Database Management Systems

(COP 5725)

Fall 2021

Instructor: Dr. Markus Schneider

TA: Kyuseo Park

Homework 4

Name:	Qinxuan Shi
UFID:	8351-8162
Email Address:	qinxuan.shi@ufl.edu.

Pledge (Must be signed according to UF Honor Code)

On my honor, I have neither given nor received unauthorized aid in doing this assignment.

Qinxuan Shi
Signature

For scoring use only:

	Maximum	Received
Exercise 1	30	
Exercise 2	25	
Exercise 3	35	
Exercise 4	10	
Total	100	

1 Exercise 1

1. [5 points] Use the Armstrong axioms to prove the soundness of the Union rule. If A → B and A → C holds, then A → BC holds.

Now we have already know that $A \rightarrow B$ and $A \rightarrow C$. Firstly, using Augmentation rule to first FD, we can get $AA \rightarrow AB$, which is $A \rightarrow AB$ since $AA = A$. Secondly, also use Augmentation rule to second FD, we can get $AB \rightarrow BC$. Finally, we use Transitivity rule, and we can know that $A \rightarrow BC$. Because the Augmentation rule and Transitivity rule have been shown that they are sound and complete, so the Union rule can also fulfill the Soundness.

2. [4 points, 2 points each] Given the set F = A→B, AB→C, AC→BD of functional dependencies, prove the following dependencies by using the Armstrong axioms.

(1) A→ABC

Augmentation rule: $A \rightarrow B \Longrightarrow A \rightarrow AB$ and $AB \rightarrow C \Longrightarrow AB \rightarrow BC$

Transitivity rule: $AB \rightarrow BC$, $A \rightarrow AB \Longrightarrow A \rightarrow BC$

Then, we use Augmentation rule for $A \rightarrow BC$, we finally get $AA \rightarrow ABC$, which is actually $A \rightarrow ABC$

(2) AD→BCD

Augmentation rule: $A \rightarrow B \Longrightarrow AD \rightarrow BD$ and $A \rightarrow B \Longrightarrow A \rightarrow AB$

Transitivity rule: $AB \rightarrow C$, $A \rightarrow AB \Longrightarrow A \rightarrow C$

Augmentation rule: $A \rightarrow C \Longrightarrow AD \rightarrow CD$

Augmentation rule: $AD \rightarrow BD \Longrightarrow AD \rightarrow ABD$ and $AD \rightarrow CD \Longrightarrow ABD \rightarrow BCD$

Transitivity rule: $AD \rightarrow ABD$, $ABD \rightarrow BCD \Longrightarrow AD \rightarrow BCD$

3. [6 points] Consider a relation schema R(X, Y, Z) with the functional dependencies XY→Z and Z→X. Can we conclude that Y→XZ holds? If yes, please argue why. If no, please argue why not by giving a counterexample.

We cannot conclude that Y→XZ holds. It's easy to know that $Y^+ = Y$, and $XZ \not\subseteq Y$, so we cannot get the statement.

For example, suppose that X represents students name and all of these names are distinct, Y represents gender, Z represents student ID as UFID. Since names of students are all different, it is easy to know that $X \rightarrow Z$ holds and $XY \rightarrow Z$ holds too. we can also know that $Z \rightarrow X$ holds. However, to the statement $Y \rightarrow XZ$, it is impossible for the single gender attribute to determine name and ID functionally. As a result, we cannot conclude $Y \rightarrow XZ$ holds with the functional dependencies that given by the problem.

4. [5 points] Consider the relation schema R(A, B, C, D, E, F) and the set of functional dependencies F = A→B, A→C, CD→E, CD→F, B→E. Infer at least five new FDs by using five different Armstrong's axioms and derived inference rules. (Please do not include the trivial ones such as A→A in your answer.) Show each step.

