Quantitative Macroeconomics Homework 5

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1 Exercise 1. Factor Input Misallocation

1.1

We simulated 10,000,000 observations with the use of $numpy.random.multivariate\_normal$ function. In our exercise the mean of k is equal to 1. Hence we know that $E(k) = e^{mean-0.5\cdot 1} = 1$, so the mean of ln(k) will be equal to -0.5.
Then we plot the joint density in logs:

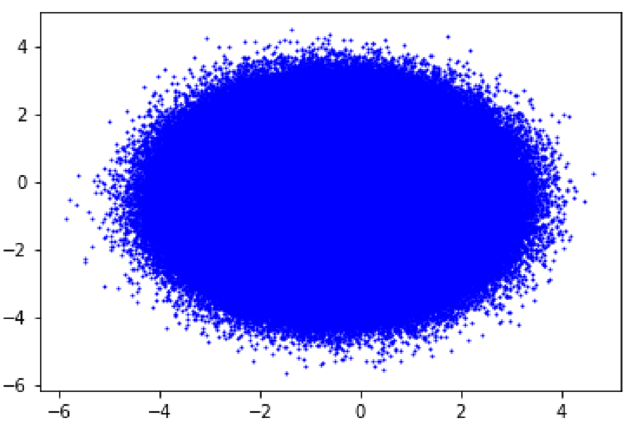

Figure 1: Joint density in logs

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And in levels:

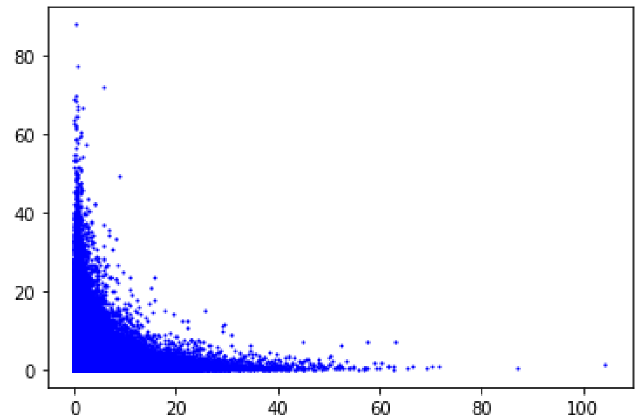

Figure 2: Joint density in levels

1.2

Now we compute firms output y_i for each of our observations. In our sample data the average $y_i = 0.787$ and the total output is $Y = 7,866M$.

1.3

We computed optimal capital as if the most productive firms receive the biggest amount of capital. Then the averrage output is $y_i = 1.00008$ and the total output is $Y = 10,001M$.

1.4

Now we compare the optimal allocations against the data. The sum of all k_i (K) is obviously the same, but capital is allocated differently.

1.5

Thanks to the reallocation we gave following output gains:

Total gain	Percentage gain	Gain per capita
$2,134M$	27.13%	0.213

Table 1: Production gain for zero correlation

1.6

1.6.1 For correlation equal to 0.5:

The plot with the joint density in logs:

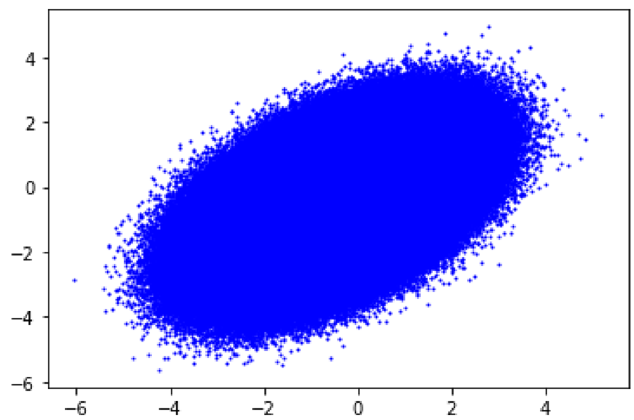

Figure 3: Joint density in logs for correlation=0.5

And in levels:

