AMATH 301 Exam 1 Version 4

Name: Brooks Dennett

Friday, February 16, 2024

Student ID: 2264027

Instructions: Write neatly and show all work. Unless it is stated otherwise, in order to receive full credit, you must show your work and carefully justify your answers. Please circle your final answer. The number in parentheses next to the problem number is the point value of the problem. Besides the one side of a 8.5 by 11 inch note sheet, notes, textbooks, or calculators may not be used on this exam. Please verify that this exam consists of 5 problems on the front and back of 5 pages. Any work on scratch paper will not be graded.

This exam is worth 60 points.

Good Luck!

Pledge: By signing your name to this exam you are agreeing to the following statement:

I pledge that the work on this exam is mine only - I received no aid from outside sources nor did I obtain answers in ways that violate the instructions on the exam. I also pledge that I did not provide aid to any other student taking the exam.

Signature:

1. Consider the following python code:

Code 1:

```python
import numpy as np

v = np.array([-5,2,-3,4])

u = np.array([-7,8,0,1])

w = np.array([4,9,16,25])
```

State what each of the following code would return if ran in the python console after running the code above: (you do not need to justify/show your work)

(a) (2 points) v[1:3]

[2, -3]

(b) (3 points) u/(2*v)

$\frac{-7}{2(-5)}$, $\frac{8}{2(2)}$, $\frac{0}{2(-3)}$, $\frac{1}{2(4)}$

$\left[\frac{7}{10} , \frac{8}{4} , 0 , \frac{1}{8} \right]$

(c) (2 points) np.sqrt(w)

[2, 3, 4, 5]

(d) (2 points) Write the code (using array slicing) that extracts the first two values from v.

v[0:2]

(e) (2 points) Write the code (using array slicing) that extracts the last two values from u.

v[-2:0]

(f) (3 points) Use numpy functions and numpy constants to write the code that computes $e^{\sqrt{2\pi^2}} / \sin(10)$.

np.exp(np.sqrt(2*((np.pi)**2)))/np.sin(10)

2. Consider the following python code:

Code 2:

```
import numpy as np
n = 3
m = 4
A = np.zeros((n,m))
for i in range(n):
    for j in range(m):
        A[i,j] = (i+1)*(j+2)
```

(a) (4 points) If this code is run in python, write out the elements of A. You may write A as a matrix or as a numpy array.

3x4

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 4 & 6 & 8 & 10 \\ 6 & 9 & 12 & 15 \end{bmatrix}$$

i=0 j=0 = 2
i=1 j=0 4
i=2 j=0 6
i=0 j=1 3
i=0 j=2 4
i=0 j=3 5
i=1 j=1 6
i=1 j=2 8
i=1 j=3 = 10
i=2 j=1 = 9
i=2 j=2 = 12
i=2 j=3 = 15

(b) (2 points) Write a code that modifies the 3nd row of A to be a row of zeros. (you do not need to justify/show your work)

A[2,0] = np.zeros((n,m))

(c) (4 points) If the code below is run in python, write out the elements of A. You may write A as a matrix or as a numpy array.

Code 3:

```
import numpy as np
n = 3
m = 4
A = np.zeros((n,m))
for i in range(n):
    for j in range(m):
        A[i,j] = (i+1)*(j+2)

for k in range(n):
    if A[k,0] < 3:
        A[k,0] = 1
    elif A[k,0] < 5:
        A[k,0] = -1
    else:
        A[k,0] = 0
```

3x4

A[0,0] = 2 = 1

A[1,0] = 4 = -1

A[2,0] = 6 = 0

$$\boxed{A = \begin{bmatrix} 1 & 3 & 4 & 5 \\ -1 & 6 & 8 & 16 \\ 0 & 9 & 12 & 15 \end{bmatrix}}$$

3. Let **A** be the following matrix:

$$\mathbf{A} = \begin{array}{c}5\\4\\\cancel{3}\\2\\1\end{array}\begin{bmatrix}5 & 1 & 0 & 2 & 0\\0 & 3 & 0 & 10 & 6\\0 & 0 & 0 & 2 & 0\\0 & 0 & 4 & 0 & 1\\0 & 0 & 0 & 0 & 2\end{bmatrix}\begin{array}{c}\\\\3\\2\\\\\end{array}$$

We would like to solve a system of linear equations, $\mathbf{Ax} = \mathbf{b}$, for a 5x1 vector of unknowns $\mathbf{x}$, given a 5x1 vector of known constant values, $\mathbf{b}$. The following questions concern the setup of the system; you will not be solving the system in this problem.

a.) (3 points) Naive Gaussian elimination will not work on this system. In a single sentence, explain why.

Unable to solve w/ out pivoting be. the 4 in row 2 prevents back subbing.

b.) (3 points) If two rows of **A** are swapped, we will be able to solve this system via either forward or backward substitution. Please identify which two rows must be swapped and write down the matrix resulting from swapping these two rows. Do not swap more than two rows, i.e. three rows must remain unchanged after this operation. You may assume that row numbering begins at 1, thus row 1 is the row containing the numbers $(0, 0, 0, 0, 2)$.

swap rows 2 and 3

$$\begin{bmatrix}5 & 1 & 0 & 2 & 0\\0 & 3 & 0 & 10 & 6\\0 & 0 & 4 & 0 & 1\\0 & 0 & 0 & 2 & 0\\0 & 0 & 0 & 0 & 2\end{bmatrix}$$

c.) (3 points) Would the system found by swapping two rows be solvable by forward substitution or backward substitution? Please explain which, and how you know, in a single sentence. Do not solve the system.

backwards substitution because you solve for x_5 then x_4 and so on up to x_1.

4. Consider the system of equations $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{x}$ is a vector of unknowns and

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix}, \quad \text{and} \quad \mathbf{b} \begin{bmatrix} 2 \\ 10 \\ 4 \end{bmatrix}.$$

$R_2 = R_2 - 3R_1$
$R_3 = R_3 - 2R_1$

(a) (9 points) Find the LU decomposition for the matrix $\mathbf{A}$. Make sure to list all of your elementary matrices.

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \qquad M_1 A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & -2 & 1 \end{bmatrix}$$

$R_3 = R_3 + R_1$

$$M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad \boxed{M_2 M_1 A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} = U}$$

$$L = M_1^{-1} M_2^{-1}$$

$$M_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \qquad M_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$M_1^{-1} M_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}} = L$$

(b) (6 points) Use forward substitution and backward substitution to solve the system $\mathbf{A}\mathbf{x} = \mathbf{b}$.

