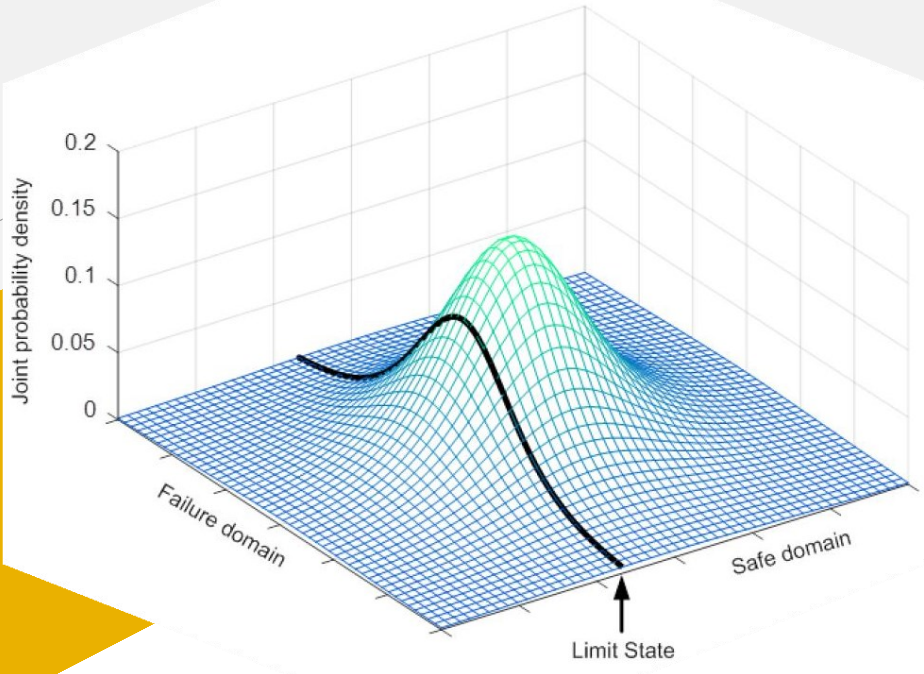

An Introduction to Risk and Reliability Analysis in Coastal Engineering Designs

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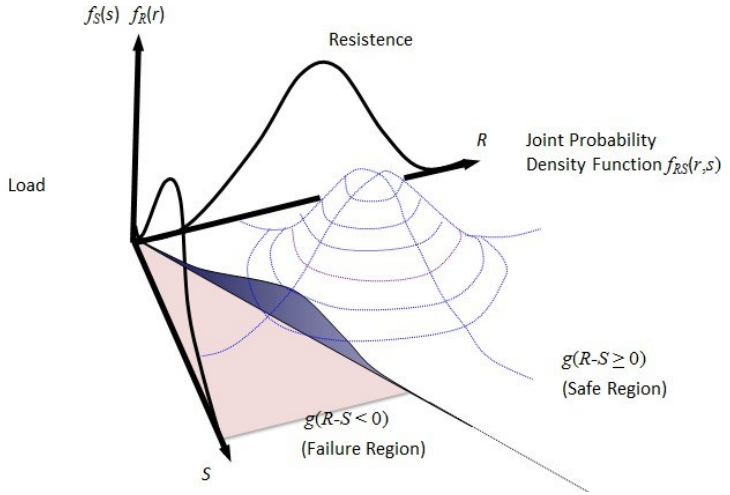
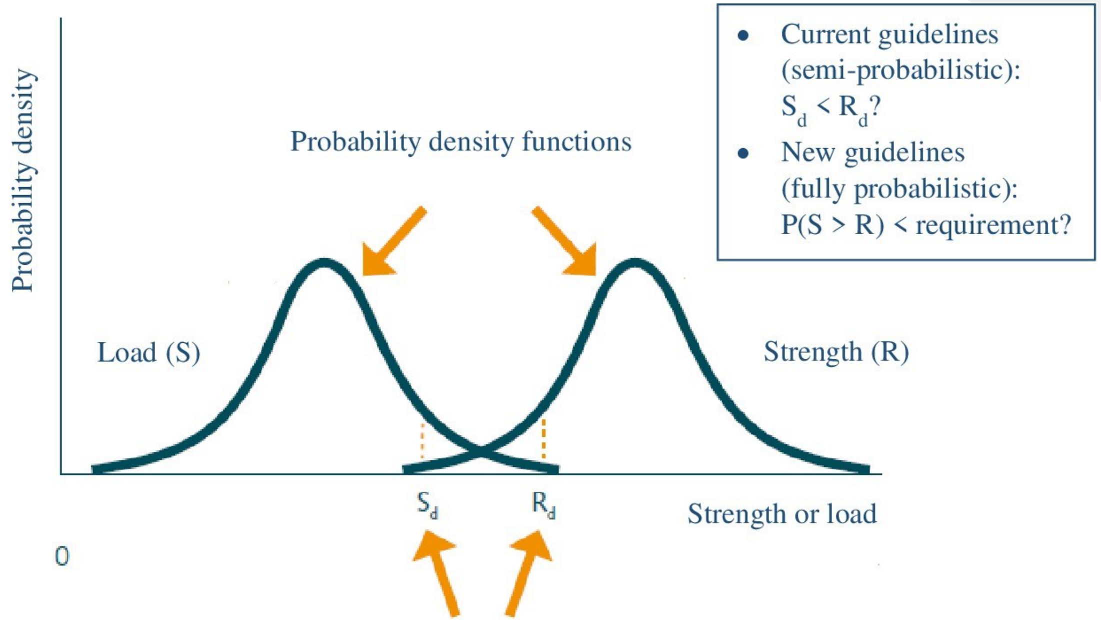
Risk

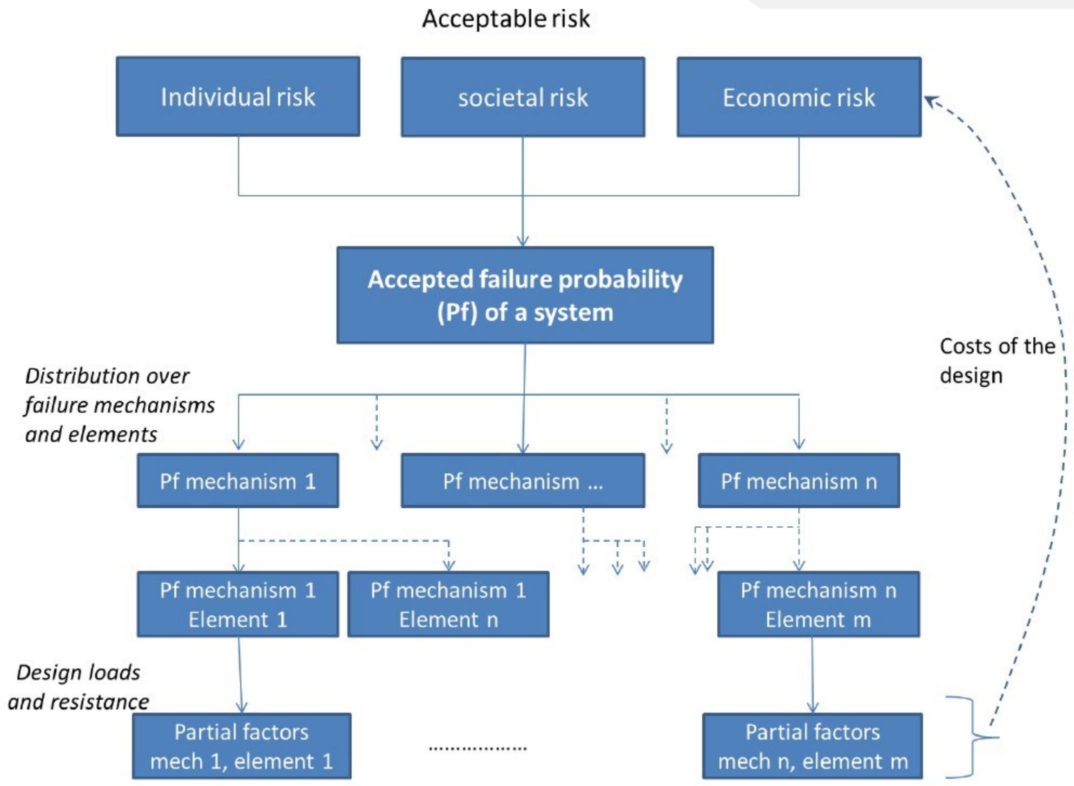
- Risk refers to the combination of **probability** and **consequences** of undesired events.
- Almost all activities in life are characterized by some level of risk.



