P module uptake

the demand for P is created by the difference between the current P content of the plant and an Optimum content. This latter decreases as the development of the plant proceeds

Mathematically, the potential P uptake rate is simulated as follows:

The daily rate of P uptake is controlled either by crop demand or by its ability to take up P from the soil. The daily crop demand for P is the difference between the actual plant P content and the optimum P content for that day. This is mathematically expressed as follows:

$$P_{DEM} = WPT * X_{P_{plant}} - P_{W_{plant}} \quad (29)$$

P_{DEM}: The total P demand rate of the plant (in M P/L^2/T) = $R_{PU_{pot}}$: Potential P uptake rate (in M P/L^2/T)

WPT is the accumulated crop dry matter (in M /L^2). (practically, it is the sum of accumulated plant dry matter for seed, leaves, stems and roots (i.e. $WPT = WSO + WLV + WST + WRT$).

$P_{W_{plant}}$ the actual P crop content (in M P/L^2).

$X_{P_{plant}}$ the optimal P concentration dependent on development plant stage (in M P/M $_{\text{dry matter}}$) and could be calculated as follows:

$$X_{P_{plant}} = b_{p1} + b_{p2} * e^{(-b_{p3} * HUI)} \quad (30)$$

HUI: Heat unit index (-). It is given as follows:

$$HUI = \frac{\sum_{t_0}^{t} HU * \Delta t}{PHU} = \frac{\sum_{t_0}^{t} max\left(0, T - T_b\right) * \Delta t}{PHU} \quad (31)$$

Where:

PHU: potential heat unit of crop (°C)

t: actual time (day)

t_0: initial time corresponding to crop emergence (day)

Δt: time step (day)

T: average daily temperature (°C)

T_b: crop specific base temperature below which crop growth ceases (°C)

b_{p1}, b_{p2}, b_{p3}: crop parameters expressing optimal P content dependent on plant development stage.

The potential P uptake rate is further divided in a diffusive and convective fraction. Given the fact that most of P plant demand (> 93 %) is supplied by "Diffusion" and not by "mass flow or convection". Source: Table 3.2 (page 59) from book Growth and Mineral Nutrition of Field Crops.

The potential diffusive P uptake rate, $R_{PU_{diff}}$ (in M P/L^2/T) from the rooting depth (z_{max}) should then be calculated as follows:

$$R_{PU_{diff}} = \sum_i R_{PU\,diff_i} = \frac{2*\pi*RDENS_i*Road*Diff_P*z_i*\omega_i*P_{Lab_i}}{D_0*z_i} \quad (23)$$

Where:

$RDENS_i$: the root density at the layer "*i*" (in L/L^3)

$Road$: mean root radius (L)

$Diff_P$: P diffusion coefficient (L^2/T) in soil solution :

P_{Lab_i}: labile P concentration (M P/L^2)

z_i : the rooting depth at layer i (L)

ω_i: volumetric water content in i soil layer (L_w^3/L^3 . (a variable calculated by the water transport module)

D_0 : the travel distance resistance between the bulk soil solution and the root (L). it is usually in the range of 0.1-15 mm

Assuming that the amount of P supplied by mass flow ($R_{PU_{conv}}$) should not exceed a maximum threshold, for example 5% of the total potential uptake rate of P ($i.e.\ R_{PU_{MAX_{conv}}} = 5\% * R_{PU_{pot}}$), the potential P uptake rate by mass flow, $R_{PU_{conv}}$ (in M P/L^2/T), from the whole rooting depth (z_{max}) can then be deduced as follows:

$$R_{PU_{conv}} = min\left(5\% * R_{PU_{pot}}, R_{PU_{pot}} - R_{PU_{diff}}\right) \quad (24)$$

Hence, based on aforementioned equations, the actual rate of P plant uptake, R_{pu}(in M P/L^2/T), is calculated as follows:

$$R_{PU} = R_{PU_{conv}} + R_{PU_{diff}} \quad (25)$$

Remark: The principal forms for P uptake by plant are $H2PO4^-$, $HPO4^{-2}$. The diffusion coefficient "$Diff_P$" for $H2PO4^-$ in soil solution is 2.4 x10^{-11} (cm^2/sec). Source: Table 3.3 (page 60) from book Growth and Mineral Nutrition of Field Crops.

Remark: To meet dimension analysis, the equation **23** was adapted from equations reported in WAVE model (page 108). (probably need to be verified again!)