

# P module

A module for the simulation of phosphorus fluxes in soil and plant system

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## 1. Model structure:

Units: length (L), mass (M), Time (T=day)

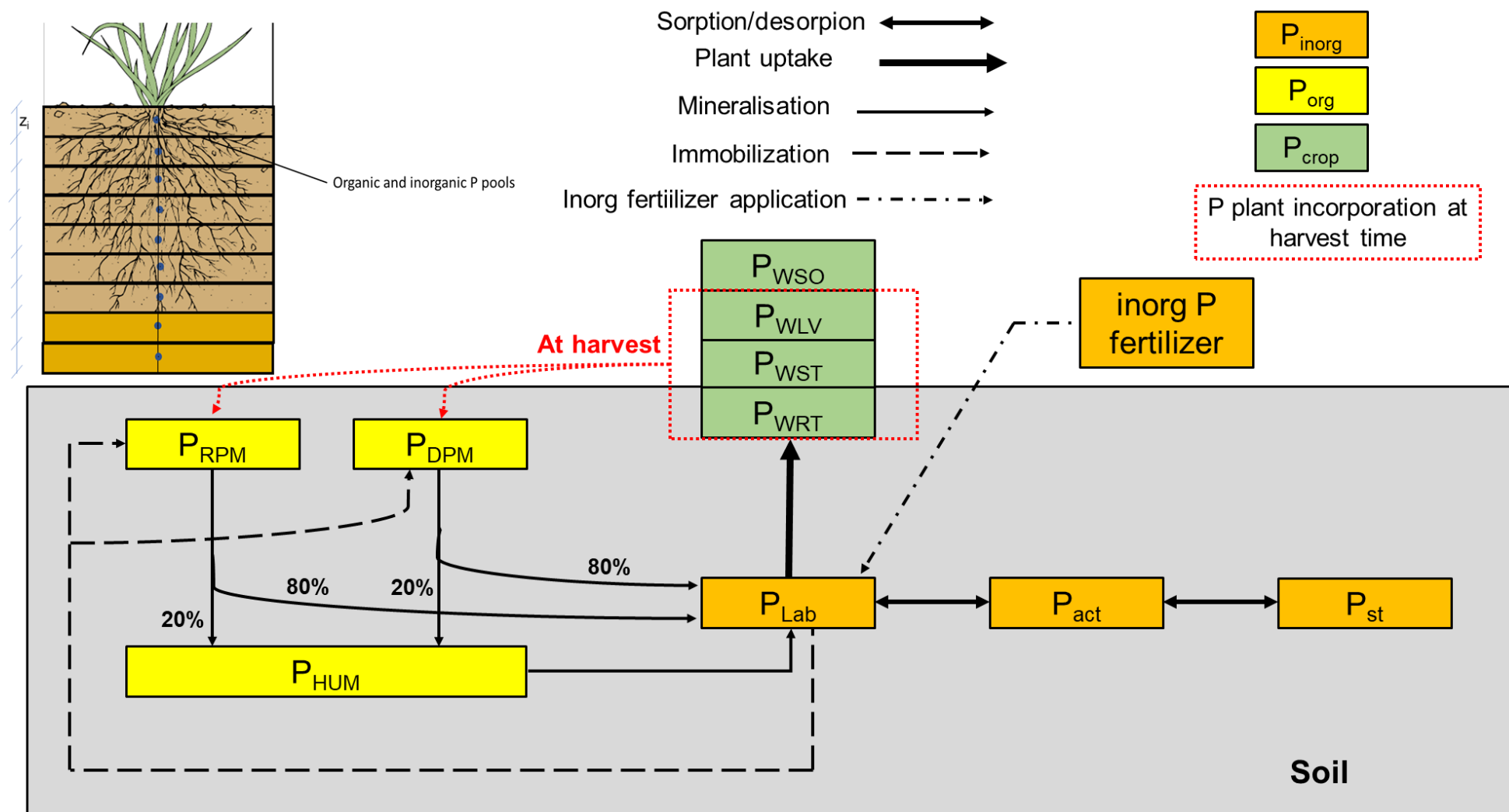
- The plant dry matter is divided into ( $M_{\text{dry matter}}/L^2$ ):
  - “ $WLV$ ”: Leaves
  - “ $WST$ ”: Stems
  - “ $WRT$ ”: Roots
  - “ $WSO$ ”: Seeds or storage organs

These time variant functions (variables) are all calculated by the crop growth module.

