Chapter 3

Data and Signals

Note

To be transmitted, data must be transformed to electromagnetic signals.

3-1   ANALOG AND DIGITAL

Data can be analog or digital. The term analog data refers to information that is continuous; digital data refers to information that has discrete states. Analog data take on continuous values. Digital data take on discrete values.

Topics discussed in this section:
- **Analog and Digital Data**
- **Analog and Digital Signals**
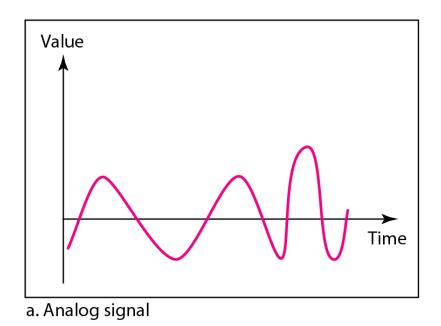- **Periodic and Nonperiodic Signals**
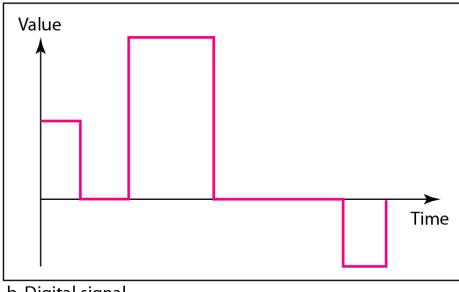
Analog and Digital Data

- Data can be analog or digital.
- Analog data are continuous and take continuous values.
- Digital data have discrete states and take discrete values.

Analog and Digital Signals

- Signals can be analog or digital.
- Analog signals can have an infinite number of values in a range.
- Digital signals can have only a limited number of values.

Figure 3.1 *Comparison of analog and digital signals*

a. Analog signal

b. Digital signal

3-2   PERIODIC ANALOG SIGNALS

In data communications, we commonly use periodic analog signals and nonperiodic digital signals.

Periodic analog signals can be classified as simple or composite. A simple periodic analog signal, a sine wave, cannot be decomposed into simpler signals. A composite periodic analog signal is composed of multiple sine waves.

Topics discussed in this section:

- **Sine Wave**
- **Wavelength**
- **Time and Frequency Domain**
- **Composite Signals**
- **Bandwidth**
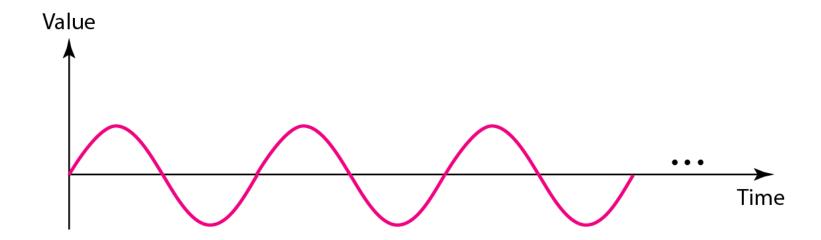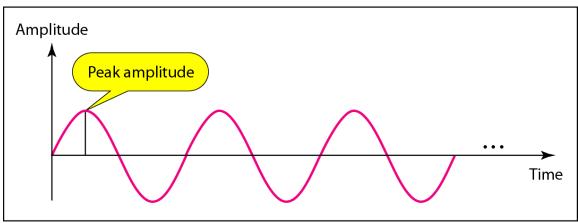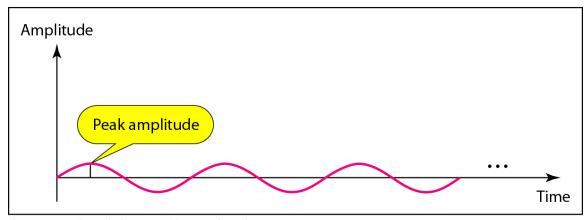
Figure 3.2 *A sine wave*

