DSA 8420 Spring 2025
Homework 3

Due February 4, 2025

1. (32 points) Solve the following linear programs graphically. For each problem, sketch the feasible region; state the set of optimal solutions and the optimal value if they exist; briefly explain the reason if no optimal solution exists.

(a)

$$
\begin{aligned}
\min \ z = \quad & x_1 - 4x_2 \\
\text{s.t.} \quad & x_1 + x_2 \leq 12 \\
& -2x_1 + x_2 \leq 4 \\
& x_2 \leq 8 \\
& x_1 - 3x_2 \leq 4 \\
& x_1,\ x_2 \geq 0
\end{aligned}
$$

(b)

$$
\begin{aligned}
\min \ z = \ & 4x_1 + 5x_2 \\
\text{s.t.} \quad & 3x_1 + 2x_2 \leq 24 \\
& x_1 \geq 5 \\
& 3x_1 - x_2 \leq 6 \\
& x_1,\ x_2 \geq 0
\end{aligned}
$$

(c)

$$\min\ z = -x_1 + 2x_2$$
$$\text{s.t.} \quad -2x_1 + x_2 \leq 2$$
$$2x_1 + 5x_2 \geq 10$$
$$x_1 - 4x_2 \leq 2$$
$$x_1,\ x_2 \geq 0$$

(d)

$$\max\ z = 6x_1 + 8x_2$$
$$\text{s.t.} \quad x_1 + 4x_2 \leq 16$$
$$3x_1 + 4x_2 \leq 24$$
$$3x_1 - 4x_2 \leq 12$$
$$x_1,\ x_2 \geq 0$$

2. (18 points) Transform the following linear programs into standard form.

(a)

$$\min\ z = 2x_1 - 3x_2 + 5x_3 + x_4$$
$$\text{s.t.} \quad -x_1 + 3x_2 - x_3 + 2x_4 \leq -12$$
$$5x_1 + x_2 + 4x_3 - x_4 \geq 10$$
$$3x_1 - 2x_2 + x_3 - x_4 = -8$$
$$x_1,\ x_2,\ x_3,\ x_4 \geq 0$$

(b)

$$\min\ z = x_1 - x_2 + x_3$$
$$\text{s.t.} \quad x_1 + 2x_2 - x_3 \leq 3$$
$$x_1 - x_2 - x_3 \leq -2$$
$$x_1 - x_2 = 10$$
$$x_1 \geq 0,\ x_2 \leq 0$$