- The Phosphorus P in each of the vegetation compartment is defined as follows ( $M P/L^2$ ):
  - “ $P_{WLV}$ ”: P in leaves
  - “ $P_{WST}$ ”: P in stems
  - “ $P_{WRT}$ ”: P in roots
  - “ $P_{WSO}$ ”: P in seeds or storage organs

The soil profile is divided into layers with variable thickness:

- The soil organic matter of each soil layer “ $i$ ” is divided into 3 pools ( $M \text{ om}/L^2$ ):
  - “ $DPM_i$ ”: Fresh decomposable plant material pool
  - “ $RPM_i$ ”: Fresh resistant plant material
  - “ $Hum_i$ ”: Stable organic materials
- The soil phosphorus in each soil layer “ $i$ ” is divided into ( $M P/L^2$ ):
  - “ $P_{Lab_i}$ ”: Labile inorganic P
  - “ $P_{act_i}$ ”: Active inorganic P
  - “ $P_{st_i}$ ”: stable inorganic P
  - “ $P_{DPM_i}$ ”: P in “DPM, $i$ ” pool
  - “ $P_{RPM_i}$ ”: P in “RPM, $i$ ” pool
  - “ $P_{Hum_i}$ ”: P in “Hum, $i$ ” pool



## 2. Model fluxes rates:

For a given soil layer  $i$ , the exchange of mass fluxes between model pools are given as follows:

### 2.1. Rapid adsorption of inorganic P:

$$M_{La} = \left( P_{Lab} - P_{act} * \frac{Fl}{1 - Fl} \right) * F_{im} * F_{it} \quad (1)$$

Where:

$P_{Lab}, P_{act}$ : the labile and active P pools respectively (in  $\text{M P/L}^2$ )

$Fl$ : P sorption coefficient defined as the fraction of fertilizer remaining in the labile pool after the rapide initial phase of P sorption was complete (unitless -). Its value depends on the soil and fertilizer properties. This value can be determined experimentally with a regression analysis by estimating the slope of the linear relationship between labile pool and added fertilizer  $P_f$  to a given soil (Source Jones et al, 1984).

When  $M_{La}$  is positive, a time constant of  $1 \text{ day}^{-1}$  is applied to describe the rate of P movement from  $P_{Lab}$  to  $P_{act}$ ,  $R_{La}$  (in  $\text{M P/L}^2/\text{T}$ ). The flow reverse  $R_{aL}$  proceeds when  $M_{La}$  is negative. Since reverse flow is supposed to be much slower, **Eq 1** is multiplied by a time constant equals to  $0.1 \text{ day}^{-1}$  (Source APEX\_EPIC repport,2008). Mathematically, this can be expressed as follows:

$$\begin{cases} \text{if } M_{La} > 0 \rightarrow \begin{cases} R_{La} = 1 * M_{La} \\ R_{aL} = 0 \end{cases} \\ \text{if } M_{La} \leq 0 \rightarrow \begin{cases} R_{La} = 0 \\ R_{aL} = 0.1 * M_{La} \end{cases} \end{cases} \quad (2)$$

**Remark:** unlike APEX\_EPIC repport,2008, Jones et al, 1984 assumed that both the direct and reverse flows were proceeding with the same time constant (i.e.  $R_{La} = 0.1 * M_{La}$ ;  $R_{aL}=0$ )

$F_{im}$ : soil moisture modifying factor (-). Its value should be in the range  $[0,1]$  and is calculated as follows:

$$\begin{cases} \frac{\omega_i}{\omega_{0.03}} & \text{if } F_{im} \leq 1 \\ 1 & \text{if } F_{im} > 1 \end{cases} \quad (3)$$

Where:

$\omega_{0.03}$ : Volumetric soil water at -0.03 MP (its value would be taken from the soil -water retention curve which is a characteristic property of a given soil) [need to be determined]

$\omega_i$ : Volumetric soil water at the soil layer  $i$  ( $L_w^3/L^3$ ). (a variable calculated by the water transport module)

$F_{it}$ : soil temperature modifying factor (-). Its value is given as follows:

$$F_{it} = e^{(0.115*T-2.88)} \quad (4)$$

Where  $T$  is soil temperature ( $^{\circ}\text{C}$ ) at the soil layer  $i$ . Unlike  $F_{im}$ ,  $F_{it}$  is allowed to exceed 1 when  $T > 25$ .

**Remark:** unlike Jones et al 1984, the version of Eq 1 reported in “APEX\_EPIC repport,2008” did not include any of the both modifying factors  $F_{im}, F_{it}$  (see Eq 238, page 43).

**Remark:** With respect to P fertilizer: The P of inorganic fertilizer “ $P_f$ ” is not a model variable state. It is assumed to be in a soluble form that is mixed uniformly to a depth specified by the user and which contributes to the labile pool “ $P_{Lab}$ ”:

- Sharply et al 1984 have reported a set of equations for calculating  $Fl$  (see Table 4) based on physical and chemical soil properties such as  $CaCo_3$  (%), P labile ( $\mu g/kg$ ), pH, soil clay content (%), soil organic content (%) and base saturation (%). However, these equations cannot be used for soil data outside the ranges mentioned in Table 1.
- APEX\_EPIC repport,2008 has also reported another set of equations for calculating  $Fl$  (see Eqs 239, 240, 241, 242 in pages 43-44).
- A comparison between the two sets of equations showed a slight difference between them.

## 2.2. Slow adsorption of inorganic P:

$$M_{as} = k_{as} * (4 * P_{act} - P_{st}) \quad (5)$$

Where:

$P_{act}, P_{st}$ : the active mineral and stable mineral P pools respectively (in  $M P/L^2$ )

With respect to **Eq 5**, there is actually a slight difference between APEX\_EPIC repport,2008 and Jones et al, 1984.

