3.7 Particle horizon and event horizon. Calculate both the particle horizon and the event horizon in the following two spatially flat cosmological models:

- Einstein-de Sitter model ($\Omega_M = 1$);
- De Sitter model ($\Omega_\Lambda = 1$). [Careful, as this model does not have a Big Bang in the past.]

So we know that for particle horizon we calculate the farthest distance from which light can reach us with

$$d_{ph} = \int_0^{t_0} \frac{dt}{a(t)}$$ and in single component

flat universe this comes out to

$$d_{ph} = \frac{2}{1+3w} H_0^{-1}$$

in einstein de sitter model $w = 0$

$$d_{ph} = 2H_0^{-1} = \frac{2}{H} c = \frac{2}{7/3 \times 10^{-18}} 3\times10^8 = \boxed{2.57\times10^{26}\ m = d_{ph} \text{ for } \Omega_m = 1}$$

$$H_0 = 70 \frac{km}{s} \cdot \frac{1}{Mpc} \cdot \frac{1\ mpc}{3\times10^{19}\ km} = 21\times10^{-18}$$

For event horizon

$$\int_{t_0}^{\infty} \frac{dt}{a(t)} \qquad a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} \qquad t^{2} \quad \frac{1}{t}$$

$$\int_{t_0}^{\infty} \left(\frac{t_0}{t}\right)^{\frac{2}{3(1+w)}} dt = \left(t^0\right)^{\frac{2}{3(1+w)}} \int_{t_0}^{\infty} t^{-\frac{2}{3(1+w)}} dt = \left(t^0\right)^{\frac{2}{3(1+w)}} \left[\frac{3(1+w)}{1+3w}\, t^{\frac{1+3w}{3(1+w)}}\right]_{t_0}^{\infty}$$

$= \infty$

$$\boxed{d_{eh} = \infty \text{ for } \Omega_m = 1}$$

event horizon

b)

$$\int_{-\infty}^{t_0} \left(\frac{t}{t_0}\right)^{-\frac{2}{3(1+w)}} dt =$$

$$\frac{H^2}{H_0^2} = a^{-3(1+w)} \qquad w=-1$$

$$\frac{H^2}{H_0^2} = 1$$

$$H = H_0 \Rightarrow \frac{d\ln a}{dt} = H_0 \Rightarrow d\ln a = H_0 dt$$

$$= H_0 dt$$

$$\ln\left(\frac{a_0}{a(t)}\right) = H_0(t_0 - t)$$

$$-\ln(a(t)) = H_0(t_0-t)$$

$$\ln(a(t)) = H_0(t-t_0) \qquad a(t) = e^{H(t-t_0)}$$

$$a(t) = e^{H_0(t-t_0)}$$

$$\int_{-\infty}^{t_0} \frac{dt}{e^{H(t-t_0)}} = \int_{-\infty}^{t_0} e^{H(t_0-t)} dt = -\int_{-\infty}^{t_0} e^{Hu} du = -\left[\frac{1}{H} e^{Hu}\right]_{-\infty}^{t_0}$$

$u = t_0 - t$

$du = -dt$

$$-1/H \left[e^{H(t_0-t)}\right]_{-\infty}^{t_0} = -1/H + 1/H\, e^{H(t_0+\infty)} = \boxed{\infty = \text{part horizon for desitter}}$$

Particle horizon : ∞

$$\text{event horizon}: \int_{t_0}^{\infty} e^{H(t_0-t)} dt = -\int_{t_0}^{\infty} e^{Hu} du = -1/H \left[e^{H(t_0-t)}\right]_{t_0}^{\infty}$$

$$-1/H \left[e^{H(t_0-\infty)} - e^{H(t_0-t_0)}\right] = -1/H[0-1] = 1/H$$

$$\boxed{\text{event horizon} = 1/H_0}$$

Problem 4.6

a) $\Gamma = n\sigma v = \frac{1}{m^3} m^2 \, m/s = 1/s$ $\qquad \Gamma = G_N^2 T^n$

Γ interaction rate

$G_N = \frac{1}{m_{pl}^2} = [1/kg^2]$

$G_N^2 = [1/kg^4]$

But with natural units, mass is in energy

$G_N^2 = [1/eV^4]$

$\Gamma = [1/s] = [E] = [1/E^4][E]^n$

$\boxed{n = 5}$

b) $H \simeq \left(\frac{8\pi\rho_R}{3m_{\rm Pl}{}^2}\right)^{1/2} = 1.66 g_*^{1/2}\frac{T^2}{m_{\rm Pl}}$ (in radiation-dominated era),

decoupling $\Gamma << H$

$\Gamma = H$

$G_N^2 T^5 = 1.66 g_*^{1/2} \frac{T^2}{m_{pl}}$

$T^3 = \frac{1.66 g_*^{1/2}}{m_{pl} G_N^2} \rightarrow T = \sqrt[3]{\frac{1.66 g_*^{1/2}}{m_{pl} G_N^2}}$

$T = \sqrt[3]{1.66 g_*^{1/2} m_{pl}^3}$

in the book $g_* = 106.75$ for high T

$$\sqrt[3]{1.66\,(106.25)^{1/2}\,(10^{28})^3} \qquad m_{pl} = 10^{19}\ GeV = 10^{28}\ eV$$

$T \sim 10^{28}\ eV$ order of mag

c) it is a boson so g_{*} and g_{*S} are;

g_{*S}^{i} : $g = 106.25$

g_{*S}^{f} : $g\left(\frac{T_g}{T_\gamma}\right)^3 = 3.94$

$$106.25\left(\frac{T_y}{T_\gamma}\right)^3 = 3.94$$

$$\frac{T_g}{T_\gamma} = \sqrt[3]{\frac{3.94}{106.25}} \qquad T_y = 0.3\,T_\gamma = 0.3\,(6.6\times10^{-4})$$

$$\boxed{T_y \approx 1.98\times10^{-4}\ eV}$$

d) so this means that at higher energies like g_{*S}^{i} the multiplets should be 8 time larger so from part c) : $T_y = \sqrt[3]{\frac{3.94}{106.25}}\,T_\gamma$ there should be an extra factor of 8 on $g_{*S}^{i} = 106.25$ so;

$$T_g = \sqrt[3]{\frac{3.94}{8(106.25)}}\, T_\gamma = T_g = \frac{1}{2}\sqrt{\frac{3.94}{106.25}}\, T_\gamma$$

So the temp at decoupling is less by a factor of 1/2.

e) $\Gamma = n\sigma v \qquad \Gamma \approx G_N^2 T^5$

$n\sigma v \approx G_N^2 T^5$

natural unit c=1

$n\sigma \approx G_N^2 T^5$

number density

$n = \frac{1}{\sigma} G_N^2 T^5$

from lecture / book $\sigma \simeq G_N^2 T^2$

$$n = \frac{G_N^2 T^5}{G_N^2 T^2} = T^3$$

$n = T^3$ remember $T_g = 1.98 \times 10^{-4}\,\text{ev}$

$n \approx 10^{-12}\,\text{ev} \qquad 1\,\text{ev} \approx 10^5 \frac{1}{m}$

$$n \approx 10^{-12}\,\text{ev}\, \frac{(10^5)^3}{1\,\text{ev}} \approx 10^3\, 1/m^3 \approx 10^{-3}\, 1/cm^3$$