



# An evaluation of empirically-based models for predicting energy performance of vapor-compression water chillers

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## ARTICLE INFO

### Article history:

Received 4 February 2010

Received in revised form 21 April 2010

Accepted 3 May 2010

### Keywords:

Water chiller

Performance

Energy consumption

Model

Accuracy

Comparison

## ABSTRACT

This paper presents an evaluation of six empirically-based models for predicting water chiller energy performance using over 1000 chiller data sets from chiller manufacturers and field measurements. The data sets comprise three broad classifications, including (1) constant condenser and constant chilled water flow, (2) constant condenser and variable chilled water flow, and (3) variable condenser and variable chilled water flow. The regression parameters for each performance model are obtained using least squares method. The criteria for evaluating the predictive ability of models are based on the coefficient of variation of root-mean-square error (CV). Results show that among the six empirically-based performance models for water chillers in this study, the bi-quadratic regression model (CV = 2.2%) and the multivariate polynomial regression model (CV = 2.25%) have the best prediction accuracy for all kinds of data sets. The results of this study can be used as a reference for selecting empirically-based models for the purposes of energy analysis, performance prediction, evaluation of energy-efficiency improvements, and fault detection and diagnosis of water chillers.

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## 1. Introduction

Vapor-compression water chillers have been widely used to cool water or secondary coolant for air-conditioning and refrigerating applications in both commercial and industrial fields. Fig. 1 shows that the main components of a vapor-compression water chiller include compressor and its driver, condenser, throttling device, and evaporator (liquid cooler). The coefficient of performance (COP) for water chillers is defined as the ratio of the evaporator cooling capacity to the compressor input power. Practically, it is more convenient to express chiller performance COP in terms of readily measured water-side data rather than refrigerant-side data. This is especially true for the purposes of performance prediction, evaluation of energy-efficiency improvements, and fault detection and diagnosis of water chillers [1–8]. Measurable water-side data include condenser inlet and outlet water temperatures, evaporator inlet and outlet water temperatures, and condenser and evaporator water flowrates. Therefore, developing chiller performance model by using water-side data has been a subject of many studies over the last decade.

Previous empirically-based models for water chillers can generally be classified into two categories: gray-box (semi-empirical) and black-box (empirical) models [3,9]. In a gray-box approach,

the functional form of the chiller model allows the parameter estimates be traced to actual physical principles that govern the performance of the water chillers being modeled. Model parameters or fitting coefficients are determined using a regression method applied to a set of training data obtained from chiller manufacturers, laboratory and field measures. Examples of gray-box studies include Gordon et al. [10], Ng et al. [11], and Lee [5]. The functional form of black-box models is developed by either statistical or non-statistical methods. However, the estimated model parameters of the models have no physical interpretations. Black-box models often require less time and effort to develop and use compared to gray-box approaches, but they cannot be used to extrapolate performance beyond the data range for which they were developed [9]. A number of researchers have used a black-box approach to predict performance of water chillers [1,2,4,12,13].

A review of the literature shows that comprehensive comparison studies on empirically-based model for predicting performance of water chillers are still lacking. To fill this gap, this study evaluates the suitability of empirically-based performance models for water chillers available in the literature. The empirically-based performance models selected in the study include the (1) simple linear regression model [13]; (2) bi-quadratic regression model [12]; (3) multivariate polynomial regression model [2,4,14]; (4) Gordon–Ng universal model [15]; (5) Gordon–Ng simplified model [16]; and (6) Lee's simplified model [5]. This study uses over 1000 chiller data sets from major chiller manufacturers and field measurement to test these models in previous studies. The data

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## Nomenclature

<b>B</b>	vector of estimated parameters $\beta_i$ of the regression models	$\hat{Y}$	vector of observations on the estimated dependent variable
COP	coefficient of performance	$Z$	95% confidence interval
CV	coefficient of variation of root-mean-square error, defined by Eq. (11).	<b>Greek symbols</b>	
CV <sub>overall</sub>	overall coefficient of variation of root-mean-square error, defined by Eq. (14).	$\varepsilon$	vector of errors between target and fitted outputs
<b>E</b>	vector of residual	$\beta$	estimated parameters
$N$	total number of data sets	<b>Subscripts</b>	
$n$	number of data sets	$ci$	water inlet to condenser
$\dot{Q}_e$	chiller cooling capacity (kW)	$i$	independent variable ( $i = 1, \dots, m$ )
RMSE	root-mean-square error, defined by Eq. (12).	$m$	measured
$T$	temperature (K or °C)	$p$	predicted
$V$	mass flow rate ( $\text{kg s}^{-1}$ )	$wi$	water inlet to evaporator
$\dot{W}_c$	electrical power input to compressor	$wo$	water outlet from evaporator
$X$	vector of the independent variables		
$Y$	vector of observations on the true dependent variable		

sets comprise three broad groups, including (1) constant condenser and constant chilled water flow, (2) constant condenser and variable chilled water flow, and (3) variable condenser and variable chilled water flow. The regression parameters or fitting coefficients for each empirically-based model are estimated using the least squares method. The criteria for evaluating model suitability are based on the coefficient of variation of root-mean-square error (CV), an assessment for predictive ability. The results obtained in this work could be served as a reference when selecting an empirically-based performance model for vapor-compression water chiller in the purposes of energy analysis, improvements evaluation, and FDD strategy development.

