# Homework 1

#### January 25, 2021

```
[2]: import os
     path = r'C:\Users\srsid\Documents\Machine Learning\Boston'
     os.chdir(path)
     os.listdir()
[2]: ['.ipynb_checkpoints',
      'Boston data_description.txt',
      'Boston.csv',
      'Homework 1 Problems.docx',
      'Homework 1.ipynb',
      '~$mework 1 Problems.docx']
[3]: import pandas as pd
     Boston_Housing = pd.read_csv('Boston.csv')
     Boston_Housing
[3]:
             CRIM
                     ZN
                         INDUS
                                CHAS
                                         NOX
                                                 RM
                                                      AGE
                                                              DIS
                                                                   RAD
                                                                           TAX
          0.00632
                   18.0
                          2.31
                                  0.0
                                      0.538
                                              6.575
                                                     65.2
                                                           4.0900
                                                                    1.0
                                                                        296.0
     0
          0.02731
                    0.0
                          7.07
                                 0.0 0.469
                                              6.421
                                                     78.9
                                                           4.9671
                                                                        242.0
     1
                                                                   2.0
     2
          0.02729
                    0.0
                          7.07
                                 0.0 0.469 7.185
                                                     61.1
                                                           4.9671
                                                                   2.0
                                                                        242.0
     3
          0.03237
                    0.0
                          2.18
                                 0.0
                                      0.458
                                              6.998
                                                     45.8
                                                           6.0622
                                                                   3.0
                                                                        222.0
          0.06905
                    0.0
                          2.18
                                 0.0 0.458
                                                     54.2
                                                           6.0622
                                                                        222.0
     4
                                              7.147
                                                                   3.0
                                                ...
     501 0.06263
                    0.0 11.93
                                      0.573
                                                     69.1
                                 0.0
                                              6.593
                                                           2.4786
                                                                   1.0
                                                                        273.0
     502 0.04527
                    0.0 11.93
                                 0.0 0.573
                                                     76.7
                                              6.120
                                                           2.2875
                                                                   1.0
                                                                        273.0
     503 0.06076
                    0.0 11.93
                                 0.0
                                      0.573
                                              6.976
                                                     91.0
                                                           2.1675
                                                                   1.0
                                                                        273.0
                                                     89.3
                    0.0 11.93
                                      0.573
     504 0.10959
                                 0.0
                                              6.794
                                                           2.3889
                                                                   1.0
                                                                        273.0
     505
         0.04741
                    0.0 11.93
                                 0.0 0.573
                                              6.030
                                                     80.8
                                                           2.5050
                                                                   1.0
                                                                        273.0
                          LSTAT MDEV
          PTRATIO
                        В
     0
             15.3 396.90
                            4.98
                                  24.0
     1
             17.8
                   396.90
                            9.14 21.6
     2
             17.8
                            4.03
                                  34.7
                  392.83
     3
             18.7
                   394.63
                            2.94
                                  33.4
             18.7
     4
                   396.90
                            5.33
                                  36.2
              •••
                             •••
                            9.67
                                  22.4
     501
             21.0
                   391.99
     502
             21.0 396.90
                            9.08 20.6
```

```
      503
      21.0
      396.90
      5.64
      23.9

      504
      21.0
      393.45
      6.48
      22.0

      505
      21.0
      396.90
      7.88
      11.9
```

[506 rows x 14 columns]

# [4]: Boston\_Housing.info()

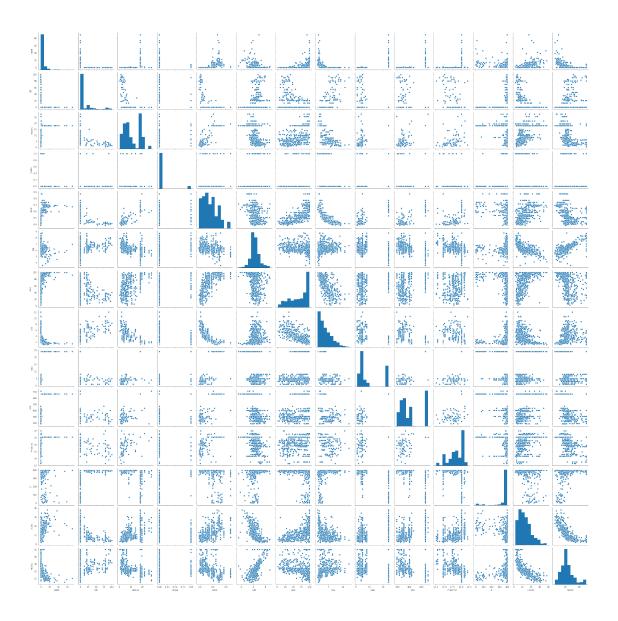
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 506 entries, 0 to 505
Data columns (total 14 columns):

| #  | Column  | Non-Null Count | Dtype   |
|----|---------|----------------|---------|
|    |         |                |         |
| 0  | CRIM    | 506 non-null   | float64 |
| 1  | ZN      | 506 non-null   | float64 |
| 2  | INDUS   | 506 non-null   | float64 |
| 3  | CHAS    | 506 non-null   | float64 |
| 4  | NOX     | 506 non-null   | float64 |
| 5  | RM      | 506 non-null   | float64 |
| 6  | AGE     | 506 non-null   | float64 |
| 7  | DIS     | 506 non-null   | float64 |
| 8  | RAD     | 506 non-null   | float64 |
| 9  | TAX     | 506 non-null   | float64 |
| 10 | PTRATIO | 506 non-null   | float64 |
| 11 | В       | 506 non-null   | float64 |
| 12 | LSTAT   | 506 non-null   | float64 |
| 13 | MDEV    | 506 non-null   | float64 |
|    |         |                |         |

dtypes: float64(14) memory usage: 55.5 KB

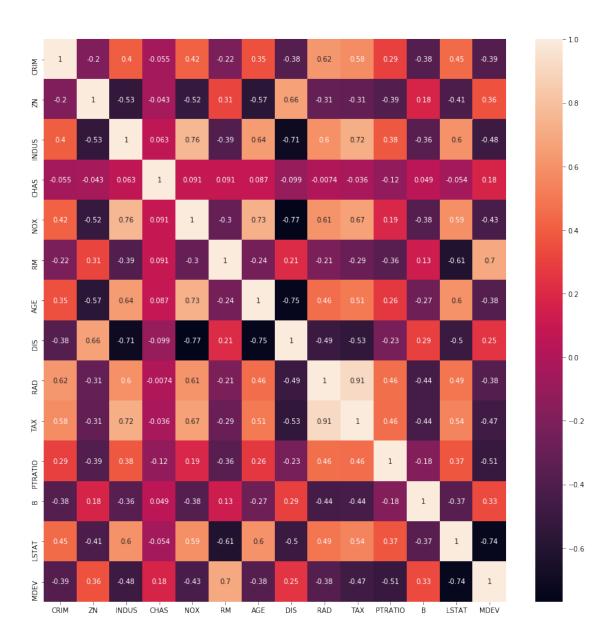
```
[5]: import seaborn as sns
```

[5]: <seaborn.axisgrid.PairGrid at 0x299f1a29808>

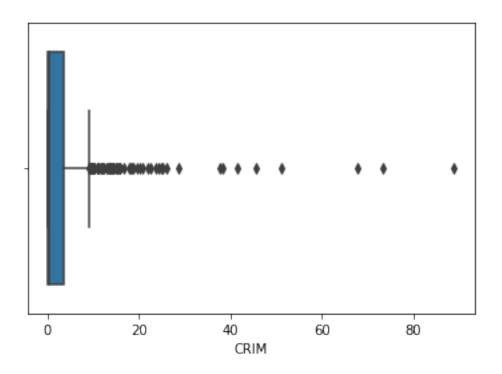


```
[6]: import matplotlib.pyplot as plt
plt.figure(figsize=(15,15))
sns.heatmap(Boston_Housing.corr(), annot=True)
```

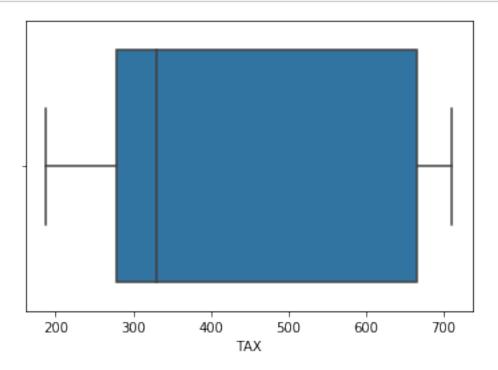
[6]: <matplotlib.axes.\_subplots.AxesSubplot at 0x299ecc7c488>



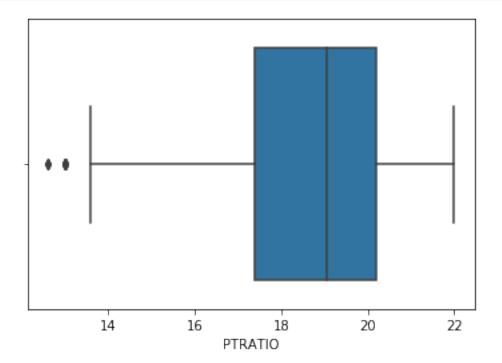
[7]: ax = sns.boxplot(x="CRIM", data=Boston\_Housing)



# [8]: ax = sns.boxplot(x="TAX", data=Boston\_Housing)



```
[9]: ax = sns.boxplot(x="PTRATIO", data=Boston_Housing)
```



```
[10]: Chas = Boston_Housing[Boston_Housing['CHAS']==1]
      Chas['CHAS'].count()
[10]: 35
[11]: Median = Boston_Housing['MDEV']
      Median.describe()
[11]: count
               506.000000
     mean
                22.532806
      std
                 9.197104
     min
                 5.000000
      25%
                17.025000
      50%
                21.200000
      75%
                25.000000
      max
                50.000000
      Name: MDEV, dtype: float64
[12]: Boston_Housing.describe()
```

