AMATH 301 Exam 2 Version 3

Name: Key

Student ID:

Instructions: Write neatly and show <u>all</u> work. Unless it is stated otherwise, in order to receive full credit, you must show your work and carefully justify your answers. Please circle your final answer. The number in parentheses next to the problem number is the point value of the problem. Besides the one side of a 8.5 by 11 inch note sheet, notes, textbooks, or calculators may not be used on this exam. Please verify that this exam consists of 4 problems and 1 extra credit problem, on the front and back of 4 pages. Any work on scratch paper will not be graded.

This exam is worth 40 points.

Good Luck!

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I pledge that the work on this exam is mine only - I received no aid from outside sources nor did I obtain answers in ways that violate the instructions on the exam. I also pledge that I did not provide aid to any other student taking the exam.

Signature:

1. Consider the function

$$f(x) = x^3 - 4.$$

It is possible to use Newton's method and the bisection method to approximate the root of this function.

(a) (3 points) If the initial guess is $x_0 = 1$, calculate x_1 using Newtons method.

$f(1) = 1 - 4 = -3$

$f'(x) = 3x^2, \quad f'(1) = 3$

$\rightarrow x_1 = 1 - \frac{-3}{3} = 1 + 1 = 2$

(b) (3 points) With an interval of $[0, 2]$ and an initial midpoint of $x_0 = \frac{1}{2}(0 + 2) = 1$, calculate x_1 using the bisection method.

$f(0) = -4$

$f(1) = -3$

$f(2) = 8 - 4 = 4$

$\rightarrow \frac{1+2}{2} = \frac{3}{2} = x_1$

(c) (4 points) Determine how many steps of the bisection method from part b, are needed to determine the root with an error of at most $(1/2) * 10^{-9}$.
hint: $\frac{\ln(10^{-9})}{\ln(2)} = -29.8974$ rounded to 4 decimal places.

$$\frac{\ln(b-a) - \ln(2 \cdot \frac{1}{2} \cdot 10^{-9})}{\ln(2)} = \frac{\ln(2)}{\ln(2)} - \frac{\ln(10^{-9})}{\ln(2)}$$

$\approx 1 + 29.8974 = 30.8974$

$\rightarrow n = 31$

2. (a) (8 points) Consider the following table of data

x	1	4	9
$f(x)$	2	3	4

Find the polynomial of degree 2 or less that interpolates the data above. You can find this polynomial using either Lagrange or Newton form, and you do not need to simplify your answer.

x	$f[\,]$	$f[\,,\,]$	$f[\,,\,,\,]$
1	2	$\frac{1}{3}$	
4	3		$\dfrac{\frac{1}{5}-\frac{1}{3}}{8} = -\frac{1}{60}$
9	4	$\frac{1}{5}$	

$$p_2(x) = 2 + \tfrac{1}{3}(x-1) - \tfrac{1}{60}(x-1)(x-4)$$

(b) (2 points) Now consider the table of data

x	-1	1
$f(x)$	0	2

The polynomials $p(x) = x + 1$, $q(x) = x(x + 1)$, and $r(x) = \frac{1}{4}(x + 1)^3$ interpolate the data (you do not need to verify this). Explain why this does not violate the uniqueness part of the theorem on existence of polynomial interpolation.

Same as version 1

3. Consider the data table:

x	-1	0	1	2	3
$f(x)$	$\frac{1}{2}$	1	2	4	8

(a) (3 points) This code below uses the forward difference formula to approximate the $f'(-1)$, $f'(0)$, $f'(1)$, and $f'(2)$ and assigns these approximations to the first four entries of the variable forward_diff. Assuming this code is run in python, write the variable forward_diff as python output or as a vector.

