Markov-Switching Model (MSM)

```
import numpy as np
import pandas as pd
import statsmodels.api as sm
from scipy import stats
from matplotlib import cm, pyplot as plt
from datetime import datetime
from io import BytesIO
from dateutil.relativedelta import relativedelta
import quandl

import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import requests
import statsmodels.api as sm
import seaborn as sns
import joblib

import yfinance as yf
import plotly.express as px
```

```
data_df = pd.read_pickle('/data/workspace_files/data_df92.pickle')
rf_df = pd.read_pickle('/data/workspace_files/data_rf.pickle')
```

```
data_df = pd.merge_asof(data_df, rf_df, right_index=True, left_index=True)
```

```
# Split the data on sets
train_set = data_df.iloc[data_df.index < pd.to_datetime('2018-01-01 00:00:00')]
test_set = data_df.iloc[data_df.index >= pd.to_datetime('2017-12-22 00:00:00')]
train_ind = list(data_df.index).index(test_set.index[0]) + 1
print("train set size is ", train_set.shape[0], ", test set size is ", test_set.shape[0])
```

```
train set size is  1351 , test set size is  211
```

```
basepath = '/data/workspace_files/'

filename = 'Equity_Risk_Premium.csv'
df_ER = pd.read_csv(basepath+filename)
df_ER.index = pd.to_datetime(df_ER['date'])
df_ER = df_ER.drop(columns={'date'})
df_ER['Real_Bond_Yield'] = pd.to_numeric(df_ER['Real_Bond_Yield'])

filename = 'Avg_Investor_Holding.csv'
df_IH = pd.read_csv(basepath+filename)
df_IH.set_index('date', inplace=True)
df_IH.index = pd.to_datetime(df_IH.index)
```

Model

```
mod_hamilton = sm.tsa.MarkovAutoregression(
    train_set.future_return,
    k_regimes = 3,
    order = 1, switching_ar=True,
    exog_tvtp=train_set.VIX, #switching_exog=True,
    # trend='ct',
    # switching_trend=True,