APEX\_EPIC repport,2008 (see Eq243, P44): When  $M_{as}$  is positive, the rate of P movement from  $P_{act}$  to  $P_{st}$ ,  $R_{as}$  (in  $M P/L^2/T$ ), has a time constant  $k_{as}$  that is a function of  $Fl$  as given by the following equation:

$$k_{as} = e^{(-1.77*Fl-7.05)} \quad (6)$$

for non-calcareous soil and a  $k_{as} = 0.0076 \text{ day}^{-1}$  for calcareous soil.

The flow reverse  $R_{sa}$  proceeds when  $M_{as}$  is negative. Since reverse flow is much slower, **Eq 5** is multiplied by 0.1. Mathematically, this can be expressed as follows:

$$\begin{cases} \text{if } M_{as} > 0 \rightarrow \begin{cases} R_{as} = M_{as} \\ R_{sa} = 0 \end{cases} \\ \text{if } M_{as} \leq 0 \rightarrow \begin{cases} R_{as} = 0 \\ R_{sa} = 0.1 * M_{as} \end{cases} \end{cases} \quad (7)$$

unlike APEX\_EPIC repport,2008, Jones et al 1984 assumed that both the direct and reverse flows were proceeding with the same time constant (i.e.  $R_{as} = M_{as}$ ;  $R_{sa}=0$ ). Here, for calcareous soil, the value of the time constant  $k_{as}$  was found to range between 0.00073 and 0.00079  $\text{day}^{-1}$  with a default value equals to 0.00076  $\text{day}^{-1}$  (it is not 0.0076  $\text{day}^{-1}$ ). For non-calcareous soil,  $k_{as}$ , can be calculated using **Eq 6**.

**Remark:** for the best of my knowledge, the version of **Eq 1 & Eq 5** reported by APEX\_EPIC repport,2008 is used in the phosphorus fortran code

### 2.3. *P mineralization/immobilization rates:*

The model considers two sources of mineralization: the fresh organic P pools associated with crop residues as microbial biomass and the stable organic P associated with soil humus.

**Remark:** the equations from 8 to 20 were all taken from Jones et al 1984. Some were modified to be adapted with the fact that the P fresh organic pool, in the new P module, had been split into two pools:  $P_{DPM}$  (corresponding to the amount of organic P in the decomposable plant material organic pool "DPM") and  $P_{RPM}$  (corresponding to the amount of organic P associated with the resistant plant material "RPM" organic pool).

The P immobilization rate depends on i) the decomposition rate of organic residues in the considered layer " $R_{or}$ ", ii) the gross demand (uptake), " $R_{upr}$ ", of the decomposing residues for P and iii) the availability of " $P_{Lab}$ " to supply

#### 2.3.1. *Decomposition rates of organic pools:*

$$R_{DPM} = k_{DPM} * DPM * (F_{ot} * F_{om})^{0.5} * \min(F_{cn}, F_{cp}) \quad (8)$$

Where:

$R_{DPM}$ : rate of decomposition of DPM organic pool (in M organic matter/L<sup>2</sup>/T).

$k_{DPM}$ : time constant (1/day).

With respect to Eq 8, the original equation reported by Jones et al 1984 included  $O_r$  and  $k_{or}$  rather than  $DPM$  and  $k_{DPM}$  respectively.  $O_r$  was representing the organic matter in crop residue plus microbial biomass. Depending on  $\frac{O_r}{O_{rx}}$  ratio (-) which represents the ratio between instantaneous  $O_r$  (in M organic matter/L<sup>2</sup>) and initial  $O_{rx}$  (in M organic matter/L<sup>2</sup>) content of organic matter in crop residue plus microbial biomass, the  $k_{or}$  value in (1/day) was calculated as follows:

$$\begin{cases} 0.8 & \text{if } \frac{O_r}{O_{rx}} > 0.8 \\ 0.05 & \text{if } 0.1 < \frac{O_r}{O_{rx}} < 0.8 \\ 0.0095 & \text{if } \frac{O_r}{O_{rx}} < 0.1 \end{cases}$$

**Remark:** As a starting value and similarly to the Nitrogen module, we suggest setting  $k_{DPM}$  to 0.0274 day<sup>-1</sup>, a value taken from RothC model.

$F_{ot}$ : temperature reduction factor (-). Its value ranges between 0 & 1 and is determined using the following equation:

$$F_{ot} = \frac{0.9 * T}{T + e^{(7.63 - 0.312 * T)}} + 0.1 \quad (9)$$

Where  $T$  is soil temperature (°C) at the soil layer  $i$ .

$F_{om}$ : moisture reduction factor (-). Its value ranges between 0 & 1 and is calculated by the same equation used for  $F_{im}$  (i.e. using Eq 3  $\rightarrow F_{om} = F_{im}$ ).

$F_{cp}$ : reduction factor linked to C/P ratio (-). Its value in the layer " $i$ " is calculated using the following equation:

$$F_{cp} = e^{\left[ \frac{-0.693 * \left( \frac{C_{DPM}}{P_{DPM} + P_{Lab}} - 200 \right)}{200} \right]} \quad (10)$$

Where  $C_{DPM}$  is the carbon content of  $DPM$  pool of the layer “i” and supposed to be 40 % of  $DPM$  (i.e.  $C_{DPM} = 0.4 * DPM$ ). One can note here that according to **Eq 10**, the C/P ratio is a limited factor for organic matter decomposition (i.e.  $F_{cp} < 1$ ) if only  $\frac{C_{DPM}}{P_{DPM} + P_{Lab}}$  ratio was higher than 200.

