On-policy Control with Approximation

Episodic Semi-gradient Control

- Parameterized action-value function with update target approximation being the Monte Carlo return or any n-step Sarsa returns, for one-step Sarsa

$$\mathbf{w}_{t+1} \doteq \mathbf{w}_t + \alpha \Big[R_{t+1} + \gamma \hat{q}(S_{t+1}, A_{t+1}, \mathbf{w}_t) - \hat{q}(S_t, A_t, \mathbf{w}_t) \Big] \nabla \hat{q}(S_t, A_t, \mathbf{w}_t).$$

- Policy improvement: ε-greedy

Episodic Semi-gradient Sarsa for Estimating $\hat{q} \approx q_*$

Input: a differentiable action-value function parameterization $\hat{q} : \mathcal{S} \times \mathcal{A} \times \mathbb{R}^d \to \mathbb{R}$
Algorithm parameters: step size $\alpha > 0$, small $\varepsilon > 0$
Initialize value-function weights $\mathbf{w} \in \mathbb{R}^d$ arbitrarily (e.g., $\mathbf{w} = \mathbf{0}$)

Loop for each episode:
    $S, A \leftarrow$ initial state and action of episode (e.g., ε-greedy)
    Loop for each step of episode:
        Take action A, observe R, S'
        If S' is terminal:
            $\mathbf{w} \leftarrow \mathbf{w} + \alpha\big[R - \hat{q}(S, A, \mathbf{w})\big]\nabla\hat{q}(S, A, \mathbf{w})$
            Go to next episode
        Choose A' as a function of $\hat{q}(S', \cdot, \mathbf{w})$ (e.g., ε-greedy)
        $\mathbf{w} \leftarrow \mathbf{w} + \alpha\big[R + \gamma\hat{q}(S', A', \mathbf{w}) - \hat{q}(S, A, \mathbf{w})\big]\nabla\hat{q}(S, A, \mathbf{w})$
        $S \leftarrow S'$
        $A \leftarrow A'$

Semi-gradient n-step Sarsa

- N-step return as the update target

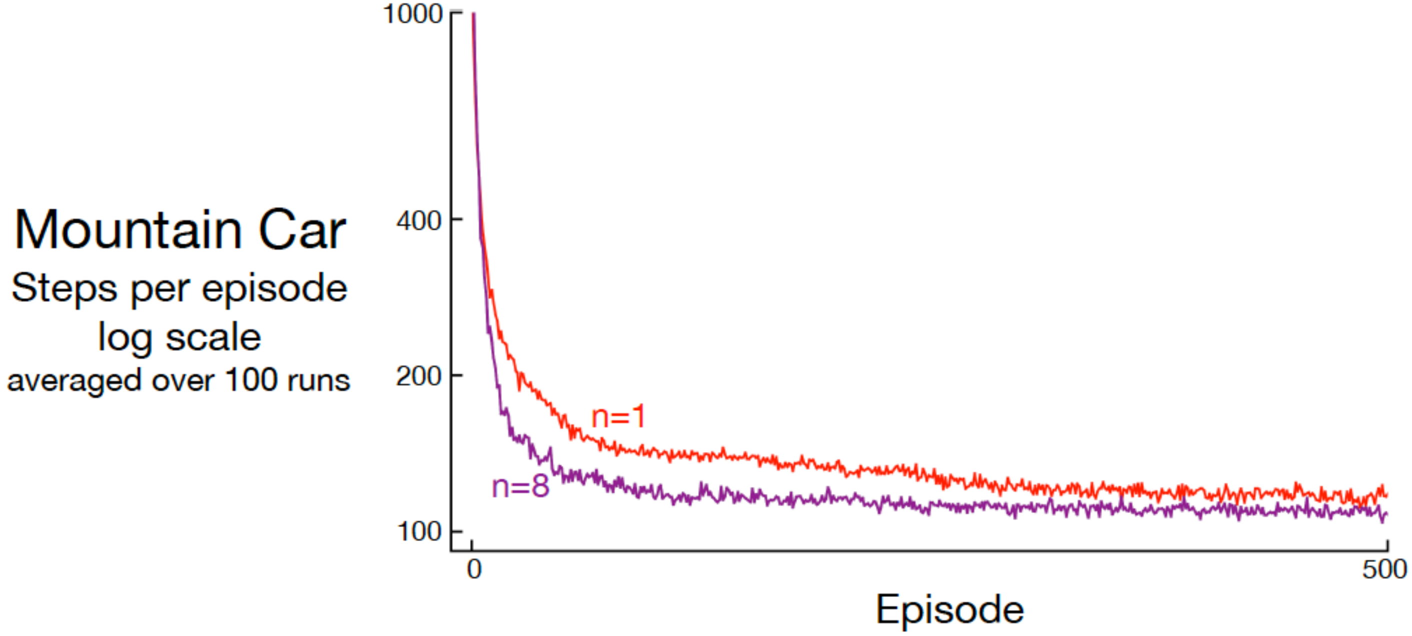
$$G_{t:t+n} \doteq R_{t+1}+\gamma R_{t+2}+\cdots+\gamma^{n-1}R_{t+n}+\gamma^n\hat{q}(S_{t+n},A_{t+n},\mathbf{w}_{t+n-1}), \quad t+n<T, \tag{10.4}$$

