TWISTED HOLOGRAPHY AND KOSZUL DUALITY

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1. From physical hologrpahy to Koszul duality

The first lecture was given by Kevin Costello.

1.1. **Motivation.** The most basic version of holography asserts the following

Conjecture 1.1 (Maldacena-Witten)**.** Type IIB string theory on $AdS_5 \times S^5$ is equivalent to $\mathcal{N} = 4$ super Yang-Mills for $G = U(N)$ on S^4 in the limit where $N \to \infty$.

Here, AdS_5 refers to five dimensional Lorentzian hyperbolic space $\mathbb{H}^5$. The S^4 on the gauge theory side arises as the boundary of $\mathbb{H}^5$.

Above, "conjecture" is meant in a physical sense as neither side of the correspondence has been formulated mathematically. Even at a physical level of rigour, understanding the above statement has some key difficulties:

- The gauge theory side must be defined nonperturbatively.
- The $AdS_5 \times S^5$ background is a Ramond-Ramond background and there are technical obstructions to understanding string theory in such backgrounds.

The goal of this seminar is to look at *twists*, or supersymmetry protected subsectors, of both sides, and examine the duality there. Such twists are holomorphic-topological field theories. We can attach familiar mathematical objects such as $\mathbb{E}_n$ algebras, vertex algebras, categories, and more to such field theories and attempt to formulate holography mathematically in terms of such objects. For example:

- To twists of 4d $\mathcal{N} = 4$ theories, we can attach categories of boundary conditions of compactifications to two dimensions - for particular twists, these recover Geometric Langlands categories. We can also attach framed $\mathbb{E}_4$-algebras, and vertex algebras.
- Twists of type II string theory admit descriptions in terms of topological strings, which can be described by certain Calabi-Yau categories and invariants thereof.

Thus, we expect a mathematical codification of holography at the level of twists to uncover novel relationships between the above structures.

1.2. **Twisting Supersymmetric Field Theories.**

Definition 1.2. A supersymmetric quantum field theory in dimension n is a quantum field theory carries an action of the super poincare algebra $(\Pi S \oplus \mathbb{R}^n) \ltimes \mathfrak{so}(n)$.

Above S is a spin representation in n dimensions. There's also an action of the commutant $G_R \subset \mathrm{End}(S)$ of $\mathrm{Spin}(n)$, the so-called R-symmetry group.

Twisting consists of a two step procedure.

(1) Choose a subgroup $H \subset \mathrm{Spin}(n)$ and a homomorphism $\rho : H \to G_R$. Letting $\mathrm{SISO}(n)$ denote the group exponentiating the super-poincare algebra, we replace $\mathrm{SISO}(n) \times G_R$ with the twisted product $\mathrm{SISO}(n) \times_\rho G_R$. That is, we use the action of R-symmetry to change the spin of the fields. Locally, this is a very mild modification of the theory - we've decided to treat some of the scalars of the theory as e.g. components of a 1-form.

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(2) Choose a $Q \in S$ that is a scalar under the action of $\mathrm{SISO}(n) \times_\rho G_R$ such that $[Q,Q]=0$ and add Q to the BRST diferential of the QFT. This typically simplifes the theory drastically - it renders the action of Q exact translations on the theory homotopically trivial.

If $Im[Q,-] = \mathbb{C}^n$ then all translations act trivially, and the result is a topological field theory. For reasons of supersymmetric linear algebra, $Im[Q,-]$ is always coisotropic. If it is of the minimal dimension $\lceil n/2 \rceil$, then the twist is holomorphic. In general we find a theory that is a mixture of holomorphic and topological.

More homotopically, a choice of Q gives an action of an abelian superalgebra $\Pi\mathbb{C}$ on the theory. Adding Q to the differential is a model for the derived invariants for this action. The result is a module over $\mathcal{O}(B\Pi\mathbb{C}) = \mathbb{C}[\![t]\!]$. If we have a $\mathbb{C}^\times$ action where Q has weight 1, then we have a Rees family. The twisted theory lives over the generic point.

