Thermal simulation

olhartin@asu.edu

Thermal simulation

- There are a range of types of thermal problems

  1) steady state or time dependent

     steady state is the solution it would settle to after infinite time

  2) heat transfer formula or heat equation

  3) 1, 2 or 3 dimensions

     very often one dimension dominates so you can approximate and make the calculation much shorter

Heat Equation

- The heat equation describes how temperature diffuses through a material over time
- Heat equation has the form

  where K is the coefficient of thermal diffusivity

$$\kappa\nabla^2 T = \frac{dT}{dt}$$

- A system obeys this equation if heat is not added or subtracted at the boundaries, and is neither created or destroyed within the system
- T is temperature and is a function of space and time, in 1 dimension this might be T(x,t)
- Under the condition – we can use separation of variables we can separate T(x,t) = X(x) T(t)
  - the temperature dependence in x is only described by X(x) and the temperature dependence with time is only described by T(t)
  - Now we can solve for X(x) and T(t) independently

Separable Solution

- $T(0,0)=T_1$, $T(0,L)=T_2$

$$\kappa\nabla^2 T = \frac{dT}{dt}$$

$$\kappa\nabla^2 X\mathrm{T} = \mathrm{T}\kappa\nabla^2 X = \frac{dX\mathrm{T}}{dt} = X\frac{d\mathrm{T}}{dt}$$

$$\kappa\frac{X''}{X} = \frac{T'}{T} = -\alpha$$

$$X'' = \frac{-\alpha}{\kappa}X, \quad X(x) = A\sin\left(\sqrt{\frac{\alpha}{\kappa}}x\right) + B\cos\left(\sqrt{\frac{\alpha}{\kappa}}x\right)$$

$$T' = -\alpha T, \quad T(t) = Ce^{-\alpha t} + De^{\alpha t}$$

Separable Solution

- At time t=0
- $T(0,0)=T_1$, $X(0,L)=T_2$
- $X(0)=BT_1$, $X(L)=CT_2$
- The time function describes how this
- changes with time

$$X(0) = B\cos\left(\sqrt{\frac{\alpha}{\kappa}}0\right) = T_1, \quad B = T_1$$

$$X(L) = A\sin\left(\sqrt{\frac{\alpha}{\kappa}}L\right) + T_1\cos\left(\sqrt{\frac{\alpha}{\kappa}}L\right) = T_2$$

$$A = \frac{T_2 - T_1\cos\left(\sqrt{\frac{\alpha}{\kappa}}L\right)}{\sin\left(\sqrt{\frac{\alpha}{\kappa}}L\right)}$$

Heat Transfer Formula

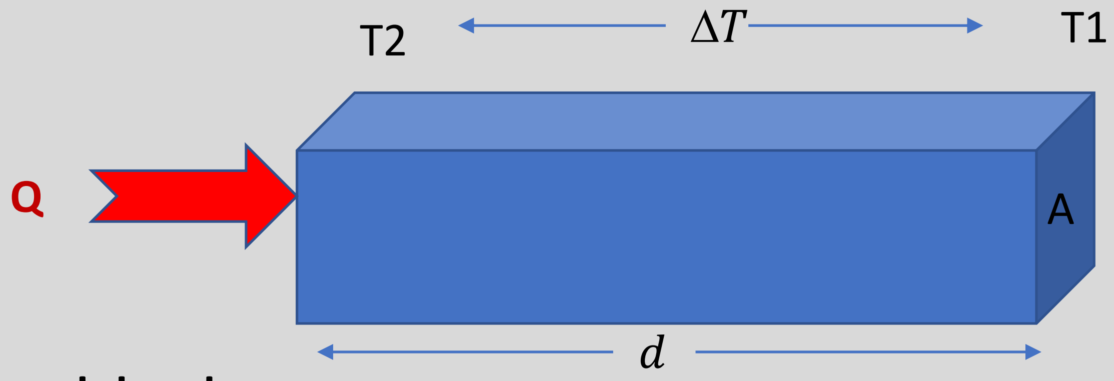
- Those assumptions don't always hold, in some cases we have heat added to the system or dissipated to the environment
- We need an equation that includes terms representing these interactions with the environment

Consider

$$\kappa\nabla^2 T + \frac{\kappa}{\lambda} g_v + \frac{\kappa}{\lambda}\nabla\lambda\nabla T = \frac{dT}{dt}$$

