CSE 211: Non-deterministic Finite Automaton

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Class 4

Outline

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What is NFA

Non-Deterministic Finite Automaton (NFA)

- For a single input symbol, transition from the current state to one or more states
- Transition from current state to one or more states without consuming any input symbol (in other word, transition consuming ϵ)

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Advantage

- Easier and simpler state diagram
- More intuitive

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Advantage

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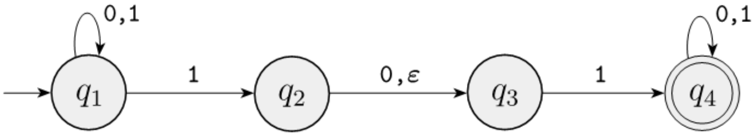
Disadvantage

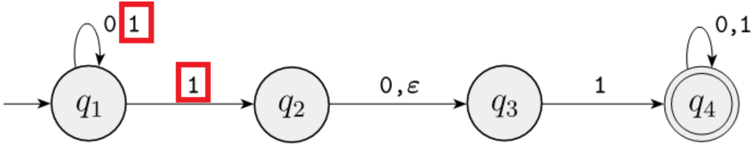
- Harder computation for decision-making

NFA Example



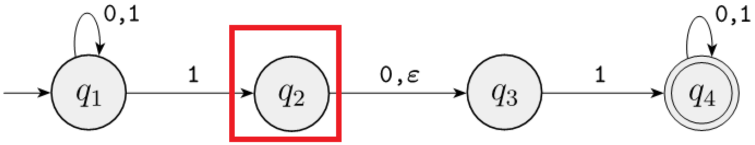
- NFA N_1 has four states

NFA Example



- NFA N_1 has four states
- For input 1, $q_1 \rightarrow q_1$ OR $q_1 \rightarrow q_2$

NFA Example



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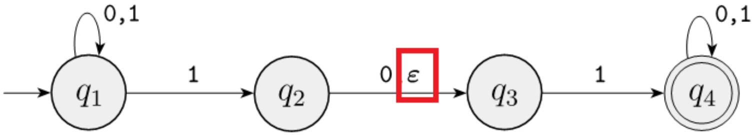
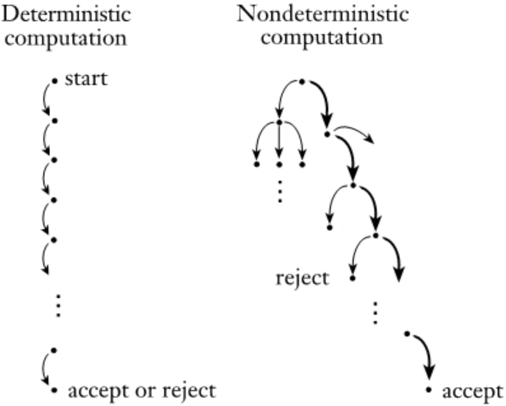


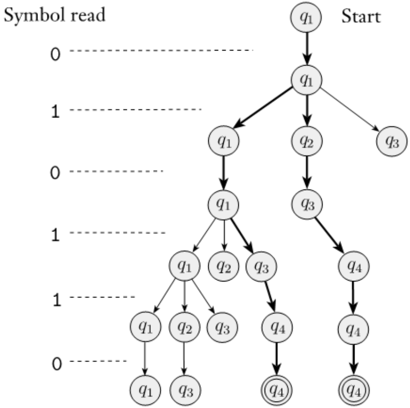
- NFA N_1 has four states
- For input 1, $q_1 \rightarrow q_1$ OR $q_1 \rightarrow q_2$
- State q_2 has only transition for input 0 BUT not for 1
- N_1 can transit $q_2 \rightarrow q_3$ without any input!

Examples of NFA (Contd.)

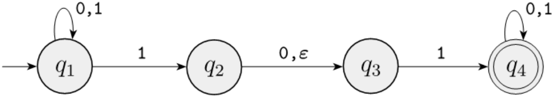


NFA Decision Tree

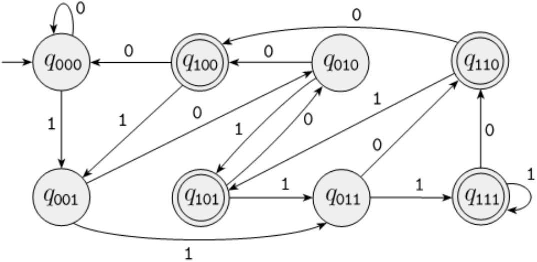
Examples of NFA (Contd.)



NFA Decision Tree for N_1 for computation of 010110



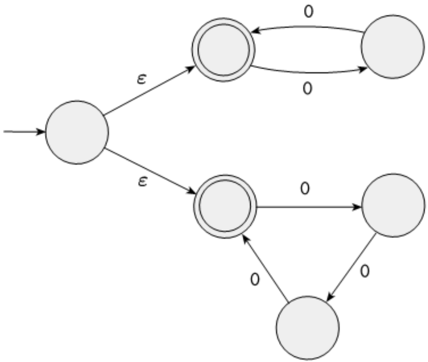
NFA N_1



Equivalent DFA D_1

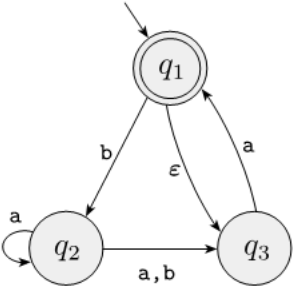
Examples of NFA (Contd.)

