Predicting APPLE Stock Prices using the GARCH Model

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Executive summary

Our primary purpose is to research the Apple stock market and anticipate the future stock value to determine long-term profit. The research is based on our observational data from Yahoo Finance, and we built a model based on our findings.

We use exploratory data analysis to see different components of time series such as seasonality and trend. We found that the data has an upward trend and changing variance. Therefore, we transformed that data using the **logarithm** function. To make the data stationary we took the **difference** of the logarithm of the data.

We then analyzed the transformed data using **sample ACF, sample PACF, and sample EACF**. We found out that the distribution of the data deviates from normal distribution.Looking at the time series plot of the data, periods of increased variation sprinkled through the time series was spotted. This indicated the presence of a **GARCH** model. We use GARCH model to forecast the next next 365 trading days of the Apple stock closing prices. A GARCH model uses values of the past squared observations and past variances to model the variance at time t.

To estimate the orders of the GARCH model we observed the EACF of the data absolute and squared values. The EACF of the absolute value of the data suggested GARCH(1,1) model. However, the EACF of the squared values suggested GARCH(1,3) or GARCH(2,3) model. We found that all the coefficients of GARCH(1,1) and GARCH(2,3) model are significant. However, all the coefficients of GARCH(1,3) model was not significant. We used **Ljung-Box test** to check the existence of auto-correlation in the time series.

The graph of the standardized residuals from fitted GARCH(1,1) model suggested no particular tendency in the standardized residuals. Sample ACF of squared and absolute standardized residuals appeared to

independently and identically distributed. The Q-Q plot displayed a straight-line pattern, indicating that the majority of the data is normally distributed with fat tails at both ends. All of the numbers between the boundary in the ACF plot of squared residuals suggested that there was no association. We found that the GARCH(1,1) model had the least AIC value. Therefore, GARCH(1,1) model was found to be the best fit for this data. Finally, we forecast the closing prices of the next 10 trading days.

Introduction

The stock market has become one of the most crucial financial systems. Due to poor rates of return on investment, traditional investment institutions such as banks are no longer appealing to investors. Investing in stocks is getting increasingly popular in today's culture. Almost everyone is drawn to the prospect of multiplying one's money in a relatively short period and achieving frequently above-average returns. As a result, corporations that sell stocks get money to invest in their growth. According to efficient market theory, the stock market reflects all existing information, digesting and absorbing new data quickly through buying and selling mechanisms. One of the few ways a corporation may influence its stock valuation is to produce an innovative, revenue-generating product or service. The return or loss for investors may be tremendous when Wall Street values are correct or incorrect. This is because predicting the influence of a globally dispersed product on a company's earnings and stock is a Herculean task. The history of Apple's stock is a great example of how this works.

Apple Inc. is a leading manufacturer of personal computers, peripherals, and consumer electronics, including the iPod digital music player, iPad tablet, iPhone smartphone, and "Apple Watch," for the business, creative, education, government, and consumer sectors. Operating systems, utilities, languages, development tools, and database applications are also available. The foreign, enterprise and education sectors have been Apple's fastest-growing segments in recent years, and while we expect the rate of growth to reduce as the company grows larger and more mature, we still see significant potential in these areas. Even though the corporation continues to dominate worldwide marketplaces, several of its new introductions have failed to move the stock. Our major purpose is to research the stock market of Apple Inc. Most of us have probably put money into the stock market. We don't know what the market's future holds. We lose and win regularly. We believe it is luck, but it is dependent on the market's time series analysis. If we understand the definition of the stock market and how it operates, we will have a better understanding of the stock market and will be more likely to invest in any stock market. Our primary purpose is to research the Apple stock market and anticipate the future stock value to determine long-term profit. The research is based on our observational data from Yahoo Finance, and we built a model based on our findings. The research is based on observation. Stock price time series are essential for stock price forecasting. The most extensively utilized research approach is time series analysis, which is a fundamental tool for predicting stock values. Other methods for studying and forecasting the Apple stock market exist. We used kurtosis and Ljung Box test.

Data collection Techniques

We gathered data for the project from Yahoo Finance (https://finance.yahoo.com/quote/AAPL/key-statistics?p=AAPL). It has several sections for the apple stock, such as summaries, charts, corporate outlook, conversion, statistics, historical data, and so on. We focused on historical datasets to get an understanding of Apple's daily stock prices. It provides Apple stock statistics going back to the year 2000. We selected the dataset from 2012 to 2022 for this project, which includes around 2517 records, with columns containing opening, closing, highest, and lowest stock prices. The data were filtered to exclude null and irrelevant values.

Data Analysis/Interpretation Page

```
data <- read.csv("/Users/subharaut/Desktop/AAPL.csv",header=TRUE,stringsAsFactors=FALSE)
library(TSA)
library(fGarch)
library(forecast)
```

```
## Warning: package 'forecast' was built under R version 4.0.5
```

```
library(RcmdrMisc)
```

```
## Warning: package 'RcmdrMisc' was built under R version 4.0.5
```

```
library(astsa)
library(tseries)
```

```
plot.ts(data$Close)
```

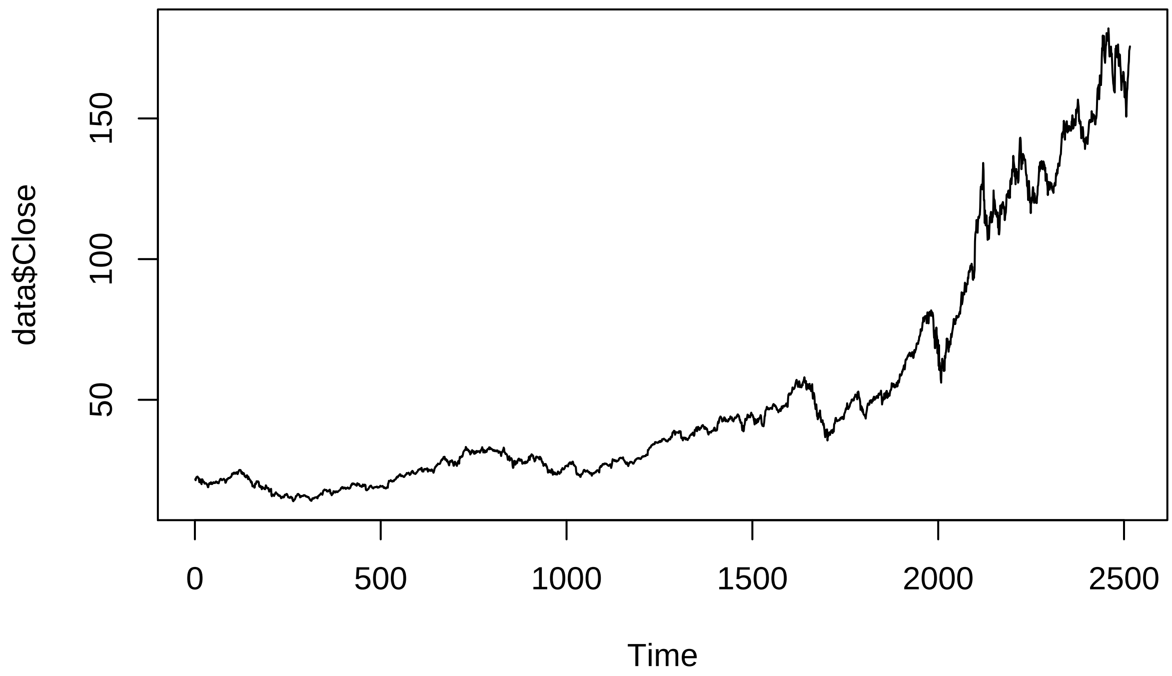


Figure 1: Above is the data time plot of the apple stock which shows an increasing trend.

#Make Data Stationary Based on our research, we observed that the data has trend and changing variance. So we took log but trend was still there. Taking difference of logarithm of the data can removed the trend and can make the variance constant. It is multiplied by 100 so that percentage changes in the stock can be calculated.

```
Apple<- diff(log(data$Close))*100
plot.ts(Apple)
```

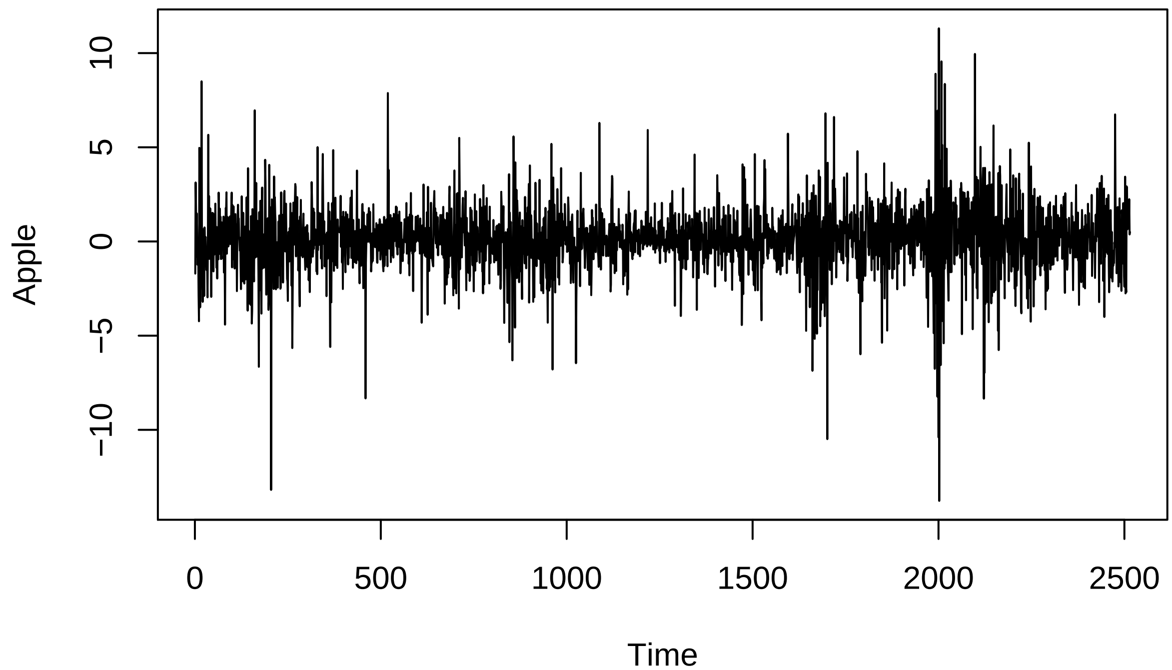
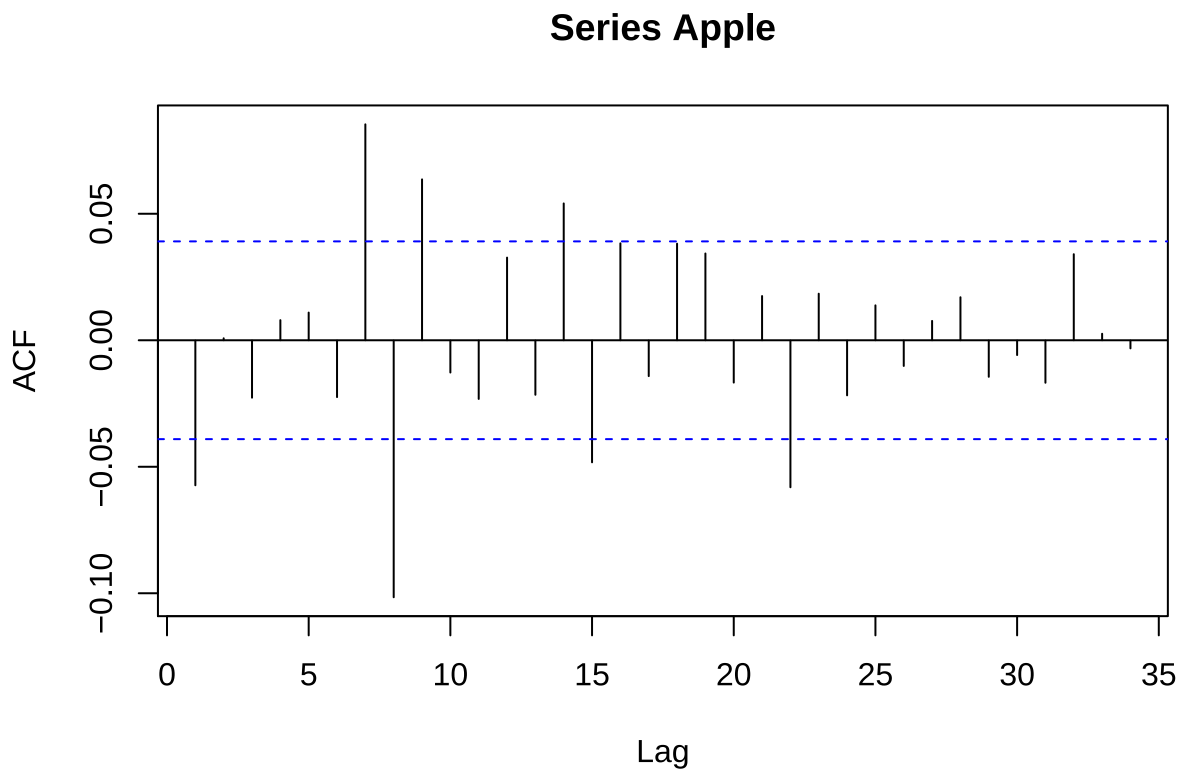


Figure 2: Time plot after applying logarithm and difference.

```
acf(Apple)
```




```r
pacf(Apple)
```

