DSA 8420 HW3

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Contents

1. Solve the following linear programs graphically. For each problem, sketch the feasible region; state the set of optimal solutions and the optimal value if they exist; briefly explain the reason if no optimal solution exists.

```
library(ggplot2)
library(lpSolve)

# using lpSolve lib
plot_lp <- function(obj, constraints, directions, rhs, maximize = FALSE, title) {
  solution <- lp(direction = ifelse(maximize, "max", "min"),
                 objective.in = obj,
                 const.mat = constraints,
                 const.dir = directions,
                 const.rhs = rhs)

  x1 <- seq(0, max(rhs), length.out = 100)
  x2 <- seq(0, max(rhs), length.out = 100)
  grid <- expand.grid(x1, x2)
  colnames(grid) <- c("x1", "x2")

  feasible <- apply(grid, 1, function(x) {
```

```
    all(constraints %*% x <= rhs)
  })

  grid$feasible <- feasible

  plot <- ggplot(data = grid, aes(x = x1, y = x2)) +
    geom_tile(aes(fill = feasible), alpha = 0.4) +
    scale_fill_manual(values = c("TRUE" = "lightgreen", "FALSE" = "white")) +
    geom_abline(slope = -obj[1]/obj[2], intercept = 0, color = "red") +
    labs(title = title, x = "x1", y = "x2") +
    theme_minimal()

  return(list(plot = plot, solution = solution))
}
```

1a.

min z = x1−4x2

such that...