INDUS

506.000000

11.136779

CHAS

506.000000

0.069170

NOX

506.000000

0.554695

RM

506.000000

6.284634

[12]:

count

mean

CRIM

506.000000

3.593761

ZN

506.000000

11.363636

```
std
               8.596783
                           23.322453
                                         6.860353
                                                      0.253994
                                                                   0.115878
                                                                               0.702617
      min
               0.006320
                            0.000000
                                         0.460000
                                                      0.000000
                                                                   0.385000
                                                                               3.561000
      25%
               0.082045
                            0.000000
                                         5.190000
                                                      0.000000
                                                                   0.449000
                                                                               5.885500
      50%
               0.256510
                            0.000000
                                         9.690000
                                                      0.000000
                                                                   0.538000
                                                                               6.208500
      75%
               3.647422
                           12.500000
                                        18.100000
                                                      0.000000
                                                                   0.624000
                                                                               6.623500
              88.976200
                          100.000000
                                        27.740000
                                                      1.000000
                                                                   0.871000
                                                                               8.780000
      max
                     AGE
                                  DIS
                                              RAD
                                                           TAX
                                                                    PTRATIO
                                                                                       В
                                                                                          \
             506.000000
                          506.000000
                                       506.000000
                                                    506.000000
                                                                             506.000000
                                                                506.000000
      count
              68.574901
                            3.795043
                                         9.549407
                                                    408.237154
                                                                  18.455534
                                                                             356.674032
      mean
      std
              28.148861
                            2.105710
                                         8.707259
                                                    168.537116
                                                                   2.164946
                                                                              91.294864
      min
               2.900000
                            1.129600
                                         1.000000
                                                    187.000000
                                                                  12.600000
                                                                               0.320000
      25%
              45.025000
                            2.100175
                                         4.000000
                                                    279.000000
                                                                  17.400000
                                                                             375.377500
      50%
              77.500000
                            3.207450
                                         5.000000
                                                    330.000000
                                                                  19.050000
                                                                             391.440000
      75%
              94.075000
                            5.188425
                                        24.000000
                                                    666.000000
                                                                  20.200000
                                                                             396.225000
      max
             100.000000
                           12.126500
                                        24.000000
                                                    711.000000
                                                                  22.000000
                                                                             396.900000
                   LSTAT
                                 MDEV
      count
             506.000000
                          506.000000
                           22.532806
              12.653063
      mean
      std
               7.141062
                            9.197104
      min
               1.730000
                            5.000000
      25%
               6.950000
                           17.025000
      50%
              11.360000
                           21.200000
      75%
              16.955000
                           25.000000
      max
              37.970000
                           50.000000
[13]: Boston_min = Boston_Housing.describe().iloc[3]
      Boston_max = Boston_Housing.describe().iloc[7]
      Boston_range = Boston_max-Boston_min
      print(Boston_range)
     CRIM
                  88.96988
     ZN
                 100.00000
     INDUS
                  27.28000
     CHAS
                   1.00000
     NOX
                   0.48600
     RM
                   5.21900
     AGE
                  97.10000
     DIS
                  10.99690
     RAD
                  23.00000
     TAX
                 524.00000
     PTRATIO
                   9.40000
                 396.58000
     LSTAT
                  36.24000
     MDEV
                  45.00000
     dtype: float64
```

```
[14]: from statistics import median
      Lowest = Boston_Housing[Boston_Housing['MDEV'] < Boston_Housing['MDEV'].</pre>
       →median()]
      Lowest.min()
[14]: CRIM
                  0.0136
      ZN
                  0.0000
      INDUS
                   1.6900
      CHAS
                  0.0000
      NOX
                  0.3850
      RM
                  4.1380
      AGE
                  18.5000
     DIS
                   1.1370
      RAD
                   1.0000
      TAX
                 188.0000
     PTRATTO
                  14.7000
      В
                  0.3200
     LSTAT
                  5.5700
      MDEV
                   5.0000
      dtype: float64
[15]: Seven = Boston_Housing[Boston_Housing['RM'] >= 7]
      Seven['RM'].count()
[15]: 64
[16]: Eight = Boston_Housing[Boston_Housing['RM'] >= 8]
      print(Eight['RM'].count())
      print(Eight['AGE'].mean())
      Eight
     13
     71.53846153846153
[16]:
             CRIM
                     ZN
                         INDUS
                                CHAS
                                          NOX
                                                 RM
                                                      AGE
                                                              DIS
                                                                    RAD
                                                                           TAX \
      97
          0.12083
                    0.0
                          2.89
                                 0.0
                                      0.4450
                                              8.069
                                                     76.0 3.4952
                                                                    2.0
                                                                         276.0
      163 1.51902
                                      0.6050
                                              8.375
                                                           2.1620
                                                                    5.0
                                                                         403.0
                    0.0
                         19.58
                                  1.0
                                                     93.9
      204 0.02009
                   95.0
                          2.68
                                 0.0 0.4161
                                              8.034
                                                     31.9 5.1180
                                                                    4.0
                                                                         224.0
                                                     78.3 2.8944
      224 0.31533
                    0.0
                          6.20
                                 0.0
                                      0.5040
                                              8.266
                                                                    8.0
                                                                         307.0
      225 0.52693
                     0.0
                          6.20
                                      0.5040
                                              8.725
                                                     83.0 2.8944
                                                                    8.0
                                                                         307.0
                                 0.0
      226 0.38214
                     0.0
                           6.20
                                 0.0 0.5040
                                              8.040
                                                     86.5 3.2157
                                                                    8.0 307.0
      232 0.57529
                                 0.0 0.5070
                                              8.337
                                                     73.3 3.8384
                                                                    8.0
                                                                         307.0
                    0.0
                          6.20
      233 0.33147
                    0.0
                          6.20
                                 0.0 0.5070 8.247
                                                     70.4 3.6519
                                                                    8.0
                                                                         307.0
      253 0.36894 22.0
                          5.86
                                 0.0 0.4310 8.259
                                                      8.4 8.9067
                                                                    7.0 330.0
                                 0.0 0.6470 8.704
      257 0.61154
                   20.0
                          3.97
                                                     86.9 1.8010
                                                                    5.0
                                                                         264.0
                   20.0
                                              8.398
                                                     91.5 2.2885
                                                                    5.0 264.0
      262 0.52014
                          3.97
                                 0.0 0.6470
      267 0.57834
                   20.0
                          3.97
                                 0.0
                                      0.5750
                                              8.297
                                                     67.0
                                                           2.4216
                                                                    5.0
                                                                         264.0
      364 3.47428
                     0.0
                         18.10
                                  1.0
                                      0.7180
                                              8.780
                                                     82.9 1.9047
                                                                    24.0
                                                                         666.0
```

```
PTRATIO B LSTAT MDEV
      18.0 396.90 4.21 38.7
97
      14.7 388.45 3.32 50.0
163
204
      14.7 390.55 2.88 50.0
224
      17.4 385.05 4.14 44.8
     17.4 382.00 4.63 50.0
225
226
     17.4 387.38 3.13 37.6
     17.4 385.91 2.47 41.7
232
233
     17.4 378.95 3.95 48.3
     19.1 396.90 3.54 42.8
253
257
     13.0 389.70 5.12 50.0
      13.0 386.86 5.91 48.8
262
267
     13.0 384.54 7.44 50.0
364
      20.2 354.55 5.29 21.9
```

```
[17]: import statsmodels.formula.api as smf
ZN = smf.ols('CRIM ~ ZN', data = Boston_Housing).fit()
ZN.summary()
```

# [17]: <class 'statsmodels.iolib.summary.Summary'>

#### OLS Regression Results

| Dep. Variable:    | CRIM             | R-squared:          | 0.040    |
|-------------------|------------------|---------------------|----------|
| Model:            | OLS              | Adj. R-squared:     | 0.038    |
| Method:           | Least Squares    | F-statistic:        | 20.88    |
| Date:             | Sat, 23 Jan 2021 | Prob (F-statistic): | 6.15e-06 |
| Time:             | 21:44:21         | Log-Likelihood:     | -1795.8  |
| No. Observations: | 506              | AIC:                | 3596.    |
| Df Residuals:     | 504              | BIC:                | 3604.    |
| D 4 14 1 7        |                  |                     |          |

Df Model: 1
Covariance Type: nonrobust

| =========    |         | ======== |          | =========     | .======= | ========  |
|--------------|---------|----------|----------|---------------|----------|-----------|
|              | coef    | std err  | t        | P> t          | [0.025   | 0.975]    |
| Intercept    | 4.4292  | 0.417    | 10.620   | 0.000         | 3.610    | 5.249     |
| ZN           | -0.0735 | 0.016    | -4.570   | 0.000         | -0.105   | -0.042    |
| =========    |         |          |          |               |          | =======   |
| Omnibus:     |         | 568.     | 366 Durb | in-Watson:    |          | 0.862     |
| Prob(Omnibus | s):     | 0.       | 000 Jarq | ue-Bera (JB): |          | 32952.356 |
| Skew:        |         | 5.       | 270 Prob | (JB):         |          | 0.00      |
| Kurtosis:    |         | 41.      | 103 Cond | . No.         |          | 28.8      |
| =========    |         |          |          |               |          |           |

#### Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly

```
specified.
```

11 11 11

```
[18]: INDUS = smf.ols('CRIM ~ INDUS', data = Boston_Housing).fit()
INDUS.summary()
```

[18]: <class 'statsmodels.iolib.summary.Summary'>

#### OLS Regression Results

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| Dep. Variable:    | CRIM             | R-squared:          | 0.164    |
|-------------------|------------------|---------------------|----------|
| Model:            | OLS              | Adj. R-squared:     | 0.162    |
| Method:           | Least Squares    | F-statistic:        | 98.58    |
| Date:             | Sat, 23 Jan 2021 | Prob (F-statistic): | 2.44e-21 |
| Time:             | 21:44:21         | Log-Likelihood:     | -1760.9  |
| No. Observations: | 506              | AIC:                | 3526.    |
| Df Residuals:     | 504              | BIC:                | 3534.    |
|                   |                  |                     |          |

Df Model: 1
Covariance Type: nonrobust

|                    | =======           |                |                 |                |                 |                 |
|--------------------|-------------------|----------------|-----------------|----------------|-----------------|-----------------|
|                    | coef              | std err        | t               | P> t           | [0.025          | 0.975]          |
| Intercept<br>INDUS | -2.0509<br>0.5068 | 0.668<br>0.051 | -3.072<br>9.929 | 0.002<br>0.000 | -3.362<br>0.407 | -0.739<br>0.607 |
| ========           |                   |                |                 |                |                 | =======         |
| Omnibus:           |                   | 585.           | .528 Durk       | oin-Watson:    |                 | 0.990           |
| Prob(Omnibus       | ):                | 0.             | .000 Jaro       | ue-Bera (JB)   | :               | 41469.710       |
| Skew:              |                   | 5.             | .456 Prob       | (JB):          |                 | 0.00            |
| Kurtosis:          |                   | 45.            | .987 Cond       | l. No.         |                 | 25.1            |
| =========          | =======           | ========       | ========        | .========      | ========        | ========        |

#### Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[19]: <class 'statsmodels.iolib.summary.Summary'>