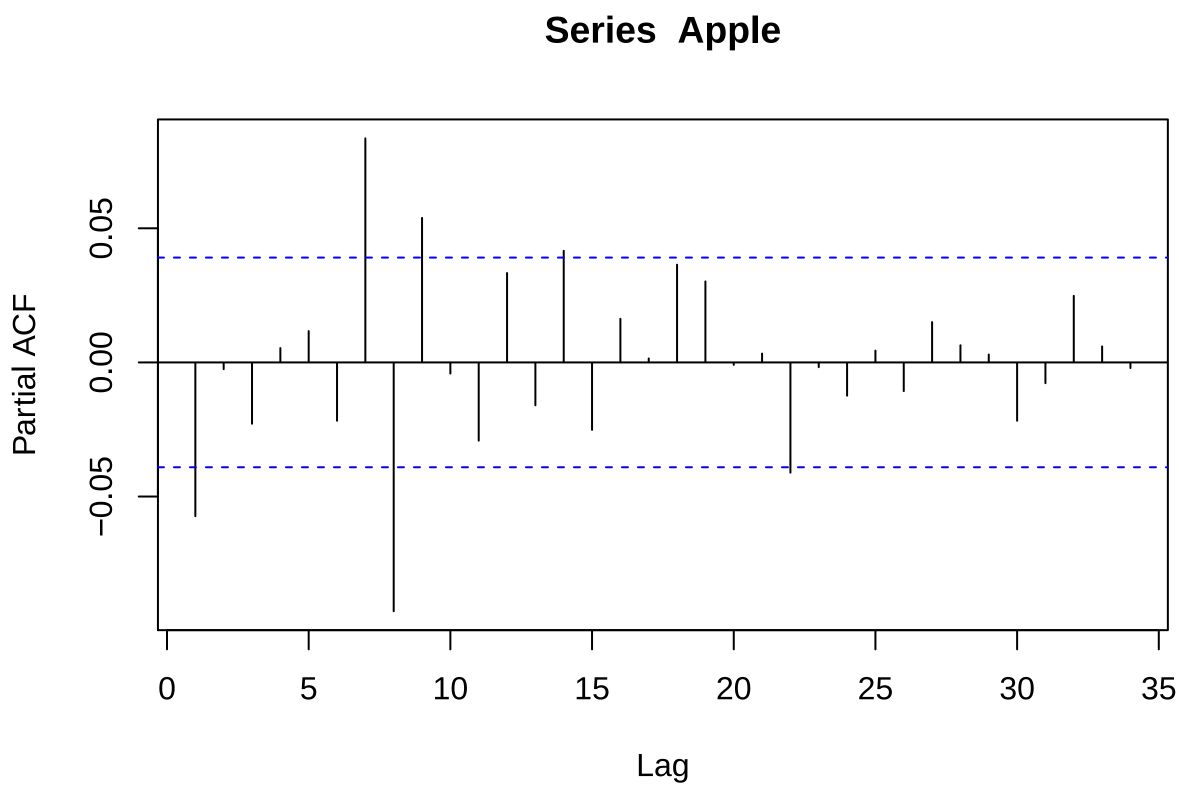
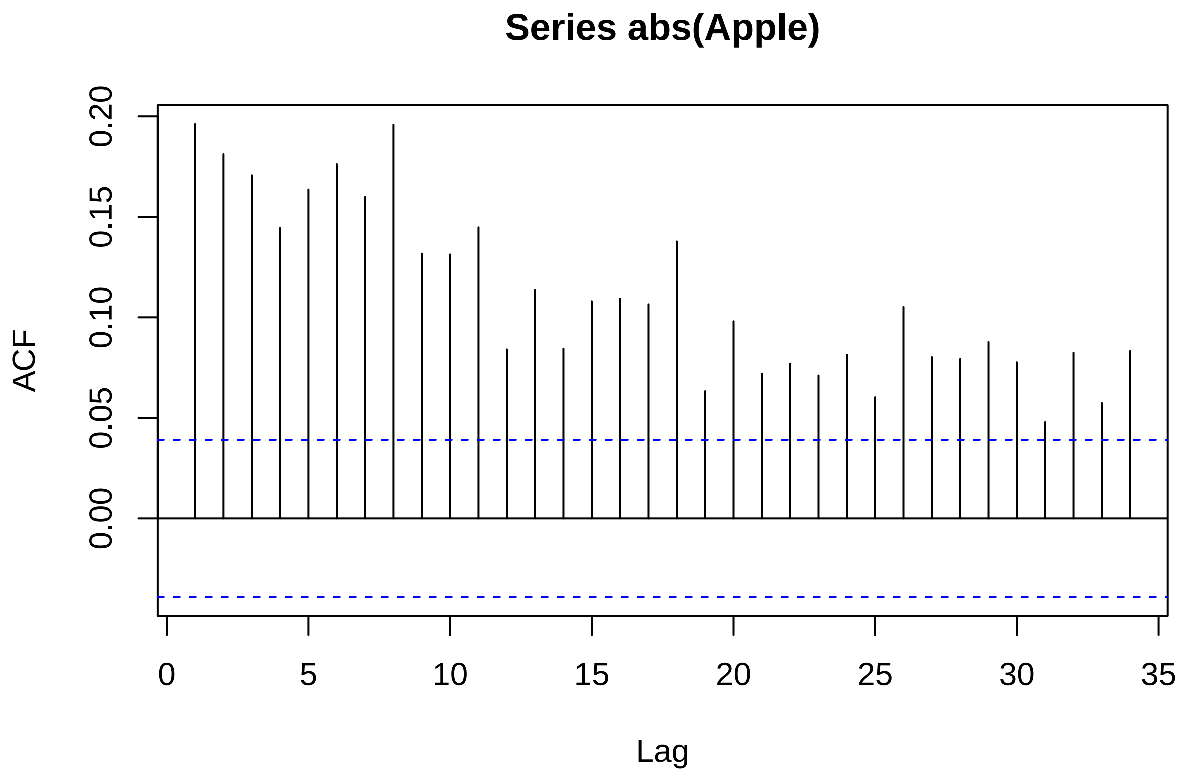


Figure 3: ACF, PACF plot of the data obtained after applying logarithm and difference.

```
acf(abs(Apple))
```




```
pacf(abs(Apple))
```

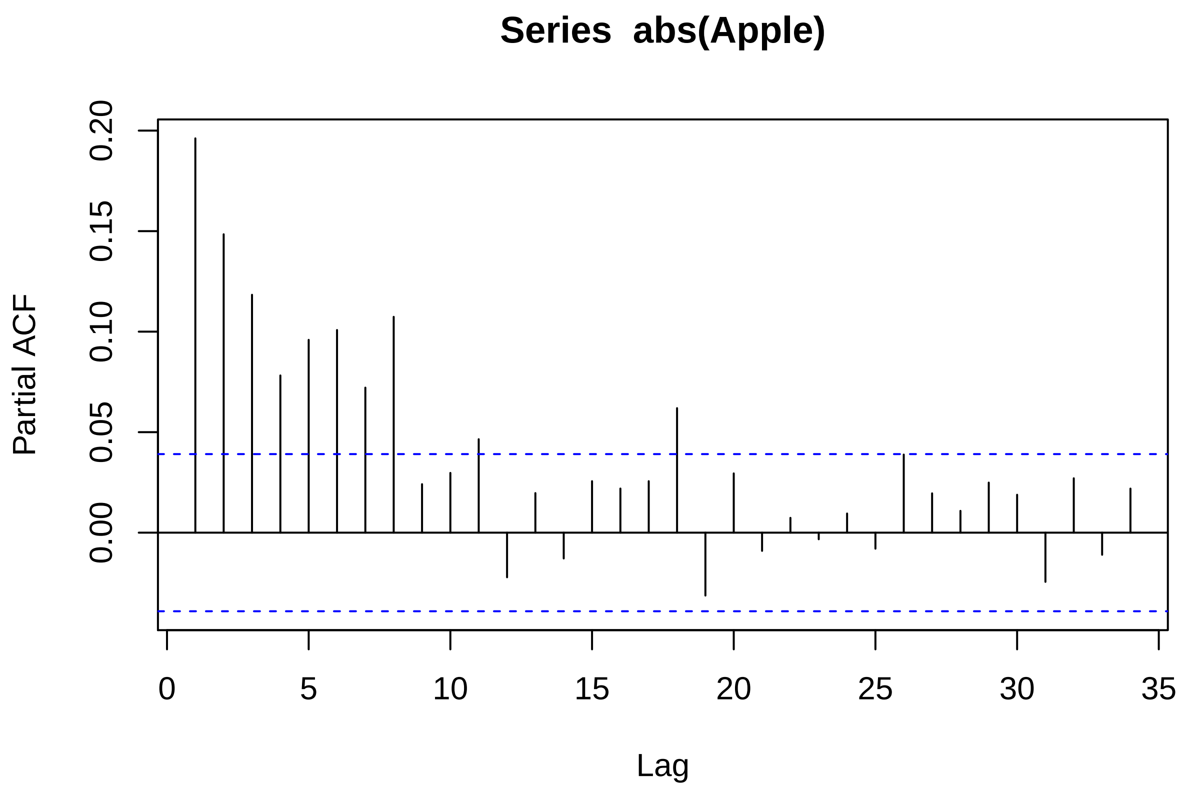
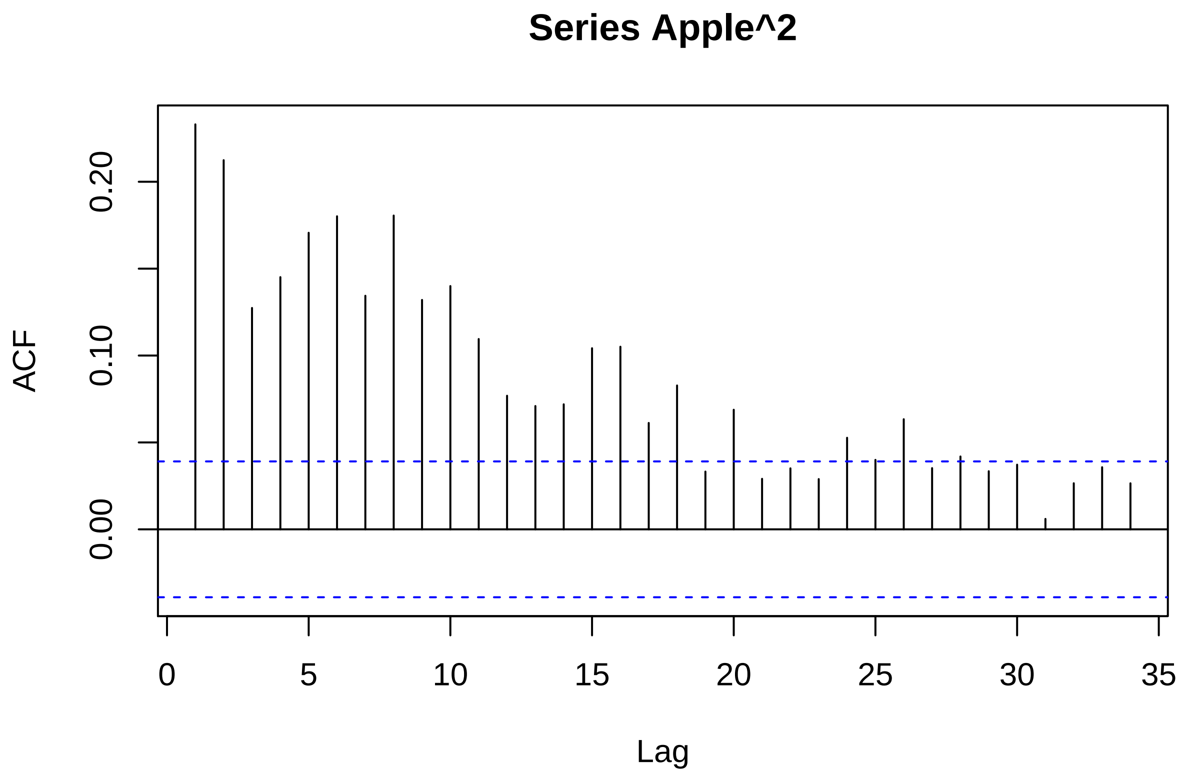


Figure 4: ACF, PACF plot of the Absolute value of data

```r
acf(Apple^2)
```




```
pacf(Apple^2)
```

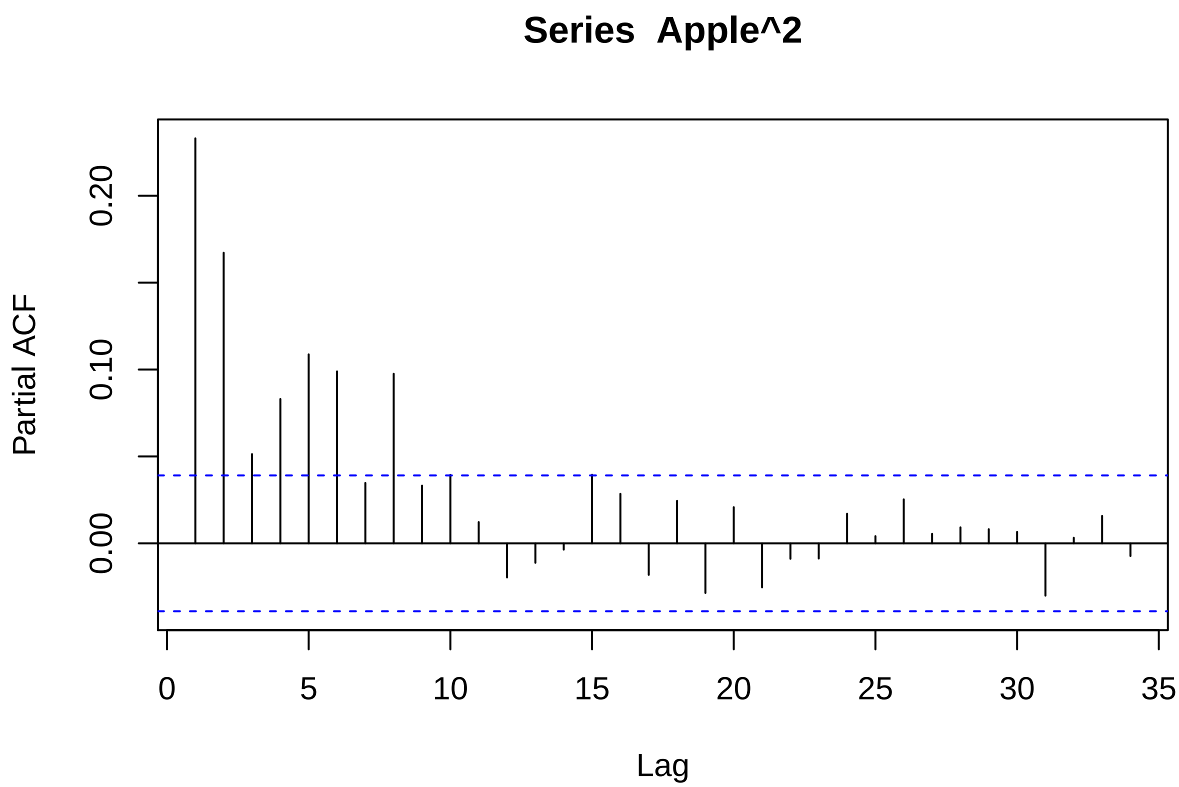
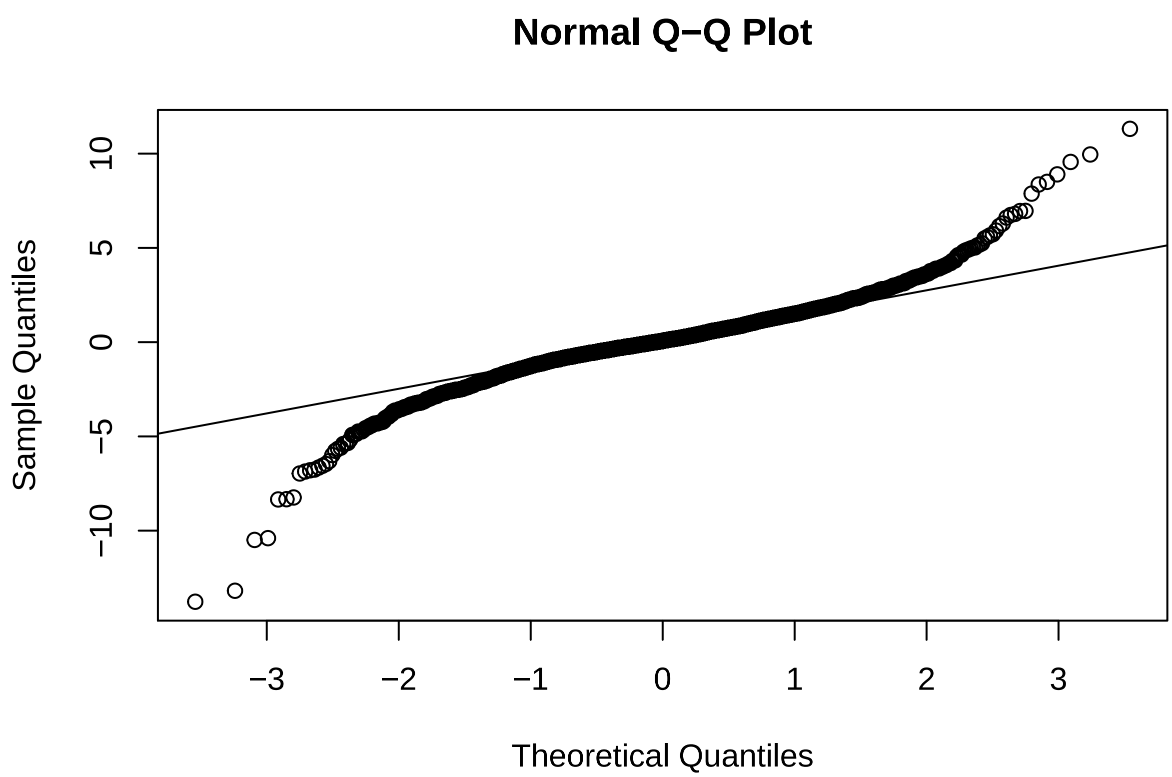