(1) Transitivity rule: $A \rightarrow B$, $B \rightarrow E \Longrightarrow A \rightarrow E$

(2) Augmentation rule: $A \rightarrow C \Longrightarrow AB \rightarrow BC$

(3) Union rule: $A \rightarrow B$, $A \rightarrow C \Longrightarrow A \rightarrow BC$

(4) Decomposition rule: we can know that $AB \rightarrow BC$ from (2), then we can get $AB \rightarrow B$ and $AB \rightarrow C$

(5) Pseudotransitivity rule: $A \rightarrow C$, $CD \rightarrow E \Longrightarrow AD \rightarrow E$

(6) Pseudotransitivity rule: $A \to C$, $CD \to F \Longrightarrow AD \to F$

(7) Reflexivity rule: since $D \subseteq CD$, we can get $CD \to D$

5. [10 points] Assume we have a set F = A→B, C→D of functional dependencies for a relation schema R(A, B, C, D). Write down all the functional dependencies of the closure F+ of F and count them.

Firstly, we can get the set of all sets that are subsets of R(A,B,C,D):

{∅,A,B,C,D,AB,AC,AD,BC,BD,CD,ABC,ABD,ACD,BCD,ABCD}

Then, we compute the attribute closures of all subsets except ∅.

A^+: AB

$\Longrightarrow A \to A, A \to B, A \to AB$ (3FDs)

B^+: B

$\Longrightarrow B \to B$ (1FDs)

C^+: CD

$\Longrightarrow C \to C, C \to D, C \to CD$ (3FDs)

D^+: D

$\Longrightarrow D \to D$ (1FDs)

AB^+: AB

$\Longrightarrow AB \to A, AB \to B, AB \to AB$ (3FDs)

AC^+: ABCD

$\Longrightarrow AC \to A, AC \to B, AC \to C, AC \to D, AC \to AB, AC \to AC, AC \to AD, AC \to BC, AC \to BD, AC \to CD, AC \to ABC, AC \to ABD, AC \to ACD, AC \to BCD, AC \to ABCD$ (15FDs)

AD^+: ABD

$\Longrightarrow AD \to A, AD \to B, AD \to D, AD \to AB, AD \to AD, AD \to BD, AD \to ABD$ (7FDs)

BC^+: BCD

$\Longrightarrow BC \to B, BC \to C, BC \to D, BC \to BC, BC \to BD, BC \to CD, BC \to BCD$ (7FDs)

BD^+: BD

$\Longrightarrow BD \to B, BD \to D, BD \to BD$ (3FDs)

CD^+: CD

$\Longrightarrow CD \to C, CD \to D, CD \to CD$ (3FDs)

ABC^+: ABCD

$\Longrightarrow ABC \to A, ABC \to B, ABC \to C, ABC \to D, ABC \to AB, ABC \to AC, ABC \to AD, ABC \to BC, ABC \to BD, ABC \to CD, ABC \to ABC, ABC \to ABD, ABC \to ACD, ABC \to BCD, ABC \to ABCD$ (15FDs)

ABD^+: ABD

$\Longrightarrow ABD \to A, ABD \to B, ABD \to D, ABD \to AB, ABD \to AD, ABD \to BD, ABD \to ABD$ (7FDs)

ACD^+: ABCD

$\Longrightarrow ACD \to A, ACD \to B, ACD \to C, ACD \to D, ACD \to AB, ACD \to AC, ACD \to AD, ACD \to BC, ACD \to BD, ACD \to CD, ACD \to ABC, ACD \to ABD, ACD \to ACD, ACD \to BCD, ACD \to ABCD$ (15FDs)

BCD^+: BCD

$\Longrightarrow BCD \to B, BCD \to C, BCD \to D, BCD \to BC, BCD \to BD, BCD \to CD, BCD \to BCD$ (7FDs)

$ABCD^+$: ABCD

$\Longrightarrow ABCD \to A, ABCD \to B, ABCD \to C, ABCD \to D, ABCD \to AB, ABCD \to AC, ABCD \to AD, ABCD \to BC, ABCD \to BD, ABCD \to CD, ABCD \to ABC, ABCD \to ABD, ABCD \to ACD, ABCD \to BCD, ABCD \to ABCD$ (15FDs)

To conclude, the closure F^+ of F has 105 elements.

2 Exercise 2

1. [5 points] Consider the relation schema R = (A, B, C, D, E, F, G, H) with the set of functional dependencies K = A→B, B→G, AC→D, DF→E, FG→BH. Show for each of the following FDs whether they can be inferred from K.

(1) ABD→ACE

Compute ABD^+, and we can get ABD^+:ABDG. $ACE \not\subseteq ABDG$, so $ABD \to ACE$ cannot be inferred from K.

