# S&DS 238 Problem Set #9

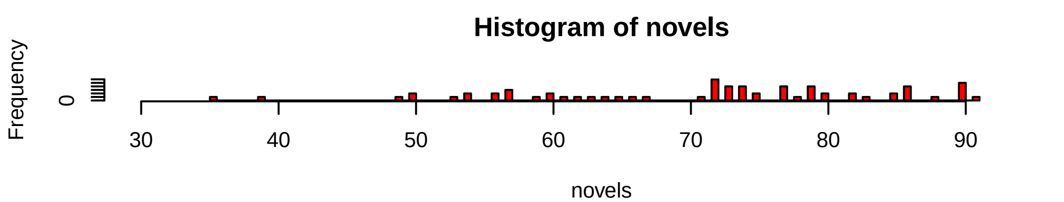
Shaun Radgowski

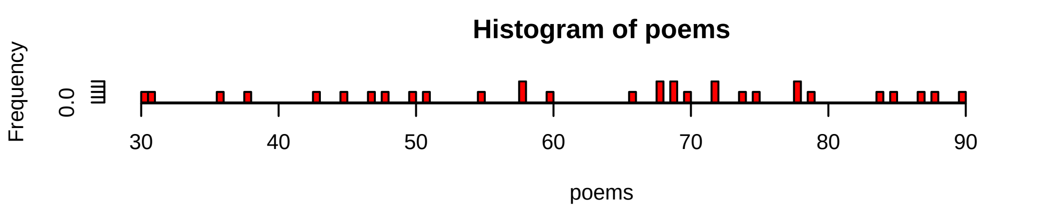
November 14, 2020

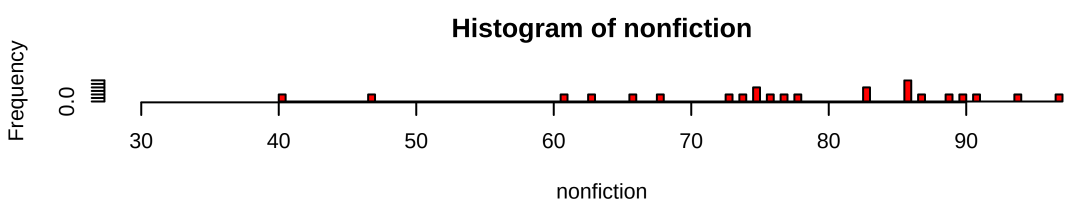
## Problem 1

### (1a)

```r
d <- read.csv("http://www.stat.yale.edu/~jtc5/238/data/cost-of-the-muse.csv")
novels <- d[d$Type1 == 1, 3]
poems <- d[d$Type1 == 2, 3]
nonfiction <- d[d$Type1 == 3, 3]

par(mfrow=c(3, 1))
rg <- range(d[, 3])
hist(novels, 100,  col="red", xlim=rg)
hist(poems, 100,  col="red", xlim=rg)
hist(nonfiction, 100,  col="red", xlim=rg)
```