# OLS Regression Results

Dep. Variable: CRIM R-squared: 0.003
Model: OLS Adj. R-squared: 0.001
Method: Least Squares F-statistic: 1.546
Date: Sat, 23 Jan 2021 Prob (F-statistic): 0.214

| Df Model:<br>Covariance Ty   | -                 | 21:44:21<br>506<br>504<br>1<br>nonrobust | AIC:<br>BIC:     | kelihood:      |                 | -1805.3<br>3615.<br>3623.          |
|--|-------------------|--|------------------|----------------|-----------------|------------------------------------|
| ========   | coef              | std err                                  | t                | P> t           | [0.025          | 0.975]                             |
| Intercept<br>CHAS  | 3.7232<br>-1.8715 | 0.396<br>1.505                           | 9.404<br>-1.243  | 0.000<br>0.214 | 2.945<br>-4.829 | 4.501<br>1.086                     |
| Omnibus: Prob(Omnibus) Skew: Kurtosis:   | ):<br>======      | 562.698<br>0.000<br>5.205<br>39.818      | Jarque<br>Prob(J |                |                 | 0.822<br>30864.755<br>0.00<br>3.96 |
| Warnings:<br>[1] Standard<br>specified.  | Errors ass        | ume that the c                           | ovariance        | e matrix of    | the errors      | is correctly                       |
| 11 11 11   |                   |  |                  |                |                 |                                    |
|  |                   | <mark>OX',</mark> data = Bo              | ston_Hous        | sing).fit()    |                 |                                    |
| : NOX = smf.ols  | )                 | ib.summary.Sum                           | mary'>           |                |                 |                                    |
| : NOX = smf.ols<br>NOX.summary()<br>: <class 'stats<="" td=""><td>)</td><td></td><td>mary'&gt;</td><td></td><td></td><td></td></class> | )                 |  | mary'>           |                |                 |                                    |

[20]

[20]

\_\_\_\_\_\_

\_\_\_\_\_

std err

P>|t| [0.025

Kurtosis: 46.776 Cond. No. 11.3 \_\_\_\_\_\_

#### Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

11 11 11

```
[21]: RM = smf.ols('CRIM ~ RM', data = Boston_Housing).fit()
      RM.summary()
```

[21]: <class 'statsmodels.iolib.summary.Summary'>

#### OLS Regression Results

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| Dep. Variable:    | CRIM             | R-squared:          | 0.048    |
|-------------------|------------------|---------------------|----------|
| Model:            | OLS              | Adj. R-squared:     | 0.046    |
| Method:           | Least Squares    | F-statistic:        | 25.62    |
| Date:             | Sat, 23 Jan 2021 | Prob (F-statistic): | 5.84e-07 |
| Time:             | 21:44:21         | Log-Likelihood:     | -1793.5  |
| No. Observations: | 506              | AIC:                | 3591.    |
| Df Residuals:     | 504              | BIC:                | 3600.    |

Df Model: 1 Covariance Type: nonrobust

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|                                       | coef               | std err        | t                    | P> t   | [0.025           | 0.975]                             |
|---------------------------------------|--------------------|----------------|----------------------|--|------------------|------------------------------------|
| Intercept<br>RM                       | 20.5060<br>-2.6910 | 3.362<br>0.532 | 6.099<br>-5.062      | 0.000<br>0.000                               | 13.901<br>-3.736 | 27.111<br>-1.646                   |
| Omnibus: Prob(Omnibus Skew: Kurtosis: | 3):                |                | 000 Jarq<br>361 Prob | in-Watson:<br>ue-Bera (JB)<br>(JB):<br>. No. | :                | 0.883<br>36966.825<br>0.00<br>58.4 |

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

11 11 11

```
[22]: AGE = smf.ols('CRIM ~ AGE', data = Boston_Housing).fit()
      AGE.summary()
```

[22]: <class 'statsmodels.iolib.summary.Summary'>

\_\_\_\_\_ Dep. Variable: CRIM R-squared: 0.123 Model: OLS Adj. R-squared: 0.121 Method: Least Squares F-statistic: 70.72 Date: Sat, 23 Jan 2021 Prob (F-statistic): 4.26e-16 21:44:21 Log-Likelihood: Time: -1772.9No. Observations: 506 AIC: 3550. Df Residuals: 504 BIC: 3558. Df Model: 1 Covariance Type: nonrobust \_\_\_\_\_\_ coef std err t P>|t| [0.025 \_\_\_\_\_\_ -3.7527 0.944 0.1071 0.013 -3.974 0.000 Intercept -5.608 -1.8988.409 0.000 0.082 0.132 \_\_\_\_\_ Omnibus: 575.090 Durbin-Watson: 0.960 Prob(Omnibus): 0.000 Jarque-Bera (JB): 36851.412 Skew: 5.331 Prob(JB): 0.00 Kurtosis: 43.426 Cond. No. 195. \_\_\_\_\_ [1] Standard Errors assume that the covariance matrix of the errors is correctly specified. 11 11 11 [23]: DIS = smf.ols('CRIM ~ DIS', data = Boston\_Housing).fit() DIS.summary() [23]: <class 'statsmodels.iolib.summary.Summary'>

| =========         |        |  | ===== |            |                  | =======    | ======= |
|-------------------|--------|--|-------|------------|------------------|------------|---------|
| Dep. Variable     | e:     |  | CRIM  | R-sq       | uared:           |            | 0.143   |
| Model:            |        |  | OLS   | Adj.       | R-squared:       |            | 0.141   |
| Method:           |        | Least Squares                          |       |            | atistic:         |            | 83.97   |
| Date:             |        | Sat, 23 Jan 2021 Prob (F-statistic):   |       | :          | 1.27e-18         |            |         |
| Time:             |        | 21:                                    | 44:21 | Log-       | Likelihood:      |            | -1767.1 |
| No. Observations: |        | 506                                    |       | AIC:       |                  |            | 3538.   |
| Df Residuals:     | ;      | 504                                    |       | BIC:       |                  |            | 3547.   |
| Df Model:         |        |  | 1     |            |                  |            |         |
| Covariance Ty     | ype:   | nonr                                   | obust |            |                  |            |         |
| =========         | coei   | ====================================== | ===== | =====<br>t | ========<br>P> t | <br>Γ0.025 | 0.975]  |
|                   |        |  |       |            |                  |            |         |
| Intercept         | 9.4489 | 0.731                                  | 12    | 2.934      | 0.000            | 8.014      | 10.884  |

| DIS                   | -1.5428 | 0.168            | -9.163 0.000                    | -1.874 | -1.212             |
|-----------------------|---------|------------------|---------------------------------|--------|--------------------|
| Omnibus: Prob(Omnibus | ·)·     | 577.090<br>0.000 | Durbin-Watson:<br>Jarque-Bera ( | -      | 0.957<br>37542.100 |
| Skew:                 | 5).     | 5.357            | Prob(JB):                       | JD).   | 0.00               |
| Kurtosis:             |         | 43.815<br>       | Cond. No.                       |        | 9.32               |

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

11 11 11

[24]: <class 'statsmodels.iolib.summary.Summary'>

# OLS Regression Results

| Dep. Variable:    | CRIM             | R-squared:          | 0.387    |
|-------------------|------------------|---------------------|----------|
| Model:            | OLS              | Adj. R-squared:     | 0.386    |
| Method:           | Least Squares    | F-statistic:        | 318.1    |
| Date:             | Sat, 23 Jan 2021 | Prob (F-statistic): | 1.62e-55 |
| Time:             | 21:44:21         | Log-Likelihood:     | -1682.3  |
| No. Observations: | 506              | AIC:                | 3369.    |
| Df Residuals:     | 504              | BIC:                | 3377.    |
| Df Model:         | 1                |                     |          |
| a                 | •                |                     |          |

Covariance Type: nonrobust

| ========     |         |         | =======   | ========     | ======== | =======   |
|--------------|---------|---------|-----------|--------------|----------|-----------|
|              | coef    | std err | t         | P> t         | [0.025   | 0.975]    |
|              |         |         |           |              |          |           |
| Intercept    | -2.2709 | 0.445   | -5.105    | 0.000        | -3.145   | -1.397    |
| RAD          | 0.6141  | 0.034   | 17.835    | 0.000        | 0.546    | 0.682     |
| ========     | ======= |         | ========  | ========     | =======  | =======   |
| Omnibus:     |         | 654.    | 232 Durbi | n-Watson:    |          | 1.336     |
| Prob(Omnibus | 3):     | 0.      | 000 Jarqu | e-Bera (JB): |          | 74327.568 |
| Skew:        |         | 6.      | 441 Prob( | JB):         |          | 0.00      |
| Kurtosis:    |         | 60.     | 961 Cond. | No.          |          | 19.2      |
| =========    | ======= |         |           |              | ======== | ========  |

#### Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

11 11 11

```
[25]: TAX = smf.ols('CRIM ~ TAX', data = Boston_Housing).fit()
    TAX.summary()
```

[25]: <class 'statsmodels.iolib.summary.Summary'>

#### OLS Regression Results

| Dep. Variable:    | CRIM             | R-squared:          | 0.336    |
|-------------------|------------------|---------------------|----------|
| Model:            | OLS              | Adj. R-squared:     | 0.335    |
| Method:           | Least Squares    | F-statistic:        | 254.9    |
| Date:             | Sat, 23 Jan 2021 | Prob (F-statistic): | 9.76e-47 |
| Time:             | 21:44:21         | Log-Likelihood:     | -1702.5  |
| No. Observations: | 506              | AIC:                | 3409.    |
| Df Residuals:     | 504              | BIC:                | 3418.    |
|                   |                  |                     |          |

Df Model: 1
Covariance Type: nonrobust

| =========        | =======           | ========       | =======         | =========    | :======== | ========        |
|------------------|-------------------|----------------|-----------------|--------------|-----------|-----------------|
|                  | coef              | std err        |                 | t P> t       | [0.025    | 0.975]          |
| Intercept<br>TAX | -8.4748<br>0.0296 | 0.818<br>0.002 | -10.36<br>15.96 |              |           | -6.868<br>0.033 |
| =========        |                   | ========       |                 | ========     |           | ========        |
| Omnibus:         |                   | 634            | 1.003 Du        | rbin-Watson: |           | 1.252           |
| Prob(Omnibus     | 3):               | C              | ).000 Ja        | rque-Bera (3 | JB):      | 63141.063       |
| Skew:            |                   | 6              | 5.134 Pr        | ob(JB):      |           | 0.00            |
| Kurtosis:        |                   | 56             | 6.332 Cc        | nd. No.      |           | 1.16e+03        |
|                  |                   | ========       |                 | ========     |           | ========        |