1.2.1. *The Ω-deformation.* A variant of this construction involves taking instead a supercharge Q that squares to an infinitessimal rotation. We can then perform an equivariant cohomology construction - we can take Q cohomology at the S^1 fixed points. If a theory on $\mathbb{R}^2 \times \mathbb{R}^2 \times \cdots$ is deformed by a supercharge Q whoes square rotates the two planes with speeds $\varepsilon_1, \varepsilon_2$, we will denote the spacetime by $\mathbb{R}^2_{\varepsilon_1} \times \mathbb{R}^2_{\varepsilon_2}$.

Example 1.3. There is a holomorphic-topological twist of 4d $\mathcal{N}=4$ super Yang-Mills on a product of Riemann surfaces $\Sigma \times C$ which is topological on Σ and holomorphic on C. This twist was first studied by Kapustin, and provides a home for some familiar objects in geometric representation theory.

Indeed, compactifying the theory on C yields a 2d TQFT whose category of boundary conditions is $\mathrm{Coh}(\mathrm{Higgs}_G C)$. This category featuers in the conjectural Dolbeault Geometric Langlands which posits an equivalence of categories

$$\mathrm{Coh}(\mathrm{Higgs}_G C) \cong \mathrm{Coh}(\mathrm{Higgs}_{\check{G}} C).$$

Let's specialize to the case $\Sigma = \mathbb{R}^2$ and $C = \mathbb{C}$. We may subject the theory to an Ω-deformation along $\mathbb{R}^2$, to get a holomorphic field theory on $\mathbb{C}$. The algebra of observables of the result is a chihral algebra on $\mathbb{C}$. Explicitly, it is given by the BRST reduction of chiral differential operators on $\mathfrak{g}$ by G acting by the adjoint action, or equivalently chiral differential operators on the quotient stack $\mathfrak{g}/G$.

Even more explicitly, the VOA is given by the BRST reduction of the VOA generated by fields X_a, Y_a where a is a lie algebra index, with OPE

$$X_a(0)Y_a(z) \sim \delta_{ab}\frac{1}{z}.$$

This example fits in the larger program of [] of attaching VOAs to 4d $\mathcal{N}=2$ superconformal field theories; this is the output of their construction applied to an $\mathcal{N}=4$ theory.

Example 1.4. Next we consider a topologically twisted Ω-deformed version of the worldvolume theory of M2 branes. The resulting theory is succinctly described as ADHM quantum mechanics. This is the theory whose algebra of operators is the quantum Hamiltonian reduction of $T^*(\mathfrak{gl}_N \oplus \mathbb{C}^N)$ by GL_N. This algebra is also known as the spherical double affine hecke algebra, which is a deformation quantization of $Hilb^N(\mathbb{C}^2)$. This can also be thought of as the output of the Coulomb branch construction of Braverman-Finkelberg-Nakajima [] for the ADHM quiver, where are we using that this quiver is self 3d-mirror.

Thus in both examples, twisting and Ω-deformation allows us to extract familliar objects of geometric representation theory from supersymmetric field theories. Our goal will be to realize these objects in the large N limit in terms of something gravitational nature.

1.3. **Twisting Supergravity.** In supergravity, the supersymmetries are gauged. The fields are a quotient stack where we quotient by the action of a huge supergroup rougly given by $C^\infty(\mathbb{R}^n, \mathrm{SISO}(n))$. Thus, the fields of the theory live over the space of maps $R^n \to B\mathrm{SISO}(n)$; this classifying stack is the analog of the moduli of vacua in gauge theory.