Figure 3.3 *Two signals with the same phase and frequency, but different amplitudes*
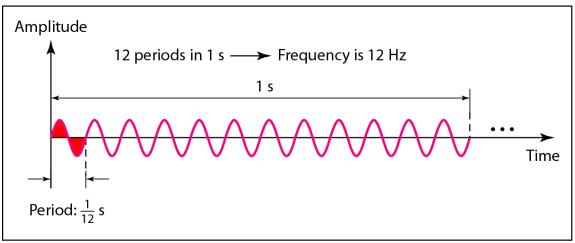

a. A signal with high peak amplitude

b. A signal with low peak amplitude

Note

Frequency and period are the inverse of each other.

$$f = \frac{1}{T} \qquad \text{and} \qquad T = \frac{1}{f}$$

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Figure 3.4 *Two signals with the same amplitude and phase, but different frequencies*
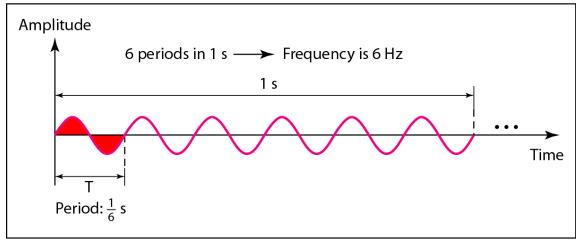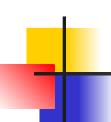

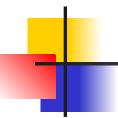a. A signal with a frequency of 12 Hz

b. A signal with a frequency of 6 Hz

Table 3.1 *Units of period and frequency*

Unit	Equivalent	Unit	Equivalent
Seconds (s)	1 s	Hertz (Hz)	1 Hz
Milliseconds (ms)	10^{-3} s	Kilohertz (kHz)	10^{3} Hz
Microseconds (µs)	10^{-6} s	Megahertz (MHz)	10^{6} Hz
Nanoseconds (ns)	10^{-9} s	Gigahertz (GHz)	10^{9} Hz
Picoseconds (ps)	10^{-12} s	Terahertz (THz)	10^{12} Hz

Example 3.1

The power we use at home has a frequency of 60 Hz. The period of this sine wave can be determined as follows:

$$T = \frac{1}{f} = \frac{1}{60} = 0.0166 \text{ s} = 0.0166 \times 10^3 \text{ ms} = 16.6 \text{ ms}$$

Example 3.2

The period of a signal is 100 ms. What is its frequency in kilohertz?

Solution

First we change 100 ms to seconds, and then we calculate the frequency from the period (1 Hz = 10^{-3} kHz).

$$100 \text{ ms} = 100 \times 10^{-3} \text{ s} = 10^{-1} \text{ s}$$

$$f = \frac{1}{T} = \frac{1}{10^{-1}} \text{ Hz} = 10 \text{ Hz} = 10 \times 10^{-3} \text{ kHz} = 10^{-2} \text{ kHz}$$

Frequency

- Frequency is the rate of change with respect to time.
- Change in a short span of time means high frequency.
- Change over a long span of time means low frequency.

Note

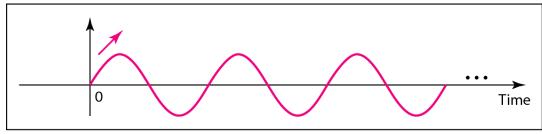If a signal does not change at all, its frequency is zero.
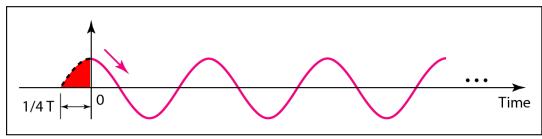If a signal changes instantaneously, its frequency is infinite.

Note

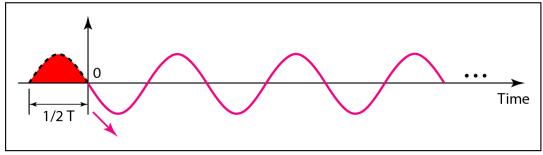**Phase describes the position of the waveform relative to time 0.**

Figure 3.5 Three sine waves with the same amplitude and frequency, but different phases
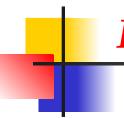

a. 0 degrees

b. 90 degrees

c. 180 degrees

Example 3.3

A sine wave is offset 1/6 cycle with respect to time 0. What is its phase in degrees and radians?

