HW6

Saturday, March 6, 2021 7:02 PM

1. $A = \begin{bmatrix} -2 & 0 \\ 2 & -2 \\ 0 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(a)

$$Ax = b \implies \begin{bmatrix} -2 & 0 \\ 2 & -2 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{cases} -2x_1 = 1 \\ 2x_1 - 2x_2 = 2 \\ 2x_2 = 3 \end{cases} \implies \left[\begin{array}{cc|c} -2 & 0 & 1 \\ 2 & -2 & 2 \\ 0 & 2 & 3 \end{array}\right] \xrightarrow{R_3 = R_2 + R_3} \begin{bmatrix} -2 & 0 & 1 \\ 2 & -2 & 2 \\ 2 & 0 & 5 \end{bmatrix}$$

$$\xrightarrow{R_3 = R_1 + R_3} \begin{bmatrix} -2 & 0 & 1 \\ 2 & -2 & 2 \\ 0 & 0 & 6 \end{bmatrix}$$

$\hookrightarrow 0 \neq 6$

There is no solution to this system.

(b)

$$A = \begin{bmatrix} -2 & 0 \\ 2 & -2 \\ 0 & 2 \end{bmatrix} = U \Sigma V^*$$

$$A^* = \begin{bmatrix} -2 & 2 & 0 \\ 0 & -2 & 2 \end{bmatrix}$$

$$AA^* = \begin{bmatrix} -2 & 0 \\ 2 & -2 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -2 & 2 & 0 \\ 0 & -2 & 2 \end{bmatrix} = \begin{bmatrix} -2(-2)+0(0) & -2(2)+(-2)(0) & -2(0)+0(2) \\ 2(-2)+(-2)(0) & 2(2)+(-2)(-2) & 2(0)+(-2)(2) \\ 0(-2)+2(0) & 0(2)+2(-2) & 0(0)+2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 4 \end{bmatrix}$$

$$\det \begin{bmatrix} 4-\lambda & -4 & 0 \\ -4 & 8-\lambda & -4 \\ 0 & -4 & 4-\lambda \end{bmatrix} = (4-\lambda) \begin{vmatrix} 8-\lambda & -4 \\ -4 & 4-\lambda \end{vmatrix} + 4 \begin{vmatrix} -4 & -4 \\ 0 & 4-\lambda \end{vmatrix} + 0$$

$$= (4-\lambda)\big((8-\lambda)(4-\lambda) - (-4)(-4)\big) + 4\big(-4(4-\lambda) - 0(-4)\big)$$

$$= (4-\lambda)(32 - 8\lambda - 4\lambda + \lambda^2 - 16) + 4(-16 + 4\lambda)$$

$$= (4-\lambda)(\lambda^2 - 12\lambda + 16) + 16\lambda - 64$$

$$= 4\lambda^2 - 48\lambda + 64 - \lambda^3 + 12\lambda^2 - 16\lambda + 16\lambda - 64$$

48, -12, -4, -16

$$= -\lambda^3 + 16\lambda^2 - 48\lambda = -\lambda(\lambda^2 - 16\lambda + 48)$$

$$-\lambda(\lambda-12)(\lambda-4) = 0$$

$$\Rightarrow \lambda = 0, 4, 12$$

$\underline{\lambda = 0}$

$$\begin{bmatrix} 4 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 4 \end{bmatrix} \xrightarrow[\frac{1}{4}R_3]{\frac{1}{4}R_1,\ \frac{1}{4}R_2} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \xrightarrow{R_2 = R_1 + R_2} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3 = R_2 + R_3} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} v_1 - v_2 = 0 \\ v_2 - v_3 = 0 \\ 0 = 0 \end{cases}$$

$$v_1 = v_2 = v_3$$

$$v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} t$$

$\underline{\lambda = 4}$

$$\begin{bmatrix} 0 & -4 & 0 \\ -4 & 2 & -4 \\ 0 & -4 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{bmatrix} \xrightarrow{R_1 = R_1 - R_2} \begin{bmatrix} 2 & -3 & 2 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{bmatrix}$$