Figure 5: ACF, PACF plot of the Square value of data

```
qqnorm(Apple)
qqline(Apple)
```




```r
shapiro.test(Apple)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  Apple
## W = 0.93117, p-value < 2.2e-16
```

```r
kurtosis(Apple)
```

```
## [1] 6.593444
## attr(,"method")
## [1] "excess"
```

Figure 6: Q-Q plot

The data has become stable with a constant mean, as seen in Figure 2. The ACF and PACF of the acquired data can be used to validate this. From the Figure 3, we can observe that the data has little correlation. From the Figure 4 and Figure 5, it can be observed that there is a significant autocorrelation. This is an evidence that the stock data is not independently and identically distributed. A Q-Q plot can be used to analyse how the data is distributed. From the Figure 6, it can be observed that the data is not normally distributed and the p-value of shapiro-test is less than 0.05 indicates that the data deviates from the normal distribution. The kurtosis is a positive value(6.593444) confirming that the distribution is a heavy tailed distribution.

#Find Best Model Fit We discovered that the data is stationary and not independently and identically distributed after analyzing the converted data. So, in this section, we'll try to fit GARCH models and determine which one is the best.

To find the P and Q of GARCH, observed the EACF of the data's absolute and squared values.

```
eacf(abs(Apple))
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x x x x x x x x x x  x  x  x
## 1 x o o o o o o x o o x  x  x  o
## 2 x x o o o o o x x o o  o  o  o
## 3 x x x o o o o x o o o  o  o  o
## 4 x x x x o o o x o o o  o  o  o
## 5 x x x x x o o x o o o  o  o  o
## 6 x x o x x x o x o o o  o  o  o
## 7 x x o x x x x o o o o  o  o  o
```

    Table 1 : EACF of the Absolute value of the data

```
eacf(Apple^2)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x x x x x x x x x x  x  x  x
```

```
## 1 x x x o o x x x o o o  o  o  o
## 2 x x x o o o o o o o o  o  o  o
## 3 x x x o o o o o o x o  o  o  o
## 4 x x o x o x o o o x o  o  o  o
## 5 x x o x x x o o o o o  o  o  o
## 6 x x x o x x o o o o o  o  o  o
## 7 x x x o x x o o o o o  o  o  o
```

```
  Table 2 : EACF of the square value of the data
```

From the Table 1, the EACF of the absolute data suggest that a GARCH(1,1) model may be appropriate. From the Table 2, the EACF of the squared data is hard to read to understand which model may be appropriate. May be it suggest the GARCH(1,3) or GARCH(2,3) model.

Now need to check the p values in the diagnostic log to see if the models are enough to reflect the data.

```
garch1_1=garch(x=Apple,order=c(1,1))
```

```
##
##  ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     2.911292e+00     1.000e+00
##      2     5.000000e-02     1.000e+00
##      3     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   NPRELDF
##      0    1  2.681e+03
##      1    4  2.669e+03  4.63e-03  2.94e-02  3.2e-02  2.3e+03  1.8e-01  3.41e+01
##      2    6  2.667e+03  7.74e-04  7.94e-04  2.0e-03  5.5e+00  1.8e-02  1.48e+00
##      3    7  2.664e+03  9.85e-04  1.00e-03  4.5e-03  3.0e+00  3.7e-02  1.11e+00
##      4    8  2.662e+03  1.04e-03  1.65e-03  9.4e-03  1.8e+01  7.4e-02  7.52e-01
##      5    9  2.659e+03  9.84e-04  1.02e-03  1.1e-02  3.0e+00  7.4e-02  4.15e-01
```

```
##      6    10  2.655e+03  1.65e-03  1.53e-03  2.4e-02  2.0e+00  1.5e-01  3.89e-01
##      7    11  2.642e+03  4.60e-03  5.12e-03  5.8e-02  2.0e+00  3.0e-01  4.96e-01
##      8    12  2.630e+03  4.56e-03  5.65e-03  6.1e-02  2.0e+00  3.0e-01  3.33e-01
##      9    15  2.586e+03  1.67e-02  1.69e-02  3.0e-01  1.7e+00  9.9e-01  4.65e-01
##     10    17  2.568e+03  7.22e-03  6.01e-03  1.4e-01  1.9e+00  3.0e-01  2.21e-01
##     11    18  2.556e+03  4.42e-03  1.40e-02  4.4e-01  2.0e+00  6.0e-01  5.60e+00
##     12    20  2.548e+03  3.11e-03  1.52e-02  3.2e-02  2.3e+00  6.0e-02  2.39e-01
##     13    21  2.535e+03  5.06e-03  6.19e-03  4.2e-02  2.0e+00  6.0e-02  8.15e-01
##     14    23  2.533e+03  1.09e-03  1.41e-03  2.1e-02  3.1e+00  3.0e-02  5.41e-02
##     15    24  2.529e+03  1.50e-03  1.82e-03  3.2e-02  2.0e+00  6.1e-02  2.06e-01
##     16    25  2.527e+03  7.45e-04  1.36e-03  3.6e-02  2.0e+00  6.1e-02  1.61e-01
##     17    26  2.525e+03  7.32e-04  1.63e-03  2.7e-02  1.9e+00  6.1e-02  2.05e-02
##     18    27  2.522e+03  1.17e-03  2.66e-03  3.3e-02  1.9e+00  6.1e-02  6.00e-02
##     19    28  2.522e+03  1.16e-04  7.18e-04  3.3e-02  1.6e+00  6.1e-02  3.78e-03
##     20    30  2.522e+03  1.82e-05  1.10e-03  1.1e-02  1.9e+00  2.1e-02  3.55e-03
##     21    31  2.521e+03  4.79e-04  5.40e-04  5.6e-03  1.8e+00  1.1e-02  6.18e-04
##     22    33  2.521e+03  1.81e-06  2.18e-06  1.0e-03  1.5e+00  2.5e-03  7.02e-06
##     23    37  2.521e+03  3.93e-07  6.47e-07  6.0e-05  3.8e+00  1.4e-04  2.12e-06
##     24    39  2.521e+03  1.08e-07  2.23e-07  2.8e-04  2.2e+00  5.6e-04  6.19e-07
##     25    40  2.521e+03  2.70e-07  2.31e-07  4.1e-04  0.0e+00  1.0e-03  2.31e-07
##     26    42  2.521e+03  5.03e-09  1.71e-08  4.7e-05  1.4e+00  9.7e-05  2.11e-08
##     27    45  2.521e+03  1.82e-10  4.86e-10  2.6e-06  1.9e+00  5.4e-06  3.35e-09
##     28    56  2.521e+03  9.02e-16  1.13e-17  1.3e-13  9.5e+07  2.2e-13  2.89e-09
##     29    59  2.521e+03 -7.22e-16  2.26e-19  2.6e-15  4.6e+09  4.4e-15  2.88e-09
##
##  ***** FALSE CONVERGENCE *****
##
##  FUNCTION     2.520683e+03   RELDX        2.580e-15
##  FUNC. EVALS      59         GRAD. EVALS      29
##  PRELDF       2.261e-19      NPRELDF      2.875e-09
##
##     I      FINAL X(I)        D(I)          G(I)
##
```

```
##      1    1.695690e-01     1.000e+00    -1.272e-01
##      2    1.030409e-01     1.000e+00     2.064e-02
##      3    8.444827e-01     1.000e+00    -1.808e-03
```

```r
summary(garch1_1)
```

```
##
## Call:
## garch(x = Apple, order = c(1, 1))
##
## Model:
## GARCH(1,1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -6.61467 -0.44009  0.04066  0.61752  6.07776
##
## Coefficient(s):
##     Estimate  Std. Error  t value Pr(>|t|)
## a0   0.16957     0.02055    8.253 2.22e-16 ***
## a1   0.10304     0.01161    8.872  < 2e-16 ***
## b1   0.84448     0.01572   53.708  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Diagnostic Tests:
##  Jarque Bera Test
##
## data:  Residuals
## X-squared = 1543.1, df = 2, p-value < 2.2e-16
##
##
##  Box-Ljung test
```

```
##
## data:  Squared.Residuals
## X-squared = 0.27825, df = 1, p-value = 0.5979
```

```r
AIC(garch1_1)
```

```
## [1] 9667.788
```

We can see that the all coefficients of GARCH(1,1) model are significant. The p-value of Box-Ljung test is 0.5979 which is greater than 0.05. There it can be considered as a good model. The AIC is 9667.788

```r
garch1_3=garch(x=Apple,order=c(1,3))
```

```
##
##  ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     2.587816e+00     1.000e+00
##      2     5.000000e-02     1.000e+00
##      3     5.000000e-02     1.000e+00
##      4     5.000000e-02     1.000e+00
##      5     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   NPRELDF
##      0    1  2.620e+03
##      1    3  2.611e+03  3.68e-03  1.34e-02  1.3e-02  3.5e+03  1.0e-01  2.36e+01
##      2    5  2.608e+03  8.90e-04  8.94e-04  1.6e-03  1.0e+01  1.0e-02  8.08e+00
##      3    7  2.604e+03  1.46e-03  1.46e-03  3.3e-03  2.0e+00  2.0e-02  1.63e+00
##      4   10  2.604e+03  2.53e-05  2.53e-05  7.0e-05  1.9e+02  4.0e-04  1.76e+00
##      5   12  2.604e+03  5.02e-05  5.02e-05  1.4e-04  2.4e+01  8.0e-04  1.56e+00
##      6   14  2.604e+03  9.89e-05  9.89e-05  2.8e-04  1.3e+01  1.6e-03  1.55e+00
##      7   16  2.604e+03  1.95e-05  1.95e-05  5.6e-05  2.4e+02  3.2e-04  1.55e+00
```

```
##      8   18  2.604e+03  3.89e-06  3.89e-06  1.1e-05  1.2e+03  6.4e-05  1.52e+00
##      9   20  2.604e+03  7.78e-06  7.78e-06  2.3e-05  1.5e+02  1.3e-04  1.51e+00
##     10   23  2.604e+03  1.55e-07  1.55e-07  4.5e-07  3.0e+04  2.6e-06  1.51e+00
##     11   25  2.604e+03  3.11e-07  3.11e-07  9.0e-07  3.7e+03  5.1e-06  1.51e+00
##     12   27  2.604e+03  6.22e-07  6.22e-07  1.8e-06  1.9e+03  1.0e-05  1.51e+00
##     13   29  2.604e+03  1.24e-07  1.24e-07  3.6e-07  3.7e+04  2.0e-06  1.51e+00
##     14   32  2.604e+03  2.49e-09  2.49e-09  7.2e-09  1.9e+06  4.1e-08  1.51e+00
##     15   34  2.604e+03  4.97e-09  4.97e-09  1.4e-08  2.3e+05  8.2e-08  1.51e+00
##     16   36  2.604e+03  9.95e-09  9.95e-09  2.9e-08  1.2e+05  1.6e-07  1.51e+00
##     17   38  2.604e+03  1.99e-09  1.99e-09  5.8e-09  2.3e+06  3.3e-08  1.51e+00
##     18   40  2.604e+03  3.98e-10  3.98e-10  1.2e-09  1.2e+07  6.6e-09  1.51e+00
##     19   42  2.604e+03  7.96e-11  7.96e-11  2.3e-10  1.9e+00  1.3e-09 -3.89e-02
##     20   44  2.604e+03  1.59e-10  1.59e-10  4.6e-10  7.2e+06  2.6e-09  1.51e+00
##     21   46  2.604e+03  3.18e-11  3.18e-11  9.3e-11  1.9e+00  5.2e-10 -3.89e-02
##     22   48  2.604e+03  6.37e-12  6.37e-12  1.9e-11  1.9e+00  1.0e-10 -3.89e-02
##     23   50  2.604e+03  1.27e-11  1.27e-11  3.7e-11  1.9e+00  2.1e-10 -3.89e-02
##     24   52  2.604e+03  2.55e-11  2.55e-11  7.4e-11  1.9e+00  4.2e-10 -3.89e-02
##     25   55  2.604e+03  5.09e-13  5.09e-13  1.5e-12  1.9e+00  8.4e-12 -3.89e-02
##     26   57  2.604e+03  1.01e-13  1.02e-13  3.0e-13  1.9e+00  1.7e-12 -3.89e-02
##     27   60  2.604e+03  5.24e-16  2.04e-15  5.9e-15  1.9e+00  3.4e-14 -3.89e-02
##     28   62  2.604e+03  8.56e-15  4.07e-15  1.2e-14  2.8e+11  6.7e-14  1.50e+00
##     29   64  2.604e+03  5.24e-16  8.15e-16  2.4e-15  1.9e+00  1.3e-14 -3.88e-02
##     30   65  2.604e+03  8.73e-16  1.63e-15  4.7e-15  1.9e+00  2.7e-14 -4.01e-02
##     31   66  2.604e+03 -3.84e+06  3.26e-15  9.5e-15  1.9e+00  5.4e-14 -3.85e-02
##
##  ***** FALSE CONVERGENCE *****
##
##  FUNCTION     2.603894e+03   RELDX        9.487e-15
##  FUNC. EVALS      66         GRAD. EVALS      31
##  PRELDF       3.259e-15      NPRELDF     -3.854e-02
##
##      I      FINAL X(I)        D(I)          G(I)
##
```

```
##       1     2.565431e+00      1.000e+00      5.799e+01
##       2     9.506078e-02      1.000e+00      1.503e+01
##       3     1.078212e-01      1.000e+00      2.365e+01
##       4     8.930446e-02      1.000e+00      1.736e+01
##       5     3.415119e-14      1.000e+00      1.433e+02
```

```r
summary(garch1_3)
```

```
##
## Call:
## garch(x = Apple, order = c(1, 3))
##
## Model:
## GARCH(1,3)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -7.61315 -0.40323  0.03544  0.56604  5.45059
##
## Coefficient(s):
##     Estimate  Std. Error  t value Pr(>|t|)
## a0 2.565e+00   1.981e-01   12.950  < 2e-16 ***
## a1 9.506e-02   2.085e-02    4.559 5.15e-06 ***
## a2 1.078e-01   1.866e-02    5.778 7.56e-09 ***
## a3 8.930e-02   2.127e-02    4.199 2.68e-05 ***
## b1 3.415e-14   7.047e-02    0.000        1
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Diagnostic Tests:
##  Jarque Bera Test
##
## data:  Residuals
```

```
## X-squared = 2774.3, df = 2, p-value < 2.2e-16
##
##
##  Box-Ljung test
##
## data:  Squared.Residuals
## X-squared = 0.21886, df = 1, p-value = 0.6399
```

```
AIC(garch1_3)
```

```
## [1] 9834.536
```