Code 1:

```
import numpy as np
x = np.array([-1,0,1,2,3.0])
y = np.array([1/2,1,2,4,8])
n = x.shape[0]
forward_diff = np.zeros(n)
for i in range(n-1):
    forward_diff[i] = (y[i+1] - y[i])/(x[i+1] - x[i])
```

$$\left[\frac{1-\frac{1}{2}}{0-(-1)}, \frac{2-1}{1-0}, \frac{4-2}{2-1}, \frac{8-4}{3-2}, 0\right]$$

$$= \left[\frac{1}{2}, 1, 2, 4, 0\right]$$

(b) (3 points) Consider the code below and add to it the code that saves the backward difference approximations for $f'(0)$, $f'(1)$, $f'(2)$, and $f'(3)$ to the last four elements of the variable backward_diff

Code 2:

```
import numpy as np
x = np.array([-1,0,1,2,3.0])
y = np.array([1/2,1,2,4,8])
n = x.shape[0]
forward_diff = np.zeros(n)
for i in range(n-1):
    forward_diff[i] = (y[i+1] - y[i])/(x[i+1] - x[i])
backward_diff = np.zeros(n)
```

Write your code here:

```
for i in range(1,n):
    backward_diff[i] = (y[i]-y[i-1])/(x[i]-x[i-1])
```

(c) (2 points) Using array slicing, write the code that assigns the middle three elements of forward_diff to the variable dx1 and assigns the middle three elements of backward_diff to the variable dx2.
Write your code here:

Same as Version 1

(d) (2 points) For the given data, one way to approximate $f'(0)$, $f'(1)$, and $f'(2)$ using the central difference formula is to average the forward and backward difference approximations for $f'(0)$, $f'(1)$, and $f'(2)$. Write the code that assigns the average of dx1 and dx2 (from part c) to the variable central_diff .
Write your code here:

Same as version 1

4. (a) (6 points) Approximate the integral

$$\int_1^3 \frac{1}{x+1}\,dx,$$

with the left-hand rule, right-hand rule, and the midpoint rule using the partition

$$P = \{1, 2, 3\}.$$

You do not need to compute the trapezoid rule approximation.

LHR: $1\left(\frac{1}{2}+\frac{1}{3}\right) = \frac{3}{6}+\frac{2}{6} = \frac{5}{6}$

RHR: $1\left(\frac{1}{3}+\frac{1}{4}\right) = \frac{4}{12}+\frac{3}{12} = \frac{7}{12}$

MPR: $1\left(\frac{1}{3/2}+\frac{1}{5/2}\right) = \frac{2}{3}+\frac{2}{5} = \frac{10}{15}+\frac{6}{15} = \frac{16}{15}$

(b) (4 points) Determine the number of sub-intervals n that guarantees an error no larger than $\epsilon = 10^{-6}$ when the trapezoid rule is used to approximate the integral from part a.

hint: $\sqrt{\frac{1}{6} \times 10^6} = 408.248$ rounded to three decimal places

$f(x) = (x+1)^{-1}$

$f'(x) = -(x+1)^{-2}$

$f''(x) = 2(x+1)^{-3}$

$f'''(x) = -6(x+1)^{-4}$

$= \frac{-6}{(x+1)^4}$ no C.P. in $[1,3]$

$f''(1) = \frac{2}{2^3} = \frac{1}{4}$

$f''(3) = \frac{2}{4^3} = \frac{2}{64} = \frac{1}{32}$

$\rightarrow \frac{1}{12}(3-1)^3 \frac{1}{n^2} |f''(\xi)|$

$\leq \frac{1}{12} 2^3 \frac{1}{n^2} \frac{1}{2^2} < 10^{-6}$

$\rightarrow \frac{1}{6} \cdot \frac{1}{n^2} < 10^{-6}$

$\rightarrow n > \sqrt{\frac{1}{6} \cdot 10^6}$

$= 408.248$

$\rightarrow n = 409$

5. Consider the Initial Value Problem

$$\frac{dx}{dt} = t(x+1), \quad x(0) = 2.$$

(a) (2 points extra credit) Verify that the solution to this initial value problem is the function

$$x(t) = 3e^{t^2/2} - 1.$$

L.H.S: $\frac{d}{dt}\left(3e^{t^2/2} - 1\right) = 3te^{t^2/2}$ ✓

R.H.S: $t\left(3e^{t^2/2} - 1 + 1\right) = 3te^{t^2/2}$ ✓

I.C. $x(0) = 3e^{0} - 1 = 3 - 1 = 2$ ✓

(b) (2 points extra credit) With a step size of $h = \frac{1}{4}$ Approximate $x(\frac{1}{2})$ using Euler's Method.

