> *# Вариант 8*
 # Задание 1. Получить разложение в тригонометрический ряд Фурье.
 # Создать пользовательскую функцию, которая осуществляет построение триг. ряда Фурье.
 # Построить и сделать анимацию в одной системе координат графиков частичных сумм ряда и его суммы.

> $with(LinearAlgebra):$
$with(plots):$

> $fourier\_series := \mathbf{proc}(function, l, low, high)$ **description** "Convert a function to Fourier series" :

$\mathbf{local}\ a_0 := simplify\left(\frac{1}{l}\cdot int(function(x), x = low..high)\right):$

$\mathbf{local}\ a_n := simplify\left(\frac{1}{l}\cdot int\left(function(x)\cdot\cos\left(\frac{\mathrm{Pi}\cdot n\cdot x}{l}\right), x = low..high\right)\right):$

$\mathbf{local}\ b_n := simplify\left(\frac{1}{l}\cdot int\left(function(x)\cdot\sin\left(\frac{\mathrm{Pi}\cdot n\cdot x}{l}\right), x = low..high\right)\right):$

$\mathbf{local}\ S_m := m \rightarrow \frac{a_0}{2} + sum\left(a_n\cdot\cos\left(\frac{\mathrm{Pi}\cdot n\cdot x}{l}\right) + b_n\cdot\sin\left(\frac{\mathrm{Pi}\cdot n\cdot x}{l}\right), n = 1..m\right):$

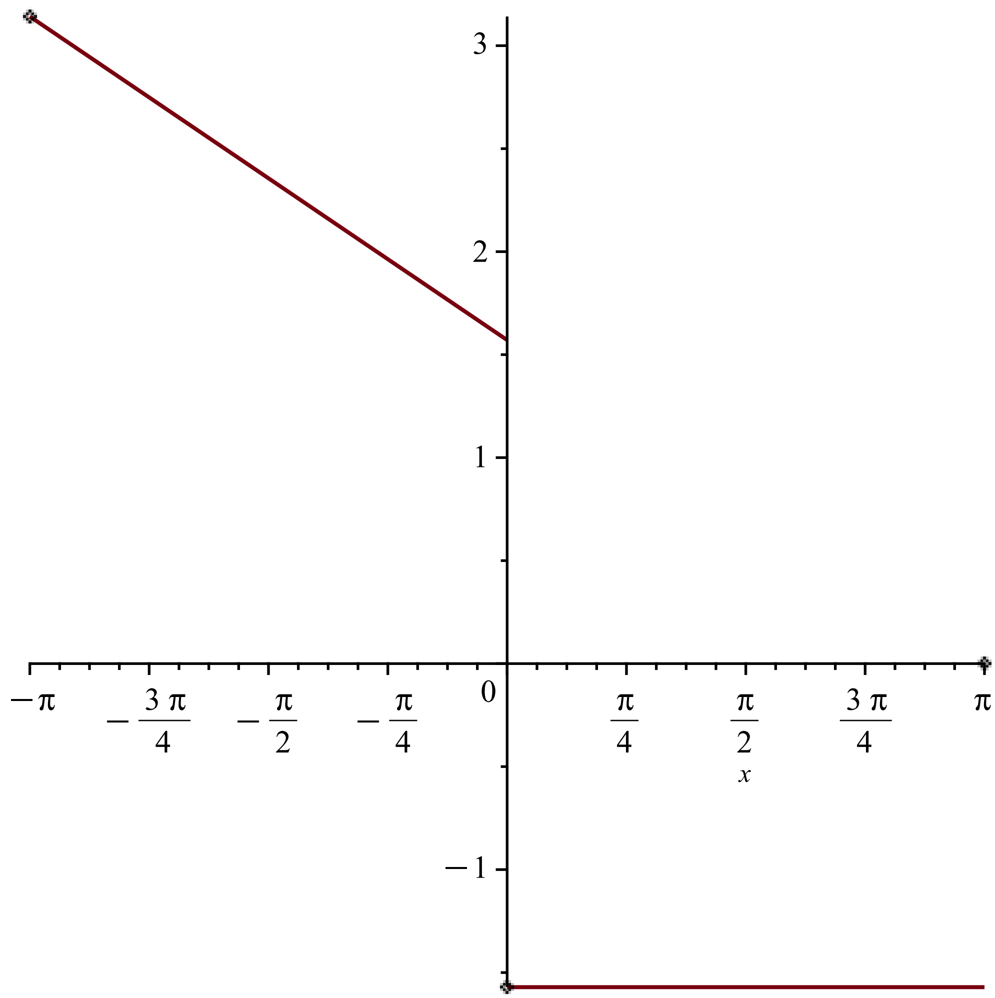
$\mathbf{return}\ S_m(10000):$

end proc:

> $f := x \rightarrow piecewise\left(-\mathrm{Pi} \le x < 0, \frac{\mathrm{Pi} - x}{2}, 0 \le x < \mathrm{Pi}, -\frac{\mathrm{Pi}}{2}\right);$

$$f := x \mapsto \begin{cases} \frac{\pi}{2} - \frac{x}{2} & -\pi \le x < 0 \\ -\frac{\pi}{2} & 0 \le x < \pi \end{cases} \quad \mathbf{(1)}$$

> $plot(f(x), x = -\mathrm{Pi}..\mathrm{Pi}, discont = true);$



> $a_0 := simplify\left(\frac{1}{\text{Pi}} \cdot int(f(x), x = -\text{Pi}..\text{Pi})\right);$

$$a_0 := \frac{\pi}{4} \tag{2}$$

> $a_n := simplify\left(\frac{1}{\text{Pi}} \cdot int(f(x) \cdot \cos(n \cdot x), x = -\text{Pi}..\text{Pi})\right)$ assuming $n :: posint;$

$$a_n := \frac{(-1)^n - 1}{2\pi n^2} \tag{3}$$

> $b_n := simplify\left(\frac{1}{\text{Pi}} \cdot int(f(x) \cdot \sin(n \cdot x), x = -\text{Pi}..\text{Pi})\right)$ assuming $n :: posint;$

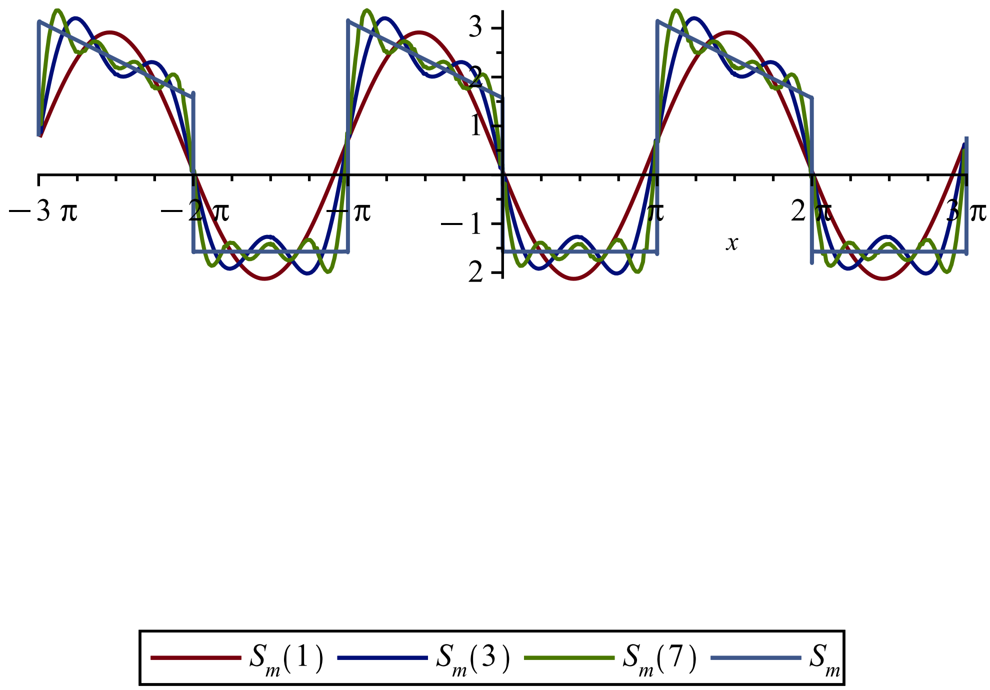
$$b_n := \frac{3(-1)^n - 2}{2n} \tag{4}$$

> $S_m := m \rightarrow \frac{a_0}{2} + sum\left(a_n \cdot \cos(n \cdot x) + b_n \cdot \sin(n \cdot x), n = 1..m\right);$

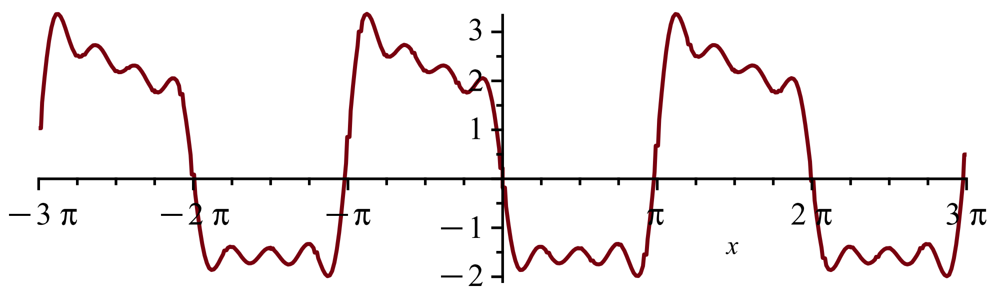
$$S_m := m \mapsto \frac{a_0}{2} + \sum_{n=1}^{m} \left(a_n \cdot \cos(n \cdot x) + b_n \cdot \sin(n \cdot x)\right) \tag{5}$$

