Introduction to Probability

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A. Introduction

- The term **probability** refers to the study of randomness and uncertainty.
- In any situation in which one of a number of possible outcomes may occur, the discipline of probability provides methods for quantifying the chances, or likelihoods, associated with the various outcomes.
- The language of probability is constantly used in an informal manner in both written and spoken contexts.
- Examples include such statements as
  1. "It is likely that Mamuns average GPA will increase by this semester,"
  2. "There is a 50–50 chance that the CSE department will win the game,"
  3. "There will probably be at least one section of that course offered next year," and
  4. "It is expected that at least 20,000 cricket math tickets will be sold."

B. SOME BASIC CONCEPTS

Random experiment:

Random experiment is one whose results depend on chance that is the result cannot be predicted.

- Tossing of coins, throwing of dice are some examples of random experiments.

Sample space:

Sample space is the collection of all possible outcomes of a random experiment is denoted by S.

- For example, when a coin is tossed, the sample space is $S = \{H, T\}$. H and T are the sample points of the sample space S.

Mutually exclusive events:

Two or more events are said to be mutually exclusive, when the occurrence of any one event excludes the occurrence of the other event. Mutually exclusive events cannot occur simultaneously.

- For example when a coin is tossed, either the head or the tail will come up. Therefore the occurrence of the head completely excludes the occurrence of the tail. Thus getting head or tail in tossing of a coin is a mutually exclusive event.

Equally likely events:

Equally likely events are events that have the same theoretical probability (or likelihood) of occurring.

- Getting a 1, 2 or 3 on the toss of a die and getting a 4, 5 or 6 on the toss of a die are equally likely events, since the probabilities of each event are equal.

Exhaustive events:

Events are said to be exhaustive when their totality includes all the possible outcomes of a random experiment.

- For example, while throwing a die, the possible outcomes are {1, 2, 3, 4, 5, 6} and hence the number of cases is 6.

C. Quantifying probability:

1. **The classical definition of probability :** If a random experiments results in 'n' exhaustive, mutually exclusive and equally likely cases, out of which 'm' are favorable to the occurrence of an event A, then the ratio $\frac{m}{n}$ is called the probability of occurrence of event A, denoted by $P(A)$, is given by

$$P(A) = \frac{m}{n} = \frac{\text{Number of cases/outcomes favourable to the event A}}{\text{Total number of exhaustive cases/oucomes}}$$

This is known as ***classical*** or ***priori probability***.

1. **Empirical/Posterior/Statistical frequency definition:** If an event occurs $\boldsymbol{m}$ times out of $\boldsymbol{n}$, its relative frequency is $\boldsymbol{m/n}$.
   - In the limiting case, when $\boldsymbol{n}$ becomes sufficiently large it corresponds to a number which is called the probability of that event.
   - In symbol, $P(A) = \underset{n \to \infty}{limit} \frac{m}{n}$.

D. The Axioms of Probability

1. The probability of any event ranges from zero to one. i.e.
   $$0 \le P(A) \le 1.$$
2. The probability of the entire space is 1. i.e.
   $$P(S) = 1.$$
3. If $A_1, A_2, \dots$ is a sequence of mutually exclusive events in S, then
   $$P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$$

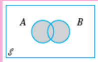
E. Rules of probability

1. Addition Rules:

Case I: If two events A and B are mutually exclusive, the probability of the occurrence of either A or B is the sum of individual probabilities of A and B. i.e.

$P(A \cup B) = P(A) + P(B) \quad [Since\ P(A \cap B) = 0]$

Case II: If two events A and B are not-mutually exclusive, the probability of the event that either A **or** B or both occur is given as



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2. Rule Complement events:

For an event A and its complement A', $P(A') = 1 - P(A)$. If $P(pass) = 0.85$ then $P(fail) = 0.15$.

3. Conditional Probability rule:

If two events A and B are dependent, then the conditional probability of B given A is

$$P(B|A) = \frac{P(A \cap B)}{P(A)}; \quad P(A) \neq 0$$

Similarly the conditional probability of A given B is given as

$$P(A|B) = \frac{P(A \cap B)}{P(B)}; \quad P(B) \neq 0$$

4. Rule of Independent events:

If two events A and B are independent, the probability that both of them occur is equal to the product of their individual probabilities i.e.

$$P(A \cap B) = P(A).P(B)$$

F. Interpretation of statistical statements in terms of set theory:

$S \Rightarrow Sample\ Space$

$\bar{A} \Rightarrow A\ does\ not\ occur$

$A \cup \bar{A} = S \Rightarrow P(A \cup \bar{A}) = 1$

$A \cap B = \emptyset \Rightarrow A\ and\ B\ are\ mutually\ exclusive \Rightarrow P(A \cap B) = 0$

$A \cup B \Rightarrow$ A or B occurs or both A and B occur (At least one of the A or B occurs)

$A \cap B \Rightarrow$ Both the events A and B occur

$\bar{A} \cap \bar{B} \Rightarrow$ Neither A nor B occurs

$A \cap \bar{B} \Rightarrow$ Event A occurs and B does not occur

$\bar{A} \cap B \Rightarrow$ Event A does not occur and B occur

Understanding the basic concepts and theorem with example:

One hundred and seventy (170) companies from the JSE were randomly selected and classified by sector and size. Table shows the cross-tabulation table of joint frequencies for the two categorical random variables 'sector' and 'company size'.

What is the probability (or percentage) that a randomly selected company will be:

a. A Mining company (A)?
b. A Medium company(Y)?
c. Classified as both Financial and Large?
d. Classified as both Financial and Retail?
e. Classified as Medium or Retail?
f. NOT a Small Company?
g. A Service Company given that it is Large?
h. A Medium and be a Financial is independent?
i. A medium but NOT a Mining?
j. Neither Service NOR Small?

Sector	Company Size			Row Total
	Small(X)	Medium(Y)	Large(Z)	
Mining(A)	3	8	30	41
Financial(B)	9	21	42	72
Service(C)	10	6	8	24
Retail(D)	14	13	6	33
Column Total	36	48	86	170

Exercise

1. **a.** A batch of 50 parts contains six defects. If two parts are drawn randomly one at a time without replacement, what is the probability that both parts are defective?
   b. If this experiment is repeated, with replacement, what is the probability that both parts are defective?
2. The probability that an integrated circuit chip will have defective etching is 0.06, the probability that it will have a crack defect is 0.03 and the probability that it has both defects is 0.02.
   a. What is the probability that newly manufactured chip will have either an etching or a crack defect?
   b. What is the probability that a newly manufactured chip will have neither defect?
3. Suppose that after 10 years of service, 40% of computers have problems with motherboards (MB), 30% have problems with hard drives (HD), and 15% have problems with both MB and HD. What is the probability that a 10-year old computer still has fully functioning MB and HD?

Exercise

4. A new computer virus can enter the system through e-mail or through the internet. There is a 30% chance of receiving this virus through e-mail. There is a 40% chance of receiving it through the internet. Also, the virus enters the system simultaneously through e-mail and the internet with probability 0.15. What is the probability that the virus does not enter the system at all?
5. Among employees of a certain firm, 70% know C/C++, 60% know Fortran, and 50% know both languages. What portion of programmers
   a. does not know Fortran?
   b. does not know Fortran and does not know C/C++?
   c. knows C/C++ but not Fortran?
   d. knows Fortran but not C/C++?
   e. If someone knows Fortran, what is the probability that he/she knows C/C++ too?
   f. If someone knows C/C++, what is the probability that he/she knows Fortran too?