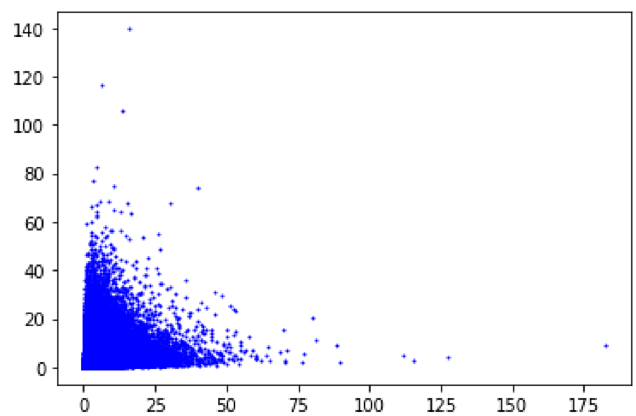

Figure 4: Joint density in levels for correlation=0.5

Firms total output is: 8,863M.
Firms total optimal output is: 9,993M.

And the production gains are:

Total gain	Percentage gain	Gain per capita
$1,129M$	12.74%	0.113

Table 2: Production gain for correlation=0.5

1.6.2 For correlation equal to -0.5:

The plot with the joint density in logs:

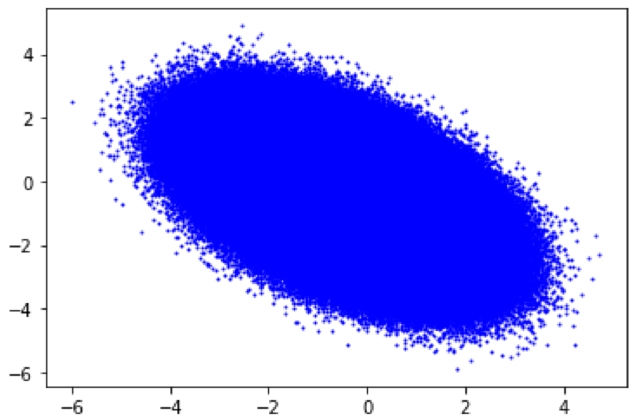

Figure 5: Joint density in logs for correlation=-0.5

And in levels:

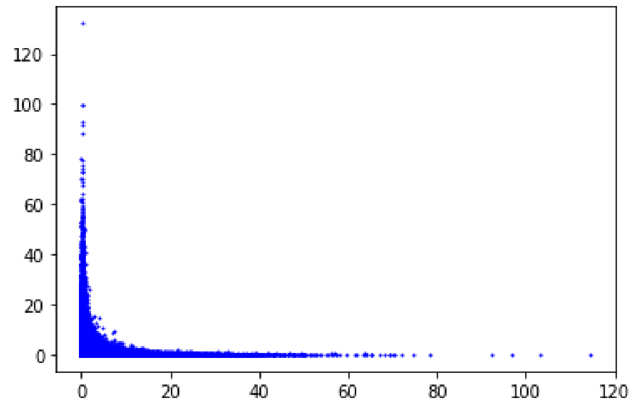

Figure 6: Joint density in levels for correlation=-0.5

Firms total output is: $6,977M$
Firms total optimal output is: $9,999M$

In this case, thanks to reallocation we gained almost 50% bigger output. Production gains are as follows:

Total gain	Percentage gain	Gain per capita
$3,022M$	43.32%	0.302

Table 3: Production gain for correlation=-0.5

2 Exercise 2: Higher Span of Control

The plots for each correlation levels are the same as in Exercise 1. The only thing that is changing is γ, now equal to 0.8.

2.1 Zero correlation

Firms total output is: 8,523M.
Firms total optimal output is: 10.0M.

And the production gains are:

Total gain	Percentage gain	Gain per capita
$,1478M$	17.35%	0.148

Table 4: Production gain - zero correlation

2.2 For correlation equal to 0.5

Firms total output is: 9,229M.
Firms total optimal output is: 9,998M.

In this case gains from reallocation are not that significant and the production are as follows:

Total gain	Percentage gain	Gain per capita
$0,768M$	8.33%	0.077

Table 5: Production gain for correlation=0.5

2.3 For correlation equal to -0.5

Firms total output is: 7,866M.
Firms total optimal output is: 9,999M.

And the production gains are:

Total gain	Percentage gain	Gain per capita
$2,134M$	27.12%	0.213

Table 6: Production gain for correlation=-0.5

3 Exercise 3: From Complete Distributions to Random Samples