$x(t+h) \approx x(t) + h f(t, x(t))$

$x(\frac{1}{4}) \approx 2 + \frac{1}{4}(0(2+1)) = 2$

$x(\frac{1}{2}) \approx 2 + \frac{1}{4}(\frac{1}{4}(2+1)) = 2 + \frac{3}{16}$

$= \frac{32}{16} + \frac{3}{16}$

$= \frac{35}{16}$

AMATH 301 Exam 2 Version 1

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Instructions: Write neatly and show all work. Unless it is stated otherwise, in order to receive full credit, you must show your work and carefully justify your answers. Please circle your final answer. The number in parentheses next to the problem number is the point value of the problem. Besides the one side of a 8.5 by 11 inch note sheet, notes, textbooks, or calculators may not be used on this exam. Please verify that this exam consists of 4 problems and 1 extra credit problem, on the front and back of 4 pages. Any work on scratch paper will not be graded.

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Signature:

1. Consider the function

$$f(x) = x^2 - 5.$$

It is possible to use Newton's method and the bisection method to approximate the roots of this function.

(a) (3 points) If the initial guess is $x_0 = 2$, calculate x_1 using Newtons method.

$f'(x) = 2x$

$f(2) = 4 - 5 = -1$

$f'(2) = 4$

$x_1 = 2 - \frac{-1}{4} = 2 + \frac{1}{4} = \boxed{\frac{9}{4}}$

(b) (3 points) With an interval of $[2, 3]$ and an initial midpoint of $x_0 = \frac{1}{2}(2+3) = 5/2$, calculate x_1 using the bisection method.

$f(2) = -1$

$f(5/2) = \frac{25}{4} - 5 = \frac{25}{4} - \frac{20}{4} = \frac{5}{4}$

$f(3) = 9 - 5 = 4$

$\cancel{x_1 = \frac{5/2 + 3}{2} = \frac{5/2 + 6/2}{2}}$

$\cancel{x_1 = \frac{11}{4}}$

$x_1 = \frac{5/2 + 4/2}{2} = \frac{9}{4}$

(c) (4 points) Determine how many steps of the bisection method are needed to determine the root with an error of at most $(1/2) * 10^{-3}$.
hint: $\frac{\ln 10^{-3}}{\ln 2} = -9.9658$ rounded to 4 decimal places

$$\frac{\ln(b-a) - \ln(2\varepsilon)}{\ln(2)} < n$$

$$\frac{\ln(3-2) - \ln(10^{-3})}{\ln(2)} < n$$

$$\frac{-\ln(10^{-3})}{\ln(2)} < n$$

$n > 9.9658$

$\rightarrow n = 10$

2. (a) (8 points) Consider the following table of data

x	1	4	9
$f(x)$	1	2	3

Find the polynomial of degree 2 or less that interpolates the data above. You can find this polynomial using either Lagrange or Newton form, and you do not need to simplify your answer.

x	$f[\ ]$	$f[\ ,\ ]$	$f[\ ,\ ,\ ]$
1	1		
4	2	$1/3$	$\frac{1/5 - 1/3}{8} = -\frac{1}{60}$
9	3	$1/5$	

$$P_2(x) = 1 + \frac{1}{3}(x-1) - \frac{1}{60}(x-1)(x-4)$$

(b) (2 points) Now consider the table of data

x	0	1
$f(x)$	0	1

The polynomials $p(x) = x$, $q(x) = x^2$, and $r(x) = -x(x-2)$ also interpolate the data (you do not need to verify this). Explain why this does not violate the uniqueness part of the theorem on existence of polynomial interpolation.

there are 2 data points and thus only a degree 1 (or less) interpolating polynomial is unique.

3. Consider the data table:

x	-1	0	1	2	3
$f(x)$	$\frac{1}{2}$	1	3	9	27

(a) (3 points) This code below uses the backwards difference formula to approximate the $f'(0)$, $f'(1)$, $f'(2)$, and $f'(3)$ and assigns these approximations to the last four entries of the variable backward_diff. Assuming this code is run in python, write the variable backward_diff as python output or as a vector.

