Introduction

- What is Linear Programming (LP)?
  - Programming: a series of actions that are planned to be done.
    - dates back to WW II
    - also known as Linear Optimization.

  - An example

minimize/maximize

eg
$$\begin{aligned}
\min\quad & 2x_1 - x_2 + 4x_3 && \text{— objective (function)} \\
\text{s.t.}\quad & x_1 + x_2 + x_4 \le 2 \\
& 3x_2 - x_3 = 5 \\
& x_3 + x_4 \ge -3 \\
& x_1 \ge 0 \\
& x_4 \le 0
\end{aligned}$$

Subject to

constraints $\begin{cases} \text{inequality} \\ \text{equality} \end{cases}$

x_1, x_2, x_3, x_4 — (decision) variables

All related functions are linear.

eg
$$\begin{aligned}
\min\quad & xyz \\
\text{s.t.}\quad & -x \le 1 \\
& -y \le 5 \\
& -z \le 0
\end{aligned}$$

Is this an LP?

- LP terms

  - Solution: A solution is a point in the underlying space $\mathbb{R}^n$.

  - feasibility: A solution is feasible if it satisfies all constraints. The set of all feasible solutions is called the feasible region. Any solution that does not satisfy all constraints is infeasible.

    A problem is feasible if its feasible region is nonempty. A problem is infeasible if its feasible region is empty.

  - optimality. A solution x^* is optimal to a maximization (resp. minimization) problem if and only if

    1. x^* is feasible
    2. $f(x^*) \geq f(x)$ (resp. $f(x^*) \leq f(x)$) for all feasible x, where f is the objective function.

    x^*: optimal solution, optimum (maximum / minimum)

    $f(x^*)$: optimal value, maximum / minimum value

  - boundedness: A maximization (resp. minimization) problem is bounded if there exists $M \in \mathbb{R}$ such that $f(x) \leq M$ (resp. $f(x) \geq M$) for all feasible x, where f is the objective function. (Note that this is different from the boundedness of the feasible region.)

Remark: All of the inequality constraints in LP are defined with "$\leq$" instead of "$<$".

- Beyond LP
  - Based on function type: Quadratic programming, Nonlinear programming, Convex programming
  - Based on variable type: Integer programming, Mixed integer programming
  - Based on data type: Stochastic programming
  - Based on objective quantity: Multi-objective programming
  - Based on problem scale: Large-scale optimization

  etc.