10.1) $\hat{u} = \frac{\sum_{i=1}^{N} x_i}{N} =$ $\quad Var = \langle(\hat{u}-\tilde{u})^2\rangle = \left\langle\left(\frac{1}{N}\sum_i^N x_i - \frac{1}{N}\sum_i^N \bar{x}_i\right)^2\right\rangle = \frac{1}{N^2}\left\langle\left(\sum_i^N x_i - \sum_i^N \bar{x}_i\right)^2\right\rangle$

$\tilde{u} = \frac{\sum_{i=1}^{N}\int_{-\infty}^{\infty} x_i P(x_i)dx_i}{N} = \frac{1}{N}\sum_{i=1}^{N}\bar{X}_i$

$= \frac{1}{N^2}\left\langle\left(\sum x_i - \bar{x}_i\right)^2\right\rangle =$

$Var(x) = \langle(x-\bar{x})^2\rangle \qquad Var\left(\sum x_i\right) = \sum x_i$

$Var(x+y) = \langle(x+y-(\overline{x+y}))^2\rangle = \langle((x-\bar{x})+(y-\bar{y}))^2\rangle =$

$\langle(x-\bar{x})^2 + 2(x-\bar{x})(y-\bar{y}) + (y-\bar{y})^2\rangle$

$\langle(x-\bar{x})^2\rangle + \langle 2(x-\bar{x})(y-\bar{y})\rangle + \langle(y-\bar{y})^2\rangle$

$\sigma_x^2 + \sigma_y^2 + \langle 2(x-\bar{x})(y-\bar{y})\rangle$

$\langle xy - x\bar{y} - \bar{x}y + \bar{x}\bar{y}\rangle$

$\langle xy\rangle - \bar{x}\bar{y} - \bar{x}\bar{y} + \bar{x}\bar{y}$

$\downarrow$ since indep

$\langle x\rangle\langle y\rangle$

$0 = \langle 2(x-\bar{x})(y-\bar{y})\rangle$

So as long as x & y

are indep $\langle(x-\bar{x})(y-\bar{y})\rangle$

$\frac{1}{N^2}\left\langle\left(\sum x_i - \bar{x}_i\right)^2\right\rangle$, therefore all cross terms like $(x_i-\bar{x}_i)(x_j-\bar{x}_j)$ will have

$\frac{1}{N^2}\left\langle\sum_i^N(x_i-\bar{x}_i)^2 + \sum_{i\neq j}(x_i-\bar{x}_i)(x_j-\bar{x}_j)\right\rangle$ a expectation of zero. $\langle(x_i-\bar{x}_i)(x_j-\bar{x}_j)\rangle$ since

$\frac{1}{N^2}\left[\sum_i^N\langle(x_i-\bar{x}_i)^2\rangle + \sum_{i\neq j}\langle(x_i-\bar{x}_i)(x_j-\bar{x}_j)\rangle\right]$ all x_i's are indep

$\frac{1}{N^2}\sum_i^N\sigma^2 = \frac{1}{N^2}N\sigma^2 = \frac{\sigma^2}{N}$ = variance of $\hat{u}$

Problem 10.3

$$P(\Upsilon) = \frac{1}{2^{n/2}\,\Gamma(n/2)}\,\Upsilon^{n/2-1}\,e^{-\Upsilon/2}$$

$$\int_0^{\Delta\chi^2} P(\Delta\chi^2)\,d\Delta\chi^2 = 0.6827,\ 0.9845,\ 0.9973$$

I used this mathematica code

```
In[25]:= n = 2
         f[x] := (x)^(n/2 - 1) * (Exp[-x/2]) / (2^(n/2) * (Gamma[n/2]))
Out[25]= 2

In[27]:= g[y] := Integrate[f[x], {x, 0, y}]

In[29]:= Solve[g[y] == .6827, y]
         ··· Solve: Inverse functions are being used by Solve, so some solutions may not be found; use
             Reduce for complete solution information.
Out[29]= {{y → 2.29582}}
```

But I changed $n=2$ to $n=3$ and changed the value in "Solve[g(y)...."

to find the following values

Prob	2-Par	3-Par	
68.27%	2.296	3.527	
95.45%	6.18	8.025	
99.73%	11.83	14.16	