## 2. Description of empirically-based performance models for water chillers

The selection of a performance model is based on the following considerations: (1) model prediction accuracy, (2) training data requirements (number of sensor points and length), (3) effort needed to calibrate or train the model, (4) generality of the model, (5) computational requirements, and (6) the ability to physically interpret the model coefficients, i.e., their physical relevance [3,17]. This study consider six empirically-based models: (1) simple linear regression model [13]; (2) bi-quadratic regression model [12]; (3) multivariate polynomial regression model [2,4,14]; (4) Gordon–Ng universal model [11,15]; (5) Gordon–Ng simplified model [16]; and (6) Lee’s simplified model [5]. The first three models are black-box models, while the other three models are gray-box models. The main difference between these two types of models is that the estimated parameters of gray-box models may be interpreted physically, but the black-box models cannot [2]. The following sections briefly introduce the individual characteristics for each model.

### 2.1. Simple linear regression model (SL model)

The simplest empirical model for predicting chiller’s COP is the black-box simple linear regression model [13]. The input variables for this model include the chiller cooling capacity, the evaporator inlet water temperature, and the condenser inlet water temperature. The three parameters in the regression model are linear, but they have no physical meaning. The functional form is shown as follows.

$$\text{COP} = \beta_1 \dot{Q}_e + \beta_2 T_{wi} + \beta_3 T_{ci} \quad (1)$$

### 2.2. Bi-quadratic regression model (BQ model)

The bi-quadratic regression model [12–13] has only two independent variables, the chiller cooling capacity and condenser inlet water temperature, and nine regression parameters with no physical relevance. This model was originally developed to determine the input work to chiller compressor, and was applied to manufacturer’s performance data sets of several kinds of chillers (centrifugal, screw, and reciprocating) with good prediction accuracy. Swider [13] utilized this BQ model to predict chiller performance with the following functional form.

$$\frac{1}{\text{COP}} = \beta_0 + \beta_1 \frac{1}{\dot{Q}_e} + \beta_2 \dot{Q}_e + \beta_3 \frac{T_{ci}}{\dot{Q}_e} + \beta_4 \frac{T_{ci}^2}{\dot{Q}_e} + \beta_5 T_{ci} + \beta_6 \dot{Q}_e T_{ci} + \beta_7 T_{ci}^2 + \beta_8 \dot{Q}_e T_{ci}^2 \quad (2)$$

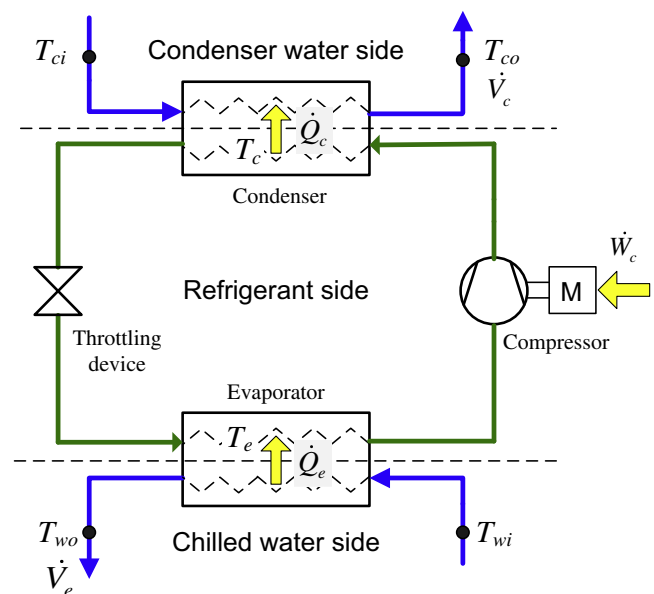


Fig. 1. Schematic of a vapor-compression water chiller.

### 2.3. Multivariate polynomial regression model (MP model)

The multivariate polynomial regression model [2,4,14] is similar to the BQ Model, but uses the evaporator inlet water temperature as additional independent variable, and has 10 regression parameters. This empirically-based model is a black-box model, and was previously used by the DOE (Department Of Energy, USA) software to serve as a performance prediction model for water chillers. The MP model takes the following form.

$$\text{COP} = \beta_0 + \beta_1 \dot{Q}_e + \beta_2 T_{wi} + \beta_3 T_{ci} + \beta_4 \dot{Q}_e^2 + \beta_5 T_{wi}^2 + \beta_6 T_{ci}^2 + \beta_7 \dot{Q}_e T_{wi} + \beta_8 \dot{Q}_e T_{ci} + \beta_9 T_{wi} T_{ci} \quad (3)$$