(2) BFG→BEFG

Compute BFG^+, and we can get BFG^+:BFGH. $BEFG \not\subseteq BFGH$, so $BFG \to BEFG$ cannot be inferred from K.

(3) ABF→ABDG

Compute ABF^+, and we can get ABF^+:ABFGH. $ABDG \not\subseteq ABFGH$, so $ABF \to ABDG$ cannot be inferred from K.

(4) CEG→BCEF

Compute CEG^+, and we can get CEG^+:CEG. $BCEF \not\subseteq CEG$, so $CEG \to BCEF$ cannot be inferred from K.

2. [5 points] Consider the relation schema R(A, B, C, D, E, F, G, H) with functional dependencies F = A→C, AC→E, D→EH, F→G and G = A→BCE, AD→CFG, D→A, DE→GH, F→D. Are the two sets F and G equivalent? Show each step.

Firstly, we show every FD in F can be inferred from G.

F has the left-hand sides A, AC, D and F.

With respect to G we calculate $A^+, AC^+, D^+ and F^+$, and we get $A^+ = ABCE$, $AC^+ = ABCE$, $D^+ = ABCDEFGH$, $F^+ = ABCDEFGH$.

We check whether the right-hand sides of the FDs in F are in the respective attribute closures just computed for their left-hand sides:

$A \to C : C \subseteq A^+$ holds, $AC \to E : E \subseteq AC^+$ holds, $D \to EH : EH \subseteq D^+$ holds, $F \to G : G \subseteq F^+$ holds.

Secondly, we show every FD in G can be inferred from F.

G has the left-hand sides A, AD, D, DE and F.

With respect to F we calculate $A^+, AD^+, D^+, DE^+ and F^+$, and we get $A^+ = ACE$, $AD^+ = ACDEH$, $D^+ = DEH$, $DE^+ = DEH$, $F^+ = FG$.

We check whether the right-hand sides of the FDs in G are in the respective attribute closures just computed for their left-hand sides:

$A \to BCE : BCE \not\subseteq A^+$, $AD \to CFG : CFG \not\subseteq AD^+$, $D \to A : A \not\subseteq D^+$, $DE \to GH : GH \not\subseteq DE^+$, $F \to D : D \not\subseteq F^+$.

We can know that G covers F while F cannot cover G, as a result, two sets F and G are not equivalent.

3. [5 points] Consider the relation schema R(A, B, C, D, E, F) with the functional dependencies K = A→B, BD→E, AC→F, DE→C. Which of the following attribute sets is a key? Show each step.

(1) ABCE

$ABCE^+$:= ABCE,

$A \to B$, $A \subseteq ABCE^+ \implies ABCE^+$:= ABCE

$BD \to E$, $BD \not\subseteq ABCE^+ \implies ABCE^+$:= ABCE

$AC \to F$, $AC \subseteq ABCE^+ \implies ABCE^+$:= ABCEF

$DE \to C$, $DE \not\subseteq ABCE^+ \implies ABCE^+$:= ABCEF

Because $ABCE^+ \neq R$, ABCE is not a key.

(2) ABDF

$ABDF^+$:= ABDF,

$A \to B$, $A \subseteq ABDF^+ \implies ABDF^+$:= ABDF

$BD \to E$, $BD \subseteq ABDF^+ \implies ABDF^+$:= ABDEF

$AC \to F$, $AC \not\subseteq ABDF^+ \implies ABDF^+$:= ABDEF

$DE \to C$, $DE \subseteq ABDF^+ \implies ABDF^+$:= ABCDEF

$AC \to F$, $AC \subseteq ABDF^+ \implies ABDF^+$:= ABCDEF

Because $ABDF^+ = R$, ABDF is a super key.

Then we will figure out if there exist any L⊂ABDF→R

We find that AD^+:= ABCDEF, so ABDF is just a super key, not a key.

(3) BEF

BEF^+:= BEF,

$A \to B$, $A \nsubseteq BEF^+ \implies BEF^+$:= BEF

$BD \to E$, $BD \nsubseteq BEF^+ \implies BEF^+$:= BEF

$AC \to F$, $AC \nsubseteq BEF^+ \implies BEF^+$:= BEF

$DE \to C$, $DE \nsubseteq BEF^+ \implies BEF^+$:= BEF

Because $BEF^+ \neq R$, ABCE is not a key.

