CSE 211 (Theory of Computation)

Context Free Languages

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Context-Free Grammars

Sipser, 2.1, p-102

- Grammar, G_1.
- $$A \rightarrow 0A1$$
  $$A \rightarrow B$$
  $$B \rightarrow \#$$

Context-Free Grammars

Sipser, 2.1, p-102

- Grammar, G_1.
- $$A \rightarrow 0A1$$
  $$A \rightarrow B$$
  $$B \rightarrow \#$$
- - substitution rules, also called productions.
  - variables.
  - terminals.
  - start variable.

Context-Free Grammars

Sipser, 2.1, p-102

$$A \rightarrow 0A1$$
$$A \rightarrow B$$
$$B \rightarrow \#$$

- Grammar G_1 generates the string $000\#111$.
- The sequence of substitutions to obtain a string is called a derivation.

Context-Free Grammars

Sipser, 2.1, p-102

$$A \rightarrow 0A1$$
$$A \rightarrow B$$
$$B \rightarrow \#$$

- A derivation of string $000\#111$ in grammar G_1 is

$$\begin{aligned} A &\Rightarrow 0A1 \\ &\Rightarrow 00A11 \\ &\Rightarrow 000A111 \\ &\Rightarrow 000B111 \\ &\Rightarrow 000\#111 \end{aligned}$$

$$
\begin{aligned}
A &\Rightarrow 0A1 \\
&\Rightarrow 00A11 \\
&\Rightarrow 000A111 \\
&\Rightarrow 000B111 \\
&\Rightarrow 000\#111
\end{aligned}
$$

- You may also represent the same information pictorially with a parse tree.

Context-Free Grammars — *continued*

Sipser, Figure 2.1, p-103

$$
\begin{aligned}
A &\Rightarrow 0A1 \\
&\Rightarrow 00A11 \\
&\Rightarrow 000A111 \\
&\Rightarrow 000B111 \\
&\Rightarrow 000\#111
\end{aligned}
$$

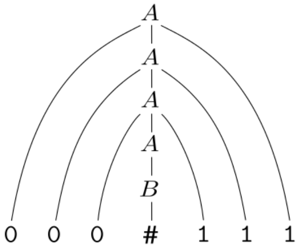

FIGURE 2.1
Parse tree for 000#111 in grammar G_1

- Let us consider the language of palindromes.

$$
\begin{array}{llll}
1. & P & \rightarrow & \epsilon \\
2. & P & \rightarrow & 0 \\
3. & P & \rightarrow & 1 \\
4. & P & \rightarrow & 0P0 \\
5. & P & \rightarrow & 1P1
\end{array}
$$

A context-free grammar for palindromes

Context-Free Grammars

Sipser, 2.1, p-102

$$
\begin{aligned}
\langle\text{SENTENCE}\rangle &\rightarrow \langle\text{NOUN-PHRASE}\rangle\langle\text{VERB-PHRASE}\rangle \\
\langle\text{NOUN-PHRASE}\rangle &\rightarrow \langle\text{CMPLX-NOUN}\rangle \mid \langle\text{CMPLX-NOUN}\rangle\langle\text{PREP-PHRASE}\rangle \\
\langle\text{VERB-PHRASE}\rangle &\rightarrow \langle\text{CMPLX-VERB}\rangle \mid \langle\text{CMPLX-VERB}\rangle\langle\text{PREP-PHRASE}\rangle \\
\langle\text{PREP-PHRASE}\rangle &\rightarrow \langle\text{PREP}\rangle\langle\text{CMPLX-NOUN}\rangle \\
\langle\text{CMPLX-NOUN}\rangle &\rightarrow \langle\text{ARTICLE}\rangle\langle\text{NOUN}\rangle \\
\langle\text{CMPLX-VERB}\rangle &\rightarrow \langle\text{VERB}\rangle \mid \langle\text{VERB}\rangle\langle\text{NOUN-PHRASE}\rangle \\
\langle\text{ARTICLE}\rangle &\rightarrow \texttt{a} \mid \texttt{the} \\
\langle\text{NOUN}\rangle &\rightarrow \texttt{boy} \mid \texttt{girl} \mid \texttt{flower} \\
\langle\text{VERB}\rangle &\rightarrow \texttt{touches} \mid \texttt{likes} \mid \texttt{sees} \\
\langle\text{PREP}\rangle &\rightarrow \texttt{with}
\end{aligned}
$$