### 2.4. Gordon–Ng universal model (GNU model)

The Gordon–Ng universal model [11,15] is a simple and analytical performance model of chiller, based on the laws of thermodynamics and heat transfer. The model applies to both unitary and large chillers operating under steady-state conditions. Evaluations by Reddy and Anderson [2], Jiang and Reddy [3], and Sreedharan and Haves [17] have shown this model to be very accurate for a large number of chiller types and sizes. This model correlates the chiller COP (dependent variable) with three input independent variables: the water inlet temperature to condenser, water inlet temperature to evaporator, and the cooling capacity of evaporator. The GNU model is a three-regression-parameter model that takes the following functional form:

$$\frac{T_{wi}}{T_{ci}} \left( 1 + \frac{1}{\text{COP}} \right) - 1 = \beta_1 \frac{T_{wi}}{\dot{Q}_e} + \beta_2 \frac{T_{ci} - T_{wi}}{T_{ci} \dot{Q}_e} + \beta_3 \frac{\dot{Q}_e}{T_{ci}} \left( 1 + \frac{1}{\text{COP}} \right) \quad (4)$$

### 2.5. Gordon–Ng simplified model (GNS model)

The Gordon–Ng simplified model [16] has three fitting coefficients that can be directly determined through linear regression. Each of these coefficients represents a physical property of the chiller. Its functional form is as follows.

$$\frac{1}{\text{COP}} = -1 + \frac{T_{ci}}{T_{wo}} + \frac{1}{\dot{Q}_e} \left[ -\beta_1 + \beta_2 T_{ci} - \beta_3 \frac{T_{ci}}{T_{wo}} \right] \quad (5)$$

The GNS model was utilized in the ASHRAE Research Project 827 [18] and is one of the chiller models recommended by ASHRAE Guideline 14. Phelan et al. [19] validated this model for centrifugal chillers with field data, and Gordon and Ng [10] validated it for reciprocating chillers using manufacturers' data.

### 2.6. Lee's simplified model (LS model)

Lee [5] developed this model, based on the first and second laws of thermodynamics and the NTU- $\epsilon$  method of heat exchanger, to predict the performance of a screw water chiller under various operating conditions. Lee conducted a series of experiments to verify this model, and the results indicate that a comparison between the predicted value and the measured data yields an RMSE (absolute root-mean-square-error) of around 0.0524, and an R-RMSE (relative root-mean-square-error) of about 1.43%. This model has the similar functional form as the GNS model, and has three input variables, three-regression-parameters with physical significance. The model takes the following form.

$$\frac{1}{\text{COP}} = -1 + \frac{T_{ci}}{T_{wi}} + \frac{1}{\dot{Q}_e} \left[ -\beta_1 + \beta_2 T_{ci} - \beta_3 \frac{T_{ci}}{T_{wi}} \right] \quad (6)$$

## 3. Description of chiller data sets

To evaluate the suitability of various performance models for water chillers, this study has analyzed three broad classes of data sets: (1) constant condenser and constant chilled water flow, (2) constant condenser and variable chilled water flow, and (3) variable condenser and variable chilled water flow. A total of 1033 data sets came from laboratory test data and in-situ field-measured data.

Table 1 provides details of centrifugal chiller data sets including the data grouping, the designation for chiller unit, the variance in temperatures and flow rates, and the amount of data. The symbol "C" indicates that the parameter is constant while "V" means that the parameter is variable. Table 2 provides details on the variation range of the model input variables for chiller data sets, including the cooling water inlet temperature, chilled water inlet and outlet temperatures, flow rate, and loading fraction for capacity modulation, etc. The plot of COP vs. cooling capacity for all data sets adopted in the study is shown in Fig. 2. Based on the uncertainty calculation method proposed by Moffat [20], the uncertainties of cooling capacity, power consumption, and COP are 2.83%, 1.0%, and 3%, respectively.

As shown in Table 1, Group I data consists of the operating data from six water chillers with constant condenser and constant chilled water flow rates. The loading fraction of chiller capacity varies from 10% to 100%. The rated chiller capacities range from 500 to 1250 RT. The Group II data includes the data sets from three units of 500 to 1000 RT water chillers. These chillers all meet cooling capacity requirement through regulating the chilled water flow

**Table 1**  
Details of chiller data sets.

Group	Chiller unit	Cooling-water side			Chilled-water side			Data sets	Remarks (RT)
		$T_{ci}$	$T_{co}$	$V_c$	$T_{wi}$	$T_{wo}$	$V_w$		
I <sup>a</sup>	A	V <sup>c</sup>	V	C <sup>c</sup>	V	C	C	10	500 <sup>d</sup>
	B	V	V	C	V	C	C	10	1000
	C	V	V	C	V	C	C	10	1250
	D	C	V	C	V	C	C	10	500
	E	C	V	C	V	C	C	10	1000
	F	C	V	C	V	C	C	10	1250
II <sup>a</sup>	G	V	V	C	C	C	V	96	500
	H	V	V	C	C	C	V	94	800
	I	V	V	C	C	C	V	87	1000
III <sup>b</sup>	J	V	V	V	V	V	V	312	1250
	K	V	V	V	V	V	V	384	1250

<sup>a</sup> Laboratory test data.

