QoI Symptotics - Timeliness Example

Abstract—**Explanation using delay instead of bandwidth:**

I. Explanation

Instead of looking at bandwidth, which we have been doing and which is the basis for the previous symptotics work, flipping the problem a bit might provide a more intuitive look, but we'll see that the result is the same. We look at a single flow that must complete in a timeliness of T seconds. We will say that the flow has a QoI requirement that results in the need for B bits, fragmented into packets of size P bits, to be sent from each source to each destination. The rate of each link is W bits per second.

Now, we have two contributors to delay. The first contributor is the end to end delay incurred by sending the B bits across the entire path. Here, assume a pipeline that To approximate this delay, let us define a term W_{eff} to describe the rate that each node along the path can use to serve this particular flow. This effective rate is the channel rate divided by the contention factor, CF, which describes the fraction of the channel unusable because of contention with neighbors, and also divided by the transit factor, TF, which accounts for the fraction of the bandwidth used to serve other flows interleaved in the node's queue:

$$W_{eff} = \frac{W}{CF * TF} \tag{1}$$

This first delay, then, is

$$\frac{B}{W_{eff}} = \frac{B * CF * TF}{W} \tag{2}$$

Now, the second delay that exists is from the multi hop propagation. This delay is simply the time for a single packet to traverse the path length. Here, scheduling can make a difference as in the case of TDMA. A node cannot forward a packet from the flow until it receives that packet from the last hop. In the direction in which nodes are scheduled with slots $1-2-3-1-2-3$, each successive node receives a packet on the time slot before it is scheduled, resulting in no extra delay. In the opposite direction, where nodes are scheduled $1-3-2-1-3-2$, the third node in line, scheduled for time slot 2 cannot transmit until after it receives a packet in time slot 3. If we think of a flow starting at time slot 1, then this node cannot transmit until the fifth slot. The first time slot 2 is scheduled, this flow cannot be served because of order constraints. Overall, every other slot is wasted, resulting is what we will call a Scheduling Factor, or SF, of 2 in that direction.

I'll make a few notes on this point. First, in a loaded network, the nodes can and will serve other flows while awaiting the arrival of packets in this flow of focus. That utilized bandwidth does not, however, preclude this SF impact on delay for this flow. Any node cannot serve this flow until it is received. Second, this delay is only accounted for once per flow because all other packets are pipelined. Imagine here either the first or last packet in a flow. All other packets' delay is captured by the previous, end to end delay.

The multi-hop propagation delay, then, is:

$$\frac{k * P}{W/SF} \tag{3}$$

If we put these together, then we can give a relation for a network that will successfully achieve this flow's data and timeliness requirements:

$$T \geq \frac{B}{W/(CF * TF)} + \frac{k * P}{W/SF} \tag{4}$$

Rearranging this inequality to put into terms of rate, we get the following, which is in a similar format to a simplified symptotics framework equation

$$W - CF * TF * \frac{B}{T} - SF * k * \frac{P}{T} \geq 0 \tag{5}$$

II. Example Figures

Figures 1-4 provide simple examples of the delay in practice. Each figure labels the two components of delay for a single flow, F_1, which we assume consists of only 2 packets. In all of the Figures, each node has scheduled one slot of a three slot frame, so $CF = 3$. Figure 1 exhibits the first example outlined above in which no additional delay occurs due to scheduling, i.e., $SF = 1$. In Figure 2, though, the second example with $SF = 2$ is exhibited. Here, we see the multi-hop propagation requires twice the number of slots because every other slot is unused in this flow's propagation. Note that these unused slots may be used by the nodes to transmit packets from a different flow, so the bandwidth may not be wasted, but since it cannot be used for flow F_1, the delay for this flow is still impacted.

Figures 3 and 4 provide more insight into the impact of other flows on the delay. For simplicity, we only show one extra flow F_2. In both of these pictures, the transit factor is now doubled ($TF = 2$), and, therefore, the end-to-end portion of the delay is doubled.

