5.1) We know number density of neutrinos today is

$$n \approx 336\ cm^{-3} = 3.36 \times 10^2 \times 10^6\ m^3 = 3.36 \times 10^8\ m^3$$

interaction rate can be

$$\Gamma = n \sigma v$$

$v = c = 3 \times 10^8\ m/s$, The cross section σ will be the area of your thumb which is about 1cm by 2cm. ish which we will approximate as $\sigma = O(cm^2) = O(10^{-4})\ m^2$

Putting it together

$$\Gamma = (3.36 \times 10^8)(3 \times 10^8)(10^{-4}) = (3.36 \times 3)\, 10^{12} \approx 10^{13}$$

So approximately 10^{13} neutrinos pass your thumb.

8.2) $\Omega_1 = \frac{\rho_1}{\rho_{crit}}$ $\Omega_2 = \frac{\rho_2}{\rho_{crit}}$

ρ_{crit} = constant

$\frac{\Omega_1}{\rho_1} = \frac{\Omega_2}{\rho_2}$, but remember $\rho \propto \frac{1}{a^3}$

$\Omega_1 a_1^3 = \Omega_2 a_2^3$

$\frac{a_1^3}{a_2^3} = \frac{\Omega_2}{\Omega_1}$

$a_1 = 10^{-27}$

$a_2 = e^N \approx$

$\Omega_2 = 10^{-6}$

$\Omega_1 = \frac{10^{94}\ TeV/m^3}{\rho_{crit}}$

$$\rho_{0\,unit} \approx 10^{-26}\ kg/m^3 \cdot 5\times10^{35}\ \frac{eV}{kg} = 10^{9}\ eV/m^3$$

$$\Omega_1 = \frac{10^{94}\ TeV/m^3}{10^{9}\ eV/m^3 \cdot \frac{1\ TeV}{10^{12}\ eV}} = \frac{10^{94}\ TeV/m^3}{10^{-3}\ TeV/m^3} = 10^{97}$$

$$\frac{a_1^3}{a_2^3} = \frac{\Omega_2}{\Omega_1} \rightarrow \frac{(10^{-27})^3}{e^{3N}} = \frac{10^{-6}}{10^{97}}$$

$$e^{3N} = \frac{10^{97}}{10^{-6}}\,10^{-81} = 10^{103}\,10^{-81} = 10^{22}$$

$$e^{3n} = 10^{22}$$

$$3n = \ln 10^{22}$$

$$n = \frac{1}{3}\ln 10^{22} = 16.89$$

So there are approx 16.89 efolds