<sup>b</sup> Field test data.

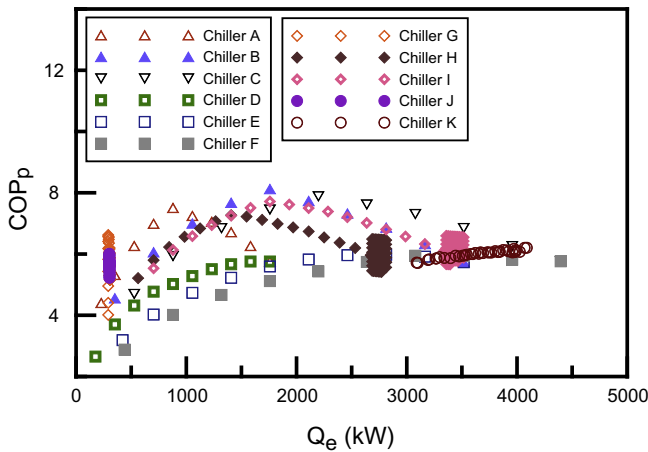
<sup>c</sup> The symbol "C" denotes keeping constant and the symbol "V" denotes variable.

<sup>d</sup> Rated cooling capacity for chillers.

**Table 2**

Variation range of the model input variables for chiller data sets.

Group	Unit	$T_{ci}$ (°C)	$T_{wi}$ (°C)	$T_{wo}$ (°C)	$V_w$ (CMS)	Loading fraction (%)
I	A	18.3–32.2	7.9–12.8	7.2	75.5	13–100
	B	18.3–32.2	7.8–12.8		150.0	10–100
	C	18.3–32.2	7.9–12.8		189	12–100
	D	32.2	7.8–12.8		75.0	10–100
	E		7.9–12.8		150.5	12–100
	F		7.8–12.8		187.5	10–100
II	G	18.3–34.0	12.0	7.0	14.2–83.7	17–100
	H	18.3–34.0			26.8–134.0	20–100
	I	18.3–34.0			33.5–167.4	20–100
III	J	27.6–30.6	9.8–10.8	5.4–5.9	163.3–207.5	67–99
	K	27.1–31.0	9.8–10.3	5.7–5.9	178.2–229.8	63–98

**Fig. 2.** The plot of COP vs. chiller cooling capacity for all data sets.

rate while maintaining a constant condenser water flow rate. The Group III data is taken on site from a 1250 RT centrifugal water chiller during different operating period. To accommodate the loading regulation of chiller unit, the cooling water flow rate, the chilled water flow rate, the inlet and outlet temperatures of the cooling water, and the inlet and outlet temperatures of the chilled water change constantly.

Overall, the chiller data sets used in the study were selected to cover a wide range of operating conditions, which is a general prerequisite for identifying sound models.

#### 4. Least-squares regression methods

The general functional form for an empirically-based performance model for water chillers can be written in the form of Eq. (7).

$$y_i = (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}) + \varepsilon_i = \hat{y}_i + \varepsilon_i \quad (7)$$

$$i = 1, \dots, n$$

where  $y$  is the true dependent variable to  $p$  independent variables,  $\hat{y}$  is the estimated dependent variable,  $x$  is the independent variable,  $\beta$  is the unknown regression coefficients or parameters to be estimated, and  $\varepsilon$  is the residual. The subscript  $i$  denotes the observational unit from which the observations on  $y$  and the  $p$  independent variables were taken. The second subscript designates the independent variable. The sample size is denoted with  $n$ ,  $i = 1, \dots, n$ , and  $p$  denotes the number of independent variables.

In Eq. (7), there are  $(p + 1)$  parameters to be estimated when the model includes the intercept. For convenience, we use  $m = (p + 1)$ .

In the article we assume that  $n > m$ . Four matrices are needed to express the regression model in matrix notation:

- $Y$ : the  $n \times 1$  column vector of observations on the true dependent variable  $y_i$ ;
- $\hat{Y}$ : the  $n \times 1$  column vector of observations on the estimated dependent variable  $\hat{y}_i$ ;
- $X$ : the  $n \times m$  matrix consisting of a column of ones, which is labeled 1, followed by the  $p$  column vectors of the observations on the independent variables;
- $B$ : the  $m \times 1$  vector of parameters to be estimated from the data; and
- $E$ : the  $n \times 1$  vector of residual.

With these definitions, the empirical model can be written in matrix notation as follows:

$$Y = XB + E = \hat{Y} + E \quad (8)$$

or

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}_{(n \times 1)} = \begin{bmatrix} 1 & x_{11} & x_{12} & x_{13} & \cdots & x_{1p} \\ 1 & x_{21} & x_{22} & x_{23} & \cdots & x_{2p} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & x_{n3} & \cdots & x_{np} \end{bmatrix}_{(n \times m)} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix}_{(m \times 1)} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}_{(n \times 1)} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix}_{(n \times 1)} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}_{(n \times 1)} \quad (9)$$

Thus, there are  $n$  unknowns in this general representation. To determine these unknowns,  $n$  pairs of  $(x_i, y_i)$  data must be given. The residual  $\varepsilon$  is the difference between the prediction value  $\hat{y}_i$  and the data value  $y_i$  at the same value of the independent variable. The curve fit then becomes a problem of finding the  $m$  values of  $\beta$  from the known information. One method of estimation is ordinary least squares (OLS) that minimize the sum of squared residual to obtain estimate parameters,

$$\text{Minimize } E = \frac{1}{2} (Y - XB)^2 \quad (10)$$

This study uses Mathematic 6.0 software to solve the matrix and to obtain the best fit estimates parameters.

