Homework 1

January 12, 2024

This is the second homework assignment for AMATH 301 Winter 2024. There are no homework problems in our textbook associated with appendix D so the written exercises will be presented on this PDF file.

For the written exercises, you should upload a scanned PDF to Gradescope and then follow the prompts given by Gradescope to assign certain pages of your PDF document to the correct problems.

For the coding exercises, you will be prompted to upload your python files directly to Gradescope.

The course syllabus found on Canvas has information on how homework is graded and how homework should be presented and submitted. Please let me or the TAs know if you have any questions or concerns.

This assignment is due on Thursday, January 18 at 11:59pm.

Written Assignment

1. Let

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix},$$

and

$$\mathbf{D} = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}.$$

Either compute the following expressions or give the reason why they are undefined:

a.) $\mathbf{B} - \frac{1}{2}\mathbf{A}$

b.) $\mathbf{AC}$

c.) $\mathbf{CD}$

d.) $\mathbf{A} + 3\mathbf{B}$

e.) $2\mathbf{C} - 3\mathbf{x}$

f.) $\mathbf{DB}$

g.) $\mathbf{xC}$

h.) $\mathbf{Cx}$

i.) $\mathbf{A}^T + \mathbf{B}^T$

j.) $\mathbf{A}^T + 2\mathbf{C}^T$

k.) $(\mathbf{CD})^T$

2. a.) In general, matrix multiplication does not commute, i.e. given two matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{AB} \neq \mathbf{BA}$. Verify this by calculating $\mathbf{AB}$ and $\mathbf{BA}$ if

$$\mathbf{A} = \begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}.$$

b.) Let

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & 9 \\ -3 & k \end{bmatrix}.$$

Find the value(s) of k, if any exist, that allow $\mathbf{AB} = \mathbf{BA}$

3. One application of systems of linear equations is in the study of heat transfer. More specifically, we can determine the steady-state temperature distribution of a thin plate when the temperature around the boundary is known.

Consider figure 1 shown at the end of this document, which shows a cross-section of a metal beam, with negligible heat flow in the direction perpendicular to the plate. Let the T_1, T_2, T_3 and T_4 denote the temperatures at the four interior nodes of the mesh. The temperature at a node is approximately equal to the average of the four nearest nodes (i.e. the nodes to the left, above, right, and below.)

Use this information to derive a linear system satisfied by the unknown temperatures T_1, T_2, T_3 and T_4. You do not need to solve the resulting system. *hint: The equation corresponding to the first node is*

$$T_1 = (10 + 20 + T_2 + T_3)/4,$$

or equivalently

$$4T_1 - T_2 - T_3 = 30.$$

Coding Assignment

1. Using the functions numpy.exp(), numpy.sqrt(), numpy.tan(), and numpy.arcsin() along with the constants numpy.pi and numpi.e write a script that computes

a.) $\sqrt{1+\tan e^{(2+\pi/2)}}$

b.) $\tan\left(e^3+\arcsin\left(\pi/21\right)\right)$.

Assign your answer from part a as the variable ans1 and assign your answer from part b as the variable ans2.

2. Using the numpy.sin() and numpy.cos() functions along with numpy.pi, write a script that creates the 1D array

$$[\cos(0), \sin(\pi), \sin(\pi/3), \cos(\pi/3)],$$

and assign it to ans1. Additionally, assign the following variables to

a.) $\text{ans2} = (\pi/2) * \text{ans1}$

b.) $\text{ans3} = \sin(\text{ans1})$

c.) $\text{ans4} = \text{ans2} * \text{ans3}$

d.) Now use the : slicing operator to create an array that consists of the first two elements of ans3 and assign it to ans5. Create another array that is the last two elements of ans4 and assign it to ans6. Finally, compute ans5-ans6 and assign it to ans7.

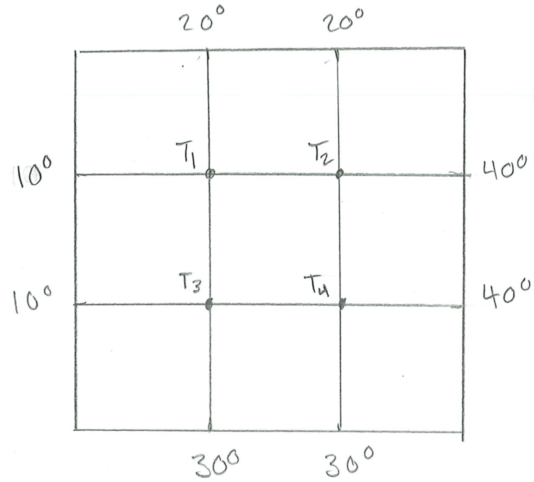

Figure 1: figure for problem 1.