## (1b)

```
lik <- function(th) {
  mu1 <- th[1]; sig1 <- th[2]; mu2 <- th[3]; sig2 <- th[4]; mu3 <- th[5]; sig3 <- th[6]
  return(
    prod(dnorm(x=novels, mean=mu1, sd=sig1)) *
    prod(dnorm(x=poems, mean=mu2, sd=sig2)) *
    prod(dnorm(x=nonfiction, mean=mu3, sd=sig3))
  )
}

prior <- function(th) {
  mu1 <- th[1]; sig1 <- th[2]; mu2 <- th[3]; sig2 <- th[4]; mu3 <- th[5]; sig3 <- th[6]
  return(
    dunif(mu1, min = 0, max = 100) *
    dunif(mu2, min = 0, max = 100) *
    dunif(mu3, min = 0, max = 100) *
    dunif(sig1, min = 0, max = 100) *
    dunif(sig2, min = 0, max = 100) *
    dunif(sig3, min = 0, max = 100)
  )
}

post <- function(th) {
  if((th[2] < 0) | (th[4] < 0) | (th[6] < 0)) {
    return(0)
  }

  return(prior(th) * lik(th))
}

nit <- 100000
results <- matrix(0, nrow = nit, ncol = 6)
th <- c(70, 10, 70, 10, 70, 10)   # Initial value
results[1, ] <- th
for(i in 2:nit) {
  cand <- th + rnorm(6)
  ratio <- post(cand)/post(th)
  lik(th)
  u <- runif(n = 1, min = 0, max = 1)
  if(u < ratio){
    th <- cand
  }
  results[i,] <- th
}

r <- data.frame(results)
names(r) <- c("mu1", "sig1", "mu2", "sig2", "mu3", "sig3")

quantile(r[,1], prob=c(0.025, 0.5, 0.975))
```

```
##     2.5%      50%    97.5%
## 68.17678 71.45977 74.70351
```

```r
quantile(r[,2], prob=c(0.025, 0.5, 0.975))
```

```
##     2.5%      50%    97.5%
## 11.18235 13.21035 15.92576
```

```r
quantile(r[,3], prob=c(0.025, 0.5, 0.975))
```

```
##     2.5%      50%    97.5%
## 56.80276 63.12158 69.71787
```

```r
quantile(r[,4], prob=c(0.025, 0.5, 0.975))
```

```
##     2.5%      50%    97.5%
## 14.01155 17.70353 23.44678
```

```r
quantile(r[,5], prob=c(0.025, 0.5, 0.975))
```

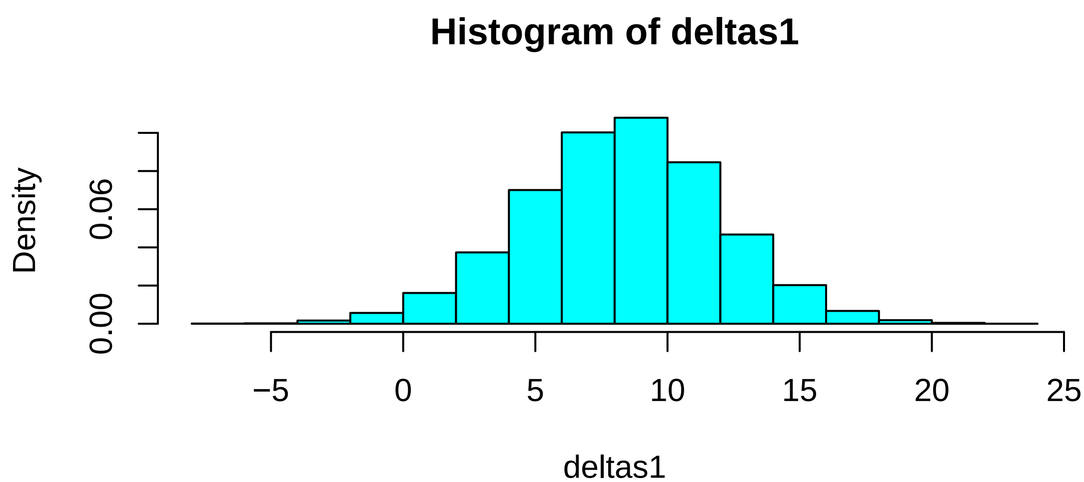
```
##     2.5%      50%    97.5%
## 70.84469 76.87117 83.08590
```

```r
quantile(r[,6], prob=c(0.025, 0.5, 0.975))
```

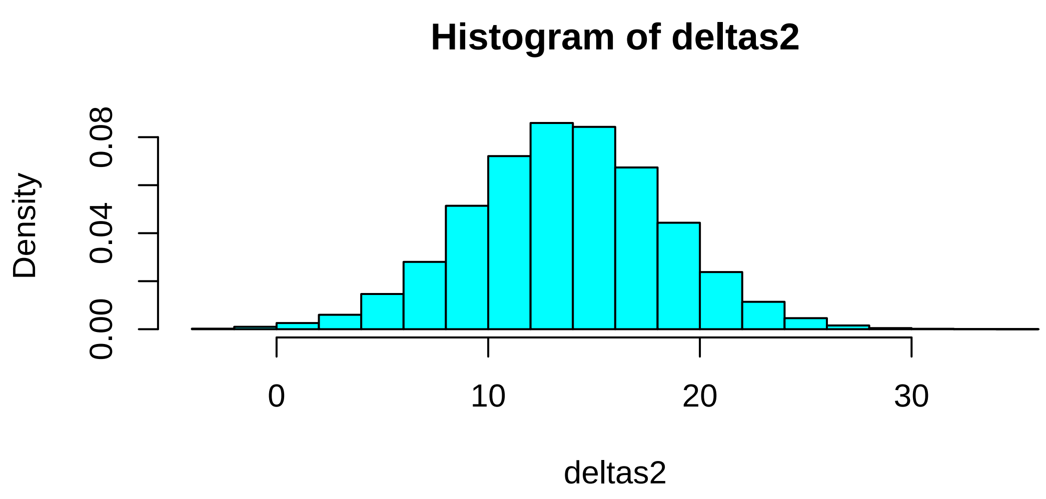
```
##     2.5%      50%    97.5%
## 11.10928 14.64732 20.49193
```