##### 4.1. Coefficient of variation of root-mean-square error (CV)

For various empirically-based performance models, prediction accuracy is an important indicator to examine the predicting capabilities of the model. This study uses the coefficient of variation of root-mean-square error (CV) to indicate how well a regression model fits the observations or the predictions, defined as follows.

$$CV = \frac{RMSE}{|(\sum_{i=1}^n y_i)/n|} \times 100 \quad (11)$$

where RMSE is the root-mean-square error of a model identified from data, and is defined as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n}} \quad (12)$$

#### 4.2. Confidence interval (CI)

The confidence interval is especially useful for detecting shifts in the process mean or variability using statistical process control charts [13]. This study selects a confidence interval of 95%. The narrower this interval is, the better the prediction accuracy will be. The 95% confidence interval is well approximated by the confidence interval CI,

$$CI = y_i \pm 1.96 \times RMSE \quad (13)$$

### 5. Results and discussion

This study presents an evaluation of the predictive ability of six empirically-based performance models for water chillers using operating data from chillers under a wide range of operating conditions. The results of this study are shown in Figs. 3–6 and in Table 3. By comparing the six empirical models in terms of differences between the predicted and measured performance of water chillers, Fig. 3 shows that the coefficient of variation of root-mean-square error (CV) can be used as an assessment index for predictive abilities of the empirical models. Fig. 4 shows a comparison of the CV values of the six empirical models for all cases of water chillers. Fig. 5 shows the average CV values of six chiller empirical models for analyzing the different group data sets. Fig. 6 shows the overall average CV values of performance prediction for all data produced by the six empirical models, used to explain the overall suitability of the models. Table 3 displays the CV values for all cases studied in this work.

#### 5.1. Criteria for determining the predictive quality of empirical models

Fig. 3 depicts a comparison between the predicted COP by the six empirical models for Group F-data and the measured COP. The horizontal axis represents the measured performance of water chiller, while the vertical axis represents the predicted performance. Each circle in the figure represents one set of data prediction results. The central diagonal solid line represents the most ideal prediction, where predicted values are equivalent to measured values. The dotted lines (one on each side of the diagonal solid line) represent a 95% confidence interval; smaller distances between the two intervals represent more accurate model predictions. CV represents the coefficient of the variation of root-mean-square error; a smaller CV value represents higher accuracy. It can be seen from Fig. 3a–f that models with narrower confidence intervals also have smaller CV values, suggesting higher predictive accuracy. The same situation can be seen in other sets of data. Consequently, the CV values obtained by empirical models for data analysis can be used as an assessing index for the regression quality and predictive accuracy of the models. Reddy and Claridge [21] also reported that for determining energy improvements involving a baseline regression model, the CV value is the better measure to consider since it has a direct bearing on the uncertainty of the energy improvements determined. Hydeman et al. [1] suggested that empirical models with accuracies of 3–5% CV for performance pre-

diction are acceptable for component models. The CV values of the six models for all case analyses are shown in detail in Table 3.

#### 5.2. Comparison of predictive ability of empirical models for all chiller cases (data sets A–K)

Fig. 4 compares the CV values obtained by the six empirical models in predicting the operating performance for all chiller cases (data sets A–K). From left to right, the horizontal axis is labeled with the six empirical models SL, BQ, MP, GNU, GNS, and LS, while the vertical axis shows the CV values. A careful comparison of the CV values of the models in terms of performance prediction shows that the CV values of the BQ, MP, and GNU models are fewer than 5% for all case predictions. The SL and LS models each had three cases where the CV value exceeded 5%, while 3/4 of GNS cases had CV values over 5%. Possible reasons for the results described above are discussed below, focusing primarily on the basis of empirical models.

A comparison of the CV values of the SL model for all chiller cases shows that, except for the D, E, and F-data sets, CV values were uniformly fewer than 5%. As the SL model expresses the chiller performance (COP) as a function of the chilled water inlet temperature ( $T_{wi}$ ), cooling water outlet temperature ( $T_{ci}$ ), and chiller cooling capacity ( $Q_e$ ), the chilled water inlet temperature in the D, E, and F data sets were held fixed values during measuring, reducing the input independent variables of the SL model by one and causing this model to suffer lower accuracy.

The CV values of the BQ, MP, and GNU models for all cases were under 5%, showing that these empirical models enjoyed excellent predicting ability. The BQ and MP models are black-box models. The functional form of the BQ model is a regression polynomial composed of two independent variables and nine estimated parameters. The MP model is a regression polynomial composed of three independent variables and ten regression coefficients. Other models involved polynomials consisting of three independent variables and three regression coefficients. Consequently, generally speaking, the BQ and MP models possessed higher predicting capability.