    switching_variance=True
)
mod_hamilton_prediciton = sm.tsa.MarkovAutoregression(
    test_set.future_return,
    k_regimes = 3,
    order = 1, switching_ar=True,
    exog_tvtp=test_set.VIX, #switching_exog=True,
    switching_variance=True
)
res_hamilton = mod_hamilton.fit()
```

```
/opt/python/envs/default/lib/python3.8/site-packages/statsmodels/base/model.py:604: ConvergenceWarning: Maximum Likelihood optimizatio
  warnings.warn("Maximum Likelihood optimization failed to "
```

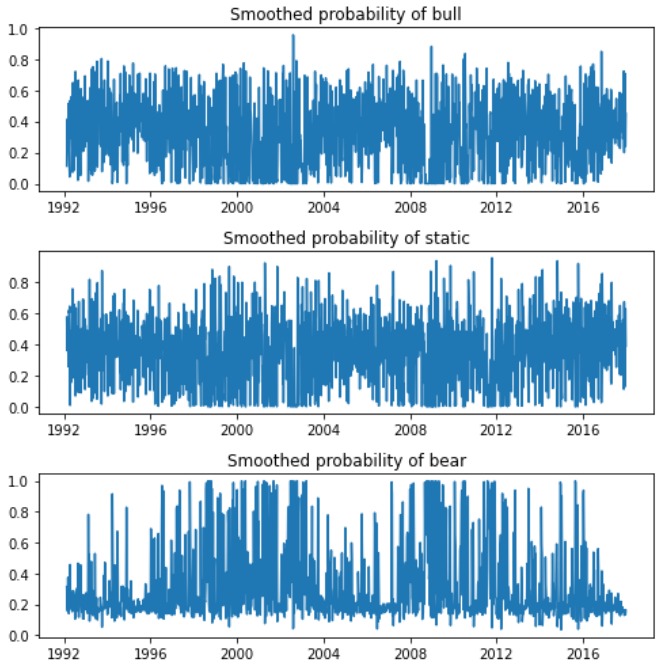
```
res_hamilton.summary()
```

Markov Switching Model Results

```
res_hamilton.params
```

```
mod_hamilton.predict_conditional(res_hamilton.params)[:,:,0]
```

```
fig, axes = plt.subplots(3, figsize=(7, 7))

ax = axes[0]
ax.plot(res_hamilton.smoothed_marginal_probabilities[0])
ax.set(title="Smoothed probability of bull")

ax = axes[1]
ax.plot(res_hamilton.smoothed_marginal_probabilities[1])
ax.set(title="Smoothed probability of static")

ax = axes[2]
ax.plot(res_hamilton.smoothed_marginal_probabilities[2])
ax.set(title="Smoothed probability of bear")

fig.tight_layout()
```

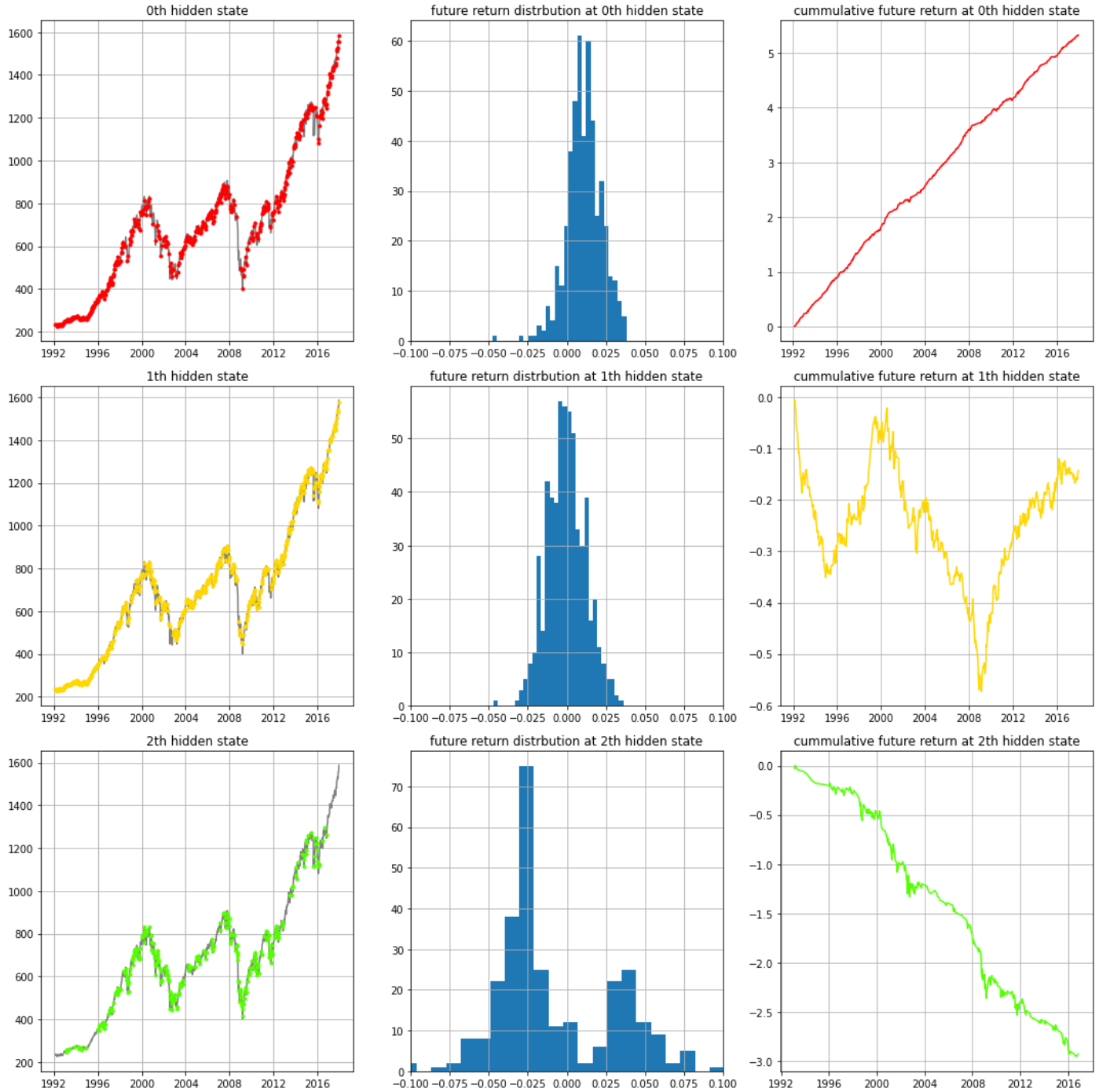



```
res_hamilton.smoothed_marginal_probabilities.idxmax(axis=1).value_counts()
```

```
def plot_hidden_states(res_hamilton, data, column_price):
    plt.figure(figsize=(15, 15))
    fig, axs = plt.subplots(3, 3, figsize = (15, 15))
    colours = cm.prism(np.linspace(0, 1, 3))
    hidden_states = res_hamilton.smoothed_marginal_probabilities.idxmax(axis=1)
    print(hidden_states.value_counts())

    for i, (ax, colour) in enumerate(zip(axs, colours)):
        mask = hidden_states == i
        ax[0].plot(data.index, data[column_price], c = 'grey')
        ax[0].plot(data.index[mask], data[column_price][mask], '.', c = colour)