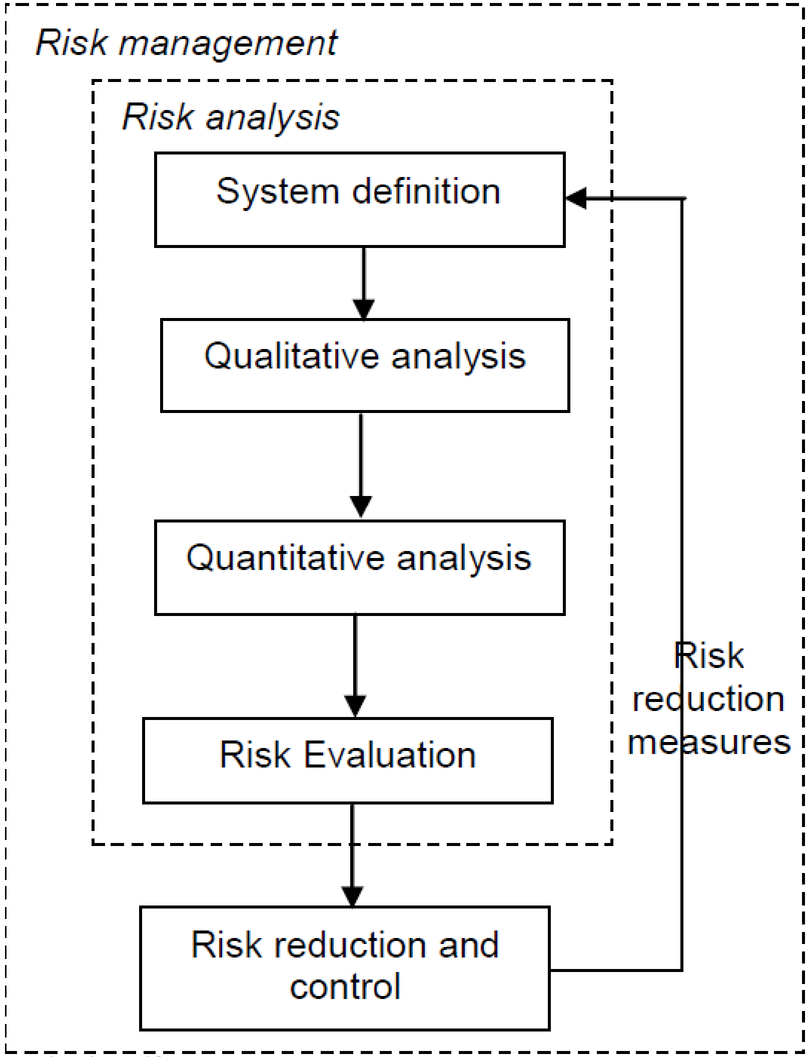


Probabilistic design

- **General definition:** the relationship between <u>safety</u> standards and engineering <u>design.</u>
- **Overall objective:** to design (and maintain) systems with an acceptable risk level in an optimal way.



Schematic view of steps in risk assessment and risk management



Qualitative analysis

- **Goal:** gain insight, as complete as possible, into all possible undesired events and their consequences.
- **Failure:** when a system or part of it no longer fulfils one or more desired functions.
- **Limit state:** a condition of a structure beyond which it no longer fulfils the relevant design criteria.
- **Ultimate limit state (ULS):** if exceeded, failure or collapse of a system or structure occurs.
- **Serviceability limit state (SLS):** if exceeded, leads to temporary and/or partial failure.

Quantitative analysis

- The **probabilities** and **consequences** of the defined undesired events are determined in this step.
- Limit state **Z** (by considering the resistance **R** and the loads **S**):

$$\mathbf{Z = R - S}$$

- Failure occurs when **R < S** , so when **Z < 0** .
- **Failure probability:**

$$\mathbf{P(Z < 0) = P(S > R)}$$

- There are several techniques for computing the probability of failure.

Formulation for limit state design

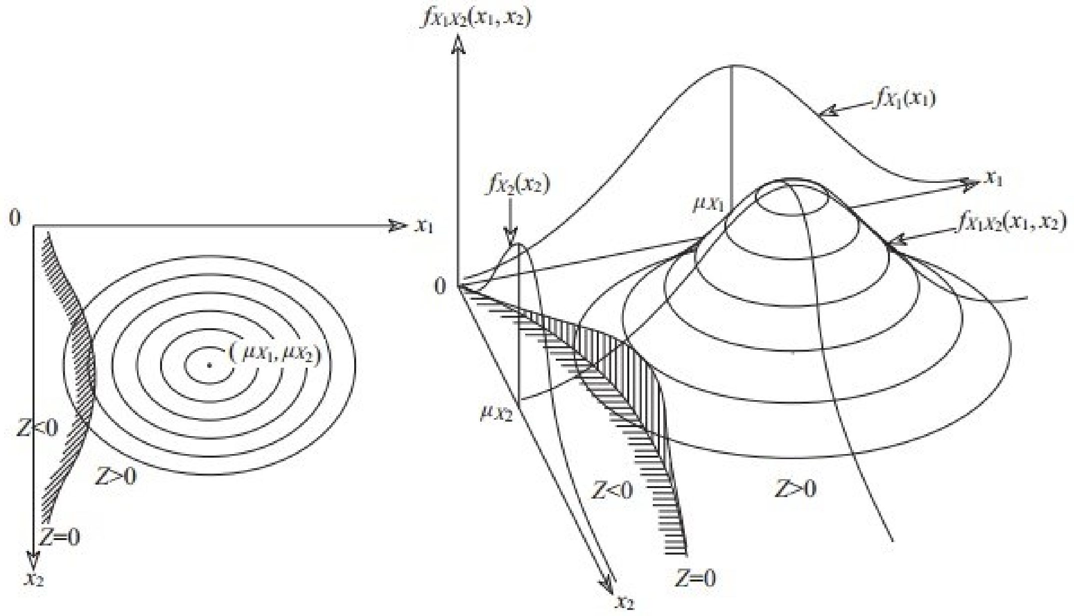
- General formulation:

$$g(\underline{X}) = Z = 0$$

- where the vector $\underline{X}$ consists of n basic variables such as:
  - material properties
  - actions (loads)
  - geometrical properties
  - model uncertainties.
- For all basic variables one has to consider an appropriate **probabilistic** model.
- Negligible variation in time or space: one can consider that variable as **deterministic**.

Failure probability

$$P_f = \Pr[G<0] = \iint_{D_f} f_{x_1,x_2}(x_1,x_2)\,dx_1\,dx_2 \Rightarrow \boxed{P_f = \Phi(-\beta)}$$



- β: reliability index
- The probability of survival (or the reliability) is defined as:

$$P_s = 1 - P_f$$

Reliability methods

Level 0 methods:

- Deterministic design

Level I methods (semi-probabilistic design):

- The uncertain parameters are modelled by **one characteristic value** for load and resistance.
- for example in codes based on the **partial coefficients (γ's)** concept.

Reliability methods

Level II methods (approximation):

- The uncertain parameters are modelled by the **mean** values and the **standard deviations**, and by the **correlation coefficients** between the stochastic variables.
- The stochastic variables are implicitly assumed to be **normally distributed**.

Level III methods (numerical):

- The uncertain quantities are modelled by their **joint distribution** functions.
- The probability of failure calculated exactly, e.g. by numerical integration.

Reliability methods

Level IV methods (risk-based):

- In these methods the consequences (cost) of failure are also taken into account and the **risk (consequence multiplied by the probability of failure**) is used as a measure of the reliability.
- In this way different designs can be compared on an economic basis taking into account **uncertainty**, **costs** and **benefits**.

