Math 3450 **Homework #2**
Winter 2021 **January 13**

This assignment is due by 11:59pm on Friday, January 22^{nd}. *Detail all work* for complete credit. Students may work together, but each student must individually write up their own code and solution set.

1. **(15 points)** Write code that implements the bisection method and Newton's method to find apporximate zeros of a function of a single variable. These codes should have the form `f(x)`, `Df(x)`, `Bisection(a,b,N)`, and `Newton(x0,N)`.

2. **(35 points)** Consider the following functions.

$$f(x) = x^5 - 2$$

$$g(x) = \mathrm{e}^x - 3x^2$$

$$h(x) = x - \tan(x)$$

   (a) Use your bisection method code to find the real roots of $f(x)$ and $g(x)$. Include your choices of a and b and a discussion of the convergence.

   (b) Use your Newton's method code to find the real roots of $f(x)$ and $g(x)$. Include your choice of x_0 and a discussion of the convergence.

   (c) Use your Newton's method code to find all five roots of $f(x)$. Include your choices of x_0.

   (d) Use your Newton's method code to find the smallest positive root of $h(x)$. Use your Newton's method code to find the root of $h(x)$ that is closest to $x = 100$. Explain the differences between these two cases.

3. **(15 points)** Write code that implements Newton's method for a function of two (or more) variables. These codes should have the form `F(x)`, `DF(x)`, and `NewtonVec(x0,N)`. Note that here `x` and `x0` should be vectors of length two (or more).

4. **(15 points)** Use your Newton's method code to find all roots of the following system.

$$x^2 + y^2 - 2x - 2y + 1 = 0$$

$$x + y - 2xy = 0$$

Bonus (8 points) Use BFGS to find all roots of the system given in problem #4.