Quantum Dots & Wires - Summary

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1 Elzeman's Lectures

1.1 Motivation

Core point It is not about building the best qubit using different approaches, but about where each specific qubit excels.

Advantages of optically active dots and wires
- Allows for light-matter interface

2 Transport

2.1 Conductance of Ballistic conductor between reflectorless contacts

Conductance Describes how easy it is for a current to travel through a material. Classically given by

$$R = \sigma \frac{vA}{L} \tag{1}$$

Derivation of current Start with the current $I = env$ where n is the number of particles and v is the velocity of the particles. This holds for uniform electron gas of n electrons.

Consider then $+k$ modes. We denote the current I^+ and the distribution they follow f^+. We could do the same for $-k$ modes, but we shall do so later. Let the electrons be distributed according to

$$n = \frac{f^+(E)}{L} \tag{2}$$

where L is the length of the conductor. Then, the current over all k modes will be

$$I^{+} = \frac{e}{L}\sum_{k} v f^{+}(E) = \frac{e}{L}\sum_{k} \frac{1}{\hbar}\frac{\partial E}{\partial k} f^{+}(E) \tag{3}$$

where

$$v = \frac{1}{\hbar}\frac{\partial E}{\partial k} \tag{4}$$

Then, we must convert the sum into an integral. This is done as follows:

$$\sum_{k} \to 2 \times \frac{L}{2\pi}\int \mathrm{d}k \tag{5}$$

Question: It is not clear to me how this conversion happens. I presume that you need L to cancel out the units of dk.

Thus we obtain

$$I^{+} = \frac{2e}{h}\int_{\epsilon}^{\infty} f^{+}(E)\mathrm{d}E \tag{6}$$

where ϵ

3 Mark Buitelaar's Lectures - Quantum Transport

3.1 Introduction

Quantum Dot a 0-dimensional structure which contains one hole and/or electron. It can e.g. be used for quantum information processing. By making use of quantum properties, an electron can be captured and manipulated in a quantum dot.

Quantum Wire a 1-dimensional structure, often a carbon nanotube, where quantum effects influence the electron transport properties.

Some examples of quantum dots and wires include

- Graphene
- Carbon nanotubes
- Si/SiGe heterostructures
- Si nanowires

- GaAs based quantum dots
- InAs quantum dots

Probing properties is often done by simply connecting the quantum wire or dot to an external voltage and then measuring the other electrical properties.

States in a waveguide which we have studied well can be used to model the behaviour of quantum states inside a wire, have energy

$$E_n(k_x) = \frac{\hbar^2 k_x^2}{2m} + \frac{\pi^2\hbar^2}{2m}\left(\frac{n_y^2}{a^2} + \frac{n_z^2}{b^2}\right) \tag{7}$$

where a is the width of the waveguide in the z direction and b in the y direction.

3.2 Conductance from transmission

Ballistic conductor has no resistivity from the scattering of electrons. A ballistic conductor differs from a superconductor by the absence of the Meissner effect. The only scattering process that occurs is when electrons scatter off the walls or the ceiling of the conductor, otherwise they follow Newtons 2nd law of motion.

Properties:
- $T(E) = 1$
-

Boltzmann transport equation can be used to write down an expression for the current. We find

$$I = \frac{2e}{h}\int_{-\infty}^{\infty} f^+(E)M(E)T(E)\mathrm{d}E \tag{8}$$

where $f^+(E)$ is the deviation form the equilibrium distribution (it is the perturbation), $M(E)$ is the number of propagating modes in the channel and $T(E)$ is the transmission probability, which for a ballistic conductor has $T = 1$.

Ballistic conductor current if we insert the Fermi energy μ_1 and μ_2 in the integral limits, we find that

$$I = \frac{2e^2}{h}M\frac{\mu_1 - \mu_2}{e} \tag{9}$$

where we (I think) have assumed that $M(E)$ is independent of the energy.

Current formula derivation Start by considering a single mode $M = 1$. The occupation of the states with k is determined by the Fermi function $f^+(E)$ (which I presume is the Fermi-Dirac distribution function). Approximate the electrons in the conductor to be a free-moving gas. If the gas is uniform and has a density of n electrons per unit length, they will carry a current if all electrons move with velocity v (modulated by the Fermi distribution). The current is then given by

$$I = env \tag{10}$$

Let the electron density of a single $+k$ state be $n = 1/L$, where L is the length of the conductor. Then, the current is given by

$$I = \frac{e}{L} \sum_k v f^+(E) \tag{11}$$

We can then obtain the velocity from the dispersion relation, such that

$$v = \frac{1}{\hbar} \frac{\partial E}{\partial k} \tag{12}$$

and by turning the sum into an integral, through the following way

$$\sum_k \to 2 \frac{L}{2\pi} \int \mathrm{d}k \tag{13}$$

Question: How can I justify this transformation?

$$I = \frac{2e}{h} \int_{\epsilon}^{+\infty} f^+(E) \mathrm{d}E \tag{14}$$

Contact Conductance from the above expression for the current, we identity

$$G_C = \frac{2e^2}{h} M \tag{15}$$

which is the conductance. This is modified for a non-ballistic conductor by multiplying it by the transmission probability T. M

Note that the contact conductance increases in steps as the number of modes M. Note also that the resistance does not scale with length or area of the conductor.