Solution

We know that 1 complete cycle is 360°. Therefore, 1/6 cycle is

$$\frac{1}{6} \times 360 = 60° = 60 \times \frac{2\pi}{360} \text{ rad} = \frac{\pi}{3} \text{ rad} = 1.046 \text{ rad}$$
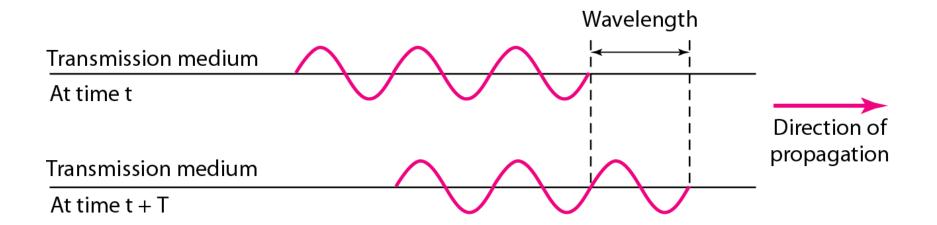
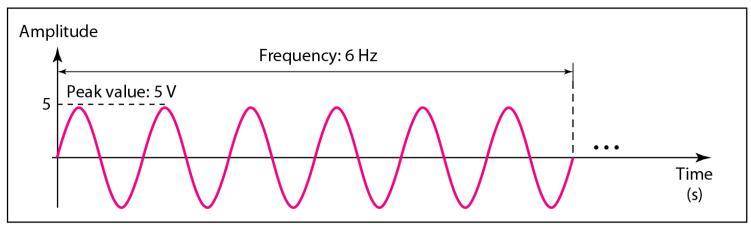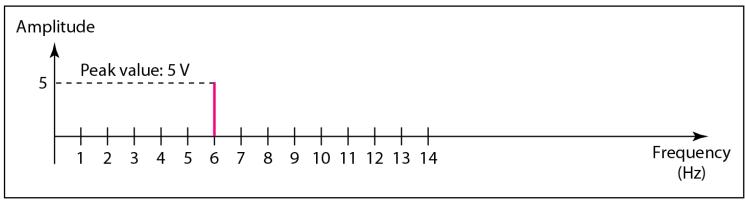# Figure 3.6 *Wavelength and period*

Figure 3.7 *The time-domain and frequency-domain plots of a sine wave*

a. A sine wave in the time domain (peak value: 5 V, frequency: 6 Hz)

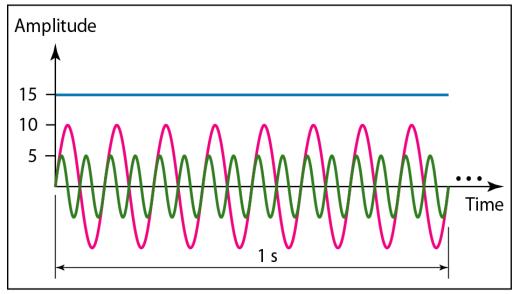b. The same sine wave in the frequency domain (peak value: 5 V, frequency: 6 Hz)

A complete sine wave in the time domain can be represented by one single spike in the frequency domain.
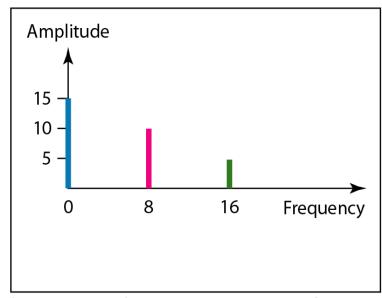
Example 3.7

The frequency domain is more compact and useful when we are dealing with more than one sine wave. For example, Figure 3.8 shows three sine waves, each with different amplitude and frequency. All can be represented by three spikes in the frequency domain.