**Remark:** concerning **Eq 10**, the original equation reported by **Jones et al 1984** included  $C_{or}$  &  $P_{or}$  (i.e. the carbon & organic P in soil crop residue plus microbial biomass) rather than  $C_{DPM}$  and  $P_{DPM}$ . Our choice to substitute them here reflects, indeed, the fact that the fresh soil organic pool has been split into two pools in the current version of P module.

$F_{cn}$ : reduction factor linked to C/N ratio (-). Its value in the layer “i” is calculated using the following equation :

$$F_{cn} = e^{\left[ \frac{-0.693 * \left( \frac{C_{DPM}}{N_{DPM} + N_{inor}} - 25 \right)}{25} \right]} \quad (11)$$

Where  $N_{DPM}$  and  $N_{inor}$  are respectively the organic N of  $DPM$  pool and inorganic N (i.e.  $NH_4 + NO_3$ ) in the soil layer “i”. one can note here that the C/P ratio is a limited factor for organic matter decomposition (i.e.  $F_{cn} < 1$ ) if only  $\frac{C_{DPM}}{N_{DPM} + N_{inor}}$  ratio was higher than 25.

**Remark:** concerning **Eq 11**, the original equation reported by **Jones et al 1984** included  $C_{or}$  &  $N_{or}$  (i.e. the carbon & organic N in soil crop residue plus microbial biomass) rather than  $C_{DPM}$  and  $N_{DPM}$ . Our choice to substitute them here reflects, indeed, the fact that the fresh soil organic pool has been split into two pools in the current version of P module.

**In a similar way and after substituting each  $DPM$  in equations 8, 10 and 11 by  $RPM$** , the decomposition rate of  $RPM$  organic pool is calculated as follows:

$$R_{RPM} = k_{RPM} * RPM * (F_{ot} * F_{om})^{0.5} * \min(F'_{cn}, F'_{cp}) \quad (12)$$

Where:

$R_{RPM}$ : rate of decomposition of  $RPM$  organic pool (in M organic matter/L<sup>2</sup>/T).

$k_{RPM}$ : time constant (1/day). Its value reported in RothC model was 0.00082 day<sup>-1</sup>

$F'_{cp}$ : reduction factor linked to C/P ratio (-). Its value in the layer “i” is calculated using the following equation :

$$F'_{cp} = e^{\left[ \frac{-0.693 * \left( \frac{C_{RPM}}{P_{RPM} + P_{Lab}} - 200 \right)}{200} \right]} \quad (13)$$

Where  $C_{RPM}$  is the carbon content of  $RPM$  pool of the layer “i” and supposed to be 40 % of  $RPM$  (i.e.  $C_{RPM} = 0.4 * RPM$ )

$F'_{cn}$ : reduction factor linked to C/N ratio (-). Its value in the layer “i” is calculated using the following equation :

$$F'_{cn} = e^{\left[ \frac{-0.693 * \left( \frac{C_{RPM}}{N_{RPM} + N_{inor}} - 25 \right)}{25} \right]} \quad (14)$$

Where  $N_{RPM}$  and  $N_{inor}$  are respectively the organic N of  $RPM$  pool and inorganic N (i.e.  $NH_4+NO_3$ ) in the layer “i”.

**Remark:** the aforementioned equations ( i.e. Eqs from 8 to 14) describe the decomposing of soil fresh organic pool(s) using the equations reported by Jones et al 1984 ( but with some modifications !). However, the code AgroC should already include equations that describe this process and these equations are mostly were taken from Rothc model. The question that should emerge here: which set of equations should we use?

### 2.3.2. The gross demand (uptake) rate of organic decomposing for P:

The gross demand rate of the decomposing "DPM" for P ( $R_{upr,DPM}$  in  $M P/L^2/T$ ) is dependent on the rate of the corresponding organic residue decomposition,  $R_{DPM}$ , the ratio  $\frac{P_m}{O_m}$  between the microbial P concentration  $P_m$  in ( $kg P/ha$ ) and the organic matter in microbial biomass  $O_m$  ( $kg organic matter/ha$ ), the efficiency of conversion of organic residue to microbial biomass  $P1_{DPM}$  [ $P1_{DPM} = 0.4(-)$ ], and the carbon/organic matter ratio of the residue  $P2_{DPM}$  [ $P2_{DPM} = 0.4 (-)$ ]

$$R_{upr,DPM} = P1_{DPM} * P2_{DPM} * R_{DPM} * \frac{P_m}{O_m} \quad (15)$$

The ratio  $\frac{P_m}{O_m}$  (-) is given as follows: 
$$\begin{cases} 0.02 & \text{if } P_{Lab} > 10 \text{ kg P/ha} \\ else & \\ 0.01 + 0.001 * P_{Lab} & \end{cases}$$

In a similar way and after substituting each  $DPM$  in equation 15 by  $RPM$ , the gross demand rate of the decomposing "RPM" for P ( $R_{upr,RPM}$  in  $M P/L^2/T$ ) is given as follows:

$$R_{upr,RPM} = P1_{RPM} * P2_{RPM} * R_{RPM} * \frac{P_m}{O_m} \quad (16)$$

**Remark:** the values of  $P2_{DPM}$ ,  $P2_{RPM}$  would be set to 0.4 but those of  $P1_{DPM}$ ,  $P1_{RPM}$  would need to be adjusted by inverse modeling ‘!!?’.