Recall that for a lie algebra $\mathfrak{g}$, we can think of the classifying stack $B\mathfrak{g}$ as the space of solutions to the Maurer-Cartan equation in $\mathfrak{g}$, modulo gauge. The set of solutions to the Maurer-Cartan equation in the supertranslation algebra $\mathbb{R}^n \oplus \Pi S$ is simply the collection of square zero supercharges, and is called the *nilpotence variety* []. This variety is closely related to classical objects such as the space of pure spinors studied by Cartan.

We conclude that supergravity contains a field that looks like a map

$$c_\alpha : \mathbb{R}^n \to \text{ nilpotence variety } /\mathrm{Spin}(n).$$

This is the ghost for gauged supersymmetries - since supersymmetries are fermionic, this ghost is bosonic.

Definition 1.5 (Costello-Li)**.** Twisted supergravity is supergravity in a vacuum where the bosonic ghost takes a nonzero VEV.

That is, twisted supergravity is simply a vacuum for ordinary supergravity.

To see how this simplifies the theory, note that there are terms in the supergravity action of the form $c_\alpha g \psi^*$ where g is the metric and ψ^* is the antifield to the gravitino. This term enforces gauge invariance under local supersymmetry transformations that mix the metric and the gravitino. If c has a VEV this gives a mass term allowing us to integrate out certain components of the metric and gravitino. Alternatively, if c_{alpha} has a mass, the above term then generates a new differential in the BRST complex which leads to cohomological cancellations.

Note that a priori this procedure seems very different from twisting a supersymmetric field theory. We can reformulate twisting a supersymmetric field theory to similarly to the above. We can enlarge the space of fields of a gauge theory by adding global supersymmetries as ghosts by hand, and turn on a constant value for these ghosts.

A key feature of twisted supergravity is that worldvolume theories of branes in twisted supregravity backgrounds are naturally twists of the supersymmetric field theories one finds as worldvolume theories in the physical string. The examples in the previous subsection will arise from worldvolume theories of branes in suitable twisted supergravity backgrounds.

Example 1.6. In the early 90s, it was discovered that the topological string on a CY3 embeds in the type II string. Originally, this was expressed by saying that certain quantities in the type II string can be computed in the topological string. That is, the topological string computes type II string amplitude in the presence of the so-called self-dual graviphoton background. In modern language, this can be expressed by saying that the topological string arises from Ω-deforming a twist of the string.

More precisely, the IIB string on $\mathbb{R}^2_\varepsilon \times \mathbb{R}^2_{-\varepsilon} \times X$ where X is a CY3 gives the B-model on X. Similarly, the IIA string on $\mathbb{R}^2_\varepsilon \times \mathbb{R}^2_{-\varepsilon} \times X$ gives the A-model on X.

Example 1.3 arises from considering a stack of D3 branes in the above Ω-deformed twist of IIB, wrapping $\mathbb{R}^2_{-\varepsilon} \times C$ where $C \subset X$ is a holomorphic curve.

Example 1.7. We can also consider an Ω-deformation of a $G2 \times \mathrm{SU}(2)$ twist of 11d supergravity; we denote the relevant background by $\mathbb{R}^2_{\varepsilon_1} \times \mathbb{R}^2_{\varepsilon_2} \times \mathbb{R}^2_{\varepsilon_3} \times \mathbb{R} \times \mathbb{C}^2$ where $\sum \varepsilon_i = 0$.

It turns out that this can be described as a 5d noncommutative Chern-Simons theoy that is partially holomorphic and topological. The fundamental field is a partial gauge field

$$A \in \Omega^1(\mathbb{R} \times \mathbb{C}^2)/(dz_1, dz_2).$$

The action is given by

$$\frac{1}{\varepsilon_3} \int dz_1 dz_2 \frac{1}{2} A dA + \frac{1}{3} A * A * A$$

where $*$ denotes the Moyal product.

