Mathematics for Linear Regression

Standard Deviation $\sigma = \frac{\sqrt{\sum_{i=1}^{n}(x_i-\mu)^2}}{n-1}$

Variance = $\left(\frac{x_i-\mu}{\sigma}\right)$

Probability Distribution Function (PDF) = $\frac{1}{2\pi\sigma}\left(e^{-\frac{1}{2}\left[\frac{(x-\mu)}{\sigma}\right]^2}\right)$

Normalization = $\left(\frac{x_i-\mu}{Max_i - Min_i}\right)$

Simple linear Regression with one variable:

Linear Regression = $Y = a_0 + a_1 x + e$

$Y = a_0 + a_1 x$

$e = Y - a_0 - a_1 x$

$$Sum\ of\ error\ squares\ (S_r) = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (Y_i - a_0 - a_1 x)^2$$

$\frac{\partial s_r}{\partial a_0} = 0 \rightarrow -2\sum(y_i - a_0 - a_1 x_i) = 0 \rightarrow \sum Y_i = n a_0 + a_1 \sum x_i$

$\frac{\partial s_r}{\partial a_1} = 0 \rightarrow -2\sum(y_i - a_0 - a_1 x_i)x_i = 0 \rightarrow \sum Y_i x_i = a_0 \sum x_i + a_1 \sum x_i^2$

$$\begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum Y_i \\ \sum Y_i x_i \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum Y_i \\ \sum Y_i x_i \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \frac{1}{n\sum x_i^2 - \sum x_i \sum x_i} \begin{bmatrix} \sum x_i^2 & -\sum x_i \\ -\sum x_i & n \end{bmatrix} \begin{bmatrix} \sum Y_i \\ \sum Y_i x_i \end{bmatrix} = \frac{1}{n\sum x_i^2 - \sum x_i \sum x_i} \begin{bmatrix} \sum x_i^2 \sum Y_i - \sum x_i \sum Y_i x_i \\ -\sum x_i \sum Y_i + n \sum Y_i x_i \end{bmatrix}$$

$$a_0 = \frac{\sum x_i^2 \sum Y_i - \sum x_i \sum Y_i x_i}{n\sum x_i^2 - \sum x_i \sum x_i}$$

$$a_1 = \frac{-\sum x_i \sum Y_i + n \sum Y_i x_i}{n\sum x_i^2 - \sum x_i \sum x_i}$$

Linear Regression with Two variables:

$Y = a_0 + a_1 x_1 + a_2 x_2 + e$

$e = Y - a_0 - a_1 x_1 - a_2 x_2$

$$Sum\ of\ error\ squares\ (S_r) = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (Y_i - a_0 - a_1 x_{1i} - a_2 x_{2i})^2$$

$\frac{\partial s_r}{\partial a_0} = 0 \rightarrow -2\sum(Y_i - a_0 - a_1 x_{1i} - a_2 x_{2i}) = 0 \rightarrow \sum Y_i = n a_0 + a_1 \sum x_{1i} + a_2 \sum x_{2i}$

$\frac{\partial s_r}{\partial a_1} = 0 \rightarrow -2\sum(Y_i - a_0 - a_1 x_{1i} - a_2 x_{2i}) x_{1i} = 0 \rightarrow \sum Y_i x_{1i} = a_0 \sum x_{1i} + a_1 \sum x_{1i}^2 + a_2 \sum x_{2i} x_{1i}$

$\frac{\partial s_r}{\partial a_2} = 0 \rightarrow -2\sum(Y_i - a_0 - a_1 x_{1i} - a_2 x_{2i}) x_{2i} = 0 \rightarrow \sum Y_i x_{2i} = a_0 \sum x_{2i} + a_1 \sum x_{1i} x_{2i} + a_2 \sum x_{2i}^2$

$$\begin{bmatrix} n & \sum x_{1i} & \sum x_{2i} \\ \sum x_{1i} & \sum x_{1i}^2 & \sum x_{2i} x_{1i} \\ \sum x_{2i} & \sum x_{1i} x_{2i} & \sum x_{2i}^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum Y_i \\ \sum Y_i x_{1i} \\ \sum Y_i x_{2i} \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} n & \sum x_{1i} & \sum x_{2i} \\ \sum x_{1i} & \sum x_{1i}^2 & \sum x_{2i} x_{1i} \\ \sum x_{2i} & \sum x_{1i} x_{2i} & \sum x_{2i}^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum Y_i \\ \sum Y_i x_{1i} \\ \sum Y_i x_{2i} \end{bmatrix}$$