Heat flow

- Imagine a block of homogeneous material of area - A
- κ - thermal conductivity (W/mK)
- Q - rate of heat flow
- d – thickness of the block
- ΔT – temperature difference across the block



The rate that heat is conducted through a material is proportional to the area normal to the heat flow, and the temperature gradient along the heat flow path

$$Q = \kappa A \Delta T / d$$

Thermal Resistance

- R - Thermal resistance

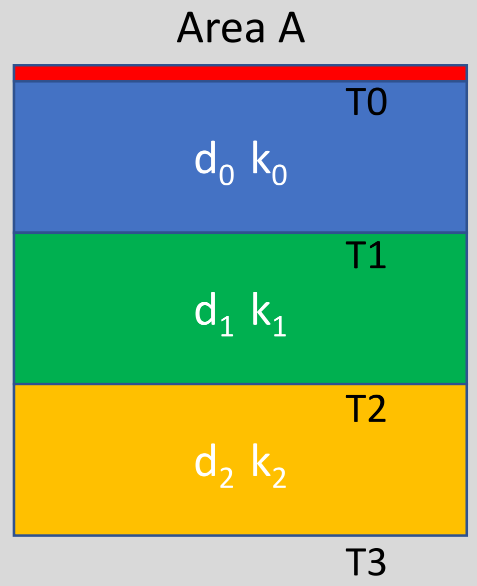
$R = A\,\Delta T/Q$

So $R = d/k$

$\theta = R_{material} + R_{contact}$

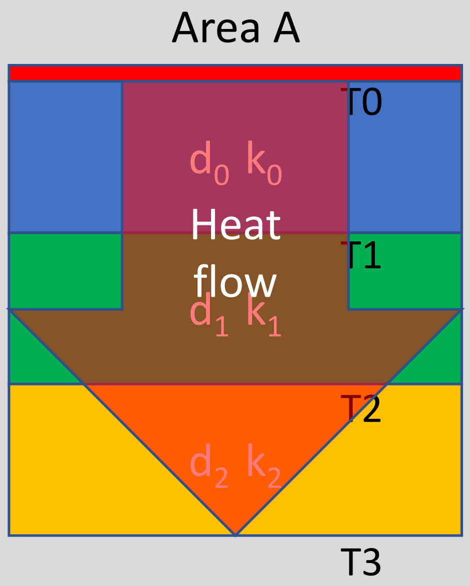
We can solve for the temperature of each interface between these layers

- Consider a stack of three materials of thickesses d_i and thermal conductivity k_i
- Assume that the heat comes from components on the top surface which dissipate heat
- Assume for simplicity that this component covers area A at the top



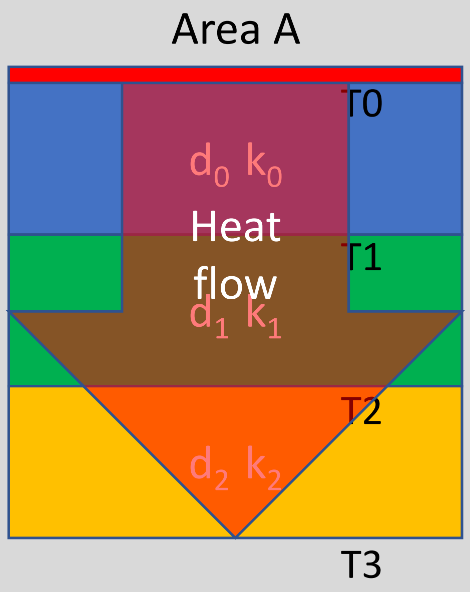
We can solve for the temperature of each interface between these layers

- Consider a stack of three materials of thicknesses d_i and thermal conductivity k_i
- Assume all heat flows uniformly (because it is homogeneous) through the material from top to bottom, no heat flows out the side
- This is a Dirchelet boundary condition at top and bottom and VonNewman on the sides --- check this