#### Warnings:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 1.16e+03. This might indicate that there are strong multicollinearity or other numerical problems.
- [26]: PTRATIO = smf.ols('CRIM ~ PTRATIO', data = Boston\_Housing).fit()
  PTRATIO.summary()
- [26]: <class 'statsmodels.iolib.summary.Summary'>

| Dep. Variable: | CRIM             | R-squared:          | 0.083    |
|----------------|------------------|---------------------|----------|
| Model:         | OLS              | Adj. R-squared:     | 0.081    |
| Method:        | Least Squares    | F-statistic:        | 45.67    |
| Date:          | Sat, 23 Jan 2021 | Prob (F-statistic): | 3.88e-11 |
| Time:          | 21:44:21         | Log-Likelihood:     | -1784.1  |

| No. Observation Df Residuals: Df Model: Covariance Type   |   | 504<br>1<br>bust       |   |   |                     | 3572.<br>3581.   |  |  |  |  |  |
|---|---|------------------------|---|---|---------------------|------------------|--|--|--|--|--|
|   | coef  | std err                |   | t   | P> t                | [0.025           | 0.975]   |  |  |  |  |
| Intercept -: PTRATIO  | 17.5307   | 3.147                  | -5  | .570<br>.758                                    | 0.000               | -23.714<br>0.812 | -11.347<br>1.477   |  |  |  |  |
| Omnibus: Prob(Omnibus): Skew: Kurtosis:   | Prob(Omnibus):       0.000       Jarque-Bera (JB):       34373.378         Skew:       5.256       Prob(JB):       0.00         Kurtosis:       41.985       Cond. No.       160. |                        |   |   |                     |                  |  |  |  |  |  |
| Warnings: [1] Standard Errors assume that the covariance matrix of the errors is correctly specified. """                 |   |                        |   |   |                     |                  |  |  |  |  |  |
| B = smf.ols('Cl<br>B.summary()  | RIM ~ B',   | data = Bo              | ston_H  | ousing  | g).fit()            |                  |  |  |  |  |  |
| : <class 'statsmo<="" td=""><td>odels.iol</td><td></td><td>.Summa:</td><td></td><td>esults</td><td></td><td></td></class> | odels.iol   |                        | .Summa:   |   | esults              |                  |  |  |  |  |  |
| Dep. Variable: Model: Method: Date: Time: No. Observation Df Residuals: Df Model: Covariance Type                         | Sa<br>ns:   | Least Squ<br>t, 23 Jan | CRIM<br>OLS<br>ares<br>2021<br>4:21<br>506<br>504 | R-squ<br>Adj.<br>F-sta<br>Prob<br>Log-I<br>AIC: | ared:<br>R-squared: |                  | 0.142<br>0.141<br>83.69<br>1.43e-18<br>-1767.2<br>3538.<br>3547. |  |  |  |  |
| =========   |   |                        |   |   | P> t                |                  |  |  |  |  |  |
| Intercept :   | -0.0355   | 0.004                  | -9  | .148  | 0.000               | -0.043           | -0.028   |  |  |  |  |
| Omnibus:  |   |                        |   |   | n-Watson:           |                  | 1.001  |  |  |  |  |

[27]

[27]

Prob(Omnibus):

Skew:

Kurtosis:

5.543 Prob(JB):

46.932

0.000 Jarque-Bera (JB):

Cond. No.

43282.465

1.49e+03

0.00

\_\_\_\_\_\_

#### Warnings:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 1.49e+03. This might indicate that there are strong multicollinearity or other numerical problems.

```
[28]: LSTAT = smf.ols('CRIM ~ LSTAT', data = Boston_Housing).fit()
LSTAT.summary()
```

[28]: <class 'statsmodels.iolib.summary.Summary'>

#### OLS Regression Results

| ============      |                  |                     |          |
|-------------------|------------------|---------------------|----------|
| Dep. Variable:    | CRIM             | R-squared:          | 0.205    |
| Model:            | OLS              | Adj. R-squared:     | 0.203    |
| Method:           | Least Squares    | F-statistic:        | 129.6    |
| Date:             | Sat, 23 Jan 2021 | Prob (F-statistic): | 7.12e-27 |
| Time:             | 21:44:21         | Log-Likelihood:     | -1748.2  |
| No. Observations: | 506              | AIC:                | 3500.    |
| Df Residuals:     | 504              | BIC:                | 3509.    |

Df Model: 1
Covariance Type: nonrobust

| =========                             | =======           | ========       | ========         | :=======   |                 | ========                           |
|---------------------------------------|-------------------|----------------|------------------|--|-----------------|------------------------------------|
|                                       | coef              | std err        | t                | P> t   | [0.025          | 0.975]                             |
| Intercept<br>LSTAT                    | -3.2946<br>0.5444 | 0.695<br>0.048 | -4.742<br>11.383 | 0.000<br>0.000                                   | -4.660<br>0.450 | -1.930<br>0.638                    |
| Omnibus: Prob(Omnibus Skew: Kurtosis: | ):                | 0              | .000 Jaro        | oin-Watson:<br>que-Bera (JB)<br>o(JB):<br>l. No. | ):              | 1.184<br>49637.173<br>0.00<br>29.7 |