$Lz = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \\ 4 \end{bmatrix}$$

$z_1 = 2$

$3(2) + 1(z_2) = 10$

$z_2 = 4$

$2(2) - 1(4) + z_3 = 4$

$z_3 = 4$

$Ux = z$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$$

$x_3 = 2$

$2x_2 + 1(2) = 4$

$x_2 = 1$

$1x_1 - x_2 + 2(2) = 2$

$x_1 - 1 + 4 = 2$

$x_1 = -1$

$$\boxed{\begin{aligned} x_1 &= -1 \\ x_2 &= 1 \\ x_3 &= 2 \end{aligned}}$$

5. (12 points) Consider the following system of equations, $\mathbf{Ax} = \mathbf{b}$:

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}.$$

Using the Jacobi iteration method, compute $\mathbf{x}^{(2)}$. Use the inital guess

$$\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \qquad C_L + C_U = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$D^{-1} = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$$

$$D^{-1}(C_L + C_U) = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1/4 & 0 \\ 1/3 & 0 & -1/3 \\ 0 & -1/4 & 0 \end{bmatrix}$$

$$D^{-1}b = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix}$$

$$X^{(1)} = \begin{bmatrix} 0 & 1/4 & 0 \\ 1/3 & 0 & -1/3 \\ 0 & -1/4 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix} = \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix}$$

$$X^{(2)} = \begin{bmatrix} 0 & 1/4 & 0 \\ 1/3 & 0 & -1/3 \\ 0 & -1/4 & 0 \end{bmatrix} \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix} + \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix} \Longrightarrow$$

$$\begin{bmatrix} 0 + 1/4\left(\frac{4}{3}\right) + 0 \\ 1/3(3/4) + 0 + -1/3(5/4) \\ 0 + -1/4(4/3) + 0 \end{bmatrix} = \begin{bmatrix} 4/12 \\ 3/12 + -5/12 \\ -4/12 \end{bmatrix} = \begin{bmatrix} 1/3 \\ -1/6 \\ -1/3 \end{bmatrix} + \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix}$$

$$\begin{bmatrix} 4/12 + 9/12 \\ -2/12 + 16/12 \\ -4/12 + 15/12 \end{bmatrix} = \boxed{\begin{bmatrix} 13/12 \\ 14/12 \\ 11/12 \end{bmatrix}}$$

AMATH 301 Exam 1 Version 4

Friday, February 16, 2024

Name: Joshell Flores Vega

Student ID: 7131676

Instructions: Write neatly and show all work. Unless it is stated otherwise, in order to receive full credit, you must show your work and carefully justify your answers. Please circle your final answer. The number in parentheses next to the problem number is the point value of the problem. Besides the one side of a 8.5 by 11 inch note sheet, notes, textbooks, or calculators may not be used on this exam. Please verify that this exam consists of 5 problems on the front and back of 5 pages. Any work on scratch paper will not be graded.

This exam is worth 60 points.

Good Luck!

Pledge: By signing your name to this exam you are agreeing to the following statement:

I pledge that the work on this exam is mine only - I received no aid from outside sources nor did I obtain answers in ways that violate the instructions on the exam. I also pledge that I did not provide aid to any other student taking the exam.

Signature: ____

1. Consider the following python code:

Code 1:

```
import numpy as np

v = np.array([-5,2,-3,4])

u = np.array([-7,8,0,1])

w = np.array([4,9,16,25])
```

[-10, 4, -6, 8]

State what each of the following code would return if ran in the python console after running the code above: (you do not need to justify/show your work)

(a) (2 points) v[1:3]

[2, -3]

(b) (3 points) u/(2*v)

$\left[\frac{7}{10}, 2, 0, \frac{1}{8}\right]$

(c) (2 points) np.sqrt(w)

[2, 3, 4, 5]

(d) (2 points) Write the code (using array slicing) that extracts the first two values from v.

v [0:2]

(e) (2 points) Write the code (using array slicing) that extracts the last two values from u.

u [2:]

(f) (3 points) Use numpy functions and numpy constants to write the code that computes $e^{\sqrt{2\pi^2}} / \sin(10)$.

(np.e ** (np.sqrt(2*(np.pi**2)))) / np.sin(10)

2. Consider the following python code:

Code 2:

```python
import numpy as np
n = 3
m = 4
A = np.zeros((n,m))
for i in range(n):
    for j in range(m):
        A[i,j] = (i+1)*(j+2)
```

1 · 2 2 5

(a) (4 points) If this code is run in python, write out the elements of A. You may write A as a matrix or as a numpy array.

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 4 & 6 & 8 & 10 \\ 6 & 9 & 12 & 15 \end{bmatrix}$$

(b) (2 points) Write a code that modifies the 3nd row of A to be a row of zeros. (you do not need to justify/show your work)

A[2] = 0

(c) (4 points) If the code below is run in python, write out the elements of A. You may write A as a matrix or as a numpy array.

Code 3:

```
import numpy as np
n = 3
m = 4
A = np.zeros((n,m))
for i in range(n):
    for j in range(m):
        A[i,j] = (i+1)*(j+2)

for k in range(n):
    if A[k,0] < 3:
        A[k,0] = 1
    elif A[k,0] < 5:
        A[k,0] = -1
    else:
        A[k,0] = 0
```

$$\begin{bmatrix} 2 & 3 & 4 & 5 \\ 4 & 6 & 8 & 10 \\ 6 & 9 & 12 & 15 \end{bmatrix}$$

(0-2)

k=0
A[0,0]=1

k=1
A[1,0]=-1

k=2
A[2,0]=0

$$A = \begin{bmatrix} 1 & 3 & 4 & 5 \\ -1 & 6 & 8 & 10 \\ 6 & 9 & 12 & 15 \end{bmatrix}$$

3. Let $\mathbf{A}$ be the following matrix:

$$\mathbf{A} = \begin{bmatrix} 5 & 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 10 & 6 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}.$$

We would like to solve a system of linear equations, $\mathbf{Ax} = \mathbf{b}$, for a 5x1 vector of unknowns $\mathbf{x}$, given a 5x1 vector of known constant values, $\mathbf{b}$. The following questions concern the setup of the system; you will not be solving the system in this problem.

a.) (3 points) Naive Gaussian elimination will not work on this system. In a single sentence, explain why. row 3 has a pivot on x_4 when it should be on x_3 for naive Gaussian elim to work.

b.) (3 points) If two rows of $\mathbf{A}$ are swapped, we will be able to solve this system via either forward or backward substitution. Please identify which two rows must be swapped and write down the matrix resulting from swapping these two rows. Do not swap more than two rows, i.e. three rows must remain unchanged after this operation. You may assume that row numbering begins at 1, thus row 1 is the row containing the numbers $(0, 0, 0, 0, 2)$.

rows 3 and 4 are swapped

$$\begin{bmatrix} 5 & 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 10 & 6 \\ 0 & 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

c.) (3 points) Would the system found by swapping two rows be solvable by forward substitution or backward substitution? Please explain which, and how you know, in a single sentence. Do not solve the system.

Forward Sub. This is currently in the form $Ax=b$ & not in LU decomp; A Naive Gauss algorithm would be best suited

4. Consider the system of equations $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{x}$ is a vector of unknowns and

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix}, \quad \text{and} \quad \mathbf{b} \begin{bmatrix} 2 \\ 10 \\ 4 \end{bmatrix}.$$

(a) (9 points) Find the LU decomposition for the matrix $\mathbf{A}$. Make sure to list all of your elementary matrices.

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 10 \\ 4 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}}_{M_1} \underbrace{\begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & -2 & 1 \end{bmatrix}}_{M_1A}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}}_{M_2} \underbrace{\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & -2 & 1 \end{bmatrix}}_{M_1A} = \underbrace{\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}}_{M_2M_1A = U}$$

$$M_1 M_1^{-1} = I \qquad \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$M_1^{-1} M_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} = L$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\cancel{A} \quad \begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

(b) (6 points) Use forward substitution and backward substitution to solve the system $\mathbf{Ax} = \mathbf{b}$.