### (1c)

```r
deltas1 <- r[,1] - r[,3]
deltas2 <- r[,5] - r[,3]
hist(deltas1, col=5, prob=T)
```



```r
hist(deltas2, col=5, prob=T)
```




```r
quantile(deltas1, prob=c(0.025, 0.975))
```

```
##      2.5%     97.5%
##  0.8399253 15.5079762
```

```r
quantile(deltas2, prob=c(0.025, 0.975))
```

```
##     2.5%     97.5%
##  4.501772 22.837328
```

### (1d)

```r
# Probability of mu2 < mu1 < mu3
sum(r[,3] < r[,1] & r[,1] < r[,5])/nit
```

```
## [1] 0.92283
```

```r
# Other orderings:
sum(r[,1] < r[,3] & r[,3] < r[,5])/nit
```

```
## [1] 0.01396
```

```r
sum(r[,1] < r[,5] & r[,5] < r[,3])/nit
```

```
## [1] 0.00083
```

```r
sum(r[,3] < r[,5] & r[,5] < r[,1])/nit
```

```
## [1] 0.06077
```

```r
sum(r[,5] < r[,1] & r[,1] < r[,3])/nit
```

```
## [1] 0.00049
```

```r
sum(r[,5] < r[,3] & r[,3] < r[,1])/nit
```

```
## [1] 0.00107
```

It seems that the second most likely ordering is $\mu_2 < \mu_3 < \mu_1$, and the third most likely ordering is $\mu_1 < \mu_2 < \mu_3$.

# Problem 2

## (2a)

```r
library(rjags)
```

```
## Loading required package: coda
```

```
## Linked to JAGS 4.3.0
```

```
## Loaded modules: basemod,bugs
```

```r
mymodel <- "
  model{
    for(i in 1:67) {
      novels[i] ~ dnorm(mu1, tau1)
    }
    for(i in 1:32) {
      poems[i] ~ dnorm(mu2, tau2)
    }
    for(i in 1:24) {
      nonfiction[i] ~ dnorm(mu3, tau3)
    }

    mu1 ~ dunif(0, 100)
    mu2 ~ dunif(0, 100)
    mu3 ~ dunif(0, 100)
    sig1 ~ dunif(0, 100)
    sig2 ~ dunif(0, 100)
    sig3 ~ dunif(0, 100)
    tau1 <- 1/(sig1^2)
    tau2 <- 1/(sig2^2)
```

```r
    tau3 <- 1/(sig3^2)
  }
"

jm <- jags.model(file=textConnection(mymodel),
  data=list(novels=novels, poems=poems, nonfiction=nonfiction),
  inits=list(mu1=50, sig1=10, mu2=50, sig2=10, mu3=50, sig3=10))
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 123
##    Unobserved stochastic nodes: 6
##    Total graph size: 139
##
## Initializing model
```

```r
cs <- coda.samples(jm, c("mu1", "sig1", "mu2", "sig2", "mu3", "sig3"), nit)
s <- as.data.frame(cs[[1]])

quantile(s[,1], prob=c(0.025, 0.5, 0.975))
```

```
##     2.5%      50%    97.5%
## 68.23338 71.44308 74.66846
```

```r
quantile(s[,2], prob=c(0.025, 0.5, 0.975))
```

```
##     2.5%      50%    97.5%
## 56.86144 63.18390 69.49094
```

```r
quantile(s[,3], prob=c(0.025, 0.5, 0.975))
```

```
##     2.5%      50%    97.5%
## 70.80163 76.87970 83.03378
```

```r
quantile(s[,4], prob=c(0.025, 0.5, 0.975))
```

```
##     2.5%      50%    97.5%
## 11.23250 13.21944 15.90323
```

```r
quantile(s[,5], prob=c(0.025, 0.5, 0.975))
```