Context-Free Grammars

Sipser, 2.1, p-102

$$
\begin{aligned}
\langle \text{SENTENCE} \rangle &\Rightarrow \langle \text{NOUN-PHRASE} \rangle \langle \text{VERB-PHRASE} \rangle \\
&\Rightarrow \langle \text{CMPLX-NOUN} \rangle \langle \text{VERB-PHRASE} \rangle \\
&\Rightarrow \langle \text{ARTICLE} \rangle \langle \text{NOUN} \rangle \langle \text{VERB-PHRASE} \rangle \\
&\Rightarrow \texttt{a} \ \langle \text{NOUN} \rangle \langle \text{VERB-PHRASE} \rangle \\
&\Rightarrow \texttt{a boy} \ \langle \text{VERB-PHRASE} \rangle \\
&\Rightarrow \texttt{a boy} \ \langle \text{CMPLX-VERB} \rangle \\
&\Rightarrow \texttt{a boy} \ \langle \text{VERB} \rangle \\
&\Rightarrow \texttt{a boy sees}
\end{aligned}
$$

Formal Definition of a Context-Free Grammar

Sipser, Definition 2.2, p-104

DEFINITION 2.2

A ***context-free grammar*** is a 4-tuple (V, Σ, R, S), where

1. V is a finite set called the ***variables***,
2. Σ is a finite set, disjoint from V, called the ***terminals***,
3. R is a finite set of ***rules***, with each rule being a variable and a string of variables and terminals, and
4. $S \in V$ is the start variable.

Formal Definition of a Context-Free Grammar — *continued*

- If u, v, and w are strings of variables and terminals, and $A \to w$ is a rule of the grammar, we say that uAv yields uwv, written $uAv \Rightarrow uwv$.
- Say that u derives v, written $u \overset{*}{\Rightarrow} v$, if $u = v$ or if a sequence $u_1, u_2, \ldots, u_k$ exists for $k \geq 0$ and $u \Rightarrow u_1 \Rightarrow u_2 \Rightarrow \cdots \Rightarrow u_k \Rightarrow v$.
- The language of the grammar is $\{w \in \Sigma* \mid S \Rightarrow w\}$.

Example

Sipser, Example 2.3, p-105

- $G_3 = (\{S\}, \{a, b\}, R, S)$.
- The set of rules (R), R, is $S \rightarrow aSb \mid SS \mid \epsilon$.

Example

Sipser, Example 2.4, p-105

- $G_4 = (V, \Sigma, R, \texttt{<EXPR>})$.
- V is $\{\texttt{<EXPR>}, \texttt{<TERM>}, \texttt{<FACTOR>}\}$,
- and Σ is $\{a, +, \times, (,)\}$.
- The rules are,

$$
\begin{aligned}
\texttt{<EXPR>} &\rightarrow \texttt{<EXPR>} + \texttt{<TERM>} \mid \texttt{<TERM>} \\
\texttt{<TERM>} &\rightarrow \texttt{<TERM>} \times \texttt{<FACTOR>} \mid \texttt{<FACTOR>} \\
\texttt{<FACTOR>} &\rightarrow (\texttt{<EXPR>}) \mid a
\end{aligned}
$$

Formal Definition of a Context-Free Grammar

Sipser, Figure 2.5, p-105

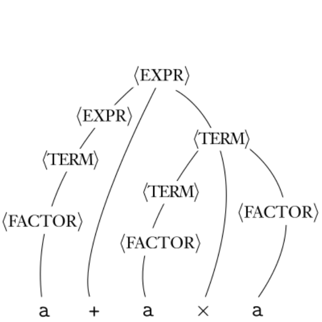

FIGURE 2.5
Parse trees for the strings a+a×a and (a+a)×a