$Ux = z$

$Lz = b$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 3 & 1 & 0 & 10 \\ 2 & -1 & 1 & 4 \end{array}\right]$$

$z_1 = 2$

$z_2 = 10 - 6 = 4$

$z_3 = 4 - 4 + 4 = 0$

$$z = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 2 & 0 \end{array}\right]$$

$x_3 = 0$

$x_2 = 4/2 = 2$

$x_1 = 2/-2 = -1$

$$x = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$$

5. (12 points) Consider the following system of equations, $\mathbf{Ax} = \mathbf{b}$:

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}.$$

Using the Jacobi iteration method, compute $\mathbf{x}^{(2)}$. Use the inital guess

$$\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$Q = D$

$$D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$x^k = (C_L + C_u)x^{(k-1)} + D^{-1}b$$

$$D^{-1} = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$$

$$x^k = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} x^{(k-1)} + \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$C_L = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$x^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_u = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x^1 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix} = \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix}$$

$$D^{-1}b = \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix}$$

$$x^2 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix} + \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix}$$

$$= \begin{bmatrix} -4/3 \\ 2/4 \\ 4/3 \end{bmatrix} + \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix} = \boxed{\begin{bmatrix} -5/12 \\ 22/12 \\ 31/12 \end{bmatrix}}$$

16 9

6 16

16 15

AMATH 301 Exam 1 Version 4 Name: Elijah Espineli

Friday, February 16, 2024 Student ID: espineli

Instructions: Write neatly and show all work. Unless it is stated otherwise, in order to receive full credit, you must show your work and carefully justify your answers. Please circle your final answer. The number in parentheses next to the problem number is the point value of the problem. Besides the one side of a 8.5 by 11 inch note sheet, notes, textbooks, or calculators may not be used on this exam. Please verify that this exam consists of 5 problems on the front and back of 5 pages. Any work on scratch paper will not be graded.

This exam is worth 60 points.

Good Luck!

Pledge: By signing your name to this exam you are agreeing to the following statement:

I pledge that the work on this exam is mine only - I received no aid from outside sources nor did I obtain answers in ways that violate the instructions on the exam. I also pledge that I did not provide aid to any other student taking the exam.

Signature: ______

1. Consider the following python code:

Code 1:

```
import numpy as np

v = np.array([-5,2,-3,4])

u = np.array([-7,8,0,1])

w = np.array([4,9,16,25])
```

State what each of the following code would return if ran in the python console after running the code above: (you do not need to justify/show your work)

(a) (2 points) v [1:3]

[2 , -3]

(b) (3 points) u/(2*v)

(-7, 8, 0, 1) / (-10, 4, -6, 8)

[$\frac{7}{10}$, 2 , 0 , $\frac{1}{8}$]

as decimals

(c) (2 points) np.sqrt(w)

[2, 3, 4, 5]

(d) (2 points) Write the code (using array slicing) that extracts the first two values from v.

v[:2]

(e) (2 points) Write the code (using array slicing) that extracts the last two values from u.

u[-2:]

(f) (3 points) Use numpy functions and numpy constants to write the code that computes $e^{\sqrt{2\pi^2}}/\sin(10)$.

(np.exp(np.sqrt(2*(np.pi**2))))/(np.sin(10))

2. Consider the following python code:

Code 2:

```
import numpy as np
n = 3
m = 4
A = np.zeros((n,m))
for i in range(n):
    for j in range(m):
        A[i,j] = (i+1)*(j+2)
                     3
```

(a) (4 points) If this code is run in python, write out the elements of A. You may write A as a matrix or as a numpy array.

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 4 & 6 & 8 & 10 \\ 6 & 9 & 12 & 15 \end{bmatrix}$$

(b) (2 points) Write a code that modifies the 3nd row of A to be a row of zeros. (you do not need to justify/show your work)

```
for j in range(m):
    A[2, j] = 0
```

or

```
A[2, :] = 0
```

(c) (4 points) If the code below is run in python, write out the elements of A. You may write A as a matrix or as a numpy array.

Code 3:

```
import numpy as np
n = 3
m = 4
A = np.zeros((n,m))
for i in range(n):
    for j in range(m):
        A[i,j] = (i+1)*(j+2)

for k in range(n):
    if A[k,0] < 3:
        A[k,0] = 1
    elif A[k,0] < 5:
        A[k,0] = -1
    else:
        A[k,0] = 0
```

$$\begin{array}{cc} & m \\ n & \begin{bmatrix} 2 & 3 & 4 & 5 \\ 4 & 6 & 8 & 10 \\ 6 & 9 & 12 & 15 \end{bmatrix} \end{array}$$

$$\boxed{\begin{bmatrix} 1 & 3 & 4 & 5 \\ -1 & 6 & 8 & 10 \\ 0 & 9 & 12 & 15 \end{bmatrix}}$$

3. Let $\mathbf{A}$ be the following matrix:

$$\mathbf{A} = \begin{bmatrix} 5 & 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 10 & 6 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}.$$

We would like to solve a system of linear equations, $\mathbf{Ax} = \mathbf{b}$, for a 5x1 vector of unknowns $\mathbf{x}$, given a 5x1 vector of known constant values, $\mathbf{b}$. The following questions concern the setup of the system; you will not be solving the system in this problem.

a.) (3 points) Naive Gaussian elimination will not work on this system. In a single sentence, explain why.

Row 4 column 3 canot be row reduced only using Gaussian elimination.

b.) (3 points) If two rows of $\mathbf{A}$ are swapped, we will be able to solve this system via either forward or backward substitution. Please identify which two rows must be swapped and write down the matrix resulting from swapping these two rows. Do not swap more than two rows, i.e. three rows must remain unchanged after this operation. You may assume that row numbering begins at 1, thus row 1 is the row containing the numbers $(0, 0, 0, 0, 2)$.

2nd & 3rd rows swap assuming that row 1 is the bottom.

$$\begin{bmatrix} 5 & 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 10 & 6 \\ 0 & 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

c.) (3 points) Would the system found by swapping two rows be solvable by forward substitution or backward substitution? Please explain which, and how you know, in a single sentence. Do not solve the system.

backwards, this is because we have a pivot in each row, so going backwards, we are solving 1 equation w/ 1 variable every time.

4. Consider the system of equations $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{x}$ is a vector of unknowns and

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix}, \quad \text{and} \quad \mathbf{b} \begin{bmatrix} 2 \\ 10 \\ 4 \end{bmatrix}.$$

(a) (9 points) Find the LU decomposition for the matrix $\mathbf{A}$. Make sure to list all of your elementary matrices.

Mult = 3

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix} \quad ② = 2 - 3① \quad \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 2 & -4 & 5 \end{bmatrix}$$

mult = 2

$$③ = ③ - 2① \quad \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & -6 & 1 \end{bmatrix}$$

mult = -3

$$③ = ③ + 3② \quad \boxed{\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 4 \end{bmatrix} = U}$$

$$\boxed{L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -3 & 1 \end{bmatrix}}$$

check:

$A = LU$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix}$$

✓ ☺

(b) (6 points) Use forward substitution and backward substitution to solve the system $\mathbf{Ax} = \mathbf{b}$.