x1 + x2 <= 12

−2x1 + x2 <= 4

x2 <= 8

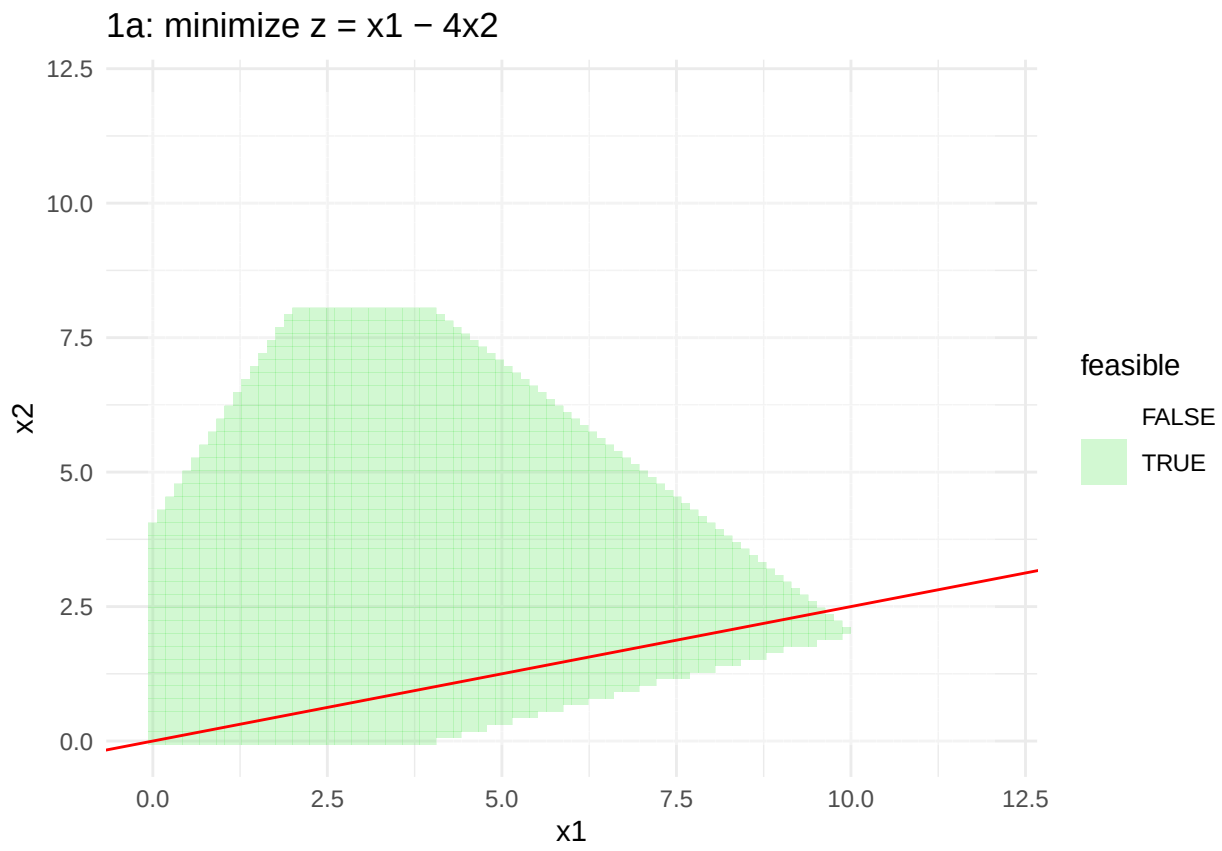
x1 −3x2 <= 4

x1, x2 >= 0

```
obj1a <- c(1, -4)
con1a <- matrix(c(1, 1, -2, 1, 0, 1, 1, -3), nrow = 4, byrow = TRUE)
dir1a <- c("<=", "<=", "<=", "<=")
rhs1a <- c(12, 4, 8, 4)

result1a <- plot_lp(obj1a, con1a, dir1a, rhs1a, title = "1a: minimize z = x1 - 4x2")
result1a
```

```
## $plot
```




```
##
## $solution
## Success: the objective function is -30
```

1b.

min z = 4x1+5x2

such that...