An NFA which all strings of the form 0^k where k is a multiple of 2 or 3



Examples of NFA (Contd.)

An NFA which all strings of the form $\epsilon, a, baba, baa, \ldots$
But does not accept $b, bb, babba$ etc.



Formal Defintion of NFA

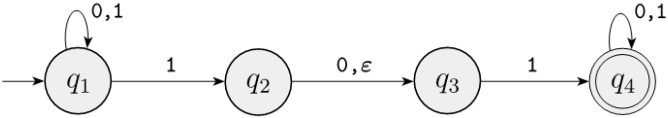
NFA

a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where:

1. Q is a finite set of states
2. σ is a finite alphabet
3. $\delta : Q \times \Sigma_\epsilon \to \mathcal{P}(Q)$
4. $q_0 \in Q$ is the start-state
5. $F \subseteq Q$ is the accept-state set

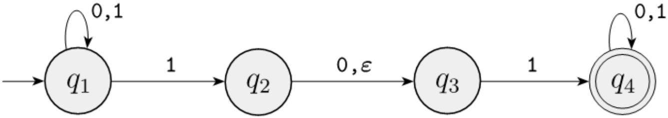
Examples of NFA (Contd.)

Give the formal definition of for the following NFA:



Examples of NFA (Contd.)

Give the formal definition of for the following NFA:



- $Q = \{q_1, q_2, q_3, q_4\}$
- $\Sigma = \{0, 1\}$
- start-state, $q_0 = q_1$
- accept-states, $F = \{q_4\}$

δ is:

	0	1	ϵ
q_1	$\{q_1\}$	$\{q_1, q_2\}$	$\varnothing$
q_2	$\{q_3\}$	$\varnothing$	$\{q_3\}$
q_3	$\varnothing$	$\{q_4\}$	$\varnothing$
q_4	$\{q_4\}$	$\{q_4\}$	$\varnothing$

Computation by NFA

Formal Definition of Computation by NFA

$N = (Q, \Sigma, \delta, q_0, F)$ is an NFA, w is a string of Σ
N accepts w if $w = y_1 \, y_2 \ldots y_m$ where each $y_i \in \Sigma_\epsilon$ and sequence of states $r_0, r_1, \ldots, r_m$ each exists in Q such that:

1. $r_0 = q_0$
2. $r_{i+1} \in \delta(r_i, y_{i+1})$ for $m = 0, 1, \ldots, m-1$ and
3. $r_m \in F$

Equivalence of NFA and DFA

Equivalence of Machines

Two machines are equivalent if they recognize the same languages

Theorem 1.39

Every NFA has an equivalent DFA

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Proof Idea

Convert the NFA into an equivalent DFA that simulates it. Issues are:

- How will you simulate the NFA by the DFA?
- How will you keep track of the input and branches of computation as the input is processed?
- For a k-state NFA, the DFA may have to remember 2^k states!
- What will be the start-state, accept-state and transition function of the DFA?

NFA has an equivalent DFA ... proof contd.

Let $N = (Q, \Sigma, \delta, q_0, F)$ be the NFA
and $M = (Q', \Sigma, \delta', q_0', F')$ be the equivalent DFA. Both N and M recognizes the same language A.

Case I: There is no ϵ arrow in the NFA N.

1. $Q' = \mathcal{P}(Q)$, because every state of M is a set of states of N
2. $q_0' = \{q_0\}$
3. $F' = \{R \in Q' |\ R$ contains an accept-state of $N\}$
4. For $R \in Q'$ and $a \in Q'$
   $\delta'(R, a) = \{q \in Q |\ q \in \delta(r, a)$ for some $r \in R\}$
   $\delta'(R, a) = \bigcup_{r \in R} \delta(r, a)$ set of all states reachable from all states of R for input a according to the δ of N

NFA=DFA (Contd.)

NFA has an equivalent DFA . . . proof contd.

Case II: There are ϵ arrow in the NFA N.

- For any state R of M, $E(M)$: collection of states reachable from any state of set R going along the ϵ-arrows, $R \subseteq Q$
- Formally, $E(R) = \{q |\, q$ is reachable from R by traveling along 0 or more ϵ arrows $\}$
- Replacing $\delta(r, a)$ by $E(\delta(r, a))$
  $\delta'(R, a) = \{q \in Q |\, q \in E(\delta(r, a))$ for some $r \in R\}$
- $q_0' = E(\{q_0\})$

Does the construction of M works correctly?

NFA=DFA (Contd.)

NFA has an equivalent DFA ... proof contd.

