Digital Image Processing (CSE/ECE 478)

Lecture # 15: Image restoration

Avinash Sharma

Center for Visual Information Technology (CVIT),

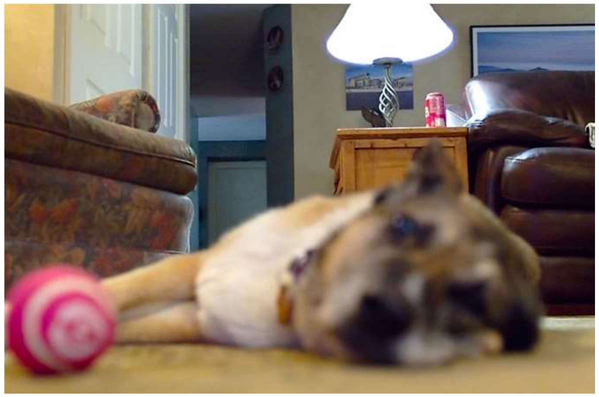
IIIT Hyderabad

Today’s Class

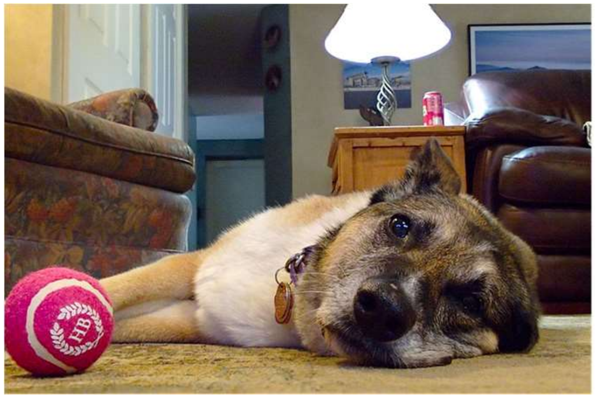
- Degradation/restoration model
- Restoration (in presence of noise only)
- Modelling degradation
- Restoration in presence of both noise and degradation

Examples

Near focus



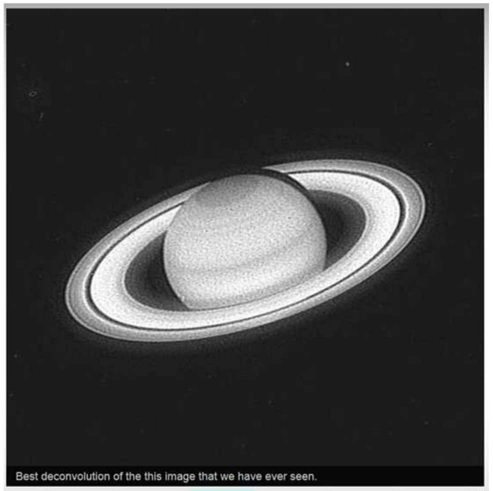
Far focus



Full depth of field

Interesting read: Light Field Cameras

Examples



Courtesy: NASA

Examples

Single Image Haze Removal [He et al. CVPR 2009]

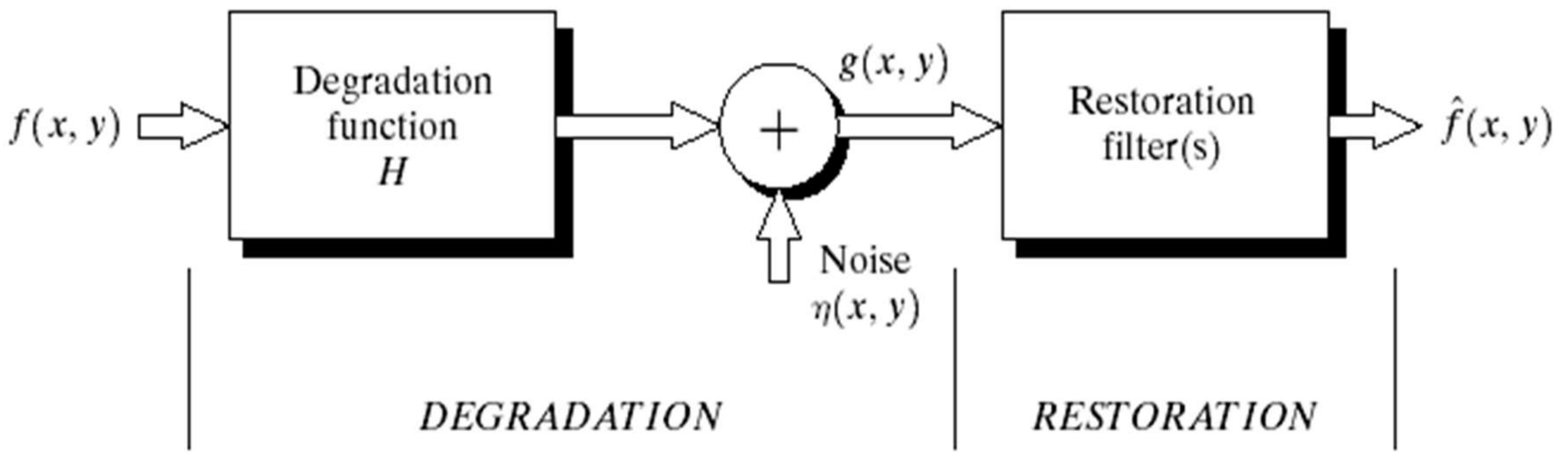
Image Restoration

$$f \longrightarrow \boxed{H} \longrightarrow g$$

Inverse problems

Known	**Problem type**
H, g	Recovery
g	Blind recovery
g, H partially	Semi blind recovery
f, g	System identification

Model of Image Degradation/Restoration



If H is linear, position invariant process,

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)\, F(u, v) + N(u, v)$$

Noise based Degradation

- Assuming H is identity, model reduces to:

$$g(x, y) = f(x, y) + \eta(x, y)$$

$$G(u, v) = F(u, v) + N(u, v)$$

First we discuss easier problem of degradation only due to noise.

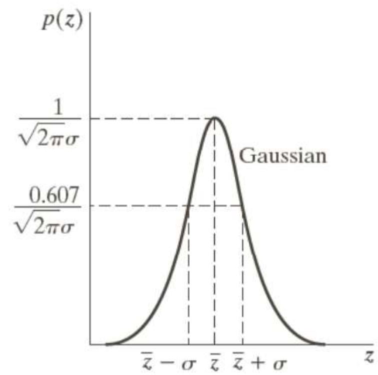
Noise based Degradation

- Source of noise
  - Image acquisition (digitization): sensor noise, light levels, atmospheric turbulance
  - Image transmission
- Spatial properties of noise
  - **Statistical behavior** of the gray-level values of pixels
  - Noise parameters, correlation with the image
- Frequency properties of noise
  - Fourier spectrum
  - E.g., **white** noise (a constant Fourier spectrum)
- *We assume noise is independent of spatial coordinates and uncorrelated with image*.

