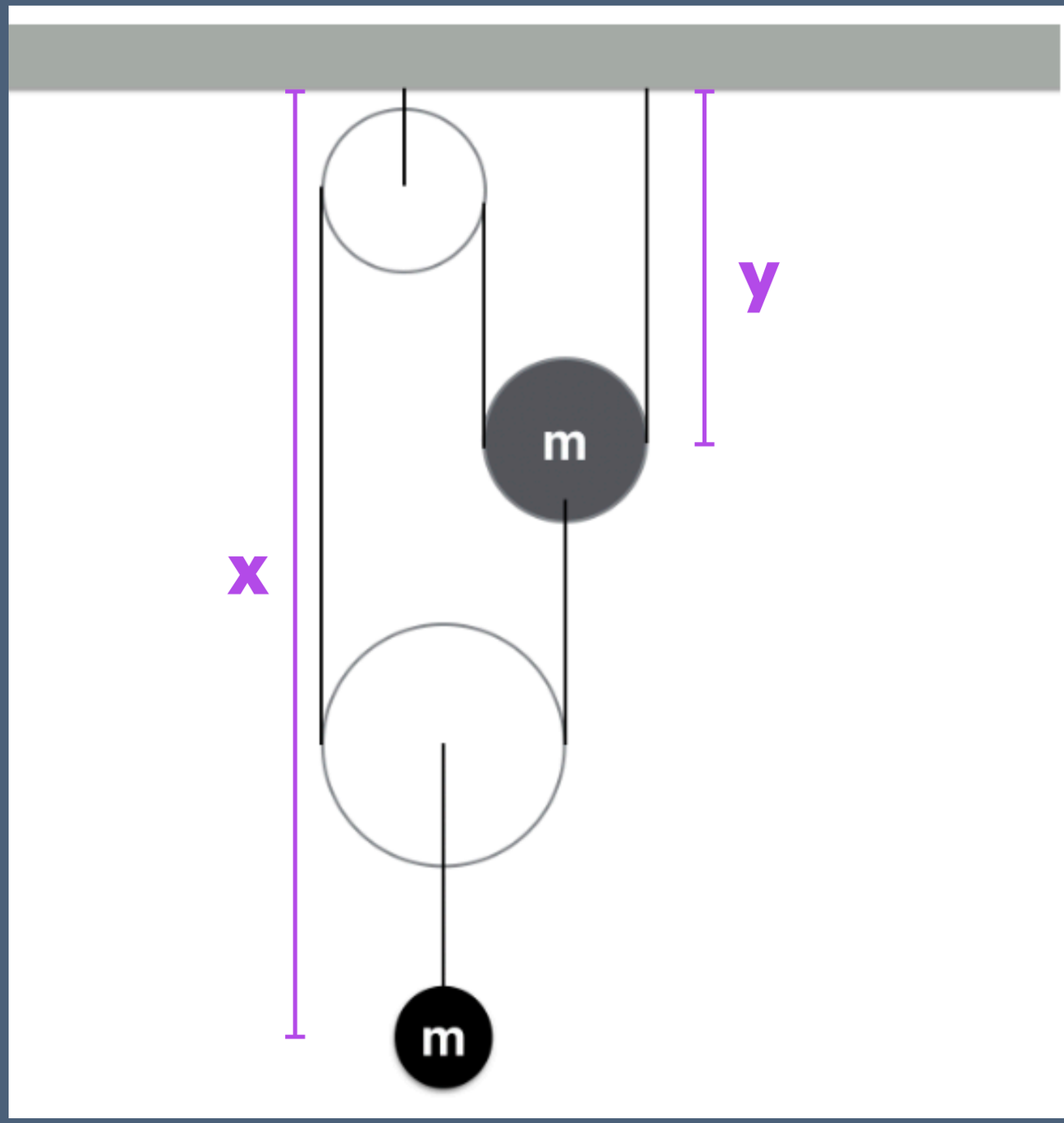


Lagrangian vs Newtonian

What's the difference?

- 2 equations vs 3 equations
- No need to solve for tension (forces of constraint in general)
- Based on energy rather than free-body diagram

BUT you still need to carefully construct your constraints!



Today's problems

- 7.10 - geometric constraints due to normal force (also conic sections again?)
- 7.11 - Imposed constraint (time-dependent driving)
- CANCELLED: 7.12