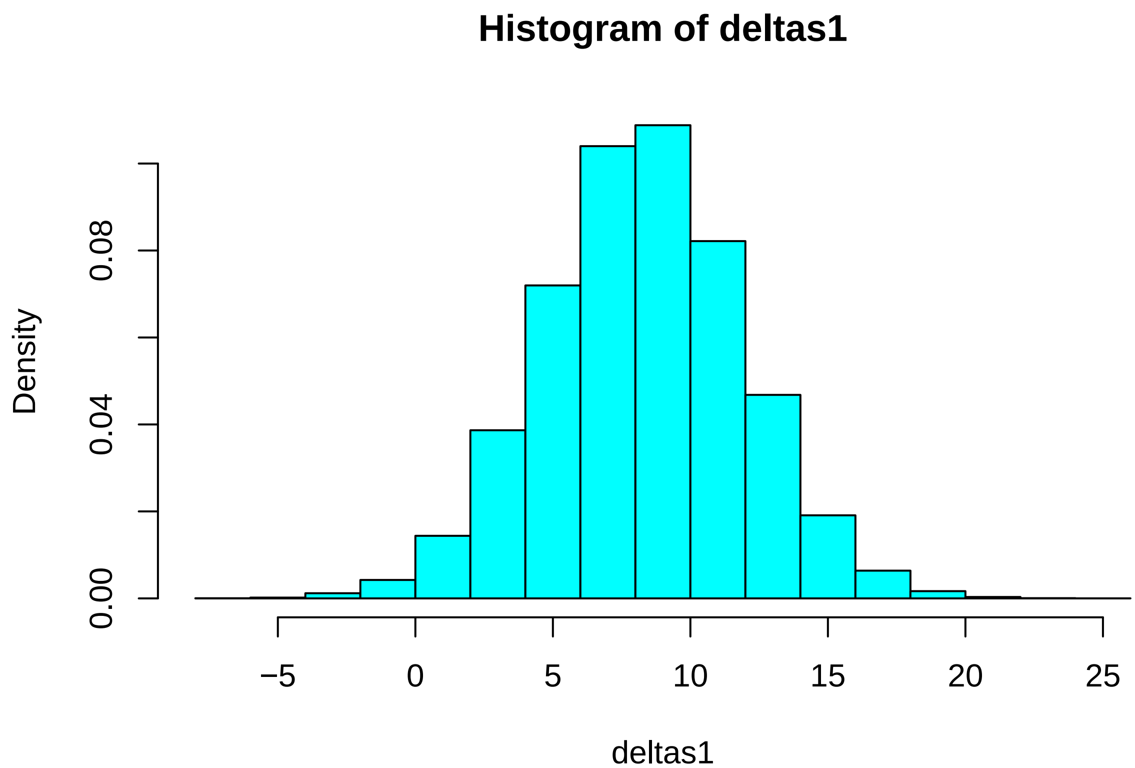
```
##     2.5%      50%    97.5%
## 14.05437 17.79097 23.47244
```

```r
quantile(s[,6], prob=c(0.025, 0.5, 0.975))
```