3x1 +2x2 <= 24

x1 >= 5

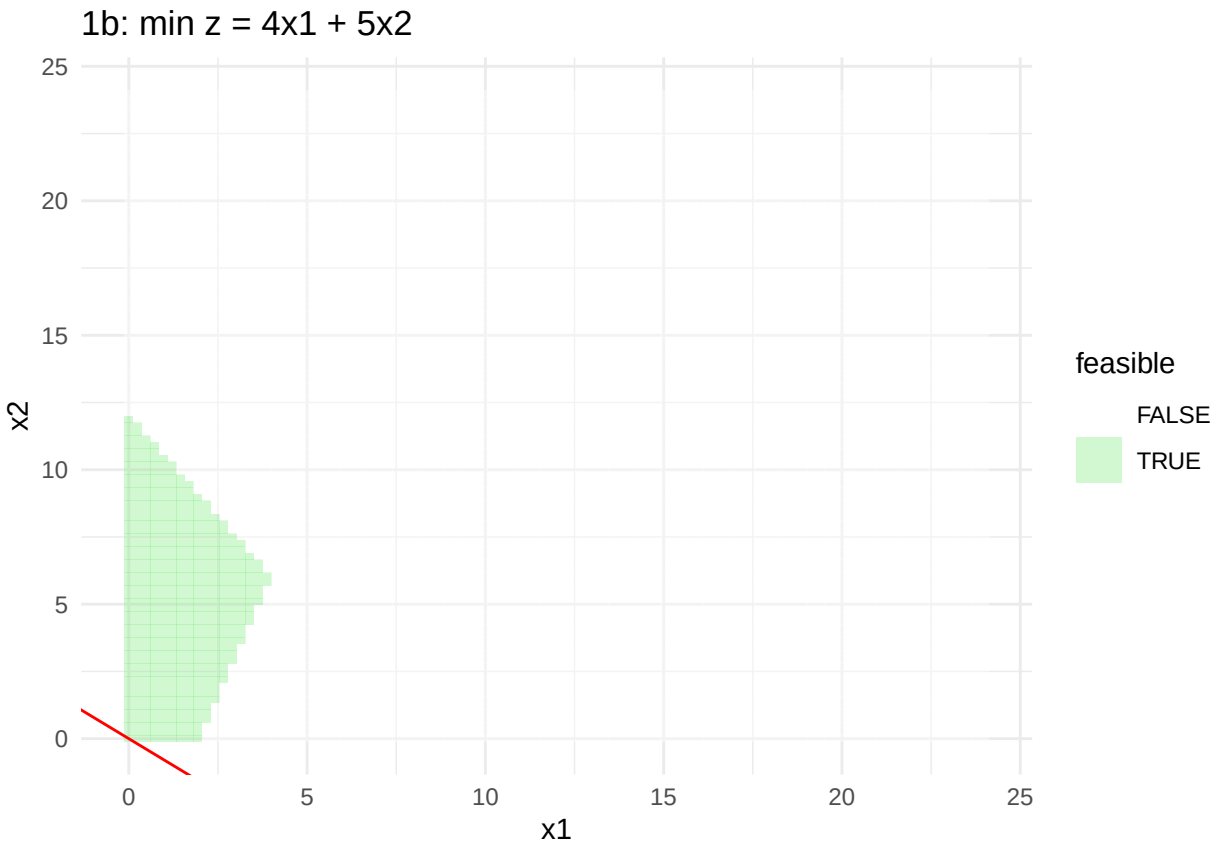
3x1 − x2 <= 6

x1, x2 >= 0

```
obj1b <- c(4, 5)
con1b <- matrix(c(3, 2, -1, 0, 3, -1), nrow = 3, byrow = TRUE)
dir1b <- c("<=", ">=", "<=")
rhs1b <- c(24, 5, 6)

result1b <- plot_lp(obj1b, con1b, dir1b, rhs1b, title = "1b: min z = 4x1 + 5x2")
result1b
```

```
## $plot
```




```
##
## $solution
## Error: no feasible solution found
```

1c.

min z = −x1+2x2

so that...