(4) ACDE

$ACDE^+$:= ACDE,

$A \to B$, $A \subseteq ACDE^+ \implies ACDE^+$:= ABCDE

$BD \to E$, $BD \subseteq ACDE^+ \implies ACDE^+$:= ABCDE

$AC \to F$, $AC \subseteq ACDE^+ \implies ACDE^+$:= ABCDEF

$DE \to C$, $DE \subseteq ACDE^+ \implies ACDE^+$:= ABCDEF

Because $ACDE^+ = R$, ACDE is a super key.

Then we will figure out if there exist any L⊂ACDE→R

A^+:= AB, C^+:= C, D^+:= B, E^+:= E, AC^+:= ABCF, AD^+:= ABCDEF, AE^+:= ABE, CD^+:= CD, DE^+:= CDE, CE^+:= CE, ACD^+:= ABCDEF, ADE^+:= ABCDEF, ACE^+:= ABCEF, CDE^+:= CDE

We find that AD^+:= ABCDEF, ACD, ADE can also dependent R functionally, so ACDE is just a super key, not a key.

4. [10 points] Consider the relation schema R(A, B, C, D, E, F) with the set of functional dependencies F = A→BC, CD→E, B→D, E→A. By using the algorithm for calculating the attribute closure provided in the lecture slides, calculate the closure of the following attributes.

(1) AD

AD^+:= AD,

$A \to BC$, $A \subseteq AD^+ \implies AD^+$:= ABCD

$CD \to E$, $CD \subseteq AD^+ \implies AD^+$:= ABCDE

$B \to D$, $B \subseteq AD^+ \implies AD^+$:= ABCDE

$E \to A$, $E \subseteq AD^+ \implies AD^+$:= ABCDE

After the second loop, we see soon that no FD from F can increase AD^+, we get AD^+:= ABCDE

(2) ACE

ACE^+:= ACE,

$A \to BC$, $A \subseteq ACE^+ \implies ACE^+$:= ABCE

$CD \to E$, $CD \nsubseteq ACE^+ \implies ACE^+$:= ABCE

$B \rightarrow D$, $B \subseteq ACE^+ \implies ACE^+$:= ABCDE

$E \rightarrow A$, $E \subseteq ACE^+ \implies ACE^+$:= ABCDE

Second loop:

$CD \rightarrow E$, $CD \subseteq ACE^+ \implies ACE^+$:= ABCDE

After the third loop, we see soon that no FD from F can increase ACE^+, we get ACE^+:= ABCDE

3 Exercise 3

1. [15 points] Find a minimal cover for the relation R(A, B, C, D, E, F, G) with the set K = A→B, C→A, BD→E, ADE→B, E→F of functional dependencies. Show each step.

Step 1: F_c:= {A→B, C→A, BD→E, ADE→B, E→F}

Step 2:

$BD \rightarrow E$ has more than one attribute on left-hand side

To check weather D can be removed, we compute whether $E \subseteq \mathit{CalculateAttributeClosure}(F_c, B)$ holds

This is not the case since $B^+ = B$ and $E \nsubseteq B$ holds

To check weather B can be removed, we compute whether $E \subseteq \mathit{CalculateAttributeClosure}(F_c, D)$ holds

This is not the case since $D^+ = D$ and $E \nsubseteq D$ holds

Hence, B and D can not be removed, so F_c:= {A→B, C→A, BD→E, ADE→B, E→F}

$ADE \rightarrow B$ also has more than one attribute on left-hand side

To check weather D can be removed, we compute whether $B \subseteq \mathit{CalculateAttributeClosure}(F_c, AE)$ holds

This is the case since $AE^+ = ABEF$ and $B \subseteq ABEF$ holds

Hence, D can be removed, so F_c:= {A→B, C→A, BD→E, AE→B, E→F}

To check weather E can be removed, we compute whether $B \subseteq \mathit{CalculateAttributeClosure}(F_c, A)$ holds

This is the case since $A^+ = AB$ and $B \subseteq AB$ holds

Hence, E can be removed, so F_c:= {A→B, C→A, BD→E, E→F}

Step 3:

To check weather B can be removed from $A \rightarrow B$, we compute whether $B \subseteq \mathit{CalculateAttributeClosure}(\{A \rightarrow \emptyset, C \rightarrow A, BD \rightarrow E, E \rightarrow F\}, A)$ holds