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- $q'_0 = E(\{q_0\})$

Does the construction of M works correctly?
At every step of computation on an input, M enters a state that corresponds to the subset of states that N could be in at that point.

NFA=DFA (Contd.)

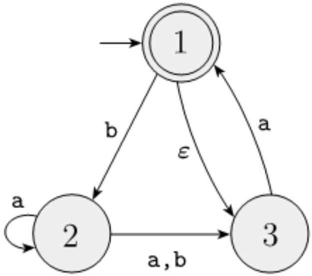
Corollary 1.40

A language is regular if and only if some non-deterministic finite automaton recognizes it

Proof

- A language is regular if some NFA recognizes it
  If some NFA recognizes the language, so does an equivalent DFA. Hence, the language is regular.
- A language is regular only if some NFA recognizes it
  If a language is regular, a DFA recognizes it. A DFA is also an NFA. So, an NFA also recognizes the regular language.

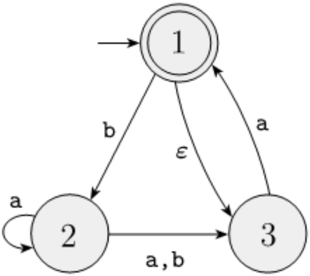
Examples of Conversion from NFA to DFA



$DFA = (Q, \Sigma, \delta, q_0, F)$

- $Q = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}, \}$
- $\Sigma = \{a, b\}$
- $q_0 = E(\{1\}) = \{1, 3\}$
- $F = \{$ All states of Q that have a final state of $N\}$
  $=\{\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$

Examples of Conversion from NFA to DFA (Contd.)

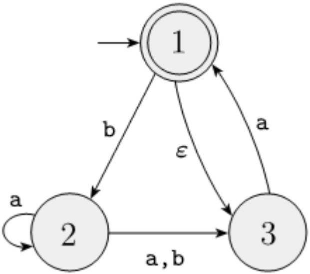


Transition Function δ=?

- On input a: $\{2\} \rightarrow \{2,3\}$, on input b: $\{2\} \rightarrow \{3\}$
- On input a: $\{1\} \rightarrow \varnothing$, on input b: $\{1\} \rightarrow \{2\}$
- On input a: $\{3\} \rightarrow \{1,3\}$, on input b: $\{3\} \rightarrow \varnothing$
- On input a: $\{1,2\} \rightarrow \{2,3\}$, on input b: $\{1,2\} \rightarrow \{2,3\}$
- Similarly, for rest of the states ...

Correction to class lecture

attention: follow ϵ after applying $\delta()$, not before!



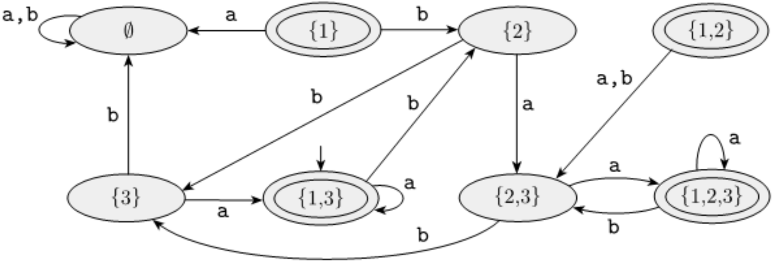
$\delta'(\{1\}, a) = \varnothing$

INCORRECT!, $\delta'(E(\{1\}), a) = \delta'(\{1, 3\}, a) = \{1\}$

But, $\delta'(\{3\}, a) = E(\{1\}) = \{1, 3\}$

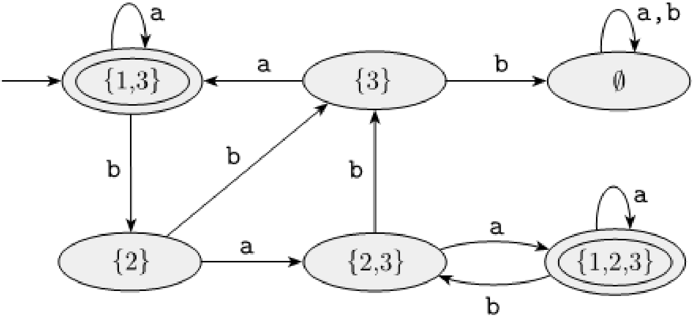
Examples of Conversion from NFA to DFA (Contd.)

After conversion . . .



Examples of Conversion from NFA to DFA (Contd.)

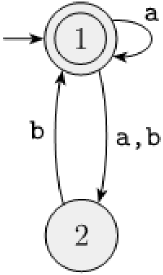
After conversion and simplification . . .



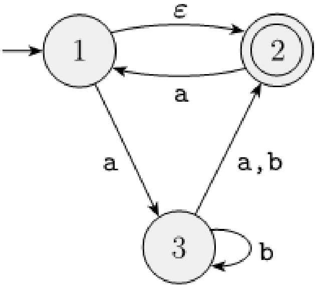
Dropping the states from which there is no outgoing arrow

Try Yourself

Convert the following NFA into the equivalent DFA



(a)



(b)

Question?