Measures of Dispersion

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Definition

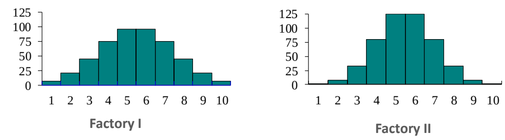
- *Measures of dispersion* are descriptive statistics that describe how similar a set of observations are to each other.
- The more similar the observations are to each other, the lower the measure of dispersion will be
- The less similar the observations are to each other, the higher the measure of dispersion will be
- In general, the more spread out a distribution is, the larger the measure of dispersion will be

Example

For Example, the daily wages in Taka of seven workers of two factories are given in below table. Which of the distributions of wages has the larger dispersion?

Wages of factory I	**142**	**143**	**150**	**150**	**153**	**153**	**155**	**157**
Wages of factory II	**122**	**140**	**150**	**150**	**150**	**154**	**159**	**175**

Example of Dispersion



- The upper distribution has more dispersion because the wages are more spread out.
- That is, they are less similar to each other.

Methods of Measures of Dispersion

1. ABSOLUTE MEASURES

- Range
- Standard Deviation
- Quartile Deviation
- Mean Deviation

2. RELATIVE MEASURES

- Co-efficient of range
- Co-efficient of quartile deviation
- Co-efficient of mean deviation
- Co-efficient of standard deviation
- Co-efficient of variation

Methods of Measures of Dispersion

Absolute measures of variation are expressed in the same statistical unit in which the original data are given such as rupees, kilograms, tones etc.

These values may be used to compare the variation in two or more than two distributions provided the variables are expressed in the same units and have almost the same average value.

Relative Measures of Variation are pure number and independent of the unit of measurement and express in percentage.

The Range

- The *range* is defined as the difference between the largest score in the set of data and the smallest score in the set of data,

$$R = X_n - X_1$$

Where,

X_n = the largest observation

X_1 =the smallest observation

- What is the range of the following data: 4 8 1 6 6 2 9 3 6 9

Solution: $X_1 = 1, \quad X_n = 9$

Hence, $R = X_n - X_1 = 9 - 1 = 8$

Variance

Variance is defined as the average of the square deviations:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$

First, it says to subtract the mean from each of the observations
- This difference is called a *deviate* or a *deviation score*.
- The deviate tells us how far a given score is from the typical average, score.
- Thus, the deviate is a measure of dispersion for a given score.

- Variance is the mean of the squared deviation observations.
- The larger the variance is, the more the observations deviate, on average, away from the mean.
- The smaller the variance is, the less the observations deviate, on average, from the mean.

Computational Formula of Variance

When calculating variance, it is often easier to use a computational formula which is algebraically equivalent to the definitional formula:

$$\sigma^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{N}}{N} = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$

Computational Formula Example

X	X^2	X-μ	(X-μ)^2
9	81	2	4
8	64	1	1
6	36	-1	1
5	25	-2	4
8	64	1	1
6	36	-1	1
Σ = 42	Σ = 306	Σ = 0	Σ = 12

Computational Formula Example

$$\sigma^2 = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N}$$

$$= \frac{306 - \frac{42^2}{6}}{6}$$

$$= \frac{306 - 294}{6}$$

$$= \frac{12}{6}$$

$$= 2$$

$$\sigma^2 = \frac{\Sigma(X-\mu)^2}{N}$$

$$= \frac{12}{6}$$

$$= 2$$

Variance of a Sample

Because the sample mean is not a perfect estimate of the population mean, the formula for the variance of a sample is slightly different from the formula for the variance of a population:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

s^2 is the sample variance, x is a score, $\bar{x}$ is the sample mean, and n is the number of observations.

Standard Deviation

- When the deviate observations are squared in variance, their unit of measure is squared as well.
- E.g. If people's weights are measured in pounds, then the variance of the weights would be expressed in pounds^2 (or squared pounds)
- Since squared units of measure are often awkward to deal with, the square root of variance is often used instead.
- The standard deviation is the square root of variance.

$$Standard\ deviation\ = \sqrt{variance}$$
$$Variance\ = (standard\ deviation)^2$$