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \\ 4 \end{bmatrix}$$

$$1x_1 - x_2 + 2x_3 = 2$$
$$2x_2 + x_3 = 10$$
$$4x_3 = 4$$

$$x_3 = 1$$

$$1x_1 - x_2 + 2 = 2$$
$$2x_2 + 1 = 10$$
$$2x\,x_2 = 9$$
$$x_2 = \frac{9}{2}$$
$$1x_1 - \frac{9}{2} + 2 = 2$$
$$x_1 = \frac{9}{2}$$

$$\boxed{x = \begin{bmatrix} 9/2 \\ 9/2 \\ 1 \end{bmatrix}}$$

5. (12 points) Consider the following system of equations, $\mathbf{A}\mathbf{x} = \mathbf{b}$:

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}.$$

Using the Jacobi iteration method, compute $\mathbf{x}^{(2)}$. Use the inital guess

$$\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$x^{k} = -D^{-1} T x^{(k-1)} + D^{-1} b \qquad -D^{-1} = \begin{bmatrix} -\frac{1}{4} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{4} \end{bmatrix}$$

$$x^{(1)} = \begin{bmatrix} -1/4 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/4 \end{bmatrix} \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + D^{-1} b \qquad T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{3}{4} \\ -\frac{4}{3} \\ -\frac{5}{4} \end{bmatrix}$$

$$x^{(2)} = \begin{bmatrix} -1/4 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -\frac{3}{4} \\ -\frac{4}{3} \\ -\frac{5}{4} \end{bmatrix} + \begin{bmatrix} -\frac{3}{4} \\ -\frac{4}{3} \\ -\frac{5}{4} \end{bmatrix}$$

$$x^{(2)} = \begin{bmatrix} 0 & -1/4 & 0 \\ -1/3 & -1 & 1/3 \\ 0 & 1/4 & 0 \end{bmatrix} \begin{bmatrix} -\frac{3}{4} \\ -\frac{4}{3} \\ -\frac{5}{4} \end{bmatrix} + \begin{bmatrix} -\frac{3}{4} \\ -\frac{4}{3} \\ -\frac{5}{4} \end{bmatrix}$$

$$x^{(2)} = \begin{bmatrix} \frac{1}{3} \\ \frac{3}{12} + \frac{16}{12} - \frac{5}{12} \\ -\frac{1}{3} \end{bmatrix} + \begin{bmatrix} -\frac{9}{12} \\ -\frac{16}{12} \\ -\frac{15}{12} \end{bmatrix} = \boxed{\begin{bmatrix} -\frac{5}{12} \\ -\frac{1}{6} \\ -\frac{19}{12} \end{bmatrix}}$$

9

AMATH 301 Exam 1 Version 4

Friday, February 16, 2024

Name: AARYAN SHAH

Student ID: 2164957

Instructions: Write neatly and show all work. Unless it is stated otherwise, in order to receive full credit, you must show your work and carefully justify your answers. Please circle your final answer. The number in parentheses next to the problem number is the point value of the problem. Besides the one side of a 8.5 by 11 inch note sheet, notes, textbooks, or calculators may not be used on this exam. Please verify that this exam consists of 5 problems on the front and back of 5 pages. Any work on scratch paper will not be graded.

This exam is worth 60 points.

Good Luck!

Pledge: By signing your name to this exam you are agreeing to the following statement:

I pledge that the work on this exam is mine only - I received no aid from outside sources nor did I obtain answers in ways that violate the instructions on the exam. I also pledge that I did not provide aid to any other student taking the exam.

Signature: A

1. Consider the following python code:

Code 1:

```
import numpy as np

v = np.array([-5,2,-3,4])

u = np.array([-7,8,0,1])

w = np.array([4,9,16,25])
```

State what each of the following code would return if ran in the python console after running the code above: (you do not need to justify/show your work)

(a) (2 points) v [1:3]

[2 , -3]

(b) (3 points) u/(2∗v)

$$\frac{u}{2v} = \frac{-7, 8, 0, 1}{-10, 4, -6, 8} = \left[\frac{7}{10}, 2, 0, \frac{1}{8}\right]$$

(c) (2 points) np.sqrt(w)

[2, 3, 4, 5]

(d) (2 points) Write the code (using array slicing) that extracts the first two values from v.

v[:2]

(e) (2 points) Write the code (using array slicing) that extracts the last two values from u.

u[-2:]

(f) (3 points) Use numpy functions and numpy constants to write the code that computes $e^{\sqrt{2\pi^2}} / \sin(10)$.

np.exp(np.sqrt(2*(np.pi**2))) / np.sin(10)

2. Consider the following python code:

Code 2:

```
import numpy as np
n = 3
m = 4
A = np.zeros((n,m))
for i in range(n):
    for j in range(m):
        A[i,j] = (i+1)*(j+2)
```

(a) (4 points) If this code is run in python, write out the elements of A. You may write A as a matrix or as a numpy array.

$$A = \begin{bmatrix} 6 & 9 & 12 & 15 \\ 8 & 12 & 16 & 20 \\ 10 & 15 & 20 & 25 \end{bmatrix}$$

(b) (2 points) Write a code that modifies the 3nd row of A to be a row of zeros. (you do not need to justify/show your work)

~~A[3,:] = np.zeros~~

A = np.zeros(3,m)

(c) (4 points) If the code below is run in python, write out the elements of A. You may write A as a matrix or as a numpy array.

Code 3:

```
import numpy as np
n = 3
m = 4
A = np.zeros((n,m))
for i in range(n):
    for j in range(m):
        A[i,j] = (i+1)*(j+2)

for k in range(n):
    if A[k,0] < 3:
        A[k,0] = 1
    elif A[k,0] < 5:
        A[k,0] = -1
    else:
        A[k,0] = 0
```

$$A = \begin{bmatrix} 1 & 3 & 4 & 5 \\ -6 & 8 & 10 & 12 \\ 0 & 9 & 12 & 15 \end{bmatrix}$$

3. Let $\mathbf{A}$ be the following matrix:

$$\mathbf{A} = \begin{bmatrix} 5 & 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 10 & 6 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}.$$

We would like to solve a system of linear equations, $\mathbf{Ax} = \mathbf{b}$, for a 5x1 vector of unknowns $\mathbf{x}$, given a 5x1 vector of known constant values, $\mathbf{b}$. The following questions concern the setup of the system; you will not be solving the system in this problem.

a.) (3 points) Naive Gaussian elimination will not work on this system. In a single sentence, explain why.

This is because we would need to swap rows here, since the element above 4 in row 3 is a zero, and we need to obtain an upper triangular matrix.

b.) (3 points) If two rows of $\mathbf{A}$ are swapped, we will be able to solve this system via either forward or backward substitution. Please identify which two rows must be swapped and write down the matrix resulting from swapping these two rows. Do not swap more than two rows, i.e. three rows must remain unchanged after this operation. You may assume that row numbering begins at 1, thus row 1 is the row containing the numbers $(0, 0, 0, 0, 2)$.

We would swap row 4 (0, 0, 4, 0, 1) and row 3 (0, 0, 0, 2, 0).

$$\text{resulting matrix } A = \begin{bmatrix} 5 & 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 10 & 6 \\ 0 & 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

c.) (3 points) Would the system found by swapping two rows be solvable by forward substitution or backward substitution? Please explain which, and how you know, in a single sentence. Do not solve the system.

Backward substitution because this matrix is in upper triangular form.