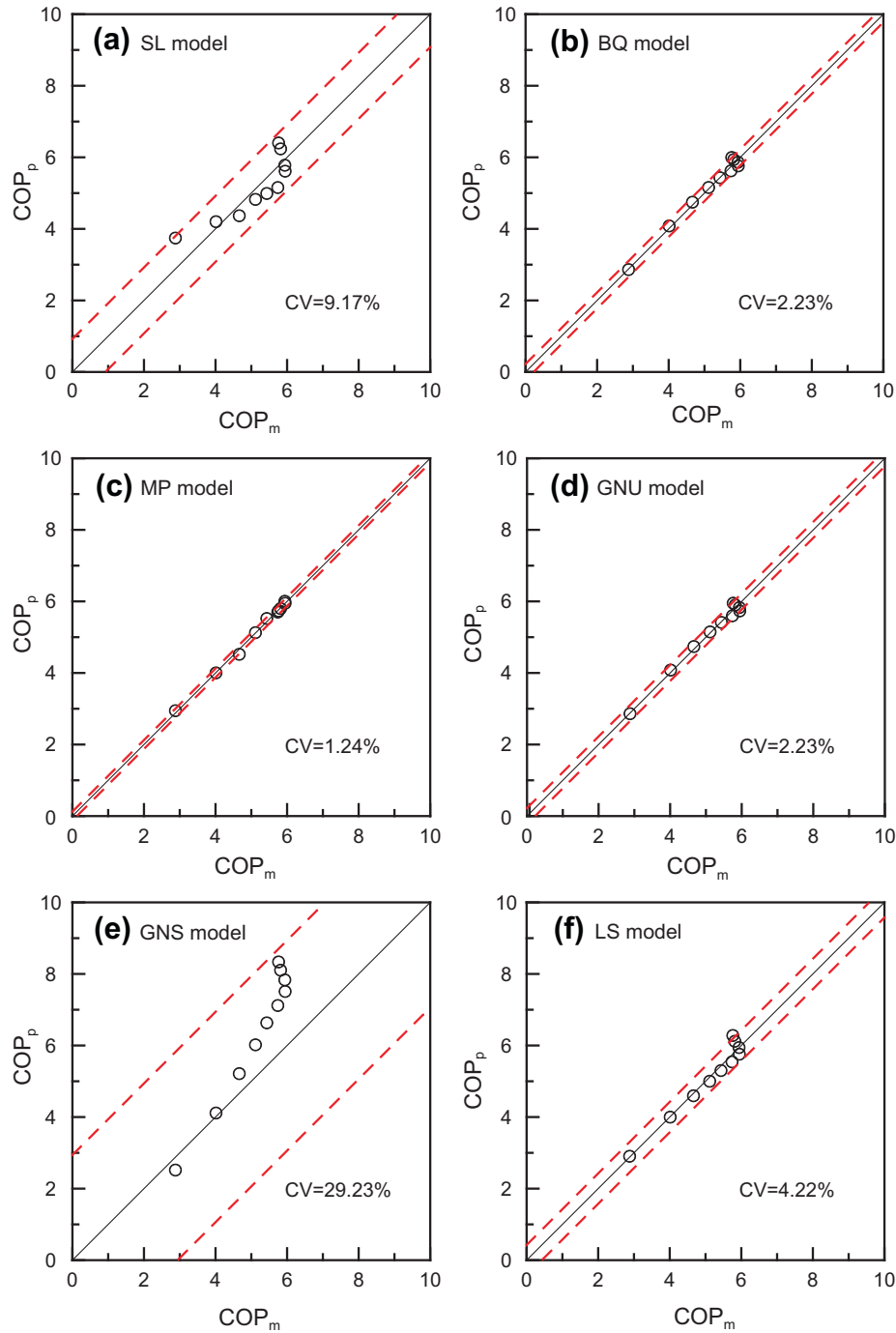
Though the initial CV values produced by the GNU model for each case fell within 5%, average CV values were high compared to those of the BQ and MP models; the calculation process is also more complex. However, as the GNU model is considered a gray-box model, the three regression coefficients of the model all have physical interpretations [15]; the BQ and MP models do not have this capability.

The CV values of the GNS model exceeded 5% for the prediction results of all cases other than the J and K data sets. Some cases, such as the A, B, C, D, E, and F data sets, had CV values in excess of 15%. These high CV values were caused by reasons similar to those in the SL model. It can be seen from Eq. (5) that the GNS model expresses water chiller COP as a polynomial consisting of outlet chilled water temperature ( $T_{wo}$ ), inlet water cooling temperature ( $T_{ci}$ ), and cooling capability. When the A, B, C, D, E, and F data sets included  $T_{wo}$  as a constant value, the input independent variables of the model were reduced to two, leading to lower predicting capabilities. In the D, E, and F data sets,  $T_{wo}$  and  $T_{ci}$  are keeping constant, leaving only one effective independent variable for the model, significantly reducing predicting accuracy.