\_\_\_\_\_\_

#### Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
[29]: MDEV = smf.ols('CRIM ~ MDEV', data = Boston_Housing).fit()
MDEV.summary()
```

[29]: <class 'statsmodels.iolib.summary.Summary'>

#### OLS Regression Results

| Dep. Variab  | le:              |                  |          | (           | CRIM            | R-sq          | uared:                  |           | 0.149    |
|--------------|------------------|------------------|----------|-------------|-----------------|---------------|-------------------------|-----------|----------|
| Model:       |                  |                  |          |             | OLS             | Adj.          | R-squared:              | 0.147     |          |
| Method:      |                  | Least Squares    |          |             | ares            | F-st          | atistic:                |           | 88.15    |
| Date:        |                  | Sat, 23 Jan 2021 |          |             | 2021            | Prob          | (F-statistic):          |           | 2.08e-19 |
| Time:        |                  |                  |          | 21:4        | 4:21            | Log-          | Likelihood:             |           | -1765.3  |
| No. Observat | tions:           |                  | 506 AIC: |             |                 |               |                         | 3535.     |          |
| Df Residuals | 3:               |                  |          |             | 504             | BIC:          |                         |           | 3543.    |
| Df Model:    |                  |                  |          | 1           |                 |               |                         |           |          |
| Covariance 7 | Covariance Type: |                  | n        | onrol       | bust            |               |                         |           |          |
| ========     | coe              | =====<br>f s     | td       |             | =====           | _             | P> t                    | [0.025    | 0.975]   |
| Intercept    | 11.720           | <br>2            | 0.       |             | 12              |               | 0.000                   | 9.884     | 13.557   |
| MDEV         | -0.360           | 3                | 0.       | 038         | -9              | . 389         | 0.000                   | -0.436    | -0.285   |
| Omnibus:     | ======           | =====            |          | ====<br>559 | ======<br>. 282 | Durb          | =========<br>in-Watson: | :======   | 1.000    |
| Prob(Omnibus | 3):              | 0.000            |          |             |                 | ue-Bera (JB): |                         | 32809.507 |          |
| Skew:        | ٠,٠              |                  |          |             | .114            | -             | (JB):                   |           | 0.00     |
| ~110W.       |                  |                  |          | J           |                 | 1100          | (02).                   |           | 0.00     |

#### Warnings:

Kurtosis:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Cond. No.

64.5

11 11 11

# [30]: Boston\_Housing.iloc[:,1:]

| [30]: |     | ZN   | INDUS | CHAS | NOX   | RM    | AGE  | DIS    | RAD | TAX   | PTRATIO | \ |
|-------|-----|------|-------|------|-------|-------|------|--------|-----|-------|---------|---|
|       | 0   | 18.0 | 2.31  | 0.0  | 0.538 | 6.575 | 65.2 | 4.0900 | 1.0 | 296.0 | 15.3    |   |
|       | 1   | 0.0  | 7.07  | 0.0  | 0.469 | 6.421 | 78.9 | 4.9671 | 2.0 | 242.0 | 17.8    |   |
|       | 2   | 0.0  | 7.07  | 0.0  | 0.469 | 7.185 | 61.1 | 4.9671 | 2.0 | 242.0 | 17.8    |   |
|       | 3   | 0.0  | 2.18  | 0.0  | 0.458 | 6.998 | 45.8 | 6.0622 | 3.0 | 222.0 | 18.7    |   |
|       | 4   | 0.0  | 2.18  | 0.0  | 0.458 | 7.147 | 54.2 | 6.0622 | 3.0 | 222.0 | 18.7    |   |
|       |     | •••  |       | •••  |       |       | •••  |        |     |       |         |   |
|       | 501 | 0.0  | 11.93 | 0.0  | 0.573 | 6.593 | 69.1 | 2.4786 | 1.0 | 273.0 | 21.0    |   |
|       | 502 | 0.0  | 11.93 | 0.0  | 0.573 | 6.120 | 76.7 | 2.2875 | 1.0 | 273.0 | 21.0    |   |
|       | 503 | 0.0  | 11.93 | 0.0  | 0.573 | 6.976 | 91.0 | 2.1675 | 1.0 | 273.0 | 21.0    |   |
|       | 504 | 0.0  | 11.93 | 0.0  | 0.573 | 6.794 | 89.3 | 2.3889 | 1.0 | 273.0 | 21.0    |   |
|       | 505 | 0.0  | 11.93 | 0.0  | 0.573 | 6.030 | 80.8 | 2.5050 | 1.0 | 273.0 | 21.0    |   |
|       |     |      |       |      |       |       |      |        |     |       |         |   |

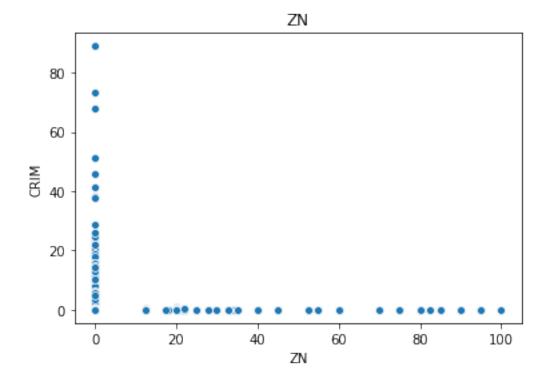
41.099

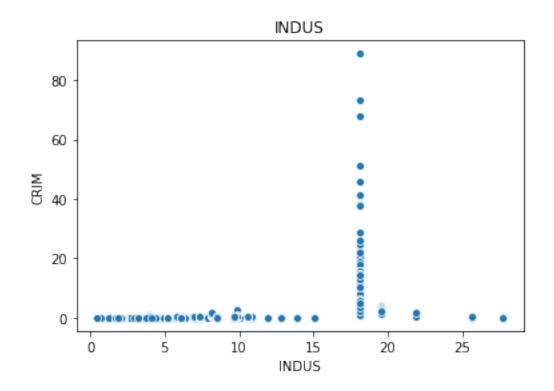
B LSTAT MDEV

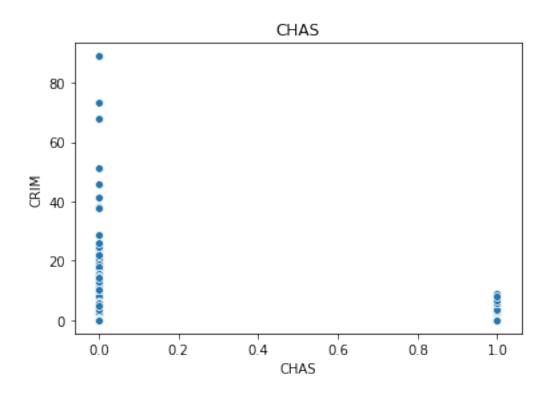
- 0 396.90 4.98 24.0
- 1 396.90 9.14 21.6
- 2 392.83 4.03 34.7

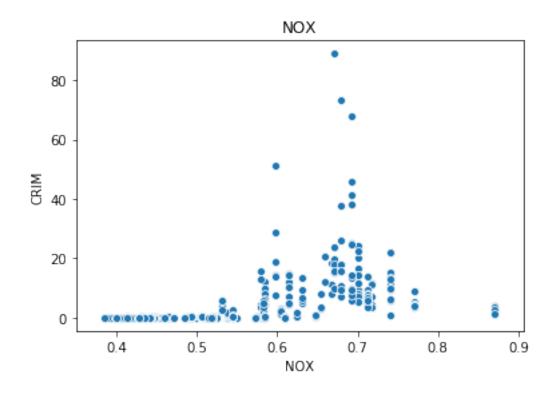
```
394.63
3
             2.94 33.4
4
     396.90
             5.33 36.2
. .
             9.67
                   22.4
501
    391.99
502 396.90
             9.08 20.6
503 396.90
             5.64 23.9
504
    393.45
             6.48 22.0
505
    396.90
             7.88 11.9
```

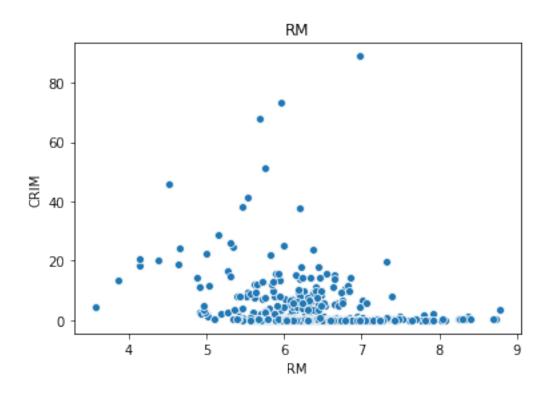
[506 rows x 13 columns]

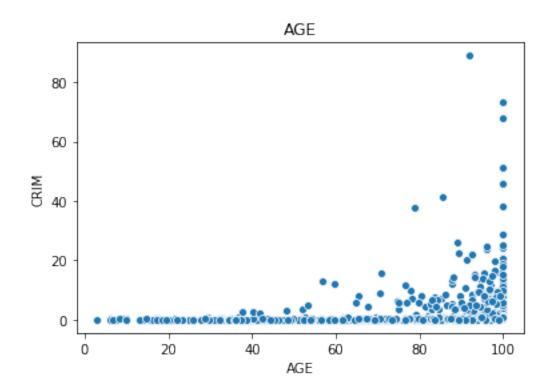


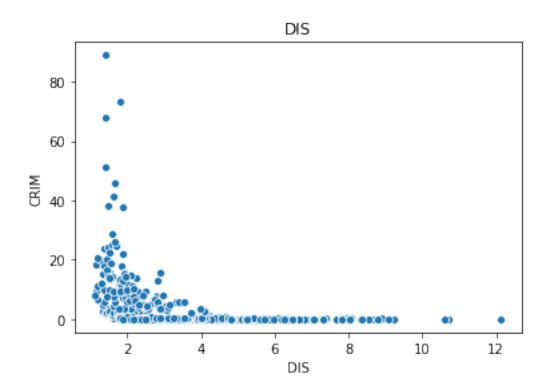


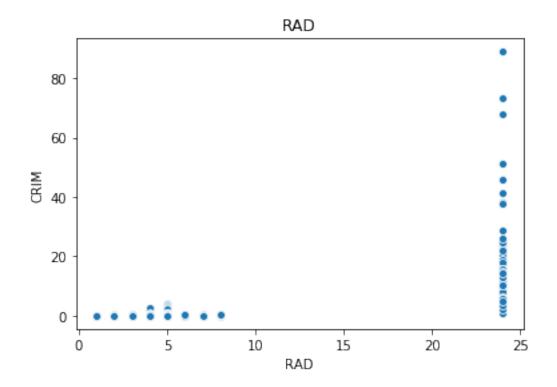


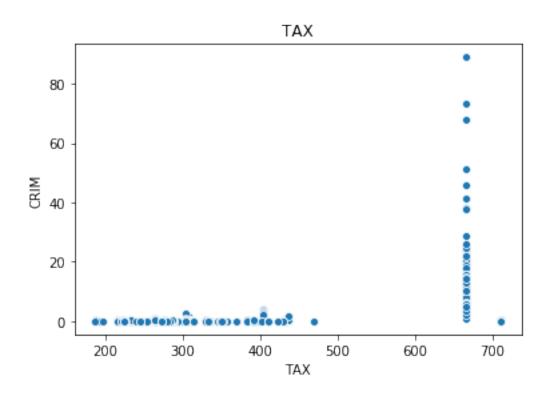


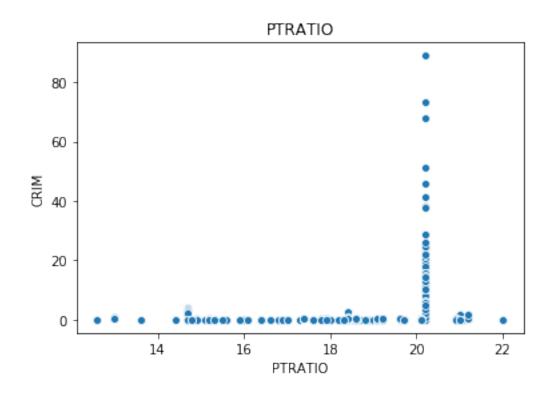


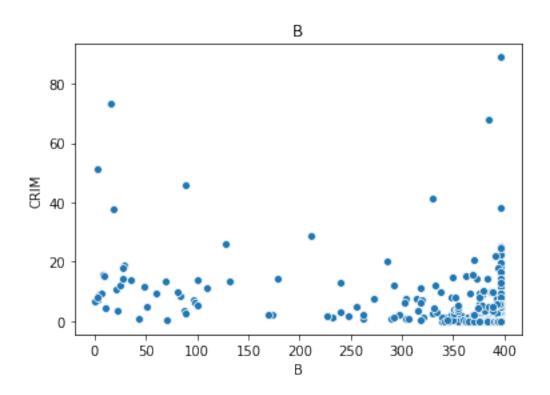


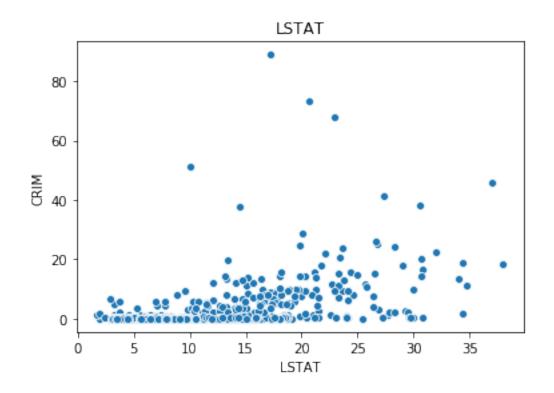


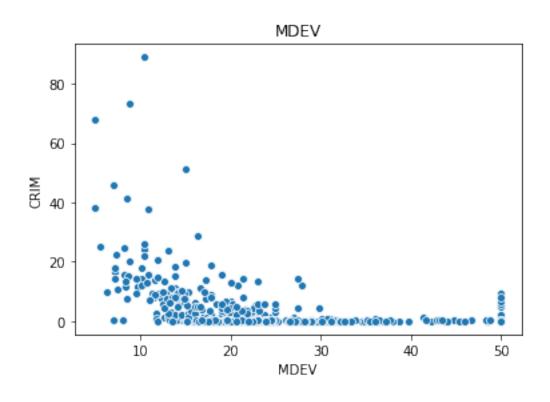












```
[32]: predictors = ' + '.join(Boston_Housing.columns.difference(['CRIM']))

Multiple_Regression = smf.ols('CRIM ~ {}'.format(predictors),data = 
Boston_Housing).fit()

print(Multiple_Regression.summary())

Multiple_Regression.params
```

#### OLS Regression Results

| ==========        |                  |                     | ========== |
|-------------------|------------------|---------------------|------------|
| Dep. Variable:    | CRIM             | R-squared:          | 0.448      |
| Model:            | OLS              | Adj. R-squared:     | 0.434      |
| Method:           | Least Squares    | F-statistic:        | 30.73      |
| Date:             | Sat, 23 Jan 2021 | Prob (F-statistic): | 2.04e-55   |
| Time:             | 21:44:23         | Log-Likelihood:     | -1655.7    |
| No. Observations: | 506              | AIC:                | 3339.      |
| Df Residuals:     | 492              | BIC:                | 3399.      |
| Df Model:         | 13               |                     |            |

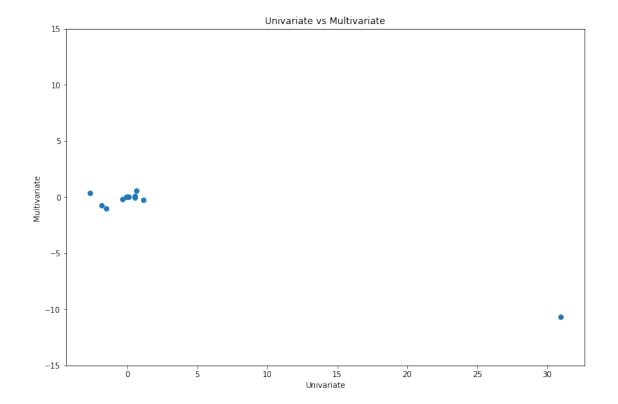
Covariance Type: nonrobust

| ========    | ======== |         |            |              | =======  |           |
|-------------|----------|---------|------------|--------------|----------|-----------|
|             | coef     | std err | t          | P> t         | [0.025   | 0.975]    |
|             |          |         |            |              |          |           |
| Intercept   | 17.4184  | 7.270   | 2.396      | 0.017        | 3.135    | 31.702    |
| AGE         | 0.0020   | 0.018   | 0.112      | 0.911        | -0.033   | 0.037     |
| В           | -0.0069  | 0.004   | -1.857     | 0.064        | -0.014   | 0.000     |
| CHAS        | -0.7414  | 1.186   | -0.625     | 0.532        | -3.071   | 1.588     |
| DIS         | -0.9950  | 0.283   | -3.514     | 0.000        | -1.551   | -0.439    |
| INDUS       | -0.0616  | 0.084   | -0.735     | 0.463        | -0.226   | 0.103     |
| LSTAT       | 0.1213   | 0.076   | 1.594      | 0.112        | -0.028   | 0.271     |
| MDEV        | -0.1992  | 0.061   | -3.276     | 0.001        | -0.319   | -0.080    |
| NOX         | -10.6455 | 5.301   | -2.008     | 0.045        | -21.061  | -0.230    |
| PTRATIO     | -0.2787  | 0.187   | -1.488     | 0.137        | -0.647   | 0.089     |
| RAD         | 0.5888   | 0.088   | 6.656      | 0.000        | 0.415    | 0.763     |
| RM          | 0.3811   | 0.616   | 0.619      | 0.536        | -0.829   | 1.591     |
| TAX         | -0.0037  | 0.005   | -0.723     | 0.470        | -0.014   | 0.006     |
| ZN          | 0.0449   | 0.019   | 2.386      | 0.017        | 0.008    | 0.082     |
| ========    |          |         |            |              |          |           |
| Omnibus:    |          | 662.2   | 271 Durbin | ı-Watson:    |          | 1.515     |
| Prob(Omnibu | s):      | 0.0     | 000 Jarque | e-Bera (JB): |          | 82701.666 |
| Skew:       |          | 6.5     | 544 Prob(J | IB):         |          | 0.00      |
| Kurtosis:   |          | 64.2    | 248 Cond.  | No.          |          | 1.58e+04  |
| ========    | ======== |         |            |              | ======== |           |

#### Warnings:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 1.58e+04. This might indicate that there are strong multicollinearity or other numerical problems.

