Math 3450 **Written HW #4**
Winter 2021 **February 3**

This assignment is due by 11:59pm on Monday, February 8^{th}. *Detail all work* for complete credit. Students may work together on this assignment, but each student must individually write up their own code and solution set.

1. **(40 points)** Consider the function $f(x) = x^2$ on the interval $x \in [-1, 1]$.
   (a) Determine the *exact* Fourier series expansion of $f(x)$ on the given interval. Simplify your answer.
   (b) Plot $f(x)$ along with its $N = 1$ and $N = 4$ approximate Fourier series on the same axes. Discuss.
   (c) Create a discrete representation of $f(x)$ on the given interval using $N = 128$ data points. Take an FFT of this data to obtain the approximate Fourier coefficients. Compare these approximate Fourier coefficients to the exact Fourier coefficients obtained in part (a). BE CAREFUL!!!
   (d) Do you prefer the method used in part (a) or (c)? Justify your preference.

2. **(35 points)** Consider the functions $g(x) = \cos(2x)$ and $h(x) = \cos(14x)$ on the interval $x \in [-\pi, \pi]$.
   (a) Plot $g(x)$ and $h(x)$ on the same axes. Briefly describe.
   (b) Create a discrete representation of $g(x)$ on the given interval using $N = 16$ data points. Take an FFT of this data and explain.
   (c) Create a discrete representation of $h(x)$ on the given interval using $N = 16$ data points. Take an FFT of this data and explain. Be sure to relate your answer here to your answer in part (b).
   (d) Plot $g(x)$ and $h(x)$ on the same axes (again). On this plot, add labels to all $N = 16$ points that you used in the FFT in part (c). What is special about these points? Explain and discuss.

3. **(25 points)** The data in the file named `FourierData.out` on our Canvas page contains the Fourier coefficients of a function $m(x)$ defined on the interval $x \in [-10, 10]$.
   (a) Plot the function $m(x)$. (Note: Do not plot the Fourier coefficients of $m(x)$.)
   (b) Determine the formula for the function $m(x)$. (Hint: There is a pattern.)