It is natural to wonder in what sense this describes a gravitational theory. Expanding the action gives

$$\int dz_1 dz_2 A dA + A \partial_{z_1} A \partial_{z_2} A + \cdots .$$

Let's analyze just the written terms. On $\mathbb{C}^2$ a deformation of the canonical holomorphic symplectic structure is given by $A \in \Omega^{0,1}(\mathbb{C}^2)$ satisfying the equation

$$\bar{\partial} A + \frac{1}{2} \varepsilon_{ij} \partial_{z_i} A \partial_{z_j} A = 0$$

. This equation is the Maurer-Cartan equation for the Beltrami differential $\varepsilon^{ij} \partial_{z_i} A \partial_{z_j}$. Expanding the action in components of the gauge field gives

$$\int dz_1 dz_2 A_t dA_{\bar{z}_i} + A_t \partial_{z_k} A_{\bar{z}_i} \partial_{z_l} A_{\bar{z}_j} + \cdots .$$

Varying with respect to A_t yields exactly the above Maurer-Cartan equation. Thus, we find that solutions to the equation of motion describe $\mathbb{R}$-families of holomorphic symplectic structures on $\mathbb{C}^2$. This is some particular class of deformations of a metric with SU(2) holonomy.

Example 1.4 arises as the worldvolume theory of M2 branes wrapping $\mathbb{R}^2_{\varepsilon_i} \times \mathbb{R}$.

Thus, we wish to make the following matches.

- The large N limit of the spherical DAHA and quantities in 5d noncommutative Chern-Simons
- Chiral differential operators on the adjoint quotient stack $\mathfrak{g}/G$ and quantities in the IIB string on $\mathbb{R}^2_\varepsilon \times \mathbb{R}^2_{-\varepsilon} \times X$.

1.4. **Global Symmetry Algebras.** The relation we are after is a litle involved, involving koszul duality of boundary operators. As a first consistency check though, we can extract certain lie algebras from both sides and check that they match. In physical AdS/CFT this is very simple.

The gravitational lie algebra $\mathfrak{g}_{grav}$ consists of gauge transformations that fix the metric. These are precisely isometries of $AdS_5 \times S^5$ some ferminoic symmetries. These will match with superconformal symmetries in the gauge theory. Explicitly, the isometries of $AdS_5 \times S^5$ are given by $\mathrm{SO}(6) \times \mathrm{SO}(4,2)$ The first factor is the R-symmetry of 4d $\mathcal{N} = 4$ and the second factor is the conformal symmetries of S^4.

However, in the twisted setting, these algebras all receive infinite dimensional enhancements, and such a check carries more content.

Example 1.8. Let's work in the setting of example 1.7 and consider the gauge transformations of 5d Chern-Simons that fix the zero gauge field. The gauge transformations act by

$$A \mapsto A + d\chi + \bar{\partial}\chi + [\chi, A]$$

where the last term denotes the Moyal commutator. Therefore, the gauge transformations that preserve the zero gauge field consist of holomorphic functions on $\mathbb{C}^2$ with the Moyal commutator. Equivalently, this is the algebra of differential operators on $\mathbb{C}$, $\mathrm{Diff}(\mathbb{C})$. This lie algebra is a small deformation of the algebra of hamiltonian diffeomorphisms of $\mathbb{C}^2$, which is the so-called w_∞ algebra.

Let's now try and arrive at this algebra from ADHM quantum mechanics. We wish to compute the quantum hamiltonian reduction of $T^*(\mathfrak{gl}_N \oplus \mathbb{C}N)$ in the large N limit. We begin by describing functions on the classical hamiltonian reduction. To do so, we wish to look at level sets of the moment map and quotient by gauge. Let us choose coordinates $I_i \in \mathbb{C}^N$, $J^i \in (\mathbb{C}^N)^*$, $X^i_j, Y^i_j \in \mathfrak{gl}_N$. The moment map equation is given by $[X, Y] + IJ = c$ where c is some generic element of $\mathfrak{gl}_N$.