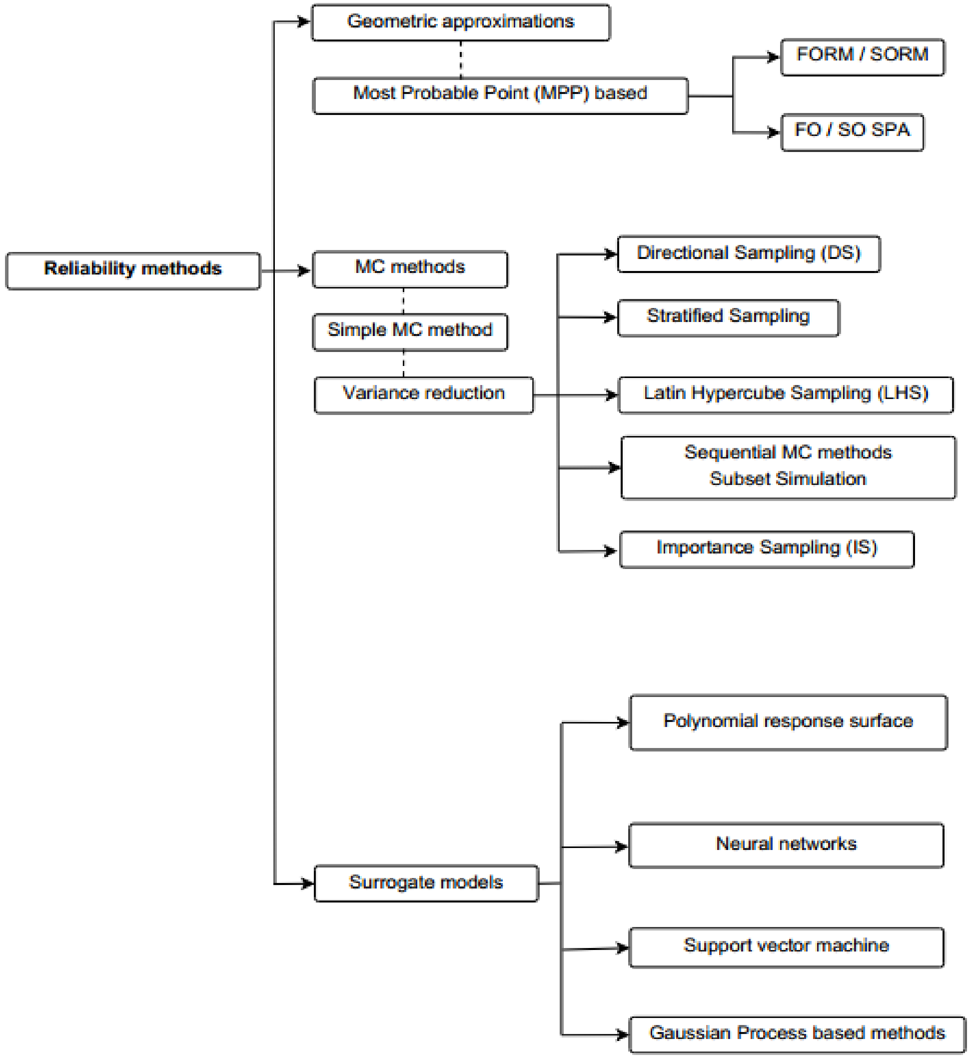
Level III Methods

- Methods of level III evaluate the following integral **explicitly**:

$$P_f = \int_{g(\underline{X})<0} f_{\underline{X}}(\underline{x})\,d\,\underline{x}$$

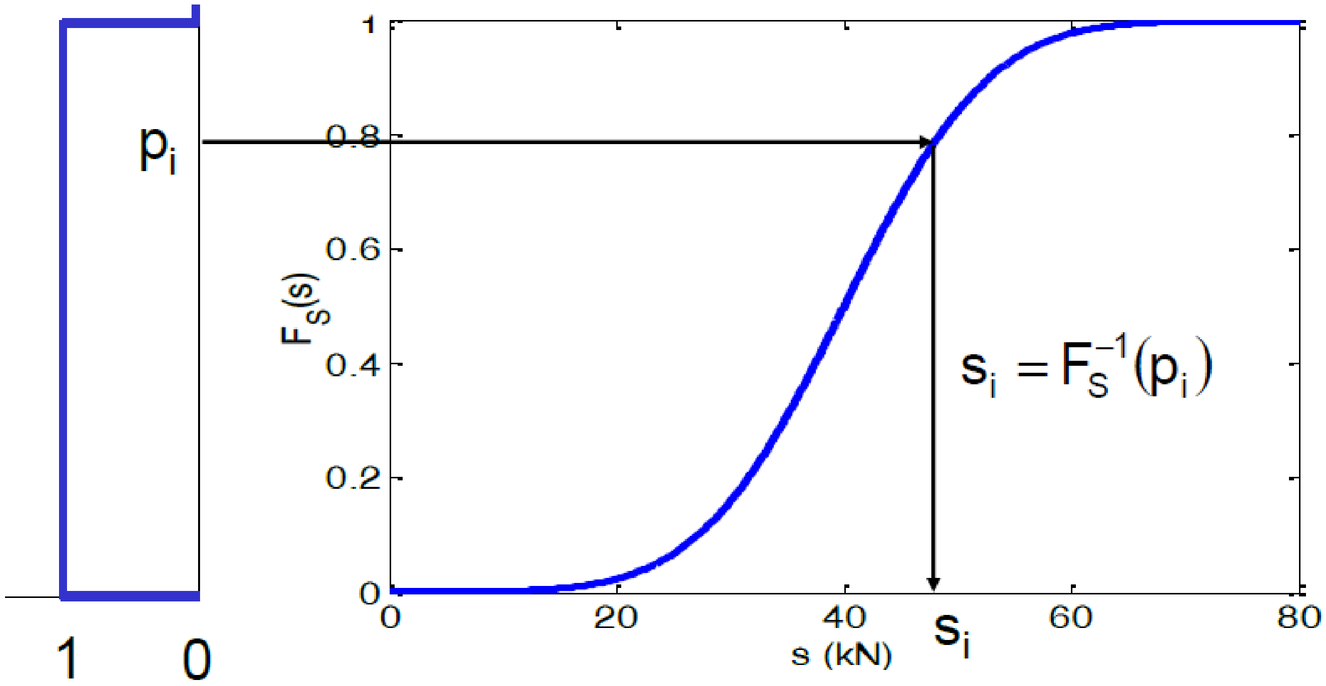
- Direct calculation of this integral is rather difficult in **n > 2**.
- In these case, we use simulation-based techniques, like Monte-Carlo Simulation (MCS)

Reliability methods



Monte-Carlo Simulation (MCS)

- Drawing random numbers from a uniform probability density function between zero and one.



Monte-Carlo Simulation (MCS)

- Drawing samples from joint probability distribution function:

$$F_{\overrightarrow{X}}(\overrightarrow{X}) = F_{X_1}(X_1)F_{X_2|X_1}(X_2 \mid X_1) \ \ldots \ F_{X_m|X_1,X_2,\ldots,X_{m-1}}(X_m \mid X_1, X_2, \ldots, X_{m-1})$$

$$X_1 = F_{X_1}^{-1}\left(X_{u_1}\right)$$

$$X_2 = F_{X_2|X_1}^{-1}\left(X_{u_2} \middle| X_1\right)$$

.

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$$X_m = F_{X_m|X_1,X_2,\ldots,X_{m-1}}^{-1}\left(X_{u_m} \middle| X_1, X_2, \ldots, X_{m-1}\right)$$

Independent variables:

$$X_i = F_{X_i}^{-1}(X_{u_i})$$

Monte-Carlo Simulation (MCS)

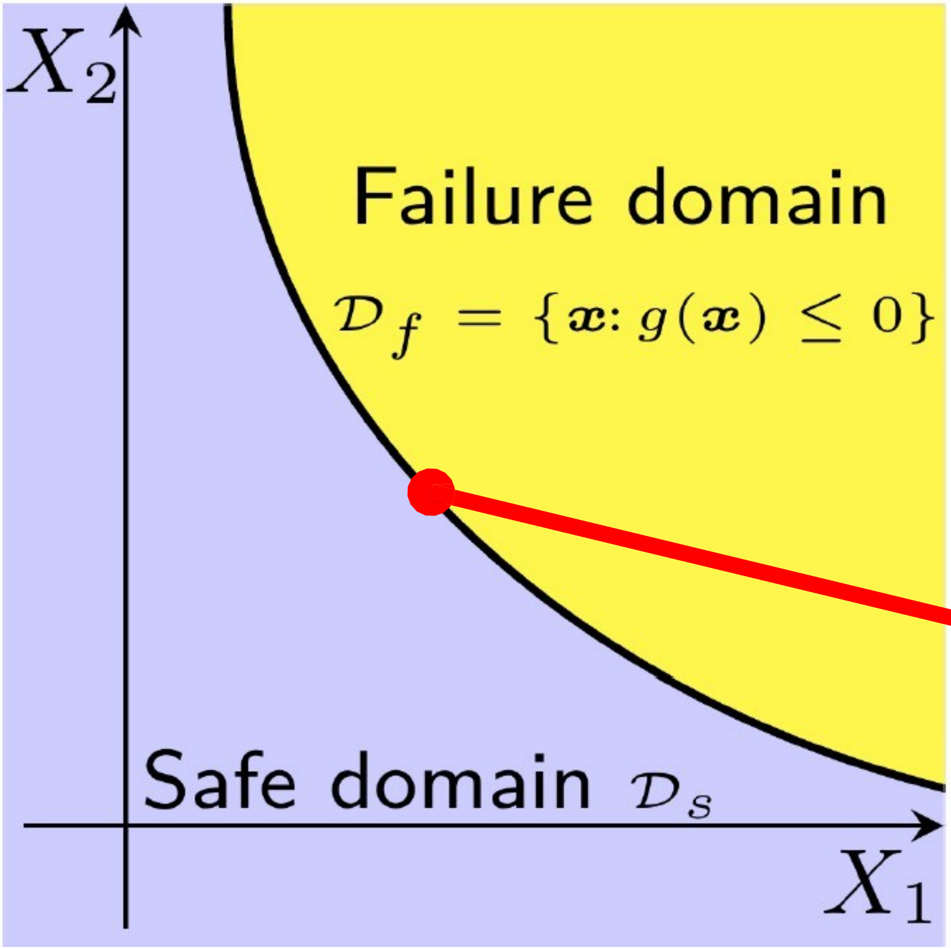
$$P_f = \frac{N_f}{N}$$

- N_f number of failure (g < 0)
- N number of simulations
- In case $N \to \infty$ one obtains the failure probability P_f.
- Criterion for the proper selection of N:

$$N \cong \frac{1}{\delta^2 P_f} \qquad \delta = \sqrt{\frac{1-P_f}{NP_f}}$$

- δ is the target coefficient of variation (relative error)