4. Consider the system of equations $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{x}$ is a vector of unknowns and

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix}, \quad \text{and} \quad \mathbf{b} \begin{bmatrix} 2 \\ 10 \\ 4 \end{bmatrix}.$$

(a) (9 points) Find the LU decomposition for the matrix $\mathbf{A}$. Make sure to list all of your elementary matrices.

$A = LU$

$MA = U$

$L = M^{-1}$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ & & \\ & & \end{bmatrix}$$

(b) (6 points) Use forward substitution and backward substitution to solve the system $\mathbf{Ax} = \mathbf{b}$.

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & 4 & 5 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 10 \\ 4 \end{bmatrix}$$

$$[A|b] = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 3 & -1 & 7 & 10 \\ 2 & 4 & 5 & 4 \end{array}\right]$$

$$R_3 \to R_3 - 2R_1 \qquad \begin{bmatrix} 1 & -1 & 2 & 2 \\ 3 & -1 & 7 & 10 \\ 0 & 6 & 1 & 0 \end{bmatrix}$$

5. (12 points) Consider the following system of equations, $\mathbf{A}\mathbf{x} = \mathbf{b}$:

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}.$$

Using the Jacobi iteration method, compute $\mathbf{x}^{(2)}$. Use the inital guess

$$\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \quad x^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Jacobi iteration $\Rightarrow$ $x^k = -D^{-1} T x^{k-1} + D^{-1} b$

$$D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \quad D^{-1} = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$x^1 = \begin{bmatrix} -1/4 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$= \underset{3\times 3}{\begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}} = \begin{bmatrix} 1/4(3) + 0(4) + 0(5) \\ 0(3) + 1/3(4) + 0(5) \\ 0(3) + 0(4) + \frac{1}{4}(5) \end{bmatrix}$$

$$x^1 = \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix}$$

contd.

$$x^2 = \underbrace{\begin{bmatrix} -1/4 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix}} + \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1/4(0) + 0(1) + 0(0) & -1/4(1) & 0 \\ 0(0) + (-1/3)(1) + 0(0) & 0 & -1/3(-1) \\ 0(0) + 0(0) + (-1/4)(0) & (-1/4)(-1) & 0 \end{bmatrix} \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix}$$

$$x^2 = \begin{bmatrix} 0 & -1/4 & 0 \\ -1/3 & 0 & 1/3 \\ 0 & 1/4 & 0 \end{bmatrix}_{3\times 3} \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix}_{3\times 1} + \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix}$$

$$= \begin{bmatrix} -1/4(4/3) \\ -1/3(3/4) + 1/3(5/4) \\ 1/4(4/3) \end{bmatrix} + \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix}$$

$$\frac{3}{4}\,{\scriptstyle\times 3} - \frac{1}{3}\,{\scriptstyle\times 4} \qquad \frac{9-4}{12}$$

$$\frac{1}{6} + \frac{4}{3} \qquad \frac{1+8}{6}$$

$$\cancel{-\frac{1}{4}\,{\scriptstyle\frac{\times 3}{\times 3}} + \frac{5}{12} \qquad \frac{5-3}{12} = \frac{2}{12}} = \frac{1}{6}$$

$$= \begin{bmatrix} -1/3 \\ 1/6 \\ 1/3 \end{bmatrix} + \begin{bmatrix} 3/4 \\ 4/3 \\ 5/4 \end{bmatrix} = \begin{bmatrix} 5/12 \\ 3/2 \\ 19/12 \end{bmatrix}$$

$$\frac{1}{3} + \frac{5}{4} \qquad \frac{4+15}{12}$$

$$x^2 = \begin{bmatrix} 5/12 \\ 3/2 \\ 19/12 \end{bmatrix}_{3\times 1}$$

AMATH 301 Exam 1 Version 4

Friday, February 16, 2024

Name: Dane Grassy

Student ID: 2232552

Instructions: Write neatly and show all work. Unless it is stated otherwise, in order to receive full credit, you must show your work and carefully justify your answers. Please circle your final answer. The number in parentheses next to the problem number is the point value of the problem. Besides the one side of a 8.5 by 11 inch note sheet, notes, textbooks, or calculators may not be used on this exam. Please verify that this exam consists of 5 problems on the front and back of 5 pages. Any work on scratch paper will not be graded.

This exam is worth 60 points.

Good Luck!

Pledge: By signing your name to this exam you are agreeing to the following statement:

I pledge that the work on this exam is mine only - I received no aid from outside sources nor did I obtain answers in ways that violate the instructions on the exam. I also pledge that I did not provide aid to any other student taking the exam.

Signature: Dane Grassy

1. Consider the following python code:

Code 1:

```
import numpy as np

v = np.array([-5,2,-3,4])

u = np.array([-7,8,0,1])

w = np.array([4,9,16,25])
```

State what each of the following code would return if ran in the python console after running the code above: (you do not need to justify/show your work)

(a) (2 points) v[1:3]

[2, -3]

(b) (3 points) u/(2*v)

$\left[\frac{-7}{-10}, \frac{8}{4}, \frac{0}{-6}, \frac{1}{8}\right]$ $\left[\frac{7}{10}, 2, 0, \frac{1}{8}\right]$

(c) (2 points) np.sqrt(w)

[2, 3, 4, 5]

(d) (2 points) Write the code (using array slicing) that extracts the first two values from v.

index 0, 1

```
V[0:2]
```

(e) (2 points) Write the code (using array slicing) that extracts the last two values from u.

```
U[-2:]
```

(f) (3 points) Use numpy functions and numpy constants to write the code that computes $e^{\sqrt{2\pi^2}} / \sin(10)$.

X = np.exp(np.sqrt(2*(np.pi.

~~X = np.~~

```
x = (np.e ** (np.sqrt(2*(np.(pi)**2)))) / np.sin(10)
```

2. Consider the following python code:

Code 2:

```
import numpy as np
n = 3
m = 4
A = np.zeros((n,m))
for i in range(n):
    for j in range(m):
        A[i,j] = (i+1)*(j+2)
```

2 3·2

1·2 1·3

3 | 4 5

2·0

(a) (4 points) If this code is run in python, write out the elements of A. You may write A as a matrix or as a numpy array.

3x4

$$\begin{bmatrix} 2 & 3 & 4 & 5 \\ 4 & 6 & 8 & 10 \\ 6 & 9 & 12 & 15 \end{bmatrix}$$

(b) (2 points) Write a code that modifies the 3nd row of A to be a row of zeros. (you do not need to justify/show your work)

for i inrange(m):
    A[2,i]=0

(c) (4 points) If the code below is run in python, write out the elements of A. You may write A as a matrix or as a numpy array.

Code 3:

```
import numpy as np
n = 3
m = 4
A = np.zeros((n,m))
for i in range(n):
    for j in range(m):
        A[i,j] = (i+1)*(j+2)

for k in range(n):
    if A[k,0] < 3:
        A[k,0] = 1
    elif A[k,0] < 5:
        A[k,0] = -1
    else:
        A[k,0] = 0
```

X

$$\cancel{\begin{bmatrix} 6 & 8 & 10 & 12 \\ 9 & 12 & 15 & 18 \\ 12 & 16 & 20 & 24 \end{bmatrix}}$$

$[0,0] = 2$

$$\begin{bmatrix} 2 & 3 & 4 & 5 \\ 4 & 6 & 8 & 10 \\ 6 & 9 & 12 & 15 \end{bmatrix}$$

Modifys column zero

$$\begin{bmatrix} 1 & 3 & 4 & 5 \\ -1 & 6 & 8 & 10 \\ 0 & 9 & 12 & 15 \end{bmatrix}$$

3. Let $\mathbf{A}$ be the following matrix:

$$\mathbf{A} = \begin{bmatrix} 5 & 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 10 & 6 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}.$$

We would like to solve a system of linear equations, $\mathbf{Ax} = \mathbf{b}$, for a 5x1 vector of unknowns $\mathbf{x}$, given a 5x1 vector of known constant values, $\mathbf{b}$. The following questions concern the setup of the system; you will not be solving the system in this problem.

a.) (3 points) Naive Gaussian elimination will not work on this system. In a single sentence, explain why.

