Sarah Mahl

MATH 3450 Intro to Numerical Methods

1/13/20

HW 1

1. (a)

Graphical user interface, text

Description automatically generated with medium confidence

(i)

Values

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| N | RER | TrapR | SimpR | MCInt |
| 10 | 0.5397698629191174 | 0.49162442739682866 | 0.3967612299940129 | 0.45256837978226294 |
| 100 | 0.6078596436599307 | 0.6004099148730263 | 0.48551008658691286 | 0.6121120821980628 |
| 1,000 | 0.6148479686501794 | 0.6140726849811773 | 0.4962558660336726 | 0.6045395609718119 |
| 10,000 | 0.6155486020209662 | 0.6154707658845877 | 0.4973517943016773 | 0.6139072154855745 |
| 100,000 | 0.6156186833667056 | 0.615610896670629 | 0.49746160373630116 | 0.6136743902616315 |
| 1,000,000 | 0.6156256916833373 | 0.6156249129829005 | 0.4974725868496859 | 0.6157506107621863 |

Error

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| N | RER | TrapR | SimpR | MCInt |
| 10 | 0.0758566074668967 | 0.1240020429891855 | 0.2188652403920012 | 0.16305880906037512 |
| 100 | 0.0077668267360834 | 0.01521655551298784 | 0.1301163837991013 | 0.003514388187951356 |
| 1,000 | 0.0007785017358347 | 0.00155378540483686 | 0.1193706043523415 | 0.01108690941420221 |
| 10,000 | 0.0000778683650479 | 0.00015570450142634 | 0.1182746760843368 | 0.001719254900439671 |
| 100,000 | 0.0000077870193085 | 0.00001557371538518 | 0.118164866649713 | 0.001952080124382638 |
| 1,000,000 | 0.0000007787026768 | 0.00000155740311369 | 0.1181538835363282 | 0.0001241403761721438 |

(ii) For the integral of *tan(x)* on the interval [0,1], both the methods for the right

endpoint rule and the trapezoidal rule show that they have linear convergence. This is

because the error bound decreases by a factor of 10 as N increases by a factor of 10. For

some N, the Monte Carlo method also seems to have linear convergence, but because

each estimate has a random element, this is not accurate enough to describe its error.

Lastly, the convergence for Simpson’s method is unclear, but it does seem to be

converging towards 0.497, so the error gets smaller and smaller as the approximation

approaches that value. This could suggest linear convergence.

(b)

Text, letter

Description automatically generated

(i)

Values

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| N | RER | TrapR | SimpR | MCInt |
| 10 | 13.344958883979656 | 13.66670438588245 | 14.512238576921714 | 12.537002207647637 |
| 100 | 13.069153414578787 | 13.086960178954612 | 13.914197650749108 | 12.560678803511973 |
| 1,000 | 13.056422847065326 | 13.058074137681794 | 13.87296627452752 | 13.014694134247673 |
| 10,000 | 13.054716894810435 | 13.054880742005379 | 13.868631627378734 | 13.052827456570437 |
| 100,000 | 13.054567177294025 | 13.054583549206658 | 13.868212844011694 | 13.062817831785441 |
| 1,000,000 | 13.054552693947075 | 13.054554331010284 | 13.868171298910866 | 13.05487052588389 |

Error

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| N | RER | TrapR | SimpR | MCInt |
| 10 | 0.2904078470805764 | 0.6121533489833713 | 1.457687540022635 | 0.517548829251442 |
| 100 | 0.0146023776797079 | 0.032409142055533 | 0.859646613850028 | 0.4938722333871 |
| 1,000 | 0.00187181016624649 | 0.003523100782715 | 0.818415237628441 | 0.039856902651406 |
| 10,000 | 0.00016585791135526 | 0.0003297051062993 | 0.814080590479655 | 0.001723580328642 |
| 100,000 | 0.0000161403949459 | 0.0000325123075786 | 0.813661807112615 | 0.008266794886361 |
| 1,000,000 | 0.0000016570479957 | 0.0000032941112042 | 0.813620262011787 | 0.000319488984811 |

(ii) Much like the error for the integral of *tan(x)* for the right endpoint rule and the

trapezoid rule*,* the convergence of the above function when found using the right

endpoint rule and the trapezoid rule is linear. This is because the error decreases by a

factor of 10 as N increases by a factor of 10. Both Simpson’s method and the Monte

Carlo method have similar results, where the results found using Simpson’s method

seem to converge at 13.868, and the error for the Monte Carlo method suggests linear

convergence but this cannot be stated as a fact.