Level II Methods

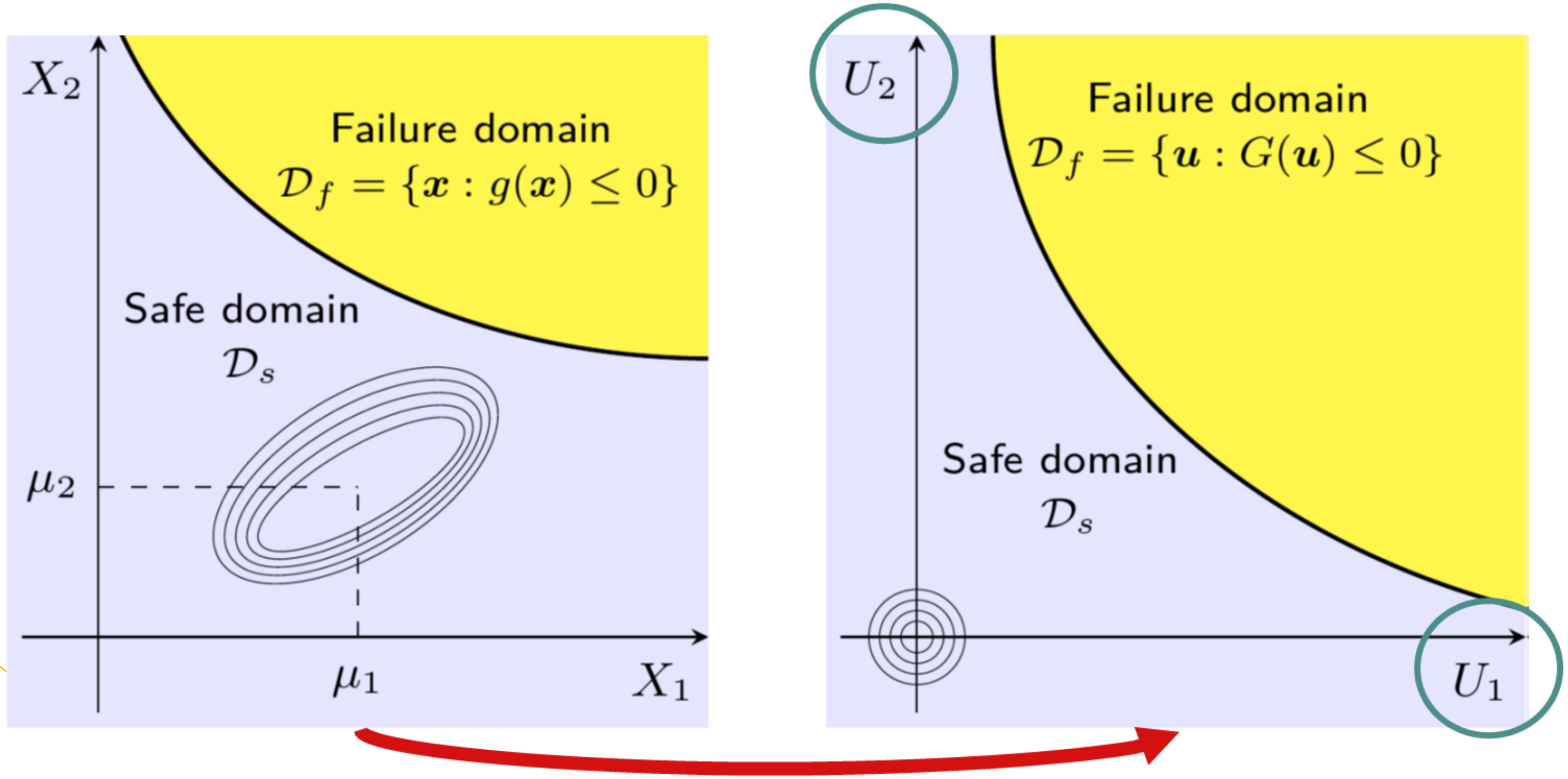


$$P_f = \mathbb{P}(g(X) \leq 0)$$

$$\mathbf{g(x) = 0}$$

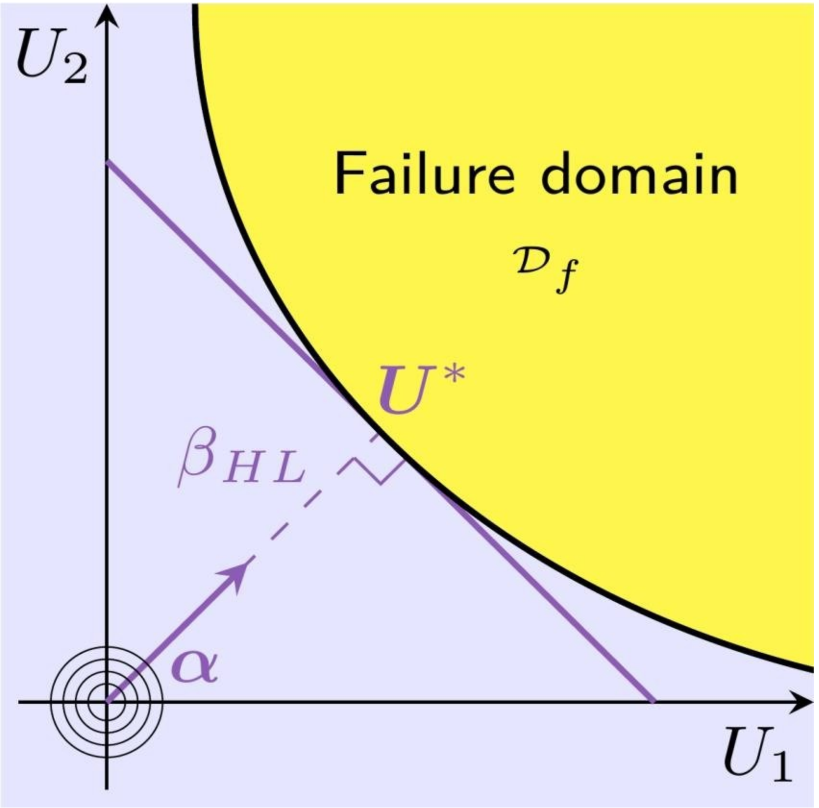
Level II Methods

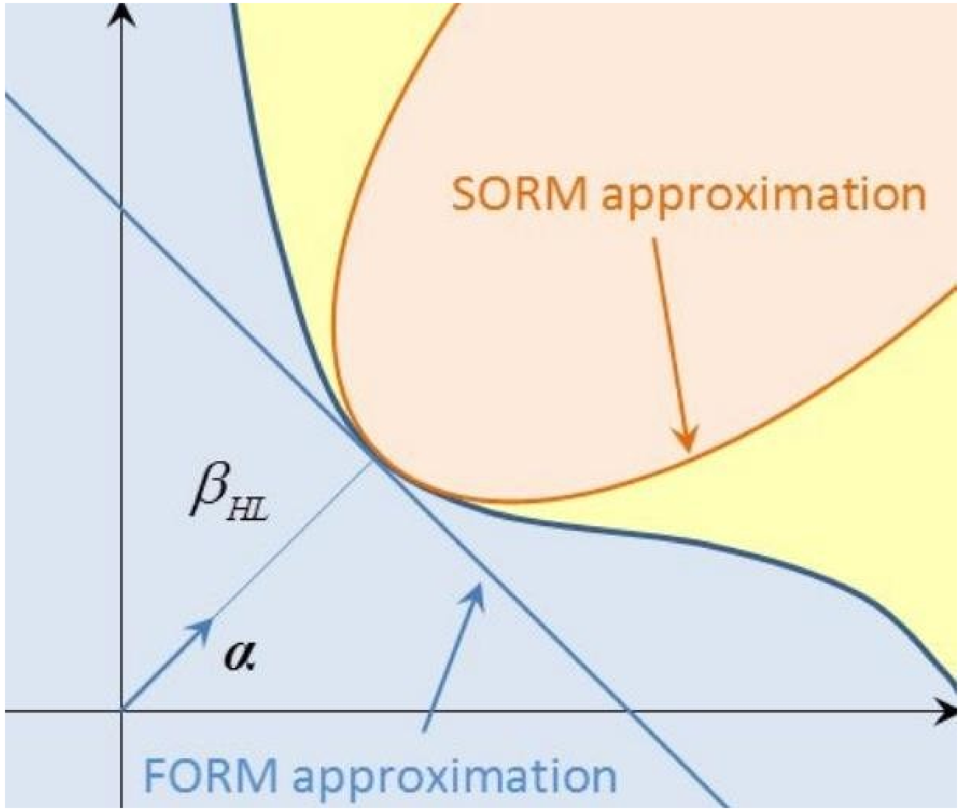
FORM method



Level II Methods

FORM method





Level II Methods

Reliability Index According to Hasofer-Lind

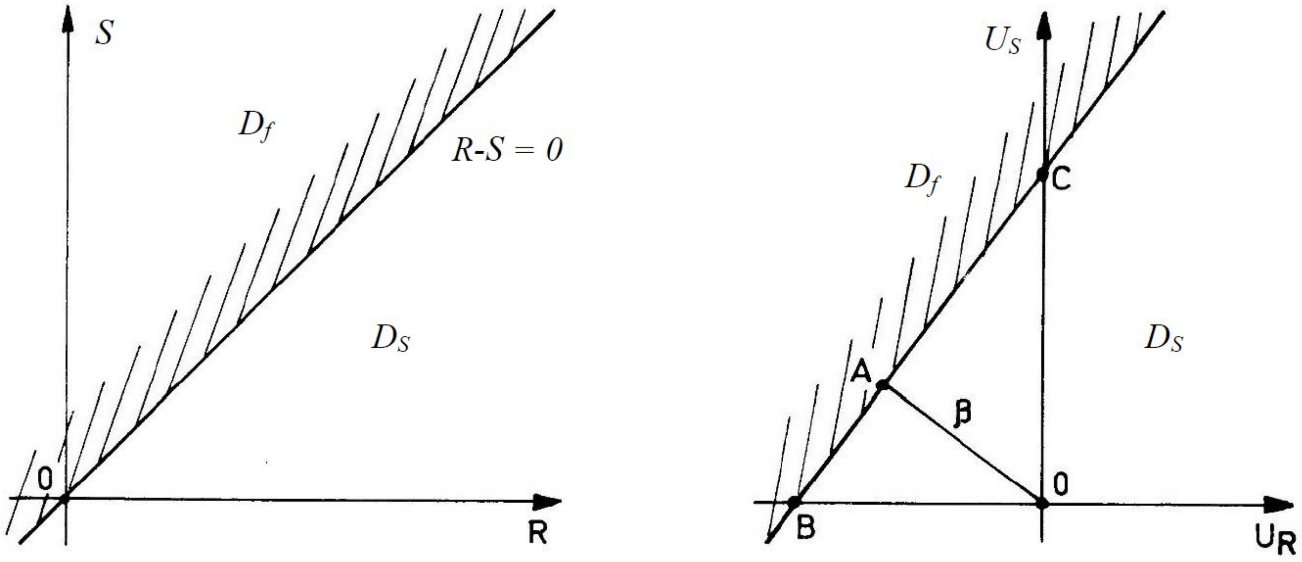
- Proposed by Hasofer and Lind (1974)
- Consider uncorrelated normally distributed variables
- First, normalize the basic variables X_i using:

$$U_i = \frac{X_i - \mu_i}{\sigma_i} \qquad \mu_i = \mathrm{E}[X_i] \text{ and } \sigma_i^2 = \mathrm{Var}[X_i]$$

- In case of normalized basic variables U_i it holds that $E[U_i] = 0$, and $Var[U_i] = 1$.
- The limit state equation becomes $g(\underline{U}) = 0$ in the n-dimensional U-space.

Level II Methods
Reliability Index According to Hasofer-Lind

- The reliability index **β** is equal to the **shortest distance** from the **origin** to the **surface described by g(U) = 0** in the space of the normalized basic variables.

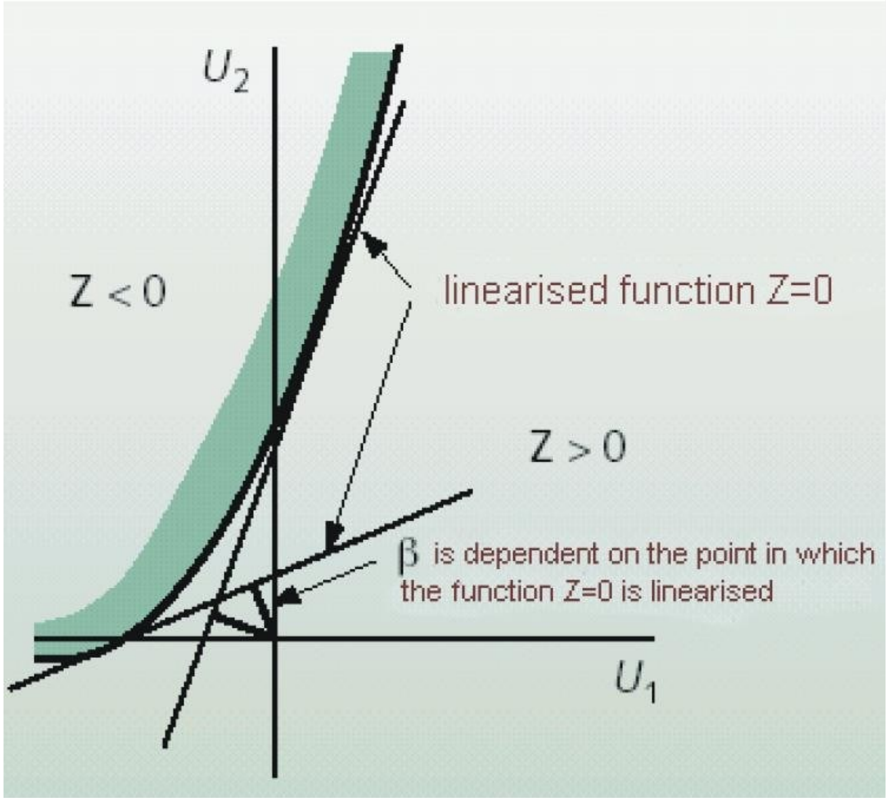


Level II Methods

Reliability Index According to Hasofer-Lind