> $fseries := fourier\_series(f, \text{Pi}, -\text{Pi}, \text{Pi}):$

```
points := plot([[0, Pi/2], [0, -Pi/2], [-Pi, Pi], [Pi, 0]], style = point) :
plot([S_m(1), S_m(3), S_m(7), fseries], x = -3·Pi..3·Pi, legend = ['S_m(1)','S_m(3)','S_m(7)','S_m']);
```




```
> for i from 1 to 7 do
  Ris[i] := plot([S_m(i)], x = -3·Pi..3·Pi) :
  end do:
  display([seq(Ris[i], i = 1..7)], insequence = true);
```



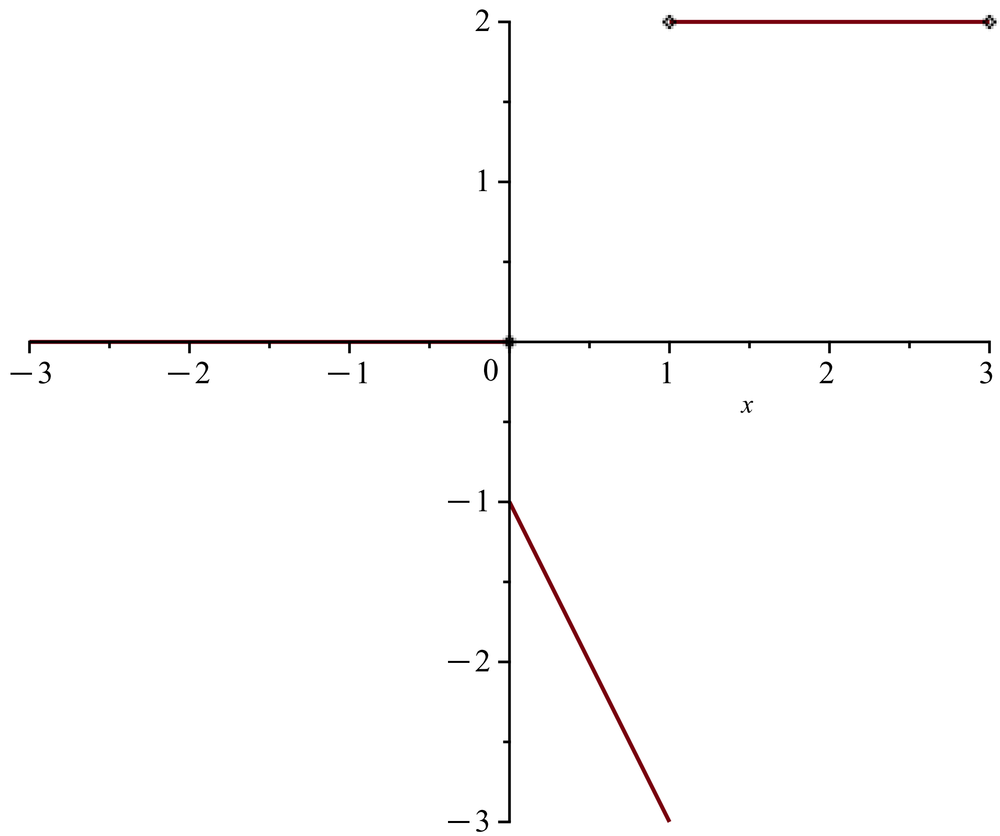
> *# Задание 2*

> *# Разложите в ряд Фурье x2 -периодическую функцию y=f(x),*
 # заданную на промежутке (0, x1) формулой y=ax+b, а на $[x1, x2] - y = c$.
 # Модифицировать процедуру
 # Построить в одной системе координат графики частичных сумм S1(x), S3(x), S7(x) ряда и его суммы S(x)
 # на промежутке $[-2\,x2, 2\,x2]$
. Сравнить полученный результат с *графиком порождающей функции на главном периоде.*
 # Анимировать процесс построения графиков сумм ряда, взяв в качестве параметра порядковый номер частичной суммы.

> $f := x \rightarrow piecewise(0 < x < 1, -2\,x - 1, 1 \le x \le 3, 2);$

$$f := x \mapsto \begin{cases} -2 \cdot x - 1 & 0 < x < 1 \\ 2 & 1 \le x \le 3 \end{cases} \qquad \textbf{(6)}$$

> $plot(f(x), x = -3\,..3, discont = true);$



> $l := \frac{3}{2}$:

> $a_0 := simplify\left(\frac{1}{l} \cdot int(f(x), x = 0..3)\right);$

$$a_0 := \frac{4}{3} \tag{7}$$

> $a_n := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), x = 0..3\right)\right)$ assuming $n :: posint$;

$$a_n := \frac{-5\,\pi\,n \sin\left(\frac{2\,n\,\pi}{3}\right) - 3\cos\left(\frac{2\,n\,\pi}{3}\right) + 3}{\pi^2\,n^2} \tag{8}$$

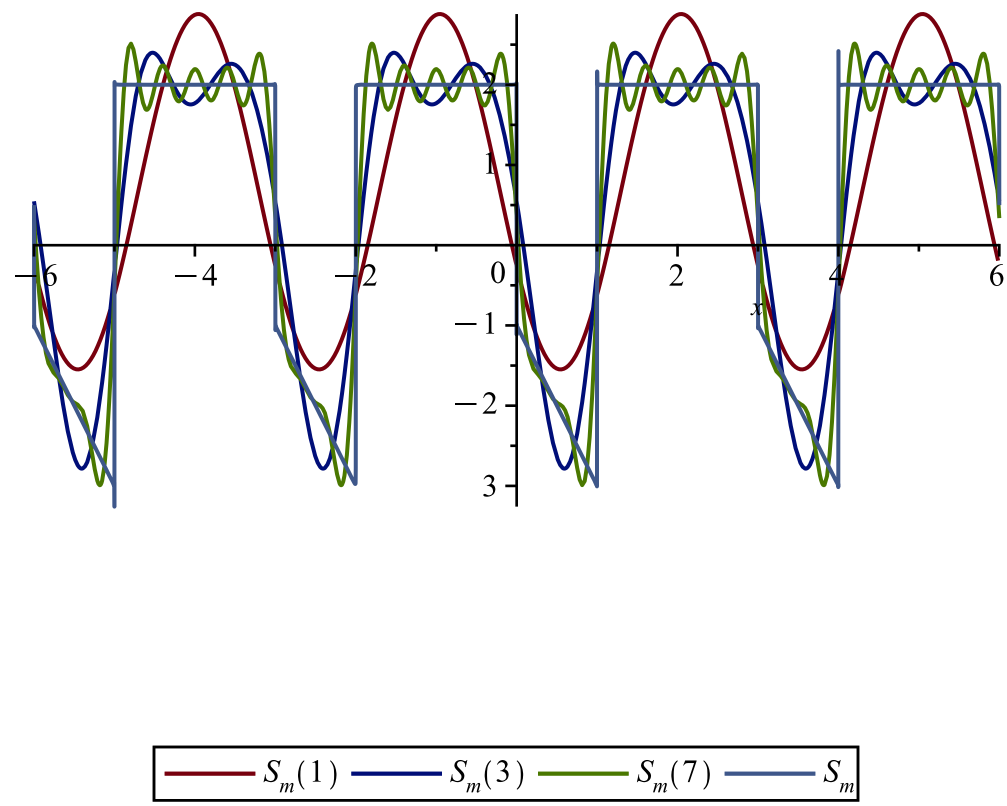
> $b_n := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), x = 0..3\right)\right)$ assuming $n :: posint$;

$$b_n := \frac{5\,\pi\,n \cos\left(\frac{2\,n\,\pi}{3}\right) - 3\,n\,\pi - 3\sin\left(\frac{2\,n\,\pi}{3}\right)}{\pi^2\,n^2} \tag{9}$$

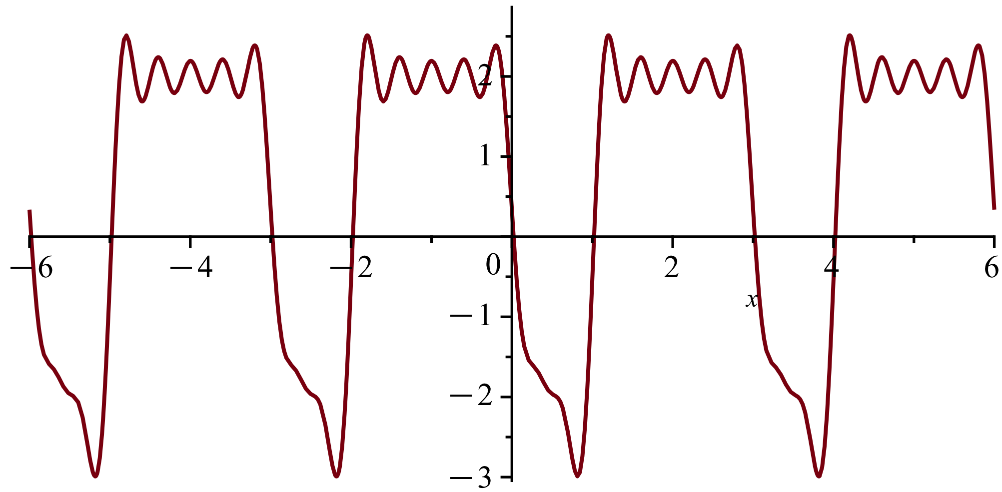
> $S_m := m \rightarrow \frac{a_0}{2} + sum\left(a_n \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right) + b_n \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), n = 1 \,..m\right);$

$$S_m := m \mapsto \frac{a_0}{2} + \sum_{n=1}^{m} \left(a_n \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right) + b_n \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right)\right) \tag{10}$$