Figure 3.8 *The time domain and frequency domain of three sine waves*

a. Time-domain representation of three sine waves with
   frequencies 0, 8, and 16

b. Frequency-domain representation of
   the same three signals

Signals and Communication

- A single-frequency sine wave is not useful in data communications

- We need to send a composite signal, a signal made of many simple sine waves.

- According to Fourier analysis, any composite signal is a combination of simple sine waves with different frequencies, amplitudes, and phases.

Composite Signals and Periodicity

- If the composite signal is periodic, the decomposition gives a series of signals with discrete frequencies.

- If the composite signal is nonperiodic, the decomposition gives a combination of sine waves with continuous frequencies.

Example 3.4

Figure 3.9 shows a periodic composite signal with frequency f. This type of signal is not typical of those found in data communications. We can consider it to be three alarm systems, each with a different frequency. The analysis of this signal can give us a good understanding of how to decompose signals.
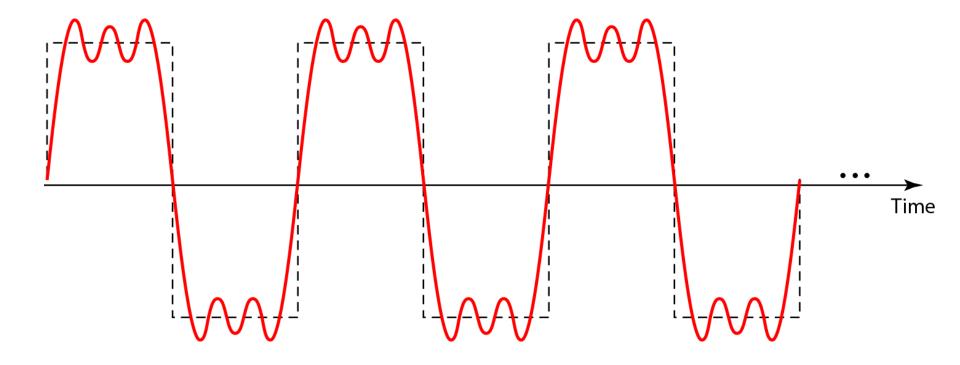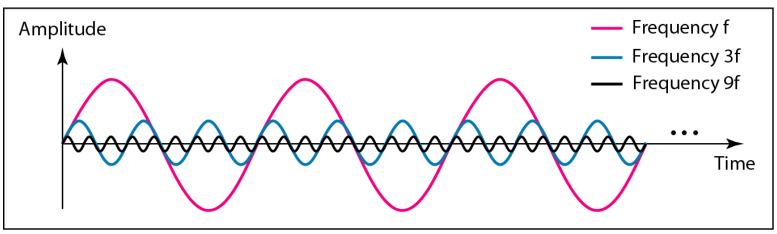
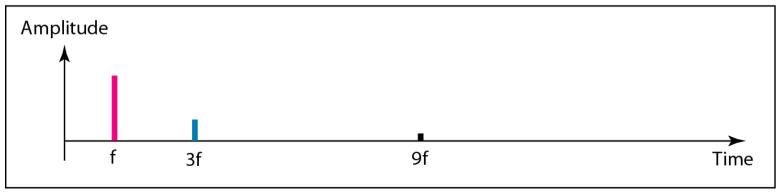# Figure 3.9 *A composite periodic signal*

Figure 3.10 Decomposition of a composite periodic signal in the time and frequency domains
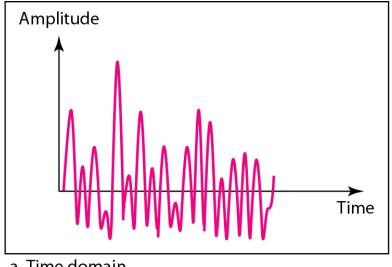

a. Time-domain decomposition of a composite signal

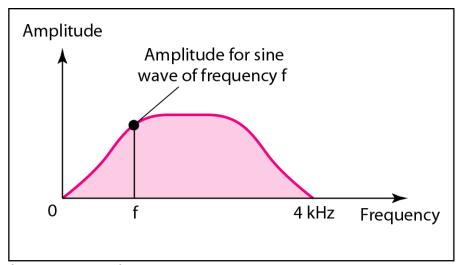b. Frequency-domain decomposition of the composite signal

Example 3.5

Figure 3.11 shows a nonperiodic composite signal. It can be the signal created by a microphone or a telephone set when a word or two is pronounced. In this case, the composite signal cannot be periodic, because that implies that we are repeating the same word or words with exactly the same tone.