- The definition of the reliability index according to Hasofer and Lind (1974) does not depend on whether or not the reliability function is **linear**.

$$\beta = \min_{Z=0}\left(\sqrt{U_1^2 + U_2^2}\right)$$



Level II Methods

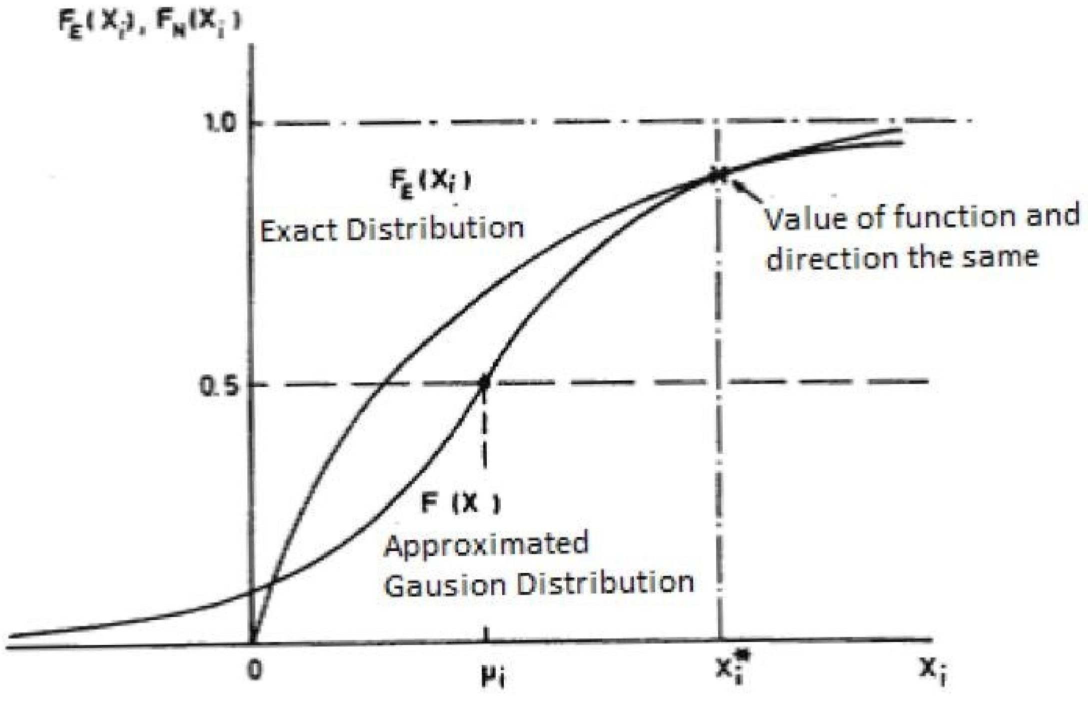
Reliability Index According to Hasofer-Lind

- The closest point to the origin is called **design point**.
- The design point is the point of the limit state equation with the highest probability density, hence in literature one often mentions this as the "**most probable failure point**".
- Finding the design point is **an iterative process**, for which several methods are available. Two methods are:
  1. Method 1: transformation to normal variables
  2. Method 2: direct iteration based on the limit state function

Level II Methods

Reliability Index According to Hasofer-Lind

- The **independent non-normally distributed** base variables have to be transformed to normally distributed base variables.
- Apply Rackwitz-Fiessler algorithm (1977).
- This transformation **assumes** that the values of the real and the approximated probability density function and probability distribution function are equal in the **design point**.