        ax[0].set_title("{0}th hidden state".format(i))
        ax[0].grid(True)

        ax[1].hist(data["future_return"][mask], bins = 30)
        ax[1].set_xlim([-0.1, 0.1])
        ax[1].set_title("future return distrbution at {0}th hidden state".format(i))
        ax[1].grid(True)

        ax[2].plot(data["future_return"][mask].cumsum(), c = colour)
        ax[2].set_title("cummulative future return at {0}th hidden state".format(i))
        ax[2].grid(True)

    plt.tight_layout()
```

```
plot_hidden_states(res_hamilton, data_df[:train_ind][1:], 'Close')
```

```
1    578
0    479
2    293
dtype: int64


<Figure size 1080x1080 with 0 Axes>
```




```
states_map = {"bear": 2,
              "bull": 0,
              "static": 1}

signal_map = {v: k for k, v in states_map.items()}
```

```
def state_predictor(params, model):
    conditional_prediciton = model.predict(params=params, conditional=True).T
    model_prediction = model.predict(params=params)
    s = np.abs(conditional_prediciton.T - [model_prediction] * 3)
    return s.argmin(axis=0)

test_set_res = test_set.copy()[1:]
test_set_res['signal'] = state_predictor(res_hamilton.params, mod_hamilton_prediciton)
test_set_res.signal.value_counts()
```

```
train_set.drop(columns=['VIX'], inplace=True)
vix_df = pd.read_pickle('/data/workspace_files/data_vix.pickle')
train_set = pd.merge_asof(train_set, vix_df / 100, right_index=True, left_index=True)
test_set_res.drop(columns=['VIX'], inplace=True)
test_set_res = pd.merge_asof(test_set_res, vix_df / 100, right_index=True, left_index=True)
```

```
/opt/python/envs/default/lib/python3.8/site-packages/pandas/core/frame.py:4906: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-
  return super().drop(
```

```