Based on the two previous equations mentioned above, the total gross demand (uptake) of decomposing residues for P ( $R_{upr}$  in  $M P/L^2/T$ ) is given as follows:

$$R_{upr} = R_{upr,DPM} + R_{upr,RPM} \quad (17)$$

### 2.3.3. The gross mineralization rate of P:

The gross mineralization rate of P from decaying  $DPM$  pool,  $R_{P,DPM}$  (in  $M P/L^2/T$ ), is given as follows:

$$R_{P,DPM} = k_{DPM} * P_{DPM} * (F_{ot} * F_{om})^{0.5} * \min(F_{cn}, F_{cp}) \quad (18)$$

where  $k_{DPM}$ ,  $F_{ot}$ ,  $F_{om}$ ,  $F_{cn}$  and  $F_{cp}$  are similar to those of Eq 8 and are calculated by the same equations.

Similarly, the gross mineralization of P from decaying  $RPM$  pool,  $R_{P,RPM}$  (in  $M P/L^2/T$ ), is given as follows:



$$R_{P,RPM} = k_{RPM} * P_{RPM} * (F_{ot} * F_{om})^{0.5} * \min(F'_{cn}, F'_{cp}) \quad (19)$$

where  $k_{RPM}$ ,  $F_{ot}$ ,  $F_{om}$ ,  $F'_{cn}$  and  $F'_{cp}$  are similar to those of **Eq 12** and are calculated by the same way.

the P module assumes that 80 % of  $R_{P,RPM}$  flux (or  $R_{P,DPM}$ ) is mineralized into  $P_{Lab}$  while the remaining part (i.e. 20 %) is incorporated into  $P_{Hum}$

The rate of P mineralization from the stable (humified) organic matter pool (in  $M P/L^2/T$ ), is given as follows:

$$R_{PHum} = k_{Hum} * P_{Hum} * \min(F_{om}, F_{ot}) \quad (20)$$

Where:

$k_{Hum}$ : time constant (1/day). the original equation reported by Jones et al 1984 has set the value of this parameter to  $0.0003 \text{ day}^{-1}$ . However, and similarly to Nitrogen module, we suggest setting this parameter to  $0.0000348 \text{ day}^{-1}$  (a value taken from RothC model).

**Remark:** Another way for modeling the mineralization/immobilization of P organic pools is substituting the equations from 8 to 20 by the set of equations reported by APEX\_EPIC report, 2008 (see Eqs 236 and 237, pages 42-43). Similarly, the model considered two sources of P mineralization: P of crop residues that was modelled by only one pool and the stable organic P pool. The equation described the mineralization of the stable P pool was quite similar to that reported by Jones et al 1984 whereas that describing the mineralization of the fresh organic P pool showed some differences. The main difference was in the equations used to calculate the effect of temperature, soil water content and C/P ratio. (see Eq 179, page 36). However, I could not figure out the equations describing the immobilization of inorganic P: indeed, in the list of notation given at the end of rapport (page 104), we can read WIP (P immobilization rate) but nothing about P immobilization was mentioned in the main text. (need to be verified again). The question that should be answered here which one of the two sets of equations need to be modified and used in our model?

#### 2.4. Plant P:

*Except the equations from 23 to 26, the equations here were all taken from Wave model (chapter 6)*

To include P uptake by plant, similar methodology as for N was employed. This module assumes that the demand for P is created by the difference between the current P content of the plant and an Optimum content. This latter decreases as the development of the plant proceeds (Optimum P contents are stoichiometry parameters that need to be identified for each investigated plant and at its different development stages).

The module assumes that the P taken up is divided between shoot and root in proportion to their relative demand. Seeds demand for P is met by the vegetative part of the plant, thus increasing the demand of P from the soil. As tissue become depleted, translocation to the seeds becomes retarded and the demand for P in the seeds is not met, resulting in seeds with a lower P content.

#### **To calculate the uptake:**

- 1- First step: Calculate potential p Uptake (by two methods)
- 2- Second step: Calculate diffusive and convective
- 3- Third step: Calculate the total real uptake

#### **First step: Calculate potential p Uptake (by two methods)**

Mathematically, the potential P uptake rate is calculated by one of the two methods:

**Method 1:** using equations derived from the Wave model (in chapter 6): here, the daily potential P uptake rate,  $R_{PU\_pot}$ , is supposed to be equivalent to the sum of the P demand rate of leaves, stems and roots.

$$R_{PU\_pot} = P_{DLV} + P_{DST} + P_{DRT} \quad (21)$$

And

$$\left. \begin{aligned} P_{DLV} &= \frac{WLV * XPLV - P_{WLV}}{taup} \\ P_{DST} &= \frac{WST * XPST - P_{WST}}{taup} \\ P_{DRT} &= \frac{WRT * XPRT - P_{WRT}}{taup} \end{aligned} \right\} \quad (22)$$

Where:

$R_{PU\_pot}$ : Potential P uptake rate (in  $M P/L^2/T$ )

$P_{DLV}$ ,  $P_{DST}$  and  $P_{DRT}$  are the P demand rate of leaves, stems and roots respectively (in  $M P/L^2/T$ )

$WLV$ ,  $WST$  and  $WRT$  are the accumulated plant dry matter (in  $M/L^2$ ) for leaves, stems and roots respectively (variables calculated by crop growth module).