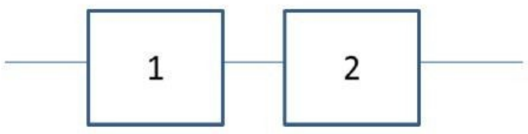
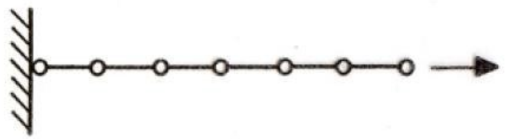
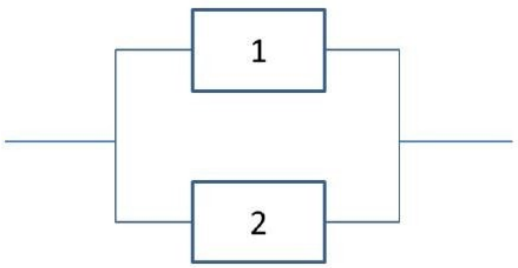
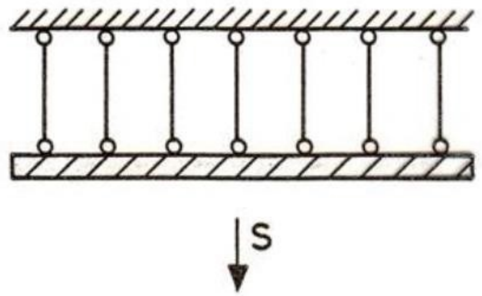


Level II Methods
Reliability Index According to Hasofer-Lind

- **Dependent random base variables:**
- Have to be transformed to independent variables.
- If there is a **clear functional relation** between the variables, it is often possible to formulate the reliability function in such a way that variables are eliminated.
- In many cases the relation between the variables is not known exactly and **statistical dependence is involved**. In this case the base variables can be transformed. A general transformation method is **the Rosenblatt-transformation**.

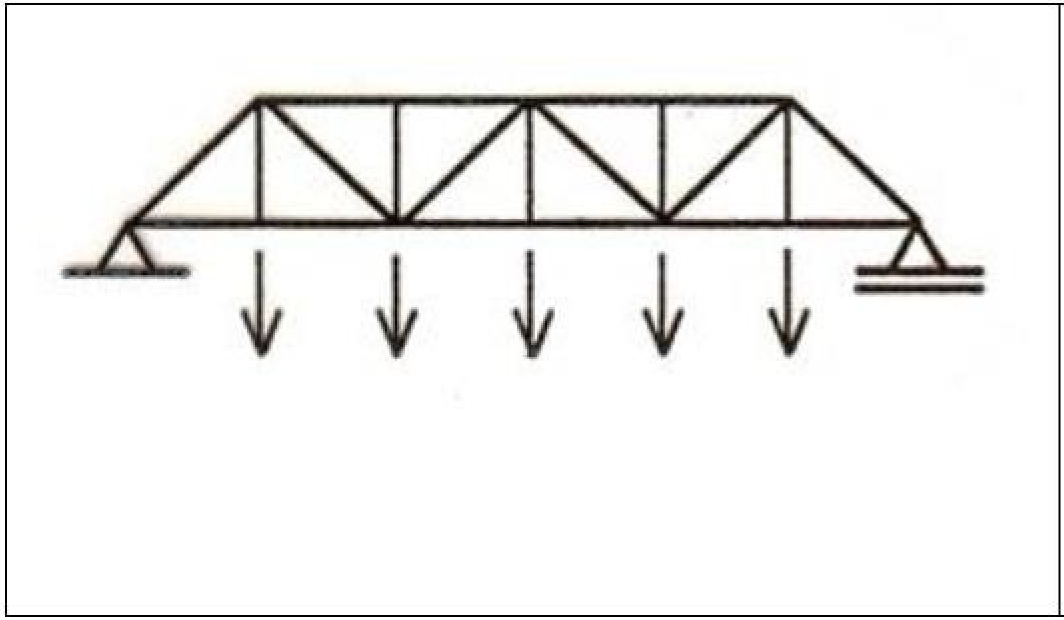
Reliability of systems

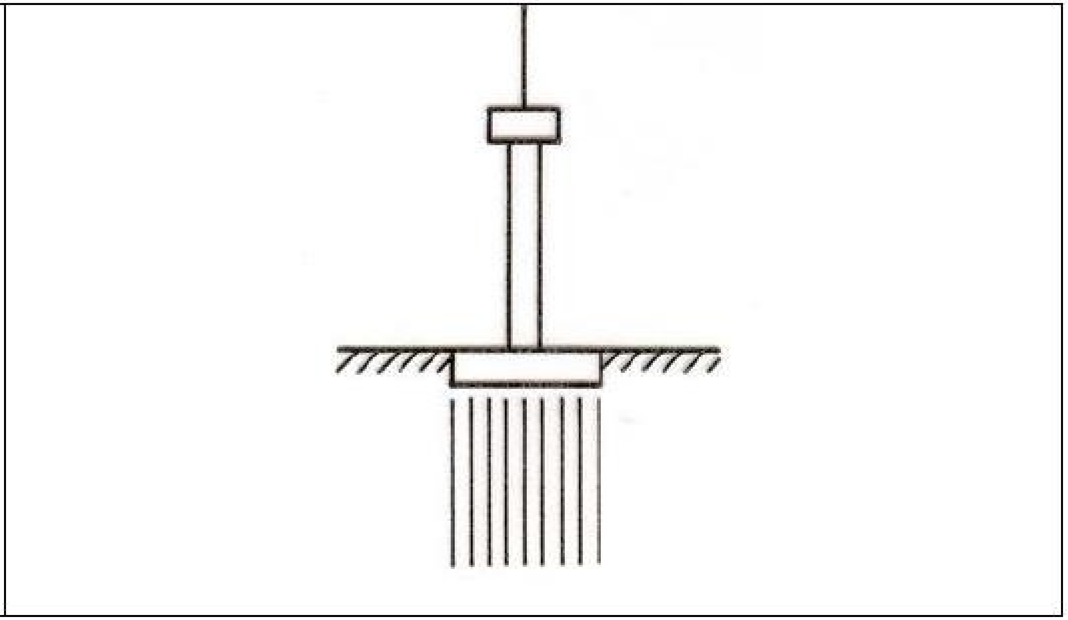
- Within a **series** system, failure of a single element will always lead to the failure of the entire system.
- Within a **parallel** system, failure of one element can be compensated by another element.