$$\xrightarrow{R_2 = R_1 + R_2} \begin{bmatrix} 2 & -3 & 2 \\ 0 & -2 & 0 \\ 0 & -2 & 0 \end{bmatrix} \xrightarrow[-\frac{1}{2}R_2,\ \frac{1}{2}R_1]{R_3 = R_2 - R_3} \begin{bmatrix} 1 & -3/2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3/2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{cases} v_1 - \frac{3}{2}v_2 + v_3 = 0 \\ v_2 = 0 \\ 0 = 0 \end{cases}$$

$$v_2 = 0 \Rightarrow v_1 = -v_3$$

$$v = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} t$$

$\underline{\lambda = 12}$

$$\begin{bmatrix} -8 & -4 & 0 \\ -4 & -4 & -4 \\ 0 & -4 & -8 \end{bmatrix} \xrightarrow[-\frac{1}{4}R_3]{-\frac{1}{4}R_1,\ -\frac{1}{4}R_2} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_1 = R_1 - R_2} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\xrightarrow{R_2 = R_1 - R_2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -2 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow[-R_2]{R_3 = R_2 + R_3} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{cases} v_1 - v_3 = 0 \\ v_2 + 2v_3 = 0 \\ 0 = 0 \end{cases}$$

$$v_1 = v_3, \quad v_2 = -2v_3$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{cases} v_1 - v_3 = 0 \\ v_2 + 2v_3 = 0 \\ 0 = 0 \end{cases}$$

$$v_1 = v_3 , \quad v_2 = -2v_3$$

$$v = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix};$$

$$A^* A = \begin{bmatrix} -2 & 2 & 0 \\ 0 & -2 & 2 \end{bmatrix}\begin{bmatrix} -2 & 0 \\ 2 & -2 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2(-2)+2(2)+0(0) & -2(0)+2(-2)+0(2) \\ 0(-2)+(-2)(2)+0(2) & 0(0)+(-2)(-2)+2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix}$$

$$\det \begin{bmatrix} 8-\lambda & -4 \\ -4 & 8-\lambda \end{bmatrix} = (8-\lambda)(8-\lambda) - (-4)(-4)$$

$$= 64 - 8\lambda - 8\lambda + \lambda^2 - 16$$

$$= \lambda^2 - 16\lambda + 48$$

$$= (\lambda - 12)(\lambda - 4)$$

$$\Rightarrow \lambda = 12, 4$$

48, −12, −4, −16

$\underline{\lambda = 4}$

$$\begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \xrightarrow{R_2 = R_1 + R_2} \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} v_1 - v_2 = 0 \\ 0 = 0 \end{cases}$$

$$v_1 = v_2$$

$$v = \begin{bmatrix} 1 \\ 1 \end{bmatrix};$$

$\underline{\lambda = 12}$

$$\begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \xrightarrow{R_2 = R_1 - R_2} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} v_1 + v_2 = 0 \\ 0 = 0 \end{cases}$$

$$v_1 = -v_2$$

$$v = \begin{bmatrix} 1 \\ -1 \end{bmatrix};$$

$A = U \Sigma V^*$

- U = unit length eigenvectors of AA^* in columns, $M \times M$
- Σ = diagonal elements are square roots of eigenvalues of $A^* A$, $M \times N$
- V = unit length eigenvectors of $A^* A$ in columns, $N \times N$

$\mathcal{U}$/

$$v = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\to \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$\to \sqrt{1^2+0^2+(-1)^2} = \sqrt{2}$

$\to \sqrt{1^2+(-2)^2+1^2} = \sqrt{6}$

$$\mathcal{U} = \begin{bmatrix} 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \\ -2/\sqrt{6} & 0 & 1/\sqrt{3} \\ 1/\sqrt{6} & -1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix}$$

Σ/ $\lambda = 12, 4$

$$\Sigma = \begin{bmatrix} \sqrt{12} & 0 \\ 0 & \sqrt{4} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2\sqrt{3} & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$$