> $fseries := fourier\_series(f, l, 0, 3):$
$points := plot([[1, 2], [3, 2], [0, 0]], style = point):$
$plot\left([S_m(1), S_m(3), S_m(7), fseries], x = -6 \,..6, legend = ['S_m(1)', 'S_m(3)', 'S_m(7)', 'S_m']\right);$



> **for** i **from** 1 **to** 7 **do**
$Ris[i] := plot\left([S_m(i)], x = -6 \,..6\right):$
end do:
$display([seq(Ris[i], i = 1 \,..7)], insequence = true);$



\> # *Задание 3*

 # Для графически заданной на промежутке функции как комбинации квадратичной и линейной постройте три разложения в тригонометрический ряд Фурье, считая, что функция определена:

 # − на полном периоде;

 # − на полупериоде (является четной);

 # − на полупериоде (является нечетной).

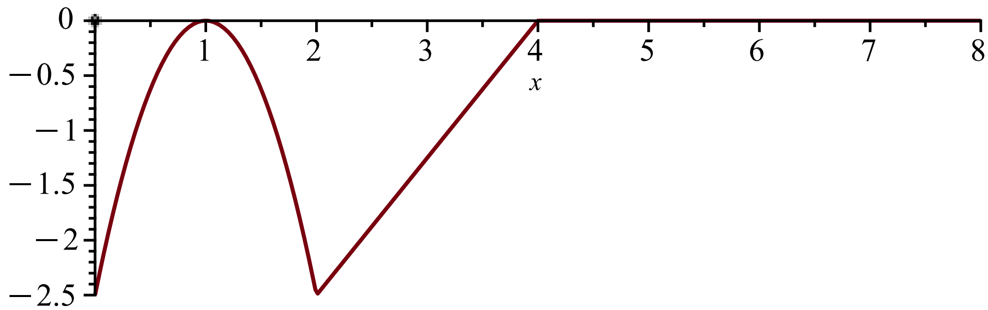
 # Убедитесь в правильности результата, проведя расчеты в системе Maple.

 # Постройте графики сумм полученных рядов на промежутке, превышающем длину заданного в 3 раза. Сравните с графиками порождающих их функций.

\> $f := x \to piecewise\left(0 < x < 2, -\frac{5}{2}(x-1)^2, 2 < x < 4, \frac{5}{4}x - 5\right);$

$$f := x \mapsto \begin{cases} -\frac{5 \cdot (x-1)^2}{2} & 0 < x < 2 \\ \frac{5 \cdot x}{4} - 5 & 2 < x < 4 \end{cases} \quad \textbf{(11)}$$

\> $plot(f(x), x = 0..8, discont = true);$



> $l := 2$:

> $a_0 := simplify\left(\frac{1}{l} \cdot int(f(x), x = 0\,..4)\right);$

$$a_0 := -\frac{25}{12} \tag{12}$$

> $a_n := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), x = 0\,..4\right)\right)$ assuming $n :: posint;$

$$a_n := \frac{5\,(-3 - 5\,(-1)^n)}{2\,\pi^2\,n^2} \tag{13}$$

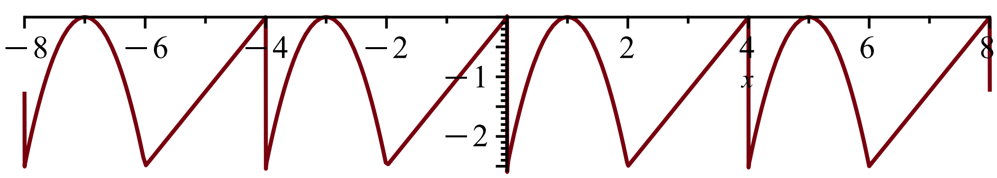
> $b_n := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), x = 0\,..4\right)\right)$ assuming $n :: posint;$

$$b_n := \frac{5\,\left(8 - \pi^2\,n^2 - 8\,(-1)^n\right)}{2\,\pi^3\,n^3} \tag{14}$$

> $S_m := m \rightarrow \frac{a_0}{2} + sum\left(a_n \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right) + b_n \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), n = 1\,..m\right);$

$$S_m := m \mapsto \frac{a_0}{2} + \sum_{n=1}^{m} \left(a_n \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right) + b_n \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right)\right) \tag{15}$$

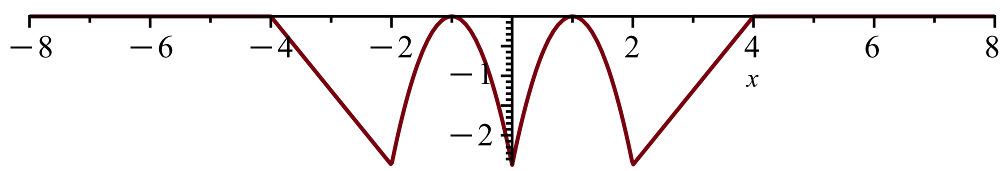
> $fseries := fourier\_series(f, l, 0, 4)$:
> $plot(fseries, x = -8..8, discont = true)$;



> $f := x \to piecewise\left(-4 < x < -2, -\frac{5}{4}x - 5, -2 < x < 0, -\frac{5}{2}(x+1)^2, 0 < x < 2, -\frac{5}{2}(x-1)^2, 2 < x < 4, \frac{5}{4}x - 5\right)$;

$$f := x \mapsto \begin{cases} -\frac{5 \cdot x}{4} - 5 & -4 < x < -2 \\ -\frac{5 \cdot (x+1)^2}{2} & -2 < x < 0 \\ -\frac{5 \cdot (x-1)^2}{2} & 0 < x < 2 \\ \frac{5 \cdot x}{4} - 5 & 2 < x < 4 \end{cases} \quad \textbf{(16)}$$

> $plot(f(x), x = -8..8)$;



> $l := 4$

> $a_0 := simplify\left(\frac{1}{l} \cdot int(f(x), x = -4..4)\right);$

$$a_0 := -\frac{25}{12} \tag{17}$$

> $a_n := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \cos\left(\frac{\text{Pi} \cdot n \cdot x}{l}\right), x = -4..4\right)\right)$ assuming $n :: posint;$

$$a_n := \frac{10\,\pi\,(-1)^n\,n - 50\,\pi\,n\cos\left(\frac{\pi\,n}{2}\right) - 40\,\pi\,n + 160\sin\left(\frac{\pi\,n}{2}\right)}{n^3\,\pi^3} \tag{18}$$

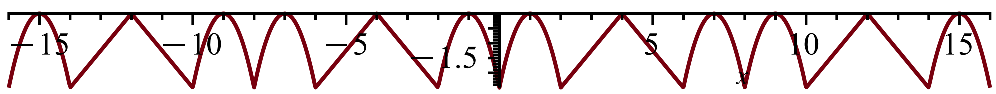
> $b_n := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \sin\left(\frac{\text{Pi} \cdot n \cdot x}{l}\right), x = -4..4\right)\right)$ assuming $n :: posint;$

$$b_n := 0 \tag{19}$$

> $S_m := m \rightarrow \frac{a_0}{2} + sum\left(a_n \cdot \cos\left(\frac{\text{Pi} \cdot n \cdot x}{l}\right) + b_n \cdot \sin\left(\frac{\text{Pi} \cdot n \cdot x}{l}\right), n = 1..m\right);$

$$S_m := m \mapsto \frac{a_0}{2} + \sum_{n=1}^{m}\left(a_n \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right) + b_n \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right)\right) \tag{20}$$

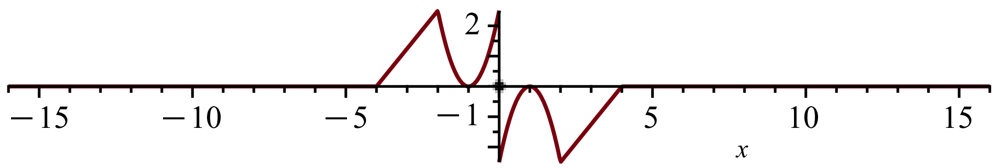
> $fseries := fourier\_series(f, l, -4, 4) :$
$plot(fseries, x = -16..16, discont = true);$



> $f := x \rightarrow piecewise\left(-4 < x < -2, \frac{5}{4}x + 5, -2 < x < 0, \frac{5}{2}(x+1)^2, 0 < x < 2, -\frac{5}{2}(x - 1)^2, 2 < x < 4, \frac{5}{4}x - 5\right);$

$$f := x \mapsto \begin{cases} \frac{5 \cdot x}{4} + 5 & -4 < x < -2 \\ \frac{5 \cdot (x+1)^2}{2} & -2 < x < 0 \\ -\frac{5 \cdot (x-1)^2}{2} & 0 < x < 2 \\ \frac{5 \cdot x}{4} - 5 & 2 < x < 4 \end{cases} \tag{21}$$

> $plot(f(x), x = -16..16, discont = true);$



> $l := 4$

$$l := 4 \tag{22}$$

> $a_0 := simplify\left(\frac{1}{l} \cdot int(f(x), x = -4 .. 4)\right);$

$$a_0 := 0 \tag{23}$$

> $a_n := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), x = -4 .. 4\right)\right)$ assuming $n :: posint;$

$$a_n := 0 \tag{24}$$

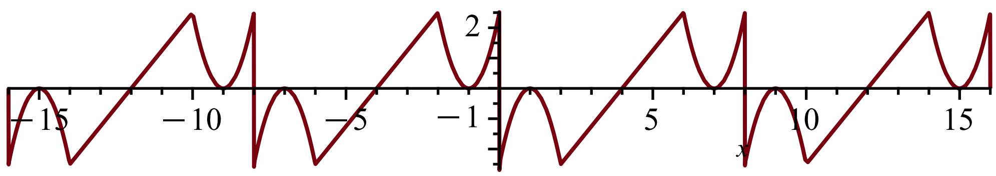
> $b_n := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), x = -4 .. 4\right)\right)$ assuming $n :: posint;$

$$b_n := \frac{-5\, n^2\, \pi^2 - 50\, n\, \pi \sin\left(\frac{n\, \pi}{2}\right) - 160 \cos\left(\frac{n\, \pi}{2}\right) + 160}{\pi^3\, n^3} \tag{25}$$