Type	System representation	Simple example (structural engineering)
Series		
Parallel		

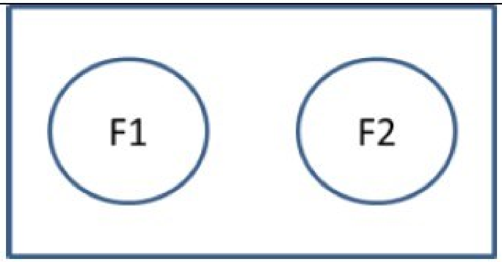
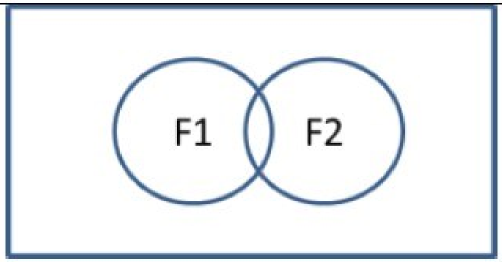
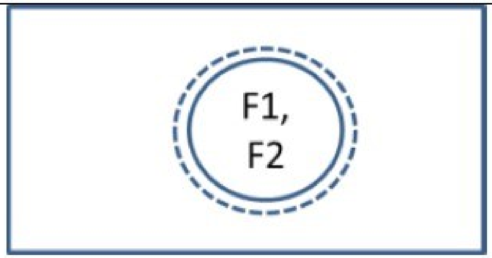
Reliability of systems

- Examples of a series and parallel system: a **bridge** (series system; left); and a **pile foundation** (parallel system, right)





Series system

Case	Mutually exclusive	Independent	Dependent
Correlation coefficient. $\rho_{Z1,Z2}$ =	-1	0	1
Venn diagram			
System failure probability $P(F)$	$P(F_1)+P(F_2)$	$P(F_1)+P(F_2)-P(F_1)\cdot P(F_2)$	$Max(P(F_1,P(F_2))$

Series system

- Bounds proposed by Ditlevsen (1977)

$$Max\left(\Phi(-\beta_1)\Phi(-\beta_2^*);\Phi(-\beta_1^*)\Phi(-\beta_2)\right) \le \boxed{P(F_1 \cap F_2)} \le \Phi(-\beta_1)\Phi(-\beta_2^*) + \Phi(-\beta_1^*)\Phi(-\beta_2)$$

in which:

$$\beta_1 = -\Phi^{-1}(P(F_1)),\ \text{so } P(F_1) = \Phi(-\beta_1)$$

$$\beta_2 = -\Phi^{-1}(P(F_2))$$

$$\beta_1^* = \frac{\beta_1 - \rho\beta_2}{\sqrt{1-\rho^2}}$$

$$\beta_2^* = \frac{\beta_2 - \rho\beta_1}{\sqrt{1-\rho^2}}$$

ρ is the correlation coefficient between F_1 and F_2.

Series system

- Three failure mechanisms

Lower bound:

$$P(F) = P(F_1) + P(F_2) - P(F_1 \cap F_2) + P(F_3) - P(F_1 \cap F-) - P(F_2 \cap F_3)$$

Upper bound:

$$P(F) = P(F_1) + P(F_2) - P(F_1 \cap F_2) + P(F_3) - Max\{ P(F_1 \cap F_3), P(F_2 \cap F_3) \}$$

Series system

- n failure mechanisms

$$P(F) \le \sum_i P_i - \sum_{i \ge 2} \max_{j<i} P_{ij}$$

$$P(F) \ge \sum_i P_i - \sum_{i \ge 2} \sum_{j<i} P_{ij}$$

In which:

$$P_i = P\{Z_i < 0\} = \Phi(-\beta_i)$$

$$\mathrm{P}_{ij} = P\left\{Z_i < 0 \text{ and } Z_j < 0\right\} = \Phi(-\beta_i)\Phi(-\beta_j{}^*) + \Phi(-\beta_i{}^*)\Phi(-\beta_j)$$

Parallel system

- The bounds for a system with multiple identical elements with each failure probability P_i are as follows:

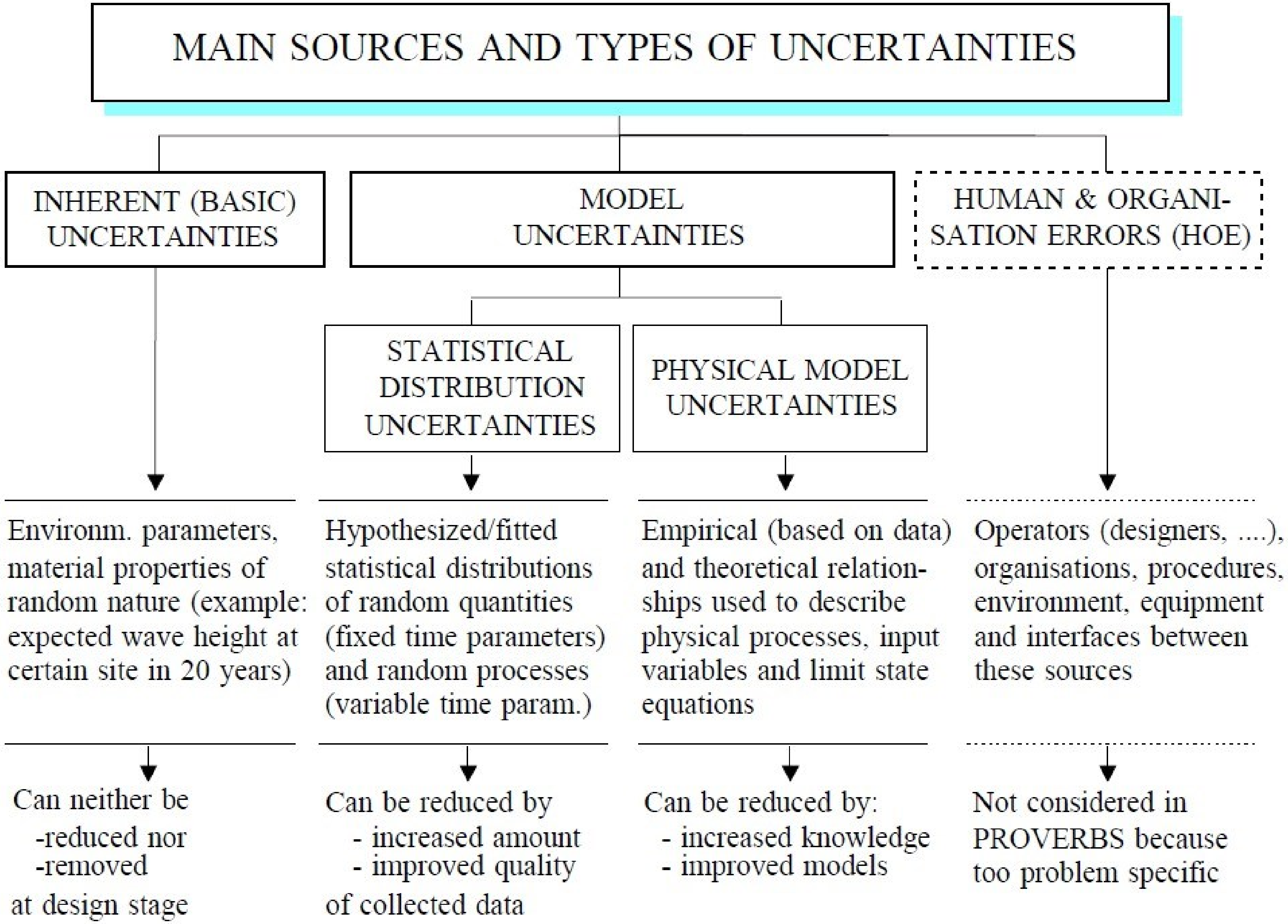
$$0 \le P_f \le Min(P_i)$$

- The **lower** bound is found for a situation in which failures are **mutually exclusive**.
- The **upper** bound is valid for a case in which the failure are **fully dependent**.

Reliability-based Design Optimization

Sources and types of uncertainties

- Ref.: PROVERBS



Why RBDO?

- Design under uncertainties
- Optimal balance between performance and cost
- Increasing safety
- The RBDO solution is basically achieved by jointly **performing a reliability analysis** and **solving an optimization problem**.

General formulation of a RBDO problem

$$\boldsymbol{d}^{*} = \arg\min_{\boldsymbol{d} \in \mathbb{D}} \mathfrak{c}\left(\boldsymbol{d}\right) \quad \text{subject to:} \quad \begin{cases} \boxed{\mathfrak{f}_j\left(\boldsymbol{d}\right) \leq 0,} & \{j = 1, \ldots, s\}, \\ \boxed{\mathbb{P}\left(g_k\left(\boldsymbol{X}\left(\boldsymbol{d}\right), \boldsymbol{Z}\right) \leq 0\right) \leq \bar{P}_{f_k},} & \{k = 1, \ldots, n\}. \end{cases}$$

Soft constraints

Hard constraints

Cost function

- **Design variables** ($\boldsymbol{d}$): to be optimized
- $\boldsymbol{X}$: a set of random variables indexed on the design variables which may represent **manufacturing tolerances**
- Z: **environmental variables** which are parameters that may be random but cannot be controlled by the designer, e.g. the loading

General formulation of a RBDO problem

- **Soft constraints:** are simple functions that bound the design space.
- **Hard constraints:** are limit-state functions which describe the performance of the system.