```
[32]: Intercept
                   17.418362
      AGE
                    0.002011
      В
                   -0.006855
      CHAS
                   -0.741435
                   -0.994992
     DIS
      INDUS
                   -0.061576
     LSTAT
                    0.121269
     MDEV
                   -0.199218
      NOX
                  -10.645500
      PTRATIO
                   -0.278731
      RAD
                    0.588838
      RM
                    0.381070
      TAX
                   -0.003746
      ZN
                    0.044919
      dtype: float64
[33]: import matplotlib.pyplot as plt
      X = [AGE.params[1], B.params[1], CHAS.params[1],DIS.params[1],INDUS.params[1],
       →LSTAT.params[1], MDEV.params[1], NOX.params[1], PTRATIO.params[1], RAD.
       →params[1], RM.params[1], TAX.params[1], ZN.params[1]]
      y = Multiple_Regression.params[1::1]
      #Source: https://stackoverflow.com/questions/52340435/
       \rightarrow extract-regressions-coefficient-from-statsmodels
[34]: plt.figure(figsize = (12,8))
      plt.ylim(-15,15)
      plt.scatter(X,y)
      plt.title('Univariate vs Multivariate')
      plt.xlabel("Univariate")
      plt.ylabel("Multivariate")
[34]: Text(0, 0.5, 'Multivariate')
```



| [35]: | Bost | on_Housin | ıg    |         |      |       |       |      |        |     |       |   |
|-------|------|-----------|-------|---------|------|-------|-------|------|--------|-----|-------|---|
| [35]: |      | CRIM      | ZN    | INDUS   | CHAS | NOX   | RM    | AGE  | DIS    | RAD | TAX   | \ |
|       | 0    | 0.00632   | 18.0  | 2.31    | 0.0  | 0.538 | 6.575 | 65.2 | 4.0900 | 1.0 | 296.0 |   |
|       | 1    | 0.02731   | 0.0   | 7.07    | 0.0  | 0.469 | 6.421 | 78.9 | 4.9671 | 2.0 | 242.0 |   |
|       | 2    | 0.02729   | 0.0   | 7.07    | 0.0  | 0.469 | 7.185 | 61.1 | 4.9671 | 2.0 | 242.0 |   |
|       | 3    | 0.03237   | 0.0   | 2.18    | 0.0  | 0.458 | 6.998 | 45.8 | 6.0622 | 3.0 | 222.0 |   |
|       | 4    | 0.06905   | 0.0   | 2.18    | 0.0  | 0.458 | 7.147 | 54.2 | 6.0622 | 3.0 | 222.0 |   |
|       |      | •••       |       |         | •••  |       |       | •••  |        |     |       |   |
|       | 501  | 0.06263   | 0.0   | 11.93   | 0.0  | 0.573 | 6.593 | 69.1 | 2.4786 | 1.0 | 273.0 |   |
|       | 502  | 0.04527   | 0.0   | 11.93   | 0.0  | 0.573 | 6.120 | 76.7 | 2.2875 | 1.0 | 273.0 |   |
|       | 503  | 0.06076   | 0.0   | 11.93   | 0.0  | 0.573 | 6.976 | 91.0 | 2.1675 | 1.0 | 273.0 |   |
|       | 504  | 0.10959   | 0.0   | 11.93   | 0.0  | 0.573 | 6.794 | 89.3 | 2.3889 | 1.0 | 273.0 |   |
|       | 505  | 0.04741   | 0.0   | 11.93   | 0.0  | 0.573 | 6.030 | 80.8 | 2.5050 | 1.0 | 273.0 |   |
|       |      |           |       |         |      |       |       |      |        |     |       |   |
|       |      | PTRATIO   | İ     | B LSTAT | MDE  | :V    |       |      |        |     |       |   |
|       | 0    | 15.3      | 396.9 | 0 4.98  | 24.  | 0     |       |      |        |     |       |   |
|       | 1    | 17.8      | 396.9 | 9.14    | 21.  | 6     |       |      |        |     |       |   |
|       | 2    | 17.8      | 392.8 | 3 4.03  | 34.  | 7     |       |      |        |     |       |   |
|       | 3    | 18.7      | 394.6 | 3 2.94  | 33.  | 4     |       |      |        |     |       |   |
|       | 4    | 18.7      | 396.9 | 0 5.33  | 36.  | 2     |       |      |        |     |       |   |
|       |      | •••       | •••   |         |      |       |       |      |        |     |       |   |
|       | 501  | 21.0      | 391.9 | 9.67    | 22.  | 4     |       |      |        |     |       |   |

```
502 21.0 396.90 9.08 20.6
503 21.0 396.90 5.64 23.9
504 21.0 393.45 6.48 22.0
505 21.0 396.90 7.88 11.9
[506 rows x 14 columns]
```

```
[37]: import numpy as np
      ZN2 = smf.ols(formula = 'CRIM \sim ZN + np.power(ZN,2) + np.power(ZN,3)', data = ___
      →Boston_Housing).fit()
      INDUS2 = smf.ols(formula = 'CRIM ~ INDUS + np.power(INDUS,2) + np.
      →power(INDUS,3)', data = Boston_Housing).fit()
      CHAS2 = smf.ols(formula = 'CRIM ~ CHAS + np.power(CHAS,2) + np.power(CHAS,3)', __

data = Boston_Housing).fit()
      NOX2 = smf.ols(formula = 'CRIM ~ NOX + np.power(NOX ,2) + np.power(NOX ,3)', __

data = Boston_Housing).fit()
      RM2 = smf.ols(formula = 'CRIM ~ RM + np.power(RM ,2) + np.power(RM ,3)', data = __
      →Boston_Housing).fit()
      AGE2 = smf.ols(formula = 'CRIM ~ AGE + np.power(AGE ,2) + np.power(AGE ,3)', u
      →data = Boston_Housing).fit()
      DIS2 = smf.ols(formula = 'CRIM ~ DIS + np.power(DIS,2) + np.power(DIS ,3)', __