> $S_m := m \rightarrow \frac{a_0}{2} + sum\left(a_n \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right) + b_n \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), n = 1 .. m\right);$

$$S_m := m \mapsto \frac{a_0}{2} + \sum_{n=1}^{m} \left(a_n \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right) + b_n \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right)\right) \tag{26}$$

> *fseries* := *fourier_series*(*f*, *l*, −4, 4) :
plot(*fseries*, *x* = −16 ..16, *discont* = *true*);

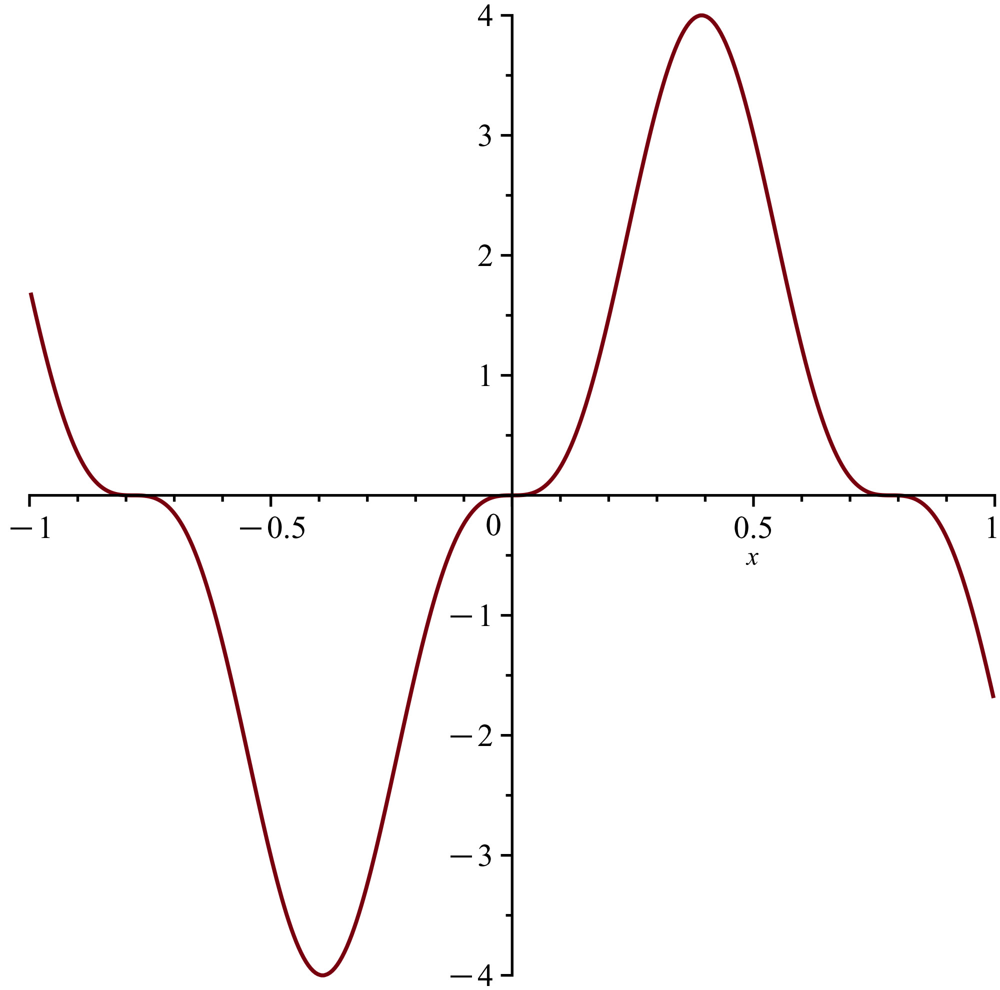


> # *Задание 4*
Разложите функцию в ряд Фурье по многочленам Лежандра и
#\` \`*Чебышёва на промежутке* [−*1, 1*] . *Создайте пользовательские процедуры, осу* −;
ществляющие построение частичной суммы ряда для абсолютно интегрируе −
мой функции по этим ортогональным полиномам.

> *with*(*orthopoly*) :
$f := x \rightarrow 4 \cdot \sin^3(4\,x)$;

$$f := x \mapsto 4 \cdot \sin(4 \cdot x)^3 \qquad \textbf{(27)}$$

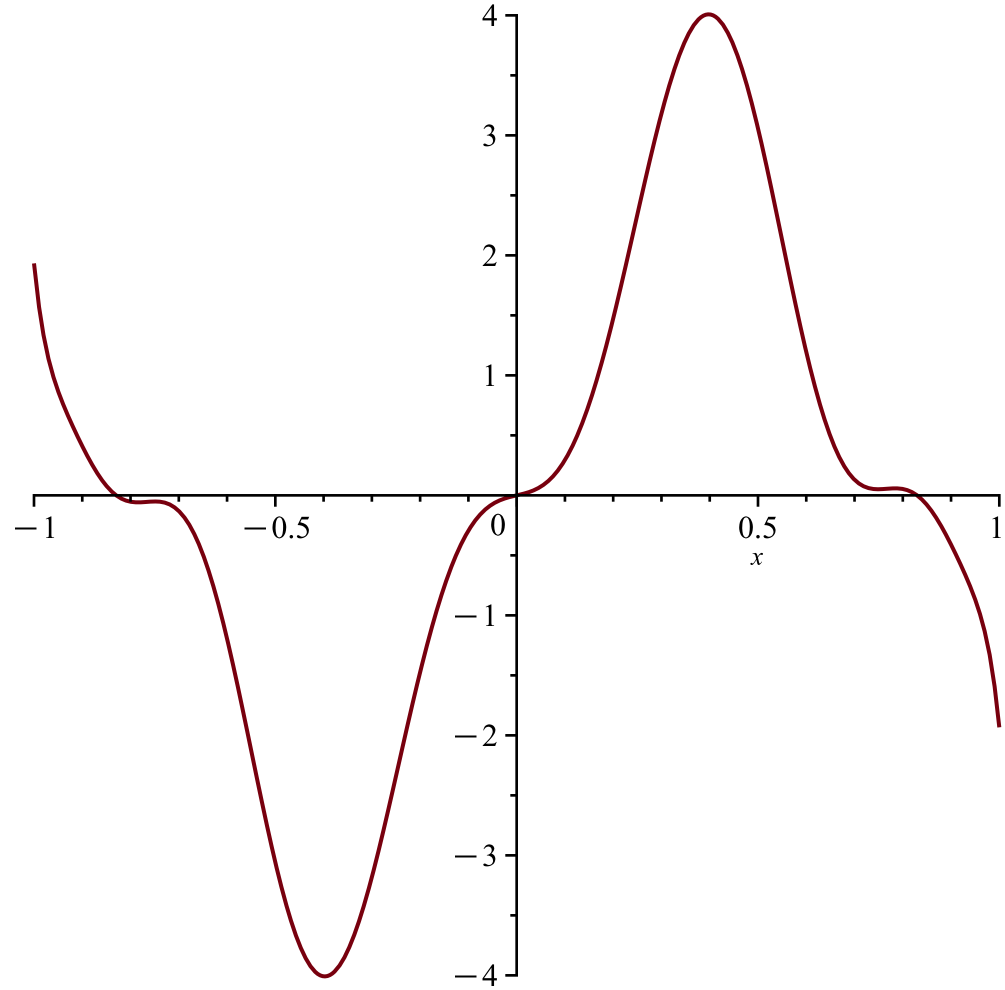
> *plot*(*f*(*x*), *x* = −1 ..1, *discont* = *true*);