Lemma 1.9. The algebra of functions on the classical hamiltonian reduction is generated by monomials of the form IX^rY^sJ. These generators are algebraically independent if $r + s < N$.

Note that these monomials are clearly GL_N invariant - the function takes a vector, acts on it by a bunch of matrices, and then pairs with a covector. The fact that these expressions generate the algebra is a consequence of the moment map relation and classical invariant theory. The independence of these generators is trickier.

A consequence of the constraint for independence is that in the large N limit, all generators are independent. Thus, we find that functions on the symplectic quotient are given by $\mathrm{Sym}(\mathcal{O}(\mathbb{C}^2))$, under the identification $IX^rY^sJ \mapsto z_1^r z_2^s$.

The quantum hamiltonian reduction will be a flat deformation of the above. The result will be a quantum deformation of $U(\mathrm{Diff}\mathbb{C})$. To see this we stipulate that the coordinates X, Y, I, J are now elements of a Weyl algebra:

$$[X^i_j, Y^k_l] = \hbar\delta^i_j\delta^k_l, \qquad [I_i, J^j] = \hbar\delta^j_i.$$

We now wish to compute commutators between the same monomials we found previously.

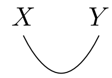
Write each monomial in index-ful notation as

$$IX^rY^sJ = I_{i_0}X^{i_0}_{i_1}X^{i_1}_{i_2}\cdots Y^{i_{r+s}}_{i_{r+s+1}}J^{i_{r+s+1}}.$$

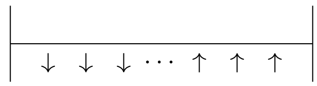
The above commutation relations stipulate that

$$X^i_j = \hbar\frac{\partial}{\partial Y^j_i}, \qquad I_i = \hbar\frac{\partial}{\partial J^i}.$$

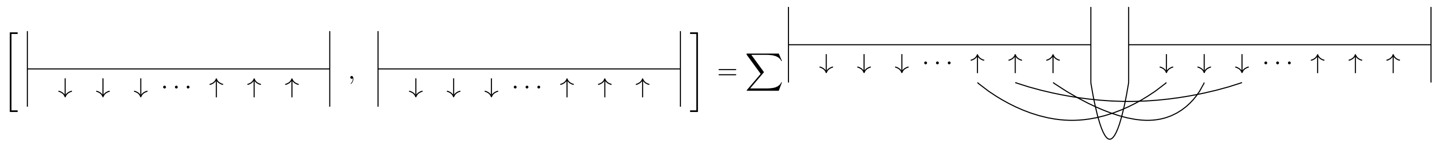
Let's introduce the diagrammatic notation



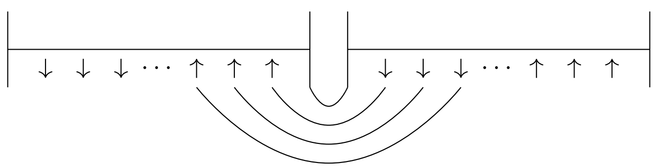
to denote the commutator between X and Y which yields a factor of $\hbar$. We diagrammatically write the monomial IX^rY^sJ as



where there are r up arrows depicting the factors of X and s down arrows depicting the factors of Y. In terms of this notation, the commutator is expressed diagrammatically as



where the sum is over all possible contractions. The right hand-side of the expression can then be simplified using the moment map relation. However, the dominant term in the large N limit is simple - it comes from the diagram where all the adjacent indices are contracted.



This recovers precisely the Lie bracket on $\mathrm{Diff}(\mathbb{C})$. Thus, we arrive at the statement that the large N quantum Hamiltonian reduction of the ADHM quiver is $U\mathrm{Diff}(\mathbb{C})$. More precisely, we find the central quotient of $U\mathrm{Diff}(\mathbb{C})$ where the identity operator in $\mathrm{Diff}(\mathbb{C})$ is identified with N. Indeed, this follows from taking the trace of the moment map relation $JI + [X, Y] = 1$.