Because there is a pivot in row 4, not row 3.

b.) (3 points) If two rows of $\mathbf{A}$ are swapped, we will be able to solve this system via either forward or backward substitution. Please identify which two rows must be swapped and write down the matrix resulting from swapping these two rows. Do not swap more than two rows, i.e. three rows must remain unchanged after this operation. You may assume that row numbering begins at 1, thus row 1 is the row containing the numbers $(0, 0, 0, 0, 2)$.

$R_3 \Leftrightarrow R_4$

$$\begin{bmatrix} 5 & 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 10 & 6 \\ 0 & 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

c.) (3 points) Would the system found by swapping two rows be solvable by forward substitution or backward substitution? Please explain which, and how you know, in a single sentence. Do not solve the system.

Backwards sub, as we know the matrix is upper triangular, and we know X_5 already, as is a lone pivot, so if we had b we could back sub. for the rest.

4. Consider the system of equations $\mathbf{A}\mathbf{x} = \mathbf{b}$ where $\mathbf{x}$ is a vector of unknowns and

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix}, \quad \text{and} \quad \mathbf{b} \begin{bmatrix} 2 \\ 10 \\ 4 \end{bmatrix}.$$

(a) (9 points) Find the LU decomposition for the matrix $\mathbf{A}$. Make sure to list all of your elementary matrices.

$$L = \begin{bmatrix} 0 & 0 & 0 \\ 1/3 & 0 & 0 \\ 1/2 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix} \quad 3R_1 - R_2 \to R_2 \quad \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 2 & -4 & 5 \end{bmatrix}$$

$R_3 - 2R_1 \to R_3$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & -2 & 5 \end{bmatrix} \quad -4+2$$

$R_3 + R_2 \to R_3$

$$U = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 6 \end{bmatrix}$$

(b) (6 points) Use forward substitution and backward substitution to solve the system $\mathbf{Ax} = \mathbf{b}$.

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \\ 4 \end{bmatrix}$$

$A = LU \quad LUX = B$

$UX = Y$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = Y$$

$1x_1 - 1x_2 + 2x_3 = 30$

$0x_1 + 2x_2 + x_3 = -11$

$6x_3 = 0$

$\boxed{x_3 = 0}$

$x_2 =$

$2x_2 + 0 = -11$

$\boxed{x_2 = -5.5}$

$$\begin{bmatrix} 0 & 0 & 0 \\ 1/3 & 0 & 0 \\ 1/2 & 1 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \\ 4 \end{bmatrix}$$

0

$1/3 x_1 = 10 \quad x_1 = 30$

$1/2 x_1 + x_2 = 4$

$15 + x_2 = 4 \quad x_2 = -11$

$$\begin{bmatrix} 30 \\ -11 \\ 0 \end{bmatrix}$$

$x_1 - (-5.5) + 2(0) = 30$

$x_1 + 5.5 = 30$

$\boxed{x_1 = 24.5}$

5. (12 points) Consider the following system of equations, $\mathbf{A}\mathbf{x} = \mathbf{b}$:

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}.$$

Using the Jacobi iteration method, compute $\mathbf{x}^{(2)}$. Use the inital guess

$$\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$x^{k} = D^{-1}(C_L + C_U)x^{k-1} + D^{-1}b$$

$$D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \quad D^{-1} = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} \quad C_L = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$C_U = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D^{-1}\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1/4 & 0 \\ -1/3 & 0 & 1/3 \\ 0 & 1/4 & 0 \end{bmatrix}$$

$$x^{1} = \begin{bmatrix} 0 & -1/4 & 0 \\ -1/3 & 0 & 1/3 \\ 0 & 1/4 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & -1/4 & 0 \\ -1/3 & 0 & 1/3 \\ 0 & 1/4 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$x^{1} = \begin{bmatrix} -1 \\ 2/3 \\ 1 \end{bmatrix} \qquad 3\begin{bmatrix} 0 \\ -1/3 \\ 0 \end{bmatrix} + 4\begin{bmatrix} -1/4 \\ 0 \\ 1/4 \end{bmatrix} + 5\begin{bmatrix} 0 \\ 1/3 \\ 0 \end{bmatrix}$$

$$x^{2} = \begin{bmatrix} 0 & -1/4 & 0 \\ -1/3 & 0 & 1/3 \\ 0 & 1/4 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ 2/3 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 2/3 \\ 1 \end{bmatrix} \qquad \left(\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 5/3 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2/3 \\ 1 \end{bmatrix}\right)$$

$$-1\begin{bmatrix} 0 \\ -1/3 \\ 0 \end{bmatrix} + 2/3\begin{bmatrix} -1/4 \\ 0 \\ 1/4 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 1/3 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 2/3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/3 \\ 0 \end{bmatrix} + \begin{bmatrix} -2/12 \\ 0 \\ 2/12 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/3 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 2/3 \\ 1 \end{bmatrix}$$

$$\boxed{x^{2} = \begin{bmatrix} -14/12 \\ 4/3 \\ 14/12 \end{bmatrix}}$$

C

AMATH 301 Exam 1 Version 4 Name: Kody Pham

Friday, February 16, 2024 Student ID: 2370652

Instructions: Write neatly and show all work. Unless it is stated otherwise, in order to receive full credit, you must show your work and carefully justify your answers. Please circle your final answer. The number in parentheses next to the problem number is the point value of the problem. Besides the one side of a 8.5 by 11 inch note sheet, notes, textbooks, or calculators may not be used on this exam. Please verify that this exam consists of 5 problems on the front and back of 5 pages. Any work on scratch paper will not be graded.

This exam is worth 60 points.

Good Luck!

Pledge: By signing your name to this exam you are agreeing to the following statement:

I pledge that the work on this exam is mine only - I received no aid from outside sources nor did I obtain answers in ways that violate the instructions on the exam. I also pledge that I did not provide aid to any other student taking the exam.