Noise Models

- Gaussian (normal) Noise
  - widely used due to mathematical convenience

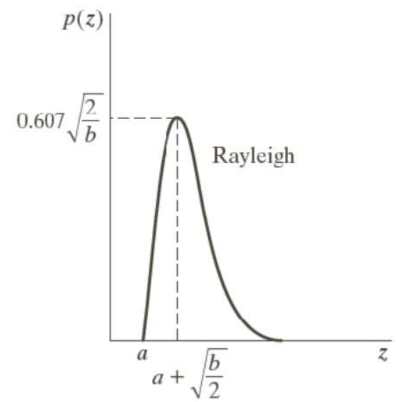
$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\bar{z})^2/2\sigma^2}$$



- Reyleigh Noise

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

Mean: $\bar{z} = a + \sqrt{\pi b/4}$ Variance: $\sigma^2 = \frac{b(4-\pi)}{4}$



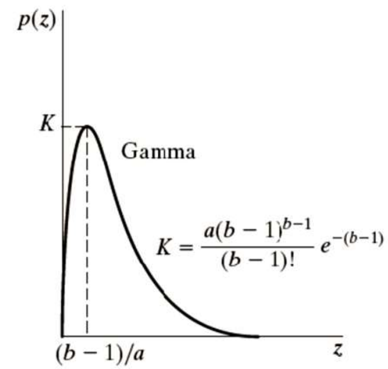
Noise Models

- Erlang (Gamma) Noise

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

$a > 0$

b positive integer

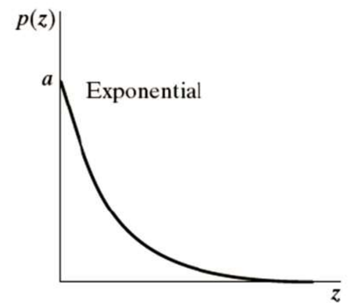


Mean: $\bar{z} = \frac{b}{a}$ Variance: $\sigma^2 = \frac{b}{a^2}$

- Exponential Noise

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

$a > 0$

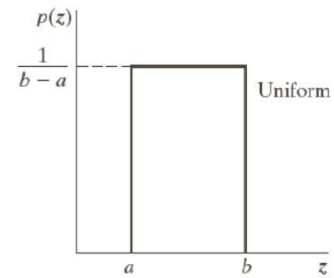


Noise Models

- Uniform Noise

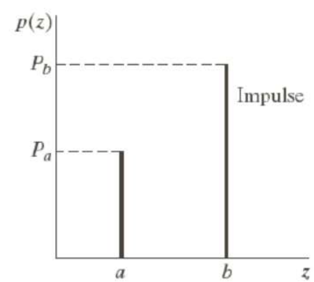
$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Mean: } \bar{z} = \frac{a+b}{2} \qquad \text{Variance: } \sigma^2 = \frac{(b-a)^2}{12}$$



- Impulse (salt-and-pepper) Noise

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$



If either $\boldsymbol{P_a}$ or $\boldsymbol{P_b}$ is zero, it is called *unipolar.*
Otherwise, it is called bipoloar.

Noise Models

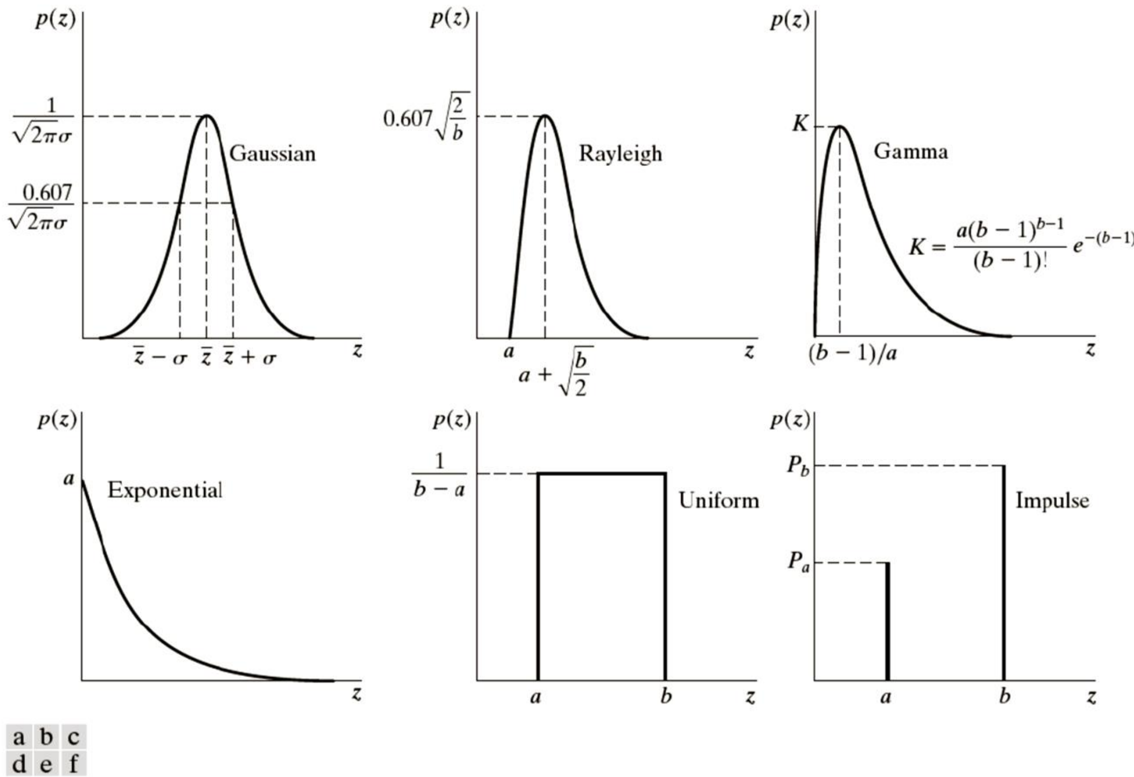
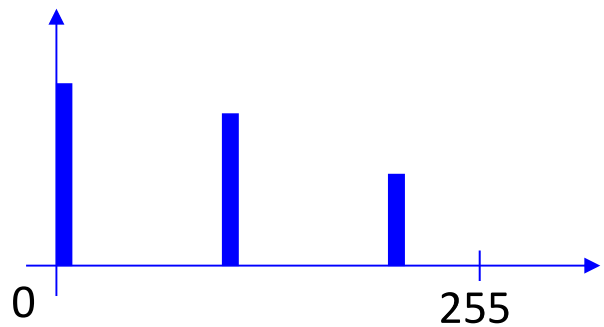


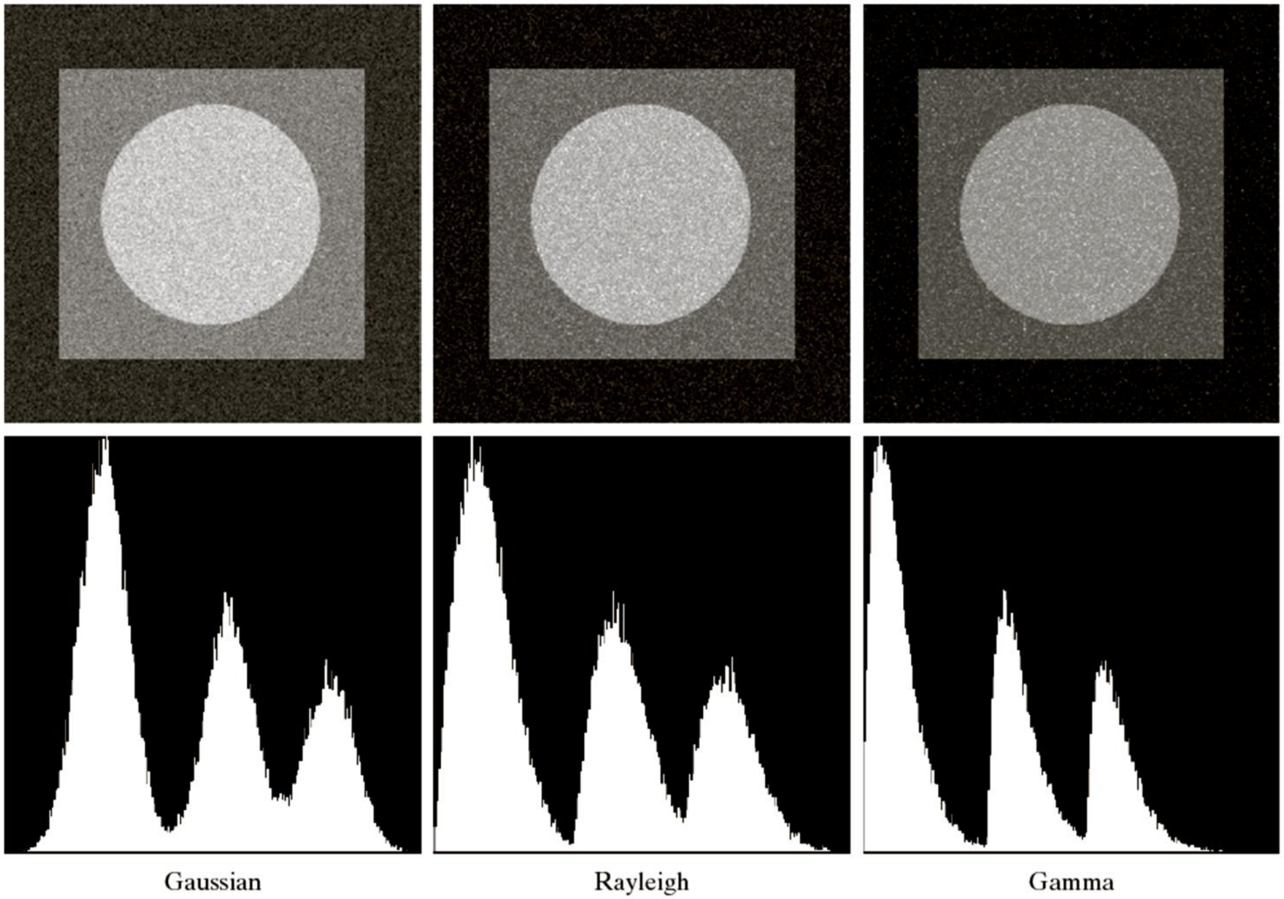
FIGURE 5.2 Some important probability density functions.