Other than the G, H, and I data sets, the LS model produced CV values under 5% for every case. As the independent variable of inlet water cooling temperature ( $T_{wi}$ ) is held fixed in data sets G, H, and I, the predicted CV value of this model was higher in those cases.

It can be seen from the above discussion that, of the six water chiller performance prediction empirical models used in this paper, the BQ, MP, and GNU models had the best prediction capacities; these models had prediction CV values under 5% for all cases.





**Fig. 3.** Comparison between measured and predicted COP for the case of F-data sets.

### 5.3. Comparison of predicting capabilities of empirical models for data types (Groups I–III)

Based on the characteristic of flowrate variation of water chillers in operation, the data used for empirical models in this paper can be broadly categorized as: (1) constant condenser and constant chilled water flow, (2) constant condenser and variable chilled water flow, and (3) variable condenser and variable chilled water flow, as shown in Table 1. The average predicting accuracy of the six empirical models for the three data groups are shown in Fig. 5. The applicability of the empirical models is examined below based on these data types.

The first type of water chiller data, including the six sets of A–F was obtained in conditions where condenser water and chilled water flow were kept constant. It can be seen from Fig. 5 and Table 3 that the MP model had the best ( $CV = 0.79\%$ ) predicting accuracy for this type of data, followed by the BQ model ( $CV = 1.05\%$ ). The GNS model clearly performed the worst ( $CV = 26.21\%$ ). A possible reason for the MP model having the best predicting accuracy was that the input independent variables of the MP model are  $T_{ci}$ ,  $T_{wi}$ , and  $Q_e$  and that the functional form of the MP model was a polynomial with ten regression coefficients, leading the MP model to be more accurate in predicting COP than other models. Though the BQ model only had two independent variables, its function form

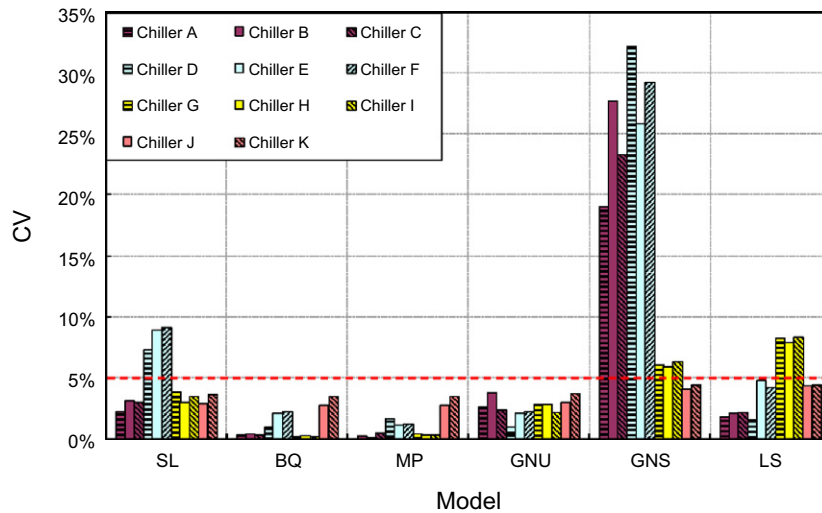


Fig. 4. A comparison of the CV of six models for all chiller cases.

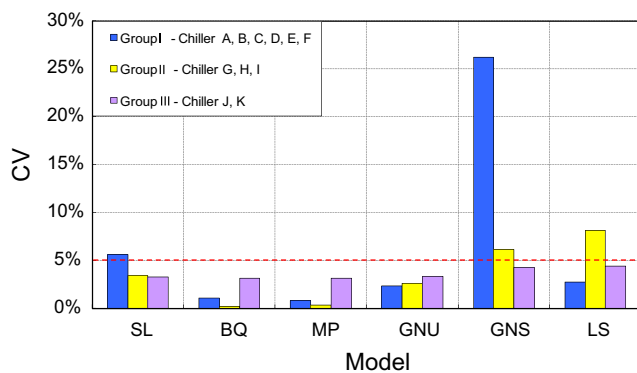


Fig. 5. The average CV values of the six empirical models for the three data groups.

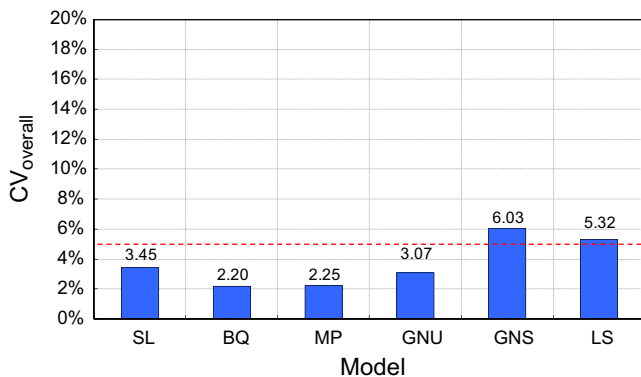


Fig. 6. Comprehensive comparison of overall predicting accuracy.

was a regression equation with nine regression coefficients, allowing it good predicting capabilities. The independent variables of the GNS model were  $T_{ci}$ ,  $T_{wo}$ , and  $Q_e$ ; the  $T_{ci}$  and  $T_{wo}$  of this type of data were held fixed, leaving only one independent variable for this model. As a result, the GNS model performed badly when making predictions for the A, B, C, D, E, and F sets of data. It can be seen that models with greater numbers of regression coefficients enjoy better performance predicting abilities in general.