We can see that the all coefficients of GARCH(1,3) model are significant. The p-value of Box-Ljung test is 0.6399 which is greater than 0.05. There it can be considered as a good model. The AIC is 9834.536

```
garch2_3=garch(x=Apple,order=c(2,3))
```

```
##
##  ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     2.426077e+00     1.000e+00
##      2     5.000000e-02     1.000e+00
##      3     5.000000e-02     1.000e+00
##      4     5.000000e-02     1.000e+00
##      5     5.000000e-02     1.000e+00
##      6     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   NPRELDF
##      0    1  2.616e+03
##      1    3  2.603e+03  4.63e-03  1.39e-02  1.3e-02  3.6e+03  1.0e-01  2.51e+01
##      2    5  2.601e+03  9.79e-04  1.02e-03  1.5e-03  1.2e+01  1.0e-02  1.29e+01
```

```
##      3    7  2.597e+03  1.46e-03  1.46e-03  3.1e-03  2.0e+00  2.0e-02  1.38e+00
##      4    9  2.597e+03  2.24e-04  2.24e-04  6.9e-04  1.3e+01  4.0e-03  1.45e+00
##      5   11  2.596e+03  4.26e-05  4.26e-05  1.4e-04  6.3e+01  8.0e-04  1.30e+00
##      6   13  2.596e+03  8.44e-06  8.44e-06  2.8e-05  3.1e+02  1.6e-04  1.25e+00
##      7   15  2.596e+03  1.68e-05  1.68e-05  5.6e-05  4.0e+01  3.2e-04  1.24e+00
##      8   17  2.596e+03  3.33e-05  3.33e-05  1.1e-04  2.1e+01  6.4e-04  1.24e+00
##      9   19  2.596e+03  6.61e-06  6.61e-06  2.2e-05  3.9e+02  1.3e-04  1.23e+00
##     10   21  2.596e+03  1.32e-06  1.32e-06  4.5e-06  2.0e+03  2.6e-05  1.23e+00
##     11   23  2.596e+03  2.64e-06  2.64e-06  9.0e-06  2.5e+02  5.1e-05  1.22e+00
##     12   25  2.596e+03  5.27e-07  5.27e-07  1.8e-06  4.9e+03  1.0e-05  1.22e+00
##     13   27  2.596e+03  1.05e-06  1.05e-06  3.6e-06  6.2e+02  2.0e-05  1.22e+00
##     14   29  2.596e+03  2.11e-07  2.11e-07  7.2e-07  1.2e+04  4.1e-06  1.22e+00
##     15   31  2.596e+03  4.21e-07  4.21e-07  1.4e-06  1.5e+03  8.2e-06  1.22e+00
##     16   34  2.596e+03  8.43e-09  8.43e-09  2.9e-08  3.1e+05  1.6e-07  1.22e+00
##     17   37  2.596e+03  1.69e-10  1.69e-10  5.7e-10  1.5e+07  3.3e-09  1.22e+00
##     18   39  2.596e+03  3.37e-10  3.37e-10  1.1e-09  1.9e+06  6.6e-09  1.22e+00
##     19   41  2.596e+03  6.74e-11  6.74e-11  2.3e-10  1.9e+00  1.3e-09 -3.33e-02
##     20   44  2.596e+03  5.39e-10  5.39e-10  1.8e-09  1.2e+06  1.0e-08  1.22e+00
##     21   47  2.596e+03  1.08e-11  1.08e-11  3.7e-11  1.9e+00  2.1e-10 -3.33e-02
##     22   49  2.596e+03  2.16e-12  2.16e-12  7.4e-12  1.9e+00  4.2e-11 -3.33e-02
##     23   51  2.596e+03  4.31e-12  4.31e-12  1.5e-11  1.9e+00  8.4e-11 -3.33e-02
##     24   53  2.596e+03  8.64e-13  8.63e-13  2.9e-12  1.9e+00  1.7e-11 -3.33e-02
##     25   55  2.596e+03  1.73e-12  1.73e-12  5.9e-12  1.9e+00  3.4e-11 -3.33e-02
##     26   57  2.596e+03  3.45e-13  3.45e-13  1.2e-12  1.9e+00  6.7e-12 -3.33e-02
##     27   59  2.596e+03  6.90e-13  6.90e-13  2.4e-12  1.9e+00  1.3e-11 -3.33e-02
##     28   61  2.596e+03  1.38e-12  1.38e-12  4.7e-12  1.9e+00  2.7e-11 -3.33e-02
##     29   63  2.596e+03  2.75e-13  2.76e-13  9.4e-13  1.9e+00  5.4e-12 -3.33e-02
##     30   65  2.596e+03  5.41e-14  5.52e-14  1.9e-13  1.9e+00  1.1e-12 -3.33e-02
##     31   67  2.596e+03  1.08e-13  1.10e-13  3.8e-13  1.9e+00  2.1e-12 -3.33e-02
##     32   69  2.596e+03  2.22e-13  2.21e-13  7.5e-13  1.9e+00  4.3e-12 -3.33e-02
##     33   72  2.596e+03 -3.85e+06  4.42e-15  1.5e-14  1.9e+00  8.6e-14 -3.33e-02
##
##  ***** FALSE CONVERGENCE *****
```

```
##
##  FUNCTION     2.596224e+03   RELDX        1.506e-14
##  FUNC. EVALS      72         GRAD. EVALS      33
##  PRELDF       4.418e-15      NPRELDF     -3.328e-02
##
##      I     FINAL X(I)        D(I)          G(I)
##
##      1    2.402682e+00     1.000e+00     5.060e+01
##      2    9.520392e-02     1.000e+00    -4.175e+00
##      3    1.064172e-01     1.000e+00     6.851e+00
##      4    9.377566e-02     1.000e+00     4.725e-02
##      5    5.613532e-14     1.000e+00     1.125e+02
##      6    1.720384e-02     1.000e+00     5.042e+01
```

```r
summary(garch2_3)
```

```
##
## Call:
## garch(x = Apple, order = c(2, 3))
##
## Model:
## GARCH(2,3)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -7.73134 -0.41016  0.03588  0.57611  5.53240
##
## Coefficient(s):
##     Estimate  Std. Error  t value Pr(>|t|)
## a0 2.403e+00   2.028e-01   11.848  < 2e-16 ***
## a1 9.520e-02   2.044e-02    4.658 3.20e-06 ***
## a2 1.064e-01   1.930e-02    5.513 3.53e-08 ***
## a3 9.378e-02   2.182e-02    4.298 1.73e-05 ***
```

```
## b1 5.614e-14   8.936e-02    0.000    1.000
## b2 1.720e-02   3.224e-02    0.534    0.594
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Diagnostic Tests:
##  Jarque Bera Test
##
## data:  Residuals
## X-squared = 2751.2, df = 2, p-value < 2.2e-16
##
##
##  Box-Ljung test
##
## data:  Squared.Residuals
## X-squared = 0.15059, df = 1, p-value = 0.698
```

```
AIC(garch2_3)
```

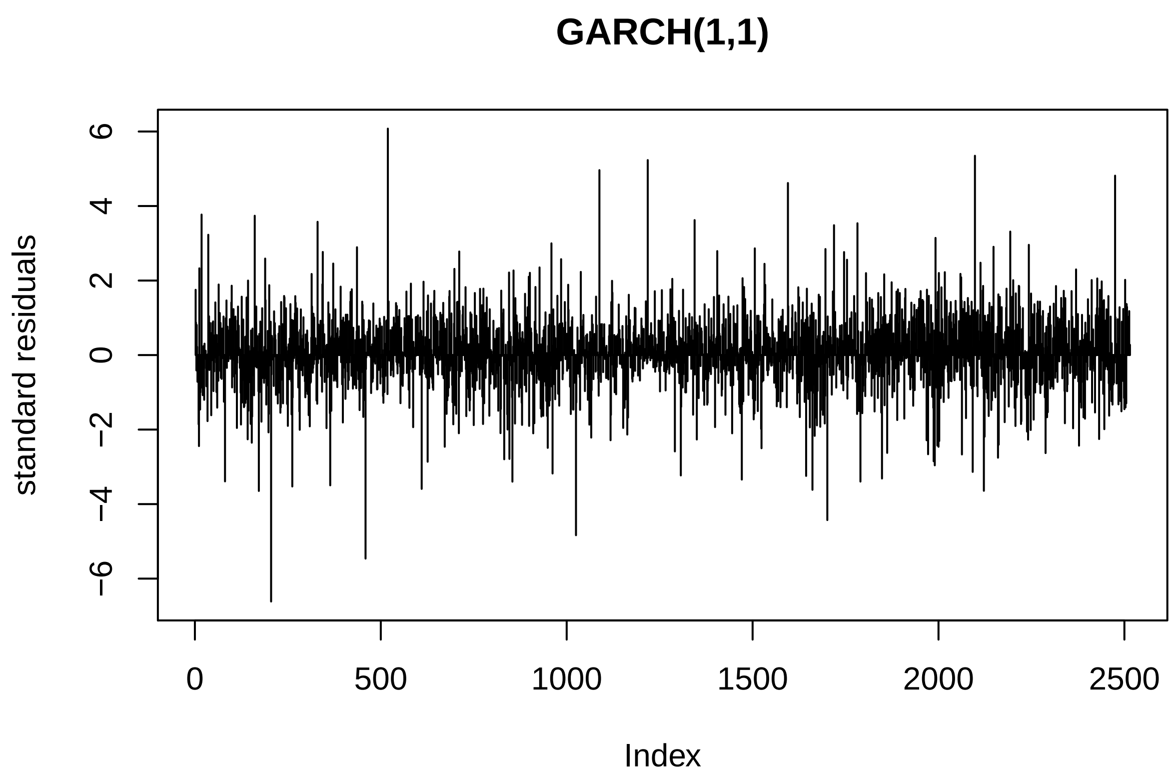
```
## [1] 9821.196
```

We can see that the all coefficients of GARCH(1,3) model are not significant so it can not be considered as a good model. The p-value of Box-Ljung test is 0.698 which is greater than 0.05. The AIC is 9821.196

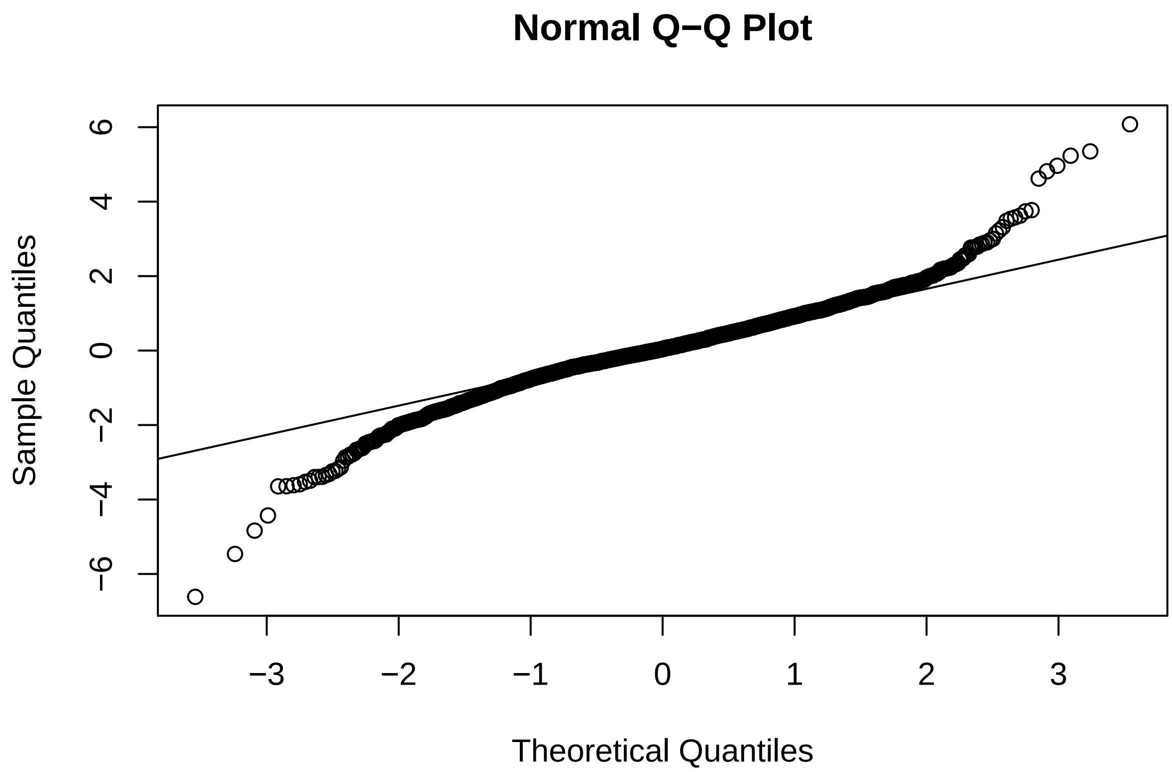
We found that GARCH(1,1) has the lowest AIC value here. Hence we choose the GARCH(1,1) model as the best model.