1.5. **Backreaction.** A key aspect of physical holography that has not appeared in our twisted story so far is the appearance of geometries like $AdS_m \times S^n$. Let's recall how this arises in the physical picture. This involves the following steps

(1) Consider some supergravity theory on $\mathbb{R}^n$ with a brane on $\mathbb{R}^d \subset \mathbb{R}^n$. The brane deforms the action to first order by the inclusion of a source term - this is some curved deformation of the L_∞ algebra describing the gravitational theory.
(2) Solve the equations of motion in the presence of this source term; a solution is called the field sourced by the brane. Among such solutions is a metric that is singular along the locus of the brane $\mathbb{R}^d$. The complement of the brane $\mathbb{R}^n \setminus \mathbb{R}^d$ with this metric is called the *black brane geometry.*
(3) Take the *near horizon limit.* This involves zooming in near the location of the brane. The result is $AdS_{d+1} \times S^{n-d-1}$.

We can repeat this proceudre in the twisted theory. The last step won't be necessary in the twisted setting as the theories will always be topological in the radial direction transverse to the brane.

Example 1.10. Let's spell out the above in the 5d chern-simons theory of examples 1.4, 1.7. We consider a stack of $NM2$ branes along the topological direction; this introduces the source term $N\int_{\mathbb{R}} A$ to the action. The equation of motion in the presence of this term, to linear order, simply reads $(d + \bar{\partial})A = N\delta_{\mathbb{R}}$. This is solved by the Bochner-Martinelli kernel

$$A = N\frac{\bar{z}_1 d\bar{z}_2 - \bar{z}_2 d\bar{z}_1}{\|z\|^4}$$

. Thus, the complement of the brane, $\mathbb{R} \times (\mathbb{C}^2 \setminus \{0\})$, with this closed string field is our analog of $AdS_2 \times S^3$.

We now wish to study boundary operators for the compactification of the theory on $S^3 \subset \mathbb{C}^2 \setminus \{0\}$. The effective 2d theory obtained by compactification is 2d BF theory for the lie algebra $\mathrm{Diff}(\mathbb{C})$, on the half space

$\mathbb{R} \times \mathbb{R}_{\geq 0}$. Here, $R_{\geq 0}$ is the radial direction in $\mathbb{C}^2 \setminus \{0\}$; let us choose a coordinate r. The algebra we wish to study is the algebra of boundary operators for a Dirichlet boundary condition at $r = \infty$. It is easy to see that the algebra of tree level boundary operators for BF theory with lie algebra $\mathfrak{g}$ is the universal enveloping algebra $U(\mathfrak{g})$. Alternatively, we can view the BF theory as a Poisson σ-model and appeal to Kontsevich's work on deformation quantization.

Thus in this example, we find (at least at tree level) that the algebra of operators on a large number of M2 branes, which we computed in example 1.8 agrees with boundary local operators in the compactification of supergavity on a sphere linking the branes. This is what is expected from physical AdS/CFT.

Recall that in 1.8 we actually found a central quotient of $U(\mathrm{Diff}(\mathbb{C}))$. On the gravitational side, this is a consequence of the backreaction.

In the previous example, the effect of the backreaction was very mild. In other examples, it will give a central extension of the gravitational algebra, rather than a central quotient. Physically, the difference stems from the following two approaches to the backreaction:

(1) Take the mode corresponding to the backreaction as dynamical. This happens when we consider branes electrically coupled to the gravitational theory. Then the boundary algebra will have a ceentral element corresponding to N.
(2) Take the mode corresponding to the backreaction to be non dynamical. This happens when we consider branes magnetically coupled to the gravitational theory. In this case, N enters as an extra parameter, i.e. the algebra of operators gets an N-dependent central extension.