train_set_res = pd.merge_asof(train_set.copy()[1:], df_ER[['Equity Risk Premium']], left_index=True, right_index=True)
train_set_res = pd.merge_asof(train_set_res, df_IH[['Avg. Stock Holding']], left_index=True, right_index=True)
train_set_res['signal'] = np.array(res_hamilton.smoothed_marginal_probabilities).argmax(axis=1)
```

```
risk_aversion = dict()
risk_premium = dict()
for regime in ['bull', 'bear', 'static']:
    risk_aversion[regime] = 3 / train_set_res.VIX.mean() * train_set_res[train_set_res.signal==states_map[regime]].VIX.mean()
    # risk_aversion[regime] = 1 / train_set_res[train_set_res.signal==states_map[regime]]['Avg. Stock Holding'].mean()
    # risk_premium[regime] = train_set_res[train_set_res.signal==states_map[regime]]['Equity Risk Premium'].mean() - train_set_res[tra
    risk_premium[regime] = train_set_res[train_set_res.signal==states_map[regime]]['future_return'].mean() * 52 - train_set_res[train_
```

```
(risk_aversion, risk_premium)
```

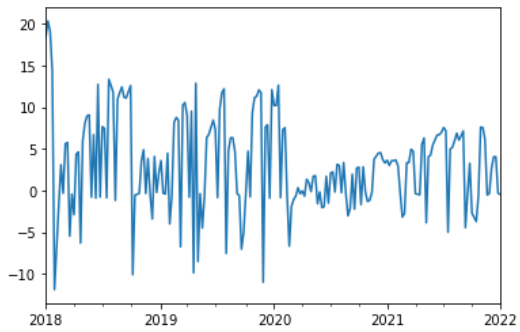
```
({'bull': 2.835931188138735,
  'bear': 3.7231633660694152,
  'static': 2.769380786545341},
 {'bull': 0.5504972425280706,
  'bear': -0.5455112454439829,
  'static': -0.03876103316590226 4})
```

```
def mean_variance_weight(vol, signal):
    variance = vol**2
    weight= np.divide((1/risk_aversion[signal_map[signal]]) * (risk_premium[signal_map[signal]]), variance)
    return weight
```

```
test_set_res['weight'] = [mean_variance_weight(vol, int(signal)) for i, (vol, signal) in test_set_res[['VIX','signal']].iterrows()]
```

```
test_set_res['weight'].plot()
```

```
<AxesSubplot:>
```



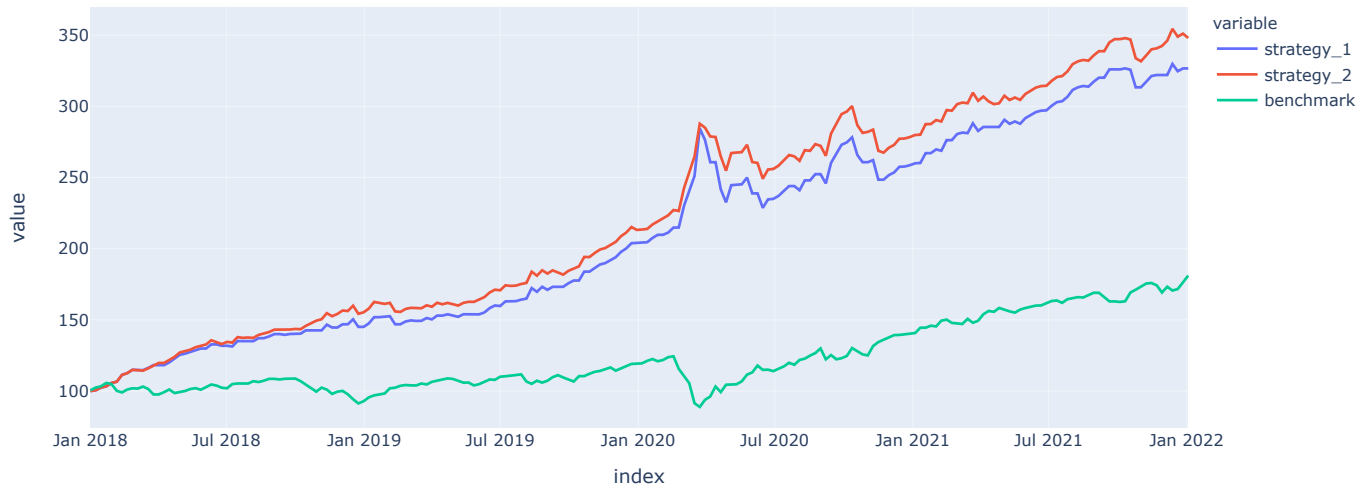
```
test_set_res['weight'] = np.minimum(np.maximum(test_set_res['weight'], -1),1)
```

```
initial_capital = 100
test_set_res['strategy_1'] = 0.