Figure 3.11 *The time and frequency domains of a nonperiodic signal*

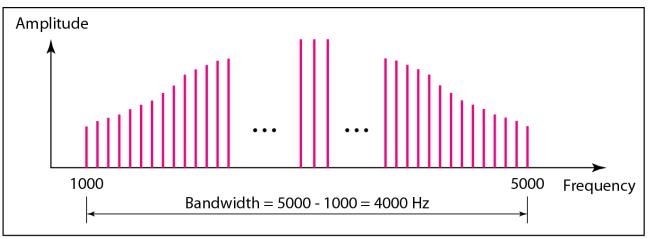

a. Time domain
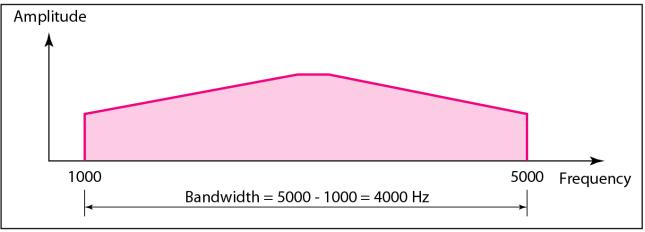
b. Frequency domain

Bandwidth and Signal Frequency

- The bandwidth of a composite signal is the difference between the highest and the lowest frequencies contained in that signal.

Figure 3.12 *The bandwidth of periodic and nonperiodic composite signals*

a. Bandwidth of a periodic signal

b. Bandwidth of a nonperiodic signal

Example 3.6

If a periodic signal is decomposed into five sine waves with frequencies of 100, 300, 500, 700, and 900 Hz, what is its bandwidth? Draw the spectrum, assuming all components have a maximum amplitude of 10 V.
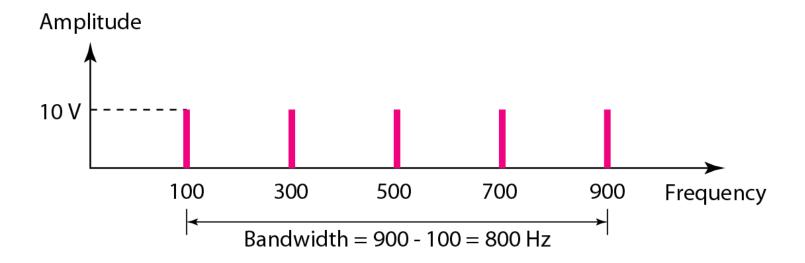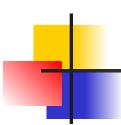
Solution

Let f_h be the highest frequency, f_l the lowest frequency, and B the bandwidth. Then

$$B = f_h - f_l = 900 - 100 = 800 \text{ Hz}$$

The spectrum has only five spikes, at 100, 300, 500, 700, and 900 Hz (see Figure 3.13).

Figure 3.13 *The bandwidth for Example 3.6*

Example 3.7

A periodic signal has a bandwidth of 20 Hz. The highest frequency is 60 Hz. What is the lowest frequency? Draw the spectrum if the signal contains all frequencies of the same amplitude.
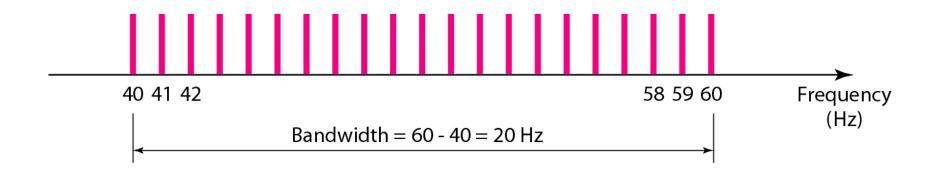
Solution

Let f_h be the highest frequency, f_l the lowest frequency, and B the bandwidth. Then

$$B = f_h - f_l \implies 20 = 60 - f_l \implies f_l = 60 - 20 = 40 \text{ Hz}$$

The spectrum contains all integer frequencies. We show this by a series of spikes (see Figure 3.14).

