Homework 3

January 19, 2024

This is the third homework assignment for AMATH 301 Winter 2024. This homework assignment is about our introduction to python.

For the written exercises, you should upload a scanned PDF to Gradescope and then follow the prompts given by Gradescope to assign certain pages of your PDF document to the correct problems.

For the coding exercises, you will be prompted to upload your python files directly to Gradescope.

The course syllabus found on Canvas has information on how homework is graded and how homework should be presented and submitted. Please let me or the TAs know if you have any questions or concerns.

This assignment is due on Thursday, January 25 at 11:59pm.

Written Assignment

For this week's written assignment, you will be asked to analyze two sets of python code. Try to answer the questions without writing the code in python. It is good practice for a potential coding problem that could appear on Exam 1 or Exam 2. Of course, you can double-check your analysis by writing the code in python.

1. Consider the following python code:

Code 1:

```
import numpy as np

n = 5

x = np.arange(1,n+1)

m = 0

for k in x:
    m = m + k**2

print(m)
```

   a.) Explain in a sentence or two what is being coded in the first three lines of code that come after import numpy as np.

   b.) Explain in a sentence or two what is being coded in the following three lines.

   c.) What is the value of m after the code is finished running?

2. Consider the piecewise polynomial function:

$$y = \begin{cases} (x+3)^3 & x < -1 \\ -x^2 + 9 & -1 \leq x < 1 \\ -x + 9 & 1 \leq x < 2 \\ -\frac{x^2}{4} + 8 & x \geq 2 \end{cases}.$$

   Harry the Husky, who is the mascot for the University of Washington, Seattle, has written the following code to determine the value of the piecewise polynomial above for a given value of x.

Code 2:

```
x = .75

if x < -1:
    y = (x+3)^3
elif x < 1:
    y = -x**2 + 9
if x < 2:
    y = -x+9
else x >= 2:
    y = -(1/4)*x**2
```

a.) There are three mistakes in Harry's code, and he has asked you to help him fix his code. Find the three mistakes and state what changes need to be made for the code to function properly.

Note: There may be more than one way to correct Harry's mistakes. Pick your favorite way to correct their mistakes.

b.) Once the mistakes are fixed, state which parts of the code will run and which will be ignored by python, explain your answer for each portion of the code containing an if, elif, or else statement.

Coding Assignment

1. Let

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix},$$

and

$$\mathbf{D} = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}.$$

You may have noticed that these are the same matrices from question 1 of the written assignment for homework 2. Write a python script that does the followng:

a.) Assigns each of the matrices to a variable with the same variable name as the matrices listed above. For example: you should save x = np.array([[-5],[3]]).

b.) Computes $\mathbf{B} - \frac{1}{2}\mathbf{A}$ and assigns it to the variable ans1.

c.) $\mathbf{CD}$ and assign it to the variable ans2.

d.) $\mathbf{A} + 3\mathbf{B}$ and assign it to the variable ans3.

e.) $\mathbf{DB}$ and assign it to the variable ans4.

f.) $\mathbf{Cx}$ and assign it to the variable ans5.

g.) $\mathbf{A}^T + \mathbf{B}^T$ and assign it to the variable ans6.

h.) $(\mathbf{CD})^T$ and assign it to the variable ans7.

2. Using a nested for loop, write a python script that creates the following 12×15 matrix.

$$\mathbf{A} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \cdots & \frac{1}{12} & \cdots & \frac{1}{15} \\ 2 & \frac{2}{2} & \frac{2}{3} & \cdots & \frac{2}{12} & \cdots & \frac{2}{15} \\ 3 & \frac{3}{2} & \frac{3}{3} & \cdots & \frac{3}{12} & \cdots & \frac{3}{15} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ 12 & \frac{12}{2} & \frac{12}{3} & \cdots & \frac{12}{12} & \cdots & \frac{12}{15} \end{bmatrix}.$$

Note that $a_{ij} = i/j$.

a.) Assign the matrix above to the variable A1.

b.) Assign a copy of A1 to the variable A2. Modify the 4th row of A2 to be a row of zeros.

c.) Use array slicing to assign the submatrix made up of rows 3, 4, and 5 and the last 4 columns of A1 to the variable A3.

d.) Assign the row 2 of A1 to the variable A4. Make sure that A4 is a 2D array of shape (1,15).

3. The third order Taylor polynomial centered at $x = 0$ approximating $f(x) = e^x$ is

$$p_3(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}.$$

Write a python script that creates the plot with the following features:

(a) Create a python script that assigns x to the 1D array containing 100 evenly spaced points between -2 and 2. One option is to use:

x = np.linspace(-2,2,100).

(b) Use matplotlib.pyplt to create a plot of $p_3(x) = 1+x+\frac{x^2}{2}+\frac{x^3}{6}$ and $f(x) = e^x$. Make sure that your plot has a legend that labels $p_3(x)$ and $f(x)$ correctly, has the label "x" on the x axis, "y" on the y-axis, and has the title "Taylor Approximation".

(c) Repeat parts a and b using x = np.linspace(-5,5,100).

(d) In one or two sentences, describe the difference in the two plots from parts b and c.