−2x1 + x2 <= 2

2x1 +5x2 >= 10

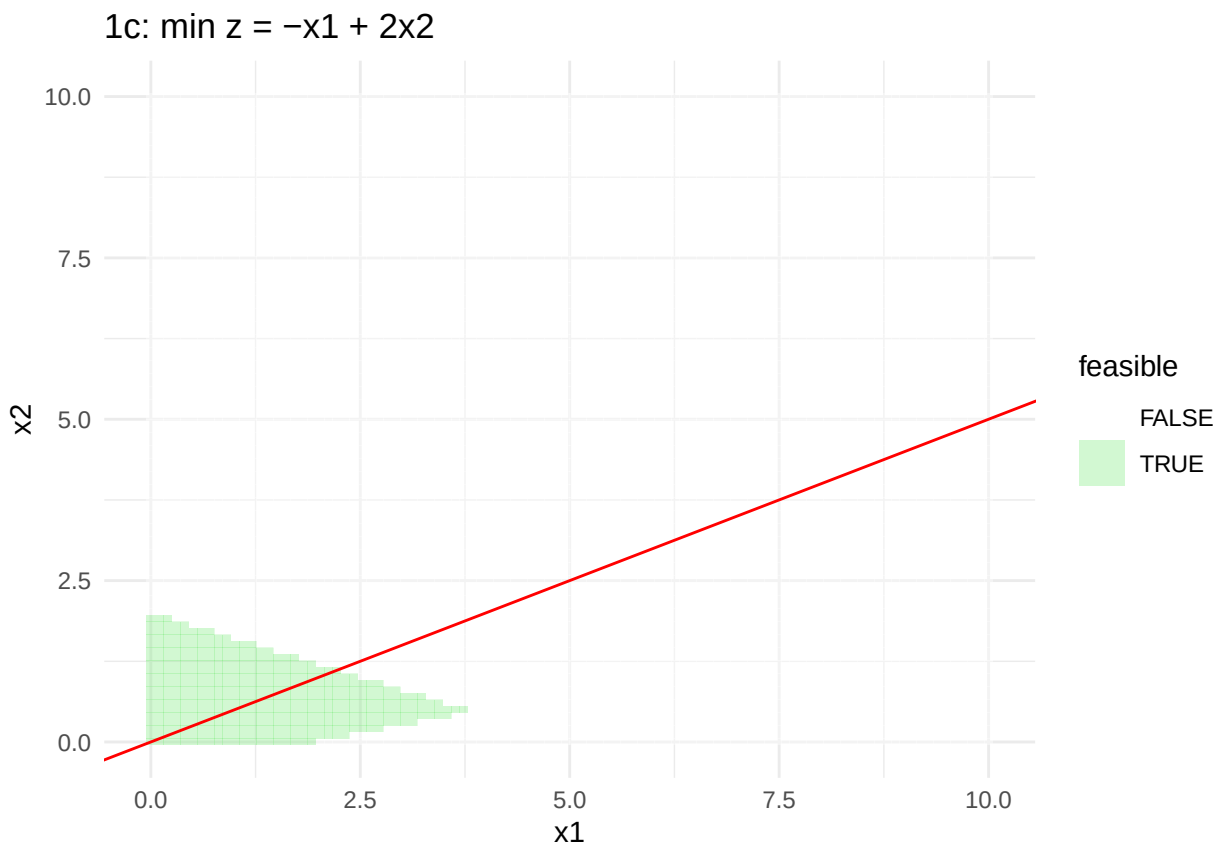
x1 −4x2 <= 2

x1, x2 >+ 0

```
obj1c <- c(-1, 2)
con1c <- matrix(c(-2, 1, 2, 5, 1, -4), nrow = 3, byrow = TRUE)
dir1c <- c("<=", ">=", "<=")
rhs1c <- c(2, 10, 2)
```

```
result1c <- plot_lp(obj1c, con1c, dir1c, rhs1c, title = "1c: min z = -x1 + 2x2")
result1c
```

```
## $plot
```




```
##
## $solution
## Error: status 3
```

1d.

max z = 6x1+8x2

such that..