This is not the case since $A^+ = A$ and $B \nsubseteq A$ holds

Hence, B can not be removed, so F_c:= {A→B, C→A, BD→E, E→F}

To check weather A can be removed from $C \rightarrow A$, we compute whether $A \subseteq \mathit{CalculateAttributeClosure}(\{A \rightarrow B, C \rightarrow \emptyset, BD \rightarrow E, E \rightarrow F\}, C)$ holds

This is not the case since $C^+ = C$ and $A \not\subseteq C$ holds

Hence, B can not be removed, so F_c:= {A→B, C→A, BD→E, E→F}

To check weather E can be removed from $BD \to E$, we compute whether $E \subseteq CalculateAttributeClosure(\{A \to B, C \to A, BD \to \emptyset, E \to F\}, BD)$ holds

This is not the case since $BD^+ = BD$ and $E \not\subseteq BD$ holds

Hence, E can not be removed, so F_c:= {A→B, C→A, BD→E, E→F}

To check weather F can be removed from $E \to F$, we compute whether $F \subseteq CalculateAttributeClosure(\{A \to B, C \to A, BD \to E, E \to \emptyset\}, E)$ holds

This is not the case since $E^+ = E$ and $F \not\subseteq E$ holds

Hence, F can not be removed, so F_c:= {A→B, C→A, BD→E, E→F}

Step 4: We obtain F_c:= {A→B, C→A, BD→E, E→F}

Step 5:

a: standard form, F_c is in this form

b: nonstandard form, F_c is in this form

As a result, F_c:= {A→B, C→A, BD→E, E→F}

2. [10 points] Find a standard form of minimal cover for the relation R(A, B, C, D, E, F, G, H) with the set K = A→BC, B→CE, A→E, AC→H, D→B of functional dependencies. Show each step.

Step 1: F_c:= {A→BC, B→CE, A→E, AC→H, D→B}

Step 2:

$AC \to H$ has more than one attribute on left-hand side

To check weather A can be removed, we compute whether $H \subseteq CalculateAttributeClosure(F_c, C)$ holds

This is not the case since $C^+ = C$ and $H \not\subseteq C$ holds

Hence, A can not be removed, so F_c:= {A→BC, B→CE, A→E, AC→H, D→B}

To check weather C can be removed, we compute whether $H \subseteq CalculateAttributeClosure(F_c, A)$ holds

This is the case since $A^+ = ABCEH$ and $H \subseteq ABCEH$ holds

Hence, C can be removed, so F_c:= {A→BC, B→CE, A→E, A→H, D→B}

Step 3:

To check weather B can be removed from $A \to BC$, we compute whether $B \subseteq CalculateAttributeClosure(\{A \to C, B \to CE, A \to E, A \to H, D \to B\}, A)$ holds

This is not the case since $A^+ = ACEH$ and $B \not\subseteq ACEH$ holds

Hence, B can not be removed, so F_c:= {A→BC, B→CE, A→E, A→H, D→B}

To check weather C can be removed from $A \rightarrow BC$, we compute whether $C \subseteq CalculateAttributeClosure(\{A \rightarrow B, B \rightarrow CE, A \rightarrow E, A \rightarrow H, D \rightarrow B\}, A)$ holds

This is the case since $A^+ = ABCEH$ and $C \subseteq ABCEH$ holds

Hence, C can be removed, so F_c:= {A→B, B→CE, A→E, A→H, D→B}

To check weather B can be removed from $A \rightarrow B$, we compute whether $B \subseteq CalculateAttributeClosure(\{A \rightarrow \emptyset, B \rightarrow CE, A \rightarrow E, A \rightarrow H, D \rightarrow B\}, A)$ holds

This is not the case since $A^+ = AEH$ and $B \not\subseteq AEH$ holds

Hence, B can not be removed, so F_c:= {A→B, B→CE, A→E, A→H, D→B}

To check weather C can be removed from $B \rightarrow CE$, we compute whether $C \subseteq CalculateAttributeClosure(\{A \rightarrow B, B \rightarrow E, A \rightarrow E, A \rightarrow H, D \rightarrow B\}, B)$ holds

