Math 3450 **Homework #3**
Winter 2021 **January 22**

This assignment is due at 11:59pm on Wednesday, January 27^{th}. <u>*Detail all work*</u> for complete credit. Students may work together on this project, but each student must individually write up his/her own codes and solution set.

1. **(30 points)** Write codes that implement Jacobi and Gauss-Seidel iteration to solve linear systems of the form $\mathbf{Ax} = \mathbf{b}$. These codes should have the form `Jacobi(A,b,x0,iterations,tol)` and `GaussSeidel(A,b,x0,iterations,tol)`.

2. **(30 points)** Consider the linear system

$$\mathbf{Ax} = (a \cdot \mathcal{I} + \mathcal{R})\mathbf{x} = \mathbf{b},$$

where a is a positive constant, $\mathcal{I}$ is the N-by-N identity matrix, and $\mathcal{R}$ is an N-by-N matrix of uniformly distributed random numbers between zero and one.

   (a) Let $N = 22$, $a = 10$, and $\mathbf{b} = \langle 1, 2, 3, \ldots, N \rangle^T$. How many iterations of Jacobi's method are required in order for the solution to converge using a tolerance of 10^{-8}? What about for the Gauss-Seidel method? (Since there are random matrices involved, your answers may be ranges of iterations.)

   (b) How does the answer to part (a) change as the value for N changes? Explain the trends you observe.

   (c) How does the answer to part (a) change as the value for a changes? Explain the trends you observe.

3. **(40 points)** Consider the linear system

$$\mathbf{Wx} = \mathbf{p},$$

where $\mathbf{W}$ and $\mathbf{p}$ are the matrix and vector, respectively, from our Canvas page.

   (a) Solve the system via (if possible)

      i. The Jacobi method

      ii. The Gauss-Seidel method

      iii. The LU decompositon (use the default Julia/Python routine)

      iv. The inverse of $\mathbf{W}$ (use the default Julia/Python routine)

      v. Using the "backslash operator", $\mathbf{W} \backslash \mathbf{p}$.

   (b) How do the solutions obtained with the various methods compare?

   (c) Approximately, how long does each method take?

   (d) What are pros and cons of each method?