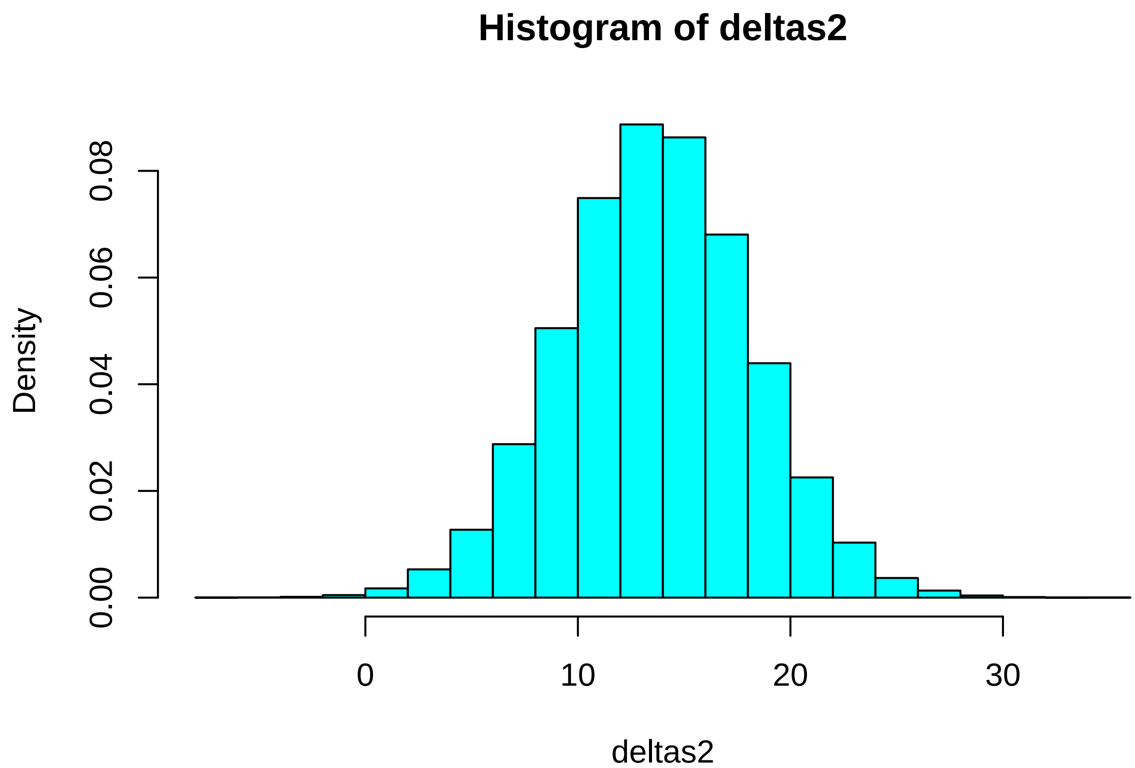
```
##     2.5%      50%    97.5%
## 11.14625 14.63275 20.44779
```

**(2b)**

```
deltas1 <- s[,1] - s[,2]
deltas2 <- s[,3] - s[,2]
hist(deltas1, col=5, prob=T)
```




```
hist(deltas2, col=5, prob=T)
```




```
quantile(deltas1, prob=c(0.025, 0.975))
```

```
##      2.5%     97.5%
##  1.188981 15.378575
```

```
quantile(deltas2, prob=c(0.025, 0.975))
```

```
##      2.5%     97.5%
##  4.990541 22.498423
```

**(2c)**

```
# Probability of mu2 < mu1 < mu3
sum(s[,2] < s[,1] & s[,1] < s[,3])/nit
```

```
## [1] 0.93002
```

```
# Other orderings:
sum(s[,1] < s[,2] & s[,2] < s[,3])/nit
```

```
## [1] 0.01076
```

```
sum(s[,1] < s[,3] & s[,3] < s[,2])/nit
```

```
## [1] 0.00031
```

```
sum(s[,2] < s[,3] & s[,3] < s[,1])/nit
```

```
## [1] 0.05786
```

```
sum(s[,3] < s[,1] & s[,1] < s[,2])/nit
```

```
## [1] 0.00023
```

```
sum(s[,3] < s[,2] & s[,2] < s[,1])/nit
```

```
## [1] 0.00082
```

Again, $\mu_2 < \mu_1 < \mu_3$ is the most common, with $\mu_2 < \mu_3 < \mu_1$ coming in second and $\mu_1 < \mu_2 < \mu_3$ coming in third. This confirms that the results from the two methods ("from scratch" and using JAGS) are similar.