Signature: ______________________

1. Consider the following python code:

Code 1:

```
import numpy as np

v = np.array([-5,2,-3,4])

u = np.array([-7,8,0,1])

w = np.array([4,9,16,25])
```

State what each of the following code would return if ran in the python console after running the code above: (you do not need to justify/show your work)

(a) (2 points) v[1:3]

$\boxed{[-5, 2, -3]}$

(b) (3 points) u/(2*v)

[-7, 8, 0, 1]

2v = [-10, 16, -6, 2]

$\boxed{[7/10, 1/2, 0, 1/2]}$

(c) (2 points) np.sqrt(w)

$\boxed{[\sqrt{4}, 3, 4, 5]}$

(d) (2 points) Write the code (using array slicing) that extracts the first two values from v.

```
x = v[1:2]
```

(e) (2 points) Write the code (using array slicing) that extracts the last two values from u.

```
x = u[-1:-2]
```

(f) (3 points) Use numpy functions and numpy constants to write the code that computes $e^{\sqrt{2\pi^2}} / \sin(10)$.

```
x = np.exp((np.sqrt(2*np.pi**2)))/ np.sin(10)
```

2. Consider the following python code:

Code 2:

```
import numpy as np
n = 3
m = 4
A = np.zeros((n,m))
for i in range(n):
    for j in range(m):
        A[i,j] = (i+1)*(j+2)
```

(a) (4 points) If this code is run in python, write out the elements of A. You may write A as a matrix or as a numpy array.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 3 & 4 \\ 3 & 2 & 3 & 4 \end{bmatrix}$$

$\boxed{\text{np.array}([[1, 2, 3], [1, 2, 3, 4]]}$

(b) (2 points) Write a code that modifies the 3nd row of A to be a row of zeros. (you do not need to justify/show your work)

For i in range n

np.array [0]

(c) (4 points) If the code below is run in python, write out the elements of A. You may write A as a matrix or as a numpy array.

Code 3:

```
import numpy as np
n = 3
m = 4
A = np.zeros((n,m))
for i in range(n):
    for j in range(m):
        A[i,j] = (i+1)*(j+2)

for k in range(n):
    if A[k,0] < 3:
        A[k,0] = 1
    elif A[k,0] < 5:
        A[k,0] = -1
    else:
        A[k,0] = 0
```

$$\begin{bmatrix} 1 & 0 & 3 & 2 \\ 4 & 1 & 2 & 0 \\ 3 & 5 & 1 & 2 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$

3. Let $\mathbf{A}$ be the following matrix:

$$\mathbf{A} = \begin{bmatrix} 5 & 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 10 & 6 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}.$$

We would like to solve a system of linear equations, $\mathbf{Ax} = \mathbf{b}$, for a 5x1 vector of unknowns $\mathbf{x}$, given a 5x1 vector of known constant values, $\mathbf{b}$. The following questions concern the setup of the system; you will not be solving the system in this problem.

a.) (3 points) Naive Gaussian elimination will not work on this system. In a single sentence, explain why.

Naive Gaussian will not work because it cannot swap, only Gaussian can pivot.

b.) (3 points) If two rows of $\mathbf{A}$ are swapped, we will be able to solve this system via either forward or backward substitution. Please identify which two rows must be swapped and write down the matrix resulting from swapping these two rows. Do not swap more than two rows, i.e. three rows must remain unchanged after this operation. You may assume that row numbering begins at 1, thus row 1 is the row containing the numbers $(5, 1, 0, 2, 0)$.

Rows 3 and 4 need to swap.

$$\begin{bmatrix} 5 & 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 10 & 6 \\ 0 & 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

c.) (3 points) Would the system found by swapping two rows be solvable by forward substitution or backward substitution? Please explain which, and how you know, in a single sentence. Do not solve the system.

Would use backwards sub because it is already in the lower triangle, through that can solve for x values.

4. Consider the system of equations $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{x}$ is a vector of unknowns and

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix} \text{ and } \mathbf{b} \begin{bmatrix} 2 \\ 10 \\ 4 \end{bmatrix}.$$

(a) (9 points) Find the LU decomposition for the matrix $\mathbf{A}$. Make sure to list all of your elementary matrices.

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix} \begin{bmatrix} 1 & \# & \# \\ 3 & 1 & \# \\ \# & \# & 1 \end{bmatrix} \quad R_2 - 3R_1$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 2 & -4 & 5 \end{bmatrix} \begin{bmatrix} 1 & \# & \# \\ 3 & 1 & \# \\ 2 & \# & 1 \end{bmatrix} \quad R_3 - 2R_1$$

$-4-(-2)$

$2.$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix} \quad R_3 - (-2)R_1$$

$-2 \quad 2 \quad -4$

$$\boxed{\underset{U}{\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}} \underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}}}$$

$$\begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & 1 \\ -2 & 0 & 3 \end{bmatrix} \quad 2$$

(b) (6 points) Use forward substitution and backward substitution to solve the system $\mathbf{Ax} = \mathbf{b}$.

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 3 & -1 & 7 & 10 \\ 2 & -4 & 5 & 4 \end{array}\right] R_2 - 3R_1 \qquad 3 \;\; -3 \;\; 6 \;\; 6$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 2 & 1 & 4 \\ 2 & -4 & 5 & 4 \end{array}\right] R_3 - 2R_1 \qquad 2 \;\; -2 \;\; 4 \;\; 4$$

$$\left[\begin{array}{cccc} 1 & -1 & 2 & 2 \\ 0 & & & \end{array}\right.$$

5. (12 points) Consider the following system of equations, $\mathbf{Ax} = \mathbf{b}$:

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}.$$

Using the Jacobi iteration method, compute $\mathbf{x}^{(2)}$. Use the inital guess

$$\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$4x_1 + 1x_2 + 0x_3 = 3$ $\quad$ $x_1 = \frac{3 - 1x_2}{4}$

$1x_1 + 3x_2 - 1x_3 = 4$ $\quad$ $x_2 = \frac{4 - 1x_1 - 1x_3}{3}$

$0x_1 - 1x_2 + 4x_3 = 5$ $\quad$ $x_3 = \frac{5 + 1x_2}{4}$

$$x^2 = \begin{bmatrix} 1/4 \\ 1/3 \\ 1/4 \end{bmatrix}$$

AMATH 301 Exam 1 Version 4

Name: MARTIN JOHNSON

Friday, February 16, 2024

Student ID: 2231573

Instructions: Write neatly and show all work. Unless it is stated otherwise, in order to receive full credit, you must show your work and carefully justify your answers. Please circle your final answer. The number in parentheses next to the problem number is the point value of the problem. Besides the one side of a 8.5 by 11 inch note sheet, notes, textbooks, or calculators may not be used on this exam. Please verify that this exam consists of 5 problems on the front and back of 5 pages. Any work on scratch paper will not be graded.

This exam is worth 60 points.

Good Luck!

Pledge: By signing your name to this exam you are agreeing to the following statement:

I pledge that the work on this exam is mine only - I received no aid from outside sources nor did I obtain answers in ways that violate the instructions on the exam. I also pledge that I did not provide aid to any other student taking the exam.

Signature: Martin Johnson

1. Consider the following python code:

Code 1:

```
import numpy as np

v = np.array([-5,2,-3,4])
               0  1  2  3
u = np.array([-7,8,0,1])

w = np.array([4,9,16,25])
```

State what each of the following code would return if ran in the python console after running the code above: (you do not need to justify/show your work)

(a) (2 points) v [1:3]

[2, -3]

(b) (3 points) u/(2*v)

$$\frac{[-7,8,0,1]}{[-10,4,-6,8]} = \boxed{\left[\frac{7}{10}, 2, 0, \frac{1}{8}\right]}$$

(c) (2 points) np.sqrt(w)

[2, 3, 4, 5]

(d) (2 points) Write the code (using array slicing) that extracts the first two values from v.

V[0:2]

(e) (2 points) Write the code (using array slicing) that extracts the last two values from u.

U[-2:]

(f) (3 points) Use numpy functions and numpy constants to write the code that computes $e^{\sqrt{2\pi^2}} / \sin(10)$.