V/

$$v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\to \sqrt{1^2+1^2} = \sqrt{2}$

$\to \sqrt{1^2+(-1)^2} = \sqrt{2}$

$$V = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$A = \mathcal{U}\Sigma V^*$

$$= \begin{bmatrix} 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \\ -2/\sqrt{6} & 0 & 1/\sqrt{3} \\ 1/\sqrt{6} & -1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix} \begin{bmatrix} 2\sqrt{3} & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}^*$$

$$\boxed{A = \begin{bmatrix} 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \\ -2/\sqrt{6} & 0 & 1/\sqrt{3} \\ 1/\sqrt{6} & -1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix} \begin{bmatrix} 2\sqrt{3} & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}}$$

(c) $A^+ = (U\Sigma V^*)^+ = V\Sigma^+ U^T$

$$U^T = \begin{bmatrix} 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}, \quad \Sigma^+ = \begin{bmatrix} 1/2\sqrt{3} & 0 & 0 \\ 0 & 1/2 & 0 \end{bmatrix}$$

$$A^+ = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/2\sqrt{3} & 0 & 0 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}$$

$Ax = b \Rightarrow x = A^+ b$

$$x = \underset{2\times 2}{\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} 1/2\sqrt{3} & 0 & 0 \\ 0 & 1/2 & 0 \end{bmatrix}} \underset{3\times 3}{\begin{bmatrix} 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}}\left(\frac{1}{2\sqrt{3}}\right) + \frac{1}{\sqrt{2}}(0) & \frac{1}{\sqrt{2}}(0) + \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right) & 0 \\ \frac{-1}{\sqrt{2}}\left(\frac{1}{2\sqrt{3}}\right) + \frac{1}{\sqrt{2}}(0) & \frac{-1}{\sqrt{2}}(0) + \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right) & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{2}} & 0 \\ \frac{-1}{2\sqrt{6}} & \frac{1}{2\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2\sqrt{6}}\left(\frac{1}{\sqrt{6}}\right) + \frac{1}{2\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right) + 0\left(\frac{1}{\sqrt{3}}\right) & \frac{1}{2\sqrt{6}}\left(\frac{-2}{\sqrt{6}}\right) + \frac{1}{2\sqrt{2}}(0) + 0\left(\frac{1}{\sqrt{3}}\right) & \frac{1}{2\sqrt{6}}\left(\frac{1}{\sqrt{6}}\right) + \frac{1}{2\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right) + 0\left(\frac{1}{\sqrt{3}}\right) \\ \frac{-1}{2\sqrt{6}}\left(\frac{1}{\sqrt{6}}\right) + \frac{1}{2\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right) + 0\left(\frac{1}{\sqrt{3}}\right) & \frac{-1}{2\sqrt{6}}\left(\frac{-2}{\sqrt{6}}\right) + \frac{1}{2\sqrt{2}}(0) + 0\left(\frac{1}{\sqrt{3}}\right) & \frac{-1}{2\sqrt{6}}\left(\frac{1}{\sqrt{6}}\right) + \frac{1}{2\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right) + 0\left(\frac{1}{\sqrt{3}}\right) \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{12} + \frac{1}{4} & -\frac{1}{6} & \frac{1}{12} - \frac{1}{4} \\ -\frac{1}{12} + \frac{1}{4} & \frac{1}{6} & -\frac{1}{12} - \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{1}{6} & -\frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3}(1) - \frac{1}{6}(2) - \frac{1}{6}(3) \\ \frac{1}{6}(1) + \frac{1}{6}(2) - \frac{1}{3}(3) \end{bmatrix} = \begin{bmatrix} \frac{1}{3} - \frac{1}{3} - \frac{3}{6} \\ \frac{1}{6} + \frac{1}{3} - 1 \end{bmatrix} = \boxed{\begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}}$$

(d) $\begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$ is the vector of minimal length that satisfies