The second type of data (Group II) included the three sets of G, H, and I; this type of data was obtained under conditions where condenser water flow and chilled water inlet and outlet temperatures

remained constant. Comparison of the predicting capability of the various models for this type of data shows that the BQ model (CV = 0.19%) had the greatest predicting accuracy; the GNS (CV = 6.11%) and LS models (CV = 8.16%) both exceeded 5%. The BQ model required only the two independent variables  $T_{ci}$  and  $Q_e$  and was therefore not affected by the impact of constant chilled water inlet and outlet temperatures ( $T_{wi}$  and  $T_{wo}$ ) and flow volumes. The GNS and LS models (CV = 8.16%) were affected by the constancy of chilled water input and output temperature, causing their associated CV values to be higher.

The third type of data (Group III) included the J and K water chiller sets and represented actual measured data; condenser water and chilled water flow were variable in this type of data. It can be seen from Fig. 5 and Table 3 that the BQ model (CV = 3.10%) had the highest predicting ability, followed by the MP model (3.12%), while the LS model (CV = 4.40%) had the poorest performance. Further examination of the CV values shows the difference between the best and the worst models is not as significant as in other types of data. While the data in this category was subject to change, fluctuations were not large, as shown in Table 2. In addition, the independent variables of the six models were not subject to the impact of set data values. Consequently, the predicting capacity of these models for this type of data was generally higher.

As can be seen, the CV predicting performance values for the various types of data using the BQ, MP, and GNU models fell within the acceptable range of 5%. Aside from its CV value exceeding 5% for Group I, the SL model's predicting ability is applicable to all other data types. The GNS model is only suitable for performance predictions of Group III data. The LS model was applicable to all data types aside from Group II data.

#### 5.4. Comprehensive evaluation of empirically-based performance models for water chillers

The comprehensive predicting capabilities of the six water chiller predicting empirical models examined in this study for the three main types of operating data are indicated by overall CV as shown in Fig. 6. The overall CV for each model can be calculated as following equation.

$$CV_{overall} = \frac{1}{N} \sum_{j=1}^3 CV_j \times n_j \quad (14)$$

where  $CV_j$  represents the mean CV for group  $j$ ;  $n_j$  represents the number of data sets for group  $j$ ;  $N$  represents the total number of

**Table 3**

Overall assessments of the predictive abilities of six empirically performance models for water chillers. CV(%).

Group		Model					
		SL	BQ	MP	GNU	GNS	LS
I	A	2.26	0.30	0.25	2.63	19.04	1.77
	B	3.12	0.37	0.05	3.76	27.67	2.08
	C	2.99	0.33	0.45	2.39	23.28	2.14
	D	7.33	0.98	1.68	0.99	32.18	1.57
	E	8.92	2.08	1.09	2.05	25.85	4.74
	F	9.17	2.23	1.24	2.23	29.23	4.22
II	G	3.88	0.16	0.43	2.86	6.08	8.25
	H	2.91	0.19	0.32	2.76	5.91	7.88
	I	3.50	0.20	0.34	2.20	6.34	8.36
III	J	2.90	2.69	2.74	2.94	4.04	4.36
	K	3.64	3.52	3.51	3.71	4.46	4.44

data sets;  $j$  represents the number of the data classification. Of all of the models, the BQ model (CV = 2.20%), a black-box model, had the best predicting capability, followed by the MP model (CV = 2.25%); the GNS model (CV = 6.03%) and LS model (CV = 5.32%), a grey box model, had the poorest performance. If having a CV within 5% is used as an evaluation standard, the five models other than the GNS model have acceptable predicting capacities.

The results discussed in the previous sections revealed that both the BQ and the MP models are appropriate for all data types with favorable predicting accuracy. From a practical point of view, these results can be referenced when selecting an empirically-based performance model of liquid chiller for the purpose of performance prediction, evaluation of energy-efficiency improvements, and fault detection and diagnosis of water chillers.

## 6. Conclusion

The objective of this study is to evaluate the suitability of six empirically-based models with rich chiller data sets. Results show that (1) the CV value can serve as an overall indicator of for model accuracy of water chiller, and (2) except for the GNS and LS model, the models all have an acceptable ranges of CV value for all kinds of data sets. This is especially true for the bi-quadratic regression model (BQ model), which has the best prediction accuracy with a CV of 2.20%, followed by the multivariate polynomial regression model (MP model) with a CV of 2.25%. The results of this study can be referenced when selecting an empirically-based performance model of water chiller for the purpose of performance prediction, evaluation of energy-efficiency improvements, and fault detection and diagnosis of water chillers.

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