An alternative formulation using the reliability index

$$\boldsymbol{d}^* = \arg\min_{\boldsymbol{d}\in\mathbb{D}} \mathfrak{c}\left(\boldsymbol{d}\right) \quad \text{subject to: } \begin{cases} \mathfrak{f}_j\left(\boldsymbol{d}\right) \leq 0, & \{j = 1, \ldots, s\}, \\ \bar{\beta}_k - \beta_k\left(\boldsymbol{X}\left(\boldsymbol{d}\right), \boldsymbol{Z}\right) \leq 0, & \{k = 1, \ldots, n\}, \end{cases}$$

$$\bar{\beta}_k = \Phi^{-1}\left(1 - \bar{P}_{f_k}\right) \text{ and } \beta_k = \Phi^{-1}\left(1 - P_{f_k}\right)$$

- are the target and structural reliability indices of the *k*-th limit-state
- Φ is the standard Gaussian cumulative distribution

Solution of a RBDO problem

Two-level approach

- Nested loops
- The outer loop explores the design space
- The inner one computes the corresponding failure probability
- **Two classical approaches**:

1. Reliability index approach
2. Performance measure approach

- **RIA:** uses FORM in the inner loop
  - Hasofer-Lind-Rackwitz-Fiessler (HLRF) and its improved version (iHLRF)
  - low numerical efficiency
  - easy to implement.
- **PMA:** inner loop consists of an inverse FORM analysis
  - searches for minimum performance target point (MPTP).

Solution of a RBDO problem

Mono-level approach

- Avoiding the reliability analysis at each iteration of the optimization process
- The problem is converted into an **equivalent single loop deterministic** process
- Enforcing the **Karush-Kuhn-Tucker optimality conditions** of the reliability analysis as additional constraints

Decoupled approach

- Sequentially solving a deterministic optimization problem followed by a reliability analysis
- Sequential Optimization and Reliability Assessment (SORA)
- Converts the probabilistic constraint into an **equivalent deterministic constraint** using the minimum performance target point

RBDO solution using surrogate models

- Avoid expensive model evaluations
- For highly non-linear or when multiple design points exist
- For real-world problems using time-consuming high-fidelity computational models (e.g. finite element).

- Gaussian process a.k.a. Kriging
- Polynomial chaos expansions
- Polynomial chaos-Kriging
- Low-rank approximation
- Support vector machines

Optimization algorithms

- Interior-point (see *fmincon* in MATLAB);
- Sequential quadratic programming (see *fmincon* in MATLAB);
- Genetic algorithm (see *ga* in MATLAB);
- Constrained (1 + 1)-CMA-ES (Arnold and Hansen (2012);
- Hybrid algorithms, which refine the solution identified by a genetic algorithm or (1+1)-CMA-ES by an additional gradient-based minimization.

UQLab intro

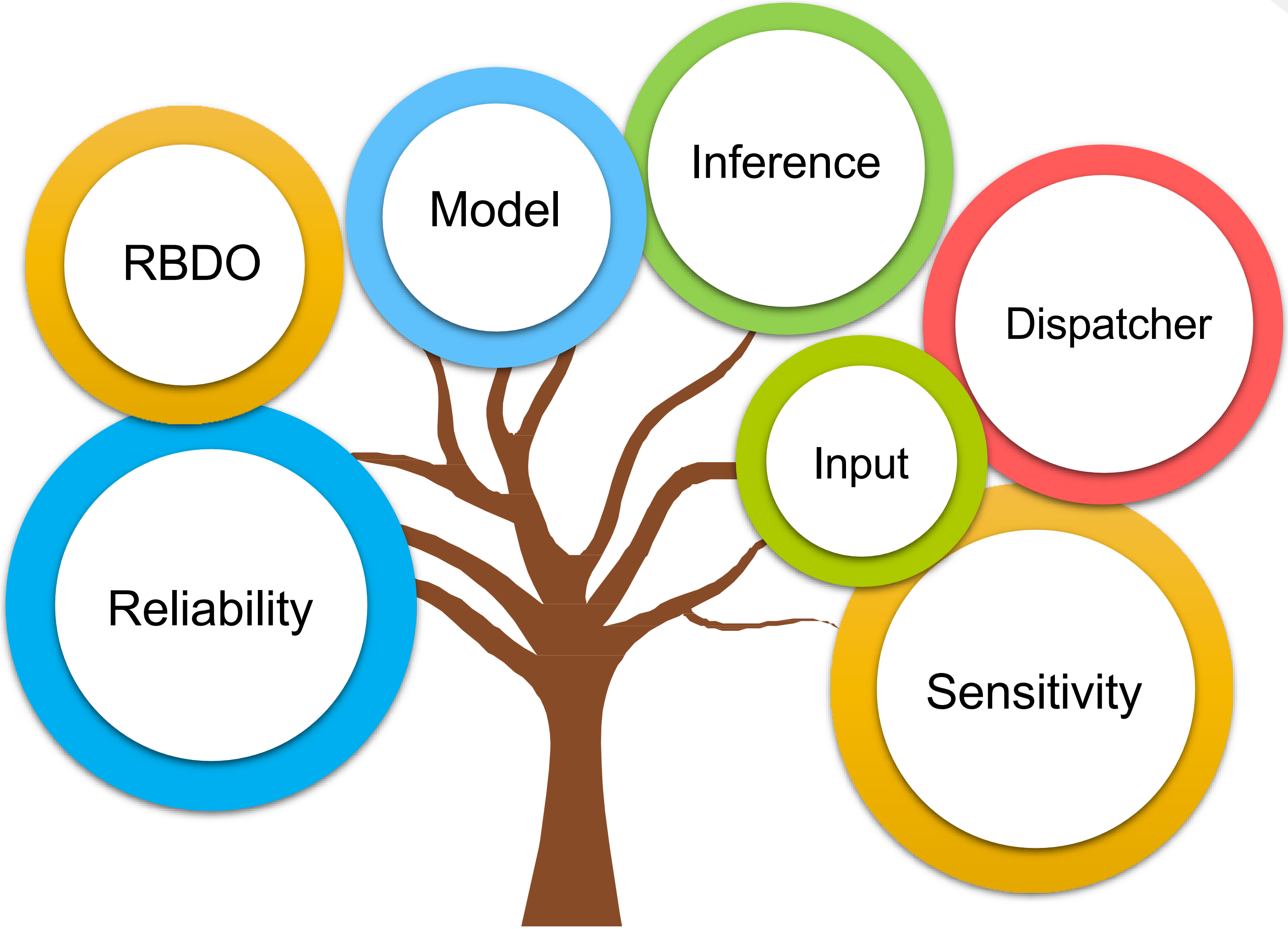
- MATLAB®-based Uncertainty Quantification framework
- Highly optimized open source algorithms
- Fast learning curve for beginners
- Modular structure, easy to extend
- Academic/ Commercial/ Group
- More that 3000 academic users

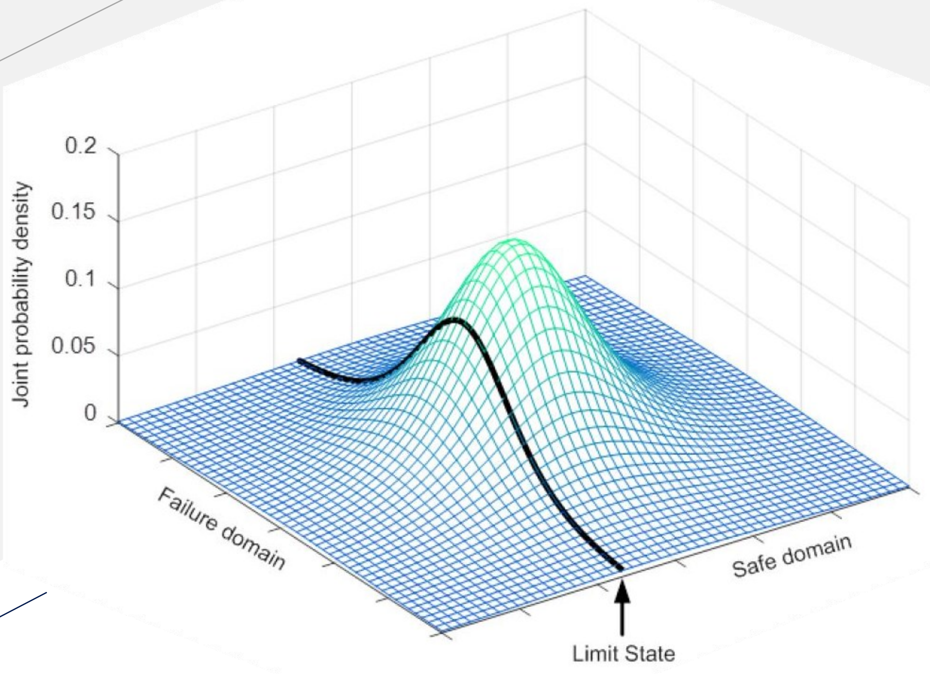
UQWorld

- Connect with fellow uncertainty quantification (UQ) practitioners.
- Discuss the practice of UQ in science and engineering, use cases, and best practices.
- Share and discuss your problem, experience, and expertise.
- News, updates, and other resources.

- www.uqworld.org
- Sign up using any email address.

UQLab modules





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