2.6

$a(t_0) \approx 1$

a) $1+z = \frac{a(t_0)}{a(t_1)}$

$$\frac{dz}{dt_0} = \frac{d}{dt_0}\left[\frac{a(t_0)}{a(t_1)}\right] = \frac{\frac{da(t_0)}{dt_0}a(t_1) - a(t_0)\frac{da(t_1)}{dt_0}}{(a(t_1))^2}$$

$$\frac{da(t_0)}{dt_0}a(t_1) - a(t_0)\frac{da(t_1)}{dt_1}\cdot\frac{dt_1}{dt_0}$$

$$\int_{t_e}^{t_e+\lambda_e/c}\frac{dt}{a(t)} = \int_{t_0}^{t_0+\lambda_0/c}\frac{dt}{a(t)} \qquad (2.37)$$

$$\int_{t_1}^{t_1+\lambda_1/c}\frac{dt}{a(t)} = \int_{t_0}^{t_0+\lambda_0/c}\frac{dt}{a(t)} \qquad \underline{\frac{a(t_0)}{a(t_e)} = \frac{\lambda_0}{\lambda_e}}$$

$$\frac{d[a(t_0)]\,a(t_e) - a(t_0)\,d[a(t_e)]}{(a(t_e))^2} = 0$$

$$\frac{da(t_0)}{dt_0}dt_0\,a(t_e) - a(t_0)\frac{da(t_e)}{dt_e}dt_e = 0$$

$$\frac{dt_0}{dt_e}\left[\frac{da(t_0)}{dt_0}a(t_e)\right] = a(t_0)\frac{da(t_e)}{dt_e}$$

$$\frac{dt_0}{dt_e} = \frac{a(t_0)}{a(t_e)}\underbrace{\frac{da(t_e)}{dt_e}}_{\dot{a}(t_e)} \cdot \underbrace{\frac{dt_0}{da(t_0)}}_{\frac{1}{\dot{a}(t_0)}}$$

$$\frac{dt_0}{dt_1} = \frac{a(t_0)}{\dot{a}(t_0)}\frac{\dot{a}(t_e)}{a(t_e)} = H_e/H_0$$ or in our notation $= \boxed{H_1/H_0 = \frac{dt_0}{dt_1}}$

i dont think this is as useful and ebtikal directed me to the correct way :

$$\frac{a(t_0)}{a(t_e)} = \frac{\lambda_0}{\lambda_e} = \frac{dt_0}{dt_e}$$

$$\frac{\frac{da(t_0)}{dt_0} a(t_1) - a(t_0) \frac{da(t_1)}{dt_0}}{(a(t_1))^2} = \frac{\frac{da(t_0)}{dt_0} a(t_1) - a(t_0) \frac{da(t_1)}{dt_1} \cdot \frac{dt_1}{dt_0}}{(a(t_1))^2}$$

$$= \frac{\dot{a}(t_0)\, a(t_1) - a(t_0)\, \dot{a}(t_1) \frac{\lambda_1}{\lambda_0}}{(a(t_1))^2} = \frac{dz}{dt_0}$$

$$\frac{\dot{a}(t_0)}{a(t_1)} - \frac{\cancel{a(t_0)}}{\cancel{a(t_1)}} \cdot \frac{\dot{a}(t_1)}{a(t_1)} \cancel{\frac{\lambda_1}{\lambda_0}} = \frac{dz}{dt_0}$$

$$\frac{\dot{a}(t_0)}{a(t_1)} - H(t_1)$$

$$\frac{a(t_0)}{a(t_1)} \frac{\dot{a}(t_0)}{a(t_0)} - H(t_1) = \frac{dz}{dt_0}$$

$$(1+z) H(t_0) - H(t_1) = \frac{dz}{dt_0}$$

b) $z = O(1)$ $\quad (1+z) H(t_0) - H(t_1) = \frac{dz}{dt_0}$

$H(t_0) \approx H(t_1)$

$$\underbrace{H(t_0)}_{O(10)} \underbrace{\left[(1+z) - \frac{H(t_1)}{H(t_0)} \right]}_{1-O(2)} = \frac{dz}{dt_0} \sim O(10^7)$$

$10^1 \cdot (365 \times 24 \times 3600$

$10^2 \times 10^1 \times 10^3$

10^6

$\text{km/s/Mpc} \cdot \frac{1}{10^{19}\text{km}}$ $\quad 10^{-18}$

$z = 10^8$

$$O(10^{-18})\, O(10^{0}) = \frac{dz}{dt_0} \sim O(10^{7})$$

$$\boxed{dz \approx 10^{-11}}$$

in order to measure a $dz \approx 10^{-11}$ we need to be able to measure the wavelength of light at more precisely

$$1+z = \frac{\lambda_o}{\lambda_e} \Rightarrow dz = \frac{1}{\lambda_e} d\lambda_o$$

$$d\lambda_o = \lambda_e dz$$

$$R = \frac{\lambda_e}{d\lambda_o} = \frac{\lambda_e}{\lambda_e dz} = \frac{1}{dz} \approx 10^{11}$$

So the spectograph resolution R must be around $R = 10^{11}$

if we can measure $\frac{dz}{dt_0}$ then we can determine the Hubble Constant at time emmission

3.4

uploaded to Github repos