test_set_res['strategy_2'] = 0.
test_set_res['strategy_1'].iloc[0] = initial_capital
test_set_res['strategy_2'].iloc[0] = initial_capital

v_t = initial_capital
### Strategy 1
for i in range(test_set_res.shape[0]-1):
    if test_set_res['signal'].iloc[i] == states_map['bull']:
        v_t = v_t * (1 + test_set_res['future_return'].iloc[i])
    if test_set_res['signal'].iloc[i] == states_map['bear']:
        v_t = v_t * (1 - test_set_res['future_return'].iloc[i])
    test_set_res['strategy_1'].iloc[i+1] = v_t

v_t = initial_capital
### Strategy 2
for i in range(test_set_res.shape[0]-1):
    v_t = v_t * (1 + test_set_res['future_return'].iloc[i] * test_set_res['weight'].iloc[i])
    test_set_res['strategy_2'].iloc[i+1] = v_t
```

```
/opt/python/envs/default/lib/python3.8/site-packages/pandas/core/indexing.py:1732: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-
  self._setitem_single_block(indexer, value, name)
```

```
test_set_res['benchmark'] = 100 * (test_set_res['future_return'] + 1).cumprod()
```

```
px.line(test_set_res[['strategy_1', 'strategy_2', 'benchmark']])
```




```
# test_set_new = pd.concat([train_set_res, spy_vix_data[['VIX','RF']].resample('W').last()],axis=1).dropna()

test_set_new = test_set_res.copy()
```

```
columns = ['strategy_1','strategy_2','benchmark']

df_summary = pd.DataFrame()
final = {}

for column in columns:
    df_returns = test_set_new[column].pct_change()
    s = {}

    yearly_multiplier = 52 #weekly
    year_count = relativedelta(test_set_new['RF'].index[-1], test_set_new['RF'].index[0]).years
    rf = (1 + test_set_new['RF'] / yearly_multiplier).prod() ** (1 / year_count) - 1
    s['Mean Return'] = np.exp(np.log(test_set_new[column]/test_set_new[column].shift(1)).mean() * yearly_multiplier) - 1
    s['Volatility'] = df_returns.std() * np.sqrt(yearly_multiplier)
    s['Skewness'] = df_returns.skew()
    s['Kurtosis'] = df_returns.kurtosis()
    s['Maximum Drawdown'] = (test_set_new[column] / test_set_new[column].cummax()-1).min()
    s['Sharpe Ratio'] = (s['Mean Return'] - rf) / s['Volatility']
    s['Semi-Deviation'] = df_returns[df_returns < df_returns.mean()].std()*np.sqrt(yearly_multiplier)
    s['1-month Autocorrelation'] = df_returns.autocorr(lag=1)
    s['3-month Autocorrelation'] = df_returns.autocorr(lag=3)
    s['1-year Autocorrelation'] = df_returns.autocorr(lag=12)
    s
    final[column] = s

df_summary = pd.DataFrame.from_dict(final, orient='index').round(4).T
# df_summary.to_csv('res.csv')
df_summary
```

	strategy_1	strategy_2	benchmark

```
## writing output of models
rename_map = {
        'bull' : 1,
        'bear' : -1,
        'static' : 0
    }
replace_map = {v : rename_map[k] for k, v in states_map.items()}
train_set_res.signal.replace(replace_map, inplace=True)
test_set_res.signal.replace(replace_map, inplace=True)

train_set_res[['signal']].to_pickle('/data/workspace_files/output/MSM_train.pickle')
test_set_res[['signal']].to_pickle('/data/workspace_files/output/MSM_test.pickle')
```