→data = Boston_Housing).fit()
      RAD2 = smf.ols(formula = 'CRIM ~ RAD + np.power(RAD ,2) + np.power(RAD ,3)',
      →data = Boston_Housing).fit()
      TAX2 = smf.ols(formula = 'CRIM ~ TAX + np.power(TAX ,2) + np.power(TAX ,3)',
      →data = Boston_Housing).fit()
      PTRATIO2 = smf.ols(formula = 'CRIM ~ PTRATIO + np.power(PTRATIO ,2) + np.
      →power(PTRATIO ,3)', data = Boston_Housing).fit()
      B2 = smf.ols(formula = 'CRIM ~ B + np.power(B ,2) + np.power(B ,3)', data = 11
      →Boston_Housing).fit()
      LSTAT2 = smf.ols(formula = 'CRIM ~ LSTAT + np.power(LSTAT ,2) + np.power(LSTAT_
      →,3)', data = Boston_Housing).fit()
      MDEV2 = smf.ols(formula = 'CRIM ~ MDEV + np.power(MDEV ,2) + np.power(MDEV_L
      →,3)', data = Boston_Housing).fit()
```

### [38]: ZN2.summary()

[38]: <class 'statsmodels.iolib.summary.Summary'>

#### OLS Regression Results

0.058 Dep. Variable: CRIM R-squared: Model: OLS Adj. R-squared: 0.052 Method: Least Squares F-statistic: 10.24 Date: Sat, 23 Jan 2021 Prob (F-statistic): 1.49e-06 Time: 21:45:41 Log-Likelihood: -1791.1

| No. Observation Df Residuals: Df Model: Covariance Type |                | 506<br>502<br>3<br>nonrobust | AIC:<br>BIC:                          |                         |                           | 3590.<br>3607.                    |
|---|----------------|------------------------------|---------------------------------------|-------------------------|---------------------------|-----------------------------------|
| 0.975]  | coef           | std err                      | t                                     | P> t                    | [0.025                    |                                   |
| Intercept 5.670 ZN -0.115 np.power(ZN, 2) 0.014         |                | 0.433<br>0.110<br>0.004      | 11.133<br>-3.008<br>1.670             | 0.000<br>0.003<br>0.096 | 3.969<br>-0.546<br>-0.001 |                                   |
| np.power(ZN, 3)<br>2.41e-05                             | -3.753e-05<br> | 3.14e-05                     | -1.196                                | 0.232                   | -9.92e-05                 |                                   |
| Omnibus: Prob(Omnibus): Skew: Kurtosis:                 |                | 41.672                       | Jarque-Bera<br>Prob(JB):<br>Cond. No. | a (JB):                 | 1.                        | 0.879<br>86.468<br>0.00<br>89e+05 |

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 1.89e+05. This might indicate that there are strong multicollinearity or other numerical problems.

# [39]: INDUS2.summary()

[39]: <class 'statsmodels.iolib.summary.Summary'>

| ======================================= |                  |                     | ======================================= |
|---|------------------|---------------------|---|
| Dep. Variable:                          | CRIM             | R-squared:          | 0.257                                   |
| Model:                                  | OLS              | Adj. R-squared:     | 0.252                                   |
| Method:                                 | Least Squares    | F-statistic:        | 57.86                                   |
| Date:                                   | Sat, 23 Jan 2021 | Prob (F-statistic): | 3.88e-32                                |
| Time:                                   | 21:46:17         | Log-Likelihood:     | -1731.0                                 |
| No. Observations:                       | 506              | AIC:                | 3470.                                   |
| Df Residuals:                           | 502              | BIC:                | 3487.                                   |
| Df Model:                               | 3                |                     |   |
| Covariance Type:                        | nonrobust        |                     |   |

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 2.47e+04. This might indicate that there are strong multicollinearity or other numerical problems.

# [40]: CHAS2.summary()

[40]: <class 'statsmodels.iolib.summary.Summary'>

| ======================================= |   |                     | ======================================= |
|---|---|---------------------|---|
| Dep. Variable:                          | CRIM                                    | R-squared:          | 0.003                                   |
| Model:                                  | OLS                                     | Adj. R-squared:     | 0.001                                   |
| Method:                                 | Least Squares                           | F-statistic:        | 1.546                                   |
| Date:                                   | Sat, 23 Jan 2021                        | Prob (F-statistic): | 0.214                                   |
| Time:                                   | 21:46:33                                | Log-Likelihood:     | -1805.3                                 |
| No. Observations:                       | 506                                     | AIC:                | 3615.                                   |
| Df Residuals:                           | 504                                     | BIC:                | 3623.                                   |
| Df Model:                               | 1                                       |                     |   |
| Covariance Type:                        | nonrobust                               |                     |   |
| ======================================= | ======================================= |                     | ======================================= |
| ====                                    |   |                     |   |
|   | coef std er                             | r t P> t            | [0.025                                  |
| 0.975]                                  |   |                     |   |

| Intercept                 | 3.7232   | 0.396   | 9.404       | 0.000    | 2.945     |
|---------------------------|----------|---------|-------------|----------|-----------|
| 4.501                     |          |         |             |          |           |
| CHAS                      | -0.6238  | 0.502   | -1.243      | 0.214    | -1.610    |
| 0.362                     | 0.0000   | 0 500   | 4 040       | 0.044    | 4 640     |
| np.power(CHAS, 2)         | -0.6238  | 0.502   | -1.243      | 0.214    | -1.610    |
| np.power(CHAS, 3)         | -0.6238  | 0.502   | -1.243      | 0.214    | -1.610    |
| 0.362                     | 0.0250   | 0.502   | 1.243       | 0.214    | 1.010     |
| ===========               |          | ======= | ========    | ======== | ========= |
| Omnibus:                  |          | 562.698 | Durbin-Wats | on:      | 0.822     |
| <pre>Prob(Omnibus):</pre> |          | 0.000   | Jarque-Bera | (JB):    | 30864.755 |
| Skew:                     |          | 5.205   | Prob(JB):   |          | 0.00      |
| Kurtosis:                 |          | 39.818  | Cond. No.   |          | 3.42e+32  |
| ===========               | ======== | ======= | =========   | ======== | ========= |

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The smallest eigenvalue is 4.39e-63. This might indicate that there are strong multicollinearity problems or that the design matrix is singular.

#### [41]: NOX2.summary()

[41]: <class 'statsmodels.iolib.summary.Summary'>

| ==========        | ======== | =======  | ========      | ======= |         | === |
|-------------------|----------|----------|---------------|---------|---------|-----|
| Dep. Variable:    |          | CRIM     | R-squared:    |         | 0.2     | 292 |
| Model:            |          | OLS      | Adj. R-square | ed:     | 0.2     | 288 |
| Method:           | Least    | Squares  | F-statistic:  |         | 69      | .14 |
| Date:             | Sat, 23  | Jan 2021 | Prob (F-stat: | istic): | 1.94e-  | -37 |
| Time:             | :        | 21:46:36 | Log-Likeliho  | od:     | -1718   | 3.6 |
| No. Observations: |          | 506      | AIC:          |         | 344     | ł5. |
| Df Residuals:     |          | 502      | BIC:          |         | 346     | 32. |
| Df Model:         |          | 3        |               |         |         |     |
| Covariance Type:  | n        | onrobust |               |         |         |     |
|                   |          |          |               |         |         |     |
| ====              |          |          |               |         |         |     |
|                   | coef     | std err  | t             | P> t    | [0.025  |     |
| 0.975]            |          |          |               |         |         |     |
|                   |          |          |               |         |         |     |
|                   |          |          |               |         |         |     |
| Intercept         | 230.1421 | 33.734   | 6.822         | 0.000   | 163.864 |     |
| 296.420           |          |          |               |         |         |     |

| NOX -1264                               | 1.1021 170.860 | -7.398       | 0.000   | -1599.791  |
|---|----------------|--------------|---------|------------|
| -928.414                                |                |              |         |            |
| np.power(NOX, 2) 2223                   | 3.2265 280.659 | 7.921        | 0.000   | 1671.816   |
| 2774.637                                |                |              |         |            |
| np.power(NOX, 3) -1232                  | 2.3894 149.687 | -8.233       | 0.000   | -1526.479  |
| -938.300                                |                |              |         |            |
| ======================================= |                |              | ======= | ========== |
| Omnibus:                                | 612.604        | Durbin-Watso | on:     | 1.159      |
| <pre>Prob(Omnibus):</pre>               | 0.000          | Jarque-Bera  | (JB):   | 52872.508  |
| Skew:                                   | 5.824          | Prob(JB):    |         | 0.00       |
| Kurtosis:                               | 51.705         | Cond. No.    |         | 1.36e+03   |
| ======================================= |                |              | ======= |            |

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 1.36e+03. This might indicate that there are strong multicollinearity or other numerical problems.  $\dots$

# [42]: RM2.summary()

[42]: <class 'statsmodels.iolib.summary.Summary'>

| OLS Regression Results    |          |  |             |                 |          |        |
|---------------------------|----------|--|-------------|-----------------|----------|--------|
| ============              | =======  | =======                                |             | =======         | =======  |        |
| Dep. Variable:            |          | $\mathtt{CRIM}  \mathtt{R-squared}$ :  |             |                 |          | 0.068  |
| Model:                    |          | OLS                                    | Adj. R-squa | red:            |          | 0.063  |
| Method:                   | Leas     | t Squares                              | F-statistic | :               |          | 12.29  |
| Date:                     | Sat, 23  | Jan 2021                               | Prob (F-sta | tistic):        | 9.0      | 06e-08 |
| Time:                     |          | 21:46:42                               | Log-Likelih | ood:            | -:       | 1788.2 |
| No. Observations:         |          | 506                                    | AIC:        |                 |          | 3584.  |
| Df Residuals:             |          | 502                                    | BIC:        |                 |          | 3601.  |
| Df Model:                 |          | 3                                      |             |                 |          |        |
| Covariance Type:          |          | nonrobust                              |             |                 |          |        |
| ===                       | coef     | ====================================== | <br>t       | =======<br>P> t | [0.025   |        |
| 0.975]                    |          | sta err                                |             |                 |          |        |
|                           |          |  |             |                 |          |        |
| Intercept<br>238.545      | 111.9002 | 64.460                                 | 1.736       | 0.083           | -14.744  |        |
| 230.545<br>RM<br>22.759   | -38.7040 | 31.284                                 | -1.237      | 0.217           | -100.167 |        |
| np.power(RM, 2)<br>14.300 | 4.4655   | 5.005                                  | 0.892       | 0.373           | -5.369   |        |

| Kurtosis:                |         | 45.461  | Cond. No.   |       | 5.36e+04  |
|--------------------------|---------|---------|-------------|-------|-----------|
| Skew:                    |         | 5.484   | Prob(JB):   |       | 0.00      |
| Prob(Omnibus):           |         | 0.000   | Jarque-Bera | (JB): | 40548.719 |
| Omnibus:                 |         | 586.445 | Durbin-Wats | on:   | 0.919     |
| ===========              | ======= |         |             |       |           |
| np.power(RM, 3)<br>0.348 | -0.1694 | 0.264   | -0.643      | 0.521 | -0.687    |

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 5.36e+04. This might indicate that there are strong multicollinearity or other numerical problems.

# [43]: AGE2.summary()

[43]: <class 'statsmodels.iolib.summary.Summary'>

| Dep. Variable: Model: Method: Date: Time: No. Observations: Df Residuals: Df Model: Covariance Type:                | OLS Adj. R-s Least Squares F-statis Sat, 23 Jan 2021 Prob (F- 21:46:44 Log-Like 506 AIC: 502 BIC: 3 |         | Prob (F-stat<br>Log-Likeliho<br>AIC: | istic): | 0.17<br>0.16<br>34.8<br>1.76e-2<br>-1758<br>3524<br>3541 | 67<br>86<br>20<br>. 2 |
