Measures of Central Tendency

MD. PEAR HOSSAIN

LECTURER OF STATISTICS

BSMRSTU, BANGLADESH

Measure of central tendency

Central tendency: In every data set all observations have a tendency of tending to central value. This phenomenon is known as central tendency.

Measure of central tendency: A measure of central tendency or an average is a single representative expression of complicated and complex data.

Common measures of central tendency:

i.   Mean (Arithmetic, Geometric and Harmonic)

ii.   Median

iii.   Mode.

In summery

i.   **Mean**: Average value of the given data.

ii.   **Median**: The middle most value of the given data set that divide the entire data set into two equal parts.

iii.   **Mode**: Observation that occurs maximum times.

The **sample mean** $\bar{x}$ of observations $x_1, x_2, \ldots, x_n$ is given by

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

The numerator of $\bar{x}$ can be written more informally as Σx_i, where the summation is over all sample observations.

The Mean

Example of Mean

Example: The monthly income (in 000' Tk's) of 10 persons working in a firm is as follows:

14.87   14.93   15.02   14.46   14.75   14.92   15.72
15.16   14.68   14.89

Find average monthly income.

Solution: We have

$$n = 10, \sum_{i=1}^{10} x_i = 149.40$$

Hence,

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{149.40}{10} = 14.94$$

Therefore, average monthly income 14.94 thousands taka.

Merits of Mean

❑ It is easy to understand

❑ It is easy to compute

❑ It is based on all the observations

❑ It is rigidly defined

❑ It is capable of further algebraic treatment

❑ It is less affected by sampling fluctuation.

Limitations of Mean

➢ It is affected by extreme values

➢ It cannot be computed in case of open-ended class interval of a frequency distribution

➢ It is not a good measure of central tendency in case of highly skewed distribution

➢ It cannot be calculated for qualitative data.

➢ It cannot be found graphically.

Example

A contractor employs three types of worker say male, female and children. To male worker he pays Tk. 125 per day, to female worker Tk. 100 per day and to child worker Tk. 75 per day. The number of male, female and child workers hired by the contractor are 15, 25 and 35 respectively. What is the average wage per day paid by the contractor?

Daily wage example

The simple arithmetic mean of the wage is

$$\bar{x} = \frac{125 + 100 + 75}{3} = 100 \ \text{Tk. per day.}$$

It is incorrect!!!

Why???

When to use Weighted Mean?

One of the limitations of the arithmetic mean is that it gives equal importance to all the observations. But there are cases where the relative importance of all the different observations is not the same. When that is so, we compute weighted mean.

What Weighted Mean is?

Suppose $x_1, x_2, ..., x_k$ are k values of a variable X whose relative importance are measured by the weights $w_1, w_2, ..., w_k$ respectively, then the weighted arithmetic mean is defined as

$$\bar{x}_w = \frac{\sum_{i=1}^{k} w_i x_i}{\sum_{i=1}^{k} w_i}$$

Daily wage example

Worker	Wage per day : (x_i)	No. of workers (w_i)	$w_i x_i$
Male	125	15	
Female	100	25	
Children	75	35	
Total			

$$\bar{x}_w = \frac{\sum_{i=1}^{k} w_i x_i}{\sum_{i=1}^{k} w_i} =$$

Median

The median is the measure of central tendency which appears in the "middle" of an ordered sequence of values. That is, half of the observations in a set of data are lower than it and half of the observations are greater than it.

Calculation of Median

Step 1: Arrange all the observations in ascending or descending order of magnitude.

Step 2: Find $\frac{1}{2}(n+1)$ where n is number of observation.

Step 3: If $\frac{1}{2}(n+1)$ is integer then $\frac{1}{2}(n+1)$th observation median.

or

Step 4: If $\frac{1}{2}(n+1)$ is not integer then consider the integer part and the next integer number.

Step 5: Take an average of the observation correspond to position of integer numbers.

Example of Median

The marks obtained by 12 students out of 50 are:

25, 20, 23, 32, 40, 27,30, 25, 20, 10, 15, 41

Calculate Median.