x1 +4x2 <= 16

3x1 +4x2 <= 24

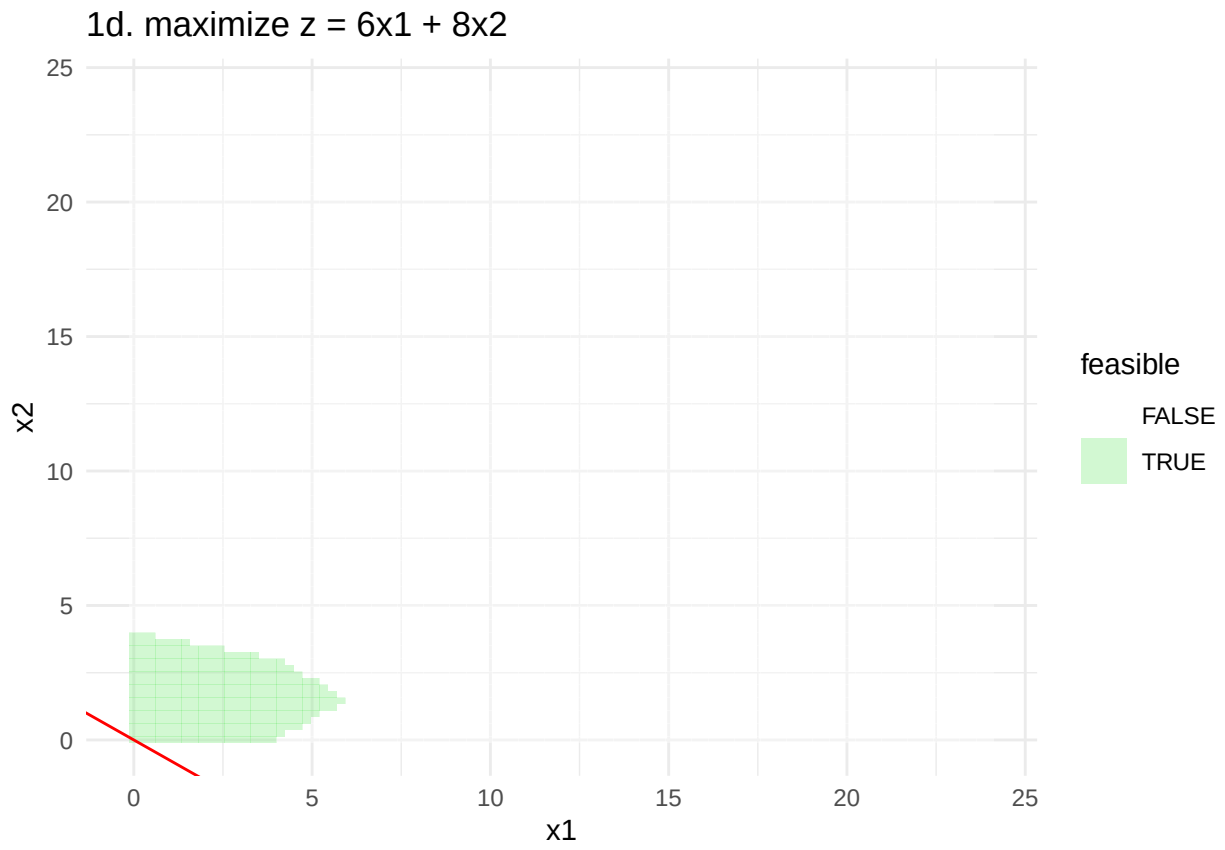
3x1 −4x2 <= 12

x1, x2 >=0

```r
obj1d <- c(6, 8)
con1d <- matrix(c(1, 4, 3, 4, 3, -4), nrow = 3, byrow = TRUE)
dir1d <- c("<=", "<=", "<=")
rhs1d <- c(16, 24, 12)

result1d <- plot_lp(obj1d, con1d, dir1d, rhs1d, maximize = TRUE, title = "1d. maximize z = 6x1 + 8x2")
result1d
```

```
## $plot
```




```
##
## $solution
## Success: the objective function is 48
```

---

2. Transform following to standard form.

```r
# LP into std
convert_to_std <- function(obj, constraints, directions, rhs, vars_nonnegative, title) {
  cat("\n", title, "\n")
```

```
  # Ensure RHS is positive
  for (i in 1:length(rhs)) {
    if (rhs[i] < 0) {
      constraints[i, ] <- -constraints[i, ]
      rhs[i] <- -rhs[i]
      if (directions[i] == "<=") {
        directions[i] <- ">="
      } else if (directions[i] == ">=") {
        directions[i] <- "<="
      }
    }
  }

  num_constraints <- nrow(constraints)
  slack_vars <- diag(num_constraints)
  std_constraints <- cbind(constraints, slack_vars)
  std_obj <- c(obj, rep(0, num_constraints))

  cat("Objective function: \n")
  print(std_obj)

  cat("Standard form constraint: \n")
  print(std_constraints)

  cat("RHS: \n")
  print(rhs)

  cat("Vars: \n")
  print(vars_nonnegative)
}
```

2a.

min z = 2x1−3x2+5x3+ x4

so that...

−x1 +3x2 − x3+2x4 <=−12

5x1 + x2+4x3− x4 >= 10

3x1 −2x2 + x3− x4 = −8

x1, x2, x3, x4 >= 0

```
obj2a <- c(2, -3, 5, 1)
con2a <- matrix(c(-1, 3, -1, 2,
                  5, 1, 4, -1,
                  3, -2, 1, -1), nrow = 3, byrow = TRUE)
dir2a <- c("<=", ">=", "=")
rhs2a <- c(-12, 10, -8)
vars2a <- c("x1 >= 0", "x2 >= 0", "x3 >= 0", "x4 >= 0")
```

```r
convert_to_std(obj2a, con2a, dir2a, rhs2a, vars2a, title = "2a standard form:")
```

```
##
##  2a standard form:
## Objective function:
## [1]  2 -3  5  1  0  0  0
## Standard form constraint:
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7]
## [1,]    1   -3    1   -2    1    0    0
## [2,]    5    1    4   -1    0    1    0
## [3,]   -3    2   -1    1    0    0    1
## RHS:
## [1] 12 10  8
## Vars:
## [1] "x1 >= 0" "x2 >= 0" "x3 >= 0" "x4 >= 0"
```

2b.

min z = x1−x2 +x3

constraints:

x1 +2x2 −x3 <= 3

x1 −x2 −x3 <= −2

x1 −x2 = 10

x1 >= 0

x2 <= 0

```r
obj2b <- c(1, -1, 1)
con2b <- matrix(c(1, 2, -1,
                  1, -1, -1,
                  1, -1, 0), nrow = 3, byrow = TRUE)
dir2b <- c("<=", "<=", "=")
rhs2b <- c(3, -2, 10)
vars2b <- c("x1 >= 0", "x2 <= 0")

convert_to_std(obj2b, con2b, dir2b, rhs2b, vars2b, title = "2b standard form:")
```

```
##
##  2b standard form:
## Objective function:
## [1]  1 -1  1  0  0  0
## Standard form constraint:
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    2   -1    1    0    0
## [2,]   -1    1    1    0    1    0
## [3,]    1   -1    0    0    0    1
## RHS:
```

```
## [1]  3  2 10
## Vars:
## [1] "x1 >= 0" "x2 <= 0"
```