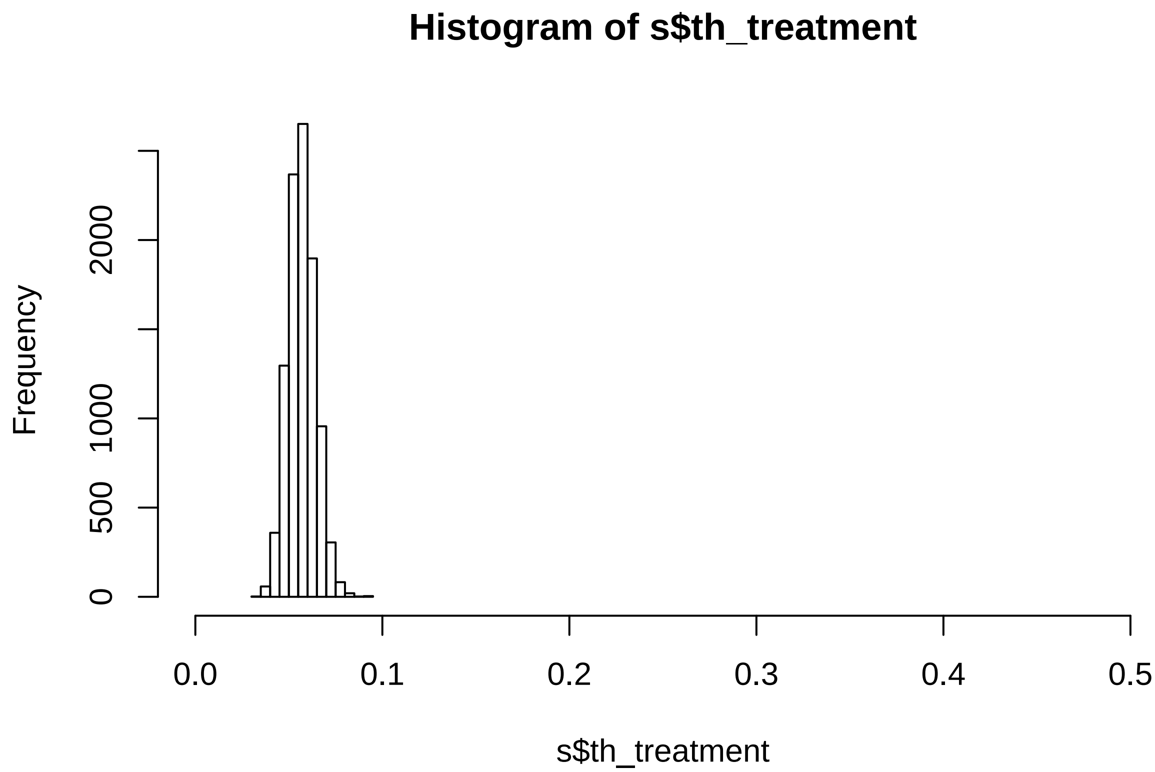
# Problem 3

## (3a)

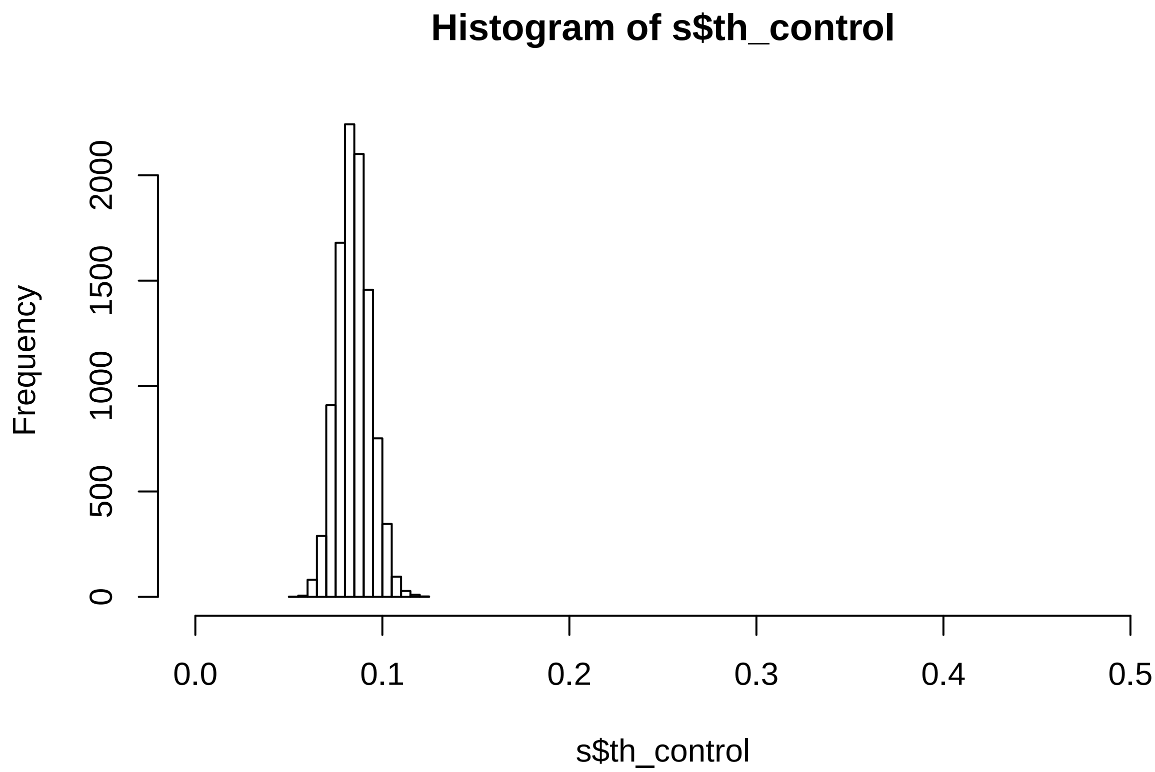
```
treatments <- 56
controls <- 84
mymodel <- "
  model {
    treatments ~ dbin(th_treatment, 1000)
    controls ~ dbin(th_control, 1000)
    th_treatment ~ dunif(0, 1)
    th_control ~ dunif(0, 1)
  }
"

jm <- jags.model(textConnection(mymodel), data=list(treatments=treatments, controls=controls))
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 2
##    Unobserved stochastic nodes: 2
##    Total graph size: 7
##
## Initializing model
```

```r
cs <- coda.samples(jm, c("th_treatment", "th_control"), 10000)
s <- as.data.frame(x=cs[[1]])
hist(s$th_treatment, xlim=c(0, 0.5))
```




```r
hist(s$th_control, xlim=c(0, 0.5))
```




### (3b)

```
quantile(s[,1], prob=c(0.025, 0.975))
```

```
##      2.5%     97.5%
## 0.0683488 0.1027466
```

```
quantile(s[,2], prob=c(0.025, 0.975))
```

```
##       2.5%      97.5%
## 0.04353467 0.07205569
```

### (3c)

```
reduction = ((s[,1] - s[,2]) * 100)/s[,1]
quantile(reduction, prob=c(0.025, 0.975))
```

```
##      2.5%     97.5%
##  7.547761 51.950546
```

This result is similar to the [8.2, 52.6] confidence interval described in the study's abstract.

