Homework 8

This homework assignment is about numerical integration, the left-hand rule, the right-hand rule, the midpoint rule, and the trapezoid rule.

For the written exercises, you should upload a scanned PDF to Gradescope and then follow the prompts given by Gradescope to assign certain pages of your PDF document to the correct problems.

For the coding exercises, you will be prompted to upload your python file directly to Grade-scope. Please make sure you submit only one file and that your variables are assigned to the correct variable names.

The course syllabus found on Canvas has information on how homework is graded and how homework should be presented and submitted. Please let me or the TAs know if you have any questions or concerns.

This assignment is due on Sunday, March 10 at 11:59pm.

Written Assignment

• From section 5.1 complete problems 2, 8, 9, 11, 23, 31. For problems number 2 and number 8 also approximate the integral using the left-hand rule, the right-hand rule, and the midpoint rule.

Coding Assignment

For this coding assignment, you will be asked to write a python function that approximates

$$\int_{a}^{b} f(x) \, \mathrm{d}x$$

With the trapezoid rule. The function should have four arguments: 1.) the function you would like to calculate the definite integral of, 2.) the left endpoint of the interval, 3.) the right endpoint of the interval, 4.) the number of sub-intervals used to approximate the definite integral. There are more than one way to create this function, but the file called left_hand_rule.py can be used as a reference to create your python file containing the trapezoid rule function.

Once you have written your trapezoid rule function, use it to approximate the following integrals:

1. It is possible to show that the length of a curve is

$$\int_a^b \sqrt{1 + (f'(x))^2} \, \mathrm{d}x,$$

where f(x) is a function whose graph is the curve on the interval $a \le x \le b$.

(a) Use your trapezoid rule function to have python compute the arc length of the curve described by the function $f(x) = x^{-1}$ on the interval $1 \le x \le 2$ with 100 sub-intervals. Save your answer to the variable called TR1a. Note that $f'(x) = -x^{-2}$ so the integral we want to approximate is:

$$\int_{1}^{2} \sqrt{1 + x^{-4}} \, \mathrm{d}x$$

- (b) Repeat part a using 10000 sub-intervals. Save your answer to the variable called TR1b
- 2. Major league baseball rules state that a baseball must weigh 5 to 5.25 ounces (0.142 to 0.149 kg). The Seattle Mariners (the local major league baseball team) have asked you to estimate the number of baseballs that they buy, that will be fit for use according to the major league baseball rules. The weight of a baseball follows a normal distribution

has an average wight of 5.15 ounces with a standard deviation of 0.08 ounces. One of the statisticians you are working with states that the probability of that the ball satisfies the rules of the mlb is

$$\int_{5}^{5.25} \frac{1}{(0.08)\sqrt{2\pi}} e^{-\frac{(x-5.15)^2}{2(.08)^2}} \,\mathrm{d}x$$

but does not know how to calculate this definite integral.

- (a) Use your trapezoid rule function to approximate the definite integral with 100 sub-intervals and save it to the variable called TR2a
- (b) repeat part a using 10000 sub intervals and save it to the variable called TR2b