Estimating Noise Models

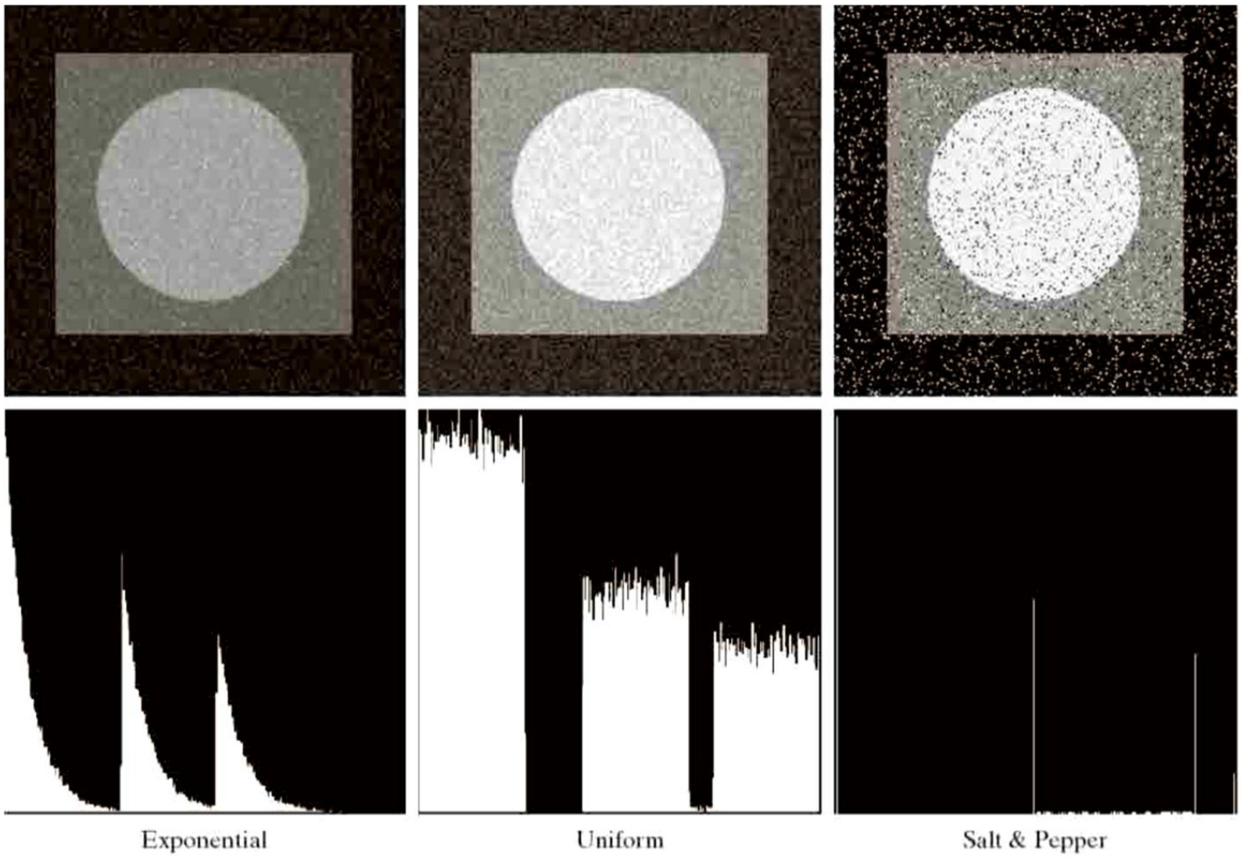
FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.



Estimating Noise Models

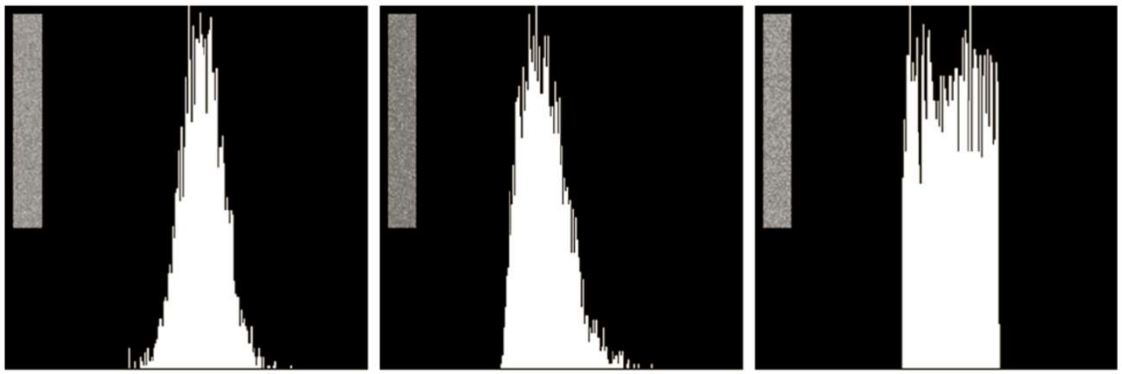


Estimating Noise Models



Estimating the noise parameters

- Consider a strip from the image



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Restoration (in presence of noise only)

Using Spatial filters:

- Mean filters

Arithmetic mean filter

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

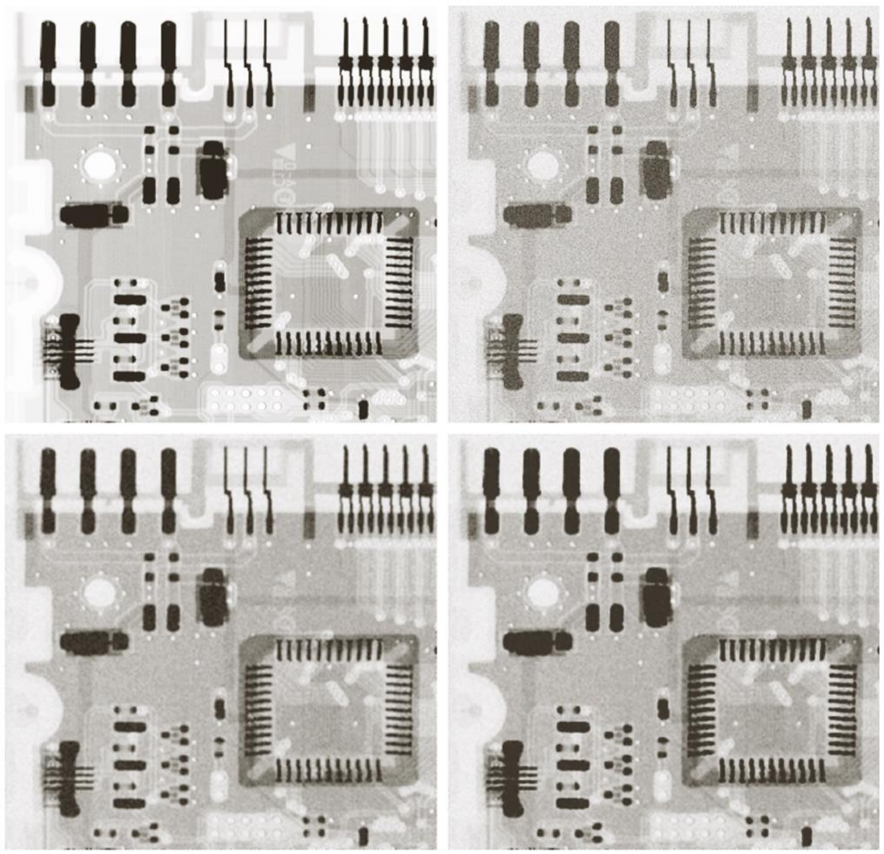
Geometric mean filter

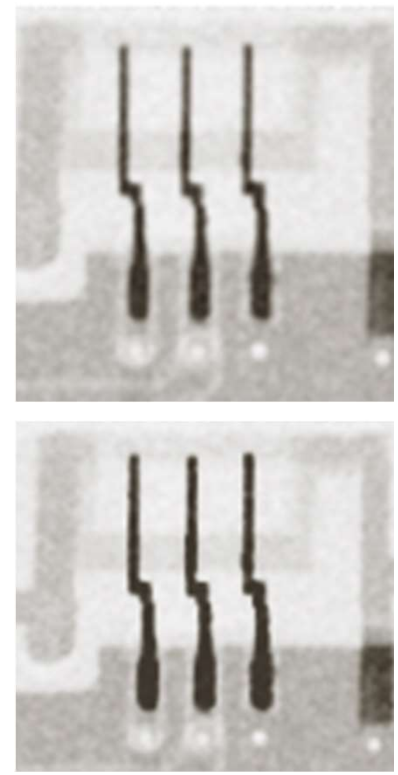
$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Restoration (in presence of noise only)

a b
c d

FIGURE 5.7
(a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)





Restoration (in presence of noise only)

- Mean filters

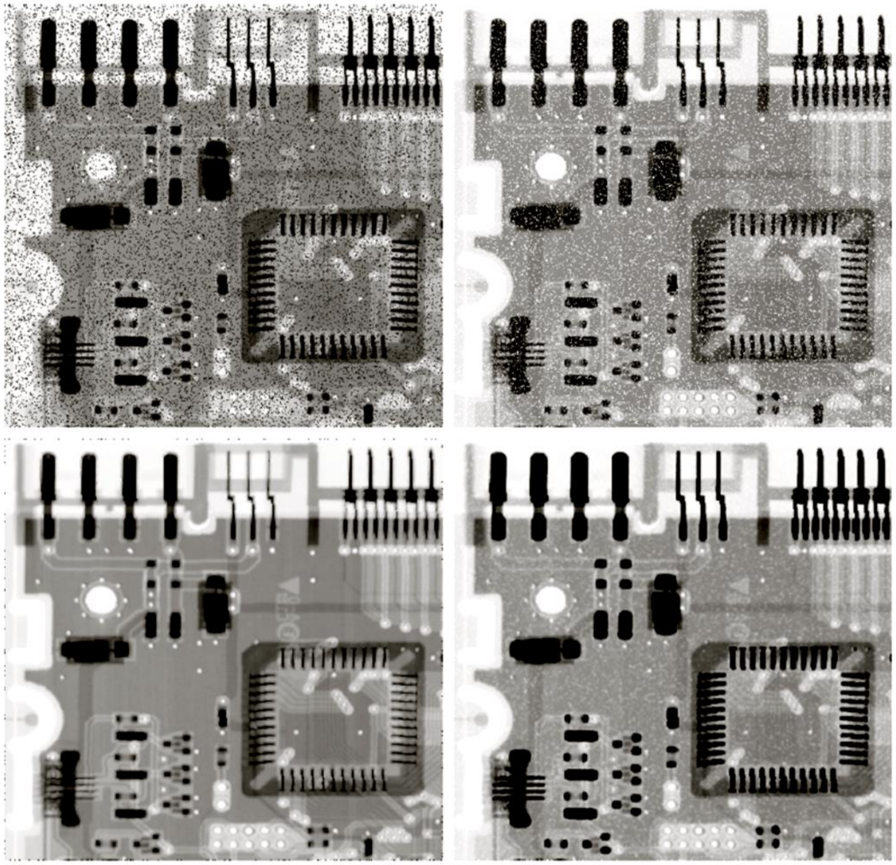
Harmonic mean filter $$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Works well for salt noise or Gaussian noise, but fails for pepper noise

Contraharmonic mean filter $$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q = *order* of the filter
Good for salt-and-pepper noise.
Eliminates pepper noise for $Q > 0$ and salt noise for $Q < 0$
NB: cf. arithmetic filter if $Q = 0$, harmonic mean filter if $Q = -1$

Restoration (in presence of noise only)



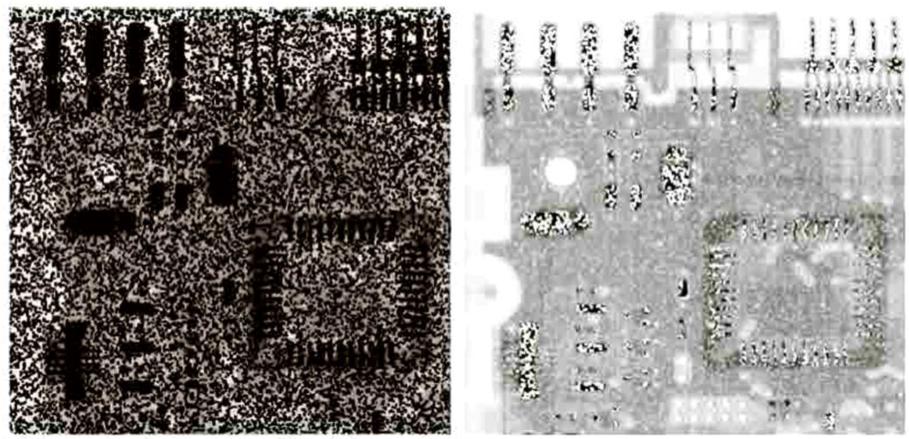
a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Restoration (in presence of noise only)

a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.