$P_{WLV}$ ,  $P_{WST}$  and  $P_{WRT}$  are the actual P content (in  $M P/L^2$ ) in leaves, stems and roots respectively.

$XPLV$ ,  $XPST$  and  $XPRT$  are stoichiometric parameters representing the optimum levels of P content in leaves, stems and roots respectively dependent on development plant stage (in  $M P/M_{dry\ matter}$ ). **The values of these stoichiometric parameters need to be investigated in literature.**

$taup$ : time period (in day) during which the P demand of the plant can be satisfied. This parameter was set to 1 day in N uptake module (i.e. daily crop demand).

**In Method 2** the P demand is based on the equation reported by APEX\_EPIC report, 2008. Compared to method 1, the main advantage is that the total P demand do not need data on the optimum P content for each plant part as function of crop development stage. (i.e. root, stem, leaves, seed). Instead, only data on the optimal P content in the whole plant as a function of crop development is needed here.

$$R_{PU_{pot}} = P_{DEM} \quad (23)$$

The daily crop demand for P,  $P_{DEM}$ , is the difference between the actual plant P content and the optimum P content for that day. This is mathematically expressed as follows:

$$P_{DEM} = WPT * X_{P_{plant}} - P_{W_{plant}} \quad (24)$$

$P_{DEM}$ : The total P demand rate of the plant (in M P/L<sup>2</sup>/T)

$WPT$  is the accumulated crop dry matter (in M /L<sup>2</sup>). (practically, it is the sum of accumulated plant dry matter for seed, leaves, stems and roots (i.e.  $WPT = WSO + WLW + WST + WRT$ ).

$P_{W_{plant}}$  the actual P crop content (in M P/L<sup>2</sup>).

$X_{P_{plant}}$  the optimal P concentration dependent on development plant stage (in M P/M dry matter) and could be calculated as follows:

$$X_{P_{plant}} = b_{p1} + b_{p2} * e^{(-b_{p3} * HUI)} \quad (25)$$

$HUI$ : Heat unit index (-). It is ranging from 0 at planting to 1.0 at physiological maturity and is given as follows:

$$HUI = \frac{\sum_{t_0}^t HU * \Delta t}{PHU} = \frac{\sum_{t_0}^t \max(0, T - T_b) * \Delta t}{PHU} \quad (26)$$

Where:

$PHU$ : potential heat unit of crop (°C)

$t$ : actual time (day)

$t_0$ : initial time corresponding to crop emergence (day)

$\Delta t$ : time step (day)

$T$ : average daily temperature (°C)

$T_b$ : crop specific base temperature below which crop growth ceases (°C)

$b_{p1}, b_{p2}, b_{p3}$ : crop parameters expressing optimal P content dependent on plant development stage.

Or from Wave:  $HUI = S_t / (S_{t,crit,1} + S_{t,crit,2})$

**Remark 1:** by using method 2, we suppose here that the P content in soil is big enough so that the daily rate of P uptake depends only on the crop demand. **A hypothesis that could not be really true!!**

**Remark 2:** Equation 26 is used in Apex model for simulating crop growth. The same equation or similar equation may be probably included in AgroC code.

## Second step: Calculate diffusive and convective

The potential P uptake rate is further divided in a diffusive and convective fraction. Given the fact that most of P plant demand (> 93 %) is supplied by “Diffusion” and not by “mass flow or convection”.

Source: Table 3.2 (page 59) from book Growth and Mineral Nutrition of Field Crops.

The potential diffusive P uptake rate,  $R_{PU\_diff}$  (in M P/L<sup>2</sup>/T) from the rooting depth ( $z_{max}$ ) should then be calculated as follows:

$$R_{PU\_diff} = \sum_i R_{PU\_diff_i} = \frac{2 * \pi * RDENS_i * Road * Diff_P * z_i * \omega_i * P_{Lab_i}}{D_0 * z_i} \quad (27)$$

Where:

$RDENS_i$ : the root density at the layer “i” (in L/L<sup>3</sup>)

$Road$ : mean root radius (L)

$Diff_P$ : P diffusion coefficient (L<sup>2</sup>/T) in soil solution :

$P_{Lab_i}$ : labile P concentration (M P/L<sup>2</sup>)

$z_i$  : the rooting depth at layer i (L)

$\omega_i$ : volumetric water content in i soil layer (L<sup>3</sup>/L<sup>3</sup>) . (a variable calculated by the water transport module)

$D_0$  : the travel distance resistance between the bulk soil solution and the root (L). **it is usually in the range of 0.1-15 mm**

