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Final Exam (Linear Alg. Question)

1) Model as sys of linear eqns

Let x = grad students
y = Lecturers
z = Professors

$\Rightarrow$ Need to have 33 sections

$$1x + 4y + 2z = 33$$

$\Rightarrow$ Total Salary = 815 k

$$15x + 50y + 90z = 815$$

$\Rightarrow$ # of grad students ($=x$)

$$x = \frac{1}{2}(4y + z)$$

$$2x = 4y + z$$

$$2x - 4y - z = 0$$

Sys of linear Eqn's:

$$\begin{aligned} x + 4y + 2z &= 33 \\ 15x + 50y + 90z &= 815 \\ 2x - 4y - z &= 0 \end{aligned}$$

2) Sys. of Linear Eq's in Matrix Form.

$$\begin{bmatrix} 1 & 4 & 2 & 33 \\ 15 & 60 & 90 & 815 \\ 2 & -1 & -1 & 0 \end{bmatrix}$$

3) Solve Matrix

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 33 \\ 15 & 50 & 90 & 815 \\ 2 & -1 & -1 & 0 \end{array}\right]$$

$R2 = R2 - 15(R1)$

$R3 = R3 - 2(R1)$ $\longrightarrow$

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 33 \\ 0 & -10 & 60 & 320 \\ 0 & -9 & -5 & -66 \end{array}\right]$$

Pivot now -10

$R3 = R3 - (0.9)(R2)$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 4 & 2 & 33 \\ 0 & -10 & 60 & 320 \\ 0 & 0 & -59 & -354 \end{array}\right]$$

$R3 = \left(\frac{-1}{59}\right)(R3)$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 4 & 2 & 33 \\ 0 & -10 & 60 & 320 \\ 0 & 0 & 1 & 6 \end{array}\right]$$

3) $\left[\begin{array}{ccc|c} 1 & 4 & 2 & 33 \\ 0 & -10 & 60 & 320 \\ 0 & 0 & 1 & 6 \end{array}\right]$

A) Back-Prop R2

$R2 = R2 - 60(R3)$

$= [0, -10, 60, 320] - 60[0, 0, 1, 6]$

$= [0, -10, 0, -40]$

$R2 = \left(\frac{-1}{10}\right)(R2)$

$= \left(\frac{-1}{10}\right)[0, -10, 0, -40]$

$= [0, 1, 0, 4]$

B) Repeat for R1

$R1 = R1 - 2(R3)$

$= [1, 4, 2, 33] - [0, 0, 2, 12]$

$= [1, 4, 0, 21]$

$R1 = R1 - 4(R2)$

$= [1, 4, 0, 21] - [0, 4, 0, 16]$

(3) $\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 6 \end{array}\right]$

4) Write solution to system

x (grad students) $= 5$

y (lecturers) $= 4$

z (Professors) $= 6$

5) The college must hire 5 grad students, 4 Lecturers, and 6 Professors in order to cover 33 sections, keep in the \$815k budget, and assert the # of grad students to be half the number of Lecturers and Professors combined.