Filtering pepper noise with a 3x3 contraharmonic filter $\boldsymbol{Q = -1.5}$

Filtering salt noise with a 3x3 contraharmonic filter $\boldsymbol{Q = 1.5}$

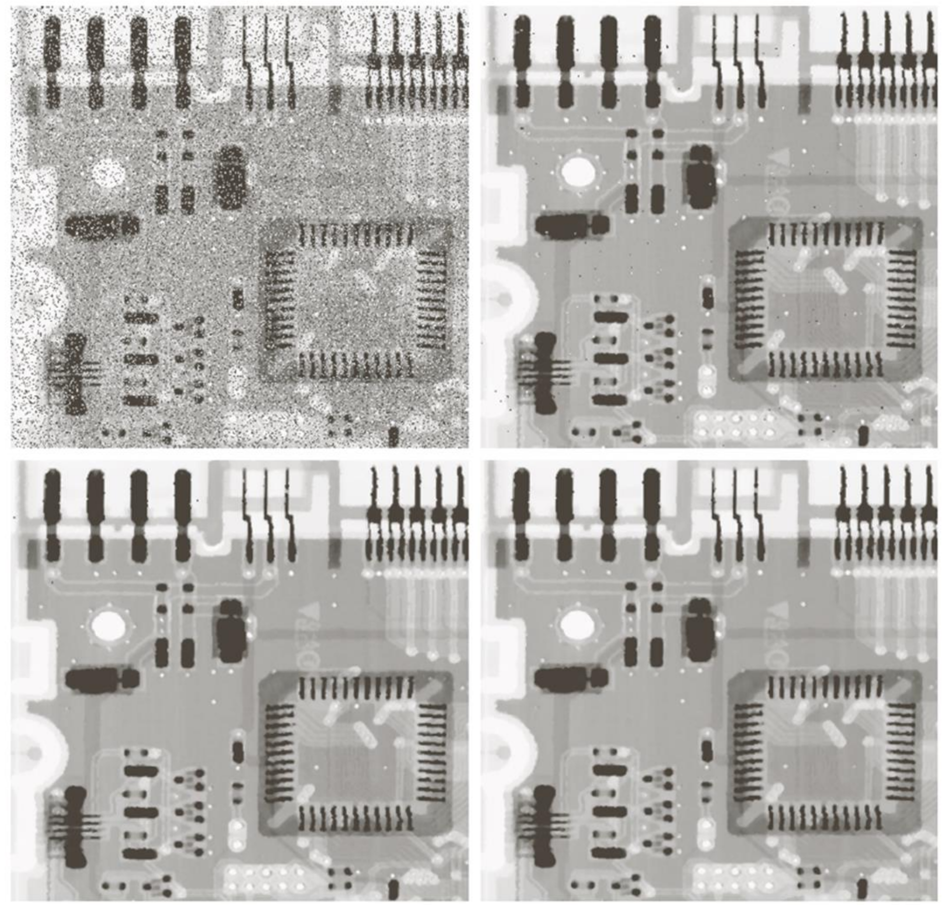
Restoration (in presence of noise only)

- Order-statistic Filter

Median filter

a b
c d

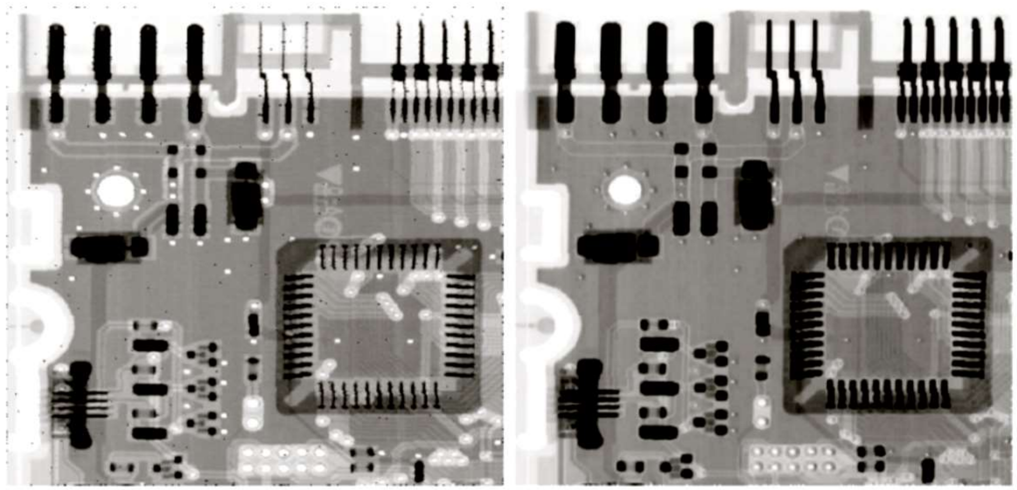
FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.



Restoration (in presence of noise only)

- Max, Min filters

Removing pepper noise | Removing salt noise



Max filter | Min filter

a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3. (b) Result of filtering 5.8(b) with a min filter of the same size.

Restoration (in presence of noise only)

- Midpoint filter

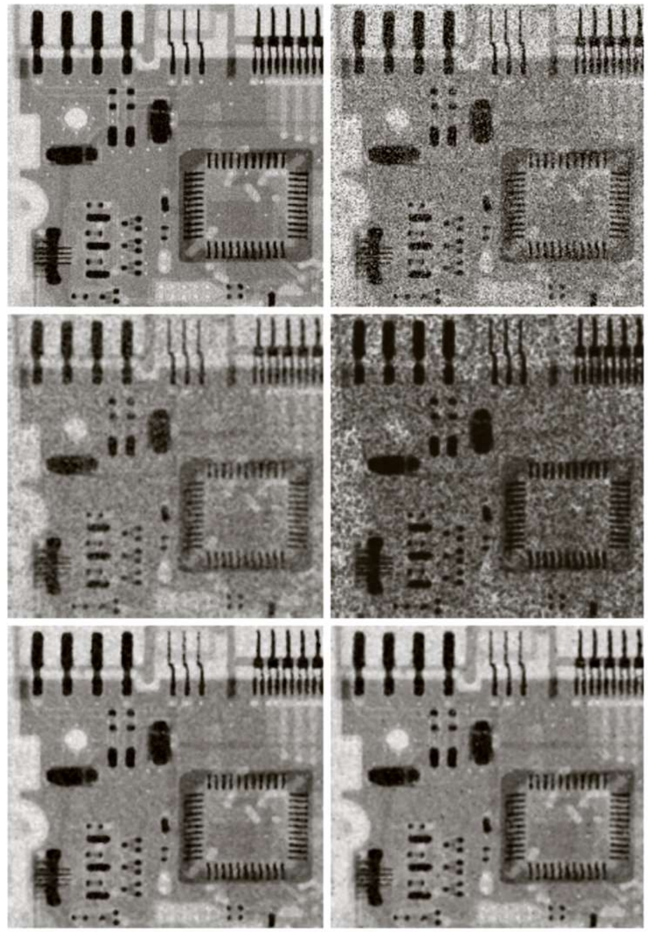
$$\hat{f}(x,y) = \frac{1}{2}\left[\max\{g(s,t)\}_{(s,t)\in S_{xy}} + \min\{g(s,t)\}_{(s,t)\in S_{xy}}\right]$$

- Alpha trimmed filter

$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

Where g_r represents the image *g* in which the *d/2* lowest and *d/2* highest intensity values in the neighbourhood S_{xy} were deleted

Restoration (in presence of noise only)



Original
(Corrupted with Gaussian noise with 0 mean and 800 variance)

