Quantitative Macroeconomics
Part 2
Project 2

Sebastian A. Roy*

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1 Problem a

In this problem we solve the household problem analytically. The assumptions that are taken here are $T = \infty$ that cash on hand is defined as $X_t = A_t + Y_t$. Then the sequential problem is rewritten in the recursive way:
$W(X) = \max_C\{u(C) + \beta E_t W(X')\}$
s.t.
$X' = (X - C) * (1 + r) + Y'$
$S = X - C >= 0$
$Y = P\epsilon$

1.1

We define the de-trended variables $x = \frac{X}{P}$, $c = \frac{C}{P}$ and $y = \frac{Y}{P} = \epsilon$. To show that the dynamic budget constraint can be rewritten as:

$x' = (x - c)\frac{1+r}{1+g} + \epsilon'$

we begin with using the original dynamic budget constrant defined as $X' = (X - C)(1 + r) + Y'$.
Now we divide both sides by P and get $\frac{X'}{P} = (x - c)(1 + r) + \frac{Y'}{P}$.
We also know that $Y = P\epsilon$ and therefore $Y' = P'\epsilon' = P\epsilon'(1 + g)$ and that leads to $P' = P(1 + g)$.
After dividing both sides by $(1 + g)$ we get that $\frac{X'}{P(1+g)} = (x - c)\frac{1+r}{1+g} + \frac{Y'}{P(1+g)}$ that after transformation leads to $x' = (x - c)\frac{1+r}{1+g} + \epsilon'$, which is exactly the rewritten version of the dynamic budget constraint that we are looking for.

*In collaboration with Jakub Bławat, ZuzannaBrzóśka-Klimek, and Szymon Wieczorek

1.2

This time we are supposed to transform the recursive household problem into a following form:
$V(x) = \max_c\{u(c) + \beta(1+g)^{1-\theta}E_tV(x')\}$
s.t.
$x' = (x-c)\frac{1+r}{1+g} + \epsilon'$
$s = x - c >= 0$

Using the transformation made in **1.1**, we can reformulate the recursive problem in the following way:
$W(X) = \max_C\{u(C) + \beta E_tW(X')\}$
s.t.
$x' = (x-c)\frac{1+r}{1+g} + \epsilon'$
$S = X - C >= 0$, which after dividing by P becomes:
$s = x - c >= 0$

If the objective function $W(X)$ holds for the following set of constraints, we should be able to scale all the arguments by $\frac{1}{P}$ at the same time. Then we get:
$W(x) = \max_c\{u(c) + \beta E_tW(\frac{X'}{P})\}$
$= \max_c\{u(c) + \beta E_tW(\frac{X'(1+g)}{P'})\}$
$= \max_c\{u(c) + \beta(1+g)^{1-\theta}E_tW(x')\}$
subject to:
$x' = (x-c)\frac{1+r}{1+g} + \epsilon'$
$s = x - c >= 0$

1.3

Here we have to explain several questions about the transformations we performed in **1.1** and **1.2** and their usefulness. The following points have answers to each of the corresponding questions in the problem.

1. These transformations are useful, because we get rid of the influence of the population change (variables are defined in the per capita terms) as well as of the economic growth. That simplifies the recursive solution of the problem.

2. Utility function has a form of constant relative risk aversion (CRRA) and it allows to represent households' risk aversion in the decision making process corresponding to choice of consumption and savings.

3. The formula $\beta(1+g^{-\theta}(1+r) < 1$ is a part of Euler equation that shows the relation between current consumption and consumption in the next period. This assumption assures that households wouldn't postpone their consumption to the infinite period and prefer to consume now than in the future.

4. Assume contrary that we allow s to be lower than 0. Then we can end up in one of the two situations: either cash on hand is negative and we and up with $c_{T>t*} = 0$ from some period t* or households end up having infinite debt.

2 Problem b

2.1

The code starts with the definitions of parameters and settings such as number of periods. Then the grid is being set. In the next step income shocks are generated with the use of functions from CompEcon Toolbox for Matlab. After that the policy function is being solved. Firstly the initial guess for consumption policy is set and then the iteration starts. After that we update the initial guess. And the end we check the convergence and we stop when we reach initially defined error (to1 in the code). Then the code generates the plot with consumption policy. After that we start the simulation to obtain evaluation errors (that is maximum and mean Euler equation errors). Finally the code plots consumption, cash-on-hand and Euler equation error over-time.