Code 1:

```
import numpy as np
x = np.array([-1,0,1,2,3.])
y = np.array([1/2,1,2,4,8])
n = x.shape[0]
backward_diff = np.zeros(n)
for i in range(1,n):
    backward_diff[i] = (y[i] - y[i-1])/(x[i] - x[i-1])
```

$$\left[0, \frac{1-1/2}{0-(-1)}, \frac{2-1}{1-0}, \frac{4-2}{2-1}, \frac{8-4}{3-2}\right]$$

$$= [0, 1/2, 1, 2, 4]$$

(b) (3 points) Consider the code below and add to it the code that saves the forward difference approximations for $f'(-1)$, $f'(0)$, $f'(1)$, and $f'(2)$ to the first four elements of the variable forward_diff

Code 2:

```
import numpy as np
x = np.array([-1,0,1,2,3.])
y = np.array([1/2,1,2,4,8])
n = x.shape[0]
backward_diff = np.zeros(n)
for i in range(1,n):
    backward_diff[i] = (y[i] - y[i-1])/(x[i] - x[i-1])
forward_diff = np.zeros(n)
```

Write your code here:

```
for i in range(n-1):
    forward_diff[i] = (y[i+1]-y[i])/(x[i+1]-x[i])
```

(c) (2 points) Using array indexing, write the code that assigns the middle three elements of forward_diff to the variable dx1 and assigns the middle three elements of backward_diff to the variable dx2.
Write your code here:

```
dx1 = forward_diff[1:n-1]
dx2 = backward_diff[1:n-1]
```

(d) (2 points) For the given data, one way to approximate $f'(0)$, $f'(1)$, and $f'(2)$ using the central difference formula is to average the forward and backward difference approximations for $f'(0)$, $f'(1)$, and $f'(2)$. Write the code that assigns the average of dx1 and dx2 (from part c) to the variable central_diff .
Write your code here:

```
central_diff = (1/2)*(dx1 + dx2)
```

4. (a) (6 points) Approximate the integral

$$\int_1^3 \frac{1}{x} dx,$$

with the left-hand rule, right-hand rule, and the midpoint rule using the partition

$$P = \{1, 2, 3\}.$$

(you do not need to compute the trapizoid rule approximation.)

LHR: $1(1 + \frac{1}{2}) = \frac{3}{2}$

RHR: $1(\frac{1}{2} + \frac{1}{3}) = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

MPR: $1(\frac{2}{3} + \frac{2}{5}) = \frac{10}{15} + \frac{6}{15} = \frac{16}{15}$

(b) (4 points) Determine the number of sub-intervals n that guarantees and error no larger than $\epsilon = 10^{-3}$ when the trapezoid rule is used to approximate the integral from part a.
hint: $\sqrt{\frac{4}{3}10^3} = 36.515$ rounded to three decimal places

$$\left|\int_1^3 \frac{1}{x} dx - T\right| = \frac{1}{12}(3-1)^3 \frac{1}{n^2} |f''(\xi)|$$

$f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -x^{-2}$

$f''(x) = 2x^{-3}$

$\leq \frac{8}{12} \frac{1}{n^2} \cdot 2 = \frac{4}{3} \frac{1}{n^2} < 10^{-3}$

$\rightarrow n > \sqrt{\frac{4}{3} 10^3} \approx 36.515$

$\rightarrow n = 37$

$f'''(x) = -6x^{-4} = \frac{-6}{x^4}$ → no critical points in $[1, 3]$

$\rightarrow f''(1) = \frac{2}{1^3}$, $f''(3) = \frac{2}{3^3} = \frac{2}{27}$

max

5. Consider the Initial Value Problem

$$\frac{dx}{dt} = 2tx^2, \quad x(0) = 1.$$

(a) (2 points extra credit) Verify that the solution to this initial value problem is the function

$$x(t) = \frac{1}{1-t^2}.$$

LHS: $\frac{d}{dt}\left(\frac{1}{1-t^2}\right) = (-1)-2t(1-t^2)^{-2} = \frac{2t}{(1-t^2)^2}$

RHS: $2t\left(\frac{1}{1-t^2}\right) = \frac{2t}{1-t^2}$

I.C. $x(0) = \frac{1}{1-0^2} = 1$ ✓

(b) (2 points extra credit) With a step size of $h = \frac{1}{4}$ Approximate $x(\frac{1}{2})$ using Euler's Method.

