Math 3450 **Written HW #6**
Winter 2021 **March 1**

This assignment is due at NOON on Wednesday, March 10^{th}. *Detail all work* for complete credit. Students may work together on this assignment, but each student must individually write up their own code and solution set.

1. **(25 points)** Consider the system

$$Ax = \begin{bmatrix} -2 & 0 \\ 2 & -2 \\ 0 & 2 \end{bmatrix} x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = b$$

   For this problem you are NOT allowed to use any intrinsic SVD, SVD-like, or pseudoinverse-like commands.

   (a) What is the solution of this system?

   (b) What is the singular value decomposition of A? (Make sure that the columns of the matrices are ordered properly.)

   (c) Use your answer to part (b) to find the pseudoinverse of A and the least squares solution of $Ax = b$.

   (d) What is the minimal value of $||Ax - b||_2^2$?

2. **(15 points)** Write a function named `MySVD` that takes in a real M-by-N matrix A and returns the singular value decomposition of A in matrices U, Σ, and V. (Make sure that the columns of the matrices are ordered properly.)

3. **(25 points)** Write a function named `MyTruncatedSVD` that takes in a real M-by-N matrix A and a real number s and returns the SVD approximation to A in which all singular values less than s (and the associated singular vectors) have been deleted.

4. **(35 points)** Write code named `MyTruncatedSVD` that takes in an image file and a real number s and creates the SVD approximation to the image in which all singular values less than s (and the associated singular vectors) have been deleted. Assume that the number of singular values is equal for the red, green, and blue components of the image. Then, use this code to answer the following questions.

   (a) Create an SVD approximation to your favorite album cover. Submit the original album cover and your SVD approximation.

   (b) What is the largest value of s that allows your code to reasonably accurately reproduce the original album cover?

   (c) Provide a *quantitative* justification for your answer to part (b).