Example 1.11. Let's work in the example of 1.6. Consider the B-model on $\mathbb{C}^3$ with a stack of N branes along $\mathbb{C}$. For concreteness, let's fix coordinates z, w_1, w_2 and take the brane to wrap the $w_i = 0$ plane. The fundamental field of the theory is a Beltrami differential

$$\mu \in \Omega^{0,1}(\mathbb{C}, T\mathbb{C}^3) \cong \Omega^{2,1}(\mathbb{C}^3)$$

where the last identity uses the Calabi-Yau structure. The inclusion of the brane deforms the action to leading order by a term of the form $N \int_{\mathbb{C}} \partial^{-1}\mu$. This is a magnetic coupling, and can be thought of as a Wess-Zumino type term which tells us to integrate μ over a three cycle whose boundary is the brane $\mathbb{C}$. Solutions to the equation of motion in this background are given by

$$\mu = N \frac{\bar{w}_1 d\bar{w}_2 - \bar{w}_2 d\bar{w}_1}{\|w\|^4} \partial_z.$$

Lemma 1.12 (Costello-Gaiotto)**.** $\mathbb{C} \times (\mathbb{C}^2 \setminus \{0\})$ deformed by this Beltrami differential is $\mathrm{SL}_2\mathbb{C}$.

Proof. We wish to determine functions that are holomorphic with respect to the deformed complex structure and show that they satisfy the relations of the algebra of functions on $\mathrm{SL}_2\mathbb{C}$. Since the beltrami differential only has a ∂_z component, the functions w_1, w_2 are still holomorphic. The function z is no longer holomorphic, but the functions

$$v_1 = zw_1 - N\frac{\bar{w}_2}{\|w\|^2}, \qquad v_2 = zw_2 + N\frac{\bar{w}_1}{\|w\|^2}$$

are. These coordinates collectively satisfy

$$v_2 w_1 - v_1 w_2 = N.$$

□

As in the previous example we can try to match boundary operators for this gravitational theory with large N chiral differential operators on the adjoint quotient stack. In [] Costello and Gaiotto compare the global symmetry algebras of the theories.

The global symmetry algebra $\mathfrak{g}_{grav}$ of the B-model on $\mathrm{SL}_2\mathbb{C}$ is generated by

$$X \in \mathrm{Vect}_0(\mathrm{SL}_2\mathbb{C}), \qquad f, g \in \Pi\mathcal{O}(\mathrm{SL}_2(\mathbb{C})).$$

Here $\mathrm{Vect}_0(\mathrm{SL}_2\mathbb{C})$ denotes holomorphic divergence free vector fields. The lie structure involves three brackets. Two divergence vector fields bracket by the lie derivative, and a divergence free vector field acts on a function by differentiation. There is a natural bracket on the functions by saying that

$$[f, g] = (\partial f \wedge \partial g) \vee \Omega^{-1}$$

where Ω^{-1} denoets the inverse of the Calabi-Yau volume form.

Let A_N denote the mode algebra of chiral differential operators on the quotient stack $\mathfrak{gl}_N/\mathrm{GL}_N$.

Theorem 1.13 (Costello-Gaiotto)**.** As $N \to \infty$ there is an embedding $U\mathfrak{g}_{grav} \hookrightarrow A_\infty$. The image of $\mathfrak{g}_{grav}$ under this embedding consists of single trace modes preserving the vacuum at 0 and ∞.

The backreaction is essential in establishing this result. In this example, the coupling of branes is magnetic, so we cannot treat the backreaction as dynamical.

Note that in this example the universal enveloping algebra of the gravitational symmetry algebra is much smaller than the algebra of modes for the gauge theory local operators. This is expected, the latter should be the same size as the mode algebra of boundary local operators in supergravity, and $U\mathfrak{g}_{grav}$ is much smaller than this. It was in a sense a coincidence that in example 1.8 $U\mathfrak{g}_{grav}$ recovered the entire algebra of boundary operators in 5d Chern-Simons.

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