#Model Diagnostic of GARCH(1,1) In this section, we will check whether the GARCH(1,1) model is supported by data or not.

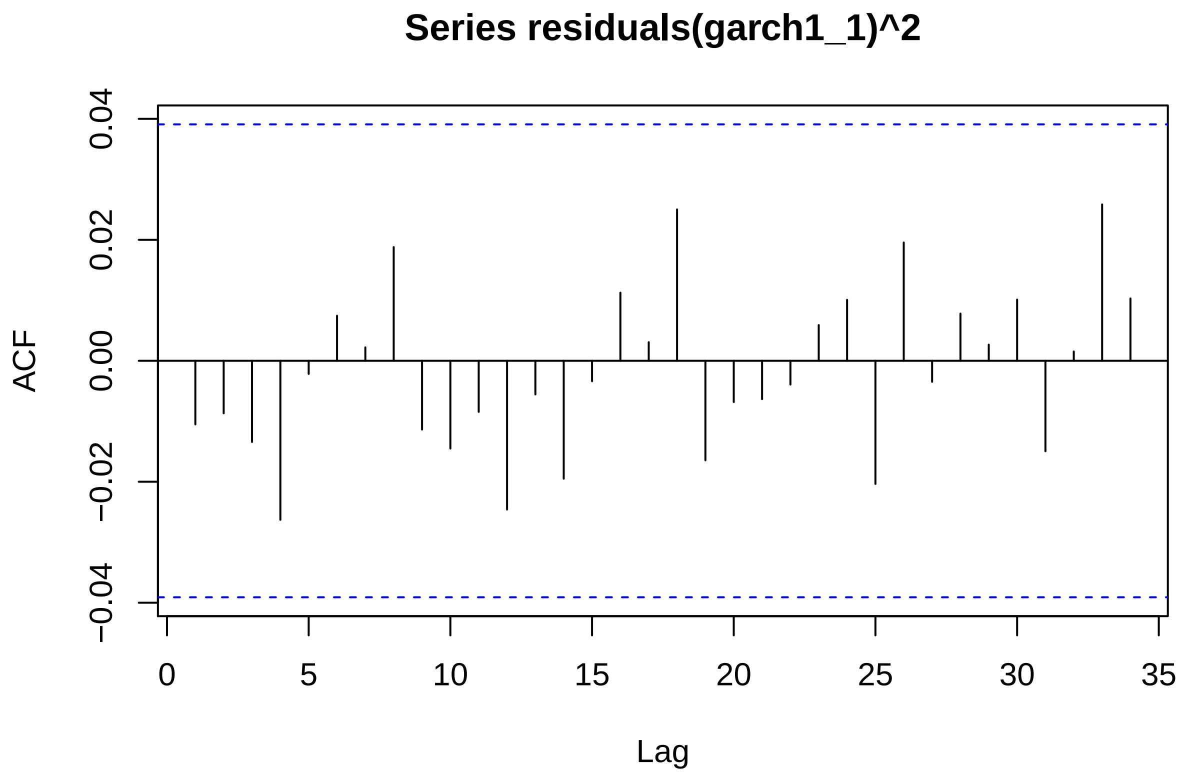
```
plot(residuals(garch1_1),type='h',main='GARCH(1,1)',ylab='standard residuals')
```



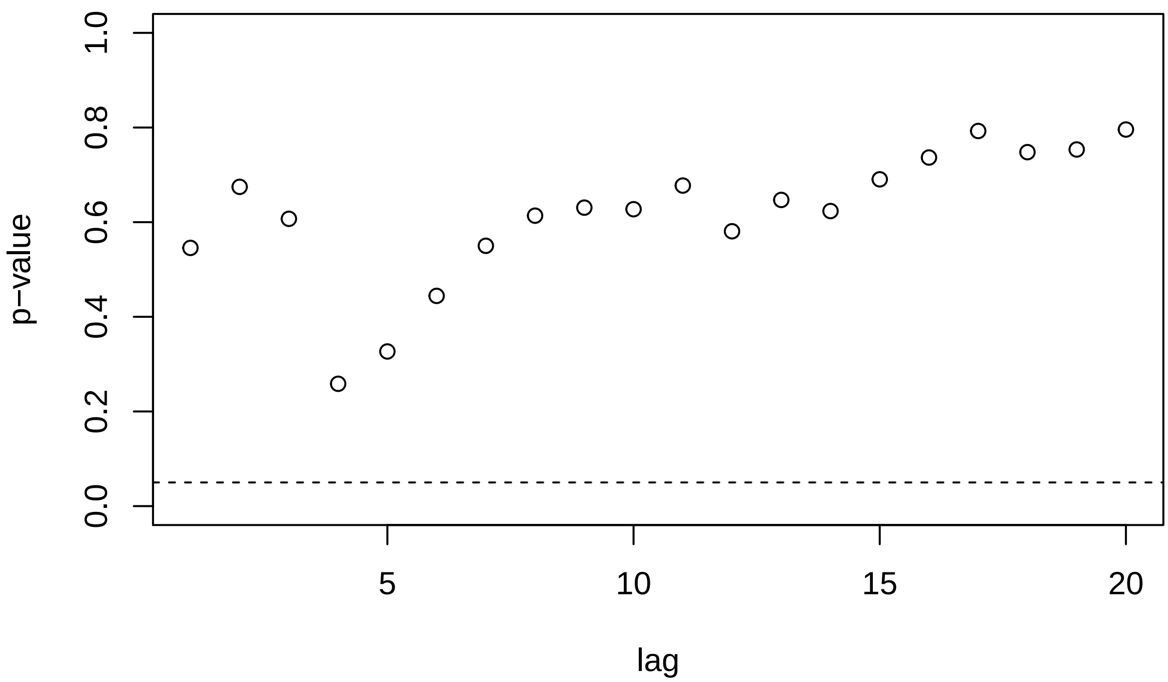

```
qqnorm(residuals(garch1_1));qqline(residuals(garch1_1))
```



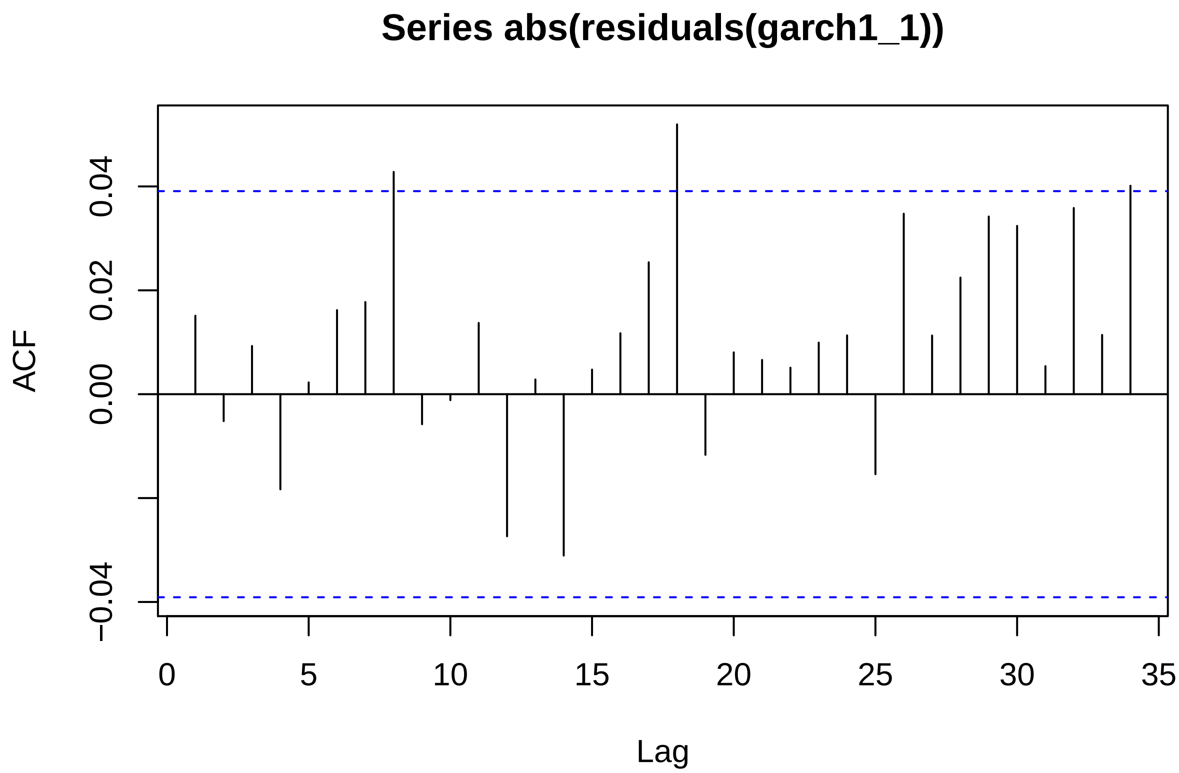

```
acf(residuals(garch1_1)^2,na.action=na.omit)
```




```
gBox(garch1_1,method='squared')
```




```
acf(abs(residuals(garch1_1)),na.action=na.omit)
```




```
gBox(garch1_1,method='absolute')
```

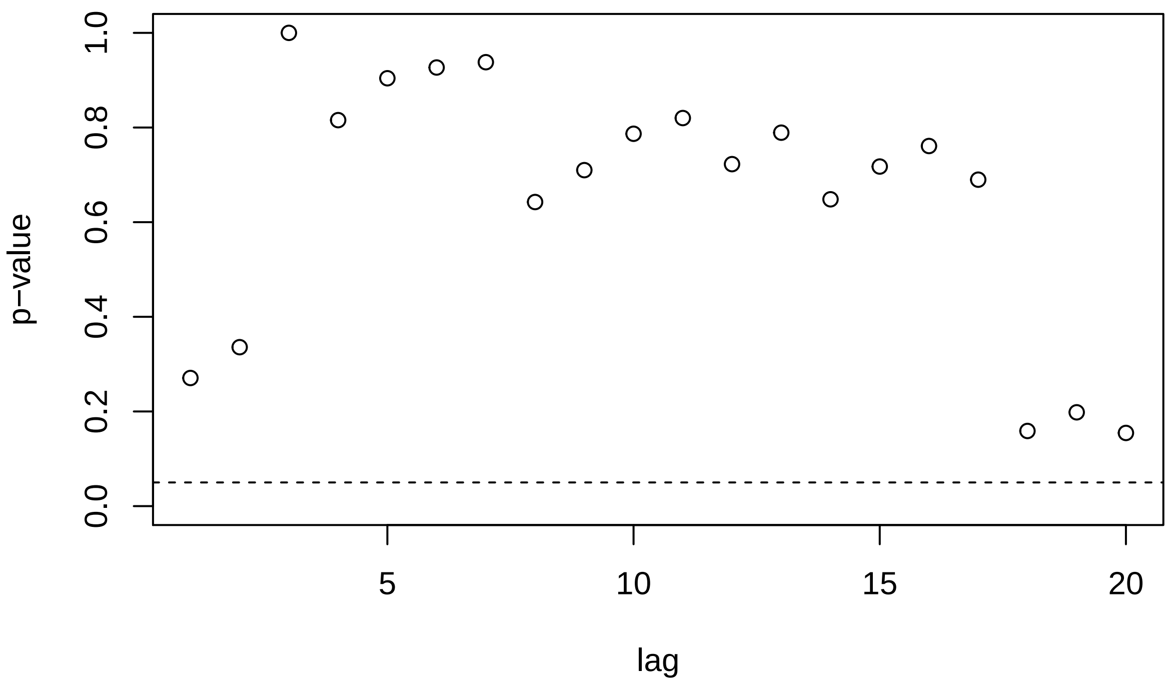


Figure 7: Model Diagnostic plot of GARCH(1,1)