# Problem 4

## (4a)

The uniform prior is the same as Beta(1, 1), meaning $\theta^{\alpha-1}\theta^{\beta-1}$ with $\alpha = 1, \beta = 1$. The likelihoods are thus $L(\theta_{treatment}) \propto \theta^{56}(1-\theta)^{944}$, $L(\theta_{control}) \propto \theta^{84}(1-\theta)^{916}$.

Thus, the exact posterior distribution for $\theta_{treatment}$ is $\theta_{treatment}^{56}(1-\theta_{treatment})^{944}$, and the exact posterior distribution for $\theta_{control}$ is $\theta_{control}^{84}(1-\theta_{control})^{916}$.

## (4b)

```r
qbeta(p=c(.025, .975), shape1=85, shape2=917)
```

```
## [1] 0.06838831 0.10284123
```

```r
qbeta(p=c(.025, .975), shape1=57, shape2=945)
```

```
## [1] 0.04340988 0.07203633
```

These match the results from Problem 3.

## (4c)

```r
sample1 <- rbeta(10000, shape1=85, shape2=917)
sample2 <- rbeta(10000, shape1=57, shape2=945)
reduction = ((sample1 - sample2) * 100)/sample1
quantile(reduction, prob=c(0.025, 0.975))
```

```
##      2.5%     97.5%
##  7.563521 52.174891
```

This result is again similar to the [8.2, 52.6] confidence interval described in the study's abstract, matching the results from 3c.