(c)

Text

Description automatically generated with medium confidence

(i)

Values

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| N | RER | TrapR | SimpR | MCInt |
| 10 | -2.7902947984e-16 | -0.585613243111825 | -0.390408828741216 | -1.859364924080084 |
| 100 | -1.87309437938e-15 | -0.000778996433168 | -0.000519330955446 | 0.4173616057729628 |
| 1,000 | -2.76132215794e-15 | -7.7927016532079e-7 | -5.1951344449961e-7 | 0.019608291066884 |
| 10,000 | -1.76612081649e-13 | -7.794493141049e-10 | -5.196914499383e-10 | 0.0288208680467364 |
| 100,000 | -3.40989718708e-12 | -4.189169769684e-12 | -3.928598586437e-12 | 0.0226624734824559 |
| 1,000,000 | 1.593084244876e-13 | 1.585291453757e-13 | 1.621303528089e-13 | -0.005682969822917 |

Error

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| N | RER | TrapR | SimpR | MCInt |
| 10 | 2.7902947984e-16 | 0.585613243111825 | 0.3904088287412167 | 1.8593649240800845 |
| 100 | 1.87309437938e-15 | 0.0007789964331686737 | 0.0005193309554464 | 0.4173616057729628 |
| 1,000 | 2.76132215794e-15 | 7.792701653207967e-7 | 5.1951344449961e-7 | 0.019608291066884 |
| 10,000 | 1.76612081649e-13 | 7.794493141049161e-10 | 5.196914499383e-10 | 0.0288208680467364 |
| 100,000 | 3.40989718708e-12 | 4.1891697696848e-12 | 3.928598586437e-12 | 0.0226624734824559 |
| 1,000,000 | 1.593084244873e-13 | 1.585291453757991e-13 | 1.621303528089e-13 | 0.0056829698229178 |

(ii) The error for the right endpoint rule doesn’t follow much of a pattern, but because

the original integral equals 0, the error is equal to the absolute value of the approximated value, so this could explain the lack of a pattern. Still, the expected values

should have converged to 0 as N increases, but this did not happen. Both the trapezoid

rule and Simpson’s method have similar patterns, however they also don’t seem to

follow a specified pattern of convergence. They do show that the approximated values

converge to 0, but as N increases by a factor of 10, the error does not decrease with a

constant factor. Of course, the error for the Monte Carlo method is randomized, so

there isn’t a pattern of convergence.



Text

Description automatically generated

Values

|  |  |  |
| --- | --- | --- |
| N | TrapR2D | MCInt2D |
| 10 | 0.1936571701757493 | -0.0075858657607652735 |
| 100 | 0.0056461510566304645 | -0.01570217941327648 |
| 1,000 | 0.00044404024901894743 | -0.009396402361080785 |
| 10,000 | 4.382904793381145e-05 | -0.01036588352952029 |

Error

|  |  |  |
| --- | --- | --- |
| N | TrapR2D | MCInt2D |
| 10 |  |  |
| 100 |  |  |
| 1,000 |  |  |
| 10,000 |  |  |

1. My values for both TrapR2D and MCInt2D don’t converge to the value found with Mathematica. The values with TrapR2D seem to be converging to 0, while the values found with MCInt2D are closer than those found with TrapR2D, but they are all negative. Regardless, the errors for both methods are fairly constant, so I’m assuming there’s a problem with my code.

A picture containing diagram

Description automatically generated