Original + salt and pepper noise P_a = 0.1 P_b = 0.1

5x5 Arithmetic mean filter

5x5 Geometric mean filter

Median filter

Alpha Trimmed filter d=5

Adaptive mean filtering

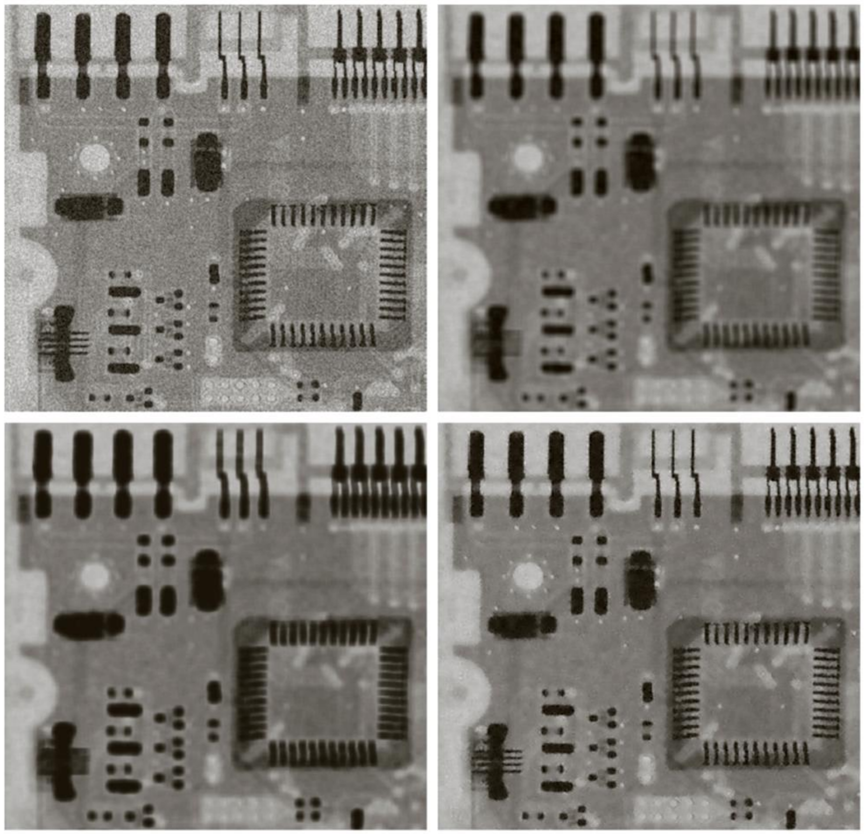
- We can benefit from noise variance estimation
- Adapted to the behavior based on the **statistical characteristics** of the image inside the filter region S_{xy}
- Improved performance v/s increased complexity

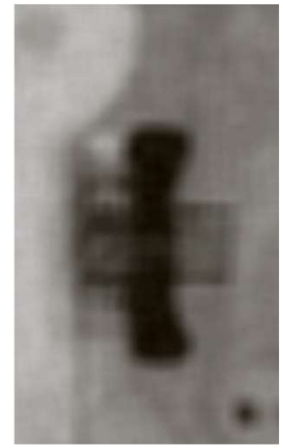
$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

Adaptive mean filtering

a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.





Restoration (in presence of noise only)

- Band pass/reject

a b
c d

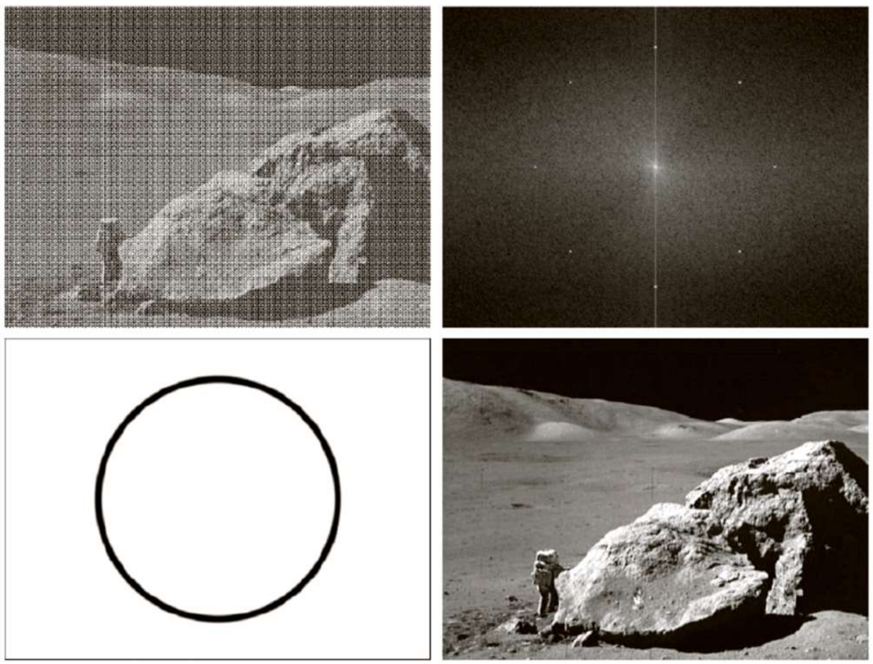
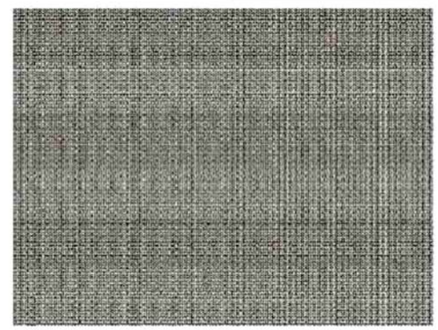


FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



Restoration (in presence of noise only)

- Notch pass/reject

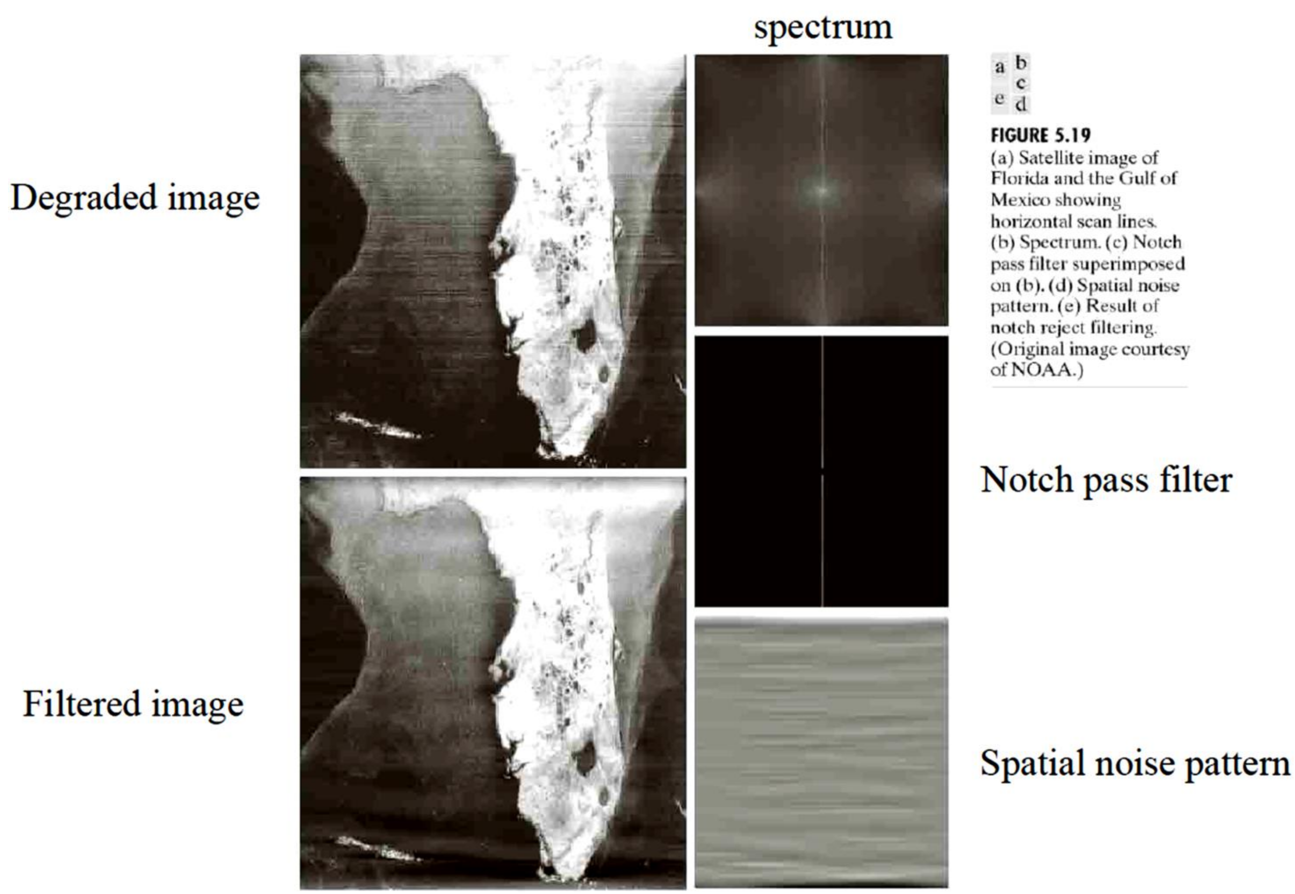
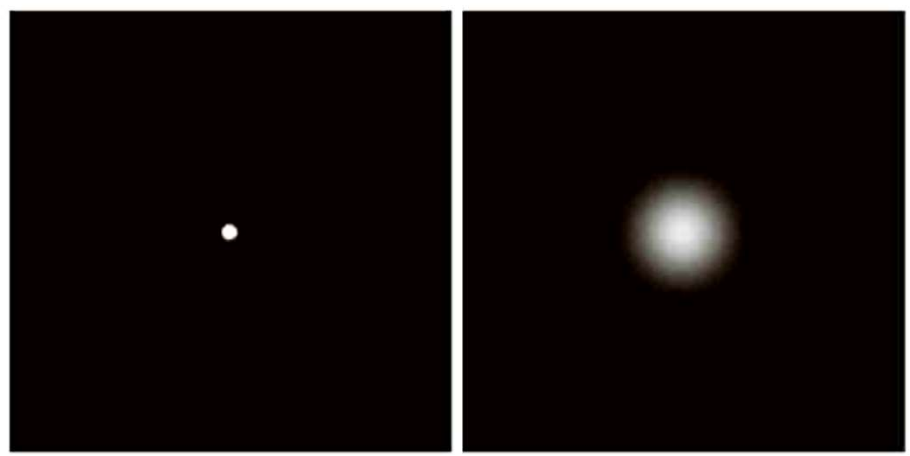