This is not the case since $B^+ = BE$ and $C \not\subseteq BE$ holds

Hence, C can not be removed, so F_c:= {A→B, B→CE, A→E, A→H, D→B}

To check weather E can be removed from $B \rightarrow CE$, we compute whether $E \subseteq CalculateAttributeClosure(\{A \rightarrow B, B \rightarrow C, A \rightarrow E, A \rightarrow H, D \rightarrow B\}, B)$ holds

This is not the case since $B^+ = BC$ and $E \not\subseteq BC$ holds

Hence, E can not be removed, so F_c:= {A→B, B→CE, A→E, A→H, D→B}

To check weather E can be removed from $A \rightarrow E$, we compute whether $E \subseteq CalculateAttributeClosure(\{A \rightarrow B, B \rightarrow CE, A \rightarrow \emptyset, A \rightarrow H, D \rightarrow B\}, A)$ holds

This is the case since $A^+ = ABCEH$ and $E \subseteq ABCEH$ holds

Hence, E can be removed, so F_c:= {A→B, B→CE, A→∅, A→H, D→B}

To check weather H can be removed from $A \rightarrow H$, we compute whether $H \subseteq CalculateAttributeClosure(\{A \rightarrow B, B \rightarrow CE, A \rightarrow \emptyset, D \rightarrow B\}, A)$ holds

This is not the case since $A^+ = ABCE$ and $H \not\subseteq ABCE$ holds

Hence, H can not be removed, so F_c:= {A→B, B→CE, A→∅, A→H, D→B}

To check weather B can be removed from $D \rightarrow B$, we compute whether $B \subseteq CalculateAttributeClosure(\{A \rightarrow B, B \rightarrow CE, A \rightarrow \emptyset, A \rightarrow H, D \rightarrow \emptyset\}, D)$ holds

This is not the case since $D^+ = D$ and $B \not\subseteq D$ holds

Hence, B can not be removed, so F_c:= {A→B, B→CE, A→∅, A→H, D→B}

Step 4: We obtain F_c:= {A→B, B→CE, A→H, D→B}

Step 5:

a: standard form, F_c:= {A→B, B→C, B→E, A→H, D→B}

3. [10 points] Find a minimal cover for the relation R(A, B, C, D, E, F) with the set K= A→D, AC→DE, B→F, D→CE of functional dependencies. Show each step.

Step 1: F_c:= {A→D, AC→DE, B→F, D→CE}

Step 2:

$AC \to DE$ has more than one attribute on left-hand side

To check weather A can be removed, we compute whether $DE \subseteq CalculateAttributeClosure(F_c, C)$ holds

This is not the case since $C^+ = C$ and $DE \not\subseteq C$ holds

Hence, A can not be removed, so F_c:= {A→D, AC→DE, B→F, D→CE}

To check weather C can be removed, we compute whether $DE \subseteq CalculateAttributeClosure(F_c, A)$ holds

This is the case since $A^+ = ACDE$ and $DE \subseteq ACDE$ holds

Hence, C can be removed, so F_c:= {A→D, A→DE, B→F, D→CE}

Step 3:

To check weather D can be removed from $A \to D$, we compute whether $D \subseteq CalculateAttributeClosure(\{A \to \emptyset, A \to DE, B \to F, D \to CE\}, A)$ holds

This is the case since $A^+ = ACDE$ and $D \subseteq ACDE$ holds

Hence, D can be removed, so F_c:= {A→∅, A→DE, B→F, D→CE}

To check weather D can be removed from $A \to DE$, we compute whether $D \subseteq CalculateAttributeClosure(\{A \to \emptyset, A \to E, B \to F, D \to CE\}, A)$ holds

This is not the case since $A^+ = AE$ and $D \not\subseteq AE$ holds

Hence, D can not be removed, so F_c:= {A→∅, A→DE, B→F, D→CE}

To check weather E can be removed from $A \to DE$, we compute whether $E \subseteq CalculateAttributeClosure(\{A \to \emptyset, A \to D, B \to F, D \to CE\}, A)$ holds

This is the case since $A^+ = ADCE$ and $E \subseteq ADCE$ holds

Hence, E can be removed, so F_c:= {A→∅, A→D, B→F, D→CE}

To check weather D can be removed from $A \to D$, we compute whether $D \subseteq CalculateAttributeClosure(\{A \to \emptyset, B \to F, D \to CE\}, A)$ holds