(d) $\begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$ is the vector of minimal length that satisfies

$$\begin{bmatrix} -2 & 0 \\ 2 & -2 \\ 0 & 2 \end{bmatrix} x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$Ax-b = \begin{bmatrix} -2 & 0 \\ 2 & -2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2(-\frac{1}{2})+0(-\frac{1}{2}) \\ 2(-\frac{1}{2})-2(-\frac{1}{2}) \\ 0(2)+2(-\frac{1}{2}) \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

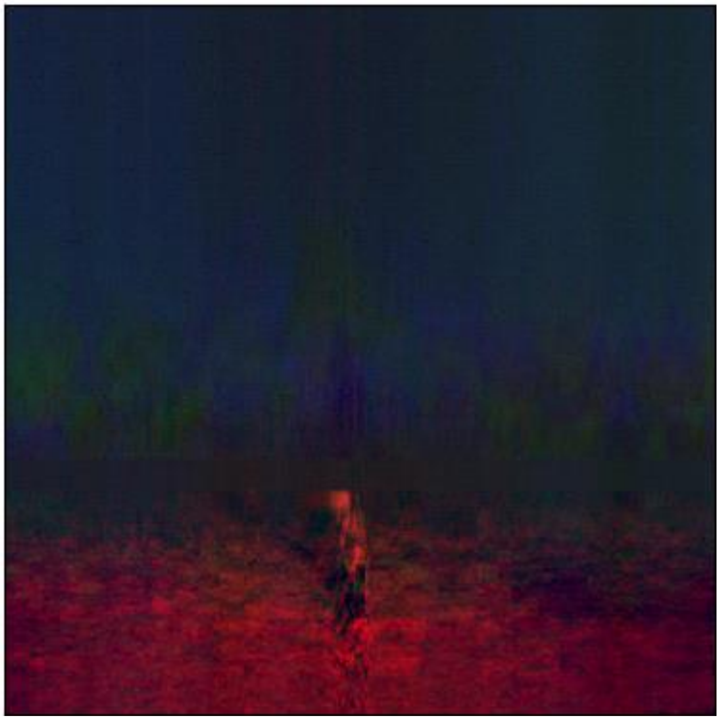
$$= \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ -4 \end{bmatrix}$$

$$\|Ax-b\|_2^2 = \sqrt{0^2+(-2)^2+(-4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

The minimal value of $\|Ax-b\|_2^2$ is $\boxed{2\sqrt{5}}$.

4. (a) Below is the original image of Phoebe Bridger's album *Punisher*.

Below is the approximation created with my function MyTruncatedSVDImage, however I ran into some problems when trying to get a better approximation. I've indicated on my code where I think the problems might be and was unable to fix, but basically, the image below was my result regardless of the singular values I took out (except for s = 500, which removes all singular values and the resulting image is black). I tried s = 5 at first, then s = 0.1, 0.01, 0.001, and so forth since most of the singular values of this image are below 50 (indicated on the plot of singular values in part c).