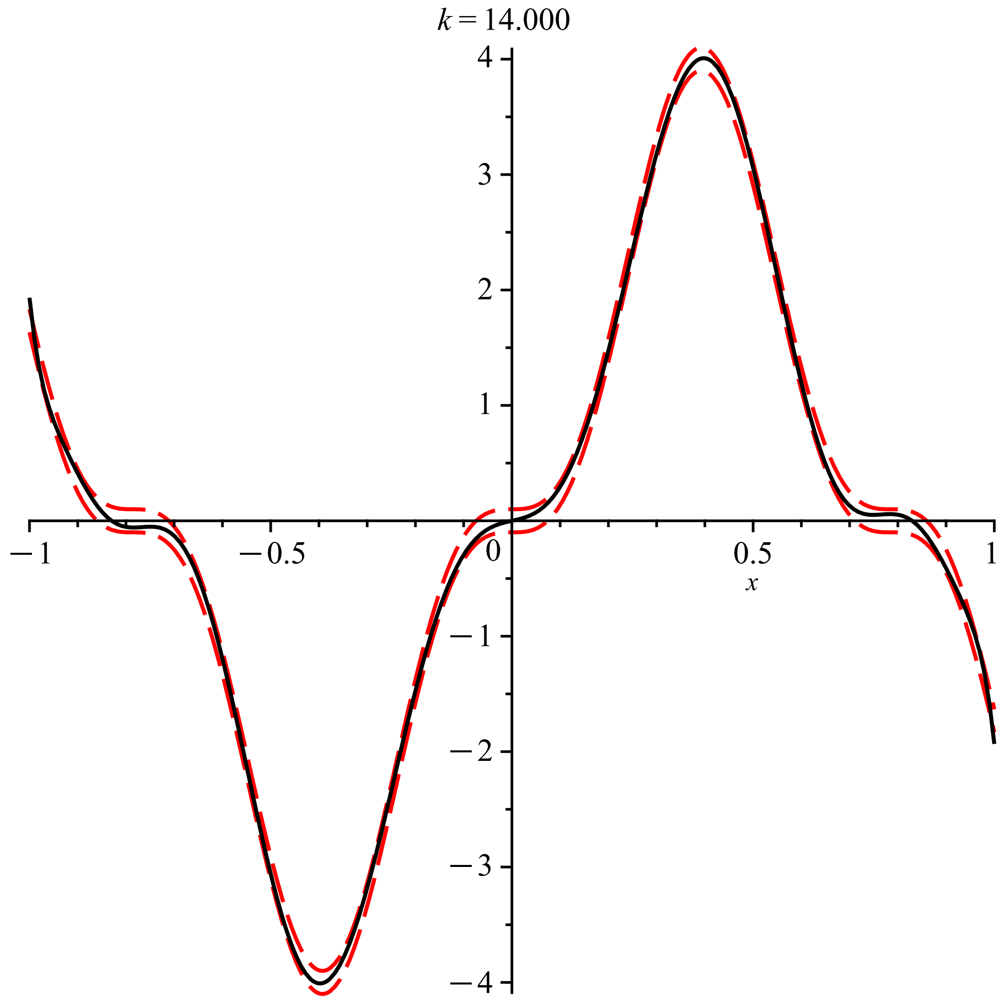
> $legendre\_polynomial\_series := \textbf{proc}(g)$ **description**
"Convert a function to a series using Legendre polynomials" :
$\textbf{return}\ \left(k \rightarrow sum\left(\dfrac{int(g(x) \cdot P(n, x), x = -1 ..1)}{int\left(P^2(n, x), x = -1 ..1\right)} \cdot P(n, x), n = 0 ..k\right)\right);$
end proc:

> $lsum := legendre\_polynomial\_series(f)$:

> $plot(lsum(14), x = -1 ..1);$



> $animate(plot, [[f(x) - 0.1, f(x) + 0.1, lsum(k)], x = -1..1, linestyle = [3, 3, 1], color = [red, red, black]], k = 0..14);$

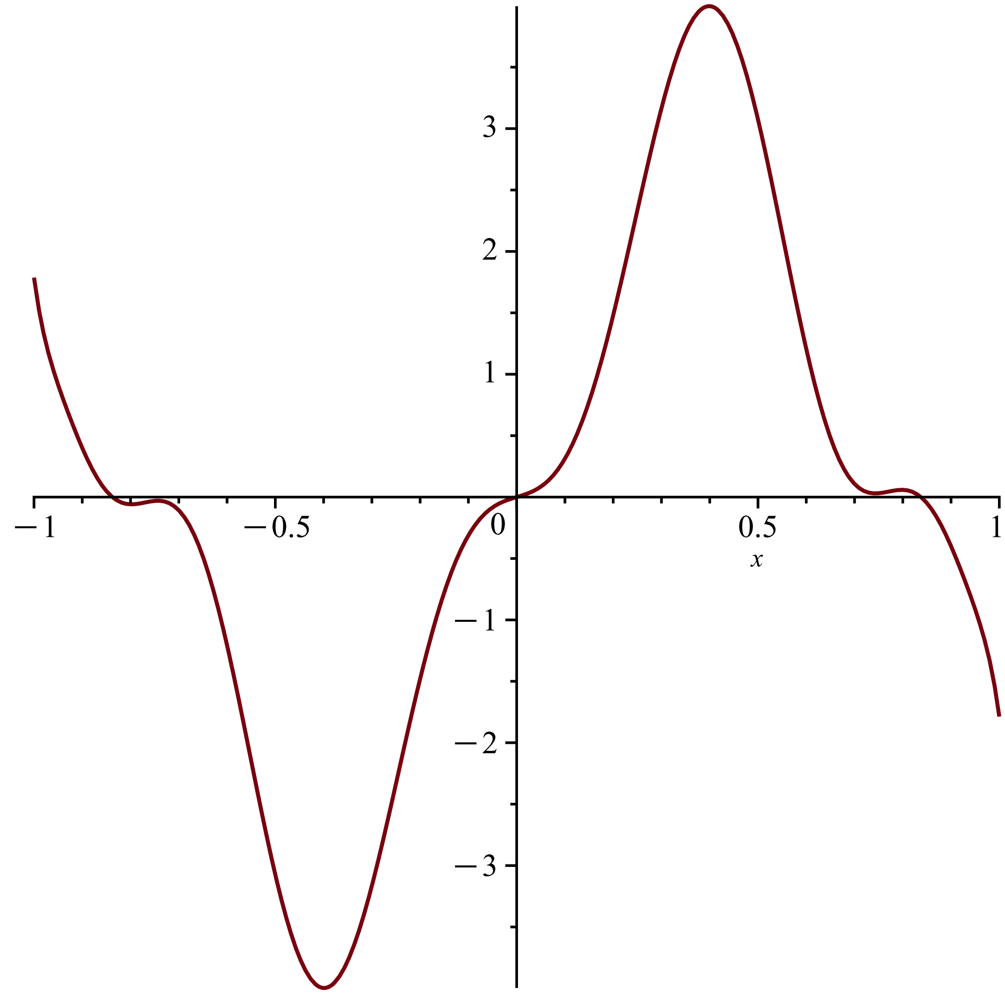


> *chebyshev_polynomial_series* := **proc**(y) **description**
"Convert a function to a series using Chebyshev polynomials" :

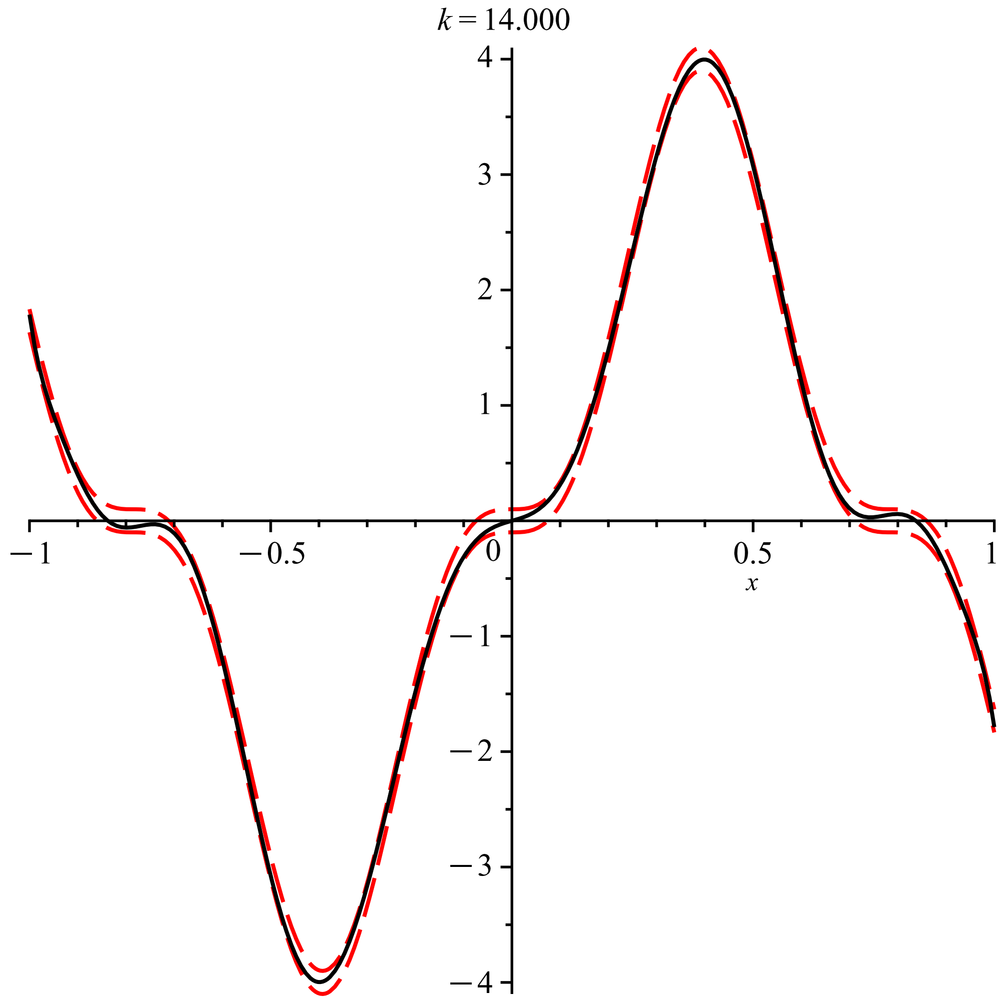
$$\textbf{return}\left(k \rightarrow sum\left(\frac{int\left(\frac{y(x)\cdot T(J,x)}{\sqrt{1-x^2}}, x=-1..1\right)}{int\left(\frac{T^2(J,x)}{\sqrt{1-x^2}}, x=-1..1\right)}\cdot T(J,x), J=0..k\right)\right);$$

end proc:

> $csum := chebyshev\_polynomial\_series(f)$:
$plot(csum(14), x=-1..1)$;



> $animate(plot, [[f(x) - 0.1, f(x) + 0.1, csum(k)], x = -1..1, linestyle = [3, 3, 1], color = [red, red, black]], k = 0..14);$