$x(t+h) \approx x(t) + h f(t, x(t))$

$x(\frac{1}{4}) \approx 1 + \frac{1}{4}(2 \cdot 0 \cdot 1) = 1$

$x(\frac{1}{2}) \approx 1 + \frac{1}{4}(2 \cdot \frac{1}{4} \cdot 1) = 1 + \frac{2}{16} = 1 + \frac{1}{8} = \frac{9}{8}$

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Signature:

1. Consider the function

$$f(x) = x^3 - 3x^2 + 1.$$

It is possible to use Newton's method and the bisection method to approximate the roots of this function.

(a) (3 points) If the initial guess is $x_0 = \frac{1}{2}$, calculate x_1 using Newtons method.

$f(\frac{1}{2}) = \frac{1}{8} - \frac{6}{8} + \frac{8}{8} = \frac{3}{8}$ $\quad x_1 = \frac{1}{2} - \frac{3}{8}\left(-\frac{4}{9}\right) = \frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2}$

$= \frac{1}{2} + \frac{1}{6}$

$x_1 = \frac{2}{3}$

$f'(x) = 3x^2 - 6x$, $f'(\frac{1}{2}) = \frac{3}{4} - \frac{6}{2} = \frac{3}{4} - \frac{12}{4} = -\frac{9}{4}$

(b) (3 points) With an interval of $[0, 1]$ and an initial midpoint of $x_0 = \frac{1}{2}(0+1) = \frac{1}{2}$, calculate x_1 using the bisection method.

$f(0) = 1$

$f(\frac{1}{2}) = \frac{3}{8}$

$f(1) = -3$

$x_1 = \frac{\frac{1}{2} + 1}{2} = \frac{\frac{3}{2}}{2} = \frac{3}{4}$

(c) (4 points) Determine how many steps of the bisection method from part b, are needed to determine the root with an error of at most $(1/2) * 10^{-5}$.
hint: $\frac{\ln(10^{-5})}{\ln(2)} = -16.6096$ rounded to 4 decimal places and recall that $\ln(1) = 0$.

$$\frac{\ln(1-0) - \ln(2\cdot\frac{1}{2}\cdot 10^{-5})}{\ln(2)} = \frac{-\ln(10^{-5})}{\ln(2)} \approx 16.6096$$

$\rightarrow n = 17$

2. (a) (8 points) Consider the following table of data

x	4	9	16
$f(x)$	2	3	4

Find the polynomial of degree 2 or less that interpolates the data above. You can find this polynomial using either Lagrange or Newton form, and you do not need to simplify your answer.

x	$f[\ ]$	$f[\ ,\ ]$	$f[\ ,\ ,\ ]$
4	2	$\frac{1}{5}$	
9	3		$\frac{\frac{1}{7}-\frac{1}{5}}{12} = -\frac{1}{210}$
16	4	$\frac{1}{7}$	

$$p_2(x) = 2 + \frac{1}{5}(x-4) - \frac{1}{210}(x-4)(x-9)$$

(b) (2 points) Now consider the table of data

x	0	1
$f(x)$	1	2

The polynomials $p(x) = x+1$, $q(x) = -(x-1)^2+2$, and $r(x) = x^2+1$ interpolate the data (you do not need to verify this). Explain why this does not violate the uniqueness part of the theorem on existence of polynomial interpolation.

Same as version 1.

3. Consider the data table:

x	-1	0	1	2	3
$f(x)$	$\frac{1}{4}$	1	4	16	64

(a) (3 points) This code below uses the forward difference formula to approximate the $f'(-1)$, $f'(0)$, $f'(1)$, and $f'(2)$ and assigns these approximations to the first four entries of the variable forward_diff. Assuming this code is run in python, write the variable forward_diff as python output or as a vector.

