Quantum Error Correction - Lecture 7

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1 Topological Codes

In this section, we shall introduce the notion of a topological code. This is the most viable candidate because, as we shall see, it has a pretty good threshold value.

Previously, we have seen that increasing the robustness is basically equivalent to increasing the distance of the code. Let us look closer at this notion. Start with the n-qubit repetition code

$$|0\rangle_L = |0\rangle^{\otimes n} \tag{1}$$

$$|1\rangle_L = |1\rangle^{\otimes n} \tag{2}$$

For a number $m = n - 1$ generators, we have

$$Z_1 Z_2, Z_2 Z_3 \ldots Z_{n-1} Z_n \tag{3}$$

The product of all generators gives

$$Z_1 \otimes I \ldots \otimes I \otimes Z_n \tag{4}$$

This is no an independent generator. We can work with a graphical representation of a qubit. Let us imagine it as a one-dimensional grid where the qubits are represented by edges.

2 Products of Stabiliser Generators

We say that the three generators A, B, C are not independent if

$$AB = C \tag{5}$$

Alternatively, an equivalent definition states that $\{A_1, A_2 \dots A_n\}$ are independent if and only if there is no product of A_j's that generates the identity.

3 The Toric Code

See handout.