(np.e ** (np.sqrt(2*(np.PI**2)))) /(np.sin(10))

2. Consider the following python code:

Code 2:

```
import numpy as np
n = 3
m = 4
A = np.zeros((n,m))
for i in range(n):
    for j in range(m):
        A[i,j] = (i+1)*(j+2)
```

3 x 5

(a) (4 points) If this code is run in python, write out the elements of A. You may write A as a matrix or as a numpy array.

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \; j+2 \\ i+1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} & \begin{bmatrix} 2 & 3 & 4 & 5 \\ 4 & 6 & 8 & 10 \\ 6 & 9 & 12 & 15 \end{bmatrix} \end{array}$$

(b) (2 points) Write a code that modifies the 3nd row of A to be a row of zeros. (you do not need to justify/show your work)

A[2]= [0,0,0,0]

(c) (4 points) If the code below is run in python, write out the elements of A. You may write A as a matrix or as a numpy array.

Code 3:

```
import numpy as np
n = 3
m = 4
A = np.zeros((n,m))
for i in range(n):
    for j in range(m):
        A[i,j] = (i+1)*(j+2)

for k in range(n):
    if A[k,0] < 3:
        A[k,0] = 1
    elif A[k,0] < 5:
        A[k,0] = -1
    else:
        A[k,0] = 0
```

SAME AS PREVIOUS $A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 4 & 6 & 8 & 10 \\ 6 & 9 & 12 & 15 \end{bmatrix}$

ALL EDITING FIRST COLUMN

$$\boxed{\begin{bmatrix} 1 & 3 & 4 & 5 \\ -1 & 6 & 8 & 10 \\ 0 & 9 & 12 & 15 \end{bmatrix}}$$

3. Let **A** be the following matrix:

$$\mathbf{A} = \begin{bmatrix} 5 & 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 10 & 6 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ \; \end{matrix}$$

We would like to solve a system of linear equations, $\mathbf{Ax} = \mathbf{b}$, for a 5x1 vector of unknowns $\mathbf{x}$, given a 5x1 vector of known constant values, $\mathbf{b}$. The following questions concern the setup of the system; you will not be solving the system in this problem.

a.) (3 points) Naive Gaussian elimination will not work on this system. In a single sentence, explain why.

YOU NEED TO GET TO UPPER TRIANGULAR, BUT YOU CANNOT ZERO OUT THE 4 @ (4, 3) BECAUSE YOU HAVE ZEROS ABOVE, YOU NEED TO SWAP ROWS.

b.) (3 points) If two rows of **A** are swapped, we will be able to solve this system via either forward or backward substitution. Please identify which two rows must be swapped and write down the matrix resulting from swapping these two rows. Do not swap more than two rows, i.e. three rows must remain unchanged after this operation. You may assume that row numbering begins at 1, thus row 1 is the row containing the numbers $(0, 0, 0, 0, 2)$.

$$\begin{bmatrix} 5 & 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 10 & 6 \\ 0 & 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} \qquad 3 \leftrightarrow 4$$

c.) (3 points) Would the system found by swapping two rows be solvable by forward substitution or backward substitution? Please explain which, and how you know, in a single sentence. Do not solve the system.

BACKWARD SUBSTITUTION,

X_5 IS EASY TO SOLVE (ONE CONSTANT) WHICH YOU WILL USE TO SOLVE X_4 AND SO ON. GOING BACKWARDS UP X.

4. Consider the system of equations $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{x}$ is a vector of unknowns and

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \text{and} \quad \mathbf{b} \begin{bmatrix} 2 \\ 10 \\ 4 \end{bmatrix}.$$

(a) (9 points) Find the LU decomposition for the matrix $\mathbf{A}$. Make sure to list all of your elementary matrices.

$$\overset{U}{\begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix}} \quad \overset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$R_2 \leftarrow R_2 - 3R_1$
$R_3 \leftarrow R_3 - 2R_1$

$$\overset{U}{\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & -2 & 1 \end{bmatrix}} \quad \overset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}}$$

$R_3 \leftarrow R_3 - (-1)R_2$

$$\overset{U}{\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}} \quad \overset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}}$$

$1 - 3 + 4$ $\quad 1 + 3 - 4 \cancel{0}$

$-2 + 4 = 2$

$$\overset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}} \overset{U}{\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}} \overset{?}{=} \begin{bmatrix} 1 & -1 & 2 \\ 3 & -1 & 7 \\ 2 & -4 & 5 \end{bmatrix} \checkmark$$

(b) (6 points) Use forward substitution and backward substitution to solve the system $\mathbf{A}\mathbf{x} = \mathbf{b}$.

$$Lz = b$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \\ 4 \end{bmatrix}$$

$$z_1 = 2$$

$$3(2) + z_2 = 10$$

$$z_2 = 4$$

$$2(\cancel{2}) - 4 + z_3 = 4$$

$$z_3 = 4$$

$$z = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$$

$$Ux = z$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$$

$$x_3 = 2$$

$$2x_2 + 2 = 4$$

$$x_2 = 1$$

$$x_1 - (1) + 2(2) = 2$$

$$x_1 + 3 = 2$$

$$x_1 = -1$$

$$\boxed{x = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}}$$

5. (12 points) Consider the following system of equations, $\mathbf{A}\mathbf{x} = \mathbf{b}$:

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}.$$

Using the Jacobi iteration method, compute $\mathbf{x}^{(2)}$. Use the inital guess

$$\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$X_1^{(k+1)} = \frac{1}{4}\left[3 - (1\cdot 0 + 0\cdot 0)\right] = \frac{3}{4}$$

$$X_2^{(1)} = \frac{1}{3}\left[4 - (1\cdot 0 + -1\cdot 0)\right] = \frac{4}{3}$$

$$X_3^{(1)} = \frac{1}{4}\left[5 - (0\cdot 0 + -1\cdot 0)\right] = \frac{5}{4}$$

$$X^{(1)} = \begin{bmatrix} \frac{3}{4} \\ \frac{4}{3} \\ \frac{5}{4} \end{bmatrix}$$

$$X_1^{(2)} = \frac{1}{4}\left[3 - \left(1\cdot\frac{4}{3} + 0\cdot\frac{5}{4}\right)\right] = \frac{5}{12}$$

$$\frac{9}{3} - \frac{4}{3} = \frac{5}{3}\cdot\frac{1}{4}$$

$$X_2^{(2)} = \frac{1}{3}\left[4 - \left(1\cdot\frac{3}{4} + -1\cdot\frac{5}{4}\right)\right] = \frac{3}{2}$$

$$\frac{3}{4} - \frac{5}{4} = -\frac{1}{2}$$

$$\frac{1}{3}\cdot\frac{9}{2} = \frac{9}{6} = \frac{3}{2}$$

$$X_3^{(2)} = \frac{1}{4}\left[5 - \left(0\cdot\cancel{\frac{3}{4}} + -1\cdot\frac{4}{3}\right)\right] = \frac{19}{12}$$

$$5 + \frac{4}{3}$$

$$\frac{1}{4}\cdot\frac{19}{3} = \frac{19}{12}$$

$$\boxed{X^{(2)} = \begin{bmatrix} \frac{5}{12} \\ \frac{3}{2} \\ \frac{19}{12} \end{bmatrix}}$$