2.2

Before the change of theta our function converged in 64 iterations. The consumption policy was then:

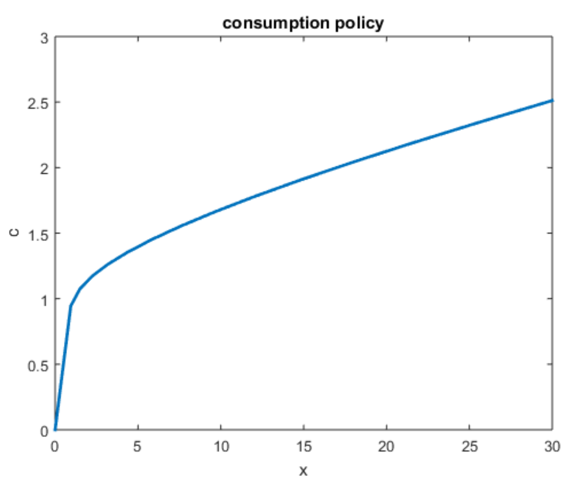

Figure 1: Consumption policy function for $\theta = 1$

After the increase of theta it took 73 iterations and the consumption policy function looked like:

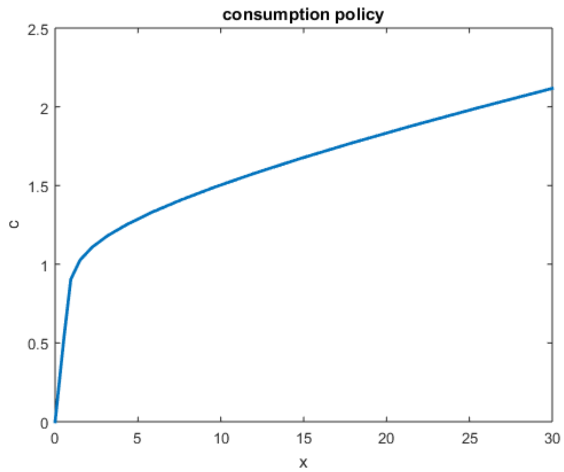

Figure 2: Consumption policy function for $\theta = 4$

As we see on the graphs above values of the consumption policy function slightly decreased after the change of theta parameter from 1 to 4, but the shape of the function is still the same.

After the changing the variance of the income shocks the policy function converged in 67 iterations. The maximum Euler equation error increased from around 0.06 to 0.346, the mean also increased from 0.0074 to 0.0149. Borrowing constraint binds in slightly smaller number of cases.

2.3

Before the change, the mean Euler equation error was equal to 0.0074. It took 64 iterations and 31 seconds to converge. The plot with Euler equation looked like:

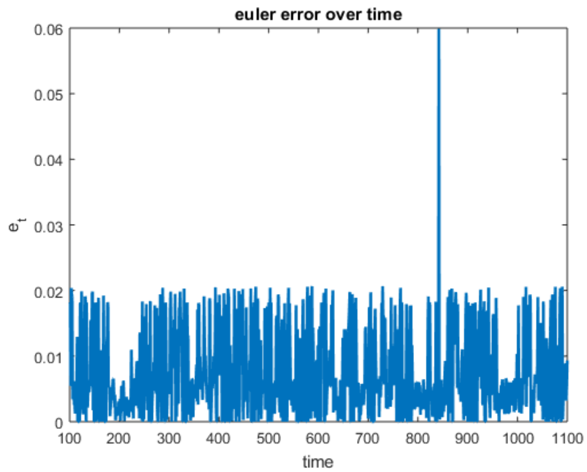

Figure 3: Euler equation error for grid=20

After the change of the grid, the mean Euler equation error increased to almost 0.0117. It took 57 iterations and 71 seconds to converge. So the number of iterations was smaller then before , but overall it was more time consuming. The plot with Euler equation looked like:

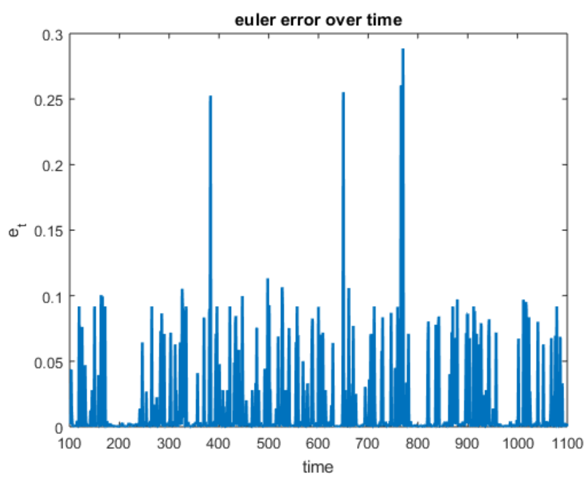

Figure 4: Euler equation error for grid=50

After we increase θ to 4 the mean Euler equation error increased even more to almost 0.02342. The plot is then:

