Exam 1 Practice Problems

This is a list of problems that wil help you prepare for Exam 1. They are optional and will not be graded.

1. For each of the following pairs of matrices, determine whether it is possible to multiply the first matrix times the second. If it is possible, preform the multiplication.

   a.) $\begin{bmatrix} 3 & 5 & 1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 4 & 1 \end{bmatrix}$

   b.) $\begin{bmatrix} 4 & -2 \\ 6 & -4 \\ 8 & -6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

   c.) $\begin{bmatrix} 1 & 4 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 4 & 5 \end{bmatrix}$

   d.) $\begin{bmatrix} 4 & 6 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$

   e.) $\begin{bmatrix} 4 & 6 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 4 & 1 & 6 \end{bmatrix}$

   f.) $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 4 & 5 \end{bmatrix}$

2. For the following matrices

$$\mathbf{A} = \begin{bmatrix} 3 & 1 & 4 \\ -2 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix}, \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 1 & 1 \\ 2 & -4 & 1 \end{bmatrix}$$

   a.) $2\mathbf{A}$

b.) $\mathbf{A} + \mathbf{B}$

c.) $2\mathbf{A} - 3\mathbf{B}$

d.) $(2\mathbf{A})^T - (3\mathbf{B})^T$

e.) $\mathbf{AB}$

f.) $\mathbf{BA}$

g.) $\mathbf{A}^T\mathbf{B}^T$

h.) $(\mathbf{BA})^T$

3. Consider the following code that is intended to be used to calculate and print the quantity:

$$\sqrt{2 + e^{(\ln(1+\sin(\pi/5)))^2}}.$$

Code 1:

```
1 import numpy as np
2
3 x = np.sqrt(2 + np.exp(np.log(1+sin(np.pi/5))**2))
4
5 print(x)
```

There is one error in this code on line 3. Find it and then write a corrected version of line 3.

4. Consider the following code run in python:

Code 2:

```
import numpy as np

v = np.linspace(0,3.75,6)

u = np.arange(3,4.1,1/6)

w = np.array([1.0,2,4,5,6,7])
```

a.) List the variable type and shape of the variables v,u,w.

b.) Using array slicing, give the code that assigns the element 2 through element 5 of v to the variable x

c.) If the code

Code 3:

```
print(w**2 - np.ones(w.shape[0])
```

is executed in the python console, what is the output?

d.) Using the np.sin() and np.cos() functions, give the code that assigns an array to the variable y. The elements of y are the sine of the elements of w plus the cos of the elements of v.

5. Consider the following python code:

Code 4:

```
1  import numpy as np
2  n = 5
3  m = 4
4
5  def create_matrix(n,m):
6      A = np.zeros((n,m))
7      for i in range(n):
8          for j in range(m):
9              den = (i+1)*(j+1)
10             A[i,j] = 1/den
11     return A
12
13 A = create_matrix(n,m)
```

This code defines a function named "create_matrix()" on lines 4 through 11. Then on line 13 the function is called and assigned to the variable A

a.) If this code is run in python, write out the entries of A. You may write this as a matrix or using NumPy array notation.

b.) If this code is run in python, then if one were to run

Code 5:

```
print(den)
```

in the python console, explain why python would return an error.

c.) If this code is run in python, there will be three variables defined globally, in other words three variables would appear in your variable explorer if you were using Spyder. For each variable, list the variable name, variable type, size of the variable.

d.) Using array slicing, give the code that assigns an array to the variable x. This array must be an array that contains the last two rows and the last 3 columns of A.

e.) Using array slicing, give the code that assigns an array to the variable y. This array should have size (5,1) containing the 2nd column of A.

6. Consider the following code:

Code 6:

```
import numpy as np

A = np.array([[3,1,4],[-2,0,1],[1,2,2]])

B = np.array([[1,0,2],[-3,1,1],[2,-4,1]])
```

Give the code that calculates each matrix operation in problem 2.

For problems 7-9, apply naive Gaussian elimination (only the row replacement and row multiplication elementary row operations) to solve the system $\mathbf{Ax} = \mathbf{b}$. If naive Gaussian elimination fails, solve the system using the additional pivoting row operation.