Assuming that the amount of P supplied by mass flow ( $R_{PU\_conv}$ ) should not exceed a maximum threshold, for example 5% of the total potential uptake rate of P (*i.e.*  $R_{PU\_MAX\_conv} = 5\% * R_{PU\_pot}$ ), the potential P uptake rate by mass flow,  $R_{PU\_conv}$  (in M P/L<sup>2</sup>/T), from the whole rooting depth ( $z_{max}$ ) can then be deduced as follows:

$$R_{PU\_conv} = \min(5\% * R_{PU\_pot}, R_{PU\_pot} - R_{PU\_diff}) \quad (28)$$

## Third step: Calculate the total real uptake

Hence, based on aforementioned equations, the actual rate of P plant uptake,  $R_{pu}$  (in M P/L<sup>2</sup>/T), is calculated as follows:

$$R_{PU} = R_{PU\_conv} + R_{PU\_diff} \quad (29)$$

**Remark:** The principal forms for P uptake by plant are H<sub>2</sub>PO<sub>4</sub><sup>-</sup>, HPO<sub>4</sub><sup>-2</sup>. The diffusion coefficient “ $Diff_P$ ” for H<sub>2</sub>PO<sub>4</sub><sup>-</sup> in soil solution is 2.4 x10<sup>-11</sup> (cm<sup>2</sup>/sec). Source: Table 3.3 (page 60) from book Growth and Mineral Nutrition of Field Crops.

**Remark:** To meet dimension analysis, the equation 27 was adapted from equations reported in WAVE model (page 108). (**probably need to be verified again!**)

Allocation the P uptake rate in the different plant components is given as follows:

$$\left. \begin{aligned} R_{P_{WLV}} &= R_{PU} * \frac{P_{DLV}}{P_{DEM}} - P_{DSWO} * \frac{WLV}{WLV + WST} \\ R_{P_{WST}} &= R_{PU} * \frac{P_{DST}}{P_{DEM}} - P_{DSWO} * \frac{WST}{WLV + WST} \\ R_{P_{WRT}} &= R_{PU} * \frac{P_{DRT}}{P_{DEM}} \end{aligned} \right\} \quad (30)$$

Where:

$R_{P_{WLV}}$ ,  $R_{P_{WST}}$  and  $R_{P_{WRT}}$  are rates of P flows to leaves, stem and root respectively (in  $M P/L^2/T$ )

$P_{DSWO}$  is the P demand for seeds (in  $M P/L^2/T$ ) that is calculated as follows:

$$P_{DSWO} = \frac{WSO * XPSO - P_{WSO}}{\tau_{aup}} * Redfact \quad (31)$$

$WSO$ : the accumulated plant dry matter (in  $M /L^2$ ) for seeds (variable calculated by crop growth model).

$XPSO$ : the optimum level of P content in seeds (in  $M P/M$  dry matter). (a stoichiometric parameter that its value need to be investigated in literature)

$P_{WSO}$ : the accumulated P content in seeds (in  $M P/L^2$ )

$Redfact$ : reduction factor (-) that is calculated as follows:

$$\left. \begin{aligned} Redfact &= 1 - \sqrt{1 - PRED} \\ 0 \leq PRED &= \frac{APCL - RLPCL}{RMPCL - RLPCL} \leq 1 \end{aligned} \right\} \quad (32)$$

Where:

$APCL$ : the actual P fraction in the leaves (in  $M P/M$  dry matter). [ $APCL = \frac{P_{WLV}}{WLV}$  ]

$RMPCL$ : the leaf P threshold concentration for unrestricted growth (in  $M P/M$  dry matter). [its value needs to be investigated in literature. In N module, this parameter was assumed to be equals to the half of the optimum N content in leaves. (a stoichiometric parameter)]

$RLPCL$ : the leaf P threshold concentration below which there is no growth any more (in  $M P/M$  dry matter). [its value needs to be investigated in literature. In N module, this parameter was assumed to be equals to 0.005 kg P/kg dry matter]. (a stoichiometric parameter). Jones et al 1984 reported that in maize and sorghum the minimum shoot P concentrations are approximately half the optimum concentrations.

## Parameters values:

### 1- Bp1, bp2, bp3

Parameter	Corn	Wheat	Rice	sunflower	soybeans	Barley	Potato	Grass	winter wheat	summer whe	Dwheat	Oats
bp1 p uptake at emergence	0.0062	0.0084	0.006	0.0063	0.0074	0.0084	0.006	0.006	0.0084	0.0084	0.0084	0.0084
bp2 p uptake at 0.5 maturity	0.0023	0.0032	0.003	0.0029	0.0037	0.0032	0.0025	0.0022	0.0032	0.0032	0.0032	0.0032
bp3 p uptake at maturity	0.0018	0.0019	0.0018	0.0023	0.0035	0.0019	0.0012	0.0018	0.0019	0.0019	0.0019	0.0019
bpi, bp2 5 and bp3 are crop parameters expressing P concentration												
BP1 Normal fraction of P in crop biomass at emergence												
BP2 Normal fraction of P in crop biomass at midseason												
BP3 Normal fraction of P in crop biomass at maturity												