Linear Regression with 3 variables:

$Y = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 + e$

$e = Y - a_0 - a_1 x_1 - a_2 x_2 - a_3 x_3$

$$Sum\ of\ error\ squares\ (S_r) = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (Y_i - a_0 - a_1 x_{1i} - a_2 x_{2i} - a_3 x_{3i})^2$$

$\frac{\partial s_r}{\partial a_0} = 0 \rightarrow -2\sum(Y_i - a_0 - a_1 x_{1i} - a_2 x_{2i} - a_3 x_{3i}) = 0 \rightarrow \sum Y_i = n a_0 + a_1 \sum x_{1i} + a_2 \sum x_{2i} + a_3 \sum x_{3i}$

$\frac{\partial s_r}{\partial a_1} = 0 \rightarrow -2\sum(Y_i - a_0 - a_1 x_{1i} - a_2 x_{2i} - a_3 x_{3i}) x_{1i} = 0 \rightarrow$

$$\sum Y_i x_{1i} = a_0 \sum x_{1i} + a_1 \sum x_{1i}^2 + a_2 \sum x_{2i} x_{1i} + a_3 \sum x_{3i} x_{1i}$$

$\frac{\partial s_r}{\partial a_2} = 0 \rightarrow -2\sum(Y_i - a_0 - a_1 x_{1i} - a_2 x_{2i} - a_3 x_{3i}) x_{2i} = 0 \rightarrow$

$\sum Y_i x_{2i} = a_0 \sum x_{2i} + a_1 \sum x_{1i} x_{2i} + a_2 \sum x_{2i}^2 + a_3 \sum x_{3i} x_{2i}$

$\frac{\partial s_r}{\partial a_3} = 0 \rightarrow -2\sum(Y_i - a_0 - a_1 x_{1i} - a_2 x_{2i} - a_3 x_{3i}) x_{3i} = 0 \rightarrow$

$\sum Y_i x_{3i} = a_0 \sum x_{3i} + a_1 \sum x_{1i} x_{3i} + a_2 \sum x_{2i} x_{3i} + a_3 \sum x_{3i}^2$

$$\begin{bmatrix} n & \sum x_{1i} & \sum x_{2i} & \sum x_{3i} \\ \sum x_{1i} & \sum x_{1i}^2 & \sum x_{2i}\, x_{1i} & \sum x_{3i}\, x_{1i} \\ \sum x_{2i} & \sum x_{1i}x_{2i} & \sum x_{2i}^2 & \sum x_{3i}x_{2i} \\ \sum x_{3i} & \sum x_{1i}x_{3i} & \sum x_{2i}x_{3i} & \sum x_{3i}^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \sum Y_i \\ \sum Y_i x_{1i} \\ \sum Y_i x_{2i} \\ \sum Y_i x_{3i} \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} n & \sum x_{1i} & \sum x_{2i} & \sum x_{3i} \\ \sum x_{1i} & \sum x_{1i}^2 & \sum x_{2i}\, x_{1i} & \sum x_{3i}\, x_{1i} \\ \sum x_{2i} & \sum x_{1i}x_{2i} & \sum x_{2i}^2 & \sum x_{3i}x_{2i} \\ \sum x_{3i} & \sum x_{1i}x_{3i} & \sum x_{2i}x_{3i} & \sum x_{3i}^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum Y_i \\ \sum Y_i x_{1i} \\ \sum Y_i x_{2i} \\ \sum Y_i x_{3i} \end{bmatrix}$$