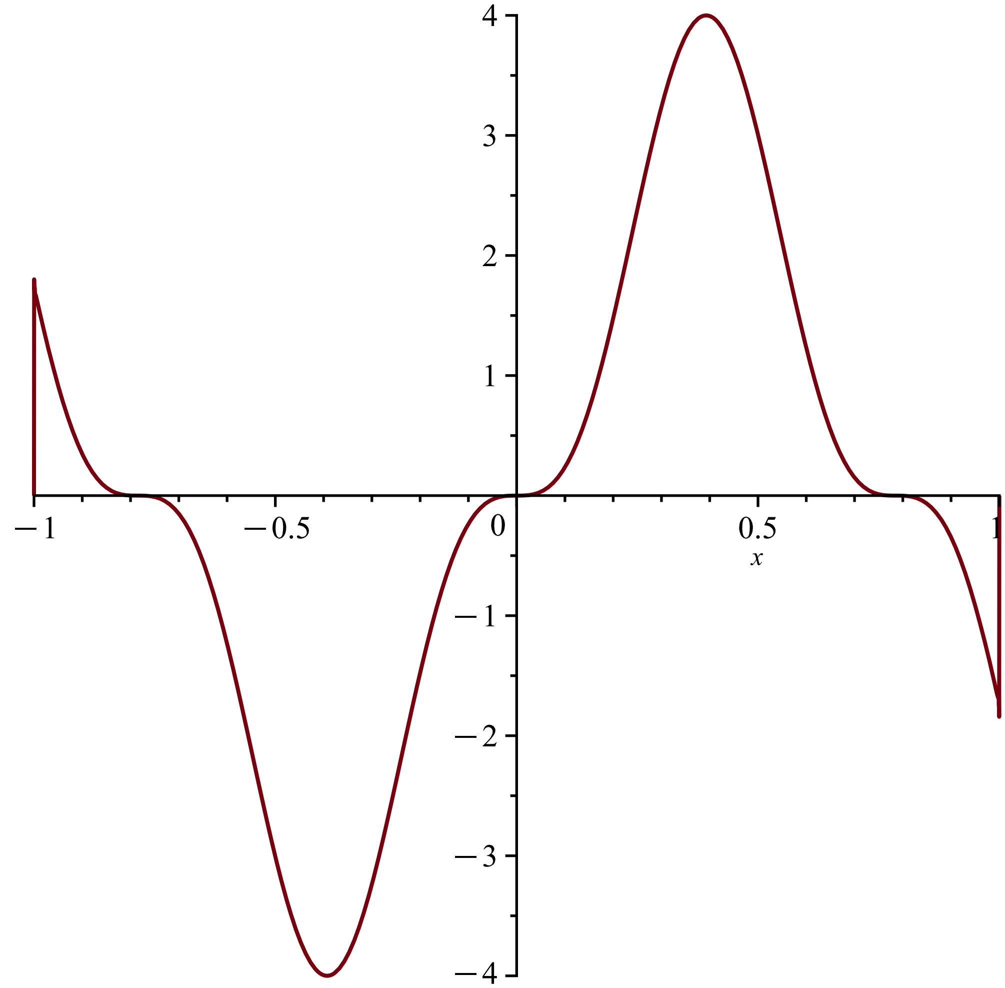
> $b_n := simplify(2\cdot int(f(x)\cdot\sin(\mathrm{Pi}\cdot n\cdot x), x = 0..1))$ assuming $n :: posint$;

$$b_n := -\frac{6\, n\, \pi \left(\sin(4)\, \pi^2\, n^2 - \frac{\sin(12)\, \pi^2\, n^2}{3} - 144\, \sin(4) + \frac{16\, \sin(12)}{3}\right) (-1)^n}{\pi^4\, n^4 - 160\, n^2\, \pi^2 + 2304} \quad \mathbf{(28)}$$

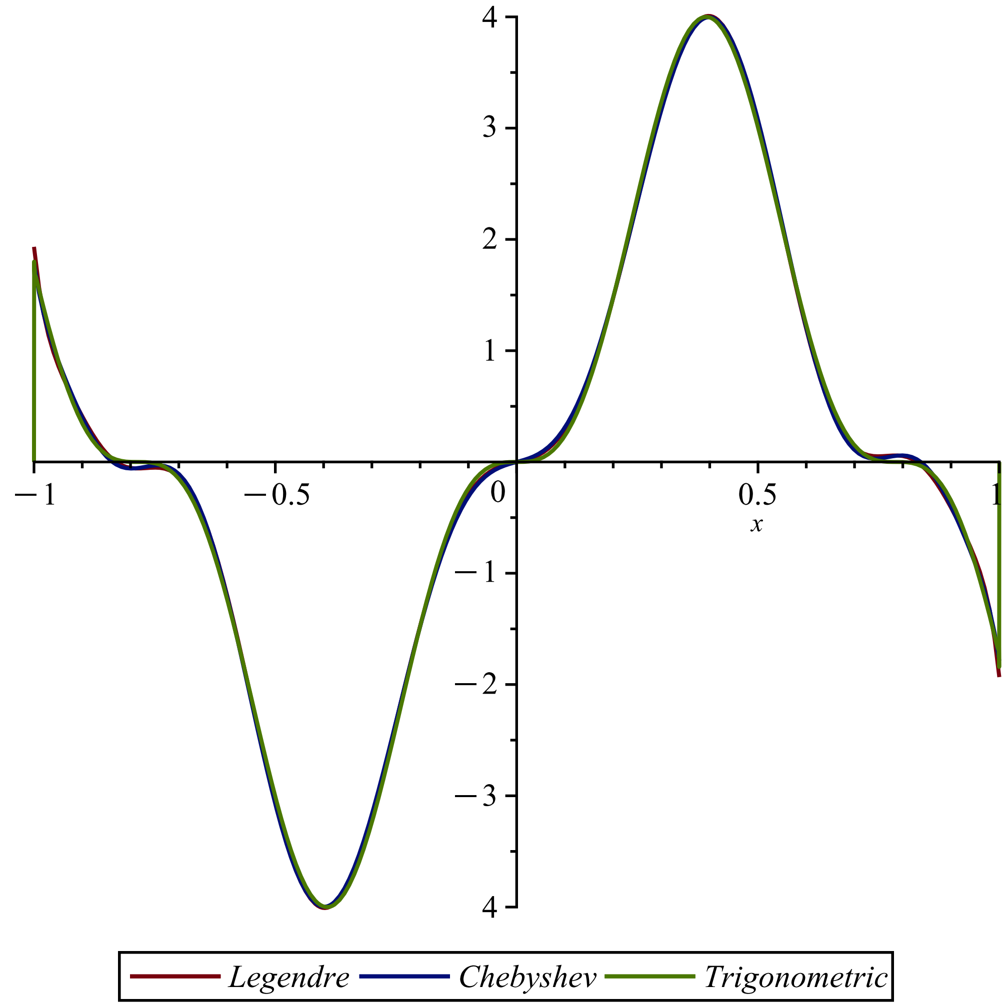
> $S_m := k \rightarrow sum(b_n\cdot\sin(\mathrm{Pi}\cdot n\cdot x), n = 1..k)$;

$$S_m := k \mapsto \sum_{n=1}^{k} b_n\cdot\sin(\pi\cdot n\cdot x) \quad \mathbf{(29)}$$

> $plot(S_m(10000), x = -1..1)$;



> $plot\left(\left[lsum(14), csum(14), S_m(10000)\right], x=-1..1, legend=\left['Legendre','Chebyshev', 'Trigonometric'\right]\right);$