We can solve for the temperature of each interface between these layers

- Consider pseudo code that describes this phenomenon



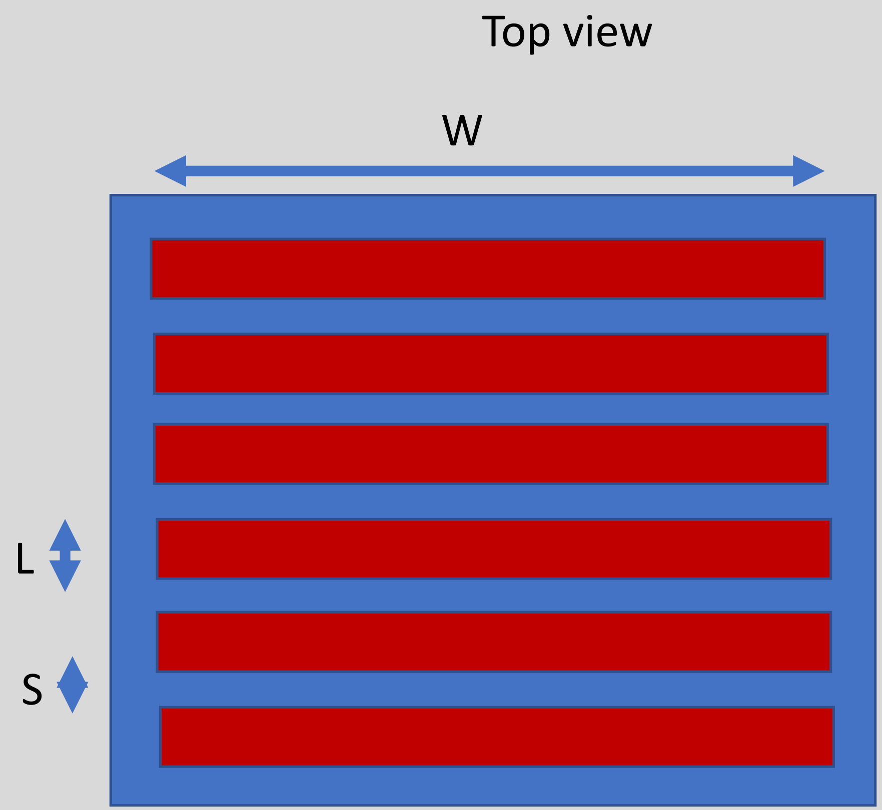
for i=2 to 0 by -1 // go through each layer

$R_i = d_i/(k_i A)$ // calculate thermal resistivity of each layer i

$T_i = T_{i+1} + QR_i$ // Calculate the temperature of each layer interface

Use this simple model as an approximation

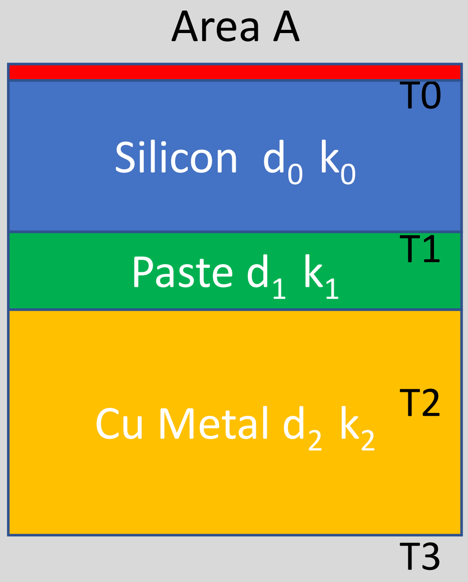
- Consider a MOSFET power transistor driving an external load which requires current
- In   EEE 202 we learned that the output impedance of this transistor should be the conjugate of the load impedance for maximum power transfer to the load
  - The consequence is that half the power is dissipated in the part
- Consider a MOSFET made with N=6 fingers, W= 100um wide and each L = 5um
- Vdd = 3 Volts, and the output impedance of each finger is R = 300 ohm
- Pdiss = N(V_{dd}^2/R)
- Area = (W+2S)*((L+S)*N+S)



Apply this for specific device design

- Layer 0 is Silicon with thickness of 5 microns, and thermal conductivity k_0 =
- Layer 1 is a thermal paste 25 microns thick with a thermal conductivity of k_1 =
- Layer 2 is the metal flag of thickness 1000 um and thermal conductivity k_2 =
- T3 = 90 deg C
- We want to keep the device at less than 150 deg C
- This creates a design limitation where

  T0 < 150 deg C



Python code

- Make sure you use consistent units and pay careful attention to dimensional analysis
- Build a table of temperatures as a function of Vdd
- We anticipate threshold and transconductance to be heavily dependent on temperature