Advanced Quantum Information - All Exercises

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Contents

1 Effect of transposition on eigenvalues

Exercise: Do the eigenvalues of a matrix A change under transposition $A \to A^T$?

2 Ensemble Ambiguity

Exercise: Write the ensemble

$$\rho = \frac{1}{2} \left(|0\rangle \langle 0| + |1\rangle \langle 1| \right) = \frac{\mathbb{I}}{2} \tag{1}$$

in terms of the $|+\rangle, |-\rangle$-basis.

3 Physicality of Linear Maps

Exercise: Why does it follow that all linear maps are physical?

4 The ultimate winner of the CHSH game

Exercise: Show that we can win the CHSH game 100% of the time using PR-boxes.

5 Proof of T'sirleon's bound

Exercise: Go through and understand each line of the proof of T'sirleon's bound in Watrous' notes.

6 Extensivity of entanglement monotone

Exercise: Show that the entropy of a subsystem $S(\rho_A)$ is extensive.

7 Optimality of entanglement distillation and concentration

Exercise: Calculate the rate of E_D and E_C and show that they are optimal.

8 Probability of error for the bit-flip channel

Exercise: Given that the Hamming distance for this channel is $H = np$, where p is the probability of a bit-flip, calculate the probability of error for this channel. Solve this by looking at the Hamming weight, and ultimately show that the capcity obeys

$$C \leq I(X^n; Y^n) \leq nI(X; Y) \tag{2}$$

9 Uniqueness of Stinespring dilation

Exercise: Show that Stinespring dilation is unique up to a unitary transformation on the reference state.

Answer:

10 The power of the Referee

Exercise: Can the referee R who holds the purifying system ρ_R change the systems of Alice and Bob?

Answer: I would suspect that this has to do with quantum steering, but might only be possible with the addition of classical communication.

11   The Data Processing Inequality

Exercise: Given a classical-quantum state,

$$\sum_x p_x \left| x \right\rangle \left\langle x \right| \otimes \rho_B \tag{3}$$

show that if we wish to send $XB \to XY$, it follows that

$$I(X;B) \geq I(;Y) \tag{4}$$

where $IX;Y)$ is a measure of how well we did.

12   Holevo Bound

Exercise: Show that $I(X;B)$ is the Holevo bound χ.

13   The completeness relation for Kraus operators

Exercise: Come up with a channel that satisfies

$$\sum_k A_k A_k^\dagger = \mathbb{I} \tag{5}$$

but not

$$\sum_k A_k^\dagger A_k = \mathbb{I} \tag{6}$$

14   The pretty good measurement

Exercise: Outline the framework of a pretty good measurement.

15   Character of an entanglement monoton

Exercise: Show that an entanglement monoton satisfies

$$E_D \leq E \leq E_C \tag{7}$$

where E_D denotes entanglement distillation and E_C denotes entanglement concentration.

16 Resources used for state merging

Exercise: Given a state merging protocol, where Alice wants to send her share of the state to Bob, how many ebits does Alice and Bob need to share in order to be able to send the state?

17 State merging using a GHZ state

exercise: Given that Alice, Bob and the Referee share the pure state

$$|\psi\rangle_{GHZ} = \frac{1}{\sqrt{2}} \left(|000\rangle + |111\rangle \right) \tag{8}$$

what happens if Alice measures her state and gets outcome $|-\rangle$?

18 Sending the state

Exercise: Given the state merging protocol, show that Alice has sent the state.

Answer: We will have to check the fidelity of the sent state with respect to the original state.

19 Super-dense coding

Exercise: Show that we can send two classical bits using one ebit.