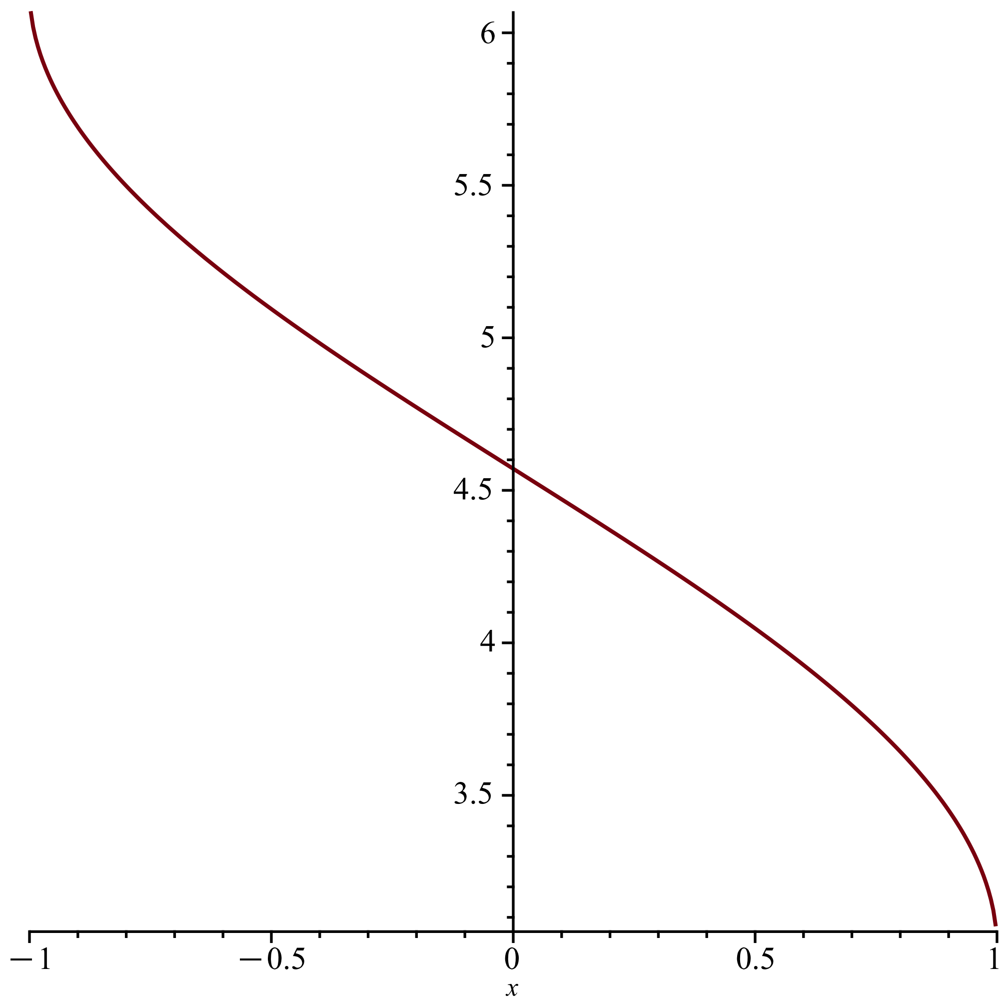


>

> $f := x \rightarrow \arccos(x) + 3;$

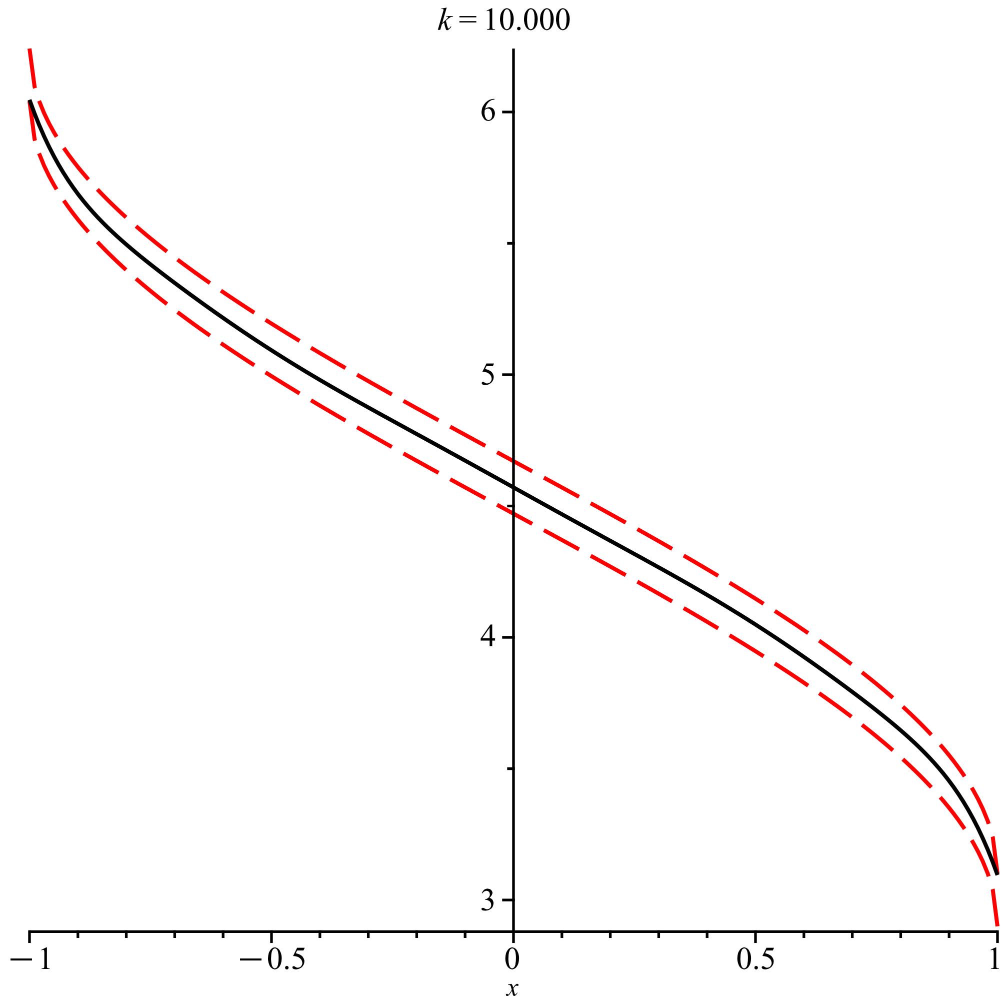
$$f := x \mapsto \arccos(x) + 3 \quad \textbf{(30)}$$

> $plot(f(x), x = -1..1, discont = true);$

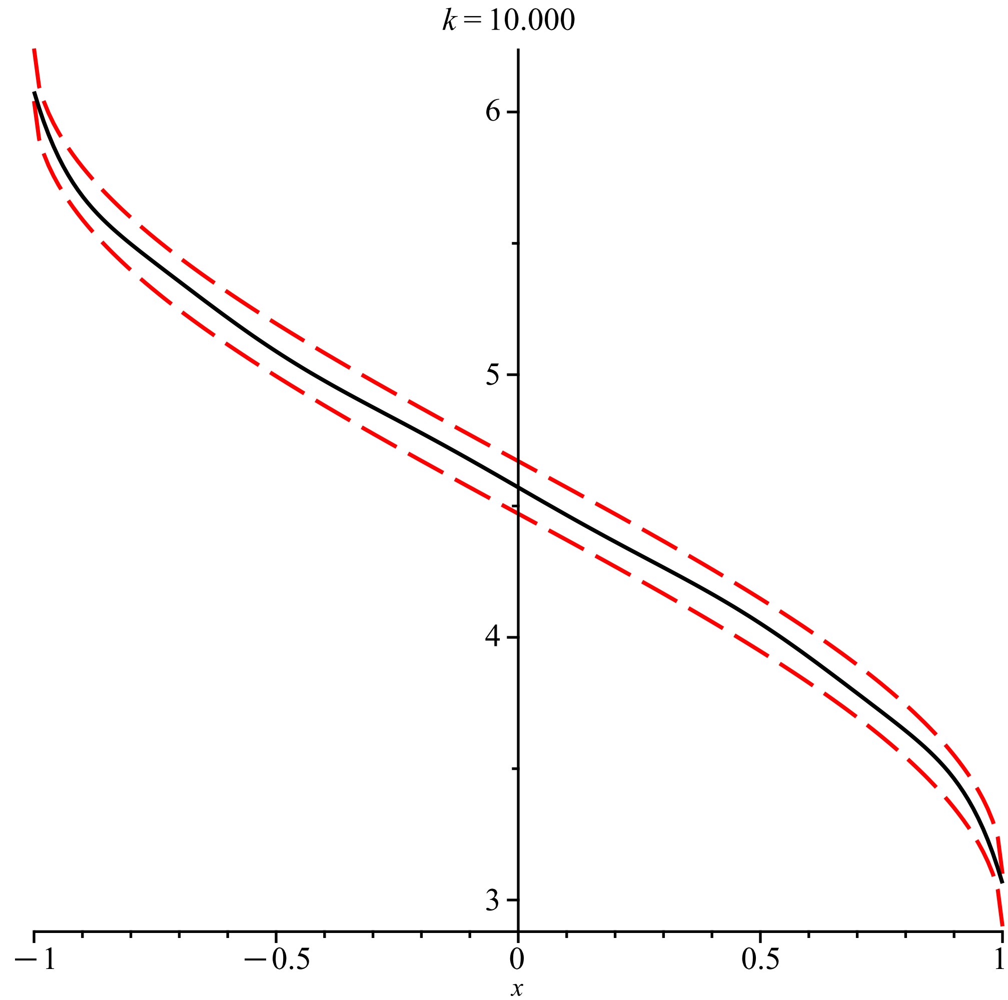


> $lsum := legendre\_polynomial\_series(f)$:

> $animate(plot, [[f(x) - 0.1, f(x) + 0.1, lsum(k)], x = -1..1, linestyle = [3, 3, 1], color = [red, red, black]], k = 0..10)$;



> $csum := chebyshev\_polynomial\_series(f)$:
> $animate(plot, [[f(x) - 0.1, f(x) + 0.1, csum(k)], x = -1..1, linestyle = [3, 3, 1], color = [red, red, black]], k = 0..10);$



> $a_0 := int(f(x), x=-1..1);$

$$a_0 := \pi + 6 \tag{31}$$

> $a_n := simplify(\ int(f(x) \cdot \cos(\mathrm{Pi} \cdot n \cdot x), x=-1..1))$ assuming $n :: posint;$