7.
$$\mathbf{A} = \begin{bmatrix} 8 & 1 & -1 \\ -1 & 7 & -2 \\ 2 & 1 & 9 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 7 \\ 4 \\ 12 \end{bmatrix}$$

8.
$$\mathbf{A} = \begin{bmatrix} 3 & -5 & -5 \\ 5 & -5 & -2 \\ 2 & 3 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -37 \\ -17 \\ 32 \end{bmatrix}$$

9.
$$\mathbf{A} = \begin{bmatrix} 6 & 2 & 2 \\ 6 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$$

10. Consider the system $\mathbf{Ax} = \mathbf{b}$ where

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 0 & 3 & 1 & 2 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 1 & -2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 5 \\ -1 \\ 11 \\ -5 \end{bmatrix}$$

This is nearly an upper triangular system in the sense that $\mathbf{A}$ is one elementary row operation from being upper triangular.

a.) What is the row operation involving row 3 and row 4 that transforms $\mathbf{A}$ into an upper triangular system?

b.) What is the corresponding elementary row matrix that corresponds to the elementary row operation from part a.

c.) preform this row operation on the augmented system $[\mathbf{A}|\mathbf{b}]$.

d.) Use backwards substitution to solve the resulting system.

11. Below is a code that includes a function that solves an upper triangular system using backward substitution. The function is named backward_sub(U,z) and the argument U is an upper triangular matrix represented in python as a 2d array and z is a vector represented as a 1d array:

Code 7:

```
1  import numpy as np
2
3  def backward_sub(U,z):
4
5      U = U.astype("float")
6      z = z.astype("float")
7      n = U.shape[0]
8      x = np.zeros(n)
9      x[n-1] = z[n-1]/U[n-1,n-1]
10
11     for i in range(n-2,-1):
12         sum = z[i]
13         for j in range(i+1,n):
14             sum = sum - U[i,j]*x[j]
15         x[i] = sum/U[i,i]
16     return x
17
18 U = np.array([[1,3,-2],[0,1,-1],[0,0,1]])
19 z = np.array([0,1,2])
20
21 x = backward_sub(U,z)
```

a.) There is a mistake with line 11 of the code. Specifically, the optional step size argument needs to be added in the range function. What is the step size that make this function produce the correct output (the output that produces the solution to the upper triangular system)?

b.) If we consider the corrected backward substitution function with arguments U and z given in lines 18 and 19. If the function is executed from lines 4 to 10 write what x is either as python output or as a vector.

c.) After one pass through the outer loop on lines 11 through 16 (i.e. when i = n-2) what is the value of the variable sum?

In problems 12 and 13, find the LU decomposition of the given matrix. Make sure to list all of your elementary matrices.

12.

$$\mathbf{A} = \begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}$$

13.

$$\mathbf{A} = \begin{bmatrix} 2 & -6 & 4 \\ -4 & 8 & 0 \\ 0 & -4 & 6 \end{bmatrix}$$

14. Use forward substitution and backward substitution to solve the system $\mathbf{Ax} = \mathbf{b}$ if $\mathbf{A}$ is the matrix from problem 12 and

$$\mathbf{b} = \begin{bmatrix} -7 \\ 5 \\ 2 \end{bmatrix}$$

15. Use forward substitution and backward substitution to solve the system $\mathbf{Ax} = \mathbf{b}$ if $\mathbf{A}$ is the matrix from problem 13 and

$$\mathbf{b} = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix}$$

In problems 16-19 use the matrices given in Section 8.4 Computer Exercises (pg 423) problem 1c and 1d.

16. Disregard the book's directions for 1c and instead compute the $\mathbf{x}^{(2)}$ using Richardson iteration for the matrix $\mathbf{A}$ and $\mathbf{b}$ listed in problems 1c.

17. Disregard the book's directions for 1d and instead compute the $\mathbf{x}^{(2)}$ using Jacobi iteration for the matrix $\mathbf{A}$ and $\mathbf{b}$ listed in problems 1c.

18. Repeat problem 16 for the matrix $\mathbf{A}$ and $\mathbf{b}$ listed in problems 1d.

19. Repeat problem 17 for the matrix $\mathbf{A}$ and $\mathbf{b}$ listed in problems 1d.