In Figure 7, The graph of the standardized residuals from fitted GARCH(1,1) model suggests no particular tendency in the standardized residuals. Sample ACF of squared and of absolute standardized residuals seems to close to independently and identically distributed. The Q-Q plot displays a mostly straight-line pattern, indicating that the majority of the data is normal distribution with fat tails at both ends, which is not a big concern. All of the numbers between the boundary in the ACF plot of squared residuals suggest that there is no association. P-value display looks great. The p-values are likewise higher, implying that the squared residuals and absolute standardized residuals are uncorrelated across time and, as a result, the standardized residuals are possibly independent. So the GARCH(1,1) model provides good fit here.

```
fgarch.fit <- garchFit(~ garch(1,1), data = Apple, include.mean = FALSE)
```

```
##
## Series Initialization:
##  ARMA Model:                arma
##  Formula Mean:              ~ arma(0, 0)
```

```
##  GARCH Model:               garch
##  Formula Variance:          ~ garch(1, 1)
##  ARMA Order:                0 0
##  Max ARMA Order:            0
##  GARCH Order:               1 1
##  Max GARCH Order:           1
##  Maximum Order:             1
##  Conditional Dist:          norm
##  h.start:                   2
##  llh.start:                 1
##  Length of Series:          2515
##  Recursion Init:            mci
##  Series Scale:              1.798546
##
## Parameter Initialization:
##  Initial Parameters:          $params
##  Limits of Transformations:   $U, $V
##  Which Parameters are Fixed?  $includes
##  Parameter Matrix:
##                      U           V params includes
##     mu     -0.46142492   0.4614249    0.0    FALSE
##     omega   0.00000100 100.0000000    0.1     TRUE
##     alpha1  0.00000001   1.0000000    0.1     TRUE
##     gamma1 -0.99999999   1.0000000    0.1    FALSE
##     beta1   0.00000001   1.0000000    0.8     TRUE
##     delta   0.00000000   2.0000000    2.0    FALSE
##     skew    0.10000000  10.0000000    1.0    FALSE
##     shape   1.00000000  10.0000000    4.0    FALSE
##  Index List of Parameters to be Optimized:
##  omega alpha1  beta1
##      2      3      5
##  Persistence:                  0.9
##
```

```
##
## --- START OF TRACE ---
## Selected Algorithm: nlminb
##
## R coded nlminb Solver:
##
##   0:     3367.9217: 0.100000 0.100000 0.800000
##   1:     3366.5910: 0.0856269 0.0998874 0.793375
##   2:     3360.6851: 0.0842511 0.113717 0.800948
##   3:     3359.2769: 0.0700058 0.120435 0.802507
##   4:     3358.1898: 0.0661151 0.124497 0.817300
##   5:     3357.0206: 0.0569911 0.116388 0.827374
##   6:     3356.6422: 0.0548224 0.106497 0.839538
##   7:     3356.6295: 0.0544267 0.106279 0.839320
##   8:     3356.6242: 0.0542674 0.106255 0.839795
##   9:     3356.6192: 0.0536899 0.105840 0.840501
##  10:     3356.6066: 0.0535977 0.104537 0.842023
##  11:     3356.6027: 0.0523688 0.104313 0.843592
##  12:     3356.6009: 0.0524991 0.102545 0.844528
##  13:     3356.5994: 0.0526062 0.102664 0.844648
##  14:     3356.5982: 0.0524266 0.102750 0.844673
##  15:     3356.5982: 0.0523685 0.102916 0.844769
##  16:     3356.5978: 0.0522990 0.102933 0.844699
##  17:     3356.5977: 0.0523392 0.102994 0.844631
##  18:     3356.5977: 0.0523463 0.103019 0.844604
##  19:     3356.5977: 0.0523459 0.103019 0.844611
##  20:     3356.5977: 0.0523492 0.103020 0.844605
##  21:     3356.5977: 0.0523492 0.103020 0.844605
##
## Final Estimate of the Negative LLH:
##  LLH:  4832.849    norm LLH:  1.92161
##     omega    alpha1     beta1
## 0.1693374 0.1030198 0.8446046
```

```
##
## R-optimhess Difference Approximated Hessian Matrix:
##            omega    alpha1     beta1
## omega  -10021.84 -15402.44 -22776.78
## alpha1 -15402.44 -40342.15 -45590.61
## beta1  -22776.78 -45590.61 -60802.61
## attr(,"time")
## Time difference of 0.01784587 secs
##
## --- END OF TRACE ---
##
##
## Time to Estimate Parameters:
##  Time difference of 0.07276511 secs

## Warning: Using formula(x) is deprecated when x is a character vector of length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.
```

```
fgarch.fit
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = Apple, include.mean = FALSE)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x7fc92d60a030>
##  [data = Apple]
##
## Conditional Distribution:
##  norm
```

```
##
## Coefficient(s):
##   omega   alpha1    beta1
## 0.16934  0.10302  0.84460
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## omega    0.16934     0.03111    5.444 5.21e-08 ***
## alpha1   0.10302     0.01530    6.734 1.66e-11 ***
## beta1    0.84460     0.02078   40.647  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -4832.849    normalized:  -1.92161
##
## Description:
##  Thu Apr 28 20:25:14 2022 by user:
```

```
fgarch.fit@sigma.t
```

```
##    [1] 1.799998 1.790344 1.971714 1.937848 1.845768 1.809810 1.718249 1.679073
##    [9] 1.601142 1.538080 1.734425 2.135025 2.562327 2.391599 2.501871 2.468997
##   [17] 2.307196 2.254935 3.451231 3.200644 2.980549 2.954789 2.748411 2.567887
##   [25] 2.406447 2.433197 2.286272 2.142311 2.012171 1.895969 1.803244 1.774952
##   [33] 1.707427 1.674232 1.855590 1.754348 2.464175 2.314978 2.300950 2.174554
##   [41] 2.047672 2.007386 1.928921 1.821613 1.965361 1.862038 1.762002 1.740715
##   [49] 1.651901 1.644300 1.647155 1.593360 1.537406 1.472043 1.421529 1.513868
##   [57] 1.453601 1.400821 1.423020 1.392758 1.485160 1.427337 1.381726 1.369463
##   [65] 1.564016 1.565718 1.541934 1.577932 1.522578 1.518237 1.484993 1.439411
```

```
##  [73] 1.416441 1.403080 1.357430 1.313629 1.275970 1.313062 1.379912 1.333500
##  [81] 1.302027 1.899661 1.793683 1.791733 1.780169 1.883416 1.791294 1.697685
##  [89] 1.666087 1.624789 1.551197 1.484785 1.425962 1.374496 1.395061 1.349258
##  [97] 1.307229 1.300213 1.393996 1.582545 1.574697 1.627547 1.580229 1.509713
## [105] 1.565660 1.497150 1.437531 1.458444 1.403653 1.431998 1.397305 1.378881
## [113] 1.346895 1.554741 1.490574 1.497650 1.567015 1.547130 1.531293 1.469466
## [121] 1.411805 1.370064 1.326195 1.356120 1.543189 1.530166 1.655001 1.714947
## [129] 1.671099 1.592692 1.596985 1.540499 1.628413 1.709780 1.628519 1.572900
## [137] 1.636715 1.561616 1.514845 1.634534 1.578564 1.623552 1.945143 2.218973
## [145] 2.336611 2.193959 2.092945 1.988703 1.930169 1.822162 2.034753 1.962908
## [153] 1.852857 2.241580 2.412653 2.321469 2.187045 2.051643 1.962796 1.971314
## [161] 1.862069 2.844667 2.660717 2.480057 2.381687 2.440131 2.294719 2.151054
## [169] 2.049620 1.940924 1.831286 1.824307 2.745719 2.605047 2.567482 2.404130
## [177] 2.351974 2.204920 2.142026 2.356960 2.275880 2.321346 2.220769 2.100825
## [185] 1.979614 1.865992 1.818971 1.726380 1.674523 2.115227 2.224961 2.125262
## [193] 2.192340 2.065106 1.943897 1.901709 1.838536 1.750215 2.028381 2.168655
## [201] 2.417768 2.270165 2.133441 2.026447 1.994163 4.631074 4.343932 4.077038
## [209] 3.816351 3.532819 3.274077 3.039832 2.937610 2.946711 2.739400 2.660060
## [217] 2.521383 2.376875 2.367456 2.215164 2.077163 2.002961 1.886241 1.947702
## [225] 1.847548 1.779867 1.782160 1.746181 1.686590 1.620075 1.741782 1.830376
## [233] 1.923169 1.860578 1.796999 1.704095 1.681707 1.745666 1.656254 1.606863
## [241] 1.737454 1.860115 1.760459 1.678116 1.596851 1.654740 1.579690 1.518444
## [249] 1.588186 1.660273 1.877039 1.774666 1.690037 1.638101 1.597499 1.541596
## [257] 1.476452 1.559039 1.494090 1.472413 1.601755 1.603535 2.374238 2.384160
## [265] 2.232891 2.195173 2.143219 2.012894 1.909059 1.928177 2.065845 2.154249
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## [1737] 1.375816 1.329813 1.293365 1.297244 1.305109 1.278237 1.246092 1.230894
## [1745] 1.260408 1.231628 1.626449 1.591289 1.525812 1.503853 1.500518 1.475563
## [1753] 1.439964 1.413722 1.790257 1.824122 1.769786 1.710519 1.650194 1.571981
## [1761] 1.516554 1.469377 1.485756 1.442864 1.389532 1.358660 1.406978 1.360366
## [1769] 1.328400 1.315960 1.277555 1.245483 1.216356 1.342432 1.305633 1.272873
## [1777] 1.322511 1.284147 1.283643 1.258816 1.228854 1.354342 2.020918 1.913821
## [1785] 1.849359 1.818659 1.931891 1.822529 1.759261 1.763398 2.547390 2.429872
## [1793] 2.302651 2.160470 2.035949 2.170267 2.125786 2.104003 2.052714 1.934799
## [1801] 1.829960 1.738176 1.657912 1.683802 1.634114 1.937706 1.898627 1.852782
## [1809] 1.944117 1.878222 1.812572 1.718919 1.632458 1.573173 1.515283 1.632582
## [1817] 1.558623 1.512327 1.453642 1.398247 1.435600 1.543290 1.476842 1.448376
## [1825] 1.510441 1.459915 1.428164 1.375804 1.488288 1.441607 1.422804 1.390743
## [1833] 1.365023 1.353943 1.315299 1.289680 1.305983 1.357543 1.500638 1.460750
## [1841] 1.404361 1.378411 1.336607 1.329379 1.296543 1.417551 1.536090 1.622769
## [1849] 2.317709 2.251372 2.135346 2.123864 2.032039 1.914031 2.244055 2.315863
## [1857] 2.173668 2.172609 2.123111 1.994111 1.909873 1.803016 2.285193 2.223708
## [1865] 2.116239 1.999488 1.958685 1.846981 1.808950 1.795755 1.810858 1.714346
## [1873] 1.634103 1.602111 1.829349 1.732335 1.761134 1.678475 1.600753 1.556710
## [1881] 1.511513 1.523937 1.466982 1.417872 1.451753 1.406045 1.365138 1.516850
## [1889] 1.456195 1.619972 1.568296 1.741860 1.652871 1.618592 1.588061 1.576083
## [1897] 1.725479 1.638932 1.563215 1.500036 1.444043 1.397930 1.457543 1.403226
## [1905] 1.419663 1.369077 1.380873 1.371927 1.524176 1.459954 1.576244 1.753464
## [1913] 1.676404 1.595332 1.522864 1.483951 1.427216 1.397800 1.349232 1.341850
```
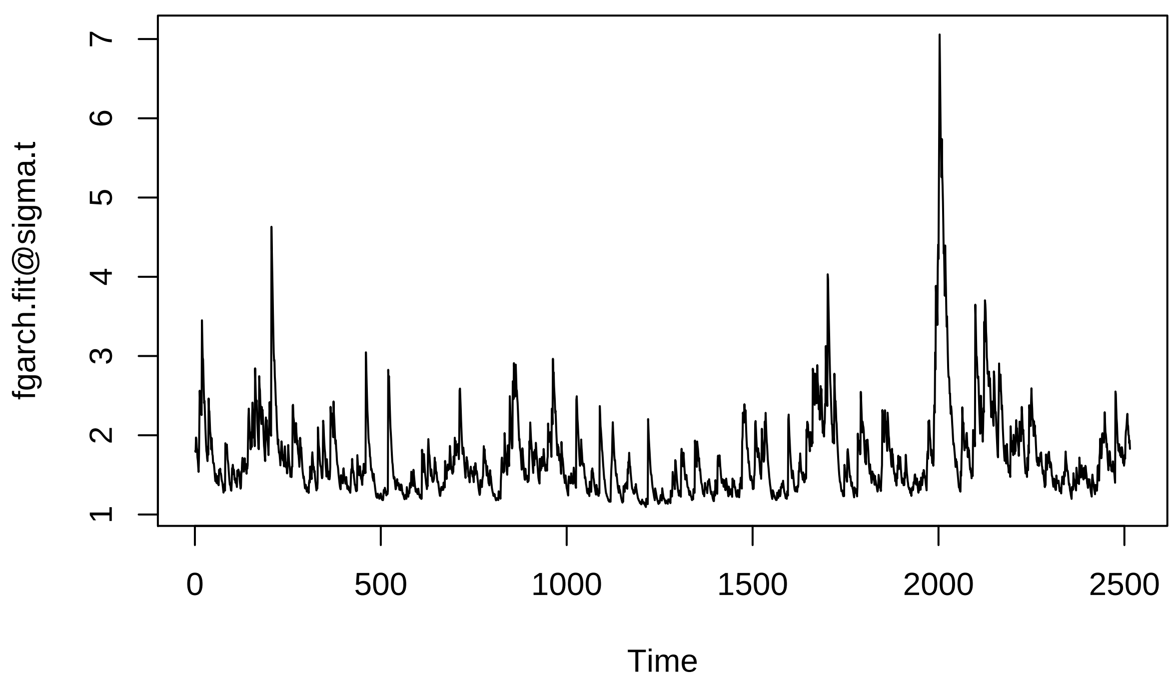

```
## [1921] 1.319002 1.335075 1.304163 1.270975 1.294199 1.266811 1.235137 1.330070
## [1929] 1.314088 1.345819 1.305419 1.322110 1.406817 1.385812 1.417713 1.498000
## [1937] 1.506459 1.456394 1.426579 1.376550 1.399131 1.455847 1.401229 1.354092
## [1945] 1.311109 1.275008 1.346211 1.304198 1.415477 1.364447 1.333351 1.313581
## [1953] 1.466635 1.443754 1.412355 1.370038 1.420218 1.525789 1.463167 1.561372
## [1961] 1.555248 1.493742 1.487792 1.470978 1.429863 1.381744 1.343745 1.304982
## [1969] 1.589144 1.761819 1.798087 1.703587 2.177028 2.044527 2.187852 2.068852
## [1977] 1.980831 1.936207 1.832703 1.744684 1.818718 1.736571 1.648168 1.677946
## [1985] 1.661421 1.615631 1.707042 2.251748 2.382243 2.284237 3.047029 2.830428
## [1993] 3.885782 3.741040 3.756033 3.633869 3.392119 4.108740 4.405731 4.225011
## [2001] 5.136534 5.970315 7.057701 6.644367 6.171665 5.692170 5.653955 5.257757
## [2009] 5.738825 5.293116 5.151920 4.942567 4.649498 4.293269 4.329787 4.035508
## [2017] 3.760302 4.395321 4.077492 3.856122 3.575135 3.369563 3.501386 3.257384
## [2025] 3.032422 2.851008 2.736373 2.740278 2.709666 2.527094 2.529353 2.360787
## [2033] 2.269740 2.365682 2.311952 2.226035 2.134924 2.060969 1.966066 1.882340
## [2041] 1.899601 1.862311 1.798527 1.747398 1.669400 1.599808 1.699914 1.626221
## [2049] 1.668887 1.606053 1.546075 1.495267 1.441248 1.387064 1.339884 1.356303
## [2057] 1.320990 1.293922 1.288594 1.544102 1.489593 1.743323 1.844178 2.352805
## [2065] 2.218319 2.116890 2.158485 2.026424 1.907297 1.809879 1.903463 1.920681
## [2073] 1.900490 1.843668 2.010864 2.029630 1.928676 1.820655 1.723084 1.842531
## [2081] 1.745466 1.813455 1.722176 1.636303 1.566136 1.586975 1.531293 1.519125
## [2089] 1.456941 1.552490 1.550485 1.485893 2.066351 1.944729 1.982133 1.941729
## [2097] 1.930065 1.861357 3.647991 3.470905 3.223365 2.993047 2.991188 2.891328
## [2105] 2.728473 2.719573 2.741764 2.614479 2.437920 2.279527 2.151523 2.020068
## [2113] 2.027834 2.498722 2.363968 2.226923 2.132099 2.039064 1.919304 2.103907
## [2121] 2.340866 2.291077 3.431692 3.180610 3.703180 3.650722 3.544138 3.310328
## [2129] 3.213231 2.981989 2.933319 2.775501 2.783179 2.762460 2.620148 2.803458
## [2137] 2.629625 2.721598 2.645491 2.477874 2.363420 2.227160 2.338368 2.394802
## [2145] 2.425881 2.330530 2.181202 2.119924 2.805755 2.750081 2.560782 2.392519
## [2153] 2.282274 2.293111 2.188039 2.060005 1.961899 1.859909 1.758139 1.721787
## [2161] 2.233756 2.397320 2.906385 2.702680 2.564743 2.715774 2.766205 2.576934
## [2169] 2.489403 2.326546 2.379517 2.226487 2.087206 1.981530 1.882829 1.816237
## [2177] 1.727085 1.677410 1.866818 1.802689 1.723675 1.643954 1.703261 1.889257
```

```
## [2185] 1.786846 1.693322 1.619796 1.593291 1.529704 1.614144 1.586265 1.530231
## [2193] 1.474618 2.113515 1.985561 1.883815 1.851926 1.795094 1.924189 1.829474
## [2201] 1.748432 2.005862 1.937186 1.847815 1.764871 1.856206 1.798604 2.027184
## [2209] 2.190909 2.073548 2.090659 1.965450 1.923267 1.879588 1.830334 1.740415
## [2217] 1.950470 2.172052 2.101421 2.160395 2.028366 1.924983 2.146060 2.357480
## [2225] 2.267154 2.133471 2.019045 2.068502 1.950618 1.839625 1.752849 1.669093
## [2233] 1.589372 1.518598 1.545526 1.585328 1.539338 1.473854 1.716638 1.630807
## [2241] 1.559671 1.874845 1.772929 2.378735 2.325454 2.316967 2.228264 2.116711
## [2249] 2.412510 2.592429 2.435781 2.335895 2.199592 2.203897 2.106395 1.989951
## [2257] 2.177063 2.047731 2.124911 2.008024 1.998779 1.887172 1.790089 1.696489
## [2265] 1.660515 1.689457 1.621605 1.717489 1.633082 1.613733 1.656112 1.702925
## [2273] 1.673759 1.769081 1.773970 1.783593 1.691980 1.616664 1.596427 1.526662
## [2281] 1.510029 1.557068 1.492031 1.433791 1.394019 1.345818 1.392358 1.369562
## [2289] 1.758034 1.668435 1.639256 1.566012 1.716028 1.647265 1.765848 1.768610
## [2297] 1.791380 1.723033 1.675888 1.594738 1.662164 1.652519 1.629729 1.554084
## [2305] 1.486394 1.481609 1.432832 1.382231 1.350308 1.365059 1.452227 1.396633
## [2313] 1.364685 1.323640 1.309945 1.310448 1.492351 1.446837 1.397591 1.407341
## [2321] 1.395763 1.420152 1.427164 1.376275 1.331911 1.293398 1.320000 1.332545
## [2329] 1.300377 1.266023 1.382419 1.415408 1.479074 1.450877 1.456222 1.406653
## [2337] 1.379910 1.536833 1.478193 1.490594 1.677138 1.795111 1.708298 1.651887
## [2345] 1.618752 1.546322 1.555976 1.539253 1.480443 1.422257 1.372380 1.386627
## [2353] 1.342120 1.300496 1.279992 1.246291 1.221818 1.197279 1.347426 1.305685
## [2361] 1.340070 1.313516 1.521180 1.459208 1.439775 1.423884 1.371897 1.353753
## [2369] 1.322440 1.303541 1.590422 1.542549 1.483108 1.443701 1.395737 1.434603
## [2377] 1.418951 1.384380 1.719124 1.637384 1.590235 1.530967 1.466850 1.529679
## [2385] 1.620481 1.548966 1.576134 1.521086 1.457356 1.441897 1.588438 1.530819
## [2393] 1.496248 1.458663 1.614256 1.604324 1.544005 1.505685 1.446299 1.391573
## [2401] 1.375152 1.336084 1.445724 1.411513 1.412098 1.443706 1.393318 1.345821
## [2409] 1.314550 1.276309 1.251652 1.225871 1.437462 1.503460 1.452945 1.415676
## [2417] 1.399998 1.355512 1.313713 1.287995 1.255665 1.373578 1.327775 1.366427
## [2425] 1.321485 1.299913 1.368251 1.601904 1.621296 1.548634 1.483561 1.428056
## [2433] 1.720271 1.775145 1.956153 1.847125 1.757830 1.709512 1.761796 2.010192
## [2441] 2.026101 1.909353 2.008684 2.006681 1.907149 2.013802 2.290729 2.155268
```

```
## [2449] 2.039901 2.013001 1.957018 1.848711 1.894074 1.798287 1.703206 1.632279
## [2457] 1.559669 1.688857 1.657234 1.799381 1.787715 1.694000 1.610298 1.626300
## [2465] 1.552411 1.607806 1.542545 1.598082 1.670778 1.624345 1.602455 1.537095
## [2473] 1.516546 1.453334 1.400802 2.552188 2.521117 2.353433 2.213157 2.144563
## [2481] 2.015636 1.902456 1.889758 1.804400 1.872648 1.887064 1.782994 1.842307
## [2489] 1.742985 1.792036 1.724157 1.735767 1.849716 1.827848 1.778243 1.686078
## [2497] 1.646620 1.699181 1.616151 1.652583 1.752009 1.704109 1.959867 2.048455
## [2505] 2.077672 2.135865 2.214381 2.270164 2.136531 2.113368 2.004014 1.999490
## [2513] 1.901395 1.934137 1.828447
```

```r
plot.ts(fgarch.fit@sigma.t)
```




```r
plot.ts(fgarch.fit@residuals)
```