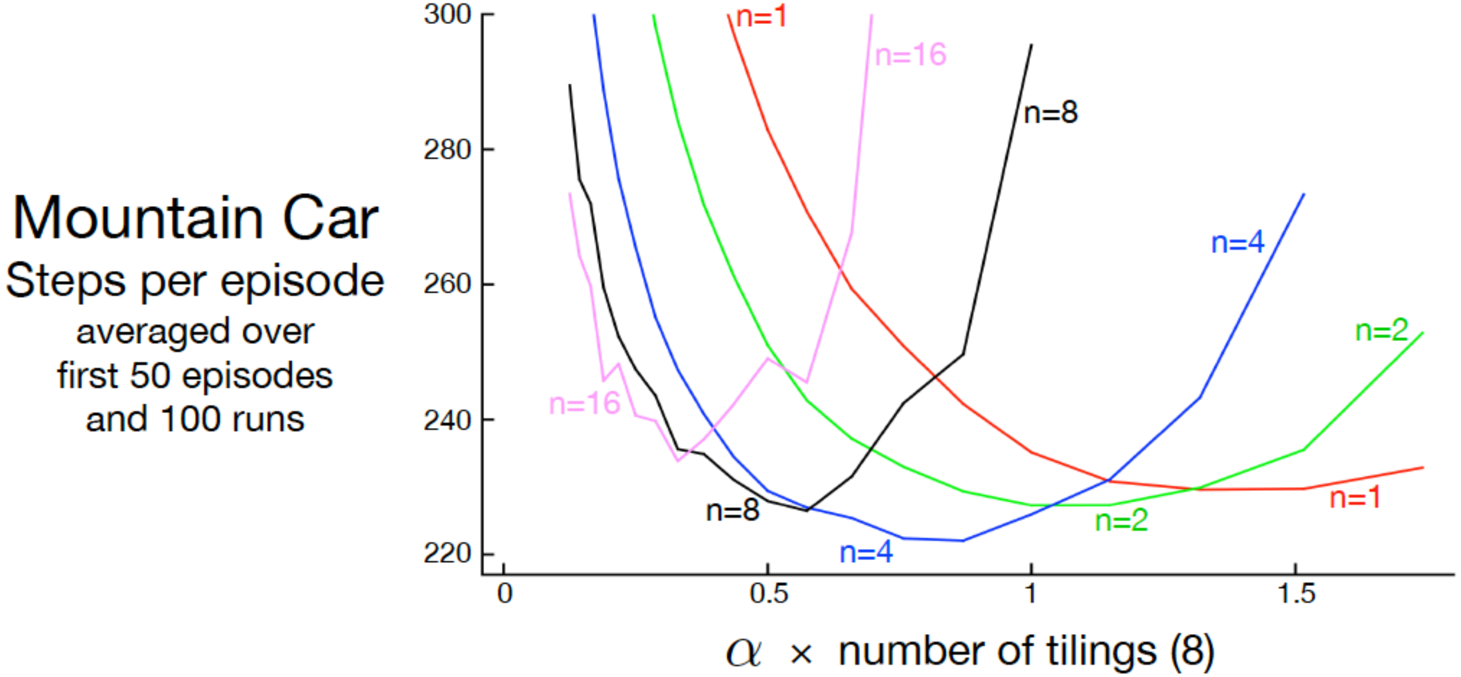
with $G_{t:t+n} \doteq G_t$ if $t+n \geq T$, as usual. The n-step update equation is