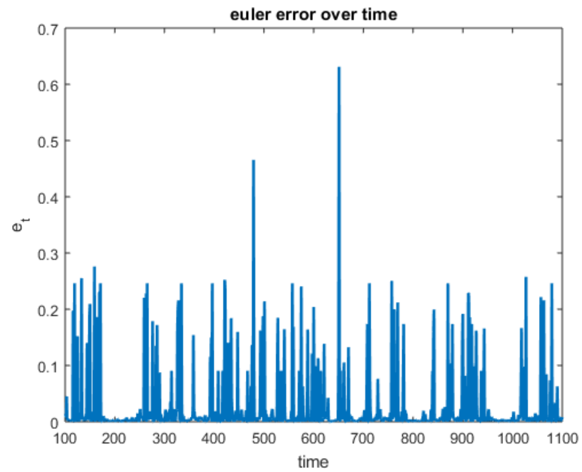

Figure 5: Euler equation error for grid=50 with $\theta = 4$

3 Problem c

4 Problem d

5 Problem e

6 Problem f

Pierre-Olivier Gourinchas and Jonathan A. Parker in their *Consumption over the Life Cycle* (from now on, CoLC) develop an outstandingly complex structural OLG-class model with income volatility, and painstakingly calibrate its parameters with cutting-edge econometrics methods. We are going to discuss the paper in following points:

1. Aims and overview of the model
2. Model structure
3. Calibration and estimation
4. Key findings

6.1 Aims and overview of the model

CoLC starting point is following: arguably, households do not actually smooth their consumption which, contrarily, depends heavily on income dynamics. It is also pointed out that in presence of non-deterministic income, standard log-linearised Euler equation no longer serves as an efficient tool for description of households consumption decisions. In order to study those issues, CoLC build a structural OLG model which includes bequest and borrowing constraint. Then they calibrate it - the income part with PSID data, the expenditure part with CEX (Consumer Expenditure Survey).

6.2 Model structure

All in all, Bellmann equation for the CoLC problem is given in a following way:

$$V_\tau(X_\tau, P_\tau, Z_\tau) = \max \mathbb{E}_\tau \Big(\sum_{t=\tau}^{T} \beta^{t-\tau} v(Z_t) \frac{C_t^{1-\rho}}{1-\rho} + \beta^{T+1-\tau} \kappa v(Z_{T+1})(X_{T+1} + hP_{T+1})^{1-\rho} \Big) \tag{1}$$

Under conditions:

$$X_{t+1} = R(X_t - C_t) + Y_{t+1} \tag{2}$$

$$X_{T+1} \geq 0 \tag{3}$$

$$Y_t = P_t U_t \tag{4}$$

$$P_t = G_t P_{t-1} N_t \tag{5}$$

Equations 4 and 5 provide insightful decomposition of income and generate income volatility. Transitory shock U_t is a log-normal random variable.

6.3 Calibration and estimation

As noted before, calibration and estimation in CoLC is a two-step process. First, income and expenditure profiles are parametrised with CEX data for households of different occupation and education status. Then income volatility is measured by means of the PSID data. Then, after parametrisaion, consumption is simulated with Simulated Method of Moments (SMM) in order to make it age-dependent only.

Such a procedure is needed to ensure that individual characteristics might vary across lifetime. For example, as CoLC argue, marginal utility of consumption positively correlates with family size, while consumption growth may shrink when children grow up and leave their family home.

6.4 Key findings

Starting from the most theoretically significant result, CoLC argue that - under income volatility - it is crucial to distinguish between two saving motives - precautionary and retirement/bequest. This is so because up to 70% of young agents wealth comes from precautionary savings (and were it not for income risk, those agents would actually smooth their consumption by borrowing , that is, transferring their future consumption into present), while after reaching age of 40 they turn into retirement- and bequest-driven saving. CoLC describe it as a transition from a *buffer-stock* type of a household into a *certainty-equivalent life cycle hypothesis* type.

CoLC succeeds in replicating standard hump-shaped lifecycle consumption curve, which is characterised by a positive correlation between consumption and income in early years of life.

Finally, CoLC provides robust estimates of the household preferences parameters, including:

- 4-4.5% discount rate
- 0.5-1.4 RRA coefficient
- 607% marginal propensity to consume from liquid assets after retirement