Exercice 3.6

Ex. 3.6 Show that the ridge regression estimate is the mean (and mode) of the posterior distribution, under a Gaussian prior $\beta \sim N(0, \tau \mathbf{I})$, and Gaussian sampling model $\mathbf{y} \sim N(\mathbf{X}\beta, \sigma^2 \mathbf{I})$. Find the relationship between the regularization parameter λ in the ridge formula, and the variances τ and σ^2.

By Bayes theorem:

$$p(\beta | y) = \frac{P(Y|\beta)\, P(\beta)}{p(y)}$$

on - $p(y|\beta) = N(X\beta, \sigma^2)$ est la vraisemblance

- $P(\beta) = N(0, \tau I)$ est le prior
- $P(y) \Rightarrow$ cste de normalisation

En utilisant le log on obtient:

$$\ln\left(P(\beta|y)\right) = -\frac{1}{2}\left(\frac{(y-X\beta)^T(y-X\beta)}{\sigma^2} + \frac{\beta^T\beta}{\tau}\right) + c$$

On pose $\lambda = \frac{\sigma^2}{\tau}$, on sait que c est une cste indépendante de β, max sur β pour $p(\beta|y)$ revient à :

$$\boxed{\min_{\beta} (y - X\beta)^T (y - X\beta) + \lambda \beta^T \beta}$$

[illegible]