FIGURE 5.19 (a) Satellite image of Florida and the Gulf of Mexico showing horizontal scan lines. (b) Spectrum. (c) Notch pass filter superimposed on (b). (d) Spatial noise pattern. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

Estimation of degradation function

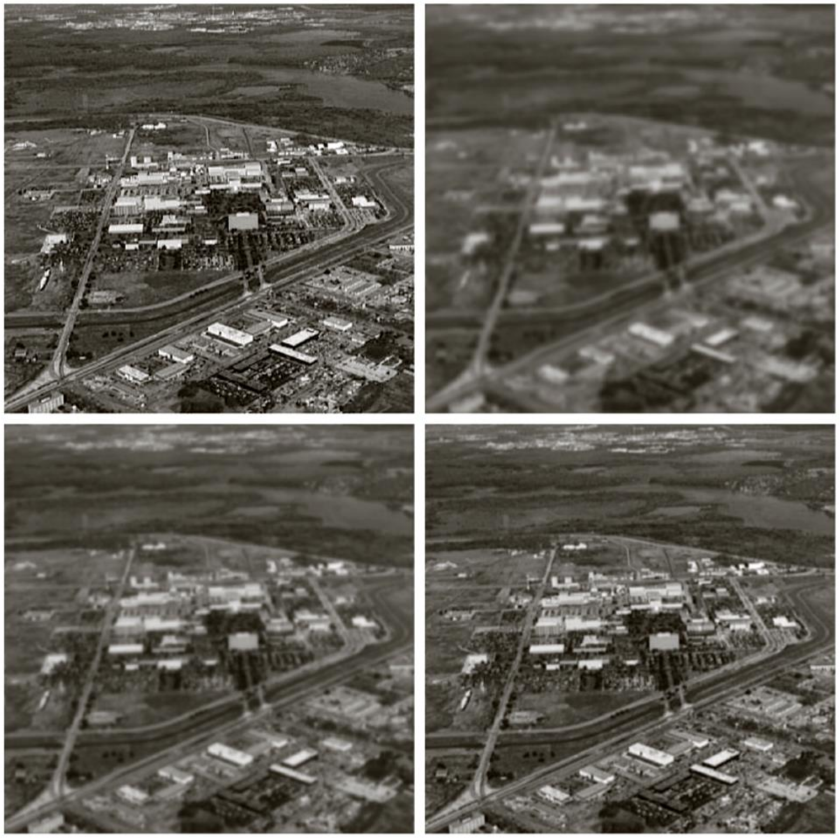
- Three main ways:
  - Observation → look, find, iterate
  - Experimentation → important idea for calibration
  - Mathematical modelling



a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

Estimation by Modeling (atmospheric turbulence)



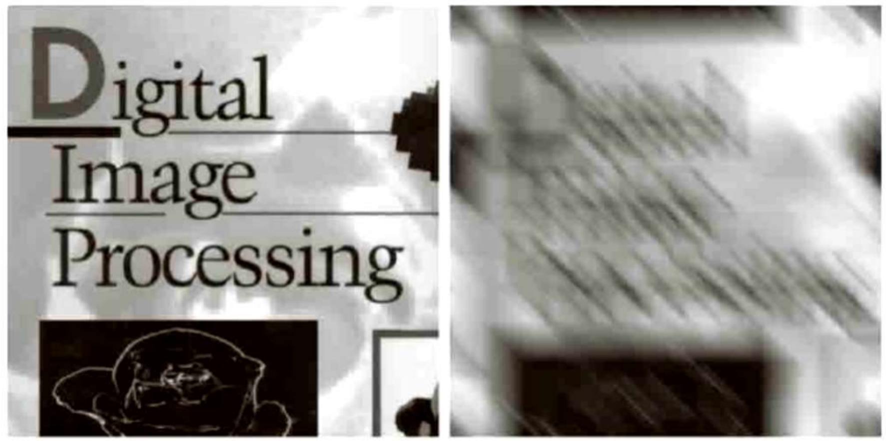
a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

Degradation model proposed by Hufnagel and Stanley [1964] based on the physical characteristics of atmospheric turbulence:

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

Estimation by Modeling (uniform motion blurring)



$$g(x,y)=\int_0^T f\left[x-x_0(t),y-y_0(t)\right]dt$$

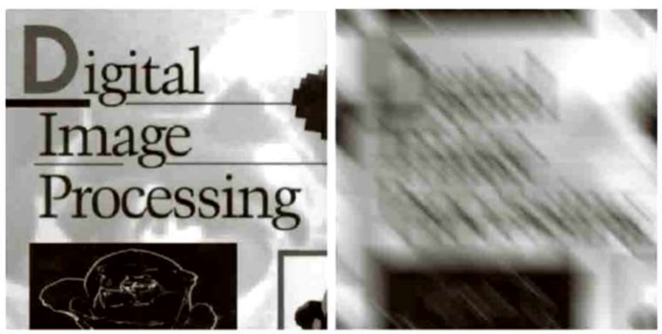
$$G(u,v)=F(u,v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt \qquad H(u,v)=\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

Estimation by Modeling (uniform motion blurring)