Code 1:

```
import numpy as np
x = np.array([-1,0,1,2,3.0])
y = np.array([1/4,1,4,16,64])
n = x.shape[0]
forward_diff = np.zeros(n)
for i in range(n-1):
    forward_diff[i] = (y[i+1] - y[i])/(x[i+1] - x[i])
```

$$\left[\frac{1-1/4}{0-(-1)}, \frac{4-1}{1-0}, \frac{16-4}{2-1}, \frac{64-\cancel{16}}{3-2}, 0\right]$$

$$= \left[3/4, 3, 12, 48, 0\right]$$

(b) (3 points) Consider the code below and add to it the code that saves the backward difference approximations for $f'(0)$, $f'(1)$, $f'(2)$, and $f'(3)$ to the last four elements of the variable backward_diff

Code 2:

```
import numpy as np
x = np.array([-1,0,1,2,3.0])
y = np.array([1/4,1,4,16,64])
n = x.shape[0]
forward_diff = np.zeros(n)
for i in range(n-1):
    forward_diff[i] = (y[i+1] - y[i])/(x[i+1] - x[i])
backward_diff = np.zeros(n)
```

Write your code here:

for i in range(1,n):

~~forward2diff[i]=~~

backward-diff[i]=(y[i]-y[i-1])/([x[i]-x[i-1]])

(c) (2 points) Using array slicing, write the code that assigns the middle three elements of forward_diff to the variable dx1 and assigns the middle three elements of backward_diff to the variable dx2.
Write your code here:

Same as version 1

(d) (2 points) For the given data, one way to approximate $f'(0)$, $f'(1)$, and $f'(2)$ using the central difference formula is to average the forward and backward difference approximations for $f'(0)$, $f'(1)$, and $f'(2)$. Write the code that assigns the average of dx1 and dx2 (from part c) to the variable central_diff .
Write your code here:

Same as Version 1

4. (a) (6 points) Approximate the integral

$$\int_0^2 x^3\,dx,$$

with the left-hand rule, right-hand rule, and the midpoint rule using the partition

$$P = \{0, 1, 2\}.$$

You do not need to compute the trapezoid rule approximation.

LHR: $1(0^3+1^3) = 1$

RHR: $1(1^3+2^3) = 9$

MPR: $1\left(\left(\frac{1}{2}\right)^3+\left(\frac{3}{2}\right)^3\right) = \frac{1}{8}+\frac{27}{8} = \frac{28}{8} = \frac{14}{4} = \frac{7}{2}$

(b) (4 points) Determine the number of sub-intervals n that guarantees and error no larger than $\epsilon = 10^{-3}$ when the trapezoid rule is used to approximate the integral from part a.
hint: $\sqrt{8 \times 10^3} = 89.443$ rounded to three decimal places

$f(x) = x^3$

$f'(x) = 3x^2$

$f''(x) = 6x$

$f'''(x) = 6$ no C.P. in $[0,2]$

$\frac{1}{12}(2)^3\frac{1}{n^2}|f''(\xi)| < 10^{-3}$ $\to$

$f''(0) = 0$

$f''(2) = 12$ max

$\frac{1}{12}\, 8\, \frac{1}{n^2}|f''(\xi)| < 8\frac{1}{n^2} < 10^{-3}$

$\to n > \sqrt{8\cdot 10^3} \approx 89.433$

$\to n = 90$

5. Consider the Initial Value Problem

$$\frac{dx}{dt} = -2tx, \quad x(0) = 2.$$

(a) (2 points extra credit) Verify that the solution to this initial value problem is the function

$$x(t) = 2e^{-t^2}.$$

LHS: $\frac{d}{dt}(2e^{-t^2}) = -4e^{-t^2}$

RHS: $-2t(2e^{-t^2}) = -4e^{-t^2}$ ✓

I.C.: $X(0) = 2e^{-0^2} = 2$ ✓

(b) (2 points extra credit) With a step size of $h = \frac{1}{4}$ Approximate $x(\frac{1}{2})$ using Euler's Method.

$X(\frac{1}{4}) \approx 2 + \frac{1}{4}(-2 \cdot 0 \cdot 2) = 2$

$X(\frac{1}{2}) \approx 2 + \frac{1}{4}(-2 \cdot \frac{1}{4} \cdot 2) = \cancel{2} \; 2 - \frac{1}{4} = \frac{7}{4}$