$$\mathbf{w}_{t+n} \doteq \mathbf{w}_{t+n-1} + \alpha\left[G_{t:t+n} - \hat{q}(S_t,A_t,\mathbf{w}_{t+n-1})\right]\nabla\hat{q}(S_t,A_t,\mathbf{w}_{t+n-1}), \qquad 0 \leq t < T. \tag{10.5}$$

Mountain car

- Grid tilings used to convert the two continuous state variables (position and velocity) to binary features (linear combination)
- Exploration from optimistic action value initialization





Average Reward: A New Problem Setting for Continuing Tasks

- No discounting (commonly considered in the classical theory of DP)
- Average reward following a policy
  - Steady state distribution μ_π assumed

  to exist for any π and is independent of S_0

  = ergodicity

$$
\begin{aligned}
r(\pi) &\doteq \lim_{h\to\infty} \frac{1}{h} \sum_{t=1}^{h} \mathbb{E}[R_t \mid S_0, A_{0:t-1} \sim \pi] \\
&= \lim_{t\to\infty} \mathbb{E}[R_t \mid S_0, A_{0:t-1} \sim \pi], \\
&= \sum_s \mu_\pi(s) \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a) r,
\end{aligned}
$$

- Differential returns: differences between rewards and the average reward

$$
G_t \doteq R_{t+1} - r(\pi) + R_{t+2} - r(\pi) + R_{t+3} - r(\pi) + \cdots .
$$

Differential value functions

- Differential value functions and Bellman equations

$$v_\pi(s) = \sum_a \pi(a|s) \sum_{r,s'} p(s', r|s, a)\Big[r - r(\pi) + v_\pi(s')\Big],$$

$$q_\pi(s, a) = \sum_{r,s'} p(s', r|s, a)\Big[r - r(\pi) + \sum_{a'} \pi(a'|s') q_\pi(s', a')\Big],$$

$$v_*(s) = \max_a \sum_{r,s'} p(s', r|s, a)\Big[r - \max_\pi r(\pi) + v_*(s')\Big], \text{ and}$$

$$q_*(s, a) = \sum_{r,s'} p(s', r|s, a)\Big[r - \max_\pi r(\pi) + \max_{a'} q_*(s', a')\Big]$$

- Differential TD errors

$$\delta_t \doteq R_{t+1} - \bar{R}_t + \hat{v}(S_{t+1}, \mathbf{w}_t) - \hat{v}(S_t, \mathbf{w}_t),$$

and

$$\delta_t \doteq R_{t+1} - \bar{R}_t + \hat{q}(S_{t+1}, A_{t+1}, \mathbf{w}_t) - \hat{q}(S_t, A_t, \mathbf{w}_t),$$

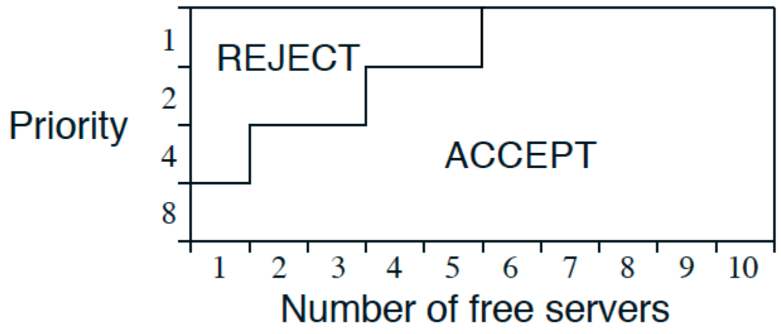
Differential semi-gradient Sarsa for estimating $\hat{q} \approx q_*$

