Shape of the Distribution

MD. PEAR HOSSAIN
LECTURER OF STATISTICS
BSMRSTU, BANGLADESH

Skewness

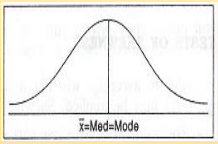
- The term skewness refers to the lack of symmetry.
- The lack of symmetry in a distribution is always determined with reference to a normal distribution, which is always symmetrical.
- The skewness of a frequency distribution can be in three different shapes:
  a. Symmetrical Distribution
  b. Positively Skewed Distribution
  c. Negatively Skewed Distribution

a) Symmetrical Distribution

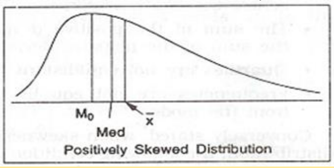
It is also known as normal. The spread of the frequencies is the same on both sides of the center point of the curve.

Example: Height, weight, exam score etc.

In symmetric distribution Mean=Median=Mode



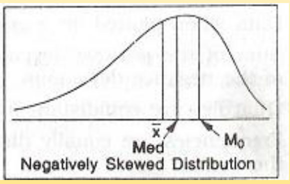
b) Positively Skewed Distribution



- In this distribution there is a long tail to the right.
- In P.S distribution, Mean>Median>Mode

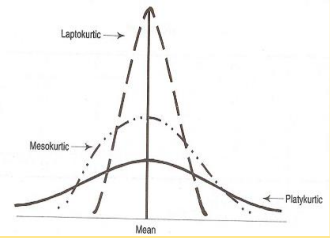
c) Negatively Skewed Distribution:

- In this distribution there is a long tail to the left.
- In N.S distribution, Mean<Median<Mode



Kurtosis

- The flatness or peakedness of a curve is known as kurtosis.
- It is also defined as the degree of peakedness of a distribution; usually taken in relation to a normal distribution.
- There are three types of kurtosis
  a) Leptokurtic: *Leptokurtic curve is a more peaked than the normal curve.*
  b) Mesokurtic: *Mesokurtic curve is neither too much flattened nor too much peaked. In fact, this is the frequency curve of a normal distribution.*
  c) Platykurtic: *Platykurtic is a relatively flat curve.*



Measure of Skewness and Kurtosis using Moment

- The r[th] central moment is denoted by μ_r and defined as: $\mu_r = \frac{\sum(X-\bar{X})^r}{n}$.
- Replacing $r = 1,2,3,4$ we will have 1st, 2nd, 3rd and 4th central moments:
  - 1st Central moment: $\mu_1 = \frac{\sum(X-\bar{X})}{n} = 0$
  - 2nd Central Moment: $\mu_2 = \frac{\sum(X-\bar{X})^2}{n}$
  - 3rd Central Moment: $\mu_3 = \frac{\sum(X-\bar{X})^3}{n}$
  - 4th Central Moment: $\mu_4 = \frac{\sum(X-\bar{X})^4}{n}$

For i) **Skewness,** calculate, $\gamma_1 = \sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}} =$; where $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$

Decision:

If $\boldsymbol{\gamma_1} = 0$; Distribution is Symmetric

If $\boldsymbol{\gamma_1} > 0$; Distribution is Positively Skewed

If $\boldsymbol{\gamma_1} < 0$; Distribution is Negatively Skewed.

For ii) **Kurtosis** calculate $\beta_2 = \frac{\mu_4}{\mu_2^2}$

Decision:

If $\beta_2 = 3$; Distribution is Mesokurtic

If $\beta_2 > 3$; Distribution is Leptokurtic

If $\beta_2 < 3$; Distribution is Platykurtic

Example

Example Comment about Skewness and Kurtosis of the following data:
- 9 2 8 10 13 18

Solution:

	x	$(X-\bar{X})$	$(X-\bar{X})^2$	$(X-\bar{X})^3$	$(X-\bar{X})^4$
	9	-1	1	-1	1
	2	-8	64	-512	4096
	8	-2	4	-8	16
	10	0	0	0	0
	13	3	9	27	81
	18	8	64	512	4096
Total	$\sum X$ =60	$\sum(X-\bar{X})$ =0	$\sum(X-\bar{X})^2$ =142	$\sum(X-\bar{X})^3$ =18	$\sum(X-\bar{X})^4$ =8290
			$\mu_2 = 23.67$	$\mu_3 = 3$	$\mu_4 = 1381.67$

Skewness: $\gamma_1 = \frac{\mu_3}{\sqrt{\mu_2^3}} = 0.026 > 0;$ *indicates Positively Skewed Distribution.*

Kurtosis: $\beta_2 = \frac{\mu_4}{\mu_2^2} = 2.47 < 3;$ *indicates Platykurtic Distribution.*