### 2- Optimum p values in each part:

Stem : 0.1

Grain: 0.4

Root: 0.07

Leaves: 0.03

### Summary of P module equations:

The following equations should be written for each soil layer i:  $C(t) = C(t - dt) + dt * \frac{dC}{dt}$

labile inorganic P pool  $P_{Lab}$  (in M P/L<sup>3</sup>)

$$P_{Lab}(t) = P_{Lab}(t - dt) + dt * \frac{dP_{Lab}}{dt}$$

$$\frac{dP_{Lab}}{dt} = -R_{La} + R_{aL} + 0.8 * R_{P,DPM} + 0.8 * R_{P,RPM} + R_{PHum} - R_{upr} - R_{PU}$$

Active inorganic P pool  $P_{act}$  (in M P/L<sup>3</sup>)

$$P_{act}(t) = P_{act}(t - dt) + dt * \frac{dP_{act}}{dt}$$

$$\frac{dP_{act}}{dt} = R_{La} - R_{aL} - R_{as} + R_{sa}$$

Stable inorganic P pool  $P_{st}$  (in M P/L<sup>3</sup>)

$$P_{st}(t) = P_{st}(t - dt) + dt * \frac{dP_{st}}{dt}$$

$$\frac{dP_{st}}{dt} = R_{as} - R_{sa}$$

Organic P in DPM pool  $P_{DPM}$  (in M P/L<sup>3</sup>)

$$P_{DPM}(t) = P_{DPM}(t - dt) + dt * \frac{dP_{DPM}}{dt}$$

$$\frac{dP_{DPM}}{dt} = -R_{P,DPM} + R_{upr,DPM}$$

Organic P in RPM pool  $P_{RPM}$  (in M P/L<sup>3</sup>)

$$P_{RPM}(t) = P_{RPM}(t - dt) + dt * \frac{dP_{RPM}}{dt}$$

$$\frac{dP_{RPM}}{dt} = -R_{P,RPM} + R_{upr,RPM}$$

Organic P in Hum pool  $P_{Hum}$  (in M P/L<sup>3</sup>)

$$P_{Hum}(t) = P_{Hum}(t - dt) + dt * \frac{dP_{Hum}}{dt}$$

$$\frac{dP_{Hum}}{dt} = 0.2 * R_{P,DPM} + 0.2 * R_{P,RPM} - R_{PHum}$$

DPM organic pool  $DPM$  (in M om/L<sup>2</sup>) [similar equation may already be present in the code !]

$$DPM(t) = DPM(t - dt) + dt * \frac{dDPM}{dt}$$

$$\frac{dDPM}{dt} = -R_{DPM}$$

RPM organic pool  $RPM$  (in M om/L<sup>2</sup>) [a similar equation may already be present in the code !]

$$RPM(t) = RPM(t - dt) + dt * \frac{dRPM}{dt}$$

$$\frac{dRPM}{dt} = -R_{RPM}$$

And the following equations calculate the accumulation P content in different plant components

Accumulation P in root  $P_{WRT}$  (in M P/L<sup>2</sup>)

$$P_{WRT}(t) = P_{WRT}(t - dt) + dt * \frac{dP_{WRT}}{dt}$$

$$\frac{dP_{WRT}}{dt} = R_{P_{WRT}}$$

Accumulation P in stem  $P_{WST}$  (in M P/L<sup>2</sup>)

$$P_{WST}(t) = P_{WST}(t - dt) + dt * \frac{dP_{WST}}{dt}$$

$$\frac{dP_{WST}}{dt} = R_{P_{WST}}$$

Accumulation P in leaves  $P_{WLV}$  (in M P/L<sup>2</sup>)

$$P_{WLV}(t) = P_{WLV}(t - dt) + dt * \frac{dP_{WLV}}{dt}$$

$$\frac{dP_{WLV}}{dt} = R_{P_{WLV}}$$



### Some points regarding initial values of P module pools

- With respect to organic P pools:  $P_{DPM}, P_{RPM}, P_{Hum}$

At  $t=0$ , The amount of organic P present in each soil layer would be distributed among these three pools  $DPM, RPM, Hum$  in proportion to their sizes.

At harvest time, plant P is divided into roots, leaves, stem and grains fractions. The P of roots, stem and leaves are added to the fresh crop residue pools (i.e.  $P_{DPM}, P_{RPM}$ ). We can use the  $DPM/RPM$  ratio reported in Roth which showed that 59 % and 41 % were respectively the proportions of  $DPM$  and  $RPM$  in the incorporated plant materials.

When no measurements of organic P content in soil are available. We can predict it using the equations reported by Sharply et al 1984 (see table 3) that related the organic P to total N and pH of soil.

**Caution:** equations reported by Sharply et al 1984 cannot be used for soil data outside the ranges mentioned in Table 1.

- With respect to the labile inorganic P pool:  $P_{Lab}$

This pool can be quantified by extraction with anion exchange resin. However, this method is time Consuming. Instead, Sharply et al 1984 have reported several equations (see table 2) that related the labile P pool to the measurements of P soil determined with one of these three common extractants {Olson P [ $NaHCO_3$ ], Barry P [ $NH_4F+HCl$ ], Double acide P [ $HCl+H_2SO_4$ ] }.

Reference:

1.