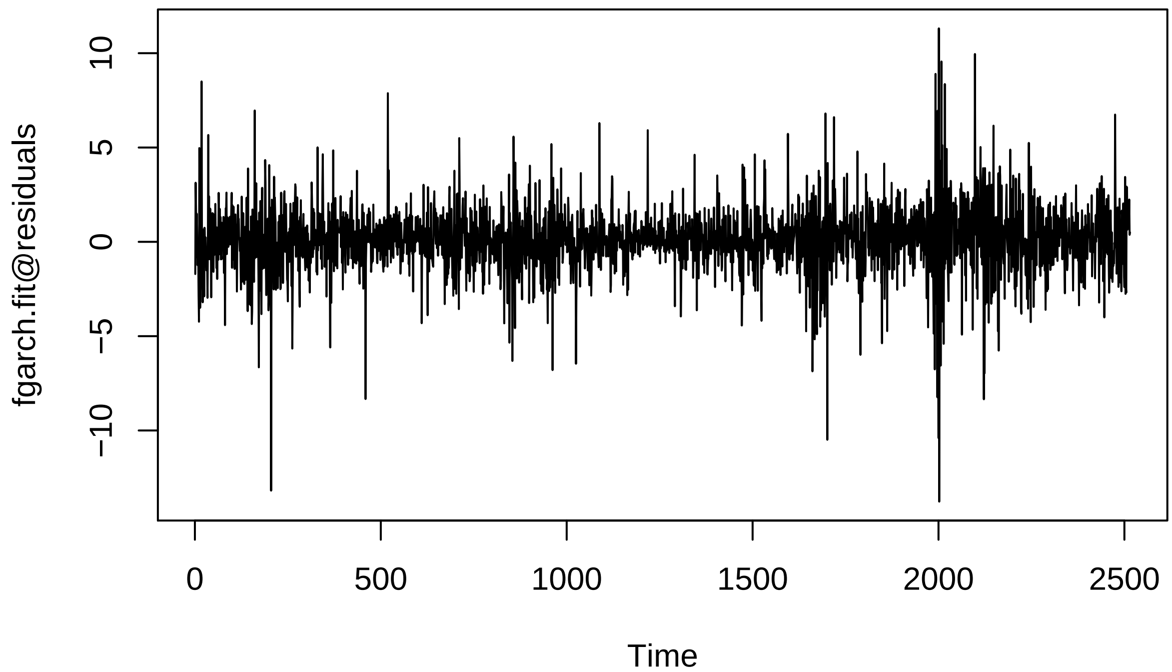


Figure 8: fitted model

Also From the p-value of coefficients, we can see that the all coefficients of GARCH(1,1) model are significant. So we conclude that GARCH(1,1) model is the best model for our time series.