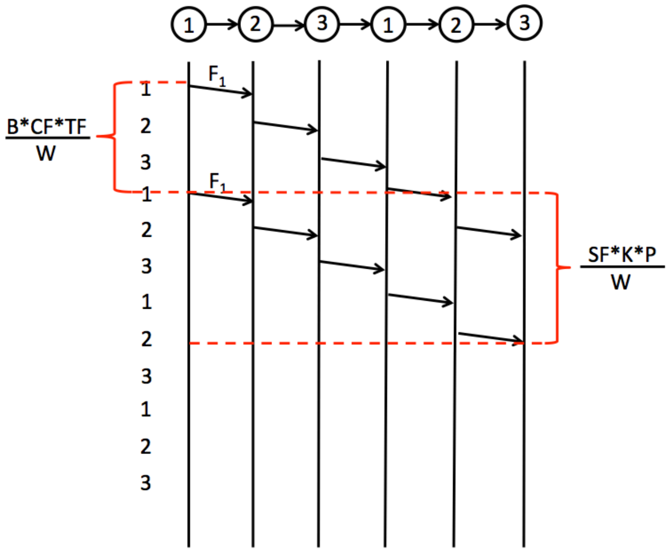

Fig. 1. TF = 1, CF = 3, SF = 1

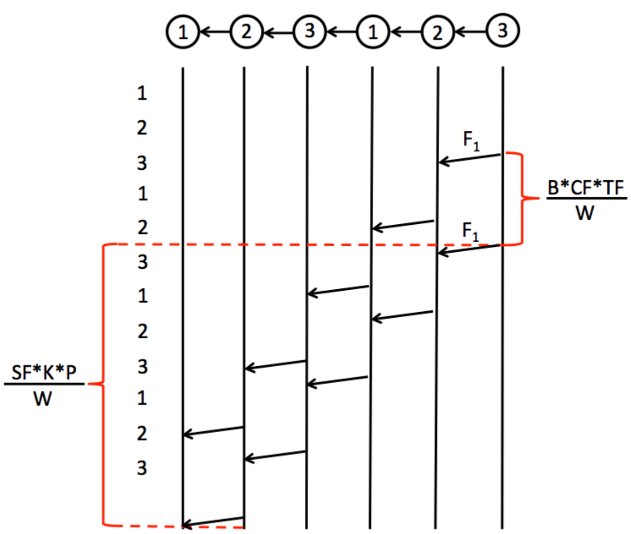

Fig. 2. TF = 2, CF = 3, SF = 2

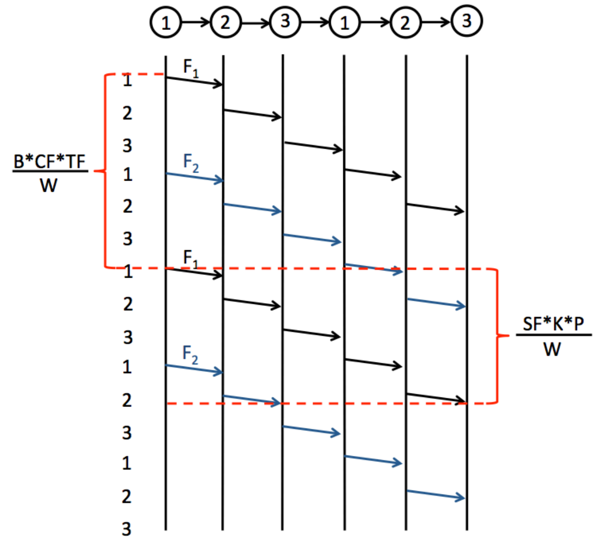

Fig. 3. TF = 1, CF = 3, SF = 2

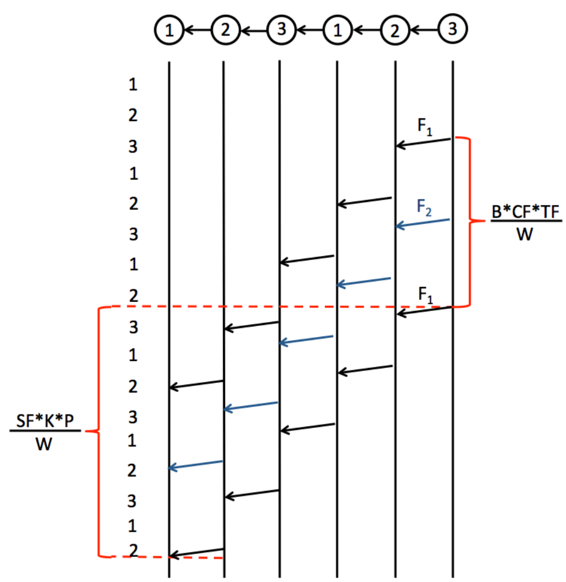

Fig. 4. TF = 2, CF = 3, SF = 2