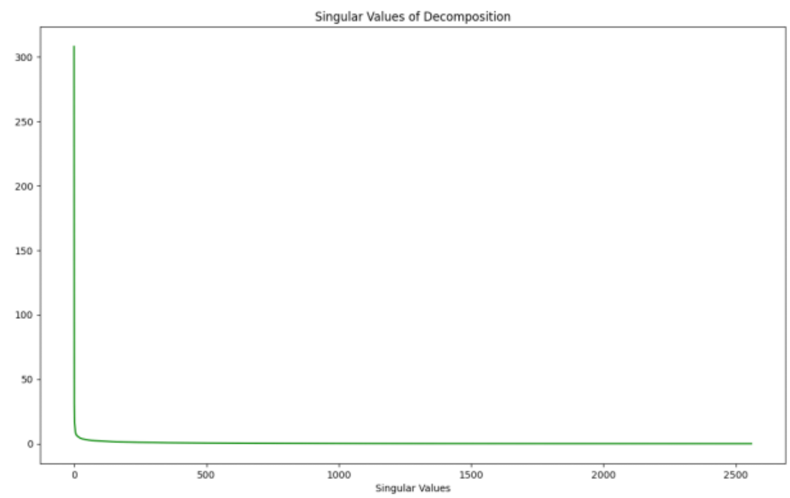


(b) The largest value for s that would be able to reasonably recreate the original image would be the value that keeps the image's singular values that contribute the most to the image's definitive features while deleting those that don't contribute as much to the image's colors. For example, because most of the singular values for the image of *Punisher* are very small numbers, s could be set at that limit and cut out the extremely small singular values, thus keeping the singular values that contributed to the image's picture more significantly. For this reason, I believe that the largest value for s that would be able to produce an accurate approximation to my original image would be 0.1. (This is explained in more detail in part c below).

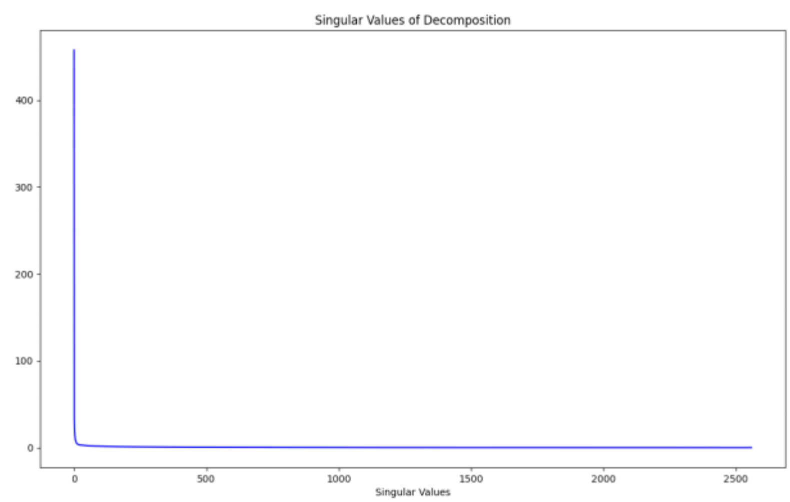
(c) If we plot singular values for the red, green, and blue bands of the original images, we can see that the majority of the singular values for all three bands are very small and converge to zero. These plots are given below. This helped to give me a general idea of what to set my value for s at.

This is the plot of the singular values for the red band of the image.

Below are the singular values for the green band.

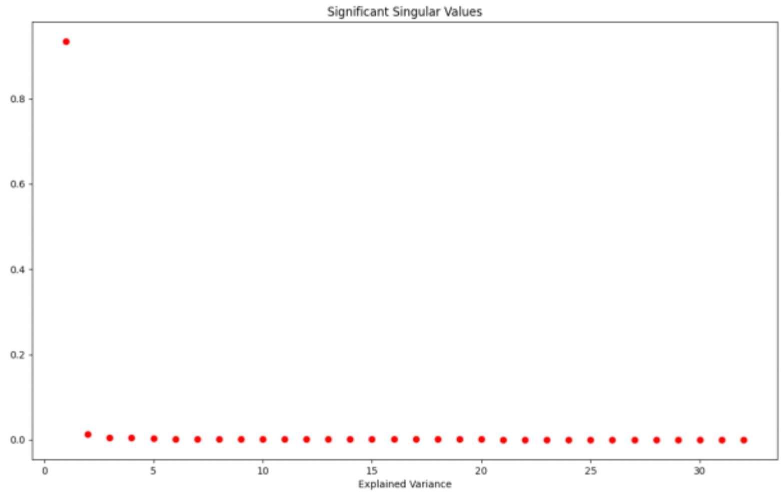


Lastly, here are the singular values for the blue band.

