Math 3450 **Homework #1**
Winter 2021 **January 8**

This assignment is due at the start of class on Wednesday, January 13^{th}. *Detail all work* for complete credit. Students may work together, but each student must individually write up their own code and solution set.

1. **(20 points)** Write code that implements the right-endpoint rule, trapezoid rule, Simpson's rule, and Monte Carlo to approximate the integral of a function $f(x)$ over the interval $x \in [a, b]$ using N subintervals. These codes should have the form `f(x)`, `RER(a,b,N)`, `TrapR(a,b,N)`, `SimpR(a,b,N)`, and `MCInt(a,b,N)`.

2. **(35 points)** Consider the following integrals.

   (a)
   $$\int_0^1 \tan(x)\ dx$$

   (b)
   $$\int_{-5}^{5} \left|x - \sqrt{2}\right|^{1/3}\ dx$$

   (c)
   $$\int_{-\pi}^{\pi} \sin\left(2\pi\cos(x)\right)\ dx$$

   i. Fill in the following tables for each of the integrals.

Values

N	RER	TrapR	SimpR	MCInt
10				
100				
1,000				
10,000				
100,000				
1,000,000				

Errors

N	RER	TrapR	SimpR	MCInt
10				
100				
1,000				
10,000				
100,000				
1,000,000				

   ii. For each integral, discuss the convergence of each method. Explain.

3. **(20 points)** Write code to approximate the value of the following integral using the trapezoid rule and Monte Carlo. Create tables similar to those above using $N = 10,\ 100,\ 1000$, and $10,000$ subintervals in each dimension. Discuss convergence of both methods. The codes should have the form `f2D(x,y)`, `TrapR2D(a,b,c,d,Nx,Ny)`, and `MCInt2D(a,b,c,d,Nx,Ny)`.
   $$\int_0^1 \int_2^3 \sin(x^2 + y^3)\ dxdy$$

4. **(25 points)** Prove the order of convergence for the Simpson's rule approximation to an integral of a smooth bounded function on a finite interval.

Bonus (10 points) The right-endpoint rule uses constants, the trapezoid rule uses lines, and Simpson's rule uses quadratics on subintervals to approximate integrals. Write code that uses cubics to approximate integrals.