Figure 3.14 *The bandwidth for Example 3.7*

40 41 42   58 59 60   Frequency (Hz)
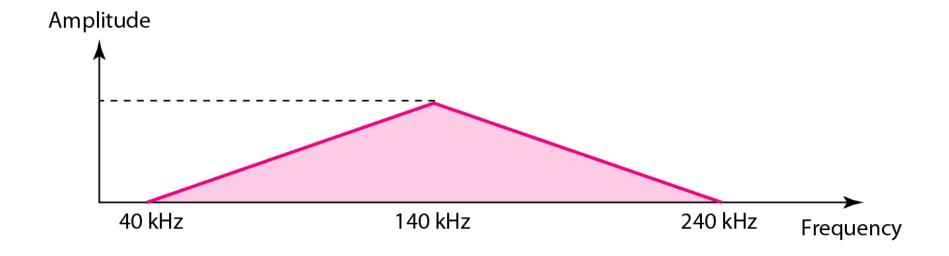
Bandwidth = 60 − 40 = 20 Hz

Example 3.8

A nonperiodic composite signal has a bandwidth of 200 kHz, with a middle frequency of 140 kHz and peak amplitude of 20 V. The two extreme frequencies have an amplitude of 0. Draw the frequency domain of the signal.

Solution

The lowest frequency must be at 40 kHz and the highest at 240 kHz. Figure 3.15 shows the frequency domain and the bandwidth.

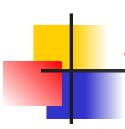# Figure 3.15 *The bandwidth for Example 3.8*

Example 3.9

An example of a nonperiodic composite signal is the signal propagated by an AM radio station. In the United States, each AM radio station is assigned a 10-kHz bandwidth. The total bandwidth dedicated to AM radio ranges from 530 to 1700 kHz. We will show the rationale behind this 10-kHz bandwidth in Chapter 5.

Example 3.10

Another example of a nonperiodic composite signal is the signal propagated by an FM radio station. In the United States, each FM radio station is assigned a 200-kHz bandwidth. The total bandwidth dedicated to FM radio ranges from 88 to 108 MHz. We will show the rationale behind this 200-kHz bandwidth in Chapter 5.

Example 3.11

Another example of a nonperiodic composite signal is the signal received by an old-fashioned analog black-and-white TV. A TV screen is made up of pixels. If we assume a resolution of 525 × 700, we have 367,500 pixels per screen. If we scan the screen 30 times per second, this is 367,500 × 30 = 11,025,000 pixels per second. The worst-case scenario is alternating black and white pixels. We can send 2 pixels per cycle. Therefore, we need 11,025,000 / 2 = 5,512,500 cycles per second, or Hz. The bandwidth needed is 5.5125 MHz.

Fourier Analysis

Fourier analysis is a tool that changes a time domain signal to a frequency domain signal and vice versa.

Fourier Series

- Every composite periodic signal can be represented with a series of sine and cosine functions.

- The functions are integral harmonics of the fundamental frequency "f" of the composite signal.

- Using the series we can decompose any periodic signal into its harmonics.

Fourier Series

Fourier series

$$s(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(2\pi nft) + \sum_{n=1}^{\infty} B_n \cos(2\pi nft)$$

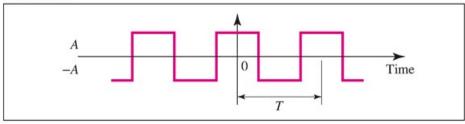$$A_0 = \frac{1}{T}\int_0^T s(t)\, dt \qquad A_n = \frac{2}{T}\int_0^T s(t) \cos(2\pi nft)\, dt$$

$$B_n = \frac{2}{T}\int_0^T s(t) \sin(2\pi nft)\, dt$$

Coefficients

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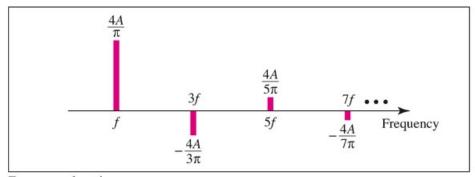# Examples of Signals and the Fourier Series Representation