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

putting, $x_0(t) = at/T$ **and** $y_0(t) = bt/T$

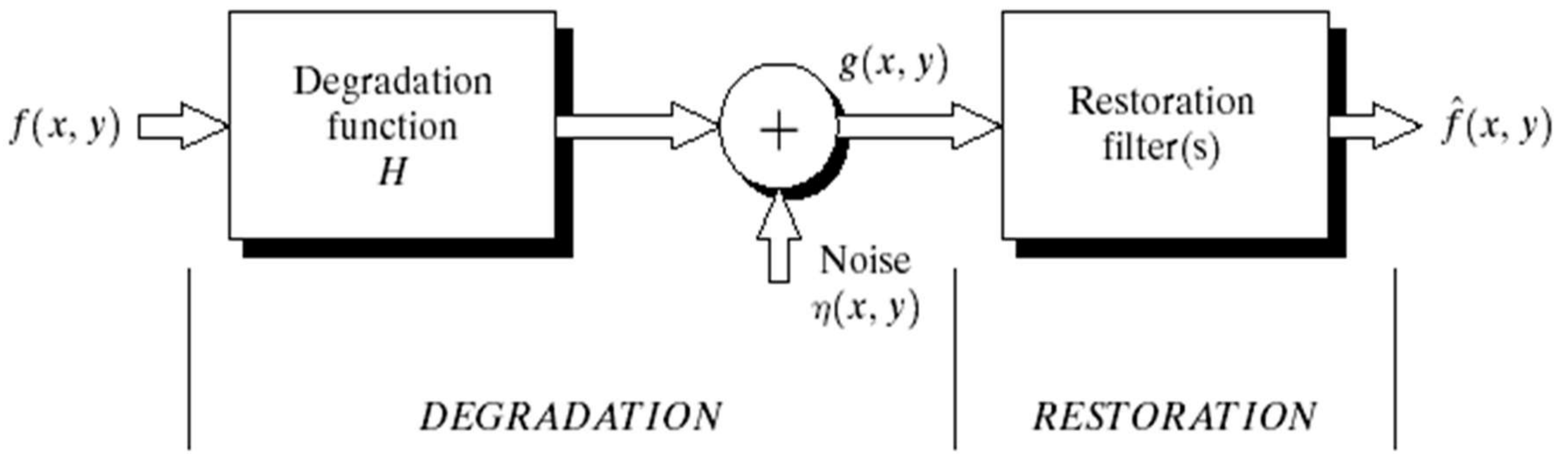
$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin[\pi(ua+vb)]\, e^{-j\pi(ua+vb)}$$



a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Model of Image Degradation/Restoration



$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)\, F(u, v) + N(u, v)$$

Recovering image (in presence of both Noise and degradation)

- Inverse Filtering: Even if we know the degradation function we cannot recover the undegraded image!!

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v) \quad \Rightarrow \quad \hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Two problems:

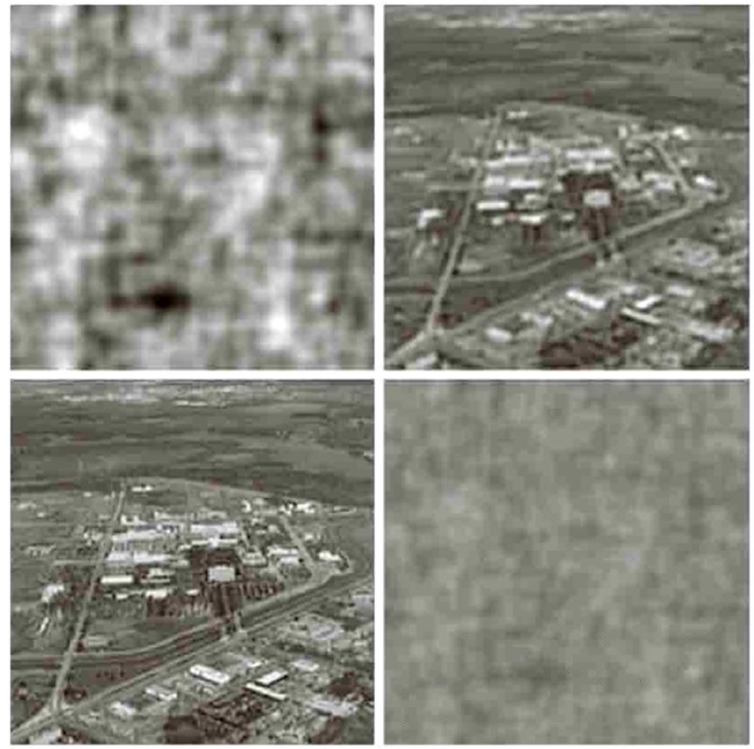
1. N(u,v) is a random function whose Fourier transform is not known
2. If degradation has zero or small values → N(u,v)/H(u,v) will dominate

Recovering image (in presence of both Noise and degradation)

Degraded Image
(with known model)

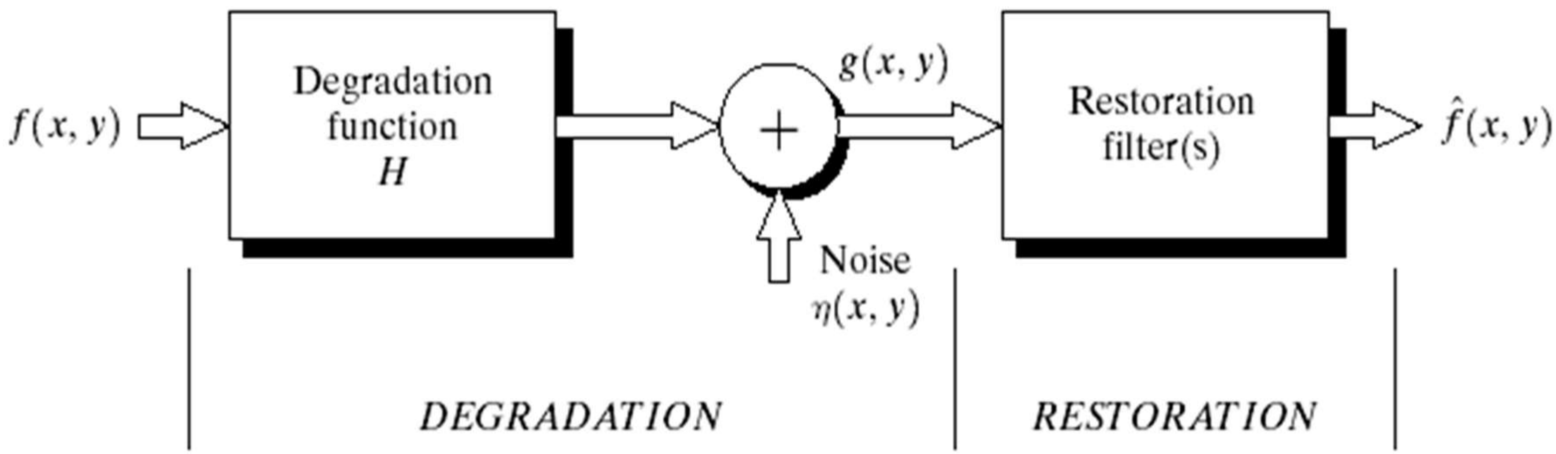
a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.



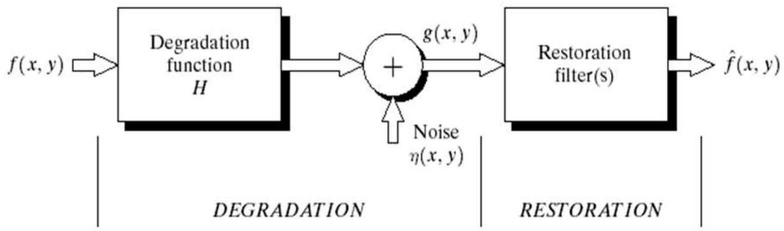
No explicit provision for handling noise!

Weiner filter



$$e^2 = E\{(f - \hat{f})^2\}$$

Weiner filter



$$e^2 = E\{(f - \hat{f})^2\}$$

The minimum of the error function e is given by:

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right] G(u,v)$$

$S_\eta(u,v) = |N(u,v)|^2 =$ Power spectrum of the noise (autocorrelation of noise)

$S_f(u,v) = |F(u,v)|^2 =$ Power spectrum of the undegraded image

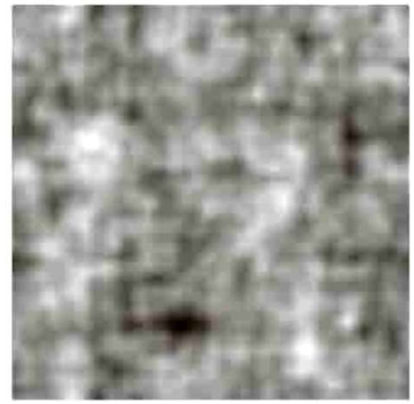
Weiner filter

- When two spectrums are not known or cannot be estimated, the equation is approximated as:

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \right] G(u,v)$$

Weiner filter

Radially limited
inversefiltering



Full Inverse filtering

Wiener filtering

Weiner filter