|---|---|---------|--------------------------------------|---------|--|-----------------------|
| 0.975]  | coef  | std err | t                                    | P> t    | [0.025   |                       |
| Intercept 2.884 AGE 0.641 np.power(AGE, 2) -8.25e-05 np.power(AGE, 3) 9.88e-05 ==================================== |   | 0.186   | 1.471<br>-1.987                      | 0.142   | -0.092<br>-0.014   | ==                    |
| Omnibus:  |   | 577.859 | Durbin-Watso                         | n:      | 1.02   | 27                    |

| <pre>Prob(Omnibus):</pre> | 0.000  | Jarque-Bera (JB): | 39629.126 |
|---------------------------|--------|-------------------|-----------|
| Skew:                     | 5.342  | Prob(JB):         | 0.00      |
| Kurtosis:                 | 45.018 | Cond. No.         | 4.74e+06  |

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 4.74e+06. This might indicate that there are strong multicollinearity or other numerical problems.

# [44]: DIS2.summary()

[44]: <class 'statsmodels.iolib.summary.Summary'>

| OLS Regression Results    |          |           |                  |         |           |  |
|---------------------------|----------|-----------|------------------|---------|-----------|--|
| Dep. Variable:            |          | CRIM      | R-squared:       |         | 0.276     |  |
| Model:                    |          | OLS       | Adj. R-squar     | ed:     | 0.272     |  |
| Method:                   | Least    | Squares   | F-statistic:     |         | 63.74     |  |
| Date:                     | Sat, 23  | Jan 2021  | Prob (F-stat     | istic): | 6.20e-35  |  |
| Time:                     |          | 21:46:51  | Log-Likeliho     | od:     | -1724.4   |  |
| No. Observations:         |          | 506       | AIC:             |         | 3457.     |  |
| Df Residuals:             |          | 502       | BIC:             |         | 3474.     |  |
| Df Model:                 |          | 3         |                  |         |           |  |
| Covariance Type:          |          | nonrobust |                  |         |           |  |
| ====                      | coef     | std err   | t                | P> t    | [0.025    |  |
| 0.975]                    |          | sta err   |                  | P> U    |           |  |
|                           |          |           |                  |         |           |  |
| Intercept<br>34.759       | 29.9496  | 2.448     | 12.235           | 0.000   | 25.140    |  |
| DIS<br>-12.104            | -15.5172 | 1.737     | -8.931           | 0.000   | -18.931   |  |
| np.power(DIS, 2)<br>3.129 | 2.4479   | 0.347     | 7.061            | 0.000   | 1.767     |  |
| np.power(DIS, 3) -0.078   | -0.1185  | 0.020     | -5.802           | 0.000   | -0.159    |  |
| Omnibus:                  | =======  | 577.986   | <br>Durbin-Watso | n:      | 1.133     |  |
| Prob(Omnibus):            |          | 0.000     |                  |         | 42441.952 |  |
| Skew:                     |          |           | Prob(JB):        | •       | 0.00      |  |
| Kurtosis:                 |          | 46.592    | Cond. No.        |         | 2.10e+03  |  |
|                           |          |           |                  | ======= | ========  |  |

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 2.1e+03. This might indicate that there are strong multicollinearity or other numerical problems.

### [45]: RAD2.summary()

# [45]: <class 'statsmodels.iolib.summary.Summary'>

### OLS Regression Results

| Dep. Variable: Model: Method: Date: Time: No. Observations: Df Residuals: Df Model: Covariance Type: | OLS A Least Squares F Sat, 23 Jan 2021 P 21:46:58 L 506 A 502 B |                                     | Log-Likelihood:<br>AIC:<br>BIC:                       |              | 0.396<br>0.392<br>109.5<br>1.47e-54<br>-1678.7<br>3365.<br>3382. |
|--|---|-------------------------------------|---|--------------|--|
| 0.975]   | coef  |                                     |   | P> t         | [0.025   |
| Intercept 3.435  | -0.6050   | 2.057                               | -0.294  | 0.769        | -4.645   |
| RAD<br>2.569   | 0.5122  | 1.047                               | 0.489   | 0.625        | -1.545   |
| np.power(RAD, 2)   | -0.0750   | 0.149                               | -0.504  | 0.615        | -0.368   |
| np.power(RAD, 3)   | 0.0032  |                                     | 0.699   | 0.485        | -0.006   |
| Omnibus: Prob(Omnibus): Skew: Kurtosis:  |   | 657.375<br>0.000<br>6.487<br>61.881 | Durbin-Watso<br>Jarque-Bera<br>Prob(JB):<br>Cond. No. | on:<br>(JB): | 1.349<br>76643.757<br>0.00<br>5.43e+04                           |

#### Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 5.43e+04. This might indicate that there are strong multicollinearity or other numerical problems.

#### [46]: TAX2.summary()

[46]: <class 'statsmodels.iolib.summary.Summary'>

| OLS Regression Results   |                              |  |  |                         |  |  |  |
|--|------------------------------|--|--|-------------------------|--|--|--|
| Dep. Variable: Model: Method: Date: Time: No. Observations: Df Residuals: Df Model: Covariance Type: | Least<br>Sat, 23             | CRIM<br>OLS<br>Squares<br>Jan 2021<br>21:47:03<br>506<br>502<br>3<br>conrobust | R-squared: Adj. R-squar F-statistic: Prob (F-stat Log-Likeliho AIC: BIC: | ed:<br>istic):<br>od:   | 0.365<br>0.361<br>96.10<br>3.69e-49<br>-1691.3<br>3391.<br>3407. |  |  |
| 0.975]   | coef                         | std err  | t  |                         | [0.025   |  |  |
| Intercept 42.307 TAX 0.036 np.power(TAX, 2) 0.001  | 19.0705<br>-0.1524<br>0.0004 | 11.827<br>0.096<br>0.000   | 1.612<br>-1.589<br>1.476   | 0.107<br>0.113<br>0.141 | -4.166<br>-0.341<br>-0.000                                       |  |  |
| np.power(TAX, 3)<br>1.53e-07   |                              | 1.89e-07   | -1.158   | 0.247                   | -5.91e-07  |  |  |
| Omnibus: Prob(Omnibus): Skew: Kurtosis:  |                              | 642.369<br>0.000<br>6.249<br>58.786  | Durbin-Watso Jarque-Bera Prob(JB): Cond. No.                             | (JB):                   | 1.292<br>68905.900<br>0.00<br>6.16e+09                           |  |  |

#### Warnings:

<sup>[1]</sup> Standard Errors assume that the covariance matrix of the errors is correctly specified.

<sup>[2]</sup> The condition number is large, 6.16e+09. This might indicate that there are strong multicollinearity or other numerical problems.

# [47]: PTRATIO2.summary()

[47]: <class 'statsmodels.iolib.summary.Summary'>

### OLS Regression Results

| ULS Regression Results   |         |                                  |                              |   |                         |  |  |
|--|---------|----------------------------------|------------------------------|---|-------------------------|--|--|
| Dep. Variable: Model: Method: Date: Time: No. Observations: Df Residuals: Df Model: Covariance Type: | 21:     |                                  | Adj<br>F-st<br>Prob          |   |                         | 0.112<br>0.107<br>21.21<br>5.99e-13<br>-1775.9<br>3560.<br>3577. |  |
| 0.975]   | coef    | std                              | err                          | t   | P> t                    | [0.025   |  |
| Intercept 782.135 PTRATIO -27.487 np.power(PTRATIO, 2) 7.764   |         | 27<br>1                          | . 823<br>. 649<br>. 609      | 3.023<br>-2.959<br>2.862                          | 0.003<br>0.003<br>0.004 | 1.444  |  |
| np.power(PTRATIO, 3) -0.023  |         |                                  | .031                         | -2.724  | 0.007                   | -0.145   |  |
| Omnibus: Prob(Omnibus): Skew: Kurtosis:  | 57<br>4 | 2.978<br>0.000<br>5.303<br>3.050 | Durk<br>Jaro<br>Prok<br>Cond | oin-Watson:<br>que-Bera (JB):<br>o(JB):<br>d. No. |                         | 0.949<br>36189.609<br>0.00<br>3.02e+06                           |  |

#### Warnings:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 3.02e+06. This might indicate that there are strong multicollinearity or other numerical problems.

# [48]: B2.summary()

[48]: <class 'statsmodels.iolib.summary.Summary'>

#### OLS Regression Results

| Dep. Variable: Model: Method: Date: Time: No. Observations Df Residuals: Df Model: Covariance Type | Sat, 2     | CRIM<br>OLS<br>ast Squares<br>23 Jan 2021<br>21:47:08<br>506<br>502<br>3<br>nonrobust | <pre>Prob (F-statistic):</pre> |                 | 0.144<br>0.139<br>28.14<br>7.83e-17<br>-1766.8<br>3542.<br>3558. |        |
|--|------------|---|--------------------------------|-----------------|--|--------|
| 0.975]   | coef       | std err   | t                              | P> t            | [0.025   | =====  |
|  |            |   |                                |                 |  |        |
| Intercept 22.531   | 17.9898    | 2.312   | 7.782                          | 0.000           | 13.448   |        |
| B<br>0.026   | -0.0845    | 0.056   | -1.497                         | 0.135           | -0.196   |        |
| np.power(B, 2)   | 0.0002     | 0.000   | 0.760                          | 0.447           | -0.000   |        |
| np.power(B, 3) - 5.7e-07   | -2.895e-07 | 4.38e-07  | -0.661                         | 0.509           | -1.15e-06  |        |
| Omnibus:   |            | 589.534   | <br>Durbin-Wat                 | =======<br>son: | =======  | 0.990  |
| Prob(Omnibus):   |            | 0.000   | Jarque-Bera (JB):              |                 | 42752.655  |        |
| Skew:  |            | 5.512   | Prob(JB):                      |                 | 0.00   |        |
| Kurtosis:  |            | 46.661  | Cond. No.                      |                 | 3.   | 59e+08 |

# Warnings:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 3.59e+08. This might indicate that there are strong multicollinearity or other numerical problems.

#### [49]: LSTAT2.summary()

[49]: <class 'statsmodels.iolib.summary.Summary'>

| Dep. Variable: | CRIM | R-squared:      | 0.214 |
|----------------|------|-----------------|-------|
| Model:         | OLS  | Adi. R-squared: | 0.210 |

| Method: Date: Time: No. Observations: Df Residuals: Df Model: Covariance Type:                | Sat, 23 Ja<br>23 | an 2021                  | F-statistic: Prob (F-stat Log-Likeliho AIC: BIC: | istic):        | 45.67<br>4.13e-26<br>-1745.0<br>3498.<br>3515. |
|---|------------------|--------------------------|--|----------------|--|
| 0.975]  | coef             | std er                   | r t  | P> t           | [0.025   |
| Intercept 5.076 LSTAT 0.502   | 1.0836           | 2.03                     |  | 0.594<br>0.375 | -2.909<br>-1.328                               |
| <pre>np.power(LSTAT, 2) 0.112 np.power(LSTAT, 3) 0.000 ================================</pre> | 0.0530           | 0.03                     |  | 0.079          | -0.006<br>-0.002                               |
| Omnibus: Prob(Omnibus): Skew: Kurtosis:   |                  | 0.000<br>5.717<br>51.941 | Prob(JB):<br>Cond. No.                           | (JB):          | 1.239<br>53255.699<br>0.00<br>5.20e+04         |

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 5.2e+04. This might indicate that there are strong multicollinearity or other numerical problems.

#### [50]: MDEV2.summary()

[50]: <class 'statsmodels.iolib.summary.Summary'>

| Dep. Variable:    | CRIM             | R-squared:                     | 0.416    |
|-------------------|------------------|--------------------------------|----------|
| Model:            | OLS              | Adj. R-squared:                | 0.413    |
| Method:           | Least Squares    | F-statistic:                   | 119.2    |
| Date:             | Sat, 23 Jan 2021 | <pre>Prob (F-statistic):</pre> | 2.65e-58 |
| Time:             | 21:47:15         | Log-Likelihood:                | -1670.0  |
| No. Observations: | 506              | AIC:                           | 3348.    |

| Df Residuals: 502 | BIC: | 3365. |
|-------------------|------|-------|
|-------------------|------|-------|

Df Model: 3
Covariance Type: nonrobust

| covariance Type.                        |           | oni obust                           |   |       |  |
|---|-----------|-------------------------------------|---|-------|--|
| 0.975]                                  | coef      | std err                             | t   | P> t  | [0.025                                 |
| <br>Intercept<br>59.553                 | 52.9386   | 3.366                               | 15.725                                      | 0.000 | 46.325                                 |
| MDEV<br>-4.222                          | -5.0774   | 0.435                               | -11.668                                     | 0.000 | -5.932                                 |
| np.power(MDEV, 2                        | 0.1551    | 0.017                               | 8.995                                       | 0.000 | 0.121                                  |
| np.power(MDEV, 3                        | ) -0.0015 | 0.000                               | -7.277                                      | 0.000 | -0.002                                 |
| Omnibus: Prob(Omnibus): Skew: Kurtosis: |           | 568.100<br>0.000<br>5.084<br>49.259 | Durbin-Wats Jarque-Bera Prob(JB): Cond. No. |       | 1.360<br>47296.533<br>0.00<br>3.67e+05 |

#### Warnings:

<sup>[1]</sup> Standard Errors assume that the covariance matrix of the errors is correctly specified.

<sup>[2]</sup> The condition number is large, 3.67e+05. This might indicate that there are strong multicollinearity or other numerical problems.