Time domain

$A_0 = 0$

$A_n = \begin{cases} \dfrac{4A}{n\pi} & \text{for } n = 1, 5, 9, \dots \\ \dfrac{-4A}{n\pi} & \text{for } n = 3, 7, 11, \dots \end{cases}$
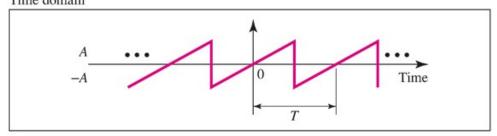
$B_n = 0$

$$s(t) = \frac{4A}{\pi} \cos(2\pi ft) - \frac{4A}{3\pi} \cos(2\pi 3ft) + \frac{4A}{5\pi} \cos(2\pi 5ft) - \frac{4A}{7\pi} \cos(2\pi 7ft) + \cdots$$

Frequency domain
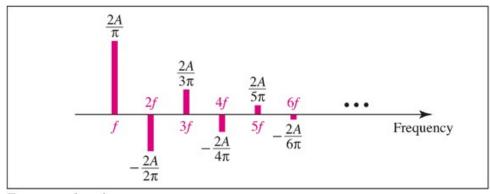
3.46

Sawtooth Signal

Time domain

$$A_0 = 0 \qquad A_n = 0 \qquad B_n = \begin{bmatrix} \dfrac{2A}{n\pi} & \text{for } n \text{ odd} \\ -\dfrac{2A}{n\pi} & \text{for } n \text{ even} \end{bmatrix}$$

$$s(t) = \frac{2A}{\pi} \sin(2\pi ft) - \frac{2A}{2\pi} \sin(2\pi 2ft) + \frac{2A}{3\pi} \sin(2\pi 3ft) - \frac{2A}{4\pi} \sin(2\pi 4ft) + \bullet\bullet\bullet$$
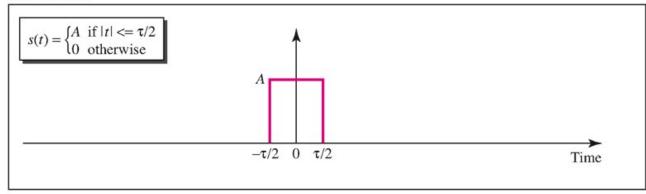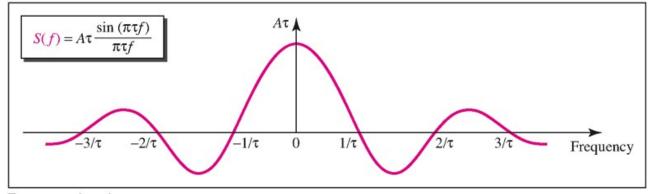

Frequency domain

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Fourier Transform

- Fourier Transform gives the frequency domain of a nonperiodic time domain signal.

Example of a Fourier Transform

Time domain

$$s(t) = \begin{cases} A & \text{if } |t| <= \tau/2 \\ 0 & \text{otherwise} \end{cases}$$

$$S(f) = A\tau \frac{\sin(\pi\tau f)}{\pi\tau f}$$

Frequency domain

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Inverse Fourier Transform

$$S(f) = \int_{-\infty}^{\infty} s(t)e^{-j2\pi ft}\, dt$$

Fourier transform

$$s(t) = \int_{-\infty}^{\infty} S(f)e^{j2\pi ft}\, dt$$

Inverse Fourier transform

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Time limited and Band limited Signals

- A time limited signal is a signal for which the amplitude $s(t) = 0$ for $t > T_1$ and $t < T_2$

- A band limited signal is a signal for which the amplitude $S(f) = 0$ for $f > F_1$ and $f < F_2$