2 years

```
predict(fgarch.fit, n.ahead =1000)
```

```
##      meanForecast meanError standardDeviation
## 1               0  1.737538          1.737538
## 2               0  1.740761          1.740761
## 3               0  1.743811          1.743811
## 4               0  1.746696          1.746696
## 5               0  1.749425          1.749425
```

```
## 6                 0  1.752007          1.752007
## 7                 0  1.754451          1.754451
## 8                 0  1.756764          1.756764
## 9                 0  1.758952          1.758952
## 10                0  1.761024          1.761024
## 11                0  1.762985          1.762985
## 12                0  1.764841          1.764841
## 13                0  1.766598          1.766598
## 14                0  1.768261          1.768261
## 15                0  1.769836          1.769836
## 16                0  1.771327          1.771327
## 17                0  1.772739          1.772739
## 18                0  1.774076          1.774076
## 19                0  1.775342          1.775342
## 20                0  1.776540          1.776540
## 21                0  1.777676          1.777676
## 22                0  1.778751          1.778751
## 23                0  1.779769          1.779769
## 24                0  1.780733          1.780733
## 25                0  1.781647          1.781647
## 26                0  1.782512          1.782512
## 27                0  1.783331          1.783331
## 28                0  1.784107          1.784107
## 29                0  1.784842          1.784842
## 30                0  1.785539          1.785539
## 31                0  1.786198          1.786198
## 32                0  1.786823          1.786823
## 33                0  1.787415          1.787415
## 34                0  1.787976          1.787976
## 35                0  1.788507          1.788507
## 36                0  1.789010          1.789010
## 37                0  1.789487          1.789487
## 38                0  1.789939          1.789939
```

```
## 39              0  1.790367          1.790367
## 40              0  1.790772          1.790772
## 41              0  1.791156          1.791156
## 42              0  1.791520          1.791520
## 43              0  1.791865          1.791865
## 44              0  1.792192          1.792192
## 45              0  1.792501          1.792501
## 46              0  1.792794          1.792794
## 47              0  1.793072          1.793072
## 48              0  1.793335          1.793335
## 49              0  1.793585          1.793585
## 50              0  1.793821          1.793821
## 51              0  1.794045          1.794045
## 52              0  1.794257          1.794257
## 53              0  1.794458          1.794458
## 54              0  1.794649          1.794649
## 55              0  1.794829          1.794829
## 56              0  1.795000          1.795000
## 57              0  1.795162          1.795162
## 58              0  1.795316          1.795316
## 59              0  1.795461          1.795461
## 60              0  1.795599          1.795599
## 61              0  1.795730          1.795730
## 62              0  1.795853          1.795853
## 63              0  1.795971          1.795971
## 64              0  1.796082          1.796082
## 65              0  1.796187          1.796187
## 66              0  1.796287          1.796287
## 67              0  1.796381          1.796381
## 68              0  1.796471          1.796471
## 69              0  1.796556          1.796556
## 70              0  1.796636          1.796636
## 71              0  1.796713          1.796713
```

```
## 72                  0  1.796785           1.796785
## 73                  0  1.796853           1.796853
## 74                  0  1.796918           1.796918
## 75                  0  1.796980           1.796980
## 76                  0  1.797038           1.797038
## 77                  0  1.797093           1.797093
## 78                  0  1.797145           1.797145
## 79                  0  1.797195           1.797195
## 80                  0  1.797242           1.797242
## 81                  0  1.797286           1.797286
## 82                  0  1.797329           1.797329
## 83                  0  1.797369           1.797369
## 84                  0  1.797406           1.797406
## 85                  0  1.797442           1.797442
## 86                  0  1.797476           1.797476
## 87                  0  1.797509           1.797509
## 88                  0  1.797539           1.797539
## 89                  0  1.797568           1.797568
## 90                  0  1.797595           1.797595
## 91                  0  1.797621           1.797621
## 92                  0  1.797646           1.797646
## 93                  0  1.797669           1.797669
## 94                  0  1.797691           1.797691
## 95                  0  1.797712           1.797712
## 96                  0  1.797732           1.797732
## 97                  0  1.797751           1.797751
## 98                  0  1.797769           1.797769
## 99                  0  1.797786           1.797786
## 100                 0  1.797802           1.797802
## 101                 0  1.797817           1.797817
## 102                 0  1.797831           1.797831
## 103                 0  1.797845           1.797845
## 104                 0  1.797858           1.797858
```

```
## 105            0  1.797870         1.797870
## 106            0  1.797882         1.797882
## 107            0  1.797893         1.797893
## 108            0  1.797903         1.797903
## 109            0  1.797913         1.797913
## 110            0  1.797922         1.797922
## 111            0  1.797931         1.797931
## 112            0  1.797939         1.797939
## 113            0  1.797947         1.797947
## 114            0  1.797955         1.797955
## 115            0  1.797962         1.797962
## 116            0  1.797969         1.797969
## 117            0  1.797975         1.797975
## 118            0  1.797981         1.797981
## 119            0  1.797987         1.797987
## 120            0  1.797993         1.797993
## 121            0  1.797998         1.797998
## 122            0  1.798003         1.798003
## 123            0  1.798007         1.798007
## 124            0  1.798012         1.798012
## 125            0  1.798016         1.798016
## 126            0  1.798020         1.798020
## 127            0  1.798024         1.798024
## 128            0  1.798027         1.798027
## 129            0  1.798031         1.798031
## 130            0  1.798034         1.798034
## 131            0  1.798037         1.798037
## 132            0  1.798040         1.798040
## 133            0  1.798042         1.798042
## 134            0  1.798045         1.798045
## 135            0  1.798047         1.798047
## 136            0  1.798050         1.798050
## 137            0  1.798052         1.798052
```

```
## 138              0  1.798054          1.798054
## 139              0  1.798056          1.798056
## 140              0  1.798058          1.798058
## 141              0  1.798059          1.798059
## 142              0  1.798061          1.798061
## 143              0  1.798063          1.798063
## 144              0  1.798064          1.798064
## 145              0  1.798066          1.798066
## 146              0  1.798067          1.798067
## 147              0  1.798068          1.798068
## 148              0  1.798069          1.798069
## 149              0  1.798071          1.798071
## 150              0  1.798072          1.798072
## 151              0  1.798073          1.798073
## 152              0  1.798074          1.798074
## 153              0  1.798075          1.798075
## 154              0  1.798075          1.798075
## 155              0  1.798076          1.798076
## 156              0  1.798077          1.798077
## 157              0  1.798078          1.798078
## 158              0  1.798079          1.798079
## 159              0  1.798079          1.798079
## 160              0  1.798080          1.798080
## 161              0  1.798080          1.798080
## 162              0  1.798081          1.798081
## 163              0  1.798082          1.798082
## 164              0  1.798082          1.798082
## 165              0  1.798083          1.798083
## 166              0  1.798083          1.798083
## 167              0  1.798083          1.798083
## 168              0  1.798084          1.798084
## 169              0  1.798084          1.798084
## 170              0  1.798085          1.798085
```

```
## 171                0  1.798085           1.798085
## 172                0  1.798085           1.798085
## 173                0  1.798086           1.798086
## 174                0  1.798086           1.798086
## 175                0  1.798086           1.798086
## 176                0  1.798086           1.798086
## 177                0  1.798087           1.798087
## 178                0  1.798087           1.798087
## 179                0  1.798087           1.798087
## 180                0  1.798087           1.798087
## 181                0  1.798088           1.798088
## 182                0  1.798088           1.798088
## 183                0  1.798088           1.798088
## 184                0  1.798088           1.798088
## 185                0  1.798088           1.798088
## 186                0  1.798089           1.798089
## 187                0  1.798089           1.798089
## 188                0  1.798089           1.798089
## 189                0  1.798089           1.798089
## 190                0  1.798089           1.798089
## 191                0  1.798089           1.798089
## 192                0  1.798089           1.798089
## 193                0  1.798089           1.798089
## 194                0  1.798090           1.798090
## 195                0  1.798090           1.798090
## 196                0  1.798090           1.798090
## 197                0  1.798090           1.798090
## 198                0  1.798090           1.798090
## 199                0  1.798090           1.798090
## 200                0  1.798090           1.798090
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## 647              0  1.798091          1.798091
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## 650              0  1.798091          1.798091
## 651              0  1.798091          1.798091
## 652              0  1.798091          1.798091
## 653              0  1.798091          1.798091
## 654              0  1.798091          1.798091
## 655              0  1.798091          1.798091
## 656              0  1.798091          1.798091
## 657              0  1.798091          1.798091
## 658              0  1.798091          1.798091
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## 660              0  1.798091          1.798091
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## 665              0  1.798091          1.798091
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## 671              0  1.798091           1.798091
## 672              0  1.798091           1.798091
## 673              0  1.798091           1.798091
## 674              0  1.798091           1.798091
## 675              0  1.798091           1.798091
## 676              0  1.798091           1.798091
## 677              0  1.798091           1.798091
## 678              0  1.798091           1.798091
## 679              0  1.798091           1.798091
## 680              0  1.798091           1.798091
## 681              0  1.798091           1.798091
## 682              0  1.798091           1.798091
## 683              0  1.798091           1.798091
## 684              0  1.798091           1.798091
## 685              0  1.798091           1.798091
## 686              0  1.798091           1.798091
## 687              0  1.798091           1.798091
## 688              0  1.798091           1.798091
## 689              0  1.798091           1.798091
## 690              0  1.798091           1.798091
## 691              0  1.798091           1.798091
## 692              0  1.798091           1.798091
## 693              0  1.798091           1.798091
## 694              0  1.798091           1.798091
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## 710              0  1.798091          1.798091
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## 714              0  1.798091          1.798091
## 715              0  1.798091          1.798091
## 716              0  1.798091          1.798091
## 717              0  1.798091          1.798091
## 718              0  1.798091          1.798091
## 719              0  1.798091          1.798091
## 720              0  1.798091          1.798091
## 721              0  1.798091          1.798091
## 722              0  1.798091          1.798091
## 723              0  1.798091          1.798091
## 724              0  1.798091          1.798091
## 725              0  1.798091          1.798091
## 726              0  1.798091          1.798091
## 727              0  1.798091          1.798091
## 728              0  1.798091          1.798091
## 729              0  1.798091          1.798091
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## 736              0  1.798091          1.798091
## 737              0  1.798091          1.798091
## 738              0  1.798091          1.798091
## 739              0  1.798091          1.798091
## 740              0  1.798091          1.798091
## 741              0  1.798091          1.798091
## 742              0  1.798091          1.798091
## 743              0  1.798091          1.798091
## 744              0  1.798091          1.798091
## 745              0  1.798091          1.798091
## 746              0  1.798091          1.798091
## 747              0  1.798091          1.798091
## 748              0  1.798091          1.798091
## 749              0  1.798091          1.798091
## 750              0  1.798091          1.798091
## 751              0  1.798091          1.798091
## 752              0  1.798091          1.798091
## 753              0  1.798091          1.798091
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## 761              0  1.798091          1.798091
## 762              0  1.798091          1.798091
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## 765              0  1.798091           1.798091
## 766              0  1.798091           1.798091
## 767              0  1.798091           1.798091
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## 769              0  1.798091           1.798091
## 770              0  1.798091           1.798091
## 771              0  1.798091           1.798091
## 772              0  1.798091           1.798091
## 773              0  1.798091           1.798091
## 774              0  1.798091           1.798091
## 775              0  1.798091           1.798091
## 776              0  1.798091           1.798091
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## 778              0  1.798091           1.798091
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## 780              0  1.798091           1.798091
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## 782              0  1.798091           1.798091
## 783              0  1.798091           1.798091
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## 785              0  1.798091           1.798091
## 786              0  1.798091           1.798091
## 787              0  1.798091           1.798091
## 788              0  1.798091           1.798091
## 789              0  1.798091           1.798091
## 790              0  1.798091           1.798091
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## 797              0  1.798091           1.798091
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## 798             0  1.798091         1.798091
## 799             0  1.798091         1.798091
## 800             0  1.798091         1.798091
## 801             0  1.798091         1.798091
## 802             0  1.798091         1.798091
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## 807             0  1.798091         1.798091
## 808             0  1.798091         1.798091
## 809             0  1.798091         1.798091
## 810             0  1.798091         1.798091
## 811             0  1.798091         1.798091
## 812             0  1.798091         1.798091
## 813             0  1.798091         1.798091
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## 818             0  1.798091         1.798091
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## 822             0  1.798091         1.798091
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## 845              0  1.798091          1.798091
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## 864             0  1.798091          1.798091
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## 887             0  1.798091          1.798091
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## 890             0  1.798091          1.798091
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## 899              0  1.798091          1.798091
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## 901              0  1.798091          1.798091
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## 903              0  1.798091          1.798091
## 904              0  1.798091          1.798091
## 905              0  1.798091          1.798091
## 906              0  1.798091          1.798091
## 907              0  1.798091          1.798091
## 908              0  1.798091          1.798091
## 909              0  1.798091          1.798091
## 910              0  1.798091          1.798091
## 911              0  1.798091          1.798091
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## 913              0  1.798091          1.798091
## 914              0  1.798091          1.798091
## 915              0  1.798091          1.798091
## 916              0  1.798091          1.798091
## 917              0  1.798091          1.798091
## 918              0  1.798091          1.798091
## 919              0  1.798091          1.798091
## 920              0  1.798091          1.798091
## 921              0  1.798091          1.798091
## 922              0  1.798091          1.798091
## 923              0  1.798091          1.798091
## 924              0  1.798091          1.798091
## 925              0  1.798091          1.798091
## 926              0  1.798091          1.798091
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## 930            0  1.798091        1.798091
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## 934            0  1.798091        1.798091
## 935            0  1.798091        1.798091
## 936            0  1.798091        1.798091
## 937            0  1.798091        1.798091
## 938            0  1.798091        1.798091
## 939            0  1.798091        1.798091
## 940            0  1.798091        1.798091
## 941            0  1.798091        1.798091
## 942            0  1.798091        1.798091
## 943            0  1.798091        1.798091
## 944            0  1.798091        1.798091
## 945            0  1.798091        1.798091
## 946            0  1.798091        1.798091
## 947            0  1.798091        1.798091
## 948            0  1.798091        1.798091
## 949            0  1.798091        1.798091
## 950            0  1.798091        1.798091
## 951            0  1.798091        1.798091
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## 963              0  1.798091          1.798091
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## 971              0  1.798091          1.798091
## 972              0  1.798091          1.798091
## 973              0  1.798091          1.798091
## 974              0  1.798091          1.798091
## 975              0  1.798091          1.798091
## 976              0  1.798091          1.798091
## 977              0  1.798091          1.798091
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## 980              0  1.798091          1.798091
## 981              0  1.798091          1.798091
## 982              0  1.798091          1.798091
## 983              0  1.798091          1.798091
## 984              0  1.798091          1.798091
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## 986              0  1.798091          1.798091
## 987              0  1.798091          1.798091
## 988              0  1.798091          1.798091
## 989              0  1.798091          1.798091
## 990              0  1.798091          1.798091
## 991              0  1.798091          1.798091
## 992              0  1.798091          1.798091
## 993              0  1.798091          1.798091
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## 996             0  1.798091          1.798091
## 997             0  1.798091          1.798091
## 998             0  1.798091          1.798091
## 999             0  1.798091          1.798091
## 1000            0  1.798091          1.798091
```

Figure 9: predict fitted model

Finally, We used a basic mathematical concept "Standard deviation" to measure the volatility in the market. We noticed that the value of a standard deviation goes on increasing slowly and there is no such high swings and low swings between trading ranges. Moreover,There is a narrow spread between the trading values which means standard deviation is low and hence, the volatility is low.Thus, we can say that the value of the apple stock converges as the time passes. After some point reaches to a constant value of 1.7980912

#(3) Data Analysis/Interpretation Page There are various properties to the offered Apple stock prices, including open, high, low, close, and adjusted prices. The adjusted closing stock values are used for stock price analysis because they provide a better picture of the stock's entire value. We can observe that the data is not stationary when we look at the time plot. Although the stock fluctuates, we can observe an increasing trend factor when looking at the entire data. The model's variance isn't constant either. With a higher stock value, there is more variety in the stock data. The equities appear to be volatile based on the time plot. The volatility clustering of the stock data occurs when the conditional variance of the stock data varies.

Based on our research, we observed that the data has trend and changing variance. So we took log but trend was still there. So we took difference of logarithm of the data to removed the trend and to make make the variance constant. It is multiplied by 100 so that percentage changes in the stock can be calculated.The data has become stable with a constant mean, as seen in Figure 2. From the Figure 3, we can observe that the data has little correlation. From the Figure 4 and Figure 5, it can be observed that there is a significant autocorrelation. This is an evidence that the stock data is not independently and identically distributed. From the Figure 6, it can be observed that the data is not normally distributed and the p-value of shapiro-test is less than 0.05 indicates that the data deviates from the normal distribution. The kurtosis is a positive value(6.593444) confirming that the distribution is a heavy tailed distribution. We discovered that the data is stationary and not independently and identically distributed after analyzing the converted data.

We also performed the Ljung Box's test to test the independence of the stock return prices. H0: there exists no autocorrelation. H1: there exists autocorrelation. From Ljung Box Tests, we observe that there is no autocorrelation as the p-values>0.05 and hence we cant reject the null hypothesis. So there is no autocorrelation and hence they behave as white noice process.

We used GARCH model here. To find the P and Q of GARCH, we observed the EACF of the data's absolute and squared values. From the Table 1, the EACF of the absolute data suggest that a GARCH(1,1). From the Table 2, the EACF of the squared data suggest the GARCH(1,3) or GARCH(2,3) model. We checked the P-values in the diagnostic log to see if the models are enough to reflect the data. We can see that the all coefficients of GARCH(1,1) model are significant. The p-value of Box-Ljung test is 0.5979 which is greater than 0.05. So GARCH(1,1) can be considered as a good model. The AIC is 9667.788. We saw that the all coefficients of GARCH(1,3) model are significant. The p-value of Box-Ljung test is 0.6399 which is greater than 0.05. So GARCH(1,3) can be considered as a good model. The AIC is 9834.536. We saw that the all coefficients of GARCH(1,3) model are not significant so it can not be considered as a good model. The p-value of Box-Ljung test is 0.698 which is greater than 0.05. The AIC is 9821.196.

We compared these three model and found the GARCH(1,1) has the lowest AIC value here. Hence we choose the GARCH(1,1) model as the best model. In Figure 7, The graph of the standardized residuals from fitted GARCH(1,1) model suggests no particular tendency in the standardized residuals. Sample ACF of squared and of absolute standardized residuals seems to close to independently and identically distributed. The Q-Q plot displays a mostly straight-line pattern, indicating that the majority of the data is normal distribution with fat tails at both ends, which is not a big concern. All of the numbers between the boundary in the ACF plot of squared residuals suggest that there is no association. P-value display looks great. The p-values are likewise higher, implying that the squared residuals and absolute standardized residuals are uncorrelated across time and, as a result, the standardized residuals are possibly independent. So the GARCH(1,1) model provides good fit here.

Conclusion

Time Series Analysis is a popular method of predicting the future values of time series based on various past behaviors. Such predictions are made through the study of underlying patterns in the time-series data. ARIMA (Autoregressive Integrated Moving Average) model is a commonly used time series model in this process. However, some other models like ARCH and GARCH models have become popular in the analysis of financial time series data. In this project, we worked on predicting the future value of APPLE stock using the GARCH mode.