MATH189R

Lecture 1: The Mathematics of Big Data

1/28/20

Lecturer: Prof. Weiqing Gu *Scribe: Sidhant Rastogi*

1 Review of Multivariable Calculus

$$\begin{aligned} f : I \subseteq \mathbb{R} \to \mathbb{R} &\quad \text{(Single variable function)} \\ f : \Omega \subseteq \mathbb{R}^n \to \mathbb{R} &\quad (\delta = f(x, y)) \\ f : \mathbb{R} \to \mathbb{R}^m &\quad \text{(Curve)} \\ f : \mathbb{R}^n \to \mathbb{R}^m &\quad \text{(Vector field)} \end{aligned}$$

Ex. of curve:

$$f : \mathbb{R} \to \mathbb{R}^3, t \mapsto (p(t), v(t), g(t))$$

Ex. of Vector Field:

$$f : \mathbb{R}^2 \to \mathbb{R}^3, (\phi, \theta) \mapsto (\cos\phi\cos\theta, \cos\phi\sin\theta, \sin\phi)$$

1.1 Taking derivatives

For $f : \mathbb{R}^n \to \mathbb{R} \quad [(x_1, \dots, x_n) \mapsto f(x_1, \dots, x_n)]$, the gradient is defined as

$$\nabla f := (\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n})$$

Note that $\nabla f : \mathbb{R}^n \to \mathbb{R}^n$. The directional derivative, then, is simply

$$D_{\boldsymbol{v}} f = \boldsymbol{v} \cdot \nabla f$$

For $f : \mathbb{R}^n \to \mathbb{R}^m \quad [(x_1, \dots, x_n) \mapsto (f_1(x_1, \dots, x_n), f_2(x_1, \dots, x_n), \dots, f_m(x_1, \dots, x_n))]$, the derivative is defined as the matrix of gradients

$$\begin{bmatrix} \nabla f_1 \\ \vdots \\ \nabla f_m \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$