Input: a differentiable action-value function parameterization $\hat{q} : \mathcal{S} \times \mathcal{A} \times \mathbb{R}^d \to \mathbb{R}$
Algorithm parameters: step sizes $\alpha, \beta > 0$
Initialize value-function weights $\mathbf{w} \in \mathbb{R}^d$ arbitrarily (e.g., $\mathbf{w} = \mathbf{0}$)
Initialize average reward estimate $\bar{R} \in \mathbb{R}$ arbitrarily (e.g., $\bar{R} = 0$)

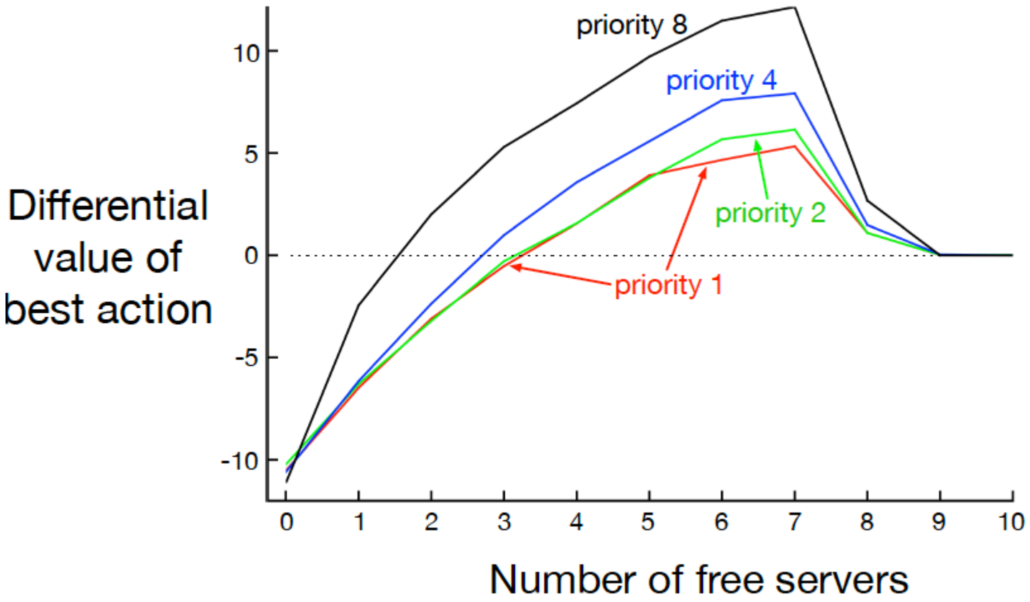
Initialize state S, and action A
Loop for each step:
  Take action A, observe R, S'
  Choose A' as a function of $\hat{q}(S', \cdot, \mathbf{w})$ (e.g., ε-greedy)
  $\delta \leftarrow R - \bar{R} + \hat{q}(S', A', \mathbf{w}) - \hat{q}(S, A, \mathbf{w})$
  $\bar{R} \leftarrow \bar{R} + \beta\delta$
  $\mathbf{w} \leftarrow \mathbf{w} + \alpha\delta\nabla\hat{q}(S, A, \mathbf{w})$
  $S \leftarrow S'$
  $A \leftarrow A'$

An Access-Control Queuing Task example

- 10 servers, 4 priorities (equal to reward), action = accept or reject head of the queue (never ending with random order), each server becomes free with probability 0.06
- Differential semi-gradient one-step Sarsa



POLICY



VALUE FUNCTION

Deprecating the Discounted Setting

- For continuous control tasks with function approximation, the average of the discounted returns is: $r(\pi)/(1-\gamma),$
  - I.e. proportional to the average reward (with the same ordering of all policies)
- γ has no effect due to symmetry
  - Each time step is exactly the same as every other and every reward will appear exactly once in each position in some return
- For control with FA we have lost the policy improvement theorem