This is not the case since $A^+ = A$ and $D \not\subseteq A$ holds

Hence, D can not be removed, so F_c:= {A→∅, A→D, B→F, D→CE}

To check weather F can be removed from $B \to F$, we compute whether $F \subseteq CalculateAttributeClosure(\{A \to \emptyset, A \to D, B \to \emptyset, D \to CE\}, B)$ holds

This is not the case since $B^+ = B$ and $F \not\subseteq B$ holds

Hence, F can not be removed, so F_c:= {A→∅, A→D, B→F, D→CE}

To check weather C can be removed from $D \to CE$, we compute whether $C \subseteq CalculateAttributeClosure(\{A \to \emptyset, A \to D, B \to F, D \to E\}, D)$ holds

This is not the case since $D^+ = DE$ and $C \not\subseteq DE$ holds

Hence, C can not be removed, so F_c:= {A→∅, A→D, B→F, D→CE}

To check weather E can be removed from $D \to CE$, we compute whether $E \subseteq CalculateAttributeClosure(\{A \to \emptyset, A \to D, B \to F, D \to C\}, D)$ holds

This is not the case since $D^+ = CD$ and $E \not\subseteq CD$ holds

Hence, E can not be removed, so F_c:= {A→∅, A→D, B→F, D→CE}

Step 4: We obtain F_c:= {A→D, B→F, D→CE}

Step 5:

a: standard form, F_c:= {A→D, B→F, D→C, D→E}

b: nonstandard form, F_c:= {A→D, B→F, D→CE}

4 Exercise 4

1. [5 points] Consider the relation schema R(A, B, C, D, E, F, G, H, I) with the set of functional dependencies K = {B→G, A→D, DE→F, G→BD}. List all candidate keys of R in a systematic manner (do not use Armstrong's Axioms) and explain how you determine them. Show each step.

Step 1 - Isolated attributes: CHI

Step 2 - Attributes only on the left side of any FD: AE

Step 3 - Attributes only on the right side of any FD: F

Step 4 - Union of the attributes from step 1 and step 2: ACEHI

Step 5 - Computation of the closure of the attributes from step 4: $ACEHI^+ = ACDEFHI$. This does not include all attributes from R

Step 6 - Attributes that can be found on both sides of the FDs: BDG

Step 7 - Computation of the closure of the attributes from step 4 and every combination of attributes from step 6

We construct the power set of the set BDG: {B, D, G, BD, BG, DG, BDG}

We consider the sets with six attributes by adding the sets with one element to ACEHI:

$AECHIB^+ = ABCEHI^+ = ABCDEFGHI = R$

$AECHID^+ = ACDEHI^+ = ACDEFHI \subset R$

$AECHIG^+ = ACEGHI^+ = ABCDEFGHI = R$

We have found the candidate keys ABCEHI, ACEGHI by adding one-element sets to ACEHI.

We consider the sets with seven attributes by adding the sets with two elements to ACEHI:

$AECHIBD^+ = ABCEHID^+ = ABCEHI^+ = R$

$AECHIBG^+ = ABCEHIG^+ = ABCEHI^+ = R$

$AECHIDG^+ = ACEGHID^+ = ACEGHI^+ = R$

We see that we only get superkeys that properly contain candidate keys when we add one-element sets to ACEHI.

When adding BDG to ACEHI, we obtain ACEHIBDG+ = ABCDEGHI+ = R, which is the default superkey; it contains any candidate key found.

In summary, the candidate keys are: ABCEHI, ACEGHI.

2. [5 points] Consider the relation schema R(A, B, C, D, E, F, G, H) with the set of functional dependencies K = {A→B, B→DE, F→H, G→CE}. Determine all candidate keys of R in a systematic manner (do not use the Armstrong's Axioms) and explain how you determine them. Show each step.

Step 1 - Isolated attributes: ∅

Step 2 - Attributes only on the left side of any FD: AFG

Step 3 - Attributes only on the right side of any FD: CDEH

Step 4 - Union of the attributes from step 1 and step 2: AFG

Step 5 - Computation of the closure of the attributes from step 4: $AFG^+ = ABCDEFGH$. This includes all attributes from R

As a result, the attributes from step 4 form the only candidate key, so the candidate key is AFG.