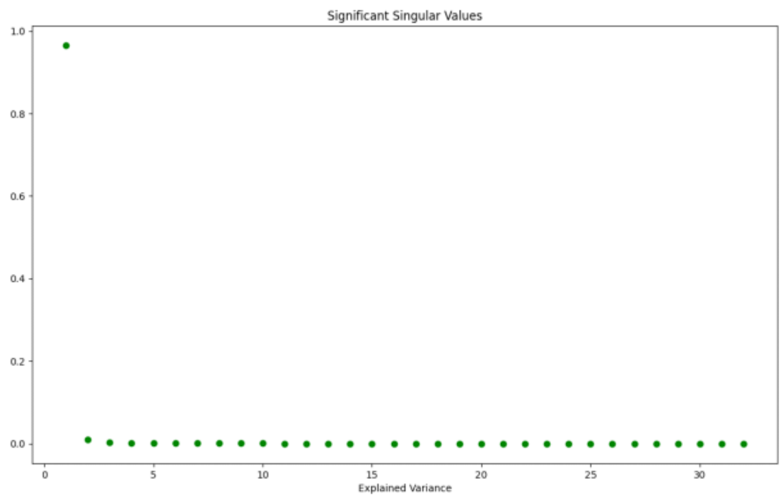


As explained previously, these plots help to provide a general idea for what the value for s might be. To quantitatively determine what to set s at, I used explained variance (code from "Image Reconstruction"). Explained variance can be used to determine which singular values have the most impact on the image for each band in the image. Because the image is 2560x2560, this means that the size of the Σ matrix is also 2560. To better see the variation in data, I plotted 1.25% of the singular values, or 32 singular values. 100% of the singular values closely resembles the plots of the singular values for each band, provided above.

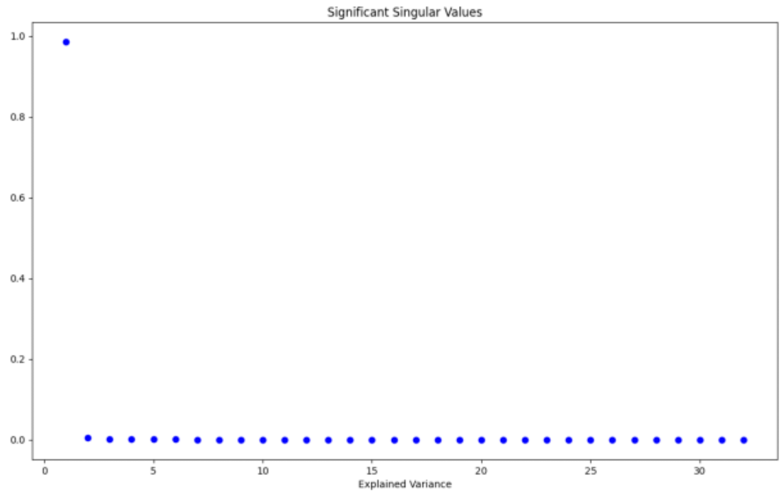
Below is the explained variance for the singular values of the red band of the image.



This next plot provides the explained variance for the singular values of the green band of the image.



And last is the plot of the explained variance for the blue band.



Because a majority of the values in each plot fall under 0.1 with one plotted around 1.0, this is why I would expect s = 0.1 to reasonably recreate my original image. Of course, because I had some issues with my code, I was unable to test this, but this is where I would start. From all six plots, it could also be interesting to try the reverse of what's happening in MyTruncatedSVDImage, where all singular values less than s are being omitted. This would be where all singular values greater than s are omitted. I propose this because a majority of the singular values for my image are close to zero or extremely small values, while only a few are larger numbers. This could end up removing all vibrant colors from the image, though, and defeat the purpose of recreating the original image. I'd also like to mention that the reason most of the singular values are zero is because my original photo has a lot of black and dark colors in it. I'm not sure what the relationship between color vibrancy and its singular value is or if there even is one, but by comparing my results to other images, this could potentially be investigated.

Source:

"Image Reconstruction using Singular Value Decomposition (SVD) in Python. (2020, January 20). Retrieved from https://cmdlinetips.com/2020/01/image-reconstruction-using-singular-value-decomposition-svd-in-python/