$$a_n := 0 \tag{32}$$

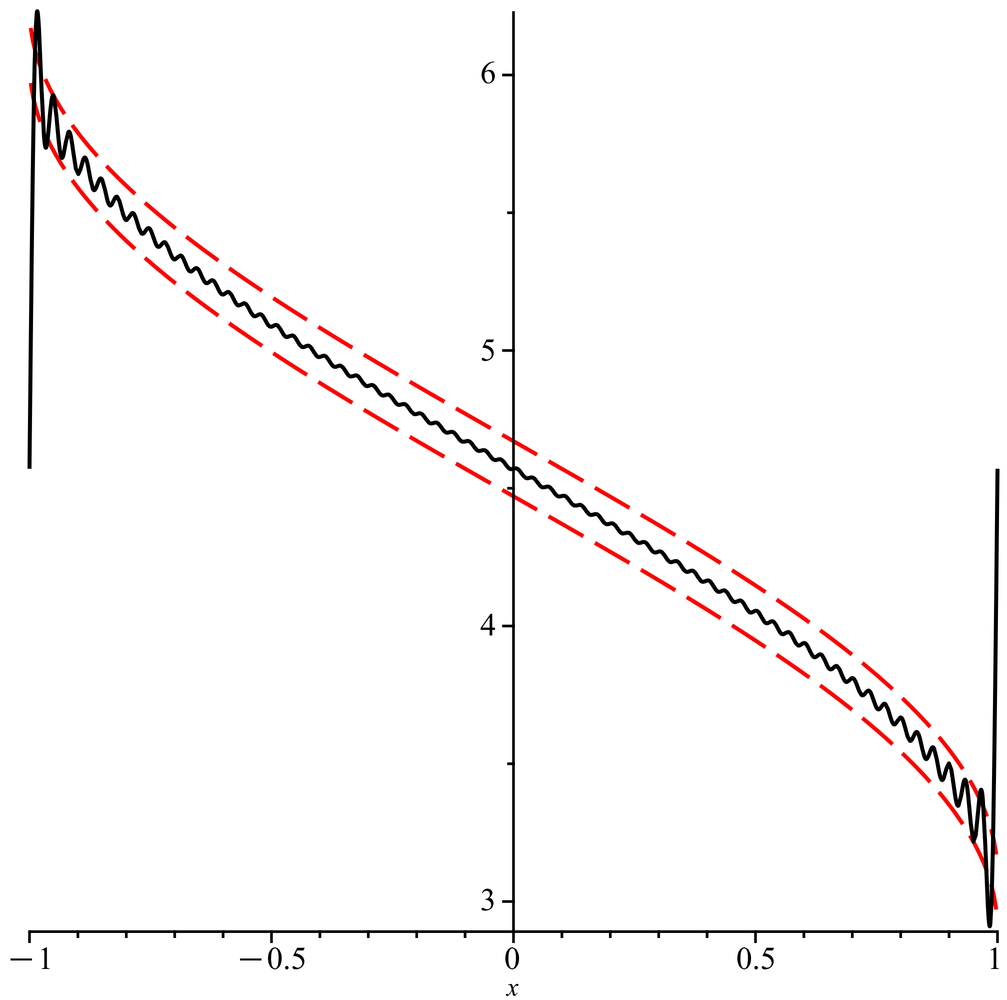
> $b_n := simplify(\ int(f(x) \cdot \sin(\mathrm{Pi} \cdot n \cdot x), x=-1..1))$ assuming $n :: posint;$

$$b_n := \int_{-1}^{1} (\arccos(x) + 3) \sin(\pi\, n\, x)\, \mathrm{d}x \tag{33}$$

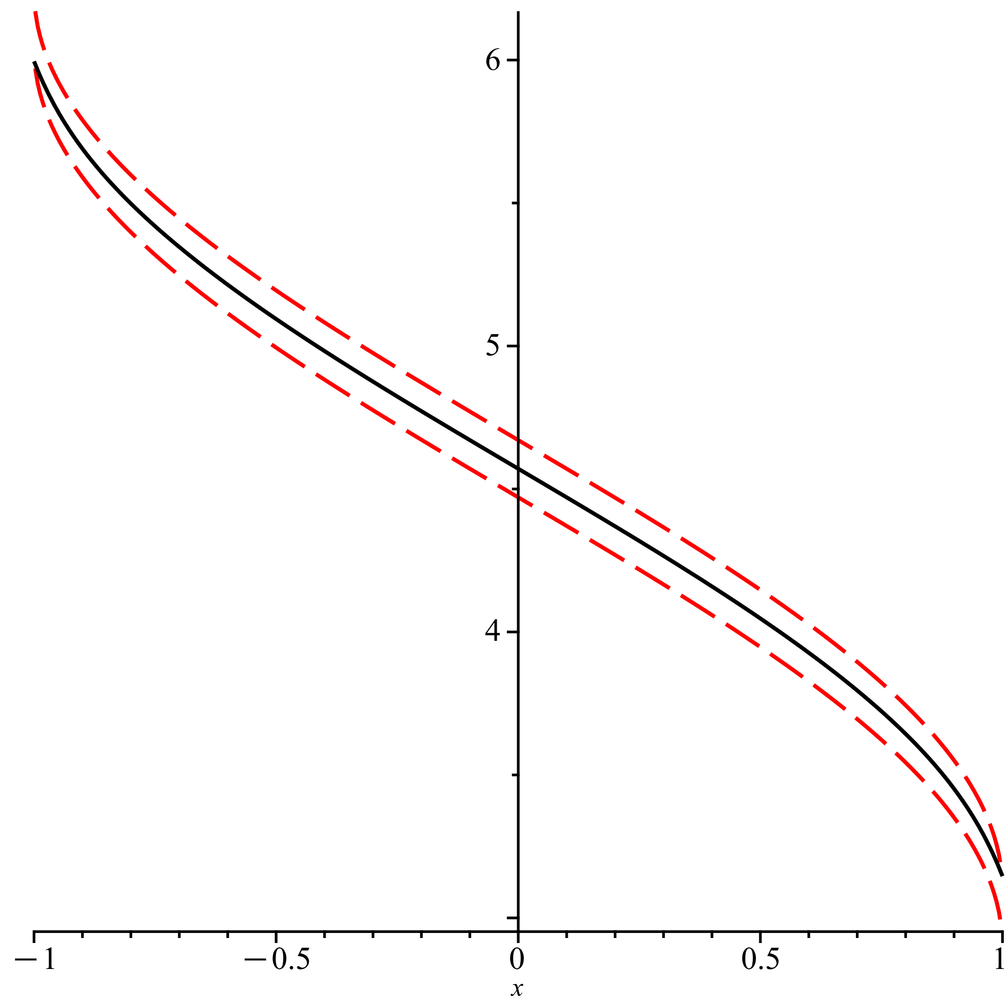
> $S_m := k \rightarrow \frac{a_0}{2} + sum\left(b_n \cdot \sin(\mathrm{Pi} \cdot n \cdot x), n=1..k\right);$

$$S_m := k \mapsto \frac{a_0}{2} + \left(\sum_{n=1}^{k} b_n \cdot \sin(\pi \cdot n \cdot x)\right) \tag{34}$$

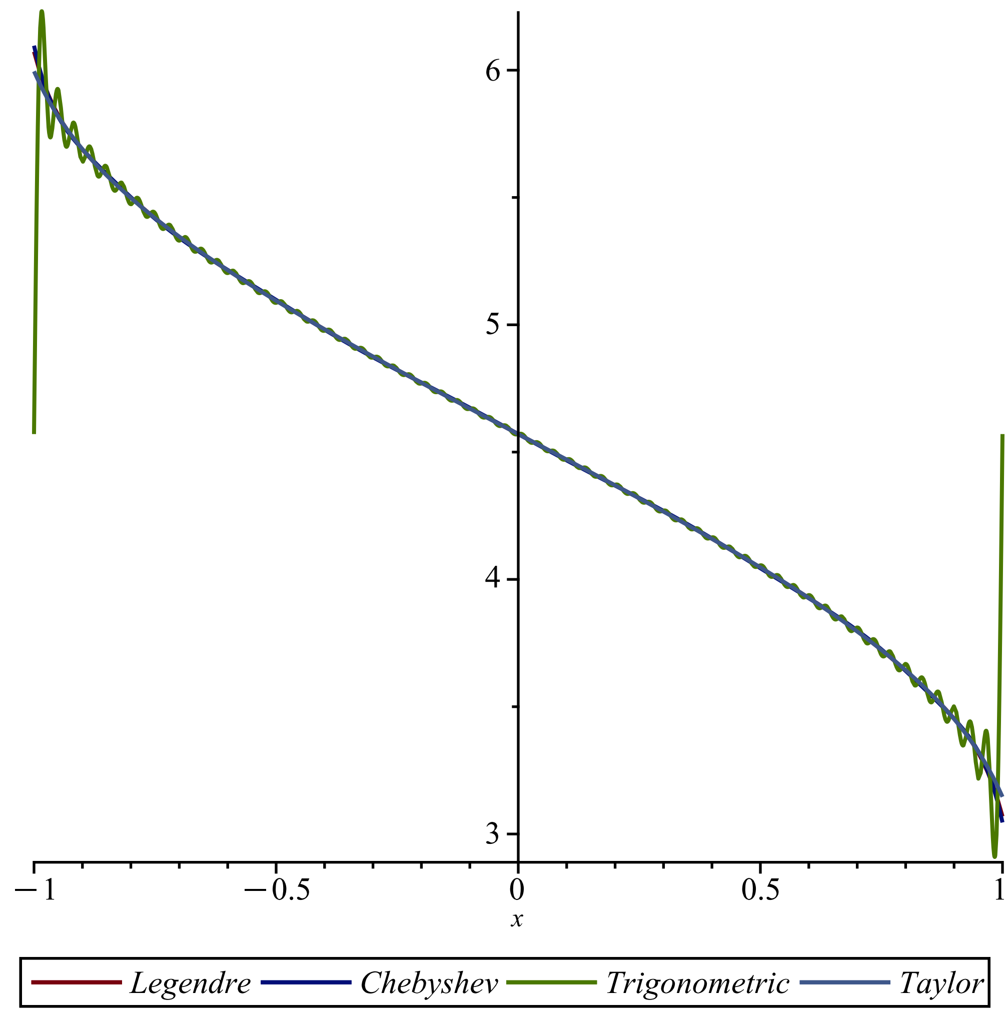
> $plot\left(\left[f(x) - 0.1, f(x) + 0.1, S_m(60)\right], linestyle = [3, 3, 1], color = [red, red, black]\right);$



> $tsum := k \rightarrow convert(taylor(f(x), x=0, k), polynom):$
$plot([f(x) - 0.1, f(x) + 0.1, tsum(30)], linestyle = [3, 3, 1], color = [red, red, black]);$



> $plot\left(\left[lsum(14), csum(14), S_m(60), tsum(30)\right], x=-1..1, legend=\left['Legendre','Chebyshev','Trigonometric','Taylor'\right]\right);$



6
5
4
3
−1
−0.5
0
0.5
1
x
Legendre
Chebyshev
Trigonometric
Taylor

>