Question: Then how do we determine the number of modes? Surely this is a property of the material itself?

Contact Resistance the resistance is the inverse of the conductance. We find that

$$R = G_C^{-1} = \frac{h}{2e^2 M} \sim 12.9k\Omega/M \tag{16}$$

Difference in Fermi energy as can be seen for the formula for the current, we find that the current flow depends on the difference in Fermi energy. All electrons will move around randomly, but only when we have this energy difference will the electrons strive towards a lower equilibrium energy, and therefore move, which in turn carries a current.

Transverse modes also called subbands are available to electrons in the conductor. Because of the Pauli exclusion principle, only one electron (with the same spin) can occupy the band at one time.

Each subband has its own dispersion relation with a cutoff energy, $\epsilon_N = E(N, k = 0)$ below which it cannot propagate. Basically, one the mode reaches this energy, the energy just keeps increasing without any changes to k.

Number of transverse modes can be obtained by counting the number of modes below the cutoff energy.

$$M(E) = \sum_N \theta(E - \epsilon_N) \tag{17}$$

where θ is the Heaviside step-function, and N is the number of electrons.

Fermi energy is given by

$$E_F = \frac{\hbar^2 k_F^2}{2m} \tag{18}$$

3.3 Conductance from Transmission

Landauer formula is given by

$$G = \frac{2e^2}{h} MT \tag{19}$$

This is the general formula for conductance, where we have taken into account the number of modes M.

Resistance in general resistance is given by

$$G^{-1} = \frac{h}{2e^2M} + \frac{h}{2e^2M}\frac{1-T}{T} \tag{20}$$

where we write the formula in this way because the transmission probability adds.

Conductance with reflection Consider having a conductor between two leads. We can calculate the net current I_2 by calculating the current difference:

$$I_2 = I_1^+ - I_1^- \tag{21}$$

Now, consider that I_1^+ is the incoming current, and I_1^- is the outgoing current, which is reflected back from the unsuccessful transmissions. We find

$$I_1^+ = \frac{2e}{h}MT(\mu_1 - \mu_2) \tag{22}$$

$$I_1^- = \frac{2e}{h}M(1-T)(\mu_1 - \mu_2) \tag{23}$$

Thus the difference is

$$I = I_1^+ - I_1^- = \frac{2e}{h}MT(\mu_1 - \mu_2) \tag{24}$$

Electrochemical potential For a conductor connected to two contacts, we can divide it up into areas. If there exists a difference in electrochemical potential $(\mu_1 - \mu_2)$, it will drop in the middle of the conductor.

Coherent conductor I have found no definition for this, but assume that a conductor is coherent if we see quantum behaviour. Basically, the phenomena happen on such large (or long) scales that we can observe them without them decohering.

However, it could equally well be that coherent refers to the way some other phenomena works.

Coherent conductor interference Consider two barriers with a conductor in-between them. As an electron travels across the distance, it picks up a phase $e^{i\chi}$. An incoming electron will reflect a number of times between the barriers before being transmitted.

Insert derivation of probabilities from problem sheet.

3.4 Nanotube quantum dot

In this section, we describe and island lying between a source and a gate. Electrons from the source will hop onto this island, which in turn acts like a capacitor.

The equivalent circuit for this situation is described by two capacitors C and C_g for the gate in series.

Conductance behaviour with temperature goes approximately as

$$G_{peak} \sim \frac{1}{T} \tag{25}$$

Charging energy is given by

$$E_C = \frac{e^2}{C} \tag{26}$$

where C is the total capacitance of the quantum dot.

Charge on the island is quantised. We find

$$-q_2 + q_1 = eN \tag{27}$$

where q_2 and q_1 are the respective charge on the island.

Total electrostatic energy is the energy accumulated in the capacitors and the work done by the voltage source to transfer the charge q_2 to the gate electrode.

Question: What role does q_1 play? Is it the charge that is already on the island?

The energy is

$$E_{el} = \frac{1}{2}\left(\frac{q_1^2}{C} + \frac{q_2^2}{C_g}\right) - q_2 V \tag{28}$$

Capacitor charge-voltage relationship is

$$CV = q \tag{29}$$

Thus, for the island and the gate,

$$CV_1 = q_1 \tag{30}$$

$$C_g V_2 = q_2 \tag{31}$$

Addition energy is the charge in the electrochemical potential as electrons are added to a quantum dot. It is given by

$$\Delta E_{add} = \mu(N+1) - \mu(N) = \frac{e^2}{C_\Sigma} + E^K_{M+1} - E^K_M \qquad (32)$$

where

For spin degeneracy, $E^K_{M+1} - E^K_M$ can be zero.

Electrochemical potential of quantum dot The electrochemical potential μ is the difference between the two potentials. Think of the lift between the two energy levels. μ is different depending which energy level is already occupied. We have

$$\mu(N+1) = E(N+1) - E(N) \qquad (33)$$

That is, we compare the energy for the two different levels.